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Ramond-Ramond Field Transformation

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Abstract

We find that the mixture of Ramond-Ramond fields and Neveu-Schwarz two form are transformed as Majorana spinors under the T-duality group $O(d,d)$. The Ramond-Ramond field transformation under the group $O(d,d)$ is realized in a simple form by using the spinor representation. The Ramond-Ramond field transformation rule obtained by Bergshoeff et al. is shown as a specific simple example. We also give some explicit examples of the spinor representation.

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1 Introduction

The transformation of the fields from the Neveu-Schwarz (NS) sector under T-duality is well established. The Ramond-Ramond (RR) field transformation was first given in [1]. The authors in [1] got the RR field transformation by identifying the same RR fields and RR moduli in $d = 9$ supergravity coming from both ten dimensional type IIA and type IIB supergravity theories compactified down to nine dimensions. Unfortunately this method gives only a specific T-duality transformation, namely Buscher's T-duality transformation [2]. It is hard to get the generalized T-duality group $O(d, d)$ transformations by this method. Recently, Hassan derived the RR field transformation under $SO(d, d)$ group by working on the worldsheet theory [3]. The RR field transformation under Buscher's T-duality was also discussed by Cvetič, Lü, Pope and Stelle using the Green-Schwarz formalism [4]. If we compactify the $d = 9$ supergravity further down to lower dimensions, we know that the lower dimensional solution has $O(d, d, R)$ transformation and how the NS-NS fields which are assumed to be independent of d coordinates transform under this group [5]. Therefore, we need to find the RR field transformation under the general $O(d, d, R)$ group.

The RR fields transform as the Majorana-Weyl spinors of $SO(d, d)$ [6]. RR fields transforming as the spinors of $O(d, d)$ group is discussed in more detail from the algebraic decomposition of U duality group in [7] [2]. The spinor representation idea was further developed by Fukuma, Oota and Tamaka [8]. It is not the RR potentials that transform as the Majorana-Weyl spinor of $SO(d, d)$; it is the mixed fields of RR potentials and NS-NS two form that transform as the Majorana-Weyl spinor of $SO(d, d)$. However, since the full T-duality group is $O(d, d)$, we expect to use the Majorana spinor representation of $O(d, d)$. Note also that $SO(d, d)$ transformations cannot interchange type IIA and type IIB theories. In this paper, we use RR fields to construct spinors of $O(d, d)$ explicitly. As a simple application we use the Majorana spinor representation to show the RR field transformations between type IIA and type IIB under T-duality. By using the Majorana spinor and the tensor representation of $O(d, d, R)$ group, we can get more general solution generating rules.

We define the RR potentials $C_{p+1} = (1/(p + 1)!)\, C_{\mu_1 \dots \mu_{p+1}}\, dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{p+1}}$. Following the definition given by [8], we define the new mixed fields as

$$\begin{array}{ll} D_0 \equiv C_0, & D_1 \equiv C_1, \\ D_2 \equiv C_2 + B_2 \wedge C_0, & D_3 \equiv C_3 + B_2 \wedge C_1, \\ D_4 \equiv C_4 + \frac{1}{2} B_2 \wedge C_2 + \frac{1}{2} B_2 \wedge B_2 \wedge C_0. & \end{array} \tag{1}$$

The RR field strengths are $F = e^{-B} \wedge dD$[8] [9], here

$$D \equiv \sum_{p=0}^{4} D_p, \quad F \equiv \sum_{p=1}^{5} F_p. \tag{2}$$

More explicitly, we have

$$\begin{array}{ll} F_1 = dD_0, & F_2 = dD_1, \\ F_3 = dD_2 - B_2 \wedge dD_0, & F_4 = dD_3 - B_2 \wedge dD_1, \\ F_5 = dD_4 - B_2 \wedge dD_2 + \frac{1}{2} B_2 \wedge B_2 \wedge dD_0. & \end{array} \tag{3}$$

We also use the convention

$$\int d^d x \sqrt{-g} |F_p|^2 = \int d^d x \frac{\sqrt{-g}}{p!} g^{\mu_1 \nu_1} \cdots g^{\mu_p \nu_p} F_{\mu_1 \mu_p} F_{\nu_1 \nu_p}. \tag{4}$$

[2] The author thanks S. Ferrara for pointing out this reference.

