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Noncommutativity in linear dilaton background

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Abstract

We consider quantization of open string theories in linear dilaton and constant anti-symmetric tensor backgrounds and discuss the noncommutativity of space-time coordinates arising in such theories, including their relationship with light-like noncommutativity as well as backgrounds with null isometries. It is argued that the results can also be understood using space-time equations of motion of the string modes. We then present $N = 2$ supersymmetric generalization of these theories and the associated noncommutative structure.

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The study of string propagation in background fields has been an active area of research for a long time [1]. One such consistent set of background includes a general target space metric, antisymmetric tensor field and a linear dilaton. String theories with linear dilaton background [2] are interesting from the point of view of D-branes [3, 4], as well as conformal field theory (CFT) using their free field realizations. They also play a role in understanding the cosmological implications of string theory[5] and their applications to quantum gravity, through the constructions of noncritical string theories [6, 7, 8, 9, 10], where the role of 2-dimensional gravity becomes important. Thus such theories are important to study, in order to have a better understanding of gravity in general, and also in constructing interesting physical systems with nontrivial quantum gravity effects. Recently, D-branes in linear dilaton background have been studied, where the corresponding Dirichlet boundary state has been constructed [3, 4]. The aim of this paper is to further study the linear dilaton background in the context of D-branes. In particular, our aim is to analyze the non-commutative structure [11] arising in such background when a constant antisymmetric tensor field is turned on along these D-branes.

We now consider open strings in d dimensional space-time in the presence of a constant Neveu-Schwarz antisymmetric tensor field B and a dilaton Φ. The world sheet action is given by [12],

$$\begin{aligned} S &= \frac{1}{4\pi\alpha'}\int_{\Sigma} d^2z[\sqrt{\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} + \epsilon^{ab}\partial_a X^\mu \partial_b X^\nu B_{\mu\nu} - \frac{1}{2}\alpha'\sqrt{\gamma}R^{(2)}\Phi(X)] - \\ & \frac{1}{4\pi\alpha'}\oint_{\partial\Sigma} d\tau[\alpha' k\Phi(X)]. \end{aligned} \quad (1)$$

Here we have taken the background gauge fields to be zero. $R^{(2)}$ is the two-dimensional curvature scalar and k is the extrinsic curvature of the boundary. We take the world sheet metric to be Minkowskian and $\epsilon^{01} = 1$. The d dimensional index μ runs from $0, \ldots, d-1$. This world sheet action in conformal gauge reduces to,

$$S_0 = \frac{1}{4\pi\alpha'}\int d^2z\left[g_{\mu\nu}\partial_a X^\mu \partial^a X^\nu + \epsilon^{ab}B_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + \alpha'\partial_a\rho\nabla_\mu\Phi\partial^a X^\mu\right]. \quad (2)$$

where, ρ is the conformal mode of the metric. We assume that the dilaton Φ is of the form $\Phi = Q_\mu X^\mu$, i.e., a linear function of the coordinates where Q is the background charge. One also takes $Q^2 \equiv Q_\mu Q^\mu = \frac{(26-d)}{24\alpha'}$, in order to cancel the conformal anomaly in the quantized theory. We will further restrict ourselves to the gauge $\rho = 0$. One can then directly apply the procedure of [13] to quantize the theory. Equations of motion, boundary conditions, as well as the canonical momenta remain unchanged with respect to the case when $Q_\mu = 0$ and $d = 26$. However, since the energy-momentum tensor acquires an extra term due to the coupling of the dilaton to the worldsheet curvature scalar, the Virasoro generators are also modified accordingly with terms depending on the background charge Q.

