Unit 1: Further on Sets

> *In Grade 7, you were introduced to basic definitions and operations involving sets. The concept of a set is fundamental in modern mathematics and is used in almost every branch of the subject, including defining relations and functions. This unit will delve deeper into definitions, operations, and applications of sets.*

Unit Outcomes

By the end of this unit, you should be able to:

- Explain facts about sets.
- Describe sets in different ways.
- Define operations on sets.
- Demonstrate set operations using Venn diagrams.
- Apply rules and principles of set theory for practical situations.

Unit Contents

This unit covers the following topics:

- 1.1 Sets and Elements
- 1.2 Set Description
- 1.3 The Notion of Sets (Empty, Finite, Infinite, Equal, Equivalent, Universal, Subset, Proper Subset)
- 1.4 Operations on Sets (Union, Intersection, Complement, Difference, Symmetric Difference, Cartesian Product)
- 1.5 Application (Number of Elements of Union)

1.1 Sets and Elements

> *This section introduces the fundamental concept of a set, its elements, and how to determine membership within a set.*

Definition of a Set

- A **set** is a collection of **well-defined objects** or **elements**.
- **Well-defined** means that for any given object, it is possible to definitively determine whether that object is a member of the set or not.

Examples of Well-defined vs. Not Well-defined Sets

- **Well-defined:**
  - Collection of students in your class.
  - Collection of consonants of the English alphabet.
  - The set of students in your class is well-defined because the members are clearly known.
- **Not Well-defined:**
  - Collection of beautiful girls in your class.
  - Collection of hardworking teachers in a school.
  - The collection of kind students in your school is not well-defined because "kind" is subjective and makes it difficult to list members.

Elements and Set Membership

- If a is an element of set A, we say "a belongs to A".
- The Greek symbol $\in$ (epsilon) denotes "belongs to". We write $a \in A$.
- If b is not an element of set A, we write $b \notin A$, read as "b does not belong to set A" or "b is not a member of set A".

Example 1: Set Membership

- Consider G as a set of vowel letters in the English alphabet.
  - $a \in G$ (a belongs to G)
  - $o \in G$ (o belongs to G)
  - $i \in G$ (i belongs to G)

- But $b \notin G$ (b does not belong to G).

Example 2: Applying Set Membership Symbols

- Suppose A is the set of positive even numbers. This set includes $\{2, 4, 6, 8, \ldots\}$.
  - $4 \in A$ (4 is a positive even number).
  - $5 \notin A$ (5 is not an even number).
  - $-2 \notin A$ (-2 is not a *positive* even number).
  - $0 \notin A$ (0 is not a *positive* even number).

Important Notes on Set Notation

- Sets are typically denoted by **capital letters** (e.g., A, B, C, X, Y, Z).
- Elements of a set are typically represented by **small letters** (e.g., a, b, c, x, y, z).

1.2 Set Description

> *Sets can be described in various ways to clearly communicate their elements. This section covers the three primary methods: Verbal, Listing (Complete & Partial), and Set-builder.*

i) Verbal Method (Statement Form)

- In this method, the well-defined description of the elements of the set is written in an ordinary English language statement form (in words).

Examples of Verbal Method

- The set of whole numbers greater than 1 and less than 20.
- The set of students in this mathematics class.

ii) Listing Methods

a) Complete Listing Method (Roster Method)

- In this method, all elements of the set are completely listed.
- The elements are separated by commas and enclosed within set braces, `{ }`.

Examples of Complete Listing Method

- The set of all even positive integers less than 7 is described as $\{2, 4, 6\}$.
- The set of all vowel letters in the English alphabet is described as $\{a, e, i, o, u\}$.

b) Partial Listing Method

- This method is used when listing all elements of a set is difficult or impossible, but the elements can be clearly indicated by listing a few representative elements that fully describe the set's pattern.
- Three dots (`...`) are used to indicate continuation of the pattern.

Examples of Partial Listing Method

- The set of natural numbers less than 100: $A = \{1, 2, 3, \ldots, 99\}$.
  - The three dots after element 3 and the comma indicate that the elements continue in that manner up to 99.
- The set of whole numbers: $\mathbb{W} = \{0, 1, 2, 3, \ldots\}$.

iii) Set Builder Method (Method of Defining Property)

- The set-builder method describes a set by stating a property that its members must satisfy.
- The format is: `{representative_element | condition_to_satisfy}`.
  - The vertical line `|` or a colon `:` means "such that".

