# Gauge Symmetry Breaking through the Hosotani Mechanism in Softly Broken Supersymmetric QCD

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## Abstract

Gauge symmetry breaking through the Hosotani mechanism (the dynamics of nonintegrable phases) in softly broken supersymmetric QCD with $N_F^{fd}$ flavors is studied. For $N =$ even, there is a single $SU(N)$ symmetric vacuum state, while for $N =$ odd, there is a doubly degenerate $SU(N)$ symmetric vacuum state in the model. We also study generalized supersymmetric QCD by adding $N_F^{adj}$ numbers of massless adjoint matter. The gauge symmetry breaking pattern such as $SU(3) \rightarrow SU(2) \times U(1)$ is possible for appropriate choices of the matter content and values of the supersymmetry breaking parameter. The massless state of the adjoint Higgs scalar is also discussed in the models.

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# 1 Introduction

Gauge symmetry breaking through the Hosotani mechanism [1, 2] (the dynamics of nonintegrable phases) is one of the remarkable phenomena in physics with extra dimensions. Component gauge fields for compactified directions, which are dynamical degrees of freedom and cannot be gauged away, can develop vacuum expectation values, and the gauge symmetry is broken dynamically. The existence of the zero mode for the component gauge field is crucial for the mechanism. Quantum effects shift the zero mode to induce the gauge symmetry breaking, reflecting the topology of the extra dimension.

The vacuum expectation values, which are nothing but the constant background fields, are also related with the eigenvalues (phases) of the Wilson line integrals along the compactified direction, and the gauge symmetry breaking corresponds to the nontrivial Wilson line integral. One can discuss the gauge symmetry breaking patterns of the theory by studying the effective potential for the phases [2].

Since the pioneering work by Hosotani [1], the dynamics of the nonintegrable phases has been studied in various models [2]–[7], namely, nonsupersymmetric gauge models. It has been known that the gauge symmetry breaking patterns depend on matter content, i.e., the number, the boundary conditions of the fields, and the representation under the gauge group of matter.

In this paper, following the author's works [8, 9], we study the gauge symmetry breaking patterns in supersymmetric $SU(N)$ gauge theory with $N_F^{fd}$ numbers of massless fundamental matter (supersymmetric QCD) defined on $M^3 \otimes S^1$. Here $M^3, S^1$ are three-dimensional Minkowski space-time and a circle, respectively. And we also study generalized supersymmetric QCD (supersymmetric QCD with massless adjoint matter).

The dynamics of the nonintegrable phases determines the vacuum structure of the theory. If we, however, introduce the matter multiplets, the vacuum expectation values of the squark fields in the multiplets also become the order parameters for gauge symmetry breaking. We assume that the gauge coupling constant $g$ is small and ignore $O(g^2)$ contributions to the effective potential. In this approximation, there exist flat directions of the potential parametrized by the vacuum expectation values of the squark field. In order to concentrate on the dynamics of the nonintegrable phases, we take the trivial "point" on the flat direction, where all the vacuum expectation values of the squark fields vanish.

If the theory has supersymmetry, one cannot discuss the dynamical breaking of gauge symmetry based on perturbation theory because the perturbative effective potential for the nonintegrable phases vanishes due to the supersymmetry. One must break the supersymmetry in order to obtain nonvanishing effective potential[1]. We resort to the Scherk-

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[1]This is not the case where the gauge charge such as the gauged $U(1)_R$ in supergravity models distinguishes bosons and fermions in a supermultiplet. In this case supersymmetry is broken spontaneously

Schwarz mechanism [11, 12], which is a natural candidate to break supersymmetry softly in this setup [13].

In the softly broken supersymmetric Yang-Mills theory, the $SU(N)$ gauge symmetry is not broken through the Hosotani mechanism. There are $N$ vacuum states in the model, and the vacuum has $Z_N$ symmetry. By adding $N_F^{fd}$ sets of massless fundamental matter multiplet, the model describes the softly broken supersymmetric QCD with $N_F^{fd}$ flavors. We find that in the case $N =$ even, there is a single $SU(N)$ symmetric vacuum state, while in the case $N =$ odd, there is a doubly degenerate $SU(N)$ symmetric vacuum state in the model. The degenerate two vacua are related to each other by the symmetry transformations of the effective potential. Unlike the case of the softly broken supersymmetric Yang-Mills theory, there is no $Z_2$ symmetry for the degenerate vacuum because of the fundamental matter in the model. The vacuum configurations do not depend on the values of $N_F^{fd}$ and the supersymmetry breaking parameter.

We also discuss the mass of the adjoint Higgs scalar. The scalar is originally the component gauge field for the $S^1$ direction and behaves as adjoint Higgs scalar at low energies. It acquires mass through the quantum correction in the extra dimension, and the mass is obtained by evaluating the second derivative of the effective potential at the minimum. The adjoint Higgs scalar is always massive in the softly broken supersymmetric QCD.

In the generalized supersymmetric QCD, we find that the partial gauge symmetry breaking such as $SU(2) \times U(1)$, which may be important for grand unified theory (GUT) symmetry breaking, is possible for appropriate choices of the matter content and values of the supersymmetry breaking parameter. This gauge symmetry breaking pattern is not realized until one considers both the massless adjoint and fundamental matter multiplets. We also find the massless state of the adjoint Higgs scalar within our approximation for the aforementioned gauge symmetry breaking pattern in the model.

In the next section we present the effective potentials for the nonintegrable phases of the models we study in this paper. And we determine the gauge symmetry breaking patterns in the softly broken supersymmetric QCD. The massless adjoint Higgs scalar is also discussed in the model. In section $3$ we consider the generalized supersymmetric QCD. We are, especially, interested in the gauge symmetry breaking pattern such as $SU(3) \rightarrow SU(2) \times U(1)$ and the massless state of the adjoint Higgs scalar. The final section is devoted to conclusions and discussion.

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by the Hosotani mechanism [10].

# 2 Supersymmetric QCD with $N_F^{fd}$ flavors

## 2.1 Effective potential for nonintegrable phases

We present the effective potentials for the nonintegrable phases of our models. The effective potential for the phase is computed by expanding the fields around the constant background gauge field,

$$\langle A_y \rangle \equiv \frac{1}{gL}\langle \Phi \rangle = \frac{1}{gL}\mathrm{diag}(\theta_1, \theta_2, \cdots, \theta_N) \qquad \text{with} \qquad \sum_{i=1}^{N} \theta_i = 0, \tag{1}$$

and $\theta_i$ is related to the Wilson line integral,

$$W_c \equiv \mathcal{P}\exp\left(-ig \oint_{S^1} dy \langle A_y \rangle\right) = \mathrm{diag}\left(\mathrm{e}^{-i\theta_1}, \mathrm{e}^{-i\theta_2}, \cdots, \mathrm{e}^{-i\theta_N}\right). \tag{2}$$

The residual gauge symmetry is generated by the generators of $SU(N)$ commuting with $W_c$ [2]. Following the standard technique given in the papers [1, 2], the effective potential for the softly broken $SU(N)$ supersymmetric Yang-Mills theory has been obtained as [8],

$$V_{SYM}(\theta) = \frac{-2}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i,j=1}^{N} \frac{1}{n^4} \left(\cos[n(\theta_i - \theta_j)] - \cos[n(\theta_i - \theta_j - \beta)]\right). \tag{3}$$

The nontrivial phase $\beta$, which breaks supersymmetry softly, comes from the boundary condition associated with the $U(1)_R$ symmetry on the gaugino field [8, 13].

