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Anomalies on orbifolds with gauge symmetry breaking

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Abstract

We consider the breaking of $5D$ SUSY $G = SU(N+K)$ gauge symmetry into $H = SU(N) \times SU(K) \times U(1)$ on an orbifold $S^1/(Z_2 \times Z_2')$. There appear two independent fixed points: one respects the full bulk gauge symmetry G while the other contains only the unbroken gauge symmetry H. In the model with one bulk $(N+K)$-plet, giving a K-plet as the zero mode, we show that localized non-abelian gauge anomalies appear at the fixed points: the H^3 gauge anomalies are equally distributed on both fixed points and the $H-(G/H)-(G/H)$ gauge anomalies contribute only at the fixed point with G symmetry. We also find that when we add a brane $\bar{K}$-plet in the field theoretic limit of a bulk field, the theory with the unbroken gauge group can be consistent up to the introduction of a bulk non-abelian Chern-Simons term. Moreover, for our set of bulk and brane fields, we find that a nonzero log Fayet-Iliopoulos term is radiatively generated at the fixed point with H, which gives rise to the localization of the bulk zero mode at that fixed point and modifies the massive modes without changing the KK mass spectrum.

Keywords: Gauge unified theories, Orbifolds, Gauge anomalies, Chern-Simons term, Fayet-Iliopoulos term.
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1 Introduction

Recently the orbifold unification models in the existence of extra dimensions have drawn much attention due to their simplicity in performing the gauge symmetry breaking and the doublet-triplet splitting at the same time. The unwanted zero modes appearing in the unification models are projected out by boundary conditions in the extra dimension, i.e, they get masses of order of the compactification scale. For instance, the Minimal Supersymmetric Standard Model(MSSM) fields were obtained in the 5D SUSY SU(5) model where the extra dimension is compactified on a simple orbifold $S^1/(Z_2\times Z_2')$[1, 2, 3]. The idea was also taken in the model with the 5D $SU(3)$ electroweak unification with the TeV-sized extra dimension[4, 5], the possibility of which was first considered in the context of the string orbifolds[6]. In the orbifold with gauge symmetry breaking, in general, in addition to the fixed point where the bulk gauge symmetry is operative, there exists a fixed point where only the unbroken gauge group is respected[2]: for instance, $G_{SM}=SU(3)\times SU(2)\times U(1)$ in the case with the 5D SUSY SU(5) model on $S^1/(Z_2\times Z_2')$. Therefore, we can put a multiplet(so called a brane field) at that fixed point allowed by the representation of the unbroken gauge group. In the more realisic model constructions, it has been proposed various possibilities of having incomplete multiplets located at the orbifold fixed points (or branes) with an unbroken gauge group[2, 3, 4, 5]. For instance, in the 5D SU(5) GUT on $S^1/(Z_2\times Z_2')$, it has been shown that the $s-\mu$ puzzle can be understood from the introduction of a split multiplet for **10** of the second generation[3] while the top-bottom mass hierarchy can be also explained with one Higgs in the bulk and the other Higgs at the brane[3, 5]. Moreover, introducing incomplete multiplets for the quark setor is indispensible in the 5D SU(3) electroweak unification on $S^1/(Z_2\times Z_2')$[4, 5].

However, we should be careful about the anomaly problem in introducing incomplete multiplets since localized gauge anomalies could appear on the independent orbifold boundaries[7, 8, 9, 10, 5, 11]. It has been shown[12] that the abelian anomalies coming from a bulk field in 5D are equally distributed at the fixed points by the half of its 4D anomaly while the bulk Chern-Simons term[13] plays a role in conveying localized anomaly at one fixed point to the other fixed point. So, the 4D anomaly cancellation for zero modes is sufficient for consistency. This is also the case for the 5D non-abelian theory on the orbifold with gauge symmetry breaking. As far as the zero modes obtained after compactification are of anomaly-free combination for the 4D gauge group as in the Minimal Supersymmetric Standard Model(MSSM), there was an examplary proof in 5D $SU(5)$ and $SU(3)$ orbifold unification models[5] that there is no localized anomalies irrespective of the locations of the zero modes up to the introduction of a bulk Chern-Simons term.

Besides the localized gauge anomalies, there is a possibility of having the localized Fayet-Iliopoulos terms at the fixed points for the case with a $U(1)$ factor included in the remaining gauge group after orbifold compactification[14, 9, 11, 17]. In the 5D $U(1)$ gauge theory on S^1/Z_2, it has been shown that non-vanishing localized Fayet-Iliopoulos terms are radiatively induced from a general set of charged brane and bulk fields[11]. In that case, even if there is no effective 4D FI term for the non-anomalous spectrum of zero modes, the supersymmetric condition is satisfied only if the real adjoint scalar in the vector multiplet develops a vacuum expectation value to compensate localized FI terms[15, 16, 9, 11, 17]. Particularly, for one pair of brane and bulk fields with opposite charges, *quadratically divergent* FI terms localized at fixed points give rise to not only the dynamical localization of the bulk zero mode but also make the bulk massive modes decoupled. Even in the case where quadratically divergent FI terms are cancelled locally by introducing a brane field with the half charge at each fixed point, there remain *logarithmically divergent* FI terms, which does not modify either the 4D supersymmetry or the KK spectrum of the bulk field but still gives rise to the localization of the zero mode[11].

In this paper, we do the explicit calculation of the gauge anomalies in the case with the gauge symmetry breaking on orbifolds. For our purpose of a general application, we consider the 5D SUSY $G = SU(N+K)$ gauge theory on $S^1/(Z_2 \times Z_2')$, which is reduced to the 4D SUSY $H = SU(N) \times SU(K) \times U(1)$ gauge theory at the zero mode level by orbifold boundary conditions. In this case, there are two fixed points with different gauge groups: the full gauge symmetry G at $y = 0$ and the unbroken gauge symmetry H at $y = \pi R/2$. In the existence of a bulk hypermultiplet in the fundamental representation of the bulk gauge group, we only leave a K-plet among the GUT multiplet as the zero mode by the boundary conditions consistent with the gauge symmetry breaking. Then, we obtain the localized non-abelian anomalies from a bulk fermion by decomposing the 5D fermions and gauge fields in terms of the bulk eigenmodes and using the standard results of 4D anomalies. As a result, the H^3 gauge anomalies are equally distributed at the fixed points while the $H-(G/H)-(G/H)$ gauge anomalies are located only at $y = 0$.

In addition to the bulk field giving rise to a K-plet as the zero mode, we add a $\bar{K}$-plet at the fixed point $y = \pi R/2$. The brane $\bar{K}$-plet can be realized from a bulk field of $\overline{(N+K)}$ with a negative infinite kink mass in a gauge invariant way. In the case with $N + K = 5(3)$ and $K = 2$, the matter zero modes in the model correspond to two Higgs fields in MSSM. Then, the H^3 gauge anomalies remains with opposite signs at the fixed points while the other gauge anomalies remain nonzero only at $y = 0$. Consequently, we show that all the remaining localized gauge anomalies are cancelled exactly by introducing a Chern-Simons 5-form with a jumping coefficient in the 5D action, which can be interpreted as being induced from the bulk heavy modes[13, 12, 8]. The variation of this Chern-Simons term gives rise to the 4D gauge anomalies on the boundaries, which is exactly what is needed to cancel the gauge anomalies coming from the asymmetric assigning of two 4D fermions in the opposite representations under H.

Moreover, we consider the localized FI terms in our model. We show that due to the orbifold boundary conditions breaking the bulk gauge symmetry into H, one $(N+K)$-plet gives rise to both a non-vanishing quadratically divergent FI term of one 4D K-plet and a non-vanishing log divergent FI term only at the fixed point with H. Then, the quadratically divergent FI term from the bulk field is cancelled by contribution from one brane $\bar{K}$-plet, which is consistent with the absence of gravitational mixed anomalies in this model. However, the log divergent FI term coming from the bulk field remains non-vanishing at the fixed point with a $U(1)$ factor. In the existence of the log divergent FI term, we show that the supersymmetric condition is satisfied independently of the log divergent FI term only if the $U(1)$ gauge component of the bulk real adjoint scalar takes a singular vacuum expectation value(VEV). Consequently, we find that this odd mode with a non-zero VEV dynamically localizes the bulk zero mode at the fixed point with H, so the bulk zero mode locally cancels the anomalies from the brane field. On the other hand, the wave functions of bulk massive modes are also modified without changing their tree-level mass spectrum. Thus, we argue that such a parity violation due to the odd mode is encoded in the wave functions of massive modes, which make up a parity-violating bulk Chern-Simons term to cancel the remaining Chern-Simons term. Even though our analysis of anomalies and FI terms in this paper is concentrated on the case with a particular set of fields in representations of the gauge group, the results are also applicable to the general set of brane and bulk fields with no 4D gauge anomalies.

