Identity of the imaginary-time and real-time thermal propagators for scalar bound states in a one-generation Nambu-Jona-Lasinio model

Bang-Rong Zhou*
Department of Physics,
the Graduate School of Chinese Academy of Sciences,
Beijing 100039, China

By rigorous reanalysis of the results, we have proven that the propagators at finite temperature for scalar bound states in one-generation fermion condensate scheme of electroweak symmetry breaking are in fact identical in the imaginary-time and the real-time formalism. This dismisses the doubt about possible discrepancy between the two formalisms in this problem. Identity of the derived thermal transformation matrices of the real-time matrix propagators for scalar bound states without and with chemical potential and the ones for corresponding elementary scalar particles shows similarity of thermodynamic property between the two types of particles. Only one former inference is modified, i.e. when the two flavors of fermions have unequal nonzero masses, the amplitude of the composite Higgs particle will decay instead grow in time.

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Finite temperature field theory has been extensively researched owing to its application to evolution of early universe and phase transition of the nuclear matter [1-3]. However, the demonstration of the equivalence of its two formalisms - the imaginary-time and the real-time formalism [2] -is often a puzzling problem and has been extensively concerned [4-7]. In a recent research on the Nambu-Goldstone mechanism [8] of dynamical electroweak symmetry breaking at finite temperature based one-generation fermion condensate scheme (a Nambu-Jona-Lasinio (NJL) model [9]), we calculate the propagators for scalar bound states from four-point amputated functions in the two formalisms which seem to show different imaginary parts in their denominators [10]. This difference is quite strange considering that the analytic continuation used in Ref. [10] of the Matsubara frequency in the imaginary-time formalism was made as the way leading to the causal Green functions obtained in the real-time formalism and that in the fermion bubble diagram approximation, the calculations of the four-point amputated functions in a NJL model may be effectively reduced to the ones of two-point functions (though they have been subtracted through use of the gap equation [11,12]), and it is accepted that a two-point function should be equivalent in the two formalisms. Therefore, we have to reexamine the whole calculations in Ref. [10]. We eventually find that the origin of the above difference is that we did not rigorously keep the general form of the analytic continuation and not explicitly separate the imaginary part of the zero-temperature loop integral from relevant expressions. In this paper we will use the main results of the propagators for scalar bound states obtained in Ref. [10], but correct some expressions which were not exact enough and complete rigorous derivation to the final results in both the formalisms. Unless specified otherwise, all the denotations will be the same as those in Ref. [10].

*Electronic mailing address: zhoubr@163bj.com

First we discuss the neutral scalar bound state ϕ_S^0. In the imaginary-time formalism, by means of the analytic continuation of the Matrubara frequency Ω_m of ϕ_S^0

$$-i\Omega_m \longrightarrow p^0+i\varepsilon\eta(p^0), \quad \varepsilon=0_+, \quad \eta(p^0)=p^0/|p^0| \quad (1)$$

we obtain the propagator for ϕ_S^0 [10]

$$\Gamma_I^{\phi_S^0}(p) = -i\sum_Q m_Q^2 \Big/ \sum_Q (p^2-4m_Q^2+i\varepsilon)m_Q^2[K_Q(p)+H_Q(p)-iS_Q^I(p)], \quad (2)$$

where

$$S_Q^I(p)=\eta(p^0)4\pi^2 d_Q(R)\int\frac{d^4l}{(2\pi)^4}\delta(l^2-m_Q^2)\delta[(l+p)^2-m_Q^2]\left[\sin^2\theta(l^0,\mu_Q)\eta(l^0+p^0)+\sin^2\theta(l^0+p^0,\mu_Q)\eta(-l^0)\right]. \quad (3)$$

