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Duality, gauging and superHiggs effect in string and M-theory

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Abstract

We consider no-scale extended supergravity models as they arise from string and M-theory compactifications in presence of fluxes. The special role of gauging axion symmetries for the Higgs and superHiggs mechanism is outlined.

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1 Introduction

Recently, supergravity models which admit a superHiggs mechanism whithout tree level cosmological constant have been reconsidered in the framework of superstring and M-theory compactifications in presence of fluxes [1, 2, 3, 4, 5, 6, 7, 8].

These models are a variant of the well known $AdS_5 \times S^5$ compactification [9, 10, 11] of type IIB string theory, which in the supergravity limit [12] corresponds to an SU(4) gauge $D = 5$ maximally extended supergravity [13, 14]. The non abelian nature of the gauging is closely connected to the isometries of the 5-sphere and the gauge coupling is turned on by the 5-form Ramond-Ramond flux. These theory has 32 unbroken supersymmetries in $D = 5$.

Theories with less supersymmetries naturally arise in scenarios where turning on fluxes gives rise to compactifications without cosmological constant.

These theories cover, in particular, the Scherk-Schwarz mechanism [15, 16] as originally proposed in eleven dimensional supergravity and type IIB orientifolds in presence of three-form fluxes [6, 7, 8]. In this class of theories fluxes correspond in the effective supergravity to charge couplings of axions to gauge fields

$$D_\mu \Phi^\Lambda = \partial_\mu \Phi^\Lambda + g_I^\Lambda A_\mu^I,$$

and the nature of the charge g_I^Λ very much depends of the theory under consideration. The gauge transformations are

$$\begin{aligned} A_\mu^I &\to A_\mu^I - \partial_\mu \xi^I, \\ \Phi^\Lambda &\to \Phi^\Lambda + g_I^\Lambda \xi^I. \end{aligned}$$

If the gauge group is non abelian then a possible situation is to have a “flat group” and the latter is what occurs in the Scherk-Schwarz” mechanism. This is actually what is needed if the super Higgs effect involves BPS multiplets, which must be charged under central charges which are unbroken U(1) symmetries of the theory [17].

Supersymmetry also requires that there exists a section X on the scalar manifold such that

$$g_I^\Lambda X_{\Lambda AB}^I$$

is a symmetric matrix in the R-symmetry labels $A, B = 1, \ldots N$. The gravitino mass term has the form

$$\bar{\Psi}_\mu^A g_I^\Lambda X_{\Lambda AB}^I \gamma^{\mu\nu} \Psi_\nu^\beta + \text{h.c.}.$$

String inspired models with spontaneous supersymmetry breaking in flat space [21] have been analyzed in the past [18]. The superHiggs effect in the context of string theory [19] and of brane scenarios for particle physics [20] have been explored more recently.

In N-extended supergravity the scalar potential is generated by the gauge symmetry, and if charged scalar fields are present these symmetry must be an isometry of the scalar manifold. We will call such isometries U-dualities with an abuse of language which comes from string theory [22].

Because of the particular nature of the theory, which couples the sigma models to vector fields, these isometries must have a particular action on the vectors and this is discussed in sections 2 and 3. In the subsequent sections, the examples of $N = 8$ spontaneously broken supersymmetry à la Scherk-Schwarz and the $N = 4$ theory coming from type IIB orientifolds will be discussed and their gauge theory structure emphasized. In the last section the role of translational isometries in the Higgs mechanism underlying the superHiggs effect will be considered in the special case when the scalar manifold is a symmetric space.

