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Open Branes in Space-Time Non-Commutative Little String Theory

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Abstract

We conjecture the existence of two new non-gravitational six-dimensional string theories, defined as the decoupling limit of NS5-branes in the background of near-critical electrical two- and three-form RR fields. These theories are space-time non-commutative Little String Theories with open branes. The theory with $(2, 0)$ supersymmetry has an open membrane in the spectrum and reduces to OM theory at low energies. The theory with $(1, 1)$ supersymmetry has an open string in the spectrum and reduces to 5+1 dimensional NCOS theory for weak NCOS coupling and low energies. The theories are shown to be T-dual with the open membrane being T-dual to the open string. The theories therefore provide a connection between 5+1 dimensional NCOS theory and OM theory. We study the supergravity duals of these theories and we consider a chain of dualities that shows how the T-duality between the two theories is connected with the S-duality between 4+1 dimensional NCOS theory and OM theory.

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Contents

1 Introduction

Recently, it has been discovered that the world-volume theory of a Dp-brane with a near-critical electrical NSNS B-field is a space-time non-commutative open string (NCOS) theory [1, 2]. Subsequently, it was shown that the world-volume theory of the M5-brane with a near-critical electrical three-form C-field is a non-commutative open membrane (OM) theory [3, 4][2]. OM theory has been shown [3, 4, 18] to encompasses all the $p+1$ dimensional NCOS theories

[2]For related papers about NCOS theory, OM theory and space-time non-commutativity, see [5]-[17].

with $p \leq 4$, along with their strong coupling duals. In this sense, we can see OM theory as a unified framework for all these lower dimensional theories, in much the same way as M-theory can be seen as a unified framework of lower dimensional string theories. Another close analogy to OM theory is the way that the 5+1 dimensional $(2,0)$ SCFT encompasses all of the $p+1$ dimensional Yang-Mills (YM) theories with $p \leq 4$.

However, the 5+1 dimensional NCOS theory does not appear to be directly related to OM theory. If we use the analogy to $(2,0)$ SCFT and YM theories, we know that the ultraviolet completion of the 5+1 dimensional YM theory is the 5+1 dimensional $(1,1)$ Little String Theory (LST) [19, 20][3] living on the world-volume of type IIB NS5-branes. The T-dual of the $(1,1)$ LST is the $(2,0)$ LST living on type IIA NS5-branes, and the low energy limit of this theory is the $(2,0)$ SCFT. Thus, the two 5+1 dimensional LSTs provide a relation between 5+1 dimensional YM and $(2,0)$ SCFT. This also means that we can consider the LSTs as encompassing both the $(2,0)$ SCFT and the YM theories.

In this paper, we find a relation between 5+1 dimensional NCOS and OM theory by defining two new theories which we call $(1,1)$ and $(2,0)$ Open Brane Little String Theories (OBLSTs). The $(1,1)$ OBLST is defined as the world-volume theory of N type IIB NS5-branes with a near-critical two-form RR-field, and the $(2,0)$ OBLST is defined as the world-volume theory of N type IIA NS5-branes with a near-critical three-form RR-field. The $(1,1)$ OBLST inherits the closed string from $(1,1)$ LST but has in addition the open string of 5+1 dimensional NCOS along with the space-time non-commutativity, since the decoupling limit of $(1,1)$ OBLST is in fact identical to that of 5+1 dimensional NCOS, as can be seen from type IIB S-duality. The $(2,0)$ OBLST has also the closed string of $(2,0)$ LST and in addition the open membrane of OM theory, again with a non-commutative geometry. Thus, the OBLSTs have open branes and are space-time non-commutative. For low energies the $(2,0)$ OBLST reduces to OM theory, while the $(1,1)$ OBLST reduces to 5+1 dimensional NCOS theory for weak NCOS coupling and low energies. We show that the $(1,1)$ and $(2,0)$ OBLST are related by T-duality, in the sense that a T-duality in one of the open membrane directions in $(2,0)$ OBLST gives the open string of $(1,1)$ OBLST. The $(1,1)$ and $(2,0)$ OBLST therefore provide a relation between 5+1 dimensional NCOS and OM theory, and we can consider

[3] See also [21, 22, 23] and see [24] for a brief review of LST.

them as encompassing OM theory and all the NCOS theories.

In order to explore the OBLSTs we find their supergravity duals. As part of this we also find the supergravity dual of OM theory. We subsequently examine the phase structure and thermodynamics of the supergravity duals. From this, we see that the $(1,1)$ OBLST only has an NCOS phase when the NCOS coupling is small. For strong coupling, the closed string from LST dominates. The $(2,0)$ OBLST reduces to OM theory at low energies in the supergravity description, as it should. At high energies the closed string inherited from LST dominates in both of the OBLSTs, just as for ordinary LST.

We test the consistency of our decoupling/near-horizon limits of the OBLSTs by connecting five different bound-states and their decoupling/near-horizon limits through S- and T-dualities. The chain of theories we relate is: OM-theory from M2-M5, $D=4+1$ NCOS from F1-D4, $D=5+1$ NCOS/$(1,1)$ OBLST from F1-D5, $D=5+1$ NCOS/$(1,1)$ OBLST from D1-NS5 and $(2,0)$ OBLST from D2-NS5. Since the $(2,0)$ OBLST from D2-NS5 is related directly to OM theory, we have a closed chain of bound states and limits. Thus, we can start at any point in the chain and then move on to other points. The S- and T-dualities are also seen to induce corresponding dualities in the world-volume theories.

It is important to note that instead of working in terms of decoupling limits we work mostly with near-horizon limits in this paper. The decoupling limits can easily be read off from the near-horizon limits. Therefore, when considering a particular near-horizon limit we also consider this limit as defining the theory which the corresponding near-horizon supergravity solution is dual to.

2 $(1,1)$ OBLST and $D=5+1$ NCOS theory

2.1 Introduction to $(1,1)$ OBLST

In [2] a new theory was found from the F1-D5 bound-state in the decoupling limit

$$\bar{g}_b \to \infty \quad , \qquad \bar{g}_b \bar{l}_s^2 = \text{fixed} \quad , \qquad B_{01} \to B_{01}^{\text{critical}} \tag{1}$$

where $\bar{g}_b$ is the string coupling, $\bar{l}_s$ the string length and B is the two-form NSNS field. This theory was subsequently shown to be a 5+1 dimensional theory of open strings, known as 5+1 dimensional NCOS theory, living in a space-time geometry with space and time being non-commutative.

In the following, we shall see that this theory also can be seen as a space-time non-commutative version of the $(1,1)$ LST. In fact, using type IIB S-duality we can define the same theory from the D1-NS5 bound-state in the decoupling limit

$$g_b \to 0 \quad , \qquad l_s = \text{fixed} \quad , \qquad A_{01} \to A_{01}^{\text{critical}} \tag{2}$$

where $g_b = 1/\bar{g}_b$, $l_s^2 = \bar{g}_b \bar{l}_s^2$ and A is the RR two-form field. Thus, just like for ordinary $(1,1)$ LST, the low energy gauge theory on D1-NS5, which has gauge coupling $g_{\rm YM}^2 = (2\pi)^3 l_s^2$, should have a solitonic string of tension $(2\pi)^2/g_{\rm YM}^2 = 1/(2\pi l_s^2)$. Since $g_b = 0$ in the decoupling limit the string cannot leave the brane. In order to study the behavior of this LST-string, as we will call it in this paper, at higher energies, we turn to the supergravity dual description of the theory and in particular the thermodynamics computed from this.

As we will explain in the following, for weak NCOS coupling and low energies the $(1,1)$ OBLST reduces to what we call $D = 5+1$ NCOS, being a theory of weakly coupled open strings. Thus, $D = 5+1$ NCOS can be regarded as a low energy limit of $(1,1)$ OBLST[4]. On the other hand, when the NCOS coupling is large, the LST tension is small, so we instead have a space-time non-commutative LST governing the dynamics of the theory. $(1,1)$ OBLST reduces to Yang-Mills theory when the effective Yang-Mills coupling is small.

2.2 The F1-D5 and D1-NS5 bound states

In this section we give the F1-D5 and D1-NS5 bound-states so that we can find the supergravity dual description of $(1,1)$ OBLST in the next section.

We introduce here the notation that the S-dual string couplings and string lengths are connected as $g_b = 1/\bar{g}_b$ and $l_s^2 = \bar{g}_b \bar{l}_s^2$. Our notation for the string couplings are further clarified in section 5.