Eq. (3) is somehow different from Eq. (3.29) in Ref.[10], but is more general since in its derivation the original form (1) of the analytic continuation is always kept; instead, in Ref. [10], $\eta(p^0)$ was replaced by $\eta(\omega_l-\omega_{l+p})$ or $\eta(\omega_{l+p}-\omega_l)$ or ±1 depending on the sign of the pole of p^0 in a term. As will be seen later, Eq.(3) is more suitable to the proof of equivalence of two formalisms. It is indicated that , owing to the factors $\eta(l^0+p^0)$ and $\eta(-l^0)$ in the integrand, $S^I_Q(p)$ in Eq. (3) does not contain any singularity when $p\to0$. The zero-temperature loop integral $K_Q(p)$ is complex and can be written by

$$K_Q(p)=K_{Qr}(p)+K_{Qi}(p). \tag{4}$$

By applying the residue theorem of complex l^0 integral to the first formula in Eq. (3.27) in Ref.[10] we can obtain the imaginary part of $K_Q(p)$

$$\begin{aligned} K_{Qi}(p) &= \frac{d_Q(R)}{16\pi^2}\int\frac{d^3l}{\omega_{Ql}\omega_{Ql+p}}\left[\delta(p^0+\omega_{Ql}+\omega_{Ql+p})\right. \\ &\left.+\delta(p^0-\omega_{Ql}-\omega_{Ql+p})\right]. \end{aligned} \tag{5}$$

The δ-functions in Eq.(5) ensure that $K_{Qi}(p)\neq0$ only if $p^2\geq4m_Q^2$. From Eq. (3) we can derive

$$S^I_Q(p)=R_Q(p)\sinh(\beta|p^0|/2)+K_{Qi}(p), \tag{6}$$

where $R_Q(p)$ is given by Eq.(6.5) in Ref. [10]. Hence the physical causal propagator (2) for ϕ^0_S in the imaginary-time formalism becomes

$$\begin{aligned} \Gamma_I^{\phi_S^0}(p) &= -i\sum_Q m_Q^2/\left\{[k_r+h-ir\sinh(\beta|p^0|/2)]p^2\right. \\ &\left.-4[\tilde{k}_r+\tilde{h}-i\tilde{r}\sinh(\beta|p^0|/2)]\right\}, \end{aligned} \tag{7}$$

where k_r, h, $\tilde{k}_r$, $\tilde{h}$ are defined by Eq. (3.32) and r, $\tilde{r}$ by Eq. (6.3) in Ref.[10]. On the other hand, in the real-time formalism, we must explicitly separate the imaginary part of $K_Q(p)$ as Eq.(4) and this operation was ignored in Ref. [10], thus the correct matrix propagator $\Gamma^{\phi^0_S ba}(p)$ $(b,a=1,2)$ can be obtained from Eq. (6.2) in Ref.[10] by the replacements

$$\begin{aligned} k &\to k_r,\quad \tilde{k}\to\tilde{k}_r, \\ s &\to s'=\sum_Q m_Q^2[S_Q(p)-K_{Qi}(p)]=r\cosh(\beta p^0/2), \\ \tilde{s} &\to \tilde{s}'=\sum_Q m_Q^4[S_Q(p)-K_{Qi}(p)]=\tilde{r}\cosh(\beta p^0/2) \end{aligned} \tag{8}$$

where the relation

$$S_Q(p)-K_{Qi}(p)=R_Q(p)\cosh(\beta p^0/2) \tag{9}$$

has been used. Correspondingly, we will have the replacement

$$\begin{aligned} S/R &= (p^2s-4\tilde{s})/(p^2r-4\tilde{r})\to \\ S'/R &= (p^2s'-4\tilde{s}')/(p^2r-4\tilde{r})=\cosh(\beta p^0/2). \end{aligned} \tag{10}$$

Applying Eqs. (8) and (10) to Eq. (6.10) in Ref.[10], we find that the physical propagator $\Gamma_R^{\phi^0_S}(p)$ for ϕ^0_S in the real-time formalism is identical to the one in the imaginary-time formalism expressed by Eq.(7), i.e. $\Gamma_R^{\phi^0_S}(p)=\Gamma_I^{\phi^0_S}(p)$. In addition, the thermal transformation matrix M_S which diagonalizes the matrix propagator $\Gamma^{\phi^0_S ba}(p)$ $(b,a=1,2)$ will be reduced to

$$\begin{aligned} M_S &= \begin{pmatrix}\cosh\theta_S & \sinh\theta_S\\ \sinh\theta_S & \cosh\theta_S\end{pmatrix}, \\ \sinh\theta_S &= \left[\frac{1}{\exp(\beta|p^0|)-1}\right]^{1/2}. \end{aligned} \tag{11}$$

Hence M_S is identical to the thermal transformation matrix of the matrix propagator for an elementary neutral scalar particle [2]. This fact implies that that the scalar bound state ϕ^0_S and an elementary neutral scalar particle have the same thermodynamic property.

