Super Five Brane Hamiltonian and the Chiral Degrees of Freedom

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Abstract

We construct the Hamiltonian of the super five brane in terms of its physical degrees of freedom. It does not depend on the inverse of the induced metric. Consequently, some singular configurations are physically admissible, implying an interpretation of the theory as a multiparticle one. The symmetries of the theory are analyzed from the canonical point of view in terms of the first and second class constraints. In particular it is shown how the chiral sector may be canonically reduced to its physical degrees of freedom.

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1 Introduction

One of the hopeful models to understand the origin of superstring dualities is the conjectured M-theory in eleven dimensions, where there are only two extended objects allowed by supersymmetry: the super 2-brane (supermembrane) and the super 5-brane. The supermembrane theory has been widely studied during the last years, see for example: [?], [?], [?], [?], [?], [?], [?], [?], [?], [?], [?], [?]. Nevertheless, the analysis of the covariant and hamiltonian quantum dynamics of the super 5-brane is now in its dawn, [?], [?], [?] [?], [?], between others. In particular, we would like to understand the nature of the super five brane spectrum. In 1997, a manifestly covariant action for the super 5-brane was constructed in [?]. Independently, at the same time, a non manifestly covariant action was obtained by J. Schwarz et. al. [?]. The field equations were first obtained in [?] and analyzed in [?], [?]. More recently, we analyzed some dynamical aspects for the M5-brane 'bosonic sector' [?][?], it included a complete study of the canonical structure of the bosonic sector of the M5-brane starting from the PST action in the gauge where the scalar field is fixed as the world volume time. We found a quadratic dependence on the antisymmetric field for the canonical Hamiltonian. This formulation contains second class constraints that we removed preserving the locality of the field theory in order to construct a master action with first class constraints only. The algebra of the 6 dimensional diffeomorphisms generated by the first class constraints was explicitly obtained. We constructed the nilpotent BRST charge of the theory and its BRST invariant effective theory. Finally, we obtained its physical Hamiltonian and analyzed its stability properties.

In this work we extend the analysis to the super 5-brane theory. In particular we show that the canonical lagrangian of the super 5-brane may be formulated without the assumption of the existence of the inverse of the induced metric, which is a requirement of the original PST as well as the Schwarz et.al actions. Consequently the hamiltonian formulation of our theory admits as physical configurations ones where locally the determinant of the induced metric in zero. They allow to connect disjoint 5-branes by singular 1,2,3 and 4-branes without changing the energy of the original configurations . They lead then to the interpretation of super 5-brane as a multiparticle theory, in a similar way as the existence of the string-like spikes suggest the same interpretation for the supermembrane [?], [?]. The canonical study of the PST super 5-brane action, in the gauge in which the auxiliar scalar field is equal to the world volume time, shows a mixture of first and second class constraints which includes the

expected reparametrization and kappa symmetry generators and the second class fermionic constraint, besides the first and second class constraints associated to the antisymmetric field. The canonical Hamiltonian is quadratic in the antisymmetric gauge field. It is very interesting to observe that the mixture of first and second class constraints associated to the chiral field gauge symmetry may be decoupled from the rest of the constraints. This feature allow us to remove the second class constraints of this sector and construct a master canonical action. From this supersymmetric formulation, we can recover the bosonic master formulation found in [?]. It is important to comment that the Hamiltonian for the super 5-brane was first obtained in [?] . It apparently depends on the determinant of $(g + \widetilde{\mathcal{H}})$, which depends on the fourth power of $\widetilde{\mathcal{H}}$, however, a further combinations of the terms in their constraints yields exactly the same constraints we obtain in this paper. It was raised in [?] the problem of the presence of the second class constraints in the chiral gauge field sector. In this paper we explicitly revolves that problem with the construction of our master Hamiltonian. In the last section we find the light cone gauge Hamiltonian for the theory and analyze its stability properties. The canonical analysis of the theory requires the explicit form of all terms in the Lagrangian, therefore, the explicit form of the Wess-Zumino term of the PST [?] super five brane Lagrangian is obtained as a first step.

