ANTHROPIC ESTIMATES OF THE CHARGE AND MASS OF THE PROTON [*]

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Abstract

By combining a renormalization group argument relating the charge e and mass m_p of the proton by $e^2 \ln m_p \approx -0.1\pi$ (in Planck units) with the Carter-Carr-Rees anthropic argument that gives an independent approximate relation $m_p \sim e^{20}$ between these two constants, both can be crudely estimated. These equations have the factor of 0.1π and the exponent of 20 which depend upon known discrete parameters (e.g., the number of generations of quarks and leptons, and the number of spatial dimensions), but they contain *no* continuous observed parameters. Their solution gives the charge of the proton correct to within about 8%, though the mass estimate is off by a factor of about 1000 (16% error on a logarithmic scale). When one adds a fudge factor of 10 previously given by Carr and Rees, the agreement for the charge is within about 2%, and the mass is off by a factor of about 3 (2.4% error on a logarithmic scale). If this 10 were replaced by 15, the charge agrees within 1.1% and the mass itself agrees within 0.7%.

1 Introduction

One prominent idea of recent decades is that some of the 'constants of physics' may only be constant (or nearly so) in our observable region of the universe (our 'subuniverse') but may take on a range of values in the entire universe (or 'multiverse,' though I shall eschew this terminology here). Two examples to be considered here are the electromagnetic fine structure constant, $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ (the square of the charge of the proton, e, in the Planck units $\hbar = c = G = 4\pi\epsilon_0 = 1$ I shall use here), and the gravitational fine structure constant for the proton, $\alpha_G = Gm_p^2/(\hbar c) \equiv (m_p/m_P)^2$ (the square of the mass of the proton, m_p, in Planck units). If these 'constants' vary widely over the entire universe, most of the universe may be unsuitable for observers, and hence one might expect that observed values would statistically have a narrower range. One goal of a complete theory of physics and of observership would be to calculate the statistical distribution of the observed values of such 'constants.'

Of course, we are nowhere near achieving such a goal. However, here I wish to use come crude hypotheses and arguments about the conditions that might be conducive for typical observers in order to come up with some rough values that typical observers might be expected to see for the electromagnetic and gravitational fine structure constants. (Such arguments often go under the name of anthropic arguments, but they are really about observership rather than about *anthropos*. Because these arguments can be made only very crudely at present, they are often deprecated, but I shall take the view that they are just the beginning hints of what should in principle be possible with a complete theory of the universe, including the physical conditions for observership.)

Rather remarkably, one finds that without using any observational data except for certain discrete integers (e.g., the dimension of space, the number of generations of quarks and leptons, etc.), one can get crude estimates for typical observed values of both of these parameters that are in the right ballpark to explain our particular observations. In this paper I shall show how one can get a crude estimate for the magnitude of the charge of the proton ($e \equiv \sqrt{\alpha}$ in Planck units) that differs from what we observe by only a few percent. One can also get a somewhat more crude estimate for the mass of the proton ($m_p \equiv \sqrt{\alpha_G}$ in Planck units) that differs by about 3 orders of magnitude from the value that we observe. However, since the observed value is about 19 orders of magnitude smaller than unity, on a logarithmic

scale the crude estimate is not that far off.

Furthermore, one just needs a single previously-published fudge factor of 10 to reduce the error to about 2% for the charge of the proton (or to within 1.1% error for the charge and to within 0.7% error for the mass if this 10 were replaced by 15). Of course, it is a cheat to take these revised values as predictions, but to the degree that it is not implausible that a factor of the order of 10 or 15 could arise from a more detailed calculation, it is not implausible that such a calculation could give a statistical distribution of observed values of the charge and mass of the proton such that our observations of these values would be typical. In other words, the 'errors' of the crude estimates previously mentioned, which do not use as input any observed continuous parameters like the fudge factors of the present paragraph, seem small enough not to be fatal to the form of the argument.

2   The Carter-Carr-Rees anthropic argument

We will need two independent formulas relating the charge and mass of the proton in order to deduce approximations for their values. First I shall give the Carter-Carr-Rees formula, and then in the next section I shall add my own renormalization group argument to get a new formula that one can combine with the Carter-Carr-Rees one to get actual estimates for the charge and mass of the proton.

