On the Super Higgs Effect
in Extended Supergravity

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Abstract

We consider the reduction of supersymmetry in N-extended four dimensional supergravity via the super Higgs mechanism in theories without cosmological constant. We provide an analysis largely based on the properties of long and short multiplets of Poincaré supersymmetry. Examples of the super Higgs phenomenon are realized in spontaneously broken $N = 8$ supergravity through the Scherk-Schwarz mechanism and in superstring compactification in presence of brane fluxes. In many models the massive vectors count the difference in number of the translation isometries of the scalar σ-model geometries in the broken and unbroken phase.

1 Introduction

If supersymmetry is of any relevance in Nature it must be realized in a broken phase. Since supersymmetry extended to curved spacetime becomes a gauge theory, called supergravity, supersymmetry breaking must be spontaneous and therefore the super Higgs mechanism must take place. It is then of physical interest to study the spontaneous breaking from N to N' supersymmetries. In particular, the breaking $N \to 1 \to 0$ is relevant for the hierarchy problem if supersymmetry has to solve it.

In the present paper we analyze some general features of supergravity theories in dimension $d = 4$ with scalar potentials [1, 2] allowing flat Minkowski background[1]. Given an unbroken theory with N supersymmetries, we analyze when the degrees of freedom are consistent with the existence of a broken phase, which retains N' supersymmetries. Our analysis, based on properties of massless and massive representations [4, 5] of Poincaré supersymmetry, is mostly kinematic. More constraints are expected to come from the dynamical realization of the spontaneously broken theory.

For $N \to N' \geq 3$ [6] the analysis is particularly predictive, since the massless theory is completely fixed by supersymmetry. This means that, assuming that there is a phase transition between the unbroken and the broken theories, the result of integrating out the massive modes must give as a result the only theory allowed by N' supersymmetry.

An important difference in the super Higgs mechanism occurs depending whether $N - N'$ is even or odd. This is because spin 3/2 BPS (short) massive multiplets can only occur in pairs since they carry a (BPS) charge, thus needing two of them to form a CPT invariant multiplet. So when $N - N'$ is odd at least one multiplet must be long (since in this case it is CPT invariant by itself). We will see that this condition already excludes some possibilities.

When $N' \leq 4$ there are massless matter multiplets ($\lambda_{MAX} \leq 1$) of the reduced N' supersymmetry. They can then undergo a Higgs mechanism and become massive. So, in abscence of further dynamical informations, one can predict only the maximal number of residual massless multiplets.

A general pattern emerges by studying the supergravity models which admit a spontaneously broken phase with Minkowski background (vanishing cosmological constant). The isometry group G has an abelian subalgebra that acts as translations on G/H. The broken gauge symmetries belong to this subalgebra [7, 8, 9]. This is at least true both, in the spontaneous breaking through Scherk-Schwarz [10, 11] mechanism and in the breaking

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[1]We do not consider here super Higgs effect in curved (AdS) backgrounds, which has been studied in many examples in the literature [2, 3]

$N = 4 \to N = 3$ [12, 13] through compactification in the presence of brane fluxes [14, 15, 16, 17, 18]. A detailed study of the pattern of symmetry breaking and its relation to scalar geometry will be considered elsewhere. The construction of four dimensional gauged supergravity can be found in the literature [3],[1],[19],[20]. A partial classification was given in [21].

The generalized dimensional reduction of Scherk-Schwarz has been shown to have a pure four dimensional interpretation as a gauged N=8 supergravity [24] with a "flat group" (in the notation of Scherk and Schwarz) as gauge group. Our discussion of the super Higgs effect in supergravity is limited to those superstring models or higher dimensional theories in which the spontaneous breakdown does not involve neither stringy nor Kaluza-Klein modes. In this situation, one may hope that the discussion of spontaneous supersymmetry breaking can be confined to an effective field theory which involves only a small number of degrees of freedom, both for the massless and the massive sectors. This has been shown to occur in K-K supergravities where, as an example, the de Wit – Nicolai gauged $SO(8)$ theory is a consistent truncation of M-theory on $AdS_4 \times S^7$ [22]. This is because the masses of the effective models that we consider here can be taken much smaller than the Kaluza-Klein masses or the string scale [12]. [2].

The paper is organized as follows. In Section 2. we recall the structure of massless and massive (long and short) Poincaré supermultiplets in four dimensions. In particular, we treat in detail the massive multiplets with maximum spin 3/2 which are relevant for the super Higgs effect.

In Section 3. we discuss spontaneously broken $N = 8$ supergravity. We see that some patterns of supersymmetry reduction $N \to N'$ are not allowed in the super Higgs mechanism. Then we consider all cases with $N = 8, 6 \to 2 \leq N' < N$.

In Section 4. we discuss the models which are dinamically realized through the Scherk-Schwarz mechanism and infer the relation between the Higgs breaking and the broken symmetries of the scalar geometry.

In Section 5. we discuss the relation of the mass generation of the vector bosons with the broken symmetries of the sigma model. We also consider spontaneous sypersymmetry breaking in matter coupled theories which arise in string compactifications in presence of brane fluxes.

In Section 6. we end with some concluding remarks.

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[2]Examples where this is not possible have been considered in the literature, when keeping only massless states [23, 25] or when keeping also the massive ones [26].

