2 NEWTON'S LAWS OF MOTION

2.1 FORCE

Most people have an idea of force. Consider, for instance, a book lying on a horizontal table. We know that force must be applied to the book to move it along the table. Force may be applied directly to the book by pushing it or indirectly, for example, a string attached to the book and pulling the string. Obviously the movement of the book is related to the magnitude of the force used, direction in which force is applied and also the point at which force is applied to the book. For example, if a string is attached to one edge of the book and pulled vertically the book will tilt about the opposite edge but if the string is attached to the middle of the book and pulled vertically no tilting will take place. So three factors determine the effect that a force has on a body to which it is applied.

(a) The amount, or the magnitude of the applied force.

(b) The direction is which the force is applied.

(c) The point of application of the force.

2.2 Types of Forces

There are basically three forces which are commonly encountered in mechanics.

(a) Field Forces

These are the forces in which contact between two objects is not necessary. Gravitational force between two bodies and electrostatic force between two charges are two examples of field forces. Weight (W = mg) of a body comes in this category.

(b) Contact Forces

Two bodies in contact exert equal and opposite forces on each other.

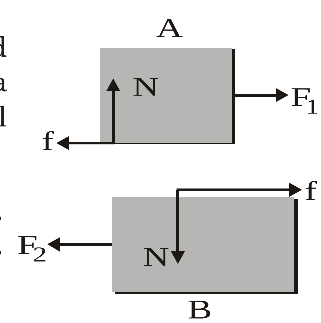
A contact force has two components. The part of the force that lies within the plane of contact is Friction, which must be overcome for the two objects to slide relative to one another along that plane. The part of the force that is perpendicular to the plane of contact is called the normal force.

Thus, there are two mutually perpendicular components of the contact force :

1. Normal reaction (N)
2. Force of Friction (f)

The net contact force = $\sqrt{N^2 + f^2}$

Consider two wooden blocks A and B being rubbed against each other.

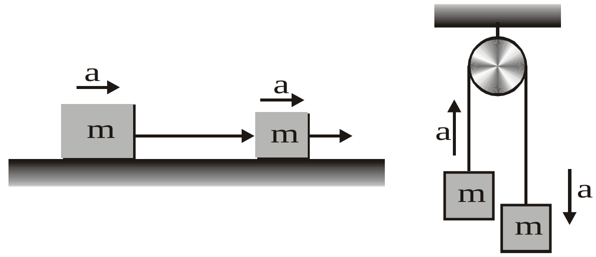


In the diagram, A is being moved to the right while B is being moved leftward. In order to see more clearly which forces act on A and which on B, a second diagram is drawn showing a space between the blocks but they are still supposed to be in contact.

In Fig. the two normal reactions each of magnitude N are perpendicular to the surface of contact between the blocks and the two frictional forces each of magnitude f act along that surface, each in a direction opposing the motion of the block upon which it acts.

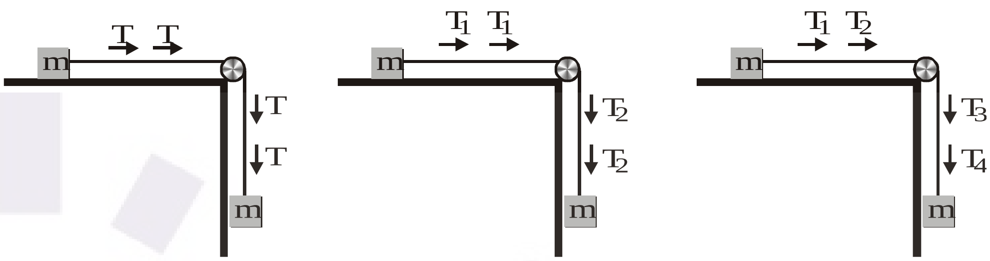
(c) **Attachment to Another Body**

Tension (T) in a string and spring force (F = kx) come in this group. Regarding the tension and string, the following three points are important to remember.



1. If a string is inextensible the magnitude of acceleration of any number of masses connected through string is always same.
2. If a string is massless, the tension in it is same everywhere. However, if a string has a mass, tension at different points will be different.
3. If there is friction between string and pulley, tension is different on two sides of the pulley, but if there is no friction between pulley and string, tension will be same on both sides of the pulley.

Last two points can be understood in diagram as follows :



Spring Force (F = kx) has been discussed in detail in the chapter of work, energy and power.

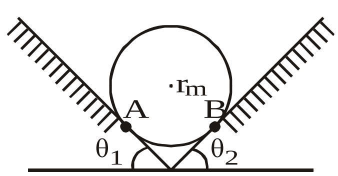
2.3 FREE BODY DIAGRAM

A free body diagram (FBD) consists of a diagrammatic representation of a single body or a sub-system of bodies isolated from its surroundings showing all the forces acting on it.

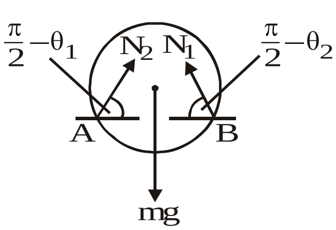
Consider, for example, a book lying on a horizontal surface.

A free body diagram of the book alone would consists of its weight (W = mg), acting through the centre of gravity and the reaction (N) exerted on the book by the surface.

Example : Draw the FBD in following cases :



Solution :

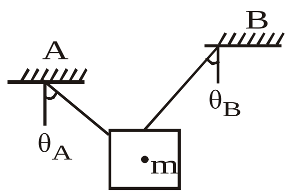


Dumb Question

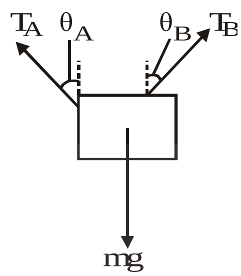
(1) Why is the normal acting like this ?

Ans. The direction of the normal force is always perpendicular to the surface of contact.

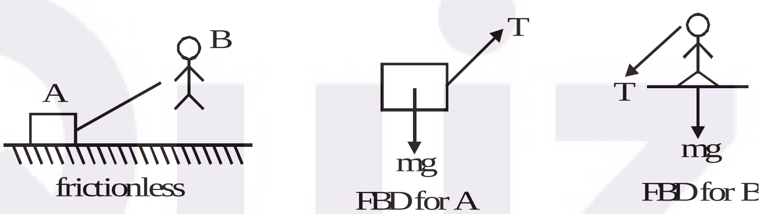
(2) Why is the tension direction like this ?



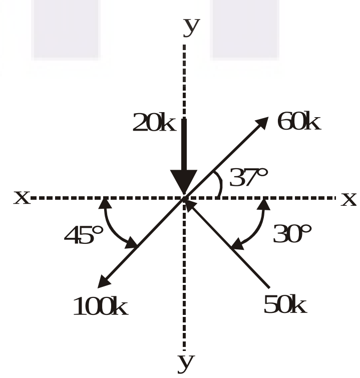
Ans. The direction of tension is always away from the point of contact.



Example : B is pulling A. If we need to draw FBD for both then ?



2.4 ALGEBRAIC FORCE RESOLUTION



Given :

Four concurrent forces with the magnitudes and geometry shown.

Determine :

(a) the vertical and horizontal components of each of the forces

(b) the resultant algebraically

(c) the equilibrium

Solution : (a) First break each of the forces into its horizontal and vertical components, either by inspection or using the algebraic method. The components can neatly be entered one by one into a chart similar to the one shown below.

Force	F_x	F_y
60	–	–
50	–	–
100	–	–
20	–	–

The 20 k force is the most straightforward,, so enter its components into the chart first. Be careful to enter the correct sign for each component or the overall result will be incorrect.

Force	F_x	F_y
60	–	–
50	–	–
100	–	–
20	0	– 20

The 60 k force can also be resolved by observation {as cos 37 $= \frac{4}{5}$, sin 37 $= \frac{3}{5}$} F_x = 4/5 (60 k) = 48 k and F_y = 3/5 (60) = 36. Enter these numbers into the chart.

The last two forces are most efficiently resolved using the algebraic equations.

Force	F_x	F_y
60	48	36
50	–	–
100	–	–
20	0	– 20

$F_x = F \cos \phi$

$Fy = F \sin \phi$

So, for the 50 k force, its components are

$F_x = -\,50 \text{ k} \cos 30 = -\,50(.866) = -\,43.3\text{k}$

$F_y = 50\text{k} \sin 30 = 50(.5) = 25\text{k}$

and for the 100k force

$F_x = -\,100 \text{ k} \cos 45 = -\,100(.707) = -\,70.7\text{k}$

$F_y = -\,100 \text{ k} \sin 45 = -\,100(.707) = -\,70.7\text{k}$

Use these components to complete the chart.

Force	F_x	F_y
60	48	36
50	– 43.3	25
100	– 70.7	– 70.7
20	0	– 20

(b) The components of the resultant are equal to the sum of the columns of the completed chart.

Force	F_x	F_y
60	48	36
50	– 43.3	25
100	– 70.7	– 70.7
20	0	– 20
R	– 66.7	– 29.7

The resultant is found using the Pythagorean Theorem :

$$= \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(-66.7)^2 + (-29.7)^2}$$

$$= 72.4 \text{ k}$$

Its angle can be found by applying

$\tan\phi$ = opposite side [F_y] / adjacent side [F_x]

$\tan\phi = -29.7 / -66$

$\tan\phi = .45$

$\phi = \tan^{-1}(0.45)$

ϕ = **24.2 degrees below the x axis down and to the left**

The quadrant can be formed by observation using the signs of the components of the resultant. In this case, both F_x and F_y are negative, so the resultant will lie in the lower left quadrant.

(c) The equilibrant will be the inverse of the resultant. Its magnitude will be the same, 72.4 k, but it will act an angle 24.2 degrees above the x axis, up and to the right.

Algorithm for Solving Problems :

Step 1 :

Define the system :

On this system you have to apply Newton's laws. A system may consist of any no of pa each component must have same acceleration.

Step 2 :

Identify the forces :

List out all the forces acting on the system.

Step 3 :

Draw FBD of the system.

Step 4 :

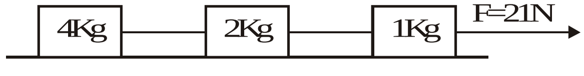
Choose Ads and write equation. Proper signs must be put with forces or acceleration.

Illustration :

In the arrangement shown in figure, the strings are light and inextensible. The surface placed is smooth. Find

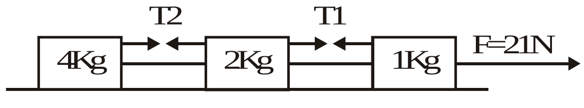
(a) **Acceleration of each block**

(b) **The tension in each string**



Solution : (a) Let a be the acceleration of each block and T_1 and T_2 be the tensions in the two strings.

Taking the two blocks and the two strings as the system.

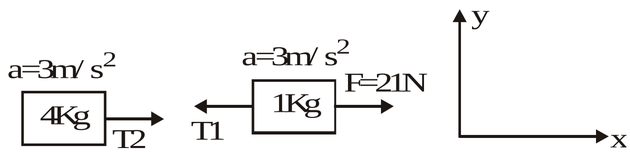


$$\sum F_x = ma_x$$

Or $21 = (4 + 2 + 1)\ a$

$a = 3 m/s^2$.

