Particle creation in an oscillating spherical cavity

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Abstract

We study the creation of massless scalar particles from the quantum vacuum due to the dynamical Casimir effect by spherical shell with oscillating radius. In the case of a small amplitude of the oscillation, to solve the infinite set of coupled differential equations for the instantaneous basis expansion coefficients we use the method based on the time-dependent perturbation theory of the quantum mechanics. To the first order of the amplitude we derive the expressions for the number of the created particles for both parametric resonance and non-resonance cases.

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1 Introduction

The Casimir effect is one of the most interesting manifestations of nontrivial properties of the vacuum state in a quantum field theory (for a review see [1, 2, 3, 4]) and can be viewed as a polarization of vacuum by boundary conditions. A new phenomenon, a quantum creation of particles (the dynamical Casimir effect) occurs when the geometry of the system varies in time. In two dimensional spacetime and for conformally invariant fields the problem with dynamical boundaries can be mapped to the corresponding static problem and hence allows a complete study (see [2, 4] and references therein). In higher dimensions the problem is much more complicated and only partial results are available. The vacuum stress induced by uniform acceleration of a perfectly reflecting plane is considered in [5]. The corresponding problem for a sphere expanding in the four-dimensional spacetime with constant acceleration is investigated by Frolov and Serebriany [6, 7] in the perfectly reflecting case and by Frolov and Singh [8] for semi-transparent boundaries. For more general cases of motion the problem of particle and energy creation is considered on the base of various perturbation methods [9]-[23](in the case of plane boundaries for more complete list of references see [20]). It has been shown that a gradual accumulation of small changes in the quantum state of the field could result in a significant observable effect. A new application of the dynamical Casimir effect has recently appeared in connection with the suggestion by Schwinger [24] that the photon production associated with changes in the quantum electrodynamic vacuum state arising from a collapsing dielectric bubble could be relevant for sonoluminescence. For the further developments and discussions of this quantum-vacuum approach see [25]-[31] and references therein.

In our previous paper [32] we studied the problem of particle creation from the quantum vacuum by a spherical shell with time-dependent radius. We have considered examples for which the mean number of particles can be explicitly evaluated in the adiabatic approximation when the squeezing effect is dominant. In the present paper the case is considered when the sphere radius performs oscillations with a small amplitude and the expressions are derived for the number of created particles to the first order of the perturbation theory.

2 Quantum scalar field inside a sphere with time-dependent radius

Consider a scalar field φ satisfying Dirichlet boundary condition on the surface of a sphere with time-dependent radius $a = a(t)$:

$$\left(\frac{\partial^2}{\partial t^2} - \Delta\right)\varphi(x,t) = 0, \qquad \varphi|_{r=a(t)} = 0. \tag{1}$$

Following Ref.[32] we expand the corresponding eigenfunctions for the interior region in a series with respect to the instantaneous basis:

$$\varphi_{lmn}(x) = \sqrt{\frac{2}{a^3(t)}}\sum_{k=1}^{\infty} q^l_{nk}(t)\frac{j_l(j_{l,k}r/a(t))}{j'_l(j_{l,k})}Y_{lm}(\theta,\varphi), \tag{2}$$

$$l = 0,1,2,..., \quad -l \le m \le l, \quad n = 1,2,..., \tag{3}$$

where (r,θ,φ) are standard spherical coordinates, $j_{l,k}$ is the n-th zero for the spherical Bessel function $j_l(z)$, $j_l(j_{l,n}) = 0$, $Y_{lm}(\theta,\varphi)$ the spherical harmonic. These functions automatically satisfy boundary condition (1). Substituting (2) into field equation (1) we arrive at an infinite set of coupled differential equations [32]

$$\ddot{q}^l_{nk} + \omega^2_{lk}(t)q^l_{nk} = 2h\sum_{p=1}^{\infty}\dot{q}^l_{np}a^l_{pk} + \dot{h}\sum_{p=1}^{\infty}q^l_{np}a^l_{pk} + h^2\sum_{p,j=1}^{\infty}q^l_{np}a^l_{pj}a^l_{kj}, \tag{4}$$

where overdot stands for the time derivative,

$$h = \frac{\dot{a}}{a}, \quad \omega_{lk}(t) = \frac{j_{l,k}}{a(t)}, \tag{5}$$

and the time dependent antisymmetric coefficients a_{nk}^{l} are determined as

$$a_{nk}^{l} = \begin{cases} 0, & k = n \\ 2j_{l,n}j_{l,k}/(j_{l,n}^{2} - j_{l,k}^{2}) & k \neq n \end{cases} . \tag{6}$$

