M-Theory

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Abstract

We construct an eleven-dimensional superspace with superspace coordinates and formulate a finite M-theory using non-anticommutative geometry. The conjectured M-theory has the correct eleven-dimensional supergravity low energy limit. We consider the problem of finding a stable finite M-theory which has de Sitter space as a natural ground state, and the problem of eliminating possible future horizons.

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1 Introduction

String duality theory has brought about the return of eleven-dimensional supergravity. The strong-coupling limit of the low-energy sector of type IIA superstring theory is eleven-dimensional supergravity. Eleven-dimensional supergravity is twenty three years old [1] and by itself does not constitute a consistent unification of quantum gravity and supersymmetry, because the ubiquitous divergences of weak field quantum gravity persist. On the other hand, superstring theory is claimed to consistently unify gravity and quantum mechanics into a finite theory of quantum gravity. The question then arises: What is the consistent strong coupling theory in eleven dimensions that contains eleven-dimensional supergravity as its low energy limit? Since M-theory does not itself contain strings, perhaps it and its low-energy limit may not have anything directly to do with string theory or D-brane theory. Since we do not know the degrees of freedom of M-theory, we must demand certain consistency requirements of M-theory:

1. It must support a finite quantum gravity theory,
2. The theory must reduce to either a massless or massive form of eleven-dimensional supergravity in a low energy limit,
3. The fermions must have a chiral low energy limit,
4. It should contain the standard model,
5. It should describe a realistic cosmology in curved spacetime.

After almost thirty years of study in supersymmetry theory, there is still no experimental evidence that it plays a role in nature. It is the *uniqueness* of eleven-dimensional supergravity that makes it such an attractive proposal for a unification of gravity and quantum mechanics and as a low energy limit of an M-theory. Standard Kaluza-Klein theories do not possess a similar uniqueness, since a large number of dimensions and group structures can potentially qualify as the constituents of a unified theory. Moreover, the indirect evidence that the coupling constants of the minimal supersymmetric standard model (MSSM) meet at a unifying energy $\sim 10^{16}$ GeV, does give some credence to the validity of the MSSM theory.

Little is known about the nature of M-theory, but it has allowed us to extend our knowledge of string theory beyond its applicability. Banks, Fischler, Shenker and Susskind [2] conjectured that the microscopic degrees of freedom of M-theory, when pictured in an infinite momentum Lorentz frame, are D_0-branes. They described the dynamics of the eleven-dimensional space by a $N\times N$ matrix quantum mechanics. They conjectured that M-theory is equivalent to a matrix quantum mechanics of $U(N)$ matrices in the $N\rightarrow\infty$ limit with a Hamiltonian that follows from reducing $9+1$-dimensional $U(N)$ super Yang-Mills theory to $0+1$ dimensions. Horava and Witten [3] developed a heterotic M-theory by compactifying eleven-dimensional theory on an S^1/Z_2 orbifold corresponding to the strong coupling limit of heterotic ten-dimensional $E_8\times E_8$ string theory. Compactifying an additional six dimensions on a Calabi-Yau 3-manifold leads, in the low energy limit, to four-dimensional $N=1$ supersymmetry theory.

In the following, we shall study possible models of M-theory which do not owe their finiteness to string theory but to an eleven-dimensional superspace that involves noncommutative as well as non-anticommutative coordinates with a $\hat f\circ\hat g$ product of operators $\hat f$ and $\hat g$ on a Hilbert superspace [4, 5, 6]. The problem of the physical necessity for chiral fermion fields is solved by an orbifold compactification with periodic boundary conditions on the fermion field operators, or, alternatively, by compactifying the finite M-theory along a noncompact direction.

Any realistic M-theory must be consistent with modern cosmological data. This means that it must describe a curved universe with positive but small cosmological constant and be consistent with the now overwhelming data supporting an accelerating universe [7]. This immediately poses a problem for M-theory (and string theory), because the standard M-theory cannot contain a positive cosmological constant. De Sitter space cannot be contained in standard eleven-dimensional supergravity or M-theory. The de Sitter superalgebra is not contained in M-theory and in a supersymmetric theory the de Sitter space solutions have zero energy and, indeed, there is no positive energy theorem. In view of these difficulties, we shall also consider a recent variant of M-theory call MM-theory (massive M-theory) [8].

Recently, another difficulty in formulating a consistent string theory or M-theory has arisen with the possibility of a future horizon, associated with an eternally accelerating universe based either on a positive cosmological constant or a quintessence model. The future horizon forbids the construction of a consistent S-matrix formalism based on asymptotic in and out states at infinity [9, 10]. Various possible solutions to this problem have been proposed [11, 12].

2 Eleven-Dimensional Superspace

Let us define an eleven-dimensional superspace with the superspace coordinates [4]:

$$\rho^M = x^M + \beta^M, \tag{1}$$

where $M = 0, 1, ...10$ and the x^M denote the classical commuting c-number coordinates of the space, $[x^M, x^N] = 0$, and β^M denote the anticommuting Grassmann coordinates, $\{\beta^M, \beta^N\} = 0$.

