Non local Thirring model with spin flipping interactions

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Abstract

We extend a non local and non covariant version of the Thirring model in order to describe a many-body system with spin-flipping interactions By introducing a model with two fermion species we are able to avoid the use of non abelian bosonization which is needed in a previous approach. We obtain a bosonized expression for the partition function, describing the dynamics of the collective modes of this system. By using the self-consistent harmonic approximation we found a formula for the gap of the spin-charge excitations as functional of arbitrary electron-electron potentials.

Keywords field theory, non local, functional bosonization, many body, two dimensional, Luttinger liquid
Pacs: 11.10.Lm, 05.30.Fk
La Plata-Th 00/14

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1 Introduction

In recent years there has been a surge of activity in the study of low-dimensional field theories. In particular, research on the one-dimensional (1d) fermionic gas has attracted a lot of attention. An interesting aspect of this system is the possibility of having a deviation from the usual Fermi-liquid behavior. This phenomenon is known as Luttinger-liquid behavior, characterized by spin-charge separation and by non universal (interaction dependent) power-law correlation functions [1]. The simplest theoretical framework that presents this feature is the Tomonaga-Luttinger (TLM) model [2], a many-body system of right and left-moving particles interacting through their charge densities. In recent papers [3] [4] [5] an alternative, field theoretical approach was developed to consider this problem. In these works a non local and non covariant version of the Thirring model was introduced, in which the fermionic densities and currents are coupled through bilocal, distance-dependent potentials. The resulting non local Thirring model (NLTM) contains the TLM as a particular case. Although it provides an elegant framework to analyze the 1d many-body problem, as it stands it is not very practical to consider magnetic properties of Luttinger liquids. This is so because in the original formulation of NLTM [3], the description of spin-flipping (SF) processes, based on non abelian bosonization, is quite involved. In this paper we show how to circumvent this problem by introducing a Thirring-like model with two fermion species. To this end we generalize the two-fermion model of Ref. [6], originally built as a local and covariant theory, to the case in which the interactions between fermionic currents are mediated by bilocal functions. From the condensed matter standpoint, our work can also be viewed as an extension of the work of Grinstein, Minnhagen and Rosengren [7] where a simplified version of the SF problem (contained in our model) was first considered. Our procedure allows to get a bosonized vacuum to vacuum functional by carefully extending previous formulations of massive [8] [9] and massive-like [10] models. The paper is organized as follows. In Section 2 we introduce the above mentioned non local two-fermion model and explain its relationship to previous non Abelian description. In Section 3 we establish an equivalence between the initial fermionic partition function and the one corresponding to a two-boson non local extension of the sine-Gordon model. In Section 4 we employ a non local version of the self-consistent harmonic approximation (SCHA) [11] in order to analyze the spectrum of the spin-density modes. In particular we give, within this approximation, a closed formula for the gap as functional of the non-contact forward-scattering (no SF) potentials. Finally, in Section 5, we summarize the main points of our investigation and gather our conclusions. In the Appendix we show how to compute the gap of the spin-charge sector by means of the harmonic approximation.

2 The model and its relationship to a previous non Abelian description

In this Section we introduce a non local version of the Thirring model which incorporates electronic spin by considering two fermion species, where each species represents a spin state. Our initial (Euclidean) action is

$$S = \int d^2x\, \bar{\Psi}^a i\partial\!\!\!/ \Psi^a - \frac{g^2}{2} \int d^2x\, d^2y\, j^a_\mu(x) V^{ab}_{(\mu)}(x-y) j^b_\mu(y)$$
$$- g_s \int d^2x\, d^2y\, \bar{\Psi}^1(x)\gamma_\mu \Psi^2(x) U_{(\mu)}(x-y) \bar{\Psi}^2(y)\gamma_\mu \Psi^1(y) \quad (1)$$

where $a, b = 1, 2$, with $1 = \uparrow, 2 = \downarrow$, the currents j^a_μ are the usual fermion currents

$$j^1_\mu = \bar{\Psi}^1 \gamma_\mu \Psi^1, \quad j^2_\mu = \bar{\Psi}^2 \gamma_\mu \Psi^2, \quad (2)$$

and the matrices $V^{ab}_{(\mu)}$ have the form

$$V_{(\mu)} = \frac{1}{2} \begin{pmatrix} V^c_{(\mu)} + V^s_{(\mu)} & V^c_{(\mu)} - V^s_{(\mu)} \\ V^c_{(\mu)} - V^s_{(\mu)} & V^c_{(\mu)} + V^s_{(\mu)} \end{pmatrix}, \quad (3)$$

