Unit 3: Elasticity and Static Equilibrium of a Rigid Body

3.1. Elasticity and plasticity

Plasticity

If a body does not regain its original size and shape after removal of deforming force, it is said to be Plastic body. And the property is called plasticity. The deformation caused is known as Plastic deformation.

Plasticity *is ability to be permanently molded.*

Plastic deformation:

- Is persistent deformation of a solid body caused by sustained force
- It happens when a great amount of tension is applied to a material.
- It is permanent and irreversible

Exercise 1

1. Plastic is:
   A, the body B, property of the body C. the deformation D. material of the body
2. Which one of the following is Plastic?
   A, rubber band B, spring C, nail D, Sponge
3. Irreversible change of size and shape is:
   A, Plasticity B, plastic C, plastic deformation D, elastic deformation

Elasticity

If a body regains its original size and shape after removal of the deforming force it is said to be elastic. And the property is called elasticity. The deformation caused is known as elastic deformation.

Exercise 2

1. Elasticity is:
   A, the body B, property of the body C. the deformation D. material of the body
2. Which one of the following objects is elastic?
   A, glass B, ruler C, nail D, Sponge
3. Temporary and reversible change of size and shape is:
   A, elastic B, elasticity C, elastic deformation D, elastic limit

Elastic limit

Maximum deforming force up to which a body retains its property of elasticity is called elastic limit of the body. Beyond elastic limit a body will not regain its original size and shape.

- Elastic limit is property of the body whereas elasticity is property of the material of the body.
- All materials are elastic up to a certain deformation then they turn plastic.
- Springs lose their springiness if subjected to large external force and tends to become plastic.
- Elastic limit of a ductile material is beginning point of elastic deformation.
- Elastic limit of a solid is utmost amount to which it may be stretched without permanently changing size or form.
- If the tension is placed beyond the elastic limit, the substance will deform plastically.

Exercise: 3

1. What will happen if metallic parts of machinery are subjected to a stress beyond elastic limit?
   A, they will be permanently deformed B, they will regain their original shape and size as the stress is removed
   C, they will not be deformed D, the deformation will be elastic
2. Elastic limit is property of _____ whereas elasticity is property of _____
   A, a material of the body, the body B, a body, material of the body
   C, a material of the body, a material of the body D. the body, the body

3. The body will regain its previous shape and size only when the deformation caused by the external force is within a certain limit. What is that limit?
   A, Elastic limit B, plastic limit C, deforming limit D, restoring limit
4. Deformation is:
   A, change of size B, change of shape C, change of form D, all

Review Exercise 3.1

I. Say true if the statement is correct and false if else.

1. An object or material is elastic if it comes back to its original shape and size when the external force is removed.
2. Solids tend to regain their original shape and size once external deforming force is removed.
3. Restoring force is external force that restores previous size and shape of a body after deformation.
4. Beyond a limit, called elastic limit, the solid does not regain its original shape and size.
5. External forces applied on an object cause deformation.

II. Match the terms under column A with their meaning Under column B

	A		B
1.	Deforming force	**A.**	hard solid object having definite size and shape
2.	Restoring force	**B.**	a body that regains its original size and shape after removal of deforming force
3.	Elasticity	**C.**	deformation that reverse when deforming force is removed
4.	Plasticity	**D.**	a force that restores original size and shape of a body
5.	Rigid body	**E.**	property of an object by which it regains its previous size and shape after removal of deforming force
6.	Plastic	**F.**	Persistent (permanent) and irreversible deformation
7.	Elastic	**G.**	a body that doesn't regain its original size and shape after removal of deforming force
8.	Elastic limit	**H.**	Maximum deforming force up to which a body retains its original size and property of elasticity
9.	Plastic deformation	**I.**	A force that causes change of shape and size
10.	Elastic Deformation	**J.**	Property of a body by which it deforms permanently

III. Choose the correct answer among the given alternatives.

1. The property of solids by virtue of which they regain their original shape and size even when the deforming forces are removed is:
   A, elastic deformation B, elasticity C, elastic D, elastic limit
2. When stress is removed, solid material will regain its original form if the stress is:
   A, beyond elastic limit B, less than elastic limit C, zero D, exerted on plastic body
3. The property of solids by virtue of which they do not regain their original shape and size, when the deforming forces are removed, is:
   A, deforming force B, plasticity C, elasticity D, elastic
4. Restoring force is:
   A, restores original form even if the stress exceeds elastic limit B, stress
   C, applied by external agent D, no answer

IV. Give short and precise answer to the following questions.

1. List some plastic materials in your locality.
2. List some elastic materials in your locality.
3. Explain deformation of an object.

3.2. Density and Specific Gravity

Density is:

- Mass of an object per unit of its volume
- A characteristic property of any pure substance
- Measure of how much mass of an object is contained in a unit volume of substance
- Ratio between mass and volume
- Represented by $\boldsymbol{\rho}$ (Greek alphabet roi)
- Given by: $\boldsymbol{Density = \frac{mass}{volume}}$ $\boxed{\boldsymbol{\rho = \frac{m}{v}}}$

  And its SI unit is Kg/m^3

Density determines compactness of substance. And it can be used in determining whether an object sinks or floats in a fluid.

Example:

1. You have a rock with a volume of 15 cm^3 and a mass of 45 g. What is its density?
2. What is mass of solid iron ball of radius 18cm?

Question:

1. Show that:
   A. 1 kilogram per meter cube is equal to 1 gram per liter (use: $1m^3 = 1000L$)
   B. 1 gram per centimeter cube is equal to thousand kilogram per meter cube
2. Find density of wooden slab of dimension 5cm * 2cm * 10cm and mass 0.5kg.

Factors that affect density

- Temperature and pressure
- Size, mass, arrangement of atoms in the substance

Exercise 4:

1. Find mass of aluminum ($\boldsymbol{\rho = 2.7 g/cm^3}$) block of dimension: 2cm * 1cm * 5cm.
2. Find height of an **iron** ($\boldsymbol{\rho = 7.8 g/cm^3}$) **cylinder** whose base area is $10cm^2$ and weighs 2kg.

