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Lorentz Transformation and Light-Like Noncommutative SYM

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Abstract

We show that combining the spatial noncommutative SYM limit and Lorentz transformation, one can obtain a well-behaved light-like noncommutative SYM limit. The light-like noncommutative SYM is unitary. When the boost velocity is finite, the resulting theory with space-time noncommutativity is unitary as well. The light-like noncommutative SYM limit can also be approached by combining the noncommutative open string theory limit and Lorentz transformation. Along this line, we obtain the supergravity dual for the light-like noncommutative SYM, which is the same as the one acquired using a different method. As a comparison, the supergravity duals for the ordinary SYM, spatial noncommutative SYM and the noncommutative open string theories are given as well, in an infinitely-boosted frame with finite momentum density, which are the decoupling limits of bound states (Dp, W), (D($p-2$), W, Dp), and (F1, W, Dp), respectively.

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1 Introduction

Over the past years there have been a lot of activities on the low energy decoupling limits of string theory involving Dp-branes. Initially, Maldacena [1] found that in a certain low energy limit (SYM limit) the closed strings and massive modes of open strings ending on the Dp-branes decouple, one is left with a field theory (super Yang-Mills theory) and such a low energy field theory with strong 't Hooft coupling has a weak coupling dual description: supergravity. In particular, string/M theory on an anti-de Sitter space (AdS) with a compact space is conjectured to be dual to a certain large N conformal field theory (CFT) which lives on the boundary of AdS [1, 2, 3, 4].

Later it was found that when the Dp-brane is put in a background with a non-zero NS B field, the worldvolume coordinates become noncommutative [5, 6, 7]. For a Dp-brane with only spatial component B field, one can also find a limit (NCSYM limit), in which the closed strings and massive modes of open strings are decoupled, and ends up with a SYM in a space-space noncommutative spacetime, which is called noncommutative SYM (NCSYM) [8]. There also exists a supergravity dual description for such spatial NCSYM's [9]-[21], from which one can investigate a lot of properties of NCSYM's. In contrast to the case of spatial component NS B field, if the Dp-brane is put in a non-zero NS B field with only a time-like component, it was found that one cannot define a low energy field theory limit [22, 23, 24]. Instead one can find a decoupling limit (NCOS limit), in which the closed strings decouple and one is left with a open string theory in a space-time noncommutative spacetime (NCOS). Since then, there have been a lot of papers appearing on the net, including [25]-[43], in which supergravity duals, the relation to the NCSYM and other related topics have been discussed.

It is argued that field theories with space-time noncommutativity may exhibit acausal behavior [25, 37], and quantum field theories on such a spacetime are not unitary [29]. However, most recently, Aharony, Gomis and Mehen [44] have shown that quantum field theories with light-like noncommutativity are unitary and they can be achieved through a certain low energy limit of string theory. Subsequently Alishahiha, Oz, and Russo (AOR)[45] have constructed the dual supergravity description of field theories with light-like noncommutativity.

In this paper we will further consider quantum field theories with light-like noncommutativity and their dual supergravity description. In order to get supergravity duals for the

SYM with light-like noncommutativity, AOR first constructed Dp-brane solutions with a non-zero light-like background NS B field and then took a usual SYM limit for that goal. To clearly see the relation of light-like NCSYM to the spatial commutative SYM and to the NCOS, we get the light-like NCSYM limit by combining the Lorentz transformation and the NCSYM decoupling limit or the NCOS decoupling limit. In sect. 2 we consider the Lorentz transformation, the NCSYM limit and the NCOS limit, and show that a well-behaved light-like NCSYM limit is obtained. The case for the finite boost velocity is also discussed. Along this line, in sect. 3 we re-derive the supergravity dual for the light-like NCSYM. In sect. 4 we consider the Lorentz transformation, along other directions, of the supergravity duals for the ordinary SYM (OSYM), the spatial NCSYM and the NCOS, thereby we obtain the supergravity duals for the OSYM, the spatial NCSYM and the NCOS in an infinitely-boosted frame with finite momentum density. The conclusion is included in sect. 5.

