Supersymmetry breaking and 4 dimensional string models

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Abstract

A family of superpotentials is constructed which may be relevant to supersymmetry breaking in 4 dimensional (0,1) heterotic string models. The scale of supersymmetry breaking, as well as the coupling constant, would be stable and could not run away to zero.

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1 Introduction

In this paper we will study a family of superpotentials which may be relevant to supersymmetry (susy) breaking in 4 dimensional (0,1) heterotic string models, and which generalise those given in [1,2]. By 4 dimensional string models we have in mind that the extra string degrees of freedom are subject to twisted boundary conditions in such a way that no 'breathing' modes for extra dimensions appear in the level zero spectrum [3]. Stability is a feature of such $D = 4$ string models, for which decompactification cannot take place. There is a fixed, arbitrary gauge coupling, related to string topology through the genus expansion. There may be a possible restriction of this coupling constant required for the consistent propagation of knotted and linked strings, which are features which appear when D is reduced to 4, with strings producing orbifold-like singularities in space-time. The origin of susy breaking is often ascribed to the gaugino condensation mechanism [4,5]. This relies on a variation of the gauge coupling with a scalar field, which is therefore absent in these models. Consequently, there would need to be susy breaking by some fundamental rather than composite Goldstone supermultiplet. If this is possible, there would be benefits in avoiding problems with the stability of the gauge hierarchy and a shallow potential well which could lead to decompactification during inflation [6]. A vanishing 4 dimensional cosmological constant also seems more natural in a model without extra dimensions. For the present, we will consider the scalar fields only for simplicity. A vanishing cosmological constant may be viewed as a balance between local and global susy breaking. This can be described geometrically as a requirement that the field-space vector formed by all auxiliary fields be null. In terms of the geometry of the chiral scalar fields, this is equivalent to the superpotential G having a critical slope of $\sqrt{3}$ in field space. The polarization of positive and negative energy auxiliary fields due to this local/global susy breaking is much greater than the energy due to non-cancellation between bosons and fermions from global susy breaking. A small adjustment of the balance between gravitational and other auxiliaries would cancel the residue to keep $\Lambda = 0$. We will assume that such a balancing mechanism applies. A small failure of the balance mechanism could give a small Λ, as may be the actual case in cosmology, but we will not attempt to understand this here.

In $N = 1$ susy, we may make a decomposition of the chiral fields into the Goldstino z and other fields s_i,where z,s_i are orthogonal at the vacuum expectation values z_0, s_0,

$$\frac{\partial^2 G}{\partial z \partial s_i^*}(z_0, s_0) = \frac{\partial G}{\partial s_i}(z_0, s_0) = 0 \tag{1}$$

We propose that the Goldstino scalar z can be approximated by the spin 0 part of the dilaton supermultiplet, which may be regarded as scalar and pseudoscalar components of the graviton, as we can form a massive gravitino multiplet, or extended graviton multiplet by taking the full content of the spin $(1)_L \otimes (1 \oplus 1/2)_R$ heterotic level zero fields. Our motivation is that the dilaton multiplet in $D = 4$ is naturally present, and has a flat potential, or critical-slope superpotential (implementing the local/global balance principle) in the absence of other sources of susy breaking [7]. The pseudoscalar graviton mass is zero, protected by a gauge invariance, and therefore the pseudoscalar part of z will be a pseudo-Goldstone boson with a relatively long range. Mixings between the gravitational and other scalars will be suppressed by factors of $1/m_{3/2}$, since the contributions of different superfields to the Goldstone supermultiplet would be in proportion to their auxiliary components. CP violation may give a small scalar component to the pseudo-Goldstone boson, but we will neglect this below. The situation in realistic models where there are also terms in V from the gauge auxiliary D fields is more complicated, but providing that the local/global balance principle applies, there will again be zero cosmological constant with a small mixing $O(1/m_{3/2})$ between z and other superfields.

2 Derivation of the Superpotentials

Decomposing z into scalar and pseudoscalar parts, we have $z = x + \mathrm{i}y$ where

$$dy = *(dB + \omega_{GS}) + O(1/m_{3/2}) \tag{2}$$

(ω_{GS} is the Green-Schwarz 3-form) so that under the transformation $y \rightarrow y + \epsilon$, $\delta G = \epsilon O(G/m_{3/2})$ or

$$\frac{\partial G(z)}{\partial y} = O(G/m_{3/2}), \ \frac{\partial G(z)}{\partial y}(z_0, s_0) = 0 \tag{3}$$

i.e. $G(z) = G(x) + O(G/m_{3/2})$. For any function $h(z)$ which depends only on x we have

$$\frac{\partial h(x)}{\partial z} = \frac{\partial h(x)}{\partial z^*} = 1/2\frac{dh}{dx} \equiv \frac{h'}{2} \tag{4}$$

so that the potential for x becomes (neglecting for now the contribution from other fields)

$$V(x) = \mathrm{e}^G(\frac{G'^2}{G''} - 3) = \frac{\mathrm{e}^G}{G''}(G'^2 - 3G''). \tag{5}$$