The non-extremal F1-D5 bound-state has the string frame metric [26, 25]

$$\begin{aligned} ds^2 \;\; = \;\; & H^{-1/2}\Big[D\Big(-f dt^2 + (dx^1)^2\Big) + (dx^2)^2 + \cdots + (dx^5)^2\Big] \\ & + H^{1/2}\Big[f^{-1} dr^2 + r^2 d\Omega_3^2\Big] \end{aligned} \tag{3}$$

the dilaton

$$e^{2\phi} = H^{-1} D \tag{4}$$

[4] This was also discussed in [25].

and potentials

$$\begin{aligned} B_{01} &= \sin\hat{\theta}\coth\hat{\alpha}DH^{-1} &(5)\\ A_{2345} &= -\tan\hat{\theta}H^{-1} &(6)\\ A_{012345} &= -\frac{1}{\cos\hat{\theta}}\coth\hat{\alpha}(H^{-1}-1) &(7)\end{aligned}$$

with $B_{\mu\nu}$ being the NSNS two-form field, $A_{\mu\nu\rho\sigma}$ being the RR four-form field and $A_{\mu\nu\rho\sigma\kappa\lambda}$ being the RR six-form field. We also define

$$\begin{aligned} f &= 1-\frac{r_0^2}{r^2} &(8)\\ H &= 1+\frac{r_0^2\sinh^2\alpha}{r^2} &(9)\\ D^{-1} &= \cosh^2\theta-\sinh^2\theta H^{-1} &(10)\end{aligned}$$

We use the two sets of variables θ, α and $\hat{\theta}$, $\hat{\alpha}$ related by

$$\sinh^2\alpha = \cos^2\hat{\theta}\sinh^2\hat{\alpha} \quad , \qquad \cosh^2\theta = \frac{1}{\cos^2\hat{\theta}} \tag{11}$$

Using charge quantization of the N D5-branes we get

$$r_0^2\sinh\alpha\sqrt{\sinh^2\alpha+\cosh^{-2}\theta} = \frac{\bar{g}_b\bar{l}_s^2 N}{\cosh\theta} = \frac{l_s^2 N}{\cosh\theta} \tag{12}$$

We now use type IIB S-duality on the F1-D5 solution (3)-(5). This gives the D1-NS5 solution

$$\begin{aligned} ds^2 &= D^{-1/2}\Big[D\Big(-f dt^2+(dx^1)^2\Big)+(dx^2)^2+\cdots+(dx^5)^2\\ &\quad +H\Big(f^{-1}dr^2+r^2 d\Omega_3^2\Big)\Big] &(13)\end{aligned}$$

$$e^{2\phi} = HD^{-1} \tag{14}$$

$$\begin{aligned} A_{01} &= -\sin\hat{\theta}\coth\hat{\alpha}DH^{-1} &(15)\\ A_{2345} &= -\tan\hat{\theta}H^{-1} &(16)\end{aligned}$$

2.3 Supergravity description of $(1,1)$ OBLST

The near-horizon limit of the F1-D5 bound-state is [2, 25]

$$\bar{l}_s\to 0 \quad , \quad \bar{g}_b\bar{l}_s^2 = \text{fixed} \quad , \quad \tilde{r} = \frac{\sqrt{b}}{\bar{l}_s}r \quad , \quad \tilde{r}_0 = \frac{\sqrt{b}}{\bar{l}_s}r_0 \quad , \quad b = \bar{l}_s^2\cosh\theta \tag{17}$$

$$\tilde{x}^i = \frac{\bar{l}_s}{\sqrt{b}} x^i \ , \ i = 0, 1 \quad , \qquad \tilde{x}^j = \frac{\sqrt{b}}{\bar{l}_s} x^j \ , \ j = 2, ..., 5 \tag{18}$$

where we use the notation $x^0 = t$. We have

$$L^2 = \tilde{r}_0^2 \sinh^2 \alpha = \bar{g}_b \bar{l}_s^2 N = l_s^2 N \tag{19}$$

We get the near-horizon solution [25]

$$\begin{aligned} ds^2 &= \frac{\bar{l}_s^2}{b} H^{-1/2} \Big[H \frac{\tilde{r}^2}{L^2} \Big(-f d\tilde{t}^2 + (d\tilde{x}^1)^2 \Big) + (d\tilde{x}^2)^2 + \cdots + (d\tilde{x}^5)^2 \\ &\quad + H \Big(f^{-1} d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2 \Big) \Big] \end{aligned} \tag{20}$$

$$\bar{g}_b^2 e^{2\phi} = \frac{l_s^4}{b^2} \frac{\tilde{r}^2}{L^2} \tag{21}$$

$$B_{01} = \frac{\bar{l}_s^2}{b} \frac{\tilde{r}^2}{L^2} \tag{22}$$

with

$$f = 1 - \frac{\tilde{r}_0^2}{\tilde{r}^2} \quad , \qquad H = 1 + \frac{L^2}{\tilde{r}^2} \tag{23}$$

Using S-duality, the near-horizon limit of D1-NS5 is

$$g_b \to 0 \ , \quad l_s = \text{fixed} \ , \quad \tilde{r} = \frac{\sqrt{b}}{\sqrt{g_b} l_s} r \ , \quad \tilde{r}_0 = \frac{\sqrt{b}}{\sqrt{g_b} l_s} r_0 \ , \quad b = g_b l_s^2 \cosh \theta \tag{24}$$

$$\tilde{x}^i = \frac{\sqrt{g_b} l_s}{\sqrt{b}} x^i \ , \ i = 0, 1 \quad , \qquad \tilde{x}^j = \frac{\sqrt{b}}{\sqrt{g_b} l_s} x^j \ , \ j = 2, ..., 5 \tag{25}$$

The limit (24)-(25) gives the near-horizon solution[5]

$$\begin{aligned} ds^2 &= H^{-1/2} \frac{L}{\tilde{r}} \Big[H \frac{\tilde{r}^2}{L^2} \Big(-f d\tilde{t}^2 + (d\tilde{x}^1)^2 \Big) + (d\tilde{x}^2)^2 + \cdots + (d\tilde{x}^5)^2 \\ &\quad + H \Big(f^{-1} d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2 \Big) \Big] \end{aligned} \tag{26}$$

$$g_b^2 e^{2\phi} = \frac{b^2}{l_s^4} \frac{L^2}{\tilde{r}^2} \tag{27}$$

$$A_{01} = -\frac{g_b l_s^2}{b} \frac{\tilde{r}^2}{L^2} \tag{28}$$

We now give the mapping from our supergravity parameters to the parameters of $(1,1)$ OBLST. The $(1,1)$ OBLST lives on a non-commutative space-time with the commutator $[\tilde{t}, \tilde{x}^1] = ib$. The energy coordinate u is related to the

[5] The string metric does not go to zero in our notation since we define the string metric via e^{ϕ} instead of $g_b e^{\phi}$.

rescaled radial coordinate $\tilde{r}$ as $u = \tilde{r}/b$. As we discuss in the following, the $(1,1)$ OBLST has three different phases: A weakly coupled Yang-Mills phase, a weakly coupled NCOS phase and a phase with LST strings. There are special parameters for each of these phases.

The open string coupling of the 5+1 dimensional NCOS is [2][6]

$$\tilde{g} = \frac{\bar{g}_b \bar{l}_s^2}{b} = \frac{l_s^2}{b} \tag{29}$$

The tension of the open string is $1/b$. The Yang-Mills coupling constant of the 5+1 dimensional Yang-Mills theory is $g_{\rm YM}^2 = (2\pi)^3 \tilde{g} b$. This gives the effective YM coupling constant $g_{\rm eff}^2 = (2\pi)^3 \tilde{g} N b u^2$. The LST-string has the tension $1/(2\pi l_s^2)$.

We now consider the phase structure of $(1,1)$ OBLST in terms of phase diagrams with the energy coordinate u as variable. The three possible phase diagrams for $N \gg 1$ are depicted in figure 1-3. For $N \sim 1$ we have instead only two phase diagrams.

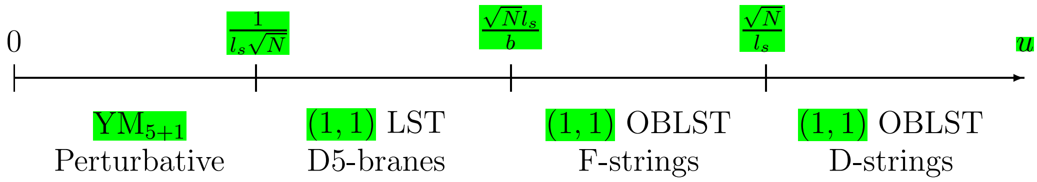

Figure 1: Phase diagram for $(1,1)$ OBLST with $1/N \ll \tilde{g} \ll 1$.

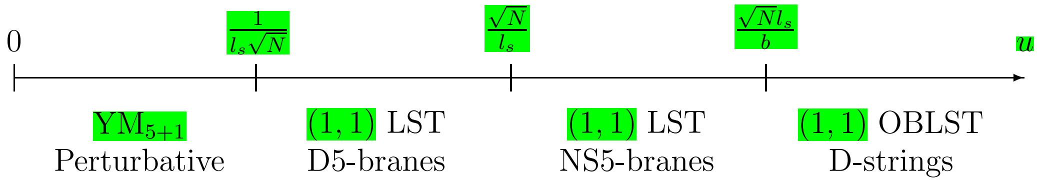

Figure 2: Phase diagram for $(1,1)$ OBLST with $\tilde{g} \gg 1$.

We observe that the supergravity dual of $(1,1)$ OBLST reduces to the supergravity dual of $(1,1)$ LST given in [27, 28] when $u \ll \sqrt{N} l_s / b$.

We consider first $\tilde{g} N \gg 1$ which gives the two possible phase diagrams depicted in figure 1 and 2. We have three transition points. At $g_{\rm eff}^2 \sim 1$, which

[6]For convinience we call $\tilde{g} = G_o^2$ the NCOS open string coupling where G_o is the NCOS open string coupling of [2].

is equivalent to $u \sim 1/(l_s\sqrt{N})$, we flow from a perturbative YM description to a near-horizon D5-brane description. At $g_b e^{\phi} \sim 1$, which is equivalent to $u \sim \sqrt{N}/l_s$, we go either from a D5 to a NS5 description, or from a delocalized F-string to a delocalized D-string description. At $u \sim L/b = \sqrt{N} l_s/b$ we flow from the ordinary $(1,1)$ LST to $(1,1)$ OBLST, and we go either from a D5 to a delocalized F-string description, or from a NS5 to a delocalized D-string description.

Figure 3: Phase diagram for $(1,1)$ OBLST with $\tilde{g} \ll 1/N$.

The third possible phase diagram, depicted in figure 3, has $\tilde{g}N \ll 1$. At energies $u \ll 1/\sqrt{b}$ we have a weakly coupled 5+1 dimensional NCOS theory description, which reduces to perturbative YM theory at low energies. At $u \sim 1/\sqrt{b}$ we flow to a delocalized F-string description and at $u \sim \sqrt{N}/l_s$ we flow to a delocalized D-string description.

