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SUSY 3D Georgi - Glashow model at finite temperature

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Abstract

We study the finite-temperature properties of the supersymmetric version of (2+1)D Georgi-Glashow model. As opposed to its nonsupersymmetric counterpart, the parity symmetry in this theory at zero temperature is spontaneously broken by the bilinear photino condensate. We find that as the temperature is raised, the deconfinement and the parity restoration occur in this model at the same point $T_c = g^2/8\pi$. The transition is continuous, but is not of the Ising type as in nonsupersymmetric Georgi-Glashow model, but rather of the Berezinsky-Kosterlitz-Thouless type as in Z_4-invariant spin model.

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During the last two years it has been realized that the finite-temperature structure of weakly interacting 3D non-Abelian gauge theories can be analysed exactly. It has also been found that these theories exhibit many phenomena similar to what one expects to find in 4D QCD and thus are useful solvable toy models for the study of the deconfining dynamics at finite temperature. In particular, various properties of the deconfining phase transition in the $SU(2)$ Georgi-Glashow model have been understood. The order of the phase transition as well as the universality class have been established explicitly without recourse to universality arguments, and the dynamics of the phase transition was given a simple interpretation in terms of restoration of the magnetic symmetry [1]. In subsequent work the effects of instantons at high temperature have been understood in detail, the dynamics of the deconfining transition has been related to the properties of confining strings, and the analysis has also been extended to the $SU(N)$ Georgi-Glashow model at $N > 2$ [2]. Also some interesting analogies between the mechanism of the deconfining transition in 2+1 dimensions and the chiral-symmetry restoration in QCD have been suggested [3]. These results have been reviewed and summarized in [4]. Recently, it has also been shown that the presence of heavy dynamical fundamental quarks turns the second-order deconfining transition into analytic but rather fast crossover [5]. Finally, the effects of variability of the Higgs-field mass, as well as the effects of light fundamental fermions on the monopole interaction have been studied in Ref. [6].

In the present note, we continue this line of investigation and consider the supersymmetric generalization of the 3D Georgi-Glashow model at finite temperature. The interest in this model is that it contains adjoint fermions whose masslessness is protected by the discrete parity symmetry. At zero temperature, the parity is spontaneously broken via a nonvanishing photino condensate. Thus, at finite temperature one may anticipate two phase transitions – one related to the vanishing of the photino condensate and the other one due to deconfinement. These two transitions could either be distinct and happen at different temperatures, or could coincide. In this respect the model is similar to QCD with adjoint quarks, where a similar question can be asked about the (non-)coincidence of deconfinement and restoration of discrete chiral symmetry.

The Lagrangian of supersymmetric Georgi-Glashow (SGG) model contains the bosonic fields of the non-supersymmetric Georgi-Glashow (GG) model, that are the light photon, the heavy $W^{\pm}$ vector bosons and the massive Higgs field, as well as their superpartners – photino, winos and Higgsino. It was shown in [7] that just like in the GG model the monopole effects render the photon massive, although the mass in this case is parametrically smaller, since it is due to the contribution from a two-monopole sector, rather than a single-monopole sector as in the GG model. Since supersymmetry is not broken, the low-energy sector of the theory contains in addition to the photon, the light photino and is described by the supersymmetric sine-Gordon model. Its Euclidean action in the superfield notation reads [1]

$$S = -\int d^3x d^2\theta \left[\frac{1}{2}\Phi \bar{D}_\alpha D_\alpha \Phi + \xi \cos\left(g_m \Phi\right)\right]. \tag{1}$$

In this equation, the scalar supermultiplet and supercovariant derivatives have the form

[1] We adopt here the notations of Ref. [8], in particular $\int d^2\theta\bar{\theta}\theta = 1$.

$$\Phi(\mathbf{x},\theta) = \chi + \bar{\theta}\lambda + \frac{1}{2}\bar{\theta}\theta F, \quad D_\alpha = \frac{\partial}{\partial\bar{\theta}_\alpha} - (\hat{\partial}\theta)_\alpha, \quad \bar{D}_\alpha = \frac{\partial}{\partial\theta_\alpha} - (\bar{\theta}\hat{\partial})_\alpha. \tag{2}$$

