




# 3-D INTERACTING CFTs AND GENERALIZED HIGGS PHENOMENON IN HIGHER SPIN THEORIES ON AdS

L. Girardello , M. Porrati <sup>b,c</sup>, A. Zaffaroni 

 *Università di Milano-Bicocca and INFN  
Piazza della Scienza 3, I-20126 Milano (Italy)*

 *Department of Physics, New York University  
4 Washington Pl., New York NY 10012 (USA)*

 *Scuola Normale Superiore  
Piazza dei Cavalieri 7, I-56126 Pisa (Italy)*

## Abstract

We study a duality, recently conjectured by Klebanov and Polyakov, between higher-spin theories on  $AdS_4$  and  $O(N)$  vector models in 3-d. These theories are free in the UV and interacting in the IR. At the UV fixed point, the  $O(N)$  model has an infinite number of higher-spin conserved currents. In the IR, these currents are no longer conserved for spin  $s > 2$ . In this paper, we show that the dual interpretation of this fact is that all fields of spin  $s > 2$  in  $AdS_4$  become massive by a Higgs mechanism, that leaves the spin-2 field massless. We identify the Higgs field and show how it relates to the RG flow connecting the two CFTs, which is induced by a double trace deformation.

---

e-mail: girardello@mib.infn.it, massimo.porrati@nyu.edu, alberto.zaffaroni@mib.infn.it

# 1 Introduction

Theories with an infinite number of massless higher-spin gauge fields (HS) have a long story. Recently, they have been reexamined by several authors [1]. One of the reasons for this resurgence of interest is that these theories are candidates for a semi-classical treatment of the small tension limit of string theory. The important observation that higher-spin theories can be consistently formulated in Anti-de-Sitter space [2] also suggests that they are useful in the context of the AdS/CFT correspondence. Weakly coupled gauged theories contain an infinite number of almost-conserved currents that may be described by a dual HS theory in Anti de Sitter space. While our understanding of the description of the weak coupling limit of four dimensional YM in terms of higher-spin theories is still elusive, some progress has been made for certain three dimensional conformal field theories [3]. The specific example of ref. [3] deals with three-dimensional  $O(N)$  vector models. The singlet sector of the  $O(N)$  theories contains an infinite number of conserved currents of even spin in the large  $N$  limit. In [3], it was conjectured that the singlet sector of this theory is dual to that of the higher spin theories in  $AdS_4$  studied by Vasiliev [4], which consists of a single Regge trajectory of even spin<sup>1</sup>. Two different three-dimensional conformal field theories were considered in [3], the free  $O(N)$  model and the infrared fixed point that can be obtained by perturbing the free theory with a relevant double trace operator. Following the general description of RG flows induced by double-trace operators [6, 7, 8], the authors of [3] also conjectured that both theories are described by the same Vasiliev Lagrangian, but with a difference consisting in the choice of boundary condition for a certain field. As shown in [7], and further studied in [9, 10], the dual description of the RG flow induced by double-trace deformation of the boundary CFT is unusual. Instead of changing the 4-d background, this flow leaves the geometry unchanged, at least at tree level in the bulk theory, but it changes the boundary behavior of a certain bulk field.

This raises the question that we want to address in this paper. When the bulk Lagrangian is dual to the  $O(N)$  model at the IR fixed point, the higher spin currents are conserved only in the large  $N$  limit. Because of the standard relation between conformal dimension in 3-d, and mass in  $AdS_4$ , an infinite number of higher spins should become massive when  $1/N$  corrections are included. All spins should instead remain massless in the description of the free UV theory. This raises an interesting puzzle. As we said, the two CFTs are described by the same bulk Lagrangian and they only differ by a choice of boundary condition of a certain field. How can such a change of boundary conditions in a (scalar) field induce masses for all particles of spin higher than 2? The answer to this question turns out to be surprisingly similar to a case recently studied by one of the

---

<sup>1</sup>Further work on the subject can be found in [5].

authors [11]; namely, a graviton coupled to conformal matter in  $AdS_4$ . There, one can show that, when matter is given non-standard boundary conditions, it can form a *bound state* that acts as the Goldstone vector for the spin 2 field. In other words, in that case the graviton gets a mass through a one-loop effect. In this note, we show that a similar mechanism can give mass to all higher-spin fields in the dual of the  $O(N)$  model at the IR fixed point. The mechanism is intrinsically one-loop in the bulk theory. That explains naturally why the masses of the higher-spin fields are  $O(1/N)$ . Differently from the case studied in ref. [11], here the boundary conditions of the bulk fields leave the spin-2 field massless, at the fixed points of the the double-trace RG flow. We finally show that mass generation can only occur, for spin  $s > 2$ , when the  $AdS_4$  theory is dual to the  $O(N)$  model at the IR fixed point.