In order to understand these phase diagrams, it is useful first to consider the thermodynamics of the supergravity description. The near-horizon solutions (20)-(22) and (26)-(28) give the leading order thermodynamics [25]

$$T = \frac{1}{2\pi l_s \sqrt{N}} \quad , \qquad S = \sqrt{N} \frac{\tilde{V}_5}{(2\pi)^4} \frac{1}{b^2 l_s^3} \tilde{r}_0^2 \tag{30}$$

$$E = \frac{\tilde{V}_5}{(2\pi)^5} \frac{1}{b^2 l_s^4} \tilde{r}_0^2 \quad , \qquad F = 0 \tag{31}$$

This thermodynamics describes $(1,1)$ OBLST for $u \gg u_{\mathrm{SG}}$, where $u_{\mathrm{SG}} = 1/(l_s\sqrt{N})$ for $\tilde{g}N \gg 1$ and $u_{\mathrm{SG}} = 1/\sqrt{b}$ for $\tilde{g}N \ll 1$, since the string corrections to the thermodynamics are small in this region. The Hagedorn temperature of ordinary $(1,1)$ LST is

$$T_{\mathrm{LST}} = \frac{1}{2\pi l_s \sqrt{N}} \tag{32}$$

So, we have that $T \sim T_{\mathrm{LST}}$ for $u \gg u_{\mathrm{SG}}$. This suggest that the LST-strings dominates the dynamics of $(1,1)$ OBLST for $u \gg u_{\mathrm{SG}}$, since the thermodynamics (30)-(31) has the same leading order Hagedorn behaviour as ordinary $(1,1)$ LST [29, 30, 31]. That the LST-strings live on a space-time non-commutative geometry can be seen by the fact that the critical behavior of the entropy at very high energies are different, as shown in [25]. For ordinary $(1,1)$ LST we have that [31, 32] $S(T) \propto (T_{\rm LST} - T)^{-1}$ while for $(1,1)$ OBLST we have [25] $S(T) \propto (T_{\rm LST} - T)^{-2/3}$.

That the LST-string dominates for $u \gg u_{\rm SG}$ is not in contradiction with the existence of open strings in $(1,1)$ OBLST, as we now explain.

Consider first the case $1/N \ll \tilde{g} \ll 1$ with phase diagram depicted in figure 1. This case corresponds to strongly coupled open strings, since $\tilde{g}N \gg 1$. Though the LST-strings are not lighter in this case, there is the other energy scale $1/(l_s\sqrt{N})$ in LST which is connected with LST Hagedorn behavior. As suggested in [29, 31], this could be a LST-string scale connected with fractional strings. The LST-modes corresponding to this scale are clearly lighter than the open string modes, thus explaining why the LST-string modes dominates for $u \gg 1/(l_s\sqrt{N})$.

The second case has $\tilde{g} \gg 1$ which also corresponds to strongly coupled open strings. The phase diagram is depicted in figure 2. We have that $1/b \gg 1/l_s^2$ thus the LST-strings are lighter than the open strings and are therefore expected to dominate, which is confirmed the thermodynamics.

The final case has $\tilde{g}N \ll 1$ corresponding to the phase diagram in figure 3. Since we have that $1/b \ll 1/(Nl_s^2)$ the open strings of NCOS theory are much lighter than the LST-modes connected with LST Hagedorn behaviour and we should therefore expect them to dominate the dynamics. For the NCOS Hagedorn temperature $T_{\rm NCOS}$ we know that $T_{\rm NCOS} \sim 1/\sqrt{b}$. Thus, we have that $T_{\rm NCOS} \ll T_{\rm LST}$. Clearly, the NCOS Hagedorn temperature is not limiting, and we should have a Hagedorn phase transition at a certain energy $u_{\rm NCOS}$. Since we have just shown that for $u \gg 1/\sqrt{b}$ we had $T \sim T_{\rm LST}$, we get that $u_{\rm NCOS} \lesssim 1/\sqrt{b}$. Thus, the reason that the LST-strings can dominate at high energies, even though they are heavier than the open strings, is that the open strings have been subject to a Hagedorn transition at these energies. We note that the lower dimensional NCOS theories exhibit similar behaviour in the sense that, when they are weakly coupled, the supergravity dual describes them only when the temperature are above the NCOS Hagedorn temperature and a Hagedorn phase transition has occured [25].

In summary, we have learned that at weak NCOS coupling $\tilde{g}N \ll 1$ the $(1,1)$ OBLST has an NCOS phase for energies $u \ll 1/\sqrt{b}$, with LST-strings dominating at higher energies, as depicted in figure 3. For strong NCOS coupling $\tilde{g}N \gg 1$ we have perturbative YM for low energies and LST-strings at high energies as depicted in figure 1 and 2. Thus, in this sense one can say that strongly coupled 5+1 dimensional NCOS theory gives a space-time non-commutative version of $(1,1)$ LST.

2.4 Branes in $(1,1)$ OBLST

In the ordinary $(1,1)$ LST we have besides the LST-string with tension $1/(2\pi l_s^2)$ the d0, d2 and d4 branes [23]. These origins from having open D1, D3 and D5 branes stretching between NS5-branes.

In the $(1,1)$ OBLST we still have the LST-string, but now the D-string stretching between two D1-NS5 bound-states induces an open string. We note that the zero modes of the open D-string is what gives the Yang-Mills theory at low energies, which fits with the picture that the NCOS theory at low energies reduce to Yang-Mills theory.

Since there is not any electric field on the ends of an open D3-brane stretching between D1-NS5 bound-states we expect that we still have the same d-membrane in $(1,1)$ OBLST as in $(1,1)$ LST, but presumably now moving in a space-time non-commutative geometry. Also the d4-brane seem to be part of $(1,1)$ OBLST.

In other words, only the open D-brane for which the potential goes to its critical value, gives an open brane in the world-volume theory. The rest of the spectrum is unchanged.

3 Supergravity dual of OM theory

In this section we find and study the supergravity dual of OM theory. We find the OM supergravity dual by uplifting the 4+1 dimensional NCOS supergravity dual. Apart from being interesting in its own right, it is also important to understand the OM theory near-horizon limit in order to understand the decoupling and near-horizon limit of $(2,0)$ OBLST.

The near-horizon and decoupling limits of the M2-M5 bound state have previously been studied in [33, 34, 35, 36, 37, 38, 39, 3, 4].

3.1 The M2-M5 and F1-D4 bound states

In this section we describe the supergravity solutions that we use for OM theory and 4+1 dimensional NCOS theory.

The non-extremal M2-M5 brane bound-state has the metric [40]

$$
\begin{aligned}
ds^2_{11} &= (\hat{H}\hat{D})^{-1/3}\Big[-fdt^2+(dx^1)^2+(dx^2)^2+\hat{D}\Big((dx^3)^2+(dx^4)^2+(dx^5)^2\Big) \\
&\quad +\hat{H}\Big(f^{-1}dr^2+r^2d\Omega_4^2\Big)\Big] \qquad (33)
\end{aligned}
$$

and the three- and six-form potentials

$$
\begin{aligned}
C_{012} &= -\sin\hat{\theta}\hat{H}^{-1}\coth\hat{\alpha} \qquad (34)\\
C_{345} &= \tan\hat{\theta}\hat{D}\hat{H}^{-1} \qquad (35)\\
C_{012345} &= \cos\hat{\theta}\hat{D}(\hat{H}^{-1}-1)\coth\hat{\alpha} \qquad (36)
\end{aligned}
$$

We have

$$
f = 1-\frac{r_0^3}{r^3} \quad , \qquad \hat{H} = 1+\frac{r_0^3\sinh^2\hat{\alpha}}{r^3} \qquad (37)
$$

$$
\hat{D}^{-1} = \cos^2\hat{\theta}+\sin^2\hat{\theta}\hat{H}^{-1} \qquad (38)
$$

The charge quantization for N M5-branes gives

$$
r_0^3\cosh\hat{\alpha}\sinh\hat{\alpha} = \frac{\pi N l_p^3}{\cos\hat{\theta}} \qquad (39)
$$

In the new variables

$$
\sinh^2\alpha = \cos^2\hat{\theta}\sinh^2\hat{\alpha} \quad , \qquad \cosh^2\theta = \frac{1}{\cos^2\hat{\theta}} \qquad (40)
$$

we have

$$
\begin{aligned}
ds^2_{11} &= (HD)^{-1/3}\Big[D\Big(-fdt^2+(dx^1)^2+(dx^2)^2\Big)+(dx^3)^2+(dx^4)^2+(dx^5)^2 \\
&\quad +H\Big(f^{-1}dr^2+r^2d\Omega_4^2\Big)\Big] \qquad (41)
\end{aligned}
$$

$$
\begin{aligned}
H &= 1+\frac{r_0^3\sinh^2\alpha}{r^3} \qquad (42)\\
D^{-1} &= \cosh^2\theta-\sinh^2\theta H^{-1} \qquad (43)
\end{aligned}
$$

We note that $\hat{H}=HD^{-1}$, $H=\hat{H}\hat{D}^{-1}$ and $D=\hat{D}^{-1}$.

We now dimensionally reduce the M2-M5 on the electric circle with the coordinate x^2. This gives the F1-D4 bound-state solution. The relation between the eleven dimensional metric ds^2_{11} and the ten-dimensional string-frame metric ds^2_{10} and dilaton e^{ϕ} is

$$
ds^2_{11} = e^{-\frac{2}{3}\phi}ds^2_{10}+e^{\frac{4}{3}\phi}(dx^2)^2 \qquad (44)
$$

Thus, we get the metric

$$ds_{10}^2 = H^{-1/2}\Big[D\Big(-fdt^2+(dx^1)^2\Big)+(dx^3)^2+(dx^4)^2+(dx^5)^2 + H\Big(f^{-1}dr^2+r^2d\Omega_4^2\Big)\Big] \tag{45}$$

the dilaton

$$e^{2\phi} = H^{-1/2}D \tag{46}$$

and the NSNS two-form potential

$$B_{01} = -\sin\hat{\theta}\coth\hat{\alpha}DH^{-1} \tag{47}$$

This solution coincides with the one given in [25].