2 Massless and Massive Representations of extended supersymmetry in $d = 4$

We make here a short review of massless and massive representations of the N extended super Poincaré algebra in a space time of dimension 4 and signature 2 [4, 5]. The odd part of the super Lie algebra is arranged as a direct sum of N Weyl (chiral) complex spinor representations. A $\mathbb{C}$-linear basis of the odd generators is

$$\{Q_\alpha^i\}_{i=1,\dots N;\ \alpha=1,2}. \tag{1}$$

We identify $\mathbb{C}^N$ with its dual by means of the sesquilinear form in $\mathbb{C}^N$, $B(u,v) = u^\dagger v$. Then, the complex conjugates of (1) are denoted as

$$(Q_\alpha^i)^* = \bar{Q}_{\dot\alpha i}. \tag{2}$$

(1) and (2) span the odd part of the superalgebra over the real numbers. The (anti) commutation relations are

$$\begin{aligned}
\{Q_\alpha^i, \bar{Q}_{\dot\alpha j}\} &= 2\sigma_{\alpha\dot\alpha}^\mu p_\mu \delta_j^i \\
\{Q_\alpha^i, Q_\beta^j\} &= \epsilon_{\alpha\beta} Z^{ij} \\
\{\bar{Q}_{\dot\alpha i}, \bar{Q}_{\dot\beta j}\} &= \epsilon_{\dot\alpha\dot\beta} \bar{Z}_{ij}
\end{aligned} \tag{3}$$

where p_μ is the translation generator and Z^{ij} are bosonic central generators organized in an antisymmetric matrix. This is to be completed with the transformation of Q_α^i under the generators of the Lorentz group $M_{\mu\nu}$ as spinor representations and the commutation relations of the Poincaré generators among themselves. The automorphism group of the algebra is $\mathrm{U}(N)$, Q and $\bar{Q}$ transforming in the fundamental ($\mathbf{N}$) and antifundamental ($\bar{\mathbf{N}}$) representations of $\mathrm{U}(N)$ respectively and Z^{ij} in the two fold antisymmetric representation. There is also a discrete automorphism, the CPT symmetry under which the generators of the algebra transform as

$$Q_\alpha^i \to i(Q_\alpha^i)^* = i\bar{Q}_{\dot\alpha i}, \qquad Z^{ij} \to -(Z^{ij})^* = -\bar{Z}_{ij}.$$

The unitary representations of this superalgebra are obtained using the method of induced representations. One considers the orbit of the Lorentz group on the dual space to the translations. The orbits are given by the value of the invariant $p^2 = m^2$. For $m > 0$ the little group is $\mathrm{SU}(2)$ and for $m = 0$ it is $E(2)$, from which we take representations which are non trivial only for the compact subgroup $U(1)$. The representation of the full Poincaré superalgebra is achieved by building at each point of the orbit a fiber which is

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a direct sum of representations of the little group, the odd generators mixing the different representation spaces.

We will take as conventions $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$ and

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Massless representations To see what is the fiber at one point of the orbit we take the point $p^\mu = (E, 0, 0, -E)$ (rest frame). The central charges must be set to zero in order to obtain a unitary representation. The algebra (3) becomes

$$\begin{aligned} \{Q^i, \bar{Q}_j\} &= 2 \begin{pmatrix} 0 & 0 \\ 0 & 2E \end{pmatrix} \delta^i_j \\ \{Q^i_\alpha, Q^j_\beta\} &= 0 \end{aligned} \tag{4}$$

We see that the generators Q^i_1, $\bar{Q}_{1j}$ form an abelian superalgebra that decouples from the rest. The nontrivial anticommutation relations are of N creation and N annihilation operators. So the representations are constructed by giving a vacuum state $|\Omega\rangle_\lambda$

$$Q^i_2 |\Omega\rangle_\lambda = 0$$

with helicity (representation of the little group) λ and acting on it with the creation operators $\bar{Q}_{2i}$.

$$|i_1, \ldots i_k\rangle_\lambda = \frac{1}{k!(2\sqrt{E})^k} \bar{Q}_{2i_k} \cdots \bar{Q}_{2i_1} |\Omega\rangle_\lambda.$$

Each creation operator lowers the helicity of the state by $1/2$. The state at level k has helicity $\lambda - k/2$ and is in the k-fold antisymmetric representation of U(N). The representation has 2^N states. The helicities range from λ to $\lambda - N/2$. Since by CPT $\lambda \to -\lambda$, a CPT conjugate representation is obtained with a vacuum $|\Omega\rangle_{N/2-\lambda}$. The direct sum is CPT invariant and has dimension 2^{N+1}. Notice that CPT also changes the k-fold antisymmetric representation of U(N), $[k]$ by its complex conjugate $[\bar{k}] \approx [N-k]$. There is one case where the CPT doubling is not required. This happens when $\lambda = N/2 - \lambda$ (so N is necessarily even) and the representation $[N/2]$ (corresponding to the spin 0 state) is real. The last condition requires N to be a multiple of 4.

We can also consider a vacuum which is a non trivial representation R of the automorphism group U(N). The vacuum will then be labelled by $|\Omega\rangle_{\lambda,R}$,

and the helicity states will be in the tensor product representation $[k] \otimes R$. The CPT conjugate representation will then be $|\Omega\rangle_{N/2-\lambda, \bar{R}}$.

¿From the above it follows that in massless multiplets the helicity range is $|\Delta\lambda| = N/2$ so that if the maximum helicity λ_{MAX} of the multiplet is $|\lambda_{MAX}| \leq 2$ then necessarily $N \leq 8$, if $|\lambda_{MAX}| \leq 1$ then $N \leq 4$ and if $|\lambda_{MAX}| \leq 1/2$ then $N \leq 2$.

We report in Tables 1 and 2 the massless representations for $N \leq 8$ [3] with $\lambda_{MAX} \leq 2$. A couple of states of helicity $\pm\lambda$ are denoted as (λ), the number in front is their multiplicity. Before the doubling, the multiplicity is the dimension of the representation $[k]$ of U(N), $\binom{N}{k}$. After the doubling the multiplicity is the dimension of the representation $[k] \oplus [4\lambda - k]$.

