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AdS$_2$ D-Branes in Lorentzian AdS$_3$

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Abstract

The boundary states for AdS$_2$ D-branes in Lorentzian AdS$_3$ space-time are presented. AdS$_2$ D-branes are algebraically defined by twisted Dirichlet boundary conditions and are located on twisted conjugacy classes of SL(2,R). Using free field representation of symmetry currents in the SL(2,R) WZNW model, the twisted Dirichlet gluing conditions among currents are translated to matching conditions among free fields and then to boundary conditions among the modes of free fields. The Ishibashi states are written as coherent states on AdS$_3$ in the free field formalism and it is shown that twisted Dirichlet boundary conditions are satisfied on them. The tree-level amplitude of propagation of closed strings between two AdS$_2$ D-branes is evaluated and by comparing which to the characters of $\widehat{\mathtt{sl}}$(2,R) Kac-Moody algebra it is shown that only states in the principal continuous series representation of $\widehat{\mathtt{sl}}$(2,R) Kac-Moody algebra contributes to the amplitude and thus they are the only ones that couple to AdS$_2$ D-branes. The form of the character of $\widehat{\mathtt{sl}}$(2,R) principal continuous series and the boundary condition among the zero modes are used to determine the physical boundary states for AdS$_2$ D-branes.

1 Introduction

Three dimensional Anti-de Sitter space-time (AdS_3) is the most accessible example in the quest to understand properties of string theory on curved space-times with curved time. AdS_3 is the covering space of the non-compact SL(2,R) group manifold and hence one can employ SL(2,R) Wess-Zumino-Novikov-Witten (WZNW) model in order to formulate string theory on AdS_3. There has been much work [1]–[8] on string theory on SL(2,R) group manifold or on its covering space AdS_3. The motivation to understand the string motion in the background of two dimensional SL(2,R)/R black hole [2],[9] and three dimensional BTZ black hole [10] also contributed to the motivation to study SL(2,R) WZNW model.

From the start SL(2,R) WZNW model presented several new challenges unseen in the formulation of string theory in flat space-time. The most important of these is the unitarity problem. In flat space-time the negative norm states due to time dimension can be eliminated successfully by using Virasoro constraints and a no ghost theorem can be proved [11]. However, in SL(2,R) WZNW model it is found that even after imposing the Virasoro constraints some negative norm states still remain in the spectrum. To resolve this problem of unitarity and to get rid of ghosts two main proposal are made. 1) The eigenvalue of the compact generator of SL(2,R), j, and the WZNW level, k, need to be restricted [2],[3] as in the case of SL(2,R)/U(1) coset model [12]. 2) The effect of zero modes of free fields in the free field formulation of SL(2,R) WZNW model should be carefully taken into account [5]. According to the first proposal, the closed string states belong to the highest and lowest weight discrete as well as principal continuous series representations of $\widehat{\mathtt{sl}}(2,R)$[1]. However, due to the mentioned extra conditions the closed strings in highest and lowest weight discrete series representations cannot be excited above the first excited level. Also only the lowest level states are allowed form the principal continuous series representation. In [8] this proposal is developed by also taking into account the contribution of possible CFT on direct product spaces so as to make the total central charge equal to the critical value 26. Then, considering also the effect of spectral flow, one finds a finite number of excited states. However if the contribution of other CFT is taken to be zero, i.e. taking SL(2,R) WZNW model by itself critical, formalism gives very a few excited string states [13]. These arguments and being the restrictions on j and k artificial makes this proposal not very attractive to us. The second proposal requires the closed string states to belong only to the principal continuous series representation of $\widehat{\mathtt{sl}}(2,R)$ supplemented with zero modes. In this spectrum there are no negative norm states and the string can be excited to any arbitrary level [5]. In [7] this idea is further supported by showing that the vertex operators for the lowest level closed string states, written for finite value of WZNW level k, indeed approach to the correct vertex operators in the flat $\mathcal{M}_3$ space-time, obtained as the limit of AdS_3 space-time as $k \to \infty$. Mass-shell condition in AdS_3 space-time is also shown to become the correct mass-shell

[1]Notation: SL(2,R) stands for the group, $\mathtt{sl}(2,R)$ for the Lie algebra and $\widehat{\mathtt{sl}}(2,R)$ for the Kac-Moody algebra.

condition in flat space-time in that limit. The presence of all the zero modes is important in getting the correct mass-shell condition in the flat limit. Setting any of the zero modes to zero results in an incorrect expression.

The conclusions of [5],[7] is supported in the present paper by considering the coupling of closed strings to AdS_2 D-branes in Lorentzian AdS_3. D-branes in flat string theory are defined through Dirichlet conditions at the open string end-points and they carry non-vanishing vector field on their world volume which couples to those open string end-points [14]. Formulation of Dirichlet and Neumann boundary conditions for the open string theories on general group manifolds has been done in [15] by using the analogy between Kac-Moody symmetry currents of WZNW model and the primary fields of the string theory in flat space-time. The analogs of Dirichlet and Neumann conditions among Kac-Moody symmetry currents are called as gluing conditions and from the representation of currents in terms of group element it is found that classically D-branes are located on the regular or twisted conjugacy classes of that group [15]. The specific classical theory of D-branes on non-compact SL(2,R) group manifold is discussed in [16],[17]. In the case of SL(2,R) WZNW model the possible gluing conditions are checked in [18] to find out if they can be obtained from the variation of the WZNW action. We review this analysis in section 2.2. Quantum theory of AdS_2 and other D-branes in Euclidean AdS_3 (SL(2,C)/SU(2) coset WZNW model) are discussed by several authors [19]–[24]. Different proposals for one-point functions and boundary states for AdS_2 D-branes in Euclidean AdS_3 space were presented in these works.

It has been hoped that the theory of D-branes in Lorentzian AdS_3 can be obtained by performing Wick rotation in the theory in Euclidean AdS_3 [19]–[24]. However, we find it safer to formulate the theory in Lorentzian AdS_3 without making a Wick rotation at the very beginning. One of the reasons of this is that as it is discussed in [4] the Wick rotation in a curved space-time is more elaborate and it might not work as in the case of flat space-time. Other than this, we note that an important difference between Euclidean and Lorentzian theories were already pointed out in the closed string theory case in respective backgrounds. The vertex operators written in the Lorentzian AdS_3 in SL(2,R) WZNW model [7] contains an extra non-trivial factor compared to the ones written in the Euclidean AdS_3 in SL(2,C)/SU(2) WZNW model [25]. Similarly we claim that the elements (boundary states, correlation functions, etc.) of the theory of D-branes in Euclidean AdS_3 may not be, after Wick rotation, the correct elements of the theory of D-branes in Lorentzian AdS_3. Therefore our strategy is to formulate the theory first in Lorentzian AdS_3 and then, as needed, Wick rotate to Euclidean AdS_3 in order to calculate some physical quantities, but not to formulate the theory first in Euclidean AdS_3 and then try to get Lorentzian theory by Wick rotating back to Lorentzian AdS_3. In this paper we will obtain the boundary states for AdS_2 D-branes directly in Lorentzian AdS_3.

