Additional considerations in the definition and renormalization of non-covariant gauges

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Abstract

In this work, we pursue further consequences of a general formalism for non-covariant gauges developed in an earlier work (hep-th/0205042). We carry out further analysis of the additional restrictions on renormalizations noted in that work. We use the example of the axial gauge $A_3 = 0$. We find that if multiplicative renormalization together with ghost-decoupling is to hold, the "prescription-term" (that defines a prescription) cannot be chosen arbitrarily but has to satisfy certain non-trivial conditions (over and above those implied by the validity of power counting) arising from the WT identitites associated with the residual

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gauge invariance. We also give a restricted class of solutions to these conditions.

The Yang-Mills theory in gauges other than the Lorentz gauges have been a subject of wide research [1, 2, 3]. These gauges have been used in a variety of Standard Model calculations and in formal arguments in gauge theories [1, 2] (as well as in string theories). As compared to the covariant gauges, these gauges have, however, not been fully developed [4]. Recently, an approach that gives the *definition* of non-covariant gauges in a Lagrangian path-integral formulation, which moreover is compatible with the Lorentz gauges by construction, has been given [5] and exploited [6] in various contexts such as those related to the axial, planar and the Coulomb gauges. A general path-integral framework, suggested by these results, that attempts to treat all these gauges formally but *rigorously* and hopefully *completely* (i.e. including the treatment of all their problems) was presented recently [7]. Several new observations regarding these gauges were recently made from such a framework by simple and direct considerations [7]. This work presents further results regarding the nature of the renormalization in axial gauges based on this formulation and the results in [7].

It was suggested in [7] that many of the ways of defining non-covariant gauges including the one based on ref. [5] can be formulated as a special case of the path-integral[1]

$$W[J,K,\overline{K};\xi,\overline{\xi}] = \int D\phi \; exp\{iS_{eff}[A,c,\overline{c},\psi] + \varepsilon O[\phi] + source - terms\} \quad (1)$$

[1] In the following, we use ϕ to generically denote all fields.

obtained by including *an* ϵ-term[2]. We recognize that in order that (1) is mathematically well-defined, this ϵ-term must, in particular, break the residual gauge invariance[3] completely. In addition, to keep the discussion general enough and to cover many of the ways suggested for dealing with these gauges, we do not necessarily limit ϵ to have dimension two in the following, nor do we restrict O to have local nature[4].

We note that the various prescriptions, say the Leibbrandt-Mandelstam (L-M) prescription for the light-cone gauges and the CPV for axial gauges etc, can be understood[5] as special cases of (1) [with rather complicated nonlocal O] and thus the following discussion should include these as special cases (For more details, see ref. [11]). Generally, the axial poles are treated by giving a way of interpreting the poles. They amount to replacing the naive propagator (with $\lambda \to 0$)

$$\frac{-i}{k^2}\left[g_{\mu\nu} - \frac{k_\mu \eta_\nu + k_\nu \eta_\mu}{\eta.k} + \frac{(\eta^2 + \lambda k^2)k_\mu k_\nu}{(\eta.k)^2}\right] \tag{2}$$

[that is obtained from the action by inverting the quadratic form in it for $k^2 \neq 0;\ \eta.k \neq 0$] by a modified propagator valid for all k. The latter, in turn, can be obtained by inverting the quadratic form in a modified action in A, that formally differs from the original action by quadratic $O(\varepsilon A^2)$ terms [11]; where ϵ is a parameter

[2]We may often require an ϵ-term of the form $\varepsilon \int d^4x O[A, c, \bar{c}; \varepsilon]$; i.e. with an ϵ-dependent O.

[3]A definition of the generalized residual gauge-invariance in the BRS-space has been given in [7].

[4]We do not however imply that *any* such ϵ-term will necessarily be appropriate to define a gauge theory *compatible with* the Lorentz gauges. Existence (and construction) of an ϵ-term which will serve *this purpose* is already known however. See e.g. [5] and 5^{th} of ref. [6].

