

# An $SL(2, \mathbb{R})$ -covariant, first order, $\kappa$ -supersymmetric action for the $D5$ -brane<sup>†</sup>

Pietro Fré and Leonardo Modesto

Dipartimento di Fisica Teorica, Università di Torino,  
& INFN - Sezione di Torino  
via P. Giuria 1, I-10125 Torino, Italy

## Abstract

The new first order, rheonomic,  $\kappa$ -supersymmetric formalism recently introduced by us for the world-volume action of the  $D3$  brane is extended to the case of  $D5$  branes. This extension requires the dual formulation of the Free Differential Algebra of type IIB supergravity in terms of 6-form gauge potentials which was so far missing and is given here. Furthermore relying on our new approach we are able to write the  $D5$  world volume action in a manifestly  $SL(2, \mathbb{R})$  covariant form. This is important in order to solve the outstanding problem of finding the appropriate boundary actions of  $D3$ -branes on smooth ALE manifolds with twisted fields. The application of our results to this problem is however postponed to a subsequent publication.

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# 1 Introduction

In a recent paper [1], the present authors introduced a **new first order formalism** that is able of generating second order world volume actions of the Born–Infeld type for  $p$ -branes. This construction allows for a description within the standard framework of rheonomy of those  $\kappa$ -supersymmetric boundary actions that function as sources for supergravity  $p$ -solutions in the bulk. Furthermore in this approach the  $SL(2, \mathbb{R})$  duality symmetry can be manifest at all levels. These features are very much valuable and quite essential in order to discuss various questions relative to the microscopic interpretation of classical  $p$ -solutions like the  $D3$ -brane with flux on smooth ALE manifolds [2] and other issues in the general quest of the gauge/gravity correspondence. The new formulation which, in our opinion, is particularly compact and elegant is based on the introduction of an additional auxiliary field, besides the world volume vielbein, and on the enlargement of the local symmetry from the Lorentz group to the general linear group:

$$SO(1, d-1) \xrightarrow{\text{enlarged}} GL(d, \mathbb{R}) \quad (1.1)$$

Within this new formalism in [1] we constructed the first order version of the  $\kappa$ -supersymmetric action for a  $D3$ -brane. This choice was not random rather it was rooted in the main motivations to undertake such a new construction. Indeed we needed a suitable formulation of the  $D3$ -brane action holding true on a generic background in order to apply it to the aforementioned  $D3$ -brane bulk solution with flux [2] or to more complicated ones. However, as for the the solution in [2], the task was not exhausted once we had the  $D3$ -brane action. Indeed this latter accounts only for the source of the Ramond-Ramond  $A^{[4]}$ -form with self dual field strength but not for the source of the doublet  $\{B^{[2]}, C^{[2]}\}$  of 2-forms whose flux is trapped on the homology 2-cycles of the transverse ALE manifold. The source of such fields can only be accounted for by the presence of 5-branes whose world-volume sweeps both the world volume of the  $D3$ -brane and the ALE 2-cycles. Hence we need to extend our first order formalism also to 5-branes and this is what we do in the present paper.

It was noted already in [3, 2, 4] that the a  $D5$  brane wrapped on a 2-cycle of the ALE manifold cannot be the direct source of the smooth, fractional brane like solution of [2]. The reason is simple. In the solution of [2] the dilaton field is constant and therefore there is no source for its field equation. On the other hand the  $D5$ -brane couples to the dilaton and contributes such a source. It is only in the orbifold limit that such a coupling vanishes together with the vanishing of the homology cycle trapping the flux of the 2-forms. In order to reproduce the correct boundary action for the smooth supersymmetric boundary solution of [2] we need a 5-brane that has vanishing coupling to the dilaton field. As we are going to show in a next coming publication [5] such a brane exists and it is a mixture of  $D5$ -branes on which closed **fundamental strings** can annihilate and of  $D5'$ -branes where such annihilation occurs for closed  **$D$ -strings**. To describe the world-volume action of such an object we need an explicitly  $SL(2, \mathbb{R})$ -covariant formulation of the 5 brane that depends on the charge vector:

$$q^\Lambda = \left( \begin{array}{l} q^1 = \text{charge with respect to } B^{[2]} \text{ (Neveu Schwarz 2-form)} \\ q^2 = \text{charge with respect to } C^{[2]} \text{ (Ramond 2-form)} \end{array} \right) \quad (1.2)$$

In this paper using the first order rheonomic formalism of [1] and relying on a dual formulation of type IIB supergravity we construct the required  $SL(2, \mathbb{R})$  covariant 5-brane action.

## 2 Type IIB supergravity with dual 6-form potentials

In the appendix *B* of [1] we gave a short but comprehensive summary of type IIB supergravity in the rheonomic approach. This summary was entirely based on the original papers by Castellani and Pesando [6, 7] but it also contained some new useful results, in particular the transcription of the curvatures from the complex  $SU(1,1)$  basis to the real  $SL(2, \mathbb{R})$  basis and a comparison of the supergravity field equations as written in the rheonomy approach and as written in string text-books like Polchinsky's [11].

In order to deal with 5-branes we need to include also the 7-form field strengths that are dual to the 3-form field strengths originally used in [6, 7] and summarized in [1]. This amounts to extending the original Free Differential Algebra of type IIB supergravity with the 6-form generators that are associated with the new cohomology classes of such an algebra. This is the exact analogue of the procedure which leads to the extended Free Differential Algebra of M-theory [8] containing both the 3-form (source of  $M2$ -branes) and the 6-form (source of the  $M5$ -branes). This latter arises by including the new generator associated with the cohomology 7-cocycle of the original Free Differential Algebra of M-theory in its 3-form formulation [9].

In our case the new complete form of the curvatures is the following one:

$$R^{\underline{a}} = \mathcal{D}V^{\underline{a}} - i\bar{\psi} \wedge \Gamma^{\underline{a}}\psi \quad (2.1)$$

$$R^{\underline{ab}} = d\omega^{\underline{ab}} - \omega^{\underline{ac}} \wedge \omega^{\underline{db}} \eta_{\underline{cd}} \quad (2.2)$$

$$\rho = \mathcal{D}\psi \equiv d\psi - \frac{1}{4}\omega^{\underline{ab}} \wedge \Gamma_{\underline{ab}}\psi - \frac{1}{2}iQ \wedge \psi \quad (2.3)$$

$$\mathcal{H}_{[3]}^{\alpha} = \sqrt{2}dA_{[2]}^{\alpha} + 2i\Lambda_{+}^{\alpha}\bar{\psi} \wedge \Gamma_{\underline{a}}\psi^{*} \wedge V^{\underline{a}} + 2i\Lambda_{-}^{\alpha}\bar{\psi}^{*} \wedge \Gamma_{\underline{a}}\psi \wedge V^{\underline{a}} \quad (2.4)$$

