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Coupling of Rolling Tachyon to Closed Strings

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Abstract

We study the late time behavior of the boundary state representing the rolling tachyon constructed by Sen. It is found that the coupling of the rolling tachyon to massive modes of the closed string grows exponentially as the system evolves. We argue that the description of rolling tachyon by a boundary state is valid during the finite time determined by string coupling, and that energy could be dissipated to the bulk beyond this time. We also comment on the relation between the rolling tachyon boundary state and the spacelike D-brane boundary state.

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1 Introduction

Initiated by a series of papers [1, 2, 3] by A. Sen, the dynamics of a time-dependent tachyon background has been intensely studied recently. In [1], he has presented an iterative scheme to construct a solution in Witten's Open String Field Theory that describes the tachyon rolling up and down its potential in unstable D-brane systems (see also [4, 5, 6]). In [1, 2, 4], he has also constructed a boundary state corresponding to the solution. The boundary state representing the rolling tachyon is constructed by deforming the boundary state of an unstable D-brane by an exactly marginal operator. Therefore, it is expected to define an open string background which is an exact solution in the weak coupling limit, where the back reaction of the closed string is ignored. The boundary state is also a source for closed strings. Once the boundary state is obtained, one can extract the time evolution of the source for each closed string mode. In particular, the energy momentum tensor can be obtained from the coefficients of the level $(-1,-1)$ states in the boundary state and it has been argued that the system will evolve to a pressureless gas with non-zero energy density called tachyon matter [2]. A similar behavior has been found in the context of Boundary String Field Theory [7, 8].

Tachyon matter appears to be a mysterious object in string theory. It is supposedly the decay product of the unstable D-brane. While the energy remains preserved and localized in the world-volume, there are no physical open string excitations [3, 9]. Furthermore, since the energy momentum tensor does not oscillate, tachyon matter does not seem to turn into the radiation of massless closed string modes. These features are crucial in the investigation of cosmology based on rolling tachyon [10, 11, 12]. For example, from the observation that tachyon matter does not decay into light particles, tachyon matter has been regarded as a candidate of dark matter in several works [2, 11]. It also implies the reheating problem in tachyon inflation scenarios [12] as pointed out in [13]. However, these analyses have only taken into account the behavior of the source for massless closed string modes in the rolling tachyon boundary state and have not seriously considered the effect of infinitely many massive closed string modes.

In this paper, we examine the rolling tachyon boundary state further to higher levels and compute the coefficients of these terms, which give the couplings between rolling tachyon and closed string massive modes. One might expect that the couplings would converge to zero or some finite values in the far future. We will see, however, that they become exponentially strong as the system evolves. In contrast to the ordinary case where massive modes decouple from the low energy physics, massive modes could play a significant role in our case, as the couplings soon become very large. Therefore, the analysis of the rolling tachyon using low energy effective theory such as supergravity [3] is not good enough to obtain a correct physical picture. This result also suggests that

[3]See [14, 15] for the study of the tachyon matter in the supergravity approach.

the tachyon matter could decay through the massive closed string states. So, it may affect the cosmological scenarios that involve the rolling tachyon.

This paper is organized as follows. In section 2, we review the rolling tachyon boundary state constructed in [1, 2]. In section 3, we explicitly compute the couplings to some of the closed string massive modes and show that they become large exponentially at late times. We also make an argument that the couplings to the massive closed string states blow up quite generally. In section 4, we consider the effect of the back reaction of the closed string fields and an estimate is given on the time scale in which the boundary state description is valid. Section 5 is devoted to exploring the exceptional cases in which all the couplings remain finite. We argue that the S-brane (spacelike D-brane) boundary state constructed in [16] can be obtained as a limit of the rolling tachyon boundary state. In section 6, we summarize our results and conclude with discussions.

2 Review of the Rolling Tachyon Boundary State

In this section, we review the boundary state description of the rolling tachyon constructed in [1, 2]. Throughout the paper, we use the convention $\alpha' = 1$.

Let us first consider a D25-brane in bosonic string theory. The configuration of the open string tachyon field on the D25-brane is described in the boundary CFT by turning on a boundary interaction. In [1], Sen has considered the tachyon field rolling up and down the potential as

$$T(x^0) \propto \cosh(x^0), \tag{2.1}$$

for which the boundary interaction is of the form

$$\tilde{\lambda} \int dt \, \cosh X^0(t), \tag{2.2}$$

where t is the parameter which parameterizes the boundary of the world-sheet. The advantage of considering this configuration is that the Wick rotated theory, which is described by the world-sheet action

$$S = \frac{1}{2\pi} \int d^2z \partial X \bar{\partial} X + \tilde{\lambda} \int dt \, \cos X(t), \tag{2.3}$$

is a solvable boundary conformal field theory [17, 18, 19]. Therefore, we can use some of the exact results obtained in the Wick rotated theory in the analysis of the rolling tachyon by performing inverse Wick rotation.

The boundary state for the D25-brane with the boundary interaction (2.2) takes the form

$$|B\rangle = |B\rangle_{X^0} \otimes |B\rangle_{\vec{X}} \otimes |B\rangle_{bc}, \tag{2.4}$$

where $|B\rangle_{\vec{X}}$ and $|B\rangle_{bc}$ are the usual boundary states for flat D25-branes:

$$|B\rangle_{\vec{X}} \propto \exp\left(-\sum_{n=1}^{\infty}\frac{1}{n}\alpha^i_{-n}\bar{\alpha}^i_{-n}\right)|0\rangle, \qquad (i=1,\ldots,25) \tag{2.5}$$

$$|B\rangle_{bc} \propto \exp\left(-\sum_{n=1}^{\infty}(\bar{b}_{-n}c_{-n}+b_{-n}\bar{c}_{-n})\right)(c_0+\bar{c}_0)c_1\bar{c}_1|0\rangle. \tag{2.6}$$