The plan of the paper is as follows. In section two we firstly comment on the background, then we review the analysis in [18] on the possible gluing conditions that one can obtain from the variation of the WZNW action. In section three we first review the free field formalism of [5] for SL(2,R) WZNW model and then we translate the gluing conditions among currents to matching conditions among free fields and later to the boundary conditions among modes of free fields. At the end of that section we present our ansatz for the Ishibashi states and show that boundary conditions imposed on them are satisfied. These states also preserve one half of the conformal symmetry as expected in a BCFT. In section four we find the annulus amplitude of closed strings propagating between two AdS_2 D-branes. Comparing holomorphic part of this amplitude to the characters of $\widehat{\mathtt{sl}}(2,R)$ Kac Moody algebra we deduce that the spectrum of closed strings, which couple to AdS_2 D-branes, are in the principal continuous series of $\widehat{\mathtt{sl}}(2,R)$ supplemented with zero modes in accordance with the results of [5],[7]. In the last sub-section we use the last boundary condition (47) among zero modes to write the physical boundary states. We conclude with a discussion of possible future research.

2 Closed and Open Strings in Lorentzian AdS_3

2.1 SL(2,R) WZNW Model and AdS_3 Space-Time

Closed string theory on group manifolds are analyzed through WZNW model. Having affine current algebra (which is nothing but the affinization of the Lie algebra that is associated to that group manifold) as a symmetry algebra of the model utilizes the the current algebra technics to find out the spectrum of the theory.

The WZNW action which describes the closed strings on SL(2,R) manifold is given by

$$S = \frac{k}{4\pi}\int_M d^2\sigma\, Tr\left(g^{-1}\partial_+ g\; g^{-1}\partial_- g\right) - \frac{k}{12\pi}\int_B d^3\varsigma\, Tr\left(g^{-1}dg\; g^{-1}dg\; g^{-1}dg\right). \tag{1}$$

where g is the SL(2,R) group element and $g^{-1}dg = (g^{-1}\partial_+ g)\, d\sigma^+ + (g^{-1}\partial_- g)\, d\sigma^-$ is the pull-back on the closed string worldsheet of the left invariant one-form on the SL(2,R) group manifold.

We parametrize the SL(2,R) group element in the triangular form

$$\begin{aligned} g\left(z,\bar{z}\right) &= \begin{pmatrix} 1 & 0 \\ -\,\gamma^-\left(z,\bar{z}\right) & 1 \end{pmatrix} \begin{pmatrix} e^{\frac{1}{2}\phi(z,\bar{z})} & 0 \\ 0 & e^{-\frac{1}{2}\phi(z,\bar{z})} \end{pmatrix} \begin{pmatrix} 1 & -\gamma^+\left(z,\bar{z}\right) \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} e^{\frac{1}{2}\phi} & -\gamma^+ e^{\frac{1}{2}\phi} \\ -\,\gamma^- e^{\frac{1}{2}\phi} & e^{-\frac{1}{2}\phi} + e^{\frac{1}{2}\phi}\gamma^+\gamma^- \end{pmatrix}. \end{aligned} \tag{2}$$

where all the fields $\phi\left(z,\bar{z}\right)$, $\gamma^-\left(z,\bar{z}\right)$ and $\gamma^+\left(z,\bar{z}\right)$ are real. $\gamma^\pm\left(z,\bar{z}\right)$ are lightcone combinations $\gamma^\pm = x \pm t$. With this parametrization of the group element the metric on the group manifold takes the so called Poincaré form

$$ds^2 = \frac{k\alpha'}{2}\, Tr\left(g^{-1}dg\; g^{-1}dg\right) = k\alpha'\left(\frac{1}{u^2}du^2 + u^2 d\gamma^- d\gamma^+\right) \tag{3}$$

where $u = e^{\frac{1}{2}\phi(z,\bar{z})}$ and $\sqrt{k\alpha'}$ is the radius of curvature. From the the form of the metric one sees that SL(2,R) group manifold contains one compact time-like coordinate, $t \in [0, 2\pi]$ and two non-compact space-like coordinates $x \in]-\infty, \infty[$ and $u \in [0, \infty[$. To obtain the universal covering space of SL(2,R) group manifold the time-like coordinate is uncompactified: $t \in]-\infty, \infty[$. The universal covering space is then the AdS$_3$ space-time. In the present paper we are going to analyze D-branes on this universal covering space by using the free field formalism of the SL(2,R) WZNW model. We note that we are not making global identification $x \equiv x + 2\pi$, i.e. we are not treating the case of vacuum BTZ black hole[2], but Lorentzian AdS$_3$.

2.2 Variation of the Action and Gluing Conditions

The WZNW action (1) describes only the theory of closed strings on the group manifold. This is rather obvious from the form of the action: The Wess-Zumino part of the action is an integration over the ball B for which the string worldsheet M is the boundary. Since a boundary cannot have a boundary is a classic result in differential geometry, we conclude that the WZNW action (1) describes only the closed strings. Therefore, one cannot work as in the flat Minkowski Space-time. In flat space-time, the Polyakov action describes both open and closed strings. Variation of the action results in the equations of motion for both open and closed strings and in the case of open strings also a surface term, from the vanishing condition for which one obtains the open string boundary conditions. Basically these boundary conditions are Neumann boundary conditions in all directions, which state that momentum does not flow out of the the string end-points. However, one can play with the kind of boundary condition in one or all directions. In the T-dual version of flat string theory, one finds that Neumann boundary condition turns into Dirichlet boundary condition in the direction that T-duality is performed [14]. Dirichlet boundary condition just means that the string end-points are free to move in all directions except in the direction for which there is Dirichlet boundary condition. Therefore, string end-points define a hyper-surface which is called a Dirichlet-brane (or D-brane). If T-duality is performed in several directions, the number of directions for which string end-points obey Dirichlet boundary condition increases and the number of dimensions that define D-brane decreases. One very important distinguishing property of D-branes from other hyper-surfaces is that D-branes have a vector field on their world-volume, which couples to open string end-points.

If one applies the same kind of logic to the case of string theory on a group manifold, one has to take into account the above fact that the hyper-surfaces which are claimed to be D-branes should have a vector field defined on them. Being careful with this point, Lomholt and Larsen [18] analyzed the possible boundary conditions that one can write for possible open strings on

[2]Vacuum BTZ black hole with zero mass and zero angular momentum has the same metric as (3). However, unlike the Lorentzian AdS$_3$ space-time, globally there is the identification $x \equiv x + 2\pi$. Closed string spectrum in such backgrounds is discussed in [26].