Specific Gravity

Specific Gravity is ratio of the density of the given substance to that of standard substance. Generally the standard substance is water at 4^oc, at 1atm, $\boldsymbol{\rho = 1000 kg/m^3}$

$$\boldsymbol{S.G = \frac{\rho_{substance}}{\rho_{water}}} \quad \boldsymbol{S.G = \frac{\rho_{substance}}{1000 kg/m^3}} \quad \boldsymbol{S.G = \frac{\rho_{substance}}{1 g/cm^3}}$$

→Specific gravity is unit less quantity as it is the ratio of two densities.

Example

1. A mining worker gets an unknown mineral with a volume of $20cm^3$ and a mass of 54g. Determine the density and specific gravity. What is the mineral?
2. You have a sample of granite with density 2.8 g /cm3. What is the specific gravity of your granite?

Factors that Affect Specific Gravity

- Temperature and pressure
- Size, mass, arrangement of atoms in the substance

Application of Specific Gravity

- To determine mineral content of rock
- To compare purity of newly found gem with the standard one
- In urine analysis and extraction of information of the urine.

Table 3.1: Density and specific gravity of some materials

Material type	Material name	Density		Specific gravity (Relative density)
		in $\mathbf{Kg/m^3}$	in $\mathbf{g/cm^3}$	
Gas	Helium	0.179	0.000179	0.000179
	Air	1.29	0.00129	0.00129
	Carbon dioxide	1.98	0.00198	0.00198
Liquid	Alcohol	790	0.79	0.79
	Gasoline	860	0.86	0.86
	Water (4^oc)	1000	1	1
	Mercury	13600	13.6	13.6
Solid	Glass (common)	2400 – 2800	2.4 – 2.8	2.5
	Aluminum	2700	2.7	2.7
	Iron	7860	7.86	7.86
	Copper	8920	8.92	8.92
	Silver	10500	10.5	10.5
	Uranium	19070	19.07	19.07
	Gold	19300	19.3	19.3

Review Exercise 3.2

I. Say true if the statement is correct and false if else.

1. An object made of a particular pure substance can have any size or mass, but the density will be the same for each.
2. The higher the temperature increase, the grater the temperature is.
3. Density is a characteristic property of any pure substance.
4. Density is measure of how much mass of an object is contained in a unit volume of substance
5. Density determines whether an object sinks or floats in a fluid.

II. Choose the correct answer among the given alternatives.

1. Which of the following shows density of materials in ascending order?
   A, $\boldsymbol{\rho_{solid}, \rho_{liquid}, \rho_{gas}}$ B, $\boldsymbol{\rho_{solid}, \rho_{gas}, \rho_{liquid}}$ C, $\boldsymbol{\rho_{gas}, \rho_{solid}, \rho_{liquid}}$ D, $\boldsymbol{\rho_{liquid}, \rho_{solid}, \rho_{gas}}$
2. Which of the following is false about water?
   A, an ice sinks in liquid water B, density of liquid water is slightly greater than density of ice (solid)
   C, water has the highest density at 4^oc D. the highest density of water is $1000kg/m^3$
3. A 5.4g wooden slab have dimension: 4cm * 2cm * $\frac{1}{4}cm$. What is the density in g/cm^3, density in Kg/m^3, and specific gravity, respectively?
   A, 2700, 2.7, 2700 B, 2700, 2.7, 2.7 C, 2.7, 2700, 2.7 D, 2700, 2700, 2.7
4. What is the approximate mass of air in a living room of 5 m× 5 m ×4m?
   A, 12.9kg B, 129kg C, 129g D, 1290g
5. A bar measures 25mm x 40 mm x 1 m. It has a specific gravity of 2.78. Determine its mass.
   A, 2.5kg B, 40kg C, 2.78kg D, 11.12kg
6. Density increases as:
   A. Temperature increase B. pressure increase C. Volume increase D. all

III. Solve the following word problems.

1. Suppose that a block of brass and a block of wood have exactly the same mass. If both blocks are dropped in a tank of water, which one floats and which one sink? Why?
2. Calculate the average density and specific gravity of the Earth given that the mass and radius of the Earth are $m_E = 5.98\times10^{24}$ kg and $R_E = 6.37\times10^6$ m, respectively.
3. A golden-colored cube is handed to you. The person wants you to buy it for 5000 birr, saying that is a gold nugget. You read that the density of gold is 19.3g /cm3. You measure the cube and find that it is 2 cm on each side, and weighs 40 g. What is its density? Is it gold? Should you buy it?

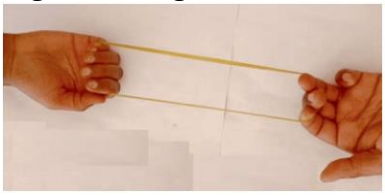
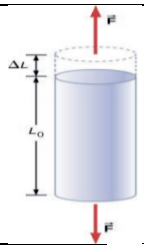
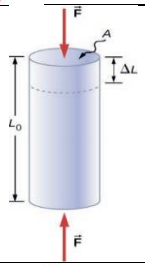
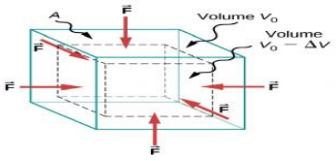
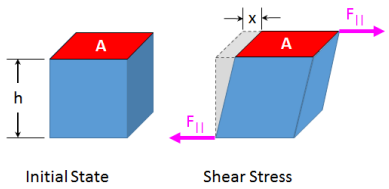
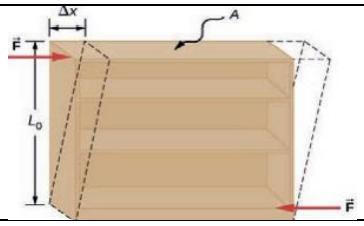
3.3. Stress and Strain

