

Generation of Neutrino Mass in a Kalb-Ramond Background in Large Extra Dimensions

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Abstract

In this paper we investigate whether spacetime torsion induced by a Kalb-Ramond field in a string inspired background can generate a mass for the left-handed neutrino. We consider an Einstein-Dirac-Kalb-Ramond lagrangian in higher dimensional spacetime with torsion generated by the Kalb-Ramond anti-symmetric field in the presence of a bulk fermion. We show that such a coupling can generate a mass term for the four dimensional neutrino after a suitable large radius compactification of the extra dimensions.

1 Introduction

Ever since Einstein-Cartan theory was proposed, space-time torsion has been considered as an integral part of any gravitational theory where the background geometry is not only characterized by curvature but also by an asymmetric part of the affine connection called space-time torsion[1]. Just as mass-energy is known to be the source of curvature in spacetime, torsion originates from spin in space-time[2]. In spite of its generality over Einstein's theory, a theory with torsion didn't draw that much attention because of the lack of experimental support possibly due to a weak value of the torsion. Moreover, it was shown that the torsion field, even if it exists, cannot couple to the electromagnetic field in a gauge invariant way. Thus there is no possibility of finding any signal of torsion in electromagnetic experiments.

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There was a revival of interest in the subject after the advent of string theory[3] where the field strength corresponding to the massless second rank antisymmetric tensor field (known as Kalb-Ramond field) in the heterotic string spectra was identified as the background spacetime torsion. The remarkable feature in such a theory was that now one could couple electromagnetism with torsion preserving the $U(1)$ gauge invariance[6]. The Chern-Simons term needed to cancel gauge anomaly compensates the gauge symmetry violating term originating from the torsion coupling. Such a theory offers possible explanations for various phenomena like the optical activity purportedly observed in the electromagnetic radiation coming from distant galactic sources,[7] presently observed accelerating phase of the Universe and others[9]. As torsion plays a crucial role in such explanations, one is tempted to propose these observations as indirect support for string theory as well as for the existence of spacetime torsion.

It has further been shown in a recent work that while torsion and curvature effects both have the same coupling with other matter fields at the Planck scale in higher dimensions, the torsion coupling becomes weak as the extra dimensions are compactified[10] via the Randall-Sundrum mechanism[11]. All these motivated us to explore whether torsion in extra dimensions could provide an explanation for the small neutrino mass predicted from neutrino oscillation experiments. We have shown earlier that the torsion coupling may indeed result in the helicity flip[12] in massive neutrinos which may suggest a possible explanation for the solar neutrino anomaly. The origin of such a mass term has been explored in several works from various points of view[18, 20]. In particular, it has been argued that bulk fermions living in higher dimensions could couple to fermionic fields living in 3+1 dimensions, for instance via the Higgs field, and generate mass[13, 14]. Here we take a new path with the input that torsion resides in the bulk and interacts with the standard model fermion on the wall of the brane as all the standard model particles are assumed to reside on the brane. Such an approach has special significance in the context of string theory where the second rank antisymmetric Kalb-Ramond field may act as the source of torsion in the background. In fact the KR field strength and torsion can be equated as we shall explain shortly. We thus explore the possibility of generating neutrino mass from the geometric property of the background spacetime. In such a scenario we compactify the extra dimensions following the scheme proposed by Arkani Hamed, Dimopolous and Dvali(ADD)[15]. Such a compactification scheme involves large extra dimensions which pull down the Planck scale near the electroweak scale thereby solving the so called hierarchy problem. In such a scenario all the standard model fields are confined on the brane where gravity resides in the bulk. The torsion field being an integral part of the geometry of spacetime like gravity is also assumed to reside in the bulk. The compactified field strength of the Kalb-Ramond field induced torsion in 4D is known to be related with the string

axion by the well known duality relation. This axion is assumed to have frozen into its vacuum expectation value during a much earlier epoch and is responsible for inducing a mass term for the left handed neutrino on the brane as well as contributions to the massive towers of Kaluza-Klein modes of the bulk fermion.