$$\mathcal{F}_{[5]} = dC_{[4]} + \frac{1}{16}i\epsilon_{\alpha\beta}\sqrt{2}A_{[2]}^{\alpha} \wedge \mathcal{H}_{[3]}^{\beta} + \frac{1}{6}\bar{\psi} \wedge \Gamma_{\underline{abc}}\psi \wedge V^{\underline{a}} \wedge V^{\underline{b}} \wedge V^{\underline{c}} \\ + \frac{1}{8}\epsilon_{\alpha\beta}\sqrt{2}A_{[2]}^{\alpha} \wedge \left( \Lambda_{+}^{\beta}\bar{\psi}\Gamma_{\underline{a}}\psi^{*} + \Lambda_{-}^{\beta}\bar{\psi}^{*}\Gamma_{\underline{a}}\psi \right) \wedge V^{\underline{a}} \quad (2.5)$$

$$\mathcal{D}\lambda = d\lambda - \frac{1}{4}\omega^{\underline{ab}}\Gamma_{\underline{ab}}\lambda - i\frac{3}{2}Q\lambda \quad (2.6)$$

$$\mathcal{D}\Lambda_{\pm}^{\alpha} = d\Lambda_{\pm}^{\alpha} \mp iQ\Lambda_{\pm}^{\alpha}. \quad (2.7)$$

$$\mathcal{H}_{[7]}^{\alpha} = dC_{[6]}^{\alpha} + \Lambda_{+}^{\alpha}\bar{\psi}\Gamma_{a_1a_2a_3a_4a_5}\psi^{*}V^{a_1a_2a_3a_4a_5} - \Lambda_{-}^{\alpha}\bar{\psi}^{*}\Gamma_{a_1a_2a_3a_4a_5}\psi V^{a_1a_2a_3a_4a_5} \\ + \frac{10}{3}\bar{\psi}\Gamma_{a_1a_2a_3}\psi V^{a_1a_2a_3}\sqrt{2}A_{[2]}^{\alpha} \\ + 40\sqrt{2}dA_{[2]}^{\alpha}C_{[4]} \quad (2.8)$$

the novelty being the expression of the  $\mathcal{H}_{[7]}^{\alpha}$  curvature. Its definition is a direct consequence of the following Fierz identities:

$$\begin{aligned} \bar{\psi}\Gamma^a\psi^{*}\bar{\psi}\Gamma^a &= 0 \\ \Gamma^a\psi\bar{\psi}^{*}\Gamma^a\psi &= 0 \\ \bar{\psi}\Gamma_{a_1a_2a_3}\psi\bar{\psi}\Gamma^{a_3} &= \bar{\psi}^{*}\Gamma_{[a_1}\psi\bar{\psi}_{a_2]}\psi^{*} \\ \bar{\psi}\Gamma_{a_1a_2a_3a_4a_5}\psi^{*}\bar{\psi}\Gamma^{a_1} &= -4\bar{\psi}\Gamma_{[a_2a_3a_4}\psi\bar{\psi}\gamma_{a_5]}\psi^{*} \\ \bar{\psi}^{*}\Gamma_{a_1a_2a_3a_4a_5}\psi\bar{\psi}\Gamma^{a_1} &= 4\bar{\psi}\Gamma_{[a_2a_3a_4}\psi\bar{\psi}^{*}\gamma_{a_5]}\psi \end{aligned} \quad (2.8)$$

From eq.s (2.7) and (2.4) one obtains the supersymmetry transformation rules of the background fields relevant in establishing the  $\kappa$ -supersymmetry transformation rules against which the 5-brane action is invariant.

### 3 Establishing the $\kappa$ -supersymmetric action of the 5-brane

Next, following the general scheme outlined in [1] we apply the new first order formalism to the case  $d = 6$  in order to derive the  $\kappa$ -supersymmetric action of a 5-brane. As extensively discussed in [1], the  $\kappa$ -supersymmetry transformations just follow, via a suitable projection, from the bulk supersymmetries as derived from supergravity, type IIB theory, in this case. The latter has a duality symmetry with respect to an  $\text{SL}(2, \mathbb{R})$  group of transformations that acts non linearly on the two scalars of massless spectrum, the dilaton  $\phi$  and the Ramond scalar  $C_0$ . Indeed these two parametrize the coset manifold  $\text{SL}(2, \mathbb{R})/\text{O}(2)$  and actually correspond to its solvable parametrization. Because of the special relevance of this issue in the context of our problem, we recall below such a solvable parametrization. This also helps to fix the notations.

$\text{SL}(2, \mathbb{R})$  Lie algebra

$$[L_0, L_{\pm}] = \pm L_{\pm} \quad ; \quad [L_+, L_-] = 2 L_0 \quad (3.1)$$

with explicit 2-dimensional representation:

$$L_0 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad ; \quad L_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad ; \quad L_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad ; \quad (3.2)$$

Coset representative of  $\text{SL}(2, \mathbb{R})/\text{O}(2)$  in the solvable parametrization

$$\mathbb{L}(\varphi, C_{[0]}) = \exp[\varphi L_0] \exp[C_{[0]} e^{\varphi} L_-] = \begin{pmatrix} \exp[\varphi/2] & 0 \\ C_{[0]} e^{\varphi/2} & \exp[-\varphi/2] \end{pmatrix} \quad (3.3)$$

where  $\varphi(x)$  and  $C_{[0]}$  are respectively identified with the dilaton and with the Ramond-Ramond 0-form of the superstring massless spectrum. The isomorphism of  $\text{SL}(2, \mathbb{R})$  with  $\text{SU}(1, 1)$  is realized by conjugation with the Cayley matrix:

$$\mathcal{C} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \quad (3.4)$$

Introducing the  $\text{SU}(1, 1)$  coset representative

$$\text{SU}(1, 1) \ni \Lambda = \mathcal{C} \mathbb{L} \mathcal{C}^{-1} \quad (3.5)$$

from the left invariant 1-form  $\Lambda^{-1} d\Lambda$  we can extract the 1-forms corresponding to the scalar vielbein  $P$  and the  $\text{U}(1)$  connection  $Q$

The  $\text{SU}(1, 1)/\text{U}(1)$  vielbein and connection

$$\Lambda^{-1} d\Lambda = \begin{pmatrix} -i Q & P \\ P^* & i Q \end{pmatrix} \quad (3.6)$$

Explicitly

$$\begin{aligned} P &= \frac{1}{2} (d\varphi - i e^{\varphi} dC_{[0]}) && \text{scalar vielbein} \\ Q &= \frac{1}{2} \exp[\varphi] dC_{[0]} && \text{U}(1)\text{-connection} \end{aligned} \quad (3.7)$$

As stressed in the introduction the 5-brane action we want to write, not only should be cast into first order formalism, but should also display manifest covariance with respect to  $\text{SL}(2, \mathbb{R})$ .

