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Bouncing and cyclic universes from brane models

Sudipta Mukherji [a,b] **and Marco Peloso** [b]*

[a] *Institute of Physics, Bhubaneswar-751 005, India*

[b] *Physikalisches Institut, Universität Bonn,*
Nussallee 12, D-53115 Bonn, Germany

mukherji@iopb.res.in, peloso@cita.utoronto.ca

We consider a D3-brane as boundary of a five dimensional charged anti de Sitter black hole. We show that the charge of the black hole induces a regular cosmological evolution for the scale factor of the brane, with a smooth transition between a contracting and an eventual expanding phase. Simple analytical solutions can be obtained in the case of a vanishing effective cosmological constant on the brane. A nonvanishing cosmological constant, or the inclusion of radiation on the brane, does not spoil the regularity of these solutions at small radii, and observational constraints such as the ones from primordial nucleosynthesis can be easily met. Fluctuations of brane fields remain in the linear regime provided the minimal size of the scale factor is sufficiently large. We conclude with an analysis of the Cardy-Verlinde formula in this set up.

*New Address: C.I.T.A., University of Toronto, 60 St. George Street, Toronto, Ontario, Canada M5S 3H8

1 Introduction

Motivated by string/M theory, the AdS/CFT correspondence, and the hierarchy problem of particle physics, brane-world models were studied actively in recent years [1]-[6]. In these models, our universe is realised as a boundary of a higher dimensional space-time (the so called bulk). In particular, a well studied example is when the bulk is an AdS space. The gravitational interaction among matter on this brane is found to be described by standard laws when one considers distance scales much larger than the inverse of the AdS mass scale [7].

In the cosmological context, embedding of a four dimensional Friedmann-Robertson-Walker (FRW) universe was also considered when the bulk is described by AdS or AdS black hole, see for example [8]-[20]. In the latter case, the mass of the black hole was found to effectively act as an (invisible) energy density on the brane with the same equation of state of radiation. In another line of interesting development, initiated in [21, 22], holographic principle was studied in the FRW universe filled with such dark radiation. Representing radiation as conformal matter and exploiting AdS/CFT correspondence, a Cardy like formula [23] for the entropy was found for the universe. This is often referred to as Cardy-Verlinde formula in recent literature. Furthermore, in [21], a cosmological entropy bound was proposed which unifies the Bekenstein bound for a limited self-gravity system and the Hubble bound for a strong-gravity system in an elegant way (for related works, see e.g. [24, 25, 26]).

In the present paper, we study the cosmology of a four dimensional brane which constitutes the boundary of a charged AdS black hole background. By introducing this extra charge parameter on the bulk, we find a host of interesting cosmological solutions for our brane universe. Their most remarkable feature is that the charge term allows for a *nonsingular* transition between a contracting phase of the scale factor of the brane, and a following expanding stage. * The cosmological evolution of this model is thus free from singularities. The latter reappear only when the bulk charge is taken to zero.

In the next section of the paper, we briefly review the charged AdS black hole background. In section 3 we then set up the Hubble equation for the scale factor of the brane, and analyse the induced four dimensional cosmological evolution. We present exact solutions in the case of a vanishing effective cosmological constant on the brane, which can be achieved through a fine tuning of the brane tension and the five dimensional cosmological constant. In the case of a flat or an open geometry, the evolution is characterized by a nonsingular and smooth transition between a contracting and an eventual expanding phase. For a closed geometry, also the expanding phase turns into contraction at later times, so that the whole evolution is cyclic. In section 4 we discuss the more physically relevant case in which radiation is present on the brane. We discuss under which conditions the system can describe a universe with a bounce followed by a radiation dominated phase compatible with the constraints coming from the successful predictions of (standard) primordial nucleosynthesis. In section 5 we discuss the cosmological evolution of fluctuations of fields on the brane. We show that they remain in the linear regime provided the minimum size of the scale factor is sufficiently large. We also comment on the more general issue of cosmological perturbations in this model. In section 6 we briefly analyse the Cardy-Verlinde

*A qualitatively similar behavior is shown by some solutions of the system described in [15].

formula in the light of the explicit solutions that we obtained. Finally, we present our conclusions.

