

# Charged Particles: A Builder's Guide<sup>1</sup>

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**Abstract:** It is sometimes claimed that one cannot describe charged particles in gauge theories. We identify the root of the problem and present an explicit construction of charged particles. This is shown to have good perturbative properties and, asymptotically before and after scattering, to recover particle modes.

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# Introduction

The title may need some explanation – after all it might seem obvious what a charged particle is. After all a free fermion field has a plane wave expansion in terms of particle creation and annihilation operators. But the fermion in QED is an interacting field and this means that it is *not* easy to describe charged particles, as the following quote from a classic paper [1] on the infra-red structure of QED makes clear: “... *the relativistic concept of a charged particle does not exist*”! The aim of this talk is to explain the problem that led to this statement (and similar ones) and to explicitly demonstrate how it can be solved.

## The Problem with Particles

Particle descriptions in field theory are based around the creation and annihilation operators in the plane wave expansion of free fields. In the standard LSZ approach to scattering, the assumption is made that the coupling constant may in some sense be ‘switched off’ at large times before and after scattering. This is fine for some theories and toy models but, unfortunately, not for our paradigm unbroken gauge theories QED and QCD. The observation of hadrons, rather than quarks or gluons, in detectors shows this for QCD. For QED the masslessness of the photon implies a long range interaction which only falls off as  $1/r$ . This is too slow to be neglected and generates the infra-red problem (see below).

Kulish and Faddeev [1] found the non-vanishing form of the asymptotic interaction Hamiltonian in QED<sup>7</sup>. They then used this to calculate the form of the Heisenberg fields at asymptotically large times,  $t$ . Since there is a residual interaction, the fields tended not to a plane wave but rather to

$$\psi^{\text{as}}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} D(p, t) \left\{ b(p, s) u^s(p) e^{-ip \cdot x} + d^\dagger(p, s) v^s(p) e^{ip \cdot x} \right\} \quad (1)$$

where  $D(p, t)$  is an  $(A_\mu)$  field dependent **distortion** factor. They then concluded that these distorted plane waves imply that it is impossible to describe charged particles in unbroken gauge theories.

This conclusion is, though, not forced upon us. Rather, since the coupling does *not* vanish asymptotically, the matter field in the Lagrangian is not gauge invariant,  $\psi \rightarrow \exp(-ie\theta)\psi$ , and may not be identified with a physical particle. Indeed it may be seen that the asymptotic form of the vector potential is such that the commutator of particle

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<sup>7</sup>This work has since been extended to theories with four point interactions [2].

creation and annihilation operators produces the correct electro-magnetic fields for a particle with the appropriate momentum. It is therefore necessary to find the (gauge invariant!) fields which do asymptotically tend to the particle creation/annihilation operators.

## Starting to Identify the Solution

Many years ago Dirac [3] proposed that the combination,  $e^{-ie\frac{\partial_i A_i}{\nabla^2}}\psi$ , should be used to describe charged particles like the electron. This was for two reasons: i) it is locally gauge invariant, and ii) it describes the matter field surrounded by a Coulombic field:

$$E_i(x_0, \mathbf{x})\psi_D(y)|0\rangle = -\frac{e}{4\pi}\frac{\mathbf{x}_i - \mathbf{y}_i}{|\mathbf{x} - \mathbf{y}|^3}\psi_D(y)|0\rangle. \quad (2)$$

The only ingredients needed to obtain this result are the fundamental equal-time commutator and the standard three dimensional Green's function for  $1/\nabla^2$ . We would say that this is a description of a matter field *dressed* by an electro-magnetic field. This idea has been taken up by various authors, see, e.g., [4–6] and references therein. Such gauge invariant fields clearly have a chance of describing physical particles, but how do we know which ones to use?

## A Systematic Approach

The extension to moving charges and, especially, QCD where our understanding is less firm, requires a more systematic approach to finding the correct *dressing*. Writing our dressed field as  $h^{-1}\psi$ , we have two requirements [7] on the dressing,  $h^{-1}$  describing a particle moving with four-velocity  $u$ :

<p>Local gauge invariance of <math>h^{-1}\psi</math>: which implies <math>h^{-1} \rightarrow h^{-1}U</math></p> <p>An additional <i>dressing equation</i>: <math>u \cdot \partial h^{-1} = -ieh^{-1}u \cdot A</math></p>
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The first requirement is a bare minimum – physical variables are gauge invariant – but the latter is new. It can be motivated by studying the form of the asymptotic interaction Hamiltonian in QED, and demanding that it should vanish at one point on the mass shell. Also, if our gauge invariant, dressed matter is to have a sharp momentum, we must demand  $u \cdot \partial(h^{-1}\Phi) = 0$ , and in the heavy effective theory the equation of motion

for the matter field has the form  $u \cdot D\Phi = 0$ . Combining these last two equations immediately gives the dressing equation.