(b) Free body diagram (showing forces in x direction) of 4 kg block and 1 kg block are shown



$$\sum F_x = ma_x$$

For 1 kg block, $F\text{-}T_1 = (1)\ (a)$

Or $21 - T_1 = (1)\ (3) = 3$

$\therefore \quad T_1 = 21\text{-}3 = 18$ N

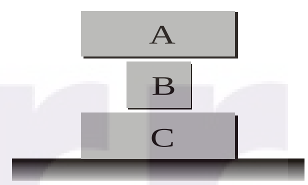
For 4 kg block,

$T_2 = (4)\ (a)$

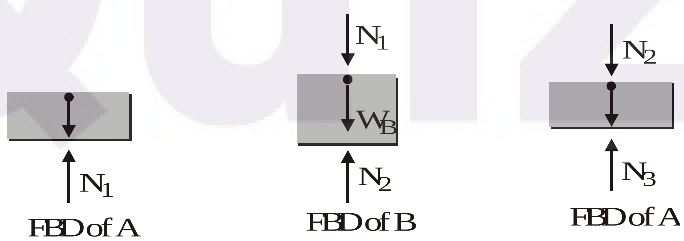
$T_2 = (4)\ (3) = 12$ N

Example 1

Three blocks A, B and C are placed one over the other as shown in figure. Draw free body diagrams of all the three blocks.



Solution : Free body diagrams of A, B and C are shown below :



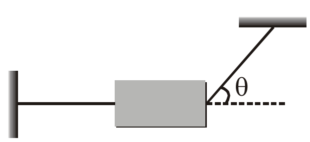
Here, N_1 = normal reaction between A and B

N_2 = normal reaction between B and C

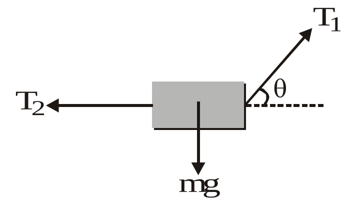
N_3 = normal reaction between C and ground.

Example 2

A block of mass m is attached with two strings as shown in figure. Draw the free body diagram of the block.



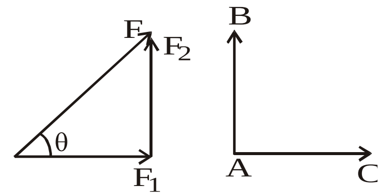
Solution : The free body diagram of the block is as shown in



Equilibrium :

When the **net resultant** of all linear and rotational forces on a body is zero, it is said to be in equilibrium. However, it's not necessary that all force on it is zero.

For this, first we would study the resolution of force into components and moment of force about a point.



In figure, $F_1 = F \cos \theta$ = component of $\vec{F}$ along AC

$F_2 = F \sin \theta$ = components of $\vec{F}$ perpendicular to AC or along AB

Finding such components is referred to as resolving a force in a pair of perpendicular directions. Note that the components of a force in a direction perpendicular to itself is zero. For example, if a force of 10 N is applied on an object in horizontal direction then its component along vertical is zero. Similarly, the component of the above force in the direction of force (horizontal) will be 10 N.

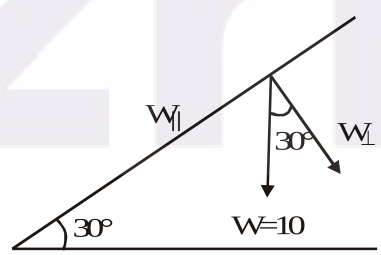
Example 3

Resolve a weight of 10 N in two directions which are parallel and perpendicular to a slope inclined at 30 to the horizontal.

Solution : Component perpendicular to the plane

$$W_{\perp} = W' \cos 30 = (10)\frac{\sqrt{3}}{2}$$

$$= 5\sqrt{3}\,\text{N} \qquad \textbf{Ans.}$$



and component parallel to the plane

$$W_{\parallel} = W' \sin 30 = (10)\left(\frac{1}{2}\right)$$

$$= 5\ \text{N} \qquad \textbf{Ans.}$$

Example 4

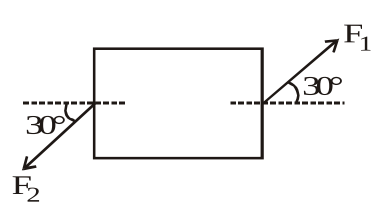
Show that the net resultant force on a given body is zero by resolving into components.

Solution : Component of force F_1 along horizontal direction is $F_1 \cos 30$ while that of F_2 is $F_2 \cos 30$. Given

$$|F_1| = |F_2|,\ F_1 \cos 30 = F_2 \cos 30$$

Hence, net force along horizontal direction is zero.

Similarly we can prove for vertical case.

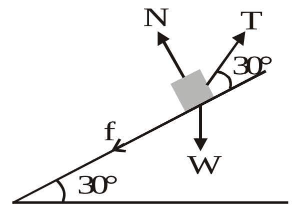


Example 5

A body is supported on a rough plane inclined at 30 to the horizontal by a string attached to the body and held at an angle of 30 to the plane. Draw a diagram showing the forces acting on the body and resolve each of these forces.

(a) **horizontally and vertically**

(b) **parallel and perpendicular to the plane**



Solution : The forces are :

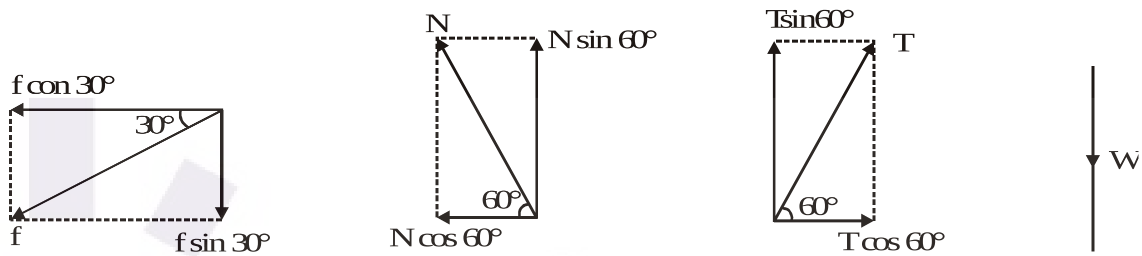
the tension in the string T

the normal reaction with the plane N

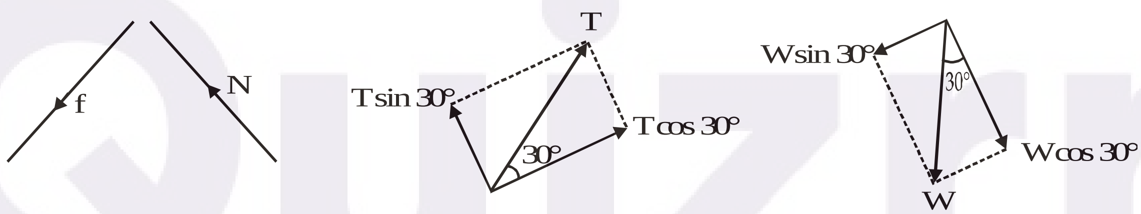
the weight of the body

and the friction f

(a) Resolving horizontally and vertically

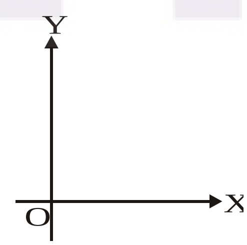


(b) Resolving parallel and perpendicular to the plane.



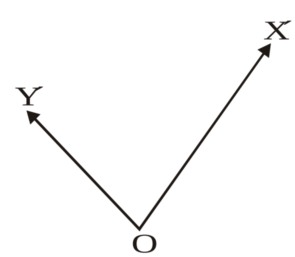
Resolving horizontally and vertically in the senses OX and OY as shown, the components are :

Force	Components	
	Parallel to OX (horizontal)	**Parallel to OY (vertical)**
f	– f cos 30	– f sin 30
N	– N cos 60	N sin 60
T	T cos 60	T sin 60
W	0	– W



Resolving parallel and perpendicular to the plane in the senses OX´ and OY´ as shown, the components are :

Force	Components	
	Parallel to OX´ (horizontal)	**Parallel to OY´ (vertical)**
f	– f	0
N	0	N
T	T cos 30	T sin 30
W	– W sin 30	– W cos 30



Moment of a Force

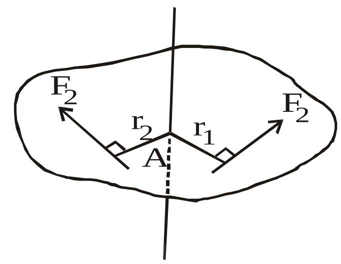
The general name given to an turning effect is **torque.** The magnitude of torque, also known as the moment of a force F is calculated by multiplying together the magnitude of the force and its perpendicular distance $r_\perp$ from the axis of rotation. This is denoted by C or τ (tau).

i.e. $C = Fr_\perp$ or $\tau = Fr_\perp$

2.7 Direction of Torque

The angular direction of a torque is the sense of the rotation it would cause.

Consider a lamina that is free to rotate in its own plane about an axis perpendicular to the lamina and passing through a point A on the lamina. In the diagram the moment about the axis of rotation of the force F_1 is F_1r_1 anticlockwise and the moment of the force F_2 is F_2r_2 clockwise. A convenient way to differentiate between clockwise and anticlockwise torques is to allocate a positive sign to one sense (usually, but not invariably, this is anticlockwise) and negative sign to the other.



With this convention, the moments of F_1 and F_2 are $+ F_1r_1$ and $- F_2r_2$ (when using a sign convention in any problem it is advisable to specify the chosen positive sense).

Zero moment

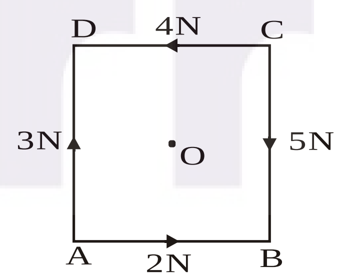
If the line of action of a force passes through the axis of rotation, its perpendicular distance from the axis is zero. Therefore, its moment about that axis is also zero.

Example 6

ABCD is a square of side 2 m and O is the centre. Forces act along the sides as shown in the diagram. Calculate the moment of each force about

(a) **an axis through A and perpendicular to the plane of square**

(b) **an axis through O and perpendicular to the plane of square.**



Solution : Taking anticlockwise moments as positive we have :

(a)

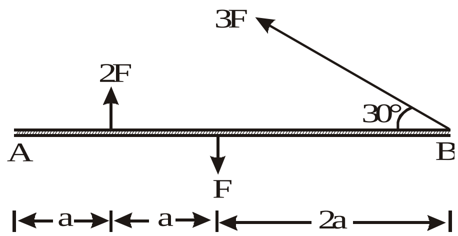
Magnitude of force	**2N**	**5N**	**4N**	**3N**
Perpendicular distance from A	0	2 m	2 m	0
Moment about A	0	– 10 N-m	+ 8 N-m	0

(b)

Magnitude of force	**2N**	**5N**	**4N**	**3N**
Perpendicular distance from O	1 m	1 m	1 m	1 m
Moment about O	+ 2 N-m	– 5 N-m	+ 4 N-m	– 3 N-m

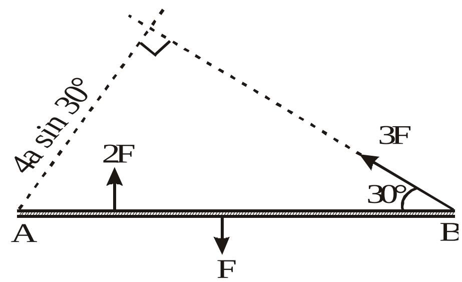
Example 7

Forces act as inclined on a rod AB which is pivoted at A. Find the anticlockwise moment of each force about the pivot.