2 Lorentz transformation and decoupling limits

To discuss a low energy decoupling limit of string theory involving Dp-branes, a good starting point is the Seiberg-Witten relation connecting the open and closed string moduli [8]:

$$
\begin{aligned}
G_{ij} &= g_{ij} - (2\pi\alpha')^2 (Bg^{-1}B)_{ij}, \\
\Theta^{ij} &= 2\pi\alpha' \left(\frac{1}{g+2\pi\alpha' B}\right)^{ij}_{A}, \\
G^{ij} &= \left(\frac{1}{g+2\pi\alpha' B}\right)^{ij}_{S}, \\
G_s &= g_s \left(\frac{\det G_{ij}}{\det(g_{ij}+2\pi\alpha' B_{ij})}\right)^{1/2},
\end{aligned}
\tag{2.1}
$$

where $(\ )_A$ and $(\ )_S$ denote the antisymmetric and symmetric parts, respectively. The open string moduli occur in the disk correlators on the open string worksheet boundaries

$$
< X^i(\tau) X^j(0) >= -\alpha' G^{ij} \ln(\tau)^2 + \frac{i}{2}\Theta^{ij}\epsilon(\tau). \tag{2.2}
$$

For simplicity of notation, we restrict ourselves here to the case of D3-branes. It is straightforward to extend to other dimensions. Suppose we have the following closed string metric:

$$ds^2 = -dx_0^2 + dx_1^2 + g(dx_2^2 + dx_3^2). \tag{2.3}$$

Here we have written down only the worldvolume coordinate part.

2.1 From the spatial NCSYM to the light-like NCSYM

When the NS B field has only nonvanishing spatial component, one has a well-defined decoupling limit, and the resulting theory is the spatial NCSYM. For example, suppose the B field has only constant component along $x_2 - x_3$ directions,

$$B_{ij} = Bdx_2 \wedge dx_3. \tag{2.4}$$

Rescaling the closed string metric g and the closed string coupling constant g_s by

$$g = (2\pi\alpha' B)^2, \quad g_s = 2\pi\alpha' BG_s, \tag{2.5}$$

and keeping B as a finite constant, in the decoupling limit $\alpha' \to 0$, we have

$$G^{ij} = \eta^{ij}, \quad \Theta^{ij} = B^{-1}(-\delta_2^i\delta_3^j + \delta_3^i\delta_2^j). \tag{2.6}$$

Thus we obtain a 3+1 dimensional SYM in a spacetime with space-space noncommutativity $\Theta^{23} = -B^{-1}$. This field theory is shown to be unitary [29].

Now we perform a Lorentz transformation for the coordinate (2.3)

$$\begin{aligned} x_0 &= \cosh\gamma x_0' - \sinh\gamma x_2', \\ x_2 &= -\sinh\gamma x_0' + \cosh\gamma x_2'. \end{aligned} \tag{2.7}$$

Using the Seiberg-Witten relation, we have

$$G^{ij} = \frac{1}{g^2 + 4\pi^2\alpha'^2 B^2} \times \begin{pmatrix} -(g^2+4\pi^2\alpha'^2B^2)\cosh^2\gamma + g\sinh^2\gamma & 0 & (g-g^2-4\pi^2\alpha'^2B^2)\cosh\gamma\sinh\gamma & 0 \\ 0 & g^2+4\pi^2\alpha'^2B^2 & 0 & 0 \\ (g-g^2-4\pi^2\alpha'^2B^2)\cosh\gamma\sinh\gamma & 0 & g\cosh^2\gamma - (g^2+4\pi^2\alpha'^2B^2)\sinh^2\gamma & 0 \\ 0 & 0 & 0 & g \end{pmatrix} \tag{2.8}$$

and the noncommutativity matrix

$$\Theta^{ij} = \frac{2\pi\alpha'}{g^2 + 4\pi^2\alpha'^2 B^2} \begin{pmatrix} 0 & 0 & 0 & -2\pi\alpha' B\sinh\gamma \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2\pi\alpha' B\cosh\gamma \\ 2\pi\alpha' B\sinh\gamma & 0 & 2\pi\alpha' B\cosh\gamma & 0 \end{pmatrix}. \tag{2.9}$$

Taking the same NCSYM limit (2.5), we reach

$$G^{ij} = \eta^{ij}, \tag{2.10}$$

and

$$\Theta^{ij} = \begin{pmatrix} 0 & 0 & 0 & -B^{-1}\sinh\gamma \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -B^{-1}\cosh\gamma \\ B^{-1}\sinh\gamma & 0 & B^{-1}\cosh\gamma & 0 \end{pmatrix}. \tag{2.11}$$

Obviously, this is a low-energy field theory limit with space-time noncommutativity when γ is finite. We will argue that field theory thus defined is unitary even though the space-time coordinates are noncommutative.

