

Accelerated Unification

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 (Dated: August, 2001)

We construct four dimensional gauge theories in which the successful supersymmetric unification of gauge couplings is preserved but accelerated by N -fold replication of the MSSM gauge and Higgs structure. This results in a low unification scale of $10^{13/N}$ TeV.

The unification of gauge couplings [1, 2] in the minimal supersymmetric standard model (MSSM) [3, 4] is one of the strongest hints for the structure of physics at and above the TeV scale. This success is so striking it seems unlikely to have arisen accidentally. Usually it is argued that this unification depends on the absence of new particles and interactions beyond those of the MSSM and a desert stretching from the electroweak scale at $v \sim 1$ TeV to the GUT scale near $M_G \sim 10^{13}$ TeV. We have of course never directly observed unified gauge couplings at such a high scale—instead we infer this unification by scaling unified couplings from M_G to the low scale v , which gives the experimentally successful relations

$$\frac{2\pi}{\alpha_3(v)} - \frac{2\pi}{\alpha_2(v)} = (b_3 - b_2)_{\text{MSSM}} \log(M_G/v) \quad (1)$$

$$\frac{2\pi}{\alpha_2(v)} - \frac{2\pi}{\alpha_1(v)} = (b_2 - b_1)_{\text{MSSM}} \log(M_G/v) \quad (2)$$

These relations are obtained by assuming the absence of any charged matter in the energy desert between v and M_G other than multiplets, like the MSSM generations, that fill out complete $SU(5)$ representations. Such representations make equal contributions to $b_{3,2,1}$ and thus affect neither the success of unification nor the scale M_G . Unification thus follows from the MSSM gauge and Higgs structure.

Successful unification is usually viewed as strong evidence for a high fundamental scale. We show that this conclusion is unwarranted: by replicating the MSSM gauge and Higgs structure N times we construct models in which the gauge couplings unify as in the MSSM but at a far lower scale, $M_U \sim 10^{13/N}$ TeV.

Imagine N copies of the MSSM gauge and Higgs structure, with fine-structure constants $\alpha_3^{(i)}, \alpha_2^{(i)}, \alpha_1^{(i)}$, where at the scale M_U $\alpha_3^{(i)} = \alpha_2^{(i)} = \alpha_1^{(i)} \equiv \alpha_0^{(i)}$. The relative running of each copy of the $G_i \equiv SU(3)_i \times SU(2)_i \times U(1)_i$ gauge couplings is as in the MSSM:

$$\frac{2\pi}{\alpha_3^{(i)}(\mu)} - \frac{2\pi}{\alpha_2^{(i)}(\mu)} = (b_3 - b_2)_{\text{MSSM}} \log(M_U/\mu) \quad (3)$$

$$\frac{2\pi}{\alpha_2^{(i)}(\mu)} - \frac{2\pi}{\alpha_1^{(i)}(\mu)} = (b_2 - b_1)_{\text{MSSM}} \log(M_U/\mu) \quad (4)$$

Now suppose that at the scale v we higgs these N copies of the MSSM gauge group down to the diagonal MSSM gauge group. The diagonal gauge couplings obtained by

tree level matching at the scale v are:

$$\frac{2\pi}{\alpha_{3,2,1}(v)} = \sum_{i=1}^N \frac{2\pi}{\alpha_{3,2,1}^{(i)}(v)} \quad (5)$$

Combining these equations we obtain the relative size of the low energy gauge couplings

$$\frac{2\pi}{\alpha_3(v)} - \frac{2\pi}{\alpha_2(v)} = (b_3 - b_2)_{\text{MSSM}} \log(M_U^N/v^N) \quad (6)$$

$$\frac{2\pi}{\alpha_2(v)} - \frac{2\pi}{\alpha_1(v)} = (b_2 - b_1)_{\text{MSSM}} \log(M_U^N/v^N) \quad (7)$$

This is identical to the MSSM result (1),(2) with

$$M_U = v \left(\frac{M_G}{v} \right)^{1/N} \sim 10^{13/N} \text{ TeV} \quad (8)$$

In equations (6),(7) we have assumed that the fields responsible for higgsing to the diagonal subgroup fill out complete $SU(5)$ multiplets and do not affect the relative running. We have also assumed that the unification scale is the same for each group G_i .