The variation of the action gives the boundary conditions at $\sigma = 0, \pi$ as

$$\partial_\sigma X^\mu + F^\mu_\nu \partial_\tau X^\nu|_{\sigma=0,\pi} = 0, \tag{3}$$

and from the equations of motion, the general solutions for the string coordinates X^μ are obtained as,

$$X^\mu = x^\mu_0 + (p^\mu_0 \tau - p^\nu_0 F^\mu_\nu \sigma) + \sum_{n\neq 0} \frac{e^{-in\tau}}{n}(ia^\mu_n \cos n\sigma - a^\nu_n F^\mu_\nu \sin n\sigma). \tag{4}$$

In flat Minkowski space ($g_{\mu\nu} = \eta_{\mu\nu}$), one has $F^\nu_\mu \equiv B^\nu_\mu$. Also, the mode expansion for the canonical momenta can be written in a form:

$$2\pi\alpha' P^\mu(\tau, \sigma) = G^\mu_\nu(a^\nu_0 + \sum_{n\neq 0} a^\nu_n e^{-in\tau} \cos n\sigma), \tag{5}$$

with G being the open string metric given by,

$$G = g - Bg^{-1}B. \tag{6}$$

The noncommutative properties and the canonical commutation relations in this theory are identical to the ones in [13] and are described by the following relations :

$$\begin{aligned}
[x^\mu_0, x^\nu_0] &= i\pi\alpha' \left[(G^{-1})^{\mu\rho} F^\nu_\rho - (G^{-1})^{\nu\rho} F^\mu_\rho\right], \\
[x^\mu_0, p^\nu_0] &= 2i\alpha'(G^{-1})^{\mu\nu}, \\
[a^\mu_m, a^\nu_n] &= 2\alpha' m (G^{-1})^{\mu\nu}\delta_{m+n,0} \\
[a^\mu_m, x^\nu_0] &= 0, \\
[a^\mu_m, p^\nu_0] &= 0.
\end{aligned} \tag{7}$$

To write down the Virasoro generators, one starts with the expression for the components of the worldsheet energy-momentum tensor. It is useful to express quantities in complex coordinates z and $\bar{z}$, where the world sheet of the open string theory corresponds to the upper half plane, Im $z > 0$. The energy momentum tensor can then be written as,

$$\begin{aligned}
T_{zz} &= \frac{1}{4\alpha'} g_{\mu\nu} \partial X^\mu \partial X^\nu + Q_\mu \partial^2 X^\mu, \\
T_{\bar{z}\bar{z}} &= \frac{1}{4\alpha'} g_{\mu\nu} \bar{\partial} X^\mu \bar{\partial} X^\nu + Q_\mu \bar{\partial}^2 X^\mu.
\end{aligned} \tag{8}$$

Using the mode expansions for the coordinates as in equation (4), one can expand T_{zz}, $T_{\bar{z}\bar{z}}$ as:

$$
\begin{aligned}
T_{zz} &= \sum_m \left(\frac{1}{4\alpha'} \sum_n G_{\mu\nu} a^\mu_{m-n} a^\nu_n + i(m+1) Q^\mu (g+B)_{\mu\nu} a^\nu_m \right) z^{-m-2}, \\
T_{\bar{z}\bar{z}} &= \sum_m \left(\frac{1}{4\alpha'} \sum_n G_{\mu\nu} a^\mu_{m-n} a^\nu_n + i(m+1) Q^\mu (g-B)_{\mu\nu} a^\nu_m \right) \bar{z}^{-m-2}.
\end{aligned} \tag{9}
$$

One can also express them in an alternative form of the mode expansion, where the Q-dependent terms appear with coefficient m rather than $(m+1)$. They can, however, be obtained from the above expressions by a shift proportional to Q^μ on α^μ_0.

Moreover, in open string theories, the conformal invariance condition on the boundary implies, a matching condition, $T_{zz} = T_{\bar{z}\bar{z}}|_{z=\bar{z}}$ (Im $z = 0$), which is derived from the requirement that there is no net flow of energy and momentum from the boundary. In our case, this constraint is satisfied by imposing the condition

$$Q^\mu B_{\mu\nu} = 0. \tag{10}$$

Also, using relations (7), it can be verified that T_{zz} as well as $T_{\bar{z}\bar{z}}$ satisfy conformal algebra with central charge $C = d + 24\alpha' Q^2$.

