"Massive" spin-2 field in de Sitter space

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In this paper we present a covariant quantization of the "massive" spin-2 field on de Sitter (dS) space. By "massive" we mean a field which carries a specific principal series representation of the dS group. The work is in the direct continuation of previous ones concerning the scalar, the spinor and the vector cases. The quantization procedure, independent of the choice of the coordinate system, is based on the Wightman-Gärding axiomatic and on analyticity requirements for the two-point function in the complexified pseudo-Riemanian manifold. Such a construction is necessary in view of preparing and comparing with the dS conformal spin-2 massless case (dS linear quantum gravity) which will be considered in a forthcoming paper and for which specific quantization methods are needed.

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I. INTRODUCTION

As recent observational data clearly favors a positive acceleration of the present universe, the de Sitter model represents an appealing first approximation of the background space-time. In two previous papers [1, 2], quantizations of "massive" spinor fields and vector fields on the dS space have been considered. The spin-2 case is of great importance since the massless tensor field (spin-2) is among the central objects in quantum cosmology and quantum gravity on dS space (dS linear quantum gravity). It has been found that the corresponding propagator (in the usual linear approximation for gravitational field) exhibits a pathological behaviour for large separated points (infrared divergence) [3–5].

On one hand this behaviour may originate from the gauge invariance of the field equation and so should have no physical consequences. Antoniadis, Iliopoulos and Tomaras [6] have shown that the large-distance pathological behavior of the graviton propagator on dS background does not manifest itself in the quadratic part of the effective action in the one-loop approximation. This means the pathological behaviour of the graviton propagator may be gauge dependent and so should not appear in an effective way as a physical quantity.

On the other hand some authors argue that infrared divergence could be exploited in order to create instability of dS space [7, 8]. Tsamis and Woodard have considered the field operator for linear gravity in dS space along the latter line in terms of flat coordinates, which cover only one-half of the dS hyperboloid [9]. Hence they have found a quantum field which breaks dS invariance, and they have examined the resulting possibility of quantum instability.

Nevertheless, a fully covariant quantization of the linear gravitational field without infrared divergence in dS space-time may reveal to be of extreme importance for further developments. It will be considered in a forthcoming paper [10]. Such a quantization requires preliminary covariant quantizations of the minimally coupled scalar field and the "massive" spin-2 field respectively.

Recently, de Vega and al. [11] have shown that, in flat coordinates (not global) on de Sitter space-time, the infrared divergence does not appear in the "massless" minimally coupled scalar field. The question of the covariant minimally coupled scalar field has been completely answered in [12] after introducing a specific Krein QFT. We have shown that the effect of that quantization, without changing the physical content of the theory, appears as an automatic renormalization of the ultraviolet divergence in the stress tensor and of the infrared divergence in the two-point function [13]. By using this method for linear gravity (the traceless rank-2 "massless" tensor field) the two-point function is

free of any infrared divergence [14]. This result has been also obtained by [15–17].

Here, we present a fully covariant quantization of the "massive" spin-2 field. Our method is based on a rigorous group-theoretical approach combined with a suitable adaptation of the Wightman-Gärding axiomatic, which is carried out in terms of coordinate independent dS waves. The whole procedure originated by [18] is based on analyticity requirements in the complexified pseudo-Riemanian manifold. The SO$(1, N)$ unitary irreducible representations (UIR) acting on symmetric, traceless and divergence-free tensor eigenfunctions of the Laplace-Beltrami operator have been investigated in [19]. Previous studies of the "massive" spin-2 field have been carried out in [20] with a specific choice of coordinate (flat coordinates) covering only one-half of the dS hyperboloid, and in [21] where the forbidden mass range for spin-2 fields has been clarified, and the null-mass limit considered. This limit has also been analyzed in [22] and recently a consistent theory for a massive spin-2 field in a general gravitational background has been presented in [23].

In section II, we describe the dS tensor field equation as an eigenvalue equation of the SO$(1, 4)$ Casimir operators. The notations and the two independent Casimir operators are introduced. It will be convenient to use ambient space notations in order to express the spin-2 field equation in terms of the coordinate independent Casimir operators. The latter carry the group-theoretical content of the theory and it will be reminded how they enable us to classify the dS group UIR [24, 25] according to two parameters p and q which behave like a spin (s) and a mass (m) in the Minkowskian limit, depending on the nature of the involved group representation.

Section III is devoted to the field equation and its solutions. The dS tensor modes are written in terms of a scalar field ϕ and a generalized polarization tensor $\mathcal{E}$

$$K_{\alpha\beta}(x) = \mathcal{E}_{\alpha\beta}(x, \xi)\phi(x).$$

As for spinor and vector fields, the tensor $\mathcal{E}(x, \xi)$ is a space-time function in dS space-time. There is a certain extent of arbitrariness in the choice of this tensor and we fix it in such a way that, in the limit $H = 0$, one obtains the polarization tensor in Minkowski space-time.

In section IV we derive the Wigthman two-point function $\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x')$. This function fulfills the conditions of : a) positiveness, b) locality, c) covariance, d) normal analyticity, e) transversality, f) divergencelessness and g) permutational index symmetries. The four conditions c), e), f), and g) allow one to associate this field with a spin-2 unitary irreducible representation of the dS group. The positivity condition permits us to construct a Hilbert space structure. The locality is related to the causality principle, which is a well defined concept in dS space. The normal analyticity allows one to view $\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x')$ as the boundary value of an analytic two-point function $W_{\alpha\beta\alpha'\beta'}(z, z')$ from

the tube domains. The analytic kernel $W_{\alpha\beta\alpha'\beta'}(z,z')$ is defined in terms of dS waves in their tubular domains. Then, the Hilbert space structure is made explicit and the field operator $\mathcal{K}(f)$ is derived. We also give a coordinate-independent formula for the unsmeared field operator $\mathcal{K}(x)$. Brief conclusion and outlook are given in section V. It is in particular asserted that the extension of our approach to "massless" tensor field (gravitational field in a dS background in the linear approximation) requires an indecomposable representation of the dS group in view of the construction of the corresponding covariant quantum field. Finally, we have detailed the classification of the unitary representation of $SO_0(1,4)$ in appendix A. In appendix B we relate our construction to the maximally symmetric bitensors introduced in Reference [26]. In appendix C and D we respectively present the "massive" vector and tensor two-point functions.

II. FIELD EQUATIONS ON DE SITTER SPACE

A. Ambient space notations and Casimir operators

The de Sitter space is a solution of the cosmological Einstein equation with positive cosmological constant Λ. It is conveniently described as a hyperboloid embedded in a five-dimensional Minkowski space

$$X_H = \{x \in \mathbb{R}^5; x^2 = \eta_{\alpha\beta}x^\alpha x^\beta = -H^{-2} = -\frac{3}{\Lambda}\}, \ \ \alpha,\beta = 0,1,2,3,4, \tag{2.1}$$

where $\eta_{\alpha\beta} = \text{diag}(1,-1,-1,-1,-1)$. The de Sitter metrics reads

$$ds^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta = g_{\mu\nu}^{dS}dX^\mu dX^\nu, \ \ \mu = 0,1,2,3,$$

where the X^μ's are 4 space-time intrinsic coordinates of the dS hyperboloid.

An immediate realization space is made of a second-rank intrinsic tensor field $h_{\mu\nu}$ satisfying the conditions of divergenceless, tracelessness, and index permutational symmetry respectively:

$$\nabla.h(X) = 0, \ \ h_\mu^\mu(X) = 0, \ \ h_{\mu\nu} = h_{\nu\mu}. \tag{2.2}$$

The wave equation for such fields propagating in de Sitter space can be written as [20]

$$\left(\Box_H + 2H^2 + m_H^2\right)h_{\mu\nu}(X) = 0, \tag{2.3}$$

where $\Box_H = \nabla_\mu\nabla^\mu$ is the d'Alembertian operator.

Let us now adopt ambient space notations (for details see [27]), namely $\mathcal{K}_{\alpha\beta}(x)$ for the field. With these notations, the relationship with unitary irreducible representations of the dS group becomes

straightforward because the Casimir operators are easy to identify. The tensor field $\mathcal{K}_{\alpha\beta}(x)$ has to be viewed as a homogeneous function of the $\mathbb{R}^5$-variables x^α with homogeneous degree λ and thus satisfies ,

$$x^\alpha \frac{\partial}{\partial x^\alpha} \mathcal{K}_{\gamma\beta}(x) = x.\partial \mathcal{K}_{\gamma\beta}(x) = \lambda \, \mathcal{K}_{\gamma\beta}(x). \tag{2.4}$$

The direction of $\mathcal{K}_{\alpha\beta}(x)$ lies in the de Sitter space if we require the condition of transversality [28]

$$x.\mathcal{K}(x) = 0. \tag{2.5}$$

With these notations, the conditions (2.2) read as

$$\bar{\partial}.\mathcal{K} = 0, \quad \mathcal{K}_\alpha^\alpha = \mathcal{K}' = 0, \quad \mathcal{K}_{\alpha\beta} = \mathcal{K}_{\beta\alpha}, \tag{2.6}$$

where $\bar{\partial}$ is the tangential (or transverse) derivative on dS space,

$$\bar{\partial}_\alpha = \theta_{\alpha\beta}\partial^\beta = \partial_\alpha + H^2 x_\alpha x.\partial, \quad \text{with} \quad x.\bar{\partial} = 0. \tag{2.7}$$

The tensor with components $\theta_{\alpha\beta} = \eta_{\alpha\beta} + H^2 x_\alpha x_\beta$ is the so-called transverse projector.

In order to express Equation (2.3) in terms of the ambient coordinates, we use the fact that the "intrinsic" field $h_{\mu\nu}(X)$ is locally determined by the transverse tensor field $\mathcal{K}_{\alpha\beta}(x)$ through

$$h_{\mu\nu}(X) = \frac{\partial x^\alpha}{\partial X^\mu} \frac{\partial x^\beta}{\partial X^\nu} \mathcal{K}_{\alpha\beta}(x(X)). \tag{2.8}$$

For instance, it is easily shown that the metric $\eta_{\mu\nu}$ corresponds to the transverse projector $\theta_{\alpha\beta}$. Covariant derivatives acting on a l-rank tensor are transformed according to

$$\nabla_\mu \nabla_\nu .. \nabla_\rho h_{\lambda_1..\lambda_l} = \frac{\partial x^\alpha}{\partial X^\mu} \frac{\partial x^\beta}{\partial X^\nu} .. \frac{\partial x^\gamma}{\partial X^\rho} \frac{\partial x^{\eta_1}}{\partial X^{\lambda_1}} .. \frac{\partial x^{\eta_l}}{\partial X^{\lambda_l}} \mathrm{Trpr}\bar{\partial}_\alpha \mathrm{Trpr}\bar{\partial}_\beta .. \mathrm{Trpr}\bar{\partial}_\gamma \mathcal{K}_{\eta_1..\eta_l}\,, \tag{2.9}$$

where the transverse projection defined by

$$(\mathrm{Trpr}\,\mathcal{K})_{\lambda_1..\lambda_l} \equiv \theta_{\lambda_1}^{\eta_1} .. \theta_{\lambda_l}^{\eta_l} \mathcal{K}_{\eta_1..\eta_l}\,,$$

guarantees the transversality in each index. Applying this procedure to a transverse second rank, symmetric tensor field, leads to

$$\begin{aligned}
\nabla_\mu \nabla_\nu h_{\rho\lambda} &= \frac{\partial x^\alpha}{\partial X^\mu} \frac{\partial x^\beta}{\partial X^\nu} \frac{\partial x^\gamma}{\partial X^\rho} \frac{\partial x^\eta}{\partial X^\lambda} \mathrm{Trpr}\bar{\partial}_\alpha \mathrm{Trpr}\bar{\partial}_\beta \mathcal{K}_{\gamma\eta} \\
&= \frac{\partial x^\alpha}{\partial X^\mu} \frac{\partial x^\beta}{\partial X^\nu} \frac{\partial x^\gamma}{\partial X^\rho} \frac{\partial x^\eta}{\partial X^\lambda} \left(\bar{\partial}_\alpha \bar{\partial}_\beta \mathcal{K}_{\gamma\eta} - H^2 \theta_{\alpha\gamma} \mathcal{K}_{\beta\eta} - H^2 \theta_{\alpha\eta} \mathcal{K}_{\beta\gamma} \right).
\end{aligned} \tag{2.10}$$

The kinematical group of the de Sitter space is the 10-parameter group $SO_0(1,4)$ (connected component of the identity in $SO(1,4)$), which is one of the two possible deformations of the Poincaré group. There are two Casimir operators

$$Q_2^{(1)} = -\frac{1}{2} L_{\alpha\beta} L^{\alpha\beta}, \qquad Q_2^{(2)} = -W_\alpha W^\alpha,$$ (2.11)

where

$$W_\alpha = -\frac{1}{8} \epsilon_{\alpha\beta\gamma\delta\eta} L^{\beta\gamma} L^{\delta\eta}, \quad \text{with 10 infinitesimal generators} \quad L_{\alpha\beta} = M_{\alpha\beta} + S_{\alpha\beta}.$$ (2.12)

The subscript 2 in $Q_2^{(1)}$, $Q_2^{(2)}$ reminds that the carrier space is constituted by second rank tensors. The orbital part $M_{\alpha\beta}$, and the action of the spinorial part $S_{\alpha\beta}$ on a tensor field $\mathcal{K}$ defined on the ambient space read respectively [29]

$$M_{\alpha\beta} = -i(x_\alpha \partial_\beta - x_\beta \partial_\alpha),$$

$$S_{\alpha\beta} \mathcal{K}_{\gamma\delta} = -i(\eta_{\alpha\gamma} \mathcal{K}_{\beta\delta} - \eta_{\beta\gamma} \mathcal{K}_{\alpha\delta} + \eta_{\alpha\delta} \mathcal{K}_{\beta\gamma} - \eta_{\beta\delta} \mathcal{K}_{\alpha\gamma}).$$ (2.13)

The symbol $\epsilon_{\alpha\beta\gamma\delta\eta}$ holds for the usual antisymmetrical tensor. The action of the Casimir operator $Q_2^{(1)}$ on $\mathcal{K}$ can be written in the more explicit form

$$Q_2^{(1)} \mathcal{K}(x) = \left(Q_0^{(1)} - 6\right) \mathcal{K}(x) + 2\eta \mathcal{K}' + 2\mathcal{S} x \partial \cdot \mathcal{K}(x) - 2\mathcal{S} \partial x \cdot \mathcal{K}(x),$$ (2.14)

In the latter, $Q_0^{(1)} = -\frac{1}{2} M_{\alpha\beta} M^{\alpha\beta}$, and the vector symmetrizer $\mathcal{S}$ is defined for two vectors ξ_α and ω_β by $\mathcal{S}(\xi_\alpha \omega_\beta) = \xi_\alpha \omega_\beta + \xi_\beta \omega_\alpha$.