We briefly recall the salient features of the ADD type models. In such models [15], the compact and Lorentz degrees of freedom can be factorized. The string scale M_s (which can be as low as tens of TeV) controls the strength of gravity in $(4+n)$ dimensions, and is related to the 4-dimensional Planck scale M_P by

$$\frac{R^n}{M_P^2} = (4\pi)^{n/2} \Gamma(n/2) M_s^{-(n+2)} \quad (1)$$

where R is the compactification radius. The current limits on the departure from Newton's law of gravity at small distances are consistent with R within a mm , for $n \geq 2$. On compactifying the extra dimensions we get a tower of Kaluza-Klein (KK) modes on the brane where we reside. Thus a massless field in the bulk in general gives rise to a massive spectrum and the density of states is given by

$$\rho(m_{\vec{n}}) = \frac{R^n m_{\vec{n}}^{(n-2)}}{(4\pi)^{n/2} \Gamma(n/2)} \quad (2)$$

where $m_{\vec{n}} = (\frac{4\pi^2 \vec{n}^2}{R^2})^{1/2}$ is the mass of a KK state with $\vec{n} = (n_1, n_2, \dots, n_n)$ [16]. Consequently, in any process (involving the graviton, for example) where a cumulative contribution from the tower is possible, a summation over the tower of fields, convoluted by the density, causes an enhancement, in spite of the suppression of individual couplings by M_P . One thus expects appreciable contributions to various processes at energies close to M_s .

In the scenario adopted by us, the source of torsion is taken to be the rank-2 antisymmetric Kalb-Ramond (KR) field B_{MN} which arises as a massless mode in heterotic string theories[3]. To understand the above statement, let us recall that the low energy effective action for the gravity and Electromagnetic sectors in D dimensions is given by

$$S = \int d^D x \sqrt{-G} \left[R(G) - \frac{1}{4} F_{MN} F^{MN} + \frac{3}{2} H_{MNL} H^{MNL} \right] \quad (3)$$

It has been shown earlier [6] that an action of the form

$$S = \int d^D x \sqrt{-G} \left[R(G, T) - \frac{1}{4} F_{MN} F^{MN} - \frac{1}{2} H_{MNL} H^{MNL} + T_{MNL} H^{MNL} \right] \quad (4)$$

reproduces the low energy string effective action if one eliminates the torsion field T_{MNL} (which is an auxiliary field) by using the equation of motion $T_{MNL} = H_{MNL}$.

Thus torsion can be identified with the rank-3 antisymmetric field strength tensor H_{MNL} which in turn is related to the KR field B_{MN} [8] as

$$H_{MNL} = \partial_{[M} B_{NL]} \quad (5)$$

with each Latin index running from 0 to 4 in a five-dimensional theory. (Greek indices, on the other hand, run from 0 to 3.) Furthermore, we use the KR gauge fixing conditions to set $B_{4\mu} = 0$. Therefore, the only non-vanishing KR field components correspond to the brane indices. These components, of course, are functions of both compact and non-compact co-ordinates.

For a spin-1/2 fermion in a spacetime with torsion, the extended Dirac Lagrangian density is given by [4, 5]:

$$\mathcal{L}_{fermion} = \bar{\psi} \left[i\gamma^\mu \left(\partial_\mu - \sigma^{\rho\beta} v_\rho^\nu g_{\lambda\nu} \partial_\mu v_\beta^\lambda - g_{\alpha\delta} \sigma^{ab} v_a^\beta v_b^\delta \tilde{\Gamma}^\alpha_{\mu\beta} \right) \right] \psi \quad (6)$$

where v_a^μ denote the tetrad connecting the curved space with the corresponding tangent space and

$$\tilde{\Gamma}^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + H^\mu_{\nu\rho}, \quad (7)$$

Γ denoting the connection without torsion.

The paper is organized as follows. In section 2, we consider a five-dimensional theory with torsion and compactify on a circle. Using the well known four dimensional duality relating the torsion field to the axion, we show that the axion vev induces a mass term for the neutrino. We generalize this mass term in higher dimensions and show that for six large extra dimensions, the value for the mass is of the order of a few electron-volts. We conclude in section 3.

