Covariant Quantization
of the Brink-Schwarz Superparticle

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ABSTRACT

The quantization of the Brink-Schwarz-Casalbuoni superparticle is performed in an explicitly covariant way using the antibracket formalism. Since an infinite number of ghost fields are required, within a suitable off-shell twistor-like formalism, we are able to fix the gauge of each ghost sector without modifying the physical content of the theory. The computation reveals that the antibracket cohomology contains only the physical degrees of freedom.

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1 Introduction

The correct quantization of systems with an infinitely reducible gauge symmetry is a long-standing problem and, recently, several new efforts (see e.g. [1]) have been made toward the construction of a covariant quantization procedure for the Green-Schwarz superstring model [2]. Indeed, the GS superstring is the most important and interesting model enjoying the feature of infinite reducibility, but since the model is very difficult to handle in its complexity, the study of simpler models provides a good test for the quantization techniques. This is essentially the reason why, in the past decades, people devoted several efforts trying to quantize the superparticle model of Brink-Schwarz-Casalbuoni type [3].

In the case of GS superstring and superparticle, there are first- and second-class constraints [4, 5, 6, 7]. The occurrence of second-class constraints arises from the fact that the Grassman momenta P_θ^α, conjugate to the fermionic variables θ_α, are non-independent phase-space variables. If, as a formal procedure, one attempts to construct Dirac brackets, treating all the fermionic constraints as if they were second class, the resulting expressions are singular. An alternative procedure would be a careful separation of first- and second-class constraints, but, in that way, a covariant quantization procedure is impossible to achieve.

In order to maintain manifest Lorentz covariance, one has to exploit the κ-symmetry [2, 8] of the model, which cancels half of the fermionic degrees of freedom realizing the matching between the bosonic and the fermionic states. Unfortunately, the κ-symmetry is a reducible local fermionic symmetry. This means that, in reality, only four degrees of freedom are effectively canceled at the first stage. Pursuing the analysis, it is easy to show that an infinite tower of ghosts is necessary to match the correct number of degrees of freedom. In terms of the Hamiltonian formalism [6], this is equivalent to the statement that the constraints are infinitely reducible: there exist not only linear vanishing combinations of constraints, but also zero modes of those relations, and so on to infinitely many levels.

Several attempts to a solution of the problem can be found in the literature. In particular, we would like to mention the idea of changing the classical constraints in order that all the second-class constraints are transformed into first-class ones [9, 10]. This essentially yields an extension of the phase-space where the κ-symmetry is gauged by means of suitable fermionic gauge fields. Other approaches to the superparticle quantization are the harmonic superspace [11] –which produces a non-local super Yang-Mills field theory– and the construction of models based on a given BRST operator which selects the correct physical subspace [12]. Nevertheless, all of these approaches share the common feature of infinite number of classical and ghost fields.

Finally, the computation of the gauge-fixed BRST characteristic cohomology [13, 14] for the superparticle model received considerable attention [12, 15, 16, 17]. It has been shown in [17] that, due to the infinitely many interacting fields, the canonical transformations performed to implement the gauge fixing turn out to be ill-defined. Moreover, as a

consequence of further gauge symmetries of the gauge-fixed action, a two-step gauge-fixing procedure is required and it may cause problems, as argued in [12].

The aim of our paper is to present a new solution of the application of the BV-BRST formalism ([18, 19]; for a review, see [5, 20, 21]) to the problem of the superparticle. In particular, we take the advantage of the existing literature to make essential steps towards the complete solution. The main issue here is the construction of a procedure to quantize the Brink-Schwarz-Casalbuoni classical action, computing the correct antibracket map and the corresponding cohomology. The present technique is based on canonical transformations which implement the gauge fixing of the nth order ghosts without affecting the $(n-1)^{\text{th}}$ order.

The structure of the superparticle model involves word-line diffeomorphisms, on-shell closure of the algebra of symmetries and an infinite tower of ghost fields. To take into account the open algebra, the BV-BRST formalism is implemented and the solution of the master equations contains quadratic terms in the antifields. On the other hand, the κ-symmetry of the model entails field-dependent transformations which produce interactions among the ghost fields of different levels and the superparticle fields. Therefore, after the gauge fixing, an infinite number of ghosts fields interact, among themselves and with the *physical* fields. In this situation, two types of problems arise: *i)* the renormalizability of the model may be lost due to an infinite number of parameters, *ii)* the computation of the gauge-fixed BRST cohomology and the definition of the physical spectrum of the model are ill-defined.

In order to solve the problem of interactions among the ghost fields of different levels, we apply the idea of P. Townsend [22] for the chiral superfield to the superparticle model. The basic idea is a suitable redefinition of ghost fields, in such a way that they appear decoupled from the rest of the theory. The complete formulation of Townsend's idea has already been discussed in [23]. There, it was shown that, in the case of the gauged, complex, linear superfield model [24], a convenient redefinition of ghost fields realizes the decoupling and the quantization can be performed. In addition, it was shown that a careful application of the technique of [24] provides a quantization procedure with the correct antibracket cohomology. The realization of Townsend's idea in the superparticle framework requires further efforts.

Following the twistor-like formulation [25], we introduce twistor variables to re-express the ghost fields in a decoupled fashion. In the literature only the on-shell twistor-like formalism is available. Unfortunately, this is not suitable for our purposes. Therefore, we extended the twistor-like formalism in order that the ghost fields can be conveniently redefined off-shell. In past years, some interesting formulations of the superparticle model have been discussed in the context of the $N = 8$ word-line supergravity framework [25]. However, we do not follow these directions and our *twisted*-ghost formalism only amounts to a redefinition of classical variables.

Even after the "twisting", the zero modes of the κ-symmetry (the ghost fields) turn out to interact with the physical fields; however, the interaction terms are proportional

to the einbein equation of motion (Virasoro constraints) and, therefore, these couplings can be eliminated by a simple canonical transformation (cf. Ref. [20]). Although the fermion generator of these transformations contains a formal sum on infinite ghost fields, it can be easily verified that each individual canonical transformation involves a finite number of fields [12, 17] and it has a well-defined inverse. In this way, the cohomology of the antibracket is not changed and these transformations do not affect the physical observables. After twisting and canonical transformations, the ghost fields are free and decoupled from the rest of the fields. In addition, the ghost fields are *off-shell* zero modes of the κ-symmetry. Comparing with [23, 24], we stress again the fact that the quantization procedure, in the presence of an infinite number of ghosts, can be performed only in the case of off-shell decoupling.

Although the technique of [24] appears very promising for the superparticle models, as it has been shown in [23], it cannot be easily implemented in the the present context. This is due to the structure of the antifield-dependent part of the action. The diagonalization of the fields necessary to achieve the decoupling among the ghosts of different levels induces new couplings in the antifield terms which cannot be removed. Therefore, in order to apply safely the quantization procedure of [23, 24], one has to reach the third order of ghost field, at least, to circumvent the problem. Unfortunately, despite several efforts, it seems that there is no way to apply the aforementioned procedure to the superparticle case. However, we can follow a similar strategy: we quantize the nth ghost field without affecting the $(n-1)^{\text{th}}$ level, but the actual realization is different.