Solution :

Magnitude of force	**2F**	**F**	**3F**
Perpendicular distance from A	a	2a	4a sin 30
Anticlockwise Moment about A	+ 2 Fa	– 2 Fa	+ 6 Fa



Coplanar Forces in Equilibrium

When an object is in equilibrium under the action of a set of two or more coplanar forces, each of three factors which comprises the possible movement of the object must be zero, i.e., the object has

(i) no linear movement along any two mutually perpendicular directions ox and oy.

(ii) no rotation about any axis.

The set of forces must, therefore, be such that

(a) the algebraic sum of the components parallel to ox is zero or $\Sigma F_x = 0$

(b) the algebraic sum of the components parallel to oy is zero or $\Sigma F_y = 0$

(c) the resultant moment about any specified axis is zero or $\Sigma \tau_{\text{any axis}} = 0$

Thus, for the equilibrium of a set of two or more coplanar forces :

$$\Rightarrow \boxed{\begin{array}{c}\sum F_x = 0 \\ \sum F_y = 0 \text{ and } \sum \tau_{\text{any axis}} = 0\end{array}}$$

Using the above three conditions, we get only three set of equations. So, in a problem number of unknowns should not be more than three.

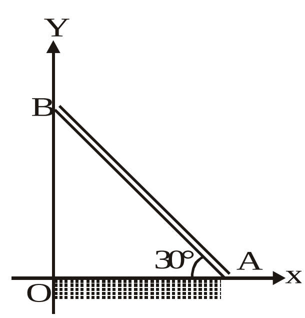
Example 8

A rod AB rests with the end A on rough horizontal ground and the end B against a smooth vertical wall. The rod is uniform and of weight W. If the rod is in equilibrium in the position shown in figure. Find :

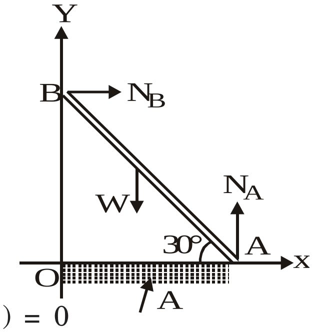
(a) **frictional force at A**

(b) **normal reaction at A**

(c) **normal reaction at B.**



Solution : Let length of the rod be 2l. Using the three conditions of equilibrium. Anticlockwise moment is taken as positive.



(i) $\sum F_x = 0 \quad \therefore \quad N_B - f_A = 0$

or $\quad N_B = f_A \quad$ ...(i)

(ii) $\sum F_y = 0 \quad \therefore \quad N_A - W = 0$

or $\quad N_A = W \quad$ ... (ii)

(iii) $\sum \tau_0 = 0 \quad \therefore \quad N_A \; (2l \cos 30\;) - N_B \; (2l \sin 30\;) - W \; (l \cos 30\;) = 0$

or $$\sqrt{3}\,N_A - N_B - \frac{\sqrt{3}}{2}W = 0$$

Solving these three equations, we get

(a) $f_A = \dfrac{\sqrt{3}}{2}W$ (b) $N_A = W$ (c) $N_B = \dfrac{\sqrt{3}}{2}W$ **Ans.**

Equilibrium of Concurrent Coplanar Forces

If an object is in equilibrium under two or more concurrent coplanar forces the algebraic sum of the components of forces in any two mutually perpendicular directions ox and oy should be zero, i.e., the set of forces must be such that :

(a) the algebraic sum of the components parallel to ox is zero, i.e., $\sum F_x = 0$

(b) the algebraic sum of the components parallel to oy is zero, i.e., $\sum F_y = 0$

Thus, for the equilibrium of two or more concurrent coplanar forces :

$$\sum F_x = 0$$

$$\sum F_y = 0$$

The third condition of zero moment about any specified axis is automatically satisfied if the moment is taken about the point of intersection of the forces. So, here we get only two equations. Thus, number of unknown in any problem should not be more than two.

2.6

Newton's laws of motion are three physical laws that form the basis for classical mechanics, directly relating the forces acting on a body to the motion of the body.

The three laws are :

First law

There exists a set of inertial reference frames relative to which all particles with no net force acting on them will move without change in their velocity. This law is often simplified as **"A body persists its state of rest or of uniform motion unless acted upon by an external unbalanced force."** Newton's first law is often referred to as the **law of inertia.** So in other words "every object in motion will stay in motion until acted upon by an outside force."

Second law

Observed from an inertial reference frame, the net force on a particle of constant mass is proportional to the time rate of change of its linear momentum :

F = d (m**v**)/dt. This law is often stated as, **"Force equals mass times acceleration (F =** ma**)"** : the net force on an object is equal to the mass of the object multiplied by its acceleration. This can also be stated as : "acceleration of a body is directly proportional to external force acting on it, while inversely proportional to its mass.

Third law

Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction. The strong form of the law further postulates that these two forces act along the same line. This law is often simplified into the sentence, "**To every action there is an equal and opposite reaction."**

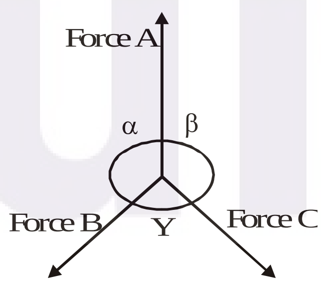
2.7 LAMI'S THEOREM

Lami's theorem in statics states that

if three coplanar forces are acting on a same point and keep it stationary, then it obeys the relation

$$\frac{A}{\sin(\gamma)} = \frac{B}{\sin(\beta)} = \frac{C}{\sin(\alpha)}$$

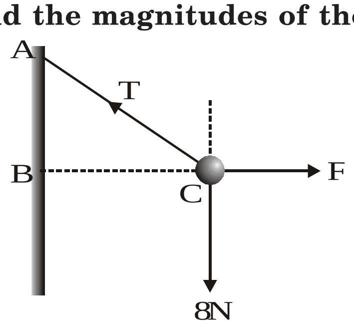
where A, B and C are the magnitude of forces acting at the point (say P), and the values of α, β, and γ are the angles directly opposite to the forces C, B and A respectively.

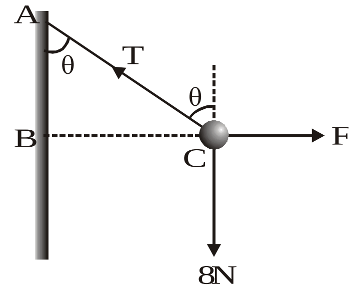


Lami's theorem is applied in static analysis of mechanical and structural systems.

Example 9

One end of a string 0.5 m long is fixed to a point A and the other end is fastened to a small object of weight 8 N. The object is pulled aside by a horizontal force F, until it is 0.3 m from the vertical through A. Find the magnitudes of the tension T in the string and the force F.





Solution : AC = 0.5 m, BC = 0.3 m

$\therefore$ AB = 0.4 m

and if $\angle BAC = \theta$. Then

$$\cos\theta = \frac{AB}{AC} = \frac{0.4}{0.5} = \frac{4}{5} \text{ and } \sin\theta = \frac{BC}{AC} = \frac{0.3}{0.5} = \frac{3}{5}$$

Here, the object is in equilibrium under three concurrent forces. So, we can apply Lami's theorem.

or
$$\frac{F}{\sin(180° - \theta)} = \frac{8}{\sin(90° + \theta)} = \frac{T}{\sin 90°}$$

or
$$\frac{F}{\sin\theta} = \frac{8}{\cos\theta} = T$$

$\therefore$
$$T = \frac{8}{\cos\theta} = \frac{8}{4/5} = 10N$$

and
$$F = \frac{8\sin\theta}{\cos\theta} = \frac{(8)(3/5)}{(4/5)} = 6N$$

Note : Mathematically a body is said to be in equilibrium if

(a) Net force acting on it is zero, i.e., $\vec{F}_{net} = 0$.

(b) Net moments of all the forces acting on it about any axis is zero. Physically the body at rest is said to be in equilibrium. (If a body is at rest just for a moment, it does not mean it is in equilibrium). For example, when a ball is thrown upwards, at highest points of its journey it momentarily comes at rest, but there it is not in equilibrium. A net force (equal to its weight) is acting downward. Due to that force it moves downwards.

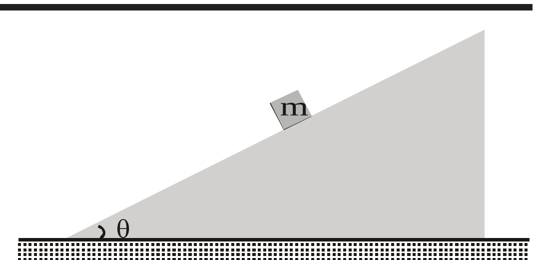
If a problem is asked on equilibrium, check whether the body is in equilibrium (permanent rest) or it is at rest just for a moment.

Now, if the body is in equilibrium, you may resolve the forces in any direction (x, y, z whatsoever). Net force on the body should be zero in all directions.

But if the body is **momentarily at rest but not in equilibrium,** see in what direction will the body moves just after few seconds. Obviously the net force on the body should point in that particular direction. Therefore components of all the forces in a direction perpendicular to the net force or perpendicular to the direction in which motion is likely to occur after few seconds should be zero.

Example 10

A block of mass m is at rest on a rough wedge as shown in figure. What is the force exerted by the wedge on the block ?



Solution : Since, the block is permanently at rest, it is in equilibrium. Net force on it should be zero. In this case only two forces are acting on the block.

(1) Weight = mg (downwards)

(2) Contact force (resultant of normal reaction and friction force) applied by the wedge on the block.

For the block to be in equilibrium these two forces should be equal and opposite.

Therefore, force exerted by the wedge on the block is mg (upwards). **Ans.**

Note :

(i) From Newton's third law of motion-force exerted by the block on the wedge is also mg but downwards.

(ii) The result can also be obtained in a different manner. The normal force on the block is N = mg cos θ and the friction force on the block is f = mg sin θ (not μ mg cos θ)

These two forces are mutually perpendicular.

$\therefore$ Net contact force would be $\sqrt{N^2 + f^2}$ or $\sqrt{(mg\cos\theta)^2 + (mg\sin\theta)^2}$ which is equal to mg.

Example 11

A body with a mass of 1.0 kg is accelerated by a force F = 2.0 N. what this body after 5.0 s of motion ?