Massive representations   In the rest frame we have that the momentum is $p_\mu = (M, 0, 0, 0)$. Q and $\epsilon\bar{Q}$ transform the same representation of the little group SU(2), so we can define Q_α^a, $\quad a = 1, \ldots 2N$

$$Q_\alpha^a = Q_\alpha^i, \ a = i = 1, \ldots N; \quad Q_\alpha^a = \epsilon^{\dot{\alpha}\dot{\beta}} Q_{\beta i}^*, \ a = N + i = N + 1, \ldots 2N.$$

This definition can be understood as a reality condition on general complex vectors Q_α^a. This condition is preserved by a transformation of the group USp(2N). The operators Q_α^a at the selected point of the orbit must satisfy the relations

$$\{Q_\alpha^a, Q_\beta^b\} = \epsilon_{\alpha\beta} \Lambda^{ab}, \qquad \Lambda = \begin{pmatrix} Z & M\mathbb{1} \\ -M\mathbb{1} & Z^* \end{pmatrix}.$$

Λ is an antisymmetric quaternionic matrix, that is, $\Lambda^* = -\Omega\Lambda\Omega$ with

$$\Omega = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}.$$

The quaternionic property is preserved by an USp(2N) transformation, which is an automorphism of the algebra in the rest frame that commutes with SU(2). In particular, this means that with a transformation $U \in$ USp(2N) we can bring Λ to a skew diagonal form $\Lambda' = U\Lambda U^T$ [27]

$$\Lambda' = \begin{pmatrix} 0_{n \times n} & \rho_{n \times n} \\ -\rho_{n \times n} & 0_{n \times n} \end{pmatrix}$$

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[3]We don't consider $N = 7$ because $N = 7$ supergravity is the same as $N = 8$ supergravity.

$$\rho_{n\times n} = \begin{pmatrix} M\mathbb{1}_{2\times2} + z_1\sigma_3 & 0\cdots & 0 \\ 0 & M\mathbb{1}_{2\times2} + z_2\sigma_3 & \cdots 0 \\ \cdots & & \\ 0 & 0 & \cdots & M\mathbb{1}_{2\times2} + z_n\sigma_3 \end{pmatrix}, \quad (5)$$

with $n = N/2$ in the even case and $n = (N-1)/2$ in the odd case. U can be interpreted as a change of basis of the Q's

$$\{Q_\alpha, Q_\beta\} = \epsilon\Lambda'. \tag{6}$$

From this it can be seen that unitary representations are obtained only if $M \geq |z_i|$ (BPS bound).

No central charges The cases $z_i = 0$ or $z_i \neq 0$ but $M > |z_i|$ are qualitatively the same. From (5) and (6) we see that we can make a rescaling of the Q's and we have an algebra with $2N$ creation and $2N$ annihilation operators. It shows explicit invariance under $SU(2) \times USp(2N)$. The vacuum state is now labeled by the spin representation of $SU(2)$, $|\Omega\rangle_J$. If $J = 0$ we have the fundamental massive multiplet with 2^{2N} states. These are organized in representations of $SU(2)$ with $J_{MAX} = N/2$. With respect to $USp(2N)$ the states with fixed $0 < J < N/2$ are arranged in the $(N - 2J)$-fold Ω-traceless antisymmetric representation, $[N - 2J]$.

The general multiplet with a spin J vacuum can be obtained by tensoring the fundamental multiplet with spin J representation of $SU(2)$. The total number of states is then $(2J + 1) \cdot 2^{2N}$.

Massive multiplets with $z_i = 0$ are called long multiplets or non BPS states. The only difference with multiplets $z_i \neq 0$ and $M > |z_i|$ is that the last ones must be doubled in order to have PCT invariance, since $z_i \to -z_i$ under PCT. We will no longer consider this case.

BPS multiplets If q of the eigenvalues z_i saturate the BPS bound, $M = |z_i|$, $i = 1, \ldots q$ then q, of the pairs creation-annihilation operators have abelian anticommutation relations and are totally decoupled, similarly to the phenomenon occuring for $m = 0$. The resulting multiplets are said to be q/N BPS. Note that $q_{MAX} = N/2$ for N even and $q_{MAX} = (N-1)/2$ for N odd. The $USp(2N)$ symmetry is now reduced to $USp(2(N - q))$.

The reduced or short multiplet has the same number of states than a long multiplet of the $N - q$ supersymmetry algebra. The fundamental multiplet, with $J = 0$ vacuum contains $2 \cdot 2^{2(N-q)}$ states with $J_{MAX} = (N - q)/2$. Note the doubling due to CPT invariance. Generic massive short multiplets can be obtained by making the tensor product with a spin J_0 representation of $SU(2)$.

In the discussion of the super Higgs and Higgs effect, massive multiplets with spin 3/2 and 1 are relevant. In Tables 3, 4 and 5 we give a list of all possible cases for $N \leq 8$. The occurence of long spin 3/2 multiplets is only possible for $N = 3, 2$ and long spin 1 multiplets for $N = 2$. In $N = 1$ there is only one type of massive multiplet (long) since there are no central charges. Its structure is

$$\left[(J_0 + \frac{1}{2}), 2(J_0), (J_0 - \frac{1}{2})\right],$$

except for $J_0 = 0$ that we have $\left[(\frac{1}{2}), 2(0)\right]$.

In the tables we will denote the spin states by (J) and the number in front of them is their multiplicity. In the fundamental multiplet, with spin $J_0 = 0$ vacuum, the multiplicity of the spin $(N-k)/2$ is the dimension of the k-fold antisymmetric Ω-traceless representation of $\mathrm{USp}(N)$. For multiplets with $J_0 \neq 0$ one has to make the tensor product of the fundamental multiplet with the representation of spin J_0. We also indicate if the multiplet is long or short.