[5]We however note some of the complications in the interpretation of *double* poles in CPV. See e.g. references [1, 2].

appearing in the pole prescription.

In this work, we wish to elaborate on one of the essential new observation made in [7] and to bring out further the power of that observation and to show that it leads to new conclusions. This observation pertains to the fact that a careful treatment of the renormalization of gauge theories, formulated by the path-integral in (1), ought also to take an account the import of the extra relations that follow from the presence of the residual gauge invariance as formulated by the IRGT[6] WT-identities in [7]. These were formulated in [7] using a generalized version of infinitesimal residual gauge invariance in the BRS space[7]. In this work, we wish to draw attention to several observations using these. We will show, in particular, that a prescription (such as those one considers commonly [1, 2]), given by any *fixed*[8] ϵO term, may lead to the IRGT WT-identities that are not compatible with the expected form of renormalization together with ghost decoupling. We shall also show that, if renormalization (in its expected form) with a given εO is possible[9] at all, we may generally *need renormalization of the prescription term* and this possibility, moreover, is not always necessarily consistent with the IRGT WT-identities. Later in this work, we shall formulate the conditions on O under which the latter interpretation becomes

[6]IRGT stands for the abbreviation of "infinitesimal residual gauge transformations" as formulated in [7].

[7]These, in particular, deal with the Green's functions with an external momentum in certain non-trivial domains (such as $\eta.k = 0$ for axial gauges) and their form is generally dependent on the specific "prescription term". The content of these *is not* covered by the *usual* BRST WT-identities. As shown in [7], however, the rigorous BRST WT-identity arising from (6), that takes into account the ϵ-term carefully, does cover IRGT WT-identities.

[8]As argued later, the usual ways of giving prescription for poles corresponds to the addition of a *fixed* term εO in the action.

[9]The renormalization scheme with a particular ϵO could, for example, be obstructed by a lack of validity of usual power counting [1, 2]

possible. As we shall later see, this observation does not look surprising when seen in the light of the present framework, where as suggested in [7], we may be required to deal with the *entire action* $S_{eff} + \varepsilon \int d^4xO$, *including the symmetry breaking term* εO while discussing renormalization. An obvious question at this point would be why one needs to care about the renormalization of the ϵ-term at all, if we are going to take the limit $\epsilon \to 0$ in the answer. This is suggested by the role of the ϵ-term and the observations made in [7] regarding it. In particular, we wish to draw attention to the fact that the limit $\epsilon \to 0$ in (1) is highly nontrivial as putting $\epsilon = 0$ in it leads to an ill-defined path-integral leading to very many unacceptable consequences [7]. We will elaborate on it further at a later stage.

We shall illustrate this point with the help of the axial gauge $A_3^\alpha = 0$. Consider the following set of defining properties and/or assumptions:

1. $A_3 = 0$: spatial axial gauge

2. Multiplicative renormalization of the type:

$$A_3 = \widetilde{Z}^{1/2} A_3^R; \quad A_\mu = Z_3^{1/2} A_\mu^R; \quad \mu = 0, 1, 2 \quad g = Z_1 g^R \tag{3}$$

   leading to renormalized Green's functions that are finite and well-defined in all momentum domains.

3. Ghost decoupling: so that we may assume that the ghost action can be taken as

$$S_{gh} \equiv \int d^4x \{ -\overline{c}^\alpha \partial_3 c^\alpha + i\varepsilon \overline{c}^\alpha c^\alpha \} \tag{4}$$

4. Path-integral formulation of axial gauges with a prescription for the gauge propagator poles implemented by a *fixed* quadratic term of the form[10]

$$- i\varepsilon \int d^4x O[A] = -i\varepsilon \int d^4x \int d^4y A^\alpha_\mu(x) a^{\mu\nu}(x,y) A^\alpha_\nu(y) \tag{5}$$

where $O[A]$ is (generally) a nonlocal operator.

In the following, we shall first show that the above set is not necessarily compatible *unless certain additional restrictions, (spelt out later) are satisfied by* $O[A]$.