Brandon Carter [1] noted that typical observers may live on planets, as we do, and that planetary formation may depend upon the formation of convective red stars. For the existence of these he deduced that

$$\alpha_G \equiv m_p^2 \overset{<}{\sim} \alpha^{12} \left(\frac{m_e}{m_p} \right)^4,$$ (1)

where m_e is the mass of the electron. Then Bernard Carr and Martin Rees [2] noted that if α_G were a lot smaller than the right hand side of Eq. (1), one might not get supernovae that are necessary for producing the elements needed for observers like us. Putting these two arguments together, one may conclude that for observers like us that live on planets and have a complex chemistry, one may need

$$\alpha_G \sim \alpha^{12} \left(\frac{m_e}{m_p} \right)^4.$$ (2)

One might alternatively write this relation as

$$\alpha_G = C \alpha^{12} \left(\frac{m_e}{m_p} \right)^4$$ (3)

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with a 'Carter constant' C that is predicted to be observed to be of the order of unity by typical observers. We observe it to be $C \approx 2.944$ in our sub-universe [3].

In their Eq. (58), Carr and Rees [2] also cited Carter [4] in noting four key coincidences of nuclear physics that may be necessary for observers like us. Together they imply what Carr and Rees write, in Eq. (45) of their paper [2], as

$$\frac{m_e}{m_p} \sim 10\,\alpha^2. \tag{4}$$

The 10 is really just an order-of-magnitude estimate for a numerical coefficient that is not predicted precisely, a 'fudge factor' as it were, so to get relations without including unpredicted factors, one might write this relation merely as

$$\frac{m_e}{m_p} \sim \alpha^2. \tag{5}$$

Alternatively, one might write this relation as

$$\frac{m_e}{m_p} = R\,\alpha^2 \tag{6}$$

with a 'Carr-Rees constant' R that is predicted to be somewhat larger than unity. To give an order of magnitude estimate, Carr and Rees say $R \sim 10$. We actually observe it to be $R \approx 10.227290$ in our subuniverse [3].

If we now combine these two relations without the numerical factors of C and R, we get what I shall call the basic Carter-Carr-Rees relation

$$\alpha_G \sim \alpha^{20}, \tag{7}$$

which is also listed explicitly as Eq. (57) of [2].

If one does include the fudge factor of 10 that Carr and Rees include in their Eq. (45), copied as Eq. (4) above, then one gets what I shall call the fudged Carter-Carr-Rees relation

$$\alpha_G \sim 10^4\,\alpha^{20}. \tag{8}$$

Alternatively, one can write

$$\alpha_G = E\,\alpha^{20} = (F\alpha)^{20}, \tag{9}$$

where $E = F^{20} = CR^4$. In our subuniverse, we observe $E \approx 32205$, or $F \approx 1.6803 \approx 2^{0.7487}$ [3]. Since F is fairly close to $2^{3/4}$, as a mnemonic device one can remember a fairly good approximation to Eq. (9) in our subuniverse as the "2-3-4" formula

$$\alpha_G \approx (2^{3/4}\alpha)^{20}. \tag{10}$$

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Thus we have one rough relation between $\alpha \equiv e^2$ and $\alpha_G \equiv m_p^2$, or between the charge e and mass m_p of the proton. To get an estimate for both values, we need another relation. This I shall provide by a crude renormalization-group (RG) analysis of supersymmetric (SUSY) Grand Unified Theory (GUT) couplings in the Minimal Supersymmetric Standard Model (MSSM).

But before I do that, I should note that the Carr-Rees paper, though preceding most of the development of the MSSM, did also propose a second relation between α and α_G based upon some arguments about a self-consistent quantum electrodynamics, namely Eq. (54) of [2], which I shall call the Carr-Rees log relation:

$$\alpha^{-1} \sim \log \alpha_G^{-1}. \tag{11}$$

Although Carr and Rees did not carry this further in their paper, it is trivial to combine Eqs. (7) and (11) [their Eqs. (57) and (54) respectively] to estimate that $\alpha^{-1} \sim 90$, which is about 66% of the value we observe in our subuniverse [3], $\alpha_{\exp}^{-1} \approx 137.036000$, and to estimate that $\alpha_G^{-1} \sim 1.2 \times 10^{39}$, about 7 times the value we observe in our subuniverse [3], $\alpha_{G\,\exp}^{-1} \approx 1.693 \times 10^{38} \approx 2^{126.993} \approx 2^{2^{2^{2^2-1}-1}-1} - 1$ (the 12th smallest Mersenne prime and largest prime found without computers, in 1876 by Edouard Lucas [5]). It is remarkable that even these very crude estimates give values for the charge and mass of the proton that are within the right ballpark for our observations.