3 Super Higgs effect in supergravity: generalities

When performing the spontaneous breaking of extended supersymmetry from N to N' in supergravity, a necessary requirement is that $N - N'$ of the original massless gravitinos must describe massive representations of the unbroken N' supersymmetries. Since massive spin 3/2 may occur both in long and short multiplets only for $N \leq 3$ (Table 3), strong constraints emerge if $N - N'$ is odd, because in this case at least one spin 3/2 multiplets must be long. This immediately excludes the cases $N = 8, 6 \to N' = 5$ and $N = 5 \to N' = 4$.

Other cases that are not possible are the ones described in the following.

$N = 6 \to N' = 3$. We first write down the decomposition of the massless, helicity 2, $N = 6$ multiplet into massless multiplets of $N = 3$.

$$\left[(2), 6(\frac{3}{2}), 16(1), 26(\frac{1}{2}), 30(0)\right] \to \left[(2), 3(\frac{3}{2}), 3(1), (\frac{1}{2})\right] +$$
$$3\left[(\frac{3}{2}), 3(1), 3(\frac{1}{2}), 2(0)\right] + 4\left[(1), 4(\frac{1}{2}), 6(0)\right].$$

The multiplets with $\lambda_{MAX} = 3/2, 1$ must be recombined into massive multiplets of $N' = 3$. From Table 3 it is easy to see that there is no combination of the long and short multiplets with spin 3/2 that can match the number of

gravitinos and vectors simultaneously. In fact, one short and one long spin 3/2 multiplets will need 14 vectors.

$N = 5 \to N' = 3$. The decomposition into massless representations is

$$\left[(2), 5(\tfrac{3}{2}), 10(1), 11(\tfrac{1}{2}), 10(0)\right] \to \left[(2), 3(\tfrac{3}{2}), 3(1), (\tfrac{1}{2})\right] +$$
$$2\left[(\tfrac{3}{2}), 3(1), 3(\tfrac{1}{2}), 2(0)\right] + \left[(1), 4(\tfrac{1}{2}), 6(0)\right].$$

We could try to combine the last two ones into two short $N = 3$ spin 3/2 multiplets, but the number of vectors is already bigger than the seven vectors at our disposal.

$N = 5 \to N' = 2$. The decomposition into massless representations is

$$\left[(2), 5(\tfrac{3}{2}), 10(1), 11(\tfrac{1}{2}), 10(0)\right] \to \left[(2), 2(\tfrac{3}{2}), (1)\right] +$$
$$3\left[(\tfrac{3}{2}), 2(1), (\tfrac{1}{2})\right] + 3\left[(1), 2(\tfrac{1}{2}), 2(0)\right] + \left[2(\tfrac{1}{2}), 4(0)\right].$$

The minimal combination of spin 3/2 multiplets that could be used is one long and two short multiplets. But the number of states with $J_z = 0$ is already 12. They should match the number of helicity states in the last three terms of the equation above, which is 10, so it is impossible.

3.1 Spontaneously broken supergravity $N = 8 \to N' = 6, 4$

We want to explore the $N = 8 \to N' = 6, 4$ spontaneous breaking of supersymmetry. In these cases there are no long spin 3/2 multiplets. In the models with $N' = 4$ there appear multiplitets with $\lambda_{MAX} = 1$, so there is a possibility of a further Higgs effect on these multiplets.

$N = 8 \to N' = 6$. The decomposition into massless multiplets is as follows,

$$\left[(2), 8(\tfrac{3}{2}), 28(1), 56(\tfrac{1}{2}), 70(0)\right] \to \left[(2), 6(\tfrac{3}{2}), 16(1), 26(\tfrac{1}{2}), 30(0)\right] +$$
$$2\left[(\tfrac{3}{2}), 6(1), 15(\tfrac{1}{2}), 20(0)\right].$$

The two massless $\lambda_{MAX} = 3/2$ multiplets can be reread as one massive, 1/2 BPS, spin 3/2 multiplet of $N' = 6$ (Table 3), so in principle the Higgs effect is possible.

$N = 8 \to N' = 4$. The decomposition into massless multiplets is as follows,

$$\left[(2), 8(\tfrac{3}{2}), 28(1), 56(\tfrac{1}{2}), 70(0)\right] \to \left[(2), 4(\tfrac{3}{2}), 6(1), 4(\tfrac{1}{2}), 2(0)\right] +$$
$$4\left[(\tfrac{3}{2}), 4(1), 7(\tfrac{1}{2}), 8(0)\right] + 6\left[(1), 4(\tfrac{1}{2}), 6(0)\right].$$

There are two types of $N = 4$ spin $3/2$ massive multiplets, both of them short with $q = 1, 2$. The number of massless vectors which are not in the gravity multiplet is (from above) 22. We have two possibilities
1. Two $q = 2$ spin $3/2$ multiplets ((16 vectors) plus six massless vector multiplets.
2. One $q = 1$ and one $q = 2$ spin $3/2$ multiplets (20 vectors) plus two massless vector multiplets.

The rest of the states also matches. The massless vector multiplets may undergo a Higgs effect. Two massless vector multiplets have the same number of states than one massive one. In case 1. we can have a Higgs effect from 6 to 4, 2 or 0 massless vectors. In case 2. we have a Higgs effect from 2 to 0.

