Tubular D3-branes and their Dualities

S. Tamaryan,[a,c*] D. K. Park[b†] and H.J.W. Müller-Kirsten[c‡]

[a] *Theory Department, Yerevan Physics Institute,*
Yerevan-36, 375036, Armenia

[b] *Department of Physics, Kyungnam University,*
Masan, 631-701, Korea

[c] *Department of Physics, University of Kaiserslautern,*
67653 Kaiserslautern, Germany

Abstract

A tubular D3-brane with electromagnetic flux is considered. It is verified that the quantized electric and magnetic charges are the F-string and D-string charges of the brane, respectively. The D3-brane with parallel electric and magnetic fields collapses to the bound state of strings. A D3-brane with nonzero Poynting vector can be viewed as expanded FD strings. A fundamental string is viewed as a collapsed D3-brane and is mapped to the (n, m) strings via electric-magnetic duality rotations. This mapping is the same as a weak-strong duality transformation and therefore reveals the equivalence of the electric-magnetic and weak-strong dualities.

[*] E-mail:sayat@moon.yerphi.am
[†] E-mail:dkpark@hep.kyungnam.ac.kr
[‡] E-mail:mueller1@physik.uni-kl.de

1 Introduction

Recently supertubes, as special tubular D2-branes, attracted considerable attention owing to a previously unknown mechanism of stabilisation by angular momentum as a result of crossed constant electric and magnetic fields [1]. In particular it was shown in ref. [1] that the finite radius of the tube results from the product of F-string and D0-brane charges, and the BPS energy is reached by a cancellation of oppositely directed Poynting and centrifugal momenta, thus requiring a nonzero angular momentum along the axis of the supertube. It was also shown that another supersymmetry preserving configuration with the same energy exists in the limit of vanishing D2-brane radius with the D0-branes aligning like beads along a IIA superstring, with vanishing angular momentum. In these two cases the directions of the E and B components of the electromagnetic field are reversed. It is evident that such a seemingly simple model immediately suggests its investigation with respect to electric-magnetic [2, 3, 4] and weak-strong or S dualities [5, 6]. Several subsequent investigations [7, 8, 9] did not venture in this direction, and this is therefore the main objective here. Earlier investigations [10, 11, 12] studied the stability of branes in relation to their possible collapse to lower dimensional ones [10, 11, 12] or the opposite, the expansion of branes into higher dimensional ones [9, 13, 14] with further investigations in refs.[14, 15]. In view of apparent physical shortcomings of the case of D2 (e.g. the Poynting momentum is a scalar), it is natural to prefer the case of D3 for the investigation since here the connection between electric-magnetic and weak-strong dualities is exposed in the fullest physical context, and at least weak-strong duality of this case has been considered from time to time [16]. In fact we shall show that weak-strong duality may be understood as the electric-magnetic duality of nonlinear electrodynamics.

Thus, in order to realise this idea, we proceed in a number of definite steps. We first demonstrate how quantised electric and magnetic charges can be connected with the numbers of F-strings and D-strings dissolved in the D3-brane. We then show that the F-string is a D3-brane with minimal electric charge. In the third step this D3-brane with minimal electric charge is mapped to the D3-brane with arbitrary electric and magnetic charges using the electric-magnetic duality rotation. Then, following this, we show that the D3-brane with arbitrary charges is a composite string in the sense of Witten[17] which can also be described as an FD string[18]. Finally the F-string is mapped to this composite string.