We are now in position to express the wave equation (2.3) by using the Casimir operators. This can be done with the help of Equation (2.10) since $Q_0^{(1)} = -H^{-2}(\bar{\partial})^2$. The d'Alembertian operator becomes

$$\Box_H h_{\mu\nu} = \nabla^\lambda \nabla_\lambda h_{\mu\nu} = -\frac{\partial x^\alpha}{\partial X^\mu} \frac{\partial x^\beta}{\partial X^\nu} \left[Q_0^{(1)} H^2 + 2H^2\right] \mathcal{K}_{\alpha\beta},$$ (2.15)

and the wave equation (2.3) is rewritten as

$$\left(Q_0^{(1)} - H^{-2} m_H^2\right) \mathcal{K}_{\alpha\beta}(x) = 0.$$ (2.16)

Finally, using formula (2.14) for the tensor field $\mathcal{K}_{\alpha\beta}(x)$ which satisfies the conditions (2.6) the field equation becomes

$$\left(Q_2^{(1)} - \left(m_H^2 H^{-2} - 6\right)\right) \mathcal{K}_{\alpha\beta}(x) = 0.$$ (2.17)

As expected, this formulation of the field equation has now a clear group-theoretical content. In fact, using the representation classification given by the eigenvalues of the Casimir operator, we will be able to identify the involved field. At this point let us clarify what we mean by "massive" spin-2 de Sitter field. Inasmuch as mass and spin are well-defined Poincaré concepts, we will consider exclusively the de Sitter elementary systems (in the Wigner sense) associated to a UIR of $SO_0(1,4)$ that admit a non-ambiguous massive spin-2 UIR of the Poincaré group at the $H = 0$ contraction limit. This contraction is performed with respect to the subgroup $SO_0(1,3)$ which is identified as the Lorentz subgroup in both relativities, and the concerned de Sitter representations are precisely those ones which are induced by the *minimal parabolic* [30] subgroup $SO(3) \times SO (1,1) \times$(a certain nilpotent subgroup), where $SO(3)$ is the space rotation subgroup of the Lorentz subgroup in both cases. This fully clarifies the concept of spin in de Sitter since it is issued from the *same* $SO(3)$.

B. "Massive" spin-2 unitary representation of the de Sitter group $SO_0(1,4)$

The operator $Q_2^{(1)}$ commutes with the action of the group generators and, as a consequence, it is constant in each unitary irreducible representation (UIR). Thus the eigenvalues of $Q_2^{(1)}$ can be used to classify the UIR's *i.e.,*

$$(Q_2^{(1)} - \langle Q_2^{(1)} \rangle) \mathcal{K}(x) = 0. \tag{2.18}$$

Following Dixmier [24] we get a classification scheme using a pair (p, q) of parameters involved in the following possible spectral values of the Casimir operators :

$$Q^{(1)} = (-p(p+1) - (q+1)(q-2)) \, I_d, \qquad Q^{(2)} = (-p(p+1)q(q-1)) \, I_d. \tag{2.19}$$

Three types of scalar, tensorial or spinorial UIR are distinguished for $SO_0(1,4)$ according to the range of values of the parameters q and p [24, 25], namely : the principal, the complementary and the discrete series. In the following, we shall restrict the list to the unitary representations which have a Minkowskian physical spin-2 interpretation in the limit $H = 0$ (for the general situation see [31] and Appendix A). The flat limit tells us that for the principal and the complementary series it is the value of p which has a spin meaning, and that, in the case of the discrete series, the only representations which have a physically meaningful Minkowskian counterpart are those with $p = q$ (details about the mathematics of the group contraction and the physical principles underlying the relationship between de Sitter and Poincaré groups can be found in [32] and [33] respectively). The spin-2 tensor representations relevant to the present work are the following :

i) The UIR's $U^{2,\nu}$ in the principal series where $p = s = 2$ and $q = \frac{1}{2} + i\nu$ correspond to the Casimir spectral values:

$$\langle Q_2^{(1)} \rangle = \nu^2 - \frac{15}{4}, \tag{2.20}$$

with parameter $\nu \in \mathbb{R}$ (note that $U^{2,\nu}$ and $U^{2,-\nu}$ are equivalent).

ii) The UIR's $V^{2,q}$ in the complementary series where $p = s = 2$ and $q - q^2 = \mu$, correspond to

$$\langle Q_2^{(1)} \rangle = q - q^2 - 4 \equiv \mu - 4, \quad 0 < \mu < \frac{1}{4}. \tag{2.21}$$

iii) The UIR's $\Pi_{2,2}^{\pm}$ in the discrete series where $q = p = s = 2$ correspond to

$$\langle Q_2^{(1)} \rangle = -6. \tag{2.22}$$

The spin-2 "massless" field in de Sitter space corresponds to the latter case in which the sign $\pm$ in $\Pi_{2,2}^{\pm}$ stands for the helicity. A forthcoming paper will be entirely devoted to this specific field.

Equation (2.17) leads to $H^2(\langle Q_2^{(1)} \rangle + 6) = m_H^2$ which enables us to write the respective "mass" relations for the three types of UIR previously described :

$$m_H^2 = \begin{cases} m_p^2 = H^2(\nu^2 + \frac{9}{4}), & \nu \geq 0 \text{ (for the principal series)}, \\[2mm] m_c^2 = H^2(\mu + 2), & 0 < \mu < \frac{1}{4} \text{ (for the complementary series)}, \\[2mm] m_d^2 = 0 \text{ (for the discrete series)}. \end{cases} \tag{2.23}$$

The spin-2 "mass" range can be represented by

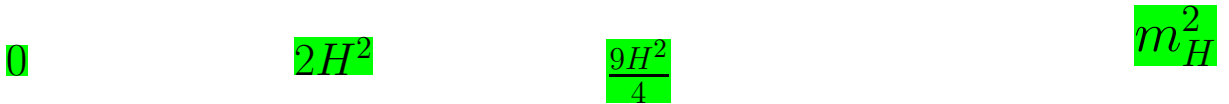$$0 \qquad\qquad 2H^2 \qquad\qquad \frac{9H^2}{4} \qquad\qquad m_H^2$$

FIG. 1: Mass range and spin-2 $\mathrm{SO}_0(1,4)$ unitary irreducible representations.

The forbidden mass range has been discussed by Higuchi in [21] and contrary to his point of view we do not consider m_H in the range of the complementary series as a "mass". This is because the complementary series with $p = 2$ is not linked to any physical representation in the Poincaré flat

limit sense. The crucial point is that m_c^2 (unlike m_p^2 !) is confined between the values 0 and $1/4$ and therefore simply vanishes in the limit $H = 0$. On the contrary, for the principal series, the contraction limit has to be understood through the constraint $m = H\nu$. The quantity m_H, supposed to depend on H, goes to the Minkowskian mass m when the curvature goes to zero. In short, we only consider as "massive" tensor fields, those ones for which the values assumed by the parameter m_H are in the range m_p which corresponds to the principal series of representations. Eq. (2.17) then gives

$$\left(\Box_H + 2H^2 + m_p^2\right) \mathcal{K}_{\alpha\beta}(x) = 0. \tag{2.24}$$

Let us recall at this point the physical content of the principal series representation from the point of view of a Minkowskian observer (at the limit $H = 0$). The principal series UIR $U^{2,\nu}, \ \nu \geq 0$, contracts toward the tensor massive Poincaré UIR's $P^<(m,2)$ and $P^>(m,2)$ with negative and positive energies respectively. Actually, the group representation contraction procedure is not unique and it has been shown that the principal series UIR can contract either toward the direct sum of the two tensor massive Poincaré UIR's [34]

$$U^{2,\nu} \xrightarrow[H\nu = m]{H \to 0, \nu \to \infty} P^<(m,2) \oplus P^>(m,2), \tag{2.25}$$

or simply (forthcoming paper)

$$U^{2,\nu} \xrightarrow[H\nu = m]{H \to 0, \nu \to \infty} P^>(m,2) \qquad \text{or} \qquad U^{2,-\nu} \xrightarrow[H\nu = m]{H \to 0, \nu \to \infty} P^<(m,2). \tag{2.26}$$

In contrast, in the massless spin-2 case, only the two aforementioned representations $\Pi_{2,2}^{\pm}$, in the discrete series with $p = q = 2$, have a Minkowskian interpretation. The representation $\Pi_{2,2}^{+}$ has a unique extension to a direct sum of two UIR's $C(3;2,0)$ and $C(-3;2,0)$ of the conformal group $SO_0(2,4)$ with positive and negative energies respectively [31, 35]. The latter restricts to the tensor massless Poincaré UIR's $P^>(0,2)$ and $P^<(0,2)$ with positive and negative energies respectively. The following diagrams illustrate these connections

$$
\begin{array}{ccccc}
 & \mathcal{C}(3,2,0) & & \mathcal{C}(3,2,0) & \leftarrow \mathcal{P}^>(0,2) \\
\Pi_{2,2}^+ \hookrightarrow & \oplus & \xrightarrow{H=0} & \oplus & \oplus \\
 & \mathcal{C}(-3,2,0) & & \mathcal{C}(-3,2,0) & \leftarrow \mathcal{P}^<(0,2),
\end{array}
\tag{2.27}
$$

$$
\begin{array}{ccccc}
 & \mathcal{C}(3,0,2) & & \mathcal{C}(3,0,2) & \leftarrow \mathcal{P}^>(0,-2) \\
\Pi_{2,2}^- \hookrightarrow & \oplus & \xrightarrow{H=0} & \oplus & \oplus \\
 & \mathcal{C}(-3,0,2) & & \mathcal{C}(-3,0,2) & \leftarrow \mathcal{P}^<(0,-2),
\end{array}
\tag{2.28}
$$

where the arrows $\hookrightarrow$ designate unique extension, and $\mathcal{P}^<_>(0,2)$ (resp. $\mathcal{P}^<_>(0,-2)$) are the massless Poincaré UIR's with positive and negative energies and positive (resp. negative) helicity.