Stress

- Stress is a quantity that describes the magnitude of forces that cause deformation.
- Stress is proportional to the force that causes deformation
- Stress is external force acting on an object per unit cross sectional area.
- Stress is given by:

$$Stress = \frac{Force}{Area} \qquad Stress = \frac{F}{A}$$

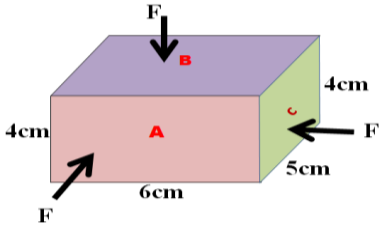
- SI unit of stress is N/m^2 = Pascal (Pa)

	Type of stress		Example	
A	Tensile stress:	Occurs when a force pulls an object.	E.g. stretching elastic band 	
B	Compressive stress:	Occurs when a force push an object and cause compression.	**E.g. compressing helical spring** 	
C	Bulk stress (volumetric stress):	Occurs when a force squeezes an object from all sides.	E.g. submarine in the depth of ocean 	
D	Shear stress:	Occurs when a deforming force acts tangentially to the surface of object's surface. E.g. painting brush		

- Shear Force: is deforming force that causes shear stress

Exercise 5

1. A circular rod of cross-sectional area $100mm^2$ has a tensile force of 100kN applied to it. Calculate the value for the stress in the rod.
2. Calculate stress each faces (A, B & C) of the box at right, if applied force is 48N,o on each.



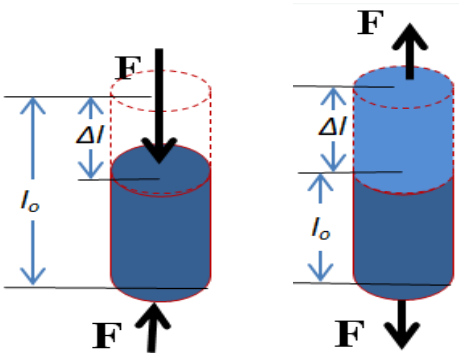
Strain:

- Strain is a dimensionless quantity that gives that gives the amount of deformation of an object or medium under stress.
- Strain is given as fractional change in: length, volume or geometry
- Strain is result of stress
- Strain is measure of degree of deformation

Table 2: Types of Strain

	Type of strain		Happens to an object	Resulted change	Equation	
A	Liner strain	Tensile	Under tensile stress	length(Increase)	$strain = \frac{change\ in\ length}{origional\ length}$	$S_{n_l} = \frac{\Delta l}{l_o}$
		Compressive	Under compressive stress	length (Decrease)	$strain = \frac{change\ in\ length}{origional\ length}$	$S_{n_l} = \frac{\Delta l}{l_o}$
B	Shear Strain		Under shear stress	Geometry (area)	$strain = \frac{tangential\ disp.}{original\ length}$	$S_{n_{sh}} = \frac{\Delta x}{l_o}$
C	Bulk Strain		Under bulk stress	volume	$strain = \frac{change\ in\ volume}{origional\ volume}$	$S_{n_v} = \frac{\Delta V}{V_o}$

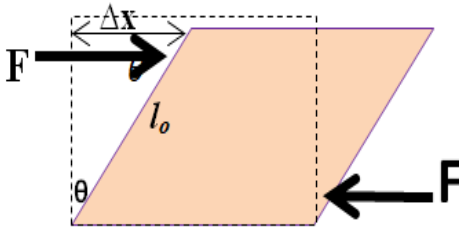
1. Linear Strain:



If on application of a longitudinal deforming force, length l_o of a body changes by Δl of then:

$$tensile\ (compressive)\ strain = \frac{\Delta l}{l_o}$$

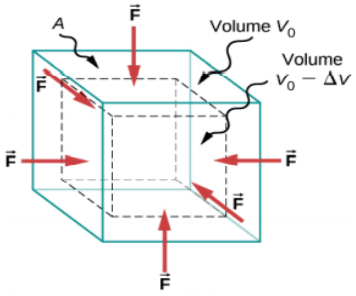
2. Shear Strain:



When the deforming forces are tangential the shearing, strain is given by the angle through which a line perpendicular to the fixed plane is turned due to deformation.

$$\text{Shear } strain = \frac{\Delta X}{l_o} = \sin\theta$$

3. Volumetric (Bulk) Strain:

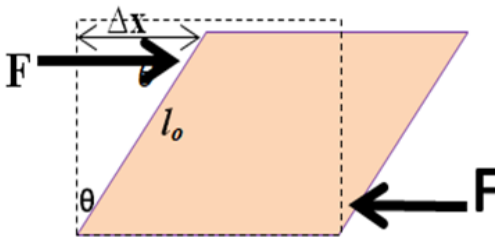


If on application of the deforming force, the volume v_o of the body.

$$Bulk\ strain = \frac{\Delta v}{v_o}$$

Exercise 6:

1. A wire is stretched 3mm by a force of 150N. Assuming the elastic limit is not exceeded; calculate the force that will stretch the wire 5 mm.
2. Find shear strain on the diagram (deformed square) at the right if
   A. L_o is 10cm and Δx is 5cm
   B. $\theta = 30^o$
3. Find θ if shear strain is 0.6
4. Find original area if the object was previously square, and strain is 0.8 and Δx is 16cm
5. Find area of the figure when strain is 1
6. A 64cm spring was to 16cm. find the strain.
7. find strain of air-filled balloon if its original volume is $100cm^3$, and its volume become $90cm^3$, after it is put under water.