We choose the conventional Lorentz invariant gauge fixing proposed in [9, 12, 15] to fix the gauge of the spinor field θ_α and for the ghost fields. In particular, following [12, 26], we take into account the Stückelberg symmetry arising in the *non-minimal* sector. The latter contains Lagrangian multipliers, anti-ghost fields and the non-minimal extra-ghost fields. Contrary to [12, 27], we use the Stückelberg symmetry at each step of the quantization procedure in such a way that all degrees of freedom are gauge fixed and their dynamics is precisely prescribed.

Along the quantization procedure, we compute the antibracket cohomology and we check that the sequence of canonical transformations does not modify the physical sector of the Hilbert space. In addition, we compute the cohomology group $H(\gamma, H(\delta))$ of the completely gauge-fixed theory –where γ and δ are defined in [13, 21]– and the result coincides with the correct physical spectrum.

The outline of the paper is the following. In section 2, we briefly review the antifield and the antibracket formalism with particular attention to the definition of physical observables. In section 3, we present some details about the strategy of the quantization procedure. In section 4, we recall the solution of the master equation in order to establish our conventions. Next, in section 5 we introduce the twistor-like variables and we define the off-shell zero modes. Finally, in section 6 the gauge fixing of the complete tower of ghost fields is implemented. Section 7 contains the computation of the cohomology and concluding remarks. An appendix details our notations.

2 Symmetries and the anti-bracket formalism

Essential ingredients of the Batalin-Vilkovisky (BV) formalism are anti-fields and anti-brackets [19, 20, 21]. For any field ϕ^A, one introduces an anti-field ϕ^*_A with opposite statistics. A ghost number is assigned to every field, in such a way that for the classical fields $gh(\phi) = 0$, the (extended) action has ghost number zero, and for all fields $gh(\phi^*_A) = -gh(\phi^A) - 1$.

The anti-bracket between two functions F and G of the fields and anti-fields is defined by

$$(F, G) = \frac{\partial_r}{\partial \phi^A} F \frac{\partial_l}{\partial \phi^*_A} G - \frac{\partial_r}{\partial \phi^*_A} F \frac{\partial_l}{\partial \phi^A} G. \tag{2.1}$$

where $\partial_{r/l}$ denote the derivative from the left or from the right. The antibrackets satisfy graded commutation, distribution, and Jacobi relations [19]. With these brackets, fields and anti-fields behave as coordinates and their canonically conjugate momenta

$$(\phi^A, \phi^B) = 0 \; ; \quad (\phi^*_A, \phi^*_B) = 0; \quad (\phi^A, \phi^*_B) = \delta^A_B. \tag{2.2}$$

The extended action $S(\phi, \phi^*)$ is a solution of the master equation

$$(S, S) = 0 \,, \tag{2.3}$$

which satisfies certain boundary conditions: 1) it coincides with the classical action $S_{\text{cl}}(\phi)$ when the ghost fields are set to zero; 2) the anti-fields ϕ^*_A are coupled to the gauge transformations of the classical fields; 3) it satisfies the *properness* requirement, i.e. the $2N \times 2N$ matrix

$$\frac{\partial_l \partial_r S}{\partial \phi^A \partial \phi^*_B}\,, \quad A, B = 1, \ldots, N$$

has rank N on the stationary surface, defined by the equations of motion $\frac{\partial S}{\partial \phi} = \frac{\partial S}{\partial \phi^*} = 0$.

Notice that there is a natural grading among the fields and the anti-fields, namely the antighost number [21]. This allows a convenient decomposition of the extended action and, accordingly, the master equation can be easily solved.

Because of the boundary conditions, a solution of the master equation does not only give the dynamics of the system, as it generates the equations of motion; it also encodes the gauge structure, as it generates the BRST transformations on a function X of fields and antifields via

$$sX = (X, S)\,.$$

The master equation implies at once that S itself is invariant under this transformation, and that s is a nilpotent differential: $s^2 = 0$. The second boundary condition ensures that s "starts as" the classical BRST differential, when acting on fields, but it is automatically

equipped with the modifications that are necessary when the classical BRST transformations do not define a nilpotent operator; this is the case of an open algebra, where the commutator of two gauge symmetries yields a gauge symmetry only on-shell.

Finally, the properness condition tells us that the extended action has N on-shell independent gauge symmetries, and this is just the number that is necessary to eliminate unphysical anti-fields and quantize only the fields. This is essential, since in the end one wants to perform a path integral only on the space of fields configurations; still, the antifields do not fall completely out of the scene at this point: they can play the role of classical sources for the BRST transformations.

It is useful, in the analysis of the cohomology of the BRST operator, to introduce another grading, called anti-field number, whose value is 0 for the fields, 1 for the anti-fields of classical fields, 2 for anti-fields of ghosts, 3 for anti-fields of ghosts for ghosts and so on. The BRST operator can be decomposed accordingly:

$$s = \delta + \gamma + \sum_{k \geq 0} s^{(k)}$$

The terms in the extended action all have non-negative antifield number, and the antibracket operation lowers it by one, so that the decomposition starts with degree -1, and the corresponding operator δ which is necessarily nilpotent is called the Koszul-Tate differential. It acts trivially on fields, while its action on an anti-field yields the equation of motion of the corresponding field:

$$\delta \phi_A^* = -\frac{\partial_l S}{\partial \phi^A} \,.$$

The next term, γ, has antifield number 0. Its action on fields is given by

$$\gamma \phi^A = (\phi^A, S)|_{\phi^*=0} \,.$$

Canonical transformations are an important part of the formalism [20]. They preserve the anti-bracket structure (2.1): calculating the anti-brackets in the old or new variables is the same, or in other words the new variables also satisfy (2.2). Therefore also the master equation $(S, S) = 0$ is preserved.

Canonical transformations from $\{\phi, \phi^*\}$ to $\{\phi', \phi'^*\}$, for which the matrix $\frac{\partial_r \phi^B}{\partial \phi'^A}\big|_{\phi'^*}$ is invertible, can be obtained from a fermionic generating function $F(\phi, \phi'^*)$ with $gh(F) = -1$. The transformations are defined by

$$\phi'^A = \frac{\partial_l F(\phi, \phi'^*)}{\partial \phi_A'^*} \,, \qquad\qquad \phi_A^* = \frac{\partial_r F(\phi, \phi'^*)}{\partial \phi^A} \,. \tag{2.4}$$

Infinitesimal canonical transformations are produced by

$$F = \phi_A'^* \phi^A + \varepsilon f(\phi, \phi'^*) \,;$$

the first term is the identity operator in the space of fields-antifields. In the following, we will have to use different types of transformations.

- Point transformations are the easiest ones. These are just fields redefinitions $\phi'^A = f^A(\phi)$. They are generated by $F = \phi'^*_A f^A(\phi)$ which thus determines the corresponding transformations of the anti-fields. The latter replace the calculations of the variations of the new variables.

- Adding the BRST transformation of a function $s\Psi(\phi)$ to the action is obtained by a canonical transformation with $F = \phi'^*_A \phi^A + \Psi(\phi)$. The latter gives

$$\phi'^A = \phi^A \; ; \qquad \phi^*_A = \phi'^*_A + \partial_A \Psi(\phi). \tag{2.5}$$

  and by means of these transformations it is possible to implement the gauge fixing conditions as a canonical change of variables.