On the other hand, when the boost velocity approaches the speed of light, that is, $\gamma \to \infty$, the Lorentz transformation (2.7) is singular and some components of the noncommutativity matrix (2.11) are divergent. But we notice that rescaling the constant B as

$$e^{\gamma}/B = b, \tag{2.12}$$

where b is a finite constant, yields a well-behaved noncommutativity matrix with space-time component,

$$\Theta^{ij} = \begin{pmatrix} 0 & 0 & 0 & -b \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -b \\ b & 0 & b & 0 \end{pmatrix}. \tag{2.13}$$

Actually, this limit is just the low energy field theory limit with light-like noncommutativity discussed in [44]. There it is shown that this theory is unitary despite the nonlocality in the time coordinate. This result is reasonable since this low-energy field theory is an infinitely-boosted limit of a unitary theory.

2.2 From the NCOS to the light-like NCSYM

Now it is clear that when the NS B field has a nonvanishing time-like component, one can define a limit, in which the closed strings are decoupled and one ends up with a noncommutative open string theory [22, 23]. For example, suppose the B field has, in the coordinate (2.3), the time-like component as

$$B_{ij} = E dx_0 \wedge dx_1. \tag{2.14}$$

Rescaling the electric field E, the closed string metric g and the closed string coupling constant g_s as

$$2\pi\alpha' E = 1 - \frac{1}{2}\frac{\alpha'}{\alpha'_{eff}}, \quad g = \frac{\alpha'}{\alpha'_{eff}}, \quad g_s = G_s\sqrt{\frac{\alpha'_{eff}}{\alpha'}}, \tag{2.15}$$

where α'_{eff} is a finite constant, in the decoupling limit $\alpha' \to 0$, one has

$$\alpha' G^{ij} = \alpha'_{eff}\eta^{ij}, \quad \Theta^{ij} = 2\pi\alpha'_{eff}(\delta^i_0\delta^j_1 - \delta_1\delta^j_0). \tag{2.16}$$

This is a NCOS limit with the open string tension $1/4\pi\alpha'_{eff}$, the coupling constant G_s and the space-time noncommutativity $\Theta^{01} = 2\pi\alpha'_{eff}$.

Now we make also the same Lorentz transformation (2.7). In this case, we find

$$G^{ij} = \begin{pmatrix} -\frac{g\cosh^2\gamma-(1-e^2)\sinh^2\gamma}{g(1-e^2)} & 0 & -\frac{(g-1+e^2)\sinh\gamma\cosh\gamma}{g(1-e^2)} & 0 \\ 0 & \frac{1}{1-e^2} & 0 & 0 \\ -\frac{(g-1+e^2)\sinh\gamma\cosh\gamma}{g(1-e^2)} & 0 & \frac{(1-e^2)\cosh^2\gamma-g\sinh\gamma}{g(1-e^2)} & 0 \\ 0 & 0 & 0 & \frac{1}{g} \end{pmatrix}, \tag{2.17}$$

where $e = 2\pi\alpha' E$, and the noncommutativity matrix

$$\Theta^{ij} = \frac{2\pi\alpha' e}{1-e^2}\begin{pmatrix} 0 & \cosh\gamma & 0 & 0 \\ -\cosh\gamma & 0 & -\sinh\gamma & 0 \\ 0 & \sinh\gamma & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{2.18}$$

Taking the same NCOS decoupling limit (2.15), we obtain

$$\alpha' G^{ij} = \alpha'_{eff}\eta^{ij}, \tag{2.19}$$

and

$$\Theta^{ij} = 2\pi\alpha'_{eff}\begin{pmatrix} 0 & \cosh\gamma & 0 & 0 \\ -\cosh\gamma & 0 & -\sinh\gamma & 0 \\ 0 & \sinh\gamma & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{2.20}$$