Here, χ is the dual-photon field (real scalar), λ is the photino field, which is the two-component Majorana spinor ($\bar{\lambda} = \lambda^T\sigma_2$), F is an auxiliary scalar field, $\hat{\partial} \equiv \gamma_i\partial_i$, and the Euclidean γ-matrices coincide with the Pauli matrices: $\vec{\gamma} = \vec{\sigma}$. The "magnetic coupling" g_m is related to the gauge coupling of the SGG model as $g_m = 4\pi/g$ and has dimensionality $[\text{mass}]^{-1/2}$. The coefficient ξ is the monopole fugacity of dimensionality $[\text{mass}]^2$ and is exponentially small. The interaction term in Eq. (1) is frequently understood as normal ordered. In this case, the fugacity in terms of the mass of the W-bosons (in the BPS limit) is $\xi \propto \exp\left(-4\pi M_W/g^2\right)$ [9, 7]. We will find it however more convenient to use the non-normal ordered form of the interaction. In this case ξ is not as small, but still has an exponential smallness $\xi \propto \exp(-S_{\text{core}})$, where S_{core} is the action of the monopole core. The action S_{core} is the contribution to the monopole action due to heavy particles – W-bosons, Higgs and their superpartners, and is a number of order $O\left(\frac{M_W}{g^2}\right)$. All results of the present note are valid to the lowest order in this parameter.

In the component notations, the action (1) can be readily rewritten up to a constant as (cf. also Ref. [7])

$$S = \int d^3x \left[\frac{1}{2}(\partial_i\chi)^2 - \frac{1}{2}\bar{\lambda}\hat{\partial}\lambda - \frac{g_m^2\zeta}{2}\left(V^2 + V^{*2}\right)\bar{\lambda}\lambda - \frac{(g_m\zeta)^2}{2}\left(V^4 + V^{*4}\right)\right], \tag{3}$$

where $\zeta = \xi/4$, and we have defined the vortex operator

$$V(x) = \exp\left(i\frac{2\pi}{g}\chi\right). \tag{4}$$

Just like the GG model, the model (3) has a magnetic Z_2 symmetry [1]. It is easiest recognized by its action on the order parameter, the vortex field:

$$V(x) \to -V(x). \tag{5}$$

Besides the magnetic Z_2 symmetry, the effective action (3) has an additional discrete parity symmetry inherited from the full SGG action,

$$V(x_1,x_2,x_3) \to iV(-x_1,x_2,x_3), \qquad \lambda(x_1,x_2,x_3) \to \sigma_3\lambda(-x_1,x_2,x_3). \tag{6}$$

The photino mass term is odd under the parity transformation (6). Thus, the photino can acquire a mass only if the parity is spontaneously broken. It is indeed easy to see that this is the case in the effective Lagrangian (3). The potential of the dual-photon field is minimized at $\langle\chi\rangle = 0$, or $\langle V\rangle = 1$. The real expectation value of V violates both the magnetic Z_2 symmetry and the parity symmetry. The spontaneous breaking of the magnetic Z_2 symmetry is synonymous with confinement [10]. The breaking of parity results in the non vanishing photino condensate

$$\left\langle\bar{\lambda}\lambda\right\rangle \sim g_m^2\zeta\Lambda. \tag{7}$$

The ultraviolet cutoff in the effective theory (3) is of course of the order of M_W – the mass of the W bosons.

