

Shadow multiplets and superHiggs mechanism

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Abstract

We discuss a general feature of Freund Rubin compactifications that was previously overlooked. It consist in a curious pairing, which we call a *shadow relation*, of completely different (in terms of spin and mass) fields of the dimensionally reduced theory. Particularly interesting is the case where the compactification preserves a certain amount of supersymmetry, giving rise to a shadowing phenomenon between whole supermultiplets of fields. In particular, there are strong suggestions about the consistency of a massive truncation of 11D supergravity to the massless modes of the graviton supermultiplet plus the massive modes of its *shadow partner*.

This fact has important consequences in the $\mathcal{N} = 2$ and $\mathcal{N} = 3$ cases, which seem to realize respectively a Higgs or a superHiggs phenomenon. In other words, we are led to reinterpret the dimensionally reduced theory as a spontaneously broken phase of some higher (super)symmetric theory.

1 Introduction

The search for consistent compactifications of higher dimensional supergravity is currently an active field of research [1], mostly due to its connection with the effort to embed brane-world scenarios in a superstring/M-theory contest [2].

It is quite common to consider several maximally supersymmetric gauged supergravities as coming from Kaluza Klein sphere compactifications of higher dimensional supergravities admitting an $AdS \times \text{Sphere}$ vacuum solution. The examples include the S^4 and S^7 compactifications of 11D supergravity and the S^5 compactification of type IIB supergravity. Generally the lower dimensional theory is obtained by *truncating* the spectrum of the higher dimensional supergravity to the massless sector, namely by setting to zero all the infinite towers of massive Kaluza Klein modes. By doing so, we are left with an interacting theory for a finite number of lower dimensional fields, which can be usually recognized as a gauged supergravity.

It is not a trivial task to show that such a truncation is consistent, which means that all the solutions of the dimensionally reduced theory are solutions of the higher dimensional one too (for instance, it took several years to show the consistency of the S^7 reduction of 11D supergravity down to $\mathcal{N} = 8$ $SO(8)$ -gauged supergravity in four dimensions [3, 4]). Indeed the full non-linear equations of motion of the higher dimensional supergravity can be regarded as the equations of motion for the lower dimensional fields only at the price of admitting a rather complex non-linear coupling of these fields with each other. This implies that in principle infinite different products of massless modes could act as sources for the massive ones. The consistency of the truncation requires that this does not happen, namely it requires that the composite currents coupled to the massive modes always contain at least one of the massive truncated modes.

As we will show, a very simple argument about the second order nature of the Kaluza Klein differential field equations leads to the conclusion that particular truncations of 11D supergravity (but the argument is sufficiently general to admit generalizations to other supergravity theories) are consistent. This requires the concept of *shadow relation* between Kaluza Klein fields that we are going to explain.

2 Shadow multiplets in Kaluza Klein theory

Following the conventions of [5] the bosonic action of 11D supergravity reads:

$$S = \frac{1}{\kappa_{11}^2} \int \mathcal{R} \det V - \frac{1}{16\kappa_{11}^2} \int F \wedge *F - \frac{1}{96\kappa_{11}^2} \int F \wedge F \wedge A , \quad (1)$$

where \mathcal{R} is the scalar curvature, V^M ($M = 0, \dots, 10$) are the vielbein, A is a three-form and F its four-form field-strength. Freund-Rubin (FR) compactifications [6] are solutions of the field equations of 11D supergravity in which the space-time \mathcal{M}_{11} has the factorized form

$$\mathcal{M}_{11} = AdS_4 \times \mathcal{M}^7 . \quad (2)$$

The only non-vanishing components of F are the 4-dimensional ones: $F_{abcd} = e \varepsilon_{abcd}$, where the parameter e sets the scale for both the 4-dimensional and the 7-dimensional cosmological constant (also \mathcal{M}^7 must be an Einstein space):

$$\mathcal{R}_{ab} = -24 e^2 \eta_{ab} , \quad \mathcal{R}_{\alpha\beta} = 12 e^2 \eta_{\alpha\beta} . \quad (3)$$

Greek letters α, β, \dots are reserved to flat seven dimensional indices, while Latin letters a, b, \dots stand for four dimensional flat indices. We denote by x the coordinates of four dimensional space, while y are the coordinates on the compact manifold \mathcal{M}^7 .

