



To Get More Information and Different Questions Join our telegram channel
 Telegram channel <https://t.me/johnson201485>
 Email: johnson201485@gmail.com

UNIT 3 and 4

MATRICES AND DETERMINANT

1. If $M = \begin{pmatrix} 0 & -1 & 0 \\ 5 & 7 & 3 \\ 2 & 9 & 4 \end{pmatrix}$, then which one of the following is equal to the determinant of M?
 A. -31 B. -8 C. -1 D. 14
2. If $A = \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ 0 & -2 \end{pmatrix}$, then which of the following is equal to $A^t + 2B$?
 A. $\begin{pmatrix} 4 & -5 \\ 1 & -2 \end{pmatrix}$ B. $\begin{pmatrix} 5 & -3 \\ 1 & 2 \end{pmatrix}$ C. $\begin{pmatrix} 4 & -1 \\ -3 & -2 \end{pmatrix}$ D. $\begin{pmatrix} 3 & -4 \\ 1 & 0 \end{pmatrix}$
3. If $\begin{pmatrix} x & -3 \\ -1 & y \end{pmatrix}$ is the inverse of $\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$, then the values of x and y respectively are:
 A. 1, 3 B. 2, 2 C. 4, 1 D. $\frac{1}{2}, -\frac{3}{2}$
4. What are the values of x and y so that $\begin{pmatrix} x & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} y & y \\ 2 & 1 \end{pmatrix}$?
 A. $x = 3, y = 2$ B. $x = 0, y = -1$ C. $x = 4, y = 5$ D. $x = -2, y = 3$
5. What would be the value of k so that the system $\begin{cases} 2x - y = 4 \\ -4x + ky = 1 \end{cases}$ has a unique solution?
 A. $k \neq 0$ B. $k = 4$ C. $k \neq 2$ D. $k = 1$
6. Let $A = \begin{pmatrix} -2 & 0 & x \\ 2y & x+y & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -y \\ 0 & 3 \\ 1-x & 2 \end{pmatrix}$ such that $A + 2B^T = 0$. Then which of the following is the value of y ?
 A. Any real number C. $-\frac{13}{2}$ D. -8
 B. 0
7. Let A and B be 3×3 matrices such that $A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & -1 & \frac{1}{2} \end{pmatrix}$ and $|B| = \frac{1}{10}$,
 Which one of the following is equal to $|2AB^T|$?
 A. 1 B. 4 C. 100 D. 400
8. What is the solution set of the following equations $\begin{cases} x + y + 2z = 1 \\ x + 2y + z = 2 \\ -2x - 2y - 4z = -2 \end{cases}$?
 A. $\{(0, 1, 0)\}$ C. $\{(3k, k-1, k) | k \in (-\infty, \infty)\}$
 B. $(-\infty, \infty)$ D. $\{(-3k, k+1, k) | k \in (-\infty, \infty)\}$
9. If $\begin{pmatrix} \alpha & 2 & \beta \\ 2 & 1 & 3 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \\ 0 \end{pmatrix}$, and the determinant of the coefficient matrix is , then the value of is equal to:
 A. 3 B. $\alpha + \beta$ C. -5α D. 5
10. Suppose $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$. If X is a 2×2 matrixes such that $AX - A^T = 2A$, then what is the value of x?
 A. $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ B. $\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$ C. $\begin{pmatrix} 3 & 6 \\ 6 & 9 \end{pmatrix}$ D. $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

11. For the system of linear equations $\begin{cases} 2x + 3y = 8 \\ x - 2y = -3 \end{cases}$, the value of x and y are:

A. $x = \frac{\begin{vmatrix} 8 & 3 \\ -3 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix}}$ and $y = \frac{\begin{vmatrix} 2 & 8 \\ 1 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix}}$

B. $x = \frac{\begin{vmatrix} 8 & 3 \\ -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix}}$ and $y = \frac{\begin{vmatrix} 2 & 8 \\ 1 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix}}$

C. $x = \frac{\begin{vmatrix} 2 & 8 \\ 1 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix}}$ and $y = \frac{\begin{vmatrix} 8 & 3 \\ -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix}}$