2 Neutrino mass from torsion

As in [14], let us start with a five-dimensional theory and consider a compactification of the fifth dimension on a circle of radius R . A five-dimensional massless fermion Ψ can be decomposed into (ψ_1, ψ_2) where ψ_1, ψ_2 are two component spinors. We consider the left-handed neutrino ν_L to be moving in four dimensions only. Consider the following effective action representing the kinetic terms for Ψ and ψ ,

$$\int d^4x dy \left[i\bar{\Psi} \gamma^\mu D_\mu \Psi + i\bar{\Psi} \gamma^5 D_5 \Psi \right] + \int d^4x dy \left[i\bar{\psi} \gamma^\mu D_\mu \psi \right] \delta(y), \quad (8)$$

where the four-dimensional $\psi = (\nu_L, N|_{y=0})$, $N = 1/\sqrt{2}(\psi_1 + \psi_2)$ containing the right-handed neutrino for reasons to be made clear later. We use the chiral representation

where the gamma matrices are given by,

$$\gamma^0 = \begin{pmatrix} 0 & -\mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad (9)$$

and $\gamma^5 = \text{diag}(\mathbf{1}, -\mathbf{1})$. Using these,

$$\int d^4x dy \{ \psi_1^\dagger i\sigma^\mu D_\mu \psi_1 + \psi_2^\dagger i\bar{\sigma}^\mu D_\mu \psi_2 - \psi_2^\dagger iD_5 \psi_1 + i\psi_1^\dagger D_5 \psi_2 \} + \int d^4x dy \delta(y) \{ \nu_L^\dagger i\sigma^\mu D_\mu \nu_L + iN^\dagger \bar{\sigma}^\mu D_\mu N \}. \quad (10)$$

Now let us introduce space-time torsion generated by the antisymmetric 3-form as explained above. We will get from the connection piece terms like

$$\int d^4x dy \frac{1}{M_s^{3/2}} \{ i\bar{\Psi} \gamma^{\mu\sigma\lambda} H_{\mu\sigma\lambda} \Psi + i\bar{\psi} \gamma^{\mu\sigma\lambda} H_{\mu\sigma\lambda} \psi \delta(y) \} \quad (11)$$

where we have made the gauge-choice $B_{4\mu} = 0$.

Using the four-dimensional duality relation for the massless antisymmetric tensor mode,

$$H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\lambda} \partial^\lambda \chi \quad (12)$$

where χ is the axion-field and is used in explaining the strong CP problem via the PQ mechanism, we get using the standard gamma-matrix relation $\epsilon_{\mu\nu\rho\lambda} \gamma^{\mu\nu} = -2i\gamma^5 \gamma^{\rho\lambda}$ ³

$$\frac{1}{M_s^{3/2}} \{ \bar{\Psi} \gamma^5 \gamma^\rho \partial_\rho \chi \Psi + \bar{\psi} \gamma^5 \gamma^\rho \partial_\rho \chi \psi \}. \quad (13)$$

Further we expand $\Psi = (\psi_1, \psi_2)$ in Kaluza-Klein modes,

$$\psi_1(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_n \psi_1(x) e^{iny/R} \quad (14)$$

$$\psi_2(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_n \psi_2(x) e^{-iny/R}. \quad (15)$$

Also let us define the following linear combinations $N(x, y) = 1/\sqrt{2}(\psi_1 + \psi_2)$, $M(x, y) = 1/\sqrt{2}(\psi_1 - \psi_2)$ whose Kaluza-Klein modes are defined as

$$N^{(n)} = \frac{\psi_1^{(n)} + \psi_2^{(n)}}{\sqrt{2}} \quad (16)$$

$$M^{(n)} = \frac{\psi_1^{(n)} - \psi_2^{(n)}}{\sqrt{2}}. \quad (17)$$

³Our conventions are such that $\gamma^{\mu_1 \mu_2 \dots} = \gamma^{\mu_1} \gamma^{\mu_2} \dots$ when $\mu_1 \neq \mu_2 \dots$ and vanishes otherwise.