Elastic Modulus

- **Modulus of Elasticity (elastic modulus): is measure of elasticity of substance. And its SI unit is N/m^2 or Pascal (Pa)**
- Elasticity modulus is independent of stress and strain but depends on nature of the material.
- Large value of elastic modulus implies that effect of stress is small.
- A small elastic modulus means that stress produces large strain and noticeable deformation. For instance: stress on a rubber band produces large strain (deformation) than the same stress on steal band of the same dimension. Because, elastic modulus of rubber is very smaller than elastic modulus of steal.

Robert Hooke (1678) obtained the strain-stress curve experimentally for number of solid substances and established law of elasticity.

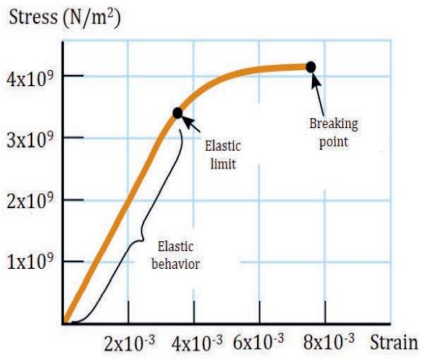
Law of elasticity states that: within elastic limit, stress is directly proportional to corresponding strain.

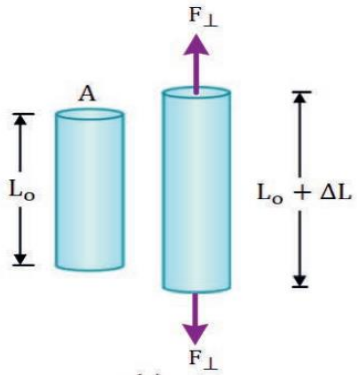
$$\boldsymbol{stress \propto strain}$$

$$\Rightarrow \boldsymbol{stress = K * strain}$$

$k = proportionality\ constant$ called Modulus of elasticity

$$\Rightarrow \boldsymbol{Elastic\ modulus = \frac{stress}{strain}}$$





Types of Elastic Modulus

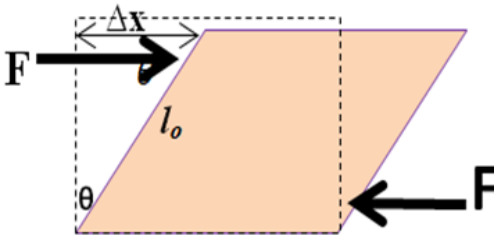
A. $\boldsymbol{Young\ modulus = \frac{Tensile\ Stress}{Tensile\ Strain} = \frac{Compressive\ Stress}{Compressive\ Strain}}$

B. $\boldsymbol{Shear\ Modulus = \frac{Shear\ Stress}{Shear\ Strain}}$

C. $\boldsymbol{Bulk\ modulus = \frac{bulk\ stress}{bulk\ srain}}$

Exercise 7:

1. A wire increases by 10^{-3} of its length when a stress of $1x108Nm^{-2}$ is applied to it. Calculate the Young modulus of the wire.
2. Find shear modulus on the diagram (deformed square) at the right if the stress 480Pa and:
   a. L_o is 10cm and Δx is 5cm
   b. $\theta = 30^o$
   c. Find θ if shear modulus is 800Pa
3. Find original area if the object was previously square, and shear modulus is 600Pa and Δx is 16cm
4. Find area of the figure when shear modulus is 480Pa



5. Find Bulk strain of a metal block under stress $100000N/m^2$, if its bulk modulus is $2.5 * 10^{10}Pa$
6. How large stress must be applied to water (bulk modulus = $2.1 * 10^9Pa$) if it is to be compressed by 0.1%
7. A copper wire has a diameter of 1mm and is 3m long. Assume elastic limit stress = $1.6 * 10^3Pa$, and ultimate stress (breaking point) = $3.4 * 10^8Pa$. Find
   A. The greatest load (weight) that can be supported by the wire without exceeding its elastic limit.
   B. The maximum load (weight) that can be supported without breaking the wire.

Review Exercise 3.3

I. Say true if the statement is correct and false if else.

1. A rubber is more elastic than steel.
2. Young modulus of perfectly rigid body is infinity.
3. A material having grater young modulus also possess greater bulk modulus.
4. Elastic moduli are due to intermolecular forces existing in the material.
5. Young modulus of a perfectly plastic body is zero.

II. Choose the correct answer among the given alternatives.

1. Young modulus is:
   A. Ratio of stress to strain C. ratio of strain to stress
   B. Force per unit area D. force per unit elongation
2. When stretched beyond elastic limit, metal rod such as steel
   A. Become elastic C. obeys hook's law
   B. has no energy D. becomes plastic
3. Which of the following is incorrect?
   A. Hooke's law is applicable only within elastic limit.
   B. Hooke's law is applicable only beyond elastic limit
   C. Young modulus is dimensionless
   D. Hooke's law is only applicable only at elastic limit.
4. The change in shape (geometry) of regular body is due to:
   A. Bulk strain B. sheering stain C. Longitudinal strain D. metallic strain
5. Change in volume of a solid body is due to:
   A. Bulk strain B. Shearing stain C. Longitudinal strain D. metallic strain
6. Change in length of a rod is due to:
   A. Bulk strain B. sheering stain C. Longitudinal strain D. metallic strain
7. A cube of aluminum side 0.1m is subjected to shearing force of 100N. The top face of the cube is displaced through 0.02cm with respect to the bottom face. The shearing strain would be:
   A. 0.02 B. 0.1 C. 0.005 D. 0.002

III. Workout!

1. A steel wire of length 6 m and diameter 0.6 mm is extended by a force of 60 N. The wire extends by 3 mm. Calculate:
   a) the applied stress.
   b) the strain on the wire.
   c) the Young Modulus of the steel
2. A metal wire 75cm long and 1.5mm in diameter stretches 0.4mm when a load of 10kg is hung on its end. Find
   a) Stress
   b) Strain
   c) The young modulus wire.
3. A chalk box has a top area of $15cm^2$ and height 3cm. when a shearing force of 0.5N is applied to the upper surface, the upper surface displaces 4mm relative to the bottom surface. Find
   a) Shear stress
   b) Shear strain
   c) The shear modulus