2 Duality and supersymmetry breaking

Extended supergravity theories enjoy beautiful covariance properties under duality rotations. Electric and magnetic field strengths undergo symplectic transformations

$$\begin{pmatrix} \mathcal{F}' \\ \mathcal{G}' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \mathcal{F} \\ \mathcal{G} \end{pmatrix}$$

with $A^T D - C^T B = \mathbb{1}$, $A^T C$ and $B^T D$ symmetric. The complexified coupling constant $\mathcal{N}_{\Lambda\Sigma}$, $(\mathcal{G}^+_\Lambda = \mathcal{N}_{\Lambda\Sigma}\mathcal{F}^{+\Sigma})$ undergo fractional transformations [23]

$$\mathcal{N}' = (C + D\mathcal{N})(A + B\mathcal{N})^{-1}.$$

In particular, if the theory has an invariance group G, it must have a symplectic action [24] on the pair $(\mathcal{F}, \mathcal{G})$. We will generically call U-duality the invariance group of a given supergravity theory, that for our purposes of gauging must be a continuous group of symmetries. As a consequence of the gauging, the fermionic variations acquire the following terms [25]

$$\begin{aligned} \delta\Psi_{A\mu} &= \cdots + \frac{S_{AB}}{2}\gamma_\mu \epsilon^B \\ \delta\lambda^I &= \cdots + N^{IA}\epsilon_A \end{aligned}$$

and the scalar potential is given by [26, 27]

$$\delta^A_B V = -3\bar{S}^{AC} S_{CB} + N^{AI} N_{BI}, \qquad N_{BI} = (N^{BI})^*.$$

Then it follows ($A = B$) that

$$V = -3 \sum_C \bar{S}^{AC} S_{AC} + N^{AI} N_{AI} \qquad \forall A$$

Flat space demands that

$$3 \sum_C \bar{S}^{AC} S_{AC} = \sum_I N^{AI} N_{AI} \qquad \forall A.$$

Furthermore, if the A_0-supersymmetry is unbroken, then

$$\sum_C \bar{S}^{A_0 C} S_{A_0 C} = 0 = N^{A_0 I} N_{A_0 I}.$$

For spontaneously broken extended supergravity, a given residual supersymmetry requires a combination of the super Higgs and Higgs effects which give further constraints on the theory.

For example, if a given theory breaks $N = 2$ to $N = 1$, there are two massive vector partners of the massive gravitino and then a Higgs effect must occur. It was shown in Ref. [28] that the condition for this to happen is that the scalar quaternionic manifold must have two translational isometries which get spontaneously broken.

For the class of models under consideration, it is actually true that all massive vector particles, which undergo a Higgs mechanism as a consequence of supersymmetry breaking correspond to spontaneously broken translational isometries of the original non linear sigma model [29, 30].

For no-scale supergravity models [31, 32], the scalar potential is positive semidefinite. In that, the contribution to the potential comes only from those spin 1/2 fields which do not participate into the sueprsymmetry breaking.

$$\langle \delta \chi^{\bar{I}} \rangle = N^{\bar{I}A} \epsilon_A = 0$$
$$V = \sum_{\bar{I}} N^{\bar{I}A} N_{\bar{I}A}$$

while there is a cancellation of the $|S^{AB}|^2$ term with the spin 1/2 fermions for which $N^{IA} \epsilon_A \neq 0$.

3 Gauging of duality symmetries

We consider $D = 4$ supergravity in absence of fluxes. Let G be the U-duality group of the theory and n the number of vectors in the theory, A_μ^Λ, $\Lambda =$

$1\dots n$. The field strengths $F^\Lambda_{\mu\nu}$ and their duals $G^{\mu\nu}_\Lambda = \partial\mathcal{L}/\partial F^\Lambda_{\mu\nu}$ together carry a linear representation of G. When G is considered as embedded in $\mathrm{Sp}(2n,\mathbb{R})$, the representation carried by $\{F^\Lambda, G_\Lambda\}$ is promoted to a representation of the full symplectic group [24].

