

# Radion-induced graviton oscillations in the two-brane world

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We study the braneworld effective action in the two-brane Randall-Sundrum model. In the framework of this essentially-nonlocal action we reveal the origin of an infinite sequence of gravitational wave modes — the usual massless one as well as the tower of Kaluza-Klein massive ones. Mixing of the modes can be interpreted as radion-induced gravitational-wave oscillations, a classical analogy to meson and neutrino oscillations. We show that these oscillations arising in M-theory-inspired braneworld setups could lead to effects detectable by gravitational-wave interferometers.

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In recent years the old Kaluza-Klein idea of a world with additional dimensions has received new impacts from string-inspired attempts to resolve the hierarchy problem [1]. Especially fruitful is the idea of implementing warped manifolds, for which the metric of the four-dimensional space-time can depend on the additional dimensions. The most popular models using warped geometries are the Randall-Sundrum (RS) models [2, 3], where our observable Universe is considered as a four-dimensional brane embedded into a five-dimensional anti-De Sitter (AdS) bulk space.

The particular properties of the AdS geometry combined with the requirement of a  $\mathbb{Z}_2$ -orbifold symmetry allow to describe the trapping of the graviton zero-mode on the four-dimensional brane [3]. Displacing the second four-dimensional brane in the bulk [2], one can propose a resolution to the hierarchy problem. However, the two-brane models have some important additional features. The most interesting effect is the existence of massive graviton modes, which have already attracted a lot of attention in the context of higher spin theories and in cosmological context [4, 5], and the nontrivial dependence of the mixing between different graviton states on the inter-brane distance called the radion field.

In the following we assume that we live on the positive-tension brane, which is motivated by considering the RS setup as a simplified model for M-theory constructions, in which the standard-model fields usually reside on this brane (cf. [6]). The effects seen by an observer on the negative-tension brane will be analyzed in [7].

In our preceding paper [8] we have studied the four-dimensional effective action in the two-brane model. This

action is a functional of two induced metrics and radion fields on the two four-dimensional branes. Using the results of [8], we shall study here the effects of radion-induced graviton oscillations in the two-brane world and investigate their phenomenological consequences.

In the RS setup [2], the braneworld effective action is induced from the five-dimensional bulk spacetime as follows. We start with the theory having the action of the five-dimensional gravitational field with metric  $G = G_{AB}(x, y)$ ,  $A = (\mu, 5)$ ,  $\mu = 0, 1, 2, 3$ , propagating in the bulk spacetime  $x^A = (x, y)$ ,  $x = x^\mu$ ,  $x^5 = y$ , and matter fields  $\phi$  confined to the two branes  $\Sigma_\pm$ ,

$$S[G, g, \phi] = S_5[G] + S_4[G, g, \phi], \quad (1)$$

where the bulk-space part of the action is given by

$$S_5[G] = \frac{1}{16\pi G_5} \int_{M^5} d^5x G^{1/2} ({}^5R(G) - 2\Lambda_5), \quad (2)$$

and the brane-part of the action is

$$S_4[G, g, \phi] = \sum_{\pm} \int_{\Sigma_{\pm}} d^4x \left( L_m(\phi, \partial\phi, g) - g^{1/2} \sigma_{\pm} + \frac{1}{8\pi G_5} [K] \right). \quad (3)$$

The branes are enumerated by the index  $\pm$  and carry induced metrics  $g = g_{\mu\nu}(x)$  and matter-field Lagrangians  $L_m(\phi, \partial\phi, g)$ . The bulk part of the action (2) is characterized by the five-dimensional gravitational constant  $G_5$  and the cosmological constant  $\Lambda_5$ , while the brane parts (3) have four-dimensional cosmological constants

$\sigma_{\pm}$ . The bulk cosmological constant  $\Lambda_5$  is negative and, therefore, is capable of generating an AdS geometry, while the brane cosmological constants play the role of brane tensions and, depending on the model, can be of either sign. The Einstein-Hilbert bulk action (2) is accompanied by terms in (3) containing the jumps of extrinsic curvatures traces  $[K]$  associated with both sides of each brane [9]. Solving the five-dimensional Einstein equations with prescribed values of the four-dimensional metric on the branes, we have obtained in [8] the tree-level effective four-dimensional action.