Examples of Set Builder Method

- Set $A = \{1, 2, 3, \ldots, 10\}$ can be described as:
  - $A = \{x \mid x \in \mathbb{N} \text{ and } x < 11\}$.
  - This is read as "A is the set of all elements x such that x is a natural number and x is less than 11."

- Let set $B = \{0, 2, 4, \ldots\}$. This can be described as:
  - $B = \{x \mid x \in \mathbb{Z} \text{ and } x \text{ is a non-negative even integer}\}$
  - OR $B = \{2x \mid x = 0, 1, 2, 3, \ldots\}$
  - OR $B = \{2x \mid x \in \mathbb{W}\}$

Common Number Sets

- **Natural Numbers ($\mathbb{N}$):** $\{1, 2, 3, \ldots\}$
- **Whole Numbers ($\mathbb{W}$):** $\{0, 1, 2, 3, \ldots\}$
- **Integers ($\mathbb{Z}$):** $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

1.3 The Notion of Sets

> *This section introduces different types of sets based on their elements and relationships between them, including empty sets, finite/infinite sets, equal/equivalent sets, universal sets, and subsets.*

Empty Set

- **Definition 1.1:** A set which does not contain any element is called an **empty set**, **void set**, or **null set**.
- The empty set is denoted mathematically by the symbol `{}` or `Ø`.

Example: Empty Set

- Let set $A = \{x \mid 1 < x < 2, x \in \mathbb{N}\}$. Then, A is an empty set, because there is no natural number between 1 and 2.

Finite Set and Infinite Set

- **Definition 1.2:**
  - A set which consists of a definite number of elements is called a **finite set**.
  - A set which is not finite is called an **infinite set**.

Example: Identifying Finite or Infinite Sets

- The set of natural numbers up to 10: $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. This is a **finite set** because it has a definite (limited) number of elements.
- The set of African countries: This is a **finite set**.
- The set of whole numbers: This is an **infinite set** because it continues indefinitely ($\{0, 1, 2, 3, \ldots\}$).

Number of Elements in a Set

- The number of elements of set A is denoted by $n(A)$.
- For example, if $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then $n(A) = 10$.

Equal Sets

- **Definition 1.3:** Two sets A and B are said to be **equal** if and only if they have exactly the same or identical elements.
- Mathematically, it is denoted as $A = B$.

Example: Equal Sets

- Let $A = \{1, 2, 3, 4\}$ and $B = \{4, 3, 2, 1\}$. Then, $A = B$ because they contain the exact same elements, regardless of order.

Equivalent Sets

- **Definition 1.4:** Two sets A and B are said to be **equivalent** if there is a one-to-one correspondence between the two sets.
- This is written mathematically as $A \leftrightarrow B$ (or $A \sim B$).

Example: Equivalent Sets

- Consider two sets $A = \{1, 2, 3, 4\}$ and $B = \{\text{Red, Blue, Green, Black}\}$.
- Set A has four elements, and set B also has four elements. Therefore, set A and set B are equivalent.

Note on Finite Equivalent Sets

- Two finite sets A and B are equivalent if and only if they have an equal number of elements, which is written as $n(A) = n(B)$.

Universal Set ($\mathbb{U}$)

- **Definition 1.5:** A **universal set** (usually denoted by U) is a set which has elements of all the related sets under discussion, without any repetition of elements.

Example: Universal Set

- Let set $A = \{2, 4, 6, \ldots\}$ (even natural numbers) and $B = \{1, 3, 5, \ldots\}$ (odd natural numbers).
- The universal set U could be the set of all natural numbers, such that $U = \{1, 2, 3, 4, \ldots\}$. This U contains all elements of set A and set B.

Subset ($\subseteq$)

- **Definition 1.6:** Set A is said to be a **subset** of set B if every element of A is also an element of B.
- Mathematically, we write this as $A \subseteq B$.
- If set A is not a subset of set B, it is written as $A \nsubseteq B$.

Example 1: Subset

- Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. Here, set A is a subset of set B, or $A \subseteq B$, since all members of set A are found in set B.

Example 2: Finding All Subsets

- For set $A = \{1, 2, 3\}$, the subsets are:
  - $\{\}$ (the empty set)
  - $\{1\}$, $\{2\}$, $\{3\}$ (single-element subsets)
  - $\{1, 2\}$, $\{2, 3\}$, $\{1, 3\}$ (two-element subsets)
  - $\{1, 2, 3\}$ (the set itself)
- The total number of subsets of set A is 8.