3.2 Spontaneously broken supergravity $N = 6 \to N' = 4, 2$

$N = 6 \to N' = 4$. The decomposition into massless representations is

$$\left[(2), 6(\tfrac{3}{2}), 16(1), 26(\tfrac{1}{2}), 30(0)\right] \to \left[(2), 4(\tfrac{3}{2}), 6(1), 4(\tfrac{1}{2}), 2(0)\right] +$$
$$2\left[(\tfrac{3}{2}), 4(1), 7(\tfrac{1}{2}), 8(0)\right] + 2\left[(1), 4(\tfrac{1}{2}, 6(0)\right].$$

The only possibility is to take an $N = 4$, $q = 2$, spin $3/2$ multiplet and two massless vector multiplets.

$N = 6 \to N' = 2$. The decomposition into massless representations is

$$\left[(2), 6(\tfrac{3}{2}), 16(1), 26(\tfrac{1}{2}), 30(0)\right] \to \left[(2), 2(\tfrac{3}{2}), (1)\right] + 4\left[(\tfrac{3}{2}), 2(1), (\tfrac{1}{2})\right] +$$
$$7\left[(1), 2(\tfrac{1}{2}, 2(0))\right] + 4\left[2(\tfrac{1}{2}), 4(0)\right].$$

For $N' = 2$ we have long and short spin $3/2$ multiplets. The only possibilities allowed are:
1. Two long multiplets and one short. Then the rest of the states arrange into three massless vector multipltes and one hypermultiplet ($\lambda_{MAX} = 1/2$).

2. Two short multiplets. The rest of the states combine into seven massless vector multiplets and two hypermultiplets.

Again, the massless multiplets can be combined into massive ones, this time with the possibility of including a mass term for the hypermultiplet.

3.3 Spontaneously broken supergravity $N = 8 \to N' = 3, 2$

As it can be seen in Table 3, for $N' = 2, 3$ there are both, long and short spin 3/2 multiplets.

$N = 8 \to N' = 3$. The decomposition into massless representations is

$$\left[(2), 8(\frac{3}{2}), 28(1), 56(\frac{1}{2}), 70(0)\right] \to \left[(2), 3(\frac{3}{2}), 3(1), (\frac{1}{2})\right] +$$
$$5\left[(\frac{3}{2}), 3(1), 3(\frac{1}{2}), 2(0)\right] + 10\left[(1), 4(\frac{1}{2}), 6(0)\right].$$

The first possibility that arises with no more than 25 vectors is to take one long and two short gravitino multiplets, with 22 vectors, so three additional vector multiplets must be present. The number of states with spin 1/2 and $J_z = 0$ also match the states with helicity $\pm 1/2$ and 0 in the massless multiplets. One can further have a Higgs effect that takes two massless vector multiplets into a massive one.

$N = 8 \to N' = 2$. The decomposition into massless representations is

$$\left[(2), 8(\frac{3}{2}), 28(1), 56(\frac{1}{2}), 70(0)\right] \to \left[(2), 2(\frac{3}{2}), (1)\right] + 6\left[(\frac{3}{2}), 2(1), (\frac{1}{2})\right]$$
$$+15\left[(1), 2(\frac{1}{2}), 2(0)\right] + 10\left[2(\frac{1}{2}), 4(0)\right].$$

For the first time we note the occurrence of a multiplet with $\lambda_{MAX} = 1/2$ (hypermultiplet). We have 27 massless vectors. We have four possible combinations of spin 3/2 multiplets: six long , four long and one short, two short and two long, and three short. Let $2n$ be the number of long gravitino multiplets; then $6 - 2n$ is the number of short gravitino multiplets, $15 - 4n$ is the number of vector multiplets and $7 - n$ is the number of hypermultiplets. Later we will see further constraints on these cases.

4 Scherk-Schwarz mechanism for $N = 8, 6$ spontaneously broken supergravity

In the previous section we have stated some necessary conditions for the super Higgs effect to take place. Of more interest is to know whether one can find dynamical models which have the two phases, with broken and unbroken symmetry. The Scherk-Schwarz generalized dimensional reduction [10, 11] provides many examples of such systems.

The Scherk-Schwarz mechanism starts by considering $N = 8$ supergravities and makes a generalized dimensional reduction ansatz to dimension 4, which generically depends on four parameters m_i, $i = 1, \ldots 4$. We obtain a family of four dimensional models, where the parameters are interpreted as gravitino masses, which means that the supersymmetry has been spontaneously broken. In fact, the gravitinos come in pairs of equal mass. One of the masses, say m_1 is set to zero in order to keep some supersymmetry [28]. If additionally $m_2 = m_3 = 0$ ($m_4 \neq 0$) then we have an $N' = 6$ model; if $m_2 = 0$ ($m_3, m_4 \neq 0$) we obtain an $N' = 4$ model and the $N' = 2$ model is obtained with only $m_1 = 0$. The masses acquired by the different states of the graviton multiplet depend on the parameters m_i. They are given in Table 6.

The $N' = 6$ model is unique (Section 3.1). The $N' = 4$ model corresponds to case 1. in Section 3.1 with a further Higgs mechanism in the vector sector. In fact, at generic values of m_3 and m_4 the model has has 2 massless and 2 massive vector multiplets and for $|m_3| = |m_4|$ we get 4 massless and 1 massive vector multiplet. The model with 6 massless vectors is not realized in this context.

For $m_i \neq 0$ and $|m_i| \neq |m_j|$, $i = 2, 3, 4$ we have $N = 8 \rightarrow N' = 2$ (Section 3.3). In this model all massive spin 3/2 multiplets are short. Then there are three massless vector multiplets, 12 short massive vector multiplets and 7 massive hypermultiplets. It corresponds to a Higgs version of the $n = 0$ model in Section 3.3.

If $|m_i| = |m_j|$, $i = 2, 3, 4$ we have the maximal number of massless vector multiplets, that is 9.