$|B\rangle_{X^0}$ is the part of the boundary state that describes the dynamics of the rolling tachyon. The corresponding boundary state in the Wick rotated theory (2.3) has been constructed in [17, 18, 19]. Since the boundary interaction in (2.3) introduces integer momenta, one can restrict oneself to the subspace spanned by the states carrying integer momenta. This allows one to work at the self-dual radius $R=1$. The boundary state in the compactified theory is given by acting an $SU(2)$ rotation on the unperturbed boundary state. The boundary state in the uncompactified theory is then obtained by projecting onto the unwinding states [17, 19]. The result is

$$|B\rangle_X = \sum_{j=0,\frac{1}{2},1,\cdots}\sum_{m=-j}^{j} D^j_{m,-m}(R)|j;m,m\rangle\rangle. \tag{2.7}$$

Here R is the $SU(2)$ rotation matrix

$$R=\left(\begin{array}{cc}\cos(\pi\tilde{\lambda}) & i\sin(\pi\tilde{\lambda})\\ i\sin(\pi\tilde{\lambda}) & \cos(\pi\tilde{\lambda})\end{array}\right), \tag{2.8}$$

and $D^j_{m,-m}(R)$ is a corresponding spin j representation matrix element. $|j;m,m\rangle\rangle$ is the Virasoro Ishibashi state built over the primary state $|j;m,m\rangle$ (see section 3.1 for more details).

In [1], the oscillator-free part and the part proportional to $\alpha_{-1}\bar{\alpha}_{-1}|0\rangle$ have been explicitly computed:

$$\begin{aligned}|B\rangle_X &= \left[1+2\sum_{n=1}^{\infty}(-\sin(\tilde{\lambda}\pi))^n\cos(nX(0))\right]|0\rangle\\ &\quad -\alpha_{-1}\bar{\alpha}_{-1}\left[\cos(2\pi\tilde{\lambda})-2\sum_{n=1}^{\infty}(-\sin(\tilde{\lambda}\pi))^n\cos(nX(0))\right]|0\rangle+\cdots.\end{aligned} \tag{2.9}$$

Performing the inverse Wick rotation $X\to -iX^0$, we obtain [1]

$$|B\rangle_{X^0} = f(X^0(0))|0\rangle+\alpha^0_{-1}\bar{\alpha}^0_{-1}\,g(X^0(0))|0\rangle+\cdots, \tag{2.10}$$

where

$$f(x^0) = \frac{1}{1+e^{x^0}\sin(\tilde{\lambda}\pi)}+\frac{1}{1+e^{-x^0}\sin(\tilde{\lambda}\pi)}-1, \tag{2.11}$$

$$g(x^0) = \cos(2\tilde{\lambda}\pi)+1-f(x^0). \tag{2.12}$$

The boundary state describing the rolling of the tachyon on unstable D-brane systems in superstring theory can also be obtained similarly. We consider a pair of D9 and $\bar{\rm D}9$ branes in type IIB theory or a single non-BPS D9-brane in type IIA theory. The D9-$\bar{\rm D}9$ brane pair is described by the boundary CFT with 2×2 Chan-Paton factors and with an appropriate GSO projection ($(-1)^F=1$ for DD and $\bar{\rm D}\bar{\rm D}$ strings, $(-1)^F=-1$ for $\rm D\bar{D}$ and $\rm \bar{D}D$ strings). A single non-BPS D9-brane is described as the orbifold of the D9-$\bar{\rm D}9$ system by $(-1)^{F_L}$, where F_L is the space-time fermion number from left-movers (see [20] for a review).

Following [2], we turn on a tachyon field of the form

$$T(x^0)\propto \cosh(x^0/\sqrt{2}), \tag{2.13}$$

for which we should assign a Chan-Paton factor σ_1 [20]. This is represented as the perturbation of the world-sheet action by the boundary interaction

$$\tilde{\lambda}\int dt\,\psi^0(t)\sinh\left(\frac{X^0(t)}{\sqrt{2}}\right)\otimes\sigma_1, \tag{2.14}$$

i.e., the integral of the corresponding zero-picture vertex operator in the σ_1 sector.

The boundary state takes the form

$$|B\rangle = |B,+\rangle - |B,-\rangle, \tag{2.15}$$

where

$$|B,\epsilon\rangle \propto |B,\epsilon\rangle_{X^0,\psi^0}\otimes|B,\epsilon\rangle_{\vec{X},\vec{\psi}}\otimes|B,\epsilon\rangle_{ghost}, \qquad (\epsilon=\pm). \tag{2.16}$$

Again, the spatial and the ghost parts take the usual expressions:

$$\begin{aligned} |B,\epsilon\rangle_{\vec{X},\vec{\psi}} &\propto \exp\left(-\sum_{n=1}^{\infty}\frac{1}{n}\alpha^j_{-n}\bar{\alpha}^j_{-n}\right)\exp\left(-i\epsilon\sum_{n=1}^{\infty}\psi^j_{-n-1/2}\bar{\psi}^j_{-n-1/2}\right)|0\rangle, \qquad (2.17)\\ |B,\epsilon\rangle_{ghost} &\propto \exp\left(-\sum_{n=1}^{\infty}(\bar{b}_{-n}c_{-n}+b_{-n}\bar{c}_{-n})\right)\\ &\quad\times\exp\left(-i\epsilon\sum_{n=1}^{\infty}(\bar{\beta}_{-n-1/2}\gamma_{-n-1/2}-\beta_{-n-1/2}\bar{\gamma}_{-n-1/2})\right)\\ &\quad\times(c_0+\bar{c}_0)c_1\bar{c}_1e^{-\phi(0)}e^{-\bar{\phi}(0)}|0\rangle. \qquad (2.18)\end{aligned}$$

We are interested in the temporal part $|B\rangle_{X^0,\psi^0}$. This part of the boundary state is obtained in a similar way to the bosonic case above. Namely, we first construct the boundary state in the Wick rotated theory, which is obtained by the analytic continuation $X^0\to iX$, $\psi^0\to i\psi$ and $\bar{\psi}^0\to i\bar{\psi}$, and inverse Wick rotate the result.