Using these in equation (13) and integrating by parts, results in mass terms

$$\sum_{n=1}^{\infty} n m N^{\dagger(n)} N^{(n)} + n m M^{\dagger(n)} M^{(n)} + n m \nu_L N^{(n)} \quad (18)$$

where,

$$m = \frac{\langle \chi \rangle R^{1/2}}{(2\pi M_s R)^{3/2}}. \quad (19)$$

we have the following four-dimensional lagrangian,

$$\mathcal{L} = \mathcal{L}_{KE} + \sum_{n=1}^{\infty} \left(mn + \frac{n}{R} \right) N^{(n)\dagger} N^{(n)} + \left(mn - \frac{n}{R} \right) M^{\dagger(n)} M^{(n)} + \left\{ \sum_{n=1}^{\infty} m_N^{(n)} \nu_L^{\dagger} N^{(n)} + \text{h.c.} \right\} \quad (20)$$

where

$$m_N^{(n)} = nm \quad (21)$$

Thus, there is no mass term for the zero-mode for \mathbf{N} . We note here that in 4 dimensions, $\langle \chi \rangle \approx f_{PQ}$ where f_{PQ} is the Peccei-Quinn scale. When the axion lives in $4+n$ dimensions, $f_{PQ} \sim \langle \chi \rangle R^{n/2}$ so that for our five-dimensional model, the numerator in our mass-formula is nothing but f_{PQ} . If an axion is a boundary field confined in 4 dimensions, the PQ scale f_{PQ} is bounded by M_s . To obtain a higher PQ scale which is demanded on astrophysical grounds, the axion field has to be a bulk field[19]. If one considers a more general model with n extra dimensions with the axion field moving in a p dimensional submanifold, the above formula generalizes to

$$m = \frac{f_{PQ}}{(2\pi M_s R)^{(n+2)/2}}. \quad (22)$$

where f_{PQ} depends on p . Thus in the basis,

$$(\nu_L, N^{(1)}, N^{(2)} \dots), \quad (23)$$

we get the following mass-matrix,

$$\mathcal{M} = \begin{pmatrix} 0 & m & 2m & 3m & \dots \\ m & m + 1/R & 0 & 0 & \dots \\ 2m & 0 & 2m + 2/R & 0 & \dots \\ \vdots & & & & \end{pmatrix}, \quad (24)$$

The eigen-values of the above matrix can be computed numerically. However one can make the following approximation. We consider the case $m \ll 1/R$, and note that the total number of modes is $O(M_s R)$. This results in the following $M_s R \times M_s R$ mass-

matrix,

$$\mathcal{M} = \begin{pmatrix} 0 & m & 2m & 3m & \cdots \\ m & 1/R & 0 & 0 & \cdots \\ 2m & 0 & 2/R & 0 & \cdots \\ \vdots & & & & \end{pmatrix}, \quad (25)$$

The eigen-values of the above matrix contain the masses of the left-handed neutrino and other Kaluza-Klein modes. Since such a matrix leads to a see-saw mechanism between ν_L and various components of \mathbf{N} , it is natural to assume that \mathbf{N} contains the right-handed neutrino. The characteristic equation whose roots yield the eigen-values for the mass-matrix in equation (25) is

$$\prod_{k=1}^{M_s R} \left(\lambda - \frac{k}{R} \right) \left(\lambda - m^2 \sum_{k=1}^{M_s R} \frac{k^2}{\lambda - \frac{k}{R}} \right) = 0. \quad (26)$$