Obviously, when the boost velocity is finite, this is a well-behaved NCOS limit. It means that the NCOS is still a NCOS with both the space-time and space-space noncommutativities after the Lorentz transformation [30]. Of course, the space-space noncommutativity can be removed using a coordinate transformation. However, when the boost velocity approaches the speed of light, $\gamma \to \infty$, from the noncommutativity matrix (2.20) one can see that in order to have a finite noncommutativity value, one has to rescale the constant α'_{eff} as

$$\alpha'_{eff} e^{\gamma} = b', \tag{2.21}$$

where b' is another finite constant. In this case, we see from (2.19) that $\alpha' G^{ij} = 0$. It means that when the boost velocity approaches the speed of light, from the NCOS limit we can define a well-behaved low energy field theory limit. Actually, this limit is also the field theory limit with light-like noncommutativity. Furthermore, equations (2.19) and (2.21) implies that in order to have a well-defined open string metric, one has to further rescale all coordinates as

$$x_i \sim e^{\gamma/2} x_i. \tag{2.22}$$

This will be used later when we discuss the supergravity dual.

2.3 Unitarity

In [29] the unitarity of quantum fields has been studied in a noncommutative spacetime. It has been found that space-like noncommutative quantum field theories are always unitary while time-like noncommutative quantum theories are not unitary. In [44] it was shown that quantum theories with light-like noncommutativity are unitary as well.

The unitarity of quantum field theories requires that the inner product $p \circ p$ be never negative [29, 44], where p is some external momentum and

$$p \circ p \equiv -p_\mu \Theta^{\mu\rho} G_{\rho\sigma} \Theta^{\sigma\nu} p_\nu \equiv p_\mu g_\theta^{\mu\nu} p_\nu \geq 0, \tag{2.23}$$

where $\Theta^{\mu\nu}$ is the noncommutativity matrix and $G_{\rho\sigma}$ is the background metric of quantum fields. Let us first consider the spatial NCSYM in 3+1 dimensions discussed above. In that case, we have

$$p \circ p = B^{-2}(p_2^2 + p_3^2), \tag{2.24}$$

which clearly indicates that the spatial NCSYM satisfies the requirement of unitarity. Therefore this theory is unitary [29]. For the finite Lorentz boost, we find

$$p \circ p = B^{-2}(p_0 \sinh\gamma + p_2 \cosh\gamma)^2 + B^{-2} p_3^2. \tag{2.25}$$

Again, the unitarity constraint is satisfied. Therefore we can conclude that the SYM with time-space noncommutativity, coming from the spatial NCSYM after Lorentz transformation, is unitary as well. This seems reasonable because the physics should be invariant after a regular coordinate transformation. Actually, the two theories are equivalent. The space-time noncommutativity in the former can be removed using the Lorentz transformation. Therefore we cannot say in general that quantum field theories with space-time noncommutativity are not unitary. Of course, the description in terms of the spatial NCSYM is simpler.

When the boost velocity approaches the speed of light, we have

$$p \circ p = b^2 (p_0 + p_2)^2, \tag{2.26}$$

which always satisfies $p \circ p \geq 0$. Thus the SYM with light-like noncommutativity is unitary. Similarly, the SYM with light-like noncommutativity coming from the NCOS limit can be proved to be unitary.

3 Supergravity duals

In the previous section we have shown that the low energy SYM with light-like noncommutativity can be obtained through combining the NCSYM limit and Lorentz transformation, or the NCOS limit and Lorentz transformation. In this section we further give evidence to support that point of view. We "derive" the supergravity duals of light-like NCSYM's through the supergravity duals of spatial NCSYM's or of NCOS's.

Let us first consider the D3-brane case. The supergravity dual of spatial NCSYM in 3+1 dimensions has been given in [9, 10], which is

$$\begin{aligned} ds^2 &= \alpha' \left[\frac{u^2}{R^2} \left(-dx_0^2 + dx_1^2 + \tilde{h}(dx_2^2 + dx_3^2) \right) + \frac{R^2}{u^2} \left(du^2 + u^2 d\Omega_5^2 \right) \right], \\ e^{2\phi} &= \hat{g}_s^2 \tilde{h}, \quad B_{23} = \frac{\alpha'}{\tilde{b}} \frac{(au)^4}{1 + (au)^4}, \end{aligned} \tag{3.1}$$