By means of Eq.(4) which is different from $K_Q(p)=K_{Qr}(p)-K_{Qi}(p)$ in Ref.[10] and Eq.(6), the equation to determine the squared mass of ϕ^0_S (Eq. (3.36) in Ref.[10]) will be changed into

$$m^2_{\phi^0_S}=p_r^2=4\left.\frac{(\tilde{k}_r+\tilde{h})(k_r+h)+\tilde{r}r\sinh^2(\beta|p^0|/2)}{(k_r+h)^2+r^2\sinh^2(\beta|p^0|/2)}\right|_{p=p_r}. \tag{12}$$

To reproduce the mass inequalities of ϕ^0_S, we must determine the sign of $R_Q(p)$ in r and $\tilde{r}$. In fact, $R_Q(p)$ given by Eq.(6.5) in Ref.[10] can be rewritten by

$$\begin{aligned} R_Q(p) &= 2\pi^2 d_Q(R)\int\frac{d^4l}{(2\pi)^4}\delta(l^2-m_Q^2)\delta[(l+p)^2-m_Q^2]\sin2\theta(l^0,\mu_Q)\sin2\theta(l^0+p^0,\mu_Q) \\ &= \frac{d_Q(R)}{32\pi^2}\int\frac{d^3l}{\omega_{Ql}\omega_{Ql+p}}\left\{\frac{\delta(p^0-\omega_{Ql}+\omega_{Ql+p})-\delta(p^0-\omega_{Ql}-\omega_{Ql+p})}{\cosh[\beta(\omega_{Ql}+\mu_Q)/2]\cosh[\beta(p^0-\omega_{Ql}-\mu_Q)/2]}-(\omega_{Ql}\to-\omega_{Ql})\right\}. \end{aligned} \tag{13}$$

The δ-functions in first equality of Eq.(13) imply that

$$R_Q(p)=0,\quad\text{when } 0\leq p^2<4m_Q^2. \tag{14}$$

Then from the second equality in Eq.(13), by means of the non-zero conditions of the four δ-functions in the integrand that $\delta(p^0-\omega_{Ql}\pm\omega_{Ql+p})$ $(\delta(p^0+\omega_{Ql}\mp\omega_{Ql+p}))\neq 0$, if $p^2\geq 4m^2$ and $p^0>0$ $(p^0<0)$; $\delta(p^0-\omega_{Ql}+\omega_{Ql+p})$ $(\delta(p^0+\omega_{Ql}-\omega_{Ql+p}))\neq 0$, if $p^2<0$ and $p^0<0$ $(p^0>0)$, we can see that when $\omega_{Ql}>\omega_{Ql+p}$ and $p^0>0$ $(p^0<0)$, only the first and the second (the third and the fourth) term are non-zero ones if $p^2\geq 4m_Q^2$, but they cancel each other after integrating over the variable $\cos\chi=\vec{l}\cdot\vec{p}/|\vec{l}||\vec{p}|$, thus there is no non-zero contribution in these cases; when $\omega_{Ql}<\omega_{Ql+p}$ and $p^0>0$ $(p^0<0)$, the non-zero terms will be the second (the third) one if $p^2\geq 4m_Q^2$ and the fourth (the first) one if $p^2<0$. Considering the signs of these terms and that $\vec{l}$ are integral variables we may reach the conclusion that $R_Q(p)<0$, if $p^2\geq 4m_Q^2$ and $R_Q(p)>0$, if $p^2<0$. In view of Eq.(14) we further have