- It is possible to redefine the symmetries by adding equations of motion ('trivial symmetries'). This is obtained by

$$F = \phi'^*_A \phi^A + \phi'^*_A \phi'^*_B h^{AB}(\phi) \,. \tag{2.6}$$

The canonical transformations leave by definition the master equation invariant, and because they are non-singular, they also preserve the properness requirement on the extended action. This point will be discussed at length in the following. Of course, in the new variables, we do not see the classical limit anymore. But the most important property is that the anti-bracket cohomology [13] (and the BRST cohomology) is not changed.

3   Strategy

Before entering into the details of the analysis of the BS superparticle [3], we illustrate the main steps of the quantization procedure of systems with an infinitely reducible gauge symmetry. In particular, we describe each single canonical redefinition of the field variables needed to bring the action in a form which is suitable for path integral quantization. The main point is to find out a correct gauge fixing procedure which fixes the gauge degrees of freedom, and to construct the physical subspace as the characteristic cohomology of the BRST operator. In the present section, we denote by Φ the "physical" variables X_μ, θ, e and P_μ (in the first order formalism), by k^α_n the ghosts of the κ-symmetry and by C_A the ghost for the remaining local and rigid symmetries. We refer to the index n as level; the zero level is associated to the fields Φ^A.

According to [15, 12, 9, 28, 16], the general solution of the master equation in the case of the BS superparticle and of the GS superstring has the form

$$
\begin{aligned}
S \;=\; & I[\Phi^A] + \Phi^*_A \left[A^{A,B}(\Phi)\, C_B + B^A_{\;\alpha}(\Phi)\, k^\alpha_1 \right] \\
& + \; C^*_A \left[D^{A,B}(\Phi, C)\, C_B + F^A_{\;\alpha\beta}(\Phi)\, k^\alpha_1\, k^\beta_1 \right]
\end{aligned}
$$

$$+ \quad k_n^{*,\alpha} \left[G_{\alpha,\beta}(\Phi)\, k_{n+1}^\beta + F_{\alpha\beta}(C)\, k_n^\beta + \hat\delta\, k_n^\alpha \right] \qquad (3.1)$$

$$+ \quad \Phi_A^* \Phi_B^* \left[H_{\alpha\beta}^{AB}(\Phi)\, k_1^\alpha k_1^\beta + J_\alpha^{AB}(\Phi)\, k_2^\alpha \right]$$

$$+ \quad C_A^* \Phi_B^* \left[M_{\alpha\,\beta}^{A\,B}(\Phi)\, k_1^\alpha\, k_2^\beta \right]$$

$$+ \quad k_n^{*,\alpha} \Phi_B^* \left[N_{\alpha,\beta}^B(\Phi)\, k_{n+2}^\beta + P_{\alpha,\beta\,\sigma}^B(C)\, k_1^\alpha k_{n+1}^\sigma + \hat{\hat\delta}\, k_n^\alpha \right] ,$$

where the terms quadratic in the anti-fields are necessary to close the algebra and to take into account the fact that the symmetry is reducible only on-shell, and $\hat\delta k_n$, $\hat{\hat\delta}k_n$ are of higher order in the ghosts. In the case of the BS superparticle there are further simplifications: *i)* the terms $F_{\alpha\beta}(C)\, k_n^\beta$, $\hat\delta\, k_n^\alpha$, and $\hat{\hat\delta}\, k_n^\alpha$ are absent since the ghosts are diffeomorphic invariant and they have no quartic interactions, *ii)* the terms with powers of the anti-fields are restricted to one type, that is when one of the anti-fields Φ^* happens to be the anti-field of the einbein e.

1. We begin the quantization procedure by introducing the "twisting" of the ghost fields. The aim of this redefinition is to decouple the ghost fields k_n^α from the physical degrees of freedom described by X_μ, P_μ and θ. For that purpose, we will follow the ideas of P. Tonwsend [22] introducing redefined ghost fields $\tilde k_n$ and showing that their dynamics is independent of the "physical" variables. In particular, the idea is to redefine the fields in such a way that $\tilde k_n^{*,\alpha} G'_{\alpha,\beta} \tilde k_{n+1}^\beta$ becomes independent of the field Φ. Consequently, provided that the gauge fixing procedure does not introduce a dependence on Φ^A, the ghost fields are decoupled from Φ^A.

   As it will be explained in the next section, in order to implement the suggested redefinition of ghost fields [22], we adopt a twistor-like (off-shell) formalism. Obviously, this change of variables can be performed by a canonical transformation. Here, we have to point out that, as recognized by [17, 12], some canonical transformations involving infinite number of fields are ill-defined and, therefore, they might change drastically the BRST cohomology. It will be our main concern to avoid any such redefinition.

2. Furthermore, although the "twisting" provides a convenient coordinatization for the ghost degrees of freedom, there are remaining couplings between ghost fields and Φ^A. After the "twisting", we have

$$k_n^{*,\alpha} G_{\alpha\beta}(\Phi)\, k_{n+1}^\beta \longrightarrow \tilde k_n^{*,\alpha} \left[G'_{\alpha\beta} + G_{\alpha\beta}^{'',A} \frac{\delta I}{\delta\Phi^A} \right] \tilde k_{n+1}^\beta . \qquad (3.2)$$

   The second term in the bracket can be eliminated by a further canonical transformation of type (2.6)

$$\Xi = \Phi_A'^* \Phi^A + \sum_n \tilde k_n^{'*,\alpha} \tilde k_n^\alpha + \Phi_A'^* \sum_n \left(\tilde k_n^{*,\alpha} G_{\alpha\beta}^{'',A} \tilde k_{n+1}^\beta \right) . \qquad (3.3)$$

7

In the present case (and in the case of GS superstring), $G_{\alpha\beta}^{'',A}\frac{\delta I}{\delta\Phi^A}$ is proportional to the equations of motion for the world-line (-sheet) metric e and, therefore, the canonical transformation Ξ involves the anti-field e^*. The latter is an anticommuting field and, thus, $(e^*)^2 = 0$. Consequently, it is easy to show that the redefinition of the fields effected by Ξ (and the corresponding inverse transformation) involves only a finite number of fields k_n. Obviously, the master equation and the classical cohomology of the theory remain unchanged by these transformations and we can thus proceed with the quantization procedure.

3. Following the BV formalism [19], one introduces a non-minimal sector of fields, that is the anti-ghosts, the extra-ghosts and the Lagrange multipliers

$$
\begin{aligned}
\bar{C}^\sigma &= \{\chi_p^q, \bar{c}_A\}\,, & \bar{C}^{*,\sigma} &= \{\chi_p^{*,q}, \bar{c}_A^*\}\,, \\
\pi^\sigma &= \{\pi_p^q, \pi^A\}\,, & q+p &= n, n \geq 1\,,
\end{aligned}
\tag{3.4}
$$

and the non-minimal part to the action

$$
S_{n.m.} = \bar{c}_A^* \, \pi^A + \sum_{p=0,q=1}^{\infty,q=p} \bar{\chi}_p^{*,q} \, \pi_p^q\,.
\tag{3.5}
$$

However, since the application of the common BV formalism for an infinitely reducible theory seems to generate wrong results [12, 17], we follow a different procedure.

4. The main problem of a complete gauge fixing lies in the fact that after having fixed the minimal and extra ghosts, the theory is still invariant under a local symmetry of Stückelberg type [26]. The latter needs a further gauge fixing. As will be shown in Sec. (6), some of the extra fields can be identified with the ghosts of Stückelberg symmetry and, therefore, they can be used to fix completely the gauge invariances. As a consequence, the non-minimal sector of ghosts and extra ghosts is partially redundant and it can be reduced to a smaller set. Moreover, the gauge fixing of the entire ghost sector is performed in a step-by-step procedure by using canonical transformations which leave the cohomology unchanged.