The fluctuations of the eleven-dimensional fields around such a background can be expanded into harmonics on the compact 7-manifold, namely eigenmodes of proper 7-dimensional covariant operators, and the linearized 4-dimensional field equations can be suitably diagonalized. The resulting general formulae were derived in [7, 8] and organized into a systematic way in [9]. Let us recall the notations and the main results of that paper. For the fluctuations h_{MN} of the metric one sets

$$h_{ab}(x, y) = \left(h_{ab}^I(x) - \frac{3}{M_{(0)^3} + 32} D_{(a} D_{b)} \left[(2 + \sqrt{M_{(0)^3} + 36}) S^I(x) \right. \right.$$

$$\begin{aligned}
& + \left(2 - \sqrt{M_{(0)^3} + 36} \right) \Sigma^I(x) \Big] + \frac{5}{4} \delta_{ab} \left[(6 - \sqrt{M_{(0)^3} + 36}) S^I(x) \right. \\
& \left. + (6 + \sqrt{M_{(0)^3} + 36}) \Sigma^I(x) \right] Y^I(y) , \tag{4}
\end{aligned}$$

$$\begin{aligned}
h_{a\beta}(x, y) &= [(\sqrt{M_{(1)(0)^2} + 16} - 4) A_a^I(x) \\
&+ (\sqrt{M_{(1)(0)^2} + 16} + 4) W_a^I(x)] Y_\beta^I(y) , \tag{5}
\end{aligned}$$

$$\begin{aligned}
h_{\alpha\beta}(x, y) &= \phi^I(x) Y_{(\alpha\beta)}^I(y) - \delta_{\alpha\beta} \left[(6 - \sqrt{M_{(0)^3} + 36}) S^I(x) \right. \\
&+ \left. (6 + \sqrt{M_{(0)^3} + 36}) \Sigma^I(x) \right] Y^I(y) . \tag{6}
\end{aligned}$$

For the fluctuations a_{MNR} of the three form field, one has

$$\begin{aligned}
a_{abc}(x, y) &= 2 \varepsilon_{abcd} D_d (S^I(x) + \Sigma^I(x)) Y^I(y) , \\
a_{ab\gamma}(x, y) &= \frac{2}{3} \varepsilon_{abcd} (D_c A_d^I(x) + D_c W_d^I(x)) Y_\gamma^I(y) , \\
a_{a\beta\gamma}(x, y) &= Z_a^I(x) Y_{[\beta\gamma]}^I(y) , \tag{7}
\end{aligned}$$

$$a_{\alpha\beta\gamma}(x, y) = \pi^I(x) Y_{[\alpha\beta\gamma]}^I(y) . \tag{8}$$

Finally, for the fluctuations of the gravitino field,

$$\begin{aligned}
\psi_a(x, y) &= \left(\chi_a^I(x) + \frac{\frac{4}{7} M_{(1/2)^3} + 8}{M_{(1/2)^3} + 8} [D_a \lambda_L^I(x)]_{3/2} \right. \\
&\quad \left. - (6 + \frac{3}{7} M_{(1/2)^3}) \gamma_5 \gamma_a \lambda_L^I(x) \right) \Xi^I(y) , \tag{9}
\end{aligned}$$

$$\psi_\alpha(x, y) = \lambda_T^I(x) \Xi_\alpha^I(y) + \lambda_L^I(x) [\nabla_\alpha \Xi^I(y)]_{3/2} . \tag{10}$$

The harmonics on \mathcal{M}^7 are grouped into seven infinite towers corresponding to as many irreducible representations of the tangent group $SO(7)$ that appear in the decomposition $4 \oplus 7$ of eleven dimensional tensors and spinors. Since $SO(7)$ has rank 3, its *irreps* are labeled by three numbers $[\lambda_1, \lambda_2, \lambda_3]$ that we take to be the Young labels¹.

