

# MODIFIED TRIPLECTIC QUANTIZATION IN GENERAL COORDINATES

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We present an extension of the previous results [1] on the quantization of general gauge theories within the BRST–antiBRST invariant Lagrangian scheme in general coordinates. Namely, we generalize [1] to the case when the base manifold of fields and antifields is a supermanifold described in terms of both bosonic and fermionic coordinates.

## 1 Introduction

Modern covariant quantization methods for general gauge theories are based on the principle of BRST [2, 3], or, more generally, BRST–antiBRST [4, 5, 6, 7] invariance. The consideration of these methods in general coordinates (using appropriate supermanifolds) appears to be very important in order to reveal the geometrical meaning of the basic objects underlying these quantization schemes.

The study of the Batalin–Vilkovisky (BV) method [2] in general coordinates was initiated by the work of Witten [8], where the geometrical meaning of the antifields, the antibracket, and the odd second-order operator  $\Delta$  was discussed. In [9], it was shown that the geometry of the BV formalism is that of an *odd* symplectic superspace, endowed with a density function  $\rho$ .

The quantization schemes based on the BRST–antiBRST symmetry involve additional basic objects. Namely, in the  $Sp(2)$ -covariant and triplectic quantization schemes one introduces  $Sp(2)$ -doublets of extended antibrackets, as well as doublets of second- and first-order operators  $\Delta^a$  and  $V^a$ , respectively. In the modified triplectic quantization, an additional  $Sp(2)$ -doublet of first-order operators  $U^a$  is required. This indicates that the geometrical formulation of these quantization methods in general coordinates, in contrast to the BV quantization, requires more complicated tools. Indeed, in this paper we show that the geometry of the  $Sp(2)$ -covariant and triplectic schemes is the geometry of an *even* symplectic superspace equipped with a density function and a flat symmetric connection (covariant derivative), while the geometry of the modified triplectic quantization also includes a symmetric structure (analogous to a metric tensor).

The paper is organized as follows. In Sect. 2, we briefly review the definitions of tensor fields, the covariant derivative, and the curvature tensor on supermanifolds [10]. In Sect. 3, we introduce the notion of a *triplectic supermanifold*, together with tensor fields and covariant derivatives defined on it. In Sect. 4, an explicit realization of the triplectic algebra of odd differential operators is suggested. In Sect. 5, we find a realization of the modified triplectic algebra and propose a suitable quantization procedure. In Sect. 6, we give a short summary and a few concluding remarks. In Appendix A, we study the connection between even (odd) non-degenerate Poisson structures and even (odd) symplectic structures on supermanifolds, and show their one-to-one correspondence. In Appendix B, the algebra of the generating operators  $\Delta^a$  and  $V^a$  is presented.

We use the condensed DeWitt notation and apply the tensor analysis of Ref. [10]. Derivatives with respect to the variables  $x^i$  are understood as acting from the left, with the notation  $\partial_i A = \partial A / \partial x^i$ . Right-hand derivatives with respect to  $x^i$  are labelled by the subscript “ $r$ ”, and the notation  $A_{,i} = \partial_r A / \partial x^i$  is used. Raising the  $Sp(2)$ -group indices is performed by the antisymmetric second rank tensor  $\varepsilon^{ab}$  ( $a, b = 1, 2$ ):  $\theta^a = \varepsilon^{ab} \theta_b$ ,  $\varepsilon^{ac} \varepsilon_{cb} = \delta_b^a$ . The Grassmann parity of any quantity  $A$  is denoted by  $\epsilon(A)$ .

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## 2 Tensor fields, covariant derivative and curvature tensor on supermanifolds

In this Section, we briefly review the tensor analysis on supermanifolds as far as it is required for the following considerations. For a comprehensive treatment, we recommend Ref. [10].

Let the variables  $x^i, \epsilon(x^i) = \epsilon_i$  be local coordinates of a supermanifold  $M$ ,  $\dim M = N$ , in the vicinity of a point  $P$ . Let the sets  $\{e_i\}$  and  $\{e^i\}$  be coordinate bases in the tangent space  $T_P M$  and the cotangent space  $T_P^* M$ , respectively. Under a change of coordinates,  $x^i \rightarrow \bar{x}^i = \bar{x}^i(x)$ , the basis vectors in  $T_P M$  and  $T_P^* M$  transform according to

$$\bar{e}_i = e_j \frac{\partial_r x^j}{\partial \bar{x}^i}, \quad \bar{e}^i = e^j \frac{\partial \bar{x}^i}{\partial x^j}. \quad (1)$$

Tensor fields of type  $(n, m)$  with rank  $n + m$  on a supermanifold in some local coordinate system  $(x) = (x^1, \dots, x^N)$  are given by a set of functions  $T^{i_1 \dots i_n}_{j_1 \dots j_m}(x)$ ,  $\epsilon(T^{i_1 \dots i_n}_{j_1 \dots j_m}) = \epsilon(T) + \epsilon_{i_1} + \dots + \epsilon_{i_n} + \epsilon_{j_1} + \dots + \epsilon_{j_m}$ , which transform under a change of coordinates,  $x^i \rightarrow \bar{x}^i$ , according to

$$\begin{aligned} \bar{T}^{i_1 \dots i_n}_{j_1 \dots j_m} &= T^{l_1 \dots l_n}_{k_1 \dots k_m} \frac{\partial_r x^{k_m}}{\partial \bar{x}^{j_m}} \dots \frac{\partial_r x^{k_1}}{\partial \bar{x}^{j_1}} \frac{\partial \bar{x}^{i_n}}{\partial x^{l_n}} \dots \frac{\partial \bar{x}^{i_1}}{\partial x^{l_1}} \\ &\times (-1)^{\left( \sum_{s=1}^{m-1} \sum_{p=1}^m \epsilon_{jp}(\epsilon_{js} + \epsilon_{ls}) + \sum_{s=1}^n \sum_{p=1}^m \epsilon_{jp}(\epsilon_{is} + \epsilon_{ks}) + \sum_{s=1}^{n-1} \sum_{p=s+1}^n \epsilon_{ip}(\epsilon_{is} + \epsilon_{ks}) \right)}. \end{aligned} \quad (2)$$