2 The Action

The D3-brane action is

$$I = -T_3 \int d^4\xi \sqrt{-det(g_{\mu\nu} + 2\pi F_{\mu\nu})}, \qquad T_3 = \frac{1}{(2\pi)^3 g_s}, \qquad (1)$$

where $g_{\mu\nu}$ is the induced metric tensor and $F_{\mu\nu}$ is the electromagnetic field strength tensor. The dilaton field is taken to be constant and target space-

time to be flat. We consider a D3-brane of cylindrical shape $\mathbb{R}^1 \otimes \mathbb{S}^2$ which does not allow a variation of the radius r in time and along the axis. This choice is conditioned by our intention to investigate the brane's dualities. The variable radius violates the electric-magnetic duality invariance and therefore we keep it constant. The brane's worldvolume is parametrized in terms of the variables (t, z, θ, φ) and the cylindrical hypersurface is given by

$$X^0 = t, \quad X^1 = z, \quad X^2 = r\cos\theta,$$
$$X^3 = r\sin\theta\cos\varphi, \quad X^4 = r\sin\theta\sin\varphi, \quad others = constant. \tag{2}$$

The induced metric is

$$ds^2 = -dt^2 + dz^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\varphi^2. \tag{3}$$

Hereafter we use bold face letters for the 3-dimensional vectors and symbols i, j, k for the space indices (z, θ, φ). The electric intensity $\mathbf{E}$ and magnetic induction $\mathbf{B}$ are defined by

$$\mathbf{E}_i = 2\pi F_{0i}, \qquad \mathbf{B}_i = 2\pi \frac{\epsilon_{ijk}}{2} F^{jk}. \tag{4}$$

The Levi-Civita form ϵ_{ijk} in curved space is defined by $\epsilon_{ijk} = \sqrt{-g}\varepsilon_{ijk}$, where $-g \equiv det(-g_{\mu\nu}) = r^4 \sin^2\theta$ and ε_{ijk} is the Levi-Civita form in flat space. The action reduces to

$$I = T_3 \int \mathcal{L}\sqrt{-g}\, dtdzd\theta d\varphi, \tag{5}$$

where the Lagrangian $\mathcal{L}$ is

$$\mathcal{L} = -\sqrt{1 + \mathbf{B}^2 - \mathbf{E}^2 - (\mathbf{E} \cdot \mathbf{B})^2}, \tag{6}$$

the brane tension is a normalization factor and the rest is the worldvolume invariant measure. We started from the D3-brane action and obtained the Non-Linear Electrodynamic(NLE) action. The reason is that we suppressed all other degrees of freedom and kept only the gauge field.

The aspects of the NLE action we are interested in are most conveniently analyzed by introducing the electric induction $\mathbf{D}$,

$$\mathbf{D} = \frac{\partial \mathcal{L}}{\partial \mathbf{E}} = \frac{\mathbf{E} + \mathbf{B}(\mathbf{E} \cdot \mathbf{B})}{\sqrt{1 + \mathbf{B}^2 - \mathbf{E}^2 - (\mathbf{E} \cdot \mathbf{B})^2}}. \tag{7}$$

Performing a Legendre transform we construct the Hamiltonian $\mathcal{H}$ as a function of $\mathbf{B}$ and $\mathbf{D}$,

$$\mathcal{H} = \sqrt{1 + \mathbf{B}^2 + \mathbf{D}^2 + |\mathbf{B} \times \mathbf{D}|^2}. \tag{8}$$

It has $SO(2)$ electric-magnetic duality rotation invariance

$$\begin{aligned}
\mathbf{D} &\rightarrow \mathbf{D}\cos\alpha - \mathbf{B}\sin\alpha, \\
\mathbf{B} &\rightarrow \mathbf{D}\sin\alpha + \mathbf{B}\cos\alpha.
\end{aligned} \tag{9}$$

There is Legendre discrete duality at $\alpha = \pi/2$ which interchanges the electric and magnetic inductions. The $SO(2)$ invariance of the Hamiltonian can be extended to $SL(2, \mathbb{R})$ invariance by including a scalar dilaton field ϕ and a pseudo-scalar axion field χ. For the time being we omit these fields for simplicity and return to this later. To complete this section we introduce the magnetic intensity $\mathbf{H}$

$$\mathbf{H} = -\frac{\partial \mathcal{L}}{\partial \mathbf{B}} = \frac{\mathbf{B} - \mathbf{E}(\mathbf{E} \cdot \mathbf{B})}{\sqrt{1 + \mathbf{B}^2 - \mathbf{E}^2 - (\mathbf{E} \cdot \mathbf{B})^2}} \tag{10}$$

and write out the equations of motion in the static case:

$$\varepsilon^{ijk} \frac{\partial \mathbf{E}_i}{\partial x^j} = 0, \qquad \varepsilon^{ijk} \frac{\partial \mathbf{H}_i}{\partial x^j} = 0, \tag{11}$$

$$\frac{\partial}{\partial x^k} \sqrt{-g}\, \mathbf{D}^k = 0, \qquad \frac{\partial}{\partial x^k} \sqrt{-g}\, \mathbf{B}^k = 0. \tag{12}$$