3.2 Near-horizon limit of OM theory from $D = 4+1$ NCOS

The near horizon limit of 4+1 dimensional NCOS is [2, 25]

$$\bar{l}_s \to 0 \quad , \quad \tilde{r} = \frac{\sqrt{b}}{\bar{l}_s}r \quad , \quad \tilde{r}_0 = \frac{\sqrt{b}}{\bar{l}_s}r_0 \quad , \quad b = \bar{l}_s^2\cosh\theta \quad , \quad \alpha = \text{fixed} \tag{48}$$

$$\tilde{x}^i = \frac{\bar{l}_s}{\sqrt{b}}x^i \ , \ i = 0,1 \quad , \quad \tilde{x}^j = \frac{\sqrt{b}}{\bar{l}_s}x^j \ , \ j = 3,4,5 \tag{49}$$

We have the open string coupling squared

$$\tilde{g} = \frac{\bar{g}_a\bar{l}_s^2}{b} \tag{50}$$

Using (48)-(49) on (45)-(47) we get [25]

$$ds_{10}^2 = \frac{\bar{l}_s^2}{b}H^{-1/2}\left[H\frac{\tilde{r}^2}{L^2}\Big(-fd\tilde{t}^2+(d\tilde{x}^1)^2\Big)+(d\tilde{x}^3)^2+(d\tilde{x}^4)^2+(d\tilde{x}^5)^2 + H\Big(f^{-1}d\tilde{r}^2+\tilde{r}^2d\Omega_4^2\Big)\right] \tag{51}$$

$$\bar{g}_a^2e^{2\phi} = \tilde{g}^2H^{-1/2}\Big(1+\frac{\tilde{r}^3}{L^3}\Big) \tag{52}$$

$$B_{01} = \frac{\bar{l}_s^2}{b}\frac{\tilde{r}^3}{L^3} \tag{53}$$

with

$$L^3 = \tilde{r}_0^3\sinh^2\alpha = \pi N\tilde{g}b^{3/2} \tag{54}$$

$$H = 1+\frac{L^3}{\tilde{r}^3} \quad , \quad f = 1-\frac{\tilde{r}_0^3}{\tilde{r}^3} \tag{55}$$

We now introduce the open membrane length scale l_m in OM theory defined by stating that the open membrane has tension $1/l_m^3$. As shown in [3, 4] we then have

$$l_m^3 = \tilde{g} b^{3/2} \tag{56}$$

This can be understod as follows. Since the radius of the electric circle with coordinate x^2 is $R_E = \bar{g}_a \bar{l}_s$ the rescaled radius is

$$\tilde{R}_E = \frac{\bar{g}_a \bar{l}_s^2}{\sqrt{b}} = \tilde{g}\sqrt{b} \tag{57}$$

where the rescaling $\tilde{R}_E = R_E \bar{l}_s / \sqrt{b}$ follows from the fact that the x^2 coordinate should scale the same way as the x^1 coordinate in (49). We can then write the relation[7]

$$\frac{1}{b} = \frac{\tilde{R}_E}{l_m^3} \tag{58}$$

where $1/b$ is the tension of the open string in NCOS. Thus, the relation (56) is the statement that the open string in NCOS is the open membrane in OM theory wrapped on a circle of radius $\tilde{R}_E$.

We now want to use (56) to write the near-horizon limit (48)-(49) in terms of the eleven dimensional variables l_m and l_p, where $l_p^3 = \bar{g}_a \bar{l}_s^3$. Using (56) we have

$$\frac{\sqrt{b}}{\bar{l}_s} = \frac{\tilde{g} b^{3/2}}{\tilde{g} b \bar{l}_s} = \frac{l_m^3}{l_p^3} \tag{59}$$

Using this together with (48)-(49) we can write the eleven dimensional near-horizon limit of OM theory as

$$l_p \to 0 \quad , \qquad \tilde{r} = \frac{l_m^3}{l_p^3} r \quad , \qquad \tilde{r}_0 = \frac{l_m^3}{l_p^3} r_0 \quad , \qquad l_m^6 = l_p^6 \cosh\theta \tag{60}$$

$$\tilde{x}^i = \frac{l_p^3}{l_m^3} x^i , \quad i = 0,1,2 \quad , \qquad \tilde{x}^j = \frac{l_m^3}{l_p^3} x^j , \quad j = 3,4,5 \tag{61}$$

This is a purely eleven dimensional near-horizon limit of OM theory, meaning that it can describe OM theory with the Lorentz symmetry $SO(1,2) \times SO(3)$.

The near-horizon limit (60)-(61) is the same limit of the M2-M5 brane bound state as in [35, 36]. Keeping r/l_p^3 fixed means that the membrane modes for open M2-branes stretching between M5-branes are kept finite.

[7] In these type of relations we ignore factors of 2π.

Using (60)-(61) on (41) and (34)-(36) we get the supergravity dual

$$\begin{aligned} ds_{11}^2 &= \frac{l_p^2}{l_m^2} H^{-2/3} \frac{L}{\tilde{r}} \Big[H \frac{\tilde{r}^3}{L^3} \Big(-f d\tilde{t}^2 + (d\tilde{x}^1)^2 + (d\tilde{x}^2)^2 \Big) \\ &\quad + (d\tilde{x}^3)^2 + (d\tilde{x}^4)^2 + (d\tilde{x}^5)^2 + H \Big(f^{-1} d\tilde{r}^2 + \tilde{r}^2 d\Omega_4^2 \Big) \Big] \end{aligned} \tag{62}$$

$$C_{012} = -\frac{l_p^3}{l_m^3} \frac{\tilde{r}^3}{L^3} \quad , \quad C_{345} = \frac{l_p^3}{l_m^3} H^{-1} \tag{63}$$

$$H = 1 + \frac{L^3}{\tilde{r}^3} \quad , \quad f = 1 - \frac{\tilde{r}_0^3}{\tilde{r}^3} \tag{64}$$

When OM theory is on an electric circle of rescaled radius $\tilde{R}_E$, the energy coordinate u is

$$u = \frac{\tilde{r}}{b} = \frac{\tilde{R}_E \tilde{r}}{l_m^3} \tag{65}$$

3.3 Phase structure of OM and $D = 4 + 1$ NCOS theory

In this section we examine the phase structure of OM theory and the 4+1 dimensional NCOS theory via their supergravity duals.

The OM theory near-horizon solution (62)-(63) is valid when the curvature in units of l_p^{-2}

$$\mathcal{C} = \left(\pi N^2 + \frac{N \tilde{r}^3}{l_m^3} \right)^{-1/3} \tag{66}$$

is small. Thus, if $N \gg 1$ this is clearly satisfied and we can describe OM theory for all energies. If N is of order 1, we instead need that $\tilde{r} \gg N^{1/3} l_m$ since we need that $\tilde{r} \gg L$.

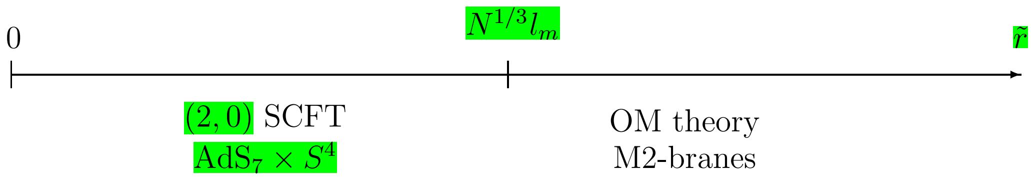

Figure 4: Phase diagram of OM theory.

From the supergravity dual (62)-(63) we see that for $\tilde{r} \ll N^{1/3} l_m$ the solution reduces to $AdS_7 \times S^4$ describing the six-dimensional $(2,0)$ SCFT [41]. Thus, for $\tilde{r} \ll N^{1/3} l_m$ OM theory reduces to $(2,0)$ SCFT. For $\tilde{r} \gg N^{1/3} l_m$ the open membrane is large enough to have an effect and the underlying non-commutative geometry is detectable. For $\tilde{r} \gg N^{1/3} l_m$ the solution is described by M2-branes delocalized in 3 directions. The phases are depicted in Figure 4.

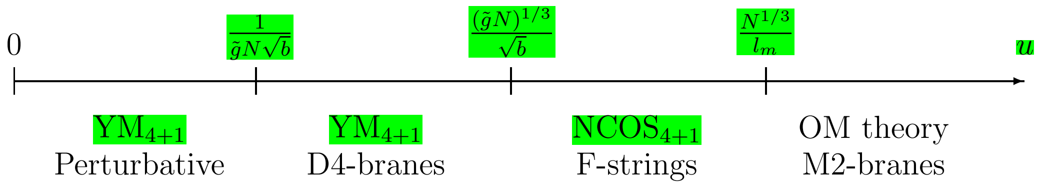

Figure 5: Phase diagram for $D = 4+1$ NCOS with $1/N \ll \tilde{g} \ll 1$.

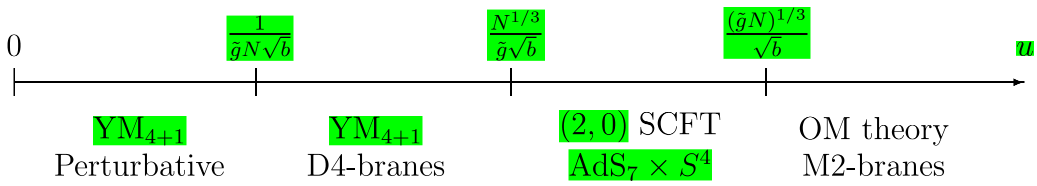

Figure 6: Phase diagram for $D = 4+1$ NCOS with $\tilde{g} \gg 1$.