For second-rank tensor fields of type  $(2, 0)$ ,  $(0, 2)$  and  $(1, 1)$ , one gets

$$\bar{T}^{ij} = T^{mn} \frac{\partial \bar{x}^j}{\partial x^n} \frac{\partial \bar{x}^i}{\partial x^m} (-1)^{\epsilon_j(\epsilon_i + \epsilon_m)}, \quad (3)$$

$$\bar{T}_{ij} = T_{mn} \frac{\partial_r x^n}{\partial \bar{x}^j} \frac{\partial_r \bar{x}^m}{\partial x^i} (-1)^{\epsilon_j(\epsilon_i + \epsilon_m)}, \quad (4)$$

$$\bar{T}^i_j = T^m_n \frac{\partial_r x^n}{\partial \bar{x}^j} \frac{\partial \bar{x}^i}{\partial x^m} (-1)^{\epsilon_j(\epsilon_i + \epsilon_m)}. \quad (5)$$

Using DeWitt's index shifting rules [10],

$$T^{ij} = (-1)^{\epsilon(T)\epsilon_i} {}^i T^j, \quad T_{ij} = (-1)^{(\epsilon(T)+1)\epsilon_i} {}_i T_j, \quad (6)$$

$$T^i_j = (-1)^{\epsilon(T)\epsilon_i} {}^i T_j, \quad T_i{}^j = (-1)^{(\epsilon(T)+1)\epsilon_i} {}_i T^j, \quad (7)$$

$$T_j{}^i = (-1)^{\epsilon_i \epsilon_j} T^i_j, \quad (8)$$

one can rewrite (3)–(5) as follows:

$${}^i \bar{T}^j = \frac{\partial_r \bar{x}^i}{\partial x^m} {}^m T^n \frac{\partial \bar{x}^j}{\partial x^n}, \quad (9)$$

$${}_i \bar{T}_j = \frac{\partial x^m}{\partial \bar{x}^i} {}_m T_n \frac{\partial_r x^n}{\partial \bar{x}^j}, \quad (10)$$

$${}^i \bar{T}_j = \frac{\partial_r \bar{x}^i}{\partial x^m} {}^m T_n \frac{\partial_r x^n}{\partial \bar{x}^j}, \quad (11)$$

$${}_i \bar{T}^j = \frac{\partial x^m}{\partial \bar{x}^i} {}_m T^n \frac{\partial \bar{x}^j}{\partial x^n}. \quad (12)$$

The unit matrix  $\delta_j^i$  is related to two tensor fields of type  $(1, 1)$ ,  ${}^i \delta_j$  and  ${}_j \delta^i$ , according to

$$\delta_j^i = {}^i \delta_j = \delta^i_j = (-1)^{\epsilon_i(\epsilon_j+1)} {}_j \delta^i = {}_j \delta^i. \quad (13)$$

In the last equality, we have used the fact that  $(-1)^{\epsilon_i(\epsilon_j+1)} = 1$  iff  $i = j$ , so that

$$\frac{\partial_r \bar{x}^i}{\partial x^k} \frac{\partial_r x^k}{\partial \bar{x}^j} = {}^i \delta_j = \delta^i_j = \delta_j^i, \quad (14)$$

$$\frac{\partial x^k}{\partial \bar{x}^j} \frac{\partial \bar{x}^i}{\partial x^k} = {}_j \delta^i = (-1)^{\epsilon_j} \delta_j^i = (-1)^{\epsilon_i(\epsilon_j+1)} \delta^i_j = \delta_j^i. \quad (15)$$

Obviously, using a tensor field of type  $(2, 0)$ ,  ${}^1T^k$ , and a tensor field of type  $(0, 2)$ ,  ${}^2T_k$ , one can construct two tensor fields of type  $(1, 1)$ ,

$${}^1T^k{}_k{}^2T_j, \quad {}^2T_k{}^1T^k{}_j, \quad (16)$$

transforming according to (11) and (12), respectively. Using a vector field  $X^i$  and a covector field  $P_i$ , one can construct a scalar field, according to

$$X^i{}_iP = (-1)^{\epsilon_i(\epsilon(P)+1)}X^iP_i. \quad (17)$$

By analogy with tensor analysis on manifolds, on supermanifolds one introduces the covariant derivative<sup>3</sup>,  $\bar{\nabla} \equiv \nabla$ , of tensor fields by the requirement that this operation should map a tensor field of type  $(n, m)$  into a tensor field of type  $(n, m + 1)$ , and that, in cases when one can introduce Cartesian coordinates, it should reduce to the usual partial derivative.

Now, let  $x^i$  be Cartesian coordinates, and  $\bar{x}^i$  be arbitrary coordinates. Let us consider a vector field  $X^i$ . Then, in the coordinate system  $(x)$  we have

$$X^i\nabla_j = X^i{}_{,j}.$$

When going over to the system  $(\bar{x})$ , by virtue of (5), the following relation holds:

$$\bar{X}^i\bar{\nabla}_j = X^m{}_{,n}\frac{\partial_r x^n}{\partial \bar{x}^j}\frac{\partial \bar{x}^i}{\partial x^m}(-1)^{\epsilon_j(\epsilon_i+\epsilon_m)}.$$

This implies

$$\bar{X}^i\bar{\nabla}_j = \bar{X}^i{}_{,j} + \bar{X}^k\Gamma^i{}_{kj}(-1)^{\epsilon_k(\epsilon_i+1)},$$

where  $\Gamma^i{}_{jk}$  are (generalized) Christoffel symbols in the superspace,

$$\Gamma^i{}_{kj} = \frac{\partial_r \bar{x}^i}{\partial x^m}\frac{\partial_r^2 x^m}{\partial \bar{x}^k\partial \bar{x}^j}, \quad (18)$$

which possess the property of generalized symmetry:

$$\Gamma^i{}_{jk} = (-1)^{\epsilon_j\epsilon_k}\Gamma^i{}_{kj}. \quad (19)$$

Similarly, the action of the covariant derivative on a covector field  $P_i$  of type  $(0, 1)$  is given by

$$\bar{P}_i\bar{\nabla}_j = \bar{P}_{i,j} + \bar{P}_k\tilde{\Gamma}^k{}_{ij},$$

with the notation

$$\tilde{\Gamma}^i{}_{jm} = (-1)^{\epsilon_m(\epsilon_j+\epsilon_k)}\frac{\partial_r^2 \bar{x}^i}{\partial x^k\partial x^n}\frac{\partial_r x^n}{\partial \bar{x}^m}\frac{\partial_r x^k}{\partial \bar{x}^j}. \quad (20)$$

Using the derivation of (14), one readily establishes the fact that

$$\tilde{\Gamma}^i{}_{jk} = -\Gamma^i{}_{jk},$$

and therefore

$$\bar{P}_i\bar{\nabla}_j = \bar{P}_{i,j} - \bar{P}_k\Gamma^k{}_{ij}.$$

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<sup>3</sup>In order to avoid cumbersome notation for signs, we use the *convention* that the covariant derivation always acts from the right. If necessary, we denote this by an arrow pointing to the left.