where $R^4 = 4\pi \hat{g}_s N$, $a^4 = \tilde{b}^2 / R^4$, $\tilde{h}^{-1} = 1 + (au)^4$, and N denotes the number of D3-branes. The spatial NCSYM has the coupling constant $g_{YM}^2 = 2\pi \hat{g}_s$. Performing the coordinate transformation (2.7) for coordinates x_0 and x_3, we have

$$\begin{aligned} ds^2 &= \alpha' \left[\frac{u^2}{R^2} \left(-dx_0^2 + dx_1^2 + \tilde{h} dx_2^2 + dx_3^2 + (\tilde{h} - 1)(\cosh\gamma dx_3^2 - \sinh\gamma dx_0)^2 \right) \right. \\ &\qquad \left. + \frac{R^2}{u^2} \left(du^2 + u^2 d\Omega_5^2 \right) \right]. \end{aligned} \tag{3.2}$$

This supergravity solution is supposed to be the supergravity dual of the spatial NCSYM after a finite Lorentz boost considered in the previous section. In this coordinate, space-time coordinates are noncommutative. When $\gamma \to \infty$, rescaling the constant $\tilde{b}$ as

$$\tilde{b} e^{\gamma} = b, \tag{3.3}$$

we have

$$ds^2 = \alpha' \left[\frac{u^2}{R^2} \left(dx_+ dx_- + dx_1^2 + dx_2^2 - \frac{b^2 u^4}{R^4} dx_-^2 \right) + \frac{R^2}{u^2} \left(du^2 + u^2 d\Omega_5^2 \right) \right], \tag{3.4}$$

where $x_\pm = x_3 \pm x_0$. The dilaton and B field become

$$e^{2\phi} = \hat{g}_s^2, \quad B_{2-} = \alpha' \frac{b u^4}{R^4}. \tag{3.5}$$

We see that this supergravity solution is just the supergravity dual of light-like NCSYM found in [45]. The light-like NCSYM has the coupling constant $g_{\rm YM}^2 = 2\pi \hat{g}_s$, the same as the one for the spatial NCSYM.

It is rather easy to extend this to other dimensions. The supergravity dual of the spatial NCSYM in $p+1$ dimensions is [19]

$$\begin{aligned} ds^2 &= \alpha' \left[\left(\frac{u}{R}\right)^{(7-p)/2} \left(-dx_0^2 + dx_1^2 + \cdots + dx_{p-2}^2 + \tilde{h}(dx_{p-1}^2 + dx_p^2) \right) \right. \\ &\qquad \left. + \left(\frac{R}{u}\right)^{(7-p)/2} \left(du^2 + u^2 d\Omega_{8-p}^2 \right) \right], \\ e^{2\phi} &= \hat{g}_s^2 \tilde{h} \left(\frac{R}{u}\right)^{(7-p)(3-p)/2}, \quad B_{p-1,p} = \frac{\alpha'}{\tilde{b}} \frac{(au)^{7-p}}{1+(au)^{7-p}}, \end{aligned} \tag{3.6}$$

where the corresponding RR fields are not exposed explicitly,

$$\tilde{h} = \frac{1}{1+(au)^{7-p}}, \quad a^{7-p} = \tilde{b}^2 / R^{7-p}, \quad R^{7-p} = \frac{1}{2}(2\pi)^{6-p} \pi^{-(7-p)/2} \Gamma[(7-p)/2] \hat{g}_s N. \tag{3.7}$$

Making a similar Lorentz transformation as (2.7) for the coordinates x_0 and x_p, taking the limit $\gamma \to \infty$ and rescaling $\tilde{b}$ as (3.3), we obtain

$$\begin{aligned} ds^2 &= \alpha' \left[\left(\frac{u}{R}\right)^{(7-p)/2} \left(dx_+ dx_- + dx_1^2 + \cdots + dx_{p-1}^2 - \frac{b^2 u^{7-p}}{R^{7-p}} dx_-^2 \right) \right. \\ &\qquad \left. + \left(\frac{R}{u}\right)^{(7-p)/2} \left(du^2 + u^2 d\Omega_{8-p}^2 \right) \right], \\ e^{2\phi} &= \hat{g}_s^2 \left(\frac{R}{u}\right)^{(7-p)(3-p)/2}, \quad B_{(p-1)-} = \alpha' \frac{b u^{7-p}}{R^{7-p}}. \end{aligned} \tag{3.8}$$

Here $x_\pm = x_p \pm x_0$. This solution gives the supergravity dual description of light-like NCSYM in $p+1$ dimensions with coupling constant $g_{\rm YM}^2 = (2\pi)^{p-2}\hat{g}_s$.

