

Recent Progress in Superstring Theory

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Abstract

Superstring theory has continued to develop at a rapid clip in the past few years. Following a quick review of some of the major discoveries prior to 1998, this talk focuses on a few of the more recent developments. The topics I have chosen to present are 1) the use of K-theory to classify conserved charges carried by D-branes; 2) tachyon condensation on unstable D-brane systems; and 3) an introduction to noncommutative field theories and their solitons.

Talk presented at the Conference *Quantization, Gauge Theory, and Strings*
dedicated to the memory of Professor Efim Fradkin
and at the *Ninth Marcel Grossmann Meeting* (MG9)

¹Work supported in part by the U.S. Dept. of Energy under Grant No. DE-FG03-92-ER40701.

1 Introduction

The purpose of this talk is to survey some of the recent progress in superstring theory that has occurred in the past few years in a way that is accessible to nonexperts. A great deal has happened, much more than can be covered in a 40-minute talk, so difficult choices needed to be made. Even for the topics that are included it is necessary to sacrifice depth for breadth. All I hope to achieve is to convey a general sense of the directions in which the subject is developing and the impressive achievements that have taken place.

The plan of this talk is as follows. First I will give a quick review of developments that took place prior to mid-1998. Then I will focus on three areas in which there has been important progress since then. The first of these concerns the classification of conserved charges for D-brane systems. As I will explain, the appropriate mathematical category for this purpose turns out to be K-theory. Next I will discuss unstable D-brane systems, which have one or more tachyon fields in the world-volume theory. We will describe the physics associated with “tachyon condensation” when the tachyon fields roll to a minimum of the appropriate potential energy function. Finally, I will introduce non-commutative geometry. This turns out to be relevant for D-brane systems that have a constant magnetic field in their world volume. Certain simplifying features associated with the strong noncommutativity limit and their implications for string theory will be discussed.

2 Survey of Prior Results

As of 1985, following some 15 years of development, five consistent superstring theories had been identified [1]. Each of these requires ten dimensions (nine spatial and one time dimension), and each is supersymmetric. The five theories are shown in the table. The two type II theories have 32 conserved supersymmetry charges, whereas each of the other three theories have 16 conserved supercharges (corresponding to a Majorana–Weyl spinor).

	Gauge Group	Chiral?
Type I	$SO(32)$	yes
Type IIA	$U(1)$	no
Type IIB	—	yes
Heterotic	$E_8 \times E_8$	yes
Heterotic	$SO(32)$	yes

Table 1. The Five Superstring Theories

Following another 10 years of development, and the “second superstring revolution,” our understanding was significantly enhanced [2]. An important ingredient was the discovery of various dualities that imply deep and subtle relationships among the five theories. In fact it became clear that they should best be viewed as five distinct quantum vacua of a single underlying theory.

Even though the underlying theory is still not fully formulated, it is clear that it is completely unique, with no adjustable dimensionless parameters. Any dimensionless numbers that characterize a particular quantum vacuum are determined by expectation values of scalar fields. In particular, the string coupling constant, g_s , is given by the expectation value of the dilaton field (which belongs to the supergravity multiplet). Each of the five theories has an interesting limit as $g_s \rightarrow \infty$. The two $SO(32)$ theories are S-dual, which means that g_s for one of them corresponds to $1/g_s$ for the other. Thus, when one theory is strongly coupled, it corresponds to the other one at weak coupling. The IIB theory is self-dual in this sense. Most remarkably, the remaining two theories — type IIA and $E_8 \times E_8$ heterotic — turn out to have an 11th dimension whose size is proportional to g_s , and thus is invisible in perturbation theory. In the limit $g_s \rightarrow \infty$ each of them gives an 11-dimensional vacuum, which is called “M-theory.” Defined in this way, M-theory is neither more fundamental nor less fundamental than the five superstring theories — it is just one more special limit in the moduli space of consistent quantum vacua. There is some confusion on this point, because the term M-theory has also been used to refer to the underlying theory, which is more fundamental.