If the sphere is asymptotically static at past and future then the in- and out- vacuum states can be defined by using the solutions for coefficients corresponding to the in- and out-modes $\varphi_{lmn}^{(\mathrm{in})}(t)$, $\varphi_{lmn}^{(\mathrm{out})}(t)$ with asymptotics

$$q_{nk}^{(\mathrm{in})l}(t) \quad \rightarrow \quad \frac{e^{-\imath\omega_{ln}^{\mathrm{in}}t}}{\sqrt{2\omega_{ln}^{\mathrm{in}}}}\delta_{nk}, \quad t \rightarrow -\infty \tag{7}$$

$$q_{nk}^{(\mathrm{out})l}(t) \quad \rightarrow \quad \frac{e^{-\imath\omega_{ln}^{\mathrm{out}}t}}{\sqrt{2\omega_{ln}^{\mathrm{out}}}}\delta_{nk}, \quad t \rightarrow +\infty, \tag{8}$$

where we use the notations

$$\omega_{ln}^{\mathrm{in}} = \frac{j_{l,n}}{a_{-}}, \qquad \omega_{ln}^{\mathrm{out}} = \frac{j_{l,n}}{a_{+}}, \qquad a_{\pm} = \lim_{t\rightarrow\pm\infty} a(t) \tag{9}$$

for the corresponding eigenfrequencies. The expansion coefficients for the in- and out-modes are related by the transformation [32]

$$q_{nk}^{(\mathrm{in})l} = \sum_{j} \left(\alpha_{nj}^{l} q_{jk}^{(\mathrm{out})l} + \beta_{nj}^{l} q_{jk}^{(\mathrm{out})l*} \right), \tag{10}$$

where α_{nj}^{l} and β_{nj}^{l} are Bogoliubov coefficients for the Bogoliubov transformation between the in- and out-modes $\varphi_{lmn}^{(\mathrm{in})}(t)$ and $\varphi_{lmn}^{(out)}(t)$. In particular, taking into account (7), in the limit $t \rightarrow +\infty$ from (10) we receive

$$q_{nk}^{(\mathrm{in})l} = \frac{1}{\sqrt{2\omega_{lk}^{\mathrm{out}}}} \left(\alpha_{nk}^{l} e^{-\imath\omega_{lk}^{\mathrm{out}}t} + \beta_{nk}^{l} e^{\imath\omega_{lk}^{\mathrm{out}}t} \right). \tag{11}$$

The mean number of out-particles produced in a given mode with quantum numbers l, m, n in the in-vacuum state is determined by the Bogoliubov coefficient β_{nk}^{l}:

$$< \mathrm{in}|N_{lmn}|\mathrm{in} >= \sum_{k=1}^{\infty} |\beta_{nk}^{l}|^{2}, \tag{12}$$

and the total number of created scalar particles is obtained by taking the sum over all the modes

$$< \mathrm{in}|N|\mathrm{in} >= \sum_{l=0}^{\infty}(2l+1) \sum_{n,k=1}^{\infty} |\beta_{nk}^{l}|^{2}. \tag{13}$$

From the form of equations (4) it follows that there are two types of effects which lead to the particle creation (see also [19]). The first one, called squeezing of the vacuum, is due to the nonstationary eigenfrequencies $\omega_{ln}(t)$ as a result of a dynamical change of the radius of the sphere and is described by the second term on the left of (4). The second one, referred as acceleration effect, is due to the motion of the boundary and comes from the terms on the right of (4). In [32] we have considered the particle creation inside a sphere in the adiabatic approximation when the squeezing effect is dominant. In the next section we will consider another approximation.