There are several solutions for the above condition between the background charges Q_μ's and the antisymmetric tensor components $B_{\mu\nu}$'s. First, if the directions along which Q_μ's are turned on, are orthogonal to the ones which have constant antisymmetric tensor couplings, conditions (10) are trivially satisfied. More interesting solutions arise when the electric and magnetic components of the B field are related as in the case of light-like noncommutativity [14]. The above condition on the background charge then implies that Q_μ is also a light-like vector. Theories with light-like noncommutativity are known to be unitary [15] and have a nice decoupling limit. In particular, the light-like noncommutativity condition reads $B_{0i} = B_{1i} \neq 0$. Noncommutativity with parallel electric and magnetic field components has also been studied in ref. [16]. Let us just consider the case where B_{12} as well as B_{02} components are nonzero. Then the light-like noncommutativity condition, eqn. (10), implies that the two corresponding background charges satisfy the condition $Q_0 = Q_1$. The dilaton is then proportional to both the time as well as space coordinates, *i.e.* $\Phi = Q_0(X^0 + X^1)$. Similar type of dilaton backgrounds have been considered before in [17]. They also appear in asymptotic limits of space-time backgrounds in various gauged WZW models. In our case, they belong to the class of space-time backgrounds with null isometries[18], and are known to be classical solutions of string theories to all orders in α'.

It remains to be seen whether the condition (10) is also a necessary one to define a consistent quantum theory. In other words, it may be interesting to understand whether the matching condition at the boundary given above can be treated as a constraint in the quantization process. In that case, equation (10) will be replaced by an appropriate physical state conditions.

We also note that the mode expansions for T_{zz} and $T_{\bar{z}\bar{z}}$, given above, are related by $B \to -B$ transformation. Such asymmetry in the form of fields and operators occurs in open string theory, even in the constant dilaton case. For example, in the case of NSR superstring in constant B background, the mode expansions of left and right-handed Neveu-Schwarz and Ramond fermion fields are obtained as [19],

$$\begin{aligned} \psi^i(z) &= \sum_r \left[g^{-1}(g - 2\pi\alpha' B)\right]^i_j b^j_r z^{-r-\frac{1}{2}}, \\ \bar{\psi}^i(\bar{z}) &= \sum_r \left[g^{-1}(g + 2\pi\alpha' B)\right]^i_j b^j_r \bar{z}^{-r-\frac{1}{2}}, \end{aligned} \tag{11}$$

where r is an integer (half-integer) in R-sector (NS-sector). Operators ∂X^μ, $\bar{\partial} X^\mu$ also have similar mode expansions[19] and correspond to the worldsheet conserved currents for the translations in the left and the right-moving sector. The worldsheet fields ψ, $\bar{\psi}$ etc. appear in vertex operators for vector fields in superstring theory [19] as a linear combination $(\psi^\mu + \bar{\psi}^\mu)e^{ik.X}$ of the left and right-moving components. For the case of Virasoro generators, however, there is an extra condition on the form of B's and Q_μ's originating from the matching condition on the boundary. One can then use the doubling trick where T_{zz} becomes holomorphic in the whole complex plane and there is then one set of Virasoro generators, given by the coefficients of the mode expansions in equation (9).

We now discuss the existence of consistent open string theory in constant B and linear dilaton backround by using certain space-time equations of motion. In particular, the equation of motion for the tachyon in the open string sector can be written in these backgrounds as [20, 12, 1],

$$(g - Bg^{-1}B)^{-1}_{\mu\nu}\nabla^\mu\nabla^\nu\Theta - \nabla^\mu\Phi\nabla_\mu\Theta + \frac{1}{\alpha'}\Theta = 0. \tag{12}$$

This is the tachyon β-function equation obtained by considering string theories in nontrivial backgrounds[12, 1]. However, one notices that for constant g, B and a linear dilaton

Φ, this equation can be reinterpreted in terms of a modification to the energy spectrum. For example, assuming a form like $\Theta = e^{ik\cdot x}$ for the open string tachyon wave function, one obtains a condition from equation (12) such as,

$$G^{\mu\nu}k_\mu k_\nu + ik_\mu Q^\mu - \frac{1}{\alpha'} = 0, \tag{13}$$

with $G = g - Bg^{-1}B$ being the open string metric. The left hand side in equation (13) can be identified with the conformal weight that appears using the Virasoro generators we obtained above and confirms the validity of our construction.