SL(2,R) group manifold. In short their analysis is as follows: They assumed that variation of open string action on SL(2,R) group manifold is just variation of closed string action (1) plus a term which comes from the coupling of the vector field to the open-string end-points on possible boundaries. The condition that the surface term should vanish and also the condition on the field strength of the vector field (namely the Jacobi condition and that the field strength should be anti-symmetric) allowed only a few possibilities for the boundary conditions that one can write for the open strings on SL(2,R) group manifold. As a result of their analysis, they found that the possible cases for Dirichlet type boundary conditions are firstly "regular" Dirichlet gluing conditions

$$J\left(z\right)=\tilde{J}\left(\bar{z}\right)\quad at\quad z=\bar{z} \tag{4}$$

and secondly so called "twisted" Dirichlet gluing conditions

$$J\left(z\right)=\omega\cdot\tilde{J}\left(\bar{z}\right)\cdot\omega\quad at\quad z=\bar{z} \tag{5}$$

where

$$\omega=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

among the currents of $\widehat{\mathtt{sl}}(2,\mathrm{R})$ Kac-Moody symmetry.

The regular and twisted gluing conditions above are both written in the open string channel at $z=\bar{z}$ where $z=e^{i(\tau+\sigma)},\ \bar{z}=e^{i(\tau-\sigma)}$. However, we can translate these gluing conditions to the closed string channel and write them as

$$\text{regular}\quad:\qquad\left(J(z)+\bar{z}^2\,\tilde{J}\left(\bar{z}\right)\right)|B\rangle=0, \tag{6}$$

$$\text{twisted}\quad:\qquad\left(J(z)+\bar{z}^2\,\omega\cdot\tilde{J}\left(\bar{z}\right)\cdot\omega\right)|B\rangle=0\quad at\quad z\cdot\bar{z}=1. \tag{7}$$

The $-\bar{z}^2$ factors in front of $\tilde{J}\left(\bar{z}\right)$ comes from the fact that as open string world-sheet with topology of a cylinder is transformed into a closed string world-sheet with the same topology one needs to interchange σ and τ. Because of the same reason the boundary in the closed string channel is at $z\cdot\bar{z}=1$. The state $|B\rangle$ in (6,7) is the boundary state which describes D-branes that constitute the boundaries for the tree-level amplitude of the propagation of the closed strings.

The result of the analysis in [18] is in tune with the previous analysis done in [16],[17]. If one expresses the symmetry currents $J_L\left(z\right)=ik\left(\partial g\right)g^{-1}$ and $\tilde{J}_R\left(\bar{z}\right)=-ikg^{-1}\left(\bar{\partial}g\right)$ in terms of fields $\gamma^-\left(z,\bar{z}\right)$, $\gamma^+\left(z,\bar{z}\right)$ and $\phi\left(z,\bar{z}\right)$ by using the representation (2) of the group element g, one finds that regular gluing condition (4) describes the hyper-surfaces

$$e^{\phi/2}\left(1+\gamma^-\gamma^+\right)+e^{-\phi/2}=\mathcal{C}, \tag{8}$$

where $\mathcal{C}$ is a arbitrary real constant, as D-branes on AdS_3. These D-branes are located on the conjugacy classes of SL(2,R) and depending on whether $|\mathcal{C}|$ is bigger, smaller or equal to one, their world-volumes have the geometry of two dimensional de Sitter space (dS_2), hyperbolic plane (H_2) or the light cone in three dimensions, respectively [16],[17]. It has been shown that none of these D-brane world-volumes correspond to a physically acceptable D-brane motion: for dS_2 D-branes the Dirac-Born-Infeld action is imaginary and thus such D-branes are "unphysical" [17], for H_2 D-branes the world-volume has Euclidean signature and the light-cone breaks up into three conjugacy classes (apex of the cone, future cone without apex and past cone without apex) with degenerate induced metric [16],[17].

Contrary to that, twisted Dirichlet gluing conditions describe physical D-branes [17] which are located on the twisted conjugacy classes of SL(2,R) [17],[18]

$$e^{\phi/2}\left(\gamma^- + \gamma^+\right) = \mathcal{C}, \tag{9}$$

where $\mathcal{C}$ is a arbitrary real constant, and have the geometry of AdS_2 space-time

$$ds^2 = \frac{1}{u^2}du^2 - u^2dt^2 \tag{10}$$

where $t = (\gamma^+ - \gamma^-)/2$.

3 Free Field Analysis

In this work our aim is to study the consequences of the twisted Dirichlet gluing conditions (5) among Kac-Moody currents and to write the boundary states for AdS_2 D-branes so as to determine the spectrum of closed strings that couple to those AdS_2 D-branes. We only analyze AdS_2 D-branes, since they are the physical ones, in the present paper. We are going to write the boundary states as coherent states on SL(2,R) group manifold. To do that we utilize the free field representation that was used in [5],[7] in the study of closed strings on AdS_3 space-time.

Therefore we now review the free-field representation of the WZNW model on SL(2,R) manifold as used in [7]. With this review we are also going to present the necessary formulas that will be used to construct the boundary states for AdS_2 D-branes on AdS_3 and fix the notation.

We write the SL(2,R) group element in the following form, separating the holomorphic and anti-holomorphic parts:

$$g\left(z,\bar{z}\right) = g_L\left(z\right) g_R\left(\bar{z}\right) \tag{11}$$

$$g_L\left(z\right) = \begin{pmatrix} e^{\frac{1}{2}X_2} & -X^+e^{\frac{1}{2}X_2} \\ -X^-e^{\frac{1}{2}X_2} & e^{-\frac{1}{2}X_2} + e^{\frac{1}{2}X_2}X^+X^- \end{pmatrix} \tag{12}$$

$$g_R\left(\bar{z}\right) = \begin{pmatrix} e^{\frac{1}{2}\tilde{X}_2} & -\tilde{X}^+e^{\frac{1}{2}\tilde{X}_2} \\ -\tilde{X}^-e^{\frac{1}{2}\tilde{X}_2} & e^{-\frac{1}{2}\tilde{X}_2} + e^{\frac{1}{2}\tilde{X}_2}\tilde{X}^+\tilde{X}^- \end{pmatrix} \tag{13}$$

This separation of holomorphic and anti-holomorphic parts is realized when one derives the equations of motion from the WZNW action. The Kac-Moody symmetry currents separate into holomorhic and anti-holomorphic parts, which we call as left and right currents. With the above representation of the SL(2,R) group element, the left (holomorphic) currents are obtained as[3]:

$$J_L(z) = ik\,(\partial g)\,g^{-1} = ik\,(\partial g_L)\,g_L^{-1} = J^a\tau^a = \begin{pmatrix} -J^2 & J^0 + J^1 \\ -J^0 + J^1 & J^2 \end{pmatrix} \qquad (14)$$

where three generators of holomorphic Kac-Moody algebra are

$$\begin{aligned} J^1(z) + J^0(z) &= P^+(z) &(15)\\ J^1(z) - J^0(z) &= -:X^-P^+X^-: -2P_2X^- + i\,(k+2)\,\partial_z X^- &(16)\\ J_2(z) &= :X^-P^+: + P_2(z). &(17)\end{aligned}$$