Solution : Given

m = 1.0 kg – mass of the body,

F = 2.0 N – force acting on the body

t = 5.0 s – time of motion of this body

From Newton's Second Law of Motion $\mathbf{F} = m\,a$ (1)

we get expression for acceleration $a = \mathbf{F}/m$ (2)

Velocity, according to general formula $\mathbf{v} = \mathbf{v}_0 + a\,t$ (3)

where $\mathbf{v}_0 = 0$, and acceleration is given by equation (2), will be

$\mathbf{v} = (\mathbf{F}/m)t$ (4)

Substituting numbers given in the problem we get

$$\vec{v} = \frac{2.0\,N}{1.0\,kg}5.0\,s = \frac{2.0\,kg\,\frac{m}{s^2}}{1.0\,kg}5.0\,s = 10.0\frac{kg.m}{s^2}.\frac{1}{kg}.s = 1$$

Example 12

The body of mass 1.0 kg has acceleration of 3.0 m/s^2 at 30 deg to the positive direction of the x **axis. What are the components along** x **and** y **axes of the net force acting on this body ?**

Given :

m **= 1.0 kg – mass of the body**

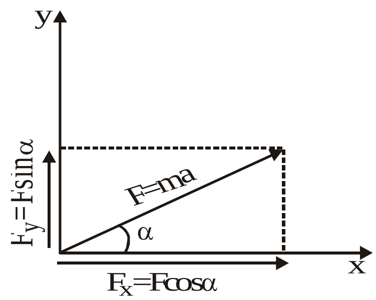
a **= 3.0 m/s^2 – acceleration**

α = 30 deg – direction of acceleration with respect to x axis

We are looking for

F_x, F_y = ? – components of net force due to acceleration of the body.

Solution : The figure below is in fact a solution to this problem.



The net force acting on the body is, according to Newton's Second law of motion,

$$\mathbf{F} = ma \qquad (1)$$

Formulas for their components along x and y axes are given on the Figure. We left the effort of substituting the numbers and finding the units to the Reader.

The final answer is :

$F_x = 2.6$ N

$F_y = 1.5$ N

Example 13

An elevator has a mass of 1400 kg. What is the tension in the supporting cable when the elevator travelling at 10 m/s is brought to rest in a distance of 40 m. Assume a constant acceleration.

Given : m **= 1400 kg – mass of elevator,**

v **= 10 m/s – initial speed of the elevator,**

D = 40 m **– distance required to stop the elevator.**

g **= 9.81 m/s^2 – gravitational acceleration, as usual is assumed to be known.**

Unknown :

T = ? – magnitude of tension in the cable while bringing the elevator to rest.

To find T we must calculate :

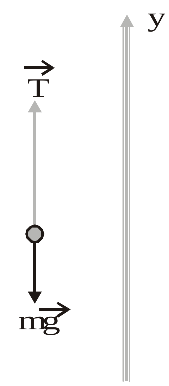
a **= ? – acceleration while stopping the elevator,**

t **= ? – time required to stop elevator.**

Solution : It is convenient to draw a free-body diagram, as in Figure below.

$\vec{T}$ is the tension in the cable of the elevator, $m\vec{g}$ is the gravity force. The resultant force is the force producing acceleration (deceleration in this case) of our elevator.

This can be written in the form of the equation



$$\vec{T} - m\vec{g} = m\vec{a} \quad (1)$$

if we chose the upward direction as positive. Solving for tension gives

$$T = m\vec{g} + m\vec{a} \quad (1a)$$

To calculate the magnitude of the tension T, we must find the magnitude a of the acceleration. It can be found from kinematics equations

$$a = v/t \quad (2)$$

$$D = vt - (1/2)a\ t^2 \quad (3)$$

Equation (2) is based on the fact that elevator final speed is zero. Equation (3) is a standard formula for distance traveled in motion with constant acceleration (negative in this case as directed opposite to the initial speed).

Solving the equations (2) and (3) with respect to acceleration a, we find

$$a = \frac{v^2}{2D} \quad (4)$$

Magnitude of tension T can be found from formula (1) taken without the vector notation (magnitude only!!)

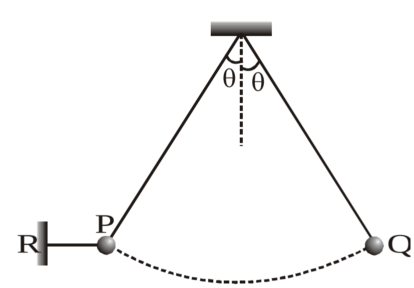
$$T = m\left(g + \frac{v^2}{2D}\right) \quad (5)$$

Substituting numbers given in the problem we get

$$T = 15484 \text{ N.}$$

Example 14

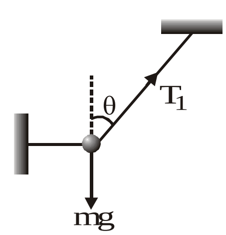
A ball of mass 1 kg is at rest in position P by means of two light strings OP and RP. The string RP is now cut and the ball swings to position Q. If θ = 45 . Find the tensions in the strings in positions OP (when RP was not cut) and OQ (when RP was cut). Take g = 10 m/s².



Solution : In the first case, ball is in equilibrium (permanent rest).

Therefore, net force on the ball in any direction should be zero.

$$\therefore \quad \left(\sum \vec{F}\right) \text{ in vertical direction} = 0$$



or $\quad T_1 \cos\theta = mg$

or $\quad T_1 = \dfrac{mg}{\cos\theta}$

Substituting m_1 = 1 kg, g = 10 m/s^2 and θ = 45 .

We get $$T_1 = 10\sqrt{2}\ \text{N} \qquad \textbf{Ans.}$$

Note : Here we deliberately resolved all the forces in vertical direction because component of the tension in RP in vertical direction is zero. Although, since, the ball is in equilibrium, net force on it in any direction is zero. But in a direction other than vertical we will have to consider component of tension in RP also, which will unnecessarily increase the calculation.

In the second case ball is not in equilibrium (temporary rest). After few seconds it will move in a direction perpendicular to OQ. Therefore, net force on the ball at Q is perpendicular to OQ, or net force along OQ = 0.

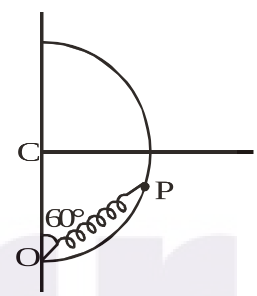
$\therefore$ $$T_2 = mg\cos\theta$$

Substituting the values, we get $$T_2 = 5\sqrt{2}\ \text{N} \qquad \textbf{Ans.}$$

Here, we can see that $$T_1 \neq T_2.$$

Example 15

A smooth semicircular wire track of radius R is fixed in a vertical plane. One end of a massless spring of natural length 3R/4 is attached to the lowest point O of the wire track. A small ring of mass m which can slide on the track is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle 60 with the vertical. The spring constant k = mg/R. Consider the instant when the ring is released.



(i) **Draw the free body diagram of the ring.**

(ii) **Determine the tangential acceleration of the ring and the normal reaction.**

(JEE 1996)

Solution : (i) CP = CO = Radius of circle (R)

$\therefore$ $\angle CPO = \angle POC = 60$

$\therefore$ $\angle OCP$ is also 60 .

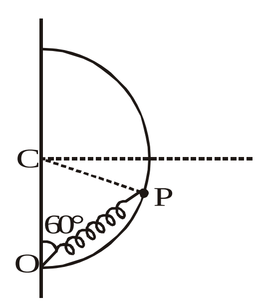
Therefore, ΔOCP is an equilateral triangle.

Hence, OP = R

Natural length of spring is 3R/4

$\therefore$ Extension in the spring,

$$x = R - \frac{3R}{4} = \frac{R}{4}$$



$\Rightarrow$ Spring force, $$F = kx = \left(\frac{mg}{R}\right)\left(\frac{R}{4}\right) = \frac{mg}{4}$$

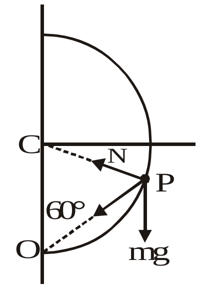
Here, $$F = kx = \frac{mg}{4}$$

and N = Normal reaction.

(ii) **Tangential acceleration, a_T :** The ring will move towards the x-axis just after the release. So net force along x-axis.

$$F_x = F \sin 60 + mg \sin 60 = \left(\frac{mg}{4}\right)\frac{\sqrt{3}}{2} + mg\left(\frac{\sqrt{3}}{2}\right)$$

$$F_x = \frac{5\sqrt{3}}{8} mg$$



Therefore, tangential acceleration of the ring.

$$a_T = a_x = \frac{F_x}{m} = \frac{5\sqrt{3}}{8} g$$

$$a_T = \frac{5\sqrt{3}}{8} g$$ **Ans.**

Normal reaction N

Net force along y-axis on the ring just after the release will be zero.

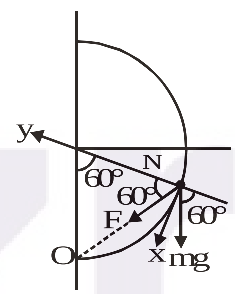
$$F_y = 0$$

$\therefore$ $N + F \cos 60 = mg \cos 60$

$\therefore$ $N = mg \cos 60 - F \cos 60$

$$= \frac{mg}{2} - \frac{mg}{4}\left(\frac{1}{2}\right) = \frac{mg}{2} - \frac{mg}{8}$$

$$N = \frac{3mg}{8}$$ **Ans.**

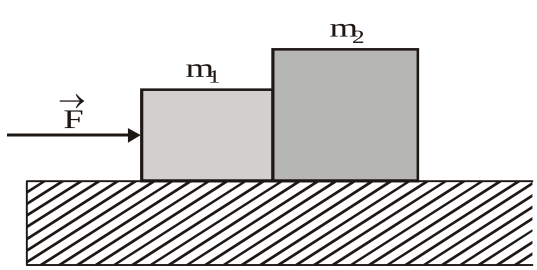


Note : Three types of equilibrium viz stable equilibrium, unstable equilibrium and neutral equilibrium will be discussed in the chapter of work, energy and power.

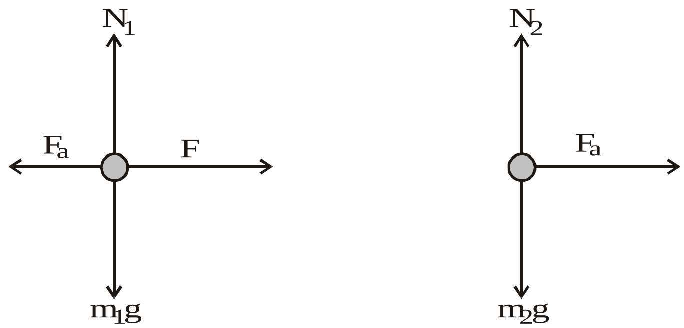
FORCE BETWEEN BLOCKS PUSHED FROM ONE SIDE

Example 16

Two blocks, m_1 = 3.0 kg are put in contact on a frictionless surface. A horizontal force F = 5.0 N is applied to one of them (see Fig. 1). (1) Find the force F_a between the two blocks. (2) Find the force F_b if the force F is applied to m_2 in the opposite direction.



Solution : Fig. 1 is drawn in such a way that it suggests that the force F is applied from the outside, but we know that this force can be considered as concentrated in the center of gravity of this block. We draw the Free Body Diagrams for these blocks. For the block m_1, it is on the left hand side of Fig. 2, and for the block m_2 on the right hand side of this Figure.