Another important lesson of the second superstring revolution was that in addition to the fundamental strings, the five superstring theories (and M-theory) also have various non-perturbative objects called p -branes. These are dynamical structures which have p extended spatial dimensions and can be idealized in many situations as being infinitely thin in the $9 - p$ (or $10 - p$) normal directions. The basic examples of p -branes are ones that carry a conserved p -brane charge that couples to a $(p + 1)$ -form gauge field. Such a field can be regarded as a generalized Maxwell field with $p + 1$ antisymmetrized indices and represented as a differential form

$$A^{(p+1)} = A_{\mu_1 \mu_2 \dots \mu_{p+1}} dx^{\mu_1} \wedge dx^{\mu_2} \wedge \dots \wedge dx^{\mu_{p+1}}. \quad (1)$$

The fact that the p -brane acts as a source for such a gauge field, with charge q , is represented

by the interaction term

$$S_{int} = q \int_Y A^{(p+1)}. \quad (2)$$

Here $A^{(p+1)}$ represents the pullback to the p -brane world-volume Y . Such a p -brane is supersymmetric, preserving half of the supersymmetry of the vacuum in which it is embedded, provided that the BPS condition,

$$q = T_p, \quad (3)$$

is satisfied. Here T_p denotes the tension of the p -brane, which is its energy density.

Let me list the BPS p -branes of the type II theories. First of all, these theories contain a two-form gauge field $B^{(2)}$ (in the NS-NS sector). The fundamental string (F1) is an electrical source of this field. Its magnetic dual, which is a source for the dual six-form potential, is called an NS5-brane. In addition, the type II theories have a collection of gauge fields $C^{(p+1)}$ (in the $R - R$ sector), for which a class of p -branes, called Dirichlet-branes (or Dp -branes) are sources. The p values that occur are even for the IIA theory and odd for the IIB theory.

The tensions of the various p -branes mentioned above are exactly determined by the BPS condition. Aside from a power of 2π , the results (in string metric) are as follows:

$$\begin{aligned} T_{F1} &= m_s^2 \\ T_{Dp} &= m_s^{p+1}/g_s \\ T_{NS5} &= m_s^6/g_s^2. \end{aligned} \quad (4)$$

Here, m_s denotes the fundamental string mass scale. It is related to the fundamental string length scale ℓ_s and the universal Regge slope α' by $m_s = \ell_s^{-1}$ and $\alpha' = \ell_s^2$. Note that the D-branes and the NS5-branes are both non-perturbative, but that the Dp -branes are much lighter at weak coupling, so that their non-perturbative effects dominate in this limit. The D-branes have played a central role in superstring theory research ever since the appearance of the breakthrough paper by Polchinski [3].

The Dirichlet branes are called such because they are hypersurfaces $Y \subset X$ on which the fundamental string can end. (X denotes the 10-dimensional spacetime.) Thus, at weak string coupling, their dynamics is determined by open-string field theory specialized to open strings that end on Y . In particular, let us consider the case of N coincident D-branes and focus on the massless states of the open strings, which are the relevant ones at low energies. In this limit one finds N^2 massless vector bosons belonging to the adjoint representation of

a $U(N)$ gauge group. Altogether, including their superpartners, one has a supersymmetric $U(N)$ gauge theory on Y .

The special case of coincident D3-branes in the type IIB theory is especially interesting, because the associated $\mathcal{N} = 4$, $U(N)$ gauge theory that gives the dynamics is superconformal. This means, in particular, that the quantum theory is finite at each order of perturbation theory, with vanishing β function. When N is large the back reaction of the branes on the spacetime geometry becomes important, and one should consider the gravitational field of the D3-branes. This gives a higher-dimensional analog of a black hole with a horizon. In particular, one finds that the near-horizon geometry is that of $AdS_5 \times S^5$, which is part of an exact solution of type IIB string theory.

These observations led Juan Maldacena in November 1997 to put forward his famous conjecture [4], which asserts that $\mathcal{N} = 4$, $d = 4$ $U(N)$ gauge theory is dual to type IIB superstring theory on $AdS_5 \times S^5$ with the following identifications:

$$\begin{aligned} g_s &\leftrightarrow g_{YM}^2 \\ R(AdS_5)/\ell_s &= R(S^5)/\ell_s \leftrightarrow (g_{YM}^2 N)^{1/4} \\ \int_{S^5} F^{(5)} &\leftrightarrow N. \end{aligned} \tag{5}$$

Here I have omitted numerical factors. $R(AdS_5)$ and $R(S^5)$ denote the radii of the respective spaces, and $F^{(5)}$ is the self-dual RR 5-form field strength of the IIB theory. Altogether, one has an equivalence of a conformally invariant field theory (CFT) and string theory in an anti de Sitter geometry, so one speaks of AdS/CFT duality.

In his paper, Maldacena also proposed several other equivalences of the same general type: namely, conjectural equivalences between conformal field theories in $p + 1$ dimensions, associated to coincident p -branes, and string theory or M-theory in a geometry that contains an AdS_{p+2} factor. The way the duality relates CFT correlation functions to AdS amplitudes was explained in papers by Gubser, Klebanov, and Polyakov [5] and by Witten [6] a few months later. This has become an enormous, and very fruitful, subject. By now, the Maldacena paper has well over 1000 citations. There is little doubt that the conjectures are correct. I will not say more about them here other than to point you to an excellent review paper [7], in case you wish to learn more.