Our paper is organized as follows. In the next section, we give an introduction to the gauge symmetry breaking on orbifolds by adopting a specific example, the 5D SUSY $SU(N+K)$ gauge theory on $S^1/(Z_2\times Z_2')$. Then, in the section 3, for this GUT orbifold, we derive the detailed expression for the localized non-abelian anomalies coming from a bulk fermion in the fundamental representation of $SU(N+K)$. The section 4 is devoted to the localization problem of a bulk fermion and the cancellation of the localized gauge anomalies coming from a 4D anomaly-free combination of bulk and brane fermions. In the next two sections, we work out with the localized Fayet-Iliopoulos terms in our model and explain their physical implication. Then, we conclude the paper in the last section.

2 Orbifold breaking of gauge symmetry

Let us consider the five-dimensional SUSY $G=SU(N+K)$ gauge theory compactified on an $S^1/(Z_2\times Z_2')$ orbifold. The fifth dimensional coordinate y is compactified to a circle $2\pi R\equiv 0$. Furthermore, the point $y=-a$ is identified to $y=a$ (Z_2 symmetry) and the point $y=(\pi R/2)+a$ is identified to $y=(\pi R/2)-a$ (Z_2' symmetry). Then, the fundamental region of the extra dimension becomes the interval $[0,\frac{\pi R}{2}]$ between two fixed points $y=0$ and $y=\frac{\pi R}{2}$.

For the two Z_2 symmetries, one can define their actions P and P' within the configuration space of any bulk field:

$$\begin{aligned}\phi(x,y) &\rightarrow \phi(x,-y)=P\phi(x,y), &(1)\\ \phi(x,y') &\rightarrow \phi(x,-y')=P'\phi(x,y') &(2)\end{aligned}$$

where $y'\equiv y+\pi R/2$. The (P,P') actions can involve all the symmetries of the bulk theory, for instance, the gauge symmetry and the R-symmetry in the supersymmetric case. In general, then, any bulk field ϕ can take one of four different Fourier expansions depending on their pair of two Z_2 parities, (i,j) as

$$\phi_{++}=\sum_{n=0}^{\infty}\sqrt{\frac{1}{2^{\delta_{n,0}}\pi R}}\phi_{++}^{(2n)}(x^\mu)\cos\frac{2ny}{R} \tag{3}$$

$$\phi_{+-}=\sum_{n=0}^{\infty}\sqrt{\frac{1}{\pi R}}\phi_{+-}^{(2n+1)}(x^\mu)\cos\frac{(2n+1)y}{R} \tag{4}$$

$$\phi_{-+}=\sum_{n=0}^{\infty}\sqrt{\frac{1}{\pi R}}\phi_{-+}^{(2n+1)}(x^\mu)\sin\frac{(2n+1)y}{R} \tag{5}$$

$$\phi_{--} = \sum_{n=0}^{\infty} \sqrt{\frac{1}{\pi R}} \phi_{--}^{(2n+2)}(x^\mu) \sin \frac{(2n+2)y}{R} \tag{6}$$

where x^μ is the 4D space-time coordinate.

The minimal supersymmetry in 5D corresponds to N=2 supersymmetry(or 8 supercharges) in the 4D N=1 language. Thus, a 5D chiral multiplet corresponds to an N=2 hypermultiplet consisting of two N=1 chiral multiplets with opposite charges. Two 4D Weyl spinors make up one 5D spinor. On the other hand, a 5D vector multiplet corresponds to an N=2 vector multiplet composed of one N=1 vector multiplet($V = (A_\mu, \lambda_1, D) \equiv V^q T^q$) and one N=1 chiral multiplet ($\Sigma = ((\Phi + iA_5)/\sqrt{2}, \lambda_2, F_\Sigma) \equiv \Sigma^q T^q$), which transforms in the adjoint representation of the bulk gauge group[2]. Upon compactification, we consider the case where one Z_2 breaks N=2 supersymmetry to N=1 while the other Z_2 breaks the bulk $G = SU(N+K)$ gauge group to its subgroup $H = SU(N) \times SU(K) \times U(1)$.

For instance, a bulk chiral multiplet $N + K$, which is composed of two chiral multiplets with opposite charges, $H = (h, \psi, F_H) \equiv (H_1, H_2)^T$ and $H^c = (h^c, \psi^c) \equiv (H_1^c, H_2^c)$, transforms under Z_2 and Z_2' identifications as

$$\begin{aligned} H(x,-y) &= \eta P H(x,y), \quad H^c(x,-y) = -\eta \tilde{H}(x,y) P^{-1} & (7) \\ H(x,-y') &= \eta' P' H(x,y'), \quad H^c(x,-y') = -\eta' H^c(x,y') P'^{-1} & (8) \end{aligned}$$

where both η and η' can take $+1$ or -1, and $P^2 = P'^2 = I_{N+K}$ where I_{N+K} is the $(N+K) \times (N+K)$ identity matrix. Then, choosing the parity matrices as

$$P = I_{N+K}, \quad P' = \mathrm{diag}(I_K, -I_N), \tag{9}$$

and with $\eta = \eta' = 1$, the corresponding N=1 supermultiplets are split as follows

$$\begin{aligned} H_1^{(2n)}: &\quad [(++); (1, K, \frac{1}{K})], \ \text{mass} = 2n/R & (10) \\ H_2^{(2n+1)}: &\quad [(+-); (N, 1, -\frac{1}{N})], \ \text{mass} = (2n+1)/R & (11) \\ H_2^{c(2n+1)}: &\quad [(-+); (\overline{N}, 1, \frac{1}{N})], \ \text{mass} = (2n+1)/R & (12) \\ H_1^{c(2n+2)}: &\quad [(--); (1, \overline{K}, -\frac{1}{K})], \ \text{mass} = (2n+2)/R & (13) \end{aligned}$$

where the brackets [ ] contain the quantum numbers of $Z_2 \times Z_2' \times SU(N) \times SU(K) \times U(1)$. Consequently, upon compactification, there appears a zero mode only from the K-plet among the bulk field components while other fields get massive.

On the other hand, the bulk gauge multiplet is transformed under the two Z_2 transformations respectively as

$$\begin{aligned} V(x,-y) &= PV(x,y)P^{-1}, & (14) \\ \Sigma(x,-y) &= -P\Sigma(x,y)P^{-1}, & (15) \\ V(x,-y') &= P'V(x,y')P'^{-1}, & (16) \\ \Sigma(x,-y') &= -P'\Sigma(x,y')P'^{-1}. & (17) \end{aligned}$$

[2] We note $D = X_3 - \partial_5 \Sigma$ and $F_\Sigma = (X_1 + iX_2)/\sqrt{2}$ in terms of the $SU(2)_R$ triplet $\vec{X}$ in $N = 2$ supersymmetry.

Therefore, with the choice for the parity matrices in the fundamental representation as Eq. (9), the $G = SU(N+K)$ gauge symmetry is broken down to $H = SU(N) \times SU(K) \times U(1)$ because P' does not commute with all the gauge generators of $SU(N+K)$: $P'T^aP'^{-1} = T^a$ and $P'T^{\hat{a}}P'^{-1} = -T^{\hat{a}}$ where $q = (a, \hat{a})$ denote unbroken and broken generators, respectively. Actually, due to the orbifold boundary conditions for the gauge fields, we get the Z_2' grading of $SU(N+K)$ as

$$[T^a, T^b] = if^{abc}T^c, \ [T^a, T^{\hat{b}}] = if^{a\hat{b}\hat{c}}T^{\hat{c}}, \ [T^{\hat{a}}, T^{\hat{b}}] = if^{\hat{a}\hat{b}c}T^c \tag{18}$$

where $f^{ab\hat{c}}$ and $f^{\hat{a}\hat{b}\hat{c}}$ is zero for the Z_2' invariance. As will be shown in the next section, the gauge anomalies in our orbifold model respect this group structure. It is interesting to see that this Z_2' graded algebra also appears in the case with the spontaneous breaking of the $SU(N+K)$ global symmetry.