On the other hand, from the previous section we see that the light-like NCSYM limit can also be achieved from the NCOS limit combining the Lorentz transformation. Now we also get the supergravity duals of the light-like NCSYM's along this line.

The supergravity dual for the 3+1 dimensional NCOS has been given in [23]. It can be described as

$$\begin{aligned} &ds^2 = \alpha' F^{1/2}\left[\frac{u^4}{R^4}\left(-dx_0^2+dx_1^2\right) + F^{-1}\left(dx_2^2+dx_3^2\right) + du^2 + u^2 d\Omega_5^2\right], \\ &F = 1+\frac{R^4}{u^4}, \quad e^{2\phi} = \hat{g}_s^2 F\frac{u^4}{R^4}, \quad B_{01} = \alpha'\frac{u^4}{R^4}, \end{aligned} \tag{3.9}$$

where R is the same as the corresponding one above. Performing a Lorentz transformation (2.7) for the coordinates x_0 and x_3, one has

$$\begin{aligned} ds^2 \;=\; & \alpha' F^{1/2}\left[\frac{u^4}{R^4}\left(-dx_0^2+dx_1^2+dx_3^2\right) + F^{-1}dx_2^2\right. \\ & \left. -\frac{u^8}{R^4(R^4+u^4)}\left(\cosh\gamma dx_3 - \sinh\gamma dx_0\right)^2 + du^2 + u^2 d\Omega_5^2\right]. \end{aligned} \tag{3.10}$$

Taking the limit $\gamma\to\infty$ and rescaling coordinates as

$$u \to u e^{-\gamma/2}, \qquad x_i \to x_i e^{\gamma/2}, \tag{3.11}$$

we reach

$$ds^2 = \alpha'\left[\frac{u^2}{R^2}\left(dx_+dx_- + dx_1^2 + dx_2^2 - \frac{u^4}{R^4}dx_-^2\right) + \frac{R^2}{u^2}\left(du^2+u^2d\Omega_5^2\right)\right], \tag{3.12}$$

and the dilaton and B field

$$e^{2\phi} = \hat{g}_s^2, \quad B_{-1} = -\alpha'\frac{u^4}{R^4}. \tag{3.13}$$

This solution is the same as (3.4) up to a constant b. The constant b is not important since it can be produced by rescaling $x_\pm$ as was pointed out in [45].

For other dimensions, the supergravity dual of NCOS is [31]

$$\begin{aligned} &ds^2 = \alpha'\left[H^{1/2}\frac{u^{7-p}}{R^{7-p}}\left(-dx_0^2+dx_1^2\right) + H^{-1/2}(dx_2^2+\cdots+dx_p^2) + H^{1/2}(du^2+u^2d\Omega_{8-p}^2)\right], \\ &e^{2\phi} = \hat{g}_s^2 H^{(3-p)/2}\left(1+\frac{u^{7-p}}{R^{7-p}}\right), \quad B_{01} = \alpha'\frac{u^{7-p}}{R^{7-p}}, \quad H = 1+\frac{R^{7-p}}{u^{7-p}}, \end{aligned} \tag{3.14}$$

Performing a Lorentz transformation on coordinates x_0 and x_p, taking the infinite boost limit $\gamma \to \infty$ and rescaling as (3.11), we finally obtain

$$
\begin{aligned}
ds^2 &= \alpha' \left[\left(\frac{u}{R}\right)^{(7-p)/2} \left(dx_+ dx_- + dx_1^2 + \cdots + dx_{p-1}^2 - \frac{u^{7-p}}{R^{7-p}} dx_-^2 \right) \right. \\
&\qquad \left. + \left(\frac{R}{u}\right)^{(7-p)/2} \left(du^2 + u^2 d\Omega_{8-p}^2 \right) \right], \\
e^{2\phi} &= \hat{g}_s^2 \left(\frac{R}{u}\right)^{(7-p)(3-p)/2}, \quad B_{-1} = -\alpha' \frac{u^{7-p}}{R^{7-p}}.
\end{aligned} \tag{3.15}
$$

Thus we arrive at again the supergravity dual of light-like NCSYM, combining the supergravity dual of NCOS and the Lorentz transformation.