2 The Super 5-Brane Action

The super 5-brane PST Lagrangian is given by

$$L = L_1 + L_2 + L_3, \tag{1}$$

with:

$$
\begin{aligned}
L_1 &= 2\sqrt{-\det M_{MN}}\ d\sigma^0 \wedge \cdots \wedge d\sigma^5 \\
L_2 &= \frac{1}{2(\partial a)^2}\widetilde{\mathcal{H}}^{MN}\mathcal{H}_{MNP}G^{PL}\partial_L a\ d\sigma^0 \wedge \cdots \wedge d\sigma^5 \\
L_3 &= \Omega_6.
\end{aligned}
\tag{2}
$$

where,

$$M_{MN} = G_{MN} + i\frac{G_{MP}G_{NL}}{\sqrt{-G(\partial a)^2}}\tilde{\mathcal{H}}^{PL},$$

$$\widetilde{\mathcal{H}}^{PL} = \frac{1}{6}\epsilon^{PLQMNR}\mathcal{H}_{MNR}\partial_Q a.$$

$$\mathcal{H}_{MNP} = H_{MNP} - b_{MNP}$$

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are the supersymmetric extensions for the Born-Infeld type term and the antisymmetric field strength $H = dB$ respectively. In components, H and B may be written as:

$$H = \frac{1}{3!} H_{MNL} d\sigma^M \wedge d\sigma^N \wedge d\sigma^L$$
$$= \frac{1}{3!} (\partial_M B_{NL} + \partial_L B_{MN} + \partial_N B_{LM}) d\sigma^M \wedge d\sigma^N \wedge d\sigma^L$$
$$B = \frac{1}{2!} B_{MN} d\sigma^M \wedge d\sigma^N$$

and

$$b = \frac{1}{6} \bar{\theta} \Gamma_{ab} d\theta [dX^a dX^b + \Pi^a dX^b + \Pi^a \Pi^b].$$

where Π^a is the SUSY-invariant Cartan form which is given by:

$$\Pi^a = \Pi_M^a d\sigma^M = (\partial_M X^a + \bar{\theta} \Gamma^a \partial_M \theta) d\sigma^M$$

and the induced supermetric by:

$$G_{MN} = \Pi_M^a \Pi_N^b \eta_{ab}$$

$a, b = 0, \cdots, 10$ are the Minkowski space-time indices while $M, N = 0, \cdots, 5$ are the world volume indices. The Wess–Zumino term Ω_6 is determined by the closed seven-form $I_7 = d\Omega_6$, where

$$I_7 = -\frac{1}{2} \mathcal{H} \wedge d\bar{\theta} \Gamma_{ab} d\theta \wedge \Pi^a \wedge \Pi^b$$
$$+ \frac{1}{60} d\bar{\theta} \Gamma_{abcdef} d\theta \wedge \Pi^a \wedge \Pi^b \wedge \Pi^c \wedge \Pi^d \wedge \Pi^e \wedge \Pi^f.$$

I_7 and Ω_6 may be expressed without the use of the induced metric G_{MN} nor the scalar field a. Moreover, the explicit expression of Ω_6 was not needed in order to prove the global supersymmetry and the local κ-symmetry of the action. In our analysis however, which will be based in the construction of the physical hamiltonian of the super 5-brane, it is required

to have an explicit expression for the Ω_6 term. It is given, up to a closed six-form, by:

$$\Omega_6 = dB \wedge b - \frac{1}{60}\bar{\theta}\Gamma_{abcde}d\theta dX^a dX^b dX^c dX^d dX^e$$
$$- \frac{1}{24}d\bar{\theta}\Gamma_{abcde}\theta \wedge d\bar{\theta}\Gamma^e\theta \wedge dX^a dX^b dX^c dX^d$$
$$+ \frac{1}{12}\bar{\theta}\Gamma_{ab}d\theta d\bar{\theta}\Gamma_{cd}\theta \wedge d\bar{\theta}\Gamma^d\theta \wedge dX^a dX^b dX^c$$
$$- \frac{1}{18}d\bar{\theta}\Gamma_{abcde}\theta \wedge d\bar{\theta}\Gamma^e\theta \wedge d\bar{\theta}\Gamma^d\theta \wedge dX^a dX^b dX^c$$
$$- \frac{1}{24}\bar{\theta}\Gamma_{cd}d\theta \wedge dX^c dX^d \bar{\theta}\Gamma_{ab}d\theta \wedge \bar{\theta}\Gamma^a d\theta \wedge \bar{\theta}\Gamma^b d\theta$$
$$+ \frac{1}{24}d\bar{\theta}\Gamma_{abcde}\theta \wedge d\bar{\theta}\Gamma^e\theta \wedge d\bar{\theta}\Gamma^d\theta \wedge d\bar{\theta}\Gamma^c\theta \wedge dX^a dX^b$$
$$+ \frac{1}{60}d\bar{\theta}\Gamma^b\theta \ d\bar{\theta}\Gamma^a\theta \wedge d\bar{\theta}\Gamma_{ab}\theta \wedge d\bar{\theta}\Gamma_{cd}\theta \wedge d\bar{\theta}\Gamma^c\theta \wedge dX^d$$
$$+ \frac{1}{60}d\bar{\theta}\Gamma_{abcde}\theta \wedge dX^a \wedge \bar{\theta}\Gamma^b d\theta \wedge \bar{\theta}\Gamma^c d\theta \wedge \bar{\theta}\Gamma^d d\theta \wedge \bar{\theta}\Gamma^e d\theta$$
$$- \frac{1}{360}d\bar{\theta}\Gamma_{abcde}\theta \wedge d\bar{\theta}\Gamma^e\theta \wedge d\bar{\theta}\Gamma^d\theta \wedge d\bar{\theta}\Gamma^c\theta \wedge d\bar{\theta}\Gamma^b\theta \wedge d\bar{\theta}\Gamma^a\theta.$$