2 The five dimensional background

In this work we will consider a 3+1 dimensional brane in a space-time described by a 5 dimensional charged AdS black hole. The background metric is thus given by [28, 29]

$$ds_5^2 = -h(a)dt^2 + \frac{da^2}{h(a)} + a^2\gamma_{ij}dx^i dx^j, \tag{1}$$

where

$$h(a) = k - \frac{\omega_4 M}{a^2} + \frac{3\omega_4^2 Q^2}{16a^4} + \frac{a^2}{L^2}. \tag{2}$$

Here, $k = 0, \pm 1$, corresponding to flat, open, or close geometries of four dimensional subspaces at any given a. M and Q are, respectively, the Arnowitt-Deser-Misner mass and charge, and L is the curvature radius of space. γ_{ij} is the metric for a constant curvature manifold M^3 with $\mathrm{Vol}(M^3) = \int d^3x\sqrt{\gamma}$. $\mathbf{G_5}$ is the five dimensional Newton's constant and $\omega_4 = 16\pi\mathbf{G_5}/3\mathrm{Vol}(M^3)$. It can be easily shown that the above metric (1) describes a charged black hole with two horizons only provided that

$$L^2 > \frac{3\,x_H^2}{\omega_4\,M - 2\,k\,x_H}\ , \tag{3}$$

where

$$x_H = \omega_4\,M\left(\sqrt{1+\frac{9\,Q^2}{16\,M^2}}-1\right)\ ,\ \ x_H = \frac{9\,\omega_4\,Q^2}{32\,M}\ ,\ \ x_H = \omega_4\,M\left(1-\sqrt{1-\frac{9\,Q^2}{16\,M^2}}\right)\ , \tag{4}$$

in the three cases $k = -1, 0, 1$, respectively. It can be also shown that, when the limit (3) is satisfied, the quantity $a_H \equiv \sqrt{x_H}$ represents a lower bound on the position of the larger of the two horizons.

The electrostatic potential difference between the horizon and infinity is given by the quantity ϕ which, following [28], we choose to be

$$\phi = \left(\frac{3}{8}\right)\frac{\omega_4 Q}{a^2}. \tag{5}$$

The entropy and the temperature associated with (1) are given by

$$\begin{aligned} \mathcal{S} &= \frac{a_H^3 \mathrm{Vol}(M^3)}{4\mathbf{G}_5}, \\ \mathcal{T} &= \frac{h'(a)}{4\pi}\Big|_{a=a_H} = \frac{4a_H^2 + 2kL^2}{4\pi L^2 a_H} - \frac{3\omega_4^2 Q^2}{32\pi a_H^5}. \end{aligned} \tag{6}$$

Let us now consider a $(3+1)$-dimensional brane with a constant tension in the background of (1). As discussed in [22], if we regard the brane as a boundary of the background AdS geometry, its location and its induced metric become time dependent. More specifically, the induced four dimensional metric describes a FRW Universe, with a scale factor evolution determined both by the five dimensional geometry and by the constituents of the brane itself. In the next two sections we discuss in details the cosmological evolution of the four dimensional metric for the geometry (1). Here we note that the boundary CFT can be determined only up to a conformal factor [30, 31]. Making use of this fact, we rescale the boundary metric in the following form

$$ds^2_{CFT} = \lim_{a\to\infty}\left[\frac{L^2}{a^2}ds_5^2\right] = -dt^2 + L^2 d\Omega_4^2. \tag{7}$$

As the thermodynamical quantities of the CFT at high temperature can be identified with those with the bulk adS black hole [32], we get the following thermodynamical variables on the boundary CFT:

$$E = \frac{LM}{a}, \quad \Phi = \frac{L\phi}{a}, \quad T = \frac{L\mathcal{T}}{a}. \tag{8}$$

These extra factors of L/a appear following the scaling in (7). The entropy S of the CFT remains the same as (6).

3 Cosmological evolution of the four dimensional brane

Consider now a brane with tension $\Lambda_{\rm br}$ in the background (1). It can be shown (see [33, 13, 16, 22, 19] for details) that an observer on the brane experiences a four dimensional FRW universe

$$ds_4^2 = -d\tau^2 + a(\tau)^2\gamma_{ij}dx^i dx^j, \tag{9}$$

with a Hubble law given by

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = -\frac{k}{a^2} + \frac{\omega_4 M}{a^4} - \frac{3\omega_4^2 Q^2}{16 a^6} + \frac{\Lambda_4}{3}\,, \tag{10}$$

where $k = +1, -1, 0$ correspond to a closed, flat or open geometry, respectively, dot denotes derivative with respect to the physical time τ on the brane, and where the effective four dimensional cosmological constant $\Lambda_4 = \Lambda_{br} - 3/L^2$ gets a contribution from the bulk cosmological constant and the brane tension Λ_{br}. By tuning these two contributions, Λ_4 can be set to zero, in which case the brane is denoted as *critical brane*. In the present section we will discuss the cosmological evolution of an empty and critical brane, while the more general cases of a *non critical* brane and of a brane filled with radiation will be discussed in the next section.