Measuring everything in units of the Freund-Rubin scale, *i.e.* setting $e = 1$, the masses of the 4-fields appearing in the Kaluza Klein expansion (eq.s 4-10) are expressed in terms of the eigenvalues $M_{[\lambda_1, \lambda_2, \lambda_3]}$ of the appropriate 7-operators as follows:

$$m_h^2 = M_{(0)^3} , \tag{11}$$

$$m_\Sigma^2 = M_{(0)^3} + 176 + 24 \sqrt{M_{(0)^3} + 36} , \tag{12}$$

$$m_S^2 = M_{(0)^3} + 176 - 24 \sqrt{M_{(0)^3} + 36} , \tag{13}$$

$$m_\phi^2 = M_{(2)(0)^2} , \tag{14}$$

$$m_\pi^2 = 16 \left(M_{(1)^3} - 2 \right) \left(M_{(1)^3} - 1 \right) , \tag{15}$$

$$m_W^2 = M_{(1)(0)^2} + 48 + 12 \sqrt{M_{(1)(0)^2} + 16} , \tag{16}$$

¹That is, for bosonic tensors with symmetry represented by a Young tableaux, λ_i is the number of boxes in the i -th row of the tableaux. For gamma-traceless irreducible spinor tensors λ^i is 1/2 plus the number of boxes.

$$m_A^2 = M_{(1)(0)^2} + 48 - 12\sqrt{M_{(1)(0)^2} + 16}, \quad (17)$$

$$m_Z^2 = M_{(1)^2(0)}, \quad (18)$$

$$m_{\lambda_L} = -\left(M_{(\frac{1}{2})^3} + 16\right), \quad (19)$$

$$m_{\lambda_T} = M_{(\frac{3}{2})(\frac{1}{2})^2} + 8, \quad (20)$$

$$m_\chi = M_{(\frac{1}{2})^3}. \quad (21)$$

The AdS relation between the mass $m_{(s)}$ and the rest energy $E_{(s)}$ of a spin s particle which, in the alternative three-dimensional conformal interpretation of the $SO(2, 3)$ group, translates into a relation with the scale dimension $\Delta = E$ of the corresponding primary conformal field (see [14] for further details), is given by²:

$$\begin{aligned} m_{(0)}^2 &= 16(E_{(0)} - 2)(E_{(0)} - 1), \\ |m_{(\frac{1}{2})}| &= 4E_{(\frac{1}{2})} - 6, \\ m_{(1)}^2 &= 16(E_{(1)} - 2)(E_{(1)} - 1), \\ |m_{(\frac{3}{2})} + 4| &= 4E_{(\frac{3}{2})} - 6. \end{aligned} \quad (22)$$

The shadowing phenomenon Having established the conventions on the Kaluza Klein expansion of the eleven dimensional fields, we are now able to discuss the shadowing phenomenon in details. As it can be seen from equations (4-10), there are infinitely many couples of fields which share the same harmonics. The masses of the fields in each couple are related through equations (11 -21) to the same eigenvalue of the corresponding harmonic. This means that fields of different type, spin and mass are nevertheless linked by a relation which determines the mass of the one as a function of the other, as shown in table 1. Furthermore, each field belongs to the same irrep of the isometry group of \mathcal{M}^7 as the shadow partner, which is just the conjugate representation of the associated harmonic.

Examples We can illustrate the basic structure of this mechanism with some simple examples that will be quite relevant in our subsequent discussion.

Let us observe that the same scalar harmonic $Y^I(y)$ is associated both to the graviton field $h_{ab}^I(x)$ of eq. (4) and to the scalar field $\Sigma^I(x)$ of eq. (5). Using the mass relations (11 -21), we see that the shadow scalar of a “parent” graviton has mass

$$m_\Sigma^2 = m_h^2 + 176 + 24\sqrt{m_h^2 + 36} \quad (23)$$

In the case of the massless graviton, $m_h^2 = 0$, corresponding to the constant harmonic $Y = 1$, its shadow scalar Σ has (squared) mass

$$m_\Sigma^2 = 320. \quad (24)$$

²The reader should be careful in comparisons with other papers and take into account that the definition of mass utilized in this paper is that of supergravity [13]. Specifically the mass squared of scalars is defined as the deviation from a conformal invariant equation, the mass of the gravitino is defined as the deviation from a Rarita Schwinger equation with supersymmetry, the mass squared of a spin one field is defined as the deviation from a gauge invariant equation.

