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Solvable $\mathcal{N}$=(4,4) Type IIA String Theory in Plane-Wave Background and D-Branes

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Abstract

We study various aspects of $\mathcal{N}$=(4,4) type IIA GS superstring theory in the pp-wave background, which arises as the compactification of maximally supersymmetric eleven-dimensional pp-wave geometry along the spacelike isometry direction. We show the supersymmetry algebra of $\mathcal{N}$=(4,4) worldsheet supersymmetry as well as non-linearly realized supersymmetry. We also give quantization of closed string and open string incorporating various boundary conditions. From the open string boundary conditions, we find configurations of D-branes which preserve half the supersymmetries. Among these we identify D4 brane configurations with longitudinal five brane configurations in matrix model on the eleven-dimensional pp-wave geometry.

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1 Introduction and Summary

Recently, there have been lots of interests in the M theory on eleven-dimensional pp-wave geometry [1]-[5] which is maximally supersymmetric by admitting 32 Killing spinors. One way to study the M theory in this background is to use matrix model [6], which seems to be particularly suitable, recalling the light-like nature of the background. Matrix model on pp-wave background [6, 7] has interesting property such as the removal of flat directions because of bosonic mass terms and therefore the moduli space reduces to the set of isolated points which represent fuzzy spheres. This matrix model has time dependent supersymmetry and thus the bosons and fermions have different masses. Using this model various kinds of nonperturbative BPS states have been identified [7]-[12]. Other aspects of the model have been discussed in [13]-[17].

Another way to study the M theory is to compactify the theory along the circle under which it becomes type IIA superstring theory, whose coupling is proportional to the compactification radius with fixed string tension. Many characteristic features, like nonperturbative BPS states, of IIA superstring theory is inherited from M theory. In turn, the study of IIA superstring theory gives some informations on 'mother' M theory. The eleven-dimensional maximally supersymmetric pp-wave geometry has various spacelike isometry, along which the M theory can be compactified[18].

One such choice of the circle direction leads to the following ten-dimensional pp-wave background[19, 20]:[1]

$$\begin{aligned} ds^2 &= -2dx^+dx^- - A(x^I)(dx^+)^2 + \sum_{I=1}^{8}(dx^I)^2 \ , \\ F_{+123} &= \mu \ , \quad F_{+4} = -\frac{\mu}{3} \ , \end{aligned} \tag{1}$$

where

$$A(x^I) = \sum_{i=1}^{4}\frac{\mu^2}{9}(x^i)^2 + \sum_{i'=5}^{8}\frac{\mu^2}{36}(x^{i'})^2 \ . \tag{2}$$

In contrast to the eleven-dimensional pp-wave geometry, this geometry admits only 24 Killing spinors. In [19] the full type IIA GS superstring action on this ten-dimensional pp-wave was constructed and their worldsheet supersymmetry has been identified as $\mathcal{N} = (4, 4)$. We would like to note that IIA GS string theories on different pp-wave geometry have been considered in [21, 22, 23].

[1]This geometry comes from the circle compactification of the maximally supersymmetric eleven dimensional pp-wave. The direction of compactification is the isometry $\partial/\partial x^9$, which is the combination of the original x^4 and x^9 coordinates. This is why we have the non-vanishing two form RR field strength. For details, see the appendix of the paper [19].

In this paper we continue the study of the (4,4) IIA superstring theory constructed in [19].[2] In what follows, we recall, the light-cone gauge fixed, IIA GS superstring action on pp-wave geometry, given in [19]. It is very complicate to get the full expression of the GS superstring action in the general background (see, for example, [24, 25]). However, in the case at hand, we can use the fact that eleven-dimensional pp-wave geometry can be thought as a special limit of $AdS_4 \times S^7$ geometry on which the full supermembrane action is constructed using coset method [26]. The full IIA GS superstring action on this geometry can be obtained by the double dimensional reduction [27] of the supermembrane action of [26] in the pp-wave limit. It turns out that, by fixing the fermionic κ-symmetry as

$$\Gamma^+\theta = 0 \ , \tag{3}$$

the action reduces to the following form:

$$\begin{aligned} S_{IIA} &= -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-h}h^{mn}\Big[-2\partial_m X^+\partial_n X^- + \partial_m X^I\partial_n X^I - A(X^I)\partial_m X^+\partial_n X^- \\ &\quad -2\partial_m X^+\bar\theta\Gamma^-\partial_n\theta + \frac{\mu}{2}\partial_m X^+\partial_n X^+\bar\theta\Gamma^-\left(\Gamma^{123}+\frac{1}{3}\Gamma^{49}\right)\theta\Big] \\ &\quad -\frac{1}{2\pi\alpha'}\int d^2\sigma\epsilon^{mn}\partial_m X^+\bar\theta\Gamma^{-9}\partial_n\theta \ , \end{aligned} \tag{4}$$

in which the Majorana fermion θ is the combination of Majorana-Weyl fermions θ^1 and θ^2 with opposite ten dimeniosional chiralities, that is, $\theta = \theta^1 + \theta^2$, and the fermion part is simply given by quadratic term. In fact, recently it has been argued in [28, 29] that for a wide class of pp-wave background, the fermionic part of GS action has only quadratic terms.

The equation of motion for X^+ is harmonic, the same as in the flat case, which allows the usual light-cone gauge fixing,

$$X^+ = \alpha' p^+ \tau \ , \tag{5}$$

where p^+ is the total momentum conjugate to X^-. The worldsheet diffeomorphism can be consistently fixed as $\sqrt{-h} = 1$, $h_{\sigma\tau} = 0$, which fix other worldsheet metric components consistently as $h_{\tau\tau} = -1$ and $h_{\sigma\sigma} = 1$.

After rescaling the fermionic coordinate as $\theta \to \theta/\sqrt{2\alpha' p^+}$, the light-cone gauge fixed action of IIA string is given by

$$\begin{aligned} S_{LC} &= -\frac{1}{4\pi\alpha'}\int d^2\sigma\Big[\eta^{mn}\partial_m X^I\partial_n X^I + \frac{m^2}{9}(X^i)^2 + \frac{m^2}{36}(X^{i'})^2 \\ &\quad +\bar\theta\Gamma^-\partial_\tau\theta + \bar\theta\Gamma^{-9}\partial_\sigma\theta - \frac{m}{4}\bar\theta\Gamma^-\left(\Gamma^{123}+\frac{1}{3}\Gamma^{49}\right)\theta\Big] \ , \end{aligned} \tag{6}$$

[2]The same model was considered in [20] in which the action was modified following our previous paper [19]. They considered the quantization of string coordinates which overlap with some materials in sections 2 and 4 in this paper. They also considered the classification of supersymmetric D-branes, which is, however, different from ours. In particular, we have got different results in D4 and D6 branes. As will be seen in section 4, the two form RR field strength should be taken care properly in the classification of D-branes.

where

$$m \equiv \mu\alpha' p^+ \tag{7}$$

is a mass parameter which characterizes the masses of the worldsheet fields. Therefore the light-cone gauge-fixed action S_{LC} is quadratic in bosonic as well as fermionic fields and thus describes a free theory much the same as in the IIB string theory [30] on the pp-wave geometry [31]. This makes accessible to study IIA string theory on pp-wave geometry which also helps to understand the M theory on pp-wave geometry.

Different characteristic feature of IIA string theory on pp-wave geometry compared to IIB case is the structure of worldsheet supersymmetry. Sixteen spacetime supersymmetries which satisfy $\Gamma^+\epsilon = 0$ are non-linearly realized on the worldsheet action. As is typical in light-cone GS superstring, the remaining eight spacetime supersymmetries, combined with appropriate kappa transformations, turn into worldsheet (4,4) supersymmetry of Yang-Mills type[19]. Furthermore, in contrast to the matrix model in eleven dimensions, this, so-called, dynamical supersymmetry is time independent. Note that, recently in [32], linearly-realized worldsheet supersymmetry of GS superstring in more general background was discussed.