III.   DE SITTER TENSOR WAVES

A.   Field Equation solution

Our aim is now to solve the "massive" spin-2 wave equation for the dS mode $\mathsf{K}(x)$

$$\left(Q_2^{(1)} - \langle Q_2^{(1)}\rangle\right)\mathsf{K}(x) = 0 \qquad \text{with} \qquad \langle Q_2^{(1)}\rangle = \nu^2 - \frac{15}{4}. \tag{3.1}$$

In ambient space notations, the most general transverse, symmetric field $\mathsf{K}_{\alpha\beta}(x)$ can be written in terms of two vector fields K, K_g and a scalar field ϕ through the following recurrence formula [29]

$$\mathsf{K} = \theta\phi + \mathcal{S}\bar{Z}_1 K + D_2 K_g, \tag{3.2}$$

with K satisfying the conditions (2.6). The symbol Z_1 denotes a constant vector and $\bar{Z}_{1\alpha} = \theta_{\alpha\beta}Z_1^{\beta}$, $x.\bar{Z}_1 = 0$. The operator D_2 is the generalized gradient $D_2 K = H^{-2}\mathcal{S}(\bar{\partial} - H^2 x)K$ which makes a symmetric transverse tensor field from the transverse vector K. The algebraic machinery valid for describing fields in anti-de Sitter space can be easily transferred *mutatis mutendis* to dS space formalism by the substitutions (see for instance [27, 29, 36]):

$$Q_s^{AdS} \longrightarrow -Q_s^{dS}, \quad (H^2)^{AdS} \longrightarrow -(H^2)^{dS}.$$

Reference [36] provides the following useful relations

$$Q_2\theta\phi = \theta Q_0\phi, \qquad Q_2 D_2 K_g = D_2 Q_1 K_g,$$

$$Q_2\mathcal{S}\bar{Z}_1 K = \mathcal{S}\bar{Z}_1(Q_1 - 4)K - 2H^2 D_2(x \cdot Z_1)K + 4\theta(Z_1 \cdot K). \tag{3.3}$$

Defining the generalized divergence $\partial_2 \cdot \mathsf{K} = \partial \cdot \mathsf{K} - H^2 x\mathsf{K}' - \frac{1}{2}\bar{\partial}\mathsf{K}'$ and $D_1 = H^{-2}\bar{\partial}$, one also has

$$\partial_2 \cdot \theta\phi = -H^2 D_1\phi, \qquad \partial_2 \cdot D_2 K_g = -(Q_1 + 6)K_g,$$

$$\partial_2 \cdot \mathcal{S}\bar{Z}_1 K = \bar{Z}_1\partial \cdot K - H^2 D_1(Z_1 \cdot K) - H^2 x(Z_1 \cdot K) + Z_1 \cdot \bar{\partial}K + 5H^2(Z_1 \cdot x)K. \tag{3.4}$$

Putting $\mathsf{K}_{\alpha\beta}(x)$ given by (3.2) into (3.1) and from the linear independence of the terms in (3.2) one gets

$$\left(Q_1 - \langle Q_1^{(1)}\rangle\right)K = 0 \quad \text{with} \quad \langle Q_1^{(1)}\rangle = \langle Q_2^{(1)}\rangle + 4\,, \tag{3.5}$$

$$\left(Q_0 - \langle Q_2^{(1)}\rangle\right)\phi = -4(Z_1 \cdot K)\,, \tag{3.6}$$

$$\left(Q_1 - \langle Q_2^{(1)}\rangle\right)K_g = 2H^2(x \cdot Z_1)K\,. \tag{3.7}$$

Note that in these formulas, $\langle Q_s^{(1)} \rangle$ corresponds to the principal series of representation with spin $\mathbf{s}$ and that K is chosen to be divergenceless. Using the equations (3.4), the divergenceless condition combined with Eq. (3.7) leads to

$$K_g = \frac{1}{\langle Q_0^{(1)} \rangle} \left[-H^2 D_1 (\phi + Z_1 \cdot K) + Z_1 \cdot \bar{\partial} K - H^2 x Z_1 \cdot K + 3H^2 x \cdot Z_1 K \right], \qquad (3.8)$$

where $\langle Q_0^{(1)} \rangle = \langle Q_2^{(1)} \rangle + 6$. Finally, the traceless condition which yields

$$\bar{\partial} \cdot K_g = -2H^2 \phi - H^2 Z_1 \cdot K, \qquad (3.9)$$

compared to the divergence of Eq. (3.8) allows to express ϕ in terms of K:

$$\phi = -\frac{2}{3} (Z_1 \cdot K). \qquad (3.10)$$

Thus, the fields K and ϕ are respectively "massive" vector field (*e.g.* transforming under the vector UIR $U^{1,\nu}$ of the principal series) [1], and "massive" scalar field (*e.g.* transforming under the scalar UIR $U^{0,\nu}$ of the principal series) [18]:

$$\left(Q_1 - \langle Q_1^{(1)} \rangle \right) K = 0, \qquad \text{and} \qquad \left(Q_0 - \langle Q_0^{(1)} \rangle \right) \phi = 0. \qquad (3.11)$$

Note that the equations for K and ϕ are compatible with the relation $\phi = -\frac{2}{3} Z_1 \cdot K$. The equations (3.8) and (3.10) show that the massive vector K determines completely the tensor field K which can now be written

$$\mathsf{K}(x) = \left(-\frac{2}{3} \theta Z_1 \cdot + \mathcal{S} \bar{Z}_1 + \frac{1}{\langle Q_0^{(1)} \rangle} D_2 [Z_1 \cdot \bar{\partial} - H^2 x Z_1 \cdot + 3H^2 x \cdot Z_1 - \frac{1}{3} H^2 D_1 Z_1 \cdot] \right) K. \qquad (3.12)$$

As explained in [1] the solutions to Eq. (3.5) are defined on connected open subsets of X_H such that $x.\xi \neq 0$, where $\xi \in \mathbb{R}^5$ lies on the null cone $\mathcal{C} = \{\xi \in \mathbb{R}^5; \ \xi^2 = 0\}$. They are homogeneous with degree $-\frac{3}{2} \mp i\nu$ on $\mathcal{C}$ and thus are entirely determined by specifying their values on a well chosen curve (the orbital basis) γ of $\mathcal{C}$. They can be written [1] as a product of a generalized polarization vector $\mathcal{E}_\alpha(x, \xi, Z_2)$ with the so-called [37](scalar) dS waves $(Hx \cdot \xi)^\sigma$ where $\sigma = -\frac{3}{2} - i\nu \in \mathbb{C}$. As such, the dS waves are multivalued and it will be explained later how suitable analyticity criteria yield univalued defined waves. The solutions to Eq. (3.5) read

$$K_\alpha(x) = \left(\frac{\sigma}{\sigma+1} \right) \mathcal{E}_\alpha(x, \xi, Z_2)(Hx.\xi)^\sigma, \quad \text{with} \quad \sigma = -\frac{3}{2} - i\nu, \qquad (3.13)$$

where Z_2 is another constant vector. Note that contrary to the Minkowskian case, the polarization tensor is a function of space-time. The simplest form of $\mathcal{E}_\alpha(x, \xi, Z_2)$ compatible with the Minkowski

polarization vector in the flat limit (see [1]) is obtained through the choice $\xi \cdot Z_2 = 0$ and reads

$$\mathcal{E}(x,\xi,Z_2) = \left(\bar{Z}_2{}^\lambda - \frac{Z_2^\lambda \cdot x}{x \cdot \xi} \bar{\xi} \right) \quad \text{with} \quad \mathcal{E}^\lambda(x,\xi,Z_2) \cdot \bar{\xi} = Z_2^\lambda \cdot \xi = 0 \,. \tag{3.14}$$

It is easy to see (flat limit) that the three Minkowski polarization four-vectors ϵ_μ^λ with $\mu = 0,1,2,3$ are linked to Z_2^λ by:

$$\lim_{H \to 0} \mathcal{E}_\alpha^\lambda(x,\xi,Z_2) = Z_{2\mu}^\lambda - \frac{Z_{24}^\lambda}{\xi_4} \xi_\mu \equiv \epsilon_\mu^\lambda. \tag{3.15}$$

We demand that the Minkowski polarization vectors satisfy the usual relations

$$\epsilon^\lambda \cdot k = 0, \quad \epsilon^\lambda \cdot \epsilon^{\lambda'} = \eta^{\lambda\lambda'}, \quad \sum_{\lambda=1}^{3} \epsilon_\mu^\lambda(k)\, \epsilon_\nu^\lambda(k) = - \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m^2} \right) \equiv \Pi_{\mu\nu}(k), \tag{3.16}$$

which is achieved if the Z_2^λ's are such that

$$Z_2^\lambda \cdot \xi = 0, \quad Z_2^\lambda \cdot Z_2^{\lambda'} = \eta^{\lambda\lambda'}, \quad \sum_{\lambda=1}^{3} Z_{2\alpha}^\lambda Z_{2\beta}^\lambda = -\eta_{\alpha\beta} \quad \text{and} \quad \sum_{\lambda=1}^{3} Z_{24}^\lambda Z_{2\mu}^\lambda = 0 \; \forall \, \mu \,. \tag{3.17}$$

These conditions are easily derived by working with a well adapted (to the flat limit) orbital basis. This basis, characterized by the values ± 1 of the component ξ_4 will be discussed later on. A remarkable feature connected with the use of ambient space notations is that with Eq. (3.17) one shows that the properties of the dS polarization vector are very similar to the Minkowskian case:

$$\sum_{\lambda=1}^{3} \mathcal{E}_\alpha^\lambda(x,\xi,Z_2)\, \mathcal{E}_\beta^\lambda(x,\xi,Z_2) = - \left(\theta_{\alpha\beta} - \frac{\bar{\xi}_\alpha \bar{\xi}_\beta}{(Hx \cdot \xi)^2} \right) \equiv \Pi_{\alpha\beta}(x,\xi) \,,$$
$$\mathcal{E}^\lambda(x,\xi,Z_2) \cdot \mathcal{E}^{\lambda'}(x,\xi,Z_2) = Z_2^\lambda \cdot \mathcal{E}^{\lambda'}(x,\xi,Z_2) = \mathcal{E}^\lambda(x,\xi,Z_2) \cdot \mathcal{E}^{\lambda'}(x',\xi,Z_2) = \eta^{\lambda\lambda'}. \tag{3.18}$$

It follows from Eq. (3.12) that the two spin-2 families of solutions to Eq. (3.1) read

$$\mathsf{K}(x) = \theta\phi + \mathcal{S}\bar{Z}_1 K + D_2 K_g \equiv \mathcal{D}(x,\partial,Z_1,Z_2)(Hx \cdot \xi)^{-\frac{3}{2}\mp i\nu}$$

where the operator $\mathcal{D}(x,\partial,Z_1,Z_2)$ is given by

$$\left(\frac{\sigma}{\sigma+1} \right) \left(-\frac{2}{3}\theta Z_1 \cdot + \mathcal{S}\bar{Z}_1 + \frac{1}{\langle Q_0^{(1)} \rangle} D_2 [\, Z_1 \cdot \bar{\partial} - H^2 x Z_1 \cdot + 3H^2 x \cdot Z_1 - \frac{1}{3}H^2 D_1 Z_1 \cdot \,] \right) \mathcal{E}(x,\xi,Z_2). \tag{3.19}$$

These spin-2 solutions can be brought into the form

$$\mathsf{K}_{\alpha\beta}(x) = a_\nu \, \mathcal{E}_{\alpha\beta}(x,\xi,Z_1,Z_2)(Hx \cdot \xi)^\sigma \quad \text{and} \quad \mathsf{K}_{\alpha\beta}^*(x) \quad \text{with} \quad a_\nu = c_\nu \left(\frac{2(\sigma-1)}{\sigma+1} \right). \tag{3.20}$$

where the $\mathcal{E}_{\alpha\beta}$'s are the generalized polarization tensor components, c_ν is a normalization constant and where we have again omitted the superscript λ. Because of the conditions $\mathsf{K}_{\alpha\beta} = \mathsf{K}_{\beta\alpha}$, $\partial.\mathsf{K} = 0$,

and $x \cdot \mathsf{K} = 0$, the 25 components $\mathcal{E}_{\alpha\beta}$ reduce to 5 independent components which correspond precisely to the $2s + 1 = 5$ degrees of freedom of a spin-2 field.

The arbitrariness due to the introduction of the constant vectors Z_1, Z_2 in our solution has partly been removed in (3.17), by comparison with the Minkowski polarization vector one eventually reaches by going to the flat limit (see 3.15). We now apply the same procedure in order to fix the value of Z_1, that is we investigate the behaviour of equation (3.19) in the $H = 0$ limit. More precisely, we show that $\mathcal{E}_{\alpha\beta}(x, \xi, Z_1, Z_2)$ contracts toward the usual Minkowski tensor polarization and takes a simple form if Z_1 is chosen to be equal to Z_2 and denoted by Z in the following. It is a matter of simple calculation to get the de Sitter polarization tensor starting with Formula (3.19):

$$\mathcal{E}_{\alpha\beta}(x, \xi, Z) \equiv \mathcal{E}_{\alpha\beta}^{\lambda\lambda'}(x, \xi) = \frac{1}{2} \left[\mathcal{S} \, \mathcal{E}_{\alpha}^{\lambda}(x, \xi) \, \mathcal{E}_{\beta}^{\lambda'}(x, \xi) - \frac{2}{3} \left(\theta_{\alpha\beta} - \frac{\bar{\xi}_{\alpha} \bar{\xi}_{\beta}}{(Hx \cdot \xi)^2} \right) \mathcal{E}^{\lambda}(x, \xi) \cdot \mathcal{E}^{\lambda'}(x, \xi) \right] ,$$

(3.21)

where $\mathcal{E}^{\lambda}(x, \xi) = \mathcal{E}^{\lambda}(x, \xi, Z_2)$. In view of (3.18) one obtains

$$\mathcal{E}_{\alpha\beta}^{\lambda\lambda'}(x, \xi) = \frac{1}{2} \left[\mathcal{S} \, \mathcal{E}_{\alpha}^{\lambda}(x, \xi) \, \mathcal{E}_{\beta}^{\lambda'}(x, \xi) + \frac{2}{3} \eta^{\lambda\lambda'} \sum_{\rho} \mathcal{E}_{\alpha}^{\rho}(x, \xi) \mathcal{E}_{\beta}^{\rho}(x, \xi) \right] .$$

