SPhT-T03/052

Massless N=1 Super Sinh-Gordon: Form Factors approach

Bénédicte Ponsot [1]

Service de Physique Théorique, Commissariat à l'énergie atomique, L'Orme des Merisiers, F-91191 Gif sur Yvette, France.

Abstract

The $N=1$ Super Sinh-Gordon model with spontaneously broken supersymmetry is considered. Explicit expressions for form-factors of operators $e^{\alpha\phi}$ of the Neveu-Schwartz sector and operators $\sigma e^{\alpha\phi}$, $\mu e^{\alpha\phi}$ of the Ramond sector are proposed.

PACS: 11.25.Hf, 11.55.Ds

1 Introduction

The SShG model can be considered as a perturbed super Liouville field theory, which lagrangian density is given by

$$\mathcal{L} = \frac{1}{8\pi}(\partial_a\phi)^2 - \frac{1}{2\pi}(\bar{\psi}\partial\bar{\psi} + \psi\bar{\partial}\psi) + i\mu b^2\psi\bar{\psi}e^{b\phi} + \frac{\pi\mu^2 b^2}{2}e^{2b\phi}.$$

with the background charge $Q = b + 1/b$. This model is a CFT with central charge

$$c_{SL} = \frac{3}{2}(1 + 2Q^2).$$

The super Sinh-Gordon model is 1+1 dimensional integrable quantum field theory with $N = 1$ supersymmetry. We consider the Lagrangian

$$\mathcal{L} = \frac{1}{8\pi}(\partial_a\phi)^2 - \frac{1}{2\pi}(\bar{\psi}\partial\bar{\psi} + \psi\bar{\partial}\psi) + 2i\mu b^2\psi\bar{\psi}\sinh b\phi + 2\pi\mu^2 b^2\cosh^2 b\phi.$$

[1] ponsot@spht.saclay.cea.fr

In this model the supersymmetry is spontaneously broken [1]: the bosonic field becomes massive, but the Majorana fermion stays massless and plays the role of Goldstino. In the IR limit, the effective theory for the Goldstino is to the lowest order the Volkov-Akulov lagrangian [2]

$$\mathcal{L}_{IR} = (\bar{\psi}\partial\bar{\psi} + \psi\bar{\partial}\psi) - \frac{4}{M^2}(\psi\partial\psi)(\bar{\psi}\bar{\partial}\bar{\psi}) + \cdots \tag{1}$$

where supersymmetry is realized non linearly. The irrelevant operator along which the Super Liouville theory flows into Ising is the product of stress-energy tensor $T\bar{T} = (\psi\partial\psi)(\bar{\psi}\bar{\partial}\bar{\psi})$, which is the lowest dimension non derivative operator allowed by the symmetries. The dots include higher dimensional irrelevant operators.
The scattering in the left-left and right-right subchannels is trivial, but not in the right-left channel. The following scattering matrices were proposed in [1]

$$S_{RR}(\theta) = S_{LL}(\theta') = -1,\ S_{RL}(\theta - \theta') = -\frac{\sinh(\theta - \theta') - i\sin\pi\nu}{\sinh(\theta - \theta') + i\sin\pi\nu}, \quad \nu \equiv b/Q.$$

For the right (left) movers the energy momentum is parametrized in terms of the rapidity variable θ (θ') by $p^0 = p^1 = \frac{M}{2}e^{\theta}$ (and $p^0 = -p^1 = \frac{M}{2}e^{-\theta'}$). The mass scale of the theory M^{-2} is equal to $2\sin\pi\nu$. The form factors[2] $F_{r,l}(\theta_1, \theta_2, \ldots, \theta_r; \theta'_1, \theta'_2, \ldots, \theta'_l)$ are defined to be matrix elements of an operator between the vacuum and a set of asymptotics states. The form factor bootstrap approach [3–5] (developed originally for massive theories, but that turned out to be also an effective tool for massless theories [6, 7]) leads to a system of linear functional relations for the matrix elements $F_{r,l}$; let us introduce the minimal form factors which have neither poles nor zeros in the strip $0 < \Im m\theta < \pi$ and which are solutions of the equations $f_{\alpha_1\alpha_2}(\theta) = f_{\alpha_1\alpha_2}(\theta + 2i\pi)S_{\alpha_1\alpha_2}(\theta),\ \alpha_i = R, L$.
Then the general form factor is parametrized as follows:

$$F^{\alpha}_{r,l}(\theta_1, \theta_2, \ldots, \theta_r; \theta'_1, \theta'_2, \ldots, \theta'_l) = \prod_{1\le i<j\le r} f_{RR}(\theta_i - \theta_j) \prod_{i=1}^{r}\prod_{j=1}^{l} f_{RL}(\theta_i - \theta'_j) \prod_{1\le i<j\le l} f_{LL}(\theta'_i - \theta'_j) Q^{\alpha}_{r,l}(\theta_1, \theta_2, \ldots \theta_r; \theta'_1, \theta'_2, \ldots, \theta'_l),$$

and the function $Q^{\alpha}_{r,l}$ depends on the operator considered through the parameter α [3].
The RR and LL scattering formally behave as in the massive case, so annihilation poles occur *only* in the RR and LL subchannel. This leads to the residue formula

$$\mathrm{Res}_{\theta_{12}=i\pi} F^{\alpha}_{r,l}(\theta_1, \theta_2, \ldots, \theta_r; \theta'_1, \theta'_2, \ldots, \theta'_l) = 2F^{\alpha}_{r-2,l}(\theta_3, \ldots, \theta_r; \theta'_1, \theta'_2, \ldots, \theta'_l)\left(1 - \prod_{j=3}^{r} S_{RR}(\theta_{2i}) \prod_{k=1}^{l} S_{RL}(\theta_2 - \theta'_k)\right), \tag{2}$$

and a similar expression in the LL subchannel. It is important to note that these equations *do not* refer to any specific operator.

[2] We refer the reader to [6] for a discussion on form factors in massless QFT.
[3] The meaning of α will be clear in the next sections.

2 Expression for form factors

The minimal form factors read explicitly:

$$f_{RR}(\theta) = \sinh\frac{\theta}{2}, \quad f_{LL}(\theta') = \sinh\frac{\theta'}{2},$$

and

$$f_{RL}(\theta) = \frac{1}{2\cosh\frac{\theta}{2}} \quad \exp\int_0^\infty \frac{dt}{t}\frac{\cosh(\frac{1}{2}-\nu)t - \cosh\frac{1}{2}t}{\sinh t\cosh t/2}\cosh t\left(1-\frac{\theta}{i\pi}\right).$$

The latter form factor has asymptotic behaviour when $\theta \to -\infty$

$$f(\theta) \sim e^{\theta/2}\left(1 + (A + A'\theta)\, e^\theta + \left(\frac{A^2}{2} + B + AA'\theta + \frac{(A')^2\theta^2}{2}\right) e^{2\theta}\right). \tag{3}$$

where $A = (1-2\nu)\cos\pi\nu - 1 + 2i\sin\pi\nu,\ A' = -\frac{2}{\pi}\sin\pi\nu,\ B = \frac{1}{2}(\cos 2\pi\nu - 1)$. The logarithmic contributions come from resonances.

The residue condition (2) written in terms of the function $Q^\alpha_{r,l}$ reads

$$\begin{aligned}\mathrm{Res}_{\theta_{12}=i\pi} Q^\alpha_{r,l}(\theta_1,\theta_2,\dots\theta_r;\theta'_1,\theta'_2,\dots,\theta'_l) &= Q^\alpha_{r-2,l}(\theta_3,\dots\theta_r;\theta'_1,\theta'_2,\dots,\theta'_l)\times(-)^{r-1}(2i)^{l+r-1}\times \\ \times\prod_{j=3}^r \frac{1}{\sinh\theta_{2j}}\left(\prod_{k=1}^l(\sinh(\theta_2-\theta'_k)+i\sin\pi\nu) - (-1)^{r+l}\prod_{k=1}^l(\sinh(\theta_2-\theta'_k)-i\sin\pi\nu)\right).\end{aligned} \tag{4}$$