In the Wick rotated theory, the boundary interaction is

$$-\tilde{\lambda}\int dt\,\psi(t)\sin\left(\frac{X(t)}{\sqrt{2}}\right)\otimes\sigma_1. \tag{2.19}$$

Since this boundary interaction is invariant under

$$X \to X + 2\pi/\sqrt{2}, \quad \psi \to -\psi, \tag{2.20}$$

and ψ appears only as byliners in the boundary state, the boundary state is a sum of states with momenta $p = \sqrt{2}n$ with integer n. Thus we can restrict ourselves to the space spanned by such states. This space is a subspace of the theory compactified at the free fermion radius $R = 1/\sqrt{2}$. Thus we can fermionize X through

$$X \equiv X_R(z) + X_L(\bar{z}), \tag{2.21}$$

$$e^{i\sqrt{2}X_R(z)} = \frac{1}{\sqrt{2}}(\xi + i\eta), \quad e^{i\sqrt{2}X_L(\bar{z})} = \frac{1}{\sqrt{2}}(\bar{\xi} + i\bar{\eta}) \tag{2.22}$$

This shows that the compactified theory admits the $SO(3)$ (or $SU(2)$) current algebra. The super-Virasoro primaries can be classified according to the $SU(2)$ quantum numbers.

Such a boundary state takes the form

$$|B, \epsilon\rangle_{X,\psi} = \sum_{j=0,1,\cdots} \sum_{n=-j}^{j} D^j_{n,-n}(R)|j;n,n;\epsilon\rangle\rangle, \tag{2.23}$$

where R is the same matrix as above, but n takes integer values here and $|j;n,n;\epsilon\rangle\rangle$ is the super-Virasoro Ishibashi state built over the super-Virasoro primary $|j,n,n\rangle$. Explicit examples will be given in section 3.2. In [2], Sen has computed the parts of the boundary state relevant to the coupling to massless modes of closed string:

$$\begin{aligned} & |B,\epsilon\rangle_{X,\psi} \\ = \ & \left(1 + 2\sum_{n=1}^{\infty}(-1)^n \sin^{2n}(\pi\tilde{\lambda}) \cos(\sqrt{2}nX(0))\right)|0\rangle - i\epsilon\psi_{-1/2}\bar{\psi}_{-1/2} \\ & \times \left(\cos(2\pi\tilde{\lambda}) - 2\sum_{n=1}^{\infty}(-1)^n \sin^{2n}(\pi\tilde{\lambda})\cos(\sqrt{2}nX(0))\right)|0\rangle + \cdots . \end{aligned} \tag{2.24}$$

The inverse Wick rotation leads to

$$|B,\epsilon\rangle_{X^0,\psi^0} = \left(\hat{f}(X^0(0)) + i\epsilon\psi^0_{-1/2}\bar{\psi}^0_{-1/2}\hat{g}(X^0(0))\right)|0\rangle, \tag{2.25}$$

where

$$\hat{f}(x^0) = \frac{1}{1+\sin^2(\pi\tilde{\lambda})e^{\sqrt{2}x^0}} + \frac{1}{1+\sin^2(\pi\tilde{\lambda})e^{-\sqrt{2}x^0}} - 1, \tag{2.26}$$

$$\hat{g}(x^0) = \cos(2\pi\tilde{\lambda}) + 1 - \hat{f}(x^0). \tag{2.27}$$

3 Computation of the Massive Mode Coupling

In this section, we compute several higher level states in the rolling tachyon boundary state constructed in [1, 2]. We will find that the couplings of the tachyon to the massive modes exponentially diverge as $x^0 \to \infty$.

3.1 Bosonic String

We write down the Virasoro Ishibashi states which are relevant in the calculation of the boundary state up to level (2,2) (see appendix A for details).

$$
\begin{aligned}
|0;0,0\rangle\rangle &= \left(1+\frac{1}{2}\alpha_{-1}^2\bar{\alpha}_{-1}^2+\cdots\right)|0\rangle, && (3.28)\\
\left|\frac{1}{2};\pm\frac{1}{2},\pm\frac{1}{2}\right\rangle\rangle &= \left(1+\alpha_{-1}\bar{\alpha}_{-1}+\frac{1}{6}(\alpha_{-1}^2\pm\sqrt{2}\alpha_{-2})(\bar{\alpha}_{-1}^2\pm\sqrt{2}\bar{\alpha}_{-2})+\cdots\right) \\
&\quad\times e^{\pm iX(0)}|0\rangle, && (3.29)\\
|1;0,0\rangle\rangle &= \left(\alpha_{-1}\bar{\alpha}_{-1}+\frac{1}{2}\alpha_{-2}\bar{\alpha}_{-2}+\cdots\right)|0\rangle, && (3.30)\\
|j;\pm j,\pm j\rangle\rangle &= \left(1+\alpha_{-1}\bar{\alpha}_{-1}+\frac{1}{2}\alpha_{-1}^2\bar{\alpha}_{-1}^2+\frac{1}{2}\alpha_{-2}\bar{\alpha}_{-2}+\cdots\right) \\
&\quad\times e^{\pm 2ijX(0)}|0\rangle \ (j\geq 1), && (3.31)\\
\left|\frac{3}{2};\pm\frac{1}{2},\pm\frac{1}{2}\right\rangle\rangle &= \left(\frac{1}{6}(\alpha_{-2}\mp\sqrt{2}\alpha_{-1}^2)(\bar{\alpha}_{-2}\mp\sqrt{2}\bar{\alpha}_{-1}^2)+\cdots\right)e^{\pm iX(0)}|0\rangle. && (3.32)
\end{aligned}
$$

Now, we are ready to compute the boundary state (2.7). Relevant matrix elements $D^j_{m,-m}(R)$ of the $SU(2)$ rotation R are given in Appendix B. Note that we have not determined possible phase factors which could appear in the above expressions (3.28)–(3.32). In [1], the phase factors for the Ishibashi states (3.28)–(3.31) have been determined by demanding that the boundary state $|B_X\rangle$ represents an array of D-branes localized at $X=(2n+1)\pi$ when $\tilde{\lambda}=1/2$ [17, 19, 21]. They can be read off from (2.9). The phase factor for (3.32) can be obtained similarly and it turns out to be $-i$.