Consequently, upon compactification, the gauge multiplets of $SU(N+K)$ are

$$V^{a(n)}: \quad [(++); (N^2-1, 1) + (1, K^2-1) + (1,1)], \ \text{mass} = 2n/R \tag{19}$$
$$V^{\hat{a}(2n+1)}: \quad [(+-); (N, K) + (\overline{N}, \overline{K})], \ \text{mass} = (2n+1)/R \tag{20}$$
$$\Sigma^{\hat{a}(2n+1)}: \quad [(-+); (N, K) + (\overline{N}, \overline{K})], \ \text{mass} = (2n+1)/R \tag{21}$$
$$\Sigma^{a(2n+2)}: \quad [(--); (N^2-1, 1) + (1, K^2-1) + (1,1)], \ \text{mass} = (2n+2)/R \tag{22}$$

where the brackets [ ] contain the quantum numbers of $Z_2 \times Z_2' \times SU(N) \times SU(K)$. Therefore, the orbifolding retains only the $SU(N) \times SU(K) \times U(1)$ gauge multiplets as massless modes $V^{a(0)}$ while the KK massive modes for unbroken and broken gauge bosons are paired up separately. Here we make an interesting observation from our parity assignments that the $G = SU(N+K)$ gauge symmetry is fully conserved at $y = 0$ while only the unbroken gauge group $H = SU(N) \times SU(K) \times U(1)$ is operative at $y = \pi R/2$. Therefore, upon the orbifold compactification, it is possible to put some incomplete multiplets transforming only under the local gauge group at $y = \pi R/2$. Actually, since the parity conservation is assumed in the Lagrangian, each component of a gauge parameter $\omega = \omega^q T^q$ has the same Z_2 parities as those of the corresponding gauge field. Therefore, in the existence of the two Z_2 symmetries, the bulk gauge transformation does not respect the full $SU(N+K)$ gauge transformation but it is restricted as follows

$$\begin{aligned} \delta A^a_M &= \partial_M \omega^a + if^{abc}A^b_M\omega^c + if^{a\hat{b}\hat{c}}A^{\hat{b}}_M\omega^{\hat{c}}, & (23)\\ \delta A^{\hat{a}}_M &= \partial_M \omega^{\hat{a}} + if^{\hat{a}\hat{b}c}A^{\hat{b}}_M\omega^c + if^{\hat{a}b\hat{c}}A^b_M\omega^{\hat{c}} & (24) \end{aligned}$$

which are consistent with the Z_2' graded algebra, eq. (18). Particularly, since $\omega^{\hat{a}}$ takes the same parities $(+,-)$ as $A^{\hat{a}}_\mu$, the gauge transformation at $y = \pi R/2$ becomes the one of the unbroken gauge group H from eq. (23).

3 Non-abelian anomalies on orbifolds with gauge symmetry breaking

A 5D fermion is not chiral in the 4D language. However, after orbifold compactification of the extra dimension, a chiral fermion can be obtained as the zero mode of a bulk non-chiral fermion. Then, the chiral fermion gives rise to the 4D gauge anomaly after integrating out the extra dimension. For the case with the 5D $U(1)$ gauge theory on S^1/Z_2[12] or $S^1/(Z_2\times Z_2')$[7], it was shown that the 4D gauge anomaly coming from a zero mode is equally distributed at the fixed points. In this section, we do the anomaly analysis in the case with the 5D $SU(N+K)$ gauge theory compactified on our gauge symmetry breaking orbifold, $S^1/(Z_2\times Z_2')$.

Let us consider a four-component bulk fermion in the fundamental representation of $SU(N+K)$. Then, the action is

$$S=\int d^4x\int_0^{2\pi R}dy\,\bar{\psi}(i\not{D}-\gamma_5 D_5-m(y))\psi \tag{25}$$

where $\not{D}=\gamma^\mu D_\mu$ and $D_M=\partial_M+iA_M$. Here $m(y)$ is a mass term for the bulk fermion and $A_M=A^q_M T^q$ is a classical non-abelian gauge field.

With the assignments of Z_2 and Z_2' parities to a $(N+K)$-plet hypermultiplet in the previous section, the fermion field transforms as

$$\psi(y)=\gamma_5 P\psi(-y),\quad \psi(y')=\gamma_5 P'\psi(-y') \tag{26}$$

where P and P' are given by Eq. (9), acting in the group space. Invariance of the action under two Z_2's gives rise to the conditions for the mass function

$$m(y)=-m(-y),\quad m(y')=-m(-y'). \tag{27}$$

And the gauge fields also transform under Z_2 as

$$A_\mu(y) = PA_\mu(-y)P^{-1},\quad A_5(y)=-PA_5(-y)P^{-1}, \tag{28}$$

and we replace $(y\to y',\, P\to P')$ for Z_2' action.

Then, with $\psi=\psi^1+\psi^2$, where 1 and 2 denotes K-plet and N-plet components respectively, the fermion field is decomposed into four independent chiral components

$$\psi^1=\psi^1_L+\psi^1_R,\quad \psi^2=\psi^2_L+\psi^2_R \tag{29}$$

where

$$\gamma_5\psi^1_{L(R)}=\pm\psi^1_{L(R)},\quad \gamma_5\psi^2_{L(R)}=\pm\psi^2_{L(R)}. \tag{30}$$

Due to the parity assignments, i.e., $(\pm,\pm)$ for $\psi^1_{L(R)}$ and $(\pm,\mp)$ for $\psi^2_{L(R)}$, we can expand each Weyl fermion in terms of KK modes

$$\psi^1_{L(R)}(x,y) = \sum_n \psi^1_{L(R)n}(x)\xi^{(\pm\pm)}_n(y), \tag{31}$$

$$\psi^2_{L(R)}(x,y) = \sum_n \psi^2_{L(R)n}(x)\xi^{(\pm\mp)}_n(y), \tag{32}$$

with

$$
\begin{aligned}
(-\partial_5 + m(y))(\partial_5 + m(y))\xi_n^{(+\pm)}(y) &= M_n^2 \xi_n^{(+\pm)}(y), && (33)\\
(\partial_5 + m(y))(-\partial_5 + m(y))\xi_n^{(-\mp)}(y) &= M_n^2 \xi_n^{(-\mp)}(y) && (34)
\end{aligned}
$$

where M_n is the nth KK mass. Here we note that ξ's make an orthonormal basis for the function on $[0, 2\pi R)$:

$$
\int_0^{2\pi R} dy\, \xi_m^{(\pm\pm)}(y)\xi_n^{(\pm\pm)}(y) = \int_0^{2\pi R} dy\, \xi_m^{(\pm\mp)}(y)\xi_n^{(\pm\mp)}(y) = \delta_{mn}, \qquad (35)
$$

$$
\int_0^{2\pi R} dy\, \xi_m^{(++)}(y)\xi_n^{(\pm\mp)}(y) = \int_0^{2\pi R} dy\, \xi_m^{(--)}(y)\xi_n^{(\pm\mp)}(y) = 0. \qquad (36)
$$