The action of the covariant derivative on second-rank tensor fields of type  $(2,0)$ ,  $(0,2)$  and  $(1,1)$  can be deduced in the same manner as follows:

$$\bar{T}^{ij}\bar{\nabla}_k = \bar{T}^{ij}_{,k} + \bar{T}^{im}\Gamma^j_{mk}(-1)^{\epsilon_m(\epsilon_j+1)} + \bar{T}^{mj}\Gamma^i_{mk}(-1)^{\epsilon_m(\epsilon_i+\epsilon_j+1)+\epsilon_i\epsilon_j}, \quad (21)$$

$$\bar{T}_{ij}\bar{\nabla}_k = \bar{T}_{ij,k} - \bar{T}_{il}\Gamma^l_{jk} - \bar{T}_{lj}\Gamma^l_{ik}(-1)^{\epsilon_j(\epsilon_i+\epsilon_l)}, \quad (22)$$

$$\bar{T}^i_j\bar{\nabla}_k = \bar{T}^i_{j,k} - \bar{T}^i_l\Gamma^l_{jk} + \bar{T}^l_j\Gamma^i_{lk}(-1)^{\epsilon_i\epsilon_j+\epsilon_l(\epsilon_i+\epsilon_j+1)}. \quad (23)$$

Similarly, one determines the action of the covariant derivative on a tensor field of any rank and type in terms of the tensor components, ordinary derivatives and Christoffel symbols. Of course, on arbitrary supermanifolds  $M$  the Christoffel symbols (i.e. connection coefficients) are not necessarily given by second-order partial derivatives with respect to the coordinates, since such a simple form arises only when local Euclidean coordinates can be introduced on  $M$ . If one chooses a coordinate system on a supermanifold, then the covariant derivative  $\bar{\nabla} = (\bar{\nabla}_i)$  is defined as a variety of differentiations with respect to separate coordinates. These differentiations are local operations acting on a scalar field  $S$  by the rule

$$S\nabla_i = S_{,i}, \quad (24)$$

on a vector field  $X^i$ , by the rule

$$X^i\nabla_j = X^i_{,j} + X^k\Gamma^i_{kj}(-1)^{\epsilon_k(\epsilon_i+1)}, \quad (25)$$

on a covector field  $P_i$ , by the rule

$$P_i\nabla_j = P_{i,j} - P_k\Gamma^k_{ij}, \quad (26)$$

and so on. If the Christoffel symbols are symmetric ones (19), then one says that on the supermanifold  $M$  a symmetric connection is defined. Here, we consider the case of symmetric connections only.

The curvature tensor  $R^i_{mjk}$  is defined in a coordinate basis [10] by the action of the commutator of covariant derivatives  $[\nabla_i, \nabla_j] = \nabla_i\nabla_j - (-1)^{\epsilon_i\epsilon_j}\nabla_j\nabla_i$  on a vector field  $X^i$  by the rule

$$X^i[\nabla_j, \nabla_k] = -(-1)^{\epsilon_m(\epsilon_i+1)}X^mR^i_{mjk}. \quad (27)$$

A straightforward calculation yields the following result:

$$R^i_{mjk} = -\Gamma^i_{mj,k} + \Gamma^i_{mk,j}(-1)^{\epsilon_j\epsilon_k} + \Gamma^i_{js}\Gamma^s_{mk}(-1)^{\epsilon_j\epsilon_m} - \Gamma^i_{ks}\Gamma^s_{mj}(-1)^{\epsilon_k(\epsilon_m+\epsilon_j)}. \quad (28)$$

The curvature tensor (28) obeys the following generalized antisymmetry:

$$R^i_{mjk} = -(-1)^{\epsilon_j\epsilon_k}R^i_{mkj}. \quad (29)$$

One readily establishes the fact that the curvature tensor (28) also obeys the Jacobi identity

$$(-1)^{\epsilon_m\epsilon_k}R^i_{mjk} + (-1)^{\epsilon_j\epsilon_m}R^i_{jkm} + (-1)^{\epsilon_k\epsilon_j}R^i_{kmj} = 0. \quad (30)$$

### 3 Triplectic supermanifolds

The supermanifolds arising within the triplectic [5] and modified triplectic [7] quantization schemes can be identified with superspaces parameterized by the variables  $x^i = (\phi^A, \bar{\phi}_A)$ ,  $i = 1, 2, \dots, N = 2n$ , where  $\phi^A$  are the (field) variables of the configuration space of a general gauge theory, and where the antifields  $\bar{\phi}_A$  are the sources of the combined BRST-antiBRST symmetry. The complete superspace of these quantization methods also involves variables  $\theta_{ia} = (\phi^*_{Aa}, \pi^{Aa})$  with Grassmann parity  $\epsilon(\theta_{ia}) = \epsilon_i + 1$ , opposite to  $x^i$ . Here, the antifields  $\phi^*_{Aa}$  are the sources of the BRST ( $a = 1$ ) and antiBRST ( $a = 2$ ) transformations, while the fields  $\pi^{Aa}$  are auxiliary ones. In the original formulation of  $Sp(2)$ -covariant quantization, these

variables are used to introduce the gauge. The character  $a$  indicates the (global)  $Sp(2)$  group index. (We remind that in Ref. [1] only the case  $\varepsilon_i = 0$  was considered.)