For 4+1 dimensional NCOS the curvature of the supergravity dual in units of l_s^{-2} is [25]

$$\mathcal{C} = \frac{b}{\tilde{r}^2} H^{-1/2} \tag{67}$$

Thus, for $\tilde{g}N < 1$ we need $\tilde{r} \gg \sqrt{b}/(\tilde{g}N)$ while for $\tilde{g}N > 1$ we need $\tilde{r} \gg \sqrt{b}$ in order for $\mathcal{C} \ll 1$.

Consider first $\tilde{g}N \gg 1$. When $\tilde{r} \sim L$ we flow from YM to NCOS with the space-time commutator $[\tilde{t}, \tilde{x}^1] = ib$. The 4+1 dimensional NCOS theory flows into OM theory when $\bar{g}_a e^{\phi} \sim 1$. This gives two possible phase diagrams, which we have depicted in the figures 5 and 6.

The case with $\tilde{g}N \ll 1$ is depicted in figure 7. This case corresponds to weakly coupled NCOS theory. The NCOS theory flows to OM theory at $u \sim N^{1/3}/l_m$.

The thermodynamics of the 4+1 dimensional NCOS theory from the supergravity dual is [25]

$$T = \frac{3}{4\pi} \frac{\sqrt{\tilde{r}_0}}{\sqrt{\pi N \tilde{g} b^{3/2}}} \quad , \qquad S = \frac{1}{12\pi^4} \tilde{V}_4 \frac{\sqrt{\pi N \tilde{g} b^{3/2}}}{\tilde{g}^2 b^4} \tilde{r}_0^{5/2} \tag{68}$$

$$E = \frac{5}{96\pi^5} \frac{\tilde{V}_4}{\tilde{g}^2 b^4} \tilde{r}_0^3 \quad , \qquad F = -\frac{1}{96\pi^5} \frac{\tilde{V}_4}{\tilde{g}^2 b^4} \tilde{r}_0^3 \tag{69}$$

$$F = -\frac{2^7 \pi^4}{3^7} N^3 \tilde{g} \sqrt{b} \tilde{V}_4 T^6 \tag{70}$$

As noted in [25] this thermodynamics is equivalent to that of ordinary 4+1 dimensional YM for large N and strong 't Hooft coupling.

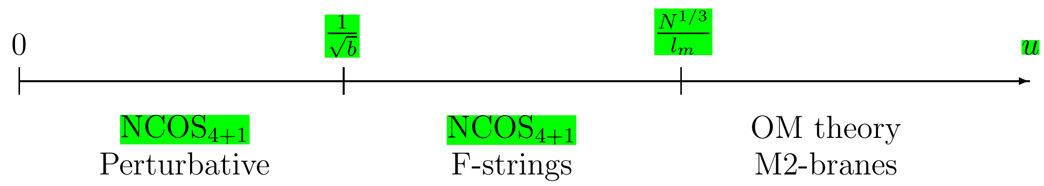

Figure 7: Phase diagram for $D = 4+1$ NCOS with $\tilde{g}N \ll 1$.

The thermodynamics of OM theory from its supergravity dual is

$$T = \frac{3}{4\pi} \frac{\sqrt{\tilde{r}_0}}{\sqrt{\pi N l_m^3}} \quad , \qquad S = \frac{1}{24\pi^5} \tilde{V}_5 \frac{\sqrt{\pi N l_m^3}}{l_m^9} \tilde{r}_0^{5/2} \tag{71}$$

$$E = \frac{5}{192\pi^6} \frac{\tilde{V}_5}{l_m^9} \tilde{r}_0^3 \quad , \qquad F = -\frac{1}{192\pi^6} \frac{\tilde{V}_5}{l_m^9} \tilde{r}_0^3 \tag{72}$$

$$F = -\frac{2^6 \pi^3}{3^7} N^3 \tilde{V}_5 T^6 \tag{73}$$

We see that the thermodynamics of OM theory is equivalent to that of $(2,0)$ SCFT for large N.

4 $(2,0)$ OBLST and OM theory

4.1 Introduction to $(2,0)$ OBLST

The $(2,0)$ OBLST is defined as the D2-NS5 bound-state in the decoupling limit

$$g_a \to 0 \quad , \qquad l_s = \text{fixed} \quad , \qquad A_{012} \to A_{012}^{\text{critical}} \tag{74}$$

We show in the following that this limit follows both from using the near-horizon/decoupling limit of OM theory found in Section 3.2 and from doing a T-duality on the $(1,1)$ OBLST. The $(2,0)$ OBLST has an LST-string and since $(2,0)$ OBLST reduces to OM theory for low energies, it also has an open membrane. The T-duality between the two OBLSTs is shown to relate the open membrane to the open string of $(1,1)$ OBLST.

At high energies the LST-string dominates and the thermodynamics has LST Hagedorn behavior. The R-symmetry is $SO(4)$ for these energies, but at low energies we get OM theory and the R-symmetry is enhanced to $SO(5)$. This we show using the supergravity dual of $(2,0)$ OBLST.

The decoupling and near-horizon limits of the D2-NS5 bound state have previously been studied in [35, 42].

4.2 D2-NS5 bound state from M2-M5 on a transverse circle

The D2-NS5 bound-state in type IIA string theory can be considered as an M2-M5 bound-state localized on a transverse circle. Thus, from the M2-M5 bound state in section 3.1 we get the metric[8]

$$
\begin{aligned}
ds_{11}^2 &= (HD)^{-1/3}\Big[D\Big(-dt^2+(dx^1)^2+(dx^2)^2\Big)+(dx^3)^2+(dx^4)^2+(dx^5)^2 \\
&\quad + H\Big(dz^2+dr^2+r^2d\Omega_3^2\Big)\Big]
\end{aligned} \tag{75}
$$

with

$$
H = 1+\sum_{n=-\infty}^{\infty}\frac{\pi N l_p^3}{(r^2+(z+2\pi n R_T)^2)^{3/2}} \tag{76}
$$

$$
D^{-1} = \cosh^2\theta - \sinh^2\theta H^{-1} \tag{77}
$$

where z is the coordinate of the transverse circle with the asymptotic radius R_T.

For $r \gg R_T$ we have

$$
H = 1+\frac{1}{\pi R_T}\frac{\pi N l_p^3}{r^2} \tag{78}
$$

We now consider the z coordinate as the eleven dimensional coordinate. Thus by the usual M/IIA correspondence we have $R_T = g_a l_s$ and $l_p^3 = R_T l_s^2$. The D2-NS5 bound-state has the string-frame metric ds_{10}^2 and the dilaton e^{ϕ} given by the formula

$$
ds_{11}^2 = e^{-\frac{2}{3}\phi}ds_{10}^2 + e^{\frac{4}{3}\phi}dz^2 \tag{79}
$$

This gives the D2-NS5 solution

$$
\begin{aligned}
ds_{10}^2 &= D^{-1/2}\Big[D\Big(-dt^2+(dx^1)^2+(dx^2)^2\Big)+(dx^3)^2+(dx^4)^2+(dx^5)^2 \\
&\quad + H\Big(dr^2+r^2d\Omega_3^2\Big)\Big]
\end{aligned} \tag{80}
$$

$$
e^{2\phi} = HD^{-1/2} \tag{81}
$$

$$
A_{012} = -\sin\hat{\theta} D H^{-1} \tag{82}
$$

$$
A_{345} = \tan\hat{\theta} H^{-1} \tag{83}
$$

[8]We write only the extremal version of this solution so in comparing with the non-extremal M2-M5 solution (41) and (34)-(36) one should use that $r_0^3 \sinh^2\alpha = \pi N l_p^3$ for $r_0 \to 0$.

4.3 Supergravity dual of $(2,0)$ OBLST

The near-horizon limit of the solution (75) is

$$l_p \to 0 \quad , \quad \tilde{r} = \frac{l_m^3}{l_p^3} r \quad , \quad \tilde{z} = \frac{l_m^3}{l_p^3} z \quad , \quad l_m^6 = l_p^6 \cosh\theta \tag{84}$$

$$\tilde{x}^i = \frac{l_p^3}{l_m^3} x^i, \quad i = 0,1,2 \quad , \quad \tilde{x}^j = \frac{l_m^3}{l_p^3} x^j, \quad j = 3,4,5 \tag{85}$$

This we obtained from the OM near-horizon limit (60)-(61) since the $(2,0)$ OBLST limit should be consistent with the OM theory limit.

The eleven dimensional supergravity dual of $(2,0)$ OBLST is therefore given by[9]

$$\begin{aligned} ds_{11}^2 \;=\; & \frac{l_p^2}{l_m^2} H^{-2/3} J^{1/3} \Big[H J^{-1} \Big(-d\tilde{t}^2 + (d\tilde{x}^1)^2 + (d\tilde{x}^2)^2 \Big) \\ & + (d\tilde{x}^3)^2 + (d\tilde{x}^4)^2 + (d\tilde{x}^5)^2 + H \Big(d\tilde{z}^2 + d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2 \Big) \Big] \end{aligned} \tag{86}$$

$$C_{012} = -\frac{l_p^3}{l_m^3} J^{-1} \quad , \quad C_{345} = \frac{l_p^3}{l_m^3} H^{-1} \tag{87}$$

$$J = \sum_{n=-\infty}^{\infty} \frac{\pi N l_m^3}{(\tilde{r}^2 + (\tilde{z} + 2\pi n \tilde{R}_T)^2)^{3/2}} \quad , \quad H = 1 + J \tag{88}$$

The limit (84)-(85) is precisely consistent with our requirements of a limit of $(2,0)$ OBLST. We need that r/R_T is finite in the near-horizon limit so since $l_p^3 = g_a l_s^3$ and $R_T = g_a l_s$ we see that the OBLST limit is $g_s \to 0$ and l_s kept fixed. But this is the limit of ordinary LST so we should have closed strings of tension $1/(2\pi l_s^2)$ in $(2,0)$ OBLST. Note that $\tilde{R}_T = l_m^3 / l_s^2$.