D. $x = \frac{\begin{vmatrix} 2 & 8 \\ 1 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix}}$ and $y = \frac{\begin{vmatrix} 8 & 3 \\ -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix}}$

12. The inverse of the matrix $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$ is ____

A. $\frac{1}{2} \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -2 & 2 & 0 \end{pmatrix}$

B. $-\frac{1}{2} \begin{pmatrix} 1 & 1 & 2 \\ -1 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix}$

C. $\frac{1}{2} \begin{pmatrix} -1 & -1 & 2 \\ -1 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix}$

D. $\frac{1}{2} \begin{pmatrix} 1 & 1 & 2 \\ -1 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix}$

13. If $\begin{pmatrix} 0 & 4 & 1 \\ 2 & 1 & 5 \\ 0 & 3 & 2 \end{pmatrix}$ and B is 3×3 matrix, such that $\det(2B) = 40$, then $\det(AB)$ is equal to ____

A. 200

B. -200

C. 50

D. -50

14. Suppose that A and B are 3×3 matrices, I is the identity matrix of order 3 such that $AB=2I$. If $\det B = |B| = 6$. What is $\det(A^T)$?

A. $\frac{1}{3}$

B. 12

C. $\frac{4}{3}$

D. 48

15. If $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ and $(2A + B)^T = A^T A$, then which one of the following is equal to B

A. $\begin{pmatrix} 1 & 0 & -2 \\ 2 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

B. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

C. $\begin{pmatrix} 8 & 0 & -4 \\ 4 & 8 & 0 \\ 0 & 0 & -4 \end{pmatrix}$

D. $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{pmatrix}$

16. Consider the system $\begin{cases} ax + y + z = 1 \\ x + 2y + 4z = 0 \\ 5x - y + z = 0 \end{cases}$. If the determinant of the coefficient matrix is, then what is the solution of the system of equations?

A. $(3\alpha, \frac{19\alpha}{2}, \frac{-11\alpha}{2})$

B. $(\frac{3}{\alpha}, \frac{19}{2}, \frac{11}{2})$

C. $3, \frac{19}{2}, \frac{-11}{2}$

D. $\frac{3}{2}, \frac{19}{2}, \frac{-9}{2}$

17. If $B = \begin{pmatrix} 0 & 1 & 2 \\ 3 & -1 & 0 \\ 5 & 2 & 4 \end{pmatrix}$ and $A^T M = 2I$, Where A a 3×3 matrix and I is the identity matrix of order 3, then what is $\det(A)$

A. 0.2

B. $\frac{4}{17}$

C. 0.8

D. $\frac{1}{17}$

18. What should be the value of k so that the system of equation $\begin{cases} x - y + z = 1 \\ -x + 5y - 4z = 1 \\ 2x + 2y - z = k \end{cases}$ has one solution?

A. 0

B. 1

C. -4

D. 4

19. Suppose $AX = b$, where A is a 3×3 matrix, $b = (b_1, b_2, b_3)^T$ and $X = (x, y, z)^T$. Which one of the following is necessarily true about this system of linear equations?

A. The system has a solution only when $\det(A) \neq 0$

B. The Cramer's rule is suitable to solve the system if two rows of A are identical.

C. If $\det(A) \neq 0$ and the second column of A is a multiple of b , then $x = 0$

D. If $b = 0$, then $X = (0, 0, 0)^T$ is the only solution of the system.

20. Let $A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 4 & -5 \\ 3 & -5 & 6 \end{pmatrix}$, then which of the following is equal to $A + A^T$

- A. $\begin{pmatrix} 4 & 6 & 4 \\ 4 & -10 & 8 \\ -10 & 4 & 12 \end{pmatrix}$ B. $\begin{pmatrix} 6 & 4 & 4 \\ 4 & 8 & -10 \\ 4 & -10 & 12 \end{pmatrix}$ C. $\begin{pmatrix} 4 & 6 & 2 \\ 4 & 8 & -10 \\ 6 & -10 & 12 \end{pmatrix}$ D. $\begin{pmatrix} 4 & 8 & -10 \\ 6 & 4 & 2 \\ 6 & -10 & 12 \end{pmatrix}$

21. Given two matrices $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$, which one of the following is NOT true about the given matrices?