By looking to the scaling with a of the different terms of eq. (10), we see that the terms proportional to M and Q behave, respectively, like the energy density of radiation and of "stiff matter" (i.e. with dominance of the kinetic energy) on the brane. In the second case, the sign is however opposite with respect to the standard situation. Since this last term dominates for sufficiently small values of a, one may expect that this sign difference could have interesting cosmological consequences. Indeed, we will see that it is crucial in allowing a nonsingular transition between a contracting and an expanding evolution of the scale factor a.

The evolution of the system can be solved exactly, as one can most simply realize by using conformal time η, defined as $d\tau = a(\eta)d\eta$. Let us first consider a closed four dimensional geometry. In this case case the solution is

$$a(\eta) = \sqrt{\frac{\omega_4 M}{2}} \left[1 - c_1 \, \cos(2\,\eta)\right]^{\frac{1}{2}}, \quad \text{with} \;\; c_1 = \sqrt{1 - \frac{3Q^2}{4M^2}}. \tag{11}$$

The reality of c_1 (i.e. $Q < 2M/\sqrt{3}$) is a necessary condition for the existence of a horizon in the five dimensional geometry, see eq. (3), which we always assume to be the case. Hence, the universe evolves periodically, with a four dimensional radius oscillating between a maximal and a minimal size given by

$$a_{\max,\min} = \sqrt{\frac{\omega_4 \, M}{2}} \left(1 \pm c_1\right)^{1/2} . \tag{12}$$

Notice that we have used the freedom in setting the origin of conformal time so that the minimal radius is reached at $\eta = n\,\pi$, where n is an integer number. From the bulk perspective, the brane starts out from the charged black hole at a distance $a_{\min}$ from the singularity and moves away up to $a_{\max}$ as it expands. At later time, it collapses again at $a = a_{\min}$.

It is instructive to compare the case at hand with the situation in which the brane is in the background of an AdS-Schwarzchild black hole. Also in the latter situation, the cosmological evolution is governed by eq. (10), but with with $Q = 0$. This, in turn, means that the time dependence of the scale factor is given by (11), with $c_1 = 1$. In this case, the scale factor starts from zero size and expands up to $a_{\max}$ before collapsing again to zero size. From the bulk point of view, the brane originates from the black hole singularity and at a later stage it collapses again into the singularity. We, therefore, see that the effect of the background charge Q is rather non-trivial, since it makes the cosmology of the model free from singularities.

For an open universe, we find the solution

$$a(\eta) = \sqrt{\frac{\omega M}{2}} \left[c_2 \, \cosh(2\,\eta) - 1\right]^{\frac{1}{2}}, \quad \text{with} \;\; c_2 = \sqrt{1 + \frac{3Q^2}{4M^2}} . \tag{13}$$

In this case, the brane is initially contracting, and then bounces to an expanding phase. Again, we have set $\eta = 0$ at the bounce. The minimal radius is given by

$$a_{\min} = \sqrt{\frac{\omega_4 \, M}{2}} \left(c_2 - 1\right)^{1/2} . \tag{14}$$

As before, $a_{\min} \to 0$ as $Q \to 0$.

Finally, in the case of a flat universe we find

$$a(\eta) = \sqrt{\frac{3\,Q^2\,\omega_4}{16\,M} + \omega_4\,M\,\eta^2}, \tag{15}$$

Also in this case we have a bouncing universe, with a minimal radius

$$a_{\min} = \sqrt{\frac{3\,Q^2\,\omega_4}{16\,M}} , \tag{16}$$

which vanishes in the limit $Q \to 0$. At late times one recovers the evolution $a(\eta) \sim \eta \sim \tau^{\frac{1}{2}}$, which is typical of a flat universe dominated by radiation.