Under the gauge $A_5 = 0$[3], inserting the mode sum of the fermion into the 5D action, we obtain

$$
\begin{aligned}
S &= \int d^4x \Bigg[\sum_n \overline{\psi_n^1}(i\partial\!\!\!/ - M_{2n})\psi_n^1 + \sum_n \overline{\psi_n^2}(i\partial\!\!\!/ - M_{2n-1})\psi_n^2 \\
&- \sum_{m,n} \Big(V_{mn}(A^a) + V_{mn}(A^i) + V_{mn}(B) + V_{mn}(A^{\hat{a}}) \Big) \Bigg] \qquad (37)
\end{aligned}
$$

where $\psi_n^1 = \psi_{Ln}^1 + \psi_{Rn}^1$ for $n > 0$ ($\psi_0^1 = \psi_{L0}^1$), $\psi_n^2 = \psi_{Ln}^2 + \psi_{Rn}^2$, and V_{mn}'s denote gauge vertex couplings. The $G = SU(N+K)$ gauge fields($A = A^q T^q$) can be decomposed into

$$
(N+K)^2 - 1 \to (N^2-1, 1) + (1, K^2-1) + (1,1) + (N,K) + (\overline{N}, \overline{K}), \qquad (38)
$$

that is, $A^a T^a (a = 1, \cdots, N^2-1)$, $A^i T^i (i = 1, \cdots, K^2-1)$, $A^{(N+K)^2-1} T^{(N+K)^2-1} \equiv BT^B$ gauge fields for the $H = SU(N) \times SU(K) \times U(1)$ group, and $A^{\hat{a}} (t^{\hat{a}})_{\alpha r} \equiv X^{\alpha r} (\alpha = 1, \cdots, N; r = 1, \cdots, K)$ gauge fields for the G/H group, respectively. Here, broken group generators are related to $t^{\hat{a}}$ as

$$
T^{\hat{a}} \equiv \begin{pmatrix} 0 & t^{\hat{a}} \\ (t^{\hat{a}})^\dagger & 0 \end{pmatrix}. \qquad (39)
$$

Then, V_{mn}'s are given by the following:

$$
\begin{aligned}
V_{mn}(A^a) &= J^{\mu a}_{mn(+-)} \mathcal{A}^{a(+-)}_{mn\mu} + J^{\mu a}_{mn(-+)} \mathcal{A}^{a(-+)}_{mn\mu} \\
V_{mn}(A^i) &= J^{\mu i}_{mn(++)} \mathcal{A}^{i(++)}_{mn\mu} + J^{\mu i}_{mn(--)} \mathcal{A}^{a(--)}_{mn\mu} \\
V_{mn}(B) &= J^{\mu B}_{mn(++)} \mathcal{B}^{(++)}_{mn\mu} + J^{\mu B}_{mn(--)} \mathcal{B}^{(--)}_{mn\mu} + J^{\mu B}_{mn(+-)} \mathcal{B}^{(+-)}_{mn\mu} + J^{\mu B}_{mn(-+)} \mathcal{B}^{(-+)}_{mn\mu} \\
V_{mn}(A^{\hat{a}}) &= J^{\mu \hat{a}}_{mn(+)} \mathcal{A}^{\hat{a}(+)}_{mn\mu} + J^{\mu \hat{a}}_{mn(-)} \mathcal{A}^{\hat{a}(-)}_{mn\mu} \qquad (40)
\end{aligned}
$$

[3]The result will be not changed in the case without a gauge condition[9]

where

$$\mathcal{A}^{a(\pm\mp)}_{mn\mu} = \int_0^{2\pi R} dy\, \xi^{(\pm\mp)}_m(y)\xi^{(\pm\mp)}_n(y) A^a_\mu(x,y), \tag{41}$$

$$\mathcal{A}^{i(\pm\pm)}_{mn\mu} = \int_0^{2\pi R} dy\, \xi^{(\pm\pm)}_m(y)\xi^{(\pm\pm)}_n(y) A^i_\mu(x,y), \tag{42}$$

$$\mathcal{B}^{(\pm\pm)}_{mn\mu} = \int_0^{2\pi R} dy\, \xi^{(\pm\pm)}_m(y)\xi^{(\pm\pm)}_n(y) B_\mu(x,y), \tag{43}$$

$$\mathcal{B}^{(\pm\mp)}_{mn\mu} = \int_0^{2\pi R} dy\, \xi^{(\pm\mp)}_m(y)\xi^{(\pm\mp)}_n(y) B_\mu(x,y), \tag{44}$$

$$\mathcal{A}^{\hat{a}(\pm)}_{mn\mu} = \int_0^{2\pi R} dy\, \xi^{(\pm\mp)}_m(y)\xi^{(\pm\pm)}_n(y) A^{\hat{a}}_\mu(x,y) \tag{45}$$

and

$$J^{\mu a}_{mn(\pm\mp)} = \overline{\psi^2_m}\gamma^\mu P_\pm T^a \psi^2_n, \qquad J^{\mu i}_{mn(\pm\pm)} = \overline{\psi^1_m}\gamma^\mu P_\pm T^i \psi^1_n, \tag{46}$$

$$J^{\mu B}_{mn(\pm\pm)} = \overline{\psi^1_m}\gamma^\mu P_\pm T^B_{K\times K} \psi^1_n, \qquad J^{\mu B}_{mn(\pm\mp)} = \overline{\psi^2_m}\gamma^\mu P_\pm T^B_{N\times N} \psi^2_n, \tag{47}$$

$$J^{\mu\hat{a}}_{mn(\pm)} = \overline{\psi^2_m}\gamma^\mu P_\pm t^{\hat{a}} \psi^1_n + \overline{\psi^1_m}\gamma^\mu P_\pm (t^{\hat{a}})^\dagger \psi^2_n \tag{48}$$

with $P_\pm = (1\pm\gamma_5)/2$. Here a decomposition of T^B is understood such as $T^B = \text{diag.}(T^B_{N\times N}, T^B_{K\times K})$. We note that the chiral current for the $SU(N+M)$ gauge symmetry is split into chiral currents coupled to the unbroken and broken gauge fields.

Applying the classical equations of motion and the standard results for the 4D chiral anomalies[12, 8, 9], we can derive the anomalies for the chiral currents classified above. By making an inverse Fourier-transformation by the convolution of the bulk eigenmodes, the 5D gauge vector current $J^{Mq} = \overline{\psi}\gamma^M T^q \psi$ is given by

$$J^{\mu a}(x,y) = \sum_{m,n}(\xi^{(+-)}_m \xi^{(+-)}_n J^{\mu a}_{mn(+-)} + \xi^{(-+)}_m \xi^{(-+)}_n J^{\mu a}_{mn(-+)}), \tag{49}$$

$$J^{\mu i}(x,y) = \sum_{m,n}(\xi^{(++)}_m \xi^{(++)}_n J^{\mu i}_{mn(++)} + \xi^{(--)}_m \xi^{(--)}_n J^{\mu i}_{mn(--)}), \tag{50}$$

$$\begin{aligned} J^{\mu B}(x,y) &= \sum_{m,n}(\xi^{(++)}_m \xi^{(++)}_n J^{\mu B}_{mn(++)} + \xi^{(--)}_m \xi^{(--)}_n J^{\mu B}_{mn(--)} \\ &+ \xi^{(+-)}_m \xi^{(+-)}_n J^{\mu B}_{mn(+-)} + \xi^{(-+)}_m \xi^{(-+)}_n J^{\mu B}_{mn(-+)}), \end{aligned} \tag{51}$$

$$J^{\mu\hat{a}}(x,y) = \sum_{m,n}(\xi^{(+-)}_m \xi^{(++)}_n J^{\mu\hat{a}}_{mn(+)} + \xi^{(-+)}_m \xi^{(--)}_n J^{\mu\hat{a}}_{mn(-)}) \tag{52}$$

and we can construct J^{5q} similarly. Consequently, it turns out that the divergence of the 5D gauge vector current is given in terms of the 4D gauge anomalies as

$$
\begin{aligned}
(D_M J^M)^a(x,y) &= f_2(y)(\mathcal{Q}^a(A)+\mathcal{Q}^a(X)), &(53)\\
(D_M J^M)^i(x,y) &= f_1(y)(\mathcal{Q}^i(A)+\mathcal{Q}^i(X)), &(54)\\
(D_M J^M)^B(x,y) &= f_1(y)(\mathcal{Q}^B_+(A)+\mathcal{Q}^B_+(X))+f_2(y)(\mathcal{Q}^B_-(A)+\mathcal{Q}^B_-(X)), &(55)\\
(D_M J^M)^{\hat a}(x,y) &= f_1(y)(\mathcal{Q}^{\hat a}_1(X)+\mathcal{Q}^{\hat a}_+(X))+f_2(y)(\mathcal{Q}^{\hat a}_2(X)+\mathcal{Q}^{\hat a}_-(X)) &(56)
\end{aligned}
$$

where

$$
\begin{aligned}
f_1(y) &= \sum_n \left[(\xi^{(++)}_n(y))^2-(\xi^{(--)}_n(y))^2\right]=\frac{1}{4}\sum_n \delta(y-\frac{n\pi R}{2}), &(57)\\
f_2(y) &= \sum_n \left[(\xi^{(+-)}_n(y))^2-(\xi^{(-+)}_n(y))^2\right]=\frac{1}{4}\sum_n (-1)^n\delta(y-\frac{n\pi R}{2}). &(58)
\end{aligned}
$$