In order to formulate the modified triplectic quantization in general coordinates, let us consider a supermanifold  $\mathcal{M}$ ,  $\dim \mathcal{M} = 3N$ , which can be locally described by coordinates  $z^\mu = (x^i, \theta_{ia})$ , with  $\epsilon(x^i) = \epsilon_i$ ,  $\epsilon(\theta_{ia}) = \epsilon_i + 1$ . Let us introduce the transformation law of  $\theta_{ia}$  under coordinate transformations ( $x \rightarrow \bar{x}$ ) in the base supermanifold  $M$ , analogous to the transformation of the basis vectors in the tangent space  $T_P M$ , namely,

$$\bar{\theta}_{ia} = \theta_{ja} \frac{\partial_r x^j}{\partial \bar{x}^i}. \quad (31)$$

Supermanifolds with such a property will be called *triplectic supermanifolds*. Then, right-hand derivatives with respect to  $\theta_{ia}$  transform like the basis vectors of the cotangent space  $T_P^* M$ ,

$$\frac{\overleftarrow{\partial}}{\partial \bar{\theta}_{ia}} = \frac{\overleftarrow{\partial}}{\partial \theta_{ja}} \frac{\partial \bar{x}^j}{\partial x^i}. \quad (32)$$

According to Sect. 2, a tensor field of type  $(n, m)$  and rank  $n + m$  is a geometric object, which, in any local coordinate system  $(x, \theta)$  on  $\mathcal{M}$ , is given by a set of functions,  $T^{i_1 \dots i_n}_{j_1 \dots j_m}(x, \theta)$ , transforming under a change of coordinates of the base manifold  $M$ ,  $(x) \rightarrow (\bar{x})$ , according to (2).

Covariant differentiation ( $\overleftarrow{\nabla}$ ) of tensor fields can be introduced using the same arguments as given above. In particular, the action of the covariant derivative on the simplest tensor fields (scalar, vector and second-rank ones) is given by the relations

$$S \nabla_i = S_{,i} + \frac{\partial_r S}{\partial \theta_{ma}} \theta_{ka} \Gamma_{mi}^k, \quad (33)$$

$$X^i \nabla_j = X^i_{,j} + X^k \Gamma_{kj}^i (-1)^{\epsilon_k(\epsilon_i+1)} + \frac{\partial_r X^i}{\partial \theta_{ma}} \theta_{ka} \Gamma_{mj}^k, \quad (34)$$

$$P_i \nabla_j = P_{i,j} - P_k \Gamma_{ij}^k + \frac{\partial_r P_i}{\partial \theta_{ma}} \theta_{ka} \Gamma_{mj}^k, \quad (35)$$

$$\begin{aligned} T^{ik} \nabla_j &= T^{ik}_{,j} + T^{im} \Gamma_{mj}^k (-1)^{\epsilon_m(\epsilon_k+1)} + T^{mk} \Gamma_{mj}^i (-1)^{\epsilon_m(\epsilon_i+\epsilon_k+1)+\epsilon_i\epsilon_k} \\ &\quad + \frac{\partial_r T^{ik}}{\partial \theta_{ma}} \theta_{na} \Gamma_{mj}^n, \end{aligned} \quad (36)$$

$$T_{ik} \nabla_j = T_{ik,j} - T_{il} \Gamma_{jk}^l - T_{lj} \Gamma_{ik}^l (-1)^{\epsilon_j(\epsilon_i+\epsilon_l)} + \frac{\partial_r T_{ik}}{\partial \theta_{ma}} \theta_{na} \Gamma_{mj}^n, \quad (37)$$

$$T^i_k \nabla_j = T^i_{k,j} - T^i_l \Gamma_{jk}^l + T^l_j \Gamma_{lk}^i (-1)^{\epsilon_i\epsilon_j+\epsilon_l(\epsilon_i+\epsilon_j+1)} + \frac{\partial_r T^i_k}{\partial \theta_{ma}} \theta_{na} \Gamma_{mj}^n. \quad (38)$$

Since, according to their introduction, the coordinates  $x^i$  and  $\theta_{ia}$  are independent of each other, (35) implies that the vectors  $\theta_{ia}$  are covariantly constant:

$$\theta_{ia} \nabla_j = 0. \quad (39)$$

Furthermore, from these relations it follows that the action of the commutator of covariant derivatives on a scalar field is given by

$$S[\nabla_i, \nabla_j] = -\frac{\partial_r S}{\partial \theta_{ma}} \theta_{na} R_{mij}^n, \quad (40)$$

where the curvature tensor  $R_{mij}^n$  has been defined in (28).

## 4 Explicit realization of triplectic algebra

The triplectic algebra defined on the triplectic supermanifold  $\mathcal{M}$  includes two sets of anticommuting and nilpotent operators of second and first order,  $\overleftarrow{\Delta}^a$  and  $\overleftarrow{V}^a$ , respectively, acting from

the right, and obeying the following algebra:

$$\Delta^{\{a}\Delta^{b\}} = 0, \quad \epsilon(\Delta^a) = 1, \quad (41)$$

$$V^{\{a}V^{b\}} = 0, \quad \epsilon(V^a) = 1, \quad (42)$$

$$V^a\Delta^b + \Delta^bV^a = 0, \quad (43)$$

where the curly bracket denotes symmetrization with respect to the enclosed indices  $a$  and  $b$ . Using the odd second-order differential operators  $\Delta^a$ , one can introduce a pair of bilinear operations  $(\ , \ )^a$  on the triplectic supermanifold  $\mathcal{M}$  by the rule

$$(F, G)^a = (-1)^{\epsilon(G)}(FG)\Delta^a - (-1)^{\epsilon(G)}(F\Delta^a)G - F(G\Delta^a). \quad (44)$$

The operations (44) possess the Grassmann parity  $\epsilon((F, G)) = \epsilon(F) + \epsilon(G) + 1$  and obey the following symmetry property:

$$(F, G)^a = -(-1)^{(\epsilon(G)+1)(\epsilon(F)+1)}(G, F)^a. \quad (45)$$

They are linear operations with respect to both arguments (see (A.3)), and obey the Leibniz rule (see (A.4)). Due to the properties (41) of the operators  $\Delta^a$ , these odd bracket operations satisfy the generalized Jacobi identity

$$(F, (G, H)^{\{a\}b\})(-1)^{(\epsilon(F)+1)(\epsilon(H)+1)} + \text{cycle}(F, G, H) \equiv 0. \quad (46)$$

According to their properties, the operations  $(\ , \ )^a$  form a set of antibrackets, as have been introduced for the first time in Ref. [4]. Therefore, if we have an explicit realization of the operators  $\Delta^a$  with the properties (41), then, according to (44), we can generate the extended antibrackets explicitly. Explicit realizations of  $\Delta^a$  are known in two cases: in Darboux coordinates [4, 5, 7] and in general coordinates when  $M$  is a flat Fedosov manifold, with bosonic variables  $x^i$  [1]. However, in Quantum Gauge Field Theory the base manifold  $M$  always requires fermionic coordinates for its description, and therefore it should be considered as a supermanifold from the beginning.

