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Open String and Morita Equivalence of the Dirac-Born-Infeld Action with Modulus Φ

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Abstract

Based on the canonical quantization of open strings ending on D-branes with a background B field, we construct the open string propagator. We demonstrate the relation between the T duality of the underlying string theory and the Morita equivalence of the interpolating general Dirac-Born-Infeld theory on a noncommutative torus in the nonzero modulus Φ sector. The general noncommutative Dirac-Born-Infeld action with the Wess-Zumino terms expressed by the background R-R fields is shown to be Morita invariant.

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Many insights of the noncommutativities in the string-M theories have been accumulated since the Yang-Mills theory on a noncommutative torus was found to describe the DLCQ of M theory on a torus with a three-form field background [1]. The Matrix theory description of the gauge theory on the noncommutative torus has been extensively studied [2] to give the BPS energy spectrum and show the Morita equivalence of it [3, 4, 5] or of the action itself [6, 7], which is argued to be related with the T-duality $SO(d, d, Z)$ of type II string theory compactified on the d-torus [7, 8]. The Morita equivalence of the BPS mass spectrum and its relation with the T-duality have been also studied by a canonical description of the Dirac-Born-Infeld (DBI) theory on a noncommutative torus [9].

On the other hand in the presence of a constant background NS-NS B field the world-volume of D-branes becomes noncommutative [10, 11, 12] and then the low energy dynamics of D-branes is naturally described by the noncommutative Yang-Mills (NCYM) theory. The quantization of open strings propagating in the background B field is directly related to the noncommutativity of D-branes, where the world-volume coordinates of D-branes, which are in the static gauge identified with the space-time coordinates of open string end-points, are shown not to commute by means of the string mode expansion method [11, 12, 13] and the Green function method [14]. The former method has been rigorously studied by Dirac's constrained quantization procedure for the mixed boundary condition [15].

Though there is in general a coupling between the open and closed strings, we can take an appropriate low energy limit in such a way that the closed strings decouple from the open strings ending on the D-branes and the resulting theory for the open strings reduces to the NCYM theory [14]. Specially through the different regularizations the equivalence between the ordinary Yang-Mills theory and the NCYM theory has been shown by comparing the ordinary DBI theory with the noncommutative DBI theory where the background B field is replaced by the noncommutativity of the world-volume coordinates. Moreover a general DBI theory interpolating the two theories has been proposed to be described by a noncommutative action including an extra modulus Φ, which is known to appear as a magnetic background in the NCYM action and show a particular transformation under the group of the Morita equivalence for the NCYM theory [5, 8]. Based on this general DBI theory the Morita transformation rules have been reproduced through the appropriate low energy limit from the T-duality of type II string theory in the zero Φ sector.

We will try to extend the results of Seiberg and Witten [14] for the interrelations among the string dynamics, the noncommutative DBI theory and the NCYM theory. In order to see whether the canonical quantization of open strings ending on D-branes in the presence of a constant background B field [12] is consistently well defined or not, we will calculate the string propagator and compare with that derived from the Green function method. Based on the interpolating DBI theory on a noncommutative torus we will study how the Morita equivalence transformation is related with the T-duality generally in the nonzero Φ sector. The invariance of the general noncommutative DBI action with modulus Φ and the Wess-Zumino (WZ) action themselves under the Morita equivalence transformation will be investigated and the transformation properties of the background R-R fields will be elucidated.