An arbitrary matrix of the Lie algebra $\mathfrak{sp}(2n,\mathbb{R})$ can be written in terms of blocks of size $n\times n$

$$X = \begin{pmatrix} a & b \\ c & -a^T \end{pmatrix}, \qquad b = b^T,\ c = c^T, \tag{1}$$

and a an arbitrary matrix of $\mathfrak{gl}(n,\mathbb{R})$. The Lie algebra $\mathfrak{sp}(2n,\mathbb{R})$ admits then the decomposition

$$\mathfrak{sp}(2n,\mathbb{R}) = \tilde{\mathfrak{g}}^0 + \tilde{\mathfrak{g}}^{+1} + \tilde{\mathfrak{g}}^{-1},$$

with

$$\tilde{\mathfrak{g}}^0 =\{ \begin{pmatrix} a & 0 \\ 0 & -a^T \end{pmatrix} \}, \quad \tilde{\mathfrak{g}}^{+1} =\{ \begin{pmatrix} 0 & 0 \\ c & 0 \end{pmatrix} \}, \quad \tilde{\mathfrak{g}}^{-1} =\{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \}.$$

So $\tilde{\mathfrak{g}}^0 \approx \mathfrak{gl}(n,\mathbb{R})$, $\tilde{\mathfrak{g}}^{+1}$ carries the representation of $\mathfrak{gl}(n,\mathbb{R})$ $\mathrm{sym}(\mathbf{n}'\otimes\mathbf{n}')$ and $\tilde{\mathfrak{g}}^{-1}$ the representation $\mathrm{sym}(\mathbf{n}\otimes\mathbf{n})$. $\mathbf{n}'$ denotes the contragradient representation of $\mathbf{n}$. The subalgebra $\mathfrak{so}(1,1)_z \subset \mathfrak{gl}(n,\mathbb{R})$ (the subindex z is to distinguish it from other subalgebras $\mathfrak{so}(1,1)$ that we will consider in the following), whose generator is the element $-\frac{1}{2}\mathbb{1}$, acts with charge $+1$ on $\tilde{\mathfrak{g}}^{+1}$ and with charge -1 on $\tilde{\mathfrak{g}}^{-1}$ (so the upper indices indicate this charge). In fact, $\mathfrak{so}(1,1)_z$ defines a grading of $\mathfrak{sp}(2n,\mathbb{R})$ and consequently $\tilde{\mathfrak{g}}^{\pm1}$ are abelian subalgebras. The matrices of the subalgebra $\tilde{\mathfrak{g}}^0 + \tilde{\mathfrak{g}}^{+1}$ have lower block-triangular form ($b=0$). The vector space carrying the fundamental representation of the symplectic algebra inherits also a grading and it decomposes as $V = V^+ \oplus V^-$. Notice that V^+ carries a representation of $\tilde{\mathfrak{g}}^0 + \tilde{\mathfrak{g}}^{+1}$.

We consider now the U-duality group G with Lie algebra $\mathfrak{g}\subset\mathfrak{sp}(2n)$. Any subalgebra of $\mathfrak{g}$ which is a subalgebra of the lower triangular matrices $\tilde{\mathfrak{g}}^0+\tilde{\mathfrak{g}}^{+1}$ will transform the field strengths (in V^+) without involving their magnetic duals. Then, the vector potentials themselves carry a linear representation of this subalgebra which will be called an *electric subalgebra* of $\mathfrak{g}$ (and generically denoted by $\mathfrak{g}_{\mathrm{el}}$). The corresponding group, G_{el}, is an *electric subgroup* of G. The gauge group $\mathrm{G}_{\mathrm{gauge}}$ is a subgroup of the G_{el} such that its action on the vector potentials is the adjoint action. As we will see, there is not a unique maximal electric subgroup of G, and this gives rise to many different gaugings of the supergravity theory.

In the next sections, we will discuss the examples of $N=8,4,$ supergravity from this new point of view.

4 $N = 8$ supergravity

The U-duality group of $N = 8$ supergravity is $\mathrm{G} = \mathrm{E}_{7,7}$ [33], which can be embedded in several ways in $\mathrm{Sp}(56, \mathbb{R})$ (there are 28 vector fields). The electric subalgebras will always be subalgebras of $\mathfrak{g}_{\mathrm{el}} \subset \tilde{\mathfrak{g}}^0 + \tilde{\mathfrak{g}}^{+1}$, being in this case $\tilde{\mathfrak{g}}^0 \approx \mathfrak{sl}(28, \mathbb{R}) + \mathfrak{so}(1,1)_z$.