We now describe an explicit model which implements this simple mechanism. The model is conveniently summarized in the “moose” (or “quiver”) diagram of figure 1:

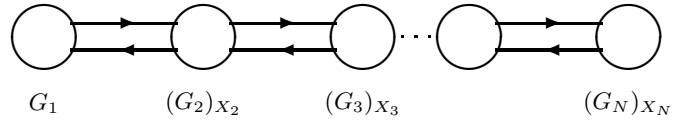


FIG. 1: A Model

Each site contains an MSSM gauge group $G_i = SU(3)_i \times SU(2)_i \times U(1)_i$ and a pair of Higgs doublets. All but the leftmost site contain an additional $U(1)_{X_i}$. These $U(1)_{X_i}$ gauge groups will be necessary to stabilize our desired pattern of symmetry breaking. The charges for the $U(1)_i$ and $U(1)_{X_i}$ will be chosen orthogonal, eliminating any mixing between the corresponding gauge bosons. Since the MSSM generations fill out complete $SU(5)$ multiplets it is not necessary to replicate them. They may be incorporated into the model in a variety of ways. We might for example have them all charged under only G_1 , or we may place them on different sites in the moose.

We assume that the couplings of the gauge group G_i unify as before at a scale M_U . The $U(1)_{X_i}$ coupling may or may not unify with those of G_i at this scale. The

links with arrows pointing from the i th site to the $(i+1)$ th site are chiral superfields F_i . These fields fill out complete multiplets of the *global* $H_i = SU(5)_i$ symmetry into which each G_i is embedded, transforming as $(\mathbf{5}, \mathbf{5})$ under the global $SU(5)_i \times SU(5)_{i+1}$. The field F_i can be represented by a 5×5 matrix:

$$\begin{pmatrix} \phi_3 & \phi_x \\ \phi_y & \phi_2 \end{pmatrix} \quad (9)$$

where under the gauge symmetries $[G_i, G_{i+1}]$ these fields transform as $\phi_3 \sim [(\mathbf{3}, 1)_{-1/3}, (\bar{\mathbf{3}}, 1)_{1/3}]$, $\phi_2 \sim [(1, \mathbf{2})_{1/2}, (1, \mathbf{2})_{-1/2}]$, $\phi_x \sim [(\mathbf{3}, 1)_{-1/3}, (1, \mathbf{2})_{-1/2}]$, $\phi_y \sim [(1, \mathbf{2})_{1/2}, (\bar{\mathbf{3}}, 1)_{1/3}]$. The F_i also have $(U(1)_{X_i}, U(1)_{X_{i+1}})$ charges $(1, -1)$. The links with arrows pointing to the left are chiral superfields \bar{F}_i transforming according to the conjugate representation. As these fields fill out complete $SU(5)$ multiplets they do not affect unification. We may also include additional vector-like fields that fill out complete multiplets under these global $SU(5)_i$ symmetries.

We include mass terms in the superpotential for the F_i, \bar{F}_i fields:

$$W = \sum_i \text{tr } \mu_i \bar{F}_i F_i \quad (10)$$

where for notational convenience we have assumed the masses of all the components of F are the same. In order to higgs down to the MSSM gauge group the fields ϕ_3 and ϕ_2 need to acquire vacuum expectation values $\phi_3^{(i)} = v_3^{(i)} \mathbf{1}_3$, $\phi_2^{(i)} = v_2^{(i)} \mathbf{1}_2$ with $v_3^{(i)}, v_2^{(i)} \sim v$. (Since these fields carry $U(1)$ charges these vevs suffice to higgs the full MSSM gauge group down to the diagonal subgroup.) We can produce these vevs in the same way that a vev for the MSSM Higgs field is triggered, via soft SUSY breaking masses $\tilde{m}_{3,2,x,y}^2$. If $\mu^2 + \tilde{m}_{3,2}^2 < 0$ and $\mu^2 + \tilde{m}_{x,y}^2 > 0$, the $\phi_{3,2}$ will acquire vevs. The stabilizing quartic potential is provided by the D-terms. With only the $SU(3)_i \times SU(2)_i \times U(1)_i$ contributions a D-flat direction with $v_3^{(i)} = v_2^{(i)}$ would not be stabilized. The $U(1)_{X_i}$ have been included to provide an additional D-term which stabilizes this direction. This higgses the theory down to the diagonal MSSM gauge group. All the $U(1)_{X_i}$ gauge bosons, together with all the fermionic components of the F, \bar{F} superfields, become massive. Thus this simple theory gives accelerated unification with no exotic states at very low energies.

If the μ_i are large compared to the SUSY breaking soft masses, no vevs are triggered. This is similar to the “ μ ” problem of the MSSM. We will simply choose the μ_i to be of order v . Since the vevs which higgs the large gauge symmetry down to the MSSM gauge group are triggered by SUSY breaking, as is electroweak symmetry breaking, these vevs are near a TeV. They can be somewhat larger since they are only stabilized by the $U(1)_X$

D-terms: a small $U(1)_X$ gauge coupling yields a parametrically larger scale. If the $U(1)_{X_i}$ gauge couplings also unify with the G_i couplings at the scale M_U , they are naturally the smallest couplings at v since they have the largest β -function.