shadow couples	mass relation
(h_{ab}^I, S^I)	$m_S^2 = m_h^2 + 176 - 24\sqrt{m_h^2 + 36}$
(h_{ab}^I, Σ^I)	$m_\Sigma^2 = m_h^2 + 176 + 24\sqrt{m_h^2 + 36}$
(A_a^I, W_a^I)	$m_w^2 = m_A^2 + 144 - 24\sqrt{m_A^2 + 4}$
(ψ_a^I, λ_L^I)	$m_{\lambda_L} = -m_\psi - 16$

Table 1: Shadow couples. The mass of each field is related to that of the shadow partner.

Using eq.s (22) this implies

$$E_\Sigma = 6 . \quad (25)$$

We conclude that in every $AdS_4 \times \mathcal{M}^7$ compactification there is always a *universal* scalar mode of conformal dimension $E = 6$ that is the shadow of the graviton. Its geometric origin is apparent from the second of equations (4): it is just the *breathing mode* corresponding to an overall dilatation of the internal manifold \mathcal{M}^7 .

As it follows by inspection of eq.s (5), to the same vector harmonic Y_α^I we associate two vector fields: one, A_a , with mass given by eq. (17), the other, W_a , with mass given by eq. (16). This is due to the fact that the linearized field equation for a 4-vector associated to an harmonic of given $M_{(1)(0)^2}$ is a second order differential equation. Hence we have two independent solutions rather than one. The very initial idea of Kaluza Klein theory is that the isometries of the internal manifold give rise to massless gauge fields in the compactified 4-dimensional theory. When we start from 11D M-theory there is a further aspect. Indeed to each Killing vector of the internal compact manifold we associate two rather than one vector fields. In addition to the KK massless gauge boson, we have its shadow massive vector. It also belongs to the adjoint representation of the isometry group, and it has fixed mass and dimension:

$$m_W^2 = 192 \Rightarrow E_W = 5 . \quad (26)$$

Finally, let us consider one more example involving fermionic fields. Comparing eq.s (9) and (10), we see that the same spinor harmonic Ξ^I appearing in the expansion of the gravitino $\psi_a(x, y)$ has as coefficients of the expansion both a spin-3/2 $\psi_a^I(x)$ and a longitudinal spin-1/2 field $\lambda_L(x)^I$. Using eq.s (19,21) the relation between the masses of the spin-3/2 and spin-1/2 modes pertaining to the same harmonic is:

$$m_\psi = -m_{\lambda_L} - 16 . \quad (27)$$

Applying eq. (27) to the case $m_{\lambda_L} = 0$, we see that each massless spin $\frac{1}{2}$ particle of this type generates a shadow massive gravitino with mass:

$$m_\psi = -16 \Rightarrow E_\psi = \frac{9}{2} . \quad (28)$$

Conversely, every massless gravitino produces a shadow massive spin-1/2 field with mass:

$$m_{\lambda_L} = -16 \Rightarrow E_{\lambda_L} = \frac{11}{2} . \quad (29)$$

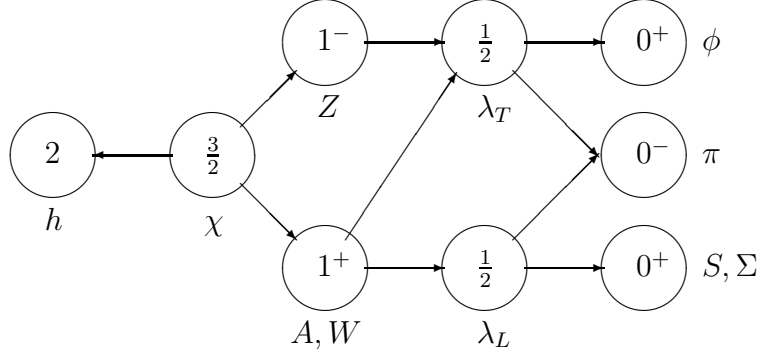


Figure 1: “Internal” supersymmetry relations between the harmonics of Kaluza Klein fields: for each pair of fields linked by an arrow the corresponding towers of harmonics can be related to each other by multiplications with a Killing spinor η^A .