Let us introduce now the functions

$$\phi(\theta_{ij}) \equiv \frac{S_{RR}}{f_{RR}(\theta_{ij})f_{RR}(\theta_{ij}+i\pi)} = \frac{2i}{\sinh\theta_{ij}}, \quad \phi(\theta'_{ij}) \equiv \frac{S_{LL}}{f_{LL}(\theta'_{ij})f_{LL}(\theta'_{ij}+i\pi)} = \frac{2i}{\sinh\theta'_{ij}}.$$

as well as

$$\Phi(\theta_i - \theta'_j) \quad \equiv \frac{S_{RL}(\theta_i-\theta'_j)}{f_{RL}(\theta_i-\theta'_j)f_{RL}(\theta_i-\theta'_j+i\pi)} = -\frac{\sinh(\theta_i-\theta'_j) - i\sin\pi\nu}{2i},$$

and

$$\tilde\Phi(\theta_i-\theta'_j) \equiv \frac{\sinh(\theta_i-\theta'_j)+i\sin\pi\nu}{2i}.$$

We assign odd Z_2-parity to both right and left-movers ($\psi_R \to -\psi_R,\quad \psi_L \to -\psi_L,\quad \phi\to\phi$) and even (odd) parity to right (left) movers under duality transformations ($\psi_R \to \psi_R,\quad \psi_L \to -\psi_L,\quad \phi \to -\phi$). In the Neveu-Schwartz sector (which consists of local operators), the operators $\cosh\alpha\phi$ and $\psi\bar\psi\sinh\alpha\phi$ have non zero matrix elements on (even,even) number of particles, whereas $\sinh\alpha\phi$ and $\psi\bar\psi\cosh\alpha\phi$ on (odd,odd) number of particles. Let us remind [8] that the Volkov-Akulov formalism is expressed in terms of a constraint superfield $\Phi(z,\bar z) = \phi(z,\bar z) + \theta\psi(z) + \bar\theta\bar\psi(\bar z) + \theta\bar\theta F(z,\bar z)$, satisfying $\Phi^2 = 0$, and for which a solution is $\phi = (1/F)\bar\psi\psi$. Hence, from the fusion rules of the critical Ising model [9] we deduce that the first set of operators renormalizes on the family of the identity and the second set on the family of the energy ϵ. As $\psi\bar\psi e^{\alpha\phi}$ is the descendant field by supersymmetry of the primary $e^{\alpha\phi}$, its form factors differ from those of $e^{\alpha\phi}$ by a multiplicative factor that does not affect the bootstrap equations and that has correct behaviour under Lorentz transformations: $F^{\psi\bar\psi e^{\alpha\phi}} = \left(\sum e^{\theta_i}\right)^{1/2}\left(\sum e^{-\theta'_j}\right)^{1/2} F^{e^{\alpha\phi}}$.
In the Ramond sector, the non local operators $\sigma e^{\alpha\phi}$ and $\mu e^{\alpha\phi}$ renormalize on the family of the spin field and disorder field respectively.

2.1 Neveu-Schwarz sector: operators $e^{\alpha\phi}$

2.1.1 Form factors of the operator $\cosh\alpha\phi$.

Because of symmetry under spin and parity, $\cosh\alpha\phi$ has non vanishing matrix elements for an even number of left and right movers. We introduce the sets $S = (1,\ldots,2r), S' = (1,\ldots,2l)$ and the first form factor[4] $Q^\alpha_{2,2} = \left(\frac{\sin\frac{\pi\nu\alpha}{b}}{\sin\pi\nu}\right)^2$. We propose

$$Q^\alpha_{2r,2l}(\theta_1,\theta_2,\ldots\theta_{2r};\theta'_1,\theta'_2,\ldots,\theta'_{2l}) = \left(\frac{\sin\frac{\pi\nu\alpha}{b}}{\sin\pi\nu}\right)^{r+l}$$
$$\sum_{\substack{T\in S,\\ \#T=r-1}}\ \sum_{\substack{T'\in S',\\ \#T'=l-1}}\ \prod_{\substack{i\in T,\\ k\in\bar T}}\phi(\theta_{ik})\prod_{\substack{i\in T',\\ k\in\bar T'}}\phi(\theta'_{ik})\prod_{\substack{i\in T,\\ k\in\bar T'}}\Phi(\theta_i-\theta'_k)\prod_{\substack{i\in T',\\ k\in\bar T}}\tilde\Phi(\theta_k-\theta'_i)$$

Such an expression satisfies the residue condition (4).