We observe that the supergravity dual of $(2,0)$ OBLST given by (86)-(87) reduces to the supergravity dual of $(2,0)$ LST given in [27, 28] when $\tilde{r} \ll N^{1/3} l_m$.

For $\tilde{r} \ll \tilde{R}_T$ we should consider (86)-(87) as an approximate description of $(2,0)$ OBLST which for small $\tilde{r}$ continuously should flow to the OM theory supergravity dual given by (62)-(63).

[9]The construction of the supergravity dual of $(2,0)$ OBLST presented here is similar to that of the D2-NS5 near-horizon solution presented in [42]. The D2-NS5 near-horizon solution of [42] describes $(2,0)$ OBLST on a magnetic circle. We thank M. Alishahiha for a discussion about this point.

When $\tilde{r} \gg \tilde{R}_T$ and $g_a e^{\phi} \ll 1$ we can use a ten-dimensional description via weakly coupled type IIA string theory. The limit (84)-(85) translates into the limit

$$g_a \to 0 \quad , \quad l_s = \text{fixed} \quad , \quad \tilde{r} = \frac{l_m^3}{g_a l_s^3} r \quad , \quad l_m^6 = g_a^2 l_s^6 \cosh\theta \tag{89}$$

$$\tilde{x}^i = \frac{g_a l_s^3}{l_m^3} x^i, \quad i = 0,1,2 \quad , \quad \tilde{x}^j = \frac{l_m^3}{g_a l_s^3} x^j, \quad j = 3,4,5 \tag{90}$$

The type IIA near-horizon solution is then

$$\begin{aligned} ds_{10}^2 \quad = \quad & H^{-1/2} \frac{L}{\tilde{r}} \Big[H \frac{\tilde{r}^2}{L^2} \Big(- d\tilde{t}^2 + (d\tilde{x}^1)^2 + (d\tilde{x}^2)^2 \Big) \\ & + (d\tilde{x}^3)^2 + (d\tilde{x}^4)^2 + (d\tilde{x}^5)^2 + H \Big(d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2 \Big) \Big] \end{aligned} \tag{91}$$

$$g_a^2 e^{2\phi} = \frac{l_m^6}{l_s^6} H^{1/2} \frac{L}{\tilde{r}} \tag{92}$$

$$A_{012} = - \frac{g_a l_s^3}{l_m^3} \frac{\tilde{r}^2}{L^2} \quad , \quad A_{345} = \frac{g_a l_s^3}{l_m^3} H^{-1} \tag{93}$$

$$L^2 = l_s^2 N \quad , \quad H = 1 + \frac{L^2}{\tilde{r}^2} \tag{94}$$

4.4 T-duality on an electric circle: From open membranes to open strings

Since we believe that $(2,0)$ OBLST has an open membrane of tension $1/l_m^3$ and that $(1,1)$ OBLST has an open string of tension $1/b$ it is natural to ask whether this is consistent with T-duality. From point of view of the bulk, T-duality on an electric circle in $(2,0)$ OBLST would give $(1,1)$ OBLST, since it takes D2-NS5 into D1-NS5. In this section we test that this is also consistent with the decoupling limits. In section 5 we develop this further and connect 5 different bound states and their decoupling limits in a duality-chain.

We take x^2 as the coordinate of the electric circle with radius R_E. From (84)-(85) and (89)-(90) we get

$$\tilde{R}_E = \frac{g_a l_s^3}{l_m^3} R_E \quad , \quad \tilde{R}_T = \frac{l_m^3}{g_a l_s^3} R_T \tag{95}$$

Since $R_T = g_a l_s$ we have

$$\tilde{R}_T = \frac{l_m^3}{l_s^2} \tag{96}$$

A T-duality in x^2 gives

$$g_b = g_a \frac{l_s}{R_E} = \frac{R_T}{R_E} = \frac{g_a^2 l_s^4}{l_m^3} \frac{1}{\tilde{R}_E} \tag{97}$$

Since l_s is fixed in both $(1,1)$ and $(2,0)$ OBLST this means that $g_b \propto g_a^2$. By comparing (24)-(25) with (89)-(90) we see that this is exactly what we need for the decoupling/near-horizon limits of $(2,0)$ and $(1,1)$ OBLST to be consistent with each other. Moreover, we see that we need

$$\frac{l_m^3}{g_a l_s^3} = \frac{\sqrt{b}}{\sqrt{g_b} l_s} \tag{98}$$

Using (97) and (98) we get that

$$\tilde{R}_E = \frac{g_a^2}{g_b} \frac{l_s^4}{l_m^3} = \frac{l_m^3}{b} \tag{99}$$

Thus, the T-duality between the decoupling limits of $(1,1)$ and $(2,0)$ OBLST requires that

$$\frac{1}{b} = \frac{\tilde{R}_E}{l_m^3} \tag{100}$$

This relation means that the open string of $(1,1)$ OBLST with tension $1/b$ is the open membrane of $(2,0)$ OBLST wrapped around the electric circle of radius $\tilde{R}_E$. Thus, the open membrane in $(2,0)$ OBLST and the open string in $(1,1)$ OBLST are related by T-duality.

We elaborate further on this in Section 5.

4.5 Phase structure and thermodynamics

As already mentioned, the $(2,0)$ OBLST has both the open membrane with tension $1/l_m^3$ as in OM theory, and also the LST-string with tension $1/(2\pi l_s^2)$. We now consider the phase structure of $(2,0)$ OBLST. We parameterize the phase diagrams with the rescaled radial coordinate $\tilde{r}$. This is not an energy coordinate, but any energy coordinate should be increasing with $\tilde{r}$ and we can therefore use it to find the succession of transition points.

We consider two possible phase diagrams depicted in figure 8 and 9. For both diagrams we have that at $\tilde{r} \sim \tilde{R}_T$ we have a transition point where for lower energies we have $SO(5)$ R-symmetry and for higher energies $SO(4)$ R-symmetry. In the supergravity solution this can be understod from the observation that at $\tilde{r} \sim \tilde{R}_T$ the radius of the S^3 in the metric (86) is of the same order as the radius $\tilde{R}_T$ of the transverse circle. Thus, at $\tilde{r} \ll \tilde{R}_T$ the supergravity dual of $(2,0)$ OBLST is in fact the supergravity dual of OM theory, given in Section 3.2.

The curvature in units of l_s^{-2} of the supergravity dual for $\tilde{r} \gg \tilde{R}_T$ is

$$\mathcal{C} = \frac{1}{N} \frac{1}{\sqrt{1 + \frac{\tilde{r}^2}{l_s^2 N}}} \tag{101}$$

For $\tilde{r} \ll \tilde{R}_T$ the curvature is given by (66). In the following we work with $N \gg 1$.

We first consider the phase diagram of figure 8. For low energies we have $(2,0)$ SCFT with $SO(5)$ as the R-symmetry group. This is described by the $\mathrm{AdS}_7 \times S^4$ supergravity dual. From Section 3.3 we know that at $\tilde{r} \sim N^{1/3} l_m$ we have OM theory, described by delocalized M2-branes. At $\tilde{r} \sim \tilde{R}_T$ the R-symmetry is broken to $SO(4)$ and we go into $(2,0)$ OBLST. However, we do not enter the weakly coupled type IIA description before the transition point $g_a e^{\phi} \sim 1$, which is at $\tilde{r} \sim \sqrt{N} l_m^6 / l_s^5$. Thus, we have either delocalized M2-branes or delocalized D2-branes describing the $(2,0)$ OBLST phase.

In the second phase diagram, depicted in figure 9, we also start at low energies with $(2,0)$ SCFT. We then proceed to the ordinary $(2,0)$ LST at $\tilde{r} \sim \tilde{R}_T$. At $\tilde{r} \sim \sqrt{N} l_m^3 / l_s^2$ we enter the weakly coupled type IIA description so that the $(2,0)$ LST is described by NS5-branes. At $\tilde{r} \sim \sqrt{N} l_s$ we enter the $(2,0)$ OBLST phase with a non-commutative space-time.

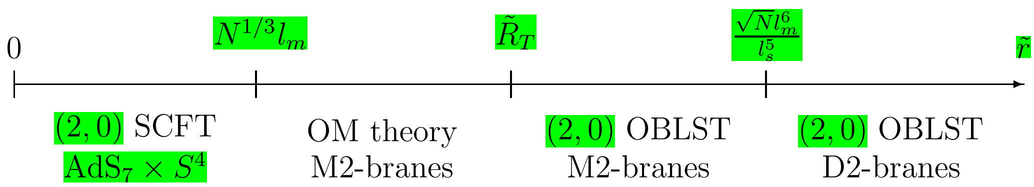

Figure 8: Phase diagram for $(2,0)$ OBLST.

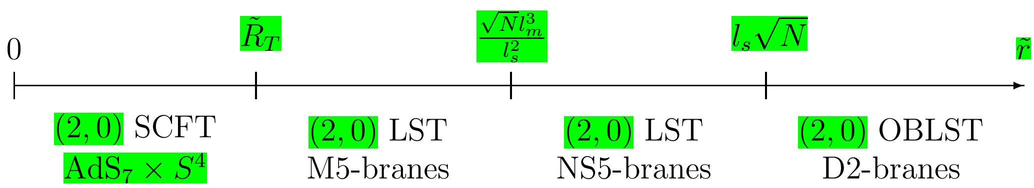

Figure 9: Phase diagram for $(2,0)$ OBLST.