The bosonic part of the classical action for an open string ending on a Dp-brane is given by

$$S = \frac{1}{4\pi\alpha'}\int_{\Sigma} d^2\sigma(g_{\mu\nu}\partial_a X^\mu \partial^a X^\nu - 2\pi\alpha' B_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu) + \oint_{\partial\Sigma} d\tau A_i(X)\partial_\tau X^i, \qquad (1)$$

where $A_i, i = 0, 1, \cdots, p$ is the U(1) gauge field living on the Dp-brane and $g_{\mu\nu}, B_{\mu\nu}, \mu = 0, 1, \cdots, 9$ represent the constant background metric and the constant background NS-NS two-form field. Since the background $B_{\mu\nu}$ field can have nonzero components only along the directions parallel to the Dp-brane, the action can be expressed as

$$S = \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma (g_{\mu\nu}\partial_a X^\mu \partial^a X^\nu - 2\pi\alpha' \mathcal{F}_{ij}\epsilon^{ab}\partial_a X^i \partial_b X^j) \tag{2}$$

in terms of the gauge invariant field strength $\mathcal{F}_{ij} = B_{ij} + F_{ij}$ with $F = dA$. The transverse string variable $X^\mu, \mu = p+1, \cdots, 9$ can be treated trivially so that we will be concerned with the longitudinal variable X^i only. The solution of X^i to the equation of motion, satisfying the mixed boundary condition $\partial_\sigma X^i - 2\pi\alpha'\partial_\tau X^j \mathcal{F}_j^i = 0$ at $\sigma = 0, \pi$, is given by

$$X^k = x_0^k + (p_0^k \tau + 2\pi\alpha' p_0^j \mathcal{F}_j^k \sigma) + \sum_{n\neq 0} \frac{e^{-in\tau}}{n}(i a_n^k \cos n\sigma + 2\pi\alpha' a_n^j \mathcal{F}_j^k \sin n\sigma). \tag{3}$$

In Ref. [12] the quantization of X^k was performed by a generalization of the canonical approach analyzing the time-averaged symplectic form on the phase space. The commutation relations for the modes were extracted as

$$\begin{aligned} &[a_m^i, a_n^j] = 2\alpha' m M^{-1ij}\delta_{m+n}, \qquad [x_0^i, p_0^j] = i2\alpha' M^{-1ij}, \\ &[x_0^i, x_0^j] = i2\pi\alpha'(M^{-1}\mathcal{F})^{ij}, \qquad [p_0^i, p_0^j] = 0, \end{aligned} \tag{4}$$

where $M_{ij} = g_{ij} - (2\pi\alpha')^2(\mathcal{F}g^{-1}\mathcal{F})_{ij}$ and $(M^{-1}\mathcal{F})^{ij}$ is abbreviation for $M^{-1ik}\mathcal{F}_{kl}g^{-1lj}$.

Making use of these results we construct the propagator of the open string position operator $X^k(\sigma,\tau)$. Here we turn to the Euclidean metric on the world sheet. By substituting $\tau = -i\tau'$ into (3) and using $z = e^{\tau' + i\sigma}$ we have the following normal mode expression

$$X^k(z) = x_0^k - \frac{i}{2}(p_0^k \ln z\bar{z} + 2\pi\alpha' p_0^j \mathcal{F}_j^k \ln\frac{z}{\bar{z}}) + i\sum_{n\neq 0}(a_n^k(z^{-n} + \bar{z}^{-n}) + 2\pi\alpha' a_n^j \mathcal{F}_j^k (z^{-n} - \bar{z}^{-n})). \tag{5}$$

The commutation relations in (4) provide the string propagator

$$\begin{aligned} &< 0|X^i(z)X^j(z')|0 >= \alpha'(-M^{-1ij}\ln z\bar{z} + 2\pi\alpha'(\mathcal{F}M^{-1})^{ij}\ln\frac{z}{\bar{z}} \\ &-\frac{1}{2}M^{-1ij}(\ln|1-\frac{z'}{z}|^2 + \ln|1-\frac{\bar{z}'}{z}|^2) + \frac{(2\pi\alpha')^2}{2}(\mathcal{F}M^{-1}\mathcal{F})^{ij}(\ln|1-\frac{z'}{z}|^2 - \ln|1-\frac{\bar{z}'}{z}|^2) \\ &-\frac{2\pi\alpha'}{2}(M^{-1}\mathcal{F})^{ij}\ln\frac{(z-z')(\bar{z}-z')}{(z-\bar{z}')(\bar{z}-\bar{z}')} + \frac{2\pi\alpha'}{2}(\mathcal{F}M^{-1})^{ij}(\ln\frac{(z-z')(z-\bar{z}')}{(\bar{z}-z')(\bar{z}-\bar{z}')} - 2\ln\frac{z}{\bar{z}})), \end{aligned} \tag{6}$$