3 The renormalization group argument

Now I shall show how to deduce a better relation between α^{-1} and $\ln \alpha_G^{-1}$ by using a renormalization group analysis of supersymmetric grand unified theories, e.g., that of William Marciano and Goran Senjanović [6].

I shall assume that there are $n_g = 3$ generations of quarks and leptons and $N_H = 2$ relatively light Higgs doublets in low-energy $SU(3) \times SU(2) \times U(1)$, which are discrete choices that themselves might by justified in the future by some anthropic argument that typical observers would see those values.

However, I shall not use any observational results of parameters that are believed to have a potentially continuous range (like the charge and mass of the proton). (More precisely, parameters like the charge and mass of the proton are not now known to be limited to discrete values, so for the sake of argument I shall assume

that they can in principle have continuous ranges, even though a future theory could conceivably show that they really are limited to certain discrete values.)

Then, to one-loop order and ignoring additive numbers like $1/4\pi$, the results of Marciano and Senjanović [6] imply that the $SU(3) \times SU(2) \times U(1)$ inverse coupling constants, and the inverse electromagnetic fine-structure constant, run with the energy scale μ, in the range $m_W < \mu < m_S$ between the weak energy scale m_W and the unification mass scale m_S, in the following way, when $n_g = 3$ and $N_H = 2$:

$$\alpha_1^{-1}(\mu) \approx \alpha_1^{-1}(m_S) + \frac{2n_g + 0.3N_H}{2\pi} \ln \frac{m_S}{\mu} = \alpha_1^{-1}(m_S) + \frac{3.3}{\pi} \ln \frac{m_S}{\mu}, \quad (12)$$

$$\alpha_2^{-1}(\mu) \approx \alpha_2^{-1}(m_S) + \frac{2n_g + 0.5N_H - 6}{2\pi} \ln \frac{m_S}{\mu} \approx \alpha_1^{-1}(m_S) + \frac{0.5}{\pi} \ln \frac{m_S}{\mu}, \quad (13)$$

$$\alpha_3^{-1}(\mu) \approx \alpha_3^{-1}(m_S) + \frac{2n_g - 9}{2\pi} \ln \frac{m_S}{\mu} \approx \alpha_1^{-1}(m_S) - \frac{1.5}{\pi} \ln \frac{m_S}{\mu}, \quad (14)$$

$$\alpha^{-1}(\mu) = \frac{5}{3}\alpha_1^{-1}(\mu) + \alpha_2^{-1}(\mu) \approx \frac{8}{3}\alpha_1^{-1}(m_S) + \frac{6}{\pi} \ln \frac{m_S}{\mu}. \quad (15)$$

Now the proton mass is set roughly by the scale μ at which the $SU(3)$ coupling constant $\alpha_3(\mu)$ becomes large, say

$$0 \sim \alpha_3^{-1}(m_p) \approx \alpha_1^{-1}(m_S) - \frac{1.5}{\pi} \ln \frac{m_S}{m_p}. \quad (16)$$

For this equation to be approximately valid despite the fact that the proton mass m_p is not in the range $m_W < \mu < m_S$ where Eqs. (12)-(15) are valid, I shall assume that on a logarithmic scale, m_p is fairly close to m_W, in comparison with a much greater logarithmic range where Eqs. (12)-(15) are valid, as is indeed the case in the subuniverse that we observe.