We shall implement the $A_3 = 0$ gauge by the use of the Nakanishi-Lautrup "b"-field. This method has also been used in the early literature on axial gauges by Kummer [10]. We write the generating functional of Green's functions of the gauge field as

$$W[J] = \int DADbDcD\overline{c} \exp\left\{iS_{eff} + \varepsilon \int d^4x O[A] + i \int d^4x\; JA\right\} \tag{6}$$

where

$$S_{eff} = S_0 - \int d^4x\; b^\alpha A^\alpha_3 + S_{gh}$$

We note that

$$\int Db \exp\left\{-i \int d^4x b^\alpha A^\alpha_3\right\} \sim \prod_{\alpha,x} \delta(A^\alpha_3(x)) \tag{7}$$

has been used in dropping the A_3-dependence in the ghost-action (4). We may do the same in $O[A]$ and assume that it has no A_3-dependence.

We now consider the following infinitesimal transformations based on the residual

[10] We do not include ghosts in O since we have assumed ghost decoupling in 3 above.

gauge-invariance of the action (without the ϵ-term) (following [7], we call them the IRGT) with $\theta^\alpha = \theta^\alpha(x_0, x_1, x_2)$

$$\begin{aligned} A^\alpha_\mu(x) &\to A^\alpha_\mu(x) + \partial_\mu\theta^\alpha - gf^{\alpha\beta\gamma}A^\beta_\mu(x)\theta^\gamma(x) \\ X^\alpha(x) &\to X^\alpha(x) - \ gf^{\alpha\beta\gamma}X^\beta(x)\theta^\gamma(x) \\ X &\equiv A_3, b, c, \overline{c}, \partial_3 c \end{aligned} \tag{8}$$

We note that under this IRGT, S_{eff} and $\overline{c}c$ are invariant. We, now, carry out the IRGT in W of (6) and equate the change to zero. We thus obtain,

$$<< \int d^4x \left\{ J^\alpha_\mu(x) D^{\alpha\gamma}_\mu - i\varepsilon\Delta O^\gamma[A] \right\} \theta^\gamma(x) >> = \ 0 \tag{9}$$

where we have expressed the change in O under (8) as

$$\int d^4x\ O \to \int d^4x\ O + \int d^4x\ \Delta O^\gamma\theta^\gamma(x) \tag{10}$$

and we have defined, for any X,

$$<< X >> \equiv \int DADbDcD\overline{c}X \exp\left\{ iS_{eff} + \varepsilon \int d^4x O[A] + i \int d^4x\ JA \right\} \tag{11}$$

In view of the fact that $\theta^\gamma = \theta^\gamma(x_0, x_1, x_2)$ can be varied arbitrarily, we find that (9) leads us to,

$$0 \ = << \int dx_3 \left\{ D^{\alpha\gamma}_\mu J^{\gamma\mu}(x) + i\varepsilon\Delta O^\alpha[A] \right\} >>$$

$$= << \int dx_3 \left\{ \sum_{\mu\neq 3} [\partial^\mu J^\alpha_\mu(x) + gf^{\alpha\beta\gamma} J^\beta_\mu(x) A^{\gamma\mu}(x)] + i\varepsilon \Delta O^\alpha[A] \right\} >>$$

$$= \int dx_3 \left\{ \sum_{\mu\neq 3} \left[\partial^\mu J^\alpha_\mu(x) W[J] - igf^{\alpha\beta\gamma} J^\beta_\mu(x) \frac{\delta W[J]}{\delta J^\gamma_\mu(x)} \right] + i\varepsilon << \Delta O^\alpha[A] >> \right\} \quad (12)$$

In the above, we have dropped the term $\sim A_3$ using the $\delta-$function in (7). We remark that, as emphasized in [7], the last term can have a finite limit as $\epsilon \to 0$ (even in tree approximation) and its presence cannot just be ignored.