whose first and second terms are the zero mode contributions. Owing to the relations $M^{-1} - (2\pi\alpha')^2 g^{-1}\mathcal{F}M^{-1}\mathcal{F}g^{-1} = g^{-1}, M^{-1}\mathcal{F}g^{-1} = g^{-1}\mathcal{F}M^{-1}$, where abbreviation is not used but the indeces are raised by metric, the propagator turns out to be

$$-\alpha'(g^{-1ij}(\ln|z-z'| - \ln|z-\bar{z}'|) + M^{-1ij}\ln|z-z'|^2 - 2\pi\alpha'(M^{-1}\mathcal{F}g^{-1})^{ij}\ln\frac{z-\bar{z}'}{\bar{z}-z'}). \tag{7}$$

In view of $M^{-1} = (g + 2\pi\alpha'\mathcal{F})^{-1}g(g - 2\pi\alpha'\mathcal{F})^{-1}$ and $M^{-1}\mathcal{F}g^{-1} = (g + 2\pi\alpha'\mathcal{F})^{-1}\mathcal{F}(g - 2\pi\alpha'\mathcal{F})^{-1}$ we note that we have obtained the same expression as that mentioned in Ref. [14], where the open string propagator is constructed as a solution to the equation of motion for the Green function satisfying the mixed boundary condition.

Seiberg and Witten have proposed a general DBI theory with an arbitrary θ and parameters G, Φ determined by a formula

$$\frac{1}{G + 2\pi\alpha'\Phi} = -\frac{\theta}{2\pi\alpha'} + \frac{1}{g + 2\pi\alpha' B}, \tag{8}$$

which implies that the open string metric G and the antisymmetric two-form Φ are expressed in terms of the closed string parameters g, B and the noncommutative parameter θ [14]. In the formula (8) expressed by the $(p+1)\times(p+1)$ matrices we assume that $B_{0i} = 0$ as well as $g_{0i} = 0$, here $i = 1, \cdots, p$ in the Lorentzian target space-time. For the Dp-brane compactified on a p-torus T^p parametrized by $x^i \sim x^i + 2\pi r$, $i = 1, \cdots, p$ with closed string metric g_{ij}, the T-duality $SO(p,p,Z)$ transformation on $E = r^2(g + 2\pi\alpha' B)/\alpha'$ is given by $E' = (aE + b)(cE + d)^{-1}$, with $c^t a + a^t c = 0, d^t b + b^t d = 0, c^t b + a^t d = 1$ where a, b, c and d are $p \times p$ matrices with integer entries. We consider the case that the θ dependence appears through the noncommutativity of the world-volume space-coordinates, so that θ^{0i} is set to zero. Therefore we have a formula (8) expressed by the $p\times p$ matrices, $G_{00} = g_{00}$ and $\Phi_{0i} = 0$ which corresponds to the treatment of Φ as the magnetic background. From the $p\times p$ matrix formula (8) using the dimensionless $p \times p$ matrix $\Theta = \theta/2\pi r^2$ we extract G and Φ as

$$\begin{aligned} G &= \frac{\alpha'}{2r^2}\frac{1}{1 + E^t\Theta}(E + E^t)\frac{1}{1 - \Theta E}, \\ \Phi &= \frac{1}{4\pi r^2}\frac{1}{1 + E^t\Theta}(2E^t\Theta E + E - E^t)\frac{1}{1 - \Theta E}. \end{aligned} \tag{9}$$

