ON THE COVARIANT QUANTIZATION OF GREEN-SCHWARZ SUPERSTRING AND BRINK-SCHWARZ SUPERPARTICLE

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The covariant quantization of the interacting Green-Schwarz [1] Superstring (GSSS) is one of the interesting problems to solve in String Theory.The formulation of Superstrings a la Green-Schwarz manifestly supersymmetric on the target manifold has several advantages over the NRS [2] formulation. In fact, the latest is only a perturbative approach to the interacting Superstring Theory. Moreover, supersymmetry on the target manifold is obtained only after summation over spin structures. However this problem for odd spin structures has not been completely understood.

The Green-Schwarz second quantized action for superstrings has only been consistently formulated in the Light Cone Gauge [3]. In spite of the fact that several attempts have been performed, the covariant formulation of the problem has not been obtained. The difficulty to start with, has been that the first class constraints associated to gauge symmetries of the first quantized theory appear mixed with second class ones and no local, Lorentz covariant, approach to extend them to a set of only first class constraints has been yet developed.The same problem appears in the covariant quantization of the Brink-Schwarz [4] Superparticle (BSSP) which describes the zero mode structure of the GSSP.

In order to circumvent the problem presented by the mixing of first and second class constraints in a covariant treatment other actions for the description of the Superparticle, following original ideas of Siegel were proposed. The so-called Siegel Superparticle, SSP [5], does not have the same number of degrees of freedom as the BSSP. It corresponds to ignore the second class constraints of the BSSP formulation, leaving only the first class constraints which may be covariantly projected from the original BSSP set of constraints. The fact that SSP has a number of degrees of freedom different from BSSP is a consequence of the non trivial restrictions imposed by the second class constraints.

The Modified Siegel Superparticles, MSSPI [6] and MSSPII [7] have the same physical spectrum as the BSSP, allowing a formulation in terms of first class constraints only. The formulations however are given in terms of irregular constraints, in distinction from the original BSSP formulation. If the constraints of a dynamical system are expressed as the kernel of a map ϕ between Banach manifolds,

$$\phi : M \rightarrow N$$

and if for some m belonging to the kernel of ϕ the induced tangent map at m, $T_m\phi$, is not surjective we call the constraint irregular.

If ϕ is a regular constraint $\phi^{-1}(0)$ always define a submanifold. If ϕ is irregular, $\phi^{-1}(0)$ is not necessarily a submanifold. For irregular constraints the Lagrange multiplier theorem does not in general apply. In fact one of the hypothesis is the regularity of the constraints. This implies that the generalized canonical quantization which introduces all constraints into the action using Lagrange multipliers is not in general valid. In fact the effective action of a constrained theory with first class irregular constraints is well defined only in a gauge implemented by a gauge condition on the associated Lagrange multipliers.

In particular this restriction allows a covariant gauge condition. However, the action on a canonical gauge is ill defined. Consequently the off-shell reduction to the physical modes, which is one of the main arguments to prove unitary of the S-matrix by functionals methods, cannot be proven.

A general discussion on the quantization for irregular systems is given in [8]. The presence of irregular constraints in all the Modified Siegel Superparticle and Superstring actions does not allow, because of the previous arguments, the direct application of the Batalin-Vilkovisky approach to quantize the problem, and so far have failed to solved the problem of the covariant quantization.

For the same reasons the off-shell nilpotent BRST charge for the Superparticle [9] has been obtained only in a indirect way which does not give any insight into the geometric structure of the theory nor any idea on the generalization to the Superstring problem.

In this work we present a direct alternative approach to the covariant quantization of the BSSP using infinite auxiliary fields. At any truncated level the theory is not Lorentz covariant. The covariance is obtained only after the introduction of the infinite auxiliary fields. It leads directly to BRST charge previously found in [9] and to the Kallosh action [10]. The formulation is based upon a general canonical approach for dynamical systems restricted by reducible first and second class constraints [11, 12]. In this approach the phase space is extended to a larger manifold where all extended constraints are first class. By an appropriate gauge fixing one may reduce the functional integral to a functional integral on the original constrained manifold, with the correct functional measure. It is an off-shell approach allowing the systematic construction of the off-shell nilpotent BRST charge and of the BRST invariant effective action.

The first order action for the ten dimensional BS superparticle is

$$S =< P_\mu \partial_\tau \chi^\mu + \overline{\xi} \not{P} \partial_\tau \xi + e P^2 > \tag{1}$$

where e is a Lagrange multiplier associated to the constraint

$$P^2 = 0 \quad . \tag{2}$$

Let η be the momenta canonically conjugate to ξ. Since the action (1) is first order in $\partial_\tau \xi$ its dynamics is restricted by,

$$\phi = \eta - \not{P} \xi = 0 \quad . \tag{3}$$

The canonical Hamiltonian action of the system is

$$S =< P_\mu \partial_\tau \chi^\mu + \overline{\eta} \partial_\tau \xi + e P^2 + \overline{\psi}(\eta - \not{P}\xi) > \tag{4}$$

where $\overline{\psi}$ are Lagrange multipliers associated to the constraints (3).