$$R_Q(p)<0 \text{ or } =0, \quad \text{if} \quad p^2\geq 0. \tag{15}$$

By this result together with $K_{Qr}(p)>0$ and $H_Q(p)>0$ [11], we will obtain from Eq.(12) the well-known mass inequalities

$$4(m_Q)^2_{\min}\leq m^2_{\phi^0_S}\leq 4(m_Q)^2_{\max}. \tag{16}$$

The determination of the sign of $R_Q(p)$ will also change the sign of the imaginary part p_i^0 of the energy of ϕ_S^0 when $0\neq m_U\neq m_D\neq 0$ obtained in Ref.[10]. In fact in present case $p_i^0\simeq b(p_r)/2p_r^0$ with

$$b(p_r)=4\left.\frac{[(\tilde{k}_r+\tilde{h})r-(k_r+h)\tilde{r}]\sinh\frac{\beta|p^0|}{2}}{(k_r+h)^2+r^2\sinh^2\frac{\beta|p^0|}{2}}\right|_{p^2=p_r^2=m^2_{\phi^0_S}}. \tag{17}$$

If we set $M_D=\alpha m_U(\alpha>0)$, then we may write the factor in Eq.(17) $f=(\tilde{k}_r+\tilde{h})r-(k_r+h)\tilde{r}=\alpha^2(1-\alpha^2)m_U^6\{[K_{Ur}(p)+H_U(p)]R_D(p)-[K_{Dr}(p)+H_D(p)]R_U(p)\}$. In view of Eqs.(14) and (15) as well as the fact that $p_r^2=m^2_{\phi^0_S}$ should obey the mass inequalities (16), so if $\alpha<1(m_D<m_U)$, then we will have $R_U(p)=0$ and obtain $f=\alpha^2(1-\alpha^2)m_U^6[K_{Ur}(p)+H_U(p)]R_D(p)<0$. Similarly, if $\alpha>1$ $(m_D>m_U)$ we will have $R_D(p)=0$ and get $f=-\alpha^2(1-\alpha^2)m_U^6[K_{Dr}(p)+H_D(p)]R_U(p)<0$. As a result, opposite to the inference in Ref. [10], we always have $b(p_r)<0$ thus $p_i^0<0$ for positive energy p_r^0. This means that when $0\neq m_U\neq m_D\neq 0$, ϕ_S^0 will decay in time instead of the conclusion that its amplitude will grow in time. This modification comes from the fact that in present calculation we have carefully separated the imaginary part $K_{Qi}(p)$ of the zero-temperature loop integral from relevant expressions e.g. $K_Q(p)$, $S_Q(p)$ and $S_Q^I(p)$ etc. and determined the sign of $R_Q(p)$. The same correction is also applicable to the case of $T\to 0$. When $T=0$, if $m_U\neq m_D$, based on the results that if $p^2<4m_Q^2$ $K_{Qi}(p)=0$ and if $p^2\geq 4m_Q^2$ $K_{Qi}>0$ obtained from Eq.(5) by direct calculation (instead of $K_{Qi}<0$ by assumption in Ref.[10]) and the similar demonstration to above, we may conclude that the amplitude of ϕ_S^0 will also decay instead grow in time.

Next we turn to the neutral pseudoscalar bound state ϕ_P^0. The discussion is almost parallel to the one of ϕ_S^0. In the imaginary-time formalism, by keeping the original form of Eq.(1) and using Eq. (6) we may change the physical causal propagator for ϕ_P^0 expressed by Eq.(4.8) in Ref.[10] into

$$\Gamma_I^{\phi_P^0}(p)=-i\frac{\sum_Q m_Q^2}{(p^2+i\varepsilon)[k_r+h-ir\sinh(\beta|p^0|/2)]}. \tag{18}$$