Compared with the given closed string parameters g and B, the NCYM theory includes the corresponding two parameters G and θ with an extra modulus Φ added, which is associated with $SO(p)$ symmetry for the relativistic generalization of some NCYM energy that is identified with the BPS mass formula of the noncommutative DBI theory [8]. Here assuming that Θ transforms by a fractional transformation as $\Theta \to \Theta' = (c + d\Theta)(a + b\Theta)^{-1}$ we examine how G and Φ transform with respect to the T-duality. There are interesting symmetric relations, $E' + E'^t = (E^t c^t + d^t)^{-1}(E + E^t)(cE + d)^{-1}$ and

$$1 - \Theta' E' = \frac{1}{a^t - \Theta b^t}(1 - \Theta E)\frac{1}{cE + d}. \tag{10}$$

Combining them with the first equation of (9) and $\Theta'^t = -\Theta'$ we derive the transformation for the open string metric

$$G' = (a + b\Theta)G(a + b\Theta)^t. \tag{11}$$

Similarly the transformed modulus Φ' is represented by

$$\Phi' = \frac{1}{4\pi r^2}(a + b\Theta)\frac{1}{1 + E^t\Theta}(2X + Y)\frac{1}{1 - \Theta E}(a + b\Theta)^t, \tag{12}$$

where $X = (E^t a^t + b^t)(c + d\Theta)(a + b\Theta)^{-1}(aE + b)$ and Y is given by $Y = E - E^t + 2Y_0$ with $Y_0 = -(E^t a^t + b^t)cE + (E^t c^t + d^t)b$. For convenience, $X = (E^t a^t + b^t)\Theta'(aE + b)$ is equivalently rewritten by

$$X = \frac{1}{2}(E^t a^t + b^t)((c + d\Theta)\frac{1}{a + b\Theta} - \frac{1}{a^t - \Theta b^t}(c^t - \Theta d^t))(aE + b) \tag{13}$$

because of $\Theta'^t = -\Theta'$, which is further expressed as $X = (E^t + (1+E^t\Theta)b^t(a^t - \Theta b^t)^{-1})(\Theta + X_0)(E+(a+b\Theta)^{-1}b(1-\Theta E))$ with $X_0 = -(c^t - \Theta d^t)b\Theta + (a^t - \Theta b^t)c$. From these expressions we first extract some terms suggested by the second equation in (9) as $2X + Y = 2E^t\Theta E + E - E^t + 2Z$, where the remaining terms are gathered by

$$\begin{aligned} Z &= E^t X_0 E + E^t(\Theta + X_0)\frac{1}{a + b\Theta}b(1 - \Theta E) \\ &+ (1 + E^t\Theta)b^t\frac{1}{a^t - \Theta b^t}(\Theta + X_0)E + Y_0 + Z_0 \end{aligned} \tag{14}$$

with $Z_0 = (1 + E^t\Theta)b^t(a^t - \Theta b^t)^{-1}(\Theta + X_0)(a + b\Theta)^{-1}b(1 - \Theta E)$.

The substitution of an identity

$$c^t - \Theta d^t = -(a^t - \Theta b^t)(c + d\Theta)\frac{1}{a + b\Theta} \tag{15}$$

into X_0 in (14) brings two kinds of cancellations in the first term against the second and third terms. As a result the first term of (14) becomes $E^t(a^t c + (a^t - \Theta b^t)d\Theta(a+b\Theta)^{-1}b\Theta)E$, whose first term is further cancelled against a term in Y_0. Since there is another cancellation between the second and third terms through $b^t(c + d\Theta)(a + b\Theta)^{-1} = (a + b\Theta)^{-1} - d^t$, the resulting third term can be expressed as $[(1 + E^t\Theta)(b^t(a^t - \Theta b^t)^{-1} - d^t b) + (a + b\Theta)^{-1}b]\Theta E + b^t cE$, whose last term is also cancelled against a term in Y_0. The remainging second term is given by $E^t[(\Theta + (a^t - \Theta b^t)c)(a + b\Theta)^{-1}b - (c^t - \Theta d^t)b\Theta(a + b\Theta)^{-1}b(1 - \Theta E)]$, which is further changed into $E^t[(a^t c + \Theta d^t a)(a + b\Theta)^{-1}b - (c^t - \Theta d^t)(1 - a(a + b\Theta)^{-1})b(1 - \Theta E)]$ where a combination $E^t(a^t c + c^t a)(a + b\Theta)^{-1}b$ vanishes. Thus we have