Constraints (3) are a combination of first and second class ones. We enlarge the phase space by introducing the following auxiliary fields

$$\Phi_1 = \eta_1 + \not{P}\xi_1 \overline{\Phi}_1 = \eta_1 - \not{P}\xi_1 \quad .$$

The enlarged constraints are in this case

$$\widetilde{\phi}_0 = \eta - \not{P}\xi + \Phi_1 \quad . \tag{5}$$

It is necessary to add the restriction

$$\overline{\Phi}_1^{\top} = 0 \quad . \tag{6}$$

where $\top$ is the transverse projection in the sense [11].

The factor $det\{\overline{\Phi}_1^{\top}, \overline{\Phi}_1^{\top}\}^{1/2}$ in the measure of the functional integral is in principle as problematic as the factor $det\{\phi^{\top}, \phi^{\top}\}^{1/2}$ in the direct approach. Nevertheless we observe that constraint (6) is equivalent to a reducible constraint

$$\overline{\Phi}_1 = 0, \;\; \not{P}\overline{\Phi}_1|_{firstclass} = \not{P}\eta_1 \equiv 0. \tag{7}$$

We iterate now the process and introduce ξ_2, η_2 and ω_2. We obtain again

$$\omega_2 = -2\;\not{P}\Phi_2 = \eta_2 + \;\not{P}\xi_2\overline{\Phi}_2 = \eta_2 - \;\not{P}\xi_2 \tag{8}$$

and we have the new constraints

$$\widetilde{\phi}_1 = \overline{\Phi}_1 + \Phi_2\overline{\Phi}_2^{\top} = 0 \tag{9}$$

For the same reason as above we take instead of (9) the reducible constraint

$$\overline{\Phi}_2 = 0 \;\; \not{P}\overline{\Phi}_2|_{firstclass} = \not{P}\eta_2 \equiv 0 \tag{10}$$

and continue the process. After ℓ steps we have

$$\widetilde{\phi}_{i-1} = \overline{\Phi}_{i-1} + \overline{\Phi}_i \quad i = 1, \cdots, \ell\overline{\Phi}_\ell^{\top} = 0 \tag{11}$$

with $\overline{\Phi}_0 \equiv \phi$.

At this level the classical action may be written in terms of the canonical variables in the form

$$S_\ell =< P_\mu \dot{x}^\mu + \sum_{i=0}^{\ell} \eta_i \dot{\xi}^i + \lambda P^2 + \sum_{i=0}^{\ell-1} \overline{\psi}^i \;\not{P}\eta_i + \sum_{i=0}^{\ell} \lambda^i \widetilde{\phi}_{i-1} > . \tag{12}$$

subject to the second class constraints

$$\overline{\Phi}_\ell^{\top} = 0.$$

Here $\widetilde{\phi}_\ell$ in (12) is irreducible,the other constraints being infinite reducible. At each level ℓ the formulation (12) is not Lorentz covariant due to the transverse projection the second equation.

In order to avoid this problem we may introduce infinite auxiliary fields. We then obtain

$$S_\infty =< P_\mu \dot{x}^\mu + \sum_{i=0}^{\infty} \eta_i \dot{\xi}^i + \lambda P^2 + \sum_{i=0}^{\infty} \overline{\psi}^i \;\not{P}\eta_i + \sum_{i=0}^{\infty} \lambda^i \widetilde{\phi}_{i-1} > . \tag{13}$$

which was first obtained by Kallosh in [10].

The effective action associated to (13) may be truncated at any level ℓ by imposing appropriate gauge fixing conditions and the effective action associated to (12) is regained. In the limit case $\not{P}$ η_i and $\widetilde{\phi}_i$ $i = 0, \cdots, \infty$ are regular infinite reducible first class constraints. This allows the systematic Batalin-Fradkin construction of the off-shell nilpotent BRST charge. The result as was shown by Kallosh in ref. [10] match with the BRST operator with the correct cohomology for the BSSP.

The superparticle action on both an $N = 1$ and $N = 2$ superworldline was discovered by Sorokin, Tkach and Volkov [13]. We shall refer them as STVSP1 and STVSP2 respectively.

The STVSP1 contains as one of the field equation the same constraint (3) as in the BSSP. The correct quantization of it follows in the same lines as we have described the BSSP.

The STVSP2 contains an extension of (3) to a more general phase space which coincides with the extension proposed in [14]. The set of constraints proposed in [14] are first class only and have two stages of reducibility. However the twistorial approach of [14] doesn't get rid of the second class constraints present in the Superstring case.

An extension of the approach presented here, for the covariant treatment of the Superparticle, has been performed which in particular generalizes the twistorial approach of [14]. Its application to the GSSS is under study. We hope to report on it soon.

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