In order to see all these more clearly, we rewrite the action S_{LC} in the 16 component spinor notation with $\theta^A = \frac{1}{2^{1/4}}\begin{pmatrix} 0 \\ \psi^A \end{pmatrix}$ (Superscript A denotes the $SO(1,9)$ chirality and takes values of 1 and 2. 1 is for the positive chirality and 2 for the negative one.), under which it becomes

$$\begin{aligned} S_{LC} \;=\; & -\frac{1}{4\pi\alpha'}\int d^2\sigma\Big[\eta^{mn}\partial_m X^I \partial_n X^I + \frac{m^2}{9}(X^i)^2 + \frac{m^2}{36}(X^{i'})^2 \\ & -i\psi^1_+\partial_+\psi^1_+ - i\psi^1_-\partial_+\psi^1_- - i\psi^2_+\partial_-\psi^2_+ - i\psi^2_-\partial_-\psi^2_- + 2i\frac{m}{3}\psi^2_+\gamma^4\psi^1_- - 2i\frac{m}{6}\psi^2_-\gamma^4\psi^1_+\Big] \;, \end{aligned} \tag{8}$$

where $\partial_\pm = \partial_\tau \pm \partial_\sigma$. Here the sign of subscript in $\psi^A_\pm$ represents the eigenvalue of γ^{1234}. In our convention, fermion has the same $SO(1,9)$ and $SO(8)$ chirality measured by Γ^9 and γ^9, respectively.

Thus, among sixteen fermionic components in total, eight with $\gamma^{12349} = 1$ have the mass of $m/6$ and the other eight with $\gamma^{12349} = -1$ the mass of $m/3$, which are identical with the masses of bosons. Therefore the theory contains two supermultiplets $(X^i, \psi^1_-, \psi^2_+)$ and $(X^{i'}, \psi^1_+, \psi^2_-)$ of worldsheet (4,4) supersymmetry with the masses $m/3$ and $m/6$, respectively.

This can be seen explicitly from the transformation laws [19] for the worldsheet (4,4) supersymmetry which are given by

$$\delta X^i \;=\; \frac{i}{\sqrt{\alpha' p^+}}(\psi^1_-\gamma^i\epsilon^1_+ + \psi^2_+\gamma^i\epsilon^2_-) \;,$$

$$
\begin{aligned}
\delta\psi^1_- &= \frac{1}{\sqrt{\alpha' p^+}}\left(\partial_- X^i\gamma^i\epsilon^1_+ - \frac{m}{3}X^i\gamma^4\gamma^i\epsilon^2_-\right) ,\\
\delta\psi^2_+ &= \frac{1}{\sqrt{\alpha' p^+}}\left(\partial_+ X^i\gamma^i\epsilon^2_- + \frac{m}{3}X^i\gamma^4\gamma^i\epsilon^1_+\right) ,
\end{aligned}
\tag{9}
$$

for the supermultiplet $(X^i, \psi^1_-, \psi^2_+)$ and

$$
\begin{aligned}
\delta X^{i'} &= \frac{i}{\sqrt{\alpha' p^+}}(\psi^1_+\gamma^{i'}\epsilon^1_+ + \psi^2_-\gamma^{i'}\epsilon^2_-) ,\\
\delta\psi^1_+ &= \frac{1}{\sqrt{\alpha' p^+}}\left(\partial_- X^{i'}\gamma^{i'}\epsilon^1_+ + \frac{m}{6}X^{i'}\gamma^4\gamma^{i'}\epsilon^2_-\right) ,\\
\delta\psi^2_- &= \frac{1}{\sqrt{\alpha' p^+}}\left(\partial_+ X^{i'}\gamma^{i'}\epsilon^2_- - \frac{m}{6}X^{i'}\gamma^4\gamma^{i'}\epsilon^1_+\right) ,
\end{aligned}
\tag{10}
$$

for the supermultiplet $(X^{i'}, \psi^1_+, \psi^2_-)$. As chirality structure of parameter for dynamical supersymmetry, we note that $\gamma^{12349}\epsilon^1_+ = -\epsilon^1_+$ and $\gamma^{12349}\epsilon^2_- = -\epsilon^2_-$.

Under the kinematical supersymmetry, the bosons do not transform:

$$
\tilde{\delta} X^I = 0 ,
\tag{11}
$$

while the fermions just shift as

$$
\begin{aligned}
\tilde{\delta}\psi^1_- &= \sqrt{2\alpha' p^+}\left(\cos\left(\frac{m}{3}\tau\right)\tilde{\epsilon}^1_- + \sin\left(\frac{m}{3}\tau\right)\gamma^{123}\tilde{\epsilon}^2_+\right) ,\\
\tilde{\delta}\psi^2_+ &= \sqrt{2\alpha' p^+}\left(\cos\left(\frac{m}{3}\tau\right)\tilde{\epsilon}^2_+ + \sin\left(\frac{m}{3}\tau\right)\gamma^{123}\tilde{\epsilon}^1_-\right) ,\\
\tilde{\delta}\psi^1_+ &= \sqrt{2\alpha' p^+}\left(\cos\left(\frac{m}{6}\tau\right)\tilde{\epsilon}^1_+ + \sin\left(\frac{m}{6}\tau\right)\gamma^{123}\tilde{\epsilon}^2_-\right) ,\\
\tilde{\delta}\psi^2_- &= \sqrt{2\alpha' p^+}\left(\cos\left(\frac{m}{6}\tau\right)\tilde{\epsilon}^2_- + \sin\left(\frac{m}{6}\tau\right)\gamma^{123}\tilde{\epsilon}^1_+\right) .
\end{aligned}
\tag{12}
$$

In this paper we study various aspects of this light-cone gauge fixed IIA superstring theory on pp-wave geometry. In section 2, we describe the quantization of closed string theory: the quantization of string coordinates and of the light-cone Hamiltonian. In section 3, we study the worldvolume supersymmetry algebra. It is shown that unlike the maximally supersymmetric case, typically of [30], the supersymmetry algebra has slightly abnormal structure, the reason for which is understood by contemplating the various chiralities of supercharges and fermionic coordinates. In section 4, we consider open superstring theory describing D-branes. Firstly, we give the mode expansions of open strings taking into account various boundary conditions. Then we find the half-BPS Dp-branes of p=2, 4, 6, 8. They should be at the origin in the transverse directions to preserve half supersymmetries.[3] The directions they can be stretched are also restricted. Among these, we identify one

[3] Similar solutions in IIB string theory on maximally supersymmetric pp-wave have been found in [33, 34, 35, 36]. See also [37, 38].

D4-brane configuration with the rotating longitudinal five brane solution in matrix model [8], whose worldvolume dynamics in low energy is governed by super Yang-Mills theory with Kahler-Chern-Simons and Myers terms [39]. Another D4-brane configuration should also have counterpart in matrix model. We give the corresponding longitudinal five brane configuration in matrix model. As will be clear, due to the nature of light-cone gauge fixing, $x^{\pm}$ satisfy the Neumann boundary condition and thus D0-brane can not be seen. In this sense as well as for the study of interaction, the study of covariant GS superstring theory on the pp-wave would be anticipated. We hope to return to this issue in the near future.

2 Quantization of Closed Strings

The Type IIA string action in the light-cone gauge, (8), is the theory of two $(4,4)$ supermultiplets. Since it is a free field theory, it can be solved exactly. In this section, we will perform quantization of the theory and obtain the light-cone Hamiltonian which gives the exact spectrum.