Supersymmetry and shadow multiplets Let us consider unitary irreducible representations of the superalgebra $Os p(\mathcal{N}|4)$, *i.e.* the supersymmetric extension of $SO(2, 3)$ with \mathcal{N} supercharges. Each of them is a supermultiplet, represented by a suitable constrained superfield, that contains fields whose spins s and dimensions E are related to each other.

These relations hint to a sort of mirror image of the supersymmetry algebra that is realized on the internal compact manifold \mathcal{M}^7 . This idea was thoroughly analyzed in [9] and traced back to the existence of (commuting) Killing spinors η^A ($A = 1, \dots, \mathcal{N}$), where \mathcal{N} is the number of preserved supersymmetries of the $AdS_4 \times \mathcal{M}^7$ compactification one considers. By means of the Killing spinors η^A , to each harmonic Y that is an eigenmode of a bosonic 7-operator one can associate another fermionic harmonic Ξ that is an eigenmode of a fermionic 7-operator with suitably related eigenvalues. These pairs of related harmonics were explicitly constructed in [9] and follow the schematic pattern given in fig. 1.

As already stressed fifteen years ago in [9], these relations are differential geometric identities on the compact 7-manifold \mathcal{M}^7 that are required by consistency with the structure of UIR.s of the superconformal group $Os p(\mathcal{N}|4)$. Because of this, it may at first sight appear that the universal mass relations analyzed in [9] do not contain further physical information besides the implications of supersymmetry. However, this is not the case, because of another aspect of eq.s (11-22), whose consequence is precisely the existence of shadow multiplets.

The point is that the relations (11-21) between masses (or conformal weights) and eigenvalues of the internal Laplacian is quadratic rather than linear. Indeed the same harmonic always plays a double role since it appears in the expansion of two quite different Kaluza Klein fields. Combined with the supersymmetry relations produced by Killing spinors, this has the curious consequence that each supersymmetry multiplet of the Kaluza

Klein spectrum is associated with another one made, so to say, by the second roots of the quadratic relations.

3 Consistency of the shadow truncation

Let us consider the 7-dimensional equation satisfied by the scalar harmonics associated to the spin two Kaluza Klein fields h_{ab}^I :

$$\square_7 Y = D^\mu D_\mu Y = M_{(0)^3} Y. \quad (30)$$

The compactness of \mathcal{M}^7 implies the positivity of \square_7 and, in particular, the fact that the lightest spin two field, i.e. the massless graviton h_{ab}^0 is associated to the constant harmonic $Y = 1$. Together with the killing spinor relations of table 1, this implies that all the harmonics associated to the fields of the massless graviton supermultiplet are given by some product of killing spinors and constant covariant tensors. The same is true, by definition, for the harmonics associated to the fields of the shadow multiplet (i.e. the shadow partner of the graviton multiplet). This implies that more general harmonics, associated to the other massive fields, cannot be obtained as products of the only Killing spinor harmonics. Hence in the complete (non-linear) 4-dimensional equations of motion, there are no non-vanishing currents (i.e. composites of the only massless and shadow fields) that could act as sources for the massive truncated fields. The conclusion is that the truncation of the Kaluza Klein modes to the massless graviton multiplet plus the shadow sector has to be consistent, independently from the specific compact manifold \mathcal{M}^7 .

Actually it is worth to stress that this is not a rigorous proof of consistency because, in principle, we do not have a definitive argument to exclude the existence of at least one field, among the higher massive modes, whose harmonic is made by a product of constant tensors and Killing spinors. If this were the case, the consistent truncation could require not to switch off this field. But this possibility seems very unrealistic. First of all because, as we have seen, the harmonics of the fields in the same supermultiplet would be related to this one by products with Killing spinors. So they all would be of the same kind and we should deal with a whole new massive multiplet to retain in the consistent truncation. Second, because the shadow multiplet fields have a really universal geometrical meaning, while the other massive ones are related to the specific features of the compactification manifold. It seems therefore really plausible that there is a deep link between the most general kind of multiplet of a supergravity theory (the massless graviton) and another universal multiplet (its shadow).