4. Give short and precise answer to the following questions.

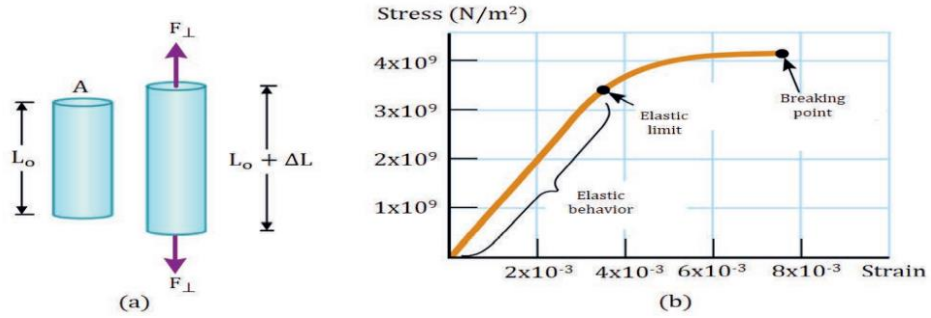
3. List some plastic materials in your local
4. Explain the terms stress, strain and Hooke's Law.
5. Explain elastic moduli with their equations

3.4. Young Modulus

Young modulus is elastic modulus when deformation is caused by either tensile or compressive stress.
Young modulus is property of the material that tells us how easily it can be starched and deform.
For a given material, magnitude of the strain produced is the same whether the stress is tensile or compressive.
Young modulus is ratio of tensile (compressive) stress to tensile (compressive) strain.

$$\textbf{\textit{Young modulus}} = \frac{\textbf{\textit{Tensile Stress}}}{\textbf{\textit{Tensile Strain}}} = \frac{\textbf{\textit{Compressive Stress}}}{\textbf{\textit{Compressive Strain}}}$$

The relationship between the tensile stress and tensile strain is linear when the object is in its elastic range.



(a). A rod of length L0 can be stretched by an amount ΔL after application of a tensile stress F⊥. (b) The stress versus strain diagram for a ductile material.

Table 3: young Modulus of Some substance			
substance	Young Modulus	Substance	Young Modulus
Tungsten	$35 * 10^{10}$Pa	Aluminum	$7.0 * 10^{10}$ Pa
Steel	$20 * 10^{10}$Pa	Glass	$6.5–7.8 * 10^{10}$ Pa
Copper	$11 * 10^{10}$ Pa	Quartz	$5.6 * 10^{10}$ Pa
Brass	$9.1 * 10^{10}$ Pa	Water	-
		Mercury	-

Review Exercise 3.4

I. Say true if the statement is correct and false if else.

1. The large young modulus implies the large elasticity
2. Any material has nonzero young modulus regardless of its physical state.
3. Young modulus depends on the nature of the material
4. Young modulus does not depend on geometric dimension.
5. Breaking strength is directly proportional to cross sectional area.

II. Choose the correct answer among the given alternatives.

1. Spring is made up of steal because and not a copper because:
   A. Elasticity of steel is greater than that of copper.
   B. Elasticity of steel is less than that of copper.
   C. Plasticity of copper is greater than that of copper.
2. If the density of the material increases then, the value of young's modulus:
   A. Increases C. first increase, then decrease
   B. Decreases D. first increase, then decrease
3. If the stress applied is equal to the young's modulus of the material of the wire, the final length of the wire is equal to:
   A. Original length of the wire. C. Half the original length of the wire.
   B. Twice the original length of the wire D. Four times the original length of the wire.

3.5. Static Equilibrium

Static equilibrium is a type of equilibrium that occurs when a body is at rest and there is no net force or net torque acting on it.

Equilibrium

- **All opposing forces are balanced**
- **Net force is zero**
- **Acceleration is zero**
- **Net torque is zero**

Static Equilibrium

- **The body is at rest.**
- **The body is neither rotating nor moving.**

Dynamic equilibrium

- **The object is moving at constant velocity.**
- **The body is in a linear or rotational motion but, its acceleration is zero. (it moves at the same speed)**

A system is at static equilibrium if it is at rest and all forces and acting on it factors influencing the object are balanced.

[Equilibrium ⇨ all opposing forces are balanced and **Static ⇨ the body is at rest]**

For a system to be in equilibrium two conditions need to be satisfied.

1. Net force acting up on the system must be zero (translational equilibrium).
2. Net torque acting up on the system must be zero (rotational equilibrium).

1. First Condition of Equilibrium (Translational Equilibrium)

First condition of equilibrium states that: for an object to remain in equilibrium, the net force acting up on it in all directions must be zero. So: $\mathbf{F_1 + F_2 + F_3 + F_3 + \ldots + F_n = 0}$

$$\Rightarrow \sum \boldsymbol{F} = \mathbf{0} \quad \Rightarrow \mathbf{F_{net} = 0}$$

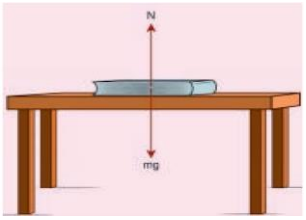
2. Second Condition of Equilibrium (Rotational Equilibrium)

The second condition of equilibrium states that, for an object to remain at equilibrium, the net external torque acting on the object must be zero.

$$\boldsymbol{\tau_{net} = 0} \quad \Rightarrow \quad \boldsymbol{\tau_{cc} = \tau_c}$$

Examples of Static Equilibrium

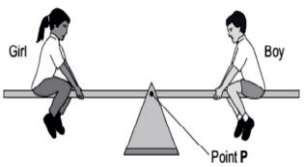
1. **Book on a Table:**



- ✓ Since the book is at rest, the opposing forces (**N** & **mg**) are equal. This implies that, the net force is zero.
- ✓ Taking a pivot at point, vector sum of the forces due to these (N & mg) forces is zero. This implies that net torque is zero.