The localized gauge anomalies $\mathcal{Q}$'s are composed of two large parts: anomalies for unbroken group components and broken group components of the 5D vector current. The anomalies for unbroken group components involve not only unbroken gauge fields

$$
\begin{aligned}
\mathcal{Q}^a(A) &= \frac{1}{32\pi^2}(D^{abc}F^b_{\mu\nu}\tilde{F}^{c\mu\nu}(x,y)+D^{abB}F^b_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y)), &(59)\\
\mathcal{Q}^i(A) &= \frac{1}{32\pi^2}D^{ijB}F^j_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y)+\frac{1}{32\pi^2}D^{ijk}F^j_{\mu\nu}\tilde{F}^{k\mu\nu}(x,y), &(60)\\
\mathcal{Q}^B_+(A) &= \frac{1}{32\pi^2}\mathrm{Tr}(T^B_{K\times K})^3F^B_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y)\\
&+ \frac{1}{64\pi^2}\mathrm{Tr}(\{T^B_{K\times K},T^i\}T^j)F^i_{\mu\nu}\tilde{F}^{j\mu\nu}(x,y), &(61)\\
\mathcal{Q}^B_-(A) &= \frac{1}{32\pi^2}\mathrm{Tr}(T^B_{N\times N})^3F^B_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y)\\
&+ \frac{1}{64\pi^2}\mathrm{Tr}(\{T^B_{N\times N},T^a\}T^b)F^a_{\mu\nu}\tilde{F}^{b\mu\nu}(x,y), &(62)\\
\mathcal{Q}^B_+(A)+\mathcal{Q}^B_-(A) &= \frac{1}{32\pi^2}(D^{BBB}F^B_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y)+D^{Bij}F^i_{\mu\nu}\tilde{F}^{j\mu\nu}(x,y)\\
&+ D^{Bab}F^a_{\mu\nu}\tilde{F}^{b\mu\nu}(x,y))\equiv\mathcal{Q}^B(A), &(63)
\end{aligned}
$$

but also broken gauge fields

$$
\begin{aligned}
\mathcal{Q}^a(X) &= \frac{1}{64\pi^2}\mathrm{Tr}(T^a t^{\hat b}(t^{\hat c})^\dagger)F^{\hat b}_{\mu\nu}\tilde{F}^{\hat c\mu\nu}(x,y)=\frac{1}{32\pi^2}D^{a\hat b\hat c}F^{\hat b}_{\mu\nu}\tilde{F}^{\hat c\mu\nu}(x,y), &(64)\\
\mathcal{Q}^i(X) &= \frac{1}{64\pi^2}\mathrm{Tr}(T^i (t^{\hat b})^\dagger t^{\hat c})F^{\hat b}_{\mu\nu}\tilde{F}^{\hat c\mu\nu}(x,y)=\frac{1}{32\pi^2}D^{i\hat b\hat c}F^{\hat b}_{\mu\nu}\tilde{F}^{\hat c\mu\nu}(x,y), &(65)\\
\mathcal{Q}^B_+(X) &= \frac{1}{64\pi^2}\mathrm{Tr}(\{T^B,(t^{\hat b})^\dagger\}t^{\hat c})F^{\hat b}_{\mu\nu}\tilde{F}^{\hat c\mu\nu}(x,y), &(66)\\
\mathcal{Q}^B_-(X) &= \frac{1}{64\pi^2}\mathrm{Tr}(\{T^B,t^{\hat b}\}(t^{\hat c})^\dagger)F^{\hat b}_{\mu\nu}\tilde{F}^{\hat c\mu\nu}(x,y), &(67)\\
\mathcal{Q}^B_+(X)+\mathcal{Q}^B_-(X) &= \frac{1}{32\pi^2}D^{B\hat b\hat c}F^{\hat b}_{\mu\nu}\tilde{F}^{\hat c\mu\nu}(x,y)\equiv\mathcal{Q}^B(X). &(68)
\end{aligned}
$$

On the other hand, the anomalies for broken group components of the 5D vector current become

$$\begin{aligned}
\mathcal{Q}_1^{\hat{a}}(X) &= \frac{1}{64\pi^2}\mathrm{Tr}((\{t^{\hat{a}},(t^{\hat{b}})^\dagger\}+\{(t^{\hat{a}})^\dagger,t^{\hat{b}}\})T^a)F^{\hat{b}}_{\mu\nu}\tilde{F}^{a\mu\nu}(x,y) \\
&= \frac{1}{32\pi^2}D^{\hat{a}\hat{b}a}F^{\hat{b}}_{\mu\nu}\tilde{F}^{a\mu\nu}(x,y), && (69) \\
\mathcal{Q}_2^{\hat{a}}(X) &= \frac{1}{64\pi^2}\mathrm{Tr}((\{t^{\hat{a}},(t^{\hat{b}})^\dagger\}+\{(t^{\hat{a}})^\dagger,t^{\hat{b}}\})T^i)F^{\hat{b}}_{\mu\nu}\tilde{F}^{i\mu\nu}(x,y) \\
&= \frac{1}{32\pi^2}D^{\hat{a}\hat{b}i}F^{\hat{b}}_{\mu\nu}\tilde{F}^{i\mu\nu}(x,y), && (70) \\
\mathcal{Q}_+^{\hat{a}}(X) &= \frac{1}{64\pi^2}\mathrm{Tr}(((t^{\hat{a}})^\dagger t^{\hat{b}}+(t^{\hat{b}})^\dagger t^{\hat{a}})T^B_{K\times K})F^{\hat{b}}_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y) \\
&= \frac{1}{64\pi^2}\mathrm{Tr}(\{T^{\hat{a}},T^{\hat{b}}\}T^B_{K\times K})F^{\hat{b}}_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y), && (71) \\
\mathcal{Q}_-^{\hat{a}}(X) &= \frac{1}{64\pi^2}\mathrm{Tr}((t^{\hat{a}}(t^{\hat{b}})^\dagger+t^{\hat{b}}(t^{\hat{a}})^\dagger)T^B_{N\times N})F^{\hat{b}}_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y) \\
&= \frac{1}{64\pi^2}\mathrm{Tr}(\{T^{\hat{a}},T^{\hat{b}}\}T^B_{N\times N})F^{\hat{b}}_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y), && (72) \\
\mathcal{Q}_+^{\hat{a}}(X)+\mathcal{Q}_-^{\hat{a}}(X) &= \frac{1}{32\pi^2}D^{\hat{a}\hat{b}B}F^{\hat{b}}_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y)\equiv\mathcal{Q}_3^{\hat{a}}(X) && (73)
\end{aligned}$$

In all the expressions for the anomalies above, we note that D^{abc} denotes the symmetrized trace of group generators

$$D^{abc}=\frac{1}{2}\mathrm{Tr}(\{T^a,T^b\}T^c) \tag{74}$$

and other D symbols with different group idices are similarly understood.