At energies somewhat larger than a TeV this theory has many new particles beyond those of the MSSM, including an $\sim N$ -fold spectrum of massive gauge bosons, Higgs particles, link fields and their superpartners.

The additional F_i, \bar{F}_i states contribute to the running of each individual gauge group G_i . We should therefore check that all the gauge couplings remain perturbative at scales between v and M_U . Since the $\alpha_3^{(i)}$ are the largest gauge couplings at each site at all scales, we need only ensure that these couplings remain perturbative. For simplicity we will assume that the MSSM generations are only charged under G_1 . The β -function coefficients for the $\alpha_3^{(i)}$ are $b_3^{(1)} = 2, b_3^{(2,\dots,N-1)} = 1$, and $b_3^{(N)} = -4$. Since all but $\alpha_3^{(N)}$ are infrared free, it is sufficient to choose the $\alpha_0^{(i)}$ perturbative for $i = 1, \dots, N-1$, and to require $\alpha_3^{(N)}(v) > 0$. Combining with (5) and running up to the scale M_U gives:

$$90 \sim \frac{2\pi}{\alpha_3(v)} + \log(M_G/v) > \frac{2\pi}{\alpha_0^{(N)}} > \frac{4}{N} \log(M_G/v) \sim \frac{120}{N} \quad (11)$$

As long as these inequalities are satisfied the theory remains weakly coupled between the scales v and M_U . These β -functions and the condition of perturbativity preclude the case where the $\alpha_0^{(i)}$ are all equal. If we had included additional vector-like multiplets transforming under G_N , the β -function coefficient $b_3^{(N)}$ would be different and could relax these constraints. For example for $b_3^{(N)} = -1$ equal $\alpha_0^{(i)}$ are possible for any value of N .

The general mechanism of accelerated unification is easily incorporated in other models. For example an even more minimal model can be obtained by using link fields that fill out complete multiplets under the “trinified” group $H_i = SU(3)_i^1 \times SU(3)_i^2 \times SU(3)_i^3$ [5, 6] rather than $SU(5)$ multiplets. A simple choice is F_i^1 transforming under $[H_i, H_{i+1}]$ as $[(\mathbf{3}, 1, 1), (\bar{\mathbf{3}}, 1, 1)]$ and $F_i^{2,3}$ defined by cyclic permutation, together with the conjugate representations $\bar{F}_i^{1,2,3}$. In addition to mass terms the superpotential contains cubic interactions $\sum_i \lambda_{1,2,3} \det F_i^{1,2,3} + \text{conjugate}$. These renormalizable interactions lift all the flat directions, eliminating the need for the $U(1)_{X_i}$ gauge groups of the previous model. SUSY breaking soft masses which destabilize $F^{1,2}$ produce the correct pattern of symmetry breaking, higgsing to the diagonal MSSM gauge group with no exotic particles at low energies. These link fields make smaller contributions to the G_i β -functions: for the MSSM generations all charged under G_1 , $b_3^{(1)} = 0, b_3^{(2,\dots,N-1)} = -3$, and $b_3^{(N)} = -6$.

Therefore all but the first group are asymptotically free, and all gauge couplings will remain perturbative at all scales above v as long as

$$\sum_{i=2}^N \frac{2\pi}{\alpha_0^{(i)}} < \frac{2\pi}{\alpha_3(v)} + 3 \log(M_G/v) \sim 150 \quad (12)$$

$$\frac{2\pi}{\alpha_0^{(N)}} > \frac{6}{N} \log(M_G/v) \sim \frac{180}{N} \quad (13)$$

$$\frac{2\pi}{\alpha_0^{(2,\dots,N-1)}} > \frac{3}{N} \log(M_G/v) \sim \frac{90}{N} \quad (14)$$

Again the case of equal $\alpha_0^{(i)}$ is precluded without the addition of extra fields charged under G_N .