(3.22)

It is easy to check that the tensor polarization (3.22) satisfies the properties $\eta^{\alpha\beta} \mathcal{E}_{\alpha\beta}(x, \xi, Z) = 0$ (tracelessness), $\xi \cdot \mathcal{E}_{\alpha\beta}(x, \xi, Z) = 0$ and the relation

$$\mathcal{E}^{\lambda\lambda'}(x, \xi) \cdot \cdot \, \mathcal{E}^{\lambda''\lambda'''}(x, \xi) = \mathcal{E}^{\lambda\lambda'}(x', \xi) \cdot \cdot \, \mathcal{E}^{\lambda''\lambda'''}(x, \xi) = \left[\eta^{\lambda\lambda''} \eta^{\lambda'\lambda'''} + \eta^{\lambda\lambda'} \eta^{\lambda''\lambda'''} \right] .$$

(3.23)

The dS tensor waves $\mathsf{K}_{\alpha\beta}(x)$ are homogeneous with degree σ on the null cone $\mathcal{C}$ and on the dS submanifold X_H characterized by $x \cdot x = -H^{-2}$ with H being constant. This is due to:

$$\mathcal{E}^{\lambda}(x, a\xi) = \mathcal{E}^{\lambda}(x, \xi) \quad \text{and} \quad \mathcal{E}^{\lambda}(ax, \xi) = \mathcal{E}^{\lambda}(x, \xi),$$

which is obvious from the definition of $\mathcal{E}^{\lambda}(x, \xi)$

$$\mathcal{E}^{\lambda}(x, \xi) = \left(\bar{Z}^{\lambda} - \frac{Z^{\lambda} \cdot x}{\xi \cdot x} \bar{\xi} \right) = \left(Z^{\lambda} - \frac{Z^{\lambda} \cdot x}{\xi \cdot x} \xi \right) .$$

(3.24)

Note that as a function of $\mathbb{R}^5$, the wave $\mathsf{K}_{\alpha\beta}(x)$ is homogeneous with degree zero ($H(x) = -1/\sqrt{-x \cdot x}$).

B. Flat limit and analytic tensor wave

It order to compute the flat limit of the polarization tensor, it is useful to precise the notion of orbital basis γ for the future null cone $\mathcal{C}^+ = \{\xi \in \mathcal{C}; \, \xi^0 > 0\}$, [37]. Let us choose a unit vector e in $\mathbb{R}^5$ and let H_e be its stabilizer subgroup in $S0_0(1, 4)$. Then two types of orbits are interesting in the present context :

(i) the spherical type γ_0 corresponds to $e \in V^+ \equiv \{x \in \mathbb{R}^5; \ x^0 > \sqrt{\| \vec{x} \|^2 + (x^4)^2}\}$, and is an orbit of $H_e \approx SO(4)$.

$$\gamma_0 = \{\xi \, ; \ e \cdot \xi = a > 0\} \cap \mathcal{C}^+ \, .$$

(ii) the hyperbolic type γ_4 corresponds to $e^2 = -1$. It is divided into two hyperboloid sheets, both being orbits of $H_e \approx SO_0(1,3)$.

The most suitable parametrization when one has in view the link with massive Poincaré UIR's is to work with the orbital basis of the second type

$$\gamma_4 = \{\xi \, \in C^+, \xi^{(4)} = 1\} \cup \{\xi \, \in C^+, \xi^{(4)} = -1\},$$

with the null vector ξ given in terms of the four-momentum $(k^0, \vec{k})$ of a Minkowskian particle of mass m

$$\xi_\pm = \left(\frac{k^0}{mc} = \sqrt{\frac{\vec{k}^2}{m^2 c^2} + 1}, \frac{\vec{k}}{mc}, \pm 1 \right) . \tag{3.25}$$

An appropriate choice of global coordinates is given by

$$\begin{cases} x^0 &= H^{-1} \sinh(HX^0) \, , \\ \vec{x} &= (H \| \vec{X} \|)^{-1} \vec{X} \cosh(HX^0) \sin(H \| \vec{X} \|) \, , \\ x^4 &= H^{-1} \cosh(HX^0) \cos(H \| \vec{X} \|) \, . \end{cases} \tag{3.26}$$

where the dS point is expressed in terms of the Minkowskian variables $X = (X_0 = ct, \vec{X})$ measured in units of the dS radius H^{-1}.

The Minkowskian limit of the dS waves at point x can be written as [18]

$$\begin{aligned} \lim_{H \to 0} (Hx \cdot \xi_-)^\sigma &= \exp[-ik \cdot X] \quad \text{(positive energy)}, \\ \lim_{H \to 0} e^{-i\pi\sigma} (Hx \cdot \xi_+)^\sigma &= \exp[ik \cdot X] \quad \text{(negative energy)}. \end{aligned} \tag{3.27}$$

Since the contraction is done with respect to the Lorentz subgroup $SO_0(1,3)$ (γ_4 is invariant under $SO_0(1,3)$) the equations (3.27) indicate that the orbital basis γ_4 can contract toward the sum of two solutions with opposite energies (see [34]).

The polarization tensor limit is easily obtained with the help of

$$\lim_{H \to 0} H^2 \sigma^2 = -m^2, \quad \lim_{H \to 0} \mathcal{E}_\alpha^\lambda(x, \xi) = \epsilon_\mu^\lambda(k) \, , \quad \lim_{H \to 0} \theta_{\alpha\beta} = \eta_{\mu\nu} \, , \quad \lim_{H \to 0} \bar{\xi}_\alpha = \frac{k_\mu}{m} \ \forall \ \xi \in \gamma_4.$$

Finally one recovers the Minkowskian massive spin-2 polarization tensor [38]:

$$\lim_{H \to 0} \mathcal{E}_{\alpha\beta}^{\lambda\lambda'}(x,\xi) = \epsilon_{\mu\nu}^{\lambda\lambda'}(k) = \frac{1}{2}\mathcal{S} \, \epsilon_{\mu}^{\lambda}(k) \, \epsilon_{\nu}^{\lambda'}(k) + \frac{1}{3}\eta^{\lambda\lambda'} \sum_{\lambda} \epsilon_{\mu}^{\lambda}(k) \, \epsilon_{\nu}^{\lambda}(k),$$

which satisfies $\eta^{\mu\nu}\epsilon_{\mu\nu}^{\lambda\lambda'}(k) = k^{\mu}\epsilon_{\mu\nu}^{\lambda\lambda'}(k) = 0$ and

$$\sum_{\lambda\lambda'} \epsilon_{\mu\nu}^{\lambda\lambda'}(k)\epsilon_{\rho\pi}^{\lambda\lambda'}(k) = \frac{1}{2}\left[\Pi_{\mu\rho}(k)\Pi_{\nu\pi}(k) + \Pi_{\nu\rho}(k)\Pi_{\mu\pi}(k)\right] - \frac{1}{3}\left[\Pi_{\mu\nu}(k)\Pi_{\rho\pi}(k)\right]. \tag{3.28}$$

Hence, we have shown that in the limit $H = 0$, $(Hx \cdot \xi)^{\sigma}$ and $\mathcal{E}_{\alpha\beta}(x,\xi,Z)$ behave like the plane wave $e^{ik \cdot X}$ and the polarization tensor in Minkowski space-time respectively.

Although the "massive" field equation solutions $\mathsf{K}_{\alpha\beta}(x)$ and $\mathsf{K}_{\alpha\beta}^{*}(x)$ are complex conjugated, they cannot be associated with the positive and negative energies respectively as in the Minkowskian situation. Actually, despite the fact that the solutions are globally defined (in a distributional sense) in dS space, the concept of energy is not (absence of global timelike killing vector field). As a result, concepts like "particle" and "antiparticle" are rather unclear and the differences between these two solutions is not really explained or understood. In terms of group representation these two solutions are equivalent, because the two representations $U^{2,\nu}$ and $U^{2,-\nu}$ are. Note that the minimally coupled scalar field requires both sets of solutions in order to achieve a covariant quantization [12]. This will certainly also be the case for the spin-2 massless field in dS space since it is constructed from a minimally coupled scalar field as it will be shown in [10].

In the present case, the "massive" free field covariant quantization can be constructed from the positive norm states alone since $\mathsf{K}_{\alpha\beta}(x)$ is closed under the group action:

$$(U(g)\,\mathsf{K})_{\alpha\beta}(x) = g_{\alpha}^{\gamma}g_{\beta}^{\delta}\mathsf{K}_{\gamma\delta}(g^{-1}x) = g_{\alpha}^{\gamma}g_{\beta}^{\delta} \, a_{\nu} \, \mathcal{E}_{\gamma\delta}(g^{-1}x,\xi,Z)(Hg^{-1}x \cdot \xi)^{\sigma} = a_{\nu} \, \mathcal{E}_{\alpha\beta}(x,g\xi,gZ)(Hx \cdot g\xi)^{\sigma}$$

$$\tag{3.29}$$

This is easily proved since the vector polarization satisfies

$$\mathcal{E}_{\alpha}(g^{-1}x,\xi,Z) = \left(Z_{\alpha} - \frac{g^{-1}x \cdot Z}{g^{-1}x \cdot \xi}\xi_{\alpha}\right) = \left(Z_{\alpha} - \frac{x \cdot gZ}{x \cdot g\xi}\xi_{\alpha}\right) = (g^{-1})_{\alpha}^{\delta}\mathcal{E}_{\delta}(x,g\xi,gZ). \tag{3.30}$$

The dS waves solutions, as functions on de Sitter space, are only locally defined since they are singular on specific lower dimensional subsets of X_H, for instance on spatial boundary defined by $x^0 = \pm x^4 \Leftrightarrow x_1^2 + x_2^2 + x_3^2 = H^{-2}$, and multivalued on dS space-time. In order to get a global definition, they have to be viewed as distributions [39] which are boundary values of analytic continuations of the solutions to tubular domains in the complexified de Sitter space $X_H^{(c)}$. The latter are defined as follows:

$$\begin{aligned} X_H^{(c)} &= \{z = x + iy \in \mathbb{C}^5; \; \eta_{\alpha\beta}z^{\alpha}z^{\beta} = (z^0)^2 - \vec{z}.\vec{z} - (z^4)^2 = -H^{-2}\} \\ &= \{(x,y) \in \mathbb{R}^5 \times \mathbb{R}^5; \; x^2 - y^2 = -H^{-2}, \; x \cdot y = 0\}. \end{aligned}$$

For an univalued determination, we must introduce the forward and backward tubes of $X_H^{(c)}$. First of all, let $T^\pm = \mathbb{R}^5 - iV^\pm$ be the forward and backward tubes in $\mathbb{C}^5$. The domain V^+ (resp. V^-) stems from the causal structure on X_H:

$$V^\pm = \{x \in \mathbb{R}^5; \ x^0 \gtrless \sqrt{\|\vec{x}\|^2 + (x^4)^2}\}. \tag{3.31}$$

We then introduce their respective intersections with $X_H^{(c)}$,

$$\mathcal{T}^\pm = T^\pm \cap X_H^{(c)}, \tag{3.32}$$

which are the tubes of $X_H^{(c)}$. Finally we define the "tuboid" above $X_H^{(c)} \times X_H^{(c)}$ by

$$\mathcal{T}_{12} = \{(z, z'); \ z \in \mathcal{T}^+, z' \in \mathcal{T}^-\}. \tag{3.33}$$

Details are given in [37]. When z varies in $\mathcal{T}^+$ (or $\mathcal{T}^-$) and ξ lies in the positive cone $\mathcal{C}^+$ the wave solutions are globally defined because the imaginary part of $(z.\xi)$ has a fixed sign and $z.\xi \neq 0$.