$$\begin{aligned} Z &= E^t(c^t b + (a^t - \Theta b^t)(c + d\Theta)\frac{1}{a + b\Theta}b + \Theta\frac{1}{a + b\Theta}b - 2\Theta d^t b)\Theta E \\ &+ E^t\Theta d^t b + (\frac{1}{a + b\Theta} - d^t b)\Theta E + d^t b + Z_0 + (1 + E^t\Theta)b^t\frac{1}{a^t - \Theta b^t}\Theta E, \end{aligned} \tag{16}$$

whose last term is cancelled against a term in $Z_0 = (E^t\Theta + 1)b^t(a^t - \Theta b^t)^{-1}(\Theta + X_0)(-E + (a + b\Theta)^{-1}(aE + b))$. The second term in (16) is simplified into $-E^t(c^t - \Theta d^t)b\Theta E$ through (15). Then the remaining Z_0 is further arranged by using (15) for X_0 into

$$\begin{aligned} Z_0 &= (E^t\Theta + 1)b^t[\frac{1}{a^t - \Theta b^t}\Theta\frac{1}{a + b\Theta}(aE + b) \\ &+ ((c + d\Theta)\frac{1}{a + b\Theta}b\Theta + c)\frac{1}{a + b\Theta}b(1 - \Theta E)]. \end{aligned} \tag{17}$$

Collecting the three terms with b on the right end in (17) and making use of

$$\frac{1}{a^t - \Theta b^t}\Theta + c = (c + d\Theta)\frac{1}{a + b\Theta}a, \tag{18}$$

we can see that they are simply expressed as $(E^t\Theta+1)b^t(c+d\Theta)(a+b\Theta)^{-1}b$. This compact expression further takes the form $(E^t\Theta+1)(a+b\Theta)^{-1}b - E^t\Theta d^t b - d^t b$ whose last two terms are cancelled out in Z (16). The three terms with E on the right end in (17) turn out to be $(E^t\Theta+1)(d^t b - 2(a+b\Theta)^{-1}b)\Theta E$ also through (18). Gathering together we arrive at a simplified expression

$$Z = (E^t\Theta+1)\frac{1}{a+b\Theta}b(1-\Theta E), \tag{19}$$

which yields

$$\Phi' = (a+b\Theta)\Phi(a+b\Theta)^t + \frac{1}{2\pi r^2}b(a+b\Theta)^t. \tag{20}$$

The $SO(p,p,Z)$ T-duality action on the closed string coupling g_s is given by $g'_s = g_s/\det(cE+d)^{1/2}$. The effective open string coupling $G_s = g_s(\det(G+2\pi\alpha'\Phi)/\det(g+2\pi\alpha' B))^{1/2}$ specified by the $(p+1)\times(p+1)$ matrices is expressed as $G_s = g_s(\det(1-\Theta E))^{-1/2}$ since $G_{00}=g_{00}, \Phi_{0i}=B_{0i}=0$. Therefore the relation (10) yields the $SO(p,p,Z)$ T-duality action on G_s

$$G'_s = \sqrt{\det(a+b\Theta)}G_s, \tag{21}$$

which further determines the transformation for the Yang-Mills gauge coupling $g_{YM} = ((2\pi)^{p-2}G_s/(\alpha')^{(3-p)/2})^{1/2}$ to be

$$g'_{YM} = g_{YM}(\det(a+b\Theta))^{\frac{1}{4}}. \tag{22}$$

The obtained expressions such as (11), (20) and (22) are just the Morita transformation rules in the NCYM theory with modulus Φ. In the $\Phi=0$ orbit the T-duality action on the closed string metric g and NS-NS B field was shown to be mapped to the Morita transformation only when the zero slope limit was taken in such a way that $E^{-1}\approx\Theta$ [14]. In our general nonzero Φ case this mapping naturally appears where we do not restrict ourselves to the zero slope limit. We would like to interpret this general mapping as a direct relation between the Morita equivalence of the noncommutative DBI theory and the T-duality.