The above equation can be re-written as

$$\lambda - m^2 \left[-\frac{1}{2} M_s R^2 (1 + 2\lambda R + M_s R) + \lambda^2 R^3 (\text{PolyGamma}(0, 1 - \lambda R) - \text{PolyGamma}(0.1 - \lambda R + M_s R)) \right] = 0 \quad (27)$$

where $\text{PolyGamma}(0, x) = \Gamma'(x)/\Gamma(x)$. Equation (28) is a transcendental equation which can be solved graphically. Instead of adopting a brute-force approach, we note that the eigen-values of the above matrix can be computed by following the procedure. Noting that usually $m \ll 1/R$, we decompose the mass-matrix into

$$\mathcal{M} = \begin{pmatrix} 0 & \cdots \\ 0 & 1/R & \cdots \\ 0 & 0 & 2/R & \cdots \\ \vdots & & & \end{pmatrix} + \begin{pmatrix} 0 & m & 2m & \cdots \\ m & 0 & \cdots \\ 2m & 0 & 0 & \cdots \\ \vdots & & & \end{pmatrix} = \mathcal{H}_0 + \mathcal{H}'. \quad (28)$$

Now we treat \mathcal{H}' as a perturbation and use standard perturbative techniques which yield corrections to the eigen-values of \mathcal{H}_0 . To second order we have,

$$E = E_0 + \langle 0 | \mathcal{H}' | 0 \rangle + \sum_i \frac{|\langle i | \mathcal{H}' | 0 \rangle|^2}{E_0 - E_i}. \quad (29)$$

This yields for the neutrino mass,

$$m_{\nu_L} = m^2 R (R M_s)^2, \quad (30)$$

which comes as a second order correction, the first order correction vanishing. Let us note here that a similar enhancement to m from the Kaluza-Klein modes was also

obtained in [14] but there the enhancement had a logarithmic dependence on RM_s instead of the quadratic dependence that we have here. As such the Kaluza-Klein enhancement in our model is substantially more than the one in [14].

The values of f_{PQ} have been computed for a variety of cases in [19] with n extra-dimensions and p being the number of dimensions in which the axion field moves. Using these results, one gets the following values for neutrino masses. For $M_s = 1 \text{ TeV}$:

| (p, n) | $R^{-1}(\text{GeV})$ | $f_{PQ}(\text{GeV})$ | $m_{\nu_L}(\text{eV})$ |
|----------|----------------------|----------------------|------------------------|
| (1,2) | 10^{-13} | 10^{10} | 10^9 |
| (2,4) | 10^{-5} | 10^{10} | 10^2 |
| (3,6) | 10^{-3} | 10^{13} | 10 |

Curiously, the case of most interest is the one with 6 extra dimensions which is what one would expect when the starting point is a superstring theory. If we are to believe that torsion does explain the existence of massive neutrinos, a theory with just 2 extra dimensions would lead to a widely different value for the neutrino mass ⁴. However we note here that for this case, $m \approx 10^{-14} \text{ GeV}$ and $1/R \approx 10^{-13} \text{ GeV}$ as a result of which the approximation that we have used *may* break down. For the other two cases, the condition $m \ll 1/R$ is indeed satisfied. We also note here that the $(n, p) = (2, 1)$ case seems to be ruled out on astrophysical grounds [19]. In fact, it seems that the number of dimensions p in which the axion lives in has to be such that $p \geq 2$. Thus the absurdly large value for the case with two extra dimensions should not be a cause for concern.

3 Conclusion

In this paper we have shown that large extra dimensions, originally proposed to resolve the gauge hierarchy problem, provides a possible mechanism to generate mass for neutrino when compactified following the ADD scheme. Spacetime torsion turns out to play the crucial role for this purpose. The bulk fermion communicates with the brane neutrino through the axion (dual to the Kalb Ramond induced torsion) coupling and the resulting axion vev determines the neutrino mass in 4D. We have determined the dependence of this mass on the number of extra dimensions where the axion resides. The magnitude of the mass turns out to match well with the present bound on neutrino mass for a certain choice of the number of extra large dimensions. This choice seems

⁴The determination of the absolute value of individual neutrino masses is an ongoing experimental problem (See for example, [17].)

to be consistent if the underlying fundamental theory was a superstring theory. Thus apart from gravity, another geometric feature of spacetime namely the torsion, in the bulk may generate mass for neutrino at a scale where the axion freezes into its vev following a large scale compactification.

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