The above identity is over and above the *usual formal* BRST-WT identity (in which no account of the ϵ-term is taken) and as pointed out in [7], the renormalization has to be compatible (or made compatible) with it. We now discuss, in the light of (12), various possibilities regarding the pole prescription treatment . Before proceeding, we shall note that

1. If $O[A]$ is a local quadratic term $\sim A^\alpha_\mu(x) A^{\alpha\mu}(x)$ then $\Delta O^\alpha[A] \sim \sum_{\mu\neq 3} \partial^\mu A^\alpha_\mu$ is linear in A. We further note that under the assumption of the multiplicative renormalization, $Z_3^{-1/2} \Delta O^\alpha[A]$ is a finite operator.

2. If $\int d^4x O[A]$ is a non-local quadratic term $\int d^4x \int d^4y \ A^\alpha_\mu(x) a^{\mu\nu}(x,y) A^\alpha_\nu(y)$ then

$$\Delta O^\alpha[A] = 2 \int d^4y \left\{ -\partial^\mu_x a^{\mu\nu}(x,y) A^\alpha_\nu(y) + gf^{\alpha\beta\gamma} A^\beta_\mu(x) a^{\mu\nu}(x,y) A^\gamma_\nu(y) \right\}$$

   and has two terms: One is linear in A and the other is quadratic in A and is moreover a composite operator. We express this, in obvious notations, as $\Delta O[A] \equiv \Delta_1 O + \Delta_2 O$.

A SPECTATOR PRESCRIPTION TERM

It is usually assumed [1, 2]that the prescription for treating the axial gauge propagator is unaffected by renormalization and so is "ϵ". Thus, in this case, we are effectively assuming that the term $\epsilon O[A]$ is unaffected during the renormalization process. We shall call this case the " spectator prescription term".

In case one above of a local quadratic $O[A]$, the renormalizations of each of the three terms in (12) has been assumed to be multiplicative with scales: $Z_3^{-1/2}$; Z_1 and $Z_3^{1/2}$. These would be compatible only if $Z_1 = 1 = Z_3$. This would, of course, contradict a non-trivial value for β-function which is (expected to be) gauge-independent and hence must be the same as the Lorentz gauges.

The discussion for the case 2 above, is a special case of the discussion given below for the " renormalized prescription term" and we shall see that it is required that $O[A]$ must satisfy certain constraints. More comments are made later.

RENORMALIZED PRESCRIPTION TERM

We shall now explore, however, another (and a more general) possibility in which the (12) is made consistent with renormalization. We shall not insist on keeping the ϵ-term fixed in form, but allow it to be modified under the renormalization process. Thus we are allowing for a "renormalization of prescription". We shall now explore the restrictions on O, under which this is possible. We assume that renormalization replaces the $\varepsilon O[A]$ term by[11] say $\varepsilon\{O[A] + \widetilde{O[A]}\}$ (where $\widetilde{O[A]}$ depends on the regularization parameter). We need not any further treat ϵ as a parameter that can be rescaled, as the definition of $\widetilde{O[A]}$ can absorb effects of such a scaling. The (12) then

[11] With the assumption of ghost-decoupling, $O[A]$ cannot mix with a $\overline{c}^{\alpha}c^{\alpha}$ like operators involving ghosts.

is replaced by the renormalized version of the (12), viz.

$$\int dx_3 \left\{ \sum_{\mu\neq 3} \left[\partial^\mu J^\alpha_\mu(x) W[J;\varepsilon] - igf^{\alpha\beta\gamma} J^\beta_\mu(x) \frac{\delta W[J;\varepsilon]}{\delta J^\gamma_\mu(x)} \right] + i\varepsilon << \Delta O^\alpha[A] + \Delta \tilde{O}^\alpha[A] >> \right\} \tag{13}$$