This covariance relies on introducing a two component charge vector  $q_\alpha$  that transforms in the fundamental representation of  $SU(1,1)$  and expresses the charges carried by the  $D5$  brane with respect to the 2-forms  $A_{[2]}^\alpha$  of bulk supergravity (both the Neveu Schwarz  $B_{[2]}$  and Ramond–Ramond  $C_{[2]}$ ). According to the geometrical formulation of type IIB supergravity summarized above we set:

$$\begin{aligned} A^\Lambda &= (B_{[2]}, C_{[2]}) & ; & \quad A^\alpha = C^\alpha \wedge A^\Lambda \\ A^{\alpha=1} &= \frac{1}{\sqrt{2}} (B_{[2]} + i C_{[2]}) & ; & \quad A^{\alpha=2} = \frac{1}{\sqrt{2}} (B_{[2]} - i C_{[2]}) \end{aligned} \quad (3.8)$$

and by definition we call  $\epsilon_{\alpha\beta} q^\beta$  the orthogonal complement of  $q_\alpha$ :

$$q_\alpha q^\alpha = 1 \quad ; \quad q_\alpha q_\beta \epsilon^{\alpha\beta} = 0 \quad (3.9)$$

In terms of these objects we write down the complete action of the  $D5$ -brane as follows:

$$\begin{aligned} \mathcal{L} &= \rho \Pi_i^a V^b \eta_{ab} \eta^{i\ell_1} \wedge e^{\ell_2} \wedge \dots \wedge e^{\ell_6} \epsilon_{\ell_1 \dots \ell_6} + \rho a_1 \Pi_i^a \Pi_j^b \eta_{ab} h^{ij} e^{\ell_1} \wedge \dots \wedge e^{\ell_6} \epsilon_{\ell_1 \dots \ell_6} \\ &+ \rho a_2 [\det(h^{-1} + \mu \mathcal{F})]^\alpha e^{\ell_1} \wedge \dots \wedge e^{\ell_6} \epsilon_{\ell_1 \dots \ell_6} \\ &+ \rho a_3 \mathcal{F}^{ij} F^{[2]} \wedge e^{\ell_3} \wedge e^{\ell_6} \epsilon_{ij\ell_3 \dots \ell_6} \\ &+ \nu F \wedge F \wedge F + a_5 q^\alpha \epsilon_{\alpha\beta} A^\beta \wedge F \wedge F + a_6 C_{[4]} \wedge F + a_7 \tilde{q}_\alpha C_{[6]}^\alpha \end{aligned} \quad (3.10)$$

where  $\rho = \rho(\phi)$  is a function of the dilaton to be determined,  $C_{[4]}$  is the 4-form potential, and  $C_{[6]}^\alpha$  are the dual potentials to the  $C_{[2]}^\alpha$  forms. The coefficients

$$\alpha = \frac{1}{4} \quad a_1 = -\frac{1}{12} \quad a_2 = -\frac{1}{3} \quad a_3 = \frac{5}{4} \quad (3.11)$$

where already determined in paper [1]. The coefficients  $a_5, a_6, \nu$  are new and they must be fixed by to be  $\kappa$ -supersymmetry. The first two are numerical, while  $\nu$  is also a function of the bulk scalars to be determined through  $\kappa$ -supersymmetry.

In the action (3.10)

$$F^{[2]} \equiv dA^{[1]} + q_\alpha A^\alpha \quad (3.12)$$

is the field strength of the world-volume gauge field and depends on the charge vector  $q^\alpha$ . The physical interpretation of  $F^{[2]}$  is as follows. By definition a  $Dp$ -brane is a locus in space-time where open strings can end or, in the dual picture, boundaries for closed string world-volumes can be located. The type IIB theory contains two kind of strings, the fundamental strings and the  $D$ -strings which are rotated one into the other by the  $SL(2, \mathbb{Z}) \subset SL(2, \mathbb{R})$  group. Correspondingly a  $D5$  brane can be a boundary either for fundamental or for  $D$ -strings or for a mixture of the two. The charge vector  $q^\alpha$  just expresses this fact and characterizes the  $D5$ -brane as a boundary for strings of  $q$ -type. Furthermore the definition (3.12) of  $F^{[2]}$  encodes the following idea: the world-volume gauge 1-form  $A^{[1]}$  is just the parameter of a gauge transformation for the 2-form  $q_\alpha A^\alpha$ , which in a space-time with boundaries can be reabsorbed everywhere except on the boundary itself. Note that if we take  $q_\alpha = \frac{1}{\sqrt{2}}(1, 1)$  we obtain :

$$q_\alpha A^\alpha = B_{[2]} \quad ; \quad q^\alpha \epsilon_{\alpha\beta} A^\beta = C_{[2]} \quad (3.13)$$

### 3.1 $\kappa$ -supersymmetry

Next we want to prove that with an appropriate choice of  $\nu, a_5, a_6$  and  $a_7$  the action (3.10) is invariant against bulk supersymmetries characterized by a projected spinor parameter. For simplicity we do this in the case of the choice  $\tilde{q}_\alpha = q_\alpha = \frac{1}{\sqrt{2}}(1, 1)$ . For other choices of the charge type the modifications needed in the prove will be obvious from its details.

To accomplish our goal we begin by writing the supersymmetry transformations of the bulk differential forms  $V^{\underline{a}}, B_{[2]}, C_{[2]}, C_{[4]}$  and  $C_{[6]}$  which appear in the action. From the rheonomic parametrizations of the curvatures recalled in [1] we immediately obtain:

$$\begin{aligned}\delta V^{\underline{a}} &= i \frac{1}{2} (\bar{\epsilon} \Gamma^{\underline{a}} \psi + \bar{\epsilon}^* \Gamma^{\underline{a}} \psi^*) \\ \delta B_{[2]} &= -2i [(\Lambda_+^1 + \Lambda_+^2) \bar{\epsilon} \Gamma_{\underline{a}} \psi^* V^{\underline{a}} + (\Lambda_-^1 + \Lambda_-^2) \bar{\epsilon}^* \Gamma_{\underline{a}} \psi V^{\underline{a}}] \\ \delta C_{[2]} &= 2 [(\Lambda_+^1 - \Lambda_+^2) \bar{\epsilon} \Gamma_{\underline{a}} \psi^* V^{\underline{a}} + (\Lambda_-^1 - \Lambda_-^2) \bar{\epsilon}^* \Gamma_{\underline{a}} \psi V^{\underline{a}}] \\ \delta C_{[4]} &= \underbrace{-\frac{1}{6}(\bar{\epsilon} \Gamma_{\underline{abc}} \psi - \bar{\epsilon}^* \Gamma_{\underline{abc}} \psi^*) V^{\underline{abc}}}_{\delta C'_{[4]}} + \frac{1}{8} [B_{[2]} \delta C_{[2]} - C_{[2]} \delta B_{[2]}]\end{aligned}\quad (3.14)$$

and :

$$\begin{aligned}\delta C_{[6]} &= g_1 C_{[4]} \delta F_{[2]} + g_2 F_{[2]} \delta C'_{[4]} + \delta C'_{[6]} \\ &= \frac{1}{\sqrt{2}} (\delta C_{[6]}^1 + \delta C_{[6]}^2) \\ &= -\frac{2}{\sqrt{2}} (\Lambda_+^1 + \Lambda_+^2) \bar{\epsilon} \Gamma_{\underline{a_1 \dots a_5}} \psi^* + \frac{2}{\sqrt{2}} (\Lambda_-^1 + \Lambda_-^2) \bar{\epsilon}^* \Gamma_{\underline{a_1 \dots a_5}} \psi V^{\underline{a_1 \dots a_5}} + \\ &\quad + 20 \sqrt{2} \delta C'_{[4]} F_{[2]} - 40 \sqrt{2} C_{[4]} \delta F_{[2]}\end{aligned}\quad (3.15)$$