where $V^c_{(\mu)}$ and $V^s_{(\mu)}$ are coupling potentials related to Solyom's [12] g_2 and g_4 forward scattering couplings. These functions are related to current-current interactions which do not flip the spins. The functions $U_{(\mu)}(x-y)$ are the couplings governing SF processes. We have kept the constants g and g_S in order to facilitate comparison with local versions of this model. For instance, the case $g_S = 0$ and $V^c_(\mu) = V^s_(\mu) = \delta^2(x-y)$ corresponds to two decoupled, usual Thirring models. The action (1) has manifest $U(1)$ chiral symmetry, i.e., it is invariant under the transformation $\Psi^a \to e^{i\gamma_5\theta}\Psi^a$, $\bar{\Psi}^a \to \bar{\Psi}^a e^{i\gamma_5\theta}$. Fermion spin conservation is also preserved in this theory.

The theory defined above is similar to the one described by ZJ [6]. There are, however, two important differences with that model. First of all, our model takes into account the possible long-range nature of the potentials, whereas ZJ's model is local. On the other hand ZJ's model does not include g_4-like terms associated to scattering processes involving just one fermionic (left or right) branch, neither for SF nor for ordinary diagrams.

Concerning the relationship of our action to previous condensed-matter inspired models, we should mention the pioneering works of Luther and Emery [13], and Grinstein, Minnhagen and Rosengren [7]. The first authors introduced the so called backscattering model. Although this system has in principle no SF, in the local limit their backscattering diagrams coincide with the ones of spin-changing processes. Grinstein et al. included from the beginning SF interactions taking into account a Coulombian potential. Thus, their model is non local, but they took the same potential for all kinds of diagrams (SF and ordinary). Besides, in order to establish the equivalence between their theory and a Coulomb gas system the authors considered again a local limit. These authors did not include g_4-like terms either.

Let us now show that the action (1) can be written in an alternative way. Consider the U(N) currents

$$J^\alpha_\mu = \bar{\Psi}\gamma_\mu \lambda^\alpha \Psi \quad (4)$$

with

$$\lambda^0 = \tfrac{1}{2} I \quad (5)$$
$$\lambda^j = t^j, \quad (6)$$

t^j being the SU(N) generators normalized according to

$$\mathrm{tr}\, t^i t^j = \tfrac{1}{2}\delta^{ij}. \tag{7}$$

With these currents, one can define a non local, chiral invariant, U(N) Gross-Neveu model, with action given by

$$S = \int d^2x\, \bar{\Psi} i\partial\!\!\!/ \Psi - \int d^2x\, d^2y\, J^\alpha_\mu(x) \mathcal{V}^{\alpha\beta}_{(\mu)}(x-y) J^\beta_\mu(y), \qquad \alpha, \beta = 0, 1, ..., N^2-1 \tag{8}$$

where $\mathcal{V}_{(\mu)}$ are $N^2 \times N^2$ symmetric matrices that weight the interaction. Taking $N = 2$, the actions (1) and (8) are equal provided that the matrices $\mathcal{V}_{(\mu)}$ can be written as

$$\mathcal{V}_{(\mu)} = \begin{pmatrix} g^2 V^c_{(\mu)} & 0 & 0 & 0 \\ 0 & g_s U_{(\mu)} & 0 & 0 \\ 0 & 0 & g_s U_{(\mu)} & 0 \\ 0 & 0 & 0 & g^2 V^s_{(\mu)} \end{pmatrix}. \tag{9}$$

This non Abelian model was considered in Ref. [3]. The bosonized effective action obtained by non Abelian bosonization led to a WZW functional, which was not easy to deal with in order to analyze the physical spectrum. In the next Sections we will show how this task is greatly simplified by starting from (1) instead of (8) and combining path-integral *Abelian* bosonization and the self consistent harmonic approximation.