Important Notes on Subsets

- Any set is a subset of itself (e.g., $A \subseteq A$).
- The empty set ($\emptyset$) is a subset of every set.
- If a finite set A has n elements, then the number of subsets of set A is 2^n.
  - In the example above, $n(A) = 3$, so $2^3 = 8$ subsets.

Proper Subset ($\subset$)

- **Definition 1.7:** If $A \subseteq B$ and $A \neq B$, then A is called the **proper subset** of set B.
- It can be written as $A \subset B$. This means A is a subset of B, but A is not equal to B (i.e., B contains at least one element not in A).

Example 1: Proper Subset

- Given sets $A = \{2, 5, 7\}$ and $B = \{2, 5, 7, 8\}$.
- Set A is a proper subset of set B, i.e., $A \subset B$, because $A \subseteq B$ and $A \neq B$. Also, observe that $B \not\subset A$.

Example 2: Finding Proper Subsets

- For set $A = \{2, 5, 7\}$, the proper subsets are:
  - $\{\}$
  - $\{2\}$, $\{5\}$, $\{7\}$
  - $\{2, 5\}$, $\{5, 7\}$, $\{2, 7\}$
- There are 7 proper subsets. (The set itself, $\{2, 5, 7\}$, is excluded).

Important Notes on Proper Subsets

- For any set A, A is not a proper subset of itself ($A \not\subset A$).
- If a finite set A has n elements, the number of proper subsets of set A is $2^n - 1$.
- The empty set ($\emptyset$) is a proper subset of any other non-empty set.
- If set A is a subset of set B ($A \subseteq B$), conversely, B is a **superset** of A, written as $B \supset A$.

1.4 Operations on Sets

> *Set operations allow us to create new sets from existing ones. This section covers the fundamental operations: Union, Intersection, Complement, Difference, Symmetric Difference, and Cartesian Product.*

Venn Diagrams

- A **Venn diagram** is a schematic or pictorial representation of the sets involved in a discussion.
- Usually, sets are represented as interlocking circles, each enclosed within a rectangle that represents the universal set.

Union and Intersection

Union of Sets ($\cup$)

- **Definition 1.8:** The **union** of two sets A and B, denoted by $A \cup B$, is the set of all elements that are either in set A or in set B (or in both sets).
- Mathematically: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

Intersection of Sets ($\cap$)

- **Definition 1.9:** The **intersection** of two sets A and B, denoted by $A \cap B$, is the set of all elements that are both in set A and in set B.
- Mathematically: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

Disjoint Sets

- Two sets A and B are **disjoint** if their intersection is the empty set, i.e., $A \cap B = \emptyset$.

Example 1: Union and Intersection with Venn Diagram

- Let $A = \{0, 1, 3, 5, 7\}$ and $B = \{1, 2, 3, 4, 6, 7\}$.
    - **Venn Diagram Illustration:** (Imagine two overlapping circles, one for A, one for B, inside a rectangle for U. The common elements {1,3,7} are in

the overlap. A-only elements {0,5} are in A's non-overlapping part. B-only elements {2,4,6} are in B's non-overlapping part.)

- $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7\}$
- $A \cap B = \{1, 3, 7\}$

Example 2: Union and Intersection of Infinite Sets

- Let $A = \{2, 4, 6, 8, 10, \ldots\}$ (positive even integers) and $B = \{3, 6, 9, 12, 15, \ldots\}$ (positive multiples of 3).
    - $A \cup B = \{x \mid x \text{ is a positive integer that is either even or a multiple of } 3\}$
        - $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, \ldots\}$
    - $A \cap B = \{x \mid x \text{ is a positive integer that is both even and a multiple of } 3\}$
        - $A \cap B = \{6, 12, 18, 24, \ldots\}$ (positive multiples of 6)

Laws of Intersection (from textbook context, implies these are general set laws)

- **Law of $\emptyset$ and U:**
    - $\emptyset \cap A = \emptyset$
    - $U \cap A = A$
- **Commutative Law:** $A \cap B = B \cap A$
- **Associative Law:** $(A \cap B) \cap C = A \cap (B \cap C)$

Complement of Sets ($'$)

- **Definition 1.10:** Let A be a subset of a universal set U. The **absolute complement** (or simply **complement**) of A, denoted by A', is defined as the set of all elements of U that are not in A.
- Mathematically: $A' = \{x \mid x \in U \text{ and } x \notin A\}$.