In the RS two-brane setup, the fifth dimension has the topology of a circle labeled by the coordinate  $y$ ,  $-d \leq y \leq d$ , with an orbifold  $\mathbb{Z}_2$ -identification of points  $y$  and  $-y$ . The branes are located at antipodal fixed points of the orbifold,  $y = y_{\pm}$ ,  $y_+ = 0$ ,  $|y_-| = d$ . They are empty, i.e.  $L_m(\phi, \partial\phi, g_{\mu\nu}) = 0$ , and their tensions are opposite in sign and fine-tuned to the values of  $\Lambda_5$  and  $G_5$ ,

$$\Lambda_5 = -6/l^2, \quad \sigma_+ = -\sigma_- = 3/(4\pi G_5 l). \quad (4)$$

Then this model admits a solution to the Einstein equations with an AdS metric in the bulk ( $l$  is its curvature radius),

$$ds^2 = dy^2 + e^{-2|y|/l} \eta_{\mu\nu} dx^\mu dx^\nu, \quad (5)$$

$0 = y_+ \leq |y| \leq y_- = d$ , and with a flat induced metric  $\eta_{\mu\nu}$  on both branes [2]. With the fine tuning (4) this solution exists for arbitrary brane separation  $d$ .

Now consider small metric perturbations  $\gamma_{AB}(x, y)$  on the background of this solution [3, 10, 11],

$$ds^2 = dy^2 + e^{-2|y|/l} \eta_{\mu\nu} dx^\mu dx^\nu + \gamma_{AB}(x, y) dx^A dx^B. \quad (6)$$

Then this five-dimensional metric *induces* on the branes two four-dimensional metrics of the form

$$g_{\mu\nu}^{\pm}(x) = a_{\pm}^2 \eta_{\mu\nu} + \gamma_{\mu\nu}^{\pm}(x), \quad (7)$$

where the scale factors  $a_{\pm} = a(y_{\pm})$  are given by  $a_+ = 1$  and  $a_- = e^{-d/l} \equiv a$ , and  $\gamma_{\mu\nu}^{\pm}(x)$  are the perturbations by which the brane metrics  $g_{\mu\nu}^{\pm}(x)$  differ from the (conformally) flat metrics of the RS solution (5). The variable  $a$  represents the modulus — the global part of the radion field determining the interbrane separation.

In [8] we have derived the effective braneworld action in terms of the four-dimensional on-brane metrics (7) and non-local matrix-valued form factors. We call the part of the action quadratic in  $h_{\mu\nu}^{\pm}(x)$  — the transverse-traceless parts of the full metric perturbations  $\gamma_{\mu\nu}^{\pm}(x)$  on the branes — the graviton sector. It reads as the following  $2 \times 2$  quadratic form,

$$S_{\text{grav}}[h_{\mu\nu}^{\pm}] = \frac{1}{16\pi G_4} \int d^4x \frac{1}{2} \mathbf{h}^T \frac{\mathbf{F}(\square)}{l^2} \mathbf{h}, \quad (8)$$

in terms of columns of the metric perturbations

$$\mathbf{h} = \begin{bmatrix} h^+ \\ h^- \end{bmatrix} \quad (9)$$

( $T$  denotes their transposition into rows and we omit from now on the tensor indices) and the special nonlocal operator  $\mathbf{F}(\square)$  (we also use  $G_4 \equiv G_5/l$ ). As was shown in Ref. [8], the operator  $\mathbf{F}(\square)$  is a complicated non-linear function of the D'Alembert operator  $\square$ , expressed by means of Bessel and Neumann functions of arguments  $l\sqrt{\square}$  and  $l\sqrt{\square}/a$ . In this paper, we study in detail its properties in the *low-energy limit*, when  $l\sqrt{\square} \ll 1$  but when  $l\sqrt{\square}/a$  can take arbitrary values in view of the smallness of the parameter  $a = e^{-d/l}$  (large interbrane distances). In this limit the kernel of the action  $\mathbf{F}(\square)$  reads

$$\mathbf{F}(\square) \approx \frac{l^2 \square}{2} \begin{bmatrix} 1 & 1/J_2[l\sqrt{\square}/a] \\ 1/J_2[l\sqrt{\square}/a] & -\frac{2}{l\sqrt{\square}a} \frac{J_1[l\sqrt{\square}/a]}{J_2[l\sqrt{\square}/a]} \end{bmatrix}. \quad (10)$$