Introducing the notation

$$G' = g(x) \tag{6}$$

we can write

$$g^2 - 3g' = f^2. \tag{7}$$

For a vanishing cosmological constant, we require that f should have a zero. The simplest ansatz with this property is essentially

$$f^2 = \alpha(g-3)^2. \tag{8}$$

Here we have used the linear transformation $x \rightarrow ax+b$ to arrange $x = 0, g = 3$ at $f = 0$ without loss of generality (these values are chosen to simplify the formulas below). The case $\alpha = 0$ gives the flat, no-scale solution

$$G = -3\ln(x), \quad V = 0. \tag{9}$$

We will now derive the solution corresponding to eq.(8). Rearranging eq.(7) gives the first order differential equation

$$\frac{dx}{dg} = \frac{1}{(1-\alpha)g^2 + 6\alpha g - 9\alpha} \tag{10}$$

with solution

$$x = \tan^{-1}(\frac{1-\alpha}{3\alpha}g + 1) - \tan^{-1}(\frac{1}{\alpha}) \tag{11}$$

giving

$$g = \frac{3\alpha}{1-\alpha}(\tan(x+c) - 1) \tag{12}$$

where

$$c = \tan^{-1}(\frac{1}{\alpha}). \tag{13}$$

Now

$$G(x) = \int_{u=0}^{x} g(u)du + G_0 \tag{14}$$

where the constant of integration G_0 gives the value of $\ln(m_{3/2}^2)$ at the potential minimum. We have

$$G = G_0 + \frac{3\alpha}{1-\alpha}(\log\sec(x+c) - \log\sec(c) - x) \tag{15}$$

and using

$$G'' = g' = \frac{3\alpha}{1-\alpha}\sec^2(x+c) \tag{16}$$

gives us, on eliminating c by using elementary identities,

$$V = 3\mathrm{e}^{G_0}\frac{(1+\alpha^2)}{1-\alpha}[\mathrm{e}^{-x}(\cos(x) - \frac{1}{\alpha}\sin(x))]^{\frac{3\alpha}{\alpha-1}}\sin^2(x). \tag{17}$$

Here x will be confined to the range where V is finite i.e. $\tan(x) < \alpha$ for $0 < \alpha < 1$. For the mass of the scalar x we have

$$m_S^2 = \frac{V''}{G''}|_{x=0} = \frac{\mathrm{e}^{G_0}}{G''}\frac{(f')^2}{G''}|_{x=0} = \alpha m_{3/2}^2 \tag{18}$$

giving the interpretation of the parameter α as $m_S^2/m_{3/2}^2$. As argued above the mass of the pseudoscalar y will be

$$m_P^2 = O(m_{3/2}^4). \tag{19}$$

The solution eq.(17) may be written implicity as a function of the sigma model parameter

$$\phi(x) = \int_{u=0}^{x} \sqrt{2G''(u)}du = \sqrt{\frac{2\alpha}{3(1-\alpha)}} \ln\left(\frac{\sec(x+c)+\tan(x+c)}{\sec(c)+\tan(c)}\right) \qquad (20)$$

with standard kinetic term. Now consider the limit $\alpha \to 1$ corresponding to the physically interesting case $m_S = m_{3/2}$. Going back to eq.(7) with $\alpha = 1$ gives

$$x = 1/2 \ln(2g/3 - 1). \qquad (21)$$

Introducing an alternative parametrization by

$$w = \exp(z), |w| = \exp(x) \qquad (22)$$

(so that y and $y + 2\pi n$ are identified), which is natural when we recall the origin of x as the 4 dimensional dilaton, gives the solution

$$V = \frac{3\mathrm{e}^{G_0}}{4}(|w|^2 \exp(|w|^2 - 1))^{3/4}(|w| - |w|^{-1})^2. \qquad (23)$$

Here the value of w is kept away from zero by the divergence in $V(0)$. In this case the sigma model field is $\phi = \sqrt{6}|w|$, so that if we write $\psi = |w|$ we can write the Lagrangian terms for ψ as

$$L_\psi = 3\sqrt{-g}((\partial\psi)^2 - \frac{\mathrm{e}^{G_0}}{4}(\psi^2 \exp\psi^2 - 1)^{3/4}(\psi - \psi^{-1})^2). \qquad (24)$$

(Here g is the usual $D = 4$ metric determinant.) An interesting feature of this model is that the scalar and pseudoscalar fields will be described by a sigma model with the flat Kahler metric

$$d\sigma^2 = 6dwdw^* = 6(d\psi^2 + \psi^2 d\theta^2). \qquad (25)$$

We will now consider the origin of the integration constant G_0, which determines $m_{3/2}$. Suppose that, near (z_0, s_0), G takes the form

$$G(z,s) = G_g(z) + G_s(s) + H(z,s) = G_g(z) + K(s) + \ln(|W(s)|^2) + H(z,s) \qquad (26)$$

solving eq.(1), where $G_g(z)$ takes the form discussed above and for the remaining chiral fields s, $K(s)$ is the Kahler potential (generating the field kinetic terms) and $W(s)$ represents the chiral scalar superfield interactions. $H(z,s)$ represents mixing terms of higher order in $1/m_{3/2}$ relative to $G(z,s)$, with $H(z_0, s_0) = 0$. We expect $K_0 \sim 0$, giving

$$\exp(G_0) \sim |W_0|^2 \qquad (27)$$

where W_0 is the minimum value of $W(s)$ - since the vacuum represents the minimum of G w.r.t. s, it also represents the minimum of $W(s)$. Now the leading renormalizable contribution gives $W \sim g_4 X_1 X_2 X_3$ where X_i may acquire expectation values by a Coleman-Weinberg mechanism well below the string tension scale. For $m_{3/2} \sim 1\text{TeV}$, the X_{i0} may come from GUT symmetry breaking, a neutrino mass see-saw mechanism or some other hidden sector. String diagrams with zero background field give $V(z) = 0$. While this may represent the $\alpha \to 0$ limit of the family of models represented by eq.(8), it may alternatively represent the limit $\exp(G_0) \to 0$, in particular with $\alpha = 1$, allowing stable supersymmetry breaking in 4 dimensional string models. For $\alpha \leq 1$, $m_{3/2}$ is prevented from running away to zero, so that it is natural in this scenario for susy breaking to survive inflation.

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