From a non-extremal version of the metric (86) the thermodynamics of $(2,0)$ OBLST for $\tilde{r} \gg \tilde{R}_T$ is found to be

$$T = \frac{1}{2\pi l_s \sqrt{N}} \quad , \quad S = \sqrt{N} \frac{\tilde{V}_5}{(2\pi)^4} \frac{1}{l_s l_m^6} \tilde{r}_0^2 \tag{102}$$

$$E = \frac{\tilde{V}_5}{(2\pi)^5} \frac{1}{l_s^2 l_m^6} \tilde{r}_0^2 \quad , \quad F = 0 \tag{103}$$

The Hagedorn temperature of $(2,0)$ OBLST is

$$T_{\mathrm{LST}} = \frac{1}{2\pi l_s \sqrt{N}} \tag{104}$$

Thus, we see that for $\tilde{r} \gg \tilde{R}_T$ we have $T \sim T_{\mathrm{LST}}$ so that the LST-strings dominate for $\tilde{r} \gg \tilde{R}_T$. As discussed in [29, 30, 31] the thermodynamics (102)-(103) exhibits leading order Hagedorn behavior, and one can calculate [43] that since the supergravity dual consist of delocalized D2-branes in the UV-region, the entropy has the critical behavior $S(T) \propto (T_{\mathrm{LST}} - T)^{-2/3}$, just as for $(1,1)$ OBLST. Thus, the critical behavior of the entropy for high energies in $(2,0)$ OBLST is different from that in $(2,0)$ LST.

From comparing the phases and thermodynamics of $(1,1)$ OBLST and $(2,0)$ we see that there are many similarities, as one would expect from T-dual theories. The LST-strings dominate the thermodynamics for high energies in both cases, and the open string and open membrane only appears as phases in the phase diagrams when they are sufficiently light.

4.6 Branes in $(2,0)$ OBLST

In the ordinary $(2,0)$ LST we have besides the LST-string with tension $1/(2\pi l_s^2)$ the d1, d3 and d5 branes[23]. These origins from having open D2, D4 and D6 branes stretching between NS5-branes.

The $(2,0)$ OBLST still have the LST-string, but the D-membrane stretching between two D2-NS5 bound-states induces an open membrane which gives OM theory at low energy. At low energies this open membrane reduces to a d1-brane in $(2,0)$ LST, or to a tensionless string in $(2,0)$ SCFT. Using similar arguments as for $(1,1)$ OBLST, we expect that the d3 and d5-branes are part of $(2,0)$ OBLST.

Thus, we see that the brane-spectra of $(1,1)$ and $(2,0)$ OBLST are related by T-duality.

5 A duality-chain of theories

In this section[10] we systematically explore how the T-dualities and S-dualities connects the various bound-states and their decoupling limits that we have been discussing in Section 2-4, in order test the consistency of these limits and also to relate the parameters of the theories. Some of the discussion has already appeared in earlier sections, but we repeat it here for clarity. The duality-chain is depicted in Figure 10.

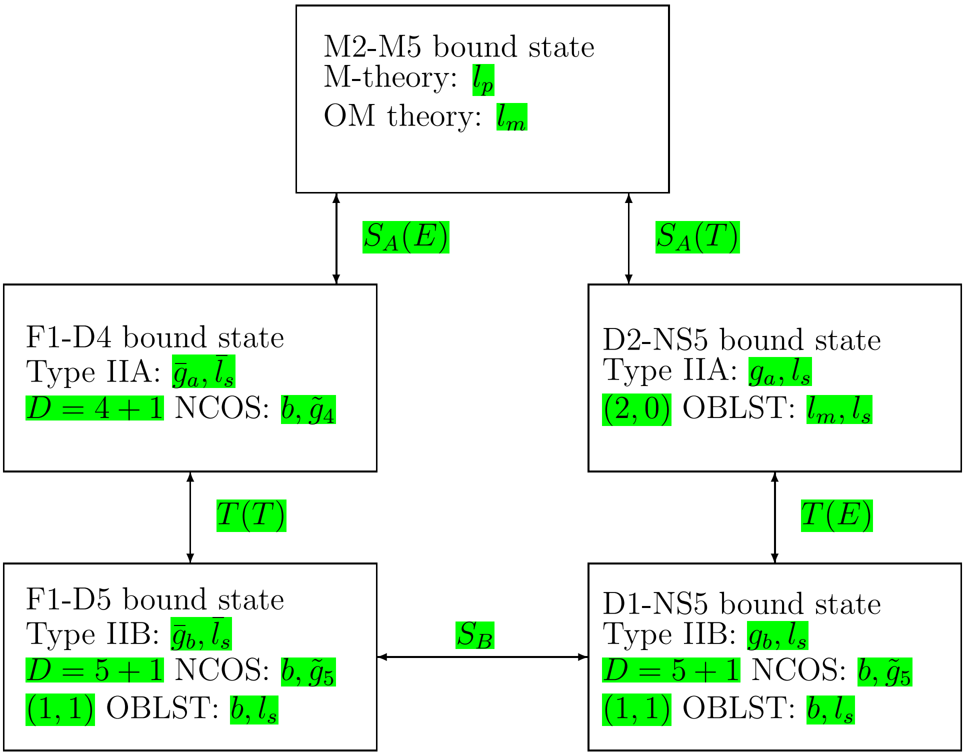

Figure 10: The chain of theories and bound-states related by S- and T-dualities. $S_A(E)$ and $S_A(T)$ means the type IIA S-duality in the electrical and transverse direction, respectively. $T(E)$ and $T(T)$ means a T-duality in the electrical and transverse direction, respectively. S_B means type IIB S-duality.

All of the bound-states in the chain can be seen as the M2-M5 bound-state on an electric circle and a transverse circle. In eleven dimensions the M2-M5 bound-state on a transverse circle is given by (75)-(76). We take x^2 to be the coordinate for the electric circle of radius R_E and z to be the coordinate for

[10]The content of this section was developed in collaboration with N. A. Obers.

the transverse circle with radius R_T.

For all the various decoupling limits we have

$$\cosh\theta \to \infty \quad , \quad \tilde{r} = \sqrt{\cosh\theta}\, r \quad , \quad \tilde{z} = \sqrt{\cosh\theta}\, z \tag{105}$$

$$\tilde{x}^i = \frac{1}{\sqrt{\cosh\theta}} x^i , \quad i = 0,1,2 \quad , \quad \tilde{x}^j = \sqrt{\cosh\theta}\, x^j , \quad j = 3,4,5 \tag{106}$$

which gives

$$\tilde{R}_E = \frac{1}{\sqrt{\cosh\theta}} R_E \quad , \quad \tilde{R}_T = \sqrt{\cosh\theta}\, R_T \tag{107}$$

Thus, all the decoupling limits are specified by the relation between $\cosh\theta$ and the parameters of M/string theory, and the parameters of the world-volume theories.

Thus, starting from the top with the M2-M5 bound-state, we have

$$\cosh\theta = \frac{l_m^6}{l_p^6} \tag{108}$$

where $1/l_m^3$ is the tension of the open membrane in OM theory and $(2,0)$ OBLST.

Choosing x^2 as the eleventh direction we go to the F1-D4 bound-state and we have

$$R_E = \bar{g}_a \bar{l}_s \quad , \quad l_p^3 = \bar{g}_a \bar{l}_s^3 \quad , \quad \cosh\theta = \frac{b}{\bar{l}_s^2} \quad , \quad \tilde{g}_4 = \frac{\bar{g}_a \bar{l}_s^2}{b} \tag{109}$$

with the NCOS open string coupling $\tilde{g}_4$ and tension $1/b$. This gives the relations

$$\tilde{R}_E = \tilde{g}_4 \sqrt{b} \quad , \quad \frac{1}{b} = \frac{\tilde{R}_E}{l_m^3} \tag{110}$$

Thus, as already mentioned in Section 3.2, this is interpreted [3, 4] as the fact that the NCOS open string in 4+1 dimensions is an open membrane in 5+1 dimensions wrapped on the electric circle of radius $\tilde{R}_E$, and for strong coupling the electric circle is large and we flow into decompactified OM theory.

Making a T-duality in the transverse direction z we go to the F1-D5 bound-state. We have

$$\bar{g}_b = \bar{g}_a \frac{\bar{l}_s}{R_T} = \frac{R_E}{R_T} \quad , \quad \cosh\theta = \frac{b}{\bar{l}_s^2} \quad , \quad \tilde{g}_5 = \frac{\bar{g}_b \bar{l}_s^2}{b} \tag{111}$$

where $\tilde{g}_5$ is the 5+1 dimensional NCOS open string coupling. We also define the T-dual radius $R'_T = \bar{l}_s^2 / R_T$ along with its rescaled version $\tilde{R}'_T$. This gives

$$\tilde{g}_5 = \tilde{g}_4 \frac{\sqrt{b}}{\tilde{R}_T} \quad , \quad \tilde{R}'_T \tilde{R}_T = b \tag{112}$$

We see that this can be interpreted as a NCOS T-duality (similar interpretations for other cases have been done in [4, 18]). Thus, the bulk T-duality on the F1-Dp bound states induces a world-volume T-duality in the NCOS theories relating the T-dual string couplings and radii by the NCOS string tension $1/b$.

The type IIB S-duality takes us into the D1-NS5 bound-state for which we have

$$g_b = \frac{1}{\bar{g}_b} \quad , \quad l_s^2 = \bar{g}_b \bar{l}_s^2 \quad , \quad \cosh\theta = \frac{b}{g_b l_s^2} \quad , \quad \tilde{g}_5 = \frac{l_s^2}{b} \tag{113}$$

as explained in Section 2.3.