## 2 The AdS/HS Correspondence

To be self-contained, in this section we briefly review the details of the correspondence conjectured in [3]. The  $O(N)$  model is formulated in terms of a three-dimensional scalar transforming in the vectorial representation of  $O(N)$ , with Lagrangian:

$$\mathcal{L} = \int d^3x \left[ \partial\phi^a \partial\phi^a + \frac{\lambda}{2N} (\phi^a \phi^a)^2 \right], \quad (1)$$

where  $a = 1, \dots, N$ . The theory has two fixed points. There is an ultraviolet, free fixed point at  $\lambda = 0$ , and an interacting infrared fixed point [12]. The free UV theory has an infinite number of conserved currents. Restricting to operators that are singlets of  $O(N)$  and single trace <sup>2</sup>, we find a conserved current for each even spin. We can schematically write it as

$$J_{\mu_1, \dots, \mu_s} = \phi^a (\overleftrightarrow{\partial})^s \phi^a - \text{traces}. \quad (2)$$

The IR theory is instead interacting and the currents in Eq. (2) are not conserved. We can reasonably assume that the only conserved current in the IR CFT is the stress-energy tensor. Among the non-singlet operator there are also other conserved currents; the Noether currents of the (global)  $O(N)$  symmetry. However, it is known [12] that the currents in Eq. (2), for  $s > 2$ , also have canonical dimension in the large  $N$  limit <sup>3</sup>. Therefore, for  $N = \infty$ , the IR theory too has an infinite number of conserved currents. The currents do acquire anomalous dimensions at order  $1/N$ . The conjecture formulated in [3] states that the singlet sector of both theories has, in the large  $N$  limit, a dual description in terms of a minimal bosonic HS theory containing one massless gauge field

<sup>2</sup>We follow the misuse of the term single trace introduced in [3].

<sup>3</sup>For more recent references on the subject see [13]

for each even spin [4]. In this correspondence,  $\Lambda$  must be identified with the inverse cosmological constant of the HS theory.

The description of the two fixed points differs only by a choice of boundary conditions. Let  $\Sigma$  be the bulk scalar dual to  $(\phi^a \phi^a)$ .  $(\phi^a \phi^a)$  has dimension 1 in the UV and dimension  $2 + O(1/N)$  in the IR [12].  $\Sigma$  is a scalar field with mass  $m^2 = -2$  at large  $\Lambda$  (in units of the cosmological constant).  $\Sigma$  is, therefore, a *conformally* coupled scalar field. The UV and IR conformal dimensions of the operator correspond, respectively, to the two roots of the equation

$$m^2 = \Delta(\Delta - 3). \quad (3)$$

The quantization of  $\Sigma$  is subtle, because both roots  $\Delta_{\pm}$  of the previous equation satisfies the unitary bound in three dimensions. The analysis of such cases has been performed in [14]. The bulk theory corresponding to the assignment  $\Delta = \Delta_{+}$  differs from the other,  $\Delta = \Delta_{-}$ , by boundary conditions. Namely, if  $\Sigma$  has asymptotic behavior

$$\Sigma \sim \alpha z^{\Delta_{-}} + \beta z^{\Delta_{+}}, \quad \Delta_{+} > \Delta_{-}, \quad (4)$$

where  $z$  is the AdS radial coordinate, the two possible quantizations are obtained by interchanging the role of  $\alpha$  and  $\beta$  [14]. In the UV,  $(\phi^a \phi^a)$  has dimension  $\Delta_{-} = 1$  and it is quantized with boundary condition  $\alpha = 0$ . In the IR  $(\phi^a \phi^a)$  has dimension  $\Delta_{+} = 2$  in the large  $\Lambda$  limit, and it is quantized with boundary condition  $\beta = 0$ .

In [3] the interesting observation was made that the two CFTs are connected by a RG flow induced by the double-trace operator  $(\phi^a \phi^a)^2$ , which, in the UV, is a dimension-two relevant operator. An analysis of flows induced by double-trace operators was carried out in [6, 7, 8]. The deformation by a double-trace operator only modifies the boundary condition for the bulk field  $\Sigma$  dual to  $(\phi^a \phi^a)$ . According to [7], the boundary condition on  $\Sigma$  to be imposed along the flow is:

$$\alpha = \lambda \beta. \quad (5)$$

We see that in the two limiting cases,  $\lambda = 0$  and  $\lambda = \infty$ , we recover the two different boundary conditions describing, respectively, the UV and the IR fixed points.