The transformations (9) can be equally described in terms of the intensities $\mathbf{E}$ and $\mathbf{H}$. They take the solution of the NLE action to another solution. Note that the duality transformations (9) are invariances of the field equations (11,12) and Hamiltonian (8) but not of the action (5).

3 Brane Charges

In this section we consider two static solutions of the NLE action (5). In the first case electric and magnetic fields are parallel, the brane carries no angular momentum generated by crossed electric and magnetic fields and collapses to the bound state of the strings. In the second case electric and magnetic fields are perpendicular and the nonzero Poynting vector prevents the collapse of the brane. The flux in the D3-brane acts as a source for the strings. It comes from an integral over the whole world-volume and it is impossible to localise the strings at a particular place in the brane's worldvolume. But they can appear as remnants when the cylindrical brane is squeezed to a line.

Using Dirac charge quantization we express the energy of the solutions as a function of corresponding quantized electric and magnetic charges n and m. In the limit of the vanishing radius r of the brane, $r \to 0$, the term proportional to the area of the surface of the cylinder ($\sim r^2$) in the expression of the energy can be neglected in leading order. Then the rest shows the strings smeared in the brane. In this way we establish that n and m are F-string and D-string charges of the brane, respectively .

Bound States of Strings

First we consider parallel uniform electric and uniform magnetic fields aligned with the z axis, the 2-form field strength being given by

$$2\pi F = E\, dt \wedge dz + Br^2 \sin\theta\, d\theta \wedge d\varphi. \tag{13}$$

The Dirac electric charge quantization condition is

$$n = \int_{S^2} \frac{\delta I}{\delta F_{0i}} ds_i = 2\pi \int \epsilon_{ijk} \frac{\delta I}{\delta \mathbf{E}_i} dx^j dx^k, \tag{14}$$

where n is an integer. The surface perpendicular to the electric field is the basesurface of the cylinder and is parametrized in terms of the variables (θ, φ). Then

$$T_3 \int D\sqrt{-g}\, d\theta d\varphi = \frac{n}{2\pi}; \qquad D = \mathbf{D}^z. \tag{15}$$

The measure of the basesurface area S_2 is

$$dS_2 = r^2 \sin\theta\, d\theta d\varphi = \sqrt{-g}\, d\theta d\varphi, \tag{16}$$

so that eq.(15) takes the form

$$T_3 \int D dS_2 = \frac{n}{2\pi}. \tag{17}$$

Eq.(17) implies that the electric induction flux through the basesurface is quantized, and as D is constant it yields

$$D = \frac{\pi n g_s}{r^2}. \tag{18}$$

This result can be also derived from M-theory [10, 19].

The magnetic flux quantization condition is

$$\int F_{\theta\varphi}\, d\theta d\varphi = 2\pi m \tag{19}$$

where m is an integer. It yields

$$B = \frac{\pi m}{r^2}. \tag{20}$$

The substitution of (18) and (20) into Hamiltonian (8) and the integration over θ, φ gives for the energy $\mathcal{E}$

$$\mathcal{E} = \sqrt{(T_3 S_2)^2 + (nT_s)^2 + (mT_1)^2} \int dz. \tag{21}$$

where $T_s = 1/2\pi$ and $T_1 = 1/2\pi g_s$ are the tensions of the F-string and D-string, respectively. To uncover the dissolved strings we consider the energy $\mathcal{E}$ in the limit $S_2 \to 0$. In pure electric case it can be presented as the mass of n fundamental strings

$$\mathcal{E}(m = 0)_{r \to 0} = nT_s \int dz. \tag{22}$$

From this asymptotoic relation we conclude that there are n fundamental strings dissolved in the worldvolume of the D3-brane, and n is the F-string

charge of the D3-brane. In the pure magnetic case the energy becomes the mass of m D-strings

$$\mathcal{E}(n=0)_{r\to 0} = mT_1 \int dz, \qquad (23)$$

with the obvious interpretation that there are m D-strings dissolved in the D3-brane, and m is the D-string charge of the D3-brane.