The force are applied to the CG (center of gravity) of mass m_1 and m_2. In the explanations and calculations we will use bold face notation for vector quantities. It is easier to handle with text editors as opposed to handwriting, where arrows are more convenient to use. The N_1 force is a reaction force to the gravity force m_1g as described by Newton's third Law of Motion. Analogically, N_2 and m_2g are equal and opposite as this law requires.

For the m_1 block (the left part of Fig. 2) the direction of force F is obvious from the text of the problem (see Fig. 1). But why is the force F_a directed to the left ? Because this is the force extended on block m_1 by block m_2. Block m_1 "pushes" block m_2 so block m_2 creates an "opposite" force according to Newton's Third Law of Motion. Stop and think it over for a moment. This is a very crucial point of all problems involving Newton's Laws. Once you understand the idea of drawing the Free Body Diagram you will solve all problems involving mechanics much faster and more efficiently.

Now it's time to write equations based on Newton's Laws. In the vertical direction

$$m_1g = \mathbf{N}$$

so we can forget about these forces in further analysis.

In the horizontal direction the resultant force exerted on m_1 is $\mathbf{F} - \mathbf{F}_a$ and this is the force accelerating block m. Therefore we can write

$$\mathbf{F} - \mathbf{F}_a = m_1 a \quad (1)$$

The FBD for m_2 shows that the only the horizontal force acting on it is the exerted by block m_1. This force has a magnitude of the F_a from the FBD on the left side if Fig. 2, but the direction is opposite. This is the force accelerating block m_2. As both blocks are in contact they must have the same acceleration a. So, for the second block the equation of motion is

$$F_a = m_2\, a \quad (2)$$

We can drop out the vector notation from these two equations as the directions are well defined on the FBD's for both blocks.

From the (2) we have

$$a = F_a/m_2$$

and substituting this acceleration into (1) we find, after a little elementary algebra,

$$F_a = F\, m_2/(m_1 + m_2)$$

And this is the answer to question (1) from the problem.

If the force F is exerted from right to left, as in part (2) of the problem, the analogical reasoning will lead to the answer.

$$F_B = F\, m_1/(m_1 + m_2)$$

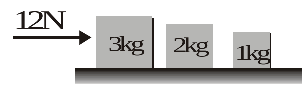
Substituting the values given in the problem we get

$$F_a = 3.0 \text{ N and } F_b = 1.2 \text{ N}$$

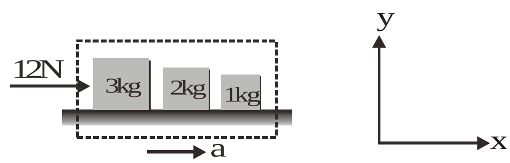
You can wonder why the force between the blocks is larger when you push from the left. This is because in that situation the block which is a kind of transmitter of force must push the larger mass (m_2) than in the second situation, when the larger block is pushing the smaller one.

Example 17

Three blocks of mass 3 kg, 2 kg and 1 kg are placed side by side on a smooth surface as shown in figure. A horizontal force of 12 N is applied on 3 kg block. Find the net force on 2 kg block.



Solution : Since, all the blocks will move with same acceleration (say a) in horizontal direction. Let us take all the blocks as a system.



Net external force on the system is 12 N in horizontal direction. Using $\sum F_x = ma_x$, we get

$$12 = (3 + 2 + 1)a = 6\,a$$

or $\quad a = \dfrac{12}{6} = 2\,\text{m/s}^2$

Now, let F be the net force on 2 kg block in x-direction, then using $\sum F_x = ma_x$ for 2 kg block, we get

$$F = (2)(2) = 4 \text{ N}$$ **Ans.**

Note : Here net force F on 2 kg block is resultant of N_1 and N_2 ($N_1 > N_2$)

where $\quad N_1$ = normal reaction between 3 kg and 2 kg block

and $\quad N_2$ = normal reaction between 2 kg and 1 kg block.

Thus, $\quad F = N_1 - N_2$

Example 18

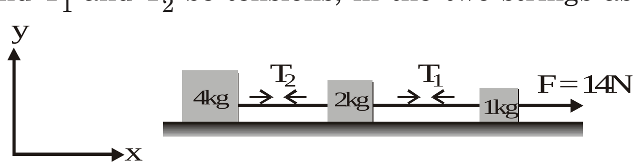
In the arrangement shown in figure. The strings are light and inextensible. The surface over which blocks are placed is smooth. Find :

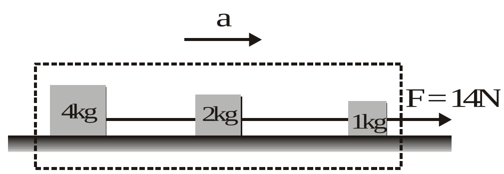
(a) the acceleration of each block

(b) the tension in each string.

Solution : Let 'a' be the acceleration of each block and T_1 and T_2 be tensions, in the two strings as shown in figure.

Taking the three blocks and the two strings as the system.



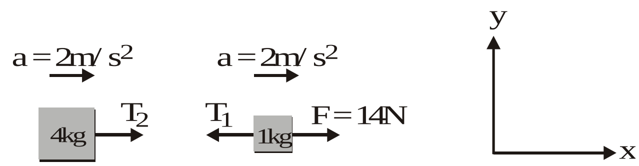


Using $\sum F_x = ma_x$

or $14 = (4 + 2 + 1)a$

or $a = \frac{14}{7} = 2\,m/s^2$ **Ans.**

(b) Free body diagram (showing the forces direction only) of 4 kg block and 1 kg block shown in figure.



Using $\sum F_x = ma_x$

For 1 kg block $F - T_1 = (1)(a)$

$14 - T_1 = (1)(2) = 2$

$\therefore$ $T_1 = 14 - 2 = 12$ N **Ans.**

For 4 kg block $T_2 = (4)(a)$

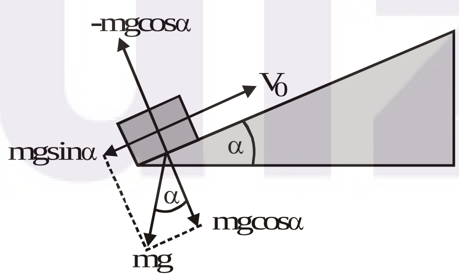
$\therefore$ $T_2 = (4)(2) = 8$ N **Ans.**

OBJECT PROJECTED UP AN INCLINED PLANE

Example 19

A block is projected up an inclined, frictionless plane with an initial speed V_0. How does the distance the block will go up the plane depend on the angle α of the incline ?

The figure below represents the situation described in this problem.



The force of gravity is decomposed into components perpendicular to the inclined plane and parallel to it. According to Newton's Third Law of Motion the "red" force is compensated by the "green" one. The force left is the one parallel to the inclined plane. We are interested in accelerations which can be obtained by dividing the forces by the mass of the block.

The distance which will be traveled by our object after being projected with an initial speed v_0 is

$$D = v_0 t - (1/2) g\sin\alpha t^2 \quad (1)$$

and the final velocity will be zero, according to formula

$$0 = v_0 - g\sin\alpha\, t \quad (2)$$

Solving this set of equations we get

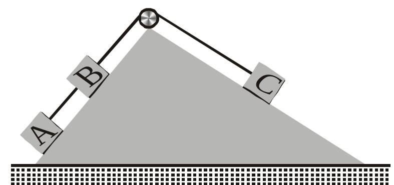
$$D = \frac{v_0^2}{2\,g\sin\alpha}$$

The distance traveled in inversely proportional to the sine of the angle of the incline and this is the answer to the question from the problem.

Notice, that this distance DOEST NOT depend on the mass of the block. If this seems strange to you, think about the moment of projecting this mass. The larger the mass the larger the force required to accelerate this mass to initial speed v_0, and here the dependence on mass is “hidden”.

Example 20

In the adjacent figure, mass of A, B and C are 1 kg, 3 kg and 2 kg respectively.



Find (a) **the acceleration of the system and**

(b) **tensions in the string**

Neglect friction ($g = 10\ m/s^2$)

Solution : (a) In this case net pulling force

$$= m_A g \sin 60 + m_B g \sin 60 - m_C g \sin 30$$

$$= (1)(10)\frac{\sqrt{3}}{2} + (3)(10)\left(\frac{\sqrt{3}}{2}\right) - (2)(10)\left(\frac{1}{2}\right)$$

$$= 24.64 \text{ N}$$

Total mass being pulled = 1 + 3 + 2 = 6 kg

$\therefore$ Acceleration of the system $a = \dfrac{24.64}{6} = 4.1\ m/s^2$

(b) For the tension in the string between A and B

F.B.D. of A

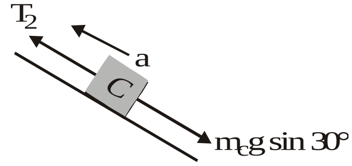
$$m_A g \sin 60 - T_1 = (m_A)(a)$$

$$\therefore \quad T_1 = m_A g \sin 60 - m_A a = m_A (g \sin 60 - a)$$

$$\therefore \quad T_1 = (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right) = 4.56 \text{ N}$$ **Ans.**

For the tension in the string between B and C

F.B.D. of C



$$T_2 - mC\ g \sin 30 = mCa$$

$$\therefore \quad T_2 = mC\ (a + g \sin 30)$$

$$\therefore \quad T_2 = 2\left[4.1 + 10\left(\frac{1}{2}\right)\right]$$

$$= 18.2 \text{ N}$$ **Ans.**

2.8 PSEUDO FORCE

Newton's first and second laws (F = ma) only hold in inertial frames of reference. An observer in an accelerating frame will note that objects seem to move from a state of rest without any apparent force acting on them. For example, a driver using his braking car as a reference frame might note that a coin on his dashboard starts gliding towards the windshield without being visibly pushed.

The easy way out is to take F = ma as a definition, and declare that a force is in fact acting on the coin, with a magnitude equal to the acceleration times mass of it. A force that is "invented" in this way so that the second law will hold in a given accelerated frame of reference is called a pseudo force. The defining characteristic of such a force is that if the same physical situation is described in an inertial frame of reference, all pseudo forces vanishes (as opposed to normal forces between objects, which stay the same no matter what frame of reference they are described in).

The most common use of pseudo forces is not linearly accelerating frames as the above car example, but rotating frames of reference. An observer in a rotating frame of reference will be subjected to two famous pseudo forces, centrifugal force and Coriolis force.

{However, study of coriolis force is not in the course of IIT-JEE}

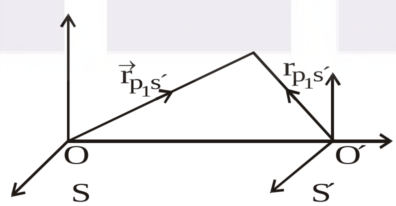
That is what is meant by the common statement that "centrifugal force is not a real force" - if the same situation is looked at from an inertial frame, the centrifugal force does not appear. That does not make it less useful, though. Many situations, like the stress in a spinning wheel, are most conveniently analysed in terms of centrifugal rather than centrifugal force.