Similarly one could start with $N = 6$ supergravity (Section 3.2). In this case the number of parameters is three and have the same interpretation as gravitino masses. The masses acquired by the helicity states of the $N = 6$ graviton multiplet are given in Table 7.

For $N' = 4$ the theory is completely fixed. The gravitinos are $\frac{1}{2}$BPS multiplets and we have two massless vector multiplets.

For $N' = 2$ the theory corresponds to the case labelled by 2. in Section 3.2. If $|m_2| = |m_3|$ there are 5 massless vectors multiplets and if $|m_2| \neq |m_3|$ there are 3.

5 Goldstone bosons and translational symmetries

When the manifold parametrized by the scalars is a coset space G/H, there is an abelian algebra of isometries that is contained in G. This algebra is the maximal abelian ideal of the solvable algebra associated to the coset space via the Iwasawa decomposition (see for example [29]). In the examples that we analyze in this section we have two models with two coset spaces, G/H corresponding to the unbroken supersymmetry model and G'/H' corresponding to the model with partial breaking of supersymmetry once the massive modes have been integrated out. We will denote by $\mathfrak{t}(G/H)$ and $\mathfrak{t}'(G'/H')$ the abelian subalgebras associated to the respective cosets (here the "$\mathfrak{t}$" stands for translational). t and t' are respectively the dimensions of these subalgebras.

If n_v and n_v' denote the number of massless vectors in each theory, we find in all the models analyzed that $t - t' = n_v - n_v'$. The solvable group obtained in the Iwasawa decomposition (now in the group instead that in the algebra) is diffeomorphic as a manifold to G/H. This parametrization has been considered in the literature to analyze U-dualities in string theory [30, 31, 32]. The generators of the maximal abelian ideal act as translations on t of the coordinates of G/H, which appear only through derivatives in the Lagrangian and which are flat directions of the scalar potential. This suggests that, as a general rule for a consistent Higgs effect, these particular coordinates are the Goldstone bosons connected to the spontaneous breaking of $\mathbb{R}^{n_v}$ to $\mathbb{R}^{n_v'}$, so they have been absorbed into the vectors that have acquired mass.

We analyze first examples that are obtained with the Scherk-Schwarz mechanism. In these cases one can prove that the above considerations are actually valid [10, 11, 33]. It would be interesting to know the cases where this rule does not hold.

$N = 8 \to N' = 6$. The coset space of the scalars in $N = 8$ supergravity is $\mathrm{E}_{7,7}/\mathrm{SU}(8)$ and the dimension of the translational subalgebra is $t = 27$ [30]. In $N' = 6$ the coset is $\mathrm{SO}^*(12)/\mathrm{U}(6)$, and $t' = 15$. So $t - t' = 12$. It is easy to see that $n_v - n_v' = 28 - 16 = 12$.

$N = 6 \to N' = 4$ For $N' = 4$ the coset is

$$\frac{SO(6, n)}{SO(6) \times SO(n)} \times \frac{SU(1, 1)}{U(1)}.$$

n is the number of vector multiplets in the theory. We take $n = 2$ (see Section 3.2). Then $t - t' = 15 - 7 = 8$. We have that $n_v - n'_v = 16 - 8 = 8$.

$N = 8 \to N' = 2$.   For $N' = 2$ we have a certain number of vector multiplets (n_1) and hypermultiples (n_2) (see Section 3.3).

The minimal model($m_i \neq m_j$, $i, j = 2, 3, 4$ in the notation of Section 4) corresponds to $n_1 = 3$ (we take $n_2 = 0$), and the coset is

$$\frac{SU(1, 1)}{U(1)} \times \frac{SU(1, 1)}{U(1)} \times \frac{SU(1, 1)}{U(1)}.$$

$t' = 3$, so $t - t' = 27 - 3 = 24$. We have that $n_v - n'_v = 28 - 4 = 24$.

The maximal model ($m_1 = m_2 = m_3$) corresponds to having $n_1 = 9$, (again we take $n_2 = 0$). The coset space is

$$\frac{SU(3, 3)}{SU(3) \times SU(3) \times U(1)}.$$

In this case $t - t' = 27 - 9 = 18$ and $n_v - n'_v = 28 - 10 = 18$.

$N = 6 \to N' = 2$.   We consider first the case with three massless vectors in the $N' = 2$ theory. Then $t - t' = 15 - 3 = 12$ and $n_v - n'_v = 16 - 4 = 12$.

The case with 5 massless vectors ($m_2 = m_3$) has coset

$$\frac{SU(1, 1)}{U(1)} \times \frac{SO(2, 4)}{SO(2) \times SO(4)}.$$

$t - t' = 15 - 5 = 10$ and $n_v - n'_v = 16 - 6 = 10$.

$N = 8 \to N' = 4$.   We take the case with two massless vector multiplets. Then the coset is

$$\frac{SO(6, 2)}{SO(6) \times SO(2)} \times \frac{SU(1, 1)}{SO(2) \times U(1)}.$$

We have $t - t' = 27 - 7 = 20$ and $n_v - n'_v = 28 - 8 = 20$.

The examples that follow can be obtained in Type IIB superstring compactified on an orientifold $T^6/\mathbb{Z}_2$ in presence of brane fluxes [12, 13] and in

certain gauged supergravity theories [9]. We want to consider the spontaneous breaking of $N = 4, 3$ supergravities down to $N' = 3, 2$. In order to have a consistent reduction it is necessary that the scalar manifold of the broken theory is a submanifold of the unbroken one [34]. For the $N' = 2$ case this is just an assumption since the effects of integrating out the massive modes could be more complicated.