Therefore, the book is at static equilibrium.

2. **A seesaw balanced by two children:**



- ✓ Since the seesaw is balanced, clockwise torque ($\boldsymbol{\tau_c}$) is equal to counterclockwise torque ($\boldsymbol{\tau_{cc}}$). This implies that net torque is zero.
- ✓ Since the seesaw is at rest, net force is zero.

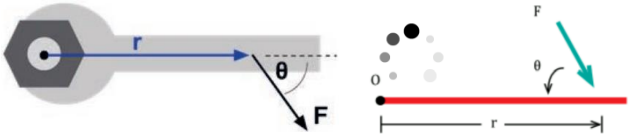
Therefore, the seesaw is at static equilibrium.

Exercise 8:
Choose the correct answer from the given alternatives

1. An object is at static equilibrium if:
   A. The net torque acting on it is zero but, net force is non-zero.
   B. The net force acting on it is zero but, the net torque may not.
   C. Both net force and net torque acting on it is zero.
   D. Neither net force, nor net torque acting on it is zero.
2. Which of the following is not at static equilibrium?
   A. a massive stationary frame hang on a wall using two cables
   B. a Freefalling object at a terminal velocity
   C. a horizontal beam supported by strut
   D. a bulb installed on a sealing
3. Which of the following is necessarily incorrect for a body at static equilibrium?
   A. There is no net torque acting on it
   B. All torques acting on it are balanced
   C. The object is at constant positive velocity
   D. The object is at rest
4. An object is at dynamic equilibrium if :
   C. There is no net torque acting on it
   D. All torques acting on it are balanced
   C. The object is at constant positive velocity
   D. The object is at rest E. all except 'D'
5. If the net torque acting on the body is zero then,
   A. The net torque is zero
   B. The body is accelerating
   C. the body is at uniform motion
   D. necessarily at rest
6. First condition of equilibrium implies that:
   A. the body must be uniformly accelerating
   B. the net torque must be zero
   C. the body must not be experiencing acceleration
   D. the body must not be experiencing torque

Torque (τ)

Torque is an amount of force that causes an object to rotate when it is applied to certain distance, r, from pivot (axis of rotation)

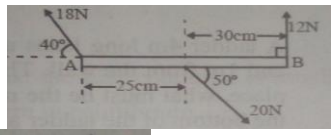


$\boldsymbol{\tau = Fr \sin \theta}$, Where: F = applied force
r = distance b/n pivot and point of application of force.
Pivot (o) = axis of rotation
θ = an angle b/n force and r

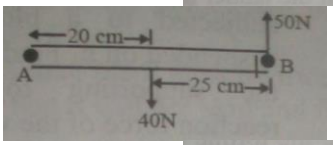
If the force (**F**) is perpendicular ($\boldsymbol{\theta = 90°}$) to radius (**r**), $\boldsymbol{\sin 90^o = 1}$, then torque, $\boldsymbol{\tau = Fr}$

Exercise: 9: solve the following questions accordingly:

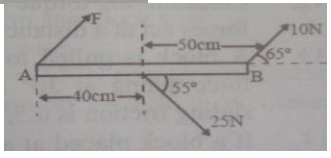
1. Two forces act on a bar as shown in the diagram. What is the net torque about:
   a) Axis A
   b) Axis B



2. Three forces acts on a bar as shown. What is a net torque about:
   a) Axis A
   b) Axis B



3. Three forces act on a bar as shown.
   a) Calculate the net torque about axis A
   b) Calculate the vertical component of F if the net torque about axis B is Zero.



General procedure for solving problems that Involve Objects at Equilibrium

1. Choose one object at a time for consideration. Make a careful free body diagram by showing all the forces acting on that object including gravity, and the point at which these forces act.
2. Choose a convenient coordinate system, and resolve the forces into their components. (F_x and F_y)
3. Using letters to represent unknowns, write down the equilibrium equations for the forces: and assuming the entire forces act in a plane.

$$\sum \boldsymbol{F_x = 0}, \qquad \sum \boldsymbol{F_y = 0}$$

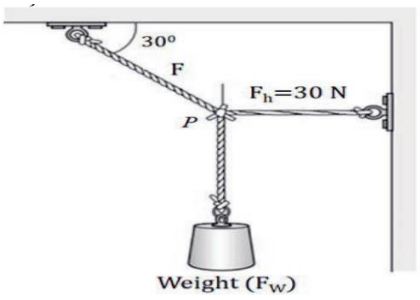
4. For the torque equation

$$\sum \boldsymbol{\tau = 0}$$

5. Solve these equations for the unknowns.

Example I.

An object shown in Figure below is in static equilibrium. The horizontal cord has a force of 30 N. Find the force F of the cord and weight F_W of the object

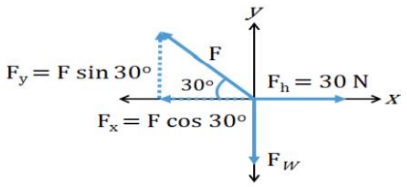


Solution:

You are given $F_W = F_x = 30$ N. What you want to find is F and F_W .

a) Drawing free body diagram:

all forces acting on the given mass are indicated on the free body diagram shown in the Figure below



b) Resolving vectors in to their components:

The only force that has components is F.