Derivation of pseudo force from triangle law of vector addition

$$\vec{r}\, p_1 s = \vec{r}\, p_1 s' + \vec{r}\, s's$$

$$\vec{r}\, p_1 s' = \vec{r}\, p_1 s - \vec{r}\, s's$$

Differentiating twice we get



$$\vec{a}\, ps' = \vec{a}\, p_1 s - \vec{a}\, s's$$

$$\vec{a}^1 = \left(\vec{a} - \vec{a}_0\right)$$

$$\left(m\vec{a} = m\vec{a} - m\vec{a}_0\right)$$

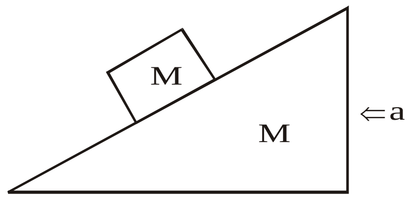
$m\vec{a}$	$= \vec{F}_{\text{external}}$	$+ \; m\left(-\vec{a}_0\right)$
[Force in non-inertial force]	[Relative to inertial frame]	[pseudo force]

$\therefore$ $\boxed{\text{Pseudo force} = m\left(-\vec{a}_0\right)}$

Example 21

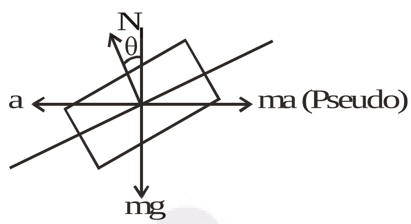
Find the horizontal acceleration that must be imparted to wedge, so that block remains stationary relative to the wedge.

Solution :



Concept : When we apply pseudo force and then analyse the block's FBD, the block should be in equilibrium.

Resolving the forces in vertical and horizontal components and solving them separately for equiliprium position after applying pseudo force



$N\cos\theta = mg$ (vertical)

$N \sin\theta = ma$ (horizontal)

$\therefore \quad a = \dfrac{mg\ \tan\theta}{m} = g\tan\theta$

($a = g \tan\theta$) must be imparted towards left to keep the body stationary relative to block.

(b) Acceleration that would make the block to loose contact with the wedge.

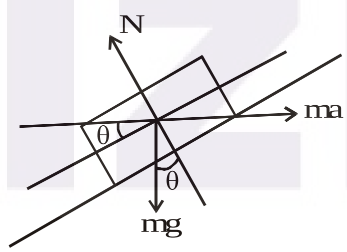
$N = (mg \cos\theta - ma \sin\theta)$

At the breaking point $N \leq 0$

$mg \cos\theta - ma \sin\theta \leq 0$

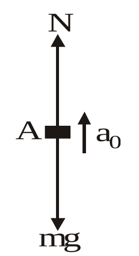
$mg \cos\theta \leq ma \sin\theta$

$\boxed{g \cot\theta \leq a}$



$\therefore$ when $a \geq g \cot\theta$ bodies loose contact.

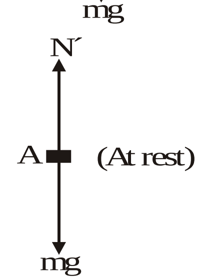
2.11 PSEUDO FORCE



Suppose a block A of mass m is placed on a lift ascending with an acceleration a_0. Let N be the normal reaction between the block and the floor of the lift. Free body diagram of A in ground frame of reference (inertial) is shown in Fig.

$\therefore \quad N - mg = ma_0$

or $\quad N = m(g + a_0)$ ...(i)



But if we draw the free body diagram of A with respect to the elevator (a non-inertial frame of reference) without applying the pseudo force, we get

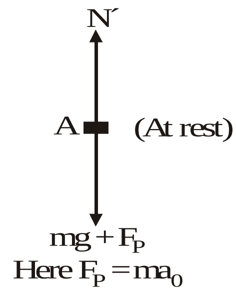
$N' - mg = 0$ or $N' = mg$ ...(ii)

Since, $N' \neq N$, either of the equations is wrong. But if we apply a pseudo force in non-inertial frame of reference, N´ becomes equal to N as shown in Fig. 3.93. Acceleration of block with respect to elevator is zero.

$\therefore \quad N' - mg - ma_0 = 0$

or $\quad N' = m(g + a_0)$ ...(iii)

$\therefore \quad N' = N]$



Pseudo force is given by $\vec{F}_P = -m\vec{a}_0$. Here, $\vec{a}_0$ is the acceleration of the non-inertial frame of reference and m the mass of the body under consideration. In the whole chapter, we will show the pseudo force by $\vec{F}_P$.

Thus, we may conclude that pseudo force is not a real force. When we draw the free body diagram of a mass, with respect to an inertial frame of reference we apply only the real forces (forces which are actually acting on the mass), but when the free body diagram is drawn from a non-inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

Note : In case of rotating frame of reference this pseudo force is called the centrifugal force when applied for centrifugal acceleration. Let us take few examples of pseudo forces.

Example 21

In the adjoining figure, the coefficient of friction between wedge (of mass M) and block (of mass m) is μ.

Find the magnitude of horizontal force F required to keep the block stationary with respect to wedge.

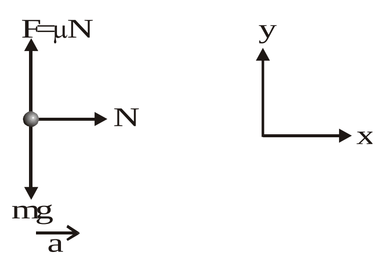
Solution : Such problems can be solved with or without using the concept of pseudo force. Let us, solve the problem by both the methods.

a = Acceleration of (wedge + block) in horizontal direction

$$= \frac{F}{M + m}$$

Inertial frame of reference (Ground)

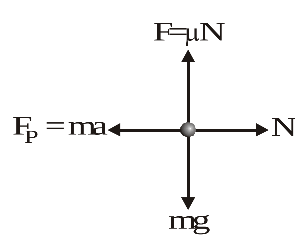
F.B.D. of block with respect to ground (only real forces have to applied)



with respect to ground is moving with an acceleration 'a'. Therefore, $\sum F_y = 0$ and $\sum F_x = ma$

Non-inertial frame of reference (Wedge)

F.B.D. of 'm' with respect to wedge (real + one pseudo force)



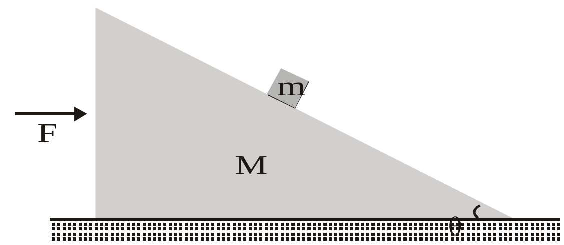
with respect to wedge block is stationary.

$\therefore \quad \sum F_x = 0 = \sum F_y$

$\therefore \quad mg = \mu N$ and $N = ma$

Example 22

All surfaces are smooth in adjoining figure. Find F such that block remains stationary with respect to wedge.

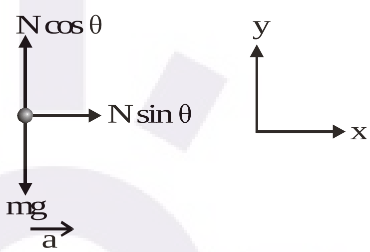


Solution : Acceleration of (block + wedge) $a = \dfrac{F}{(M+m)}$

Let us solve the problem by both the methods

From inertia frame of ref. (Ground)

F.B.D. of block w.r.t. ground (Apply real forces)



with respect to ground block is moving with an acceleration 'a'

$\therefore \; = \Sigma F_y = 0 \Rightarrow N \cos\theta = mg$ ...(i)

and $\Sigma F_x = ma \Rightarrow N \sin\theta = ma$ ...(ii)

From Eqs. (i) and (ii)

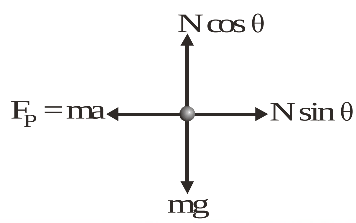
$a = g \tan\theta$

$\therefore \; F = (M + m)\, a$

$= (M + m)\, g \tan\theta$ **Ans.**

From non-inertial frame of ref. (Wedge)

F.B.D. of block w.r.t. wedge (real forces + pseudo force)



w.r.t. wedgde, block is stationary

$\therefore \; \Sigma F_y = 0 \;\Rightarrow\; N \cos\theta = mg$ ...(iii)

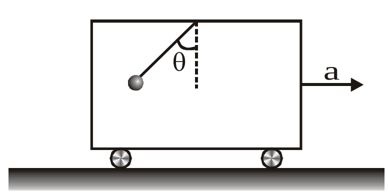
$\Sigma F_x = 0 \;\Rightarrow\; N \sin\theta = ma$ ...(iv)

From Eqs. (iii) and (iv), we will get the same result

i.e., $F = (M + m) \tan\theta$

Example 22

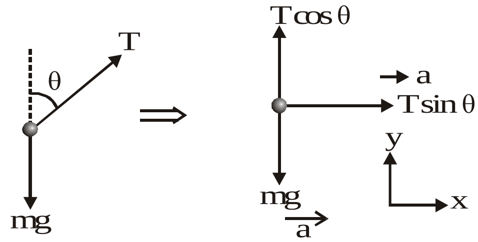
A bob of mass m **is suspended from the ceiling of a train moving with an acceleration** 'a' **as shown in figure. Find the angle θ in equilibrium position.**



Solution : This problem can also be solved by both the methods.

Inertial frame of reference (Ground)

F.B.D. of bob w.r.t. ground (only real forces)



with respect to ground bob is also moving with an acceleration 'a'

$\therefore \quad \sum F_y = 0 \quad \Rightarrow T \cos\theta = mg \qquad ...(i)$

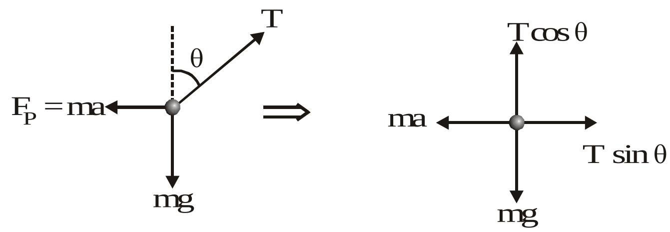
and $\quad \sum F_x = ma \quad \Rightarrow T \sin\theta = ma \qquad ...(ii)$

From Eqs. (i) and (ii), we get

$\tan\theta = \frac{a}{g}$ or $\theta = \tan^{-1}\left(\frac{a}{g}\right)$ **Ans.**

Non-inertial frame of reference (Train)

F.B.D. of bob w.r.t. train (real forces + pseudo force) :



With respect to train, bob is in equilibrium

$\therefore \quad \sum F_y = 0 \quad \Rightarrow T \cos\theta = mg \qquad ...(iii)$

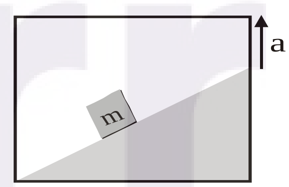
and $\quad \sum F_x = ma \quad \Rightarrow T \sin\theta = ma \qquad ...(iv)$

From Eqs. (iii) and (iv), we get the same result,

i.e. $\quad \theta = \tan^{-1}\left(\frac{a}{g}\right)$ **Ans.**

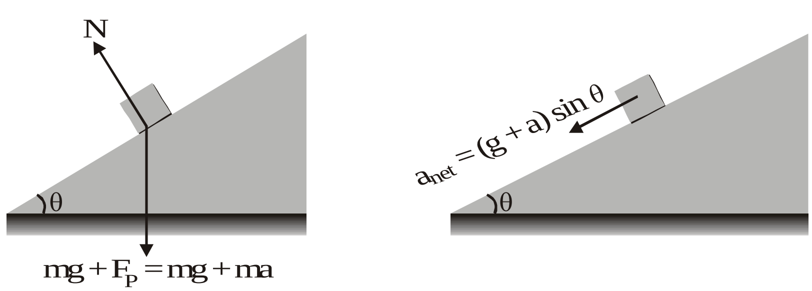
Example 24

In the adjoining figure, a wedge is fixed to an elevator moving upwards with an acceleration 'a'. A block of mass 'm' is placed over the wedge. Find the acceleration of the block with respect to wedge. Neglect friction.