2 $d = 10$ Type IIA and Type IIB Reduction to $d = 9$

The action of ten dimensional type IIA supergravity can be written as

$$\begin{aligned} S_{10}^{IIA} =& \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R(G)+4G^{MN}\partial_M\Phi\partial_N\Phi-\frac{1}{2}|H_3|^2\right] \\ &-\frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-G}\Big(|F_2|^2+|F_4|^2\Big)+\frac{1}{4\kappa_{10}^2}\int d^{10}xB_2\wedge dC_3\wedge dC_3, \end{aligned} \tag{5}$$

where $H_3 = dB_2$, $F_2 = dC_1 = dD_1$, $F_4 = dC_3 + H_3 \wedge C_1 = dD_3 - B_2 \wedge dD_1$ and the subscript number of a form denotes the degree of the form. Now we dimensionally reduce the action (5) to nine dimensions by the vielbein,

$$E_M^A = \begin{pmatrix} e_\mu^a & eA_\mu^{(1)} \\ 0 & e \end{pmatrix}, \quad E_A^M = \begin{pmatrix} e_a^\mu & -e_a^\nu A_\nu^{(1)} \\ 0 & e^{-1} \end{pmatrix}. \tag{6}$$

The dimensionally reduced nine dimensional action for the NS and R sector is

$$\begin{aligned} S_9 =& \frac{1}{2\kappa_9^2}\int d^9x\sqrt{-g}e^{-2\phi}\Big[R(g)+4g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-e^{-2}g^{\mu\nu}\partial_\mu e\partial_\nu e \\ &-\frac{1}{2}e^2|F_2^{(1)}|^2-\frac{1}{2}e^{-2}|F_2^{(2)}|^2-\frac{1}{2}|H_3^{(1)}|^2\Big] \\ &-\frac{1}{4\kappa_9^2}\int d^9x\sqrt{-g}\Big(e|F_2|^2+e^{-1}g^{\mu\nu}\partial_\mu D_x\partial_\nu D_x+e^{-1}|H_3^{(2)}|^2+e|F_4|^2\Big), \end{aligned} \tag{7}$$

where

$$e^2 = G_{xx}, \qquad g_{\mu\nu} = G_{\mu\nu} - G_{xx}A_\mu^{(1)}A_\nu^{(1)}, \tag{8a}$$

$$A_\mu^{(1)} = \frac{G_{\mu x}}{G_{xx}}, \qquad A_\mu^{(2)} = B_{\mu x} \tag{8b}$$

$$A_\mu = D_\mu - A_\mu^{(1)}D_x, \quad F_{\mu\nu}^i = \partial_\mu A_\nu^{(i)} - \partial_\nu A_\mu^{(i)}, \tag{8c}$$

$$B_{\mu\nu}^{(1)} = B_{\mu\nu} + \frac{1}{2}A_\mu^{(1)}A_\nu^{(2)} - \frac{1}{2}A_\nu^{(1)}A_\mu^{(2)}, \quad B_{\mu\nu}^{(2)} = D_{\mu\nu x}, \tag{8d}$$

$$\phi = \Phi - \ln G_{xx}/4, \qquad \mathcal{D}_{\mu\nu\rho} = D_{\mu\nu\rho}, \tag{8e}$$

$$H_3^{(1)} = dB_2^{(1)} - \frac{1}{2}(A_1^{(1)} \wedge F_2^{(2)} + A_1^{(2)} \wedge F_2^{(1)}), \tag{8f}$$

$$H_3^{(2)} = dB_2^{(2)} - B_2^{(1)} \wedge dD_x + \frac{1}{2}A_1^{(2)} \wedge A_1^{(1)} \wedge dD_x - A_1^{(2)} \wedge (F_2 + F_2^{(1)}D_x), \tag{8g}$$