4 The minimal solution of the Master Equation

The general strategy is here applied to the specific model of the BS superparticle. The analysis of the classical action, the constraints coming from the general coordinates invariance and the supersymmetry on the target space imply that the action is invariant also under the κ-symmetry. We associate to the world-line diffeomorphism the local ghost

c, and to the fermionic local κ-symmetry the infinite tower of ghosts k_n. This fixes the field content to (without considering the global ghosts for the super-Poincaré invariance)

$$
\begin{aligned}
\{\Phi^A\} &= \{e, X^\mu, \theta^\alpha\}, & \mathrm{gh}\,\Phi^A &= 0 \\
\{C^A\} &= \{c\}, & \mathrm{gh}\,C^A &= +1 \\
\{K^A\} &= \{k_n\}, & \mathrm{gh}\,k_n &= n,
\end{aligned}
\tag{4.1}
$$

and the corresponding antifields

$$
\begin{aligned}
\{\Phi_A^*\} &= \{e^*, X^{*,\mu}, \theta^{*,\alpha}\}, & \mathrm{gh}\,\Phi_A^* &= -1, \\
\{C_A^*\} &= \{c^*\}, & \mathrm{gh}\,C_A^* &= -2, \\
\{K_A^*\} &= \{k_n^*\}, & \mathrm{gh}\,k_n^* &= -n-1.
\end{aligned}
\tag{4.2}
$$

The classical action for BS superparticle [3] is given, in first order formalism, by

$$
S = P^\mu \partial X_\mu - \bar{\theta}\slashed{P}\partial\theta - \frac{1}{2}eP^2;
\tag{4.3}
$$

it is invariant under world-line reparametrizations, target-space Poincaré transformations and the kappa-symmetry transformations

$$
\begin{aligned}
\delta X^\mu &= \bar{\theta}\Gamma^\mu \slashed{P}\kappa, \\
\delta\theta &= \slashed{P}\kappa, \\
\delta e &= 4\partial\bar{\theta}\kappa,
\end{aligned}
\tag{4.4}
$$

where κ, the parameter of the transformation, is an anticommuting MW spinor, and is replaced by a commuting spinor ghost k in the BRST version of the symmetry. The κ-symmetry is closed on-shell (see e.g.[2]) and one can see easily that it is reducible. The transformation has 16 parameters, the number of components of a MW spinor in 10 dimensions, but on-shell they are not all independent, since for $\kappa = \slashed{P}\kappa'$ the symmetry is trivial due to the equation of motion $P^2 = 0$. Thus only half of the components of κ can be used to gauge away degrees of freedom of θ, and on-shell one achieves the matching between bosonic and fermionic degrees of freedom that is required by supersymmetry. The other components are zero-modes of the symmetry. There is no way to get rid of these if one wants to keep the manifest Lorentz-invariance of the model; instead, one has to consider this additional redundancy as arising from a gauge symmetry in the ghost sector, namely

$$
\delta k = \slashed{P}k_2,
\tag{4.5}
$$

where an additional ghost has been introduced, which is a MW spinor with chirality opposite to that of k. The relation between k and k_2 is the same as the one between θ and k, so the new symmetry is again reducible, a third generation ghost is required,

and so on: the symmetry is infinitely reducible. According to the BV procedure [19], the minimal solution of the master equation takes the form of an action depending on an infinite number of fields. The matching of degrees of freedom is obtained in this way: 8 components of θ are physical, the remaining 8 are canceled by half of the components of the first ghost, the remaining components of k are canceled by half of those of k_2 and so on. This can be written as

$$8 = 8 + (8 - 8) - (8 - 8) + \ldots$$

and clearly the sum makes sense only if it is performed with the parenthesis in a certain position, or if it is regularized in a proper way; it is customary to use the Euler regularization of an alternate sum:

$$\sum_{n=0}^{\infty} (-1)^n = \lim_{x \to 1} \sum_{n=0}^{\infty} (-x)^n = \frac{1}{2} \, . \tag{4.6}$$

This gives a hint that the theory must be handled very carefully to avoid problems arising from formal manipulations of non-convergent series. This kind of problems invalidate solutions given so far to the problem of covariantly quantizing the GS string and the BS superparticle, as was first pointed out by Bastianelli et al. [17].

The minimal solution of the master equation is [15]

$$
\begin{aligned}
L \;=\; & P^\mu \partial X_\mu - \bar{\theta} \slashed{P} \partial \theta - \frac{1}{2} \, e P^2 \\
& + X_\mu^* (c P^\mu + \bar{\theta} \Gamma^\mu \slashed{P} k_1) + e^* (\partial c + 4 \partial \bar{\theta} k_1) \\
& - 2 c^* \bar{k}_1 \slashed{P} k_1 + \bar{\theta}^* \slashed{P} k_1 + \sum_{p \geq 1} \bar{k}_p^* \slashed{P} k_{p+1} \\
& + 2 e^* \left[X_\mu^* (\bar{\theta} \Gamma^\mu k_2 - \bar{k}_1 \Gamma^\mu k_1) - 4 c^* \bar{k}_1 k_2 + \bar{\theta}^* k_2 + \sum_{p \geq 1} \bar{k}_p^* k_{p+2} \right]
\end{aligned} \tag{4.7}
$$

This action satisfies the requirements requested to a minimal solution. As expected, it is quadratic in the antifields since the gauge algebra is open, but there are no terms with higher powers of antifields.

5   Twisting the ghosts

The ghosts' zero mode condition, as we have seen, is field-dependent and is only satisfied on-shell. These are the two unpleasant features that we would like to eliminate, in order to get a well-defined quantization procedure. Our strategy is to look for a redefinition of the ghosts such that the new ghosts have off-shell zero modes, after possibly some further manipulations that turn out to be necessary to eliminate terms proportional to

the equations of motion. All the transformations are performed via canonical transformations, so that the master equation is satisfied at every step. The first observation to be made is that a twistor-like formulation[25] is possible for the massless particle or superparticle, in which bosonic degrees of freedom are traded for world-line spinors, and –for example in Galperin *et al.*– the kappa-symmetry is replaced by an extended world-line supersymmetry. We adopt here a formulation that differs from the previous ones in that the twistor-like variables are not regarded as fundamental, but just as (non-linear) functions of the fundamental fields.