Here the canonical momenta $P^+(z)$ and $P_2(z)$ are identified as

$$\begin{aligned} P^+(z) &= ik\partial_z X^+ e^{X_2} \\ P_2(z) &= -\frac{1}{2}ik\partial_z X_2 \end{aligned} \qquad (18)$$

Similarly, one calculates the right (anti-holomorphic) currents as

$$\tilde{J}_R(\bar{z}) = -ikg^{-1}\left(\bar{\partial}g\right) = ikg_R^{-1}\left(\bar{\partial}g_R\right) = \tilde{J}^a\tau^a = \begin{pmatrix} -\tilde{J}^2 & \tilde{J}^0 + \tilde{J}^1 \\ -\tilde{J}^0 + \tilde{J}^1 & \tilde{J}^2 \end{pmatrix} \qquad (19)$$

where three generators of anti-holomorphic Kac-Moody algebra are

$$\begin{aligned} \tilde{J}^1(\bar{z}) - \tilde{J}^0(\bar{z}) &= -\tilde{P}^-(\bar{z}) &(20)\\ \tilde{J}^1(\bar{z}) + \tilde{J}^0(\bar{z}) &= :\tilde{X}^+\tilde{P}^-\tilde{X}^+: +2\tilde{P}_2\tilde{X}^+ - i\,(k+2)\,\partial_{\bar{z}}\tilde{X}^+(\bar{z}) &(21)\\ \tilde{J}_2(\bar{z}) &= -:\tilde{X}^+\tilde{P}^-: -\tilde{P}_2(\bar{z}) &(22)\end{aligned}$$

The canonical momenta $\tilde{P}^-(\bar{z})$ and $\tilde{P}_2(\bar{z})$ are identified as

$$\begin{aligned} \tilde{P}^-(\bar{z}) &= ik\partial_{\bar{z}}\tilde{X}^- e^{\tilde{X}_2} \\ \tilde{P}_2(\bar{z}) &= -\frac{1}{2}ik\partial_{\bar{z}}\tilde{X}_2 \end{aligned} \qquad (23)$$

In (15-17) the holomorphic fields $X^-(z)$, $P^+(z)$ and $P_2(z)$ are free fields and have the following mode expansions:

$$\begin{aligned} X^-(z) &= q^- - ip^-\ln z + \sum_{n\neq 0}\frac{1}{n}\alpha_n^- z^{-n}, & \left(\alpha_n^-\right)^\dagger = \alpha_{-n}^- \qquad (24)\\ P^+(z) &= \frac{p^+}{z} + \sum_{n\neq 0}\alpha_n^+ z^{-n-1}, & \left(\alpha_n^+\right)^\dagger = \alpha_{-n}^+ \qquad (25)\\ P_2(z) &= \frac{p_2}{z} + \sum_{n\neq 0}s_n z^{-n-1}, & (s_n)^\dagger = s_{-n}\ . \qquad (26)\end{aligned}$$

[3] $\tau^a\ (a = 1,2,3)$ are the matrix representations of SL(2,R) generators:

$$\tau^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\quad \tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\quad \tau^2 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The fields (X^-, P^+) are conjugate to each other and similarly (X_2, P_2). The modes of the free fields obey the following commutation rules:

$$\begin{aligned} &[q^-, p^+] = i\,, \\ &[\alpha_n^-, \alpha_m^+] = n\,\delta_{n+m,0}\;\;, \\ &[s_n, s_m] = \left(\tfrac{k}{2}-1\right)\,n\,\delta_{n+m,0}\;\;. \end{aligned} \tag{27}$$

Likewise anti-holomorphic free-fields $\tilde{X}^+(\bar{z})$, $\tilde{P}^+(\bar{z})$ and $\tilde{P}_2(\bar{z})$ have mode expansions:

$$\tilde{X}^+(\bar{z}) \;=\; \tilde{q}^+ - i\tilde{p}^+ \ln \bar{z} + \sum_{n\neq 0} \frac{1}{n}\tilde{\alpha}_n^+ \bar{z}^{-n}, \qquad \left(\tilde{\alpha}_n^+\right)^\dagger = \tilde{\alpha}_{-n}^+ \tag{28}$$

$$\tilde{P}^-(\bar{z}) \;=\; \frac{\tilde{p}^-}{z} + \sum_{n\neq 0} \tilde{\alpha}_n^- \bar{z}^{-n-1}, \qquad \left(\tilde{\alpha}_n^-\right)^\dagger = \tilde{\alpha}_{-n}^- \tag{29}$$

$$\tilde{P}_2(\bar{z}) \;=\; \frac{\tilde{p}_2}{z} + \sum_{n\neq 0} \tilde{s}_n \bar{z}^{-n-1}, \qquad \left(\tilde{s}_n\right)^\dagger = \tilde{s}_{-n}\;. \tag{30}$$

Here the fields $(\tilde{X}^+, \tilde{P}^-)$ are conjugate to each other and similarly $\left(\tilde{X}_2, \tilde{P}_2\right)$. The commutation relations among modes are

$$\begin{aligned} &[\tilde{q}^+, \tilde{p}^-] = i\,, \\ &[\tilde{\alpha}_n^+, \tilde{\alpha}_m^-] = n\,\delta_{n+m,0}\;\;, \\ &[\tilde{s}_n, \tilde{s}_m] = \left(\tfrac{k}{2}-1\right)\,n\,\delta_{n+m,0}\;\;. \end{aligned} \tag{31}$$

The holomorphic Sugawara energy-momentum tensor takes the form (after careful ordering of operators, including zero modes, to insure hermiticity)

$$T(z) =: P^+ i\partial X^- : + \frac{1}{k-2}\left(: P_2^2 : - \frac{i}{z}\partial\left(zP_2\right) + \frac{1}{4z^2}\right) \tag{32}$$

From this expression for the energy-momentum tensor, by using the mode expansion of holomorphic free fields, we obtain the Virasoro generators in the holomorphic sector as

$$L_n = \sum_{m=-\infty}^{\infty} : \alpha_{n-m}^+ \alpha_m^- : + \frac{1}{k-2}\left[\sum_{m=-\infty}^{\infty} : s_{n-m} s_m : + i n s_n + \frac{\delta_{n,0}}{4}\right] \tag{33}$$

where $\alpha_0^+ = p^+$, $\alpha_0^- = p^-$ and $s_0 = p_2$.

The anti-holomorphic energy-momentum tensor and the Virasoro generators have the same form as the above expressions.

3.1 Matching Conditions

After this review of the free field realization of the WZNW model on SL(2,R) group manifold now we are ready to analyze the consequences of the twisted Dirichlet gluing conditions (5) among Kac-Moody currents. We will use the expressions (14) and (19) to translate the twisted gluing conditions among the symmetry currents to boundary conditions among the free fields.

The boundary conditions among the free fields are called as "matching conditions" [27]. Subsequently from the matching conditions, by using the mode expansions of the free fields, the "boundary conditions" among the modes will be derived.