We stress again that neither the dressing nor the matter field are physical on their own. *Only the combination  $h^{-1}\psi$  is locally gauge invariant and can be identified with a charged particle.*

In QED we can solve these requirements and so obtain a dressing with a rich structure. The detailed form of the dressing can be found elsewhere, but in the static limit it becomes:

$$h^{-1}(x) = \exp \left( ie \int_{-\infty}^{x^0} ds \frac{\partial^i F_{i0}(s, \mathbf{x})}{\nabla^2} \right) \exp \left( -ie \frac{\partial_i A_i}{\nabla^2} \right), \quad (3)$$

where we recognise the Dirac dressing and an additional, and separately gauge invariant structure. We call the Dirac term, and its generalisations to a moving charge, the minimal or *soft* dressing and, for reasons explained below, we call the additional structure the *phase* dressing. The minimal term is needed for gauge invariance and, as will be explained in D. McMullan's talk, the additional structure contains screening effects in QED.

## Perturbative Tests

The variables which we are proposing are necessarily non-local and non-covariant. It is thus natural to ask whether they can be used in practical work. The answer is yes, as we have shown in a detailed series of calculations [7,8]. For the purposes of showing the particle nature of our dressed fields the main point to note is that the on-shell Green's functions of these gauge invariant variables have a good pole structure. Recall that the usual on-shell Green's functions of QED, such as the fermion propagator, and the  $S$ -matrix elements have IR divergences which are of two forms: soft divergences and (imaginary) phase divergences. We have shown that this rich structure meets its match in the structure of the dressing: the soft dressing introduces new Feynman diagrams which contain and cancel the usual soft divergences, while the additional phase dressing removes the usual phase divergences. We thus end up with IR finite on-shell Green's functions. We stress that this cancellation only takes place if the velocity parameter in the dressing and the velocity of the point of the mass shell where the renormalisation takes place match. In general this requires different dressings for different legs. This good pole structure is of course a necessity if we want to be able to describe particles<sup>8</sup>.

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<sup>8</sup>It should also be noted that this good IR behaviour is not accompanied by poor UV properties: we have also seen that these fields can be multiplicatively renormalised and that their composite operator behaviour is good (they do not mix [8]).

## Charged Particles

We have seen that these variables have good perturbative properties. Now we want to return to the initial problem as raised by Kulish and Faddeev. This was, we recall, that the Lagrangian fermion does not asymptotically tend to a plane wave. Normally we would, e.g., define the large time limit

$$b(q, s, t) := \int d^3x \frac{1}{\sqrt{2E_q}} u^{\dagger s}(q) \psi(x) e^{iq \cdot x}, \quad (4)$$

to extract a particle annihilation operator. But as those authors showed, one so obtains at large times in QED up to order  $e$ :

$$b(q, s, t) = \left\{ 1 - e \int_{\text{soft}} \frac{d^3k}{(2\pi)^3 2\omega_k} \left( \frac{q \cdot a(k)}{q \cdot k} e^{-it \frac{k \cdot q}{E_q}} - \frac{q \cdot a^\dagger(k)}{q \cdot k} e^{it \frac{k \cdot q}{E_q}} \right) \right\} b(q, s), \quad (5)$$

which shows the distortion<sup>9</sup>. However, repeating this calculation with dressed matter yields a further structure from the soft dressing. Essentially the factors of  $\frac{q^\mu}{q \cdot k}$  dotting into the vector potential creation and annihilation operators in the above equation are replaced by

$$\frac{q^\mu}{q \cdot k} \rightarrow \frac{q^\mu}{q \cdot k} - \frac{V^\mu}{V \cdot k}, \quad (6)$$

where  $V^\mu = (\eta + v)^\mu (\eta - v) \cdot k - k^\mu$ , with  $u^\mu = \gamma(\eta + v)^\mu$  being the four velocity of the charged particle. It is now fairly easy to see [9, 10] that this implies that the distortion vanishes at the correct point on the mass shell. This means that *we have a particle interpretation!*

## Summary

Frivolously we may conclude that we are indeed entitled to call ourselves particle physicists. More seriously we note the following:

- The residual interaction means that the coupling in QED does not asymptotically vanish and so the matter field *on its own* is never physical.
- Trying to ignore this fact generates the IR problem.
- Taking this interaction seriously implies:

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<sup>9</sup>Actually this exponentiates and there is a further factor from the phase which we do not mention here

- we must construct *gauge invariant* (dressed) descriptions of charged particles!
- Gauge invariance is not enough: we require a *dressing equation* to describe charges with sharp momenta.
- The solutions of these two demands have gauge invariant, Green's functions which are IR finite on-shell.
- They have other good perturbative properties and we have seen that they have a particle description.

The extension of this programme to QCD is sketched in D. McMullan's contribution to these proceedings, where it is shown that the analogue of these variables give an insight into the physics of screening, anti-screening and indeed confinement.

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