The energy (21) is minimal when $r = 0$, as it should be. The brane shrinks to a line with the energy

$$\mathcal{E} = \sqrt{(nT_s)^2 + (mT_1)^2} \int dz. \qquad (24)$$

This is the bound state of (n, m) strings. The D3-brane with charges (n, m) and with no net D3-brane charge collapses to the (n, m) strings and therefore the latter can be considered as a collapsed D3-brane. The mass formula (24) for the $SL(2, \mathbb{Z})$ multiplet was derived in [17, 20]. The basic condition was the fact that the string solutions are BPS-saturating states and preserve half of supersymmetry. Here the same relation is derived from the viewpoint of the supersymmetric D3-brane. The main point is that the string bound state is considered as a collapsed superbrane. This method allows to apply Legendre discrete duality to the (n, m) strings. The latter interchanges the S-dual partners, i.e. F-strings and D-strings, resulting in (m, n) strings.

Expanded FD strings

Eq.(11,12) have nontrivial solutions with the perpendicular electric and magnetic fields. Suppose we have the electric field along the z axis and the magnetic field along meridians, so only $\mathbf{E}_z, \mathbf{D}_z, \mathbf{B}_\theta, \mathbf{H}_\theta$ components are nonzero. The eq.(12) gives a simple solution for $\mathbf{B}_\theta$

$$\mathbf{B}_\theta = \frac{B}{\sin\theta}, \qquad (25)$$

where B is an integration constant. The $\mathbf{E}_z$ components can be expressed via $\mathbf{D}_z$ and $\mathbf{B}_\theta$

$$\mathbf{E}_z^2 = \left(1 + \frac{B^2}{r^2\sin^2\theta}\right)\frac{\mathbf{D}_z^2}{1 + \mathbf{D}_z^2}. \qquad (26)$$

From the eq.(11) it follows that $\mathbf{E}_z$ can not depend on the variable θ. The r.h.s of the eq.(26) satisfies this restriction only when

$$\mathbf{D}_z = D\sin\theta, \qquad (27)$$

where the constant D is defined by

$$D^2B^2 = r^2. \qquad (28)$$

Then eq.(26) becomes $\mathbf{E}_z^2 = 1$ and we rewrite it in the invariant form

$$\mathbf{E}^2 = 1. \qquad (29)$$

This is the critical value of the electric intensity which forces the critical value of the magnetic intensity $\mathbf{H}^2 = 1$. Now we can make $SO(2)$ duality rotations and obtain more general solutions. As $\mathbf{E}$ and $\mathbf{H}$ are unit mutually perpendicular vectors, they will stay such after rotations (9). Consequently the condition (29) is right for any rotated solution and defines the true minimum of the NLE action. It has another interesting interpretation. The question whether the tubular D2-brane with the nonzero Poynting vector is supersymmetric has been clarified [1]. The condition for the preservation of 1/4 of the supersymmetry was reduced to the eq.(29). We come to the conclusion that the classical minimum of the NLE action is a BPS-saturating state.