Solution : Since, acceleration of block w.r.t. wedge (an accelerating or non-inertial frame of reference) is to be find out

F.B.D. of 'block' w.r.t. 'wedge' is shown in Fig. 3.104.



The acceleration would had been g sin θ (down the plane) if the lift were stationary or when only weight (i.e. mg) acts downwards.

Here, downward force is m(g + a)

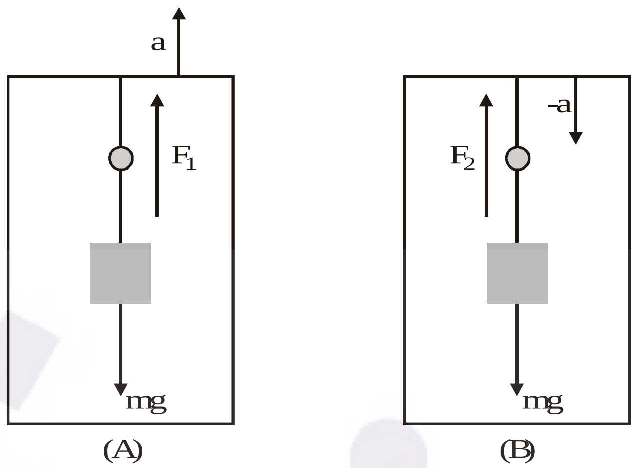
$\therefore$ Acceleration of the block (of course w.r.t. wedge) will be (g + a) sin θ down the plane. **Ans.**

FINDING THE MASS OF AN OBJECT IN AN ACCELERATING ELEVATOR

Example 24

An object is hung from a spring balance attached to the ceiling of an elevator. The balance reads F_1 = 12 N when the elevator is accelerating upward, and reads F_2 = 8 N when it is accelerating downward with acceleration of the same magnitude a. Find the mass of this object and magnitude of acceleration a. We assume that gravitational acceleration g is known.

Solution :



From the Fig. 1 we can say that the reading of the balance is simply the force exerted on mass m by the rope attached to the mass. In both cases

(A) elevator accelerates upwards

(B) elevator accelerates downwards

the force exerted by rope suspending the mass is directed upwards. The resultant force is responsible for acceleration which has the same magnitude a, but different direction in both cases. We write Newton's Second Law of motion for these two cases. As all forces are acting along the same line (vertical) we omit vector notation.

$$F_1 - mg = ma \quad (1)$$

$$F_2 - mg = -ma \quad (2)$$

The minus sign on the right hand side of Eq. 2 says that acceleration a in case (B) has the opposite direction to acceleration in case (A).

This is a set of two linear equations with two unknowns, which required only a little algebra to solve. We leave this algebra to the Reader.

The solution is :

$$a = \frac{F_1 - F_2}{F_1 + F_2} g = 1.962 \frac{m}{s^2}$$

$$m = \frac{F_1 + F_2}{2g} = 1.019\,kg$$

A MAN LOWERS HIMSELF WITH THE HELP OF A PULLEY

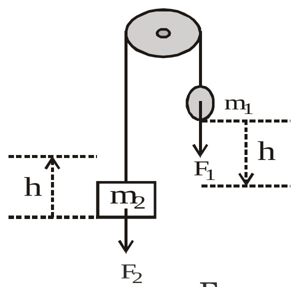
Example 25

A man with a mass of m_1 = 80 kg lowers himself by h = 10 m along a wall, while fixed to a rope that runs over a frictionless pulley to a m_2 = 70 kg sand bag.

(1) What is his final speed if he started from a state of rest ?

(2) How long does it take to "travel" this distance?

Solution : The Figure below illustrates the problem schematically. We can write following equations.



$$F_1 = m_1 g$$
$$F_2 = m_2 g$$

$F_1 > F_2$ so the resultant force FR exerted on these two masses is directed downward on the side of m_1.

$$F_R = m_1 g - m_2 g \quad (1)$$

Under action of this force BOTH masses are moving with acceleration a

$$a = \frac{F_R}{m_1 + m_2} \quad (2)$$

Substituting (1) into (2) we get

$$a = \frac{g(m_1 - m_2)}{m_1 + m_2} \quad (3)$$

The distance h traveled from the state of rest with this acceleration is

$$h = (1/2)at^2 \quad (4)$$

and the final speed

$$v = at \quad (5)$$

Equations (4) and (5) constitute a set of two linear equations with two unknowns and can be solved by a standard method. As the algebra involved in this is very simple we give here only the final solutions.

$$v = \sqrt{\frac{2hg(m_1 - m_2)}{m_1 + m_2}} \qquad t = \sqrt{\frac{2h(m_1 + m_2)}{g(m_1 - m_2)}}$$

Substituting the numbers gives

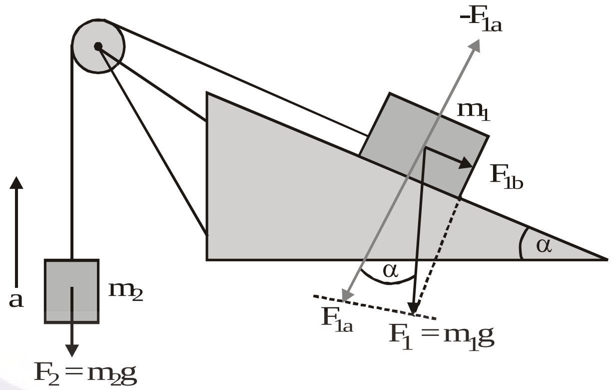
v = 3.62 m/s, t = 5.53 s.

BLOCK OF AN INCLINE

Example 26

A block of mass m_1 = 2.0 kg on a frictionless inclined plane of angle 20 deg is connected by a rope over a pulley to another block of mass m_2 = 1.0 kg. What are the magnitude and direction of the acceleration of the second block?

The figure below will help us to solve this problem.



The force m_1g, caused by gravity, can be decomposed into two forces : F_{1a} and F_{1b}. Elementary geometry and the definitions of trigonometric functions allow us to write

$$F_{1a} = m_1 g \cos \alpha \quad (1)$$

$$F_{1b} = m_1 g \sin \alpha \quad (2)$$

$$F_2 = m_2 g \quad (3)$$

The force F_{1a}, represented by the red vector is compensated, according to Newton's Third Law of Motion, by a force represented by the green vector. So we are left with forces F_{1b} and F_2 acting one against the other through a rope connecting blocks.

Let's assume that $F_{1b} = F_2 = a\,(m_1 + m_2) \quad (4)$

Substituting (2) and (3) into (4), after a little of elementary algebra we get

$$a = g\,(m_1 \sin\alpha - m_2)/(m_1 + m_2)$$

Substituting numbers given in the problem we get

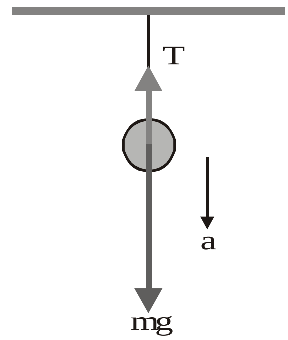
$$a = -\,1.03 \text{ m/s}^2$$

The minus sign tells us, that acceleration has direction opposite to the one chosen by us for writing equations leading to the solution of the problem. This is a general rule in all kinds of problems. Negative numerical value means the direction of the parameter found is opposite to the one which was assumed for writing equations.

LAMP IN AN ASCENDING ELEVATOR

Example 27

A lamp hangs vertically from a cord in an elevator which is descending with an downward acceleration of a = 2.0 m/s^2. The tension in the cord is T = 10.0 N. What is the mass m of this lamp?



The drawing below illustrates this situation.

There are two forces acting on our lamp.

mg – gravity force, acting downwards and,

T – tension, acting upwards.

The resultant force is responsible for acceleration a, directed downwards. The equation based on second Newton's law of motion is :

$$ma = mg - T$$

The only unknown is mass m elementary algebra leads to the solution

$$m = T(g - a)$$

Substituting in numbers results in

$$m = 1.28 \text{ kg}$$

As usual we assume knowledge of gravitational acceleration $g = 9.81 \text{ m/s}^2$.

2.9 MOMENTUM

The change of momentum of a body is proportional to the impulse impressed on the body, and happens along the straight line of which that impulse is impressed.

Using modern symbolic notation, Newton's second law can be written as

$$F_{net} = \frac{d(mv)}{dt} = m\frac{dv}{dt}$$

where F is the force vector, m is the mass of the body, v is the velocity vector and t is time.

It should be noted that, as is consistent with the law of inertia, the time derivative of the momentum is non-zero when the momentum changes direction, even if there is no change in its magnitude.

Since the mass of the system is constant, this differential equation can be rewritten in its simpler and more familiar form :

$$F = ma$$

where $a = \frac{dv}{dt}$

is the acceleration.

A verbal equivalent of this is "the acceleration of an object is proportional to the force applied, and inversely proportional to the mass of the object". In general, at slow speeds (slow relative to the speed of light), the relationship between momentum and velocity is approximately linear. Nearly all speeds within the human experience fall within this category. At higher speeds, however, this approximation becomes increasingly inaccurate and the theory of special relativity must be applied.

2.10 IMPULSE

The term impulse is closely to the second law, and historically speaking is closer to the original meaning of the law. (12) The meaning of an impulse is as follows :

An **impulse** occurs when a force **F** acts over an interval of time Δt and is given by $\int_{\Delta t} F\,dt$

$$I = \Delta P = m\Delta v$$

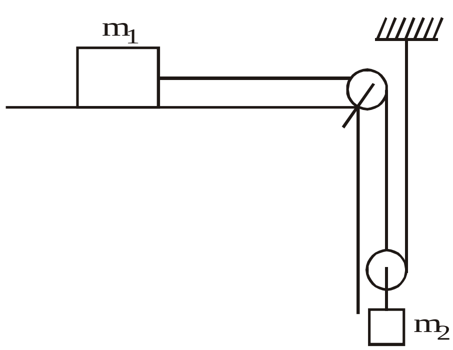
where I is the impulse, Δp is the change in momentum, m is the mass, and Δv is the change in velocity.