Consider the following decomposition of $\mathfrak{e}_{7,7}$

$$\mathfrak{e}_{7,7} = \mathfrak{sl}(8, \mathbb{R}) + \mathbf{70}. \tag{2}$$

$\mathfrak{sl}(8, \mathbb{R})$ is a maximal subalgebra of $\mathfrak{e}_{7,7}$ and $\mathbf{70}$ is an irreducible representation of $\mathfrak{sl}(8, \mathbb{R})$, the four-fold antisymmetric.

The representation $\mathbf{56}$ of $\mathfrak{e}_{7,7}$ decomposes under the subgroup $\mathfrak{sl}(8, \mathbb{R})$ as

$$\mathbf{56} \longrightarrow \mathbf{28} + \mathbf{28}',$$

where $\mathbf{28}$ and $\mathbf{28}'$ are two-fold antisymmetric representations of $\mathfrak{sl}(8, \mathbb{R})$.

The embedding $\mathfrak{e}_{7,7} \subset \mathfrak{sp}(56, \mathbb{R})$ is constructed as follows. We have that $\mathfrak{sl}(8, \mathbb{R}) \subset \mathfrak{sl}(28, \mathbb{R})$ by means of the two-fold antisymmetric representation (the $\mathbf{28}$) of $\mathfrak{sl}(8, \mathbb{R})$. With a two-fold antisymmetric tensor we can construct a four-fold antisymmetric tensor by taking the symmetrized tensor product. In this way the generators in the $\mathbf{70}$ of (2) are realized in the two-fold symmetric representation of $\mathfrak{sl}(28, \mathbb{R})$, $\mathrm{sym}(\mathbf{28} \otimes \mathbf{28})$, forming the b matrix of (1), $b_{\{[AB][CD]\}}$. Since in $\mathfrak{sl}(8, \mathbb{R})$ there is an invariant, totally antisymmetric tensor $\epsilon_{A_1 \dots A_8}$, we have another symmetric matrix

$$c^{\{[A_1A_2][A_3A_4]\}} = \frac{1}{4!} \epsilon^{A_1 \dots A_8} b_{\{[A_5A_6][A_7A_8]\}}.$$

This is the standard embedding of $\mathfrak{e}_{7,7}$ in $\mathfrak{sp}(56, \mathbb{R})$. $\mathfrak{sl}(8, \mathbb{R})$ is a maximal electric subalgebra. The gauging of different electric subalgebras of $\mathfrak{sl}(8, \mathbb{R})$ and of its contractions gives rise to all the theories described in [34, 35]. In this choice, $\mathrm{SO}(8)$ is the maximal compact electric subgroup.

We will now consider a different embedding, which is the one relevant for the Scherk–Schwarz mechanism [29]. Consider the decomposition

$$\mathfrak{e}_{7,7} = \mathfrak{e}_{6,6} + \mathfrak{so}(1,1)_k + \mathbf{27}_{-\mathbf{2}} + \mathbf{27}'_{+\mathbf{2}},$$

where $\mathfrak{e}_{6,6} + \mathfrak{so}(1,1)_k$ is a maximal reductive subalgebra. Notice that it is not a maximal subalgebra. In five dimensions the U-duality group is $\mathrm{E}_{6,6}$ and it is totally electric. If we want to see the four dimensional theory as the dimensional reduction of a five dimensional one, this is the natural decomposition to consider, and the $\mathfrak{so}(1,1)_k$ rescales the modulus of the compactification radius of the 5th dimension. The normalization of the generator of $\mathfrak{so}(1,1)_k$ has been chosen in such way that the fundamental representation decomposes as

$$\mathbf{56} \to \mathbf{27_{+1}} + \mathbf{1_{+3}} + \mathbf{27'_{-1}} + \mathbf{1_{-3}}.$$

(Note that the ratio one to three of the $\mathfrak{so}(1,1)_k$ charges is what one obtains for the relative charges of the 27 five-dimensional vectors versus the graviphoton in the standard Kaluza–Klein reduction).