Further analysis of (13) will have to be carried out under a restricted but "reasonable" set of assumptions spelt out later in various places. First of all, we shall assume that $O[A]$ is of net dimension two. We shall write, in obvious notations, $\Delta\widetilde{O[A]} \equiv \Delta_1\widetilde{O[A]} + \Delta_2\widetilde{O[A]}$; where the two pieces are respectively linear and quadratic in A[12]. We multiply the identity by $Z_3^{1/2}$ and express the equation in terms of the renormalized quantities[13]:

$$\begin{aligned} \int dx_3 \sum_{\mu\neq 3} \left[\partial^\mu J^{R\alpha}_\mu(x) W^R[J^R;\varepsilon] - iZ_1 Z_3^{1/2} g^R f^{\alpha\beta\gamma} J^{R\beta}_\mu(x) \frac{\delta W^R[J^R;\varepsilon]}{\delta J^{R\gamma}_\mu(x)} \right] \\ = -i\varepsilon Z_3 \int dx_3 << \Delta_1 O^\alpha[A^R] + \Delta_1 \tilde{O}^\alpha[A^R] >> \\ - i\varepsilon Z_3^{1/2} \int dx_3 << \Delta_2 O^\alpha[A] + \Delta_2 \tilde{O}^\alpha[A] >> \end{aligned} \tag{14}$$

Let us now discuss the above equation in the 1-loop approximation. We express $Z_3 = 1 + z_3$ etc. and look at the divergent part of (14). We find,

$$\begin{aligned} & i(z_1 + \tfrac{1}{2}z_3) g^R f^{\alpha\beta\gamma} J^{R\beta}_\mu(x) \tfrac{\delta W^R[J^R;\varepsilon]}{\delta J^{R\gamma}_\mu(x)} \\ = \; & i\varepsilon \textstyle\int dx_3 << z_3 \Delta_1 O^\alpha[A^R] + \Delta_1 \tilde{O}^\alpha[A^R] >> \end{aligned}$$

[12]This amounts to the *assumption* that the usual power counting works for the prescription at hand.

[13]We are going to assume that the renormalized Green's functions are finite functions of ϵ for ϵ in some interval $(0, \varepsilon_0)$. We require this especially since in axial gauges, it has been found that there can be finite contributions to diagrams from $\varepsilon \bullet \frac{1}{\varepsilon}$type terms (See e.g. Ref. [12]). In any case, the $\epsilon \to 0$ limit is to be taken only at the end of the calculation.

$$+\,i\varepsilon\int dx_3 << \Delta_2 O^\alpha[A] >>_i^{div} A_i^R + +i\varepsilon\tfrac{z_3}{2}\int dx_3 \Delta_2 O^\alpha[A^R] W^R[J^R]$$
$$+\,i\varepsilon\int dx_3 << \Delta_2 O^\alpha[A] >>_{mn}^{div} A_m^R A_n^R + i\varepsilon\int dx_3 << \Delta_2 \widetilde{O}^\alpha[A] >> \tag{15}$$

where we have expressed (in obvious notations) the linear and the quadratic terms in $<< \Delta_2 O[A] >>^{div}$ in one loop approximation[14]. We note that the usual BRST WT-identities, which hold when one stays away from external momenta satisfying $k.\eta = 0$, imply that, should the multiplicative renormalization as postulated be possible, we have

$$z_1 + \frac{1}{2} z_3 = 0 \tag{16}$$

We now compare the $O[A]$ and $O[A^2]$ terms on both sides :It leads us to two constraints:

$$0 = \int dx_3 << z_3 \Delta_1 O^{\alpha R}[A] + \Delta_1 \widetilde{O}^{\alpha R}[A] >> + \int dx_3 << \Delta_2 O^\alpha[A] >>_i^{div} A_i^R \tag{17}$$

$$0 = \int dx_3 \left[\frac{1}{2} z_3 \Delta_2 O^\alpha[A^R] W[0] + << \Delta_2 O^\alpha[A] >>_{mn}^{div} A_m^R A_n^R + << \Delta_2 \widetilde{O}^\alpha[A^R] >>\right] \tag{18}$$

These constraints determine the unknowns $\Delta_1 \widetilde{O}^\alpha[A^R]$ and $\Delta_2 \widetilde{O}^\alpha[A^R]$. In addition, there is the requirement that these can be written as the IRGT variation of some $\int d^4x \widetilde{O}[A]$. Moreover, this term $\varepsilon \int d^4x \widetilde{O}[A]$ when added to the action should make, say, the inverse propagator $\Gamma_{\mu\nu}(k,\varepsilon)$ in 1-loop finite. If there is a solution to these conditions, then only one can interpret this as the "renormalization of prescription".