$$F_4 = d\mathcal{D}_3 - B_2^{(1)} \wedge dD_1 + \frac{1}{2}A_1^{(1)} \wedge A_1^{(2)} \wedge dD_1 + H_3^{(2)} \wedge A_1^{(1)}, \tag{8h}$$

and x is the compactified coordinate. Here we follow the general prescription of dimensional reduction given in [10]. For example, the lower dimensional field strength comes from the higher dimensional field strength as $H_{\mu\nu\rho}^{(1)} = e_\mu^a e_\nu^b e_\rho^c E_a^M E_b^N E_c^P H_{MNP}$. The action (7) can be obtained from the type IIB supergravity in ten dimensions also if we use the following vielbein for the IIB theory [1]

$$\mathcal{E}_M^A = \begin{pmatrix} e_\mu^a & e^{-1}A_\mu^{(2)} \\ 0 & e^{-1} \end{pmatrix}, \quad \mathcal{E}_A^M = \begin{pmatrix} e_a^\mu & -e_a^\nu A_\nu^{(2)} \\ 0 & e \end{pmatrix}, \tag{9}$$

together with the following definitions,

$$e^{-2} = \mathscr{G}_{xx}, \qquad g_{\mu\nu} = \mathscr{G}_{\mu\nu} - \mathscr{G}_{xx} A_\mu^{(2)} A_\nu^{(2)}, \tag{10a}$$

$$A_\mu^{(1)} = \mathscr{B}_{\mu x}, \qquad A_\mu^{(2)} = \frac{\mathscr{G}_{\mu x}}{\mathscr{G}_{xx}}, \qquad D = D_x, \tag{10b}$$

$$A_\mu = D_{\mu x} - \mathscr{B}_{\mu x} D = D_{\mu x} - A_\mu^{(1)} D, \tag{10c}$$

$$B_{\mu\nu}^{(1)} = \mathscr{B}_{\mu\nu} - \frac{1}{2} A_\mu^{(1)} A_\nu^{(2)} + \frac{1}{2} A_\nu^{(1)} A_\mu^{(2)}, \qquad B_{\mu\nu}^{(2)} = D_{\mu\nu}, \tag{10d}$$

$$\phi = \hat{\Phi} - \ln \mathscr{G}_{xx}/4, \qquad \mathcal{D}_{\mu\nu\rho} = D_{\mu\nu\rho x}. \tag{10e}$$

The type IIB ten dimensional supergravity action we use is

$$\begin{aligned} S_{10}^{IIB} =& \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-\mathscr{G}} e^{-2\hat{\Phi}} \left[R(\mathscr{G}) + 4 \mathscr{G}^{MN} \partial_M \hat{\Phi} \partial_N \hat{\Phi} - \frac{1}{2} |\mathscr{H}_3|^2 \right] \\ & - \frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-\mathscr{G}} \left(|F_1|^2 + |F_3|^2 + \frac{1}{2} |F_5|^2 \right) + \frac{1}{4\kappa_{10}^2} \int d^{10}x \mathscr{B}_2 \wedge dC_4 \wedge dC_2, \end{aligned} \tag{11}$$

together with the self dual constraint on F_5. Now we can get Buscher's T-duality transformations [2] from Eqs. (8a)-(8e) and Eqs. (10a)-(10e) as follows

$$\tilde{g}_{xx} = \frac{1}{g_{xx}}, \quad \tilde{g}_{\mu x} = \frac{B_{\mu x}}{g_{xx}}, \quad \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{g_{\mu x} g_{\nu x} - B_{\mu x} B_{\nu x}}{g_{xx}}, \tag{12a}$$

$$\tilde{B}_{\mu x} = \frac{g_{\mu x}}{g_{xx}}, \quad \tilde{B}_{\mu\nu} = B_{\mu\nu} - \frac{B_{\mu x} g_{\nu x} - B_{\nu x} g_{\mu x}}{g_{xx}}, \tag{12b}$$

$$\tilde{\phi} = \phi - \frac{1}{2} \ln g_{xx}, \tag{12c}$$

$$\tilde{D}_x = D, \quad \tilde{D}_\mu = D_{\mu x}, \quad \tilde{D}_{\mu\nu x} = D_{\mu\nu}, \quad \tilde{D}_{\mu\nu\rho} = D_{\mu\nu\rho x}. \tag{12d}$$