The idea of the twisting is to replace, in the κ-symmetry transformation laws, the operator $\not{P}$ with another operator whose nilpotency is independent of the equations of motion. The simplest choice is to take a 2×2 matrix, and the only nilpotent matrices are the Pauli matrices σ_- and σ_+, up to a multiplicative factor. The fundamental equations that define the "twistors" λ are the following:

$$
\begin{aligned}
(\not{P})_\alpha^{\ \beta} &= \lambda_{\alpha,\underline{\alpha}i}(\sigma_- + P^2\sigma_+)^{\underline{\alpha}}_{\ \underline{\beta}}\, \delta^i_j\, \bar{\lambda}^{\,\underline{\beta}j,\beta} \\
\bar{\lambda}^{\,\underline{\beta}j,\beta}\, \lambda_{\beta,\underline{\alpha}i} &= \delta^{\underline{\beta}}_{\ \underline{\alpha}}\, \delta^j_i
\end{aligned}
\tag{5.1}
$$

where we have defined $\bar{\lambda} = C_2^{-1}\lambda^T C_{10}$. As a consequence of the second equation of (5.1), the two- and ten-dimensional charge-conjugation matrices are related:

$$
C_2 = \lambda^T C_{10}\lambda\,.
\tag{5.2}
$$

On shell, $\not{P}$ is simply replaced by σ_-; the choice of σ_+ is of course possible, and is related to the other choice by a discrete symmetry, namely $\lambda \to \lambda C_2$. From the ten-dimensional point of view, the twistors λ are commuting spinors (it is worthwhile mentioning that although $\not{P}$ is real, the twistor can be complex), but they carry also internal indices $\bar{\alpha} = 1, 2\,; i = 1, \ldots, 8$ in order to be invertible matrices, as required by the second equation in (5.1). We will never need to display explicitly the whole index structure.

Due to the anticommuting properties of the σ matrices, one can verify that if eq. (5.1) is read as the definition of a matrix $\not{P}$, that matrix satisfies $\not{P}^2 = P^2$. But we need to show that the equation can be solved, with respect to λ, for any P. It is clear from the outset that the solution can never be unique: for instance, λ can be multiplied by an orthogonal matrix O^j_i in the internal indices. It can be proved that these are the only internal transformations that leave eq. (5.1) invariant. By a rotation, we can bring P to have only the P^+ and P^- components in light-cone coordinates. Then, using the well-known iterative construction of Γ matrices starting from the two-dimensional ones, we have

$$
\Gamma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes \mathbf{1}\,, \quad
\Gamma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \otimes \mathbf{1}
$$

In this frame, Γ^+ and Γ^- can be identified with σ_+ and σ_-.

On-shell, $\not{P}$ is a nilpotent matrix. We can choose a reference frame in which P^+ is the only

non-vanishing component of P; in this frame, $\not{P}$ is proportional to Γ^+, whose kernel has a dimension equal to one half of the dimension of the whole space. The Jordan canonical form of $\not{P}$ is a matrix with diagonal 2×2 blocks of the form of σ_-, and eventually a block of zeroes, but the argument on the dimension of the kernel forbids the latter. Then $\not{P}$ can be transformed with a change of basis into $\sigma_- \otimes 1$, and this amounts to say that a solution to the twisting equation does exist.

Off-shell, $\not{P}$ is diagonalizable with eigenvalues $\pm\sqrt{P^2}$. Since $\not{P}$ is a linear combination of Γ matrices, $\mathrm{tr}\not{P} = 0$. Then its Jordan form is

$$\sqrt{P^2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5.3}$$

which is the same Jordan form of $\sigma_- + P^2\,\sigma_+$, so we conclude again that eq. (5.1) can be solved.

We have to stress that, even though the twistor is a non linear function of the momentum P, the Lorentz symmetry is linearly realized in the target space. The presence of an internal structure carried by the twisted ghosts does not affect the 10-dimensional covariance. In addition, all degrees of freedom are quantized in a covariant manner, and the physical spectrum is identified with the antibracket cohomology. In appearance, our formalism could remind the reader of a light-cone formulation of the superparticle, however the choice of $\sigma_\pm$ has been adopetd only for the internal indices $\underline{\alpha}$ of the twistors $\lambda^{\alpha,\underline{\alpha}i}$ and it is harmless for the covariance in the target space.

After the twisting, the minimal solution has the form

$$\begin{aligned}
L_{\min} &= P^\mu \partial X_\mu - \bar{\theta}\not{P}\partial\theta - \frac{1}{2}P^2(e - 2\sum \tilde{\bar{k}}^*_p \sigma_+ \tilde{k}_{p+1}) \\
&\quad + X^*_\mu(cP^\mu + \bar{\theta}\Gamma^\mu \not{P}\lambda\tilde{k}_1) + e^*(\partial c + 4\partial\bar{\theta}\lambda\tilde{k}_1) \\
&\quad - 2c^*\tilde{\bar{k}}_1\bar{\lambda}\not{P}\lambda\tilde{k}_1 + \bar{\theta}^*\not{P}\lambda\tilde{k}_1 + \sum \tilde{\bar{k}}^*_p \sigma_- \tilde{k}_{p+1} \\
&\quad + 2e^*\left[X^*_\mu(\bar{\theta}\Gamma^\mu\lambda\tilde{k}_2 - \tilde{\bar{k}}_1\bar{\lambda}\Gamma^\mu\tilde{k}_1) - 4c^*\tilde{\bar{k}}_1\tilde{k}_2 + \bar{\theta}^*\lambda\tilde{k}_2 + \sum \tilde{\bar{k}}^*_p \tilde{k}_{p+2}\right] \tag{5.4}
\end{aligned}$$

The twisted ghosts have off-shell zero modes, given by $\delta\tilde{k} = \sigma_-\tilde{k}'$ (from now on the tilde shall be understood, and all the ghosts are twisted). These zero-modes coincide on-shell with the original ones. We can now eliminate completely the coupling of the non-zero mode part of the ghosts, given by the terms $P^2\bar{k}\sigma_+ k$, simply by a redefinition of the einbein field. This is performed by means of a canonical transformation with generating function

$$F = -2e^{*\prime}\sum_{p\geq 1} \bar{k}^{*\prime}_p \sigma_+ k_{p+1}\,. \tag{5.5}$$

The trivial part of the generating function, corresponding to the identity operator, has been understood, as will be henceforth. The corresponding shifts of the fields are listed

below:

$$\begin{aligned}
e' &= e - 2\sum_{p\geq 1} \bar{k}_p^{*'}\sigma_+ k_{p+1} \\
k_p' &= k_p - 2e^*(-)^p \sigma_+ k_{p+1} \\
\bar{k}_{p+1}^* &= \bar{k}_{p+1}^{*'} - 2e^*\bar{k}_p^{*'}\sigma_+
\end{aligned} \tag{5.6}$$

These redefinitions as well as their inverses do not involve an infinite number of fields due to the presence of the nilpotent factor e^*, so they are allowed transformation in the sense discussed in [12], that is, the redefined field are not subject to any constraint. As a partial check of this statement, we shall now verify that the BRST cohomology is not affected by the transformation. The action is

$$\begin{aligned}
L_{\min} &= P^\mu \partial X_\mu - \frac{1}{2}eP^2 - \bar{\theta}\slashed{P}\partial\theta \\
&\quad + X_\mu^*(cP^\mu + \bar{\theta}\Gamma^\mu \slashed{P}\lambda k_1) + e^*(\partial c + 4\partial\bar{\theta}\lambda k_1) - 2c^*\bar{k}_1\bar{\lambda}\slashed{P}\lambda k_1 \\
&\quad + \bar{\theta}^*\slashed{P}\lambda k_1 + \sum \bar{k}_p^* \sigma_- k_{p+1} \\
&\quad + 2e^* X_\mu^*(\bar{\theta}\Gamma^\mu \lambda k_2 - \bar{k}_1\bar{\lambda}\Gamma^\mu \lambda k_1) - 8e^* c^*\bar{k}_1 \sigma_+ \sigma_- k_2 + 2e^*\bar{\theta}^*\lambda\sigma_+ \sigma_- k_2 \, ;
\end{aligned} \tag{5.7}$$

One can notice that, as was to be expected, the terms $e^* k_n^*$ have disappeared.