Collecting these together, we obtain

$$
\begin{aligned}
&|B\rangle_X \\
= \ & |0;0,0\rangle\rangle+\sum_{n=1}^{\infty}(-1)^n\sin^n(\pi\tilde{\lambda})\left(\left|\frac{n}{2};\frac{n}{2},\frac{n}{2}\right\rangle\rangle+\left|\frac{n}{2};-\frac{n}{2},-\frac{n}{2}\right\rangle\rangle\right) \\
&-\cos(2\pi\tilde{\lambda})\,|\,1;0,0\,\rangle\rangle \\
&+\sin(\pi\tilde{\lambda})(3\cos^2(\pi\tilde{\lambda})-1)\left(\left|\frac{3}{2};\frac{1}{2},\frac{1}{2}\right\rangle\rangle+\left|\frac{3}{2};-\frac{1}{2},-\frac{1}{2}\right\rangle\rangle\right)+\cdots. \qquad (3.33)
\end{aligned}
$$

After the inverse Wick rotation, we find

$$
\begin{aligned}
|B\rangle_{X^0} &= |B_0\rangle + |B_{-1;-1}\rangle \\
&\quad + |B_{-2;-2}\rangle + |B_{(-1)^2;(-1)^2}\rangle + \left|B_{(-1)^2;-2}\right\rangle + \left|B_{-2;(-1)^2}\right\rangle + \cdots .
\end{aligned} \tag{3.34}
$$

Here $|B_0\rangle$ and $|B_{-1,-1}\rangle$ are level $(0,0)$ and $(-1,-1)$ states given in (2.10).

We further find

$$
\begin{aligned}
&|B_{-2;-2}\rangle \\
&= -\frac{1}{2}\alpha^0_{-2}\bar{\alpha}^0_{-2}\int dx^0\left[-(1+\cos(2\pi\tilde{\lambda})) + 2\sin(\pi\tilde{\lambda})\cos^2(\pi\tilde{\lambda})\cosh(x^0) + f(x^0)\right]|x^0\rangle,
\end{aligned} \tag{3.35}
$$

$$
|B_{(-1)^2;(-1)^2}\rangle = \frac{1}{2}(\alpha^0_{-1})^2(\bar{\alpha}^0_{-1})^2\int dx^0\left[4\sin(\pi\tilde{\lambda})\cos^2(\pi\tilde{\lambda})\cosh(x^0) + f(x^0)\right]|x^0\rangle, \tag{3.36}
$$

and

$$
\begin{aligned}
&\left|B_{(-1)^2;-2}\right\rangle + \left|B_{-2;(-1)^2}\right\rangle \\
&= -i\sqrt{2}((\alpha^0_{-1})^2\bar{\alpha}^0_{-2} + \alpha^0_{-2}(\bar{\alpha}^0_{-1})^2)\sin(\pi\tilde{\lambda})\cos^2(\pi\tilde{\lambda})\int dx^0\sinh(x^0)\left|x^0\right\rangle .
\end{aligned} \tag{3.37}
$$

All these states contain terms proportional to either $\cosh(x^0)$ or $\sinh(x^0)$ which exponentially blow up when $|x^0|$ becomes large. This result shows that the coupling between tachyon matter and massive closed string modes will become strong at late times.

We will not explicitly compute the states of level higher than 2, but we generally expect this kind of behavior in the higher level states. Note that the primary state $|\,j;m,m\,\rangle$ is of the form

$$
|\,j;m,m\,\rangle = \mathcal{O}_{j,m}\, e^{2imX(0)}\,|\,0\,\rangle\,, \tag{3.38}
$$

where $\mathcal{O}_{j,m}$ is an operator of level (j^2-m^2, j^2-m^2). When we are interested in a level (k,k) state in the boundary state, we should take into account the Ishibashi states $|\,j;m,m\,\rangle\rangle$ with $j^2-m^2\le k$. Then, apart from the case $m=\pm j$, there are only a finite number of choices for j and m satisfying $j^2-m^2\le k$. The contribution from the infinite sum $\sum_j D^j_{\pm j,\mp j}|\,j;\pm j,\pm j\,\rangle\rangle$ managed to sum up to a harmless function like $f(x^0)$ in (2.11). But, unless there is an accidental cancellation among the other terms, the inverse Wick rotation of the contribution from momentum $2m$ states will blow up as e^{2mx^0}.