Let us introduce $N_F^{fd}$ sets of fundamental massless matter multiplet denoted by $Q(\bar{Q})$ belonging to the (anti)fundamental representation under $SU(N)$. The physical fields in $Q(\bar{Q})$ are quark $q(\bar{q})$ and squark $\phi_q(\bar{\phi}_q)$. We impose the boundary conditions associated with the $U(1)_R$ symmetry on the squark fields[2] [8], $\phi_q(\bar{\phi}_q)(x^\mu, y+L) = \mathrm{e}^{i\beta}\phi_q(\bar{\phi}_q)(x^\mu, y)$,

where we have suppressed the flavor index for the squark. It has been pointed out that the phase is common to all flavors, so that the supersymmetry breaking terms in three dimensions are flavor blind [8, 13].

In order to evaluate the effective potential for the phases, one needs the mass operators for $Q$ and $\bar{Q}$, which actually give the mass terms for the (s)quarks in three dimensions after compactifications. Since the matter multiplet $Q(\bar{Q})$ belongs to the (anti)fundamental representation under $SU(N)$ and the squark fields have the nontrivial phase $\beta$, the mass operator for $\phi_q$ and that for $\bar{\phi}_q$ have different forms[3]. On the other hand, the quark fields have no nontrivial phase, so that both $q$ and $\bar{q}$ give the same mass operators[4].

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[2] These boundary conditions are defined by the assignments of $U(1)_R$ charge on the fields based on the invariance of the action under the $U(1)_R$ transformation in the presence of the mass term $m\bar{Q}Q$. The discussion on the effective potential of the nonintegrable phases in this paper corresponds to the massless limit.

[3] This point has been overlooked in the previous paper [8].

[4] This is also clear from the fact that $q$ and $\bar{q}$ forms a Dirac spinor satisfying the periodic boundary condition.

One can read the mass operators in the covariant derivatives for the squark fields,

$$(\partial_{\hat{\mu}}\phi_q^\dagger + ig\phi_q^\dagger A_{\hat{\mu}})(\partial_{\hat{\mu}}\phi_q - igA_{\hat{\mu}}\phi_q), \qquad (\partial_{\hat{\mu}}\bar{\phi}_q + ig\bar{\phi}_q A_{\hat{\mu}})(\partial_{\hat{\mu}}\bar{\phi}_q^\dagger - igA_{\hat{\mu}}\bar{\phi}_q^\dagger). \tag{4}$$

They are obtained as

$$\left(D_3^{\phi_q}\right)^2 = -\sum_{n=-\infty}^{\infty}\sum_{i=1}^{N}\left(\frac{2\pi}{L}\right)^2\left(n - \frac{\theta_i - \beta}{2\pi}\right)^2 \qquad \text{for} \qquad \phi_q, \tag{5}$$

$$\left(D_3^{\bar{\phi}_q}\right)^2 = -\sum_{n=-\infty}^{\infty}\sum_{i=1}^{N}\left(\frac{2\pi}{L}\right)^2\left(n - \frac{-\theta_i - \beta}{2\pi}\right)^2 \qquad \text{for} \qquad \bar{\phi}_q. \tag{6}$$

Here $n$ stands for the Kaluza-Klein mode for the $S^1$ direction. That the prescription $\theta_i \to -\theta_i$ in Eq. (5) gives Eq. (6) shows the field $\bar{\phi}_q$ belongs to the antifundamental representation under $SU(N)$. We see that $\phi_q$ and $\bar{\phi}_q$ contribute to the effective potential in a different manner[5].

Following again the standard prescription, we obtain the effective potential for the phases coming from the fundamental massless matter multiplets,

$$V_{matter}^{fd}(\theta) = \frac{2N_F^{fd}}{\pi^2 L^4}\sum_{n=1}^{\infty}\sum_{i=1}^{N}\frac{1}{n^4}\left((\cos(n\theta_i) - \cos[n(\theta_i - \beta)]) + (\cos(n\theta_i) - \cos[n(\theta_i + \beta)])\right)$$

$$= \frac{2N_F^{fd}}{\pi^2 L^4}\sum_{n=1}^{\infty}\sum_{i=1}^{N}\frac{1}{n^4}\left(2\cos(n\theta_i) - \cos[n(\theta_i - \beta)] - \cos[n(\theta_i + \beta)]\right) \tag{7}$$

where the first term in Eq. (7) arises from the quarks $q, \bar{q}$, and the second and third terms come from $\phi_q$ and $\bar{\phi}_q$, respectively. By putting Eqs. (3) and (7) together, we obtain the effective potential for the softly broken supersymmetric QCD with $N_F^{fd}$ numbers of the massless fundamental matter,

$$V_{SQCD}(\theta) = V_{SYM}(\theta) + V_{matter}^{fd}(\theta)$$

$$= \frac{-2}{\pi^2 L^4}\sum_{n=1}^{\infty}\sum_{i,j=1}^{N}\frac{1}{n^4}(\cos[n(\theta_i - \theta_j)] - \cos[n(\theta_i - \theta_j - \beta)])$$

$$+ \frac{2N_F^{fd}}{\pi^2 L^4}\sum_{n=1}^{\infty}\sum_{i=1}^{N}\frac{1}{n^4}(2\cos(n\theta_i) - \cos[n(\theta_i - \beta)] - \cos[n(\theta_i + \beta)]). \tag{8}$$

As a general remark, the phase $\theta_i$ gives no physical effects at least classically, but the effect is essential at the quantum level. It should be emphasized that these effective potentials (3), (7) arise from taking into account the quantum correction in the extra dimension.

## 2.2 Gauge symmetry breaking via the Hosotani mechanism

We discuss the gauge symmetry breaking through the Hosotani mechanism based on the obtained effective potentials in the previous subsection. Before doing this, let us mention

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[5]The gauge group $SU(2)$ is an exceptional case as we will see in the section 3.

the vacuum structure of the model, which is peculiar to softly broken supersymmetric gauge theories.

Strictly speaking, the dynamics of nonintegrable phases itself does not give complete information on the vacuum structure of softly broken supersymmetric gauge theories. This is because, as noted in the Introduction, the vacuum expectation values of the squark fields $\langle\phi_q\rangle, \langle\bar\phi_q\rangle \in \mathbf{C}$ are also the order parameters for gauge symmetry breaking. If one wishes to study the entire vacuum structure, one should take into account the order parameters in addition to the nonintegrable phases. This means that one has to include the tree-level potential and one-loop corrections to the vacuum expectation values of the squark fields as well.