The physical sector of the theory is given by the zero-ghost number characteristic cohomology, that is, the cohomology of the BRST operator modulo the equations of motion. More precisely, two classical physical observables are identified if they coincide on the equations of motion; the identification is enforced by considering the cohomology of the Koszul-Tate differential at antifield number 0, i.e. $H_0(\delta)$. The BRST operator γ defined by (2) anticommutes with δ so that it defines a nilpotent operator in the cohomology of δ, and one can consider the group $H(\gamma, H_0(\delta))$. This group is actually isomorphic to $H(s)$ [14].

The relevant BRST transformations are

$$\begin{aligned}
\gamma P &= 0 \\
\gamma X &= cP + \bar{\theta}\Gamma\slashed{P}\lambda k_1 \\
\gamma\theta &= \slashed{P}\lambda k_1 \\
\gamma e &= \partial c + 4\partial\bar{\theta}\lambda k_1
\end{aligned} \tag{5.8}$$

Manifestly P is in the cohomology, and on-shell is a constant light-like vector. It is easy to verify that the vector

$$U^\mu = (X \cdot P)X^\mu - \frac{1}{2}X^2 P^\mu - \frac{1}{2}(\bar{\theta}\Gamma^{\alpha\beta\mu}\theta)X_{[\alpha}P_{\beta]}$$

is BRST invariant, is light-like on-shell, and vanishes for X proportional to P, so it accounts for 8 bosonic degrees of freedom. The fermionic variables in the cohomology are $\slashed{P}\theta$.

6 Gauge-fixing

We have now written the minimal extended action, i.e. a minimal proper solution of the master equation, in a form that is suitable for further manipulations. The next task to be performed is the gauge-fixing of the κ-symmetry. This requires the introduction of a set of non-minimal fields and non-minimal terms in the extended action. We shall follow a completely standard procedure: the non-minimal fields include extra-ghosts χ and Lagrange multipliers π, with opposite statistics, and the non-minimal terms have generically the form $\bar{\chi}^* \pi$. We shall work by steps, at each step fixing the gauge of a given level of fields. At level 0 (the classical fields), we have

$$L_0 = L_{\min} + L_{\mathrm{nm},0} = L_{\min} + \bar{\chi}_1^{1*} \pi_1^1 \, ; \tag{6.1}$$

the gauge-fixing of θ is achieved by the gauge fermion

$$\Psi_0 = \bar{\chi}_1^1 \, \partial(\bar{\lambda}\theta) \, ; \tag{6.2}$$

the canonical transformation generates new terms in the action:

$$L_0 \rightarrow L_0 + \bar{\theta} \, \lambda \partial \pi_1^1 - \partial \bar{\chi}_1^1 \bar{\lambda} \, \slashed{P} \lambda k_1 - 2e^* \partial \bar{\chi}_1^1 \, \sigma_+ \sigma_- k_2; \tag{6.3}$$

and the term proportional to the equations of motion which appears after rewriting $\slashed{P}$ in the twisted form can be eliminated by the canonical transformation

$$\Theta = 2e^{*\prime} \partial \bar{\chi}_1^1 \sigma_+ k_1 \tag{6.4}$$

that gives

$$L_0' = L_0 + \bar{\theta} \, \lambda \partial \pi_1^1 - \partial \bar{\chi}_1^1 \sigma_- k_1 + 2e^* \bar{k}_1 \sigma_+ \partial \pi_1^1 \, . \tag{6.5}$$

The propagator for the physical field θ is now invertible, as is required for a gauge-fixed action.

At level 1,

$$
\begin{aligned}
L_{\mathrm{nm},1} &= \bar{\chi}_2^{1*} \pi_2^1 \, , \\
\Psi_1 &= \bar{\chi}_2^1 \partial k_1 \, , \\
L_1' &= L_0' + (\bar{\chi}_2^{1*} - \partial \bar{k}_1) \pi_2^1 - \partial \bar{\chi}_2^1 \, \sigma_- k_2 \, ;
\end{aligned}
\tag{6.6}
$$

we have fixed the gauge of k_1. A crucial remark is now in order [12]: the ghost χ_1^1 not only has the usual and expected κ-symmetry, but also a more general one, of Stückelberg type, namely

$$\delta \chi_1^1 = \epsilon \, , \qquad \delta \pi_2^1 = \sigma_- \epsilon \, .$$

The gauge-fixed action (the part independent of the antifields) is invariant under this symmetry. We take into account this new symmetry introducing a new ghost, ω_1, and

new terms in the action. These have the form of minimal terms for the new symmetry, but it turns out that other terms are necessary to close the master equation:

$$L_{\text{St}}^{(1)} = (\bar{\chi}_1^{1*} + \bar{\pi}_2^{1*}\sigma_- + \bar{\chi}_3^{3*})\omega_1 - \bar{\chi}_2^{1*}\sigma_-\chi_3^3. \tag{6.7}$$

It can be verified that $L_1' + L_{\text{St}}^{(1)}$ satisfies the master equation, and the cohomology is not altered. This is hardly surprising, when one realizes that $L_{\text{St}}^{(1)}$ is just a non-minimal term written in transformed variables. Indeed, the non minimal term is

$$\bar{\chi}_3^{3*}\omega_1$$

and $L_{\text{St}}^{(1)}$ is recovered via the transformation

$$\Psi = (\bar{\chi}_1^{1*'} + \bar{\pi}_2^{1*'}\sigma_-)\chi_3^3.$$

With a canonical transformation we now redefine the new ghost $\omega_1 \to \omega_1 - \pi_1^1$; as a result, the non-minimal term $\bar{\chi}_1^{1*}\pi_1^1$, previously introduced, is now canceled by the shift of the first term in $L_{\text{St}}^{(1)}$. We can now proceed to fix the Stückelberg symmetry; although it allows for an algebraic fixing, we choose to use a Lorentz gauge-fixing, in order to have more uniformity in the treatment of the various extra-ghosts. We then add further non-minimal terms

$$L_{\text{nm},1}' = \bar{\chi}_2^{2*}\pi_2^2 + \bar{\chi}_3^{1*}\pi_3^1 ; \tag{6.8}$$
$$\Psi_1' = \bar{\chi}_2^2\partial\chi_1^1 + \bar{\chi}_3^1\partial k_2 ;$$

$$\begin{aligned}
L_2 &= L_1' + L_{\text{St}}^{(1)}(\omega_1 \to \omega_1 - \pi_1^1) \\
&+ (\bar{\chi}_2^{2*} - \partial\bar{\chi}_1^1)\pi_2^2 - \partial\bar{\chi}_2^2\omega_1 \\
&+ (\bar{\chi}_3^{1*} - \partial\bar{k}_2)\pi_3^1 - \partial\chi_3^1\sigma_- k_3 .
\end{aligned} \tag{6.9}$$