3 The equivalent bosonic action

We start by considering the partition function

$$\mathcal{Z} = \mathcal{N} \int \mathcal{D}\bar{\Psi}^a \mathcal{D}\Psi^a\, e^{-S}, \tag{10}$$

where $\mathcal{N}$ is a normalization constant. It is convenient to write

$$S = S_0 + S_{\mathrm{flip}}, \tag{11}$$

where

$$S_0 = \int d^2x\, \bar{\Psi}^a i\partial\!\!\!/ \Psi^a - \frac{g^2}{2} \int d^2x\, d^2y\, j^a_\mu(x) V^{ab}_{(\mu)}(x-y) j^b_\mu(y) \tag{12}$$

and

$$S_{\mathrm{flip}} = -g_s \int d^2x\, d^2y\, \bar{\Psi}^1(x) \gamma_\mu \Psi^2(x) U_{(\mu)}(x-y) \bar{\Psi}^2(y) \gamma_\mu \Psi^1(y). \tag{13}$$

The reason for this separation lies on the fact that S_0 contains all the interaction terms that have separate chiral invariance for each species of fermions (spin states). They are Thirring-like interactions and will be treated in much the same way as the massless Thirring action, i.e. they will be transformed into free, non local action terms. Concerning the second term, it has no separate

chiral invariance, and will be expanded in perturbative series like the mass term in the Thirring model.

It is easy to show that the introduction of auxiliary vector fields A^a_μ allows to write

$$\mathcal{Z} = \mathcal{N}' \int \mathcal{D}\bar{\Psi}^a \mathcal{D}\Psi^a \mathcal{D}A^a_\mu \exp\left[- \int d^2x\, \bar{\Psi}^a \left(i\partial\!\!\!/ + g A\!\!\!/^a \right) \Psi^a - S[A] - S_{\text{flip}} \right], \tag{14}$$

where $\mathcal{N}'$ is a new constant (See [3] for details) and

$$S[A] = \frac{1}{2} \int d^2x\, d^2y\, \left(V^{-1}_{(\mu)} \right)^{ab} (x-y) A^a_\mu(x) A^b_\mu(y), \tag{15}$$

with $\left(V^{-1}_{(\mu)} \right)^{ab}$ defined through the following equation:

$$\int d^2y\, \left(V^{-1}_{(\mu)} \right)^{ab} (x-y) V^{bc}_{(\mu)}(y-z) = \delta^{(2)}(x-z)\delta^{ac}. \tag{16}$$

We now decompose A^a_μ in longitudinal and transverse pieces

$$A^a_\mu(x) = \epsilon_{\mu\nu}\partial_\nu \phi^a(x) + \partial_\mu \eta^a(x), \tag{17}$$

where ϕ^a and η^a are scalar fields. We also perform a change in the fermionic fields

$$\Psi^a(x) = e^{-g[\gamma_5 \phi^a(x) - i\eta^a(x)]} \chi^a(x) \tag{18}$$

$$\bar{\Psi}^a(x) = \bar{\chi}^a(x) e^{-g[\gamma_5 \phi^a(x) + i\eta^a(x)]} \tag{19}$$

whose Jacobian, as it is well-known, yields a kinetic term for the ϕ^a fields. One thus gets

$$\mathcal{Z} = \mathcal{N} \int \mathcal{D}\bar{\chi}^a \mathcal{D}\chi^a \mathcal{D}\phi^a \mathcal{D}\eta^a e^{-S_{\text{eff}}}, \tag{20}$$

being S_{eff} a sum of three pieces:

$$S_{\text{eff}} = S_{0\text{F}} + S_{0\text{B}} + S_{\text{flip}} \tag{21}$$

where

$$S_{0\text{F}} = \int d^2x\, \left(\bar{\chi}^1 i\partial\!\!\!/ \chi^1 + \bar{\chi}^2 i\partial\!\!\!/ \chi^2 \right), \tag{22}$$

$$S_{0\text{B}} = \frac{g^2}{2\pi} \int d^2x\, \left[(\partial_\mu \phi^1)^2 + (\partial_\mu \phi^2)^2 \right] + \frac{1}{2} \int d^2x\, d^2y\, \left(V^{-1}_{(\mu)} \right)^{ab} (x-y) \times \left[\epsilon_{\mu\nu}\epsilon_{\mu\rho} \partial_\nu \phi^a(x) \partial_\rho \phi^b(y) + \partial_\mu \eta^a(x) \partial_\mu \eta^b(y) + 2\epsilon_{\mu\nu} \partial_\nu \phi^a(x) \partial_\mu \eta^b(y) \right], \tag{23}$$

and S_{flip} is the same SF interaction term already defined in equation (13). Concerning this last term, from now on we shall restrict our study to the case of contact SF interactions:

$$U(x-y)_{(0)} = U(x-y)_{(1)} = \delta^{(2)}(x-y), \tag{24}$$

and by performing a Fierz transformation followed by the chiral change defined in equations (18) and (19), we can write it in the form

$$S_{\text{flip}} = 2g_s \int d^2x \Big[e^{-2g(\phi^1-\phi^2)} \bar{\chi}^1 \frac{1+\gamma_5}{2} \chi^1 \cdot \bar{\chi}^2 \frac{1-\gamma_5}{2} \chi^2 + e^{2g(\phi^1-\phi^2)} \bar{\chi}^1 \frac{1-\gamma_5}{2} \chi^1 \cdot \bar{\chi}^2 \frac{1+\gamma_5}{2} \chi^2 \Big]. \tag{25}$$