If we approximate the electromagnetic coupling constant at zero momentum transfer, which is the quantity we have been calling α, with the electromagnetic coupling constant at energy scale $\mu = m_p$, we get from Eqs. (15) and (16) that

$$\alpha^{-1} \approx \alpha^{-1}(m_p) \approx \frac{10}{\pi} \ln \frac{m_S}{m_p}. \quad (17)$$

The next approximation we shall make is that on a logarithmic scale, the unification mass m_S is nearly the Planck mass m_P, so we can replace $\ln(m_S/m_p)$ with $\ln(m_P/m_p) = 0.5 \ln \alpha_G^{-1}$ in Eq. (17). This then gives us our new relation between the electromagnetic and gravitational fine structure constants (or between the charge and mass of the proton in Planck units, as given in the Abstract):

$$\alpha^{-1} \approx \frac{5}{\pi} \ln \alpha_G^{-1}. \quad (18)$$

One can note that if we stick with $n_g = 3$ generations of quarks and leptons but allow the number of relatively light Higgs doublets, N_H, to be different from its minimal value of 2, then we get instead

$$\alpha^{-1} \approx \frac{9 + 0.5N_H}{2\pi} \ln \alpha_G^{-1}. \tag{19}$$

Alternatively, one can replace the approximate Eqs. (18)-(19) with the exact equation

$$\alpha^{-1} = \frac{N}{2\pi} \ln \alpha_G^{-1} \tag{20}$$

(taking N to be defined by this equation) and the approximate relation

$$N \approx 9 + 0.5N_H. \tag{21}$$

Since N_H must be an even integer, the right hand side of Eq. (21) must be an integer, 10 if $N_H = 2$, whereas we can take the left hand side, N, to be defined by Eq. (20). In our subuniverse, we observe $N \approx 9.7816$ [3]. This is close enough to 10 that we might conclude that it is likely that our assumption of $N_H = 2$ is correct for our subuniverse.

Incidentally, to get some feel for the accuracy of the approximations above, one may note that at the same level of approximation and truncation of the MSSM renormalization group equations [6], one gets that the weak-mixing angle θ_W gives

$$\sin^2 \theta_W \approx 0.2, \tag{22}$$

about 13% less than the observed value 0.231 in our subuniverse [3].

4   Combining the two relations between α and α_G

Now we can combine some form of the Carter-Carr-Rees anthropic relation between α and α_G with the new MSSM renormalization group approximate Eq. (18) to derive approximate estimates for values of both α and α_G that a typical observer might be expected to see.

If we use the basic Carter-Carr-Rees relation above, Eq. (7), $\alpha_G \sim \alpha^{20}$, then we get for the electromagnetic fine structure constant α or the charge $e = \sqrt{\alpha}$ of the proton in Planck units,

$$\alpha \ln \alpha = 2e^2 \ln e \sim -\frac{\pi}{100}, \tag{23}$$

with the solution

$$\alpha^{-1} \sim 162 \approx 1.18 \, \alpha_{\exp}^{-1}. \tag{24}$$

The prediction for a typical observed value of the charge of the proton is then

$$e \sim 0.0786 \approx 1.47 \times 10^{-19} \, \text{coulomb} \approx 0.920 \, e_{\exp}, \tag{25}$$

within about 8% of the experimental value observed in our subuniverse near our present time and location, $e_{\exp} = \sqrt{\alpha_{\exp}} \approx 0.085424543 \approx 1.6021765 \times 10^{-19}$ coulomb [3].

Similarly, we can combine Eqs. (7) and (18) to get an equation for the gravitational fine structure constant α_G or the mass $m_p = \sqrt{\alpha_G}$ of the proton in Planck units,

$$\alpha_G^{1/20} \ln \alpha_G = 2 m_p^{1/10} \ln m_p \sim -\frac{\pi}{5}. \tag{26}$$

This has the solution

$$\alpha_G^{-1} \sim 1.54 \times 10^{45} \sim 2^{147} \sim 907\,000 \, \alpha_{G\,\exp}^{-1}, \tag{27}$$

or

$$m_p = \sqrt{\alpha_G} \sim 8.1 \times 10^{-23} \approx 1.76 \times 10^{-30} \, \text{kg} \approx 0.00105 \, m_{p\,\exp}. \tag{28}$$

In this case the agreement is not so good, as the estimated mass of the proton comes out to be only about 0.1% of the value we observe in our subuniverse. The large magnitude of the error can be attributed to the the large exponent of 20 in the basic Carter-Carr-Rees relation of Eq. (7), $\alpha_G \sim \alpha^{20}$. This makes it so that a relatively small error in the estimate for α can get converted into a relatively large error in α_G. However, since m_p is so small in Planck units, it might be more natural to make the comparison on a logarithmic scale, in which case one gets

$$\frac{\ln m_p}{\ln m_{p\,\exp}} \sim 1.156, \tag{29}$$

which has an error of less than 16%.