The breaking of parity leads to the non vanishing photino mass $m = 2g_m^2\zeta$. On the classical level, the mass of the photon can be read off from the photon self-interaction term. The photon and the photino are of course degenerate as a consequence of an unbroken supersymmetry. It is in fact quite amusing to see how this degeneracy is preserved on the quantum level. The simplest loop corrections are those due to summation of "bubble diagrams", or "normal-ordering" corrections. Taking those into account, the photino mass becomes

$$m = g_m^2\zeta\left\langle V^2 + V^{*2}\right\rangle = 2g_m^2\zeta \mathrm{e}^{-\frac{2\pi}{g^2}\Lambda}. \tag{8}$$

On the other hand, when we examine the normal-ordering corrections to the photon self-interaction, the result is

$$\frac{(g_m\zeta)^2}{2}\left(V^4 + V^{*4}\right) = \frac{(g_m\zeta)^2}{2}\mathrm{e}^{-\frac{8\pi}{g^2}\Lambda} : \left(V^4 + V^{*4}\right) : \ . \tag{9}$$

Thus, on the quantum level the self-interaction term gives the contribution to the photon mass which is exponentially smaller than the mass of the photino! This of course does not mean that the supersymmetry is broken, but merely that the main contribution to the photon mass comes from the diagrams containing the photino.

To calculate the photon mass to the order $O(\zeta)$, we have to find the effective potential to the order $O(\zeta^2)$. Integration over the photino yields the following $O(\zeta^2)$-contribution to the effective action:

$$-\frac{(g_m^2\zeta)^2}{32\pi^2}\int d^3x \int d^3y \frac{1}{(x-y)^4}\left[V^2(x) + V^{*2}(x)\right]\left[V^2(y) + V^{*2}(y)\right]. \tag{10}$$

The effective potential is obtained by further integration over the field χ, keeping the zero-momentum mode of χ as a fixed background. Since the integral is dominated by the distances $|x-y|$ much smaller than the inverse photon mass, we can with the exponential accuracy take $\chi(x)$ as a free massless field. We then get

$$U_{\text{eff}} = -\frac{(g_m^2\zeta)^2}{32\pi^2}\mathrm{e}^{-\frac{4\pi}{g^2}\Lambda}\left[\int d^3(x-y)\frac{1}{(x-y)^4}\mathrm{e}^{-\frac{4\pi}{g^2|x-y|}}\right]\left[V^4 + V^{*4}\right] + \text{const.} \tag{11}$$

The integral over $x-y$ is easily performed with the expected result

$$U_{\text{eff}} = -\frac{(g_m\zeta)^2}{2}\mathrm{e}^{-\frac{4\pi}{g^2}\Lambda}\left(V^4 + V^{*4}\right), \tag{12}$$

so that indeed the equality of the photon and photino masses is preserved.

An interesting property of this calculation is that the main contribution to the effective potential in the integral (11) comes from the distances of order $O\left(\frac{1}{g^2}\right)$. The contribution of large distances is suppressed by the photino propagator, while the short distances are cut off by the photon propagator in the exponential in Eq. (11). The saddle point of the integral in Eq. (11) is in fact at $|x-y| = \frac{2\pi}{g^2}$. The reason this is of some interest is that as we know from the study of the GG model at finite temperature, it is exactly in the range of temperatures $T \sim g^2$, where the interesting phase transitions occur.

Let us now turn to the study of the model at finite temperature. We will be mostly interested in temperatures of order g^2. Since this is much higher than both the photon and photino masses, the proper way to proceed is via dimensionally reduced theory of the zero Matsubara mode. To derive it, we have to integrate out the fermions and the nonzero Matsubara modes of the photon field. Technically, to the order $O(\zeta^2)$ this calculation is very similar to the one just performed. One keeps the zero Matsubara mode of χ as fixed external background and integrates over the rest of the degrees of freedom. Omitting the exponentially suppressed terms, the result is

$$S = \int d^2x \left[\frac{\beta}{2}(\partial_i \chi)^2 - \frac{\bar{\zeta}^2}{2}\left(V^4 + V^{*4}\right)\right] \tag{13}$$

with

$$\bar{\zeta}^2 = \zeta^2 \beta g_m^2 \mathrm{e}^{-\frac{4\pi}{g^2}\Lambda} \int d^3x D_\beta^2(x) \mathrm{e}^{-\frac{16\pi^2}{g^2}G_\beta(x)}. \tag{14}$$

Here, $\beta = 1/T$, $D_\beta(x)$ is the finite-temperature massless-fermion propagator, and $G_\beta(x)$ is the finite-temperature massless-boson propagator with the contribution of zero Matsubara frequency subtracted.