Now we are ready to make an expansion of the partition function taking g_s as perturbative parameter:

$$\mathcal{Z} = \mathcal{N} \int \mathcal{D}\phi^a \mathcal{D}\eta^a e^{-S_{0\text{B}}} \sum_{n=0}^{\infty} \frac{(-2g_s)^n}{n!} \int \left(\prod_{i=1}^{n} d^2x_i \right) \times \left\langle \prod_{i=1}^{n} \Big[e^{-2g(\phi^1-\phi^2)} \bar{\chi}^1 \frac{1+\gamma_5}{2} \chi^1 \cdot \bar{\chi}^2 \frac{1-\gamma_5}{2} \chi^2 + e^{2g(\phi^1-\phi^2)} \bar{\chi}^1 \frac{1-\gamma_5}{2} \chi^1 \cdot \bar{\chi}^2 \frac{1+\gamma_5}{2} \chi^2 \Big] \right\rangle_{0\text{F}} \tag{26}$$

where $\langle\,\rangle_{0\text{F}}$ means v.e.v. in a theory with action $S_{0\text{F}}$. Only mean values involving an equal number of factors of the form $\frac{1}{2}\bar{\chi}^a(1+\gamma_5)\chi^a$ and $\frac{1}{2}\bar{\chi}^a(1-\gamma_5)\chi^a$ (no sum over repeated indices is implied in these expressions) are non zero, and thus the partition function can be written in the form

$$\mathcal{Z} = \mathcal{N} \int \mathcal{D}\phi^a \mathcal{D}\eta^a \, e^{-S_{0\text{B}}} \times \sum_{n=0}^{\infty} \frac{(2g_s)^{2n}}{(n!)^2} \int \left(\prod_{i=1}^{n} d^2x_i \, d^2y_i \, e^{-2g\left[\phi^1(x_i)-\phi^2(x_i)-\phi^1(y_i)+\phi^2(y_i)\right]} \right) \times \left\langle \prod_{i=1}^{n} \bar{\chi}^1(x_i) \frac{1+\gamma_5}{2} \chi^1(x_i) \bar{\chi}^1(y_i) \frac{1-\gamma_5}{2} \chi^1(y_i) \right\rangle_{0\text{F}} \times \left\langle \prod_{i=1}^{n} \bar{\chi}^2(x_i) \frac{1-\gamma_5}{2} \chi^2(x_i) \bar{\chi}^2(y_i) \frac{1+\gamma_5}{2} \chi^2(y_i) \right\rangle_{0\text{F}}. \tag{27}$$

The next step is to introduce two local massless scalar fields ϑ^a to be associated with the free fermions $\bar{\chi}^a$ and χ^a. This trick allows to replace the fermionic mean values in the above equation by their ultraviolet regularized bosonic counterparts, which yields

$$\mathcal{Z} = \mathcal{N} \int \mathcal{D}\phi^a \mathcal{D}\eta^a \, e^{-S_{0\mathrm{B}}}$$
$$\times \sum_{n=0}^{\infty} \frac{(2g_s)^{2n}}{(n!)^2} \int \left(\prod_{i=1}^{n} d^2x_i \, d^2y_i e^{-2g\left[\phi^1(x_i)-\phi^2(x_i)-\phi^1(y_i)+\phi^2(y_i)\right]} \right)$$
$$\times \left(\frac{i\Lambda}{2\pi}\right)^{4n} \left\langle \prod_{i=1}^{n} e^{i\sqrt{4\pi}\left[\vartheta^1(x_i)-\vartheta^1(y_i)\right]} \right\rangle_{0,\vartheta^1} \left\langle \prod_{i=1}^{n} e^{i\sqrt{4\pi}\left[\vartheta^2(x_i)-\vartheta^2(y_i)\right]} \right\rangle_{0,\vartheta^2} . \quad (28)$$

We now use the fact that the infrared divergences of the ϑ^a field's propagator provide a neutrality condition for v.e.v's of vertex operators. This allows to rearrange the perturbative series in a non-trivial way. One is then led to the completely bosonized action S_{bos}

$$S_{\mathrm{bos}} = S_{0\mathrm{B}} + \int d^2x \left\{ \frac{1}{2}(\partial_\mu \vartheta^a)^2 - \frac{g_s\Lambda^2}{\pi^2} \cos\left[2ig(\phi^1-\phi^2) + \sqrt{4\pi}(\vartheta^1+\vartheta^2)\right] \right\}. \quad (29)$$