Using (5), (14) and (19) the twisted gluing conditions in open string channel at $z = \bar{z}$ become

$$-J^2 = \tilde{J}^2 \tag{34}$$

$$J^1 - J^0 = \tilde{J}^1 + \tilde{J}^0 \tag{35}$$

$$J^1 + J^0 = \tilde{J}^1 - \tilde{J}^0 \tag{36}$$

and in the closed string channel at $z \cdot \bar{z} = 1$ they become

$$\left[J^2 - \bar{z}^2\ \tilde{J}^2\right]|B\rangle = 0 \tag{37}$$

$$\left[\left(J^1 - J^0\right) + \bar{z}^2\ \left(\tilde{J}^1 + \tilde{J}^0\right)\right]|B\rangle = 0 \tag{38}$$

$$\left[\left(J^1 + J^0\right) + \bar{z}^2\ \left(\tilde{J}^1 - \tilde{J}^0\right)\right]|B\rangle = 0. \tag{39}$$

Substituting in the above gluing conditions the free field representations (15-17, 20-22) of symmetry currents we obtain the matching conditions among free fields in the open string channel as

$$P^+(z) = -\tilde{P}^-(\bar{z}), \quad P_2(z) = \tilde{P}_2(\bar{z}), \quad X^-(z) = -\tilde{X}^+(\bar{z}) \quad at \quad z = \bar{z} \tag{40}$$

and in the closed string channel they are

$$\left[P^+(z) - \bar{z}^2\ \tilde{P}^-(\bar{z})\right]|B\rangle = 0, \tag{41}$$

$$\left[P_2(z) + \bar{z}^2\ \tilde{P}_2(\bar{z})\right]|B\rangle = 0, \tag{42}$$

$$\left[X^-(z) + \tilde{X}^+(\bar{z})\right]|B\rangle = 0 \quad at \quad z \cdot \bar{z} = 1. \tag{43}$$

Next we use the mode expansions of the free fields, (24,25,26) and (28,29,30), to find the boundary conditions among the modes of free fields in the closed string channel. In terms of modes the twisted Dirichlet boundary condition reads in the closed string channel as

Zero Modes	Oscillator Modes	
$\left(p^+ - \tilde{p}^-\right)\|B\rangle = 0$	$\left(\alpha_n^+ - \tilde{\alpha}_{-n}^-\right)\|B\rangle = 0$	(44)
$\left(p_2 + \tilde{p}_2\right)\|B\rangle = 0$	$\left(s_n + \tilde{s}_{-n}\right)\|B\rangle = 0$	(45)
$\left(p^- - \tilde{p}^+\right)\|B\rangle = 0$	$\left(\alpha_n^- - \tilde{\alpha}_{-n}^+\right)\|B\rangle = 0$	(46)
$\left(q^- + \tilde{q}^+\right)\|B\rangle = 0$		(47)

3.2 Coherent Boundary States

The free field analysis of the previous subsections allows us to write the spectrum of closed strings on Lorentzian AdS$_3$ as a Fock space built on a base state which is a direct product of base states for holomorphic and anti-holomorphic sectors [5]:

$$|base\rangle = \left|p^+, p^-, p_2\right\rangle \otimes \left|\,\tilde{p}^+, \tilde{p}^-, \tilde{p}_2\right\rangle. \tag{48}$$

Then the physical states are obtained by applying creation oscillators $\alpha^+_{-n},\ \alpha^-_{-n},\ s_{-n},\ (n > 0)$ and their anti-holomorhic counterparts on this base state.

Our aim is to find conformally invariant boundary states built on the above base state. In the case of bosonic string theory on flat Minkowski space-time such states were found in the form of coherent states [28]. Hence, we have the following ansatz for the coherent boundary state:

$$|B_{p^+,p^-,p_2}\rangle \;=\; \exp\left(\sum_{n>0}\frac{1}{n}\alpha^-_{-n}\tilde{\alpha}^-_{-n} + \sum_{n>0}\frac{1}{n}\alpha^+_{-n}\tilde{\alpha}^+_{-n} - \sum_{n>0}\frac{1}{n}s^{'}_{-n}\tilde{s}^{'}_{-n}\right)\left|p^+, p^-, p_2\right\rangle \tag{49}$$

$$\langle B_{p^+,p^-,p_2}| \;=\; \left\langle p^+, p^-, p_2\right| \exp\left(\sum_{n>0}\frac{1}{n}\alpha^-_{n}\tilde{\alpha}^-_{n} + \sum_{n>0}\frac{1}{n}\alpha^+_{n}\tilde{\alpha}^+_{n} - \sum_{n>0}\frac{1}{n}s^{'}_{n}\tilde{s}^{'}_{n}\right) \tag{50}$$

where $s^{'}_n = \frac{s_n}{\sqrt{\frac{k}{2}-1}}$ and $\tilde{s}^{'}_n = \frac{\tilde{s}_n}{\sqrt{\frac{k}{2}-1}}$. In this ansatz for coherent boundary state we have taken the base state as

$$|base\rangle = \left|p^+, p^-, p_2\right\rangle \tag{51}$$

already using the twisted Dirichlet boundary conditions for zero modes (44-46). We have to check whether this coherent boundary states obey also the twisted Dirichlet boundary conditions for oscillator modes (44-46). Indeed, they satisfy:

$$(\alpha^+_n - \tilde{\alpha}^-_{-n})\,|B_{p^+,p^-,p_2}\rangle \;=\; 0 = \langle B_{p^+,p^-,p_2}|\,(\alpha^+_n - \tilde{\alpha}^-_{-n}), \tag{52}$$

$$(\alpha^-_n - \tilde{\alpha}^+_{-n})\,|B_{p^+,p^-,p_2}\rangle \;=\; 0 = \langle B_{p^+,p^-,p_2}|\,(\alpha^-_n - \tilde{\alpha}^+_{-n}), \tag{53}$$

$$(s_n + \tilde{s}_{-n})\,|B_{p^+,p^-,p_2}\rangle \;=\; 0 = \langle B_{p^+,p^-,p_2}|\,(s_n + \tilde{s}_{-n}). \tag{54}$$

In BCFT such boundary states, that obey the necessary boundary conditions, are called as Ishibashi states [29].

To have a consistent BCFT the conformal symmetry of the theory should be preserved. This means that, in the open string channel, the holomorphic and anti-holomorphic energy-momentum tensors should match at the boundary

$$\left[T\left(z\right) - \tilde{T}\left(\bar{z}\right)\right] = 0 \quad at \quad z = \bar{z}, \tag{55}$$

since the energy-momentum tensor is the generator of the conformal transformations. In the closed string channel this condition becomes a relation between holomorphic and anti-holomorphic Virasoro generators

$$(L_n - \tilde{L}_{-n})\left|B_{p^+,p^-,p_2}\right\rangle = 0 = \langle B_{p^+,p^-,p_2}|\,(L_n - \tilde{L}_{-n}). \tag{56}$$

Our coherent boundary states (49-50) also obey these conditions.