Example 1: Complement of a Set

- Let $U = \{0, 1, 2, 3, 4\}$ and $A = \{3, 4\}$.
- Then, $A' = \{0, 1, 2\}$.

Example 2: Complement and De Morgan's Law

- Let $U = \{1, 2, 3, \ldots, 10\}$ be a universal set.
- $A = \{x \mid x \text{ is a positive factor of 10 in } U\} = \{1, 2, 5, 10\}$.
- $B = \{x \mid x \text{ is an odd integer in } U\} = \{1, 3, 5, 7, 9\}$.
    - a. Find A' and B':
        - $A' = \{3, 4, 6, 7, 8, 9\}$
        - $B' = \{2, 4, 6, 8, 10\}$
    - b. Find $(A \cup B)'$ and $A' \cap B'$:
        - First, $A \cup B = \{1, 2, 3, 5, 7, 9, 10\}$.
        - So, $(A \cup B)' = \{4, 6, 8\}$.
        - From A' and B', we find $A' \cap B' = \{4, 6, 8\}$.
    - **Observation:** We observe that $(A \cup B)' = A' \cap B'$.

De Morgan's Laws

- For the complement set of $A \cup B$ and $A \cap B$:
    - **1st statement:** $(A \cup B)' = A' \cap B'$
    - **2nd statement:** $(A \cap B)' = A' \cup B'$

Difference of Sets ($-$)

- **Definition 1.11:** The **difference** between two sets A and B, denoted by $A - B$, is the set of all elements in A and not in B. This set is also called the **relative complement of A with respect to B**.
- Mathematically: $A - B = \{x \mid x \in A \text{ and } x \notin B\}$.
- The notation $A - B$ can also be written as $A \setminus B$.

Example 1: Difference of Sets

- If sets $A = \{0, 1, 2, 3, 4\}$ and $B = \{3, 4\}$.
- Then, $A - B$ or $A \setminus B = \{0, 1, 2\}$.

Example 2: Multiple Set Differences and Union

- Let U be the universal set of one-digit numbers ($\{0, 1, 2, \ldots, 9\}$).

- A be the set of even numbers (in U) = $\{0, 2, 4, 6, 8\}$.
- B be the set of prime numbers less than 10 = $\{2, 3, 5, 7\}$.
  - a. $A - B = \{0, 4, 6, 8\}$
  - b. $B - A = \{3, 5, 7\}$
  - c. $A \cup B = \{0, 2, 3, 4, 5, 6, 7, 8\}$
  - d. $U - (A \cup B) = \{1, 9\}$

Example 3: Relationships between Complement and Difference

- Using the same sets from Example 2 ($U = \{0, \ldots, 9\}$, $A = \{0, 2, 4, 6, 8\}$, $B = \{2, 3, 5, 7\}$):
  - a. $A' = \{1, 3, 5, 7, 9\}$
  - b. $U - A = \{1, 3, 5, 7, 9\}$
  - c. $B' = \{0, 1, 4, 6, 8, 9\}$
  - d. $A \cap B' = \{0, 4, 6, 8\}$
- **Observations:**
  - From a. and b., we can see that $A' = U - A$.
  - From d. and Example 2a., we can see that $A - B = A \cap B'$.

Theorem 1.1: Properties of Sets

- For any two sets A and B, each of the following holds true:
  - $(A')' = A$ (Double complement)
  - $A' = U - A$
  - $A - B = A \cap B'$
  - $A \subseteq B \iff B' \subseteq A'$ (Subset implies complement relationship)

Symmetric Difference of Two Sets ($\triangle$)

- **Definition 1.12:** For two sets A and B, the **symmetric difference** between these two sets is denoted by $A \triangle B$.
- It is defined as:
  - $A \triangle B = (A \setminus B) \cup (B \setminus A)$ (which is $(A - B) \cup (B - A)$)
  - OR $A \triangle B = (A \cup B) \setminus (A \cap B)$

- In a Venn diagram, $A \triangle B$ represents the elements that are in A or B, but not in their intersection.

Example 1: Symmetric Difference Calculation

- Consider sets $A = \{1, 2, 4, 5, 8\}$ and $B = \{2, 3, 5, 7\}$.
  - First, find $A \setminus B = \{1, 4, 8\}$.
  - Next, find $B \setminus A = \{3, 7\}$.
  - Then, $A \triangle B = (A \setminus B) \cup (B \setminus A) = \{1, 4, 8\} \cup \{3, 7\} = \{1, 3, 4, 7, 8\}$.
  - Alternatively, using the second definition:
    - $A \cup B = \{1, 2, 3, 4, 5, 7, 8\}$
    - $A \cap B = \{2, 5\}$
    - $A \triangle B = (A \cup B) \setminus (A \cap B) = \{1, 3, 4, 7, 8\}$.