3 Classification of D-brane Charges

Consider, as before, a D-brane system with world volume Y embedded in a spacetime manifold X of dimension $d = 10$ (superstrings) or $d = 26$ (bosonic strings). The surface Y will shrink to a minimal surface due to the brane tension (radiating away any excess energy). Even then, the D-brane may be unstable unless it is protected by a conserved charge. A signal of instability is when the open string spectrum on Y contains one or more tachyonic modes. Such tachyonic modes occur for the following cases, in particular:

1. All D-branes in the $d = 26$ bosonic string theory
2. Type II Dp -branes with a “wrong” p value (i.e., p odd in IIA or p even in IIB)
3. Type II D-brane + anti-D-brane systems.

Lest the terminology cause confusion, let me emphasize that there never are physical tachyons. Their apparent presence simply means that the vacuum under consideration is unstable. As for the Higgs fields in the standard model, one should like for a stable minimum of the appropriate potential. In the remainder of this section I will discuss the classification of conserved D-brane charges, and in the next section I will discuss the fate of unstable D-brane systems when the tachyon rolls to the minimum of its effective potential.

Since, as we have already indicated, the RR gauge fields that couple to D-branes can be represented as differential forms, it would be natural to guess that D-brane charges are given by cohomology classes. In simple cases this is good enough, but for some topologies it can give wrong answers. The right answer, as I will explain, is given by K-theory classes [8, 9]. This is important, because K-theory and cohomology can have different torsion groups.

To understand the physics, and the relevance of K-theory, let us consider a system of m Dp -branes and n anti- Dp -branes all of which are coincident. The gauge field configuration of the D-branes is described by a rank m vector bundle E and that of the anti-D-branes by a rank n vector bundle F . It is natural to postulate that complete annihilation of the branes and the anti-branes is possible if and only if E is topologically equivalent to F . This requires that $m = n$, but that is not sufficient by itself.

To be specific, let us consider equal numbers of coincident D9-branes and anti-D9-branes in the IIB theory for a 10-dimensional spacetime X . Note that these are spacetime-filling branes so $Y = X$. We can represent the system by a pair (E, F) , where E is the vector

bundle of the D-branes and F is the vector bundle of the anti-D-branes. Next, we define equivalence of pairs of bundles by

$$(E, F) \sim (E \oplus H, F \oplus H). \quad (6)$$

Physically, this corresponds to adding brane-antibrane pairs with vector bundles (H, H) . These branes can completely annihilate, and therefore adding them does not change any of the charges carried by the D-brane system. The equivalence relation described above defines equivalence classes $[(E, F)]$. Such an equivalence class is a K-theory class, and they form an additive group, called the K-theory group $K(X)$. Thus we learn that the conserved charges of type IIB D-branes are classified by $K(X)$. The possible values of the conserved charges are in one-to-one correspondence with the K-theory classes.

There is a similar, but somewhat different, construction for the type IIA theory [10], which I will not review here. However, I would like to remind you that in the strong coupling limit the IIA theory becomes M theory, so all these objects should have suitable 11-dimensional counterparts. The precise way in which this works is a quite technical business, which was recently analyzed by Diaconescu, Moore, and Witten [11]. The fact that everything works out properly provides additional nontrivial evidence for the consistency of the whole picture. The group E_8 enters the eleven-dimensional analysis in a surprising way, whose physical significance is unclear.

4 Tachyon Condensation

Consider a spacetime-filling D25-brane in the $d = 26$ bosonic string theory. It has a tension $T_{25} = C/g_s$, where C is a known constant. Its dynamics is described by the bosonic open string field theory that Witten constructed long ago [12]. I won't attempt to describe that theory here, but simply point out that it is cubic in the string field Φ , which is a functional of the string embedding functions $x^\mu(\sigma)$ and world-sheet ghost fields. The lowest mode of Φ is a tachyon $t(x)$, so the purely t -dependent part of the potential has the form $V(t) = -\alpha t^2 + \beta t^3$ with a local maximum at $t = 0$ and a local minimum at $t = t_0 > 0$. Note that $V(0) = 0$ and $V(t_0) = V_{min} < 0$. The string field theory contains infinitely many other scalar fields besides t , and it is really the complete potential whose extremum V_{min} is relevant. The calculation based on the cubic expression $V(t)$ only gives a first approximation to the exact result.