At this point it is interesting to observe that there is a set of changes of variables that allows to express the action in a very suggestive way. Indeed, writing

$$\theta = \frac{1}{\sqrt{2}}(\vartheta^1 + \vartheta^2) \quad (30)$$

$$\tilde{\theta} = \frac{1}{\sqrt{2}}(\vartheta^1 - \vartheta^2) \quad (31)$$

$$\phi^{1,2} = \frac{1}{\sqrt{2}}(\phi_c \pm \phi_s) \quad (32)$$

$$\eta^{1,2} = \frac{1}{\sqrt{2}}(\eta_c \pm \eta_s) \quad (33)$$

where the plus(minus) sign corresponds to the pair $\phi^1, \eta^1 (\phi^2, \eta^2)$, one sees that the field $\tilde{\theta}$ completely decouples from the others and can then be integrated, and the bosonized action becomes

$$S_{\mathrm{bos}} = S_c + S_s, \quad (34)$$

where

$$S_c = \frac{g^2}{2\pi} \int d^2x \, (\partial_\mu \phi_c)^2 + \frac{1}{2} \int d^2x \, d^2y \, \left(V^c_{(\mu)}\right)^{-1} (x-y)$$
$$\times \left[\epsilon_{\mu\nu}\epsilon_{\mu\rho}\partial_\nu\phi_c(x)\partial_\rho\phi_c(y) + \partial_\mu\eta_c(x)\partial_\mu\eta_c(y) + 2\epsilon_{\mu\nu}\partial_\nu\phi_c(x)\partial_\mu\eta_c(y)\right], \quad (35)$$

and

$$S_s = \frac{g^2}{2\pi}\int d^2x\,(\partial_\mu\phi_s)^2 + \frac{1}{2}\int d^2x\,d^2y\,\left(V^s_{(\mu)}\right)^{-1}(x-y)$$
$$\times\,[\epsilon_{\mu\nu}\epsilon_{\mu\rho}\partial_\nu\phi_s(x)\partial_\rho\phi_s(y) + \partial_\mu\eta_s(x)\partial_\mu\eta_s(y) + 2\epsilon_{\mu\nu}\partial_\nu\phi_s(x)\partial_\mu\eta_s(y)]$$
$$+\int d^2x\left[\frac{1}{2}(\partial_\mu\theta)^2 - \frac{g_s\Lambda^2}{\pi^2}\cos\left(\sqrt{8}ig\phi_s + \sqrt{8\pi}\theta\right)\right] \quad (36)$$

with the functions $\left(V^{c,s}_{(\mu)}\right)^{-1}$ defined as

$$\int d^2y\,\left(V^{c,s}_{(\mu)}\right)^{-1}(x-y)V^{c,s}_{(\mu)}(y-z) = \delta^{(2)}(x-z). \quad (37)$$

This, in turn, leads to the factorization of the partition function in the form $\mathcal{Z} = \mathcal{Z}_c\mathcal{Z}_s$. This result is a clear manifestation of the so called spin-charge separation, a typical feature of Luttinger liquids [13] [7]. $\mathcal{Z}_c$ is the partition function associated to charge density excitations. It coincides with the spinless NLTM studied in reference [3]. $\mathcal{Z}_s$ describes spin-density excitations. The action S_s corresponds to a certain non local sine-Gordon model similar to the one previously considered in [10]. In the next Section we shall derive an expression for the gap of its spectrum.

Since our main goal is to analyze the spin-charge sector, from now on we shall focus our attention to S_s. Going to momentum space (with the exception of the cosine term, whose transformed expression is not very illuminating) one has

$$S_s = \int \frac{d^2p}{(2\pi)^2}\left[\hat{\phi}(p)\hat{\phi}(-p)A(p) + \hat{\eta}(p)\hat{\eta}(-p)B(p) + \hat{\phi}(p)\hat{\eta}(-p)C(p)\right]$$
$$+\int \frac{d^2p}{(2\pi)^2}\,\frac{p^2}{2}\theta(p)\theta(-p) - \frac{g_s\Lambda^2}{\pi^2}\int d^2x\,\cos\left(\sqrt{8}ig\phi_s + \sqrt{8\pi}\theta\right) \quad (38)$$

with

$$A(p) = \frac{g^2}{2\pi} + \frac{1}{2}\left[\left(V^s_{(0)}\right)^{-1}(p)p_1^2 + \left(V^s_{(1)}\right)^{-1}(p)p_0^2\right], \quad (39)$$

$$B(p) = \frac{1}{2}\left[\left(V^s_{(1)}\right)^{-1}(p)p_1^2 + \left(V^s_{(0)}\right)^{-1}(p)p_0^2\right], \quad (40)$$

$$C(p) = \left[\left(V^s_{(0)}\right)^{-1}(p) - \left(V^s_{(1)}\right)^{-1}(p)\right]p_0p_1. \quad (41)$$