In order to confirm this interpretation we apply the above Morita transformation rules to the non-abelian DBI theory living on a noncommutative torus. The effective action of Dp-branes with the modulus two-form Φ on a p-dimensional noncommutative torus is given by

$$S = -\frac{1}{G_s(2\pi)^p\alpha'^{\frac{p+1}{2}}}\int d^{p+1}\sigma Tr_\theta\sqrt{-\det(G+2\pi\alpha'(F+\Phi))} + S_{WZ}. \tag{23}$$

The first term is regarded as an general DBI action interpolationg between the ordinary DBI one on a commutative torus with $G=g, \Phi=B, G_s=g_s$ and the commutative gauge fields, and the noncommutative DBI one expressed in terms of the open string variables with $\Phi=0$ and the noncommutative gauge fields [14]. The Tr_θ is the trace on the gauge bundle on the noncommutative torus and is regarded as the symmetric trace [16] for the non-abelian group. We assume that the WZ action S_{WZ} on a noncommutative torus takes the same form as that on a commutative torus [17]. So it is represented by $S_{WZ} = \int Tr_\theta(\sum C^{(n)})e^{2\pi\alpha' F}$ in terms of a pullback of the n-form R-R potential $C^{(n)}$, where on the exponential F only appears and the background B field is replaced by the noncommutativity θ. In the DBI action the square root part is decomposed into $\sqrt{\det(G+2\pi\alpha'(F+\Phi))_{ij}}(-(G_{00}-(2\pi\alpha')^2F_{0i}(G+$

$2\pi\alpha'(F+\Phi))^{-1ij}F_{j0}))^{1/2}$, where $i=1,\cdots,p$ and we have taken account of $\Phi_{0i}=G_{0i}=0$. Under the Morita equivalence transformation on a p-dimensional noncommutative torus the magnetic component of the gauge field strength F_{ij} is transformed as $F_{ij}\to((a+b\Theta)F(a+b\Theta)^t-\frac{1}{2\pi r^2}b(a+b\Theta)^t)_{ij}$ according to (20), while the electric component F_{0i} is transformed as $F_{0i}\to(F(a+b\Theta)^t)_{0i}$. These transformation rules are argued in the investigation of the Morita equivalence in the context of the NCYM theory [4, 5, 6, 7]. We can take $G_{00}=g_{00}$ to be unchanged since the T-duality transformation is performed on the directions of the p-torus. The transformation of the trace Tr_θ is given by $Tr_\theta\to(\det(a+b\Theta))^{-1/2}Tr_\theta$ [9]. Under these transformation rules together with (11), (20) and(21) we can see that the general interpolating DBI action is invariant.

For the WZ action we consider the $D5$-brane theory for concreteness and write down

$$\begin{aligned} S_{WZ} &= \int d^6\sigma Tr_\theta \epsilon^{ilklm}(\frac{2\pi\alpha'}{24}F_{0i}C_{jklm}+\frac{(2\pi\alpha')^2}{4}F_{0i}F_{jk}C_{lm}+\frac{(2\pi\alpha')^3}{8}F_{0i}F_{jk}F_{lm}C \\ &+ \frac{1}{5!}C_{0ijklm}+\frac{2\pi\alpha'}{12}C_{0ijk}F_{lm}+\frac{(2\pi\alpha')^2}{8}C_{0i}F_{jk}F_{lm}). \end{aligned} \tag{24}$$