Note that in writing the above transformations we have neglected all terms involving the dilatino field. This is appropriate since the background value of all fermion fields is zero. The gravitino 1-form  $\psi$  is instead what we need to keep track of. Proving  $\kappa$ -supersymmetry is identical with showing that all  $\psi$  terms cancel against each other in the variation of the action. Relying on (3.14) the variation of the W.Z.T term is as follows:

$$\begin{aligned}\delta(\nu F \wedge F \wedge F + a_5 C_{[2]} \wedge F \wedge F + a_6 C_{[4]} \wedge F + a_7 C_{[6]}) &= \\ = 3\nu F F \delta F - \frac{3g_1 a_7}{16} F F \delta C_{[2]} + (g_2 - g_1) a_7 F \delta C'_{[4]} + a_7 \delta C'_{[6]}\end{aligned}\quad (3.16)$$

for  $a_5 = -\frac{g_1 a_7}{16}$ ,  $a_6 = -g_1 a_7$ ,  $g_1 = -40\sqrt{2}$  and  $g_2 = 20\sqrt{2}$ .

And with such a choice the complete variation of the Lagrangian under a supersymmetry transformation of arbitrary parameter is:

$$\begin{aligned}\delta \mathcal{L} &= \delta \mathcal{L}_\psi + \delta \mathcal{L}_{\psi^*} \\ \delta \mathcal{L}_\psi &= [-5! i (h^{-1})^{ij} (\bar{\epsilon} \gamma_i \psi) + (\mu_3 \mathcal{F}^{ij} + \mu_4 \widetilde{F F}^{ij}) (\bar{\epsilon}^* \gamma_i \psi) + \\ &\quad + (g_1 - g_2) \frac{a_7}{3} \widetilde{F}^{klmj} (\bar{\epsilon} \gamma_{klm} \psi) + a_7 \mu_6 (\bar{\epsilon}^* \widetilde{\gamma}^j \psi)] \Omega_j^{[5]} \\ \delta \mathcal{L}_{\psi^*} &= [-5! i (h^{-1})^{ij} (\bar{\epsilon}^* \gamma_i \psi^*) + (\mu_1 \mathcal{F}^{ij} + \mu_2 \widetilde{F F}^{ij}) (\bar{\epsilon} \gamma_i \psi^*) + \\ &\quad - (g_1 - g_2) \frac{a_7}{3} \widetilde{F}^{klmj} (\bar{\epsilon}^* \gamma_{klm} \psi^*) + a_7 \mu_5 (\bar{\epsilon} \widetilde{\gamma}^j \psi^*)] \Omega_j^{[5]}\end{aligned}\quad (3.17)$$

where :

$$\begin{aligned}\mu_1 &= -4! i a_3 (\Lambda_+^1 + \Lambda_+^2) ; \quad \mu_2 = 4[-3\nu i (\Lambda_+^1 + \Lambda_+^2) - \frac{3}{16} g_1 a_7 (\Lambda_+^1 - \Lambda_+^2)] \\ \mu_3 &= -4! i a_3 (\Lambda_-^1 + \Lambda_-^2) ; \quad \mu_4 = 4[-3\nu i (\Lambda_-^1 + \Lambda_-^2) - \frac{3}{16} g_1 a_7 (\Lambda_-^1 - \Lambda_-^2)]\end{aligned}\quad (3.18)$$

Recalling eq.s (3.5) and (3.3) the above eq.s (3.18) become:

$$\begin{aligned}\mu_1 &= -5! i e^{\phi/2} ; \quad \mu_2 = \frac{3}{4} g_1 a_7 e^{-\frac{\phi}{2}} \\ \mu_3 &= -5! i e^{\phi/2} ; \quad \mu_4 = -\frac{3}{4} g_1 a_7 e^{-\frac{\phi}{2}}\end{aligned}\quad (3.19)$$

where we have chosen:

$$\nu = \frac{g_1 a_7}{16} C_{[0]} = \frac{g_1 a_7}{16} \text{Re} \mathcal{N} \quad (3.20)$$

In the above equation we have introduced the complex kinetic matrix which would appear in a  $d = 4$  gauge theory with scalars sitting in  $\text{SU}(1, 1)/\text{U}(1)$  and determined by the classical Gaillard–Zumino general formula <sup>1</sup> applied to the specific coset:

$$\mathcal{N} = i \frac{\Lambda_-^1 - \Lambda_-^2}{\Lambda_-^1 + \Lambda_-^2} \Rightarrow \begin{cases} \text{Re} \mathcal{N} = C_0 \\ \text{Im} \mathcal{N} = e^{-\phi} \end{cases} \quad (3.21)$$

It is convenient to rewrite the full variation (3.17) of the Lagrangian in matrix form in the 2-dimensional space spanned by the fermion parameters  $(\epsilon, \epsilon^*)$  :

$$\delta \mathcal{L} = \delta \mathcal{L}_\psi + \delta \mathcal{L}_{\psi^*} = (\bar{\epsilon}, \bar{\epsilon}^*) A \begin{pmatrix} \psi \\ \psi^* \end{pmatrix} \quad (3.22)$$

$$A_p = \begin{pmatrix} -5! i \gamma_p - 20\sqrt{2} a_7 \widetilde{F}^{klmj} h_{jp} \gamma_{klm} & (-5! i e^{\frac{\phi}{2}} \mathcal{F}^{ij} \gamma_i - 30\sqrt{2} a_7 e^{-\frac{\phi}{2}} \widetilde{F} \widetilde{F}^{ij} \gamma_i - \frac{2}{\sqrt{2}} a_7 e^{\frac{\phi}{2}} \widetilde{\gamma}^j) h_{jp} \\ (-5! i e^{\frac{\phi}{2}} \mathcal{F}^{ij} \gamma_i + 30\sqrt{2} a_7 e^{-\frac{\phi}{2}} \widetilde{F} \widetilde{F}^{ij} \gamma_i + \frac{2}{\sqrt{2}} a_7 e^{\frac{\phi}{2}} \widetilde{\gamma}^j) h_{jp} & -5! i \gamma_p + 20\sqrt{2} a_7 \widetilde{F}^{klmj} h_{jp} \gamma_{klm} \end{pmatrix} \quad (3.23)$$

where  $A = A_p \Omega_{[5]}^p$ , and  $e^i \wedge e^j \wedge e^k \wedge e^l \wedge e^m \equiv \epsilon^{ijklmp} \Omega_p^{[5]}$  denotes the quadruplet of five-volume forms