The charge quantization requires the compact length of the variable z. Therefore we assume that the z direction is compactified by a circle with the circumference l. We denote by R the compactification radius, then $l = 2\pi R$. The quantization repeats the procedure of the previous case resulting in

$$D = \frac{4ng_s}{r^2}, \qquad B = \frac{m}{R}. \tag{30}$$

The energy expression takes the form

$$\mathcal{E} = \int \sqrt{(rT_3S_2')^2 + (4nT_sR)^2 \sin^2\theta + \frac{(mT_1r)^2}{\sin^2\theta} + \frac{4n^2m^2}{\pi^2r^2}} \, \sin\theta \, d\theta. \tag{31}$$

where $S_2' = 2\pi rl$ is the azimuthal lateral area of the brane. The first term under the square root is the energy of the brane with no electromagnetic flux. The second term presents n dissolved F-strings which is easy to see by setting $m = 0$ and taking the limit $r \to 0$.

$$\mathcal{E}(m = 0)_{r \to 0} = nT_sl. \tag{32}$$

In the pure magnetic case the energy is

$$\mathcal{E}(n = 0) = mT_1 \int \sqrt{\left(\frac{Rr^2}{m}\right)^2 \sin^2\theta + r^2} \, d\theta \tag{33}$$

In the limit $r \to 0$ this becomes

$$\mathcal{E}(n = 0)_{r \to 0} = mT_1(\pi r). \tag{34}$$

This is a bunch of m D-strings with the length of a semicircle. Therefore m is the brane's D-string charge. The integrand in (33) represents the length measure of an elliptic arc along which the energy density is distributed and the ellipse is mapped into the azimuthal lateral face of $\mathbb{R}^1 \otimes \mathbb{S}^2$ becoming an elliptic helix with a pitch r.

The last term in (31) is generated by the Poynting vector and owing to it the energy grows infinitely when $r \to 0$. The nonzero Poynting vector $P(P \sim nm)$ stabilizes the brane at nonzero radius r_0. The substitution of (30) into (28) determines the value of the radius r_0

$$r_0{}^3 = \frac{4\pi g_s}{R} |nm|. \tag{35}$$

The brane tension forces the collapse of the brane while the momentum works oppositely. The same value r_0 can be determined from the balance condition of these two acts. Indeed, equating the first and last terms under the square root (31)

$$rT_3 S_2' = \frac{2|nm|}{\pi r} \tag{36}$$

yields the eq.(35). Therefore the D3-brane with the given charges can be regarded as an expanded FD string which is self-supported from collapse by the momentum $\mathbf{D} \times \mathbf{B}$.

Legendre discrete transformation maps the above D3-brane to the its electromagnetic dual D3-brane

$$\mathbf{D}_\theta = \frac{D}{\sin\theta}, \quad \mathbf{B}_z = B\sin\theta; \qquad |BD| = r. \tag{37}$$

The quantization of the electromagnetic charges gives rise to the S-dual picture, i.e. D-strings are aligned along the cylinder axis and F-strings are wound around them helically. One should note that the precise form of the helix does not have any meaning because the worldvolume reparametrization invariance allows to transmute it to another form. What is definite, is that F and D strings are wound around each other and form a 3-dimensional braid. Necessarily they carry nonzero angular momentum, otherwise the system collapses to the bound state of the strings.

4 Dualities

The weak-strong duality may be realized as a NLE electric-magnetic duality on the D3-brane worldvolume theory. In this section we envisage the possibility that this conception is presented in the precise form. In the case of constant flux in the *flat direction* of the tubular D3-brane we explicitly demonstrate the equivalence of these dualities.

Equations of motion of the NLE action (5) and Hamiltonian (8) are invariant under $SO(2)$ electric-magnetic duality rotations. It has been shown that this invariance becomes $SL(2,\mathbb{R})$ symmetry by including dilaton and axion fields [2, 3, 4]. The Lagrangian takes the form [3, 5, 6]

$$\mathcal{L}' = e^{-\phi}\mathcal{L} + \chi(\mathbf{E} \cdot \mathbf{B}). \tag{38}$$

It defines the electric induction $\mathbf{D}'$

$$\mathbf{D}' = e^{-\phi}\mathbf{D} + \chi\mathbf{B} \tag{39}$$

which shows that the axion charge of the string is provided by electric flux in the worldvolume of the brane. To restore the symmetric dependence of the Hamiltonian on the inductions $\mathbf{D}'$ and $\mathbf{H}$ one has also to rescale the metric tensor.