From the above transformation rules (12a)- (12d), we have the following transformations in terms of the original RR potentials,

$$\tilde{C}_x = C, \quad \tilde{C}_\mu = C_{\mu x} + B_{\mu x} C, \quad \tilde{C}_{\mu\nu x} = C_{\mu\nu} + \frac{g_{\mu x} C_{\nu x} - g_{\nu x} C_{\mu x}}{g_{xx}}, \tag{13a}$$

$$\tilde{C}_{\mu\nu\rho} = C_{\mu\nu\rho x} - \frac{3}{2} B_{[\mu\nu} C_{\rho]x} - \frac{3}{2} B_{x[\mu} C_{\nu\rho]} - \frac{6 g_{x[\mu} B_{\nu|x|} C_{\rho]x}}{g_{xx}}. \tag{13b}$$

In general we should consider the $O(d,d,R)$ transformations. The group element Ω of $O(d,d,R)$ satisfies

$$\Omega^T J \Omega = J, \qquad J = \begin{pmatrix} 0 & \mathbb{1}_d \\ \mathbb{1}_d & 0 \end{pmatrix}. \tag{14}$$

If we put the NS sector fields in a $2d$ by $2d$ matrix

$$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix} = \begin{pmatrix} \mathbb{1} & 0 \\ B & \mathbb{1} \end{pmatrix} \begin{pmatrix} G^{-1} & 0 \\ 0 & G \end{pmatrix} \begin{pmatrix} \mathbb{1} & -B \\ 0 & \mathbb{1} \end{pmatrix}, \tag{15}$$

where $G = [G_{ij}]$ and $B = [B_{ij}]$ are $d \times d$ matrices, i and j run over the compactified or independent d coordinates. Let

$$A^{(1)}_{\mu m} = G_{\mu m}, \quad A^{(1)m}_{\mu} = G^{mn} A^{(1)}_{\mu n}, \tag{16}$$

$$A^{(2)}_{\mu m} = B_{\mu m} + B_{mn} A^{(1)n}_{\mu}, \quad \mathcal{A}^{i}_{\mu} = \begin{pmatrix} A^{(1)m}_{\mu} \\ A^{(2)}_{\mu m} \end{pmatrix}, \tag{17}$$

$$g_{\mu\nu} = G_{\mu\nu} - G_{mn} A^{(1)m}_{\mu} A^{(1)n}_{\nu}, \tag{18}$$

$$\phi = \Phi - \frac{1}{4} \ln \det(G_{mn}), \tag{19}$$

$$B_{\mu\nu} = \hat{B}_{\mu\nu} + \frac{1}{2} A^{(1)m}_{\mu} A^{(2)}_{\nu m} - \frac{1}{2} A^{(1)m}_{\nu} A^{(2)}_{\mu m} - A^{(1)m}_{\mu} B_{mn} A^{(1)n}_{\nu}, \tag{20}$$

where Φ, $G_{\mu m}$, $G_{\mu\nu}$, G_{mn}, $\hat{B}_{\mu\nu}$, $B_{\mu m}$ and B_{mn} are the original NS fields. The $O(d,d)$ transformations for the NS fields are

$$M \to \Omega M \Omega^T, \quad \mathcal{A}^{i}_{\mu} \to \Omega_{ij} \mathcal{A}^{j}_{\mu}, \quad g_{\mu\nu} \to g_{\mu\nu}, \quad \phi \to \phi, \quad B_{\mu\nu} \to B_{\mu\nu}. \tag{21}$$