The quantization begins with mode expansion of fields in the theory, which is given in terms of solutions of the field equations. We first consider the bosonic sector of the theory. The equations of motion for the bosonic coordinates X^I are read off from the action (8) as

$$\begin{aligned} &\eta^{mn}\partial_m\partial_n X^i - \left(\frac{m}{3}\right)^2 X^i = 0 \ , \\ &\eta^{mn}\partial_m\partial_n X^{i'} - \left(\frac{m}{6}\right)^2 X^{i'} = 0 \ , \end{aligned} \tag{13}$$

where the fields are subject to the periodic boundary condition,

$$X^I(\tau,\sigma+2\pi) = X^I(\tau,\sigma) \ . \tag{14}$$

The solutions of the above equations are characterized by discrete momentum, say n, in the σ direction due to the periodic boundary condition and have the form of plane-wave. For X^i, we have two mode solutions for each n, which are

$$\phi_n(\tau,\sigma) = e^{-i\omega_n\tau - in\sigma} \ , \quad \tilde{\phi}_n(\tau,\sigma) = e^{-i\omega_n\tau + in\sigma} \ , \tag{15}$$

with the wave frequency

$$\omega_n = \mathrm{sign}(n)\sqrt{\left(\frac{m}{3}\right)^2 + n^2} \ . \tag{16}$$

We note that ϕ_n ($\tilde{\phi}_n$) becomes the left-moving (right-moving) wave on the string worldsheet in the massless case, that is, when we take the mass m to vanish. As for the coordinates

$X^{i'}$, we have the same mode solutions but with $m/6$ instead of $m/3$. In order to distinguish the modes of $X^{i'}$ from those of X^i, the primed quantities will be used;

$$\phi'_n(\tau,\sigma) = e^{-i\omega'_n\tau - in\sigma} \ , \quad \tilde{\phi}'_n(\tau,\sigma) = e^{-i\omega'_n\tau + in\sigma} \ , \quad \omega'_n = \mathrm{sign}(n)\sqrt{\left(\frac{m}{6}\right)^2 + n^2} \ . \tag{17}$$

With the solutions of the equations of motion, Eq. (13), the mode expansions of the string coordinates X^I are then given by

$$\begin{aligned} X^i(\tau,\sigma) &= x^i \cos\left(\frac{m}{3}\tau\right) + \alpha' p^i \frac{3}{m} \sin\left(\frac{m}{3}\tau\right) + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{\omega_n} (\alpha^i_n \phi_n(\tau,\sigma) + \tilde{\alpha}^i_n \tilde{\phi}_n(\tau,\sigma)) \ , \\ X^{i'}(\tau,\sigma) &= x^{i'} \cos\left(\frac{m}{6}\tau\right) + \alpha' p^{i'} \frac{6}{m} \sin\left(\frac{m}{6}\tau\right) + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{\omega'_n} (\alpha^{i'}_n \phi'_n(\tau,\sigma) + \tilde{\alpha}^{i'}_n \tilde{\phi}'_n(\tau,\sigma)) \ , \end{aligned} \tag{18}$$

where x^I and p^I are center-of-mass variables defined in the usual manner, coefficients for the zero-modes, and α^I_n and $\tilde{\alpha}^I_n$ are the expansion coefficients for the non-zero modes. Here the reality of X^I requires that $\alpha^{I\dagger}_n = \alpha^I_{-n}$ and $\tilde{\alpha}^{I\dagger}_n = \tilde{\alpha}^I_{-n}$.

We now promote the expansion coefficients in the mode expansions (18) to operators and give the commutation relations between them leading to the quantization of bosonic fields. The canonical equal time commutation relations for the bosonic fields are

$$[X^I(\tau,\sigma), \mathcal{P}^J(\tau,\sigma')] = i\delta^{IJ}\delta(\sigma - \sigma') \ , \tag{19}$$

where $\mathcal{P}^J = \partial_\tau X^J / 2\pi\alpha'$ is the canonical conjugate momentum of X^J. By using these commutators, the mode expansions, (18), lead us to have the following commutation relations:

$$[x^I, p^J] = i\delta^{IJ} \ , \quad [\alpha^i_n, \alpha^j_m] = \omega_n \delta^{ij} \delta_{n+m,0} \ , \quad [\alpha^{i'}_n, \alpha^{j'}_m] = \omega'_n \delta^{i'j'} \delta_{n+m,0} \ . \tag{20}$$

Let us next turn to the fermionic sector of the theory. As introduced in the last section, the fermionic fields are split into two parts according to the $(4,4)$ supersymmetry; (ψ^1_-, ψ^2_+) and (ψ^1_+, ψ^2_-). We first consider the former case. The equations of motion for ψ^1_- and ψ^2_+ are obtained as

$$\begin{aligned} \partial_+ \psi^1_- + \frac{m}{3}\gamma^4 \psi^2_+ &= 0 \ , \\ \partial_- \psi^2_+ - \frac{m}{3}\gamma^4 \psi^1_- &= 0 \ . \end{aligned} \tag{21}$$

The non-zero mode solutions of these equations are given by using the modes, (15). For the zero mode part of the solution, we impose a condition that, at $\tau = 0$, the solution behaves just as that of massless case. The mode expansions for the fermionic coordinates are then

$$\begin{aligned}
\psi^1_-(\tau,\sigma) &= c_0\tilde{\psi}_0\cos\left(\frac{m}{3}\tau\right) - c_0\gamma^4\psi_0\sin\left(\frac{m}{3}\tau\right) \\
&\quad + \sum_{n\neq 0} c_n\left(\tilde{\psi}_n\tilde{\phi}_n(\tau,\sigma) - i\frac{3}{m}(\omega_n - n)\gamma^4\psi_n\phi_n(\tau,\sigma)\right) , \\
\psi^2_+(\tau,\sigma) &= c_0\psi_0\cos\left(\frac{m}{3}\tau\right) + c_0\gamma^4\tilde{\psi}_0\sin\left(\frac{m}{3}\tau\right) \\
&\quad + \sum_{n\neq 0} c_n\left(\psi_n\phi_n(\tau,\sigma) + i\frac{3}{m}(\omega_n - n)\gamma^4\tilde{\psi}_n\tilde{\phi}_n(\tau,\sigma)\right) ,
\end{aligned} \tag{22}$$

where $\gamma^{1234}\psi_n = \psi_n$ and $\gamma^{1234}\tilde{\psi}_n = -\tilde{\psi}_n$ for all n, and c_n are the normalization constants to be fixed in the process of quantization. As in the case of bosonic sector, the expansion coefficients ψ_n and $\tilde{\psi}_n$ are promoted to operators. By using the canonical equal time anti-commutation relations,

$$\{\psi^A_\pm(\tau,\sigma), \psi^B_\pm(\tau,\sigma')\} = 2\pi\alpha'\delta^{AB}\delta(\sigma-\sigma') , \tag{23}$$

and fixing the normalization constants as

$$c_0 = \sqrt{\alpha'} , \quad c_n = \frac{\sqrt{\alpha'}}{\sqrt{1+\left(\frac{3}{m}\right)^2(\omega_n - n)^2}} ,$$

the following anti-commutation relations between mode operators are obtained.

$$\{\psi_n, \psi_m\} = \delta_{n+m,0} , \quad \{\tilde{\psi}_n, \tilde{\psi}_m\} = \delta_{n+m,0} . \tag{24}$$

The quantization of fermionic coordinates (ψ^1_+, ψ^2_-) in the other $(4,4)$ supermultiplet proceeds along the same way with that of above case. The equations of motion for ψ^1_+ and ψ^2_- are respectively

$$\begin{aligned}
\partial_+\psi^1_+ - \frac{m}{6}\gamma^4\psi^2_- &= 0 , \\
\partial_-\psi^2_- + \frac{m}{6}\gamma^4\psi^1_+ &= 0 .
\end{aligned} \tag{25}$$