2.11 CENTRIPETAL FORCE AND CENTRIFUGAL FORCE

Centripetal force and centrifugal force, action-reaction force pair associated with circular motion. According to Newton's first law of motion, a moving body travels along a straight path with constant speed (i.e., has constant velocity) unless it is acted on by an outside force. For circular motion to occur there must be a constant force acting on a body, pushing it toward the center of the circular path. This force is the centripetal ("center-seeking") force. For a planet orbiting the sun, the force is gravitational; for an object twirled on a string, the force is mechanical; for an electron orbiting an atom, it is electrical. The magnitude F of the centripetal force is equal to the mass m of the body times its velocity squared v^2 divided by the radius r of its path : $F = mv^2/r$. According to Newton's third law of motion, for every action there is an equal and opposite reaction. The centripetal force, the action, is balanced by a reaction force, the centrifugal ("center-fleeing") force. The two forces are equal in magnitude and opposite in direction. The centrifugal force does not act on the body in motion; the only force acting on the body in motion is the centripetal force. The centrifugal force acts on the source of the centripetal force to displace it radially from the center of the path. Thus, in twirling a mass on a string, the centripetal force transmitted by the string pulls in on the mass to keep it in its circular path, while the centrifugal force transmitted by the string pulls outward on its point of attachment at the center of the path. The centrifugal force is often mistakenly though to cause a body to fly out of its circular path when it is released; rather, it is the removal of the centripetal force that allows the body to travel in a straight line as required by Newton's first law. If there were in fact a force acting to force the body out of its circular path, its path when released would not be the straight tangential course that is always observed.

2.12 CONSTRAINT RELATION

It is the equation showing the relation between the motion of different bodies of a system in which motion of one body is constrained by other bodies.



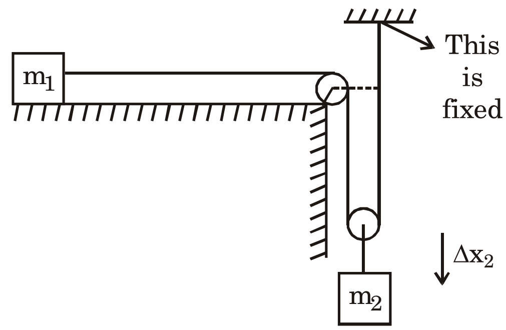
There are three approaches to derive the constraint equation.

Approach (1)

If m_2 moves by a distance Δx_2 down, the string or m, is stretched by $2(\Delta x_1)$

Since in this question one end is fixed, therefore 2x comes from string attached to m_2.

Note : If the end has not been fixed then the movement of m, will not be $2\Delta x_2$, it will depend on the movement of the unfixed end also. This will be illustrated in the coming examples.



$\therefore \quad \Delta x_1 = 2\Delta x_2$

$\Rightarrow \quad x_1 = 2x_2$

differentiating it twice for getting relation in acceleration

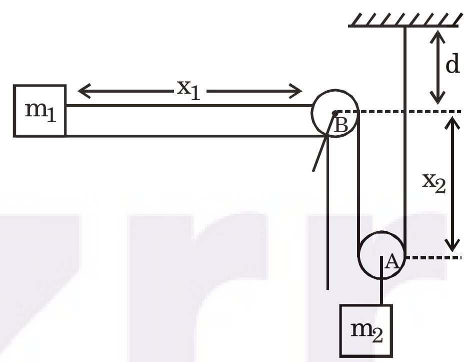
$$\boxed{a_1 = 2a_2}$$

Approach (2)

This approach is based on the concept that the length of spring remains constant (assuming that string is inextensible) we calculate the length of string

i.e. $\quad x_1 + (x_2 + x_2) + d = l$

where d is a constant length, as will not change on the movement of any block & l is also constant.



differentiating the equation twice

$$(-a_1) + 2a_2 = 0$$

$\Rightarrow \quad \boxed{a_1 = 2a_2}$

{Note : derivative of decreasing length (such as x_1 in this question) is taken as negetive, whereas for increasing length the derivative is positive}

Approach (3)

(Work-Energy Method)

Concept : Work done by the tension in string in all the frame of reference = 0

Reason : String is inextensible

i.e. $\quad$ Elastic potential energy stored in string = 0

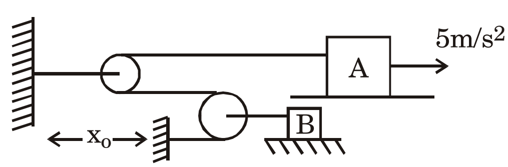
$\therefore \quad 2T(-x_2) + T(x_1) = 0$

$\Rightarrow \quad x_1 = 2x_2$

$\Rightarrow \quad \boxed{a_1 = 2a_2}$

Illustration

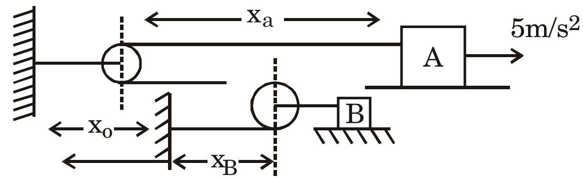
Blocks A & B are connected through light inextensible string, if block A is moved to the right with acceleration of 5 m/s^2. Find acceleration of block B.



Solution : At any distance of block A and block B from wall P/pulley be x_a & x_b respectively.

length of string

$(x_B - x_0) + (x_B) + x_A = L$ {here x_0 is constant}



differentiating

$$2\frac{dx_B}{dt} + \frac{dx_A}{dt} = 0$$

here x_a is increasing & x_B is decreasing (assuming that B is moving towards wall)

$\therefore \quad -2V_B + V_A = 0$

$\Rightarrow \quad -2a_B + a_A = 0$

$\Rightarrow \quad \boxed{a_A = 2a_B}$

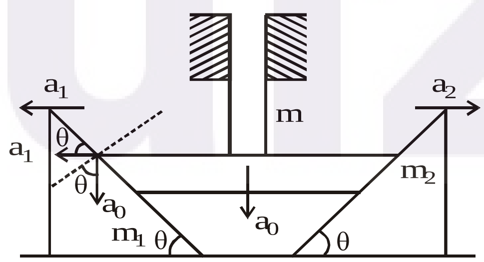
$\therefore \quad a_B = \frac{a_A}{2} = \frac{5}{2} = 2.5 \text{ m/s}^2$

Steps to sovle the motion of connected bodies

1. Separate the system into an isolated body
2. Make free body diagram of individual bodies to assess the forces.
3. Find the relation among the motion of different bodies through contraint.
4. Frame the equation of motion in suitable direction and solve.

Example 29

Find the acceleration of T-shaped rod and of wedge.



Solution : We can see from the diagram that as the T-shaped rod moves downward, the wedge with mass (m), moves to the left & wedge with mass (m_2) moves toward right.

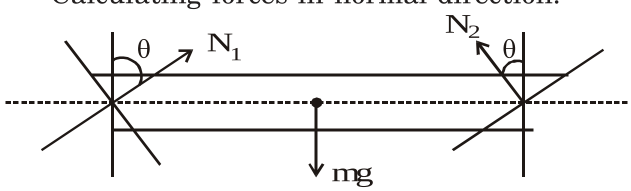
From the free body diagram of the parts first finding the relation between acceleration through constraints.

$a_0 \cos\theta = a_1 \sin\theta$

$a_0 = a_1 \tan\theta \qquad \Rightarrow \qquad a_1 = a_0 \cot\theta \qquad (1)$

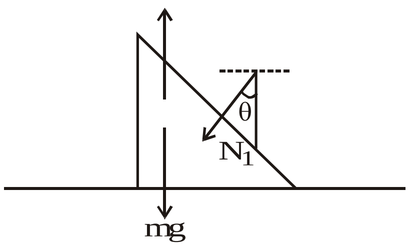
$a_2 = a_0 \cot\theta \qquad (2)$

Calculating forces in normal direction.



$mg - N_1 \cos\theta - N_2 \cos\theta = ma_0 \qquad (3)$

And finlly Calculating Values for individual wedges



$$N_1 \sin\theta = m_1 a_1 \qquad (4)$$
$$N_2 \sin\theta = m_2 a_2 \qquad (5)$$

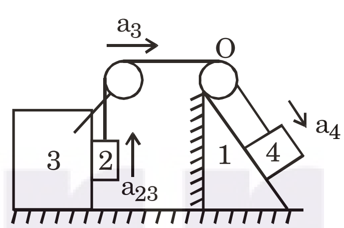
Example 30

(a) Find the acceleration of each body when the wedges are fixed.

(b) The wedges are movable.

Solution :

(a) When wedge (i) is fixed :



by logical approach

$a_4 = a_3 + a_{23}$

(Note : here a_{23} is the acceleration of block 2 with respect to block 3. This is taken because block 3 is also moving.

from point O, applying string method

$r_4 + r_3 + r_{23} = L$

differentiating

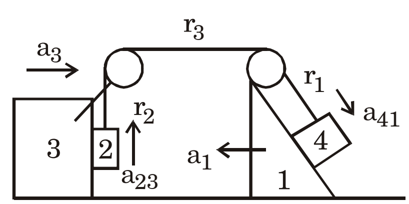
$a_4 - a_3 - a_{23} = 0$ {since r_3 & r_{23} are decreasing}

$\Rightarrow \quad \boxed{a_4 = a_3 + a_{23}}$

(b) When wedge (1) is also moving :

now in this part wedge (1) is also moving,

$\therefore$ acceleration of block 4 will be relative to that of block 1. So a_{41}.



Applying string method

Here now the component r_3 (as in the above question) will be due to 2 component (due eto movement of block 1 & 3 & that too in opposite direction.)

So indirectly for understanding we can say

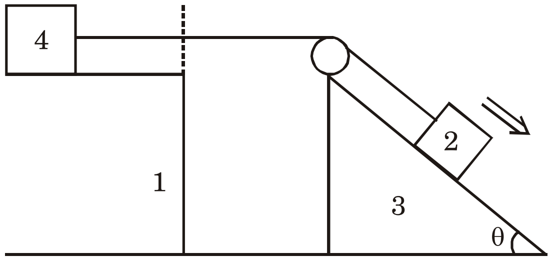
$r_3 = r_{31} + r_{33}$

$\therefore$ differentiating

$$\boxed{a_{41} = a_{23} + a_3 + a_1}$$

Example 32

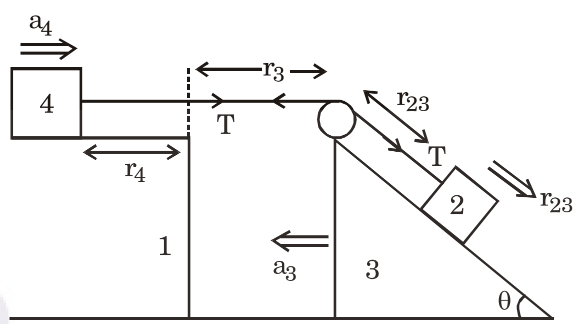
Find acceleration relation.



Solution :

(a) In such type of question follow logical approach which is the best suited.

$a_{23} = a_4 + a_3$



(b) **Applying string method**

$r_4 + r_3 + r_{23} = L$

– differentiating

$\Rightarrow a_4 - a_3 + a_{23} = 0$

(c) **Work energy method**

$T(r_4) + (T - T\cos\theta)\, r_3 + T(r_3\cos\theta - r_{23}) = 0$

Solving this we get

$\boxed{r_4 + r_3 = r_{23}}$

thus $\boxed{a_4 + a_3 = a_{23}}$