The last twisted Dirichlet boundary condition (47) will be used in the next section in order to determine the form of the "physical" boundary states and the location of AdS_2 D-branes. That is this condition will be imposed on the physical boundary states not on the Ishibashi states.

4 Physical Boundary States

4.1 Annulus Amplitude and Closed String Spectrum

Any combination of Ishibashi states (49,50) obey the twisted Dirichlet boundary conditions (44-46). However, not all the combinations can be taken as genuine boundary state. Apart from the gluing conditions (44-46) and the condition to preserve the conformal symmetry, there is an extra condition which chooses the physical boundary states among the infinitely many combinations of the Ishibashi states. The extra condition comes from the fact that the annulus amplitude in the closed string channel, which is just the tree-level amplitude of propagation of a closed string from one boundary B to the other boundary B'[4]

$$\left\langle B'_{p^{+\prime},p^{-\prime},p'_2}\right| \; e^{2\pi i\tau(L_0-\frac{c}{24})}e^{-2\pi i\bar{\tau}(\tilde{L}_0-\frac{c}{24})} \; e^{2\pi i\theta J_0^+}e^{-2\pi i\bar{\theta}\tilde{J}_0^+} \; \left|B_{p^+,p^-,p_2}\right\rangle \tag{57}$$

should be equivalent to the one-loop amplitude in the open string channel with boundary conditions B and B'. This condition is first discussed by Cardy in the case of rational conformal field theories (RCFT) in [30] and therefore it is known as Cardy condition. However, since SL(2,R) WZNW model contains infinite number of primary fields and, therefore, is not a RCFT, one cannot use the Cardy condition directly. Either one has to utilize an analog of the Cardy condition for non-RCFT as in [31] and [23] or try another way in order to determine the physical boundary states. In this paper we do not use a (modified) Cardy condition, instead we determine the physical boundary states using the condition (47) between the zero modes of X^- and $\tilde{X}^+$.[5]

However, before determining the physical boundary states with the above mentioned method we would like to derive the annulus amplitude in the free field formalism and use it to decide the spectrum of the closed strings that couple to the AdS_2 D-branes on Lorentzian AdS_3. The holomorphic part of the annulus amplitude

$$\hat{\chi}_{p^+,p^-,p_2}(\tau,\theta) = \left\langle B'_{p^{+\prime},p^{-\prime},p'_2}\right| \; q^{L_0-\frac{c}{24}} \; w^{J_0^+} \; \left|B_{p^+,p^-,p_2}\right\rangle \tag{58}$$

[4]Since in the representation of Kac-Moody currents the diagonal generator is $J^+(z)$, we are using its zero mode J_0^+ in the expression for the annulus amplitude.

[5]Similar analysis was done in the case of SU(2) WZW model in [27].

where $q = \exp(2\pi i\tau)\,,\ w = \exp(2\pi i\theta)$, can be calculated straightforwardly by separating the Virasoro generator L_0 into zero-mode and oscillator parts

$$L_0 = p^+p^- + \frac{1}{k-2}\left(p_2^2 + \frac{1}{4}\right) + \sum_{m>0}\left(\alpha^+_{-m}\alpha^-_m + \alpha^-_{-m}\alpha^+_m + s'_{n-m}s'_m\right) \tag{59}$$

where $s'_n = \frac{s_n}{\sqrt{\frac{k}{2}-1}}$ and $\tilde{s}'_n = \frac{\tilde{s}_n}{\sqrt{\frac{k}{2}-1}}$. Then one uses the coherent state methods to obtain the contribution of oscillators to the free field character (58) as

$$\prod_{n>0}\left(\frac{1}{1-q^n}\right)^3 = \frac{q^{1/8}}{\eta(\tau)^3}\,. \tag{60}$$

The contribution from zero-modes is

$$\left\langle p^{+\prime}, p^{-\prime}, p_2'\right|\ q^{\left(p^+p^- + \frac{1}{k-2}\left(p_2^2+\frac{1}{4}\right) - \frac{k}{8(k-2)}\right)}\ w^{p^+}\ \left|p^+, p^-, p_2\right\rangle = q^{\left(p^+p^- + \frac{p_2^2}{k-2} - \frac{1}{8}\right)}\ w^{p^+}. \tag{61}$$

Combining them one finds the full free-field character:

$$\hat{\chi}_{p^+,p^-,p_2}(\tau,\theta) = \frac{1}{\eta(\tau)^3}\ q^{\left(p^+p^- + \frac{p_2^2}{k-2}\right)}\ w^{p^+}. \tag{62}$$

In order to find the spectrum of the closed string states which couple to AdS$_2$ D-branes we compare the holomorhic part of the annulus amplitude (or free field character) with the characters of the irreducible representations of $\widehat{\mathtt{sl}}$(2,R). This is because if we had written down the Ishibashi states in a specific representation of $\widehat{\mathtt{sl}}$(2,R) instead of free field representation, the holomorphic part of the annulus amplitude in the closed string channel would be just the character of $\widehat{\mathtt{sl}}$(2,R) for that specific representation.[6] The free field character (62) is most closely related to the character of the principal continuous series representation, $\hat{C}^{\alpha}_j$, of $\widehat{\mathtt{sl}}$(2,R) [4]:

$$\hat{\chi}_j(\tau,\theta) = \frac{1}{\eta(\tau)^3}\ q^{\left(\frac{\rho^2}{k-2}\right)} \sum_{m=-\infty}^{\infty} w^{m+\alpha} \tag{63}$$

where $j = -\frac{1}{2} + i\rho$ and $\alpha \in [0,1)$. Therefore, we expect that the states of closed strings, which couple to the AdS$_2$ D-branes in Lorentzian AdS$_3$, belong to the principal continuous series of $\widehat{\mathtt{sl}}$(2,R). Since the free field character (62) differs from the character of principal continuous series of $\widehat{\mathtt{sl}}$(2,R), we conjecture the "consistent" boundary states[7] as sum of "coherent" boundary states as

$$|\mathcal{B}\rangle = \sum_{m\in Z}\ \sum_{r\in Z_+} \left|B_{m+\alpha,-\frac{r}{m+\alpha},p_2}\right\rangle \tag{64}$$

[6] See [22] for a form of such Ishibashi states (equ. (20) in that paper). However, even though in [22] it is claimed that only the states in the principal continuous series representation of $\widehat{\mathtt{sl}}$(2,R) are treated, the annulus amplitude (equ. (26) in [22]) is in the form of discrete series character of $\widehat{\mathtt{sl}}$(2,R), which is in contradiction with the claim made.