We can now state the basic conjecture, due to Sen [13]: *The minimum of the potential,*

where the tachyon has condensed and other scalar fields have also adjusted appropriately, corresponds to pure vacuum. Therefore

$$T_{25} + V_{min} = 0. \tag{7}$$

This means that the D-brane has completely disappeared. One has a configuration identical to what one would have if the D-brane hadn't been introduced in the first place!

Open string field theory is much too complicated to carry out analytic calculations. So, Sen and Zwiebach [14] tested Sen's conjecture numerically using a level truncation approximation scheme introduced by Kosteletzky and Samuel [15]. At leading order (the approximation based on the cubic expression in t described above) they found a value of V_{min} that is roughly 70% of the predicted answer. At the level 2, where some additional scalars and interactions are taken into account, they obtained about 95% of the predicted answer, and at level 4 they found 98.6%. Following that, Moeller and Taylor [16] included fields up to level 10, and all the associated interaction terms that the approximation scheme calls for at this order, and found 99.91% of the expected answer. The results are convincing evidence for the correctness of the conjecture, even though it is not completely clear why this approximation scheme converges so well.

Berkovits, Sen, and Zwiebach have done the analogous calculation for the unstable D9-brane in the type IIA theory [17], which is predicted to disappear when the tachyon condenses in exactly the same way. They used a version of open NS-sector superstring field theory developed by Berkovits [18]. (It avoids certain technical problems associated with picture-changing operators.) They obtained about 60% of the expected answer at leading order and 85% at the next order. So this seems to be working well, too. This calculation also applies to the annihilation of a D9 and an anti-D9-brane in type IIB [19].

This D-brane disappearance story raises some interesting questions. The basic ones are: what has happened to the original $U(N)$ gauge symmetry, and what has happened to the entire open string spectrum? The answer to both of these is that they are confined. What this means in terms of the string field theory calculation is that when all the scalar fields take the values corresponding to the minimum of the potential, the coefficients of all kinetic terms — both of the gauge fields and of all the massive string modes — become zero. This is how confinement is realized mathematically. There is some numerical evidence in support of this, but further studies would be desirable.

As an aside, let me remark that the $d = 26$ bosonic string theory also has a closed string

tachyon, so it is natural to speculate about its fate as well. However, it is not clear what is the right conjecture to test. Zwiebach has constructed a closed string field theory [20], which is much more complicated than open string field theory. (Zwiebach informs me that if there were a clear conjecture to test, he and his collaborators would find the energy to carry out the closed string field theory computations.) Moreover, it is not clear to me whether a classical computation should capture the physics correctly or if quantum effects would play an important role. The point is that extremizing the potential might also force the dilaton to a particular value. What are some possibilities? One possibility is that the effective potential has no minimum and there is no stable vacuum. Alternatively, if the effective potential does have a stable minimum, this would seem to imply that there is a negative cosmological constant of order string scale. One could also imagine more exotic possibilities. Even though the bosonic string theory is surely not relevant to the real world, it would be satisfying to understand the answer to this question.

5 Noncommutative Field Theory

A field theory with noncommutativity parameters $\theta^{\mu\nu} = -\theta^{\nu\mu}$ can be defined as a deformation of an ordinary field theory in which field multiplication is replaced by a nonlocal star product:

$$A(x) \cdot B(x) \rightarrow A * B(x), \quad (8)$$

where the definition of the star product is

$$A * B(x) = e^{\frac{i}{2}\theta^{\mu\nu}\partial_\mu\partial'_\nu} A(x)B(x')|_{x'=x}. \quad (9)$$

The star product (also called a Moyal product) is associative but noncommutative. In particular,

$$x^\mu * x^\nu - x^\nu * x^\mu = i\theta^{\mu\nu}. \quad (10)$$

Thus one has a sort of Heisenberg uncertainty principle for spacetime coordinates, and one sometimes speaks of noncommutative geometry. Combining the two uncertainty principles gives a UV/IR connection, which is a frequent theme in modern string theory.

Following important earlier work by Connes, Douglas, and A. Schwarz [21], as well as others, Seiberg and Witten [22] gave a very clear description of how noncommutative gauge theory arises on D-branes when there is a nonvanishing constant $B^{(2)}$ field in the bulk. I will

not present the formulas, but simply assert that there is a rule for getting from $B_{\mu\nu}$ in the bulk to $\theta^{\mu\nu}$ on the brane. (In the limit that B is large it is just the reciprocal matrix.)

On the brane, $\theta^{\mu\nu}$ corresponds to a background Maxwell field strength. Thus, if it has only nonzero space-space components, this corresponds to a magnetic field on the D-brane and describes nonlocal behavior. The Hamiltonian still generates unitary, causal time evolution in the usual way. Such a theory may be useful for analyzing the quantum Hall effect.