### 3 A Generalized Higgs Effect

It is intriguing to notice that the same Lagrangian gives a semi-classical description of two different fixed points, one of which is free while the other is interacting. In particular, an obvious question can be raised. Since the currents in Eq. (2) are not conserved in the IR at finite  $\Lambda$ , we expect that the corresponding higher spin fields in the bulk acquire a mass of order  $1/N$ , when quantum (loop) corrections are included. We want now to

prove that this is indeed the case. Namely, that in the bulk Lagrangian describing the IR fixed point, a generalized Higgs effect may take place, which gives mass to all fields of spin greater than two. We will also show that no Higgs effect is expected in the Lagrangian describing the UV fixed point, and that boundary conditions alone are responsible for the different behaviors of the two theories.

In AdS, a spin  $\mathbf{s}$  field can acquire a mass by “eating” a single massive field of spin  $\mathbf{s}-1$ , by a Higgs-like mechanism. To describe this phenomenon properly, recall that the representations of the 3-d conformal group,  $SO(3,2)$ , which is also the isometry group of  $AdS_4$ , are labeled by their quantum numbers under the maximal compact subgroup  $SO(3) \times U(1)$ : the spin  $\mathbf{s}$  and the conformal weight  $\Delta$ . A representation  $D(\Delta, \mathbf{s})$  satisfies a shortening condition when  $\Delta = \mathbf{s} + 1$ , and corresponds to a conserved current in the CFT and a massless field in  $AdS_4$ . A massive spin  $\mathbf{s}$  representation of the conformal group decomposes in the massless limit as [15]

$$D(\Delta, \mathbf{s}) \xrightarrow{\Delta \rightarrow \mathbf{s}+1} D(\mathbf{s}+1, \mathbf{s}) \oplus D(\mathbf{s}+2, \mathbf{s}-1). \quad (6)$$

The representation  $D(\mathbf{s}+2, \mathbf{s}-1)$  is the Goldstone field. Since in  $AdS_4$  the energy spectrum is discrete, two gauge fields can form a bound state with the quantum number of the Goldstone field (even when they are free!). In the presence of an appropriate trilinear coupling, a spin  $\mathbf{s}$  field can then acquire mass through radiative corrections. Let us stress that this phenomenon cannot occur in flat space where the spectrum is continuous. This analysis was already performed in [11] in the case of a graviton in  $AdS_4$ , coupled to a conformal scalar.

Let us denote with  $W_s \equiv W_{\mu_1, \dots, \mu_s}$  the spin  $\mathbf{s}$  gauge field. In a Lagrangian as those proposed by Vasiliev [4], we expect many trilinear couplings between gauge fields of different spin. Some involve the field  $\Sigma$  and some do not. Those without  $\Sigma$  can be schematically written as  $W^s \partial^k (W_{s_1} W_{s_2})$  with  $\mathbf{s}_1 + \mathbf{s}_2 + k = \mathbf{s}$ , with derivatives arbitrarily distributed among the gauge fields. They cannot be responsible for the Higgs mechanism. Even if the product of representations of spin  $\mathbf{s}_1$  and  $\mathbf{s}_2$  contains a mode in the representation  $D(\mathbf{s}+2, \mathbf{s}-1)$ , the latter would have wrong parity for being a Goldstone field; let us see why. Since we are interested in a one loop effect, we can neglect  $1/N$  corrections to the dimensions of our fields. All the  $W_s$  are thus massless in the large  $N$  limit and, therefore, have dimension  $\mathbf{s}+1$ . The ground state of the would-be Goldstone representation has conformal dimension  $\mathbf{s}+2$ . It is obtained from the lowest weight state of  $D(\mathbf{s}_i+1, \mathbf{s}_i)$ , which has dimension  $\mathbf{s}_i+1$ , by acting on it with  $k$  raising operators of the group  $SO(3,2)$ . Since the parity of a genuine spin  $\mathbf{s}_i$  field is  $(-1)^{s_i}$  and the parity of a raising operator is  $-1$ , this mode has parity  $P = (-1)^{s_1+s_2+k} = (-1)^{\mathbf{s}}$ , which is the wrong one for a spin  $(\mathbf{s}-1)$  gauge field.