In this Section, we extend the considerations of Ref. [5] in order to consider some possible explicit realizations of the triplectic algebra (41) – (43) on a triplectic supermanifold  $\mathcal{M}$ . To this end, we equip the base supermanifold  $M$  with a Poisson structure, i.e., with a non-degenerate *even* second-rank tensor field  $\omega^{ij}(x)$ ,  $\epsilon(\omega^{ij}) = \epsilon_i + \epsilon_j$ , obeying the property of generalized antisymmetry,

$$\omega^{ij} = -(-1)^{\epsilon_i\epsilon_j}\omega^{ji}, \quad (47)$$

and satisfying the following identities:

$$\omega^{ik}\partial_k\omega^{jn}(-1)^{\epsilon_i\epsilon_n} + \text{cycle}(i, j, n) \equiv 0. \quad (48)$$

This tensor field  $\omega^{ij}$  defines a Poisson bracket, and, due to its non-degeneracy, also a corresponding even symplectic structure on the supermanifold  $M$  (see Appendix A). At this level, the supermanifold  $M$  can be considered as either an even Poisson supermanifold or an even symplectic supermanifold.

The Poisson structure  $\omega^{ij}$  allows one to equip the triplectic supermanifold  $\mathcal{M}$  with an  $Sp(2)$ -irreducible second-rank tensor  $S_{ab}$ ,

$$S_{ab} = \frac{1}{6}\theta_{ia}\omega^{ij}\theta_{jb}, \quad \epsilon(S_{ab}) = 0, \quad (49)$$

which is invariant under changes of the local coordinates on  $\mathcal{M}$ , i.e.,  $\bar{S}_{ab} = S_{ab}$ , and is symmetric with respect to the  $Sp(2)$  indices,  $S_{ab} = S_{ba}$ .

Now, let us require that the covariant derivative  $\nabla_i$  should respect the Poisson structure  $\omega^{ij}$ :

$$\omega^{ij}\nabla_k = 0 \quad \leftrightarrow \quad \omega^{ij}_{,k} + \omega^{im}\Gamma^j_{mk}(-1)^{\epsilon_m(\epsilon_j+1)} - \omega^{jm}\Gamma^i_{mk}(-1)^{\epsilon_i\epsilon_j+\epsilon_m(\epsilon_i+1)} = 0. \quad (50)$$

From (39) and (50), it follows that the  $Sp(2)$  second-rank tensor  $S_{ab}$  is a covariantly constant,

$$S_{ab}\nabla_i = 0. \quad (51)$$

In terms of the tensor field  $\omega_{ij}$ , the inverse of  $\omega^{ij}$ , the relations (50) read

$$\omega_{ij,k} - \omega_{im}\Gamma_{jk}^m + \omega_{jm}\Gamma_{ik}^m(-1)^{\epsilon_i\epsilon_j} = 0, \quad (52)$$

which provides the covariant constancy of the differential two-form  $\omega$  (see, eq. A.14),

$$\omega\nabla = 0. \quad (53)$$

Then the supermanifold  $M$  can be considered as an even symmetric symplectic supermanifold, being a supersymmetric extension of the Fedosov manifold [11, 12].

Taking into account the relations (17) and (32), and using the covariant operators  $\overleftarrow{\nabla}_i$  and  $\overleftarrow{\partial}/\partial\theta_{ia}$ , we find that there exists a unique (up to first-order differential operators)  $Sp(2)$ -doublet of odd second-order differential operators acting as scalars on triplectic supermanifolds  $\mathcal{M}$ ,

$$\overleftarrow{\Delta}^a = (-1)^{\epsilon_i} \frac{\overleftarrow{\partial}}{\partial\theta_{ia}} \overleftarrow{\nabla}_i + \frac{1}{2}(-1)^{\epsilon_i} \frac{\overleftarrow{\partial}}{\partial\theta_{ia}} \rho_{,i}, \quad (54)$$

where  $\rho = \rho(x)$ ,  $\epsilon(\rho) = 0$ , is a scalar density on  $M$ . The operators (54) generate a doublet of operations on  $\mathcal{M}$ ,

$$(F, G)^a = (F\nabla_i) \frac{\partial G}{\partial\theta_{ia}} - (-1)^{(\epsilon(F)+1)(\epsilon(G)+1)} (G\nabla_i) \frac{\partial F}{\partial\theta_{ia}}. \quad (55)$$

These operations obviously obey all the properties of extended antibrackets, except for the Jacobi identity, which is closely related to the anticommutativity and nilpotency (41) of  $\Delta^a$ .

In order to get also that missing property we must restrict the base supermanifold somewhat. Using the operations (55) and the irreducible second-rank  $Sp(2)$ -tensor  $S_{ab}$  (49), we shall define the following  $Sp(2)$ -doublet of odd first-order differential operators  $\overleftarrow{V}_a$ :

$$\overleftarrow{V}_a = (\cdot, S_{ab})^b = \frac{1}{2} \overleftarrow{\nabla}_i \omega^{ij} \theta_{ja}, \quad (56)$$

where the relations (51) have been used for the second equality. Straightforward calculations, with allowance for the algebra of operators  $\Delta^a$  (54) and  $V^a$  (56) (see Appendix B), show that there exists a choice of the density function  $\rho$ , namely,

$$\rho = -\log \text{sdet} [\omega^{ij}], \quad (57)$$

such that the triplectic algebra (41) – (43) is fulfilled on  $\mathcal{M}$  when the base supermanifold  $M$  is a flat Fedosov superspace,

$$R^i{}_{mjk} = 0, \quad (58)$$

with the curvature tensor  $R^i{}_{mjk}$  given by (28). Therefore, we have explicitly realized the extended antibrackets (55) and the triplectic algebra (41) – (43) of the generating operators  $\Delta^a$ ,  $\overleftarrow{V}^a$ . Moreover, the operators  $V^a$  can always be considered as anti-Hamiltonian vector fields.