The matrix  $A_p$  is a tensor product of a matrices in spinor space and  $2 \times 2$  matrices in the space spanned by  $(\epsilon, \epsilon^*)$ . It is convenient to spell out this tensor product structure which is achieved by the following rewriting:

$$A_p = f_1 \gamma_p \otimes \mathbb{1} + f_2 \widetilde{F}^{klmj} h_{jp} \gamma_{klm} \otimes \sigma_3 + f_3 e^{\frac{\phi}{2}} \mathcal{F}^{ij} h_{jp} \gamma_i \otimes \sigma_1 + (f_4 \widetilde{\gamma}^j h_{jp} + f_5 e^{-\frac{\phi}{2}} \widetilde{F} \widetilde{F}^{ij} \gamma_i h_{jp}) \otimes \sigma_2 \quad (3.24)$$

where :

$$f_1 = -5! i \rho ; f_2 = -20\sqrt{2} a_7 ; f_3 = -5! i \rho ; f_4 = -\frac{2}{\sqrt{2}} i a_7 e^{\frac{\phi}{2}} ; f_5 = -30 i \sqrt{2} a_7 \quad (3.25)$$

and:

$$\widetilde{\gamma}^i \equiv \epsilon^{lmnpqi} \gamma_{lmnpq} ; \quad \widetilde{F}^{klmp} \equiv \frac{1}{2} F_{ij} \epsilon^{ijklmp} ; \quad (\widetilde{F} \widetilde{F})_{ij} \equiv \frac{1}{2} \epsilon_{ijlmnp} F^{lm} F^{pq} \quad (3.26)$$

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<sup>1</sup>For a general discussion of the Gaillard–Zumino formula see for instance [12]

now using the solution of the first order equations already determined in [1], namely

$$\begin{cases} \hat{G} = \eta \\ \mathcal{F} = \frac{1}{\mu^2} h^{-1} F \eta \\ h\eta = (\mathbb{1} - \frac{1}{\mu^2} F \eta F \eta) \left[ \det \left( \eta - \frac{1}{\mu} F \right) \right]^{-1/2} \end{cases} \quad (3.27)$$

or, in more compact form:

$$h = \left( \eta - \frac{1}{\mu^2} F^2 \right) \left[ \det \left( \eta - \frac{1}{\mu} F \right) \right]^{-1/2} \quad (3.28)$$

we can set

$$\begin{aligned} \frac{1}{\mu} &= e^{-\phi/2} = \sqrt{\text{Im} \mathcal{N}} \\ \hat{F} &\equiv \sqrt{\text{Im} \mathcal{N}} F \\ \rho &= e^{\frac{\phi}{2}} \end{aligned} \quad (3.29)$$

and we obtain:

$$\begin{aligned} e^{\phi/2} \mathcal{F}^{ij} h_{jp} &= e^{\phi/2} e^{-\phi} (F h^{-1})^{ij} h_{jp} \equiv \hat{F}^i{}_p \\ \tilde{F}^{klmp} &= e^{\frac{\phi}{2}} \hat{F}^{klmp} \\ e^{-\frac{\phi}{2}} (\widetilde{F F})_{ij} &= e^{\frac{\phi}{2}} (\hat{F} \hat{F})_{ij} \end{aligned} \quad (3.30)$$

This observation further simplifies the expression of  $A_k$  which can be rewritten as:

$$A_p = e^{\frac{\phi}{2}} [z_1 \gamma_p \otimes \mathbb{1} + z_2 \tilde{\hat{F}}^{klmj} h_{jp} \gamma_{klm} \otimes \sigma_3 + z_3 (\hat{F} \hat{F})^{ij} \gamma_i h_{jp} \otimes \sigma_2 + z_4 \hat{F}_p^i \gamma_i \otimes \sigma_1 + z_5 \tilde{\gamma}^j h_{jp} \otimes \sigma_2] \quad (3.31)$$

where :

$$z_1 = f_1 \quad ; \quad z_2 = f_2 \quad ; \quad z_3 = f_5 \quad ; \quad z_4 = f_3 \quad ; \quad z_5 = f_4. \quad (3.32)$$

We introduce :

$$\begin{aligned} \Pi_{(1)}^p &= \tilde{\hat{F}}^{klmp} \gamma_{klm} \equiv \frac{1}{2} \tilde{\gamma}^{pij} F_{ij} \\ \Pi_{(2)}^p &= (\hat{F} \hat{F})^{ip} \gamma_i \\ \Pi_{(3)}^p &= F^i{}_p \gamma_i \end{aligned} \quad (3.33)$$

and we obtain :

$$A_p = e^{\frac{\phi}{2}} [z_1 \gamma_p \otimes \mathbb{1} + z_2 \Pi_{(1)}^{(1)j} h_{jp} \otimes \sigma_3 + z_3 \Pi_{(2)}^{(2)j} h_{jp} + z_4 \Pi_{(3)}^{(3)} \otimes \sigma_1 + z_5 \tilde{\gamma}^j h_{jp} \otimes \sigma_2] \quad (3.34)$$

Just as in the case of the  $D3$ -brane discussed in [1], the proof of  $\kappa$ -supersymmetry can now be reduced to the following simple computation. Assume we have a matrix operator  $\Gamma$  with the following properties:

$$\begin{aligned} [\text{a}] \quad & \Gamma^2 = \mathbb{1} \\ [\text{b}] \quad & \Gamma A_k = A_k \end{aligned} \quad (3.35)$$



It follows that

$$P = \frac{1}{2}(\mathbb{1} - \Gamma) \quad (3.36)$$

is a projector since  $P^2 = P$  and that

$$PA_k = \frac{1}{N}(\mathbb{1} - \Gamma) A_k = 0 \quad (3.37)$$

Therefore if we use supersymmetry parameters  $(\bar{\kappa}, \bar{\kappa}^*) = (\bar{\epsilon}, \bar{\epsilon}^*)$   $P$  projected with this  $P$ , then the action is invariant and this is just the proof of  $\kappa$ -supersymmetry.

The appropriate  $\Gamma$  is the following:

$$\Gamma = \frac{1}{N}[(\omega_{[6]} + \omega_{[2]}) \otimes \sigma_2 + (\omega_{[4]} + \omega_{[0]}) \otimes \sigma_3] \quad (3.38)$$

where:

$$\begin{aligned} \omega_{[6]} &= \alpha_6 \epsilon^{ijklmn} \gamma_{ijklmn} \\ \omega_{[4]} &= \alpha_4 \epsilon^{ijklmn} \hat{F}_{ij} \gamma_{klmn} \\ \omega_{[0]} &= \alpha_0 \epsilon^{ijklmn} \hat{F}_{ij} \hat{F}_{kl} \hat{F}_{mn} \\ \omega_{[2]} &= \alpha_2 \epsilon^{ijklmn} \hat{F}_{ij} \hat{F}_{kl} \gamma_{mn} \\ N &= \left[ \det \left( \mathbb{1} \pm \hat{F} \right) \right]^{1/2} \end{aligned} \quad (3.39)$$

and the coefficients are fixed to:

$$\alpha_6 = \frac{i}{6!} ; \quad \alpha_4 = \frac{1}{24!} ; \quad \alpha_0 = \frac{1}{48} ; \quad \alpha_2 = \frac{i}{16} \quad (3.40)$$