4 NCSYM, OSYM and NCOS in the infinite-momentum frame

In order to get the supergravity dual of light-like NCSYM, we have used the Lorentz transformation along the nonisotropic direction coordinates, such as x_0 and x_3, or x_0 and x_p. In this section we consider the Lorentz transformation along the isotropic directions. For generality, we start with the supergravity dual (3.6) of spatial NCSYM in $p+1$ dimensions. Obviously, the solution (3.6) is Lorentz invariant if one performs a Lorentz coordinate transformation along one of directions x_1 to x_{p-2}. To produce a nontrivial effect, we make a Lorentz transformation along one of those directions, say x_1, for the supergravity dual of finite temperature NCSYM. The latter is [19]

$$
\begin{aligned}
ds^2 &= \alpha' \left[\left(\frac{u}{R}\right)^{(7-p)/2} \left(-f dx_0^2 + dx_1^2 + \cdots + dx_{p-2}^2 + \tilde{h}(dx_{p-1}^2 + dx_p^2) \right) \right. \\
&\qquad \left. + \left(\frac{R}{u}\right)^{(7-p)/2} \left(f^{-1} du^2 + u^2 d\Omega_{8-p}^2 \right) \right], \\
e^{2\phi} &= \hat{g}_s^2 \tilde{h} \left(\frac{R}{u}\right)^{(7-p)(3-p)/2}, \quad B_{p-1,p} = \frac{\alpha'}{\tilde{b}} \frac{(au)^{7-p}}{1+(au)^{7-p}},
\end{aligned} \tag{4.1}
$$

where

$$
\begin{aligned}
\tilde{h} &= \frac{1}{1+(au)^{7-p}}, \quad a^{7-p} = \tilde{b}^2/R^{7-p}, \\
f &= 1 - \frac{u_0^{7-p}}{u^{7-p}}, \quad R^{7-p} = \frac{1}{2}(2\pi)^{6-p}\pi^{-(7-p)/2}\Gamma[(7-p)/2]\hat{g}_s N.
\end{aligned} \tag{4.2}
$$

We now make a Lorentz transformation for coordinates x_0 and x_1 and rescale the non-extremal parameter u_0 as

$$u_0^{7-p}e^{2\gamma} = P, \tag{4.3}$$

where P is a finite constant. When $\gamma \to \infty$, we have

$$\begin{aligned} ds^2 = \alpha' \Bigg[\left(\frac{u}{R}\right)^{(7-p)/2} & \left(dx_+ dx_- + dx_2^2 + \cdots + dx_{p-2}^2 + \tilde{h}(dx_{p-1}^2 + dx_p^2) + \frac{P}{u^{7-p}} dx_-^2)\right) \\ & + \left(\frac{R}{u}\right)^{(7-p)/2} \left(du^2 + u^2 d\Omega_{8-p}^2\right)\Bigg], \end{aligned} \tag{4.4}$$

and the dilaton and B fields are kept unchanged, where $x_\pm = x_1 \pm x_0$. This is a supergravity solution with a pp-wave, and P has an interpretation as the momentum density of the wave. Let us notice that when the B field vanishes, that is $\tilde{h} = 1$, the solution (4.4) reduces to

$$\begin{aligned} ds^2 = \alpha' \Bigg[\left(\frac{u}{R}\right)^{(7-p)/2} & \left(dx_+ dx_- + dx_2^2 + \cdots + dx_{p-2}^2 + dx_{p-1}^2 + dx_p^2 + \frac{P}{u^{7-p}} dx_-^2)\right) \\ & + \left(\frac{R}{u}\right)^{(7-p)/2} \left(du^2 + u^2 d\Omega_{8-p}^2\right)\Bigg], \end{aligned} \tag{4.5}$$

and the dilaton and NS B field

$$e^{2\phi} = \hat{g}_s^2 \left(\frac{R}{u}\right)^{(7-p)(3-p)/2}, \quad B = 0. \tag{4.6}$$