In the present case, the basic solutions are given in terms of the primed ones, (17), and the mode expansions for the fields are

$$\begin{aligned}
\psi^1_+(\tau,\sigma) &= c'_0\tilde{\psi}'_0\cos\left(\frac{m}{6}\tau\right) + c'_0\gamma^4\psi'_0\sin\left(\frac{m}{6}\tau\right) \\
&\quad + \sum_{n\neq 0} c'_n\left(\tilde{\psi}'_n\tilde{\phi}'_n(\tau,\sigma) + i\frac{6}{m}(\omega'_n - n)\gamma^4\psi'_n\phi'_n(\tau,\sigma)\right) , \\
\psi^2_-(\tau,\sigma) &= c'_0\psi'_0\cos\left(\frac{m}{6}\tau\right) - c'_0\gamma^4\tilde{\psi}'_0\sin\left(\frac{m}{6}\tau\right) \\
&\quad + \sum_{n\neq 0} c'_n\left(\psi'_n\phi'_n(\tau,\sigma) - i\frac{6}{m}(\omega'_n - n)\gamma^4\tilde{\psi}'_n\tilde{\phi}'_n(\tau,\sigma)\right) ,
\end{aligned} \tag{26}$$

where $\gamma^{1234}\psi'_n = -\psi'_n$ and $\gamma^{1234}\tilde{\psi}'_n = \tilde{\psi}'_n$. Then the equal time anti-commutation relations, (23), and the following fixing of the normalization constants

$$c'_0 = \sqrt{\alpha'} \ , \quad c'_n = \frac{\sqrt{\alpha'}}{\sqrt{1 + \left(\frac{6}{m}\right)^2 (\omega'_n - n)^2}} \ ,$$

lead us to the anti-commutators between mode operators.

$$\{\psi'_n, \psi'_m\} = \delta_{n+m,0} \ , \quad \{\tilde{\psi}'_n, \tilde{\psi}'_m\} = \delta_{n+m,0} \ . \tag{27}$$

Having the quantized $(4,4)$ supermultiplets which are free, we now consider the light-cone Hamiltonian of the theory which has the quadratic form in term of the string coordinates or mode operators and is thus exact. The light-cone Hamiltonian is written as[4]

$$H_{LC} = \frac{1}{2\pi} \int_0^{2\pi} d\sigma P^- = \frac{1}{p^+} \int_0^{2\pi} d\sigma \mathcal{H} \ , \tag{28}$$

where $\mathcal{H}$ is the Hamiltonian density of the light-cone gauge fixed string action, Eq. (8), and is obtained as

$$\begin{aligned} \mathcal{H} \ &= \ \frac{1}{2}(\mathcal{P}^I)^2 + \frac{1}{2}(\partial_\sigma X^I)^2 + \frac{1}{2}\left(\frac{m}{3}\right)^2 (X^i)^2 + \frac{1}{2}\left(\frac{m}{6}\right)^2 (X^{i'})^2 \\ &\quad -\frac{i}{2}\psi^1_- \partial_\sigma \psi^1_- + \frac{i}{2}\psi^2_+ \partial_\sigma \psi^2_+ + i\frac{m}{3}\psi^2_+ \gamma^4 \psi^1_- \\ &\quad -\frac{i}{2}\psi^1_+ \partial_\sigma \psi^1_+ + \frac{i}{2}\psi^2_- \partial_\sigma \psi^2_- - i\frac{m}{6}\psi^2_- \gamma^4 \psi^1_+ \ . \end{aligned} \tag{29}$$

By plugging the mode expansions for the fields, Eqs. (18), (22), and (26), into Eq. (28), we see that the light-cone Hamiltonian becomes

$$H_{LC} = E_0 + E + \tilde{E} \ , \tag{30}$$

where E_0 is the zero mode contribution and E ($\tilde{E}$) the contribution of the non-zero modes of the type α^I_n, ψ_n, and ψ'_n ($\tilde{\alpha}^I_n$, $\tilde{\psi}_n$, $\tilde{\psi}'_n$). Each of the contributions is expressed as follows:

$$\begin{aligned} E_0 \ &= \ \frac{\pi}{p^+}\left(\left(\frac{p^I}{2\pi}\right)^2 + \left(\frac{m}{3}\right)^2 (x^i)^2 + \left(\frac{m}{6}\right)^2 (x^{i'})^2 - \frac{i}{\pi}\frac{m}{3}\tilde{\psi}_0 \gamma^4 \psi_0 + \frac{i}{\pi}\frac{m}{6}\tilde{\psi}'_0 \gamma^4 \psi'_0\right) \ , \\ E \ &= \ \frac{1}{2p^+}\sum_{n\neq 0}(\alpha^I_{-n}\alpha^I_n + \omega_n \psi_{-n}\psi_n + \omega'_n \psi'_{-n}\psi'_n) \ , \\ \tilde{E} \ &= \ \frac{1}{2p^+}\sum_{n\neq 0}(\tilde{\alpha}^I_{-n}\tilde{\alpha}^I_n + \omega_n \tilde{\psi}_{-n}\tilde{\psi}_n + \omega'_n \tilde{\psi}'_{-n}\tilde{\psi}'_n) \ . \end{aligned} \tag{31}$$

In the quantized version, the modes in the expression of Hamiltonian become operators with the commutation relations, (20), (24), and (27), and should be properly normal ordered.

[4]From now on, we set $2\pi\alpha' = 1$ for notational convenience.

For the non-zero mode or the string oscillator contributions, that is, E and $\tilde{E}$, we place operator with negative mode number to the left of operator with positive mode number as in the flat case. This is natural since E and $\tilde{E}$ become those of the string in the flat background when the mass m is taken to vanish. The normal ordered expressions of them are then given by

$$\begin{aligned} E &= \frac{1}{p^+}\sum_{n=1}^{\infty}(\alpha^I_{-n}\alpha^I_n + \omega_n\psi_{-n}\psi_n + \omega'_n\psi'_{-n}\psi'_n) \ , \\ \tilde{E} &= \frac{1}{p^+}\sum_{n=1}^{\infty}(\tilde{\alpha}^I_{-n}\tilde{\alpha}^I_n + \omega_n\tilde{\psi}_{-n}\tilde{\psi}_n + \omega'_n\tilde{\psi}'_{-n}\tilde{\psi}'_n) \ . \end{aligned} \tag{32}$$

Here we note that there is no zero-point energy because bosonic contributions are exactly canceled by those of fermions.

The zero mode contribution is the Hamiltonian for the simple harmonic oscillators and massive fermions. For the bosonic part, we introduce the usual creation and annihilation operators as

$$\begin{aligned} a^{i\dagger} &= \sqrt{\frac{3\pi}{m}}\left(\frac{p^i}{2\pi} + i\frac{m}{3}x^i\right) \ , \quad a^i = \sqrt{\frac{3\pi}{m}}\left(\frac{p^i}{2\pi} - i\frac{m}{3}x^i\right) \ , \\ a^{i'\dagger} &= \sqrt{\frac{6\pi}{m}}\left(\frac{p^{i'}}{2\pi} + i\frac{m}{6}x^{i'}\right) \ , \quad a^{i'} = \sqrt{\frac{6\pi}{m}}\left(\frac{p^{i'}}{2\pi} - i\frac{m}{6}x^{i'}\right) \ , \end{aligned} \tag{33}$$

whose commutation relations are read as, from Eq. (20),

$$[a^I, a^{J\dagger}] = \delta^{IJ} \ . \tag{34}$$

As for the fermionic creation and annihilation operators, we take the following combination of modes, the reason for which will be explained below.

$$\begin{aligned} \chi^\dagger &= \frac{1}{\sqrt{2}}(\psi_0 - i\gamma^4\tilde{\psi}_0) \ , \quad \chi = \frac{1}{\sqrt{2}}(\psi_0 + i\gamma^4\tilde{\psi}_0) \ , \\ \chi'^\dagger &= \frac{1}{\sqrt{2}}(\psi'_0 + i\gamma^4\tilde{\psi}'_0) \ , \quad \chi' = \frac{1}{\sqrt{2}}(\psi'_0 - i\gamma^4\tilde{\psi}'_0) \ , \end{aligned} \tag{35}$$

where $\gamma^{12349}\chi = -\chi$ and $\gamma^{12349}\chi' = \chi'$. From Eqs. (24) and (27), the anti-commutation relations between these operators become

$$\{\chi, \chi^\dagger\} = 1 \ , \quad \{\chi', \chi'^\dagger\} = 1 \ , \tag{36}$$

where, since each of the fermionic coordinates has four independent components although we have used the 16 component notation, the right hand sides should be understood as 4×4 unit matrix. In terms of the operators introduced above, Eqs. (33) and (35), the normal ordered zero mode contribution to the light-cone Hamiltonian is then given by

$$E_0 = \frac{m}{6p^+}(2a^{i\dagger}a^i + a^{i'\dagger}a^{i'} + 2\chi^\dagger\chi + \chi'^\dagger\chi') \ . \tag{37}$$

Here we see that E_0 has vanishing zero-point energy as in the case of string oscillator contributions. The zero-point energy of the bosonic part is evaluated as

$$\frac{1}{2}\left(4\times\frac{m}{3}+4\times\frac{m}{6}\right)=\frac{m}{6}\times 6\ ,$$

which is exactly canceled by that from the fermionic part.