We have that $\mathfrak{e}_{6,6} + \mathfrak{so}(1,1)_k \subset \mathfrak{gl}(28, \mathbb{R})$, by decomposing the fundamental representation

$$\mathbf{28} \to \mathbf{27_{+1}} + \mathbf{1_{+3}}$$

but $\mathfrak{so}(1,1)_\mathrm{k}$ does not correspond to the trace generator $\mathfrak{so}(1,1)_z$ in $\mathfrak{gl}(28, \mathbb{R})$, since all the vectors in the representation $\mathbf{28}$ have the same charge under $\mathfrak{so}(1,1)_z$, $\mathbf{27_z} + \mathbf{1_z}$. Indeed there is another subalgebra $\mathfrak{so}(1,1)_r$ in $\mathfrak{gl}(28, \mathbb{R})$ which commutes with $\mathfrak{e}_{6,6}$. This comes from the sequence of embeddings

$$\mathfrak{e}_{6,6} \subset \mathfrak{sl}(27, \mathbb{R}) \subset \mathfrak{gl}(27, \mathbb{R}) = \mathfrak{sl}(27, \mathbb{R}) + \mathfrak{so}(1,1)_r \subset \mathfrak{sl}(28, \mathbb{R})$$

corresponding to the fact that only 27 of the 28 vectors are transformed by $\mathfrak{e}_{6,6}$. The charges of the 28 vectors are $\mathbf{27_r} + \mathbf{1_{-27r}}$. Then $\mathfrak{so}(1,1)_k$ turns out to be a combination of $\mathfrak{so}(1,1)_z$ and $\mathfrak{so}(1,1)_r$.

In this setting the symplectic embedding of $\mathfrak{e}_{7,7}$ in $\mathfrak{sp}(56, \mathbb{R})$ is different from the standard one previously considered. It was explicitly worked out in [29]. Here $\mathfrak{g}^0 = \mathfrak{e}_{6,6} + \mathfrak{so}(1,1)_k$ is the block diagonal part and

$$\mathfrak{g}_{\mathrm{el}} = \mathfrak{e}_{6,6} + \mathfrak{so}(1,1)_k + \mathbf{27'_{+2}}$$

is lower block-triangular. Then, it is an electric subalgebra.

Note that in order to have a physical theory the unbroken gauge group in the $\mathfrak{g}^0$ part must belong to the maximal compact subgroup of $\mathfrak{g}^0$.

The Scherk–Schwarz [15, 16] mechanism corresponds to the gauging of an electric subgroup (a "flat group") with algebra $\mathfrak{g}_{\mathrm{el}} = \mathfrak{u}(1) \circledS \mathbf{27'_{+2}}$ (semidirect sum), where $\mathfrak{u}(1)$ is a generic element of the Cartan subalgebra of the maximal compact subgroup $\mathfrak{usp}(8)$ of $\mathfrak{e}_{6,6}$ [29]. The gauging of this electric group breaks spontaneously the supersymmetry. Partial breaking is allowed, and the unbroken supersymmetry algebra has a central charge. Central charges are $\mathfrak{u}(1)$ symmetries which belong to the CSA of G, so if they belong to $\mathfrak{g}_{\mathrm{el}}$ they must belong to the maximal compact subalgebra of $\mathfrak{g}^0 \subset \mathfrak{g}_{\mathrm{el}}$. In our case we have one central charge which is identified with the $\mathfrak{u}(1)$ factor in the semidirect sum $\mathfrak{g}_{\mathrm{el}}$.

In fact, the Scherk–Schwarz mechanism allows partial breakings $N = 8 \to N' = 6, 4, 2, 0$, and the spin 3/2 multiplets are 1/2 BPS, which means that only one central charge is present. The number of unbroken translational symmetries in the phases $N' = 6, 4, 2, 0$ is, respectively, 15,7,3,3.