Although of course it is a cheat for getting a true estimate, one can improve the estimate by using the fudged Carter-Carr-Rees relation, Eq. (8), $\alpha_G \sim 10^4 \, \alpha^{20}$, along with the MSSM renormalization group approximate Eq. (18) to get the fudged improved equations

$$\alpha \ln \left(10^{0.2} \, \alpha \right) = 2 e^2 \ln \left(10^{0.1} \, e \right) \sim -0.01 \, \pi, \tag{30}$$

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$$\alpha_G^{0.05} \ln \alpha_G = 2m_p^{0.1} \ln m_p \sim -10^{0.2} 0.2 \, \pi. \tag{31}$$

The approximate numerical solutions of these fudged equations are

$$\alpha^{-1} \sim 143.4 \approx 1.046 \, \alpha_{\exp}^{-1}, \tag{32}$$

$$e \sim 0.08351 \approx 1.566 \times 10^{-19} \, \text{coulomb} \approx 0.9775 \, e_{\exp}, \tag{33}$$

$$\alpha_G^{-1} \sim 1.35 \times 10^{39} \sim 2^{130} \sim 7.99 \, \alpha_G^{-1}{}_{\exp}, \tag{34}$$

$$m_p = \sqrt{\alpha_G} \sim 2.72 \times 10^{-20} \approx 5.92 \times 10^{-28} \, \text{kg} \approx 0.354 \, m_{p \, \exp}. \tag{35}$$

Thus the fudged equations give an estimated charge of the proton within about 2.25% of the observed value nearby in our subuniverse, and an estimated mass of the proton within a factor of 3 of the observed value. Of course, the factor of 10 in Eq. (4), which gets raised to the 4th power in Eq. (8), was just an order-of-magnitude estimate that was no doubt biased by the observed value of this factor in our subuniverse, so it is not expected to be reliable. However, it does show that if one combines my new MSSM renormalization group approximate Eq. (18) (having no continuous free parameters) with the previously-published anthropic equations with this previously-published fudge factor, one gets a remarkable agreement with the observed charge of the proton, nearly 98% of the observed value, and reasonably good agreement with the observed mass, nearly 98% of its logarithm.

More importantly, since it shows that one needs a fudge factor, like the factor of 10 in Eq. (4), that is of the same general order of magnitude as unity to get good agreement with the observations in our subuniverse, it does make it plausible to conjecture that when we gain sufficient knowledge to be able to calculate more precisely the statistical distribution of observed values of the charge and mass of the proton over all subuniverses, the values we observe in our subuniverse will turn out to be typical, even if the values can vary widely over the full range of subuniverses.

If one cheats even more and uses the MSSM renormalization group approximate Eq. (18) with the 2-3-4 formula (10), one gets an even better fit to our observations, with the charge of the proton being predicted to be 0.986 of the observed value, and the mass of the proton being predicted to be 0.760 of the observed value. (This replacement is equivalent to replacing the Carr-Rees fudge factor of 10, in Eq. (8) where it is raised to the 4th power, with $2^{15/4} \approx 13.4543$.) Since the new fudge factor of $2^{3/4}$ in the 2-3-4 formula (10) was chosen as a simple expression close to the numerical value for that equation to be true in our subuniverse, it cannot be

counted as a prediction at all. Then the main error in the 'predicted' charge and mass of the proton comes from the MSSM renormalization group approximate Eq. (18).

Alternatively, one can choose F in Eq. (9) so that when it is combined with Eq. (18) to get separate equations for α and α_G,

$$\alpha \ln (F\alpha) = -0.01\pi, \tag{36}$$

$$\alpha_G^{1/20} \ln \alpha_G = -0.2\,\pi F, \tag{37}$$

one fits the observed value of either α or α_G and then predicts the other one.