- A. $(A^T)^T = A$ C. $(kB)^T = kB^T$, if k is any scalar.
 B. $(AB)^T = A^T B^T$, provided that AB is defined D. $(A + B)^T = A^T + B^T$

22. Consider the following system of equations: $\begin{cases} ax + by = 2 \\ x + 3y + 2z = 0 \\ 2x + y + z = 0 \end{cases}$. If the determinant of the coefficient matrix is , then what is the solution set of the system?

- A. $\left\{ \left(\frac{1}{a}, \frac{1}{b}, 0 \right) \right\}$ B. $\{(1, 3, -5)\}$ C. $\{(-2, -6, 10)\}$ D. \emptyset

23. If $A = \begin{pmatrix} 3 & -2 & 8 \\ 0 & 6 & 7 \\ 0 & 4 & 5 \end{pmatrix}$, then $\det(A^T A)$ is equal to____.

- A. 12 B. 36 C. 30 D. 15

24. What is the solution set of the system $\begin{cases} x + y - z = 1 \\ x + 2y - 3z = 1 \\ 2x + 3y - 4z = 2 \end{cases}$?

- A. $\{(1 - k, 2k, k) | k \in \mathbb{R}\}$ D. \emptyset
 B. $\{(0, 2, 1)\}$
 C. $\square \square 2k \square 1, \square \square k, k \square \square | k \square \square \square$

25. If $A = \begin{pmatrix} 0 & x & 0 \\ 1 & -1 & 1 \\ 0 & y & -1 \end{pmatrix}$ $A^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 0 & 0 \\ 2 & 0 & -1 \end{pmatrix}$, then what are the values of x and y ?

- A. $x = 3, y = -2$ B. $x = \frac{2}{3}, y = \frac{1}{3}$ C. $x = -3, y = 2$ D. $x = \frac{1}{3}, y = \frac{2}{3}$

26. Let $A = \begin{pmatrix} 2 & 7 \\ 1 & 3 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$, then $(AB)^{-1}$ is equal to____

- A. $\begin{pmatrix} 4 & -3 \\ 4 & -5 \end{pmatrix}$ B. $\begin{pmatrix} -2 & 5 \\ 2 & -4 \end{pmatrix}$ C. $\begin{pmatrix} -3 & 11 \\ 1 & -3 \end{pmatrix}$ D. $\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$

27. Let $A = \begin{pmatrix} 0 & \alpha & \beta \\ 2 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}$, $b = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\det(A) = 3$, then what the solution set of the system $AX = b$?

- A. $\left\{ \left(0, \frac{1}{a}, \frac{5}{b} \right)^T \right\}$ B. $\{(6, -2, -8)^T\}$ D. \emptyset
 C. $\{(-3, 1, 4)^T\}$

28. The solution of the system of linear equation of $\begin{cases} x - 3y - z = 6 \\ 2x - 4y - 3z = 8 \\ -3x + 6y + 8z = -5 \end{cases}$ is

- A. $x = -1, y = -3, z = -2$ C. $x = 1, y = -3, z = 2$
 B. $x = -1, y = -3, z = 2$ D. $x = 1, y = 3, z = -2$

29. For any $n \times n$ square matrix A , which one of the following is true?

- A. $\det(A) = -\det(A^T)$, Where A^T is the transpose of A
 B. If k is scalar, then $\det(kA) = k^n \det(A)$
 C. If B is a matrix obtained from A by interchanging of two rows A , then $\det(A) = \det(B)$
 D. If A is invertible , then $\det(A) = \det(A^{-1})$