The tree-level potential, which arises from the covariant derivative and the quartic couplings for the squark field, is given by[6].

$$
\begin{aligned}
V_{tree} &= g^2\left(\langle\phi_q^\dagger\rangle\langle A_y\rangle^2\langle\phi_q\rangle + \langle\bar\phi_q\rangle\langle A_y\rangle^2\langle\bar\phi_q^\dagger\rangle\right) + g^2\left(\langle\phi_q^\dagger\rangle T^a\langle\phi_q\rangle - \langle\bar\phi_q\rangle T^a\langle\bar\phi_q^\dagger\rangle\right)^2 \\
&= \frac{1}{L^2}\sum_{i=1}^{N}\theta_i^2\left(|\langle\phi_{qi}\rangle|^2 + \left|\langle\bar\phi_q^i\rangle\right|^2\right) + g^2\left(\langle\phi_q^\dagger\rangle T^a\langle\phi_q\rangle - \langle\bar\phi_q\rangle T^a\langle\bar\phi_q^\dagger\rangle\right)^2,
\end{aligned}
\tag{9}
$$

where we have used Eq. (1) and $T^a(a = 1, \cdots, N^2 - 1)$ stands for the generator of $SU(N)$. Let us note that the interactions between $\langle\phi_q\rangle, \langle\bar\phi_q\rangle$ and $\theta_i$ are $O(1)$, while the self-interactions among the squarks are of order $g^2$. And the one-loop correction to the vacuum expectation values of the squark fields, which is not written explicitly, is also of order $g^2$.

If the gauge coupling $g$ is very small, then, one may ignore the $O(g^2)$ terms, so that the term which does not have the gauge coupling dependence becomes a dominant contribution to the vacuum structure of the theory. In this approximation, the total effective potential is given by

$$
V(\theta, \langle\phi_q\rangle, \langle\bar\phi_q\rangle) = \frac{1}{L^2}\sum_{i=1}^{N}\theta_i^2\left(|\langle\phi_{qi}\rangle|^2 + \left|\langle\bar\phi_q^i\rangle\right|^2\right) + V_{SQCD}(\theta),
\tag{10}
$$

where $V_{SQCD}(\theta)$ is given by Eq. (8). The relevant interaction to generate the effective potential (10) is only the gauge interaction, which is $O(1)$. That is why the total effective potential does not have the dependence on the gauge coupling.

The first term in Eq. (10), which stands for the tree-level potential, is positive semi-definite. The configuration that minimizes it is given by $\langle\phi_{qi}\rangle = \langle\bar\phi_q^i\rangle = 0$ for nonzero values of $\theta_i(i = 1, \cdots, N)$. In fact, as we will see soon, the nonzero values of $\theta_i$ are the case where the absolute minima of $V_{SQCD}(\theta)$ is realized. As a result, the tree-level potential does not affect the vacuum structure of the model in this approximation. Therefore, the vacuum structure is determined by the dynamics of the nonintegrable phases alone in this model.

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[6]Since the tree-level potential in the model is not the Higgs type potential, we do not expect the phase structures depending on the size of $S^1$ such as the ones studied in Ref. [14].

Let us now consider the effective potential $V_{SQCD}(\theta)$ in order to study the dynamics of the nonintegrable phases, i.e., gauge symmetry breaking through the Hosotani mechanism. Our strategy to find the vacuum configuration of the potential is to minimize $V_{SYM}(\theta)$ and $V_{matter}(\theta)$ separately, and we take the common configuration for both of them, which actually gives the absolute minima of the potential $V_{SQCD}(\theta)$. It has been studied [9] that the absolute minima of $V_{SYM}(\theta)$ is located at

$$\theta_i(i = 1, \cdots, N) = \frac{2\pi}{N}m, \qquad m = 0, \cdots, N-1. \tag{11}$$

The Wilson line integral just corresponds to an element of the center of $SU(N)$, so that the gauge symmetry is not broken.

It is important to note here that there are $N$ vacuum states corresponding to the values of $m$. The $N$ vacua are physically equivalent because, for example, the mass spectra on the vacua are exactly the same as each other. The fields $A_\mu, \lambda$ remain massless on the vacuum configuration (11). The vacuum has $Z_N$ symmetry. A way of looking at the $Z_N$ symmetry is to consider the gauge transformation (regular, nonperiodic) defined by

$$U^{(m)}(y) = \exp\frac{2\pi i y}{L}\begin{pmatrix} \frac{m}{N} & & & \\ & \frac{m}{N} & & \\ & & \ddots & \\ & & & -\frac{(N-1)m}{N} \end{pmatrix}. \tag{12}$$

This transformation does not change the boundary conditions of the fields $A_{\hat{\mu}}, \lambda$ because they belong to the adjoint representation under $SU(N)$. It is easy to see that the $N$ vacuum states are related to each other by this transformation.

Let us next consider the potential $V_{matter}^{fd}(\theta)$ given by Eq. (7) and find the configuration that minimizes it. This is interesting in its own light because, as we will see later, this potential corresponds to the case of the generalized supersymmetric QCD with $N_F^{adj} = 1$. The potential is recast as

$$V_{matter}^{fd}(\theta) = \frac{4N_F^{fd}}{\pi^2 L^4}\sum_{n=1}^{\infty}\sum_{i=1}^{N}\frac{1}{n^4}[1 - \cos(n\beta)]\cos(n\theta_i). \tag{13}$$

We see that the nontrivial phase $\beta$ does not affect the location of the absolute minima of the potential. In finding the minimum, let us note that the potential is invariant under[7]

$$\beta \to 2\pi - \beta. \tag{14}$$

This invariance means that the potential is symmetric under the reflection with respect to $\beta = \pi$ for fixed $\theta_i$. The region given by $0 < \beta \leq \pi$ is enough to study the potential. Moreover, the potential also possesses the invariance under

$$\theta_i \to 2\pi - \theta_i, \qquad i = 1, \cdots, N. \tag{15}$$

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[7]The potential is also invariant under $\beta \to \beta + 2\pi i k, k \in \mathbf{Z}$. This corresponds to $\lambda \to e^{2\pi i k}\lambda$.

The maximal symmetry of $V_{matter}^{fd}(\theta)$ is given by the transformations with Eqs. (14) and (15).