The WZ action for the first three terms is shown to be invariant under the Morita equivalence transformation, if we take the R-R potentials such as C, C_{lm} and C_{jklm} to transform simultaneously as

$$\begin{aligned} C &\to \frac{1}{\sqrt{\det A}}C, \qquad C_{ij} \to \frac{1}{\sqrt{\det A}}((ACA^t)_{ij}+\frac{\alpha'}{r^2}(bA^t)_{[ij]}C), \\ C_{ijkl} &\to \frac{1}{\sqrt{\det A}}(A^a_{[i}A^b_jA^c_kA^d_{l]}C_{abcd}+\frac{6\alpha'}{r^2}A^a_{[i}A^b_j(bA^t)_{kl]}C_{ab}+3(\frac{\alpha'}{r^2})^2C(bA^t)_{[ij}(bA^t)_{kl]}), \end{aligned} \tag{25}$$

where $A=a+b\Theta$ and A^a_i stands for ${A_i}^a$ and the appropriate antisymmetrization factors 6, 3 appear. The invariance of S_{WZ} for the last three terms is also shown, if we make C_{0i}, C_{0ijk} and C_{0ijklm} transform in the following way

$$\begin{aligned} C_{0i} &\to \frac{1}{\sqrt{\det A}}A^a_iC_{0a}, \qquad C_{0ijk} \to \frac{1}{\sqrt{\det A}}(A^a_{[i}A^b_jA^c_{k]}C_{0abc}+\frac{3\alpha'}{r^2}A^a_{[i}(bA^t)_{jk]}C_{0a}), \\ C_{0ijklm} &\to \frac{1}{\sqrt{\det A}}(A^a_{[i}\cdots A^e_{m]}C_{0a\cdots e}+\frac{10\alpha'}{r^2}A^a_{[i}A^b_jA^c_k(bA^t)_{lm]}C_{0abc} \\ &+ 15(\frac{\alpha'}{r^2})^2A^a_{[i}(bA^t)_{jk}(bA^t)_{lm]}C_{0a}), \end{aligned} \tag{26}$$

where the coefficients 3, 10 and 15 also show the appropriate antisymmetrization factors specified by ${}_3C_1, {}_5C_3$ and $3\cdot{}_5C_1$.

Redefinitions of the above R-R potentials as

$$\begin{aligned} \lambda^i &= \frac{1}{4!}\epsilon^{ijklm}C_{jklm},\ \lambda^{ijk}=\frac{1}{2!}\epsilon^{ijklm}C_{lm},\ \lambda^{ijklm}=\epsilon^{ijklm}C, \\ \lambda &= \frac{1}{5!}\epsilon^{ijklm}C_{0ijklm},\ \lambda^{ij}=\frac{1}{3!}\epsilon^{ijklm}C_{0klm},\ \lambda^{ijkl}=\epsilon^{ijklm}C_{0m} \end{aligned} \tag{27}$$

enable us to write the WZ action (24) to be

$$
\begin{aligned}
S_{WZ} &= \int d^6\sigma Tr_\theta(2\pi\alpha' F_{0i}\lambda^i + \frac{(2\pi\alpha')^2}{2}F_{0i}F_{jk}\lambda^{ijk} + \frac{(2\pi\alpha')^3}{8}F_{0i}F_{jk}F_{lm}\lambda^{ijklm} \\
&+ \lambda + \frac{2\pi\alpha'}{2}F_{ij}\lambda^{ij} + \frac{(2\pi\alpha')^2}{8}F_{ij}F_{kl}\lambda^{ijkl}).
\end{aligned} \tag{28}
$$

The first three terms are also invariant under the Morita transformation accompanied with

$$
\begin{aligned}
\lambda^i &\to \sqrt{\det A}(B^i_a\lambda^a + \frac{\alpha'}{2r^2}B^i_aB^j_bB^k_c\lambda^{abc}(bA^t)_{jk}) + \frac{1}{8}(\frac{\alpha'}{r^2})^2\frac{1}{\sqrt{\det A}}\lambda^{ijklm}(bA^t)_{jk}(bA^t)_{lm}, \\
\lambda^{ijk} &\to \sqrt{\det A}B^i_aB^j_bB^k_c\lambda^{abc} + \frac{\alpha'}{2r^2}\frac{1}{\sqrt{\det A}}\lambda^{ijklm}(bA^t)_{lm},\ \lambda^{ijklm} \to \frac{1}{\sqrt{\det A}}\lambda^{ijklm},
\end{aligned} \tag{29}
$$