5 $N = 4$ supergravity.

As we are going to see, a richer structure emerges in the $N = 4$ theory because in this case we can have both non abelian and abelian flat groups, depending on the particular model we consider.

Let us consider the $N = 4$ theory with $n_v + 1$ vector multiplets. The U-duality group is $\mathrm{G} = \mathrm{SO}(6, n_v + 1) \times \mathrm{SL}(2, \mathbb{R})$, embedded (in different ways) in $\mathrm{Sp}(2(6 + n_v + 1), \mathbb{R})$, so any electric subalgebra must have block diagonal part a subalgebra $\mathfrak{g}^0 \subset \mathfrak{sl}(6 + n_v + 1, \mathbb{R}) \times \mathfrak{so}(1, 1)_z$.

The standard embedding corresponds to take

$$\mathfrak{so}(6, n_v + 1) + \mathfrak{so}(1, 1)_q \subset \mathfrak{gl}(6 + n_v + 1, \mathbb{R}).$$

Here $\mathfrak{so}(1, 1)_q$ is the Cartan subalgebra of $\mathfrak{sl}(2, \mathbb{R})$ and it is identified with $\mathfrak{so}(1, 1)_z$. The only off-diagonal elements are the other two generators of $\mathfrak{sl}(2, \mathbb{R})$, say $X^{\pm}$. The electric subalgebra is then the lower triangular subalgebra $(\mathfrak{so}(6, n_v + 1) + \mathfrak{so}(1, 1)_q)Ⓢ\{X^+\}$. This embedding appears when doing the compactification of the heterotic string on T^6. (See for example the review of Ref. [36]).

We analyze now another symplectic embedding. We take $n_v = 5$ (the embedding is possible only for $n_v \geq 5$). Then we have the following decomposition

$$\mathfrak{so}(6, 6) = \mathfrak{sl}(6, \mathbb{R}) + \mathfrak{so}(1, 1)_s + \mathbf{15}'^+ + \mathbf{15}^-,$$

where $\mathbf{15}$ is the two-fold antisymmetric representation. Since $\mathfrak{sl}(n) + \mathfrak{sl}(m) \subset \mathfrak{sl}(nm)$, we have that

$$\mathfrak{sl}(6, \mathbb{R}) + \mathfrak{sl}(2, \mathbb{R}) + \mathfrak{so}(1, 1)_s \subset \mathfrak{gl}(12, \mathbb{R}).$$

The representation $(\mathbf{15}', \mathbf{1})$ is symmetric (the singlet of $\mathfrak{sl}(2, \mathbb{R})$ is the two-fold antisymmetric), so we have that $\mathbf{15}'^+ \subset \tilde{\mathfrak{g}}^+ \subset \mathfrak{sp}(24, \mathbb{R})$. This defines the symplectic embedding. The $\mathfrak{so}(1, 1)_s$ is identified with $\mathfrak{so}(1, 1)_z$ of the symplectic algebra.

The representation $\mathbf{12}$ of $\mathfrak{so}(6, 6)$ decomposes, with respect to $\mathfrak{sl}(6, \mathbb{R}) + \mathfrak{so}(1, 1)_z$, as

$$\mathbf{12} \to \mathbf{6_{+1}} + \mathbf{6_{-1}},$$

thus containing six electric and six magnetic fields, and the bifundamental of $\mathfrak{so}(6,6)+\mathfrak{sl}(2,\mathbb{R})$ decomposes as

$$(\mathbf{12},\mathbf{2}) = (\mathbf{6}_{+\mathbf{1}},\mathbf{2})_{\text{electric}} + (\mathbf{6}_{-\mathbf{1}},\mathbf{2})_{\text{magnetic}}.$$

In particular, we see that $\mathfrak{sl}(2,\mathbb{R})$ is totally electric.

The twelve vectors gauge an abelian 12-dimensional subgroup of the $\mathbf{15}'^{+}$ translations.