For example, if one uses our observed value of α^{-1}, $\alpha_{\mathrm{exp}}^{-1} \approx 137.036000$ [3], and solves Eq. (36) for

$$F = \alpha_{\mathrm{exp}}^{-1} \exp\left(-0.01\pi\alpha_{\mathrm{exp}}^{-1}\right) \approx 1.8498985 \tag{38}$$

and then inserts this into Eq. (37) to solve for $\alpha_G = m_p^2$, one gets a value for the mass of the proton that is too large by a factor of about 2.615, rather oppositely to the previous predicted values that were all too small.

On the other hand, if one uses our observed value of α_G, $\alpha_{G\ \mathrm{exp}} \approx 2^{-126.993}$ [3], and solves Eq. (37) for

$$F = -\frac{5}{\pi}\,\alpha_{G\ \mathrm{exp}}^{0.05} \ln \alpha_{G\ \mathrm{exp}} \approx 1.7179 \tag{39}$$

and then inserts this into Eq. (36) to solve for $\alpha = e^2$, one gets a value for the charge of the proton that is 0.989 of the observed value.

A possibility very similar to this last one is to replace the 10 in Eqs. (30)-(31) with 15, which is equivalent to using $F = 15^{0.2} \approx 1.7188$ in Eqs. (36)-(37). Then one gets $e \approx 0.9891\,e_{\mathrm{exp}}$ and $m_p \approx 1.0069\,m_{p\ \mathrm{exp}}$, so then the errors are less than 1.1% and 0.7% respectively.

To fit our observed values of both the charge and mass of the proton to better than 99% accuracy, we need not only a single fudge factor like F in Eq. (9), but also a second fudge factor in Eq. (18), such as the N in Eq. (20). As noted previously, if we set $N \approx 9.7816$, as well as $F \approx 1.6803$, then we can reproduce our observed values of both the charge and mass of the proton. However, with 3 generations of quarks and leptons, and the minimal number (2) of relatively light Higgs doublets, the coefficient N in Eq. (20) is predicted to be precisely 10. Thus if we did put a fudge factor into that equation, it should be somewhere else.

The main error of Eq. (18) seems to come from the approximations that on a logarithmic scale, the unification mass is close to the Planck mass and the weak energy scale is close to the proton mass. The first of these errors tends to increase the right hand side above the left hand side, and the second tends to decrease it. If we wildly conjecture that the ratios of the unification mass to the Planck mass and of the weak energy scale to the proton mass might be expected anthropically to go as some powers of the running coupling constants at those scales, then we might naïvely expect that a first correction to Eq. (18) would be to replace it with

$$\alpha^{-1} \approx \frac{5}{\pi} \ln \left(\alpha^p \alpha_G^{-1} \right) \tag{40}$$

for some exponent p that might conceivably be predictable. If we then set $F = e^q$ for another exponent q (where here and in the following equation e means the base of the natural logarithms rather than the charge of the proton), then Eq. (9) becomes

$$\alpha_G = (e^q \alpha)^{20}. \tag{41}$$

Here the factor of $5/\pi$ in Eq. (40) and the exponent of 20 in Eq. (41) are determined by discrete parameters of the crude theory used to derive these equations, but p and q are not so determined, at least not yet. The simplest version of the argument above would just suggest that p and q should both be roughly 0.

Using both fudge factors p and q in Eqs. (40) and Eq. (41), we can fit the observations in our subuniverse with $p \approx 0.390782$ and $q \approx 0.518993$.

Now the point of a complete anthropic argument would not be to predict precise values for p and q (which would give precise values for the charge and mass of the proton), but rather to predict a joint statistical distribution for their observed values over the entire universe. The simple argument given above just suggests that typical observers, if they are like us in living on planets, having complex chemistry, etc., might expect to see values of p and q that are not too different from zero (i.e., having magnitudes not very large in comparison with unity). This does fit the observations in our subuniverse, where both p and q are even somewhat smaller than unity. In this way our observations are consistent with the simple argument given above.

Of course, this consistency just means that the simple argument passed its first test in not being falsified where it conceivably could have been, but the consistency does not confirm the basic truth of the simple argument. For example, we do not yet know whether the charge and mass of the proton really do have a distribution

of different values over all subuniverses. However, it is at least encouraging that one can find a simple argument of this form that does so well in giving rough values for our observations of the charge and mass of the proton.

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