We define the de Sitter tensor wave $\mathsf{K}_{\alpha\beta}(x)$ as the boundary value of the analytic continuation to the future tube of Eq. (3.20). Hence, for $z \in \mathcal{T}^+$ and $\xi \in \mathcal{C}^+$ one gets the two solutions

$$\mathsf{K}_{\alpha\beta}(z) = a_\nu \, \mathcal{E}_{\alpha\beta}^{\lambda\lambda'}(z,\xi) \, (Hz \cdot \xi)^\sigma, \quad \text{and} \quad \mathsf{K}_{\alpha\beta}^*(z^*) = a_\nu^* \, \mathcal{E}_{\alpha\beta}^{*\lambda\lambda'}(z^*,\xi) \, (Hz \cdot \xi)^{\sigma^*}. \tag{3.34}$$

IV. TWO-POINT FUNCTION AND QUANTUM FIELD

A. The two-point function

As explained in [37], the dS axiomatic field theory is based on the Wightman two-point double tensor-valued function

$$\mathcal{W}_{\alpha\beta\alpha'\beta'}(x,x') \qquad \alpha', \beta' = 0, 1, .., 4. \tag{4.1}$$

Indeed, this kernel entirely encodes the theory of the generalized free fields on dS space-time X_H, at least for the massive case. For this, it has to satisfy the following requirements:

a) **Positiveness**

for any test function $f_{\alpha\beta} \in \mathcal{D}(X_H)$, we have

$$\int_{X_H \times X_H} f^{*\alpha\beta}(x) \mathcal{W}_{\alpha\beta\alpha'\beta'}(x,x') f^{\alpha'\beta'}(x') d\sigma(x) d\sigma(x') \geq 0, \tag{4.2}$$

where $d\sigma(x)$ denotes the dS-invariant measure on X_H [37]. $\mathcal{D}(X_H)$ is the space of functions C^∞ with compact support in X_H.

b) **Locality** for every space-like separated pair (x, x'), *i.e.* $x \cdot x' > -H^{-2}$,

$$\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x') = \mathcal{W}_{\alpha'\beta'\alpha\beta}(x', x). \tag{4.3}$$

c) **Covariance**

$$(g^{-1})^{\gamma}_{\alpha}(g^{-1})^{\delta}_{\beta}\mathcal{W}_{\gamma\delta\gamma'\delta'}(gx, gx')g^{\gamma'}_{\alpha'}g^{\delta'}_{\beta'} = \mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x'), \tag{4.4}$$

for all $g \in \mathrm{SO}_0(1, 4)$.

d) **Index symmetrizer**

$$\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x') = \mathcal{W}_{\alpha\beta\beta'\alpha'}(x, x') = \mathcal{W}_{\beta\alpha\alpha'\beta'}(x, x'). \tag{4.5}$$

e) **Transversality**

$$x^{\alpha}\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x') = 0 = x'^{\alpha'}\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x'). \tag{4.6}$$

f) **Divergencelessness**

$$\partial_x^{\alpha}\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x') = 0 = \partial_{x'}^{\alpha'}\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x'). \tag{4.7}$$

g) **Normal analyticity** $\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x')$ is the boundary value (bv) in the distributional sense of an analytic function $W_{\alpha\beta\alpha'\beta'}(z, z')$.

Concerning the last requirement, $W_{\alpha\beta\alpha'\beta'}(z, z')$ is actually maximally analytic, *i.e.* can be analytically continued to the "cut domain"

$$\Delta = \{(z, z') \in X_H^{(c)} \times X_H^{(c)} \ : \ (z - z')^2 < 0\}.$$

The Wightman two-point function $\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x')$ is the boundary value of $W_{\alpha\beta\alpha'\beta'}(z, z')$ from $\mathcal{T}_{12}$ and the "permuted Wightman function" $\mathcal{W}_{\alpha'\beta'\alpha\beta}(x', x)$ is the boundary value of $W_{\alpha\beta\alpha'\beta'}(z, z')$ from the domain

$$\mathcal{T}_{21} = \{(z, z'); \ z \in \mathcal{T}^-, z' \in \mathcal{T}^+\}.$$

Once these properties are satisfied, the reconstruction theorem [40] allows to recover the corresponding quantum field theory. Our present task is therefore to find a doubled tensor valued analytic function of the variable (z, z') satisfying the properties a) to g). Following Reference [37] (in which the construction

has been done for the scalar case), the analytic two-point function $W_{\alpha\beta\alpha'\beta'}(z,z') \equiv W^{\nu}_{\alpha\beta\alpha'\beta'}(z,z')$ is obtained from the dS tensor waves (3.34). The parameter ν refers to the principal series. The two-point function is given in terms of the following class of integral representations

$$W^{\nu}_{\alpha\beta\alpha'\beta'}(z,z') = |a_{\nu}|^2 \int_{\gamma} (Hz \cdot \xi)^{\sigma} (Hz' \cdot \xi)^{\sigma^*} \sum_{\lambda\lambda'} \mathcal{E}^{\lambda\lambda'}_{\alpha\beta}(z,\xi) \, \mathcal{E}^{*\lambda\lambda'}_{\alpha'\beta'}(z'^*,\xi) \, d\sigma_{\gamma}(\xi), \qquad (4.8)$$

where $d\sigma_{\gamma}(\xi)$ is the natural $\mathcal{C}^+$ invariant measure on γ, induced from the $\mathbb{R}^5$ Lebesgue measure [37] and the normalization constant a_{ν} is fixed by local Hadamard condition. The latter selects a unique vacuum state for quantum tensor fields which satisfies the dS field equation. In order to check wether condition a) to g) are satisfied by Eq. (4.8) let us first rewrite the two-point function in a more explicit way. This will be done by using the scalar and the vector "massive" analytic two-point functions $W^{\nu}_0(z,z')$, $W^{\nu}_1(z,z')$ (where $\mathcal{Z} = -H^2 z \cdot z'$). The latter satisfy the complex versions of the Casimir equations:

$$\left(Q_1 - \langle Q_1^{(1)} \rangle \right) W^{\nu}_1(z,z') = 0 \qquad \text{and} \qquad \left(Q_0 - \langle Q_0^{(1)} \rangle \right) W^{\nu}_0(z,z') = 0. \qquad (4.9)$$

In appendix C and in Reference [1] it is shown how $W^{\nu}_1(z,z')$ can be written in terms of the scalar analytic two-point function

$$W^{\nu}_1(z,z') = \frac{\langle Q_0 \rangle}{\langle Q_1 \rangle} \left(-\theta_{\alpha} \cdot \theta'_{\alpha'} + \frac{H^2 \sigma(\theta \cdot z') D_1'}{\langle Q_0 \rangle} + \frac{H^2 \sigma^*(\theta' \cdot z) D_1}{\langle Q_0 \rangle} + \frac{H^2 \mathcal{Z} D_1 D_1'}{\langle Q_0 \rangle} \right) W^{\nu}_0(z,z'). \quad (4.10)$$

The Wightman scalar two-point function $\mathcal{W}^{\nu}_0(x,x')$ is given by [37]

$$\mathcal{W}^{\nu}_0(x,x') = \text{bv} \, W_0(z,z') \qquad \text{with} \qquad W^{\nu}_0(z,z') = c_{\nu}^2 \int_{\gamma} (Hz \cdot \xi)^{\sigma} (Hz' \cdot \xi)^{\sigma^*} \, d\sigma_{\gamma}(\xi). \qquad (4.11)$$

The normalization constant c_{ν}^2 is determined by imposing the Hadamard condition on the two-point function. This has been done in Ref. [37] where the scalar two-point function has been rewritten in terms of the generalized Legendre function for well chosen space like separated points z and z'. It has been established that $W_0(z,z') = C_{\nu} P^{(5)}_{\sigma}(-\mathcal{Z})$ with $C_{\nu} = 2\pi^2 e^{-\pi\nu} c_{\nu}^2$ and

$$c_{\nu}^2 = \frac{H^2 e^{\pi\nu} \Gamma(-\sigma)\Gamma(-\sigma^*)}{2^5 \pi^4 m^2}. \qquad (4.12)$$

This normalization corresponds to the Euclidean vacuum [37] and $P^{(5)}_{\sigma}(\mathcal{Z})$ is the generalized Legendre function of the first kind. There are several reasons which explain the appearance of $W^{\nu}_0(z,z')$ and $W^{\nu}_1(z,z')$. First of all, both correspond to the commonly used two-point functions (see for instance reference [26]) as it is checked in Appendix C. Moreover, since the vector two-point function is written in terms of the scalar two-point function it exhibits the two building blocks of the tensor expression

which are well known and simple to manipulate. As a matter of fact, the flat limit is very easy to compute in this framework.

We have seen that the spin-2 analytic two-point function (4.8) is obtained from the tensor waves (3.34). Let us cast the latter into the more suitable form

$$\mathcal{K}(z) = \frac{a_\nu}{2} \left[\mathcal{S} \, \mathcal{E}^\lambda(z,\xi) \mathcal{E}^{\lambda'}(z,\xi) - \frac{2\sigma g^{\lambda\lambda'}}{3(\sigma-1)} \left(\theta - \frac{H^2 D_2 D_1}{2\sigma^2} \right) \right] (Hz \cdot \xi)^\sigma \,, \qquad (4.13)$$

by using the property

$$\sum_\lambda \mathcal{E}^\lambda(z,\xi) \, \mathcal{E}^\lambda(z,\xi) \, (Hz \cdot \xi)^\sigma = -\left(\theta - \frac{\bar{\xi}\bar{\xi}}{(Hz \cdot \xi)^2} \right) (Hz \cdot \xi)^\sigma = -\frac{\sigma}{\sigma-1} \left[\theta - \frac{H^2 D_2 D_1}{2\sigma^2} \right] (Hz \cdot \xi)^\sigma \,. \qquad (4.14)$$

We then simply develop the two-point function and obtain :

$$
\begin{aligned}
W^\nu(z,z') &= \frac{|a_\nu|^2}{4} \int_\gamma \mathcal{S}\mathcal{S}' \left(\sum_\lambda \mathcal{E}^\lambda(z,\xi) \, \mathcal{E}^{*\lambda}(z'^*,\xi) \right) \left(\sum_{\lambda'} \mathcal{E}^{\lambda'}(z,\xi) \, \mathcal{E}^{*\lambda'}(z'^*,\xi) \right) (Hz \cdot \xi)^\sigma (Hz' \cdot \xi)^{\sigma^*} \, d\sigma_\gamma(\xi) \\
&\quad - \frac{4}{3} \frac{\langle Q_0 \rangle}{\langle Q_1 \rangle} \left[\theta - \frac{H^2 D_2 D_1}{2\sigma^2} \right] \left[\theta' - \frac{H^2 D_2' D_1'}{2\sigma^{*2}} \right] c_\nu^2 \int_\gamma (Hz \cdot \xi)^\sigma (Hz' \cdot \xi)^{\sigma^*} \, d\sigma_\gamma(\xi) \,. \qquad (4.15)
\end{aligned}
$$

From the property

$$\sum_\lambda \mathcal{E}^\lambda(z) \mathcal{E}^{*\lambda}(z'^*) = \left[-\theta \cdot \theta' + \frac{(\theta \cdot z')\bar{\xi}'}{z' \cdot \xi} + \frac{(\theta' \cdot z)\bar{\xi}}{z \cdot \xi} + \frac{\mathcal{Z} \bar{\xi} \bar{\xi}'}{H^2 z \cdot \xi z' \cdot \xi} \right] \,, \qquad (4.16)$$

and the relation $H^2 \, D_2 \, K(x) = (\sigma-1) \, \mathcal{S} \, \bar{\xi} \, K(x) / (z \cdot \xi)$, it is clear that the analytic two-point function can be written in the general form:

$$W^\nu(z,z') = M(z,z') \, W_1^\nu(z,z') + N(z,z') \, W_0^\nu(z,z') \,. \qquad (4.17)$$

The differential operators $M(z,z')$ and $N(z,z')$ are given by

$$
\begin{aligned}
M(z,z') &= \frac{\langle Q_0 \rangle + 4}{\langle Q_0 \rangle} \left[-\mathcal{S}\mathcal{S}'\theta \cdot \theta' + \frac{H^2 \mathcal{S}(\theta \cdot z') D_2'}{\sigma^* - 1} + \frac{H^2 \mathcal{S}'(\theta' \cdot z) D_2}{\sigma - 1} + \frac{\mathcal{Z} H^2 D_2 D_2'}{(\sigma-1)(\sigma^*-1)} \right] \,, \\
N(z,z') &= \frac{4}{3} \frac{\langle Q_0 \rangle}{\langle Q_1 \rangle} \left[\theta - \frac{H^2 D_2 D_1}{2\sigma^2} \right] \left[\theta' - \frac{H^2 D_2' D_1'}{2\sigma^{*2}} \right] \,. \qquad (4.18)
\end{aligned}
$$

Eventually, the analytic tensor two-point function is given in terms of the scalar analytic two-point function by:

$$W^\nu_{\alpha\beta\alpha'\beta'}(z,z') = D(z,z') W_0^\nu(z,z') \,,$$

with $D(z,z')$ a differential operator discussed in appendix D. The boundary value of $W^\nu(z,z')$ gives the following integral representation for the Wightman two-point function:

$$\mathcal{W}(x,x') = |a_\nu|^2 \sum_{\lambda\lambda'} \int_\gamma d\sigma_\gamma(\xi) \mathcal{E}^{\lambda\lambda'}(x,\xi) \mathcal{E}^{*\lambda\lambda'}(x',\xi) \, \mathrm{bv} \, (Hz \cdot \xi)^\sigma \left(Hz' \cdot \xi \right)^{\sigma^*} \,, \qquad (4.19)$$

with

$$\mathrm{bv}\,(Hz \cdot \xi)^{\sigma}\,(Hz' \cdot \xi)^{\sigma^*} = |Hx{\cdot}\xi|^{\sigma}|Hx'{\cdot}\xi|^{\sigma^*}\big[\theta(Hx{\cdot}\xi)+\theta(-Hx{\cdot}\xi)\,e^{-i\pi\sigma}\big]\big[\theta(Hx'{\cdot}\xi)+\theta(-Hx'{\cdot}\xi)\,e^{+i\pi\sigma^*}\big].$$

(4.20)

This relation defines the two-point function in terms of global waves on the real hyperboloid X_H.