In the above expressions $\hat{\phi}(p)$, $\hat{\eta}(p)$ and $\theta(p)$ are the Fourier transforms of $\phi_s(x)$, $\eta_s(x)$ and $\theta(x)$ respectively.

4 The spectrum of the spin-charge sector

In order to obtain the approximate spectrum for the spin-charge sector of our model, we will introduce a non local version of the self-consistent harmonic

approximation [11]. Basically, this amounts to replacing the so called *true* action (36) by a *trial* action in which the cosine term is approximated as

$$-\frac{g_s\Lambda^2}{\pi^2}\cos\left(\sqrt{8}ig\phi_s+\sqrt{8\pi}\theta\right)\rightarrow\frac{\Omega^2}{2}\left(\sqrt{8}ig\phi_s+\sqrt{8\pi}\theta\right)^2 \tag{42}$$

where the parameter Ω of the trial action can be variationally determined (See the Appendix for details). Once this is done, it is straightforward to obtain the spectrum. Indeed, going to momentum space, and back to real frequencies, $p_0=i\omega$, $p_1=k$, the following equation is obtained:

$$4\pi\Omega^2+k^2(1+\frac{g^2}{\pi}V^s_{(0)})-\omega^2(1+\frac{g^2}{\pi}V^s_{(1)})=0. \tag{43}$$

Our result for Ω is

$$\Omega^2=\frac{g_s\Lambda^2}{\pi^2}e^{-4\pi I_0(\Omega)} \tag{44}$$

where

$$I_0(\Omega)=\int\frac{d^2p}{(2\pi)^2}\frac{1}{p^2+\frac{g^2}{\pi}\left(p_0^2V^s_{(1)}+p_1^2V^s_{(0)}\right)+8\pi\Omega^2} \tag{45}$$

Equation (44) is one of our main results. It gives, within the self-consistent harmonic approximation, a closed expression for the gap as functional of the potentials $V^s_{(\mu)}(p)$.

At this stage it is interesting to consider the contact potential given by

$$V^s_{(0)}(p)=V^s_{(1)}(p)=1 \tag{46}$$

In this case $I_0(\Omega)$ is infinite (See Appendix). After a suitable regularization one obtains

$$\frac{\Omega^2}{\Lambda^2}=\frac{g_s}{\pi^2}\left[1+\frac{(1+g^2/\pi)\Lambda^2}{8\pi\Omega^2}\right]^{\frac{-1}{1+g^2/\pi}}. \tag{47}$$

This equation can be easily solved for $\Lambda\gg\Omega$ and $\Omega\gg\Lambda$. The first case corresponds to $g_s\ll 1$, and the result is

$$\Omega^2=\frac{g_s\Lambda^2}{\pi^2}\left[\frac{8g_s}{\pi(1+g^2/\pi)}\right]^{\pi/g^2}. \tag{48}$$

Note that for the value $g^2=\pi$ the gap posses a linear dependence on the coupling. It is interesting to observe that for this value, the action S_s (see Appendix) corresponds to a free massive fermion, with mass proportional to g_s. This is analog to the behavior found on the Luther-Emery line [13]. Despite this similarity, the position of the line in our model differs slightly from theirs, but since the models are not exactly equal such a disagreement is not unexpected. However, we feel that the precise connection between both systems deserves a closer analysis that will be undertaken in a future work.

The second case corresponds to $g_s\gg 1$, and we obtain

$$\Omega^2=\frac{g_s\Lambda^2}{\pi^2}, \tag{49}$$

which is independent of g. This is expected because in this limit the contribution of the ordinary (without SF) terms are completely negligible.