A typical way to extract the particle content from an action with a matrix-valued kernel is its diagonalization in terms of normal modes. However, in view of the nonlocality of  $\mathbf{F}(\square)$  the number of propagating modes enormously exceeds the number of entries in the  $2 \times 2$ -matrix  $\mathbf{F}(\square)$  and they do not diagonalize the quadratic action (8) in the usual sense. The propagating modes  $\mathbf{h}_i(x) = h_i(x) \mathbf{v}_i$  are the zero modes of  $\mathbf{F}(\square)$  which solve the matrix-valued nonlocal equation  $\mathbf{F}(\square) \mathbf{h}_i(x) = 0$ . The consistency of the latter,  $\det \mathbf{F}(\square) = 0$ , yields a mass spectrum of the theory given by the roots of this equation, i.e.  $\square = m_i^2$ , so that the  $h_i(x)$  above are massive Klein-Gordon modes,  $(\square - m_i^2) h_i(x) = 0$ , and the isotopic vectors of the propagating modes  $\mathbf{v}_i$  are zero eigenvectors of  $\mathbf{F}(m_i^2)$ ,  $\mathbf{F}(m_i^2) \mathbf{v}_i = 0$ . This gives the Kaluza-Klein spectrum which contains the massless mode  $i = 0$ ,  $m_0 = 0$ , and the tower of massive modes with masses  $m_i = a j_i / l$  given in the low-energy approximation of (10),  $a \ll 1$ , by the roots of the first order Bessel function,  $J_1(j_i) = 0$ . The isotopic structure of their  $\mathbf{v}_i$  is

$$\mathbf{v}_0 = \frac{\sqrt{2}}{l} \begin{bmatrix} 1 \\ a^2 \end{bmatrix}, \quad \mathbf{v}_i = \frac{\sqrt{2}a}{l} \begin{bmatrix} 1/J_2(j_i) \\ -1 \end{bmatrix}. \quad (11)$$

The action (8) is not, however, diagonalizable in the basis of these states because under the decomposition  $\mathbf{h}(x) = \sum_{i=0} h_i(x) \mathbf{v}_i$  (with off-shell coefficients  $h_i(x)$ ) the cross terms intertwining different  $i$ -s are nonvanishing,  $\mathbf{v}_i^T \mathbf{F}(\square) \mathbf{v}_j \neq 0$ .

A crucial observation is, however, that the diagonal and nondiagonal terms of this expansion are linear and bilinear, respectively, in on-shell operators  $\square - m_i^2$ ,

$$\mathbf{v}_i^T \mathbf{F}(\square) \mathbf{v}_i = \left( \mathbf{v}_i^T \frac{d\mathbf{F}(\square)}{d\square} \mathbf{v}_i \right)_{\square=m_i^2} (\square - m_i^2), \quad (12)$$

$$\mathbf{v}_i^T \mathbf{F}(\square) \mathbf{v}_j = M_{ij}(\square) (\square - m_i^2) (\square - m_j^2), \quad i \neq j, \quad (13)$$

where higher powers of  $(\square - m_i^2)$  have been dropped in (12) and  $M_{ij}(\square)$  is nonvanishing at both  $\square = m_i^2$  and  $\square = m_j^2$ . Therefore, the nondiagonal terms of the action do not contribute to the residues of the Green's function

$\mathbf{G}(\square)$  of  $\mathbf{F}(\square)$ , which turns out to be given by direct products of the  $\mathbf{v}_i$  and their transposes,

$$\mathbf{G}(\square) = \sum_{i=0} \frac{\mathbf{v}_i \mathbf{v}_i^T}{\square - m_i^2}, \quad (14)$$

the normalization of  $\mathbf{v}_i$  being chosen in such a way as to render a unit coefficient of  $(\square - m_i^2)$  in (12).

In passing, we note that the problem of non-orthogonal physical modes  $\mathbf{v}_i$  is well known in the theory of atomic and nuclear resonances [12], where these modes are called Siegert states as opposed to the so-called Kapur-Peierls states,  $\mathbf{b}_s(\square)$ , which diagonalize the *matrix* Hamiltonian (the analogue of our  $\mathbf{F}(\square)$ ),  $\mathbf{F}(\square)\mathbf{b}_s(\square) = \lambda_s(\square)\mathbf{b}_s(\square)$ ,  $s = 1, 2$ ,  $\mathbf{b}_s^T(\square)\mathbf{b}_{s'}(\square) = \delta_{ss'}$ . In this formalism the Siegert states and their masses (energy levels) arise as zeroes of the eigenvalues,  $\lambda_s(m_i^2) = 0$ , and turn out to be related to the Kapur-Peierls states,  $\mathbf{v}_0 \sim \mathbf{b}_1(m_0^2)$ ,  $\mathbf{v}_i \sim \mathbf{b}_2(m_i^2)$ ,  $i \neq 0$ , up to some nontrivial normalization coefficients. The details of this formalism will be presented in [7].