3 Particle creation by an oscillating sphere

Below we will consider the case when the sphere radius performs small oscillations at a frequency Ω during a period T:

$$a(t) = \left\{ \begin{array}{cc} a_0[1+\varepsilon \sin(\Omega t)], & 0 \le t \le T \\ a_0, & t < 0,\ t > T \end{array} \right. , \tag{14}$$

where a_0 is the mean radius, ε is a small parameter. For this type of motion $\omega_{ln}^{\rm in} = \omega_{ln}^{\rm out} = \omega_{ln}^{(0)} \equiv j_{l,n}/a_0$. We will assume that for $t < 0$ the field is in the in-vacuum state. This means that we need to solve Eq.(4) with the initial condition

$$q_{nk}^l(t) = \frac{e^{-i\omega_{ln}^{(0)}t}}{\sqrt{2\omega_{ln}^{(0)}}}\delta_{nk}, \quad t \le 0. \tag{15}$$

To derive the particle number created we will follow the scheme developed in [17] for the case of a perfect plane cavity with vibrating walls. First of all we note that introducing new functions

$$X_{nk\pm}^l(t) = \sqrt{\frac{\omega_{lk}^{(0)}}{2}} \left(q_{nk}^l \mp i\frac{\dot{q}_{nk}^l}{\omega_{lk}^{(0)}} \right), \tag{16}$$

to the first order of ε the set of equations (4) can be replaced by a coupled first-order differential equations

$$\begin{aligned} \dot{X}_{nk\pm}^l &= \pm i\omega_{lk}^{(0)} X_{nk\pm}^l \mp i\omega_{lk}^{(0)} \varepsilon \sin(\Omega t) \left(X_{nk+}^l + X_{nk-}^l \right) \\ &\pm \frac{\varepsilon\Omega}{\sqrt{\omega_{lk}^{(0)}}} \cos(\Omega t) \sum_j a_{jk}^l \sqrt{\omega_{lj}^{(0)}} \left(X_{nj+}^l - X_{nj-}^l \right) \\ &\pm \frac{i\varepsilon\Omega^2}{\sqrt{\omega_{lk}^{(0)}}} \sin(\Omega t) \sum_j \frac{a_{jk}^l}{\sqrt{\omega_{lj}^{(0)}}} \left(X_{nj+}^l + X_{nj-}^l \right). \end{aligned} \tag{17}$$

Introducing the column vector

$$\vec{X}_n^l(t) = \left(\begin{array}{c} X_{n1-}^l \\ X_{n1+}^l \\ X_{n2-}^l \\ \vdots \end{array} \right) \tag{18}$$

this system can be written in the matrix form

$$\frac{d}{dt}\vec{X}_n^l(t) = V^{(0)}\vec{X}_n^l(t) + \varepsilon V^{(1)}\vec{X}_n^l(t). \tag{19}$$

Here the components for the matrices $V^{(0)}$ and $V^{(1)}$ are

$$V_{k\sigma,j\sigma'}^{(0)} = i\omega_{lk}^{(0)}\sigma\delta_{kj}\delta_{\sigma\sigma'}, \quad V_{k\sigma,j\sigma'}^{(1)} = \sum_{s=\pm} v_{k\sigma,j\sigma'}^s e^{is\Omega t}, \tag{20}$$

where

$$v_{k\sigma,j\sigma'}^s = -\frac{1}{2}s\sigma\omega_{lk}^{(0)}\delta_{kj} + \sigma\Omega a_{jk}^l \sqrt{\frac{\omega_{lj}^{(0)}}{\omega_{lk}^{(0)}}} \left(\frac{\sigma'}{2} + \frac{s\Omega}{4\omega_{lj}^{(0)}} \right), \tag{21}$$

and $s, \sigma, \sigma' = +, -$. To solve Eq. (19) we use a perturbation expansion

$$\vec{X}_n^l = \vec{X}_n^{(0)l} + \varepsilon \vec{X}_n^{(1)l} + \cdots, \tag{22}$$

where at the zeroth order, by taking into account initial condition (15), one has

$$X_{nk\sigma}^{(0)l} = \delta_{nk}\delta_{\sigma -} e^{-i\omega_{lk}^{(0)} t}. \tag{23}$$

The first order term can be easily found on the base of this expression:

$$X_{nk\sigma}^{(1)l} = e^{i\sigma\omega_{lk}^{(0)} t} \int_0^t dt' e^{-i\sigma\omega_{lk}^{(0)} t'} \sum_{j,\sigma'} V_{k\sigma,j\sigma'}^{(1)} X_{nk\sigma'}^{(0)l}. \tag{24}$$