4 Radiation on the brane

We now consider the more physically relevant case in which a perfect fluid with equation of state of radiation is present on the brane. We will discuss under which conditions the system can describe a universe with a bounce followed by a radiation dominated phase compatible with observations (more precisely, with the constraints coming from primordial nucleosynthesis). In presence of a perfect fluid on the brane, eq. (10) rewrites (see i.e. [13] for details)

$$H^2 = -\frac{k}{a^2} + \frac{\omega_4\, M}{a^4} - \frac{3\,\omega_4^2\, Q^2}{16\, a^6} - \frac{1}{L^2} + \frac{4\,\pi}{3\, M_p^2\, \rho_0} \left(\rho_0 + \rho_{\rm br}\right)^2 \;, \tag{17}$$

where ρ_0 denotes the tension of the brane, while the energy density $\rho_{\rm br}$ of the fluid satisfies the standard conservation equation

$$\dot{\rho}_{\rm br} + 3\,\frac{\dot{a}}{a}\,\left(1+w\right)\,\rho_{\rm br} = 0 \;, \tag{18}$$

where w is the equation of state of the fluid, $w = 1/3$ for radiation. Notice that the total energy density on the brane $\rho_0 + \rho_{\rm br}$ contributes quadratically to H^2 [36]. However, from the mixed term $\propto \rho_0\, \rho_{\rm br}$ one recovers at small energies ($\rho_{\rm br} \ll \rho_0$) a leading contribution which is linearly proportional to the energy density of the fluid [37, 38]. The overall coefficient of the last term of eq. (17) has been normalized so to reproduce the standard coefficient for the term linear in $\rho_{\rm br}$.

The evolution equation (17) can be rewritten as

$$\begin{aligned} H^2 &= -\frac{k}{a^2} + \frac{\omega_4\, M}{a^4} - \frac{3\,\omega_4^2\, Q^2}{16\, a^6} + \frac{\Lambda_4}{3} + \frac{8\,\pi}{3\, M_p^2} \left(\rho_{\rm br} + \frac{\rho_{\rm br}^2}{2\,\rho_0}\right) \;, \\ \Lambda_4 &\equiv \frac{4\,\pi}{M_p^2}\,\rho_0 - \frac{3}{L^2} \equiv \Lambda_{\rm br} - \frac{3}{L^2} \;. \end{aligned} \tag{19}$$

The four effective cosmological constant Λ_4 is obtained as a sum of the tension of the brane and of the five dimensional cosmological constant. To have a realistic cosmology, these two quantities have to be fine-tuned to cancel each other. Even if the cancellation is not perfect, the contribution from the cosmological constant term to eq. (19) is negligible at small scale factor a, so that this term does not alter the evolution of the universe close to the bounce. At large a, we can instead neglect the term proportional to Q, as well the last contribution quadratic in $\rho_{\rm br}$. Then, eq. (10) effectively describes the rather standard situation of an universe filled with radiation and cosmological constant. In the context of brane models, the cosmological evolution for $Q = \rho_{\rm br} = 0$ can be found, for example, in refs. [34, 35].

To discuss in more details the cosmological evolution across and after the bounce, let us consider eq. (19) at small scale factor. Besides the constant term Λ_4, also the curvature term

$-k/a^2$ can be neglected in eq. (19), since it is subdominant also today. Specifying the energy density on the brane to be in the form of radiation, $\rho_{\rm br} \equiv \rho_r/a^4$, we have

$$H^2 \simeq \left(\frac{8\,\pi\,\rho_r}{3\,M_p^2} + \omega_4\,M\right)\frac{1}{a^4} - \frac{3\,\omega_4^2\,Q^2}{16}\,\frac{1}{a^6} + \frac{4\,\pi\,\rho_r^2}{3\,M_p^2\,\rho_0}\,\frac{1}{a^8}\;. \tag{20}$$

The first contribution is a sum of the effective dark radiation term, coming from the mass of the five dimensional black hole, and of the radiation present on the brane. Primordial nucleosynthesis imposes that the latter term dominates over the former, since radiation is not only responsible for the expansion of the universe at early times, but also for the formation of light elements. From the observed abundances of the latter, one typically concludes [39, 40] that any nonstandard form of energy density can contribute at most as an additional neutrino species at the time of nucleosynthesis (a more constrained bound is found by combining results from nucleosynthesis with the ones inferred from anisotropies of the Cosmic Microwave Background [41]). Requiring the dark radiation term to contribute less than an additional neutrino species gives the constraint

$$\omega_4\,M \lesssim 1.1\,\frac{\rho_r}{M_p^2}\;. \tag{21}$$

In the following, we neglect the dark radiation term with respect to the the standard radiation on the brane.