This choice suffices to guarantee property [a] in the above list. An outline of the details of the calculation leading to such a result is given below. We have:

$$\begin{aligned} \Gamma^2 &= \frac{1}{N^2} \left[ \omega_{[6]}^2 + \omega_{[2]}^2 + \omega_{[6]} \omega_{[2]} + \omega_{[2]} \omega_{[6]} + \omega_{[4]}^2 \omega_{[0]}^2 + 2\omega_{[4]} \omega_{[0]} \right] \otimes \mathbb{1}_{2 \times 2} \\ &\quad + [\omega_{[6]} \omega_{[4]} - \omega_{[4]} \omega_{[6]} + \omega_{[6]} \omega_{[0]} - \omega_{[0]} \omega_{[6]}] \otimes \sigma_2 \sigma_3 = \\ &= \frac{1}{N^2} \left[ \omega_{[6]}^2 + \omega_{[2]}^2 + 2\omega_{[6]} \omega_{[2]} + \omega_{[4]}^2 \omega_{[0]}^2 + 2\omega_{[4]} \omega_{[0]} \right] \otimes \mathbb{1}_{2 \times 2} \end{aligned} \quad (3.41)$$

and by straightforward manipulations we obtain

$$\begin{aligned} \omega_{[6]}^2 &= -(\alpha_6)^2 (6!)^2 \\ \omega_{[4]}^2 &= -(4!)^2 (\alpha_4)^2 \hat{F}_{a_1 a_2} \hat{F}_{a_3 a_4} \gamma^{a_1 a_2 a_3 a_4} - 2(4!)^2 (\alpha_4)^2 Tr(\hat{F}^2) \\ \omega_{[2]}^2 &= \frac{4(\alpha_2)^2}{9\alpha_0 \alpha_4} \omega_{[4]} \omega_{[0]} - 32(\alpha_2)^2 \left[ Tr(\hat{F}^2) Tr(\hat{F}^2) - 2 Tr(\hat{F}^4) \right] \\ \omega_{[6]} \omega_{[2]} &= -2(6!) \alpha_6 \alpha_2 \hat{F}_{a_1 a_2} \hat{F}_{a_3 a_4} \gamma^{a_1 a_2 a_3 a_4} \end{aligned} \quad (3.42)$$

Using the following identities on determinants

$$\det \left( \mathbb{1} \pm \hat{F} \right) = 1 - \frac{1}{2} Tr(\hat{F}^2) + \frac{1}{8} \left[ Tr(\hat{F}^2) \right]^2 - \frac{1}{4} Tr(\hat{F}^4) + \det(\hat{F}) \quad (3.43)$$

and

$$\det(\hat{F}) = -\frac{1}{6} \text{Tr}(\hat{F}^6) - \frac{1}{48} [\text{Tr}(\hat{F}^2)]^3 + \frac{1}{8} [\text{Tr}(\hat{F}^2)] [\text{Tr}(\hat{F}^4)] = \quad (3.44)$$

$$= \left( \frac{1}{48} \epsilon_{a_1 a_2 a_3 a_4 a_5 a_6} \hat{F}^{a_1 a_2} \hat{F}^{a_3 a_4} \hat{F}^{a_5 a_6} \right) = \omega_{[0]}^2 \quad (3.45)$$

$$\left( \hat{F} \widetilde{\hat{F}} \hat{F} \right)_{ab} = -\frac{1}{12} \left( \epsilon_{a_1 a_2 a_3 a_4 a_5 a_6} \hat{F}^{a_1 a_2} \hat{F}^{a_3 a_4} \hat{F}^{a_5 a_6} \right) \mathbb{1}_{ab} = -4 \omega_{[0]} \mathbb{1}_{ab} \quad (3.46)$$

that are just consequences of  $\hat{F} = -\hat{F}^T$  being antisymmetric property [a] is proved.

Let us now turn to supersymmetry, namely to property [b] :

$$\Gamma A_p = A_p \quad (3.47)$$

We need to calculate the products of the matrices  $\omega_{[i]}$  ( $i = 6, 4, 2$ ) defined above with the gamma matrix structures appearing in the matrix  $A_p$ . Explicitly for  $\omega_{[6]}$  and omitting the simbol “ $\wedge$ ” we find:

$$\begin{aligned} \omega_{[6]} \gamma_a &= \frac{i}{5!} \tilde{\gamma}_a \\ \omega_{[6]} \Pi_{(1)}^{b_1} &= \frac{3!}{2} i F_{b_2 b_3} \gamma^{b_1 b_2 b_3} \\ \omega_{[6]} \Pi_{(2)}^{b_2} &= \frac{i}{5!} (\widetilde{F F})^{b_1 b_2} \tilde{\gamma}_{b_1} \\ \omega_{[6]} \Pi_{(3)}^{b_2} &= \frac{i}{5!} F^{b_1 b_2} \tilde{\gamma}_{b_1} \\ \omega_{[6]} \tilde{\gamma}^{b_1} &= -5! i \gamma^{b_1} \end{aligned} \quad (3.48)$$

while for  $\omega_{[4]}$  we obtain

$$\begin{aligned} \omega_{[4]} \gamma_{b_1} &= \frac{1}{4!} \tilde{F}^{a_1 a_2 a_3 a_4} \gamma_{a_1 a_2 a_3 a_4 b_1} - \frac{1}{3!} \tilde{F}_{b_1 a_2 a_3 a_4} \gamma^{a_2 a_3 a_4} \\ \omega_{[4]} \Pi_{(1)}^{b_4} &= -\frac{3}{2} F_{a_3 a_4} F_{a_5 a_6} \gamma^{b_4 a_3 a_4 a_5 a_6} - 3 F^{b_5 b_6} F^{b_3 b_4} \gamma_{b_5 b_6 b_3} - 3 \text{Tr}(F^2) \gamma^{b_4} + 6 (F^2)^{b_5}_{b_6} \gamma^{b_6} \\ \omega_{[4]} \Pi_{(2)}^{b_2} &= \frac{1}{4!} \tilde{F}_{a_1 a_2 a_3 a_4} (\widetilde{F F})_{b_1}^{b_2} \gamma^{a_1 a_2 a_3 a_4 b_1} - \frac{1}{6} \tilde{F}_{a_1 a_2 a_3 a_4} (\widetilde{F F})^{a_1 b_2} \gamma^{a_2 a_3 a_4} \\ \omega_{[4]} \Pi_{(3)}^{b_2} &= \frac{1}{4!} \tilde{F}_{a_1 a_2 a_3 a_4} F_{b_1}^{b_2} \gamma^{a_1 a_2 a_3 a_4 b_1} + \frac{1}{12} F^{a_5 a_6} F^{a_1 b_2} \gamma_{a_1 a_5 a_6} \\ \omega_{[4]} \tilde{\gamma}^{b_6} &= \frac{5!}{2} F_{b_4 b_5} \gamma^{b_6 b_4 b_5} + 5! F^{b_5 b_6} \gamma_{b_5} \end{aligned} \quad (3.49)$$