Using (23) and (20) one finds

$$X_{nk\pm}^{(1)l}(t) = \mp v_{k-,n-}^{-} E_{\pm\omega_{lk}^{(0)}+\Omega+\omega_{ln}^{(0)}}^{\mp\omega_{lk}^{(0)}} \mp v_{k-,n-}^{+} E_{\pm\omega_{lk}^{(0)}-\Omega+\omega_{ln}^{(0)}}^{\mp\omega_{lk}^{(0)}} \tag{25}$$

where we use the notation

$$E_m^{\omega_{lk}^{(0)}}(t) = \begin{cases} t e^{-i\omega_{lk}^{(0)} t}, & \text{for} \quad m = 0 \\ (i/m)\left[e^{-i(m+\omega_{lk}^{(0)})t} - e^{-i\omega_{lk}^{(0)} t}\right], & \text{for} \quad m \neq 0 \end{cases}. \tag{26}$$

For the instantaneous basis expansion coefficients this yields

$$\begin{aligned} q_{nk}^l(t) &= \frac{e^{-i\omega_{lk}^{(0)} t}}{\sqrt{2\omega_{lk}^{(0)}}}\delta_{nk} + \frac{\varepsilon}{\sqrt{2\omega_{lk}^{(0)}}}\left\{v_{k-,n-}^{-} E_{-\omega_{lk}^{(0)}+\Omega+\omega_{\mathrm{ln}}^{(0)}}^{\omega_{lk}^{(0)}} + v_{k-,n-}^{+} E_{-\omega_{lk}^{(0)}-\Omega+\omega_{ln}^{(0)}}^{\omega_{lk}^{(0)}}\right. \\ &- \left. v_{k-,n-}^{-} E_{\omega_{lk}^{(0)}+\Omega+\omega_{ln}^{(0)}}^{-\omega_{lk}^{(0)}} - v_{k-,n-}^{+} E_{\omega_{lk}^{(0)}-\Omega+\omega_{ln}^{(0)}}^{-\omega_{lk}^{(0)}}\right\} + \mathcal{O}(\varepsilon^2). \end{aligned} \tag{27}$$

This expression includes terms proportional to t which are due to the parametric resonance. In the situation $\omega_{lk}^{(0)} t \gg 1$ the resonance terms are dominant and solution (27) becomes

$$\begin{aligned} q_{nk}^l(t) &\approx \frac{e^{-i\omega_{lk}^{(0)} t}}{\sqrt{2\omega_{lk}^{(0)}}}\delta_{nk} + \frac{\varepsilon t}{\sqrt{2\omega_{lk}^{(0)}}}\left[-v_{k-,n-}^{+} e^{i\omega_{lk}^{(0)} t}\delta_{\omega_{lk}^{(0)},\Omega-\omega_{ln}^{(0)}}\right. \\ &+ \left. v_{k-,n-}^{-} e^{-i\omega_{lk}^{(0)} t}\delta_{\omega_{lk}^{(0)},\Omega+\omega_{ln}^{(0)}} + v_{k-,n-}^{+} e^{-i\omega_{lk}^{(0)} t}\delta_{\omega_{ln}^{(0)},\Omega+\omega_{lk}^{(0)}}\right]. \end{aligned} \tag{28}$$

After the time interval T the sphere is static with radius a_0 and the function $q_{nk}^l(t)$ has the form (11). From the continuity condition between the functions (11) and (28) at $t = T$ one has

$$\alpha_{nk}^l = \delta_{nk} + \varepsilon T\left[v_{k-,n-}^{-}\delta_{\omega_{lk}^{(0)},\Omega+\omega_{ln}^{(0)}} + v_{k-,n-}^{+}\delta_{\omega_{ln}^{(0)},\Omega+\omega_{lk}^{(0)}}\right] \tag{29}$$

$$\beta_{nk}^l = -\varepsilon T v_{k-,n-}^{+}\delta_{\omega_{lk}^{(0)},\Omega-\omega_{ln}^{(0)}}. \tag{30}$$

By taking into account expressions (21) and (6) one has

$$\beta_{nk}^l = -\frac{1}{2}\varepsilon T\sqrt{\omega_{ln}^{(0)}\omega_{lk}^{(0)}}\delta_{\omega_{lk}^{(0)},\Omega-\omega_{ln}^{(0)}}. \tag{31}$$

Now from Eq.(12) for the number of out-particles with the energy $\omega_{ln}^{(0)}$ and quantum numbers l, m created by the parametric resonance in the in-vacuum state we receive

$$< \mathrm{in}|N_{lmn}|\mathrm{in} >= \frac{1}{4}(\varepsilon T)^2 \sum_{k=1}^{\infty} \omega_{ln}^{(0)}\omega_{lk}^{(0)}\delta_{\omega_{lk}^{(0)},\Omega-\omega_{ln}^{(0)}}. \tag{32}$$

As we see the necessary condition for the parametric resonance is

$$\Omega = \frac{1}{a_0}\left(j_{l,q} + j_{l,p}\right) \tag{33}$$

for some values q and p.