The occurrence of the bounce places another constraint on the parameters. This is obtained by requiring that the Hubble parameter H given in eq. (20) vanishes at some finite value of a, since otherwise the term proportional to the charge of the black hole Q is always subdominant with respect to the other two terms. Thus, we must have

$$\omega_4^4\,Q^4 \gtrsim 4000\,\frac{\rho_r^3}{M_p^4\,\rho_0}\;. \tag{22}$$

If this bound is satisfied, the scale factor approximatively amounts to $a_{\rm min} \simeq 0.15\,\omega_4\,Q\,M_p/\rho_r^{1/2}$ at the bounce (the right hand side of (22) has been neglected in this estimate). Since we want a standard (radiation dominated) expansion at the time of primordial nucleosynthesis, that is when the temperature of the thermal bath is ~ 0.2 MeV, we finally have to require

$$\rho_{\rm br}\left(a_{\rm min}\right) = \frac{\rho_r}{a_{\rm min}^4} = \frac{\rho_r^3}{0.15^4\,\omega_4^4\,Q^4\,M_p^4} \gtrsim 0.2^4\,{\rm MeV}^4\;. \tag{23}$$

By combining the two expressions (22) and (23), we see that in order for the $\rho_{\rm br}^2$ term in the Hubble equation not to affect the predictions of primordial nucleosynthesis, the tension of the brane has to satisfy the lower bound

$$\rho_0^{1/4} \gtrsim 0.24\,{\rm MeV}\;. \tag{24}$$

This is a less stringent bound than the one coming by imposing that ordinary gravity is recovered in current gravity experiments, $\rho_0^{1/4} > {\rm few} \times {\rm TeV}$ [42].

5 Fluctuations of brane fields

Fluctuations about the regular backgrounds described in the previous sections are also expected to be regular. Here we discuss the simple case of fluctuations of a massless minimally coupled scalar field ϕ living on the brane, assuming that it does not contribute significantly to the cosmological evolution of the background. At the end of the section, we briefly comment on the more general issue of cosmological perturbations in this model. For definiteness, we discuss the case of a flat critical brane, while the generalization to the other cases is straightforward. The four dimensional scale factor then evolves according to eq. (15). Here we redefine

$$a\,(\eta) = a_{\rm min}\,\sqrt{1+\gamma^2\,\eta^2} \;\;,\;\; a_{\rm min} = \sqrt{\frac{3\,Q^2\,\omega_4}{16\,M}} \;,\; \gamma = \frac{4\,M}{\sqrt{3}\,Q} \;. \tag{25}$$

Consider a mode of the fluctuations $\delta\phi_k$ with a given comoving momentum k, and the rescaled mode $v_k \equiv a\,\delta\phi_k$. The latter evolves according to

$$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0 \;\;,\;\; \frac{a''}{a} = \frac{\gamma^2}{\left(1+\gamma^2\,\eta^2\right)^2} \;. \tag{26}$$

At early times, the term a''/a is negligible, and the mode behaves as a plane wave in Minkowski space. Since the field v is canonically normalized, we have

$$\delta\phi_k = v_k/a \rightarrow \frac{{\rm e}^{-i\,k\,\eta}}{\sqrt{2\,k}\,a} \;\;,\;\; \eta \rightarrow -\infty \;. \tag{27}$$

If $k \gg \gamma$, the size of the mode is always much smaller than the size of the horizon, and v always behaves as a plane wave. The case $k < \gamma$ is more interesting. As η increases, the evolution of the universe starts to be important, and the term a''/a cannot anymore be neglected in eq. (26). The two terms a''/a and k become equal at the time η_* given by

$$\gamma\,\eta_* = -\sqrt{\frac{\gamma-k}{k}}\,. \tag{28}$$

In the long wave limit (that is, neglecting k in eq. (26)) we have then

$$\delta\phi_k \simeq C_1 + \tilde{C}_2 \int^\eta \frac{d\eta'}{a^2} = C_1 + C_2\,{\rm arctg}\,(\gamma\,\eta) \;\;,\;\; |\eta| \ll |\eta_i| \;. \tag{29}$$