3 Canonical Analysis and the Master Supersymmetric Hamiltonian

In this section we consider the construction of the canonical formulation of the PST theory, in the gauge in which the auxiliar scalar field a is equal to the world volume time and the time components of the antisymmetric field $B_{0\mu}$ are zero ($\mu = 1, \cdots, 5$ denote the spatial world volume indices), and its hamiltonian. After fixing the light cone gauge and the gauge symmetries related to the antisymmetric field, we will obtain the physical hamiltonian of the theory. We will show that is possible to perform a canonical reduction to the light cone gauge, as is usual in string and supermembrane theory. Moreover, we will show that the gauge symmetry related to the antisymmetric field may also be fixed in a way which allows a canonical reduction to the physical degrees of freedom. In fact, in order to do so, we realize that the selfduality condition on the curvature of the antisymmetric field in six dimensions describes a first order propagating equation for six physical degrees of freedom. It is then natural to look for a canonical formulation where those degrees of freedom should be realized in terms of canonically conjugate fields.

We will consider the supersymmetric extension of an ADM parametrization of the metric [**?**]

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as in [?] to obtain:

$$L = 2n\sqrt{M} - \frac{1}{4}N^\rho \widehat{\mathcal{V}}_\rho + \frac{1}{2}\widetilde{H}^{\mu\nu}\partial_0 B_{\mu\nu} \tag{3}$$
$$-\widetilde{\mathcal{H}}^{\mu\nu}b_{\mu\nu 0} + \mathcal{F}(X,\theta)$$

where

$$g = \det g_{\mu\nu}, \quad g_{\mu\nu} = G_{\mu\nu}$$

$$M = 1 + \widehat{\mathcal{Y}} + \widehat{\mathcal{Z}}$$

$$\widehat{\mathcal{Y}} = \frac{1}{2}g^{-1}\widetilde{\mathcal{H}}^{\mu\nu}\widetilde{\mathcal{H}}_{\mu\nu}$$

$$\widehat{\mathcal{Z}} = \frac{1}{64}g^{-1}g^{\mu\nu}\widehat{\mathcal{V}}_\mu \widehat{\mathcal{V}}_\nu$$

and

$$\widehat{\mathcal{V}}_\mu = \epsilon_{\mu\alpha\beta\gamma\delta}\widetilde{\mathcal{H}}^{\alpha\beta}\widetilde{\mathcal{H}}^{\gamma\delta}$$

The spatial world volume indices are raised and lowered with the induced metric $g_{\mu\nu}$. It will turn out that the final expression of the hamiltonian does not require the existence of the inverse of $g_{\mu\nu}$. $\mathcal{F}(X,\theta)$ in the lagrangian density denotes the contribution from $\Omega_6 - \frac{1}{2}\epsilon^{\mu\nu\alpha\beta\rho}b_{\mu\nu\alpha}b_{\beta\rho 0}$ independent on the antisymmetric field. The latest term is the remaining part of L_2. It involve at most linear terms on the time derivative of the X^a and θ, since it is a six form constructed from dX and $d\theta$. The conjugate momenta to X^a may be directly evaluated. It is:

$$P_a = \widetilde{P}_a + f_a(X^a, \theta)$$

where

$$\widetilde{P}_a = \frac{2}{n}\sqrt{gM}\left[-\dot{\Pi}_a + N^\alpha \Pi_{a\alpha}\right] - \frac{1}{4}\widehat{\mathcal{V}}^\rho \Pi_{a\rho},$$

and

$$f_a = \frac{\delta\mathcal{F}}{\delta\dot{X}^a},$$

from which we deduce the following constraints:

$$\widehat{\Phi} = \frac{1}{2}\widetilde{P}_a\widetilde{P}^a + 2g(1 + \widehat{\mathcal{Y}}) = 0 \tag{4}$$

$$\widehat{\Phi}_\alpha = \widetilde{P}_a\Pi^a_\alpha + \frac{1}{4}\widetilde{\mathcal{V}}_\alpha = 0 \tag{5}$$

The conjugate momenta to the $B_{\mu\nu}$ will be denoted $P^{\mu\nu}$ and satisfies the constraint

$$\Omega^{\mu\nu} \equiv P^{\mu\nu} - \widetilde{H}^{\mu\nu} = 0 \qquad (6)$$

which is exactly the same constraint that appears in the bosonic 5-brane canonical formalism. The previous constraints (4) and (5) are modified with respect to the bosonic ones by the terms f_a and $b_{\mu\nu\lambda}$, while (6) remains unchanged. This property of the antisymmetric field has important consequences in the construction of the physical hamiltonian. At this point, we would like to comment that we have obtained in [?] a preliminar canonical version with

$$P^{\mu\nu} - \widetilde{\mathcal{H}}^{\mu\nu} = 0$$

that arises from the fact that the first term in Ω_6 may be written as: $-B \wedge db$. Both theories are consistent, nevertheless, the present version of this constraint is more manageable for the later analysis of the chiral degrees of freedom. This behavior can be attributed to the existence of a canonical transformation between both versions. Finally, the fermionic constraint arises directly from the evaluation of the conjugate momenta ξ to θ. It is:

$$\Psi \equiv \xi - (\bar{\theta}\Gamma^a \widetilde{P}_a - \widetilde{\mathcal{H}}^{\mu\nu}\frac{\delta b_{\mu\nu 0}}{\delta\dot{\theta}} + \frac{\delta\mathcal{F}}{\delta\dot{\theta}}) = 0 \qquad (7)$$

(4), (5), (6) and (7) are the complete set of constraints of the super 5-brane theory. The canonical hamiltonian is a linear combination of these constraints,

$$\mathbf{H} = \Lambda\widehat{\Phi} + \Lambda^\mu\widehat{\Phi}_\mu + \Lambda_{\mu\nu}\Omega^{\mu\nu} + \Psi\eta \qquad (8)$$

The set of constraints is a mixture of second and first class one, as usual for the superstring and supermembrane theories.

We may now introduce a new canonical lagrangian with a gauge group which contains as a subgroup the one generated by the first class constraints of the previous formulation (8), such that under partial gauge fixing it reduces to (8). To do so, we first replace $\widetilde{\mathcal{H}}^{\mu\nu}$ in (4), (5) and (7) by:

$$\widetilde{\mathcal{H}}^{\mu\nu} \longrightarrow \widehat{\mathcal{H}}^{\mu\nu} \equiv \frac{1}{2}(P^{\mu\nu} + \widetilde{H}^{\mu\nu}) - \widetilde{b}^{\mu\nu} \qquad (9)$$

which is a valid procedure under (6). Having done that replacement we now relax (6) into:

$$\Omega^\nu \quad \equiv \quad \partial_\mu P^{\mu\nu} = 0 \qquad (10)$$

$$\Omega^{5i} \quad \equiv \quad P^{5i} - \widetilde{H}^{5i} = 0 \quad i = 1, \cdots, 4 \qquad (11)$$

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and consider them together with:

$$\Phi = \frac{1}{2}\widetilde{P}_a\widetilde{P}^a + 2g(1 + \mathcal{Y}) = 0 \tag{12}$$

$$\Phi_\alpha = \widetilde{P}_a\Pi_\alpha^a + \frac{1}{4}\mathcal{V}_\alpha = 0 \tag{13}$$

$$\Psi \equiv \xi - (\bar{\theta}\Gamma^a\widetilde{P}_a - \widehat{\mathcal{H}}^{\mu\nu}\frac{\delta b_{\mu\nu 0}}{\delta\dot{\theta}} + \frac{\delta\mathcal{F}}{\delta\dot{\theta}}) = 0 \tag{14}$$