The duality transformations are conveniently described in terms of the complex field $\lambda = \chi + ie^{-\phi}$. They mix electric and magnetic fields [3]

$$
\begin{pmatrix} \mathbf{D'} \\ \mathbf{B} \end{pmatrix} \rightarrow S \begin{pmatrix} \mathbf{D'} \\ \mathbf{B} \end{pmatrix}
\tag{40}
$$

and induce a Möbius transformation for the λ field,

$$
\lambda \rightarrow \frac{a\lambda + b}{c\lambda + d},
\tag{41}
$$

where $S \in SL(2, \mathbb{R})$ is

$$
S = \begin{pmatrix} a & b \\ c & d \end{pmatrix}
\tag{42}
$$

with $ad - bc = 1$.

In the static case $\mathbf{D'}$ satisfies the same Gauss law and quantization condition (14). Its normalization change is caused by the replacement $1/g_s \rightarrow exp(-\phi)$. For instance, instead of eq.(18) one gets

$$
D' = \frac{\pi n}{r^2}.
\tag{43}
$$

Owing to this, $SL(2, \mathbb{Z})$ transformations can be formulated in terms of $\mathbf{D}$ and $\mathbf{B}$ introduced in the previous section

$$
\begin{pmatrix} \frac{1}{g_s}\mathbf{D} \\ \mathbf{B} \end{pmatrix} \rightarrow S \begin{pmatrix} \frac{1}{g_s}\mathbf{D} \\ \mathbf{B} \end{pmatrix}
\tag{44}
$$

for the quantized charges. We now consider a D3-brane with minimal electric charge, $n = 1$. This has to shrink to the F-string and we are actually considering a fundamental string. We will map it to the (n, m) strings for any relatively prime pair n and m. Indeed, it is well known that the Diophante equation

$$
nx - my = 1
\tag{45}
$$

always has a solution for the such a pair of integers. In fact, it has countably many solutions, because if $(x = i, y = j)$ is a solution then for any integer k $(x = km + i, y = kn + j)$ is also a solution. Then for the given pair (n, m) we have a matrix $S \in SL(2, \mathbb{Z})$

$$
S = \begin{pmatrix} n & j \\ m & i \end{pmatrix}.
\tag{46}
$$

We perform an electric-magnetic duality rotation (44) via S, i.e.

$$
\begin{pmatrix} n & j \\ m & i \end{pmatrix} \frac{\pi}{r^2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\pi}{r^2} \begin{pmatrix} n \\ m \end{pmatrix}.
\tag{47}
$$

It produced parallel electric and magnetic fields with corresponding charges n and m. We already ascertained that the D3-brane with these charges collapses to the bound state of strings. Thus electric-magnetic duality mapped

the fundamental string to the (n, m) strings which is a weak-strong duality transformation [17].

We note that eq.(45) has no solutions when n and m are not relatively prime. Consequently we can not map the fundamental string to the FD strings in this case, they are not in the same $SL(2, \mathbb{Z})$ multiplet. This means that the bound state of strings becomes unstable when n and m have a common multiplier.

5   Conclusions

The NLE static equations (11,12) admit a wider class of solutions with nonzero Poynting vector, but we did not consider these. Our justification is that these solutions do not allow the application of $SL(2, \mathbb{Z})$ tranformations. In curved subspaces SO(2) duality is maintained but other dualities, in general, are broken [4]. One can see this from the expression (30), the charges are quantized in different units and they can not be mapped to each other. There are two possibilities to get D3-brane charges quantized in the same units, that is to consider branes of the shape $\mathbb{R}^2 \otimes \mathbb{S}^1$ or $\mathbb{R}^1 \otimes \mathbb{S}^1 \otimes \mathbb{S}^1$. In both cases there are two orthogonal equivalent directions along which the electric and magnetic fields can be directed. It will restore $SL(2, \mathbb{R})$ electric-magnetic duality and describe an $SL(2, \mathbb{Z})$ multiplet of D3-branes which can be mapped to each other.

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