To continue with the analogy, we notice that the equation of motion for the gauge field 'fluctuations' in the background of constant g and B, and linear dilaton can be written (by dropping interaction terms) in the form[12, 1],

$$(g - Bg^{-1}B)^{-1}_{\lambda\nu}\nabla^\nu f^\lambda_\mu + \nabla^\nu \Phi f_{\nu\mu} = 0, \tag{14}$$

provided the condition (10) once again holds. Now, assuming the wave function for the gauge field fluctuation to be of the form: $a_\mu = \epsilon_\mu e^{ik\cdot X}$, we obtain the mass-shell condition as,

$$\tilde{h} = G^{\mu\nu}k_\mu k_\nu + ik^\mu Q_\mu = 0 \tag{15}$$

and the physical state condition reads as,

$$(k_\mu G^{\mu\nu} + Q^\nu)\epsilon_\nu = 0. \tag{16}$$

Using the expressions of the conformal generators mentioned above, we then reproduce the expressions for the conformal weight ($\tilde{h}$) in (15), as well as the physical state condition (16). We find it interesting to observe that the condition (10) appears from the point of view of CFT, as well as in the study of the space-time equations of motion. We also mention that, although the above set of equations, (12)-(16), for string modes is derived from the string effective action in commutative quantization scheme, one can argue that by restricting to the linearized fluctuations and ignoring interaction terms, one gets identical conditions in noncommutative framework as well.

We now discuss $N = 2$ (worldsheet) supersymmetric[21, 22] extension of our results in the critical string theory case. This analysis is of importance for studying the supersymmetric version of D-branes with linear dilaton background as well as in their constructions using $N=2$ minimal models [23]. To present such an extension of the above analysis, we now give the representation of $N=2$ superconformal field theories using free bosonic and fermionic fields having mode expansions of the type (4), (11), and superconformal generators are the generalizations of the ones appearing in (8, 9). We write down explicit expressions for the superconformal generators and show that they satisfy the $N=2$ operator algebra. To be specific, in this part of the paper, we first restrict ourselves to the case when g, B etc.are 2×2 matrices. We then start with the 2-point function for bosons ϕ^i and fermions ψ^i, $(i=1,2)$ that can be obtained by turning on constant B. By restricting ourselves to the boundary, these 2-point functions are:

$$\begin{aligned} \langle\phi^i(\tau_1)\phi^j(\tau_2)\rangle &= -G^{ij}ln(\tau_1-\tau_2)^2+\theta^{ij}\epsilon(\tau_1-\tau_2), \\ \langle\psi^i(\tau_1)\psi^j(\tau_2)\rangle &= -\frac{G^{ij}}{(\tau_1-\tau_2)}. \end{aligned} \tag{17}$$

with $G=g-Bg^{-1}B$, g now being a two dimensional identity matrix and $B_{ij}=b\epsilon_{ij}$. θ_{ij} are the noncommutativity parameters: $\theta_{ij}=\theta\epsilon_{ij}$, with $\theta=-\frac{b}{1+b^2}$. Now following [22], we work in a complex notation, where,

$$\begin{aligned} &A=\phi^1+i\phi^2, \qquad \bar{A}=\phi^1-i\phi^2, \\ &\psi=\psi^1-i\psi^2, \qquad \bar{\psi}=\psi^1+i\psi^2. \end{aligned} \tag{18}$$

The boundary two point functions are then given by,

$$\begin{aligned} \langle A(\tau_1)\bar{A}(\tau_2)\rangle &= -\frac{2}{1+b^2}ln(\tau_1-\tau_2)^2+\theta\epsilon(\tau_1-\tau_2), \\ \langle \bar{A}(\tau_1)A(\tau_2)\rangle &= -\frac{2}{1+b^2}ln(\tau_1-\tau_2)^2-\theta\epsilon(\tau_1-\tau_2), \end{aligned} \tag{19}$$

and

$$\langle\psi(\tau_1)\bar{\psi}(\tau_2)\rangle = -\frac{1}{(1+b^2)(\tau_1-\tau_2)} = \langle\bar{\psi}(\tau_1)\psi(\tau_2)\rangle. \tag{20}$$