2.1.2 Form factors of the trace of the stress energy tensor.

The first form factor is determined by using the Lagrangian: $Q_{2,2} = -4\pi M^2$.

$$Q_{2r,2l}(\theta_1,\theta_2,\ldots\theta_{2r};\theta'_1,\theta'_2,\ldots,\theta'_{2l}) =$$
$$-4\pi M^2\sum_{\substack{T\in S,\\ \#T=r-1}}\ \sum_{\substack{T'\in S',\\ \#T'=l-1}}\ \prod_{\substack{i\in T,\\ k\in\bar T}}\phi(\theta_{ik})\prod_{\substack{i\in T',\\ k\in\bar T'}}\phi(\theta'_{ik})\prod_{\substack{i\in T,\\ k\in\bar T'}}\Phi(\theta_i-\theta'_k)\prod_{\substack{i\in T',\\ k\in\bar T}}\tilde\Phi(\theta_k-\theta'_i)$$

Let us note that the leading infrared behavior of $F_{2,2}$ is given by $T\bar T$, which defines the direction of the flow in the IR region. To determine the subleading IR terms that appear in the expansion (1), one uses the asymptotic developpement for f_{RL} given by equation (3). For example (up to the logarithmic terms):

$$f_{RL}(\theta_1-\theta'_1)f_{RL}(\theta_1-\theta'_2)f_{RL}(\theta_2-\theta'_1)f_{RL}(\theta_2-\theta'_2)\sim e^{\theta_1+\theta_2-\theta'_1-\theta'_2}\times$$
$$\left[1+Ae^{\theta_1-\theta'_1}+\left(\frac{A^2}{2}+B\right)e^{2\theta_1-2\theta'_1}\right]\times\left[1+Ae^{\theta_1-\theta'_2}+\left(\frac{A^2}{2}+B\right)e^{2\theta_1-2\theta'_2}\right]\times$$
$$\left[1+Ae^{\theta_2-\theta'_1}+\left(\frac{A^2}{2}+B\right)e^{2\theta_2-2\theta'_1}\right]\times\left[1+Ae^{\theta_2-\theta'_2}+\left(\frac{A^2}{2}+B\right)e^{2\theta_2-2\theta'_2}\right].$$

The terms into brackets give

$$1+A(e^{\theta_1}+e^{\theta_2})(e^{-\theta'_1}+e^{-\theta'_2})+\left(\frac{A^2}{2}+B\right)(e^{2\theta_1-2\theta'_1}+e^{2\theta_1-2\theta'_2}+e^{2\theta_2-2\theta'_1}+e^{2\theta_2-2\theta'_2})$$
$$+A^2(e^{2\theta_1-\theta'_1-\theta'_2}+e^{\theta_1+\theta_2-2\theta'_1}+e^{\theta_1+\theta_2-2\theta'_2}+e^{2\theta_2-\theta'_1-\theta'_2}+2e^{\theta_1+\theta_2-\theta'_1-\theta'_2})+\cdots$$
$$=1+\frac{A}{M^2}L_{-1}\bar L_{-1}+\frac{A^2}{2M^4}L^2_{-1}\bar L^2_{-1}+\frac{B}{M^4}L_{-2}\bar L_{-2}+\cdots$$

where $L_{-1}=e^{\theta_1}+e^{\theta_2}$ and $L_{-2}=e^{2\theta_1}+e^{2\theta_2}$. So the next irrelevant operator appearing is $T^2\bar T^2$ (up to derivatives).

[4]As it is mentionned above, the bootstrap equations do not refer to any particular operator, so the dependence with respect to the parameter α is introduced by hand. We do not have so far any compelling argument to justify our choice for the α dependence of the lowest form factor, neither in the Neveu-Schwartz sector nor in the Ramond sector considered in the next section. They are simply similar to some other known cases (see *e.g.* [10]).