and finally for  $\omega_{[2]}$  we have:

$$\begin{aligned} \omega_{[2]} \gamma_{b_1} &= \frac{i}{8} (\widetilde{F F})_{a_1 a_2} \gamma_{b_1}^{a_1 a_2} - \frac{i}{4} (\widetilde{F F})_{b_1 a_2} \gamma^{a_2} \\ \omega_{[2]} \Pi_{(1)}^{b_4} &= \frac{i}{8} (\widetilde{F F})_{a_1 a_2} \tilde{F}^{b_1 b_2 b_3 b_4} \gamma_{b_1 b_2 b_3}^{a_1 a_2} - \frac{3}{4} i (\widetilde{F F})_{a_1 a_2} \tilde{F}^{a_1 b_2 b_3 b_4} \gamma_{b_2 b_3}^{a_2} - \frac{3}{4} i (\widetilde{F F})_{a_1 a_2} \tilde{F}^{a_1 a_2 b_3 b_4} \gamma_{b_3} \\ \omega_{[2]} \Pi_{(2)}^{b_2} &= \frac{i}{8} (\widetilde{F F})_{a_1 a_2} F^{b_1 b_2} \gamma_{b_1}^{a_1 a_2} - \frac{i}{4} (\widetilde{F F})_{b_1 a_2} (\widetilde{F F})^{b_1 b_2} \gamma^{a_2} \\ \omega_{[2]} \Pi_{(3)}^{b_2} &= \frac{i}{8} (\widetilde{F F})_{a_1 a_2} F^{b_1 b_2} \gamma_{b_1}^{a_1 a_2} + \frac{1}{4} [F (\widetilde{F F})]^{b_2}_{a_2} \gamma^{a_2} \\ \omega_{[2]} \tilde{\gamma}^{b_6} &= -\frac{5}{2} i (\widetilde{F F})_{a_1 a_2} \tilde{\gamma}^{a_1 a_2 b_6} \end{aligned} \quad (3.50)$$

With these relations we determine the values of the parameter  $a_7 = \frac{\sqrt{2}}{2} i$  from the supersymmetry relation [b] (3.35) and the projector (3.38).

## 4 Outlook and conclusions

In this paper we have applied the new first order formalism for  $p$ -brane world volume actions introduced by us in [1] to the case of 5 branes. The motivations for such a calculation were already discussed in the introduction and are not repeated here. We recall the essential features of the new formalism that we have adopted and that allows to reproduce the Born–Infeld second order action via the elimination of a set composed by three auxiliary fields:

- $\Pi_i^a$
- $h^{ij}$  (symmetric)
- $\mathcal{F}^{ij}$  (antisymmetric)

Distinctive properties are:

1. All fermion fields are implicitly hidden inside the definition of the  $p$ -form potentials of supergravity
2.  $\kappa$ -supersymmetry is easily proven from supergravity rheonomic parametrizations
3. The action is manifestly covariant with respect to the duality group  $\text{SL}(2, \mathbb{R})$  of type IIB supergravity.
4. The action functional can be computed on any background which is an exact solution of the supergravity bulk equations.

Of specific interest in applications are precisely the last two properties. Putting together our result we can summarize the  $D5$  brane action we have found as follows:

$$\begin{aligned}
\mathcal{L} &= \frac{1}{\sqrt{\text{Im}\mathcal{N}}} \Pi_i^a V^b \eta_{ab} \eta^{i\ell_1} \wedge e^{\ell_2} \wedge \dots \wedge e^{\ell_6} \epsilon_{\ell_1 \dots \ell_6} - \frac{1}{12 \sqrt{\text{Im}\mathcal{N}}} \Pi_i^a \Pi_j^b \eta_{ab} h^{ij} e^{\ell_1} \wedge \dots \wedge e^{\ell_6} \epsilon_{\ell_1 \dots \ell_6} \\
&\quad - \frac{1}{3 \sqrt{\text{Im}\mathcal{N}}} \left[ \det \left( h^{-1} + \frac{1}{\sqrt{\text{Im}\mathcal{N}}} \mathcal{F} \right) \right]^{\frac{1}{4}} e^{\ell_1} \wedge \dots \wedge e^{\ell_6} \epsilon_{\ell_1 \dots \ell_6} \\
&\quad + \frac{5}{4 \sqrt{\text{Im}\mathcal{N}}} \mathcal{F}^{ij} F^{[2]} \wedge e^{\ell_3} \wedge \dots \wedge e^{\ell_6} \epsilon_{ij\ell_3 \dots \ell_6} \\
&\quad - \frac{5}{2} i \text{Re}\mathcal{N} F \wedge F \wedge F + \frac{5}{2} i q^\alpha \epsilon_{\alpha\beta} A^\beta \wedge F \wedge F + 40 i C_{[4]} \wedge F + \frac{\sqrt{2}}{2} i \tilde{q}_\alpha C_{[6]}^\alpha \\
F &\equiv dA^{[1]} + q_\alpha A^\alpha \\
\mathcal{N} &= -\frac{q^\alpha \epsilon_{\alpha\beta} \Lambda_-^\beta}{q_\gamma \Lambda_-^\beta}
\end{aligned} \tag{4.1}$$

In the particular case of  $q^\Lambda = \{1, 0\}$  that implies  $q^\alpha = \frac{1}{\sqrt{2}}(1, 1)$  we have:

$$\mathcal{N} = i \frac{\Lambda_-^1 - \Lambda_-^2}{\Lambda_-^1 + \Lambda_-^2} = i e^{-\phi} + C_{[0]} \tag{4.2}$$

As many times stressed, our specific interest in the above action is given by its evaluation on the background provided by the bulk solution found in [2] which describes a  $D3$ -brane with an  $\mathbb{R}^2 \times ALE$  transverse manifold and a 2-form flux trapped on a homology 2-cycle of the ALE. Combining this with the world volume action of a  $D3$ -brane obtained in [1] we will finally obtain the appropriate source term of that exact solution which was so far missing. Alternatively by expanding (4.1) for small fluctuations around the same background we can use it as a token to explore the gauge/gravity correspondence. The key point is to reconcile this with the fact there is no running dilaton in the bulk solution of [1]. This will be achieved by an appropriate choice of the charge vector  $q^\Lambda$ , thanks to present manifestly  $SL(2, R)$ -covariant formulation. Such applications are postponed to a forthcoming paper.