If $\theta^{\mu\nu}$ has nonzero space-time components this corresponds to an electric field on the brane. This results in a non-unitary (and hence inconsistent) field theory [23]. In the string theory context various authors have argued that electric fields are okay provided they do not exceed a critical value E_{crit} . This has a simple intuitive explanation: The open string has opposite electric charges on its two ends, so when it is placed in an electric field it gets stretched. The critical field is reached when the stretching force balances the restoring force due to the string tension. Clearly, larger fields would lead to instability.

Let us now focus on the magnetic case, assuming that only $\theta^{12} = \theta$ is nonzero. Then, following Gopakumar, Minwalla, and Strominger [24], let us consider a scalar field theory with this noncommutativity parameter. Letting $z = x_1 + ix_2$, and assuming no dependence on any other coordinates, the energy of a configuration is proportional to

$$\int d^2z (\partial_z \phi \partial_{\bar{z}} \phi + V(\phi)), \quad (11)$$

where

$$V(\phi) = \frac{1}{2} m^2 \phi * \phi + \frac{1}{3} \lambda \phi * \phi * \phi + \dots \quad (12)$$

Rescaling $z \rightarrow z\sqrt{\theta}$ gives

$$\int d^2z (|\partial\phi|^2 + \theta V(\phi)), \quad (13)$$

where $*$ is now defined with $\theta = 1$. Thus, for large θ , the extrema are given simply by

$$\frac{dV}{d\phi} = 0. \quad (14)$$

Ordinarily, the solutions of $\frac{dV}{d\phi} = 0$ would just give the extrema of V . However, when V is defined using star products, there can be nontrivial solitonic solutions. The reason for this is that the equation

$$\phi_0 * \phi_0(x) = \phi_0(x), \quad (15)$$

has nontrivial solutions. For example, as one can readily verify, one solution is

$$\phi_0(x) = 2e^{-(x_1^2 + x_2^2)}. \quad (16)$$

This “wave function” corresponds to the ground-state projection operator $|0\rangle\langle 0|$ of the harmonic oscillator algebra $[z, \bar{z}] = 1$. Clearly, any projection operator would solve the same equation.

Using such a projection operator wave function ϕ_0 one can construct soliton solutions $t = t_0\phi_0(x)$ to $\frac{dV}{dt} = 0$, in the notation of the previous section. The intuitive idea is that the core of the soliton near $x_1 = x_2 = 0$ is in the false vacuum $t = 0$, whereas for large $x_1^2 + x_2^2$ it approaches the true vacuum $t = t_0$. The amazing fact about this construction is that it only requires knowing the extrema of $V(t)$ and not the precise form of the function that interpolates between them. If one accepts Sen’s conjecture, this is precisely what we do know.

Harvey et al. [25] have applied these methods to bosonic open string field theory in the manner indicated above. (Related ideas for superstrings were discussed by Dasgupta et al. [26].) They demonstrated that the ground state ϕ_0 soliton on a D25-brane describes a D23-brane (concentrated near $x_1 = x_2 = 0$) with exactly the correct tension. This construction can be iterated to give the $D(25 - 2n)$ -brane. Moreover, these branes were shown to contain tachyons with exactly the correct (imaginary) mass. The constant B field responsible for θ is pure gauge in the bulk, where the D25-brane has disappeared as a result of tachyon condensation. This suggests that even though the analysis was carried out in the limit of large θ , the result should be valid for any value of θ including $\theta = 0$. Harvey, et al. have found many other interesting results, and also Witten has made some clarifying remarks [27].

6 Concluding Remarks

We have shown that studies of D-branes have led to a deeper understanding of many issues in string theory. As you are probably aware, they have also had a big impact on subjects that I have not mentioned ranging from black hole entropy to nonperturbative properties of gauge theories.

One lesson we can learn from these developments is that open string field theory is important because it describes D-branes. When it was developed, this connection was not understood, and it was not clear what it is good for. There are some indications that to achieve background independence it will be necessary to consider it in the $N \rightarrow \infty$ limit [28]. If a nice way to define such a limit is found, it might prove to be very important.

Noncommutative field theory is a deformation of ordinary field theory, which apparently

does not destroy quantum consistency (in the magnetic case). This is a remarkable fact, which has spurred a lot of activity. Here I have focused on its use as a technical trick for analyzing certain issues associated to tachyon condensation on unstable D-brane systems.

As I hope to have convinced you, there have been many interesting and fruitful developments in recent years. I am confident that there are many more surprises to be uncovered in the coming years. We are still quite far from a complete understanding of this marvelous mathematical edifice.

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