The field  $\Sigma$  is what describes the RG flow, and it is, moreover, the only one to change under it, to leading order in  $1/N$ . So, we expect it to appear in the couplings needed to

give mass to our high-spin fields. Recall that  $\Sigma$  only has different boundary conditions at the two fixed points of the RG flow. We can easily write a trilinear coupling of the form

$$W^{\mu_1, \dots, \mu_s} W_{\mu_1, \dots, \mu_{s-2}} \partial_{\mu_{s-1}} \partial_{\mu_s} \Sigma, \quad (7)$$

where, for simplicity, we chose a specific distributions for the derivatives. Such coupling can be certainly reconstructed from the three point function of free fields in the CFT. It is also reminiscent of the equation for the conservation of the currents in Eq. (2). To see this, write the Lagrangian Eq. (1) in terms of an auxiliary field  $\sigma$ ,

$$\mathcal{L} = \int d^3x \left[ \partial \phi^a \partial \phi^a + \sigma (\phi^a \phi^a) - \frac{N \sigma^2}{2\lambda} \right], \quad (8)$$

and use the equations of motions  $\sigma = \lambda(\phi^a \phi^a)/N$ ,  $\square \phi^a = \sigma \phi^a$ . Then, the divergence of the current can be rewritten, schematically, as

$$\partial^\mu J_{\mu, \mu_1, \dots, \mu_{s-1}} \sim J_{\mu_1, \dots, \mu_{s-2}} \partial_{\mu_{s-1}} \sigma|_{ST}, \quad (9)$$

where the subscript means that the right hand side is projected on the symmetric-traceless part.

The coupling in Eq. (7) can give mass to the spin  $s$  fields, by a one-loop diagram, only when the product of the representations to which  $W_{\mu_1, \dots, \mu_{s-2}}$  and  $\Sigma$  belong contains the Goldstone representation  $D(s+2, s-1)$ . To leading order in  $1/N$ ,  $\Sigma$  has dimension  $\Delta=1$  in the UV, but dimension  $\Delta=2$  in the IR, while all the  $W_s$  have always dimension  $s+1$ . We also have [15]

$$D(s-1, s-2) \oplus D(\Delta, 0) = \sum_{S=0}^{\infty} \sum_{n=0}^{\infty} D(\Delta + S + s + n - 1, s + S - 2). \quad (10)$$

This equation shows that a mode  $D(s+2, s-1)$ , with the right quantum numbers to be the Goldstone, appears for both values of  $\Delta$ . However, it is easy to check that the candidate Goldstone has the same parity of the would-be massive field  $W_s$  only when  $\Delta=2$ <sup>4</sup>. We conclude that, only when  $\Sigma$  is quantized with conformal weight 2 in the large  $N$  limit, a Higgs mechanism is possible.

We must also check that the graviton remains massless: in a CFT, a singlet conserved current corresponding to the stress-energy tensor always exists. It was already noticed in [11] that the a graviton coupled to a conformal scalar can acquire mass only if the boundary conditions on the scalar make it belong to the reducible representation  $D(1, 0) \oplus$

---

<sup>4</sup>In the case  $\Delta=1$ , we create the lowest weights of  $D(s+2, s-1)$  by applying two raising operators,  $L^+$ , to the product of the lowest weight of  $D(s-1, s-2)$  and  $D(1, 0)$ , thus obtaining a field of parity  $P = (-1)^s$ .  $D(s+2, s-1)$  is thus a *pseudo-spin*  $(s-1)$  field.

$D(2, 0)$ . In our case, the scalar belongs to the  $D(1, 0)$  in the UV, and to the  $D(2, 0)$  in the IR, so that no Higgs mechanism is expected. We can see this explicitly from the decomposition

$$D(\Delta', 0) \oplus D(\Delta, 0) = \sum_{S=0}^{\infty} \sum_{n=0}^{\infty} D(\Delta + \Delta' + S + 2n, S), \quad (11)$$

that replaces Eq. (10) in the case  $s = 2$ . No Goldstone representation  $D(4, 1)$  is contained in this formula for  $\Delta = \Delta'$ , and  $\Delta, \Delta'$  equal to either 1 or 2.