Now we turn to calculating the Bogoliubov coefficient β^l_{nk} in the non-resonance case. By using the expressions (26) from the continuity condition between the functions (11) and (28) and their derivatives at $t = T$ after some algebra we get

$$\beta^l_{nk} = -2\varepsilon \exp\left[-iT\left(\omega^{(0)}_{ln} + \omega^{(0)}_{lk}\right)/2\right] \sum_{\sigma=+,-} v^{\sigma}_{k-,n-} e^{i\sigma\Omega T/2} \frac{\sin\left[T\left(\omega^{(0)}_{ln} + \omega^{(0)}_{lk} - \sigma\Omega\right)/2\right]}{\omega^{(0)}_{ln} + \omega^{(0)}_{lk} - \sigma\Omega} \tag{34}$$

For the number of quanta we derive from here

$$\begin{aligned} < \mathrm{in}|N_{lmn}|\mathrm{in} > \;&=\; 4\varepsilon^2 \sum_{k=1}^{\infty} \left\{ \sum_{\sigma=+,-} \left(v^{\sigma}_{k-,n-}\right)^2 \frac{\sin^2\left[T\left(\omega^{(0)}_{ln} + \omega^{(0)}_{lk} - \sigma\Omega\right)/2\right]}{\left(\omega^{(0)}_{ln} + \omega^{(0)}_{lk} - \sigma\Omega\right)^2} \right. \qquad (35) \\ &+\; \left. \frac{v^{-}_{k-,n-} v^{+}_{k-,n-} \cos(\Omega T)}{\left(\omega^{(0)}_{ln} + \omega^{(0)}_{lk}\right)^2 - \Omega^2} \left[\cos(\Omega T) - \cos\left[T\left(\omega^{(0)}_{ln} + \omega^{(0)}_{lk}\right)\right]\right] \right\}. \end{aligned}$$

Note that in the resonance case from the term with $\sigma = +$ of Eq.(34) we recover the result (30).

Up to now we have considered the scalar field with the Dirichlet boundary condition. In a similar way it is easy to generalize the corresponding results for other type of boundary conditions. For example, in the case of the Neumann scalar the number of particles created inside an spherical shell is given by expressions (32) and (35) where now $a\omega^{(0)}_{ln}$, $n = 1, 2, ...$ are zeros for the function $j'_l(z)$, $j'_l(a\omega^{(0)}_{ln}) = 0$.

4 Concluding remarks

Our interest in this paper has been in the quantum radiation by an spherical mirror with time-dependent radius given by (14) and under assumption that the amplitude of oscillation is small. To define and interpret asymptotic spaces of physical states uniquely we assumed that the oscillation of the sphere lasts a finite period of time. We consider a massless scalar field obeying the Dirichlet boundary condition on the sphere surface. To solve the corresponding equations for the spontaneous basis expansion coefficients in the short-time limit, $\varepsilon\Omega T \ll 1$, we use the method developed in [17] for the case of plane boundaries. To the first order in perturbation theory we derive the expression for the number of particles created inside the sphere in both resonance and non-resonance cases. In the resonance case the oscillation frequency is given by expression (33). The corresponding solution includes resonance terms proportional to t and oscillating parts. In the situation, when $\Omega T \gg 1$ the resonance terms are dominant, and the total number of particles created in the mode with quantum numbers lmn and with frequency $\omega^{(0)}_{ln}$ is given by (32). In this case the number of particles grows quadratically in time. In the non-resonance case the corresponding formula for the number of particles has the form (35). The generalization of these formulas for the other boundary conditions is straightforward. Another possible generalization is to perform the higher order calculations with respect to ε. We hope that the results obtained in present paper may have an application in studies of quantum radiation created by bubble formation during first order phase transitions in the Early Universe.

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