The coefficients $C_1\,,\,C_2$ can be estimated by assuming that the mode evolution is given by eq. (27) for $\eta < \eta_*$ and by eq. (29) for $\eta > \eta_*$, and by matching $\delta\phi_k$ and its derivative at η_*. This gives

$$\begin{aligned} |C_1| &\simeq \frac{1}{\sqrt{2\,\gamma}\,a_{\rm min}} \left[\frac{\gamma}{k}{\rm arctg}^2\sqrt{\frac{\gamma-k}{k}} + 2\,\sqrt{\frac{\gamma-k}{k}}\,{\rm arctg}^2\sqrt{\frac{\gamma-k}{k}} + 1\right]^{1/2} \,, \\ |C_2| &\simeq \frac{1}{\sqrt{2\,k}\,a_{\rm min}} \;. \end{aligned} \tag{30}$$

As it was expected, the spectrum of these fluctuations is not scale invariant. For very long wave modes, $k \ll \gamma$, we find (up to an overall irrelevant phase)

$$\delta\phi_k \simeq \frac{1}{\sqrt{2\,k}\,a_{\rm min}}\left[\frac{\pi}{2}+{\rm arctg}\left(\gamma\,\eta\right)\right]\;. \tag{31}$$

We see that the linear theory of the fluctuations breaks down if the background becomes singular at the bounce, $a_{\rm min} \to 0$.

The computation just reported has mainly the aim to enlight some of the features that can be expected in the more interesting study of the cosmological perturbations of this system. Although the latter is beyond the aims of the present work, some considerations are in order. The simplest approach to this problem is to study the perturbations of a four dimensional effective theory leading to eq. (10). In a four dimensional context, this can be obtained at the expense of introducing a massless field φ with negative kinetic terms. The absence of a potential term for the field φ guarantees the equation of state $w_\varphi = +1$, so that $\rho_\varphi \propto a^{-6}$, and eq. (10) can be recovered. Cosmological perturbations in this four dimensional set-up have been studied in [43], where it is also concluded that perturbations remain regular as long as the background is regular, and where a late time scale depended spectrum is also obtained (but with different k dependence wih respect to our eq. (31)). Although the results of [43] are very interesting for the general study of cosmological perturbations in bouncing models, the presence of a negative kinetic term opens several problems, and the system should be considered only as an effective one, as also mentioned in [43]. Here, we would like to point out another potential difficulty, related to the choice of initial conditions. In the standard case, it is customary to start with an adiabatic (Bunch-Davies [44]) vacuum, since - for an early adiabatic evolution of the background - it minimizes the energy of the system (for a detailed discussion, see i.e. [45]). With negative kinetic terms, the adiabatic vacuum maximizes the energy of the fluctuations, so that the choice of the initial conditions appears more problematic than in the standard case. An alternative approach is certainly a direct calculation of the perturbations in the full five dimensional theory. The formalism for the study of perturbations in brane models is at the moment under deep investigation, see for example [46, 47, 48]. However, the application of this formalism to the system discussed in this paper is at present a highly nontrivial issue.

6 Cardy-Verlinde Formula

In this section, we analyse the Cardy-Verlinde formula [21, 22] in the light of the explicit solutions found in the second section. We specify to the case of a critical brane. We start with the case of a closed universe. As discussed in section-2, the brane in this case behaves as a closed non-singular universe if Q is less than certain critical value. The radius of the brane oscillates between $a_{\rm min}$ and $a_{\rm max}$ given in eq. (12). Note that the bulk geometry has a non-singular horizon at $a = a_H$ where a_H obeys

$$1-\frac{\omega_4 M}{a_H^2}+\frac{3\omega_4^2 Q^2}{16 a_H^4}+\frac{a_H^2}{L^2}=0\;. \tag{32}$$

It is easy to check that for a given value of Q and M, the brane starts expanding at a location beyond horizon. The distance from the horizon depends critically on the Q/M ratio. The radius then increases and, as a result, it comes out of the black hole horizon. As time progresses, it then again reaches a minimum size at a point beyond horizon. As we increase this ratio, keeping other parameters fixed, $a_{\min}$ moves closer to the horizon.