which are obtained from (25) by defining $B = (A^{-1})^t$ and using B^i_a for $B^i{}_a$. The invariance for the last three terms is also provided by the following transformation properties of the redefined R-R potentials

$$
\begin{aligned}
\lambda &\to \sqrt{\det A}(\lambda + \frac{\alpha'}{2r^2}B^i_aB^j_b(bA^t)_{ij}\lambda^{ab} + \frac{1}{8}(\frac{\alpha'}{r^2})^2B^i_aB^j_bB^k_cB^l_d(bA^t)_{ij}(bA^t)_{kl}\lambda^{abcd}), \qquad (30) \\
\lambda^{ij} &\to \sqrt{\det A}(B^i_aB^j_b\lambda^{ab} + \frac{\alpha'}{2r^2}B^i_aB^j_bB^k_cB^l_d(bA^t)_{kl}\lambda^{abcd}),\ \lambda^{ijkl} \to \sqrt{\det A}B^i_aB^j_bB^k_cB^l_d\lambda^{abcd},
\end{aligned}
$$

which are derived by combining (26) and (27). Thus we have observed the Morita equivalence of the action itself for the interpolating general DBI theory, which is compared with the Morita equivalence of the BPS energy spectrum based on the canonical approaches of the DBI theory on the noncommutative two- or four- torus [9] and the NCYM theory [3, 4, 5]. The action of the Yang-Mills theory on a noncommutative three-torus with the Chern-Simon topological terms is proved to be Morita invariant in Ref.[7]. The transformation behaviors of the zero-form and two-form R-R potentials given there are now extended to (25) for the noncommutative five-torus, where the transformation property of the four-form R-R potential is obtained in a suggestive and systematic form, while the corresponding transformation rules (29) for the redefined R-R potentials are considered as an extension of those for the noncommutative three-torus presented in Ref. [5]. In Refs. [5, 7] only the space components of the R-R potentials are assumed to be nonzero. Here even if we take account of the R-R potentials with a time component the Morita equivalence of the WZ action for them is confirmed to hold separately, where the transformation of the six-form R-R potential is specified.

We have shown that the quantization of open strings ending on the D-branes with the background B field is so well constructed from the symplectic structure on the phase space that we can consistently derive the string propagator with the mixed boundary conditions. Combining the T-duality transformation for the closed string parameters with the fractional transformation for the noncommutativity parameter we can extract the Morita transformation rules for the open string parameters which characterize the interpolating noncommutative DBI action, without recourse to the low energy zero slope limit. The interpolating DBI action with nonzero modulus Φ as well as the WZ action on a noncommutative torus have been demonstrated to have the same Morita equivalence as the NCYM theory. Therefore it can be said that the T-duality transformation is directly translated to the Morita equivalence in the context of the noncommutative DBI theory, which is considered as the generalization of the indirect low-energy extraction of the Morita equivalence for the NCYM theory.

On the five-dimensional noncommutative torus we have seen that the transformed R-R potentials are expanded in terms of the same and lower rank R-R potentials with the appropriate coefficients which appear as the antisymmetrization numbers. This expansion reflects the characteristics for the couplings of the Dp-brane not only to the the R-R potential with the $(p+1)$ rank but also to the R-R potentials with the ranks lower by even numbers, accompanied with the additional and multiple interactions to the world-volume gauge field strength in the noncommutative WZ acion. We speculate that this expansion is related with the recent works about the noncommutative description of D-branes [18] where the D-branes can be expressed as a configuration of infinitely many lower dimensional D-branes. It is tempting to suspect that the application of the Morita transformation of the noncommutative DBI theory is useful to build the nontrivial bound state consisting of a Dp-brane and the D-branes with lower world-volume dimensions from a pure Dp-brane configuration.

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