The normal ordered expressions Eqs. (32) and (37) now constitute the quantum light-cone Hamiltonian, which implicitly defines the vacuum $|0\rangle$ of the quantized theory as a state annihilated by string oscillation operators with positive mode number, that is $n \geq 1$, and zero mode operators a^I, χ, and χ' defined in Eqs. (33) and (35). Actually, the vacuum defined in this paper, especially the vacuum state in the zero mode sector, is not unique but one of the possible Clifford vacua, since our theory is massive and there can be various definitions for the creation and annihilation operators. This is also the case for the IIB superstring in pp-wave background and has been discussed in [40]. However, considering the regularity of states at $\tau \to i\infty$ that has been pointed out in [41], our definition is a natural one. This is because the expansion coefficients corresponding to mode solutions diverging at $\tau \to i\infty$ in Eqs. (18), (22), and (26) must annihilate the vacuum in order to ensure the regularity of physical states constructed out from the vacuum at such Euclidean time region.

The string states are then obtained by acting creation operators on the vacuum $|0\rangle$. However, the physical states are not all possible such states but those in the subspace of states which are constrained by the Virasoro constraint imposing the invariance under the translation in σ direction. In the light-cone gauge, the Virasoro constraint is given by

$$\int_0^{2\pi} d\sigma\left(-\frac{1}{2\pi}p^+\partial_\sigma X^- +\mathcal{P}^I\partial_\sigma X^I+\frac{i}{2}\psi_+^A\partial_\sigma\psi_+^A+\frac{i}{2}\psi_-^A\partial_\sigma\psi_-^A\right)=0\ . \tag{38}$$

The integration of the first integrand vanishes trivially since p^+ is constant, and the remaining parts give us the following constraint.

$$N=\tilde{N}\ , \tag{39}$$

where N and $\tilde{N}$ are defined as

$$\begin{aligned} N &= \sum_{n=1}^{\infty} n\left(\frac{1}{\omega_n}\alpha_{-n}^i\alpha_n^i+\frac{1}{\omega_n'}\alpha_{-n}^{i'}\alpha_n^{i'}+\psi_{-n}\psi_n+\psi_{-n}'\psi_n'\right)\ , \\ \tilde{N} &= \sum_{n=1}^{\infty} n\left(\frac{1}{\omega_n}\tilde{\alpha}_{-n}^i\tilde{\alpha}_n^i+\frac{1}{\omega_n'}\tilde{\alpha}_{-n}^{i'}\tilde{\alpha}_n^{i'}+\tilde{\psi}_{-n}\tilde{\psi}_n+\tilde{\psi}_{-n}'\tilde{\psi}_n'\right)\ . \end{aligned} \tag{40}$$

Here we normal ordered the expressions and see that the normal ordering constants have canceled between bosonic and fermionic contributions.

3 Supersymmetry Algebra

The light-cone gauge fixed action (8) is invariant under the dynamical and the kinematical supersymmetry transformation. The dynamical (kinematical) supersymmetry parameter has eight (sixteen) independent components hence leading to eight (sixteen) supersymmetries. In this section, we obtain the supercharges corresponding to these supersymmetries and compute the supersymmetry algebra between them.

The charges for the supersymmetry transformations, (9), (10), (11) and (12), are derived through the standard Noether procedure. For the kinematical supersymmetry, we have four types of supercharge, $\widetilde{Q}^A_\pm$, corresponding to the transformation parameters, $\tilde{\epsilon}^A_\pm$, which are derived as

$$\begin{aligned}
\widetilde{Q}^1_- &= \sqrt{\frac{p^+}{\pi}}\int_0^{2\pi} d\sigma \left(\cos\left(\frac{m}{3}\tau\right)\psi^1_- - \sin\left(\frac{m}{3}\tau\right)\gamma^{123}\psi^2_+\right) , \\
\widetilde{Q}^2_+ &= \sqrt{\frac{p^+}{\pi}}\int_0^{2\pi} d\sigma \left(\cos\left(\frac{m}{3}\tau\right)\psi^2_+ - \sin\left(\frac{m}{3}\tau\right)\gamma^{123}\psi^1_-\right) , \\
\widetilde{Q}^1_+ &= \sqrt{\frac{p^+}{\pi}}\int_0^{2\pi} d\sigma \left(\cos\left(\frac{m}{6}\tau\right)\psi^1_+ - \sin\left(\frac{m}{6}\tau\right)\gamma^{123}\psi^2_-\right) , \\
\widetilde{Q}^2_- &= \sqrt{\frac{p^+}{\pi}}\int_0^{2\pi} d\sigma \left(\cos\left(\frac{m}{6}\tau\right)\psi^2_- - \sin\left(\frac{m}{6}\tau\right)\gamma^{123}\psi^1_+\right) .
\end{aligned} \tag{41}$$

It is now convenient to introduce combinations of these supercharges as

$$\widetilde{Q}^\pm = \widetilde{Q}^1_\pm + \widetilde{Q}^2_\mp \tag{42}$$

such that the eigenvalue of γ^{12349} or of γ^{5678} becomes manifest. The kinematical supersymmetry transformations rules of string coordinates, (11) and (12), are then obtained from

$$\tilde{\delta}\varphi = [\tilde{\epsilon}^+\widetilde{Q}^+ + \tilde{\epsilon}^-\widetilde{Q}^-, \varphi] , \tag{43}$$

where $\tilde{\epsilon}^\pm = \tilde{\epsilon}^1_\pm + \tilde{\epsilon}^2_\mp$. It is easily seen that one $(4,4)$ supermultiplet $(X^i, \psi^1_-, \psi^2_+)$ is only affected by $\widetilde{Q}^-$ and the other supermultiplet $(X^{i'}, \psi^1_+, \psi^2_-)$ only by $\widetilde{Q}^+$.

As for the dynamical supersymmetry transformation with parameters, ϵ^1_+ and ϵ^2_-, we have obtained the following two types of supercharge.

$$\begin{aligned}
Q^1_+ &= \sqrt{\frac{2\pi}{p^+}}\int_0^{2\pi} d\sigma \left(\partial_- X^i\gamma^i\psi^1_- + \partial_- X^{i'}\gamma^{i'}\psi^1_+ + \frac{m}{3}X^i\gamma^i\gamma^4\psi^2_+ - \frac{m}{6}X^{i'}\gamma^{i'}\gamma^4\psi^2_-\right) , \\
Q^2_- &= \sqrt{\frac{2\pi}{p^+}}\int_0^{2\pi} d\sigma \left(\partial_+ X^i\gamma^i\psi^2_+ + \partial_+ X^{i'}\gamma^{i'}\psi^2_- - \frac{m}{3}X^i\gamma^i\gamma^4\psi^1_- + \frac{m}{6}X^{i'}\gamma^{i'}\gamma^4\psi^1_+\right) .
\end{aligned} \tag{44}$$

Since two supercharges have different transverse $SO(8)$ chirality, that is, Q^1_+ is in $\mathbf{8_c}$ of $SO(8)$ and Q^2_- in $\mathbf{8_s}$, and each of them has four independent spinor components, they naturally represent the dynamical $(4,4)$ supersymmetry as it should be. If we now combine them so as to make the eigenvalue of γ^{12349} manifest as done in the kinematical case

$$Q^- = Q^1_+ + Q^2_- \ , \tag{45}$$

we can check that the dynamical supersymmetry transformation rules, (9) and (10), follow through the relation

$$\delta\varphi = [\epsilon^- Q^-, \varphi] \ , \tag{46}$$

where $\epsilon^- = \epsilon^1_+ + \epsilon^2_-$.