Taking into account Eqs. (14) and (15), we see that the region given by $\theta_i - \beta \geq 0$ is enough to study the potential. Thanks to this, one does not need the classification depending on the sign of $\theta_i - \beta$ when one uses the formula

$$\sum_{n=1}^{\infty} \frac{1}{n^4} \cos(nx) = \frac{-1}{48} x^2 (x - 2\pi)^2 + \frac{\pi^4}{90} \qquad (0 \leq x \leq 2\pi). \tag{16}$$

Noting an expression obtained by applying the formula (16),

$$\sum_{n=1}^{\infty} \frac{1}{n^4} (2\cos(n\theta) - \cos[n(\theta - \beta)] - \cos[n(\theta + \beta)]) = \frac{\beta^2}{24} (6\theta^2 - 12\pi\theta + \beta^2 + 4\pi^2), \tag{17}$$

we have

$$V_{matter}^{fd} = \frac{2N_F^{fd}}{\pi^2 L^4} \frac{\beta^2}{24} \left( \sum_{i=1}^{N-1} \left( 6\theta_i^2 - 12\pi\theta_i + \beta^2 + 4\pi^2 \right) + 6 \left( \sum_{i=1}^{N-1} \theta_i \right)^2 - 12\pi \sum_{i=1}^{N-1} \theta_i + \beta^2 + 4\pi^2 \right). \tag{18}$$

The extremum condition $\partial V_{matter}/\partial \theta_k (k = 1, \cdots, N-1) = 0$ yields

$$\theta_k + (\theta_1 + \cdots + \theta_{N-1}) = 0 \pmod{2\pi}, \qquad k = 1, \cdots, N-1. \tag{19}$$

The solution to Eq. (19) is obtained as $\theta_k = 2\pi q/N \ (q = 0, \cdots, N-1)$. Since $\theta_N = -\sum_{k=1}^{N-1} \theta_k = 2\pi q/N$, we finally have $\theta_i (i = 1, \cdots, N) = 2\pi q/N$.

Unlike the case of the softly broken supersymmetric Yang-Mills theory, the effective potential has different energies for different values of $q$ in the present case. The minimum of the function (17) is achieved at $\theta = \pi$. If all the $\theta_i$'s can take this value, the potential $V_{matter}^{fd}(\theta)$ is obviously minimized at $\theta_i = \pi \ (i = 1, \cdots, N)$. In fact, this is the case when $N =$ even and corresponds to $q_{even} = N/2$. For $N =$ odd, the value which is as close as possible to $\pi$ gives the lowest energy of the potential. It is given by $q_{odd}^{(1)} = (N-1)/2$, i.e., $\theta_i^{(1)} = (N-1)\pi/N$. The potential is invariant under Eq. (15), so that the configuration with $q_{odd}^{(2)} = (N+1)/2$ corresponding to $\theta_i^{(2)} = (N+1)\pi/N (= 2\pi - \theta_i^{(1)})$ gives the same energy as that for $q_{odd}^{(1)} = (N-1)/2$ and also becomes a vacuum configuration[8].

The two vacuum configurations $\theta_i^{(1)}, \theta_i^{(2)}$ are not distinct. In order to see this, let us consider the mass spectra for $\phi_q$ on the vacua $\theta_i^{(1)}, \theta_i^{(2)}$. They are given by $(n - (\theta_i^{(1)} - \beta)/2\pi)^2$ and $(n - (\theta_i^{(2)} - \beta)/2\pi)^2$ from Eq. (5). The former is reduced to the latter by the transformations with Eqs. (14) and (15) and vice versa. Since they are the symmetry transformation of the effective potential, both of the mass spectra are physically identical to each other.

The vacuum configuration for the case $N =$ odd is a doubly degenerate. There is, however, no $Z_2$ symmetry for the vacuum configurations in the present case because the

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[8]Note that the physical region of $\theta_i (i = 1, \cdots, N)$ is $0 \leq \theta_i \leq 2\pi$.

model contains the massless matter multiplet belonging to the (anti)fundamental representation under $SU(N)$. The gauge transformation with Eq. (12) changes the boundary condition of the field in the multiplet. In fact, we see that

$$\phi'_q(y + L) = \mathrm{e}^{i(\beta + \frac{2\pi}{N})}\phi'_q(y), \tag{20}$$

where $\phi'_q = U^{(m=1)}(y)\phi_q$.

We have obtained the vacuum configuration which minimizes $V^{fd}_{matter}(\theta)$ as

$$\theta_i(i = 1, \cdots, N) = \begin{cases} \pi & \cdots N = \text{even}, \\ \frac{N-1}{N}\pi, \quad \left(\text{or} \quad \frac{N+1}{N}\pi\right) & \cdots N = \text{odd}. \end{cases} \tag{21}$$

As we have noticed before, they do not depend on $N^{fd}_F$ and the supersymmetry breaking parameter $\beta$ by the Scherk-Schwarz mechanism. The vacuum configurations respect the $SU(N)$ gauge symmetry and are the parts of the center of $SU(N)$.

We are ready to find the common configuration between Eqs. (11) and (21), which gives the absolute minima of the effective potential (8). It is given by Eq. (21) obviously. We conclude that for $N =$ even, there is a single vacuum state, while for $N =$ odd, there is a doubly degenerate vacuum state in the softly broken supersymmetric QCD with $N^{fd}_F$ flavors.

Here we confirm the discussion on the tree-level potential at the beginning of this subsection. As we have studied above, the configuration that minimizes the effective potential (8) is given by the nonzero values of $\theta_i(i = 1, \cdots, N)$, so that only the vanishing vacuum expectation values of the squark fields minimize the total potential (10).

Let us now study the mass of the adjoint Higgs scalar. The scalar is originally the component gauge field for the $S^1$ direction and behaves as an adjoint Higgs scalar at low energies. It acquires mass through the quantum correction in the extra dimension. The mass is obtained by the second derivative of the effective potential (8) at the minimum,

$$\frac{\partial^2 V_{SQCD}}{\partial\theta_i\partial\theta_j} = \frac{C^{SQCD}_H}{\pi^2 L^4}M_{ij}, \qquad C^{SQCD}_H \equiv \beta^2\left(N + N^{fd}_F\right), \tag{22}$$

where the matrix $M_{ij}$ is given by

$$M_{ij} \equiv \begin{pmatrix} 2 & 1 & \cdots & \cdots & 1 \\ 1 & 2 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ 1 & \cdots & \cdots & \cdots & 2 \end{pmatrix}. \tag{23}$$

All the (off-)diagonal elements of the matrix are $2(1)$. As studied in Ref. [9], this matrix is easily diagonalized, and the mass is obtained as

$$m^2_\Phi = \frac{g^2 C^{SQCD}_H}{\pi^2 L^2}\frac{N}{2}. \tag{24}$$

The mass of the adjoint Higgs scalar is $SU(N)$ invariant, reflecting the $SU(N)$-symmetric vacuum configuration of the model. It is easy to see that there is no possibility of having $C_H^{SQCD} = 0$, so that the adjoint Higgs scalar is always massive and cannot be massless.

# 3 Supersymmetric QCD with massless adjoint matter

In this section we proceed to study the generalized version of supersymmetric QCD by introducing $N_F^{adj}$ numbers of massless adjoint matter multiplet. Let us first discuss the tree-level potential within our approximation in this model.