It is now apparent that the role of the ghost ω_1 is that of a Lagrange multiplier, which fixes the gauge of χ_2^2; thus there is no need to introduce other non-minimal terms and gauge-fixings for χ_2^2 that would bring in the action a mixing of χ_1^1 with higher-level ghosts. The first step is completed; at this point, the ghost k_2 is fixed, while χ_2^1 has the Stückelberg symmetry. As before, we introduce non-minimal terms $\bar{\chi}_4^{3*}\,\omega_2$ followed by

$$\Psi = (\bar{\chi}_2^{1*'} - \bar{\pi}_3^{1*'}\sigma_-)\chi_4^3 ,$$

that yields

$$L_{\text{St}}^{(2)} = (\bar{\chi}_2^{1*} - \bar{\pi}_3^{1*}\sigma_- + \bar{\chi}_4^{3*})\omega_2 + \bar{\chi}_3^{1*}\sigma_-\chi_4^3 ; \tag{6.10}$$

then we perform a canonical transformation

$$\begin{aligned}
\omega_2 &\to \omega_2 - \pi_2^1 + \sigma_-\chi_3^3 \\
\bar{\pi}_2^{1*} &\to \bar{\pi}_2^{1*} + \bar{\omega}_2^* \\
\bar{\chi}_3^{3*} &\to \bar{\chi}_3^{3*} - \bar{\omega}_2^*\sigma_- .
\end{aligned} \tag{6.11}$$

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It is actually sufficient to substitute in the action the shift of ω_2, since the shifts of the antifields do not generate any new term, as they cancel each other. But this canonical transformation leads also to the cancellation of the terms $\bar{\chi}_2^{1*}\pi_2^1$ and $\bar{\chi}_2^{1*}\sigma_-\chi_3^3$. The gauge-fixing is done in perfect analogy with the previous step:

$$L_{\mathrm{nm},2} = \bar{\chi}_3^{2*}\pi_3^2 + \bar{\chi}_4^{1*}\pi_4^1 \,; \tag{6.12}$$

$$\Psi_2 = \bar{\chi}_2^2\partial\chi_1^1 + \bar{\chi}_3^1\partial k_2 \,;$$

$$\begin{aligned}
L_3 &= L_2 + L_{\mathrm{St}}^{(2)}(\omega_2 \to \omega_2 - \pi_2^1 + \sigma_-\chi_3^3) \\
&+ (\bar{\chi}_3^{2*} - \partial\bar{\chi}_2^1)\pi_3^2 - \partial\bar{\chi}_3^2\omega_2 \\
&+ (\bar{\chi}_4^{1*} - \partial\bar{k}_3)\pi_4^1 - \partial\bar{\chi}_4^1\sigma_-k_4 \,.
\end{aligned} \tag{6.13}$$

Now the ghosts k_3, χ_2^1 and χ_3^2 are fixed, while χ_3^1 has a Stückelberg symmetry.

We are now able to give the algorithmic procedure for obtaining the gauge-fixed action at any level: given the action at level n, when the last ghost fixed is k_n and χ_n^1 has the Stückelberg symmetry, one has to perform the following operations:

1) introduce the non-minimal terms corresponding to the Stückelberg symmetry

$$L_{\mathrm{St}}^{(n)} = (\bar{\chi}_n^{1*} + (-)^{n+1}\bar{\pi}_{n+1}^{1*}\sigma_- + \bar{\chi}_{n+2}^{3*})\omega_n + (-)^n\bar{\chi}_{n+1}^{1*}\sigma_-\chi_{n+2}^3 \,; \tag{6.14}$$

2) redefine the Stückelberg ghost:

$$\omega_n \to \omega_n - \pi_n^1 + (-)^n\sigma_-\chi_{n+1}^3 \,; \tag{6.15}$$

this has to be done via a canonical transformation, so there are also other redefinitions which, however, do not have any effect on the action;

3) add non-minimal terms

$$L_{\mathrm{nm},n} = \bar{\chi}_{n+1}^{1*}\pi_{n+1}^1 + \bar{\chi}_{n+1}^{2*}\pi_{n+1}^2 \,; \tag{6.16}$$

4) fix the gauge with the canonical transformation

$$\Psi_n = \bar{\chi}_{n+1}^1\partial k_{n+1} + \bar{\chi}_{n+1}^2\partial\chi_n^1 \,. \tag{6.17}$$

The procedure just consists of canonical operations which can not change the cohomology. The situation at level 3 is depicted in Table 1. We find in this way that only a subset of the whole pyramid of extra-ghosts as is given e.g. in [9] is really necessary for the quantization procedure: an entire sub-pyramid with vertex in χ_3^3 can be omitted. It is easy to count the degrees of freedom and verify that the counting is correct at each step.

Table 1: Fields after the 3rd step

$$
\begin{array}{c}
\theta \\
{\scriptstyle(0)}
\end{array}
$$
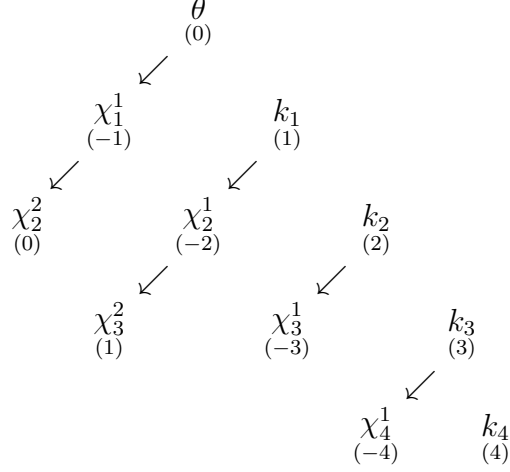
It is useful to look at the Lagrangian at a generic step n. Its form is given by

$$
\begin{aligned}
L_n \;=\;& L_{\min} + 2e^*\bar{k}_1\sigma_+\partial\pi_1^1 + \bar{\theta}\lambda\,\partial\pi_1^1 \\
&- \sum_{p=1}^{n+1}\partial\bar{\chi}_p^1\,\sigma_-\,k_p - \sum_{p=1}^{n}\partial\bar{k}_p\,\pi_{p+1}^1 - \sum_{p=1}^{n-1}\partial\bar{\chi}_p^1\,\pi_{p+1}^2 - \sum_{p=2}^{n}\partial\bar{\chi}_p^2\,\omega_{p-1} \\
&+ \sum_{p=1}^{n-1}\bar{\chi}_p^{1*}\,\omega_p + \sum_{p=2}^{n}\bar{\chi}_p^{2*}\,\pi_p^2 + \sum_{p=3}^{n}\bar{\chi}_p^{3*}\big(\omega_{p-2} - \pi_{p-2}^1 + (-)^p\sigma_-\chi_{p-1}^3\big) \\
&+ \sum_{p=2}^{n}(-)^p\bar{\pi}_p^{1*}\sigma_-\big(\omega_{p-1} - \pi_{p-1}^1\big) \\
&+ \bar{\chi}_n^{1*}\big(\pi_n^1 - \sigma_-\chi_{n+1}^3\big) + \bar{\chi}_{n+1}^{1*}\,\pi_{n+1}^1\,.
\end{aligned}
\tag{6.18}
$$

The complete action, fixed at every level of ghosts, is now obtained simply as the formal limit $n \to \infty$: all the sums in (6.18) run up to infinity, and the two terms in the last line must be dropped. The equations of motion fix all fields to constant values.

Finally, the invariance under diffeomorphisms is algebraically fixed by introducing the non-minimal fields (a Lagrangian multiplier d and the antighost b), a non-minimal term in the action and performing the canonical transformation generated by Ψ_{diff} :

$$
L_{\text{diff}} = b^*d\,, \qquad \Psi_{\text{diff}} = b\,(e-1)\,.
\tag{6.19}
$$

They form a trivial pair under BRST, namely $s\,b = d$ and $s\,d = 0$, so they do not belong to the cohomology.