3 Spinor Representation

In this section, we will show that the RR fields transform as the Majorana spinors. We can write the general group element Ω of $O(d,d,R)$ as

$$\Omega = \begin{pmatrix} \mathscr{A} & \mathscr{B} \\ \mathscr{C} & \mathscr{D} \end{pmatrix}, \tag{22}$$

with $\mathscr{A}\mathscr{B}^T + \mathscr{B}\mathscr{A}^T = \mathscr{C}\mathscr{D}^T + \mathscr{D}\mathscr{C}^T = 0$, $\mathscr{A}\mathscr{D}^T + \mathscr{B}\mathscr{C}^T = \mathscr{C}\mathscr{B}^T + \mathscr{D}\mathscr{A}^T = 1$, $\mathscr{A}$, $\mathscr{B}$, $\mathscr{C}$ and $\mathscr{D}$ are $d \times d$ matrices. We can also show that $\mathscr{D} = \mathscr{C}\mathscr{A}^{-1}\mathscr{B} + (\mathscr{A}^{-1})^T$. The $O(d,d,R)$ group can be generated by the following three matrices [11]

$$\Lambda_C = \begin{pmatrix} \mathbb{1} & 0 \\ C & \mathbb{1} \end{pmatrix}, \quad \Lambda_R = \begin{pmatrix} (R^T)^{-1} & 0 \\ 0 & R \end{pmatrix}, \quad \Lambda_i = \begin{pmatrix} -\mathbb{1} + e_i & e_i \\ e_i & -\mathbb{1} + e_i \end{pmatrix}, \quad (e_i)_{jk} = \delta_{ij}\delta_{jk}, \tag{23}$$

where $C^T = -C$, $R \in GL(d,R)$ and i, j, and $k = 1$, $\ldots$, d. The action of Λ_C shifts the NS two-form by the matrix C. Under the action of Λ_R, $G \to RGR^T$, $B \to RBR^T$. For the group $O(d,d,Z)$, we need to restrict the matrix elements to be integers.

The Dirac matrices satisfy $\{\Gamma_r,\ \Gamma_s\} = 2J_{rs}$ with r and $s = 1$, $\ldots$, $2d$. Let

$$a_i = \frac{\Gamma_{d+i}}{\sqrt{2}}, \quad a_i^\dagger = \frac{\Gamma_i}{\sqrt{2}}, \quad i = 1,\ \ldots,\ d. \tag{24}$$

Then we have $\{a_i,\ a_j^\dagger\} = \delta_{ij}\mathbb{1}$, $\{a_i,\ a_j\} = \{a_i^\dagger,\ a_j^\dagger\} = 0$. Define the vacuum to be $a_i|0\rangle = 0$, we can get the representation (Fock) space as

$$|\alpha\rangle = (a_1^\dagger)^{i_1} \cdots (a_d^\dagger)^{i_d}|0\rangle, \quad i_1,\ \ldots,\ i_d = 0 \text{ or } 1. \tag{25}$$

The spinor representation of the $O(d,d)$ group is given by

$$S(\Omega)\Gamma_s S(\Omega)^{-1} = \sum_r \Gamma_r \Omega^r{}_s. \tag{26}$$

For convenience we can define the operator corresponding to a matrix $\mathbf{\Omega}$ as

$$\mathbf{\Omega}\Gamma_s = \sum_r \Gamma_r \Omega^r{}_s \mathbf{\Omega}, \quad \mathbf{\Omega}|\beta\rangle = \sum_\alpha |\alpha\rangle S_{\alpha\beta}(\Omega). \tag{27}$$