## 5 Operators $U^a$ and quantization

The modified triplectic quantization [7], in comparison with the  $Sp(2)$ -covariant method [4], or the triplectic scheme [5], involves an additional  $Sp(2)$ -doublet of odd operators  $\overleftarrow{U}^a$  ( $\epsilon(U^a) = 1$ ), with the following properties:

$$U^{\{a} U^{b\}} = 0, \quad \Delta^{\{a} U^{b\}} + U^{\{a} \Delta^{b\}} = 0, \quad U^{\{a} V^{b\}} + V^{\{b} U^{a\}} = 0. \quad (59)$$

An invariant realization of these operators on  $\mathcal{M}$  makes us introduce a new geometrical structure on  $M$ . Namely, because  $M$  is already equipped with the symplectic structure  $\omega_{ij}$ , there exists possibility to equip the base supermanifold  $M$  also with a symmetric second-rank tensor field  $g_{ij}(x) = (-1)^{\epsilon_i \epsilon_j} g_{ji}(x)$  of type  $(0, 2)$ .

Notice that on the triplectic supermanifold  $\mathcal{M}$  there exists a vector field  $\theta_a^i$ ,

$$\theta_a^i = \omega^{ij} \theta_{ja} (-1)^{\epsilon_i}, \quad (60)$$

which, due to (39) and (50), is covariantly constant,

$$\theta_a^i \nabla_j = 0. \quad (61)$$

This vector field can be used to construct on  $\mathcal{M}$  an  $Sp(2)$ -scalar function  $S_0$ , the so-called anti-Hamiltonian, according to

$$S_0 = \frac{1}{2} \varepsilon^{ab} \theta_a^i g_{ij} \theta_b^j (-1)^{\epsilon_i + \epsilon_j}, \quad \epsilon(S_0) = 0. \quad (62)$$

The anti-Hamiltonian  $S_0$  generates the vector fields  $\overleftarrow{U}^a$

$$\overleftarrow{U}^a = (\cdot, S_0)^a = -\overleftarrow{\nabla}_i \omega^{im} g_{mn} \theta^{na} (-1)^{\epsilon_m} - \frac{1}{2} \frac{\overleftarrow{\partial}}{\partial \theta_{ia}} \theta_c^m (g_{mn} \nabla_i) \theta^{nc} (-1)^{\epsilon_n(\epsilon_i+1)+\epsilon_m}. \quad (63)$$

The conditions (59) yield the following equations for  $S_0$ :

$$S_0 U^a \equiv (S_0, S_0)^a = 0, \quad S_0 V^a = 0, \quad S_0 \Delta^a = 0. \quad (64)$$

In fact, these are equations to be fulfilled for  $g_{ij}$ . Of course, solutions of these equations always exist. For example, the covariantly constant tensor field  $g_{ij}$ ,  $g_{ij} \nabla_k = 0$ , belongs to them. We do not restrict ourselves to this special case, and just assume that equations (64) are fulfilled. Therefore, we obtain a realization of the modified triplectic algebra on  $\mathcal{M}$ , and have at our disposal all the ingredients for the quantization of general gauge theories within the modified triplectic scheme.

This quantization procedure repeats all the essential steps performed for the first time in Ref. [1], and leads to the vacuum functional

$$Z = \int dz d\lambda \mathcal{D}_0 \exp\{(i/\hbar)[W + X + \alpha S_0]\}, \quad (65)$$

where the quantum action  $W = W(z)$  and the gauge fixing functional  $X = X(z, \lambda)$  satisfy the following quantum master equations:

$$\frac{1}{2}(W, W)^a + W \mathcal{V}^a = i\hbar W \Delta^a, \quad (66)$$

$$\frac{1}{2}(X, X)^a + X \mathcal{U}^a = i\hbar X \Delta^a. \quad (67)$$

Here,  $\mathcal{D}_0$  is the integration measure,

$$\mathcal{D}_0 = (\text{sdet}[\omega^{ij}])^{-3/2}, \quad (68)$$

$\alpha$  is an arbitrary constant, and the function  $S_0$  has been introduced in (62). In (66) and (67), we have introduced operators  $\mathcal{V}^a$ ,  $\mathcal{U}^a$ , according to

$$\mathcal{V}^a = \frac{1}{2}(\alpha U^a + \beta V^a + \gamma U^a), \quad \mathcal{U}^a = \frac{1}{2}(\alpha U^a - \beta V^a - \gamma U^a). \quad (69)$$

Evidently, for arbitrary constants  $\alpha, \beta, \gamma$  the operators  $\mathcal{V}^a, \mathcal{U}^a$  obey the properties

$$\mathcal{V}^{\{a} \mathcal{V}^{b\}} = 0, \quad \mathcal{U}^{\{a} \mathcal{U}^{b\}} = 0, \quad \mathcal{V}^{\{a} \mathcal{U}^{b\}} + \mathcal{U}^{\{a} \mathcal{V}^{b\}} = 0. \quad (70)$$

Therefore, the operators  $\Delta^a, \mathcal{V}^a, \mathcal{U}^a$  also realize the modified triplectic algebra.

The integrand of the vacuum functional (65) is invariant under the BRST-antiBRST transformations defined by the generators

$$\delta^a = (\cdot, X - W)^a + \mathcal{V}^a - \mathcal{U}^a. \quad (71)$$

In the usual manner, one can prove that the vacuum functional (65), for every fixed set of parameters  $\alpha, \beta, \gamma$ , does not depend on the gauge-fixing function  $X$ .



## 6 Conclusion

In this paper, we have proposed a formulation of the modified triplectic quantization in general coordinates.

We have found an explicit realization of the modified triplectic algebra of generating operators  $\Delta^a, V^a, U^a$  on a triplectic superspace  $\mathcal{M}$ , where the base supermanifold  $M$  is a flat Fedosov superspace endowed with a symmetric structure. The proposed scheme is characterized by three free parameters,  $\alpha, \beta, \gamma$ , whose specific choice, together with the Darboux coordinates, reproduces all the known schemes of covariant quantization based on the BRST–antiBRST invariance (for details, see [1]). Every specific choice of these parameters  $\alpha, \beta, \gamma$  gives a gauge-independent vacuum functional and, therefore, a gauge independent S-matrix (see [13]).