Let us stress that our gap equation (44) provides a new tool to check the validity of different potentials [15]. This is interesting because in condensed-matter applications the assumption of short-range electron-electron interactions works well for conductors in which the screening between adjacent chains reduces the range of interactions within one chain [16]. On the other hand, as the dimensionality of the system decreases charge screening effects are expected to become less important and the long-range interaction between electrons seems to play a central role in determining the properties of the system. This assertion seems to be confirmed by experiments in GaAs quantum wires [18] and quasi-1d conductors [17]. We hope to report on numerical solutions of the gap equations for long-range potentials in a future contribution.

5 Summary

In this work we have improved a non local version of the Thirring model which provides a tractable field-theoretical description of Luttinger liquids with spin. Indeed, in the context of non local Thirring-like theories previous treatments of SF interactions led to an involved non Abelian model (See for instance [3]). Specifically we have built an action based on two fermion species which allows to take into account SF interactions in an elegant and simpler way. Although our model was inspired by the one considered in Ref.[6], it includes interaction diagrams not contained in that previous work (the so called g_4 diagrams in Solyom's terminology [12]). Besides, the theory we present has general bilocal potentials governing the interactions that do not flip spins. We parametrized this potentials in terms of the functions V^c and V^s which become associated to charge-density and spin-density dynamics respectively, once the spin-charge separation is made manifest after suitable changes of variables (See equations (32) and (33)). On the other hand, we could not get closed formulae for arbitrary distance-dependent SF potentials. Thus our analysis is only valid for local (contact) SF interactions. However, we were able to keep the general distance dependence of the ordinary (no SF) potentials up to the end of our computations. Under these conditions we got an effective bosonic action whose charge-density piece coincides with previously obtained descriptions of the forward-scattering spinless problem ([3]. Concerning the more interesting spin-charge sector, we obtained, within the framework of the SCHA, a general equation which allows to have the gap in terms of arbitrary electron-electron potentials. Using this formula we could, for instance, estimate the effect of a long-range interaction on the gap.

Acknowledgements

This work was supported by the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET) and Universidad Nacional de La Plata (UNLP), Argentina. KL has benefited from a fellowship of CONICET and the Third World Academy of Sciences (TWAS).

6 Appendix: details of the SCHA method

We shall give an sketch of the SCHA method. One usually starts from a partition function

$$\mathcal{Z}_{\rm true} = \int \mathcal{D}\mu e^{-S_{\rm true}} \tag{50}$$

where $\mathcal{D}\mu$ is a generic integration measure. An elementary manipulation leads to

$$\mathcal{Z}_{\rm true} = \frac{\int \mathcal{D}\mu e^{-(S_{\rm true}-S_{\rm trial})}\, e^{-S_{\rm trial}}}{\int \mathcal{D}\mu e^{-S_{\rm trial}}} \int \mathcal{D}\mu e^{-S_{\rm trial}} = \mathcal{Z}_{\rm trial} \left\langle e^{-(S_{\rm true}-S_{\rm trial})} \right\rangle_{\rm trial}. \tag{51}$$

for any trial action $S_{\rm trial}$. Now, by means of the property

$$\left\langle e^{-f} \right\rangle \geq e^{-\langle f \rangle}, \tag{52}$$

for f real, and taking natural logarithm in equation (51), we obtain Feynman's inequality [14]

$$\ln \mathcal{Z}_{\rm true} \geq \ln \mathcal{Z}_{\rm trial} - \langle S_{\rm true} - S_{\rm trial} \rangle_{\rm trial} \tag{53}$$

In our case, the true action is the corresponding to the spin-charge dynamics, S_s. However, before proceeding to the actual computation it is convenient to consider still another change of variables which diagonalizes the cuadratic part of S_s. This change is given by

$$\theta = \frac{ig}{\sqrt{\pi}}\,\zeta + \frac{A - \frac{C^2}{4B}}{A - \frac{C^2}{4B} - \frac{g^2p^2}{2\pi}}\,\xi \tag{54}$$

$$\eta = \frac{C}{2B}\,\zeta - \frac{i\sqrt{\pi}gCp^2}{4B\left(A - \frac{C^2}{4B} - \frac{g^2p^2}{2\pi}\right)}\,\xi + \varphi \tag{55}$$

$$\phi = -\zeta + \frac{igp^2}{\sqrt{4\pi}\left(A - \frac{C^2}{4B} - \frac{g^2p^2}{2\pi}\right)}\,\xi \tag{56}$$