$N = 4 \to N' = 3$. We consider the $N = 4$ model with six massless vector multiplets,

$$\frac{\text{SO}(6,6)}{\text{SO}(6) \times \text{SO}(6)} \times \frac{\text{SU}(1,1)}{\text{U}(1)},$$

with $t = 15$. Note that the SU(1,1) factor is not considered because SU(3,3)⊂SO(6,6) The decomposition of the massless multiplets is

$$\left[(2), 4(\frac{3}{2}), 6(1), 4(\frac{1}{2}), 2(0)\right] + 6\left[(1), 4(\frac{1}{2}), 6(0)\right] \to$$
$$\left[(2), 3(\frac{3}{2}), 3(1), 1(\frac{1}{2})\right] + 6\left[(1), 4(\frac{1}{2}), 6(0)\right]$$

In $N = 3$ the long spin 3/2 multiplet is formed by adding 3 massless vector multiplets to the $\lambda_{MAX} = 3/2$ multiplets. There remain three massless vector multiplets. The scalar manifold of the theory is

$$\frac{\text{SU}(3,3)}{\text{SU}(3) \times \text{SU}(3) \times \text{U}(1)}$$

with $t' = 9$. So we have $t - t' = 15 - 9 = 6$ an $n_v - n'_v = 12 - 6 = 6$.

$N = 4 \to N' = 2$. We take $N = 4$ supergravity with 6 massless vector multiplets. The decomposition of the graviton multiplet into massless multiplets of $N' = 2$ is

$$\left[(2), 4(\frac{3}{2}), 6(1), 4(\frac{1}{2}), 2(0)\right] \to \left[(2), 2(\frac{3}{2}), (1)\right] +$$
$$2\left[(\frac{3}{2}), 2(1), (\frac{1}{2})\right] + \left[(1), 2(\frac{1}{2}), 2(0)\right],$$

and the decomposition of the vector multiplet is

$$\left[(1) + 4(\frac{1}{2}) + 6(0)\right] \to \left[(1) + 2(\frac{1}{2}) + 2(0)\right] + \left[2(\frac{1}{2}), 4(0)\right].$$

One can form two long spin 3/2 multiplets with the 2 massless $\lambda_{MAX} = 3/2$ multiplets, four massless vectors and two massless hypermultiplets. We are left with the graviton multiplet, three massless vectors and 4 massless hypermultiplets. The coset space of this theory is

$$\frac{SO(2,2)}{SO(2) \times SO(2)} \times \frac{SO(4,4)}{SO(4) \times SO(4)} \times \frac{SU(1,1)}{U(1)},$$

with $t' = 7$. t and t' refer here to the translational isometries of the manifolds SO(n,n)/SO(n)×SO(n), with $t = 15$ for $n = 6$ and $t' = 1 + 6$ for $n = 2, 4$. So we have $t - t' = 15 - 7 = 8$ and $n_v - n'_v = 12 - 4 = 8$.

$N = 3 \rightarrow N' = 2$. We start with the $N = 3$ model with 3 vector multiplets as above. The decomposition of the graviton multiplet is

$$\left[(2), 3(\frac{3}{2}), 3(1), (\frac{1}{2})\right] \rightarrow \left[(2), 2(\frac{3}{2}), (1)\right] + \left[(\frac{3}{2}), 2(1), (\frac{1}{2})\right]$$

and the decomposition of the massless vector multiplet is

$$\left[(1), 4(\frac{1}{2}, 6(0))\right] \rightarrow \left[(1), 2(\frac{1}{2}), 2(0)\right] + \left[2(\frac{1}{2}), 4(0)\right].$$

To form a long spin 3/2 multiplet we need two massless vector multiplets and one hypermultiplet. The residual theory has then one vector multiplet and two hypermultiplets. The coset is then

$$\frac{SU(1,1)}{U(1)} \times \frac{SU(2,2)}{SU(2) \times SU(2) \times U(1)}$$

with $t' = 5$. So we have $t - t' = 9 - 5 = 4$ an $n_v - n'_v = 6 - 2 = 4$.

6 Concluding remarks

In this paper we have considered general features of the super Higgs effect in extended supergravity, based on the analysis of massless and massive representations of N extended supersymmetry in four dimensions. The same analysis could be carried out for higher dimensional theories as well. Many of these breaking patterns find a realization in the Scherk-Schwarz mechanism of supersymmetry breaking as well as in string compactifications in presence of brane fluxes. The requirement of the super Higgs effect with vanishing cosmological constant is satisfied in these models by spontaneous breakdown

of a certain number of abelian gauge isometries, related to properties of the scalar manifolds in the broken and unbroken phase. Many other situations can be studied as for example the breaking of $N = 4 \rightarrow N' = 3, 2$ in presence of an arbitrary number of matter multiplets. Also, the more interesting case of $N \rightarrow N' = 1$ has not been considered here. In the case of $N' = 2, 1$ the broken phase may have a complicated structure due to the integration of massive modes. This is so because the reduction of supersymmetries is not as predictive as in the cases with $N' \geq 3$.

Superstring compactifications in presence of brane fluxes appear to offer a general set up [12, 13] where many models of spontaneous supersymmetry breaking can be realized and an almost realistic hierarchy of scales can be obtained.