As a result, we find that a bulk fermion gives rise to the localized gauge anomalies for all gauge components of the 5D vector current. Since the broken gauge fields vanish at $y=\pi R/2$ due to their boundary conditions, the localized gauge anomalies at $y=\pi R/2$ are only $\mathcal{Q}(A)$'s, i.e., the H^3 gauge anomalies. However, at the other fixed point $y=0$, in addition to $\mathcal{Q}(A)$'s, there also appear the localized gauge anomalies $\mathcal{Q}(X)$'s, i.e., the $H-(G/H)-(G/H)$ gauge anomalies. We note that there is no anomalies of the type AAX or XXX since their anomaly coefficients automatically vanishes due to the group structure.(Here A denotes the unbroken gauge field with $(+,+)$ while X denotes the broken gauge field with $(+,-)$.) With this in mind and restricting to the region $[0,2\pi R)$, we can rewrite the divergence of the 5D vector current as

$$
\begin{aligned}
(D_M J^M)^a(x,y) &= \frac{1}{2}\bigg(\delta(y)-\delta(y-\frac{\pi R}{2})\bigg)\mathcal{Q}^a(A)+\frac{1}{2}\delta(y)\mathcal{Q}^a(X), && (75)\\
(D_M J^M)^i(x,y) &= \frac{1}{2}\bigg(\delta(y)+\delta(y-\frac{\pi R}{2})\bigg)\mathcal{Q}^i(A)+\frac{1}{2}\delta(y)\mathcal{Q}^i(X), && (76)\\
(D_M J^M)^B(x,y) &= \frac{1}{2}\bigg(\delta(y)+\delta(y-\frac{\pi R}{2})\bigg)\mathcal{Q}^B_+(A)+\frac{1}{2}\bigg(\delta(y)-\delta(y-\frac{\pi R}{2})\bigg)\mathcal{Q}^B_-(A)\\
&+ \frac{1}{2}\delta(y)\mathcal{Q}^B(X), && (77)\\
(D_M J^M)^{\hat{a}}(x,y) &= \frac{1}{2}\delta(y)\mathcal{Q}^{\hat{a}}(X) && (78)
\end{aligned}
$$

where $\mathcal{Q}^{\hat{a}}(X)\equiv\mathcal{Q}^{\hat{a}}_1(X)+\mathcal{Q}^{\hat{a}}_2(X)+\mathcal{Q}^{\hat{a}}_3(X)$.

4 Localization of a bulk field and anomaly problem

As shown in the section 2, we can freely put some brane fields consistently with the local gauge symmetries at the fixed points: a brane field at $y=0$ should be a representation of $SU(N+K)$ while a brane field at $y=\pi R/2$ should be a representation of $SU(N)\times SU(K)\times U(1)$. Since we assume that a bulk fermion gives rise to a K-plet as the zero mode and we want to have the anomaly-free theory at least at the zero mode level, we can only put a brane field of $\bar{K}$-plet at $y=\pi R/2$. This introduction of an incomplete brane multiplet is sufficient for the 4D anomaly-free theory at low energies but it could be inconsistent due to the existence of the localized gauge anomalies on the boundaries of the extra dimension. In this section, we consider the localization of a bulk fermion with a kink mass and subsequently deal with the appearing anomaly problem by using the results in the previous section.

It was shown in the literature that the localization of a bulk fermion can be realized by introducing a kink mass in the Lagrangian and even a brane fermion is possible in the limit of a kink mass being infinite[12]. In the 5D $U(1)$ gauge theory on S^1/Z_2 with a single bulk fermion, as a result of introducing an infinite kink mass, the anomaly contribution from a bulk fermion on the boundaries of the extra dimension was interpreted as the sum of contributions from a brane fermion and a parity-violating Chern-Simon term in 5D[12]. In other words, as a kink mass becomes infinite, heavy KK modes are decoupled but their effects remain as a local counterterm such as the 5D Chern-Simon term. The similar observation has been made for the non-abelian anomalies on orbifolds[5].

In our case with gauge symmetry breaking on orbifolds, however, we should be careful about the sign of a kink mass because an infinite kink mass could give rise to the localization of the unwanted bulk modes as massless modes[9, 17, 18]. For instance, a positive(negative) infinite kink mass for the even modes ($(+,+)$ and $(-,-)$) gives rise to a localization of the massless mode for $(+,+)$ at $y=0(y=\pi R/2)$. On the other hand, a positive infinite kink mass for the odd modes ($(+,-)$ and $(-,+)$) could lead to new massless modes localized at $y=0$ and $y=\pi R/2$, respectively. Suppose that there are the universal(preserving the bulk gauge symmetry) kink masses for even and odd modes, i.e., $m(y) = M\epsilon(y)I_{(N+K)\times(N+K)}$ in eq. (25) where $\epsilon(y)$ is the sign function with periodicity πR. Then, in order to avoid unwanted massless modes in the limit of the kink mass being infinite, we only have to take the sign of M to be negative. That is to say, when we introduce a bulk multiplet $\overline{(N+K)}$ with $M \to -\infty$, we obtain a massless $\bar{K}$-plet only from the $(+,+)$ mode, which is localized at $y = \pi R/2$, while other modes get decoupled from the theory. Thus, in this respect, a brane $\bar{K}$-plet is naturally realized from a bulk complete multiplet in the field theoretic limit. In this process of localization, we find that the consistency with the incomplete brane field can be guaranteed with introducing a 5D Chern-Simons term[5], which would be interpreted as the effects from the decoupled heavy modes[13, 12, 8].

When we introduce a brane $\bar{K}$-plet at $y = \pi R/2$, it gives rise to 4D gauge anomalies such as $-\mathcal{Q}^i(A)$ and $-\mathcal{Q}^B_+(A)$ at that fixed point. Therefore, with the addition of the brane $\bar{K}$-plet to a bulk $(N+K)$-plet, the divergence of the 5D vector current is changed to

$$
\begin{aligned}
(D_M J^M)^a(x,y) &= \frac{1}{2}\Big(\delta(y) - \delta(y - \frac{\pi R}{2})\Big)\mathcal{Q}^a(A) + \frac{1}{2}\delta(y)\mathcal{Q}^a(X), && (79)\\
(D_M J^M)^i(x,y) &= \frac{1}{2}\Big(\delta(y) - \delta(y - \frac{\pi R}{2})\Big)\mathcal{Q}^i(A) + \frac{1}{2}\delta(y)\mathcal{Q}^i(X), && (80)\\
(D_M J^M)^B(x,y) &= \frac{1}{2}\Big(\delta(y) - \delta(y - \frac{\pi R}{2})\Big)\mathcal{Q}^B(A) + \frac{1}{2}\delta(y)\mathcal{Q}^B(X), && (81)\\
(D_M J^M)^{\hat{a}}(x,y) &= \frac{1}{2}\delta(y)\mathcal{Q}^{\hat{a}}(X). && (82)
\end{aligned}
$$

Here we observe that the total localized gauge anomalies only involving the unbroken gauge group($\mathcal{Q}(A)$'s) appear in the combination of $(\delta(y) - \delta(y - \pi R/2))$, so their integrated gauge anomalies vanish. On the other hand, the anomalies involving broken gauge fields($\mathcal{Q}(X)$'s) remain nonzero even after integration because $\mathcal{Q}(X)$'s are nonzero only at $y = 0$. This asymmetric localization of $\mathcal{Q}(X)$'s reflects the difference between two fixed point groups. The existence of the localized gauge anomalies could make the theory with the unbroken gauge group anomalous. However, these localized gauge anomalies can be exactly cancelled with the introduction of a Chern-Simons(CS) 5-form $Q_5[A = A^q T^q]$ with a jumping coefficient in the action[5]

$$
\mathcal{L}_{CS} = -\frac{1}{96\pi^2}\epsilon(y)Q_5[A] \tag{83}
$$

where $\epsilon(y)$ is the sign function with periodicity πR and

$$
Q_5[A] = \mathrm{Tr}\Big(AdAdA + \frac{3}{2}A^3 dA + \frac{3}{5}A^5\Big). \tag{84}
$$

The parity-odd function $\epsilon(y)$ in front of Q_5 is necessary for the parity invariance because Q_5 is a parity-odd quantity according to our parity assignments for bulk gauge fields, eqs. (19)-(22). Under the gauge transformation $\delta A = d\omega + [A,\omega] \equiv D\omega$,

$$
\delta Q_5 = Q_4^1[\delta A, A] = \mathrm{str}\Big(D\omega\, d(AdA + \frac{1}{2}A^3)\Big) \tag{85}
$$

where str means the symmetrized trace and the restricted gauge transformation in eqs. (23) and (24) is understood. Then, due to the sign function in front of Q_5, the variation of the Chern-Simons action gives rise to the 4D consistent anomalies on the boundaries

$$\begin{aligned}\delta\mathcal{L}_{CS} &= \frac{1}{48\pi^2}(\delta(y)-\delta(y-\frac{\pi R}{2}))\,\epsilon^{\mu\nu\rho\sigma}\sum_{q=a,i,B}\omega^q \mathrm{str}(T^q\partial_\mu(A_\nu\partial_\rho A_\sigma+\frac{1}{2}A_\nu A_\rho A_\sigma)) \\ &+ \frac{1}{48\pi^2}\delta(y)\,\epsilon^{\mu\nu\rho\sigma}\omega^{\hat{a}}\mathrm{str}(T^{\hat{a}}\partial_\mu(A_\nu\partial_\rho A_\sigma+\frac{1}{2}A_\nu A_\rho A_\sigma)). \qquad (86)\end{aligned}$$