If again start from the top and instead choose z as the eleventh direction we go from the M2-M5 to the D2-NS5 bound-state. We have

$$R_T = g_a l_s \quad , \quad l_p^3 = g_a l_s^3 \quad , \quad \cosh\theta = \frac{l_m^6}{g_a^2 l_s^6} \tag{114}$$

Here the world-volume theory is $(2,0)$ OBLST with the parameters l_s, l_m and $\tilde{R}_E$.

Making a T-duality in the x^2 direction we obtain the D1-NS5 bound-state. As already explained in Section 4.4, we have

$$g_b = g_a \frac{l_s}{R_E} = \frac{R_T}{R_E} \quad , \quad \cosh\theta = \frac{l_m^3}{g_b l_s^2 \tilde{R}_E} \tag{115}$$

We again note that $\cosh\theta$ exactly has the right dependence on the string coupling g_b that makes us able to compare this T-dualized limit to the limit obtained in (113) by going the other way in the chain. We now define the T-dual radius $R'_E = l_s^2/R_E$ of the electric circle. Since x^2 after the T-duality is a magnetic coordinate, R'_E scales oppositely to R_E. Using this and comparing with (113) we therefore get

$$\frac{1}{b} = \frac{\tilde{R}_E}{l_m^3} \quad , \quad \tilde{R}'_E \tilde{R}_E = l_s^2 \tag{116}$$

Thus, we see that the T-duality in the bulk now induces a little-closed-string-T-duality in the OBLST, since contrary to (112) the T-duality is in terms of the length scale l_s of the closed strings in OBLST. One could speculate that this means that open strings of tension $1/(2\pi l_s^2)$ can end on the open string/membrane in OBLST. Certainly, this would make sense from the bulk point of view, since the open string/membrane origins from D1 or D2-branes stretching between the D1-NS5 or D2-NS5 bound-states.

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Note added:
While writing this paper, we received the paper [44] which also considers a supergravity dual of OM theory using a different approach than the one we have in section 3.

In the final stages of writing this paper we were made aware that N. Seiberg and A. Strominger have presented similar ideas as of this paper in their talks at Strings 2000, july 10-15. We were subsequently informed that these ideas will appear in a revised version of the paper [3] by R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger.

References

[1] N. Seiberg, L. Susskind, and N. Toumbas, "Strings in background electric field, space / time noncommutativity and a new noncritical string theory," *JHEP* **06** (2000) 021, `hep-th/0005040`.

[2] R. Gopakumar, J. Maldacena, S. Minwalla, and A. Strominger, "S-duality and noncommutative gauge theory," *JHEP* **06** (2000) 036, `hep-th/0005048`.

[3] R. Gopakumar, S. Minwalla, N. Seiberg, and A. Strominger, "OM theory in diverse dimensions," *JHEP* **08** (2000) 008, `hep-th/0006062`.

[4] E. Bergshoeff, D. S. Berman, J. P. van der Schaar, and P. Sundell, "Critical fields on the M5-brane and noncommutative open strings," `hep-th/0006112`.

[5] N. Seiberg, L. Susskind, and N. Toumbas, "Space/time non-commutativity and causality," *JHEP* **06** (2000) 044, `hep-th/0005015`.

[6] O. J. Ganor, G. Rajesh, and S. Sethi, "Duality and non-commutative gauge theory," `hep-th/0005046`.

[7] J. L. F. Barbon and E. Rabinovici, "Stringy fuzziness as the custodian of time-space noncommutativity," *Phys. Lett.* **B486** (2000) 202, `hep-th/0005073`.

[8] J. Gomis and T. Mehen, "Space-time noncommutative field theories and unitarity," `hep-th/0005129`.

[9] D. J. Gross and N. A. Nekrasov, "Monopoles and strings in noncommutative gauge theory," *JHEP* **07** (2000) 034, `hep-th/0005204`.

[10] G.-H. Chen and Y.-S. Wu, "Comments on noncommutative open string theory: V-duality and holography," `hep-th/0006013`.

[11] I. R. Klebanov and J. Maldacena, "1+1 dimensional NCOS and its U(N) gauge theory dual," `hep-th/0006085`.

[12] J. X. Lu, S. Roy, and H. Singh, "((F,D1),D3) bound state, S-duality and noncommutative open string / Yang-Mills theory," *JHEP* **09** (2000) 020, `hep-th/0006193`.

[13] J. G. Russo and M. M. Sheikh-Jabbari, "On noncommutative open string theories," *JHEP* **07** (2000) 052, `hep-th/0006202`.

[14] J. Gomis, K. Kamimura, and J. Llosa, "Hamiltonian formalism for space-time non-commutative theories," `hep-th/0006235`.

[15] O. Aharony, J. Gomis, and T. Mehen, "On theories with light-like noncommutativity," *JHEP* **09** (2000) 023, `hep-th/0006236`.

[16] S.-J. Rey and R. von Unge, "S-duality, noncritical open string and noncommutative gauge theory," `hep-th/0007089`.

[17] R.-G. Cai and N. Ohta, "(F1, D1, D3) bound state, its scaling limits and $SL(2,Z)$ duality," `hep-th/0007106`.

[18] T. Kawano and S. Terashima, "S-duality from OM-theory," `hep-th/0006225`.

[19] M. Berkooz, M. Rozali, and N. Seiberg, "Matrix description of M theory on T^4 and T^5," *Phys. Lett.* **B408** (1997) 105–110, `hep-th/9704089`.

[20] N. Seiberg, "New theories in six-dimensions and matrix description of M theory on T^5 and T^5/Z_2," *Phys. Lett.* **B408** (1997) 98–104, hep-th/9705221.

[21] R. Dijkgraaf, E. Verlinde, and H. Verlinde, "BPS quantization of the five-brane," *Nucl. Phys.* **B486** (1997) 89–113, hep-th/9604055.

[22] R. Dijkgraaf, E. Verlinde, and H. Verlinde, "5d black holes and matrix strings," *Nucl. Phys.* **B506** (1997) 121, hep-th/9704018.

[23] A. Losev, G. Moore, and S. L. Shatashvili, "M & m's," *Nucl. Phys.* **B522** (1998) 105–124, hep-th/9707250.

[24] O. Aharony, "A brief review of 'little string theories'," *Class. Quant. Grav.* **17** (2000) 929, hep-th/9911147.

[25] T. Harmark, "Supergravity and space-time non-commutative open string theory," *JHEP* **07** (2000) 043, hep-th/0006023.

[26] J. X. Lu and S. Roy, "Non-threshold (F,Dp) bound states," *Nucl. Phys.* **B560** (1999) 181, hep-th/9904129.

[27] N. Itzhaki, J. M. Maldacena, J. Sonnenschein, and S. Yankielowicz, "Supergravity and the large N limit of theories with sixteen supercharges," *Phys. Rev.* **D58** (1998) 046004, hep-th/9802042.

[28] O. Aharony, M. Berkooz, D. Kutasov, and N. Seiberg, "Linear dilatons, NS5-branes and holography," *JHEP* **10** (1998) 004, hep-th/9808149.

[29] J. M. Maldacena, "Statistical entropy of near extremal five-branes," *Nucl. Phys.* **B477** (1996) 168–174, hep-th/9605016.

[30] J. M. Maldacena and A. Strominger, "Semiclassical decay of near-extremal fivebranes," *JHEP* **12** (1997) 008, hep-th/9710014.

[31] T. Harmark and N. A. Obers, "Hagedorn behaviour of little string theory from string corrections to NS5-branes," *Phys. Lett.* **B485** (2000) 285, hep-th/0005021.

[32] M. Berkooz and M. Rozali, "Near Hagedorn dynamics of NS fivebranes, or a new universality class of coiled strings," *JHEP* **05** (2000) 040, hep-th/0005047.

[33] N. Seiberg and E. Witten, "String theory and noncommutative geometry," *JHEP* **09** (1999) 032, `hep-th/9908142`.

[34] J. M. Maldacena and J. G. Russo, "Large N limit of non-commutative gauge theories," *JHEP* **09** (1999) 025, `hep-th/9908134`.

[35] M. Alishahiha, Y. Oz, and M. M. Sheikh-Jabbari, "Supergravity and large N noncommutative field theories," *JHEP* **11** (1999) 007, `hep-th/9909215`.

[36] T. Harmark and N. A. Obers, "Phase structure of non-commutative field theories and spinning brane bound states," *JHEP* **03** (2000) 024, `hep-th/9911169`.

[37] S. Chakravarty, K. Dasgupta, O. J. Ganor, and G. Rajesh, "Pinned branes and new non Lorentz invariant theories," `hep-th/0002175`.

[38] E. Bergshoeff, D. S. Berman, J. P. van der Schaar, and P. Sundell, "A noncommutative M-theory five-brane," `hep-th/0005026`.

[39] S. Kawamoto and N. Sasakura, "Open membranes in a constant C-field background and noncommutative boundary strings," *JHEP* **07** (2000) 014, `hep-th/0005123`.

[40] J. G. Russo and A. A. Tseytlin, "Waves, boosted branes and BPS states in M-theory," *Nucl. Phys.* **B490** (1997) 121–144, `hep-th/9611047`.

[41] J. Maldacena, "The large N limit of superconformal field theories and supergravity," *Adv. Theor. Math. Phys.* **2** (1998) 231–252, `hep-th/9711200`.

[42] M. Alishahiha, "On type II NS5-branes in the presence of an RR field," `hep-th/0002198`.

[43] J. Correia, T. Harmark, and N. A. Obers. In preparation.

[44] D. S. Berman and P. Sundell, "Flowing to a noncommutative (OM) five brane via its supergravity dual," *JHEP* **10** (2000) 014, `hep-th/0007052`.