2.1.3 Form factors of the operator $\sinh \alpha\phi$.

The number of left movers and right movers is odd. The form factors are written in this case Let $S = (1, \dots, 2r+1), S' = (1, \dots, 2l+1)$. The lowest form factor is $Q^{\alpha}_{1,1} = \left(\frac{\sin \frac{\pi\nu}{b}\alpha}{\sin \pi\nu}\right)$. We propose

$$Q^{\alpha}_{2r+1,2l+1}(\theta_1, \theta_2, \dots \theta_{2r+1}; \theta'_1, \theta'_2, \dots, \theta'_{2l+1}) = \left(\frac{\sin \frac{\pi\nu}{b}\alpha}{\sin \pi\nu}\right)^{r+l+1}$$
$$\sum_{\substack{T \in S, \\ \#T = r}} \sum_{\substack{T' \in S', \\ \#T' = l}} \prod_{\substack{i \in T, \\ k \in \bar{T}}} \phi(\theta_{ik}) \prod_{\substack{i \in T', \\ k \in \bar{T}'}} \phi(\theta'_{ik}) \prod_{\substack{i \in T, \\ k \in \bar{T}'}} \Phi(\theta_i - \theta'_k) \prod_{\substack{i \in T', \\ k \in \bar{T}}} \tilde{\Phi}(\theta_k - \theta'_i).$$

We suppose that the form factors of the energy operator ϵ are given by the expression

$$Q_{2r+1,2l+1}(\theta_1, \theta_2, \dots \theta_{2r+1}; \theta'_1, \theta'_2, \dots, \theta'_{2l+1}) =$$
$$\sum_{\substack{T \in S, \\ \#T = r}} \sum_{\substack{T' \in S', \\ \#T' = l}} \prod_{\substack{i \in T, \\ k \in \bar{T}}} \phi(\theta_{ik}) \prod_{\substack{i \in T', \\ k \in \bar{T}'}} \phi(\theta'_{ik}) \prod_{\substack{i \in T, \\ k \in \bar{T}'}} \Phi(\theta_i - \theta'_k) \prod_{\substack{i \in T', \\ k \in \bar{T}}} \tilde{\Phi}(\theta_k - \theta'_i).$$

2.1.4 Remark

One can check that in the limit $\theta_1, \theta'_1 \to \pm\infty$, the cluster property is satisfied[5]: $F^{\alpha}_{r,l}(\theta_1, \dots, \theta_r, \theta'_1, \dots \theta'_l) \sim F^{\alpha}_{1,1}(\theta_1, \theta'_1) F^{\alpha}_{r-1,l-1}(\theta_2, \dots, \theta_r, \theta'_2, \dots \theta'_l)$.

2.2 Ramond sector

2.2.1 Operator $\sigma e^{\alpha\phi}$.

It has non vanishing matrix elements when the sum of left movers and right movers is odd. Let $S = (1, \dots, 2r+1), S' = (1, \dots, 2l)$. The lowest form factors are $Q^{\alpha}_{1,0} = Q^{\alpha}_{0,1} = \left(\frac{\sin \frac{\pi\nu(\alpha+b)}{2b} \sin \frac{\pi\nu(-\alpha+b)}{2b}}{\sin^2 \frac{\pi\nu}{2}}\right)^{1/2}$. We propose:

$$Q^{\alpha}_{2r+1,2l}(\theta_1, \theta_2, \dots \theta_{2r+1}; \theta'_1, \theta'_2, \dots, \theta'_{2l}) =$$
$$\left(\frac{\sin \frac{\pi\nu(\alpha+b)}{2b} \sin \frac{\pi\nu(-\alpha+b)}{2b}}{\sin^2 \frac{\pi\nu}{2}}\right)^{r+l+1/2} \sum_{\substack{T \in S, \\ \#T = r}} \sum_{\substack{T' \in S', \\ \#T' = l}} \prod_{\substack{i \in T, \\ k \in \bar{T}}} \phi(\theta_{ik}) \prod_{\substack{i \in T', \\ k \in \bar{T}'}} \phi(\theta'_{ik}) \prod_{\substack{i \in T, \\ k \in \bar{T}'}} \Phi(\theta_i - \theta'_k) \prod_{\substack{i \in T', \\ k \in \bar{T}}} \tilde{\Phi}(\theta_k - \theta'_i)$$

2.2.2 Operator $\mu e^{\alpha\phi}$ ($r+l$ even).

As it is explained in [11], there is an additional minus sign in front of the product of S matrices in the bootstrap equation (2).