Having derived the supercharges, (41) and (44), let us turn to the computation of the supersymmetry algebra between them. The basic rules for it are the commutation relations of Eqs. (19) and (23), and the $SO(8)$ Fierz identity being obtained from the usual $SO(9)$ identity.

The first algebra we consider is that between kinematical supercharges, which is simply computed as

$$\{\widetilde{Q}^\pm_\alpha, \widetilde{Q}^\pm_\beta\} = 2p^+ (h_\pm)_{\alpha\beta} \ , \tag{47}$$

where α, β are spinor indices and $h_\pm$ are projection operators onto spinor states of positive and negative eigenvalue of γ^{12349} respectively,

$$h_\pm = \frac{1}{2}(1 \pm \gamma^{12349}) \ . \tag{48}$$

For the supersymmetry algebra between kinematical and dynamical supercharges, we get

$$\begin{aligned} \{\widetilde{Q}^+_\alpha, Q^-_\beta\} &= \frac{1}{\sqrt{2}}(h_+\gamma^{i'})_{\alpha\beta} P^{i'} - \frac{1}{\sqrt{2}}\frac{\mu}{6}(h_+\gamma^{123}\gamma^{i'})_{\alpha\beta} J^{+i'} \ , \\ \{\widetilde{Q}^-_\alpha, Q^-_\beta\} &= \frac{1}{\sqrt{2}}(h_-\gamma^{i})_{\alpha\beta} P^{i} - \frac{1}{\sqrt{2}}\frac{\mu}{3}(h_-\gamma^{123}\gamma^{i})_{\alpha\beta} J^{+i} \ , \end{aligned} \tag{49}$$

where we have defined the following quantities.

$$\begin{aligned} P^i &= \int d\sigma \left(\cos\left(\frac{m}{3}\tau\right) \mathcal{P}^i + \frac{m}{3}\sin\left(\frac{m}{3}\tau\right) X^i \right) \ , \\ P^{i'} &= \int d\sigma \left(\cos\left(\frac{m}{6}\tau\right) \mathcal{P}^{i'} + \frac{m}{6}\sin\left(\frac{m}{6}\tau\right) X^{i'} \right) \ , \\ J^{+i} &= \int d\sigma \left(\frac{3}{\mu}\sin\left(\frac{m}{3}\tau\right) \mathcal{P}^i - \frac{1}{2\pi}\cos\left(\frac{m}{3}\tau\right) p^+ X^i \right) \ , \\ J^{+i'} &= \int d\sigma \left(\frac{6}{\mu}\sin\left(\frac{m}{6}\tau\right) \mathcal{P}^{i'} - \frac{1}{2\pi}\cos\left(\frac{m}{6}\tau\right) p^+ X^{i'} \right) \ . \end{aligned} \tag{50}$$

P^I and J^{+I} are the nothing but the kinematical generators for the translation in the transverse space and the rotation in the (x^-, x^I) plane respectively, which have been extensively discussed in [30].

Finally, the dynamical supersymmetry algebra is computed as

$$\{Q^-_\alpha, Q^-_\beta\} = 4\pi(h_-)_{\alpha\beta}H_{LC} - \frac{\mu}{3}(h_-\gamma^{ij}\gamma^{123})_{\alpha\beta}J^{ij} + \frac{\mu}{6}(h_-\gamma^{i'j'}\gamma^{123})_{\alpha\beta}\widehat{J}^{i'j'} \ , \tag{51}$$

where J^{ij} is the rotation generator in three dimensional space spanned by $x^{1,2,3}$ (and thus the indices i, j take values only among $1, 2, 3$), and, interestingly enough, $\widehat{J}^{i'j'}$ is one part of the rotation generator $J^{i'j'}$ in the space spanned by $x^{5,6,7,8}$. The structure of this algebra reflects the fact that the symmetry group $SO(8)$ of transverse space is broken to $SO(3) \times SO(4)$ due to the presence of the Ramond-Ramond four form and two form field strengths, F_{+123} and F_{+4}. The explicit expressions of J^{ij} for $SO(3)$ and $\widehat{J}^{i'j'}$ for $SO(4)$ are given by

$$\begin{aligned} J^{ij} &= \int d\sigma \left(X^i\mathcal{P}^j - X^j\mathcal{P}^i - \frac{i}{4}\psi^1_-\gamma^{ij}\psi^1_- - \frac{i}{4}\psi^2_+\gamma^{ij}\psi^2_+ - \frac{i}{4}\psi^1_+\gamma^{ij}\psi^1_+ - \frac{i}{4}\psi^2_-\gamma^{ij}\psi^2_- \right) \ , \\ \widehat{J}^{i'j'} &= \int d\sigma \left(X^{i'}\mathcal{P}^{j'} - X^{j'}\mathcal{P}^{i'} - \frac{i}{4}\psi^1_-\gamma^{i'j'}\psi^1_- - \frac{i}{4}\psi^2_+\gamma^{i'j'}\psi^2_+ \right) \ . \end{aligned} \tag{52}$$

In order to see what makes special thing in $x^{5,6,7,8}$ directions, let us first consider the property of $\widehat{J}^{i'j'}$. In the above expression for $\widehat{J}^{i'j'}$, there is no dependence on the fermionic coordinates ψ^1_+ and ψ^2_-, which states that those coordinates do not rotate under the action of $\widehat{J}^{i'j'}$. The full form of the rotation generator $J^{i'j'}$ which rotates ψ^1_+ and ψ^2_- as well as other string coordinates is simply given by

$$J^{i'j'} = \widehat{J}^{i'j'} - \frac{i}{4}\int d\sigma(\psi^1_+\gamma^{i'j'}\psi^1_+ + \psi^2_-\gamma^{i'j'}\psi^2_-) \ . \tag{53}$$

Since the theory we study is non-interacting, $\widehat{J}^{i'j'}$ is really a rotation generator but without affecting ψ^1_+ and ψ^2_-. The reason for why only one sub-generator rather than full of the rotation generator $J^{i'j'}$ appears in the algebra (51) may be understandable by looking at the chirality structure of the dynamical supercharges and the fermionic coordinates, which is listed in table 1. The right hand side of (51) is basically the sum of field bilinears and should respect the chirality structure of $\{Q^-, Q^-\}$. The $SO(4)$ rotation is sensitive to the chirality of fermionic coordinates measured by γ^{5678}. From the table 1, one may recognize that bilinears made of ψ^1_+ or ψ^2_- cannot contribute to the structure associated with the $SO(4)$ rotation. On the other hand, for the $SO(3)$ rotation where eigenvalues of γ^{1234} enter the story, they can contribute. This at first glance abnormal situation is basically because the supersymmetry is not maximal but $(4, 4)$. If the theory had maximal dynamical supersymmetry, we would obtain the algebra containing full of the rotation generators as was the case of IIB superstring in pp-wave background [30].