This model was investigated in Ref. [37], but from our point of view it comes from the general analysis of gauging flat groups. In this case the flat group is completely abelian, since no central charge is gauged. The theory has four independent mass parameters, the four masses of the gravitinos. This allows a partial supersymmetry breaking without cosmological constant from $N=4 \to N'=3,2,1,0$, where the massive gravitinos belong to long (non BPS) massive representations in all the cases, as it is implied by the fact that the central charge is not gauged and the fields are not charged under it.

From the analysis of the consistent truncation of $N=4 \to N'=3$ supergravity, it is known that $N=4$ supergravity coupled to 6 matter vector multiplets can indeed be consistently reduced to an $N=3$ theory coupled to 3 matter multiplets [17]. Correspondingly we have, for the scalar manifolds of such models,

$$\mathrm{SU}(3,3)/(\mathrm{SU}(3)\times\mathrm{SU}(3)\times\mathrm{U}(1)) \subset \mathrm{SO}(6,6)/(\mathrm{SO}(6)\times\mathrm{SO}(6)).$$

In fact, the Higgs effect in this theory needs the gauging of a group of dimension 12, spontaneously broken to a group of dimension 6. The scalar manifolds of the broken phases are

$$\begin{array}{cl}
\mathrm{SU}(3,3)/(\mathrm{SU}(3)\times\mathrm{SU}(3)\times\mathrm{U}(1)), & \text{for} \quad N'=3 \\
(\mathrm{SU}(1,1)/\mathrm{U}(1))\times\mathrm{SU}(2,2)/(\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)), & \text{for} \quad N'=2 \\
(\mathrm{SU}(1,1)/\mathrm{U}(1))^3, & \text{for} \quad N'=1.
\end{array}$$

These models are also analyzed in Ref. [7, 8] from another point of view. There, Type IIB supergravity is compactified on the $T_6/\mathbb{Z}_2$ orientifold with brane fluxes turned on and the same pattern of spontaneous symmetry breaking is found.

6 Translational symmetries, Goldstone bosons and Higgs effect

When the manifold parametrized by the scalars is a coset space G/H, there is an abelian algebra of isometries that is contained in G. This algebra is the maximal abelian ideal of the solvable algebra associated to the coset space via the Iwasawa decomposition (see for example [39]). In the examples that we analyze in this section we have two models with two coset spaces, G/H corresponding to the unbroken supersymmetry model and G'/H' corresponding to the model with partial breaking of supersymmetry once the massive modes have been integrated out. We will denote by $\mathfrak{t}(G/H)$ and $\mathfrak{t}'(G'/H')$ the abelian subalgebras associated to the respective cosets (here the "$\mathfrak{t}$" stands for translational). t and t' are respectively the dimensions of these subalgebras.

If n_v and n'_v denote the number of massless vectors in each theory, we find in all the models analyzed that $t - t' = n_v - n'_v$. The solvable group obtained in the Iwasawa decomposition (now in the group instead that in the algebra) is diffeomorphic as a manifold to G/H. This parametrization has been considered in the literature to analyze U-dualities in string theory [40, 41, 33]. The generators of the maximal abelian ideal act as translations on t of the coordinates of G/H, which appear only through derivatives in the Lagrangian and which are flat directions of the scalar potential. This suggests that, as a general rule for a consistent Higgs effect, these particular coordinates are the Goldstone bosons connected to the spontaneous breaking of $\mathbb{R}^{n_v}$ to $\mathbb{R}^{n'_v}$, so they have been absorbed into the vectors that have acquired mass.

We first analyze first that are obtained with the Scherk-Schwarz mechanism. In these cases one can prove that the above considerations are actually valid [17]. It would be interesting to know the cases where this rule does not hold.