Example 2: Symmetric Difference with Disjoint Sets

- Given sets $A = \{d, e, f\}$ and $B = \{4, 5, 6\}$.
  - First, find $A \setminus B = \{d, e, f\}$.
  - Next, find $B \setminus A = \{4, 5, 6\}$.
  - Hence, $A \triangle B = (A \setminus B) \cup (B \setminus A) = \{d, e, f\} \cup \{4, 5, 6\} = \{d, e, f, 4, 5, 6\}$.

Cartesian Product of Two Sets ($\times$)

- **Definition 1.13:** The **Cartesian product** of two sets A and B, denoted by $A \times B$, is the set of all **ordered pairs** (a, b) where $a \in A$ and $b \in B$.
- This can also be expressed as: $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.

Example 1: Cartesian Product Calculation

- Let $A = \{1, 2\}$ and $B = \{a, b\}$.
  - a. $A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$
  - b. $B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

- Note that $A \times B \neq B \times A$ unless $A = B$ or one of the sets is empty.

Example 2: Finding Sets from their Cartesian Product

- If $A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$.
  - Set A is the set of all first components of the ordered pairs in $A \times B$. So, $A = \{1, 2, 3\}$.
  - Set B is the set of all second components of the ordered pairs in $A \times B$. So, $B = \{a, b\}$.

1.5 Application

> *Set theory can be applied to solve practical problems, especially concerning the number of elements in combined sets.*

Number of Elements of Union of Two Sets

- For two subsets A and B of a universal set U, the following formula on the number of elements holds:
  - $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- **Special Case: Disjoint Sets**
  - If $A \cap B = \emptyset$ (meaning the sets are disjoint), then $n(A \cap B) = 0$.
  - In this case, the formula simplifies to: $n(A \cup B) = n(A) + n(B)$.
- If $A \cap B \neq \emptyset$, then elements in the intersection are counted twice when summing $n(A)$ and $n(B)$, so they must be subtracted once.

Example: Calculating Intersection Size

- Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$, and $n(A \cup B) = 36$. Find $n(A \cap B)$.
- **Solution:**
  - Using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$:
  - $36 = 20 + 28 - n(A \cap B)$
  - $36 = 48 - n(A \cap B)$
  - $n(A \cap B) = 48 - 36$

- $n(A \cap B) = 12$.

Summary

> *This summary consolidates the key definitions and concepts introduced in Unit 1: Further on Sets.*

- **Set:** A collection of **well-defined** objects or elements, meaning it's clear whether an object belongs to the set or not.
- **Set Description Methods:**
  - **Verbal method:** Describing the set using ordinary language.
  - **Complete listing method (Roster Method):** Listing all elements, separated by commas, within `{ }`.
  - **Partial listing method:** Listing a few elements followed by `...` when a complete list is difficult or impossible, but the pattern is clear.
  - **Set builder method:** Defining a set by a property its members must satisfy, using the format `{x | condition}`.
- **Empty Set ($\emptyset$ or $\{\}$):** A set containing no elements.
- **Equal Sets ($A = B$):** Two sets A and B are equal if they contain exactly the same elements.
- **Subset ($A \subseteq B$):** Set A is a subset of set B if every element of A is also an element of B.
  - Any set is a subset of itself.
  - The empty set is a subset of every set.
  - Number of subsets for a set with n elements is 2^n.
- **Proper Subset ($A \subset B$):** If $A \subseteq B$ and $A \neq B$.
  - A set is not a proper subset of itself.
  - Number of proper subsets for a set with n elements is $2^n - 1$.
  - The empty set is a proper subset of any non-empty set.
- **Universal Set (U):** A set containing all elements relevant to a particular context or discussion, without repetition.