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## A Poisson and symplectic structures on supermanifolds

Let us consider a second-rank tensor field  $\omega^{ij} = \omega^{ij}(x)$  of type  $(2, 0)$ ,  $\epsilon(\omega^{ij}) = \epsilon(\omega) + \epsilon_i + \epsilon_j$ , which is defined on a supermanifold  $M$  with local coordinates  $x^i$ ,  $\epsilon(x^i) = \epsilon_i$ . Then, let us introduce the bilinear operation

$$(A, B) = \frac{\partial_r A}{\partial x^i} (-1)^{\epsilon(\omega)\epsilon_i} \omega^{ij} \frac{\partial B}{\partial x^j} = \frac{\partial_r A}{\partial x^i} {}^i\omega^j \frac{\partial B}{\partial x^j}, \quad (\text{A.1})$$

which is invariant,  $(\bar{A}, \bar{B}) = (A, B)$ , under arbitrary coordinate transformations,  $(x) \rightarrow (\bar{x})$ . Evidently, this operation obeys the following properties:

(a) Grassmann parity

$$\epsilon(A, B) = \epsilon(A) + \epsilon(B) + \epsilon(\omega), \quad (\text{A.2})$$

(b) linearity

$$\begin{aligned} (A + C, B) &= (A, B) + (C, B), & \epsilon(A) &= \epsilon(C), \\ (A, B + D) &= (A, B) + (A, D), & \epsilon(B) &= \epsilon(D), \end{aligned} \quad (\text{A.3})$$

(c) Leibniz rule

$$\begin{aligned} (AC, B) &= A(C, B) + (A, B)C(-1)^{\epsilon(C)(\epsilon(B)+\epsilon(\omega))}, \\ (A, BD) &= (A, B)D + B(A, D)(-1)^{\epsilon(B)(\epsilon(A)+\epsilon(\omega))}. \end{aligned} \quad (\text{A.4})$$

If, in addition, the tensor field  $\omega^{ij}$  obeys the property of generalized antisymmetry,

$$\omega^{ij} = -(-1)^{\epsilon_i\epsilon_j+\epsilon(\omega)}\omega^{ji} \quad \leftrightarrow \quad {}^i\omega^j = -(-1)^{(\epsilon_i+\epsilon(\omega))(\epsilon_j+\epsilon(\omega))}{}^j\omega^i, \quad (\text{A.5})$$

then, the binary operation (A.1) has the property of

(d) generalized antisymmetry

$$(A, B) = -(-1)^{(\epsilon(A)+\epsilon(\omega))(\epsilon(B)+\epsilon(\omega))} (B, A). \quad (\text{A.6})$$

Finally, let us restrict  $\omega^{ij}$  to obey

$$\omega^{ik}\partial_k\omega^{jn}(-1)^{(\epsilon_i+\epsilon(\omega))(\epsilon_n+\epsilon(\omega))} + \text{cycle}(i, j, n) \equiv 0, \quad (\text{A.7})$$

then the operation (A.1) obeys

(e) generalized Jacobi identity

$$(A, (B, C))(-1)^{(\epsilon(A)+\epsilon(\omega))(\epsilon(C)+\epsilon(\omega))} + \text{cycle}(A, B, C) \equiv 0. \quad (\text{A.8})$$

From (A.2), (A.3), (A.4), (A.6) and (A.8), it follows that the operation (A.1) coincides with the Poisson bracket when  $\omega$  is even,  $\epsilon(\omega) = 0$ , and with the antibracket when  $\omega$  is odd,  $\epsilon(\omega) = 1$ .

Now, suppose that the tensor field  $\omega^{ij}$  is non-degenerate. Then, defining the right and left inverse matrices, labelled by  $(R)$  and  $(L)$ , respectively, according to

$${}^i\omega^k{}_k {}^{(R)}\omega_j = {}^i\delta_j, \quad {}^{(L)}\omega_k {}^k\omega^j{}_j = {}^i\delta^j{}_i, \quad (\text{A.9})$$

we find that these inverse matrices coincide:

$${}^{(R)}\omega_j = {}^{(L)}\omega_j = {}^i\omega_j, \quad \epsilon({}^i\omega_j) = \epsilon(\omega) + \epsilon_i + \epsilon_j. \quad (\text{A.10})$$

Due to (A.5), we find that the inverse matrix  ${}^i\omega_j$  has the following property of generalized symmetry:

$${}^i\omega_j = -(-1)^{\epsilon_i\epsilon_j+(\epsilon(\omega)+1)(\epsilon_i+\epsilon_j)}{}_j\omega_i \quad \leftrightarrow \quad \omega_{ij} = -(-1)^{\epsilon_i\epsilon_j}\omega_{ji}. \quad (\text{A.11})$$

We also observe the remarkable fact that the symmetry properties of the inverse matrix  $\omega_{ij}$  do not depend on the Grassmann parity of the tensor field  $\omega$ . Using the tensor field  $\omega_{ij}$  and DeWitt's index shifting rules (6), one can rewrite the relations (A.9) in the form

$$\omega^{ik} \omega_{kj} (-1)^{\epsilon_k} = \delta^i_j, \quad (-1)^{\epsilon_i} \omega_{ik} \omega^{kj} = \delta^j_i. \quad (\text{A.12})$$

In terms of  $\omega_{ij}$ , the generalized Jacobi identity (A.7) can be rewritten in the form

$$\partial_i \omega_{jk} (-1)^{\epsilon_i(\epsilon(\omega)+1+\epsilon_k)} + \text{cycle}(i, j, k) \equiv 0 \quad \leftrightarrow \quad \omega_{ij,k} (-1)^{\epsilon_i\epsilon_k} + \text{cycle}(i, j, k) \equiv 0. \quad (\text{A.13})$$

Let us now introduce a differential 2-form  $\omega$  on the supermanifold  $M$ , having the same form in both the even and odd cases:

$$\omega = \omega_{ij} dx^j \wedge dx^i, \quad dx^i \wedge dx^j = -(-1)^{\epsilon_i\epsilon_j} dx^j \wedge dx^i. \quad (\text{A.14})$$

It is invariant under a change of the local coordinates,  $\bar{\omega} = \omega$ . The external derivative of this 2-form is given by

$$d\omega = \omega_{ij,k} dx^k \wedge dx^j \wedge dx^i, \quad d^2\omega = 0. \quad (\text{A.15})$$

It is also invariant under a change of the local coordinates,  $d\bar{\omega} = d\omega$ . The requirement of closure ( $d\omega = 0$ ) leads exactly to the Jacobi identities for  $\omega_{ij}$  (A.13). Therefore, as in the case of the usual differential geometry, there exists a one-to-one correspondence between even (odd) non-degenerate Poisson supermanifolds and even (odd) symplectic supermanifolds.