30. The solution of $\begin{cases} -x + 4y - z = 1 \\ 2x - y + z = 0 \\ x + y + z = 1 \end{cases}$ using Cramer's rule is:
- A. $x = -1, y = 2, z = 5$ B. $x = \frac{1}{2}, y = \frac{2}{3}, z = \frac{2}{5}$ C. $x = -\frac{1}{5}, y = \frac{2}{5}, z = \frac{4}{5}$ D. $x = 5, y = 1, z = 2$
31. For what value of x is $\begin{vmatrix} 1 & -3 \\ x & x-2 \end{vmatrix} = x+1$
- A. 1 B. $\frac{3}{2}$ C. $\frac{1}{2}$ D. -1
32. Suppose $\begin{vmatrix} p & r & q \\ t & y & z \\ a & b & c \end{vmatrix} = 3$, which of the following is equal to $\begin{vmatrix} 3p & 3r & 2.4q \\ t & y & 0.8z \\ a & b & 0.8c \end{vmatrix}$?
- A. 2.4 B. 9 C. 1.92 D. 7.2
33. Let $\begin{pmatrix} 0 & -3 & -4 \\ k & 0 & 8 \\ 4 & -8 & 0 \end{pmatrix}$ be skew-symmetric matrix. Then what is the value of?
- A. $\frac{3}{2}$ B. -3 C. 3 D. 4
34. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 12 & \sin x & \cos x \\ 13 & \cos x & \sin x \end{bmatrix}$, then $|A|$ is
- A. $\cos 2x$ B. $-\cos 2x$ C. $\sin 2x$ D. $\cos x$
35. If A and B are square matrices of the same order and A is non- singular, then for a positive integer n , $(A^{-1}BA)^n$ is equal to
- A. $(A^{-1})^n B^n A^n$ B. $A^n B^n (A^{-1})^n$ C. $A^{-1}B^n A$ D. $n(A^{-1}BA)$
36. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ is equal to
- A. $2AB$ B. $2BA$ C. $A + B$ D. AB
37. Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ and if $AX = B$, then X equal to:
- A. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ B. $\begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$ C. $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$ D. $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$
38. If the system of equations, $x + 2y - 3z = 1$, $(k + 3)z = 3$, $(2k + 1)x + z = 0$ is inconsistent, then the value of k is
- A. -3 B. $\frac{1}{2}$ C. 0 D. 2
39. Let p a non-singular matrix $1 + p + p^2 + \dots + p^n = O$, then $p^{-1} = (O$ denotes the null matrix)
- A. p^n B. $-p^n$ C. $-(1 + p + \dots + p^n)$ D. None of these
40. For any square matrix A, AA^T is a
- A. Unit matrix C. Skew symmetric matrix
B. Symmetric matrix D. Diagonal matrix
41. If the system of equations $x + ay = 0$, $az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a is
- A. -1 B. 1 C. 0 D. No real values
42. If a matrix A is such that $4A^3 + 2A^2 + 7A + I = O$, then A^{-1} equals
- A. $(4A^2 + 2A + 7I)$ B. $-(4A^2 + 2A + 7I)$ C. $-(4A^2 - 2A + 7I)$ D. $(4A^2 + 2A - 7I)$
43. If the system of equations $x - ky - z = 0$, $kx - y - z = 0$ and $x + y - z = 0$ has a non-zero solution, then the possible value of k are
- A. -1, 2 B. 1, 2 C. 0, 1 D. -1, 1
44. If A is 3×3 matrix and B is matrixes such that are both AB and BA defined. Then find order of B.
- A. 3×2 B. 2×3 C. 2×2 D. 3×3