The exact value of $\bar{\zeta}$ is not important. It is however clear that it is positive for any finite temperature and at temperatures of interest it is in fact parametrically of the same order as at zero temperature. The issue of sign is important for the following reason. If $\bar{\zeta}^2$ were to change sign at some temperature T^*, the classical vacuum of the potential in Eq. (13) would shift from $\langle V\rangle = \pm 1$ to $\langle V\rangle = \frac{1\pm i}{\sqrt{2}}$. At this new value, we would have $\langle V^2 + V^{*2}\rangle = 0$, and parity would be restored. This would not be a deconfining transition, since $\langle V\rangle \neq 0$, and thus the magnetic Z_2 symmetry would remain broken. The change of sign of $\bar{\zeta}^2$ thus would mean that the deconfining phase transition is preceded by the parity restoration.

However, it is easy to see that this does not happen in our model. The integral in $\bar{\zeta}^2$ is explicitly positive, since for any temperatures $D_\beta(x)$ and $G_\beta(x)$ are both real functions. The use of thermal propagators effectively limits the integration region to $|x| < 2\pi\beta$. Since, as we noted above, the main contribution to the integral comes from the distances $|x| \sim 2\pi/g^2$, this means that for temperatures of interest ($T \sim g^2$) the finite part of the relevant integration region contributes, and thus the integral parametrically has the same value as at zero temperature.

As discussed in [1, 4], to study the deconfining phase transition one cannot neglect the heavy charged degrees of freedom. The thermal excitation of W bosons (and their superpartners) leads to appearance of extra operators in the high-temperature effective action. As discussed in detail in [1, 4], the most important such operator is the adjoint Abelian Polyakov line, which is the variable dual to the vortex operator. The respective complete Lagrangian is

$$S = \int d^2x \left[\frac{\beta}{2}(\partial_i \chi)^2 - \frac{\bar{\zeta}^2}{2}\left(V^4 + V^{*4}\right) - \mu\left(P^2 + P^{*2}\right)\right], \tag{15}$$

where

$$P = \exp\left(i\frac{\tilde{\chi}}{2}\right) \tag{16}$$

and $\tilde{\chi}$ is the field dual to χ:

$$i\partial_\mu \tilde{\chi} = \frac{g}{T}\epsilon_{\mu\nu}\partial^\nu \chi. \tag{17}$$

The last relation is valid modulo quantized discontinuities in the phase χ and $\tilde{\chi}$, and more properly

$$iP^*\partial_\mu P = \frac{g^2}{4\pi T}\epsilon_{\mu\nu}V^*\partial_\nu V. \tag{18}$$

The parameter μ is proportional to the fugacities of heavy charged particles – W bosons and winos:

$$\mu \propto \exp\left(-\frac{M_W}{T}\right). \tag{19}$$

The 2D models of the type of Eq. (15) have been extensively studied in the literature starting with [11]. For a recent discussion see [12]. For $T < \frac{g^2}{8\pi}$, the last term in the action, which contains the Polyakov loops, is irrelevant, and can be neglected in discussing the infrared physics. At these low temperatures the photon self-interaction term $V^4 + c.c.$ is relevant, and the vortex operator has a non vanishing expectation value. At $T = T_c = \frac{g^2}{8\pi}$ it becomes irrelevant. If not for the Polyakov loop, the theory would be in a massless phase above this temperature with the Berezinsky-Kosterlitz-Thouless transition between the phases [13]. Just like in the GG model, the transition into the massless phase corresponds to logarithmic binding of monopoles (or rather monopole pairs in the present theory) into molecules [14]. However, the Polyakov loop becomes relevant precisely at the same temperature T_c and renders the theory massive also in the high-temperature phase. The phase transition at T_c remains a continuous one. The critical conformal theory has the central charge $c = 1$ and is the theory of one massless scalar field.

