ANOMALIES FROM IMMERSIONS

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ABSTRACT

Two forms of anomalies for chiral spinors living on submanifolds of the spacetime are obtained from the integrality theorem for immersions. The first form of the chiral anomaly is the usual for chiral spinors living on D-brane and O-plane intersections, the second form is exotic.

1 Introduction

The anomaly for chiral spinors living on submanifolds of the spacetime may be computed as the index of an appropiate Dirac operator corresponding to an generalized Spin complex [1]:

$$index(D) =$$

$$(-1)^{\frac{d(d+1)}{2}} \int_M ch_\rho(V) \frac{ch(S^+_{T(M)} - S^-_{T(M)}) ch(S^+_{N(M)} - S^-_{N(M)})}{e(T(M))} Td(T(M^C))$$

The computation then produces [1]:

$$index(D) = \int_M ch_\rho(V) \wedge \frac{\hat{A}(R)}{\hat{A}(R')} \wedge e(R')$$

From the other side, the anomaly for chiral spinors living on submanifolds of the spacetime may be computed as the index of an appropiate Dirac operator twisted with the superbundle $\Xi \to \Sigma$ [2]:

$$index(D) = (-1)^{\frac{(p+1)(p+2)}{2}} \int_\Sigma ch^+(\Xi) \wedge \frac{Td(T\Sigma \bigotimes C))}{\chi(T\Sigma)}$$

Now the computation then produces [2]:

$$index(D) = \int_\Sigma ch^+(E) \wedge ch^+(\overline{E}) \wedge e^{d(N\Sigma)} \wedge \frac{\hat{A}(T\Sigma)}{\hat{A}(N\Sigma)} \wedge \chi(N\Sigma)$$

2 The integrality theorem for immersions

In this section is presented the following integrality theorem for immersions [3]:

Let X be an n-dimensional closed manifold with a transitive $G_{TX}-structure$, n=2m even. Let X be immersed into an (n+k)-dimensional spin manifold Y, such that the normal bundle ν carries a $G_{\nu}-structure$. Let $\Phi_{TX}: X \to BG_{TX}$ and $\Phi_{\nu}: X \to BG_{\nu}$ be the classifying maps for the tangent and the normal bundle.

Let $\sigma \in R(\hat{G}_{TX}, \hat{H}_{TX})$ and $V \in R(\hat{G}_{\nu})$ such that (-1,-1) acts trivially on $\sigma \cdot V$. Let $W \in K^0(X)$, then

$$\int_X ch(W) \cdot \Phi_{\nu}^*((\pi_2^*)^{-1} ch(V)) \cdot \Phi_{TX}^*(\frac{(\pi_1^*)^{-1} ch(\sigma)}{e|BG_{TX}}) \cdot \hat{A}(TX)^2 = integer$$

where, $e \in H^{2m}(BSO(2m);Q)$ is the universal Euler class, $ch : R(G) \to H^*(BG;Q)$ is the universal Chern character, and $\hat{A}(TX)$ is the total $\hat{A}-class$ of X.

For the proof of the integrality theorem for immersions, the procedure is the following [3]:

The immersions with certain properties yield structure groups for the manifolds under consideration, from such structure groups we can to obtain elliptic symbols and then the corresponding elliptic operators, finally applying the Atiyah-Singer index theorem for these elliptic operators we can to produce the integrality theorem for immersions.

3 Anomalies from immersions

Anomalies are obtained from the integrality theorem for immersions using the following structure groups for the tangent and normal bundles:

$$G_{TX} = Spin(n)$$

$$G_\nu = Spin^c(k)$$

Then [3]:

$$\Phi^*_{TX}\left(\frac{(\pi_1^*)^{-1}ch(\sigma)}{e|BG_{TX}}\right) \cdot \hat{A}(TX)^2 = \hat{A}(TX)$$

$$\Phi^*_\nu((\pi_2^*)^{-1}ch(V)) = e^{d(\nu)} \cdot e(\nu) \cdot \hat{A}(\nu)^{-1}$$

finally, applying the integrality theorem for immersions is obtained that [3]:

$$\int_X ch(W) \cdot e^{d(\nu)} \cdot e(\nu) \cdot \hat{A}(\nu)^{-1} \cdot \hat{A}(TX) = integer$$

This last expression is the usual for the chiral anomaly [1],[2].

An exotic anomaly for chiral spinors that are living on submanifolds of the spacetime is obtained according to the following procedure:

again,

$$G_{TX} = Spin(n)$$

$$G_\nu = Spin^c(k)$$

but in this case one has the following [3]:

$$\Phi^*_{TX}\left(\frac{(\pi_1^*)^{-1}ch(\sigma)}{e|BG_{TX}}\right) \cdot \hat{A}(TX)^2 = \hat{A}(TX)$$

$$\Phi^*_\nu((\pi_2^*)^{-1}ch(V)) = 2^l \cdot e^{d(\nu)} \cdot M(\nu)$$

finally applying the integrality theorem for immersions is obtained that:

$$2^l \int_X ch(W) \cdot e^{d(\nu)} \cdot M(\nu) \cdot \hat{A}(TX) = integer$$

here $M(\nu)$ is the Mayer class, it is to say, is the multiplicative class for the power series $\cosh(\frac{x}{2})$, i.e. if we write the Pontrjagin class $p(\nu)$ formally as:

$$p(\nu) = \prod_{j=1}^{l}(1 + x_j^2)$$

then

$$M(\nu) = \prod_{j=1}^{l} \cosh(\frac{x_j}{2})$$

4 Conclusions

From the anomaly:

$$index(D) = \int_{\Sigma} ch^{+}(E) \wedge ch^{+}(\overline{E}) \wedge e^{d(N\Sigma)} \wedge \frac{\hat{A}(T\Sigma)}{\hat{A}(N\Sigma)} \wedge \chi(N\Sigma)$$

the following anomalous RR coupling on the brane-antibrane system is obtained [1], [2]:

$$Y = ch^{+}(E) \wedge e^{\frac{d(N\Sigma)}{2}} \wedge \sqrt{\frac{\hat{A}(T\Sigma)}{\hat{A}(N\Sigma)}}$$

Then, the question is,the exotic anomaly is given by:

$$2^l \int_X ch(W) \cdot e^{d(\nu)} \cdot M(\nu) \cdot \hat{A}(TX) = integer$$

what is the anomalous coupling that can be obtained from such exotic chiral anomaly?

5 References

[1] C. A. Scrucca and M. Serone, Nuclear Physics B556 (1999) hep-th/9903145

[2] Richard J. Szabo , hep-th/0108043

[3] Christian Bar, Elliptic Symbols