The above method provides us with the conventional particle interpretation of the propagating modes of the nonlocal operator  $\mathbf{F}(\square)$  and their role in its Green's function (14) mediating the gravitational effect of matter sources. Amended by the matter action on the branes (cf. [8]) the effective action of the graviton sector reads

$$S[h_{\mu\nu}^\pm] = \int d^4x \left( \frac{1}{32\pi G_4} \mathbf{h}^T \frac{\mathbf{F}(\square)}{l^2} \mathbf{h} + \frac{1}{2} \mathbf{h}^T \mathbf{T} \right), \quad (15)$$

where  $\mathbf{T}$  is the column vector of the stress-energy tensors on the branes. Varying this action with respect to  $\mathbf{h}$  we obtain the linearized equations of motion, their solution  $\mathbf{h} = -8\pi G_4 l^2 \mathbf{G}_{\text{ret}} \mathbf{T}$  being expressed in terms of the retarded version of the Green's function (14).

We shall now restrict ourselves to observers living on the  $\Sigma_+$ -brane ("visible brane"). For simplicity we also restrict our attention to frequencies below the mass threshold of the second massive mode  $m_2$ . Then using the spectral representation (14) and the structure of  $\mathbf{v}_{0,1}$  given by (11) we find for  $h^+$ ,

$$h^+ = - \frac{16\pi G_4}{\square} \Big|_{\text{ret}} (T^+ + a^2 T^-) - \frac{16\pi G_4}{\square - m_1^2} \Big|_{\text{ret}} \left( \frac{a^2}{\mathcal{J}_2^2} T^+ - \frac{a^2}{\mathcal{J}_2} T^- \right), \quad (16)$$

where  $\mathcal{J}_2 \equiv J_2[lm_1/a] \approx 0.403$ . We consider astrophysical sources at  $\mathbf{x} = 0$  on both branes of equal intensity with a harmonic time dependence  $T^\pm(t, \mathbf{x}) = \mu e^{-i\omega t} \delta(\mathbf{x})$ . If the frequency of the source is above the mass threshold of the massive mode, i.e.  $\omega > m_1$ , both modes, the massless and the first massive one, are excited and produce long range gravitational waves. At distance  $r$  from the source the waves on each brane are given by a mixture of massless and massive spherical waves. On the

$\Sigma_+$ -brane this superposition is given by the sum of the contributions

$$h^+[T^+] = A e^{-i\omega t} \left( e^{i\omega r} + \frac{a^2}{\mathcal{J}_2^2} e^{i\sqrt{\omega^2 - m_1^2} r} \right), \quad (17)$$

$$h^+[T^-] = A a^2 e^{-i\omega t} \left( e^{i\omega r} - \frac{1}{\mathcal{J}_2} e^{i\sqrt{\omega^2 - m_1^2} r} \right) \quad (18)$$

of sources on the  $\Sigma_+$ - and  $\Sigma_-$ -brane, respectively, where  $A = 4 G_4 \mu / r$  is the amplitude of the massless mode produced on the  $\Sigma_+$ -brane. The amplitudes detected by a gravitational-wave interferometer are given by the absolute values of (17) and (18),

$$|h^+[T^+]| = \mathcal{A}^+ \left[ 1 - \frac{4a^2 \mathcal{J}_2^2}{(\mathcal{J}_2^2 + a^2)^2} \sin^2 \left( \frac{\pi r}{L} \right) \right]^{1/2}, \quad (19)$$

$$|h^+[T^-]| = \mathcal{A}^- \left[ 1 + \frac{4\mathcal{J}_2}{(\mathcal{J}_2 - 1)^2} \sin^2 \left( \frac{\pi r}{L} \right) \right]^{1/2}. \quad (20)$$