3.2 Superstring

We again begin by listing the super-Virasoro Ishibashi states for low-level primaries:

$$
\begin{aligned}
|0;0,0;\epsilon\rangle\rangle &= (1+i\epsilon\alpha_{-1}\psi_{-1/2}\bar{\alpha}_{-1}\bar{\psi}_{-1/2}+\cdots)|0\rangle, && (3.39)\\
|1;\pm1,\pm1;\epsilon\rangle\rangle &= [1+i\epsilon\psi_{-1/2}\bar{\psi}_{-1/2}+\alpha_{-1}\bar{\alpha}_{-1} \\
&\quad +\frac{1}{2}i\epsilon(\psi_{-3/2}\pm\alpha_{-1}\psi_{-1/2})(\bar{\psi}_{-3/2}\pm\bar{\alpha}_{-1}\bar{\psi}_{-1/2})+\cdots] \\
&\quad \times e^{\pm i\sqrt{2}X(0)}|0\rangle, && (3.40)\\
|1;0,0;\epsilon\rangle\rangle &= (i\epsilon\psi_{-1/2}\bar{\psi}_{-1/2}+\alpha_{-1}\bar{\alpha}_{-1}+i\epsilon\psi_{-3/2}\bar{\psi}_{-3/2}+\cdots)|0\rangle, && (3.41)\\
|j;\pm j,\pm j;\epsilon\rangle\rangle &= (1+i\epsilon\psi_{-1/2}\bar{\psi}_{-1/2}+\alpha_{-1}\bar{\alpha}_{-1} \\
&\quad +i\epsilon\psi_{-3/2}\bar{\psi}_{-3/2}+i\epsilon\psi_{-1/2}\alpha_{-1}\bar{\psi}_{-1/2}\bar{\alpha}_{-1}+\cdots)e^{\pm\sqrt{2}ijX(0)}|0\rangle, \\
&\quad (j=2,3,4,\cdots), && (3.42)\\
|\,2;\pm1,\pm1;\epsilon\,\rangle\rangle &= \frac{1}{2}(\psi_{-3/2}\mp\alpha_{-1}\psi_{-1/2})(\bar{\psi}_{-3/2}\mp\bar{\alpha}_{-1}\bar{\psi}_{-1/2})e^{\pm i\sqrt{2}X(0)}|0\rangle+\cdots. \\
& && (3.43)
\end{aligned}
$$

The coefficient of $|2;\pm1,\pm1;\epsilon\rangle\rangle$ is $D^2_{\pm1,\mp1}(R)$ times a phase factor which is determined to be $-i\epsilon$ as in the bosonic case. Other coefficients can be read off from (2.24). Therefore the boundary state is

$$
\begin{aligned}
& |B,\epsilon\rangle_{X,\psi} \\
= \; & |\,0;0,0;\epsilon\,\rangle\rangle+\sum_{n=1}^{\infty}(-1)^n\sin^{2n}(\pi\tilde{\lambda})\,(\,|\,n;n,n;\epsilon\,\rangle\rangle+|\,n;-n,-n;\epsilon\,\rangle\rangle) \\
& -\cos(2\pi\tilde{\lambda})\,|\,1;0,0;\epsilon\,\rangle\rangle \\
& +i\epsilon\sin^2(\pi\tilde{\lambda})\cos(2\pi\tilde{\lambda})(\,|\,2;1,1;\epsilon\,\rangle\rangle+|\,2;-1,-1;\epsilon\,\rangle\rangle)+\cdots. \qquad (3.44)
\end{aligned}
$$

After analytic continuation, we get

$$
\begin{aligned}
|B\rangle_{X^0,\psi^0} &= |B_{0;0}\rangle+|B_{-1/2;-1/2}\rangle+|B_{-1;-1}\rangle+|B_{-3/2;-3/2}\rangle \\
&\quad +|B_{-1,-1/2;-1,-1/2}\rangle+|B_{-3/2;-1,-1/2}\rangle+|B_{-1,-1/2;-3/2}\rangle+\cdots,
\end{aligned}
$$

where the first two terms are given in (2.25) and

$$
\begin{aligned}
|B_{-1;-1}\rangle &= -\alpha^0_{-1}\bar{\alpha}^0_{-1}(-\cos(2\pi\tilde{\lambda})-1+\hat{f}(X^0(0)))|0\rangle, && (3.45)\\
|B_{-3/2;-3/2}\rangle &= i\epsilon\psi^0_{-3/2}\bar{\psi}^0_{-3/2}[\,1+\cos(2\pi\tilde{\lambda})-\hat{f}(X^0(0)) \\
&\quad -\sin^2(\pi\tilde{\lambda})(\cos(2\pi\tilde{\lambda})+1)\cosh(\sqrt{2}X^0(0))]\,|\,0\,\rangle\,, && (3.46)\\
|B_{-1,-1/2;-1,-1/2}\rangle &= i\epsilon\alpha^0_{-1}\psi^0_{-1/2}\bar{\alpha}^0_{-1}\bar{\psi}^0_{-1/2}[\,\hat{f}(X^0(0)) \\
&\quad +\sin^2(\pi\tilde{\lambda})(1+\cos(2\pi\tilde{\lambda}))\cosh(\sqrt{2}X^0(0))]\,|\,0\,\rangle\,, && (3.47)
\end{aligned}
$$

and

$$
\begin{aligned}
& \left| B_{-3/2;-1/2,-1} \right\rangle + \left| B_{-1/2,-1;-3/2} \right\rangle \\
= \ & \epsilon \sin^2(\pi\tilde{\lambda})(1+\cos(2\pi\tilde{\lambda})) \\
& \times \left(\psi^0_{-3/2}\bar{\alpha}^0_{-1}\bar{\psi}^0_{-1/2} + \alpha^0_{-1}\psi^0_{-1/2}\bar{\psi}^0_{-3/2} \right) \sinh(\sqrt{2}X^0(0)) \, | \, 0 \, \rangle \, .
\end{aligned} \tag{3.48}
$$

We see that as in the bosonic case, the boundary state contains exponentially divergent couplings to massive modes.

4 On the Back Reaction of Closed Strings

So far, we have ignored the dynamics of the closed string and focused on the classical equation of motion for the open string. This analysis is only justified in the weak coupling limit. What happens when we slightly turn on the string coupling g_s? As we have shown in the previous section, even if the string coupling is small, the system with rolling tachyon will become strongly coupled to closed string massive modes at late times and we expect a large back reaction of the closed string fields. Let us make this point more explicit in this section.