[7] Here by consistent boundary states, unlike its usage in [32], we do not mean Cardy states. In order to find Cardy states we still have to use boundary condition among zero modes (47) and we will call the final result as the "physical" boundary state instead of the Cardy state, because we are not using Cardy(–like) condition in our analysis.

where we set $p^+ = m + \alpha$, $m \in Z$ and $p^+ p^- = -r$, $r \in Z_+$. The second relation originates from the monodromy considerations [5] and is also reviewed below. With this form of the consistent boundary states the holomorphic part of the annulus amplitude become

$$\hat{\chi}(\tau, \theta) = \left\langle \mathcal{B}' \right| q^{L_0 - \frac{c}{24}} w^{J_0^+} \left| \mathcal{B} \right\rangle = \frac{1}{\eta(\tau)^3} q^{\left(\frac{p_2^2}{k-2}\right)} \sum_{r \in Z_+} q^{-r} \sum_{m \in Z} w^{m+\alpha} \tag{65}$$

If one sets $p^- = 0$ and $p_2 = \rho$ one obtains the character for the principal continuous series[8]. However, as explained below one should not set $p^- = 0$. Hence, we conclude that the closed string states which couple to the AdS$_2$ D-branes in Lorentzian AdS$_3$ are in the principal continuous series of $\widehat{\mathtt{sl}}$(2,R) supplemented with the zero mode p^-.

The form of the holomorphic part of the annulus amplitude gives support to the conclusions in papers [5],[7], namely that the closed string spectrum on SL(2,R) group manifold contains only the states from the principal continuous series of $\widehat{\mathtt{sl}}$(2,R) Kac-Moody algebra supplemented with the zero mode p^-. In the anti-holomorphic sector there is also another extra degree of freedom $\tilde{p}^+$, which is equivalent to p^- in the open string channel. The effect of this extra degree of freedom in the holomorphic sector is that it contributes a new term to the mass-shell condition [5],[7]. Without p^- the eigenvalue of the Virasoro generator L_0 (Hamiltonian) on a physical state at level N is

$$L_0 \left| phys \right\rangle = \left(-\frac{j(j+1)}{k-2} + N \right) \left| phys \right\rangle . \tag{66}$$

Mass-shell condition requires that $L_0 = a \leq 1$. In order to obey this condition, since N is positive, $j(j+1)$ should be positive. This requires that only the highest and the lowest weight discrete series representations of $\widehat{\mathtt{sl}}$(2,R), for which $j(j+1) > -\frac{1}{4}$, should be taken into account. Other than these representations of $\widehat{\mathtt{sl}}$(2,R) only the lowest level states from the principal continuous series are allowed. However, in the presence of p^- the eigenvalue of L_0 is modified as

$$L_0 \left| phys \right\rangle = \left(p^+ p^- - \frac{j(j+1)}{k-2} + N \right) \left| phys \right\rangle . \tag{67}$$

Considerations of monodromy (i.e. invariance of closed string states under transformation $\sigma \to \sigma + 2\pi$ on closed string world-sheet) have shown in [5] that $p^+ p^-$ should be equal to a negative integer

$$p^+ p^- = -r, \quad r \in Z_+ \tag{68}$$

Since $p^+ p^-$ is negative and N is positive, now on shell condition $L_0 = a \leq 1$ can be satisfied with $j(j+1)$ being negative. This means that the physical states of closed strings on SL(2,R) group manifold belong to the principal continuous series of $\widehat{\mathtt{sl}}$(2,R) Kac-Moody algebra. Existence or non-existence of p^-, therefore, changes completely the spectrum of physical states. Which of

[8]Since these expressions are divergent, we are just comparing, formally, the form of the expressions.

these mass-shell conditions, and consequently which spectrum is the physical spectrum can be deduced by going to the flat limit in the SL(2,R) WZNW model. The flat limit of the model is obtained by sending the radius of curvature of AdS$_3$ to infinity. From the metric of AdS$_3$ (3) the radius of curvature is read as $R = \sqrt{k\alpha'}$. Therefore $k \to \infty$ limit in SL(2,R) WZNW model is the flat limit that we are seeking for.

It has been shown in [7] that the vertex operator for the lowest level closed string states in the momentum basis is

$$V^{r,p_2}_{p^+,p^-}(z) = Ce^{iX^- p^+}\, e^{-\frac{1}{2}X_2}\, J_{2\sigma}\left(2\sqrt{-p^+p^-}e^{-\frac{1}{2}X_2(z)}\right)\, e^{iX^+p^-} \tag{69}$$

where $r \in Z_+$, $\sigma = \sqrt{kr - p_2^2}$ and C is an overall factor. In the flat limit, $k \to \infty$, this vertex operator becomes the correct vertex operator for the lowest level closed string states in Minkowski space-time

$$\left(V^{r,p_2}_{p^+p^-}(z)\right)_{k\to\infty} = e^{i\breve{X}^-\breve{p}^+} e^{i\breve{X}^2\breve{p}^2} e^{i\breve{X}^+\breve{p}^-}\,\delta\left(\breve{p}^+\breve{p}^- + \breve{p}_2^2 + m^2\right) \tag{70}$$

where $m = \frac{\sigma}{\sqrt{k}}$ is mass, $\breve{p}^+ = \frac{1}{\sqrt{k}}p^+$, $\breve{p}^- = \frac{1}{\sqrt{k}}p^-$, $\breve{p}_2 = \frac{1}{\sqrt{k}}p_2$ are flat space-time momenta and $\breve{X}^- = \sqrt{k+2}\,X^-$, $\breve{X}^+ = \sqrt{k+2}\,X^+$, $\breve{X}_2 = \sqrt{k+2}\,X_2$ are holomorphic part of the fields on closed string world-sheet in flat Minkowski space-time. In the expression for flat vertex operator (70) we also see the mass-shell condition in flat space-time:

$$\breve{p}^+\breve{p}^- + \breve{p}_2^2 + m^2 = 0. \tag{71}$$

It is absolutely necessary to have flat momentum $\breve{p}^-$ in this expression in order to have physically acceptable motion of any relativistic particle or string in the flat Minkowski space-time. Therefore it is absolutely necessary to have its corresponding quantity p^- in the formulation of string theory in curved AdS$_3$ space-time.

In this paper we have reached this same conclusion from the viewpoint of AdS$_2$ D-branes: the correct spectrum of closed strings in Lorentzian AdS$_3$ space-time is the principal continuous series of $\widehat{\mathfrak{sl}}$(2,R) Kac-Moody algebra supplemented with zero modes.

4.2 Location of AdS$_2$ D-branes

In section 3.2 we have shown that the coherent boundary states (49,50) obey the twisted Dirichlet type boundary conditions (44-46) and therefore give a representation of Ishibashi states. These states also preserve one half of the conformal symmetry (56). The only condition that is left to be satisfied is the condition among the zero modes

$$\left(q^- + \tilde{q}^+\right)|B\rangle = 0 \tag{72}$$

which contains information about the location of AdS$_2$ D-branes in AdS$_3$ space-time. However, this condition has been derived by an algebraic analysis and thus without using any extra data on the group manifold the above condition only applies to AdS$_2$ D-brane at the unit element of SL(2,R).