45. For two invertible matrices A and B of suitable orders, the value of $(AB)^{-1}$ is
 A. $(BA)^{-1}$ B. $B^{-1} A^{-1}$ C. $A^{-1} B^{-1}$ D. $(AB')^{-1}$
46. The matrix product $AB = O$, then
 A. $A = O$ and $B = O$ B. $A = O$ or $B = O$ C. A is null matrix D. None of these
47. If $3X + 2Y = I$ and $2X - Y = O$, where I and O are unit and null matrices of order 3 respectively, then
 A. $X = (1/7)$, $Y = (2/7)$ C. $X = (1/7)I$, $Y = (2/7)I$
 B. $X = (2/7)$, $Y = (1/7)$ D. $X = (2/7)I$, $Y = (1/7)I$
48. The number of solution of the following equations $x_2 - x_3 = 1$, $-x_1 + 2x_3 = -2$, $x_1 - 2x_2 = 3$ is
 A. Zero B. One C. Two D. Infinite
49. Which of the following is not true?
 A. Every skew-symmetric matrix of odd order is non-singular
 B. If determinant of a square matrix is non-zero, then it is non-singular
 C. Ad joint of symmetric matrix is symmetric
 D. Ad joint of a diagonal matrix is diagonal
50. If a matrix A is such that $3A^3 + 2A^2 + 5A + I = 0$ then its inverse is
 A. $-(3A^2 + 2A + 5I)$ B. $3A^2 + 2A + 5I$ C. $3A^2 - 2A - 5I$ D. None of these
51. Choose the correct answer
 (a) Every identity matrix is a scalar matrix
 (b) Every scalar matrix is an identity matrix
 (c) Every diagonal matrix is an identity matrix
 (d) A square matrix whose each element is 1 is an identity matrix
52. The system of equations $x_1 - x_2 + x_3 = 2$, $3x_1 - x_2 + 2x_3 = -6$ and $3x_1 + x_2 + x_3 = -18$ has
 A. No solution B. Exactly one solution C. Infinite solutions D. None of these
53. If $A^2 - A + I = 0$, then A^{-1} =
 A. A^{-2} B. $A + I$ C. $I - A$ D. $A - I$
54. If A and B are square matrices of order 2, then $(A + B)^2$ =
 A. $A^2 + 2AB + B^2$ B. $A^2 + AB + BA + B^2$ C. $A^2 + 2BA + B^2$ D. None of these
55. The number of values of k for which the system of equations $(k+1)x + 8y = 4k$, $kx + (k+3)y = 3k - 1$ has infinitely many solutions, is
 A. 0 B. 1 C. 2 D. Infinite
56. If A and B are square matrices of order 3 such that $|A| = -1$, $|B| = 3$, then $|3AB| =$
 A. -9 B. -81 C. -27 D. 81
57. If $|A|$ denotes the value of the determinant of the square matrix A of order 3, then $|-2A| =$
 A. $-8|A|$ B. $8|A|$ C. $-2|A|$ D. None of these
58. Which one of the following is not true?
 A. Matrix addition is commutative C. Matrix multiplication is commutative
 B. Matrix addition is associative D. Matrix multiplication is associative
59. If A and B are square matrices of the same order, then
 A. $(AB)' = A'B'$ C. $AB = O$; If $|A| = 0$ or $|B| = 0$
 B. $(AB)' = B'A'$ D. $AB = O$; if $A = I$ or $B = I$
- 60.

Cramer's Rule to Solve 3×3 Systems of Linear Equations

We can solve the general system of equations,

$$a_1x + b_1y + c_1z = d_1$$

Example:

$$\begin{aligned} x + 3y + z &= 4 \\ 2x - 6y - 3z &= 10 \\ 4x - 9y + 3z &= 4 \end{aligned}$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$$x = \frac{\begin{vmatrix} 4 & 3 & 1 \\ 10 & -6 & -3 \\ 4 & -9 & 3 \end{vmatrix}}{\Delta}$$

$$y = \frac{\begin{vmatrix} 1 & 4 & 1 \\ 2 & 10 & -3 \\ 4 & 4 & 3 \end{vmatrix}}{\Delta}$$

where

$$\Delta = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -6 & -3 \\ 4 & -9 & 3 \end{vmatrix}$$

$$= 1(-45) - 2(18) + 4(-3)$$

$$= -93$$

Note: Once we have x and y , we can find z without using Cramer's Rule.

So

$$x = \frac{4(-45) - 10(18) + 4(-3)}{-93} = \frac{-372}{-93} = 4$$

$$y = \frac{1(42) - 2(8) + 4(-22)}{-93} = \frac{-62}{-93} = \frac{2}{3}$$

Using these two results, we can easily find that $z = -2$.

Checking the solution:

$$[1] (4) + 3\left(\frac{2}{3}\right) + -2 = 4$$

$$[2] 2(4) - 6\left(\frac{2}{3}\right) - 3(-2) = 10$$

$$[3] 4(4) - 9\left(\frac{2}{3}\right) + 3(-2) = 4$$

So the solution is $\left(4, \frac{2}{3}, -2\right)$.

by using the determinants:

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\Delta}$$

$$y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\Delta}$$

$$z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\Delta}$$

where

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$