As argued in [21], during the cosmological evolution, the universe goes from weakly self-gravitating to strongly self-gravitating region or otherwise. The transition occurs when the Hubble radius H^{-1} is comparable to the radius a of the universe. We see from (10), this happens when a satisfies

$$a^4 - \frac{\omega_4 M}{2} a^2 + \frac{3\omega_4^2 Q^2}{32} = 0. \tag{33}$$

It is easy to locate at which time $Ha = 1$ from our explicit solution (11). This happens when η satisfies

$$\sin\ 2\eta + \cos\ 2\eta = \frac{1}{c_1}. \tag{34}$$

During the evolution, the universe passes from weak self gravitating phase $(Ha \leq 1)$ to strong self-gravitating phase $(Ha \geq 1)$. It is easy to check that on the brane universe,

$$\begin{aligned} S_B - S_Q &\leq S \quad \text{for} \quad Ha \leq 1, \\ S_B - S_Q &\geq S \quad \text{for} \quad Ha \geq 1. \end{aligned} \tag{35}$$

Here, the Bekenstein entropy is $S_B \equiv 2\pi a E/3$, $S_Q \equiv (2\pi a/3)\phi Q/2$, and S is the Hawking-Bekenstein entropy (6) written in terms of brane Newton constant $G_4 = 2\mathbf{G_5}/L$. As discussed in [27], on the brane, the following relation (similar to Cardy-Verlinde formula) holds

$$s = \left(\frac{4\pi}{n}\right)\sqrt{\gamma(\rho - \frac{\Phi\tilde{\rho}}{2} - \frac{\gamma}{a^2})}. \tag{36}$$

In the above equation, s, ρ and $\tilde{\rho}$ are the entropy, energy and charge densities on the brane that follows from (8). Furthermore, γ represent the Casimir part of the energy density [22] and is given by

$$\gamma = \frac{3a_H^2}{8\pi G_4 a^2}, \tag{37}$$

where a is given in (11). As a is finite throughout the evolution, γ is also bounded

$$\frac{9\mathrm{Vol}(S^3)a_H^2}{32\pi^2 G_4^2 LM(1+c_1)} \leq \gamma \leq \frac{9\mathrm{Vol}(S^3)a_H^2}{32\pi^2 G_4^2 LM(1-c_1)}. \tag{38}$$

We now briefly discuss the Cardy-Verlinde formula for open and flat universe. As for $k = 0$, universe passes from weak to strong gravitational system when $Ha > \sqrt{2}$. It can be easily checked that at $Ha = \sqrt{2}$, (33) holds. From our explicit solution (15), we find that the universe reaches the strong gravity domain when $16M^2/3Q^2 > \sqrt{2}$. As for $k = -1$, the universe transits from weak to strong-gravity region when $Ha > \sqrt{3}$. This happens, from (13), when

$$\cosh\ 2\eta - \sinh\ 2\eta = \frac{1}{c_2}. \tag{39}$$

In both these cases, an equation similar to (35) is satisfied as can be found in [49]. Furthermore, the analogue of (36) is given by [49]

$$s = \left(\frac{4\pi}{n}\right)\sqrt{\gamma(\rho - \frac{\Phi\tilde{\rho}}{2} - \frac{k\gamma}{a^2})}. \tag{40}$$

The Casimir energy density has the same form as (37) except the explicit form of a_H in terms of M and Q is different in this case. We would now like to point out that, in all our solutions in section-2, since universe never goes to zero size, γ is always bounded from above. In particular,

$$\begin{aligned} \gamma &< \frac{3a_H^2}{2\pi G_4 \omega_4 M (c_2 - 1)} \quad \text{for} \quad k = -1, \\ &< \frac{M a_H^2}{2\pi G_4 Q^2 \omega_4} \quad \text{for} \quad k = 0. \end{aligned} \tag{41}$$

7 Conclusion

In this paper, we have carried out a detailed study of possible cosmological scenarios for a brane universe in a charged AdS black hole background. The four dimensional Hubble law for the scale factor of the brane gets a contribution proportional to the bulk charge with an unconventional negative sign. As a consequence, we found the cosmological evolution of the universe to be regular, with a smooth transition between a contracting and an eventual expanding phase. We presented exact cosmological solutions for a (open, flat, and closed) critical brane; the inclusion of matter on the brane modifies these analytical solutions. However, in the latter case one can easily recover an evolution characterized by a bounce followed by a radiation dominated phase compatible with observations (more precisely, with the constraints coming from primordial nucleosynthesis). We then discussed the evolution of fluctuations of fields on the brane, showing that they remain in the linear regime provided the minimal size of the scale factor is sufficiently large, which can always be achieved by appropriate choices of the parameters of the model. The spectrum of the fluctuations is however not scale invariant. In the last section, we discussed the Cardy-Verlinde formula in the above set up. The brane entropy density has a Casimir part; the energy density of this contribution is always found to be bounded from above.

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