In string field theory, the tree-level action in terms of the canonically normalized string fields looks like [22]

$$
S \sim \tilde{\psi} Q_o \tilde{\psi} + g_s^{1/2} \tilde{\psi}^3 + \tilde{\phi} Q_c \tilde{\phi} + g_s \tilde{\phi}^3 + \tilde{\phi} B(g_s^{1/2}\tilde{\psi}) + \cdots . \tag{4.49}
$$

Here $\tilde{\psi}$ is an open string field and $\tilde{\phi}$ is a closed string field. $\tilde{\phi} B(g_s^{1/2}\tilde{\psi})$ symbolically denotes the coupling among one closed string field and open string fields. The power of g_s in each term is determined from the Euler characteristic of the disk or sphere, and the powers of open and closed string fields.

The boundary state considered in the previous sections corresponds to $B(g_s^{1/2}\tilde{\psi}_0)$ and plays a role of a source for the closed string. Here, $\tilde{\psi}_0$ denotes a solution of the equations of motion for the open string obtained by turning off the closed strings. To see this structure more clearly, it is convenient to change the normalization for the open string and absorb the coupling constant as

$$
\tilde{\psi} = g_s^{-1/2} \psi, \tag{4.50}
$$

Then, the action becomes

$$
S = g_s^{-1}(\psi Q_o \psi + \psi^3) + \tilde{\phi} Q_c \tilde{\phi} + g_s \tilde{\phi}^3 + \tilde{\phi} B(\psi) + \cdots . \tag{4.51}
$$

The equation of motion for the closed string field is

$$
Q_c \tilde{\phi} + g_s \tilde{\phi}^2 + B(\psi) + \cdots = 0, \tag{4.52}
$$

which imply the BRST invariance of the boundary state $Q_c B = 0$ in the weak coupling limit. On the other hand, the equation of motion for the open string field is

$$Q_o \psi + \psi^2 + g_s \tilde{\phi} B'(\psi) + \cdots = 0, \tag{4.53}$$

which shows that the closed strings are decoupled from the open strings in the weak coupling limit.

However, once we turn on the non-zero string coupling g_s, the coupling of the rolling tachyon boundary state to the closed string massive modes exponentially grows when $x^0 \to \infty$, as we have shown in the previous section, and hence it is not consistent to ignore the dynamics of closed string at late times, even if g_s is small. In fact, since there are exponentially growing source terms for the massive closed string modes in the equations of motion (4.52), the fluctuation will also blow up exponentially. Then, according to (4.53), this back reaction will alter the equations of motion for the open string. Therefore, even if the effect of merely the lowest diverging modes are included, the boundary state description of the rolling tachyon given in the previous sections is only reliable when

$$g_s e^{|t|/l_s} \ll 1. \tag{4.54}$$

Since higher levels have couplings which grow as $e^{m|t|}$ with arbitrary m, the boundary state description is expected to be valid in the time-scale shorter than that given by this inequality.

When the time t is large enough, we need to take the closed strings into consideration, and the whole picture will drastically change. For example, from the fact that the energy momentum tensor does not oscillate, people anticipated that the tachyon matter will not decay into closed string modes. However, since the coupling to the massive modes will exponentially grow, we can expect that the tachyon matter will start to emit massive closed string modes and eventually decay into massless particles. Moreover, there is a large back reaction of closed strings, especially from the massive fields. This is in contrast to the usual static D-brane cases, in which supergravity analysis essentially gives the precise low energy description. It would be interesting to analyze the system further using the equations of motion (4.52) and (4.53).

5 Relation to the S-brane Boundary States

In section 3, we explicitly computed the coefficients of some higher level states in the rolling tachyon boundary state and found that they become large at late times. However, there are special values of $\tilde{\lambda}$ at which the couplings remain finite. In the bosonic string case, since $D^j_{m,-m}(R)$ is invariant under $\tilde{\lambda} \to \tilde{\lambda} + 2$ and $\tilde{\lambda} \to 1 - \tilde{\lambda}$, the whole system has the same symmetries. Thus we can restrict $\tilde{\lambda}$ to $-1/2 \leq$

$\tilde{\lambda} \leq 1/2$. If we take $\tilde{\lambda}$ to be $\pm 1/2$ or 0, the coefficients of $\cosh(x^0)$ and $\sinh(x^0)$ in (3.35)–(3.37) vanish. This fact can be easily understood from the known behavior of the boundary state (2.7) in the Wick rotated theory at those values of $\tilde{\lambda}$. When $\tilde{\lambda}$ is zero, the boundary state becomes a static boundary state with the Neumann boundary condition, which represents a static D25-brane with a tachyon sitting on top of the maximum of its potential. For $\tilde{\lambda} = 1/2$ or $\tilde{\lambda} = -1/2$, the boundary state (2.7) represents an array of D-branes localized at $X = (2n+1)\pi$ or $X = 2n\pi$ with $n \in \mathbf{Z}$, respectively, in the Wick rotated theory [17, 19, 21]. Therefore, we obtain

$$| B \rangle_{X;\tilde{\lambda}=\pm 1/2} = \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n}\bar{\alpha}_{-n}\right) \sum_{n\in\mathbf{Z}} (\mp)^n e^{inX(0)} \,|\, 0 \,\rangle \,. \tag{5.55}$$

The zero mode part can be formally written as

$$\sum_{n\in\mathbf{Z}} (\mp)^n e^{inX(0)} |0\rangle \quad = \quad \left(\frac{1}{1 \pm e^{iX(0)}} + \frac{1}{1 \pm e^{-iX(0)}} - 1\right) |0\rangle, \tag{5.56}$$

whose inverse Wick rotation is given as

$$\left(\frac{1}{1 \pm e^{X(0)}} + \frac{1}{1 \pm e^{-X(0)}} - 1\right) |\, 0 \,\rangle \;=\; \lim_{\tilde{\lambda}\to\pm 1/2} f(X^0(0))\, |\, 0 \,\rangle \,, \tag{5.57}$$

where $f(x^0)$ is given in (2.11). In fact, we can show that the boundary state is of the form [4]

$$| B \rangle_{X^0;\tilde{\lambda}\sim\pm 1/2} \sim \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n}\bar{\alpha}_{-n}\right) f(X(0))\, |\, 0 \,\rangle \,. \tag{5.58}$$

up to the terms which are cancelled at $\tilde{\lambda} = \pm 1/2$. As a check, we can see in (2.10) and (3.35)–(3.37) that every term which does not include $f(x^0)$ vanishes in the limit $\tilde{\lambda} \to \pm 1/2$.