In the real-time formalism, we only need in Eq. (6.13) in Ref.[10] simply to make the replacements $k\to k_r$, $s\to s'$ and $s/r\to s'/r=\cosh(\beta|p^0|/2)$ given by Eq.(8) and will obtain correct matrix propagator $\Gamma^{\phi_P^0 ba}(p)$ $(b,a=1,2)$ for ϕ_P^0. Then diagonalization of $\Gamma^{\phi_P^0 ba}(p)$ $(b,a=1,2)$ by the thermal transformation matrix M_P will lead to the physical causal propagator $\Gamma_R^{\phi_P^0}(p)$ for ϕ_P^0 which is proven to satisfy $\Gamma_R^{\phi_P^0}(p)=\Gamma_I^{\phi_P^0}(p)$, i.e. the physical causal propagator for ϕ_P^0 has identical expression in the two formalisms. In addition, the derived M_P is equal to M_S given by Eq.(11), thus the thermal transformation matrix of the matrix propagator for the neutral pseudoscalar bound state ϕ_P^0 is also the same as the one for an elementary neutral scalar particle.

Lastly we discuss the propagator for charged scalar bound states $\phi^\mp$. In the imaginary-time formalism, by the analytic continuation of the Matsubara frequency Ω_m

$$-i\Omega_m+\mu_D-\mu_U\longrightarrow p^0+i\varepsilon\eta(p^0), \quad \varepsilon=0_+, \tag{19}$$

we will obtain the physical causal propagator for ϕ^- (and ϕ^+) [10]

$$\Gamma_I^{\phi^-}(p)=-i/\left\{(p^2+i\varepsilon)\left[K_{UD}(p)+H_{UD}(p)\right]+E_{UD}(p)-i(p^2-\bar{M}^2+i\varepsilon)S^I_{UD}(p)\right\}, \tag{20}$$

where we express alternatively

$$K_{UD}(p)=\frac{1}{p^2+i\varepsilon}\frac{4d_Q(R)}{m_U^2+m_D^2}\int\frac{id^4l}{(2\pi)^4}\frac{(m_D^2-m_U^2)l\cdot p-m_U^2(p^2+i\varepsilon)}{(l^2-m_U^2+i\varepsilon)[(l+p)^2-m_D^2+i\varepsilon]} \tag{21}$$

which is actually equal to Eq.(5.25) in Ref.[10] and

$$S^I_{UD}(p) = \eta(p^0) 4\pi^2 d_Q(R) \int \frac{d^4 l}{(2\pi)^4} \delta(l^2 - m_U^2)\delta[(l+p)^2 - m_D^2] \left[\sin^2\theta(l^0, \mu_U)\eta(l^0+p^0) + \sin^2\theta(l^0+p^0, \mu_D)\eta(-l^0)\right]. \tag{22}$$

which differs from Eq.(5.28) in Ref.[10] and is the result of rigorously keeping the general form of the right-handed side of Eq.(19). By applying the residue theorem of complex l^0 integral to Eq.(21), we may find out the imaginary part of $K_{UD}(p)$

$$K_{UDi}(p) = \left[1 - \frac{p^2}{(p^2)^2 + \varepsilon^2}\bar{M}^2\right] \Delta_{UD}(p), \tag{23}$$

$$\begin{aligned} \Delta_{UD}(p) \;=\; & \frac{d_Q(R)}{16\pi^2} \int \frac{d^3 l}{\omega_{Ul}\omega_{Dl+p}} \left[\delta(p^0 + \omega_{Ul} + \omega_{Dl+p}) \right. \\ & \left. + \delta(p^0 - \omega_{Ul} - \omega_{Dl+p})\right]. \end{aligned} \tag{24}$$

Noting that when $m_U = m_D = m_Q$ we will have $K_{UDi}(p) = \Delta_{UD}(p)$ to be reduced to $K_{Qi}(p)$ in Eq.(5). If we explicitly write $K_{UD}(p) = K_{UDr}(p) + iK_{UDi}(p)$ and use the relation

$$S^I_{UD}(p) - \Delta_{UD}(p) = R_{UD}(p)\eta(p^0)\sinh\frac{\beta(p^0-\mu)}{2}, \tag{25}$$

where $R_{UD}(p)$ was given by Eq.(6.21) in Ref.[10] and $\mu = \mu_D - \mu_U \equiv \mu_{\phi^-}$ is the chemical potential of ϕ^-, then Eq.(20) will be changed into

$$\Gamma_I^{\phi^-}(p) = -i\Big/\left\{(p^2 + i\varepsilon)\left[K_{UDr}(p) + H_{UD}(p)\right] + E_{UD}(p) - i(p^2 - \bar{M}^2 + i\varepsilon)R_{UD}(p)\eta(p^0)\sinh\frac{\beta(p^0-\mu)}{2}\right\}. \tag{26}$$