If we add the massless adjoint matter, the tree-level potential becomes, ignoring the $O(g^2)$ terms and the flavor index[9].

$$V_{tree} = \frac{1}{L^2} \sum_{i=1}^{N} \theta_i^2 \left( |\langle \phi_{qi} \rangle|^2 + \left| \langle \bar{\phi}_q^i \rangle \right|^2 \right) + \frac{2}{L^2} \text{tr} \left| \left[ \langle \Phi \rangle, \ \langle \phi_q^{adj} \rangle \right] \right|^2. \tag{25}$$

The second term comes from the covariant derivative of the squark field in the adjoint representation under $SU(N)$. The total effective potential is, then, given by

$$V_{total} = \frac{1}{L^2} \sum_{i=1}^{N} \theta_i^2 \left( |\langle \phi_{qi} \rangle|^2 + \left| \langle \bar{\phi}_q^i \rangle \right|^2 \right) + \frac{2}{L^2} \text{tr} \left| \left[ \langle \Phi \rangle, \ \langle \phi_q^{adj} \rangle \right] \right|^2 + V_{GSQCD}(\theta). \tag{26}$$

$V_{GSQCD}(\theta)$ is given by

$$
\begin{aligned}
V_{GSQCD}(\theta) &\equiv V_{SQCD}(\theta) + V_{matter}^{adj}(\theta) \\
&= \frac{2N_F^{adj} - 2}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i,j=1}^{N} \frac{1}{n^4} (\cos[n(\theta_i - \theta_j)] - \cos[n(\theta_i - \theta_j - \beta)]) \\
&+ \frac{2N_F^{fd}}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i=1}^{N} \frac{1}{n^4} (2\cos(n\theta_i) - \cos[n(\theta_i - \beta)] - \cos[n(\theta_i + \beta)]),
\end{aligned} \tag{27}
$$

where the first line in Eq. (27) stands for the contributions from the supersymmetric Yang-Mills theory and $N_F^{adj}$ numbers of the massless adjoint matter [9].

Let us note that one cannot rotate $\langle \phi_q^{adj} \rangle$ into a diagonal form by utilizing the $SU(N)$ degrees of freedom because we have already used them to parametrize $\langle A_y \rangle$ as the diagonal form given by Eq. (1). The first and second terms in Eq. (26) are positive semi-definite. In order to minimize the second term in Eq. (26), $\langle \phi_q^{adj} \rangle$ have only a diagonal form. Then, it commutes with $\langle \Phi \rangle$ for any values of $\theta_i$ and yields the vanishing second term $[\langle \Phi \rangle, \ \langle \phi_q^{adj} \rangle] = 0$. Therefore, $\langle \phi_q^{adj} \rangle$ is undetermined in this approximation and parametrizes the flat direction of the potential.

In addition to $\langle \phi_q^{adj} \rangle$, the vacuum expectation values of $\phi_q$ and $\bar{\phi}_q$ can also parametrize the flat direction of the potential. If all the $\theta_i$'s take nonzero values, $\langle \phi_q \rangle = \langle \bar{\phi}_q \rangle = 0$ gives

---

[9]We have ignored the terms coming from the trilinear coupling of the chiral superfields, $QQ^{adj}Q$, by assuming that the coupling is of order $g$, hence $O(g^2)$ in the potential.

the vanishing first term in Eq. (26). In this case, there is no flat direction of the potential parametrized by $\langle \phi_q \rangle$ and $\langle \bar{\phi}_q \rangle$. This was the situation in the softly broken supersymmetric QCD. If some of $\theta_i$'s, however, take the values of zero, say, $\theta_k \neq 0(k = 1, \cdots, l < N - 1)$, the corresponding $\langle \phi_{qk} \rangle$ and $\langle \phi_q^k \rangle$ can take arbitrary values in keeping the vanishing first term and parametrize the flat direction of the potential. In our approximation ignoring the $O(g^2)$ terms, the effective potential has the flat direction in general.

In this paper, we are interested in the dynamics of the nonintegrable phases, or one can say that we study the gauge symmetry breaking in this model at the trivial "point," where all the vacuum expectation values of the squark fields $\phi_q, \bar{\phi}_q, \phi_q^{adj}$ vanish. We ignore the tree-level potential, first and second terms in Eq. (26) and focus on the effective potential $V_{GSQCD}(\theta)$ only.

Here we notice that the effective potential $V_{GSQCD}(\theta)$ is reduced to $V_{matter}^{fd}(\theta)$ for $N_F^{adj} = 1$. The contributions from the vector multiplet $(A_{\hat{\mu}}, \lambda)$ and the massless adjoint multiplet $(q^{adj}, \phi_q^{adj})$ to the constant background (1) cancel each other. This is because in four dimensions the two massless multiplets form $\mathcal{N} = 2$ supersymmetry to have the $SU(2)_R$ symmetry, so that we still have $\mathcal{N} = 1$ supersymmetry for the two multiplets even though we imposed the boundary condition associated with the $U(1)_R$ symmetry [8]. As we have already studied in the previous section, the vacuum configuration for this special case is given by Eq. (21) from the potential $V_{matter}^{fd}(\theta)$ alone. The $SU(N)$ gauge symmetry is not broken for any values of $N_F^{fd}$ and $\beta$. In order to avoid the cancellation, one needs to impose the boundary condition associated with the $SU(2)_R$ symmetry in addition to $U(1)_R$.

## 3.1  $SU(2)$ case

The effective potential (27) seems to have a simple form. It is, however, hard to study the vacuum configuration of the potential fully analytically. As we will show in the next subsection, the location of the minima of the potential changes according to the values of the phase $\beta$. The only exceptional case is the $SU(2)$ gauge group. The effective potential for the case of $SU(2)$ becomes

$$
\begin{aligned}
V_{GQSCD}(\theta) =\ & \frac{2N_F^{adj} - 2}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \Big( 2(1 - \cos(n\beta)) \\
& + 2\cos(2n\theta) - \cos[n(2\theta - \beta)] - \cos[n(2\theta + \beta)] \Big) \\
& + \frac{2 \times (2N_F^{fd})}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^4} (2\cos(n\theta) - \cos[n(\theta - \beta)] - \cos[n(\theta + \beta)]). \quad (28)
\end{aligned}
$$

Let us note that the contributions from $\phi_q$ and $\bar{\phi}_q$ to the potential (28) have the same forms. This is because $\mathbf{2}$ and $\bar{\mathbf{2}}$ of $SU(2)$ are equivalent. The $SU(2)$ gauge group is special in this sense.