[5] This type of argument has also been used before, see *e.g.* [3, 10].

- r, l even
  Let $S = (1, \ldots, 2r), S' = (1, \ldots, 2l)$ and the lowest form factor $Q^{\alpha}_{0,0} = 1$. We propose :

$$Q^{\alpha}_{2r,2l}(\theta_1, \theta_2, \ldots \theta_{2r}; \theta'_1, \theta'_2, \ldots, \theta'_{2l}) = (-i)^{r+l} \left(\frac{\sin \frac{\pi\nu(\alpha+b)}{2b} \sin \frac{\pi\nu(-\alpha+b)}{2b}}{\sin^2 \frac{\pi\nu}{2}} \right)^{r+l}$$
$$\sum_{\substack{T \in S, \\ \#T=r}} \sum_{\substack{T' \in S', \\ \#T'=l}} \prod_{\substack{i \in T, \\ k \in \bar{T}}} \phi(\theta_{ik}) e^{\frac{1}{2}\sum \theta_{ik}} \prod_{\substack{i \in T', \\ k \in \bar{T}'}} \phi(\theta'_{ik}) e^{\frac{1}{2}\sum \theta'_{ki}} \prod_{\substack{i \in T, \\ k \in \bar{T}'}} \Phi(\theta_i - \theta'_k) \prod_{\substack{i \in T', \\ k \in \bar{T}}} \tilde{\Phi}(\theta_k - \theta'_i)$$

- r, l odd
  Let $S = (1, \ldots, 2r+1), S' = (1, \ldots, 2l+1)$. The lowest form factor is $Q^{\alpha}_{1,1} = \left(\frac{\sin \frac{\pi\nu(\alpha+b)}{2b} \sin \frac{\pi\nu(-\alpha+b)}{2b}}{\sin^2 \frac{\pi\nu}{2}} \right) e^{\frac{\theta'_1 - \theta_1}{2}}$. We propose:

$$Q^{\alpha}_{2r+1,2l+1}(\theta_1, \theta_2, \ldots \theta_{2r+1}; \theta'_1, \theta'_2, \ldots, \theta'_{2l+1}) = (-i)^{r+l} \left(\frac{\sin \frac{\pi\nu(\alpha+b)}{2b} \sin \frac{\pi\nu(-\alpha+b)}{2b}}{\sin^2 \frac{\pi\nu}{2}} \right)^{r+l+1}$$
$$\sum_{\substack{T \in S, \\ \#T=r}} \sum_{\substack{T' \in S', \\ \#T'=l}} \prod_{\substack{i \in T, \\ k \in \bar{T}}} \phi(\theta_{ik}) e^{\frac{1}{2}\sum \theta_{ik}} \prod_{\substack{i \in T', \\ k \in \bar{T}'}} \phi(\theta'_{ik}) e^{\frac{1}{2}\sum \theta'_{ki}} \prod_{\substack{i \in T, \\ k \in \bar{T}'}} \Phi(\theta_i - \theta'_k) \prod_{\substack{i \in T', \\ k \in \bar{T}}} \tilde{\Phi}(\theta_k - \theta'_i)$$

2.2.3 Remarks

- Let us introduce now the operator $\mathcal{O} = \sigma e^{\alpha\phi} + \mu e^{\alpha\phi}$. Its form factors satisfy the cluster property like an exponential of a bose field

$$\mathcal{O}_{r,l}(\theta_1, \theta_2, \ldots \theta_r; \theta'_1, \theta'_2, \ldots, \theta'_l) \sim \mathcal{O}_{1,0}(\theta_1)\mathcal{O}_{r-1,l}(\theta_2, \ldots \theta_r; \theta'_1, \theta'_2, \ldots, \theta'_l) \quad \text{for} \quad \theta_1 \to \infty\,.$$

- The expressions for the form factors of $\sigma e^{\alpha\phi}$ and $\mu e^{\alpha\phi}$ give the expected leading IR behaviour [4,11,12]: $F^{IR}_{r,l}(\theta_1, \theta_2, \ldots, \theta_r; \theta'_1, \theta'_2, \ldots, \theta'_l) \sim \prod_{i<j} \tanh \frac{\theta_{ij}}{2} \tanh \frac{\theta'_{ij}}{2}$, where $r+l$ is odd for $\sigma e^{\alpha\phi}$ and even for $\mu e^{\alpha\phi}$.