and the resulting action, reads

$$S_s = \frac{1}{2}\int \frac{d^2p}{(2\pi)^2}\left\{\zeta(p)\zeta(-p)\frac{p^4}{p_0^2V_{(1)}^s + p_1^2V_{(0)}^s} + \varphi(p)\varphi(-p)\left(\frac{p_0^2}{V_{(0)}^s} + \frac{p_1^2}{V_{(1)}^s}\right)\right.$$
$$\left. + \xi(p)\xi(-p)\left[p^2 + \frac{g^2}{\pi}\left(p_0^2V_{(1)}^s + p_1^2V_{(0)}^s\right)\right]\right\} - \frac{g_s\Lambda^2}{\pi^2}\int d^2x\, \cos(\sqrt{8\pi}\xi) \tag{57}$$

where one sees that the ζ and φ fields become completely decoupled from ξ. Then it becomes apparent that the ξ-dependent piece is the relevant one to consider in the present approximation. In other words, the relevant true and trial actions are given by

$$S_{\rm true} = \int \frac{d^2p}{(2\pi)^2}\,\xi(p)\frac{F(p)}{2}\xi(-p) - \frac{g_s\Lambda^2}{\pi^2}\int d^2x\,\cos(\sqrt{8\pi}\xi) \tag{58}$$

$$S_{\rm trial} = \int \frac{d^2p}{(2\pi)^2}\,\left[\xi(p)\frac{F(p)}{2}\xi(-p) + \frac{8\pi\Omega^2}{2}\xi(p)\xi(-p)\right]. \tag{59}$$

where

$$F(p) = p^2 + \frac{g^2}{\pi}\left(p_0^2 V^s_{(1)} + p_1^2 V^s_{(0)}\right). \tag{60}$$

The parameter Ω can be determined by maximizing the right hand side of equation (53). In order to achieve this goal we first write

$$\ln Z_{\rm trial} = \ln\int \mathcal{D}\xi\,\exp\left[-\frac{1}{2}\int d^2x\,\xi(x)(\hat{A}\xi)(x)\right] = [\ln(\det\hat{A})^{-1/2}] + \text{const} \tag{61}$$

$$= -\frac{1}{2}\,\mathrm{tr}\ln\hat{A} + \text{const.} \tag{62}$$

where the operator $\hat{A}$ is defined, in Fourier space, by

$$(\hat{A}\xi)(p) = [F(p) + 8\pi\Omega^2]\xi(p). \tag{63}$$

It is then easy to get

$$\mathrm{tr}\ln\hat{A} = V\int\frac{d^2p}{(2\pi)^2}\ln[F(p) + 8\pi\Omega^2] \tag{64}$$

where V is the volume (infinite) of the whole space $\int d^2x$. On the other hand, it is straightforward to compute $\langle S_{\rm true} - S_{\rm trial}\rangle$, by following, for instance, the steps explained in ref. [10]. The result is

$$-\langle S_{\rm true} - S_{\rm trial}\rangle_{\rm trial} = V\frac{g_s\Lambda^2}{\pi^2}\exp\left[-4\pi\int\frac{d^2p}{(2\pi)^2}\,\frac{1}{F(p)+8\pi\Omega^2}\right] + V4\pi\Omega^2\int\frac{d^2p}{(2\pi)^2}\,\frac{1}{F(p)+8\pi\Omega^2}. \tag{65}$$

Finally we can gather all the terms, and write them as

$$\ln\mathcal{Z}_{\rm trial} - \langle S_{\rm true} - S_{\rm trial}\rangle_{\rm trial} = V\left[\frac{g_s\Lambda^2}{\pi^2}\exp\left(-4\pi I_0(\Omega)\right) + \frac{8\pi\Omega^2}{2}I_0(\Omega) - \frac{1}{2}I_1(\Omega)\right] + \text{const.} \tag{66}$$

where we have defined the integrals

$$I_1(\Omega) = \int\frac{d^2p}{(2\pi)^2}\ln[F(p) + 8\pi\Omega^2] \tag{67}$$

$$I_{-n}(\Omega) = \int \frac{d^2p}{(2\pi)^2} \frac{1}{[F(p) + 8\pi\Omega^2]^{n+1}} \tag{68}$$

with the formal properties

$$\frac{dI_1(\Omega)}{d\Omega} = 16\pi\Omega I_0 \tag{69}$$

$$\frac{dI_{-n}(\Omega)}{d\Omega} = -16\pi(n+1)\Omega I_{-n-1}. \tag{70}$$

Now extrimizing expression (66) with respect to Ω, and assuming that $I_{-1}(\Omega)$ is not zero (a condition that holds for most realistic potentials), we finally obtain the gap equation

$$\Omega^2 - \frac{g_s\Lambda^2}{\pi^2} e^{-4\pi I_0(\Omega)} = 0. \tag{71}$$

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