Let us now check if this kernel fulfills the conditions a) to g) required in order to get a Wightman two-point function. We recall that the existence of the latter which is requested by dS axiomatic field theory.

- The positiveness property follows from the relation

$$\int_{X_H \times X_H} f^{*\alpha\beta}(x)\mathcal{W}_{\alpha\beta\alpha'\beta'}(x,x')f^{\alpha'\beta'}(x')d\sigma(x)d\sigma(x') = |a_\nu|^2 \int_\gamma d\sigma_\gamma(\xi)\sum_{\lambda\lambda'}g^{*\lambda\lambda'}(\xi)\,g^{\lambda\lambda'}(\xi),$$

(4.21)

where

$$g^{\lambda\lambda'}(\xi) = \int_{X_H} d\sigma(x)f^{\alpha\beta}(x)\mathcal{E}^{*\lambda\lambda'}_{\alpha\beta}(x,\xi)\big[\theta(Hx \cdot \xi) + \theta(-Hx \cdot \xi)\,e^{+i\pi\sigma^*}\big]|Hx \cdot \xi|^{\sigma^*}.$$

(4.22)

The hermiticity property is obtained, by considering boundary values of the following identity

$$W_{\alpha\beta\alpha'\beta'}(z,z') = W^*_{\alpha'\beta'\alpha\beta}(z'^*, z^*),$$

(4.23)

which is easily checked on Eq. (4.8).

- In order to prove the locality condition, we use the hermiticity condition and the following relation:

$$W^*_{\alpha'\beta'\alpha\beta}(z'^*, z^*) = W_{\alpha'\beta'\alpha\beta}(z', z).$$

This easily follows from the form of the two-point function for space-like separated points given in Appendix D :

$$W^\nu(z,z') = C_\nu D(z,z')P^{(5)}_\sigma(-\mathcal{Z}) \quad \text{with} \quad D^*(z^*, z'^*) = D(z,z'),$$

and from the relation [41]

$$P^{(5)}_\sigma(-\mathcal{Z}) = P^{(5)}_{\sigma^*}(-\mathcal{Z}).$$

One finally gets

$$W_{\alpha\beta\alpha'\beta'}(z,z') = W^*_{\alpha'\beta'\alpha\beta}(z'^*, z^*) = W_{\alpha'\beta'\alpha\beta}(z', z).$$

It should be noticed that the space-like separated pair (x, x') lies in the same orbit of the complex dS group as the pairs (z, z') and (z'^*, z^*). Therefore the locality condition $\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x') = \mathcal{W}_{\alpha'\beta'\alpha\beta}(x', x)$ holds.

- The group action on the dS modes (3.29) and the independence of the integral (4.8) with respect to the selected orbital basis entail the covariance property

$$(g^{-1})^\gamma_\alpha (g^{-1})^\delta_\beta \mathcal{W}_{\gamma\delta\gamma'\delta'}(gx, gx') g^{\gamma'}_{\alpha'} g^{\delta'}_{\beta'} = \mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x'). \tag{4.24}$$

- The symmetry with respect to the indices α, β and α', β' and the transversality with respect to x and x' are guaranteed by construction. So is the divergencelessness condition.

- The analyticity properties of the tensor Wightman two-point function follow from the expression of the dS tensor waves (3.34).

Remark

A massive spin-2 two-point function had already been proposed in Ref. [20]. Although the approach we have used here is very different (in Ref. [20] the coordinates are non global, the modes have a spin-0 and spin-2 content..) it has been possible to check that our vector two-point function is in agreement with the one presented in [20]. This is of importance since it confirms for tensor fields the validity of the integral representation method (4.8) originated in [37] for the scalar case. However, explicit comparison for the spin-2 case would be a rather tedious task given the differences between both formalisms and the involved expression of the spin-2 two-point function given in [20]. It seems that one can at least say that the ambient space formalism presents the advantage of simplicity. This is again verified by performing the flat limit as it is seen in the next paragraph and this was already the case when the unitary irreducible representations had to be identified in section II.

B. The flat limit

The flat limit is straightforward to compute with the help of the orbital basis γ_4. The measure $d\sigma_{\gamma_4}(\xi)$ is chosen to be m^2 times the natural one induced from the $\mathbb{R}^5$ Lebesgue measure. This yields $d\sigma_{\gamma_4}(\xi) = d^3\vec{k}/k_0$ and the constant $|a_\nu|^2$ reads

$$|a_\nu|^2 = 4\frac{\langle Q_0 \rangle + 4}{\langle Q_1 \rangle}\left[\frac{H^2 e^{\pi\nu}\Gamma(-\sigma)\Gamma(-\sigma^*)}{2^5\pi^4 m^2}\right] = 4\frac{\langle Q_0 \rangle + 4}{\langle Q_1 \rangle}\left[\frac{H^2\nu^2 + H^2/4}{2^4\pi^3 m^2}\right]. \tag{4.25}$$

One finds the massive spin-2 Minkowski two-point function:

$$\lim_{H\to 0}\frac{1}{4}\mathcal{W}^\nu(x,x') = \frac{1}{2(2\pi)^3}\int\sum_{\lambda\lambda'}\epsilon^{\lambda\lambda'}(k)\epsilon^{\lambda\lambda'}(k)\exp(-ik(x-x'))d^3\vec{k}/k_0\,, \tag{4.26}$$

where the factor $1/4$ is due to our definition of the operators S and S'. This limit can also be computed (more explicitly) starting with Formula (4.17). The flat limit for the scalar and vector two-point functions have been computed in [1, 37], one obtains:

$$\lim_{H\to 0}\mathcal{W}_0^\nu(x,x') = \mathcal{W}^P(X,X'), \quad \lim_{H\to 0}\mathcal{W}_1(x,x') = -\left[\eta_{\mu\nu}+\frac{1}{m^2}\frac{\partial}{\partial X^\mu\partial X^\nu}\right]\mathcal{W}^P(X,X')\equiv\mathcal{W}^P_{\mu\nu}(X,X'),$$
$$\tag{4.27}$$

where $\mathcal{W}^P(X,X')$ and $\mathcal{W}^P_{\mu\nu}(X,X')$ are the scalar and vector massive Minkowskian two-point functions respectively. Under the constraint $H\nu = m$ which implies

$$\lim_{H\to 0}H^2\langle Q_s\rangle = m^2 \quad\text{and}\quad \lim_{H\to 0}H^2\sigma^2 = -m^2\,, \tag{4.28}$$

one finally gets the massive spin-2 Minkowski two-point function (see for instance [19])

$$\lim_{H\to 0}\frac{1}{4}\mathcal{W}(x,x') = +\frac{1}{3}\left[\eta_{\mu\nu}+\frac{1}{m^2}\frac{\partial}{\partial X^\mu\partial X^\nu}\right]\mathcal{W}^P_{\mu'\nu'}(X,X') - \frac{1}{2}\mathcal{S}\left[\eta_{\mu\nu'}+\frac{1}{m^2}\frac{\partial}{\partial X^\mu\partial X^{\nu'}}\right]\mathcal{W}^P_{\mu'\nu}(X,X')\,. \tag{4.29}$$

C. The quantum field

The explicit knowledge of $\mathcal{W}^\nu(x,x')$ allows us to make the QF formalism work. The tensor fields $\mathcal{K}(x)$ is expected to be an operator-valued distributions on X_H acting on a Hilbert space $\mathcal{H}$. In terms of Hilbert space and field operator, the properties of the Wightman two-point functions are equivalent to the following conditions [40]:

1. **Existence of an unitary irreducible representation of the dS group**

$$U = U^{2,\nu}, \quad (\text{and possibly } V^{2,q}),$$

2. **Existence of at least one "vacuum state" Ω**, cyclic for the polynomial algebra of field operators and invariant under the above representation of the dS group.

3. **Existence of a Hilbert space $\mathcal{H}$** with positive definite metric that can be described as the Hilbertian sum

$$\mathcal{H} = \mathcal{H}_0 \oplus [\oplus_{n=1}^\infty \mathcal{S}\mathcal{H}_1^{\otimes n}],$$

where $\mathcal{H}_0 = \{\lambda\Omega, \ \lambda\in\mathbb{C}\}$.

4. **Covariance** of the field operators under the representation U,

$$U(g)\mathcal{K}_{\alpha\beta}(x)U(g^{-1}) = g_\alpha^\gamma g_\beta^\delta \mathcal{K}_{\gamma\delta}(gx).$$

5. **Locality** for every space-like separated pair (x, x')

$$[\mathcal{K}_{\alpha\beta}(x), \mathcal{K}_{\alpha'\beta'}(x')] = 0.$$

6. **KMS condition or geodesic spectral condition** [37] which means the vacuum is defined as a physical state with the temperature $T = \frac{H}{2\pi}$.

7. **Transversality**

$$x \cdot \mathcal{K}(x) = 0.$$

8. **Divergencelessness**

$$\partial \cdot \mathcal{K}(x) = 0.$$

9. **Index symmetrizer**

$$\mathcal{K}_{\alpha\beta} = \mathcal{K}_{\beta\alpha}.$$

Given the two-point function, one can realize the Hilbert space as functions on X_H as follows. For any test function $f_{\alpha\beta} \in \mathcal{D}(X_H)$, we define the vector valued distribution taking values in the space generated by the modes $\mathsf{K}_{\alpha\beta}(x, \xi) \equiv \mathrm{bv}\, \mathsf{K}_{\alpha\beta}(z, \xi)$ by :

$$x \to p_{\alpha\beta}(f)(x) = \int_{X_H} \mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x')f^{\alpha'\beta'}(x')d\sigma(x') = \sum_{\lambda\lambda'} \int_\gamma d\sigma_\gamma(\xi)\mathsf{K}_\xi^{\lambda\lambda'}(f)\,\mathsf{K}_{\alpha\beta}^{\lambda\lambda'}(x, \xi)\,, \qquad (4.30)$$

where $\mathsf{K}_\xi^{\lambda\lambda'}(f)$ is the smeared form of the modes:

$$\mathsf{K}_\xi^{\lambda\lambda'}(f) = \int_{X_H} \mathsf{K}_{\alpha\beta}^{*\lambda\lambda'}(x, \xi)f^{\alpha\beta}(x)d\sigma(x)\,. \qquad (4.31)$$

The space generated by the $p(f)$'s is equipped with the positive invariant inner product

$$\langle p(f), p(g) \rangle = \int_{X_H \times X_H} f^{*\alpha\beta}(x)\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x')g^{\alpha'\beta'}(x')d\sigma(x')d\sigma(x)\,. \qquad (4.32)$$

As usual, the field is defined by the operator valued distribution

$$\mathcal{K}(f) = a\left(p(f)\right) + a^\dagger\left(p(f)\right)\,, \qquad (4.33)$$

where the operators $a(\mathsf{K}^{\lambda\lambda'}(\xi)) \equiv a^{\lambda\lambda'}(\xi)$ and $a^{\dagger}(\mathsf{K}^{\lambda\lambda'}(\xi)) \equiv a^{\dagger\lambda\lambda'}(\xi)$ are respectively antilinear and linear in their arguments. One gets:

$$\mathcal{K}(f) = \sum_{\lambda\lambda'} \int_{\gamma} d\sigma_{\gamma}(\xi) \left[\mathsf{K}_{\xi}^{*\lambda\lambda'}(f)\, a^{\lambda\lambda'}(\xi) + \mathsf{K}_{\xi}^{\lambda\lambda'}(f)\, a^{\dagger\lambda\lambda'}(\xi) \right]. \tag{4.34}$$

The unsmeared operator reads

$$\mathcal{K}_{\alpha\beta}(x) = \sum_{\lambda\lambda'} \int_{\gamma} d\sigma_{\gamma}(\xi) \left[\mathsf{K}_{\alpha\beta}^{\lambda\lambda'}(x,\xi)\, a^{\lambda\lambda'}(\xi) + \mathsf{K}_{\alpha\beta}^{*\lambda\lambda'}(x,\xi)\, a^{\dagger\lambda\lambda'}(\xi) \right], \tag{4.35}$$

where $a^{\lambda\lambda'}(\xi)$ satisfies the canonical commutation relations (ccr) and is defined by

$$a^{\lambda\lambda'}(\xi)|\Omega> = 0.$$

The measure satisfies $d\sigma_{\gamma}(l\xi) = l^3 d\sigma_{\gamma}(\xi)$ and $\mathsf{K}_{\alpha\beta}^{\lambda\lambda'}(x,l\xi) = l^{\sigma}\mathsf{K}_{\alpha\beta}^{\lambda\lambda'}(x,\xi)$ yields the homogeneity condition

$$a^{\lambda\lambda'}(l\xi) \equiv a(\mathsf{K}^{\lambda\lambda'}(l\xi)) = a(l^{\sigma}\mathsf{K}^{\lambda\lambda'}(\xi)) = l^{\sigma*} a^{\lambda\lambda'}(\xi).$$

The integral representation (4.35) is independent of the orbital basis γ as explained in [37]. For the hyperbolic type submanifold γ_4 the measure is $d\sigma_{\gamma_4}(\xi) = d^3\vec{\xi}/\xi_0$ and the ccr are represented by

$$\left[a^{\lambda\lambda'}(\xi), a^{\dagger\lambda''\lambda'''}(\xi') \right] = \left[\eta^{\lambda\lambda''}\eta^{\lambda'\lambda'''} + \eta^{\lambda\lambda'}\eta^{\lambda''\lambda'''} \right] \xi^0 \delta^3(\vec{\xi} - \vec{\xi'}). \tag{4.36}$$

The field commutation relations are

$$\left[\mathcal{K}_{\alpha\beta}(x), \mathcal{K}_{\alpha'\beta'}(x') \right] = 2i \mathrm{Im} \langle p_{\alpha\beta}(x), p_{\alpha'\beta'}(x') \rangle = 2i \mathrm{Im} \mathcal{W}_{\alpha\beta\alpha'\beta'}(x,x'). \tag{4.37}$$

V. CONCLUSION

In this paper we have considered the "massive" spin-2 tensor field that is associated to the principal series of the dS group $SO_0(1,4)$ with $< Q_{\nu} > = \nu^2 - \frac{15}{4}, \quad \nu \geq 0$ and corresponding to the nonzero "mass" $m_p^2 = H^2(\nu^2 + \frac{9}{4})$. In our view, the use of the "mass" concept is more forced by tradition than relevant to our analysis. The use of ambient space formalism endowed the de Sitter physics with a Minkowskian-like appearance. The main differences hold in the space time dependence of the de Sitter polarization tensor. This formalism yield simple expressions and make de Sitter QFT look almost like standard QFT in flat space time.