The operators for the three generating matrices are [6][8]

$$\mathbf{\Lambda}_C = \exp\left(\frac{1}{2}C_{ij}a_i a_j\right), \quad \mathbf{\Lambda}_i = \pm(a_i + a_i^\dagger), \tag{28}$$

$$\mathbf{\Lambda}_R = (\det R)^{-1/2} \exp\left(a_i A_i{}^j a_j^\dagger\right), \quad R = R_i{}^j = \exp(A_i{}^j), \tag{29}$$

where the repeated indices are summed. We choose $+$ sign for the $\mathbf{\Lambda}_i$ operator. The new mixed D fields form a spinor as follows: for $d = 1$,

$$\begin{gathered} \chi_\alpha = (D,\ D_x), \quad \chi_{\mu\alpha} = (D_\mu,\ D_{\mu x}), \\ \chi_{\mu\nu\alpha} = (D_{\mu\nu},\ D_{\mu\nu x}), \quad \chi_{\mu\nu\rho\alpha} = (D_{\mu\nu\rho},\ D_{\mu\nu\rho x}), \\ \cdots, \end{gathered}$$

with $|\alpha\rangle = (|0\rangle,\ a^\dagger|0\rangle)$; for $d = 2$,

$$\begin{gathered} \chi_\alpha = (D,\ D_x,\ D_y,\ D_{yx}), \\ \chi_{\mu\alpha} = (D_\mu,\ D_{\mu x},\ D_{\mu y},\ D_{\mu yx}), \\ \chi_{\mu\nu\alpha} = (D_{\mu\nu},\ D_{\mu\nu x},\ D_{\mu\nu y},\ D_{\mu\nu yx}), \\ \cdots, \end{gathered}$$

with $|\alpha\rangle = (|0\rangle,\ a_x^\dagger|0\rangle,\ a_y^\dagger|0\rangle,\ a_x^\dagger a_y^\dagger|0\rangle)$ and so on. The fields χ transform as

$$|\tilde{\chi}_{\mu_1\ldots\mu_p\alpha}\rangle = \sum_\beta S^{-1}(\Omega^T)_{\alpha\beta}|\tilde{\chi}_{\mu_1\ldots\mu_p\beta}\rangle. \tag{30}$$

For instance, the spinor representation matrix of $O(1,1)$ for Λ_i is

$$S\left((\Lambda^T)^{-1}\right) = S(\Lambda) = \Lambda = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{31}$$

From the spinor matrix (31), it is easy to get Buscher's T-duality transformations (12a)-(12d), (13a) and (13b) by combining Eqs. (21) and (30). The spinor representation of $SO(1,1)$ for $\Lambda_i\Lambda_j$ is

$$S(\Lambda^2) = \Lambda^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{32}$$

This is a trivial identity transformation. Furthermore it gives a Majorana-Weyl spinor representation.

4 More Examples

In order to discuss the solution generating transformations, we focus on $O(d)\otimes O(d)$ group in this section. We embed the $O(d)$ matrices R and S into $O(d,d)$ matrix Ω. Because the metric J of $O(d,d)$ is rotated from the diagonal metric η by

$$J = \mathscr{R}\eta\mathscr{R}, \quad \eta = \begin{pmatrix} -1\!\!1 & 0 \\ 0 & 1\!\!1 \end{pmatrix}, \quad \mathscr{R} = \frac{\sqrt{2}}{2}\begin{pmatrix} -1\!\!1 & 1\!\!1 \\ 1\!\!1 & 1\!\!1 \end{pmatrix},$$

so

$$\Omega = \mathscr{R}^{-1}\begin{pmatrix} S & 0 \\ 0 & R \end{pmatrix}\mathscr{R} = \frac{1}{2}\begin{pmatrix} R+S & R-S \\ R-S & R+S \end{pmatrix}.$$

Note that Ω is also an element of $O(2d)$, so $(\Omega^T)^{-1} = \Omega$.