Acknowledgements

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N	massless $\lambda_{MAX} = 2$ multiplet	massless $\lambda_{MAX} = 3/2$ multiplet
8	$\left[(2), 8(\frac{3}{2}), 28(1), 56(\frac{1}{2}), 70(0)\right]$	none
6	$\left[(2), 6(\frac{3}{2}), 16(1), 26(\frac{1}{2}), 30(0)\right]$	$\left[(\frac{3}{2}), 6(1), 15(\frac{1}{2}), 20(0)\right]$
5	$\left[(2), 5(\frac{3}{2}), 10(1), 11(\frac{1}{2}), 10(0)\right]$	$\left[(\frac{3}{2}), 6(1), 15(\frac{1}{2}), 20(0)\right]$
4	$\left[(2), 4(\frac{3}{2}), 6(1), 4(\frac{1}{2}), 2(0)\right]$	$\left[(\frac{3}{2}), 4(1), 7(\frac{1}{2}), 8(0)\right]$
3	$\left[(2), 3(\frac{3}{2}), 3(1), (\frac{1}{2})\right]$	$\left[(\frac{3}{2}), 3(1), 3(\frac{1}{2}), 2(0)\right]$
2	$\left[(2), 2(\frac{3}{2}), (1)\right]$	$\left[(\frac{3}{2}), 2(1), (\frac{1}{2})\right]$
1	$\left[(2), (\frac{3}{2})\right]$	$\left[(\frac{3}{2}), (1)\right]$

Table 1: Massless $\lambda_{MAX} = 2, 3/2$ multiplets.

N	massless $\lambda_{MAX} = 1$ multiplet	massless $\lambda_{MAX} = 1/2$ multiplet
8,6,5	none	none
4	$\left[(1), 4(\frac{1}{2}), 6(0)\right]$	none
3	$\left[(1), 4(\frac{1}{2}), 6(0)\right]$	none
2	$\left[(1), 2(\frac{1}{2}), 2(0)\right]$	$\left[2(\frac{1}{2}), 4(0)\right]$
1	$\left[(1), (\frac{1}{2})\right]$	$\left[(\frac{1}{2}), 2(0)\right]$

Table 2: Massless $\lambda_{MAX} = 1, 1/2$ multiplets.

N	massive spin 3/2 multiplet	long	short
8	none		
6	$2 \times \left[(\frac{3}{2}), 6(1), 14(\frac{1}{2}), 14'(0) \right]$	no	$q = 3,\ (\frac{1}{2}\text{BPS})$
5	$2 \times \left[(\frac{3}{2}), 6(1), 14(\frac{1}{2}), 14'(0) \right]$	no	$q = 2,\ (\frac{2}{5}\text{BPS})$
4	$2 \times \left[(\frac{3}{2}), 6(1), 14(\frac{1}{2}), 14'(0) \right]$	no	$q = 1,\ (\frac{1}{4}\text{BPS})$
	$2 \times \left[(\frac{3}{2}), 4(1), 6(\frac{1}{2}), 4(0) \right]$	no	$q = 2,\ (\frac{1}{2}\text{BPS})$
3	$\left[(\frac{3}{2}), 6(1), 14(\frac{1}{2}), 14'(0) \right]$	yes	no
	$2 \times \left[(\frac{3}{2}), 4(1), 6(\frac{1}{2}), 4(0) \right]$	no	$q = 1,\ (\frac{1}{3}\text{BPS})$
2	$\left[(\frac{3}{2}), 4(1), 6(\frac{1}{2}), 4(0) \right]$	yes	no
	$2 \times \left[(\frac{3}{2}), 2(1), (\frac{1}{2}) \right]$	no	$q = 1,\ (\frac{1}{2}\text{BPS})$
1	$\left[(\frac{3}{2}), 2(1), (\frac{1}{2}) \right]$	yes	no

Table 3: Massive spin 3/2 multiplets.

N	massive spin 1 multiplet	long	short
8,6,5	none		
4	$2 \times \left[(1), 4(\frac{1}{2}), 5(0)\right]$	no	$q = 2$, $(\frac{1}{2}\mathrm{BPS})$
3	$2 \times \left[(1), 4(\frac{1}{2}), 5(0)\right]$	no	$q = 1$, $(\frac{1}{3}\mathrm{BPS})$
2	$\left[(1), 4(\frac{1}{2}), 5(0)\right]$	yes	no
	$2 \times \left[(1), 2(\frac{1}{2}), (0)\right]$	no	$q = 1$, $(\frac{1}{2}\mathrm{BPS})$
1	$\left[(1), 2(\frac{1}{2}), (0)\right]$	yes	no

Table 4: Massive spin 1 multiplets.

N	massive spin 1/2 multiplet	long	short
8,6,5,4,3	none		
2	$2 \times \left[(\frac{1}{2}), 2(0)\right]$	no	$q = 1$, $(\frac{1}{2}\mathrm{BPS})$
1	$\left[(\frac{1}{2}), 2(0)\right]$	yes	no

Table 5: Massive spin 1/2 multiplets.

helicities	acquired masses	degeneracy	n. of physical modes
2	0	1	2
$\frac{3}{2}$	$|m_i|$	2	16
1	0	4	8
	$|m_i \pm m_j|$, $i < j$	2	48
$\frac{1}{2}$	$|m_i|$	6	48
	$|m_i \pm m_j \pm m_k|$, $i < j < k$	2	64
0	0	6	6
	$|m_i \pm m_j|$ $i < j$	4	48
	$|m_1 \pm m_2 \pm m_3 \pm m_4|$	2	16

Table 6: Mass spectrum of $N = 8$ supergravity.

helicities	acquired masses	degeneracy	n. of physical modes
2	0	1	2
$\frac{3}{2}$	$|m_i|$	2	12
1	0	4	8
	$|m_i \pm m_j|,\ i < j$	2	24
$\frac{1}{2}$	$|m_i|$	8	48
	$|m_1 \pm m_2 \pm m_3|$	2	4
0	0	6	6
	$|m_i \pm m_j|\ i < j$	4	24

Table 7: Mass spectrum of $N = 6$ supergravity.