Furthermore, when $p = 3$ the above solution (4.5) goes to the one obtained in [46], there the authors of [46] have studied the decoupling limit and near-horizon geometry of D3-branes (M2- and M5-branes) with a pp-wave. They got the solution (4.5) with $p = 3$, (in this case the metric is of the Kaigorodov-type and preserves 1/4 supersymmetries), and argued that this is the supergravity dual of the ordinary (3+1)-dimensional SYM in an infinitely-boosted frame with constant momentum density. In this sense, the solution (4.5) gives a generalization of [46], which describes the supergravity dual of the ordinary $(p+1)$-dimensional SYM in an infinitely-boosted frame. It is easy to show that the solution (4.5) can also be obtained by taking the decoupling limit for the (Dp, W) bound states, where W denotes a pp-wave.

Thus it is natural to regard the solution (4.4) as the supergravity dual of spatial NCSYM in an infinitely-boosted frame and such a solution (4.4) can be achieved through considering the NCSYM limit for the (D$(p-2)$, W, Dp) bound states [47, 48].

Next let us consider the finite temperature picture for the supergravity dual of NCOS in $p+1$ dimensions [31]

$$ds^2 = \alpha' \left[H^{1/2} \frac{u^{7-p}}{R^{7-p}} \left(-f dx_0^2 + dx_1^2 \right) + H^{-1/2}(dx_2^2 + \cdots + dx_p^2) + H^{1/2}(f^{-1}du^2 + u^2 d\Omega_{8-p}^2) \right],$$
$$e^{2\phi} = \hat{g}_s^2 H^{(3-p)/2} \left(1 + \frac{u^{7-p}}{R^{7-p}} \right), \quad B_{01} = \alpha' \frac{u^{7-p}}{R^{7-p}}, \quad H = 1 + \frac{R^{7-p}}{u^{7-p}}. \tag{4.7}$$

Again, making a Lorentz transformation for coordinates x_0 and x_1, and rescaling as (4.3), we have

$$\begin{aligned} ds^2 &= \alpha' \left[H^{1/2} \frac{u^{7-p}}{R^{7-p}} \left(dx_+ dx_- + \frac{P}{u^{7-p}} dx_-^2 \right) + H^{-1/2}(dx_2^2 + \cdots + dx_p^2) \right. \\ & \qquad \left. + H^{1/2}(du^2 + u^2 d\Omega_{8-p}^2) \right], \end{aligned} \tag{4.8}$$

and the dilaton and NS B field unchanged. This supergravity solution has a good interpretation as the gravity dual of NCOS in an infinitely-boosted frame. Of course, this solution can also be reached by considering the NCOS limit for (F1, W, Dp) bound states [49].

Thus we have given the supergravity duals for the OSYM, spatial NCSYM, and the NCOS in an infinitely-boosted frame by combining the SYM, NCSYM, and NCOS limits, and the Lorentz transformation, respectively. We note that these supergravity duals look like the one for light-like NCSYM in the sense that they all have the pp-wave component in solutions, but actually they are quite different.

5 Conclusions

Combining the Lorentz transformation and the spatial NCSYM limit, or the space-time NCOS limit, we have "derived" the light-like NCSYM limit. The light-like NCSYM is unitary. We have argued that the SYM with space-time noncommutativity, which comes from the Lorentz transformation of the spatial NCSYM, is unitary as well. Actually, the two theories connected through the Lorentz transformation are equivalent. Perhaps the spatial NCSYM gives a better description. Along the consideration of light-like NCSYM limit, we obtain the supergravity dual of the light-like NCSYM using the Lorentz transformation and the gravity dual of spatial NCSYM or of the NCOS, both giving the same result. These supergravity duals are the same as those given by AOR [45], where the authors first constructed the Dp-brane solution with light-like NS B field and then took a usual SYM decoupling limit.

In addition, as a comparison, we have given supergravity duals for the OSYM, spatial NCSYM, and the NCOS in an infinitely-boosted frame with finite momentum density, by using the Lorentz transformation and a certain rescaling. These supergravity duals can also be obtained through considering certain decoupling limits for the (Dp, W), (D$(p-2)$, W, Dp) and (F1, W, Dp) bound states, respectively.

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