The consistent anomalies we obtained here can be changed to the covariant anomalies[19] by regarding the covariant non-abelian gauge current J^q_μ as being redefined from a non-covariant gauge current $\tilde{J}^q_\mu$ as

$$J^q_\mu(x,y)=\tilde{J}^q_\mu(x,y)+U^q_\mu(x,y) \qquad (87)$$

where

$$\begin{aligned}U^{q=(a,i,B)}_\mu &= -\frac{1}{96\pi^2}(\delta(y)-\delta(y-\frac{\pi R}{2}))\,\epsilon^{\mu\nu\rho\sigma}\mathrm{str}(T^q(A_\nu F_{\rho\sigma}+F_{\rho\sigma}A_\nu-A_\nu A_\rho A_\sigma)) \\ U^{q=\hat{a}}_\mu &= -\frac{1}{96\pi^2}\delta(y)\,\epsilon^{\mu\nu\rho\sigma}\mathrm{str}(T^q(A_\nu F_{\rho\sigma}+F_{\rho\sigma}A_\nu-A_\nu A_\rho A_\sigma)). \qquad (88)\end{aligned}$$

Consequently, when we take into account the fact that the broken gauge fields are vanishing at $y=\pi R/2$, the CS term contributes to the anomaly for the 5D covariant gauge current as

$$\begin{aligned}(D_M J^M)^{q_1=(a,i,B)} &= -\frac{1}{64\pi^2}(\delta(y)-\delta(y-\frac{\pi R}{2}))\sum_{q_2,q_3=(b,j,B)}\mathrm{str}(T^{q_1}T^{q_2}T^{q_3})F^{q_2}_{\mu\nu}F^{q_3\mu\nu} \\ &- \frac{1}{64\pi^2}\delta(y)\sum_{q_2,q_3=\hat{a}}\mathrm{str}(T^{q_1}T^{q_2}T^{q_3})F^{q_2}_{\mu\nu}F^{q_3\mu\nu}, \qquad (89) \\ (D_M J^M)^{q_1=\hat{a}} &= -\frac{1}{64\pi^2}\delta(y)\sum_{q_2q_3=\hat{b}(a,i,B)}\mathrm{str}(T^{q_1}T^{q_2}T^{q_3})F^{q_2}_{\mu\nu}F^{q_3\mu\nu} \qquad (90)\end{aligned}$$

where $q_{1,2,3}$ run the bulk group indices. It turns out that the CS contributions to the anomalies exactly cancel the remaining localized covariant gauge anomalies on the boundaries, eq. (79)-(82).

5 Fayet-Iliopoulos terms

In our model, the only place where the $U(1)$-graviton-graviton anomalies could appear is the fixed point $y=\pi R/2$ with the local gauge group including a $U(1)$ gauge factor. As argued in the literature[5], there is no gravitational counterpart $A\wedge R\wedge R$ of the 5D Chern-Simons term since the non-abelian gauge fields propagate in the bulk. It has been shown that the gravitational anomalies at $y=\pi R/2$ indeed cancel between the bulk and brane contributions without the need of a bulk Chern-Simons term[5]. Then, since both gravitational anomalies and FI terms are proportional to the common factor $\mathrm{Tr}(q)$, where q is the $U(1)$ charge operator, it seems that the absence of the gravitational anomalies should guarantee the absence of the FI terms which could also exist at $y = \pi R/2$. This is the requirement for the stability of the 4D supersymmetric theory.

In the orbifold models with an unbroken $U(1)$, however, it has been shown that the localized FI terms can be induced from a bulk field without breaking the 4D supersymmetry[7, 9, 11, 17]. In this section, we present the explicit computation of the Fayet-Iliopoulos(FI) terms[14, 11] for our set of bulk and brane fields in our model.

The relevant part of the action for bulk(h, h^c) and brane(h_b) scalar fields for performing the FI term calculation is given by

$$\begin{aligned} S &= \int d^4x \int_0^{2\pi R} dy \bigg[|\partial_M h|^2 + |\partial_M h^{c\dagger}|^2 + g D^B (h^\dagger T^B h - h^c T^B h^{c\dagger}) \\ &+ \delta(y - \frac{\pi R}{2}) \Big(|\partial_\mu h_b|^2 + g D^B h_b^\dagger q_b h_b \Big) \bigg] \end{aligned} \tag{91}$$

where D^B imply the auxiliary field for the unbroken $U(1)$. Denoting the bulk scalar fields as $h = (h_{++}, h_{+-})^T$ and $h^c = (h_{--}, h_{-+})$, let us expand those in terms of bulk eigenmodes as

$$h_{\pm\pm}(x, y) = \sum_n h_{(\pm\pm)n}(x) \xi_n^{(\pm\pm)}(y), \tag{92}$$

$$h_{\pm\mp}(x, y) = \sum_n h_{(\pm\mp)n}(x) \xi_n^{(\pm\mp)}(y). \tag{93}$$

As in the anomaly computation, inserting the above mode expansions in the 5D action gives

$$\begin{aligned} S &= \int d^4x \bigg[\sum_{\alpha,\beta=\pm} \sum_{m,n} -h^\dagger_{(\alpha\beta)m}(x) \Big((\Box_4 + M_n)\delta_{mn} - g q_{\alpha\beta} D^{B(\alpha\beta)}_{mn}(x) \Big) h_{(\alpha\beta)n}(x) \\ &- h_b^\dagger(x) \Big(\Box_4 - g q_{--} D^B(x, y = \frac{\pi R}{2}) \Big) h_b(x) \bigg] \end{aligned} \tag{94}$$

where

$$D^{B(\alpha\beta)}_{mn}(x) = \int_0^{2\pi R} dy\, \xi_m^{(\alpha\beta)}(y) \xi_n^{(\alpha\beta)}(y) D^B(x, y) \tag{95}$$

and $\mathrm{Tr}(T^B) = K q_{++} + N q_{+-} = 0$, $q_{-+} = -q_{+-}$, $q_{--} = -q_{++}$, and the introduction of a brane $\bar{K}$-plet with $q_b = q_{--}$ is understood.

From the one-loop tadpole diagram for the KK modes of auxiliary field D^B, we can get the bulk and brane field contributions to the FI term with the cutoff Λ regularization as follows

$$F(x) = F_{bulk}(x) + F_{brane}(x) \tag{96}$$

where

$$F_{bulk}(x) = \sum_n \sum_{\alpha\beta} q_{\alpha\beta} T_n D^{B(\alpha\beta)}_{nn}(x) \tag{97}$$

with

$$T_n = ig \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - M_n^2} = \frac{g}{16\pi^2}\left(\Lambda^2 - M_n^2 \ln \frac{\Lambda^2 + M_n^2}{M_n^2}\right), \tag{98}$$

and

$$F_{brane}(x) = igKq_{--}D^B(x, y = \frac{\pi R}{2}) \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2} = \frac{gKq_{--}}{16\pi^2}\Lambda^2 D^B(x, y = \frac{\pi R}{2}). \tag{99}$$