In the limit $\tilde{\lambda} \to 1/2$, the function $f(x^0)$ also vanishes and the whole boundary state becomes zero. This implies that the system is at the closed string vacuum, which corresponds to placing the tachyon at the minimum of its potential [1]. We should be more careful in taking the limit $\tilde{\lambda} \to -1/2$, since the function $f(x^0)$ is singular at $x^0 = \pm \log(-\sin(\tilde{\lambda}\pi))$ for $-1/2 < \tilde{\lambda} < 0$. This behavior of $f(x^0)$ with negative $\tilde{\lambda}$ has been interpreted in [1] that the tachyon is rolling on the wrong side of the hill where the potential is unbounded from below. Though it is not quite clear whether it is physically meaningful to take the limit $\tilde{\lambda} \to -1/2$, here we would like to point out a suggestive relation between the rolling tachyon boundary state in this limit and the S-brane boundary state constructed in [16].

[4] At each level, an infinite number of terms from $|\, j; \pm j, \pm j \,\rangle\rangle$ sum up to $f(x^0)$, leaving a finite number of terms that vanish in the limit $\tilde{\lambda} \to \pm 1/2$.

Because of the limit $\tilde{\lambda} \to -1/2$, the summation in (5.56) should be understood as being regularized as

$$\sum_{n\in\mathbf{Z}} e^{inx} = \lim_{\epsilon\to+0} \sum_{n\in\mathbf{Z}} e^{-\epsilon|n|} e^{inx} = \lim_{\epsilon\to+0} \left(\frac{1}{1-e^{-\epsilon+ix}} + \frac{1}{1-e^{-\epsilon-ix}} - 1 \right). \tag{5.59}$$

After the inverse Wick rotation x is replaced by $-ie^{i\delta}x^0$:

$$\lim_{\epsilon,\delta\to+0} \left(\frac{1}{1-e^{-\epsilon+e^{i\delta}x^0}} + \frac{1}{1-e^{-\epsilon-e^{i\delta}x^0}} - 1 \right). \tag{5.60}$$

The limit vanishes except at $x^0 = 0$. Integrating this function with the use of residue theorem, one finds that this function equals $2\pi i \delta(x^0)$. The boundary state now becomes

$$|\, B \,\rangle_{X^0;\tilde{\lambda}\to-1/2} = 2\pi i \exp\left(-\sum_{n=1}^{\infty} \frac{1}{n} \alpha^0_{-n} \bar{\alpha}^0_{-n} \right) \delta(X^0(0)) \,|\, 0 \,\rangle\,. \tag{5.61}$$

This is nothing but the S-brane boundary state, which represents the Dirichlet boundary condition in the time direction.

Similarly, in the superstring case, the system is invariant under $\tilde{\lambda} \to \tilde{\lambda} + 1$ and $\tilde{\lambda} \to -\tilde{\lambda}$, so we can restrict $\tilde{\lambda}$ to $0 \leq \tilde{\lambda} \leq 1/2$. The system does not evolve at $\tilde{\lambda} = 0, 1/2$. When $\tilde{\lambda}$ is zero, the system is the original D9-brane system. When $\tilde{\lambda} = 1/2$, the Wick rotated NS-NS boundary state considered here represents an array of D-branes placed at $x = \frac{2\pi}{\sqrt{2}}(n + \frac{1}{2})$:

$$|B\rangle_{X,\psi;\tilde{\lambda}=1/2} = \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n} \bar{\alpha}_{-n} + i\epsilon \sum_{r=1/2}^{\infty} \psi_{-r} \bar{\psi}_{-r} \right) \sum_{n\in\mathbf{Z}} (-1)^n e^{i\sqrt{2}nX(0)} |0\rangle. \tag{5.62}$$

Now if we shift $X \to X + \pi/\sqrt{2}$ and then inverse Wick rotate, one obtains as in the bosonic case

$$\sqrt{2}\pi i |x^0 = 0\rangle \tag{5.63}$$

for the zero mode part. This gives the boundary state for a spacelike brane. This suggests that the spacelike brane corresponds to the boundary interaction

$$S = \frac{i}{2} \int dt\, \psi^0(t) \cosh\left(\frac{X^0(t)}{\sqrt{2}} \right) \otimes \sigma_1. \tag{5.64}$$

Unfortunately, comparing this with the boundary interaction before the shift in X, one sees that this corresponds to an imaginary value of the tachyon field, which should be real to be physical. Thus, we conclude that the S-brane boundary state in superstring can be formally thought of as a rolling of imaginary tachyon, though its physical relevance is unclear.

6 Conclusions and Discussions

In this paper, we analyzed the higher level states in the rolling tachyon boundary state for bosonic string as well as superstring theory. We explicitly calculated some of the coefficients of the higher level states which correspond to the coupling between the tachyon matter and the massive closed string modes and found that they include terms that blow up like $\cosh(x^0)$ or $\sinh(x^0)$. We also argued that the S-brane boundary state given in [16] can be obtained as a special limit of the rolling tachyon boundary state.