The potential (28) happens to be invariant under Eqs. (14) and (15), so that the region given by $\theta - \beta \geq 0$ is enough to study the potential, and we can apply the formula (16) to the effective potential. We obtain that

$$
\begin{aligned}
V_{GQSCD}(\theta) &= \frac{2N_F^{adj} - 2}{\pi^2 L^4} \frac{\beta^2}{24} \left( (\beta - 2\pi)^2 + 24\theta^2 - 24\pi\theta + \beta^2 + 4\pi^2 \right) \\
&\quad + \frac{2 \times (2N_F^{fd})}{\pi^2 L^4} \frac{\beta^2}{24} \left( 6\theta^2 - 12\pi\theta + \beta^2 + 4\pi^2 \right).
\end{aligned}
\tag{29}
$$

By solving the extremum condition $\partial V_{QSCD}/\partial\theta = 0$, we have

$$
\theta^{(1)} = \frac{N_F^{fd} + N_F^{adj} - 1}{N_F^{fd} + 2(N_F^{adj} - 1)} \pi.
\tag{30}
$$

The other solution, which is obtained by taking into account the invariance of the potential under Eq. (15),

$$
\theta^{(2)} = 2\pi - \theta^{(1)} = \frac{N_F^{fd} + 3(N_F^{adj} - 1)}{N_F^{fd} + 2(N_F^{adj} - 1)} \pi
\tag{31}
$$

is not distinct from the solution $\theta^{(1)}$. The squark mass spectra on the solutions are identical to each other due to Eq. (14) [and/or Eq. (15)]. There is a doubly degenerate vacuum state. The vacuum configuration breaks the $SU(2)$ gauge symmetry to $U(1)$ spontaneously.

The second derivative of the effective potential at the minimum gives the mass of the adjoint Higgs scalar as we have stated in the section 3. We find that

$$
m_\Phi^2 \equiv (gL)^2 \frac{\partial^2 V_{GSQCD}}{\partial\theta^2} = \frac{2g^2\beta^2}{\pi^2 L^2} \left( 2(N_F^{adj} - 1) + N_F^{fd} \right).
\tag{32}
$$

No massless state of the adjoint Higgs scalar appears except for $(N_F^{adj}, N_F^{fd}) = (1, 0)$, whose flavor number corresponds to the aforementioned $\mathcal{N} = 2$ supersymmetry in four dimensions.

## 3.2 $SU(3)$ case

Let us next consider the $SU(3)$ gauge group. Even in this case, we find interesting physics such as the partial gauge symmetry breaking and massless adjoint Higgs scalar, which is never observed in the models studied in Ref. [9] and the previous section.

In order to see that the vacuum configuration changes according to the values of $\beta$ by the Scherk-Schwarz mechanism, we first assume that $\beta$ is very small, but nonzero. After finding the vacuum configuration for the small values of $\beta$, we next study the vacuum configuration for $\beta = \pi$. The potential (27) for the case of $SU(3)$ is still invariant under Eq. (14), so that $0 < \beta \leq \pi$ is relevant. Then, we compare the configurations for the two cases.

We may apply the formula (16) to the potential (27) for the small values of $\beta$. We obtain that

$$
\begin{aligned}
V_{GQSCD}(\theta) &= \frac{2N_F^{adj} - 2}{\pi^2 L^4} \beta^2 \Bigg[ \frac{N}{48}(\beta - 2\pi)^2 + \frac{N(N-1)}{48}(\beta^2 + 4\pi^2) \\
&+ \frac{N}{2}\left( \sum_{i=1}^{N-1} \theta_i^2 + \sum_{1 \leq i < j \leq N-1} \theta_i \theta_j \right) - \pi \sum_{i=1}^{N-1}(N-i)\theta_i \Bigg] \\
&+ \frac{4N_F^{fd}}{\pi^2 L^4} \frac{\beta^2}{48} \Bigg[ 12\left( \sum_{i=1}^{N-1} \theta_i^2 + \Big( \sum_{i=1}^{N-1} \theta_i \Big)^2 \right) - 48\pi \sum_{i=1}^{N-1} \theta_i + N(\beta^2 + 4\pi^2) \Bigg], \quad (33)
\end{aligned}
$$

where we have used the result obtained in Ref. [9] for the first line in Eq. (27). The extremum condition $\partial V_{GSQCD}/\partial \theta_k (k = 1, \cdots, N-1) = 0$ yields

$$
\frac{1}{2\pi} \left( N(N_F^{adj} - 1) + N_F^{fd} \right) (\theta_k + (\theta_1 + \cdots + \theta_{N-1})) = N_F^{fd} + (N_F^{adj} - 1)(N - k). \quad (34)
$$

This is written in the form, denoting $d \equiv N(N_F^{adj} - 1) + N_F^{fd}$,

$$
\frac{d}{2\pi}
\begin{pmatrix}
2 & 1 & \cdots & \cdots & 1 \\
1 & 2 & & & \vdots \\
\vdots & & \ddots & & \vdots \\
\vdots & & & \ddots & \vdots \\
1 & \cdots & \cdots & \cdots & 2
\end{pmatrix}
\begin{pmatrix}
\theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \theta_{N-2} \\ \theta_{N-1}
\end{pmatrix}
= N_F^{fd}
\begin{pmatrix}
1 \\ 1 \\ 1 \\ \vdots \\ 1 \\ 1
\end{pmatrix}
+ (N_F^{adj} - 1)
\begin{pmatrix}
N - 1 \\ N - 2 \\ N - 3 \\ \vdots \\ 2 \\ 1
\end{pmatrix}, \quad (35)
$$

where the matrix on the left-hand side in Eq. (35) is the same as the one in Eq. (23). The inverse of the matrix is given by

$$
\frac{1}{N}
\begin{pmatrix}
N - 1 & -1 & \cdots & \cdots & -1 \\
-1 & N - 1 & & & \vdots \\
\vdots & & \ddots & & \vdots \\
\vdots & & & \ddots & \vdots \\
-1 & \cdots & \cdots & \cdots & N - 1
\end{pmatrix}. \quad (36)
$$

All the (off-)diagonal elements of the matrix are $N - 1(-1)$. The solution to Eq. (34) is then found to be

$$
\theta_k = \frac{N_F^{fd}}{d} \frac{2\pi}{N} + \frac{(N_F^{adj} - 1)}{d} \pi \left( N - (2k - 1) \right), \qquad k = 1, \cdots, N - 1 \quad (37)
$$

with

$$
\theta_N = -\sum_{k=1}^{N-1} \theta_k = \frac{-2\pi}{d} \frac{N - 1}{N} \left( N_F^{fd} + \frac{N}{2}(N_F^{adj} - 1) \right). \quad (38)
$$