Here $\mathcal{V}_\mu$, $\mathcal{Y}$ and $\mathcal{Z}$ now depend on $\widehat{\mathcal{H}}^{\mu\nu}$ instead of $\widetilde{\mathcal{H}}^{\mu\nu}$. We notice that (10) and (11) commute between themselves and with: (12), (13) and (14). They are then, first class constraints. Moreover under partial gauge fixing of the gauge symmetry they generate we may recover (6), and consequently the hamiltonian (8). We thus conclude that:

$$\mathbf{H} = \Lambda\Phi + \Lambda^\mu\Phi_\mu + \rho_\mu\Omega^\mu + \rho_{5\mathrm{i}}\Omega^{5\mathrm{i}} + \eta\Psi \tag{15}$$

defines a master canonical system describing the super 5-brane theory, where the constraints related to the antisymmetric field have been raised to first class ones. If we turn the fermionic coordinates off in (15), it reduces to the master bosonic hamiltonian obtained in [?]. We will show in the next section that we can use the additional gauge symmetry generated by (11) to perform a canonical reduction of the system ending up with a canonical description of the super 5-brane action in terms of the physical degrees of freedom of the antisymmetric field.

4   The Physical Hamiltonian

The constraints (12), (13) and (14) are a mixture of first and second class constraints. The first class ones generates the six dimensional difeomorphisms on the world volume, together with the κ-symmetry. We consider now the light cone gauge fixing conditions:

$$X^+ = -P_0^+\tau, \tag{16}$$

$$P^+ = \sqrt{\omega}P_0^+, \tag{17}$$

$$\Gamma^+\theta = 0 \tag{18}$$

where ω is a time independent scalar density on the world volume. In general, it is not possible to impose the condition $\omega = 1$ on the whole world volume, we prefer then to leave it explicitly in the expression. The physical consequences of the theory should be independent

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of w, which may be thought as the determinant of a metric on the world volume. In this sense, the dependence on ω is like the dependence on the metric in topological field theories, the metric appears through the gauge fixing procedure but the observables of the theory are independent of it.

The gauge fixing (16), (17), (18) together with the constraints (12), (13) and (14) allow a canonical reduction of the hamiltonian (15). In fact, the canonical conjugate pairs:

$$X^+, \quad P_+$$
$$X^-, \quad P_-$$
$$\Gamma^+\theta, \quad \xi\Gamma^-$$

may be eliminated from the above mentioned gauge fixing conditions and constraints. One is left only with the constraints:

$$\Psi\Gamma^+ = \xi\Gamma^+ - (\bar{\theta}\Gamma^I P_I - \widehat{\mathcal{H}}^{\mu\nu}\frac{\delta b_{\mu\nu 0}}{\delta\dot{\theta}} + \frac{\delta\mathcal{F}}{\delta\dot{\theta}})\Gamma^+ = 0 \tag{19}$$

and

$$\Theta_{\mu\nu} \equiv \partial_{[\mu}\left(\frac{\widetilde{P}_I\partial_{\nu]}\Pi^I}{\omega} + \frac{1}{4\omega}\mathcal{V}_{\nu]} - P_0^+\bar{\theta}\Gamma^-\partial_{\nu]}\theta\right) = 0 \tag{20}$$

where $I, J = 1, 2, \cdots, 9$.

A suitable linear combination of the left handed sides of (19) and (20) defines the volume preserving diffeomorphisms generator, the fermionic constraint $\Psi\Gamma^+ = 0$ is left as a second class one.

The light cone gauge Hamiltonian then reads:

$$\mathbf{H}_{LCG} = \frac{1}{\sqrt{\omega}}[\frac{1}{2}\widetilde{P}^J\widetilde{P}_J + 2g(1+\mathcal{Y})] + P_0^+ f^-(X^a, \theta)$$
$$+ \rho_\mu\Omega^\mu + \rho_{5i}\Omega^{5i} + \Lambda^{\alpha\beta}\Theta_{\alpha\beta} + \Psi\Gamma^+\eta \tag{21}$$