- **Union ($A \cup B$):** The set of all elements that are in A or in B (or both). Mathematically, $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.
- **Intersection ($A \cap B$):** The set of all elements that are in both A and B. Mathematically, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.
- **Difference ($A - B$ or $A \setminus B$):** The set of all elements in A but not in B. Mathematically, $A - B = \{x \mid x \in A \text{ and } x \notin B\}$. Also known as the relative complement of A with respect to B.
- **Complement (A'):** The set of all elements in the universal set U that are not in A. Mathematically, $A' = \{x \mid x \in U \text{ and } x \notin A\}$.
- **Venn Diagram:** A pictorial representation of sets, typically using interlocking circles within a rectangle (universal set).
- **Symmetric Difference ($A \triangle B$):** The set of elements that are in A or B, but not in their intersection. Defined as $(A \setminus B) \cup (B \setminus A)$ or $(A \cup B) \setminus (A \cap B)$.
- **Cartesian Product ($A \times B$):** The set of all possible ordered pairs (a, b) where $a \in A$ and $b \in B$. Mathematically, $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.
- **Number of Elements in Union:** For any two finite sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. If A and B are disjoint, $n(A \cup B) = n(A) + n(B)$.

Review Exercise

1. Express the following sets using the listing method.
    - a. A is the set of positive factors of 18
    - b. B is the set of positive even numbers below or equal to 30
    - c. $C = \{2n \mid n = 0, 1, 2, 3, \ldots\}$
    - d. $D = \{x \mid x^2 = 9\}$
2. Express the following sets using the set-builder method.
    - a. $\{2, 4, 6, \ldots\}$
    - b. $\{1, 3, 5, \ldots, 99\}$
    - c. $\{1, 4, 9, \ldots, 81\}$

3. Find all the subsets of the following sets.
    - a. $\{3, 4, 5\}$
    - b. $\{a, b\}$
4. Find $A \cup B$ and $A \cap B$ for the following.
    - a. $A = \{2, 3, 5, 7, 11\}$ and $B = \{1, 3, 5, 8, 11\}$
    - b. $A = \{x \mid x \text{ is the factor of } 12\}$ and $B = \{x \mid x \text{ is the factor of } 18\}$
    - c. $A = \{3x \mid x \in \mathbb{N}, x \leq 20\}$ and $B = \{4x \mid x \in \mathbb{N}, x \leq 15\}$
5. If $B \subseteq A$, $A \cap B' = \{1, 4, 5\}$, and $A \cup B = \{1, 2, 3, 4, 5, 6\}$, then find set B.
6. Let $A = \{2, 4, 6, 7, 8, 9\}$, $B = \{1, 3, 5, 6, 10\}$ and $C = \{x \mid x \in \mathbb{Z}, 3x + 6 = 0 \text{ or } 2x + 6 = 0\}$.
    - a) Find $A \cup B$.
    - b) Is $(A \cup B) \cup C = A \cup (B \cup C)$?
7. Suppose the universal set U be the set of one-digit numbers, and set $A = \{x \mid x \text{ is an even natural number less than or equal to } 9\}$.
   Describe each set by complete listing method:
    - a. A'
    - b. $A \cap A'$
    - c. $A \cup A'$
    - d. $(A')'$
    - e. $\emptyset \setminus U$
    - f. $\emptyset'$
    - g. U'
8. Let $U = \{1, 2, 3, 4, \ldots, 10\}$, $A = \{1, 3, 5, 7\}$, $B = \{1, 2, 3, 4\}$. Evaluate $U \setminus (A \triangle B)$.
9. Consider a universal set $U = \{1, 2, 3, \ldots, 14\}$, $A = \{2, 3, 5, 7, 11\}$, $B = \{2, 4, 8, 9, 10, 11\}$. Then, which one of the following is true?
    - A. $(A \cup B)' = \{1, 4, 6, 12, 13, 14\}$
    - B. $A \cap B = A' \cup B'$
    - C. $A \triangle B = (A \cap B)'$

- D. $A \setminus B = \{3, 5, 7\}$
- E. None

10. Let $A = \{3, 7, a^2\}$ and $B = \{2, 4, a + 1, a + b\}$ be two sets and all the elements of the two sets are integers. If $A \cap B = \{4, 7\}$, then find a and b. In addition, find $A \cup B$.
11. In a survey of 200 students in Motta Secondary School, 90 students are members of Nature club, 31 students are members of Mini-media club, 21 students are members of both clubs. Answer the following questions.
    - a. How many students are members of either of the clubs?
    - b. How many students are not members of either of the clubs?
    - c. How many students are only in Nature club?
12. A survey was conducted in a class of 100 children and it was found out that 45 of them like Mathematics whereas only 35 like Science and 10 students like both subjects. How many like neither of the subjects?
    - A) 70
    - B) 30
    - C) 100
    - D) 40