These solutions become

$$
(\theta_1, \theta_2) = \left( \frac{2}{3}\pi, \ \frac{N_F^{fd}}{3(N_f^{adj} - 1) + N_F^{fd}} \frac{2\pi}{3} \right). \quad (39)
$$

for the $SU(3)$ gauge group, which is of our interest. Except for the case of $N_F^{adj} = 1$, the configuration breaks $SU(3)$ to $U(1) \times U(1)$. Therefore, for the small values of $\beta$, the gauge symmetry is maximally broken, which still holds for the $SU(N)$ gauge group. As an example, the solutions for certain values of $N_F^{adj}$ and $N_F^{fd}$ are given by

$$
\begin{aligned}
(\theta_1, \theta_2) &= \left(\frac{2}{3}\pi, \frac{1}{6}\pi\right) \cdots (N_F^{adj}, N_F^{fd}) = (2,1), \\
&= \left(\frac{2}{3}\pi, \frac{4}{15}\pi\right) \cdots (N_F^{adj}, N_F^{fd}) = (2,2), \\
&= \left(\frac{2}{3}\pi, \frac{1}{3}\pi\right) \cdots (N_F^{adj}, N_F^{fd}) = (2,3).
\end{aligned}
\tag{40}
$$

Let us next study the vacuum configuration at $\beta = \pi$. The possible gauge symmetry breaking patterns are[10].

$$
SU(3) \to
\begin{cases}
SU(3) & \cdots \ (\theta_1, \theta_2) = (\frac{2}{3}\pi, \frac{2}{3}\pi), \\
SU(2) \times U(1) & \cdots \ (\theta_1, \theta_2) = (\pi, 0) + \text{permutations}, \\
U(1) \times U(1) & \cdots \ (\theta_1, \theta_2) = (\frac{2}{3}\pi, 0) + \text{permutations}.
\end{cases}
\tag{41}
$$

By studying the determinant of the Hessian,

$$
H_{ij} \equiv \left. \frac{\partial^2 V_{GSQCD}}{\partial \theta_i \partial \theta_j} \right|_{\beta = \pi}
\tag{42}
$$

and comparing the potential energy for the given gauge symmetry breaking pattern (41), we know the position and stability of the global minima of the effective potential. And at the same time, as we will see later, the matrix gives the information on the mass of the adjoint Higgs scalar at $\beta = \pi$. Depending on the numbers of flavors $N_F^{adj}, N_F^{fd}$, the gauge symmetry breaking patterns are different. We obtain[11]

$$
\begin{aligned}
0 < N_F^{fd} \le \frac{3}{7}(N_F^{adj} - 1) & \quad \cdots (\theta_1, \theta_2) = \left(\frac{2}{3}\pi, 0\right) + \text{permutations}, \\
(N_F^{adj} - 1) < N_F^{fd} \le 9(N_F^{adj} - 1) & \quad \cdots (\theta_1, \theta_2) = (\pi, 0) + \text{permutations}, \\
9(N_F^{adj} - 1) < N_F^{fd} & \quad \cdots (\theta_1, \theta_2) = \left(\frac{2}{3}\pi, \frac{2}{3}\pi\right).
\end{aligned}
\tag{43}
$$

The vacuum configuration at $\beta = \pi$ corresponding to our example (40) is given by $(\theta_1, \theta_2) = (\pi, 0)$ and its permutations, for which the residual gauge symmetry is $SU(2) \times U(1)$. Therefore, we observe that the vacuum configuration changes according to the values of the phase $\beta$. The configuration in Eq. (40) starts to change as $\beta$ becomes large, keeping $U(1) \times U(1)$ symmetry, and arrives at $(\theta_1, \theta_2) = (\pi, 0)$ at $\beta = \pi$, where $SU(2) \times U(1)$ symmetry is realized[12].

---

[10]We have confirmed that the configurations given by $(\theta_1, \theta_2) = (0,0), (\pi/3, \pi/3)$ do not alter our discussions.

[11]The configuration for the region $3(N_F^{adj} - 1)/7 \le N_F^{fd} < N_F^{adj} - 1$ is not given by $(\theta_1, \theta_2) = (\pi, 0)$, but is close to it and respects $U(1) \times U(1)$ symmetry.

[12]The gauge symmetry breaking pattern becomes $SU(3) \to SU(2)$ for the configuration $(\theta_1, \theta_2) = (\pi, 0)$ if we consider the nonzero values of $\langle \phi_{q2} \rangle, \langle \bar{\phi}_q^2 \rangle$.

What is interesting is that the gauge symmetry breaking pattern $SU(3) \to SU(2) \times U(1)$ cannot be realized until one considers the softly broken suppersymmetric QCD with the massless adjoint matter. Actually, as we have studied in the previous section, the gauge symmetry breaking pattern in the softly broken supersymmetric QCD and Yang-Mills theory is $SU(N) \to SU(N)$ and that the softly broken supersymmetric gauge theory with only the massless adjoint matter is $SU(N) \to U(1)^{N-1}$ [9]. This partial gauge symmetry breaking has been pointed out in the nonsupersymmetric gauge theory with both the massless adjoint and fundamental matter [15].

If we change the number of flavors, the vacuum configuration at $\beta = \pi$ also changes. For $(N_F^{adj}, N_F^{fd}) = (4, 1)$, the vacuum configuration is given by $(\theta_1, \theta_2) = (2\pi/3, \pi/15)$ from Eq. (39) for the small values of $\beta$, while at $\beta = \pi$, taking account of Eq. (43), it is given by $(\theta_1, \theta_2) = (2\pi/3, 0)$. The configuration at $\beta = \pi$ still respects $U(1) \times U(1)$ symmetry though the configurations themselves are different for the two cases.

The above observation implies that if $N_F^{adj}$ increases, then the first term in Eq. (27) dominates in the effective potential. The vacuum configuration tends to realize the maximal breaking of $SU(3)$. This is consistent with the result that the dynamics of the nonintegrable phases for the massless adjoint matter always result in the maximal breaking of $SU(N)$, i.e., $U(1)^{N-1}$ [9]. If we, instead, increase $N_F^{fd}$ for fixed $N_F^{adj}$, the vacuum configuration tends toward the original gauge symmetry. This is because the second term in Eq. (27) dominates in the effective potential for a large number of $N_F^{fd}$, and the potential has the $SU(N)$ symmetric vacuum as we have studied in the section 3.

Let us finally discuss the massless state of the adjoint Higgs scalar. To this end, we study the determinant of the Hessian for the configuration $(\theta_1, \theta_2) = (\pi, 0)$,

$$\det H \Big|_{\beta=\pi} = \left(N_F^{fd} - (N_F^{adj} - 1)\right)\left(9(N_F^{adj} - 1) - N_F^{fd}\right). \qquad (44)$$

The determinant vanishes for the case $N_F^{fd} = N_F^{adj} - 1$ or $N_F^{fd} = 9(N_F^{adj} - 1)$ except for the aforementioned $\mathcal{N} = 2$ supersymmetry. The conditions are satisfied without any fine-tuning of the parameters as long as $N_F^{adj}$ and $N_F^{fd}$ are discrete numbers. In our example, $(N_F^{adj}, N_F^{fd}) = (2, 1)$ satisfies the former condition. The vanishing determinant implies that the Hessian contains the massless mode, which is nothing but the massless adjoint Higgs scalar in our approximation[13]. The massless state of the adjoint Higgs scalar has also been pointed out in the nonsupersymmetric gauge theories [15].