3 Conclusion

Finally, we would like to say that the representations proposed for the functions $Q^{\alpha}_{r,l}$ are general enough [6] to obtain immediately the form factors of the operators $\Theta, \epsilon, \sigma, \mu$ in the Tricritical Ising model perturbed by the subenergy that defines a massless flow to the Ising model [14,15], simply by replacing S_{RL} and f_{RL} by their corresponding values [16]. We checked for a low number of particles that they indeed reproduce the results of [6] where the first form factors of Θ, σ, μ are computed in terms of symmetric polynomials, as well as with [7], where an expression for the form factors of the operator Θ for an arbitrary number of intermediate particles is proposed. In principle, it should not be difficult to extend our results to other massless models flowing to the Ising model, but where the S-matrix has a more complicated structure of resonance poles [16]. Of course it remains to prove if our choice for the α dependence of the form factors reproduces the correct UV conformal dimension of the operators.

[6] Their structure is very similar to the one found for the operator $e^{\alpha\phi}$ in the bosonic Sinh-Gordon model in [13], equ. (61).

Acknowledgments

I am grateful to Al.B. Zamolodchikov for suggesting the problem. Discussions with D. Bernard, V.A. Fateev, G. Mussardo, H. Saleur, F.A. Smirnov are acknowledged. In particular, I thank G. Delfino for his comments on the manuscript and useful discussions. Work supported by the CEA and the Euclid Network HPRN-CT-2002-00325.

References

[1] C. Ahn, C. Kim, C. Rim and Al.B. Zamolodchikov, "RG flows from Super-Liouville Theory to Critical Ising Model", Phys. Lett. **B541** (2002) 194, hep-th/0206210

[2] D.V. Volkov and V.P. Akulov, "Is the neutrino a Goldstone particle?", Phys. Lett. **B46** (1973) 109

[3] M. Karowski and P. Weisz, Nucl. Phys. **B139** (1978) 455

[4] B. Berg, M. Karowski and P. Weisz, Phys. Rev. **D19** (1979) 2477

[5] F.A. Smirnov, "Form factors in Completely Integrable Models of Quantum Field Theory", Adv. Series in Math. Phys. **14**, World Scientific 1992

[6] G. Delfino, G. Mussardo and P. Simonetti, "Correlation fonctions along a massless flow", Phys. Rev. **D51** (1995) 6620, hep-th/9410117

[7] P. Méjean and F.A. Smirnov, "Form-factors for principal chiral field model with Wess-Zuminov-Novikov-Witten term", Int. J. Mod. Phys. **A12** (1997) 3383, hep-th/9609068

[8] M. Rocek, "Linearizing the Volkov-Akulov Model", Phys. Rev. Lett. **41** (1978) 451

[9] A.A. Belavin, A.M. Polyakov and A.B. Zamolodchikov, "Infinite conformal symmetry in 2D quantum field theory", Nucl. Phys. **B241** (1984) 333

[10] A. Koubek and G. Mussardo, "On the Operator Content of the Sinh-Gordon Model", Phys. Lett. **B311** (1993) 193, hep-th/9306044

[11] V.P. Yurov and Al.B. Zamolodchikov, "Correlation functions of integrable 2D models of the relativistic field theory; Ising model", Int. J. Mod. Phys. **A6** (1991) 3419

[12] J.L. Cardy and G. Mussardo, Nucl. Phys. **B340** (1990) 387

[13] H. Babujian and M. Karowski, "Sine-Gordon breather form factors and quantum field equations", J. Phys. **A35** (2002) 9081, hep-th/0204097

[14] D.A. Kastor, E.J. Martinec and S.H. Shenker, "RG flow in $N = 1$ Discrete Series", Nucl. Phys. **B316** (1989) 590

[15] A.B. Zamolodchikov, Sov. J. Nucl. Phys. **48** (1987), 1090

[16] Al.B. Zamolodchikov, "From tricritical Ising to critical Ising by thermodynamic Bethe ansatz", Nucl. Phys. **B358** (1991) 524