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## A Rheonomic parametrization of the type IIB supergravity curvatures

In order to obtain the supersymmetry transformation rules used in the text one needs the rheonomic parametrizations of the curvatures. For simplicity we write them only in the complex basis and we disregard the bilinear fermionic terms calculated by Castellani and Pesando. We have:

$$R^{\underline{a}} = 0 \tag{A.1}$$

$$\begin{aligned} \rho = & \rho_{\underline{ab}} V^{\underline{a}} \wedge V^{\underline{b}} + \frac{5}{16} i \Gamma^{\underline{a_1-a_4}} \psi V^{\underline{a_5}} \left( F_{\underline{a_1-a_5}} + \frac{1}{5!} \epsilon_{\underline{a_1-a_{10}}} F_{\underline{a_6-a_{10}}} \right) \\ & + \frac{1}{32} \left( -\Gamma^{\underline{a_1-a_4}} \psi^* V_{\underline{a_1}} + 9 \Gamma^{\underline{a_2 a_3}} \psi^* V^{\underline{a_4}} \right) \Lambda_+^\alpha \mathcal{H}_{\underline{a_2-a_4}}^\beta \epsilon_{\alpha\beta} \\ & + \text{fermion bilinears} \end{aligned} \tag{A.2}$$

$$\mathcal{H}_{[3]}^\alpha = \mathcal{H}_{\underline{abc}}^\alpha V^{\underline{a}} \wedge V^{\underline{b}} \wedge V^{\underline{c}} + \Lambda_+^\alpha \bar{\psi}^* \Gamma_{\underline{ab}} \lambda^* V^{\underline{a}} \wedge V^{\underline{b}} + \Lambda_-^\alpha \bar{\psi} \Gamma_{\underline{ab}} \lambda V^{\underline{a}} \wedge V^{\underline{b}} \tag{A.3}$$

$$\mathcal{F}_{[5]} = F_{\underline{a_1-a_5}} V^{\underline{a_1}} \wedge \dots \wedge V^{\underline{a_5}} \tag{A.4}$$

$$\mathcal{D}\lambda = \mathcal{D}_{\underline{a}} \lambda V^{\underline{a}} + i P_{\underline{a}} \Gamma^{\underline{a}} \psi^* - \frac{1}{8} i \Gamma^{\underline{a_1-a_3}} \psi \epsilon_{\alpha\beta} \Lambda_+^\alpha \mathcal{H}_{\underline{a_1-a_3}}^\beta \tag{A.5}$$

$$\mathcal{D}\Lambda_+^\alpha = \Lambda_-^\alpha P_{\underline{a}} V^{\underline{a}} + \Lambda_-^\alpha \bar{\psi}^* \lambda \tag{A.6}$$

$$\mathcal{D}\Lambda_-^\alpha = \Lambda_+^\alpha P_{\underline{a}}^* V^{\underline{a}} + \Lambda_+^\alpha \bar{\psi} \lambda^* \tag{A.7}$$

$$R^{\underline{ab}} = R_{\underline{cd}}^{\underline{ab}} V^{\underline{c}} \wedge V^{\underline{d}} + \text{fermionic terms} \tag{A.8}$$

$$\mathcal{H}_{[7]}^\alpha = \frac{1}{3! 7!} \epsilon^{\underline{abcd_1 \dots d_7}} \mathcal{H}_{\underline{abc}}^\alpha V_{\underline{d_1}} \wedge \dots \wedge V_{\underline{d_7}} + \dots \tag{A.9}$$

(A.10)

## B Notations and Conventions

General adopted notations for first order actions are the following ones:

$$\begin{aligned}
d &= \text{dimension of the world-volume } \mathcal{W}_d \\
D &= \text{dimension of the bulk space-time } \mathcal{M}_D \\
V^a &= \text{vielbein 1-form of bulk space-time} \\
\Pi_i^a &= D \times d \text{ matrix. 0-form auxiliary field} \\
h^{ij} &= d \times d \text{ symmetric matrix. 0-form auxiliary field} \\
e^\ell &= \text{vielbein 1-form of the world-volume} \\
\underline{\eta}_{ab} &= \text{diag}\{+, \underbrace{-, \dots, -}_{D-1 \text{ times}}\} = \text{flat metric on the bulk} \\
\underline{\eta}^{ij} &= \text{diag}\{+, \underbrace{-, \dots, -}_{d-1 \text{ times}}\} = \text{flat metric on the world-volume}
\end{aligned} \tag{B.1}$$

The supersymmetric formulation of type IIB supergravity we rely on is that of Castellani and Pesando [7] that uses the rheonomy approach [10]. Hence, as it is customary in all the rheonomy constructions, the adopted signature of space-time is the **mostly minus signature**:

$$\underline{\eta}_{ab} = \text{diag} \left\{ +, \underbrace{-, \dots, -}_{9 \text{ times}} \right\} \tag{B.2}$$

The index conventions are the following ones:

$$\begin{aligned}
\underline{a}, \underline{b}, \underline{c}, \dots &= 0, 1, 2, \dots, 9 \quad \text{Lorentz flat indices in } D = 10 \\
\underline{i}, \underline{j}, \underline{k}, \dots &= 0, \dots, d \quad \text{Lorentz flat indices on the world-volume}
\end{aligned} \tag{B.3}$$

$$\begin{aligned}
\alpha, \beta, \dots &= 1, 2 \quad \text{SU}(1, 1) \text{ doublet indices} \\
A, B, C, \dots &= 1, 2 \quad \text{O}(2) \text{ indices for the scalar coset} \\
\Lambda, \Sigma, \Gamma, \dots &= 1, 2 \quad \text{SL}(2, \mathbb{R}) \text{ doublet indices}
\end{aligned} \tag{B.4}$$

For the gamma matrices our conventions are as follows:

$$\left\{ \Gamma^a, \Gamma^b \right\} = 2 \eta^{ab} \tag{B.5}$$

The convention for constructing the dual of an  $\ell$ -form  $\omega$  in  $D$ -dimensions is the following:

$$\omega = \omega_{\underline{i}_1 \dots \underline{i}_\ell} V^{\underline{i}_1} \wedge \dots \wedge V^{\underline{i}_\ell} \quad \Leftrightarrow \quad \star \omega = \frac{1}{(D - \ell)!} \epsilon_{a_1 \dots a_{D-\ell} b_1 \dots b_\ell} \omega^{\underline{b}_1 \dots \underline{b}_\ell} V^{a_1} \wedge \dots \wedge V^{a_{D-\ell}} \tag{B.6}$$

Note that we also use  $\ell$ -form components with strength one:  $\omega = \omega_{\underline{i}_1 \dots \underline{i}_\ell} V^{\underline{i}_1} \wedge \dots \wedge V^{\underline{i}_\ell}$  and not with strength  $\ell!$  as it would be the case if we were to write  $\omega = \frac{1}{\ell!} \omega_{\underline{i}_1 \dots \underline{i}_\ell} V^{\underline{i}_1} \wedge \dots \wedge V^{\underline{i}_\ell}$ . When it is

more appropriate to use curved rather than flat indices then the convention for Hodge duality is summarized by the formula:

$$\star(dx^{\mu_1} \wedge \dots dx^{\mu_n}) = \frac{\sqrt{-\det(g)}}{(10-n)!} G^{\mu_1\nu_1} \dots G^{\mu_n\nu_n} \epsilon_{\rho_1 \dots \rho_{10-n} \nu_1 \dots \nu_n} dx^{\rho_1} \wedge \dots dx^{\rho_{10-n}} \quad (\text{B.7})$$

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