[14] We are again *making an assumption* that the naive power-counting will work here also. Moreover, note that in evaluating $<< \Delta_2 O^\alpha[A] >>_{mn}^{div}$, we need to pay attention to the fact that there are *unrenormalized* coupling and fields in $\Delta_2 O^\alpha[A]$ that do contribute to the divergence.

To summarize, up-to 1-loop order, the IRGT WT-identity can be made consistent with renormalization in the assumed form by " renormalization of prescription" if :

[1] There exists an $\int d^4x\tilde{O}[A]$, such that its IRGT variation $\Delta\tilde{O}$ can be expressed as $\Delta\widetilde{O[A]} \equiv \Delta_1\widetilde{O[A]} + \Delta_2\widetilde{O[A]}$; where $\Delta_1\tilde{O}^\alpha[A^R]$ and $\Delta_2\tilde{O}^\alpha[A^R]$ satisfy the constraints (17) and (18) ;

[2] The counterterm $\varepsilon \int d^4x\tilde{O}[A]$ makes $\Gamma_{\mu\nu}(k,\varepsilon)$ finite;

[3] The usual power counting holds to this order for the renormalization of *nonlocal* operator $\Delta_2 O^\alpha[A]$.

(These spell out the sufficient conditions).

Finally, we note that the case 2 of a "spectator prescription term" is a special case of the above discussion with $\Delta\tilde{O}$ deleted. Thus, in this case, it is necessary that (17) and (18) hold with the terms $\widetilde{\Delta_1 O}$ and $\widetilde{\Delta_2 O}$ deleted.

We add some conclusions that follow from an analysis of the above conditions. The analysis of these conditions shows that:

1. Let us suppose that the (arbitrary) function $a_{\mu\nu}(x-y) = a_{\nu\mu}(y-x)$ in (5) be such that the power counting in momentum space *in terms of the external momenta* holds for the one-loop diagrams contributing to $<< \Delta_2 O^\alpha[A] >>_i$ and $<< \Delta_2 O^\alpha[A] >>_{mn}$ in the sense that the divergence in the first is a monomial in p of degree 1; and that in latter a monomial of degree zero. [Note: $\int d^4x\Delta_2 O^\alpha[A] \equiv 0$].

2. Then, in momentum space, $<< \Delta_2 O^\alpha[A] >>_i^{div}$ is of the form $p_\mu\Delta^{\mu\nu}$ (with $\Delta^{\mu\nu}$ a *constant* matrix); and the divergence *from the one-loop diagram* contributing

to $<< \Delta_2 O^{\alpha}[A] >>^{div}_{mn}$ vanishes on account of the fact that $\int d^4x \Delta_2 O^{\alpha}[A] \equiv 0$ for any $a_{\mu\nu}$.

3. In such a case, a solution to the conditions (17) and (18) exists provided $\Delta^{\mu\nu}$is symmentric for (μ,ν= 0,1,2) and is given by,

$$\int d^4x \widetilde{O[A]} = -z_3 \int d^4x O[A] + \frac{1}{2} \int d^4x \Delta'^{\mu\nu} A^{\alpha}_{\mu} A^{\alpha\mu} \tag{19}$$

   where $\Delta'_{\mu\nu} = \Delta_{\mu\nu}$ for all (μ, ν) except that $\Delta'_{3i} = \Delta_{i3}; i = 0, 1, 2$.

4. If we further assume that the divergence in $\Gamma_{\mu\nu}(p, \varepsilon)$ proportional to ϵ is , by the assumed validity of power counting in terms of external momenta, also a constant, and therefore independent of momentum p, then the condition [2] above is also satisfied by this solution.