Hence, $F_x = F\cos 30^0 = 0.86F$ and

$F_y = F\sin 30^0 = 0.5F$

c) By applying the first condition of equilibrium,

$F_x = 0$ yields $30N - F\cos 30^o = 0$

$0.86\ F = 30N$

$F = 30\ N/0.86 = 34.9\ N$

$F_y = 0$ yields $F\sin 30^o - F_W = 0$

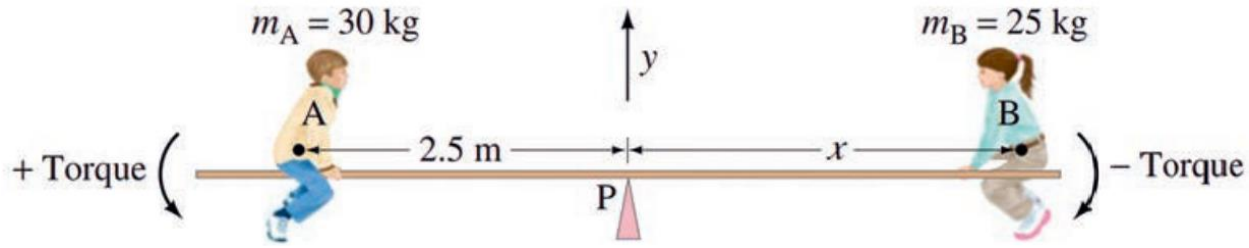
$0.5\ F = F_W$

Substituting the value of F into the above expression gives:

$F_W = 0.5 \times 34.9N =$ **17.5 N**

Example II

A uniform board of mass 'M' serves as a seesaw for two children as shown in Figure below. Child A has a mass of 30 kg and sits 2.5 m from the pivot point, P. At what distance x from the pivot must child B, of mass 25 kg, place her to balance the seesaw?



Solution:

You are given mA = 30 kg , mB = 25 kg , and xA = 2.5 m. The required quantity is the value of xB = x. You can easily solve this problem using the above stated steps.

(a) Draw free body diagram. The forces acting on the board are the forces exerted downward on it by each child, and the upward force

(b) Coordinate system. You choose y to be vertical, with positive upward, and x horizontal to the right, with origin at the pivot.

(c) Force equation. All the forces are in the y-(vertical) direction. So,

(d) Torque equation.

(e) Solve. You solve the torque equation for x and find

Review Exercise 3.5

1. Say true if the statement is correct and false if else.

1. Torque is maximum if the angle between force and radius is 90^o.
2. Acceleration of a body is zero whether it is at static equilibrium or dynamic equilibrium.
3. A body is said to be in equilibrium, it satisfies one of the conditions of equilibrium.
4. Velocity of a body remains zero, if whether it is at static equilibrium, or dynamic equilibrium.
5. Whenever force is applied on a rod at distance, r, from the pivot there will be some torque, whether the rod turns or not.

2. Match the terms under column A with their meaning Under column B

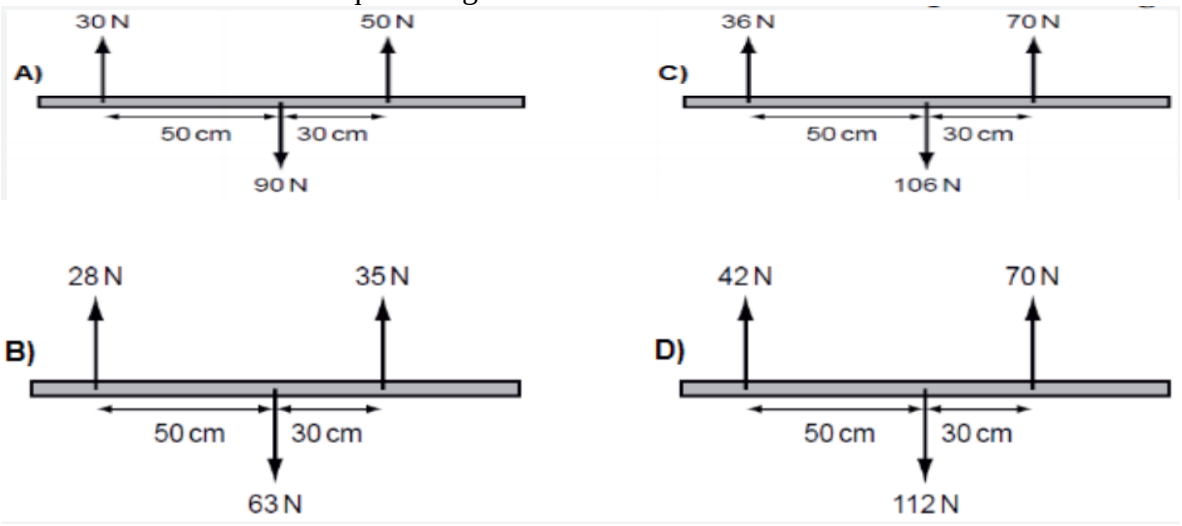
A		B	
1.	First condition of equilibrium	A.	$\boldsymbol{Fr\sin\theta}$
2.	Net force	B.	$\boldsymbol{\tau_{cc} = \tau_c = 0}$
3.	torque	C.	$\boldsymbol{\tau_{cc} = \tau_c}$
4.	Second condition of equilibrium	D.	$\boldsymbol{Fr\cos\theta}$
5.	Net torque	E.	$\boldsymbol{F_1 + F_2 + F_3 + F_3 + \ldots + F_n}$
		F	$\boldsymbol{F_1 + F_2 + F_3 + F_3 + \ldots + F_n = 0}$

3. Choose the correct answer among the given alternatives.

1. Which of the following is odd?
   A. Turning effect of force C. Twisting force
   B. Shear force D. Torque
2. Torque is affected by:
   A. Applied force C. radius
   B. Angle between radius and force D. all
3. Which of the following is correct about torque?
   A. Maximum torque is attained whenever force is applied is parallel to the radius.
   B. Maximum torque is attained whenever force is applied at a pivot point.
   C. Objects rotate (or tilt) whenever the torque applied on them is nonzero.
   D. An objects twist (or tilts) only if the net torque applied on it is nonzero.
   E. Whenever there is rotation there is net torque.
4. If radius is to the right and force is upward, then the torque will be:
   A. Clockwise C. into the page
   B. Upward D. counterclockwise
5. If radius is to the right, and force is to the left, then the torque will be:
   A. in counter clockwise C. in clockwise
   B. in upward direction D. zero
6. If radius is to the left, and force is to the down, then the torque will be:

A. Clockwise C. into the page
B. Upward D. counterclockwise

7. If radius is to the right, and force is to the down, then the torque will be:
   A. in counter clockwise C. in clockwise
   B. in upward direction D. zero
8. If radius is to the left, and force is to the upward, then the torque will be:
   A. in counter clockwise C. in clockwise
   B. in upward direction D. zero
9. If radius is 1m,left and applied force is 2N, upward then the torque is:
   A. 2Nm, counterclockwise C. 0.5Nm, counterclockwise
   B. 2Nm, clockwise D. 0.5Nm, clockwise
10. Four beams of the same length each have three forces acting on them. Which beam has both zero resultant force and zero resultant torque acting?