In the real-time formalism, it is indicated that in the expression for the matrix propagator $\Gamma^{\phi^- ba}(p)$ $(b,a=1,2)$ given by Eq.(6.19) in Ref.[10], the fact that $K_{UD}(p)$ is complex was ignored. Now if taking this into account and noting Eq.(23), we will obtain correct expression for $\Gamma^{\phi^- ba}(p)$ $(b,a=1,2)$ from Eq.(6.19) in Ref.[10] and successive modified results by means of the replacements

$$\begin{aligned} K_{UD}(p) &\to K_{UDr}(p), \\ S_{UD}(p) &\to S'_{UD}(p) = S_{UD}(p) - \Delta_{UD}(p) \end{aligned} \tag{27}$$

and the derived relation

$$S'_{UD}(p) = R_{UD}(p)\cosh[\beta(p^0-\mu)/2],$$

$$\sqrt{S'^2_{UD}(p) - R^2_{UD}(p)} = R_{UD}(p)\eta(p^0)\sinh[\beta(p^0-\mu)/2]. \tag{28}$$

It is proven that through diagonalization of $\Gamma^{\phi^- ba}(p)$ $(b,a=1,2)$ by the thermal transformation matrix M_C the resulting physical propagator $\Gamma_R^{\phi^-}(p)$ will have identical form to $\Gamma_I^{\phi^-}(p)$ in Eq.(26). This shows equivalence of the two formalisms once again. In addition, M_C will have the expression

$$M_C = \begin{pmatrix} \cosh\theta_C & e^{-\beta\mu/2}\sinh\theta_C \\ e^{\beta\mu/2}\sinh\theta_C & \cosh\theta_C \end{pmatrix}, \qquad \sinh\theta_C = \left[\frac{\theta(p^0)}{e^{\beta(p^0-\mu)}-1} + \frac{\theta(-p^0)}{e^{\beta(-p^0+\mu)}-1}\right]^{1/2}. \tag{29}$$

Eq.(29) shows that the thermal transformation matrix M_C of the matrix propagator for the charged scalar bound state ϕ^- with chemical potential μ is identical to the one for an elementary charged scalar particle with chemical potential μ (noting that M_C in Eq. (29) differs from usual one [2] by a transpose since our original definition of the matrix $\Gamma^{ba}_{\phi^-}(p)$ $(b,a=1,2)$ is just so).

In conclusion, by means of keeping general expressions of the analytic continuations of the Matsubara frequencies in the imaginary-time formalism and separating explicitly the imaginary parts of the zero-temperature loop integrals from relevant expressions e.g. $S^I_Q(p)$, $S_Q(p)$, $S^I_{UD}(p)$ and $S_{UD}(p)$ etc., we have reanalyzed the results in Ref.[10] and proven identity of the physical causal propagators for every scalar bound state in the two formalisms of thermal field theory in the one generation NJL model. This dismisses the doubt about possible discrepancy between the two formalisms in this problem. Next the derived identity between the thermal transformation matrices of the matrix propagators for scalar bound states and corresponding elementary scalar particles including the case without and with chemical potential indicates similarity of thermodynamic property between these two types of particles, even though these bound states could be linear combinations of the scalar or pseudoscalar configurations of the Q-fermions with different flavors. The reanalysis have not changed the main conclusions of the Nambu-Goldstone mechanism at finite temperature reached in Ref.[10] except that the composite Higgs ϕ^0_S will decay in time instead of its amplitude's growing in time when the two flavors of fermions have unequal nonzero masses.

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