4 Supersymmetric D-branes

In the light-cone framework, we now investigate what kinds of D-brane are possible in the theory and how amount of supersymmetry is preserved by studying the open superstring

	γ^9	γ^{1234}	γ^{5678}
Q^1_+	$-$	$+$	$-$
Q^2_-	$+$	$-$	$-$
ψ^1_-	$+$	$-$	$-$
ψ^2_+	$-$	$+$	$-$
ψ^1_+	$+$	$+$	$+$
ψ^2_-	$-$	$-$	$+$

Table 1: Eigenvalues or chiralities of dynamical supercharges and fermionic coordinates for γ^9, γ^{1234} and γ^{5678}.

ending on D-branes. Considering the tensor rank of Ramond-Ramond gauge field in the IIA string theory, one expects Dp-branes with even p ranging from 0 to 8. As is well known, however, the light-cone formulation does not give definite decision on the presence of D0-brane, since X^- always satisfies the Neumann boundary condition due to the Virasoro constraint, Eq. (38). Thus we are led to concentrate on the case of $p \geq 2$.

4.1 Quantization of Open Strings and D-branes

The equations of motion for the open superstring are the same with those of the closed string, that is, Eqs. (13), (21) and (25). In addition to them, the open string should satisfies appropriate open string boundary conditions. We let the end points of open string are at $\sigma = 0$ and $\sigma = \pi$. Since we will consider single brane in this paper, two end points should satisfy the same boundary condition. Of course, an open string may have different boundary conditions at two ends for the case of multiple D-branes.

We impose the Neumann boundary conditions on the longitudinal directions of D-brane and the Dirichlet boundary conditions on the transverse directions which are respectively

$$\partial_\sigma X^{I_N}|_{\sigma=0,\pi} = 0 \ , \quad \partial_\tau X^{I_D}|_{\sigma=0,\pi} = 0 \ , \tag{54}$$

where the index I_N (I_D) represents the Neumann (Dirichlet) direction. The mode expansions may be obtained from that of closed string, (18) by imposing these boundary conditions, which are given by

$$\begin{aligned}
X^{i_N} &= x^{i_N} \cos\left(\frac{m}{3}\tau\right) + 2\alpha' p^{i_N} \frac{3}{m} \sin\left(\frac{m}{3}\tau\right) + \sqrt{2\alpha'} i \sum_{n\neq 0} \frac{1}{\omega_n} \alpha_n^{i_N} e^{-i\omega_n \tau} \cos(n\sigma) \ , \\
X^{i'_N} &= x^{i'_N} \cos\left(\frac{m}{6}\tau\right) + 2\alpha' p^{i'_N} \frac{6}{m} \sin\left(\frac{m}{6}\tau\right) + \sqrt{2\alpha'} i \sum_{n\neq 0} \frac{1}{\omega'_n} \alpha_n^{i'_N} e^{-i\omega'_n \tau} \cos(n\sigma) \ , \\
X^{i_D} &= \sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{\omega_n} \alpha_n^{i_D} e^{-i\omega_n \tau} \sin(n\sigma) \ ,
\end{aligned}$$

$$X^{i'_D} \;=\; \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{\omega'_n}\alpha_n^{i'_D}e^{-i\omega'_n\tau}\sin(n\sigma)\ , \tag{55}$$

where $n\in\mathbf{Z}$, i_N (i_D) is the index for Neumann (Dirichlet) direction among $1,2,3,4$ and i'_N (i'_D) among $5,6,7,8$. Actually, we have obtained the mode expansions also for the case where different boundary conditions are imposed at two end points from the equations of motion (13) and the boundary conditions (54). For the Neumann boundary condition at $\sigma=0$ and the Dirichlet one at $\sigma=\pi$, they are

$$X^i=\sqrt{2\alpha'}i\sum_r\frac{1}{\omega_r}\alpha_r^ie^{-i\omega_r\tau}\cos(r\sigma)\ ,\quad X^{i'}=\sqrt{2\alpha'}i\sum_r\frac{1}{\omega'_r}\alpha_r^{i'}e^{-i\omega'_r\tau}\cos(r\sigma)\ , \tag{56}$$

and, for the case where the boundary conditions are imposed inversely, we get

$$X^i=\sqrt{2\alpha'}\sum_r\frac{1}{\omega_r}\alpha_r^ie^{-i\omega_r\tau}\sin(r\sigma)\ ,\quad X^{i'}=\sqrt{2\alpha'}\sum_r\frac{1}{\omega'_r}\alpha_r^{i'}e^{-i\omega'_r\tau}\sin(r\sigma)\ , \tag{57}$$

where $r\in\mathbf{Z}+\frac{1}{2}$. Now it is clear from the above mode expansions that Dirichlet brane is centered at the origin of the pp-wave background. We note that the same observation was given also for the IIB case in the light-cone framework [33].

For the fermionic coordinates, the boundary condition is given by

$$\psi^1_\pm\Big|_{\sigma=0,\pi}=\Omega\psi^2_\mp\Big|_{\sigma=0,\pi}\ , \tag{58}$$

where, as noted in [33], Ω is the product of gamma matrices with indices of Dirichlet directions. From the mode expansions, (22) and (26), we see that the fermionic boundary condition gives the following relations between expansion coefficients that lead us to the mode expansion for open string fermionic coordinates: for all n,

$$\tilde{\psi}_n=\Omega\psi_n\ ,\quad \tilde{\psi}'_n=\Omega\psi'_n\ . \tag{59}$$

Precise knowledge about Ω gives possible D-brane configurations in the IIA pp-wave background. If we evaluate the equations of motion for fermions, (21) and (25), at boundaries, we have the following condition for Ω:

$$\gamma^4\Omega\gamma^4\Omega=-1\ . \tag{60}$$

Since Ω relates fermionic coordinates with different $SO(8)$ chiralities and eigenvalues for γ^{1234}, it should anti-commute with γ^9 and γ^{1234} as follows.

$$\{\Omega,\gamma^9\}=0\ ,\quad \{\Omega,\gamma^{1234}\}=0\ . \tag{61}$$

Now Eqs. (60) and (61) let us know the structure of Ω: First of all, Ω should be a product of odd number of gamma matrices. Secondly, among the gamma matrices, even number of

N_D	Ω
1	γ^i
3	$\gamma^{ij}\gamma^4,\ \gamma^{i'j'}\gamma^4$
5	$\gamma^{123}\gamma^{i'j'},\ \gamma^i\gamma^{5678}$
7	$\gamma^{ij}\gamma^{45678}$

Table 2: List of Ω satisfying the condition, (60). N_D is the number of Dirichlet directions and we temporarily restrict the indices i, j to take values in $1, 2, 3$.

them should be indexed in $1, 2, 3, 4$ and hence even number of them in $5, 6, 7, 8$. Finally, since the gamma matrices are symmetric, we have $\Omega^T\Omega = 1$. With these informations, one can solve Eq. (60) rather easily and know about possible single D-brane configurations, which is listed in table 2.