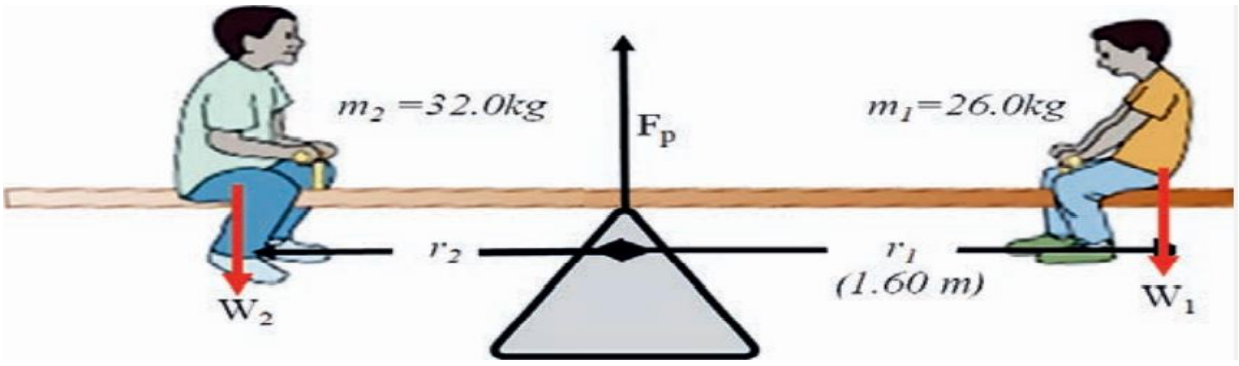


4. Give short and precise answer to the following questions.

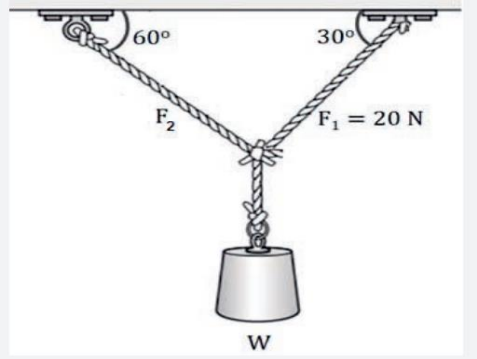
1. The two children shown in the Figure below are balanced on a seesaw of negligible mass. The first child has a mass of 26.0 kg and sits 1.60 m from the pivot.

(a) If the second child has a mass of 32.0 kg, how far is he from the pivot?

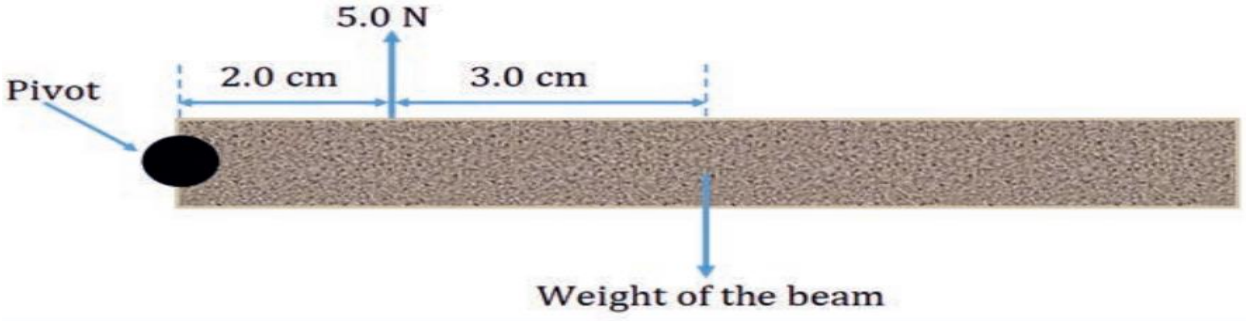
(b) What is F_p, the supporting force exerted by the pivot?



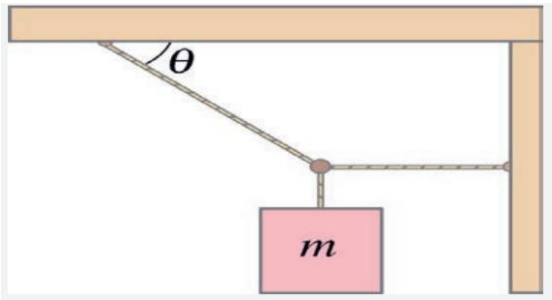
2.A force of 20 N at angle of 30^o to the horizontal and a force F_2 at an angle of 60^o to the horizontal are applied on an object as shown in the Figure below so as to make the object in equilibrium. Calculate the magnitude of the force F_2 and weight of the object.



3.A beam pivoted at one end has a force of 5.0 N acting vertically upwards on it as shown in the Figure below. What is the weight of the beam?



4.Find the tension in the two cords shown in the Figure below. Neglect the mass of the cords, and assume that the angle is 33^o and the mass m is 190kg.



5.Two wires are made of the same metal. The length of the first wire is half that of the second and its diameter is double that of the second wire. If equal loads are applied on both wires, find the ratio of the increase in their lengths?