Here  $L$  is the oscillation length of the amplitude modulation of the gravitational wave (GW),

$$L = 2\pi \left( \omega - \sqrt{\omega^2 - m_1^2} \right)^{-1} \simeq \frac{2\pi}{m_1}, \quad (21)$$

where the approximation corresponds to  $m_1 \lesssim \omega$ . The pre-factors of the amplitudes (19) and (20) are given by

$$\mathcal{A}^+ = \left( 1 + \frac{a^2}{\mathcal{J}_2^2} \right)^2 A \approx A, \quad (22)$$

$$\mathcal{A}^- = \left( 1 - \frac{1}{\mathcal{J}_2} \right)^2 a^2 A \approx 2.2 a^2 A, \quad (23)$$

where the approximations are valid in the limit  $a \ll 1$ . We find oscillations in the amplitudes of the waves from both sources. For a GW produced by  $T^+$ , Eq. (19), the oscillation is suppressed by a factor of  $a^2$  compared to the constant part of the amplitude in the limit of large brane separation,  $a \ll 1$ . The amplitude of the GW produced by  $T^-$  (20) is totally oscillating, regardless of the inter-brane distance.

We can express the oscillation length of these radion-induced gravitational wave oscillations (RIGO's) through the AdS radius  $l$  and the scale factor  $a$ ,

$$L = 2\pi j_1 l / a \approx 24 l / a, \quad (24)$$

where  $j_1 \approx 3,831$  is the first root of  $J_1$ . The oscillation length is inverse proportional to  $a$ . Graviton oscillations become observable when the oscillation length is of the same size as the arm length of a GW detector. For the ground-based interferometric detectors this requirement corresponds to  $L \sim 10^3$  m. Combining this with the constraint on the maximal AdS radius  $l$  from sub-millimeter test of gravity  $l \lesssim 10^{-4}$  m [13], we find an upper limit on the warp factor  $a \lesssim 10^{-6}$  for the oscillation length to be

detectable. Inserting this into the ratio of the amplitudes (22) and (23) we find

$$\mathcal{A}^-/\mathcal{A}^+ \lesssim 10^{-12}. \quad (25)$$

Therefore, the amplitude of a wave originating from a source on the (“hidden”)  $\Sigma_-$ -brane with oscillations which are sufficiently long to be detectable, is strongly suppressed compared to a GW stemming from a source on the  $\Sigma_+$ -brane itself. A strongly oscillating wave has to be generated by a source 12 orders of magnitude stronger than that of a weakly oscillating one in order to be of the same magnitude, which at first sight makes the detection of RIGO’s impossible. However, this requirement may actually be fulfilled in brane-world setups inspired by string- or M-theory. In these scenarios only the brane which we inhabit features the usual physics of the standard model, whereas the gauge groups and the field content of the hidden brane differ drastically from that of our world. As strongly oscillating waves have their source on the hidden brane, one might expect some exotic physics (e. g. topological defects) on that brane to be responsible for the generation of extremely strong GW’s, which are necessary to have observable RIGO’s.

RIGO’s are in principle a feature of every higher-dimensional spacetime model as there will always occur amplitude modulations in GW’s, which are a mixture of a massless mode and KK modes. However, in traditional models with flat extra dimensions, the mass of the first KK mode is so big that it will neither be produced by astrophysical sources nor lead to oscillation lengths of macroscopic size. In contrast to this, warped geometries allow KK-mode masses which are so low that they can lead to oscillations of detectable length. In particular, waves from sources on the hidden brane show strong oscillations on the visible brane. Although these waves are strongly suppressed, they may nevertheless be of large amplitude because according to M-theory the entirely different physics on the hidden brane can be responsible for the production of very strong GW’s. Even without this assumption it is conceivable to find setups which combine the requirements of small KK masses and strong amplitude modulations of GW’s. One of such setups is the Kogan-Mouslopoulos-Papazoglou version [14] of the Karch-Randall model [15] with two  $\text{AdS}_4$  branes embedded in  $\text{AdS}_5$ . In this model the massless graviton and the lightest massive one are coupled to matter with nearly equal strength and, therefore, produce strong oscillations which are unfortunately not observable because of the big oscillation length exceeding the horizon of the  $\text{AdS}$  brane. Thus, RIGO’s could be an effect which is suitable for verifying the existence of extra dimensions in the paradigm of warped braneworlds. They are a generic feature of these models, and their occurrence is not affected by modulus stabilization.

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