## B Algebra of operators $\Delta^a$ and $V^a$

Let us investigate the algebra of the operators  $\overleftarrow{\Delta}^a$  (54) and  $\overleftarrow{V}^a$  (56). Omitting the details of

the tedious calculations for operators acting on scalars on  $\mathcal{M}$ , we obtain the following results:

$$\Delta^{\{a}\Delta^{b\}} = -\frac{\overleftarrow{\partial}}{\partial\theta_{ia}}\frac{\overleftarrow{\partial}}{\partial\theta_{jb}}\frac{\overleftarrow{\partial}}{\partial\theta_{mc}}\theta_{nc}R^n_{mij}(-1)^{\epsilon_j(\epsilon_i+1)}, \quad (\text{B.1})$$

$$V^{\{a}V^{b\}} = -\frac{1}{4}\frac{\overleftarrow{\partial}}{\partial\theta_{nc}}\theta_{lc}R^l_{nim}\theta^{ia}\theta^{mb}(-1)^{\epsilon_m(\epsilon_i+1)}, \quad (\text{B.2})$$

$$\begin{aligned} 2(\Delta^a V^b + V^b \Delta^a) &= \varepsilon^{ab}\frac{\overleftarrow{\partial}}{\partial\theta_{mc}}\theta_{nc}R^n_{mij}\omega^{ij} \\ &\quad -\frac{\overleftarrow{\partial}}{\partial\theta_{ia}}\frac{\overleftarrow{\partial}}{\partial\theta_{mc}}\theta_{nc}R^n_{mij}\theta^{jb}(-1)^{\epsilon_i+\epsilon_j} \\ &\quad -\varepsilon^{ab}\overleftarrow{\nabla}_j\left(\omega^{ji}_{,i} + \frac{1}{2}\omega^{ji}\rho_{,i}\right)(-1)^{\epsilon_i} \\ &\quad +\frac{\overleftarrow{\partial}}{\partial\theta_{ma}}\left[-\Gamma^n_{mj,n}(-1)^{\epsilon_j\epsilon_n+\epsilon_n(\epsilon_m+1)} + \Gamma^n_{js}\Gamma^s_{mn}(-1)^{\epsilon_j(\epsilon_m+\epsilon_n)+\epsilon_n(\epsilon_m+1)}\right. \\ &\quad \left.+\frac{1}{2}\rho_{,mj}(-1)^{\epsilon_m} - \frac{1}{2}\rho_{,i}\Gamma^i_{mj}(-1)^{\epsilon_m}\right]\theta^{jb}(-1)^{\epsilon_j}. \end{aligned} \quad (\text{B.3})$$

Let us introduce a function  $\rho$ , using the relations

$$\omega^{ji}_{,i}(-1)^{\epsilon_i} + \frac{1}{2}\omega^{ji}\rho_{,i}(-1)^{\epsilon_i} = 0, \quad (\text{B.4})$$

which, in view of (A.12), is equivalent to

$$\rho_{,m} = 2\omega^{ji}_{,i}\omega_{jm}(-1)^{\epsilon_i+\epsilon_j}. \quad (\text{B.5})$$

To solve these equations, it is necessary to use the consequences of the Jacobi identities in the form

$$\omega_{ij}\omega^{ji}_{,m} + 2\omega^{ji}_{,i}\omega_{jm}(-1)^{\epsilon_i+\epsilon_j} = 0. \quad (\text{B.6})$$

Therefore, the function  $\rho$  must satisfy the relations

$$\rho_{,m} = -\omega_{ij}\omega^{ji}_{,m} \quad (\text{B.7})$$

and can be chosen as

$$\rho = -\log \text{sdet} [\omega^{ij}]. \quad (\text{B.8})$$

Indeed, for the variation of  $\rho$  (B.8) we have

$$\begin{aligned} \delta\rho &= -\log \text{sdet} [\omega^{ij} + \delta\omega^{ij}] + \log \text{sdet} [\omega^{ij}] \\ &= -\log \text{sdet} [\delta^i_j + (-1)^{\epsilon_i}\omega_{il}\delta\omega^{lj}] \\ &= -\text{str}[(-1)^{\epsilon_i}\omega_{il}\delta\omega^{lj}] = -\omega_{ij}\delta\omega^{ji}. \end{aligned} \quad (\text{B.9})$$

With allowance for (50) and (B.5), one can derive the following useful relation:

$$\frac{1}{2}\rho_{,m} = \Gamma^i_{mi}(-1)^{\epsilon_i(\epsilon_m+1)} = \Gamma^i_{im}(-1)^{\epsilon_i}. \quad (\text{B.10})$$

From (28), (B.4) and (B.10) one gets the final expression for the anticommutator (B.3):

$$\begin{aligned} \Delta^a V^b + V^b \Delta^a &= \frac{1}{2}\left(\varepsilon^{ab}\frac{\overleftarrow{\partial}}{\partial\theta_{mc}}\theta_{nc}R^n_{mij}\omega^{ij} - \frac{\overleftarrow{\partial}}{\partial\theta_{ia}}\frac{\overleftarrow{\partial}}{\partial\theta_{mc}}\theta_{nc}R^n_{mij}\theta^{jb}(-1)^{\epsilon_i+\epsilon_j} \right. \\ &\quad \left. - \frac{\overleftarrow{\partial}}{\partial\theta_{ma}}R^n_{mnj}\theta^{jb}(-1)^{\epsilon_n(\epsilon_m+1)+\epsilon_j}\right). \end{aligned} \quad (\text{B.11})$$

Taking this into account, we can see that when the base supermanifold  $M$  is a flat Fedosov superspace,  $R^i_{jkm} = 0$ , then on the triplectic supermanifold  $\mathcal{M}$  we have an explicit realization of the triplectic algebra (41) – (43). Simultaneously, the operations (55) satisfy the Jacobi identity (46), and therefore (55) are identified with extended antibrackets.

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