4 The superHiggs phenomenon

The conclusion about the consistency of the truncation of any supersymmetric compactification of 11D supergravity to the massless plus the shadow sectors implies that the dimensionally reduced theory is a consistent coupling of some gauged supergravity to a proper multiplet of massive fields. In particular, in the case of $\mathcal{N} = 3$ supersymmetric compactifications, independently on the internal manifold \mathcal{M}^7 , there has to be a consistent coupling of a whole massive gravitino multiplet to the $\mathcal{N} = 3$ $SO(3)$ -gauged supergravity

Spin	Energy	$SO(3)$ -Isospin	Field	Harmonic
2	3	0	h_{ab}	$Y = 1$
$\frac{3}{2}$	$\frac{5}{2}$	1	χ_a^A	$\Xi^A = \eta^A$ (Killing spinor)
1	2	1	A_a^A	$Y_\alpha^A = \frac{1}{2} \epsilon^{ABC} \bar{\eta}^B \gamma_\alpha \eta^C \equiv k_\alpha^A$
$\frac{1}{2}$	$\frac{3}{2}$	0	λ_L	$\Xi = \frac{1}{3} \epsilon^{ABC} \gamma_\alpha \eta^A \bar{\eta}^B \gamma^\alpha \eta^C$

Table 2: The massless $\mathcal{N} = 3$ graviton multiplet and its Kaluza Klein origin.

in four dimensions. Indeed the shadow partner of the $\mathcal{N} = 3$ graviton multiplet, displayed in table 2, is a massive gravitino multiplet. The only way to introduce such a multiplet in a consistent way is through the super-Higgs phenomenon³. Hence we are led to the conclusion that any $\mathcal{N} = 3$ dimensional reduction of 11D supergravity on a background of the form $AdS_4 \times \mathcal{M}^7$ actually is a broken phase of some gauged $\mathcal{N} = 4$ supergravity.

The $\mathcal{N} = 4 \rightarrow 3$ partial supersymmetry breaking in four-dimensional supergravity is well-known in the literature [15]. It can be realized by coupling three vector multiplets to the $SO(4)$ -gauged $\mathcal{N} = 4$ supergravity and giving a proper *vev* to four scalars which preserve the $SO(3)$ R -symmetry of the $\mathcal{N} = 3$ vacuum. As explicitly shown in [16], there is an upper bound on the mass acquired by the broken gravitino, which is a function of these *vevs* (the moduli of the partial breaking). The rest energy of this gravitino is given by:

$$E_\psi = \frac{3}{2} + \sqrt{\frac{8 + \rho^4 - \rho^2(8 + \rho^2) \cos 4\theta}{8 + \rho^4 - 8\rho^2 - \rho^4 \cos 4\theta}}, \quad (31)$$

where the complex variable $\rho e^{i\theta}$, $\rho < 1$, partially parametrizes the partial breaking moduli space. Hence standard $\mathcal{N} = 4$ supergravity poses the upper bound $E_\psi < 9/2$.

Now, in the case where the supergravity we are considering is realized through a dimensional reduction from 11D, the broken gravitino is nothing but the higher spin component of the shadow supermultiplet of the compactification. As we have shown in the third example of section 2 and as it is discussed in [17] for a particular compactification, the energy of such field is just $E_\psi = 9/2$, independently from the specific \mathcal{M}^7 . This means that there must exist some new way to realize the $\mathcal{N} = 4 \rightarrow 3$ supersymmetry breaking, admitting a broken gravitino energy $E_\psi \geq 9/2$, so far not considered in the literature.

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³analogously, in the $\mathcal{N} = 2$ case, where the shadow partner of the massless graviton is a massive vector multiplet, we deal with a simple Higgs phenomenon

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