The group theoretical point of view allows a systematic and complete study of the spin-2 field theory and legitimates the restriction of "massive" fields to those which carry principal series representations. Indeed, in the case of the complementary series ($< Q_\mu >= \mu - 4, \ 0 < \mu < \frac{1}{4}$), although the associated "mass" $m_c^2 = H^2(\mu + 2), \ 0 < \mu < \frac{1}{4}$ is strictly positive, the physical meaning of their carrier fields remains unclear since the $H = 0$ limits of these representations in the complementary series do not correspond to any physical representation of the Poincaré group.

Since m_p^2 and m_c^2 are strictly non zero, "massless" spin-2 fields must belong to the discrete series among which only $\Pi_{s,s}^\pm$ have a physically meaningful Poincaré limit. Now since the associated "mass" is $m_d^2 = H^2\{6 - 2(s^2 - 1)\}, \ s \geq 2$, and is expected to be real, the only possible value of s is 2 with $m_d^2 = 0$. Hence $\Pi_{2,2}^\pm$ correspond precisely to "massless" tensor fields (linear quantum gravity in dS space) in perfect agreement with the fact that on one hand these representations have non ambiguous extensions to the conformal group $SO(4,2)$ and on the other hand, the latter are precisely the unique extensions of the massless Poincaré group representations with helicity ± 2. In this case ν should be replaced by $\pm \frac{3i}{2}$ in the formulas of the present paper [14]. The projection operator D (Eq. (3.19) on the classical level) and the normalization constant c_ν^2 (Eq. (4.12) on the quantum level) then become singular. This singularity is actually due to the divergencelessness condition needed to associate the tensor field with a specific UIR of the dS group. To solve this problem, the divergencelessness condition must be dropped. Then the field equation becomes gauge invariant, *i.e.* $\mathcal{K}^{gt} = \mathcal{K} + D_2\Lambda_g$ is a solution of the field equation for any vector field Λ_g as far as $\mathcal{K}$ is. As a result, the general solutions transform under indecomposable representations of the dS group. By fixing the gauge, the field can eventually be quantized.

A second type of singularity appears. It is due to the zero mode problem of the Laplace-Beltrami operator on dS space inherited from the minimally coupled scalar field [12]. Accordingly, we feel that a Krein space quantization along the lines presented in [12] can be successfully carried out in the spin-2 massless case in dS space. This situation will be considered in a forthcoming paper [10].

Appendix A: Classification of the unitary irreducible representations of the de Sitter group $SO_0(1,4)$.

Unitary irreducible representations (UIR) of $SO_0(1,4)$ are characterized by the eigenvalues of the two Casimir operators $Q^{(1)}$ and $Q^{(2)}$ introduced in Section II. In fact the UIR's may be labelled by a pair of parameters $\Delta = (p,q)$ with $2p \in \mathbb{N}$ and $q \in \mathbb{C}$, in terms of which the eigenvalues of $Q^{(1)}$ and $Q^{(2)}$ are expressed as follows [2, 24, 25]:

$$Q^{(1)} = [-p(p+1) - (q+1)(q-2)]\mathbf{Id}, \quad Q^{(2)} = [-p(p+1)q(q-1)]\mathbf{Id}.$$

According to the possible values for p and q, three series of inequivalent representations may be distinguished: the principal, complementary and discrete series. We write s when p or q have spin meaning.

1. Principal series representations $U_{s,\nu}$, also called "massive" representations: $\Delta = (s, \frac{1}{2} + i\nu)$ with

$$
\begin{aligned}
s &= 0, 1, 2, \ldots \quad \text{and} \quad \nu \geq 0 \quad \text{or,} \\
s &= \frac{1}{2}, \frac{3}{2}, \ldots \quad \text{and} \quad \nu > 0.
\end{aligned}
$$

The operators $Q^{(1)}$ and $Q^{(2)}$ take respectively the following forms:

$$Q_1 = \left[\left(\frac{9}{4} + \nu^2\right) - s(s+1)\right]\mathbf{Id}, \qquad Q_2 = \left[\left(\frac{1}{4} + \nu^2\right)s(s+1)\right]\mathbf{Id}.$$

They are called the massive representations of the dS group because they contract toward the massive spin s representations of the Poincaré group.

2. Complementary series representations $V_{s,\nu}$: $\Delta = (s, \frac{1}{2} + \nu)$ with

$$
\begin{aligned}
s &= 0 \quad \text{and} \quad \nu \in \mathbb{R}, \ 0 < |\nu| < \frac{3}{2} \quad \text{or,} \\
s &= 1, 2, 3, \ldots \quad \text{and} \quad \nu \in \mathbb{R}, \ 0 < |\nu| < \frac{1}{2}.
\end{aligned}
$$

The operators $Q^{(1)}$ and $Q^{(2)}$ take forms:

$$Q_1 = \left[\left(\frac{9}{4} - \nu^2\right) - s(s+1)\right]\mathbf{Id}, \qquad Q_2 = \left[\left(\frac{1}{4} - \nu^2\right)s(s+1)\right]\mathbf{Id}.$$

Here, the only physical representation in the sense of Poincaré limit is the scalar case corresponding to $\Delta = (0,1)$ and also called conformally coupled massless case.

3. Discrete series $\Pi_{p,0}$ and $\Pi_{p,q}^{\pm}$: $\quad \Delta = (p,q)$ with

$$p = 1, 2, 3, \ldots \quad \text{and} \quad q = 0 \quad \text{or,}$$
$$p = \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots \quad \text{and} \quad q = p, p-1, \ldots, 1 \text{ or } \frac{1}{2}.$$

In this case, the only physical representations in the sense of Poincaré limit are those with $p = q = s$. They are called the massless representations of the dS group.

Note that the substitution $q \to (1-q)$ does not alter the eigenvalues; the representations with labels $\Delta = (p,q)$ and $\Delta = (p, 1-q)$ can be shown to be equivalent. Finally, we have pictured some of these representations in terms of p and q. The symbols $\bigcirc$ and $\blacksquare$ stand for the discrete series with semi-integer and integer values of p respectively. The complementary series is represented in the same frame by bold lines. The principal series is represented in the $\text{Re}(q) = 1/2$ plane by dashed lines.
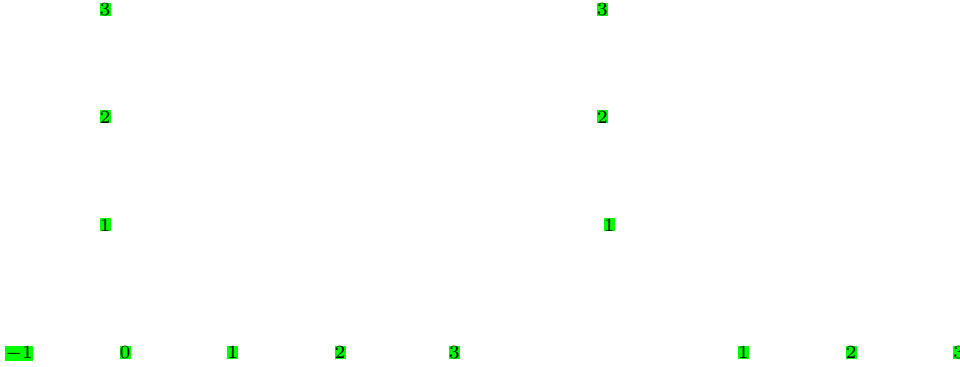

FIG. 2: $SO_0(1,4)$ unitary irreducible representation diagrams.

Appendix B: Maximally symmetric bitensors in ambient space

Following Allen and Jacobson in reference [26] we will write the two-point functions in de Sitter space (maximally symmetric) in terms of bitensors. These are functions of two points (x, x') which behave like tensors under coordinate transformations at either point. The bitensors are called maximally symmetric if they respect the de Sitter invariance.

As shown in reference [26], any maximally symmetric bitensor can be expressed as a sum of products of three basic tensors. The coefficients in this expansion are functions of the geodesic distance $\mu(x, x')$, that is the distance along the geodesic connecting the points x and x' (note that $\mu(x, x')$ can be defined

by unique analytic extension also when no geodesic connects x and x'). In this sense, these fundamental tensors form a complete set. They can be obtained by differentiating the geodesic distance:

$$n_a = \nabla_a \mu(x, x'), \qquad n_{a'} = \nabla_{a'} \mu(x, x')$$

and the parallel propagator

$$g_{ab'} = -c^{-1}(\mathcal{Z}) \nabla_a n_{b'} + n_a n_{b'} .$$

The geodesic distance is implicitly defined [37] for $\mathcal{Z} = -H^2 x \cdot x'$ by

$$\mathcal{Z} = \cosh(\mu H) \quad \text{for } x \text{ and } y \text{ timelike separated,}$$
$$\mathcal{Z} = \cos(\mu H) \quad \text{for } x \text{ and } y \text{ spacelike separated such that} \quad |x \cdot x'| < H^{-2}.$$

The basic bitensors in ambient space notations are found through:

$$\bar{\partial}_\alpha \mu(x, x'), \qquad \bar{\partial}'_{\beta'} \mu(x, x'), \qquad \bar{\partial}_\alpha \bar{\partial}'_{\beta'} \mu(x, x'),$$

restricted to the hyperboloid by

$$T_{ab'}(x, x') = \frac{\partial x^\alpha}{\partial X^a} \frac{\partial x'^{\beta'}}{\partial X'^{b'}} \, T_{\alpha\beta'} .$$

For $\mathcal{Z} = \cos(\mu H)$, one finds

$$n_a = \frac{\partial x^\alpha}{\partial X^a} \, \bar{\partial}_\alpha \mu(x, x') = \frac{\partial x^\alpha}{\partial X^a} \frac{H(\theta_\alpha \cdot x')}{\sqrt{1 - \mathcal{Z}^2}}, \qquad n_b = \frac{\partial x'^{\beta'}}{\partial X'^{b'}} \, \bar{\partial}'_{\beta'} \mu(x, x') = \frac{\partial x'^{\beta'}}{\partial X'^{b'}} \frac{H(\theta'_{\beta'} \cdot x)}{\sqrt{1 - \mathcal{Z}^2}} ,$$

and

$$\nabla_a n_{b'} = \frac{\partial x^\alpha}{\partial X^a} \frac{\partial x'^{\beta'}}{\partial X'^{b'}} \, \theta_\alpha^\sigma \theta_{\beta'}'^{\gamma'} \bar{\partial}_\sigma \bar{\partial}'_{\gamma'} \mu(x, x') = c(\mathcal{Z}) \left[\mathcal{Z} n_a n_{b'} - \frac{\partial x^\alpha}{\partial X^a} \frac{\partial x'^{\beta'}}{\partial X'^{b'}} \, \theta_\alpha \cdot \theta'_{\beta'} \right] ,$$

with $c(\mathcal{Z}) = -\dfrac{H}{\sqrt{1 - \mathcal{Z}^2}}$. For $\mathcal{Z} = \cosh(\mu H)$, n_a, $n_{b'}$ are multiplied by i and $c(\mathcal{Z})$ becomes $-\dfrac{iH}{\sqrt{1 - \mathcal{Z}^2}}$. In both cases we have

$$\frac{\partial x^\alpha}{\partial X^a} \frac{\partial x'^{\beta'}}{\partial X'^{b'}} \, \theta_\alpha \cdot \theta'_{\beta'} = g_{ab'} + (\mathcal{Z} - 1) n_a n_{b'} .$$

Appendix C: "Massive" vector two-point function

Given the important role played by the "massive" vector Wightman two-point function in the construction of the spin-2 two-point function we briefly present here a derivation of it (for details see