For example, if we take $R = -\mathbb{1} + 2e_i$, $S = -\mathbb{1}$, then we recover the T-duality Λ_i discussed before. If we choose [3]

$$S = \mathbb{1}, \qquad R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix},$$

then we have the spinor representation

$$S(\Omega) = \begin{pmatrix} \cos\frac{\theta}{2} & 0 & 0 & -\sin\frac{\theta}{2} \\ 0 & \cos\frac{\theta}{2} & \sin\frac{\theta}{2} & 0 \\ 0 & -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ \sin\frac{\theta}{2} & 0 & 0 & \cos\frac{\theta}{2} \end{pmatrix}. \tag{33}$$

For flat background with zero B field, RR fields transform the same way as the D fields. This result is consistent with that obtained in [3].

If one of the coordinate is timelike, we have

$$\Omega = \frac{1}{2}\begin{pmatrix} \eta(S+R)\eta & \eta(R-S) \\ (R-S)\eta & S+R \end{pmatrix}, \quad \mathscr{R} = \frac{1}{\sqrt{2}}\begin{pmatrix} -\eta & \mathbb{1} \\ \eta & \mathbb{1} \end{pmatrix}, \tag{34}$$

here η is the Minkowski metric, S and R are $O(d-1,1)$ matrices satisfying $S\eta S^T = \eta$ and $R\eta R^T = \eta$. For example,

$$R = S = \begin{pmatrix} \cosh\alpha & \sinh\alpha \\ \sinh\alpha & \cosh\alpha \end{pmatrix}$$

generate the boost transformation along t-x coordinates,

$$S^{-1}(\Omega_b^T) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cosh\alpha & \sinh\alpha & 0 \\ 0 & \sinh\alpha & \cosh\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{35}$$

More explicitly, the boost transformation for the RR field and B field is

$$\tilde{B}_{\mu t} = B_{\mu t}\cosh\alpha + B_{\mu x}\sinh\alpha, \quad \tilde{B}_{\mu x} = B_{\mu t}\sinh\alpha + B_{\mu x}\cosh\alpha, \tag{36a}$$

$$\tilde{C}_{\mu\ldots\nu t} = C_{\mu\ldots\nu t}\cosh\alpha + C_{\mu\ldots\nu x}\sinh\alpha, \quad \tilde{C}_{\mu\ldots\nu x} = C_{\mu\ldots\nu t}\sinh\alpha + C_{\mu\ldots\nu x}\cosh\alpha, \tag{36b}$$

$$\tilde{B}_{tx} = B_{tx}, \quad \tilde{C}_{\mu\ldots\nu tx} = C_{\mu\ldots\nu tx}, \quad \tilde{B}_{\mu\nu} = B_{\mu\nu}, \quad \tilde{C}_{\mu\ldots\nu} = C_{\mu\ldots\nu}. \tag{36c}$$

Finally let us choose[5]

$$S = \begin{pmatrix} \cosh\alpha & -\sinh\alpha \\ -\sinh\alpha & \cosh\alpha \end{pmatrix}, \quad R = \begin{pmatrix} \cosh\alpha & \sinh\alpha \\ \sinh\alpha & \cosh\alpha \end{pmatrix}. \tag{37}$$

In this case, we have

$$S^{-1}(\Omega_s^T) = \begin{pmatrix} \cosh\alpha & 0 & 0 & \sinh\alpha \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh\alpha & 0 & 0 & \cosh\alpha \end{pmatrix}. \tag{38}$$

For the background $B_{\mu\nu}=0$, $g_{11}=1$ and $g_{01}=0$, the transformations of NS-NS and RR fields are

$$\tilde{g}_{00} = \frac{g_{00}}{1+(1+g_{00})\sinh^2\alpha}, \tag{39a}$$

$$\tilde{g}_{11} = \frac{1}{1+(1+g_{00})\sinh^2\alpha}, \tag{39b}$$

$$\tilde{B}_{01} = \frac{(1+g_{00})\sinh 2\alpha}{2[1+(1+g_{00})\sinh^2\alpha]}, \tag{39c}$$

$$\tilde{g}_{\mu 0} = \frac{g_{\mu 0}\cosh\alpha}{1+(1+g_{00})\sinh^2\alpha}, \tag{39d}$$