These results suggest that the massive closed string modes play an important role in string theory, when we take into account the interaction between closed strings and open strings. The tachyon matter may decay through the massive closed string modes and this effect may become important in the cosmological scenarios using the rolling tachyon. Note that this is truly a stringy effect which cannot be seen using the low energy effective theory. It would be interesting to make a systematic analysis of the open-closed mixed system to obtain a better picture about the fate of the unstable D-brane.

It is well-known that in the presence of the space-time filling D-branes, the closed string background should be shifted in order to cancel the divergence due to the massless tadpole. This argument is usually given in the static configuration, in which the tachyon is not rolling. In the static case, the boundary state represents a constant source for the closed strings. This constant source is important for the massless fields, but not for the massive fields, since it only causes a small constant shift for the massive fields. On the other hand, in the case with time evolution, the massive fields could also receive a large back reaction. Moreover, since the boundary state carries nonzero energy, it will become possible to create some particles and transfer the energy to closed string modes. In our rolling tachyon case, since the coupling between the closed string and the open string will become large and we should take into account the huge back reaction of the closed string fields beyond the time range given in (4.54), the perturbative analysis of the string theory may not be practical. We hope we can make these issues clear in the near future.

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Note added: While preparing the paper, we received a related paper [23] in which some of the higher level states in the rolling tachyon boundary state in bosonic string theory are also calculated.

A Discrete Primaries and Ishibashi States

A.1 Bosonic String

Let us consider the theory of a single free boson X. The state $|\,j;m,m\,\rangle$ used in section 2 is a primary state of momentum $2m$ and conformal weight (j^2,j^2), where $j=0,1/2,1,\ldots$ and $m=-j,-j+1,\ldots,j$. It can be factorized into right and left moving parts as $|\,j;m,m\,\rangle = |\,j,m\,\rangle\overline{|\,j,m\,\rangle}$. The state $|\,j,m\,\rangle$ belongs to the spin j representation of the $SU(2)$ current algebra defined by

$$J^{\pm}=\oint\frac{dz}{2\pi i}\,e^{\pm 2iX_R(z)},\quad J^3=\oint\frac{dz}{2\pi i}\,i\partial X_R(z), \tag{A.1}$$

and it can be explicitly given as [24]

$$|\,j,j\,\rangle \;=\; e^{2ijX(0)}\,|\,0\,\rangle\,, \tag{A.2}$$
$$|\,j,m\,\rangle \;=\; N_{j,m}(J^-)^{j-m}\,|\,j,j\,\rangle\,, \tag{A.3}$$

where $N_{j,m}$ is the normalization constant. In practice, primaries are computed as the lowest null state in the Verma module of lower-weight primaries. Examples of the discrete primary states can be found in (3.28)-(3.32) as the first terms.

Given a primary state $|h\rangle$, the Virasoro Ishibashi state $|h\rangle\rangle$ is defined as [25]

$$|h\rangle\rangle \equiv |\,n\,\rangle\otimes\overline{U\,|\,n\,\rangle}, \tag{A.4}$$

where $\{|\,n\,\rangle\}$ is an arbitrary orthonormal basis of the space spanned by Virasoro descendant states of $|h\rangle$ and U is an antiunitary operator such that

$$UL_nU^{-1}=L_n,\;\; U|h\rangle=|h\rangle. \tag{A.5}$$

Such a state preserves half the conformal symmetries:

$$(L_n-\bar{L}_{-n})|h\rangle\rangle=0. \tag{A.6}$$

A.2 Superstring

Let us next consider the superconformal theory of (X,ψ) in the NS-NS sector. This theory has the symmetry of the NS-algebra with $\hat{c}=1$ and its antiholomorphic copy.

The super-Virasoro primaries $|j;m,m\rangle$ in section 2 are given in a similar way to the bosonic case. This time, j takes only integer values and the $SO(3)$ generators are

$$J^{\pm} = \oint \frac{dz}{2\pi i}\sqrt{2}\psi(z)e^{\pm i\sqrt{2}X_R(z)}, \quad J^3 = \oint \frac{dz}{2\pi i}\sqrt{2}\partial X_R(z), \tag{A.7}$$

which commute with super-Virasoro generators. Given a super-Virasoro primary $|h\rangle$, the super-Virasoro Ishibashi state $|h\rangle\rangle$ is defined as (A.4), where $\{|\,n\,\rangle\}$ is any orthonormal basis of the space spanned by the descendants $L_{-n_1}\cdots L_{-n_p}G_{-r_1}\cdots G_{-r_q}\,|\,h\,\rangle$. U is an antiunitary operator satisfying (A.5) as well as

$$UG_rU^{-1} = i\epsilon G_r(-1)^F, \tag{A.8}$$

where F counts the number of G_r acting on $|\,h\,\rangle$ [25]. The super-Virasoro Ishibashi state preserves half the superconformal symmetries:

$$(G_r - i\epsilon\bar{G}_{-r})\,|\,h\,\rangle\rangle = (L_n - \bar{L}_{-n})\,|\,h\,\rangle\rangle = 0. \tag{A.9}$$

B Matrix Elements for rotation R

Here, we list the relevant matrix elements of the $SU(2)$ rotation R in (2.8). See for example [19] for general matrix elements.

$$D^j_{\pm j,\mp j}(R) = (i\sin(\pi\tilde{\lambda}))^{2j} \quad (j = 0, 1/2, 1, \cdots), \tag{B.10}$$

$$D^1_{0,0}(R) = \cos(2\pi\tilde{\lambda}), \tag{B.11}$$

$$D^{3/2}_{\pm 1/2,\mp 1/2} = i\sin(\pi\tilde{\lambda})(3\cos^2(\pi\tilde{\lambda}) - 1), \tag{B.12}$$

$$D^2_{\pm 1,\mp 1}(R) = -\sin^2(\pi\tilde{\lambda})\cos(2\pi\tilde{\lambda}). \tag{B.13}$$

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