We now write down the expression for the superconformal generators, by extending the expressions for the conformal generators written earlier in equation (8), (9) to the full $N=2$ superconformal symmetry. We give these expressions in terms of conformal fields, rather than their oscillator modes. They can, however, be mapped into each other. The full set of generators in $N=2$ theories consist of the $U(1)$ generators J, two superconformal generators $G,\bar{G}$ and the conformal generator T. Their conformal weights are $1,\frac{3}{2}$ and 2

respectively. In our case, they have a form:

$$\begin{aligned}
J &= \frac{1}{2}[(1+b^2)\bar{\psi}\psi + (1+ib)\bar{\beta}\partial_\tau A - (1-ib)\beta\partial_\tau \bar{A}], \\
G &= \frac{i}{\sqrt{2}}(1+b^2)\psi\partial_\tau A - 2\sqrt{2}i\beta(1-ib)\partial_\tau\psi, \\
\bar{G} &= -\frac{i}{\sqrt{2}}(1+b^2)\bar{\psi}\partial_\tau \bar{A} + 2\sqrt{2}i\bar{\beta}(1+ib)\partial_\tau\bar{\psi}, \\
T &= -\frac{1}{4}(1+b^2)\partial_\tau A\partial_\tau \bar{A} - \frac{1}{2}(1+b^2)\partial_\tau\bar{\psi}\psi + \frac{1}{2}(1+b^2)\bar{\psi}\partial_\tau\psi + \\
&\quad \frac{\bar{\beta}}{2}(1+ib)\partial_\tau^2 A + \frac{\beta}{2}(1-ib)\partial_\tau^2\bar{A}.
\end{aligned} \tag{21}$$

Here β and $\bar{\beta}$ are complex parameters. By writing down A, $\bar{A}$ etc. using oscillator mode expansions, such as the ones in (4), and restricting to the boundary at $z = \bar{z}$, one obtains the mode expansions of the above generators which are the generalizations of the ones appearing in equation (8, 9) to the supersymmetric case. To verify that operators in equation (21) satisfy $N = 2$ algebra, one uses the two point functions in equations (19), (20). The parameter θ, appearing in these expressions does not play a role in verifying the $N = 2$ superconformal algebra which involves a time ordering of the operators on the boundary. Moreover, since Wick contraction in this context always involves at least one τ derivative, this term drops out from the computation.

We also mention that the generators in equation (21) have been obtained from the reduction of the holomorphic part of the superconformal generators to the boundary. As mentioned before, if one starts with the antiholomorphic generators, one gets expressions which are related to the ones in (21) by a transformation: $B \to -B$.

In fact this construction of $N = 2$ generators can be extended to several copies of bosonic fields $A^I, \bar{A}^{\bar{I}}$ and fermionic fields $\psi^I, \bar{\psi}^{\bar{I}}$ [‡]. The non-vanishing components of the (closed string) metric and antisymmetric tensors are: $g_{I\bar{J}}$ and $B_{I\bar{J}}$. $N = 2$ generators can be written for these cases by generalizing the expressions in equation (21) to include indices $I, \bar{I}$ over the fermions and bosons. At the same time the expression $(1+b^2)$ and $(1 \pm ib)$ appearing in equation (21) are replaced by : $G_{I\bar{J}}$ and $(g_{I\bar{J}} \pm iB_{I\bar{J}})$ respectively, with $G_{I\bar{J}}$, now being the 'open' string metric. The matching of the $N = 2$ generators on the boundary once again gives a condition involving B, and background charges $\beta, \bar{\beta}$, representing the linear dilaton in the present case: $\beta^I B_{I\bar{J}} = 0$, $\bar{\beta}^{\bar{I}} B_{\bar{I}J} = 0$.

[‡]the indices on ψ and $\bar{\psi}$ correspond to their definitions in equation (18)

We have therefore given the constructions of noncommutative open string theories in linear dilaton and constant antisymmetric tensor backgrounds. We also note that the classical action (2) discussed above, appears in general in the context of noncritical string theories[6, 7, 24, 25], where 2-d gravity couples to the matter. In view of our analysis, it will be interesting to investigate non-commutativity in these cases as well. We end with a comment that the relationship with light-like noncommutativity, as found above, deserves further study. In particular, one would also like to understand the decoupled field theory limit [14, 16, 26, 27] in the above case and the corresponding dual supergravity descriptions, as has been discussed in other theories with light-like noncommutativity [28]. It may also be interesting to understand our results in the context of little string theories [29] where the dual theory is a theory without gravity in lower dimension.

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