$$\tilde{g}_{\mu 1} = \frac{g_{\mu 1}\cosh\alpha}{1+(1+g_{00})\sinh^2\alpha}, \tag{39e}$$

$$\tilde{B}_{\mu 0} = \frac{-g_{00}g_{\mu 1}\sinh\alpha}{1+(1+g_{00})\sinh^2\alpha}, \tag{39f}$$

$$\tilde{B}_{\mu 1} = \frac{g_{\mu 0}\sinh\alpha}{1+(1+g_{00})\sinh^2\alpha}, \tag{39g}$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{(g_{\mu 0}g_{\nu 0}+g_{00}g_{\mu 1}g_{\nu 1})\sinh^2\alpha}{1+(1+g_{00})\sinh^2\alpha}, \tag{39h}$$

$$\tilde{B}_{\mu\nu} = \frac{(g_{\mu 0}g_{\nu 1}-g_{\mu 1}g_{\nu 0})\sinh\alpha\,\cosh\alpha}{1+(1+g_{00})\sinh^2\alpha}, \tag{39i}$$

$$\tilde{C} = C\cosh\alpha - C_{01}\sinh\alpha, \tag{39j}$$

$$\tilde{C}_0 = C_0, \quad \tilde{C}_1 = C_1, \quad \tilde{C}_\mu = C_\mu\cosh\alpha - C_{\mu 01}\sinh\alpha, \tag{39k}$$

$$\begin{aligned}\tilde{C}_{01} =& \frac{C_{01}[1+2(1+g_{00})\sinh^2\alpha]\cosh\alpha}{1+(1+g_{00})\sinh^2\alpha} \\ &- \frac{C[1+(1+g_{00})(\sinh^2\alpha+\cosh^2\alpha)]\sinh\alpha}{1+(1+g_{00})\sinh^2\alpha},\end{aligned} \tag{39l}$$

$$\tilde{C}_{\mu 0} = C_{\mu 0} + \frac{g_{00}g_{\mu 1}\sinh\alpha(C\cosh\alpha - C_{01}\sinh\alpha)}{1+(1+g_{00})\sinh^2\alpha}, \tag{39m}$$

$$\tilde{C}_{\mu 1} = C_{\mu 1} - \frac{C g_{\mu 0}\sinh\alpha\cosh\alpha}{1+(1+g_{00})\sinh^2\alpha} + \frac{C_{01}g_{\mu 0}\sinh^2\alpha}{1+(1+g_{00})\sinh^2\alpha}, \tag{39n}$$

$$\begin{aligned}\tilde{C}_{\mu\nu} =& C_{\mu\nu}\cosh\alpha - C_{\mu\nu 01}\sinh\alpha \\ &+ \frac{(C_{01}\sinh\alpha - C\cosh\alpha)(g_{\mu 0}g_{\nu 1}-g_{\mu 1}g_{\nu 0})\sinh 2\alpha}{2[1+(1+g_{00})\sinh^2\alpha]},\end{aligned} \tag{39o}$$

$$e^{-2\tilde{\phi}} = e^{-2\phi}[1+(1+g_{00})\sinh^2\alpha]. \tag{39p}$$

5 Discussion

The RR field transformations are very simple in terms of the new mixed fields $\boldsymbol{D}$. It is very easy to see the RR field transformations from the spinor representations. For any group element $\Omega \in O(d,d)$, we can get the spinor representation $S(\Omega)$ from Eq (26) or Eq. (27). We can introduce higher degree potentials and field strengths with some constraints as shown in [8]. With the extra potentials, the action for the RR and Chern-Simons terms can be written in a simple way. This may suggest that the $\boldsymbol{D}$ fields are the natural RR potentials. We can apply the transformation Eqs. (21) for the NS-NS fields and the transformation Eqs. (30) for the RR fields to get more general solution generating rules.

Note Added: In the second version of paper [3], Hassan gave a general transformation of $\boldsymbol{D}$ field by spinor representation and discussed the equivalence of the RR field transformations between his supersymmetry method and the spinor representation.

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