The formulas (6),(7) do not include various threshold corrections [7]. Although these corrections depend on the details of the model such as the complete mass spectrum, the general size of these effects are easily estimated. The unification relation is most conveniently expressed as a prediction for α_3 in terms of the more accurately measured couplings $\alpha_{1,2}$. The threshold corrections are similar in size to those of the MSSM, except multiplied by N . The resulting prediction for α_3 is

$$\frac{2\pi}{\alpha_3} = \frac{b_3 - b_2}{b_2 - b_1} \left(\frac{2\pi}{\alpha_2} - \frac{2\pi}{\alpha_1} \right) + \frac{2\pi}{\alpha_2} + CN \quad (15)$$

where the last term CN represents the threshold corrections and C is a constant which depends on group theory factors and the detailed mass spectrum of the theory, but is parametrically independent of N . In addition to these threshold corrections there are two-loop running corrections of comparable magnitude. These terms produce a fractional change in α_3 of size

$$\frac{\delta\alpha_3}{\alpha_3} = -CN \frac{\alpha_3}{2\pi} \simeq -\frac{CN}{60}. \quad (16)$$

Roughly speaking, since the size of the matter content under each gauge group is not dramatically different from the MSSM, these corrections are of order N times larger than the MSSM threshold corrections. Note that this means we cannot take N arbitrarily large—if N becomes too large these threshold corrections would destroy the successful MSSM prediction unless the parameters conspire to make C small, in which case the success of the MSSM would be an accident. The precise size of C and the corresponding limit on N is model-dependent. For N as small as 2, these corrections should be negligible for most reasonable choices of parameters, while for N as large as 60 the opposite is true. Intermediate values interpolate between these two extremes. The exponential dependence of M_U on N means that a low unification scale does not require a large value of N : already for N of 6, the unification scale is near 100 TeV.

Given the low unification scale (8) we might wonder about other unification phenomena. A qualitative success of the usual desert is the proximity of the GUT scale

and the Planck scale, a relation absent here. It is nevertheless possible that the Planck/string scale is M_U , with the usual phenomenology associated with a low quantum gravity scale [8]. With such a low scale, proton decay becomes an important constraint. This can be dealt with in a number of ways familiar from theories of TeV scale quantum gravity (see for example [9]).

It is also possible that the quantum gravity scale remains high, with a purely gauge theoretic unification at M_U . Proton decay can be avoided in this case as well. We might consider unifying all but the left-most group in figure 1 into $SU(5)$. By placing the MSSM generations on this first site we can enforce baryon number symmetries which prevent dangerous proton decay [10]. Or we could contemplate a model in which each G_i is unified into a trinified group.

Accelerated unification with a high fundamental scale for 4-dimensional quantum gravity allows for new physics between M_U and M_{Planck} . Unlike conventional unification which requires a desert from 1 TeV to M_G , our models allow even strongly coupled dynamics at intermediate scales. An example might be the strongly coupled CFT dynamics dual to RS type models [11].

The model of figure 1 looks very much like a deconstructed extra dimension [12, 13]. Extra-dimensional models with a low unification scale have been proposed in [14], where the power-law running of the gauge couplings in a (compactified) extra dimension provides faster unification. In straightforward deconstructions of these power-law unification models, holomorphy of the resulting 4-d supersymmetric moose model allows reliable computation of the running of the gauge couplings, even at strong coupling. The results confirm the power-law running of [14], but demonstrate UV sensitivity to the precise way in which the 5-dimensional theory is completed [15]. Power-law running does occur in the regime where the theory looks 5-dimensional. However in deconstructed examples, corrections from the scale where the theory transitions to a 4-dimensional gauge theory typically destroy the unification of the couplings. For example in two different deconstructions, which look identical below the scale at which the 5th dimension forms, the values of the high energy MSSM gauge couplings are completely different.

By contrast in our model of accelerated running, the phenomena which results in a low unification scale is occurring at energies *above* the scale where, for large N , a 5th dimension would form. The power-law running effects from below this scale are incorporated in the threshold corrections proportional to N in equation (15). For small N the accelerated unification effects, which come from running at high energies, dominate over these threshold effects.

Although inspired by deconstruction of power-law running in higher dimensional field theories, accelerated unification demonstrates a completely non-extra-

dimensional phenomenon. As emphasized in [12], “theory space” provides a much richer set of possibilities than field-theoretic extra dimensions, which arise as a special case. As in other examples [16–21] the power of deconstruction lies in its ability to construct purely four-dimensional models exhibiting surprising field-theoretic phenomena, with no higher-dimensional interpretation.

The phenomenology of accelerated unification models at the TeV scale involves a plethora of new particles beyond those of the MSSM. It is these particles at the TeV scale which are responsible for lowering the unification scale to more accessible energies. Exploration of this phenomenology offers exciting opportunities for future experiments.

We thank Ann Nelson for useful discussions. We also thank Lisa Randall for informing us of recent work on unification with an extra dimension. H.G. is supported in part by the National Science Foundation under grant number NSF-PHY/98-02709. A.G.C. is supported in part by the Department of Energy under grant number #DE-FG02-91ER-40676. N.A-H. is supported in part by the Department of Energy under Contracts DE-AC03-76SF00098, the National Science Foundation under grant PHY-95-14797, the Alfred P. Sloan foundation, and the David and Lucille Packard Foundation.

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