## 4 Conclusions

The duality between  $O(N)$  critical vector models and HS theories *à la* Vasiliev is still in its infancy. A challenge in establishing it firmly is that while the 3-d CFT is considerably simpler than in the adjoint case, the 4-d AdS dual is much more complicated than semiclassical supergravity. In this paper we furthered the study of that duality by showing how to explain a puzzling feature of the IR (interacting) fixed point of the  $O(N)$  model. There, almost all higher-spin currents that were conserved in the UV acquire anomalous dimensions. In the AdS dual, this means that almost all massless fields of the HS theory become massive. To interpret this effect as a Higgs phenomenon, one has to explain how to reconcile it with the fact that the (double trace) perturbation of the UV theory flowing into the IR fixed point does not change the AdS background, to leading order in  $1/N$ . In this paper, we showed that a radiative Higgs effect, where the Goldstone particle is composite, can solve this puzzle. We performed a group theoretical analysis showing that only particles with spin  $s > 2$  can become massive, and only at the IR fixed point. It would be interesting and important to explicitly compute the one-loop self-energy diagram for all particles in the dual HS theory [4], to check this phenomenon explicitly and quantitatively. It may also be possible to extend our analysis to a model that contains some (or all) the non-singlet currents of  $O(N)$ , or to other examples of RG flows in 3d, like those discussed in [16].

## Acknowledgements

We would like to thank D. Anselmi for fruitful conversations. A.Z. would like to thank the Scuola Normale Superiore in Pisa for its kind hospitality while this work was done. L.G. and A.Z. are partially supported by INFN and MURST under contract 2001-025492, and by the European Commission TMR program HPRN-CT-2000-00131. M.P. is supported in part by NSF grant PHY-0070787.

## References

- [1] D. Francia and A. Sagnotti, Phys. Lett. B **543**, 303 (2002) [hep-th/0207002]; P. Haggi-Mani and B. Sundborg, JHEP **0004**, 031 (2000) [hep-th/0002189]; B. Sundborg, Nucl. Phys. Proc. Suppl. **102**, 113 (2001) [hep-th/0103247]; E. Sezgin and P. Sundell, JHEP **0109**, 036 (2001) [hep-th/0105001]; A. Mikhailov, hep-th/0201019; E. Sezgin and P. Sundell, Nucl. Phys. B **644**, 303 (2002) [hep-th/0205131]; E. Sezgin and P. Sundell, JHEP **0207**, 055 (2002) [hep-th/0205132]; J. Engquist, E. Sezgin and P. Sundell, Class. Quant. Grav. **19**, 6175 (2002) [hep-th/0207101]; M. A. Vasiliev, Int. J. Mod. Phys. D **5**, 763 (1996) [hep-th/9611024]; D. Anselmi, Nucl. Phys. B **541**, 323 (1999) [hep-th/9808004]; D. Anselmi, Class. Quant. Grav. **17** 1383 (2000) [hep-th/9906167].
- [2] E. S. Fradkin and M. A. Vasiliev, Nucl. Phys. B **291**, 141 (1987); E. S. Fradkin and M. A. Vasiliev, Phys. Lett. B **189**, 89 (1987).
- [3] I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **550**, 213 (2002) [hep-th/0210114]
- [4] M. A. Vasiliev, hep-th/9910096.
- [5] T. Leonhardt, A. Meziane and W. Ruhl, hep-th/0211092.
- [6] O. Aharony, M. Berkooz and E. Silverstein, JHEP 0108:006, 2001, hep-th/0105309.
- [7] E. Witten, and AdS/CFT correspondence,” hep-th/0112258.
- [8] M. Berkooz, A. Sever and A. Shomer JHEP 0205:034, 2002, hep-th/0112264.
- [9] S. S. Gubser and I. Mitra, hep-th/0210093.
- [10] S. S. Gubser and I. R. Klebanov, hep-th/0212138.
- [11] M. Porrati, JHEP **0204**, 058 (2002) [hep-th/0112166].
- [12] K. G. Wilson and J. B. Kogut, Phys. Rept. **12**, 75 (1974).
- [13] K. Lang and W. Ruhl, Nucl. Phys. B **400** 597 (1993); A. Petkou, Annals Phys. **249**, 180 (1996), hep-th/9506116; Phys. Lett. B **359**, 101 (1995), hep-th/9410093.
- [14] I. R. Klebanov and E. Witten, Nucl. Phys. B **556**, 89 (1999) [hep-th/9905104].
- [15] W. Heidenreich, Of SO(3,2) And SO(4,2),” J. Math. Phys. **22**, 1566 (1981).
- [16] D. Anselmi, hep-th/0210123.