We may now consider a further canonical reduction related to (11). We impose the gauge fixing condition:

$$B_{5i} = 0 \tag{22}$$

that allows to eliminate the canonical conjugate pair (B_{5i}, P^{5i}) using the first class constraint (11). The remaining constraint (10) reduces then to:

$$\partial_i P^{ij} + \partial_5\widetilde{H}^{5j} = 0 \tag{23}$$

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which may be rewritten as:

$$\partial_j(P^{ij} - \widetilde{H}^{ij}) = 0 \tag{24}$$

In order to disentangle the physical degrees of freedom the antisymmetric field we may impose a further gauge fixing condition associated to the first class constraint (24). We consider:

$$B_{4\underline{a}} = 0 \tag{25}$$

where $\underline{a} = 1, 2, 3$. This is an admissible gauge fixing condition which has the interesting property that the kinetic term $P^{ij}\dot{B}_{ij}$ reduces completely. To do so, we notice that (24) may be resolved explicitly:

$$P^{ij} = \widetilde{H}^{ij} + \epsilon^{ijkl}\partial_k A_l \tag{26}$$

where A_l are the remaining degrees of freedom of the momenta P^{ij}. If we now evaluate the kinetic term we obtain:

$$\langle P^{ij}\dot{B}_{ij}\rangle = \langle \epsilon^{ijkl}\partial_5 B_{kl}\dot{B}_{ij} + \dot{A}_l\epsilon^{ijkl}\partial_k B_{ij}\rangle$$
$$= \langle \dot{A}_l\epsilon^{ijkl}\partial_k B_{ij}\rangle$$

Since (22) and (25) ensures that the first term on the right hand side member is zero. We then define:

$$P^{\underline{a}} \equiv -\epsilon^{\underline{abc}}B_{\underline{bc}}$$
$$P \equiv \epsilon^{\underline{abc}}\partial_{\underline{a}}B_{\underline{bc}}$$

they satisfy the equation:

$$\partial_{\underline{a}}P^{\underline{a}} + P = 0 \tag{27}$$

It yields:

$$\langle P^{ij}\dot{B}_{ij}\rangle = \langle \dot{A}_{\underline{a}}\partial_4 P^{\underline{a}} + \dot{A}_4 P\rangle$$

which implies that $\partial_4 P^{\underline{a}}$ and P are the conjugate momenta to $A_{\underline{a}}$ and A_4 respectively. We may perform a final canonical reduction by imposing the gauge fixing $A_4 = 0$ and eliminate its conjugate momentum P from (27). All the dependence of the hamiltonian on the antisymmetric field is through the terms:

$$\widehat{\mathcal{H}}^{ij} \equiv \frac{1}{2}(P^{ij} + \widetilde{H}^{ij}) + \tilde{b}^{ij}$$
$$\widehat{\mathcal{H}}^{5i} \equiv \widetilde{\mathcal{H}}^{5i}$$

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which appear quadratically in the hamiltonian.

We notice that:

$$\widetilde{\mathcal{H}}^{\underline{ab}} = 0$$

$$\widetilde{\mathcal{H}}^{4\underline{a}} = -\frac{1}{6}\epsilon^{\underline{abc}}\partial_5 B_{\underline{bc}} = \frac{1}{6}\partial_5 P^{\underline{a}}$$

$$\widetilde{\mathcal{H}}^{54} = -\frac{1}{6}\epsilon^{\underline{abc}}\partial_{\underline{a}} B_{\underline{bc}} = \frac{1}{6}\partial_{\underline{a}} P^{\underline{a}}$$

$$\widetilde{\mathcal{H}}^{5\underline{a}} = \frac{1}{6}\epsilon^{\underline{abc}}\partial_4 B_{\underline{bc}} = -\frac{1}{6}\partial_4 P^{\underline{a}},$$

hence the canonical lagrangian can be expressed in terms of $A_{\underline{a}}$ and $P^{\underline{a}}$ which describe the unrestricted independent degrees of freedom associated to the antisymmetric field. The explicit terms in the canonical lagrangian are:

$$\langle \dot{A}_{\underline{a}}\partial_4 P^{\underline{a}} + \widehat{\mathcal{H}}^{ij}\widehat{\mathcal{H}}^{kl}g_{ik}g_{jl} + $$
$$+ 4\widehat{\mathcal{H}}^{i5}\widehat{\mathcal{H}}^{kl}g_{ik}g_{5l} + 4\widehat{\mathcal{H}}^{i5}\widehat{\mathcal{H}}^{j5}(g_{ij}g_{55} - g_{i5}g_{j5})\rangle \tag{28}$$