Then, when we write the FI term in terms of the 5D field $D^B(x, y)$ as

$$F(x) = \int_0^{2\pi R} dy\, f(y) D^B(x, y), \tag{100}$$

we make an inverse Fourier-transformation for the auxiliary field to obtain the bulk profile for the FI term as

$$f(y) = f_{even}(y) + f_{odd}(y) + f_{brane}(y) \tag{101}$$

where

$$\begin{aligned}
f_{even}(y) &= gKq_{++} \sum_n T_n[|\xi_n^{(++)}|^2 - |\xi_n^{(--)}|^2] \\
&= \frac{gKq_{++}}{16\pi^2}\left[\frac{1}{2}\Lambda^2(\delta(y) + \delta(y - \frac{\pi R}{2})) + \frac{1}{4}\ln\frac{\Lambda}{\mu}(\delta''(y) + \delta''(y - \frac{\pi R}{2}))\right], \quad (102) \\
f_{odd}(y) &= gNq_{+-} \sum_n T_n[|\xi_n^{(+-)}|^2 - |\xi_n^{(-+)}|^2] \\
&= \frac{gNq_{+-}}{16\pi^2}\left[\frac{1}{2}\Lambda^2(\delta(y) - \delta(y - \frac{\pi R}{2})) + \frac{1}{4}\ln\frac{\Lambda}{\mu}(\delta''(y) - \delta''(y - \frac{\pi R}{2}))\right], \quad (103) \\
f_{brane}(y) &= \frac{gKq_{--}}{16\pi^2}\Lambda^2\delta(y - \frac{\pi R}{2}). \quad (104)
\end{aligned}$$

Here, prime denotes the derivative with respect to the extra dimension coordinate. Consequently, the resultant FI term is given by

$$\begin{aligned}
f(y) &= \frac{g\mathrm{Tr}(T^B)}{32\pi^2}\left[\Lambda^2(\delta(y) + \delta(y - \frac{\pi R}{2})) + \frac{1}{2}\ln\frac{\Lambda}{\mu}\delta''(y)\right] + \frac{gKq_{++}}{32\pi^2}\ln\frac{\Lambda}{\mu}\delta''(y - \frac{\pi R}{2}) \\
&\equiv c\,\delta''(y - \frac{\pi R}{2}) \quad (105)
\end{aligned}$$

where we used $\mathrm{Tr}(T^B)=0$ in the last line and

$$c\equiv\frac{gKq_{++}}{32\pi^2}\ln\frac{\Lambda}{\mu}. \tag{106}$$

We note that there is no FI term at $y=0$ with the full bulk gauge group, which is as expected because there is no $U(1)$ factor at this fixed point. Moreover, we find that there is no conventional FI term with quadratic divergence even at $y=\pi R/2$ with a $U(1)$ factor, which is consistent with the absence of mixed gravitational anomalies as argued in [5]. However, there exists a non-vanishing FI term with logarithmic divergence at $y=\pi R/2$.

6 Localization of a bulk zero mode via the log FI term

In the 5D $U(1)$ gauge theory on S^1/Z_2, it has been shown that in the presence of the localized FI terms, the supersymmetric condition is satisfied only if the real adjoint scalar in the vector multiplet develops a vacuum expectation value[15, 16, 9, 11]. Then, the localized FI terms give rise to not only the dynamical localization of the bulk zero mode but also make the bulk massive modes decoupled[11].

In our case, there remains only the log FI term at the fixed point with the unbroken gauge group. This is different from the case in the 5D $U(1)$ gauge theory where the log FI term are equally distributed at the fixed points[11]. In this section, we present the physical implication of the log FI term in our model for the localization of a bulk field and the anomaly cancellation.

The effective potential in our model with no gauge field background is written as

$$\begin{aligned}V &= \int_0^{2\pi R}dy\bigg[-\frac{1}{2}\Big(D^q+\partial_5\Phi^q+g(h^\dagger T^q h-h^cT^qh^{c\dagger})+(f(y)+\delta(y-\frac{\pi R}{2})gh_b^\dagger q_b h_b)\delta^{qB}\Big)^2\\ &+\frac{1}{2}\Big(\partial_5\Phi^q+g(h^\dagger T^q h-h^cT^qh^{c\dagger})+(f(y)+\delta(y-\frac{\pi R}{2})gh_b^\dagger q_b h_b)\delta^{qB}\Big)^2\\ &+2g^2|h^cT^qh|^2-|F^q_\Sigma+\sqrt{2}gh^cT^qh|^2+|(\partial_5+g\Phi)h|^2+|(\partial_5-g\Phi)h^{c\dagger}|^2\bigg].\end{aligned} \tag{107}$$

Then, the equation of motion for D^q and F^q_Σ and the supersymmetric condition with a zero vacuum energy give

$$0 = D^q=-\partial_5\Phi^q-g(h^\dagger T^qh-h^cT^qh^{c\dagger})-(f(y)+\delta(y-\frac{\pi R}{2})gh_b^\dagger q_bh_b)\delta^{qB}, \tag{108}$$

$$0 = F^q_\Sigma=-\sqrt{2}gh^cT^qh, \tag{109}$$

$$0 = (\partial_5+g\Phi)h=(\partial_5-g\Phi)h^{c\dagger}. \tag{110}$$

Therefore, for a gauge invariant background $\langle h\rangle=\langle h^c\rangle=\langle h_b\rangle=0$, Φ^q develops a singular VEV for the supersymmetric vacuum as

$$\langle\Phi^q\rangle(y)=-\int_0^y dyf(y)\delta^{qB}=-c\,\delta'(y-\frac{\pi R}{2})\delta^{qB} \tag{111}$$

which breaks the $Z_2 \times Z_2'$ parities spontaneously. Then, for the nonzero $\langle \Phi^B \rangle$, the shape of the zero mode is modified as

$$h^{(0)}_{++}(y) = h^{(0)}_{++}(0)\exp\bigg(- gq_{++} \int_0^y dy \langle \Phi^B \rangle \bigg). \tag{112}$$

Here we note that the delta function can be regularized as $\delta(y) = \frac{1}{2\rho}(0)$ for $|y| < \rho(|y| > \rho)$ and the normalization for the zero mode is

$$\int_0^{2\pi R} dy\, |h^{(0)}_{++}(y)|^2 = 1. \tag{113}$$

Consequently, as $\rho \to 0^+$, we obtain the wave function of the zero mode independently of the cutoff Λ as

$$|h^{(0)}_{++}(y)|^2 = \delta(y - \frac{\pi R}{2}) + \delta(y - \frac{3\pi R}{2}). \tag{114}$$

Thus, the bulk zero mode is localized exactly at the fixed point $y = \pi R/2$ due to the log FI term. On the other hand, with a nonzero $\langle \Phi^B \rangle$, the equations of motion for the massive modes are given by

$$(\partial_5^2 + gq_{\alpha\beta}\partial_5 \langle \Phi^B \rangle - g^2 q^2_{\alpha\beta} \langle \Phi^B \rangle^2 + \lambda) h_{\alpha\beta} = 0 \tag{115}$$

where λ is the mass eigenvalue. From the similar analysis in the appendix B of Ref. [11], we find that the wave functions of massive modes are also modified due to the nonzero $\langle \Phi^B \rangle$ but their mass spectrum is not changed. Because our model is supersymmetric, we obtain the same results for the fermionic zero and massive modes as in the case with the scalar modes.

As for the anomaly cancellation, the dynamically localized bulk zero mode cancels the anomalies from the brane fermion locally. On the other hand, since the anomalies coming from a bulk field is given as in eqs. (75) to (78) by the sum of zero mode and massive mode contributions irrespective of the bulk basis, the modified massive modes corresponds to a bulk Chern-Simons term, which cancel the remaining Chern-Simons term, eqs. (89) and (90).

7 Conclusion

We considered the breaking of the 5D non-abelian gauge symmetry on $S^1/(Z_2 \times Z_2')$ orbifold. Then, we presented the localized gauge anomalies coming from a bulk fundamental field through the explicit KK mode decomposition of the 5D fields. In the orbifold with gauge symmetry breaking, there are fixed points with their own local gauge symmetries. Thus, there is the possibility of embedding some incomplete multiplets at the fixed point with unbroken gauge group, which can be sometimes phenomenologically preferred. The incomplete brane multiplet we considered can be realized from a bulk muliplet in the field theoretic limit. Therefore, we have shown that the 4D anomaly combination of a brane field and a bulk zero mode does not have the localized gauge anomalies up to the addition of a Chern-Simon 5-form with some jumping coefficient, which could be regarded as the effects of the bulk heavy modes as in the abelian gauge theory on S^1/Z_2. Then, we found a nonzero log FI term at the fixed point with H, which dynamically localizes the bulk zero mode at that fixed point while the wave functions of massive modes are modified to make up a bulk Chern-Simons term.

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