Reference [1]). In addition we compare our two-point function with the one given in Reference [26]. We consider the "massive" vector Wightman two-point function which corresponds to the principal series of representation of $SO_0(1,4)$ and satisfies:

$$(Q_1 - \langle Q_1 \rangle)\, \mathcal{W}^{\nu}_{1\alpha\beta'}(x,x') = 0, \qquad \text{where} \quad \langle Q_1 \rangle = \nu^2 + \frac{1}{4} \quad \text{with} \quad \nu \in \mathbb{R}\,.$$

This bivector is obtained as the boundary value of the analytic bivector two-point function obtained with the modes (3.13):

$$W^{\nu}_{1\alpha\beta'}(z,z') = c_{\nu}^2 \frac{\langle Q_1 \rangle}{\langle Q_0 \rangle} \int_{\gamma} \sum_{\lambda} \mathcal{E}^{\lambda}_{\alpha}(z,\xi)\, \mathcal{E}^{*\lambda}_{\beta'}(z'^{*},\xi)(Hz \cdot \xi)^{\sigma}(Hz' \cdot \xi)^{\sigma^*}\, d\sigma_{\gamma}(\xi).$$

With the help of Eq. (4.16) and the relation $H^2 D_1 (Hz \cdot \xi)^{\sigma} = \bar{\xi}\,(Hz \cdot \xi)^{\sigma} / (z \cdot \xi)$ it is easy to expand the transverse bivector in terms of the analytic scalar two-point function $W_0(z,z')$:

$$W^{\nu}_1(z,z') = \frac{\langle Q_0 \rangle}{\langle Q_1 \rangle} \left(-\theta_{\alpha} \cdot \theta'_{\alpha'} + \frac{H^2 \sigma (\theta \cdot z') D'_1}{\langle Q_0 \rangle} + \frac{H^2 \sigma^* (\theta' \cdot z) D_1}{\langle Q_0 \rangle} + \frac{H^2 \mathcal{Z} D_1 D'_1}{\langle Q_0 \rangle} \right) W^{\nu}_0(z,z'). \qquad (3.1)$$

The analytic "massive" scalar two-point function is

$$W^{\nu}_0(z,z') = c_{\nu}^2 \int_{\gamma} (Hz \cdot \xi)^{\sigma}(Hz' \cdot \xi)^{\sigma^*}\, d\sigma_{\gamma}(\xi) \quad \text{with} \quad c_{\nu}^2 = H^2 e^{+\pi\nu} \Gamma(-\sigma)\Gamma(-\sigma^*)/(2^5 \pi^4 m^2)\,,$$

which satisfies:

$$(Q_0 - \langle Q_0 \rangle)\, W^{\nu}_0(z,z') = 0, \qquad \text{where} \quad \langle Q_0 \rangle = \nu^2 + \frac{9}{4} \quad \text{with} \quad \nu \in \mathbb{R}\,.$$

The choice of normalization corresponds to the Euclidean vacuum and $W_0(z,z')$ can be written as a hypergeometric function (see [37]):

$$\mathcal{W}^{\nu}_0(z,z') = C_{\nu}\, {}_2F_1\left(-\sigma, -\sigma^*; 2; \frac{1+\mathcal{Z}}{2} \right) = C_{\nu} P^5_{\sigma}(-\mathcal{Z}) \quad \text{with} \quad C_{\nu} = \frac{H^2 \Gamma(-\sigma)\,\Gamma(-\sigma^*)}{2^4 \pi^2 m^2}.$$

In order to show that our vector two-point function is the same two-point function as the one given by Allen and Jacobson in Reference [26], we develop $W^{\nu}_1(z,z')$ using essentially $\partial_{\alpha}\phi(\mathcal{Z}) = -(\theta_{\alpha} \cdot z')\, H^2 \frac{d}{d\mathcal{Z}}\phi(\mathcal{Z}).$ One finds

$$\mathcal{W}^{\nu}_{1\alpha\beta'}(x,x') = \text{bv}\, W^{\nu}_1(z,z') = \theta_{\alpha} \cdot \theta'_{\beta'} U(\mathcal{Z}) + H^2 \frac{(\theta'_{\beta'} \cdot z)\,(\theta_{\alpha} \cdot z')}{1 - \mathcal{Z}^2}\, V(\mathcal{Z})\,,$$

with

$$U(\mathcal{Z}) = -\frac{1}{\langle Q_1 \rangle} \left[Q_0 + \mathcal{Z}\frac{d}{d\mathcal{Z}} \right] W^{\nu}_0(z,z')\,, \quad V(\mathcal{Z}) = \frac{1}{\langle Q_1 \rangle} \left[3\frac{d}{d\mathcal{Z}} + \mathcal{Z}^2 \frac{d}{d\mathcal{Z}} + \mathcal{Z} Q_0 \right] W^{\nu}_0(z,z')\,,$$

where

$$Q_0 = \left(1 - \mathcal{Z}^2\right)\frac{d^2}{d\mathcal{Z}^2} - 4\mathcal{Z}\frac{d}{d\mathcal{Z}},$$

is the second order differential operator deduced from the Casimir operator expressed with the variable $\mathcal{Z}$ in place of (z, z'). The functions $U(\mathcal{Z})$ and $V(\mathcal{Z})$ satisfy the property

$$\mathcal{Z}U(\mathcal{Z}) + V(\mathcal{Z}) = \frac{3}{\langle Q_1 \rangle}\frac{d}{d\mathcal{Z}}W_0^\nu(z, z') \equiv \Lambda(\mathcal{Z}),$$

with

$$\Lambda(\mathcal{Z}) = 3H^2\frac{\Gamma(1 - \sigma)\,\Gamma(1 - \sigma^*)}{2^6\langle Q_1 \rangle\pi^2 m^2}\,{}_2F_1\left(1 - \sigma, 1 - \sigma^*; 3; \frac{1 + \mathcal{Z}}{2}\right),$$

which is solution of the equation

$$\left[Q_0 - 2\mathcal{Z}\frac{d}{d\mathcal{Z}} - 6 - \langle Q_1 \rangle\right]\Lambda(\mathcal{Z}) = 0.$$

Finally, let us write the intrinsic expression of the two-point function $\mathcal{W}_1^\nu(x, x')$ obtained as the boundary value of $W_1^\nu(z, z')$. The intrinsic expression is:

$$Q_{ab'} \equiv \frac{\partial x^\alpha}{\partial X^a}\frac{\partial x'^{\beta'}}{\partial X'^{b'}}\,\mathcal{W}_{1\alpha\beta'}^\nu(x, x').$$

Since

$$\frac{\partial x^\alpha}{\partial X^a}\frac{\partial x'^{\beta'}}{\partial X'^{b'}}\,\theta_\alpha \cdot \theta'_{\beta'} = g_{ab'} + (\mathcal{Z} - 1)n_a n_{b'}, \qquad \frac{\partial x^\alpha}{\partial X^a}\frac{\partial x'^{\beta'}}{\partial X'^{b'}}\frac{H^2(\theta'_{\beta'} \cdot x)\,(\theta_\alpha \cdot x')}{1 - \mathcal{Z}^2} = n_a n_{b'},$$

one gets

$$Q_{ab} = g_{ab'}U(\mathcal{Z}) + n_a\,n_{b'}\left(\Lambda(\mathcal{Z})\right) - U(\mathcal{Z})),$$

and in the case of $SO_0(4, 1)$:

$$Q_{ab} = -g_{ab'}U(\mathcal{Z}) - n_a\,n_{b'}\left(U(\mathcal{Z}) - \Lambda(\mathcal{Z})\right)).$$

This is the expression given by Allen and Jacobson in Ref. [20] and [26].

Appendix D: Another expression for the spin-2 Two-point function

We present another form of the spin-2 two-point function, which is useful for the proof of the locality condition. We begin with the term $M(z, z')W_1^\nu(z, z')$:

$$\frac{\langle Q_0 \rangle + 4}{\langle Q_0 \rangle}\left[-\mathcal{S}\mathcal{S}'\theta \cdot \theta' + \frac{H^2\mathcal{S}(\theta \cdot z')D_2'}{\sigma^* - 1} + \frac{H^2\mathcal{S}'(\theta' \cdot z)D_2}{\sigma - 1} + \frac{\mathcal{Z}H^2 D_2 D_2'}{(\sigma - 1)(\sigma^* - 1)}\right]W_1^\nu(z, z').$$

We rewrite this equation using the relations

$$H^2 \mathcal{S}(\theta \cdot z') D_2' W_1^\nu = -\mathcal{S}\mathcal{S}'\theta \cdot \theta' W_1^\nu + 2\theta' \mathcal{S}\theta \cdot W_1^\nu + \frac{1}{2} D_2 D_2' W_2 \,,$$

$$H^2 \mathcal{S}'(\theta' \cdot z) D_2 W_1^\nu = -\mathcal{S}\mathcal{S}'\theta \cdot \theta' W_1^\nu + 2\theta \mathcal{S}'\theta' \cdot W_1^\nu + \frac{1}{2} D_2 D_2' W_2 \,,$$

$$H^2 \mathcal{Z} D_2 D_2' W_1^\nu = -\mathcal{S}\mathcal{S}'\theta \cdot \theta' W_1^\nu + 2\theta \mathcal{S}'\theta' \cdot W_1^\nu + 2\theta' \mathcal{S}\theta \cdot W_1^\nu + D_2 D_2' \left(W_2 + H^2 \mathcal{Z} W_1^\nu \right) \,,$$

where $D_2 D_2' W_2 = 2H^2 D_2 \mathcal{S}'\left(\theta' \cdot z \right) W_1 - 4\mathcal{S}\mathcal{S}'\theta' \cdot W_1$. This is obtained by simple calculation of

$$(Q_2 - \langle Q_2 \rangle) \left(\mathcal{S}\mathcal{S}'\theta \cdot \theta' W_1^\nu + D_2 D_2' W_3 \right) = 0 \,,$$

with the help of Eq. (3.3) and where we have written $W_2 = (Q_1 - \langle Q_2 \rangle) W_3$. One gets:

$$M(z,z') W_1^\nu(z,z') = -\mathcal{S}\mathcal{S}'\theta \cdot \theta' W_1^\nu + \frac{2\theta \sigma^* \mathcal{S}'\theta' \cdot W_1^\nu}{\langle Q_0 \rangle} + \frac{2\theta' \sigma \mathcal{S}\theta \cdot W_1^\nu}{\langle Q_0 \rangle} + D_2 D_2' \left(\frac{H^2 \mathcal{Z} W_1^\nu}{\langle Q_0 \rangle} - \frac{3W_2}{2\langle Q_0 \rangle} \right) \,.$$

Now, given that

$$\mathcal{S}\theta \cdot W_1^\nu = \frac{2}{3} \left[\theta + \frac{H^2 D_2 D_1}{2\langle Q_0 \rangle} \right] (W_1^\nu)' \quad \text{and} \quad \mathcal{S}'\theta' \cdot W_1^\nu = \frac{2}{3} \left[\theta' + \frac{H^2 D_2' D_1'}{2\langle Q_0 \rangle} \right] (W_1^\nu)' \,,$$

where $(W_1^\nu)'$ is the trace of the vector two-point function given by

$$(W_1^\nu)' = \eta \cdot \cdot W_1^\nu = 3U(\mathcal{Z}) + \mathcal{Z}\Lambda(\mathcal{Z}) = -3 \frac{\langle Q_0 \rangle}{\langle Q_1 \rangle} W_0^\nu(z,z') \,.$$

We find the following form for the spin-2 two-point function :

$$\begin{aligned}
W^\nu(z,z') &= M(z,z') W_1^\nu(z,z') + N(z,z') W_0^\nu(z,z') \\
&= q \left[\theta\theta'(W_1^\nu)' - \frac{3}{2}\theta \mathcal{S}'\theta' \cdot W_1^\nu - \frac{3}{2}\theta' \mathcal{S}\theta \cdot W_1^\nu \right] - \mathcal{S}\mathcal{S}'\theta \cdot \theta' W_1^\nu + D_2 D_2' W_4 \,,
\end{aligned}$$

where $q = -\frac{4}{9} \left(\langle Q_0 \rangle - 9 \right) / \langle Q_0 \rangle$ and

$$D_2 D_2' W_4 = \frac{6\theta \mathcal{S}'\theta' \cdot W_1^\nu}{\langle Q_0 \rangle} - \frac{3H^2 D_2 \mathcal{S}'(\theta' \cdot z) W_1^\nu}{\langle Q_0 \rangle} + D_2 D_2' \left(\frac{H^2 \mathcal{Z} W_1^\nu}{\langle Q_0 \rangle} + \frac{H^4 D_1 D_1'(W_1^\nu)'}{9\langle Q_0 \rangle^2} \right) \,.$$

The two-point function can be rewritten as

$$W^\nu(z,z') = D(z,z') W_0^\nu(z,z') \,,$$

where the differential operator $D(z,z')$ obviously satisfies $D^*(z^*, z'^*) = D(z,z')$. This property serves to prove the locality condition.

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