The above solution (19) has been given under certain conditions sufficient for its existence. The main restriction on O seems to come from (1) the requirement of power-counting as enumerated above; and (2) the symmetry requirement on $\Delta^{\mu\nu}$ mentioned above in 2.

We shall note further that while the present analysis has arrived at its results using a specific form of path-integral definition of non-covariant gauges of (6), we expect an equivalent set of conclusions should follow from any other way of defining these gauges. This formalism has enabled us to see the existence of and to arrive at these conclusions in a easy and direct manner. No such analysis seems to have been carried out in the context of attempts at defining the non-covariant gauges [1, 2] in other ways.

A QUALITATIVE EXPLANATION

We shall now explain the results qualitatively. Consider the inverse propagator $\Gamma_{\mu\nu}$ for the gauge-field in one loop approximation. There is a contribution to the ϵ-dependent terms to this order. For momenta k, such that $k.\eta \neq 0$, the ϵ terms as a whole are negligible (as $\epsilon \to 0$). In this sector, the usual multiplicative renormalization does the job of making the inverse propagator finite, if ϵ-terms are ignored. Nonetheless, in the *3-dimensional* subspace $k.\eta = 0$, the quantity $k^\mu \Gamma_{\mu\nu} k^\nu$ obtained by taking the longitudinal projection of Γ has only ϵ-terms remaining (and the inverse of which tends to infinity as $\epsilon \to 0$*). These also receive divergences ; which need not generally be removed by the field-renormalization.* (Recall that there was no such subspace in the case of Lorentz gauge that needs to be worried about). One may be required to perform an extra renormalization on the ϵ-term (This may have to be checked in each case).

At this point, one may ask the justifiable question, as to whether the renormalization of the ϵ-term should matter at all, since we mean to take the ϵ to zero in the end!. Earlier, we have already made some comments based on [7]. In addition, we recall that there are several examples [1, 2] where the change of prescription has altered (1) the nature and the presence of divergences (2) value of gauge-invariant quantities[15]. This makes us strongly suspect that this sector in momentum space is important enough.

Now, S_{eff} is invariant under IRGT. Any prescription breaks the residual gauge

[15]Here, we recall that two different prescriptions ϵO and $\epsilon O'$ *may not be related by a residual gauge transformation,* and hence they need not lead to identical physical results. Moreover, neither of these need coincide with the Lorentz gauge result for analogous reasons.

invariance in a particular manner. It is not obvious that the physical quantities so calculated using it will be gauge-independent. This is controlled by the behavior of the path-integral under infinitesimal residual gauge transformations as formulated by IRGT WT-identities[16]. Under IRGT, the path-integral changes solely due to the "symmetry breaking" term ϵO in addition to the source term. The form of divergence in the variation in the source term is restricted by the *assumptions* we made in the beginning. This restriction then becomes imposed on the divergences that can arise from the variation of the ϵ-term via IRGT WT-identity (and such terms can have non-vanishing contributions as $\epsilon \to 0$ [7]). These are additional restrictions on O, and it not *a priori* obvious that they will be obeyed.

SUMMARY AND CONCLUSIONS

We shall now summarize our conclusions. We considered the formalism for non-covariant gauges presented in [7], where the "prescription" is imposed via an $\varepsilon \int d^4xO[A]$ term added to the action. We found this formalism lead us in an easy manner to an additional consideration that is required in the definition and renormalization of these gauges. We illustrated this for the $A_3 = 0$ gauge. This fact, which was brought out in [7], has been further elaborated and analyzed here. We see that the usual expectations of multiplicative renormalization together with ghost decoupling are not automatically compatible with every prescription term $\varepsilon \int d^4xO[A]$; there are additional constraints that have to be satisfied further by it (which are implied by the IRGT WT-identities). We also pointed out the need to have to deal with renormalization of ϵ-terms carefully. These considerations do not seem to have

[16] As mentioned earlier, these have been shown to be contained in the BRST-identities for the *net* action including the ϵ-term in [7].

been taken into account so far in attempts to define noncovariant gauges.

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[3] See references 5-9 for some recent works; also see the references therein.

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