7 The BRST cohomology

We will now compute the BRST cohomology for the model in the limit $n \to \infty$. At each step of our procedure, it is guaranteed that the cohomology is correct. The limit is the only operation which is not canonical; nevertheless, we will show now that the cohomology is still the correct one. The relevant parts of the BRST transformations for the fermionic sector, in the limit $n \to \infty$, are listed below:

$$s\,\theta = \slashed{P}\lambda k_1 + 2b\lambda\sigma_+\sigma_-k_2\,, \qquad\qquad s\,\theta^* = \slashed{P}\partial\theta + \lambda\partial\pi_1^1\,,$$
$$s\,k_p = \sigma_-k_{p+1}\,, \qquad\qquad s\,k_p^* = (-)^p\sigma_-\left(\partial\chi_p^1 - k_{p-1}^*\right) + \partial\pi_{p+1}^1\,,$$
$$s\,\chi_p^1 = \omega_p\,, \qquad\qquad s\,\chi_p^{1*} = \sigma_-\partial k_p + \partial\pi_{p+1}^2\,,$$
$$s\,\chi_p^2 = \pi_p^2\,, \qquad\qquad s\,\chi_p^{2*} = \partial\,\omega_{p-1}\,,$$
$$s\,\chi_p^3 = \omega_{p-2} - \pi_{p-2}^1 + (-)^p\sigma_-\chi_{p-1}^3\,, \qquad\qquad s\,\chi_p^{3*} = \sigma_-\chi_{p+1}^{3*}\,,$$
$$s\,\pi_p^1 = (-)^p\sigma_-(\omega_{p-1} - \pi_{p-1}^1)\,, \qquad\qquad s\,\pi_p^{1*} = (-)^p\left(\partial k_{p-1} + \chi_{p+2}^{3*}\right) + \sigma_-\,\pi_{p+1}^{1*}\,,$$
$$s\,\pi_p^2 = 0\,, \qquad\qquad s\,\pi_p^{2*} = (-)^p\left(\partial\chi_{p-1}^1 - \chi_p^{2*}\right)\,,$$
$$s\,\omega_p = 0\,, \qquad\qquad s\,\omega_p^* = (-)^p\left(\partial\chi_{p+1}^2 - \chi_p^{1*} - \chi_{p+2}^{3*}\right) - \sigma_-\pi_{p+1}^{1*}\,,$$

where we have omitted the terms coming from the part of the action quadratic in the antifield in eq. (5.7), as well as the terms dependent from the antifields of the bosonic sector and of the diffeomorphism sector. These terms do not change the results of the analysis. In the above list of transformations, one can recognize at once some trivial pairs, e.g. (χ_p^1, ω_p), (χ_p^2, π_p^2); then one can define a reduced cohomology, setting to zero the trivial pairs in the above transformations. Splitting the spinor fields in two components, $\kappa = (\kappa^+, \kappa^-)$, so that $\sigma_-\kappa = (\kappa^-, 0)$, one can see that also (k_p^+, k_{p+1}^-), $(\pi_p^{1-}, \pi_{p+1}^{1+})$ are trivial pairs, as well as $(\chi_p^{3-}, \chi_{p+1}^{3+})$, and finally $(\pi_1^{1+}, \chi_3^{3+})$. We conclude that the only field in the physical spectrum is $\slashed{P}\theta$. Moreover, a careful analysis of the antifield sector along the same lines shows that $\slashed{P}\partial\theta$ is actually BRST-exact, so that only its zero mode survives in the cohomology. In this way, we recover the correct superparticle spectrum.

In the limit, we obtain an action that is free, *i.e.* quadratic in all fields, except for non-polynomial interactions involving the twistor-like fields and cubic interactions involving only the physical fields, the diffeomorphism ghost , and the first κ-symmetry ghost k_1, as well as the corresponding Lagrange multipliers. All the other ghosts are decoupled, so they do not enter in the computation of any amplitude involving only external physical states.

8 Conclusions

Using the conventional BV-BRST methods, we provide the complete gauge-fixed action for the Casalbuoni-Brink-Schwarz superparticle. Despite several difficulties, by means of a suitable redefinition of the minimal sector fields in terms of twisted variables, we are able to construct a quantization procedure of the model. A direct computation of the antibracket cohomology shows that the BRST charge selects the correct physical spectrum of the theory. In addition, it is also verified that the characteristic cohomology $H(\gamma, H(\delta))$ confirms the result of the antibracket analysis.

Acknowledgements

A Notations and conventions

Our conventions are the following. X^m, $m = 0, 1, \ldots, 9$, denotes the 10d space-time coordinates and Γ^m are 10-d Dirac matrices

$$\{\Gamma^m, \Gamma^n\} = 2\eta^{mn}, \quad \text{where } \eta = (- + + \ldots +). \tag{1.1}$$

These Γ's differ by a factor of i from those of ref. [2]. For this choice of gamma matrices the massive Dirac equation is $(\Gamma \cdot \partial - M)\Psi = 0$. In fact, our conventions are such that the quantity $i = \sqrt{-1}$ will not appear in any equations. As it is quite standard, we also introduce $\Gamma_{11} = \Gamma_0 \Gamma_1 \ldots \Gamma_9$, which satisfies $\{\Gamma_{11}, \Gamma^m\} = 0$ and $(\Gamma_{11})^2 = 1$ and the totally antisymmetrized Dirac matrices $\Gamma^{mn\ldots k} = \Gamma^{[m}\Gamma^n \ldots \Gamma^{k]}$.

The Grassmann coordinates θ^α are space-time spinors and world-line scalars. They can be decomposed as $\theta = \theta_R + \theta_L$, where

$$\theta_R = \frac{1}{2}(1 + \Gamma_{11})\,\theta, \qquad \theta_L = \frac{1}{2}(1 - \Gamma_{11})\,\theta. \tag{1.2}$$

For anticommuting spinors, the rule to lower and to rise the spinor indices implies

$$\begin{aligned}
\xi^\alpha &= C^{\alpha\beta}\xi_\beta\,, \\
\frac{\partial}{\partial\theta^\alpha} &= C_{\beta\alpha}\frac{\partial}{\partial\theta_\beta}\,, \\
\bar{\theta}\chi &= \theta^\alpha\chi_\alpha = -\theta_\alpha\chi^\alpha = \chi^\alpha\theta_\alpha = \bar{\chi}\theta\,,
\end{aligned} \tag{1.3}$$

where $C^{\alpha\beta}$ is the antisymmetric charge conjugation matrix. With these conventions, the fermionic degrees of freedom of the BS superparticle are carried by a Majorana-Weyl (MW) spinor θ_R. In the same way all the κ-symmetry ghosts are MW spinors.

In general, the symmetry properties of bilinears built out of MW spinors are the following [29]:

$$\bar{\psi}\Gamma_{\alpha_1...\alpha_n}\chi = (-1)^{1+\epsilon(\psi)\epsilon(\chi)+n(n+1)/2}\ \bar{\chi}\Gamma_{\alpha_1...\alpha_n}\psi. \tag{1.4}$$

There is only one independent non-vanishing bilinear of a single MW-spinor, i.e. $\bar{\chi}\Gamma_{\alpha\beta\gamma}\chi$.

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