For comparison to the case of $\beta = \pi$, let us evaluate the second derivative of the effective potential (27) for the small values of $\beta$. The vacuum configuration in this case is given by Eq. (37) and breaks the $SU(N)$ gauge symmetry to $U(1)^{N-1}$ spontaneously.

---

[13]This vanishing determinant is modified if we consider the nonzero values of the vacuum expectation values for the squark fields.

The second derivative is calculated, using Eq. (33), as

$$\frac{\partial^2 V_{GSQCD}}{\partial \theta_i \partial \theta_j} = \frac{C_H^{GSQCD}}{\pi^2 L^4} M_{ij}, \qquad C_H^{GSQCD} \equiv \beta^2 \left( N(N_F^{adj} - 1) + N_F^{fd} \right), \qquad (45)$$

where $M_{ij}$ is given by Eq. (23). The matrix does not have the zero eigenvalue, and the coefficient $C_H^{GSQCD}$ never vanishes except for the aforementioned $\mathcal{N} = 2$ supersymmetry. Therefore, the adjoint Higgs scalar for the small values of $\beta$ is always massive and cannot be massless.

# 4   Conclusions and discussion

We have studied the gauge symmetry breaking patterns through the Hosotani mechanism (the dynamics of the nonintegrable phases) in supersymmetric QCD with $N_F^{fd}$ numbers of the massless fundamental matter and its generalized version by introducing $N_F^{adj}$ numbers of the massless adjoint matter. The supersymmetry is broken softly by the Scherk-Schwarz mechanism to give the nonvanishing effective potentials for the phases.

We have first studied the softly broken supersymmetric Yang-Mills theory. The $SU(N)$ gauge symmetry is not broken, and there are $N$ vacuum states given by Eq. (11). The $N$ vacua are physically equivalent, $Z_N$ symmetric and are related to each other by the gauge transformation with Eq. (12). The fields $A_\mu, \lambda$ remain massless on the vacuum configuration.

By introducing $N_F^{fd}$ sets of the massless fundamental matter multiplet, we have obtained the softly broken supersymmetric QCD with $N_F^{fd}$ flavors. The $SU(N)$ gauge symmetry is not broken again in this model, but the vacuum configuration itself depends on the number of color $N$. For $N =$ even, there is a single vacuum state, while for $N =$ odd, there is a doubly degenerate vacuum state. The symmetry transformations with Eqs. (14) and (15) of the effective potential relate the degenerate two vacua. The $Z_2$ symmetry is broken by the massless matter multiplet belonging to the (anti)fundamental representation under $SU(N)$. The adjoint Higgs scalar is always massive in the two models except for the case of the accidental $\mathcal{N} = 2$ supersymmetry in four dimensions.

We have also discussed the gauge symmetry breaking patterns in the generalized version of supersymmetric QCD (supersymmetric QCD with the massless adjoint matter). We have first studied the case of $SU(2)$ and found the vacuum configuration given by Eq. (30), which breaks the $SU(2)$ gauge symmetry to $U(1)$ spontaneously. There is no massless state of the adjoint Higgs scalar in this case.

In order to see how the gauge symmetry is broken through the Hosotani mechanism for higher rank gauge group, we have considered the $SU(3)$ gauge group and chosen the appropriate numbers of the flavors as a demonstration. The vacuum configuration changes according to the values of the supersymmetry breaking parameter $\beta$ by the Scherk-Schwarz

mechanism. We have explicitly shown that the vacuum configurations for small values of $\beta$ and $\beta = \pi$ are given by the different configurations, which realize the different gauge symmetry breaking patterns. It is possible to have the gauge symmetry pattern such as $SU(3) \to SU(2) \times U(1)$ for the choice given by Eq. (40) at $\beta = \pi$. This symmetry breaking pattern is peculiar to the model and is never observed in the models studied in Ref. [9] and the section 3.

We have investigated the massless state of the adjoint Higgs scalar by studying the determinant of the Hessian (42) for the small values of $\beta$ and $\beta = \pi$. We have shown that the massless adjoint Higgs scalar is impossible for the small values of $\beta$. At $\beta = \pi$, however, we have obtained the condition for the vanishing determinant of the Hessian without any fine-tuning, which implies the existence of the massless adjoint Higgs scalar in our approximation. And we have given the explicit example of the parameter choices for the massless state. It seems that in order to have the massless adjoint Higgs scalar, the partial gauge symmetry breaking such as $SU(3) \to SU(2) \times U(1)$ is necessary. Hence, the massless state is a specific feature to the generalized version of the softly broken supersymmetric QCD.

We have also discussed the tendency of a gauge symmetry breaking pattern at $\beta = \pi$ by varying the number of the flavor in the generalized supersymmetric QCD. If the number of the massless adjoint matter $N_F^{adj}$ increases for a fixed number of the fundamental matter $N_F^{fd}$, the gauge symmetry breaking patterns tend toward the maximal breaking of the original gauge symmetry, say, $U(1) \times U(1)$ in our example. On the other hand, if $N_F^{fd}$ increases for fixed $N_F^{adj}$, it tends toward the vacuum configuration respecting the original gauge symmetry, $(\theta_1, \theta_2) = (\frac{2}{3}\pi, \frac{2}{3}\pi)$ in our example.

It may be interesting to ask what gauge symmetry pattern is realized if we consider the higher rank gauge group such as $SU(5)$ in the generalized supersymmetric QCD. Taking into account the lessons in this paper, one has to select carefully the numbers of flavors $N_F^{adj}, N_F^{fd}$ in order to realize the partial gauge symmetry breaking such as $SU(5) \to SU(3) \times SU(2) \times U(1)$, which may be relevant to the mechanism of GUT symmetry breaking. We need further studies in order to determine the gauge symmetry breaking patterns for the higher rank gauge group in the model. This will be reported elsewhere.

We have assumed the gauge coupling $g$ is very small and ignored the $O(g^2)$ terms in the effective potential. In this approximation there exists the flat direction of the potential parametrized by the vacuum expectation values of the squark fields, namely $\langle \phi_q^{adj} \rangle$. We have chosen the trivial "point" corresponding to the vanishing vacuum expectation values of them, and we have studied the gauge symmetry breaking patterns through the dynamics of the nonintegrable phases alone. In order to determine the whole vacuum structure, one needs to take into account the ignored $O(g^2)$ term including the tree-level potential and one-loop corrections to the vacuum expectation values of the squark fields.

We have implicitly assumed the mass term for the squarks, from which we have defined

the boundary condition of the squark field associated with the $U(1)_R$ symmetry. We have taken the massless limit in order to study the gauge symmetry breaking patterns. It is expected that the nonvanishing mass term may also influence the gauge symmetry breaking [16]. It is important and interesting to study the effect of the mass term on the Hosotani mechanism.

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