$N = 8 \rightarrow N' = 6$. The coset space of the scalars in $N = 8$ supergravity is $\mathrm{E}_{7,7}/\mathrm{SU}(8)$ and the dimension of the translational subalgebra is $t = 27$ [40]. In $N' = 6$ the coset is $\mathrm{SO}^*(12)/\mathrm{U}(6)$, and $t' = 15$. So $t - t' = 12$. It is easy to see that $n_v - n'_v = 28 - 16 = 12$.

$N = 8 \rightarrow N' = 2$. For $N' = 2$ we have a certain number of vector multiplets (n_1) and hypermultiplets (n_2).

The minimal model($m_i \neq m_j,\ i, j = 2, 3, 4$ in the notation of Ref. [15, 16] corresponds to $n_1 = 3$ (we take $n_2 = 0$), and the coset is

$$\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}.$$

$t' = 3$, so $t - t' = 27 - 3 = 24$. We have that $n_v - n'_v = 28 - 4 = 24$.

The maximal model ($m_1 = m_2 = m_3$) corresponds to having $n_1 = 9$, (again we take $n_2 = 0$). The coset space is

$$\frac{\mathrm{SU}(3,3)}{\mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{U}(1)}.$$

In this case $t - t' = 27 - 9 = 18$ and $n_v - n_v' = 28 - 10 = 18$.

The examples that follow can be obtained in Type IIB superstring compactified on an orientifold $T^6/\mathbb{Z}_2$ in presence of brane fluxes [7, 8] and in certain gauged supergravity theories [37, 38]. We want to consider the spontaneous breaking of $N = 4, 3$ supergravities down to $N' = 3, 2$. In order to have a consistent reduction it is necessary that the scalar manifold of the broken theory is a submanifold of the unbroken one [42]. For the $N' = 2$ case this is just an assumption since the effects of integrating out the massive modes could be more complicated.

$N = 4 \to N' = 3$. We consider the $N = 4$ model with six massless vector multiplets,

$$\frac{\mathrm{SO}(6,6)}{\mathrm{SO}(6) \times \mathrm{SO}(6)} \times \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)},$$

with $t = 15$. Note that the SU(1,1) factor is not considered because SU(3,3)⊂SO(6,6) The decomposition of the massless multiplets is

$$\begin{aligned}&\left[(2), 4(\frac{3}{2}), 6(1), 4(\frac{1}{2}), 2(0)\right] + 6\left[(1), 4(\frac{1}{2}), 6(0)\right] \to \\ &\qquad \left[(2), 3(\frac{3}{2}), 3(1), 1(\frac{1}{2})\right] + 6\left[(1), 4(\frac{1}{2}), 6(0)\right]\end{aligned}$$

In $N = 3$ the long spin 3/2 multiplet is formed by adding 3 massless vector multiplets to the $\lambda_{MAX} = 3/2$ multiplets. There remain three massless vector multiplets. The scalar manifold of the theory is

$$\frac{\mathrm{SU}(3,3)}{\mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{U}(1)}$$

with $t' = 9$. So we have $t - t' = 15 - 9 = 6$ an $n_v - n_v' = 12 - 6 = 6$.

$N = 3 \to N' = 2$. We start with the $N = 3$ model with 3 vector multiplets as above. The decomposition of the graviton multiplet is

$$\big[(2), 3(\frac{3}{2}), 3(1), (\frac{1}{2})\big] \to \big[(2), 2(\frac{3}{2}), (1)\big] + \big[(\frac{3}{2}), 2(1), (\frac{1}{2})\big]$$

and the decomposition of the massless vector multiplet is

$$\big[(1), 4(\frac{1}{2}, 6(0))\big] \to \big[(1), 2(\frac{1}{2}), 2(0)\big] + \big[2(\frac{1}{2}), 4(0)\big].$$

To form a long spin 3/2 multiplet we need two massless vector multiplets and one hypermultiplet. The residual theory has then one vector multiplet and two hypermultiplets. The coset is then

$$\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(2,2)}{\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)}$$

with $t' = 5$. So we have $t - t' = 9 - 5 = 4$ and $n_v - n'_v = 6 - 2 = 4$.

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