Note that as opposed to the GG model where the value of the critical temperature depends on the Higgs mass [1, 15], in the present case T_c appears to be independent of the Higgs mass with exponential accuracy. The difference is in the fact that in the GG case both the monopole and charge induced operators where relevant at the transition point. The value of the temperature was determined by the equality of (renormalized) monopole and charge fugacities [15]. In the SUSY case, however, both operators are irrelevant at T_c, and the values of respective fugacities are of no importance as long as they are small.

The transition at T_c is clearly a deconfining transition. The magnetic Z_2 symmetry is restored, and the expectation value of the vortex operator vanishes, $\langle V\rangle = 0$. It is interesting that the parity is also restored at the same point. Above the transition, the monopole-induced photon self-interaction term is irrelevant. Thus, the infrared physics at $T > T_c$ is described by the Lagrangian

$$S = \int d^2x \left[\frac{T}{2g^2}(\partial_i\tilde{\chi})^2 - \mu\left(P^2 + P^{*2}\right)\right]. \tag{20}$$

This is exactly the same as in the nonsupersymmetric model [2]. The order parameter for parity is $\langle V^2\rangle + c.c.$. The average $\langle V^2\rangle$ was calculated in the first paper of [2] and found to be nonzero. One could therefore suspect that in the present case this average is also non vanishing and the parity remains broken at high temperature. This is however not the case. As explained in [2], to calculate $\langle V^2\rangle$ it is not sufficient to consider the infrared limit (20), but is rather necessary to include also the monopole contributions. In fact, this particular expectation value is dominated completely by the contribution of the one-monopole sector. The reason is that the insertion of V^2 into the path integral induces an (anti)vortex of the field P. Due to the Debye mass term $P^2 + c.c.$, the vortex is accompanied by a string and has a linearly infrared divergent action. In the GG model, a monopole also creates a vortex of P and thus the string emanating from V^2 can end on it. The one-monopole sector thus has a finite action in the presence of V^2 and dominates the expectation value. The situation in the present case is quite different in this respect. A single monopole gives a vanishing contribution to the partition function due to the photino zero modes. The only contributions to partition function one can consider are those originating from the action (15) in expansion in powers of ζ^2. Those are contributions of even number of monopoles and are obviously equivalent to insertions of integer powers of V^4. Each such insertion creates two rather than one strings, and thus cannot screen a single insertion of V^2. We thus conclude that all contributions to $\langle V^2 \rangle$ have linearly infrared divergent action and thus in the thermodynamic limit $\langle V^2 \rangle = 0$. The parity symmetry is therefore restored at all temperatures above T_c.

To summarize, we find that in the present model confinement disappears and parity is restored at the same critical temperature $T_c = g^2/8\pi$. The phase transition is the same as in the Z_4-invariant spin model. In fact, the dimensionally-reduced theory (15) has Z_4 global symmetry rather than the $Z_2 \otimes Z_2$ magnetic symmetry plus parity. The action eq.(15) also has a separate parity symmetry under which the vortex field does not transform. The reason for this symmetry enhancement is that the only degrees of freedom which couple the parity transformation with part of the Z_4 group ($V \to iV$) are photinos. In the absence of fermions, the original Lagrangian (3) indeed has the full Z_4 symmetry supplemented by parity. At high temperature, where the reduced theory (15) is valid, the fermions are "heavy", or better to say all their correlation functions are short range. Thus, they indeed disappear from the infrared theory, and the symmetry is effectively enhanced. It is due to this symmetry enhancement that the deconfining and parity restoring transitions happen at the same temperature. While this is an interesting phenomenon, it seems somewhat non generic. In particular, in (3+1)D gauge theory with adjoint fermions there is no reason to expect the deconfining and chiral symmetry restoring phase transitions to coincide. The physical order parameter for deconfinement is the 't Hooft loop V [16], while for chiral symmetry it is the fermionic bilinear form $\lambda\lambda$. In 3+1 dimensions, the two have very different nature. While $\lambda\lambda$ is a local field, V is a string-like object. It is thus difficult to imagine the two combining into a single order parameter as it is the case in the (2+1)D theory discussed in the present note. The lattice results indeed suggest that at least in the $SU(3)$-theory in 3+1 dimensions the two transitions are distinct [17].

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