It has been shown in [15],[17] that the twisted Dirichlet boundary condition (47) –when translated from the unit element of the group to an arbitrary point on the group manifold– requires the corresponding D-branes to be located on twisted conjugacy classes. Given the particular representation of an arbitrary group element of SL(2,R) (2), the twisted conjugacy classes of SL(2,R) are given by the condition [16],[17]

$$e^{\phi/2}\left(\gamma^{-}+\gamma^{+}\right)=\mathcal{C}, \tag{73}$$

where $-\infty<\mathcal{C}<\infty$ is a real constant. Let us analyze this condition in the free field representation of the SL(2,R) group element g (11-13). Equating expressions (2) and (11) for g we find [7]

$$\begin{aligned}
\phi\left(z,\bar{z}\right) &= X_2\left(z\right)+\tilde{X}_2\left(\bar{z}\right)+2\ln\left(1+X^{+}\left(z\right)\tilde{X}^{-}\left(\bar{z}\right)\right) && (74)\\
\gamma^{-}\left(z,\bar{z}\right) &= X^{-}\left(z\right)+\frac{e^{-X_2(z)}\tilde{X}^{-}\left(\bar{z}\right)}{1+X^{+}\left(z\right)\tilde{X}^{-}\left(\bar{z}\right)} && (75)\\
\gamma^{+}\left(z,\bar{z}\right) &= \tilde{X}^{+}\left(\bar{z}\right)+\frac{e^{-\tilde{X}_2(\bar{z})}X^{+}\left(z\right)}{1+X^{+}\left(z\right)\tilde{X}^{-}\left(\bar{z}\right)}\,. && (76)
\end{aligned}$$

Substituting the expressions for $\gamma^{-}\left(z,\bar{z}\right)$, $\gamma^{+}\left(z,\bar{z}\right)$ and $e^{\phi(z,\bar{z})/2}$ into (73) and then using the matching condition $X^{-}(z)=-\tilde{X}^{+}(\bar{z})$ at $z=\bar{z}$ in the open string channel we obtain

$$e^{-X_2(z)}\tilde{X}^{-}\left(\bar{z}\right)+e^{-\tilde{X}_2(\bar{z})}X^{+}\left(z\right)-\mathcal{C}\,e^{-X_2(z)-\tilde{X}_2(\bar{z})}=0. \tag{77}$$

In the free field approach, that we utilize, $X^{+}(z)$ and $\tilde{X}^{-}(\bar{z})$ are not free fields, but are composites of free fields [5],[7]:

$$\begin{aligned}
X^{+}\left(z\right) &= q^{+}-\frac{i}{k}\int^{z}dz'P^{+}(z')e^{-X_2(z')}, && (78)\\
\tilde{X}^{-}\left(\bar{z}\right) &= \tilde{q}^{-}-\frac{i}{k}\int^{\bar{z}}d\bar{z}'\tilde{P}^{-}(\bar{z}')\,e^{-\tilde{X}_2(\bar{z}')}. && (79)
\end{aligned}$$

Substituting these expressions into (77) and using matching conditions

$$\begin{aligned}
P^{+}\left(z\right) &= -\tilde{P}^{-}\left(\bar{z}\right),\\
P_2\left(z\right) &= \tilde{P}_2\left(\bar{z}\right)
\end{aligned}\quad\Rightarrow\quad X_2(z)=\tilde{X}_2(\bar{z})+(q_2-\tilde{q}_2)$$

we obtain

$$q_2-\tilde{q}_2=\Theta. \tag{80}$$

where Θ is a function of $q^{+},\ \tilde{q}^{-}$ and $\mathcal{C}$ and can be obtained by solving (77). In order to write the physical boundary states we have to construct appropriate combinations of consistent boundary states (64) and form a δ–function realizing the boundary condition (80) among the zero modes. This δ–function can be represented as

$$\delta(q_2 - \tilde{q}_2 - \Theta) = \int_{-\infty}^{\infty} d\rho\, e^{i\rho(q_2 - \tilde{q}_2 - \Theta)} = \int_{-\infty}^{\infty} d\rho\, e^{-i\rho\Theta}\, e^{i\rho(q_2 - \tilde{q}_2)}. \tag{81}$$

and it describes the wave function of the zero modes. Then, the physical boundary state for AdS_2 D-branes can be written as

$$|\mathcal{B}_{phys}\rangle = \int_{-\infty}^{\infty} d\rho \sum_{m\in Z} \sum_{r\in Z_+} e^{-i\rho\Theta} \left| B_{p^+=m+\alpha,\, p^-=-\frac{r}{m+\alpha},\, p_2=\rho} \right\rangle. \tag{82}$$

D-branes with this physical boundary state wrap the twisted conjugacy classes (73) in AdS_3.

5 Conclusions

In this paper we have determined the boundary states for AdS_2 D-branes in Lorentzian AdS_3. In constructing the boundary states we have utilized the free field formalism of SL(2,R) WZNW model. The gluing conditions among the symmetry currents are translated into the free field language and the boundary conditions among the modes of free fields are deduced. Ishibashi states are constructed as coherent states in the free field Fock space and the boundary conditions imposed on them are shown to be satisfied. Annulus amplitude is evaluated in the closed string channel and then it is compared to the characters of the unitary representations of $\widehat{\mathrm{sl}}(2,\mathrm{R})$ Kac-Moody algebra. It is found that only the closed strings in the principal continuous series of $\widehat{\mathrm{sl}}(2,\mathrm{R})$ couple to AdS_2 D-branes. This result is in accordance with the conclusions of [5],[7]. Then using the fact that AdS_2 D-branes are located on the twisted conjugacy classes of SL(2,R), the form of physical boundary states is determined.

At the beginning of section 4 we explained the non-usability of the Cardy procedure in the case of SL(2,R) WZNW model. The method of finding the boundary states by determining the one-point functions in BCFT [20],[23] is also not possible in the case of Lorentzian AdS_3. In [20],[23] the expressions for one-point functions are determined by solving the factorization conditions of two-point functions in Euclidean AdS_3. Two-point functions are known in SL(2,C)/SU(2) WZNW model (Euclidean AdS_3), however the correct procedure to Wick rotate them into SL(2,R) WZNW model (Lorentzian AdS_3) is still not known. Therefore, presently, utilizing such a technique in Lorentzian AdS_3 is not possible. Only after proper calculation of two-point and three-point functions in SL(2,R) WZNW model one can use BCFT techniques to determine the one-point functions and consequently the physical boundary states. Comparing the physical boundary states obtained via different methods will be a non-trivial check of the correctness of their form (82).

As a possible future research these physical boundary states can be used to calculate the boundary correlation functions. The free field formalism will help this calculations to simplify enormously. Such correlation functions correspond to physically observable quantities and they will provide more information on the properties of the theory. The regularization and the modular properties of the annulus amplitude (57,65) should also be examined thoroughly. From the regularized and modular transformed form of this amplitude it should be possible to read the open string spectrum in the Lorentzian AdS_3.

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