where

$$\widehat{\mathcal{H}}^{\underline{ab}} = \frac{1}{2}\epsilon^{\underline{abkl}}\partial_k A_l + \underline{\tilde{b}}^{\underline{ab}}$$

$$\widehat{\mathcal{H}}^{4\underline{a}} = \frac{1}{12}\partial_5 P^{\underline{a}} - \frac{1}{2}\epsilon^{\underline{abc}}\partial_{\underline{b}} A_c + \underline{\tilde{b}}^{4\underline{a}} =$$

$$\widehat{\mathcal{H}}^{54} = \frac{1}{6}\partial_{\underline{a}} P^{\underline{a}} + \underline{\tilde{b}}^{54}$$

$$\widehat{\mathcal{H}}^{5\underline{a}} = -\frac{1}{6}\partial_4 P^{\underline{a}},$$

5   Discussion and Conclusions

We constructed the physical hamiltonian of the super 5-brane theory which is still constrained by the second class fermionic constraint and by the first class volume preserving diffeomorphisms. We prefer to leave this gauge symmetry without fixing it, since it may have a relevant interpretation in terms of a noncommutative geometry. That was the case for the supermembrane. In that case the area preserving diffeomorphisms on the spatial world volume is the same as the symplectomorphisms which are very closely related to the noncommutative geometry constructed in terms of the Weyl algebra bundle [?]. In our case, the group of symplectomorphisms is contained in the volume preserving diffeomorphisms. A complete analysis of that relation seems to be very important.

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The canonical lagrangian we have obtained is expressed in terms of the covariant induced metric $g_{\mu\nu}$, we do not require the existence of the inverse contravariant metric in our construction of the hamiltonian of the super 5-brane theory. The original PST formulation as well as the Schwarz et. al. construction requires the uses of the inverse metric $g^{\mu\nu}$ in order to define their lagrangians. This property which was already discuss for the bosonic sector of the 5-brane in [?], and shown to be valid even in the realization of the algebra of diffeomorphisms in terms of the first class constraints of the theory, remains valid for the supersymmetric formulation of the 5-brane. It has the important consequence that singular configurations which annihilate the determinant of the induced metric at a neighborhood of any point on the world volume are admissible configurations of the theory. They are essential in the physical interpretation of the supermembrane as a multiparticle theory [?] [?]. In that case they are string like spikes which can change the topology of any configuration and connect disjoint membranes without changing the energy of the system. Consequently, these configurations are responsible, together with the supersymmetry of the continuos spectrum of the supermembrane. In the super 5-brane case those singular configurations may be $1, 2, 3,$ or, 4-branes which also provide the same interpretation of the theory as a multiparticle one. This property was analyzed in [?] and we have shown now that the same analysis may be extended to the supersymmetric 5-brane theory.

In order to describe the self duality property of the super 5-brane equations of motion in a canonical local formulation we propose the hamiltonian (21) together with the constraints (19), (20) and (24). This formulation may be further reduced preserving the canonical structure in terms of the conjugate pairs $(A_a, \partial_4 P^a)$, (A_4, P), however, this latest formulation becomes non local because of the term $\partial_5 P^a = \partial_5 \partial_4^{-1}(\partial_4 P^a)$. In this case we can performed a final canonical reduction by imposing the gauge fixing condition $A_4 = 0$ and eliminating its conjugate momentum P from (21) and end up with the unconstrained pair $(A_{\underline{a}}, \partial_4 P^{\underline{a}})$.

If we consider the dimensional reduction of the super 5-brane hamiltonian identifying $X^5 = \sigma^5$, and taking all other $\partial_5 \cdot = 0$ we end up with the super 4-brane hamiltonian in 10 dimensions Minkowski space. In that case the nonlocal term of the canonical formulation becomes zero and we obtain a local canonical lagrangian for the super 4-brane. The terms involving the antisymmetric field becomes now quadratic, because P_5 has to be eliminated from the con-

straints:

$$P_5 + \frac{1}{4}\mathcal{V}_5 = 0$$

hence the hamiltonian incudes the term:

$$\frac{1}{16}\mathcal{V}_5^2$$

where

$$\mathcal{V}_5 = \epsilon_{5ijkl}\left(\frac{P^{ij}}{2} + \tilde{b}^{ij}\right)\left(\frac{P^{kl}}{2} + \tilde{b}^{kl}\right)$$