We now describe the light-cone Hamiltonian and its quantization for the open superstring describing single D-brane. Since the story will be almost the same with that for the closed string case, the description will be rather brief. The light-cone Hamiltonian for the open string is given by

$$H_{LC} = \frac{1}{\pi}\int_0^\pi d\sigma P^- = \frac{2}{p^+}\int_0^\pi d\sigma \mathcal{H} \ . \tag{62}$$

where $\mathcal{H}$ is given in Eq. (29). By plugging the mode expansions of Eq. (55) for bosonic string coordinates and those of Eqs. (22) and (26) with the condition (59) for fermionic coordinates into the Hamiltonian, we get

$$H_{LC} = E_0 + E \ , \tag{63}$$

where E_0 and E are the zero mode and non-zero mode contributions respectively. Firstly, the normal ordered expression for E is read as

$$E = \frac{2}{p^+}\sum_{n=1}^{\infty}(\alpha^I_{-n}\alpha^I_n + \omega_n\psi_{-n}\psi_n + \omega'_n\psi'_{-n}\psi'_n) \ . \tag{64}$$

The zero mode contribution is given by

$$E_0 = \frac{\pi}{p^+}\left(\left(\frac{p^{I_N}}{\pi}\right)^2 + \left(\frac{m}{3}\right)^2(x^{i_N})^2 + \left(\frac{m}{6}\right)^2(x^{i'_N})^2 + \frac{i}{\pi}\frac{m}{3}\psi_0\gamma^4\Omega\psi_0 - \frac{i}{\pi}\frac{m}{6}\psi'_0\gamma^4\Omega\psi'_0\right) \ . \tag{65}$$

In contrast to the closed string case, fermionic part has no ordering ambiguity upon quantization and thus we leave it as intact. For the bosonic part, we introduce the creation and the annihilation operators,

$$\begin{aligned} a^{i\dagger} &= \sqrt{\frac{3\pi}{2m}}\left(\frac{p^i}{\pi} + i\frac{m}{3}x^i\right) \ , \quad a^i = \sqrt{\frac{3\pi}{2m}}\left(\frac{p^i}{\pi} - i\frac{m}{3}x^i\right) \ , \\ a^{i'\dagger} &= \sqrt{\frac{6\pi}{2m}}\left(\frac{p^{i'}}{\pi} + i\frac{m}{6}x^{i'}\right) \ , \quad a^{i'} = \sqrt{\frac{6\pi}{m}}\left(\frac{p^{i'}}{\pi} - i\frac{m}{6}x^{i'}\right) \ , \end{aligned} \tag{66}$$

which give the usual commutation relations,

$$[a^I, a^{J\dagger}] = \delta^{IJ} \ . \tag{67}$$

Then the zero mode contribution is finally read as

$$E_0 = \frac{m}{3p^+}\left(2a^{i_N\dagger}a^{i_N} + a^{i'_N\dagger}a^{i'_N} + i\psi_0\gamma^4\Omega\psi_0 - \frac{i}{2}\psi'_0\gamma^4\Omega\psi'_0 + e_0\right) \ , \tag{68}$$

where e_0 is the zero-point energy only coming from the bosonic part

$$e_0 = n_N + \frac{1}{2}n'_N \ . \tag{69}$$

Here n_N (n'_N) is the number of Neumann directions in $1,2,3,4$ ($5,6,7,8$) and constrained to be $n_N + n'_N = p-1$ for Dp-brane.

4.2 Supersymmetry of D-branes

In this subsection we show that the D-branes found above are half-BPS states of IIA superstring theory. We would like to find the supersymmetries preserved by the corresponding boundary conditions. Firstly, we consider the boundary condition from the dynamical supersymmetry transformation for bosonic string coordinates. For the Dirichlet directions, we require that δX^{I_D} vanish at the open string boundaries, while for the Neumann directions, $\delta\partial_\sigma X^{I_N}$ vanish at the boundaries. Then using the boundary conditions for the fermionic coordinates, (58) and noting that

$$\partial_\sigma\psi^1_\pm\Big|_{\sigma=0,\pi} = -\Omega\partial_\sigma\psi^2_\mp\Big|_{\sigma=0,\pi} \ ,$$

which is the reflection of the fact that the Euler-Lagrange equations for ψ^1 and ψ^2 are related by $\sigma \leftrightarrow -\sigma$ [42], we see that the supersymmetry variations both for the Dirichlet and Neumann directions give rise to the same condition which is

$$\gamma^I(\Omega^T\epsilon^1_+ + \epsilon^2_-) = 0 \ . \tag{70}$$

Therefore four dynamical supersymmetries generated by the supersymmetry parameters with the relation

$$\epsilon^1_+ = -\Omega\epsilon^2_- \ , \tag{71}$$

are preserved in the existence of the D-brane configurations given in the previous subsection. From the fermionic boundary condition, (58), we have the equation relating the dynamical supersymmetry variations for the fermionic coordinates,

$$\delta\psi^1_\pm\Big|_{\sigma=0,\pi} = \Omega\delta\psi^2_\mp\Big|_{\sigma=0,\pi} \ . \tag{72}$$

If we insert the above relation (71) into this equation, we have after some manipulation

$$\gamma^4\Omega\gamma^4\Omega = -1 \ , \tag{73}$$

which is nothing but the condition (60) in the previous subsection that Ω should satisfy. Thus, we see that the condition for Ω is consistent with the dynamical supersymmetry.

For the kinematical supersymmetry transformations, from

$$\tilde{\delta}\psi^1_\pm\Big|_{\sigma=0,\pi} = \Omega\tilde{\delta}\psi^2_\mp\Big|_{\sigma=0,\pi} \ , \tag{74}$$

one can easily see that the kinematical supersymmetry parameters which satisfy

$$\tilde{\epsilon}^1_\pm = \Omega\tilde{\epsilon}^2_\mp \tag{75}$$

are compatible with the boundary conditions (74), and furthermore they give exactly the same condition for Ω, (60) or (73), and thus the existence of D-brane breaks half the kinematical supersymmetry. In total, D-brane listed in table 2 preserves half the supersymmetry of the pp-wave: eight kinematical and four dynamical.

4.3 Comparison of IIA and M Theory Branes

It is interesting to compare D-branes found here with flat BPS states found in the context of matrix model in eleven-dimensional pp-wave background [8]. First of all, we do not expect to get the corresponding objects of D6 and D8 branes in the matrix model set-up[43]. Furthermore, all the D-branes found here is stretched along x^--direction, as x^--coordinate should satisfy Neumann boundary condition. As a result, D0-branes can not be seen in this formalism. This may be regarded as the limitation of light-cone gauge fixed string theory. On the other hand, in the matrix model, only the membrane which is transverse to x^- was identified, which also may be regarded as the limitation of matrix model. Henceforth, the D2-brane configuration found here has no counterpart in the matrix model and vice versa.

On the other hand, the longitudinal M5-brane in the matrix model is stretched along x^--direction and would be identified with the D4-brane configuration found here. Here in IIA string theory side, we have two different configurations for flat D4-branes, depending on the direction they are stretched. The flat D4-branes stretched along four $x^{i'}$-coordinates can be identified with rotating longitudinal M5-brane found in [8]. Note that time-dependent rotation among coordinates with Neumann boundary conditions does not change the boundary conditions themselves.

Other D4-brane solution which is stretched along two x^i-coordinates and two $x^{i'}$-coordinates should also have counterpart in matrix model, though not yet described in the literature[5].

[5]This has been found independently by Jeong-Hyuck Park and Sangheon Yi.

The corresponding longitudinal five brane solutions in the matrix model is given as follows. Note that, in the matrix model, the relevant fermion transformation law is given by

$$\delta\theta = \left(\sum_{I=1}^{9} P^I \gamma^I + \frac{i}{2} \sum_{I=1}^{9} [X^I, X^J] \gamma^{IJ} + \frac{\mu}{3R} \sum_{i=1}^{3} X^i \Pi \gamma^i - \frac{\mu}{6R} \sum_{i'=4}^{9} X^{i'} \gamma^{i'} \Pi \right) \eta \ , \tag{76}$$

where P^I is the canonical conjugate momentum of bosonic coordinate X^I. Consider the configurations

$$\begin{aligned} X^i &= x^i \cos(\mu t/3) + x^j \sin(\mu t/3) \ , \quad X^j = -x^i \sin(\mu t/3) + x^j \cos(\mu t/3) \ , \\ X^{i'} &= x^{i'} \cos(\mu t/6) - x^{j'} \sin(\mu t/6) \ , \quad X^{j'} = x^{i'} \sin(\mu t/6) + x^{j'} \cos(\mu t/6) \ , \end{aligned} \tag{77}$$

while all other coordinates vanish. The rotating longitudinal five brane solution of second type, then, is given by time-independent x^i and $x^{i'}$ satisfying the condition:

$$[x^i, x^j] = \frac{1}{2} \epsilon_{iji'j'} [x^{i'}, x^{j'}] \ . \tag{78}$$

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