Both noncommutative and non-anticommutative geometries can be unified within the superspace formalism using the $\circ$-product of two operators $\hat{f}$ and $\hat{g}$:

$$\begin{aligned}(\hat{f} \circ \hat{g})(\rho) &= \Big[\exp\Big(\frac{1}{2}\omega^{MN}\frac{\partial}{\partial\rho^M}\frac{\partial}{\partial\eta^N}\Big)f(\rho)g(\eta)\Big]_{\rho=\eta} \\ &= f(\rho)g(\rho) + \frac{1}{2}\omega^{MN}\partial_M f(\rho)\partial_N g(\rho) + O(\omega^2),\end{aligned} \tag{2}$$

where $\partial_M = \partial/\partial\rho^M$ and ω^{MN} is a nonsymmetric tensor

$$\omega^{MN} = -\tau^{MN} + i\theta^{MN}, \tag{3}$$

with $\tau^{MN} = \tau^{NM}$ and $\theta^{MN} = -\theta^{NM}$. Moreover, ω^{MN} is Hermitian symmetric $\omega^{MN} = \omega^{\dagger MN}$, where $\dagger$ denotes Hermitian conjugation. The familiar commutative coordinates of spacetime are replaced by the superspace operator relations

$$[\hat{\rho}^M, \hat{\rho}^N] = 2\beta^M\beta^N + i\theta^{MN}, \tag{4}$$

$$\{\hat{\rho}^M, \hat{\rho}^N\} = 2x^M x^N + 2(x^M\beta^N + x^N\beta^M) - \tau^{MN}. \tag{5}$$

In the limit that $\beta^M \to 0$ and $|\tau^{MN}| \to 0$, we get the familiar expression for noncommutative coordinate operators

$$[\hat{x}^M, \hat{x}^N] = i\theta^{MN}. \tag{6}$$

In the limits $x^M \to 0$ and $|\theta^{MN}| \to 0$, we obtain the Clifford algebra anticommutation relation

$$\{\hat{\beta}^M, \hat{\beta}^N\} = -\tau^{MN}. \tag{7}$$

In the following, we shall consider the simpler non-anticommutative geometry obtained in the limit $\theta^{MN} = 0$, because it alone can lead to a finite and unitary quantum field theory and quantum gravity theory [4, 5, 6]. In the non-anticommutative field theory formalism, the product of two operators $\hat{f}$ and $\hat{g}$ has a corresponding $\diamondsuit$-product

$$\begin{aligned}(\hat{f}\diamondsuit\hat{g})(\rho) &= \exp\Big(-\frac{1}{2}\tau^{MN}\frac{\partial}{\partial\rho^M}\frac{\partial}{\partial\eta^N}\Big)f(\rho)g(\eta)|_{\rho=\eta} \\ &= f(\rho)g(\rho) - \frac{1}{2}\tau^{MN}\partial_M f(\rho)\partial_N g(\rho) + O(\tau^2).\end{aligned} \tag{8}$$

3 M-theory

In previous work [13], it was shown that noncommutative quantum field theory cannot give a renormalizable quantum gravity. On the other hand, the non-anticommutative superspace quantum field theory can lead to a finite perturbative quantum gravity theory [4, 5, 6]. We shall apply this quantum field theory formalism to M-theory. We begin with the standard eleven-dimensional supergravity, describing the highest number of dimensions in which supersymmetry representations with $J \leq 2$ can exist. Its reduction to four dimensions is automatically guaranteed to give an $O(8)$ invariant supergravity theory with $N = 8$ supersymmetry.

The field content consists of the vielbein e_M^A (where A, B, C... refer to tangent space indices), a Majorana spin $\frac{3}{2}$ ψ_M, and of a completely antisymmetric gauge tensor field A_{MNP}. The metric is $(+ - - - ... -)$ and the eleven-dimensional Dirac matrices satisfy

$$\{\Gamma_A, \Gamma_B\} = -2\eta_{AB}, \tag{9}$$

where η_{AB} denotes the flat Minkowski tangent space metric. Moreover, $\Gamma^{A_1...A_N}$ denotes the product of $N\Gamma$ matrices completely antisymmetrized.

Our superspace M-theory Lagrangian, using the $\diamondsuit$-product has the form

$$\begin{aligned}\mathcal{L} = -\frac{1}{4\kappa^2}e\diamondsuit R(\omega) - \frac{i}{2}e\diamondsuit\bar{\psi}_M\diamondsuit\Gamma^{MNP}D_N\Big(\frac{\omega+\hat{\omega}}{2}\Big)\diamondsuit\psi_P - \frac{1}{48}e\diamondsuit F_{MNPQ}\diamondsuit F^{MNPQ} \\ +\frac{\kappa}{192}e\diamondsuit(\bar{\psi}_M\diamondsuit\Gamma^{MNOPQR}\psi_N + 12\bar{\psi}^P\diamondsuit\Gamma^{OR}\psi^Q)\diamondsuit(F_{PQOR} + \hat{F}_{PQOR}) \\ +\frac{2\kappa}{(144)^2}\epsilon^{O_1O_2O_3O_4P_1P_2P_3P_4MNR}F_{O_1O_2O_3O_4}\diamondsuit F_{P_1P_2P_3P_4}\diamondsuit A_{MNR},\end{aligned} \tag{10}$$

where $R(\omega)$ is the scalar contraction of the curvature tensor

$$R_{MNAB} = \partial_M\omega_{NAB} - \partial_N\omega_{MAB} + \omega_{MA}{}^C\diamondsuit\omega_{NCB} - \omega_{NA}{}^C\diamondsuit\omega_{MCB}, \tag{11}$$

and F_{MNOP} is the field strength defined by

$$F_{MNOP} = 4\partial_{[M}A_{NOP]}, \tag{12}$$

with [...] denoting the antisymmetrized sum over all permutations, divided by their number.

The covariant derivative is

$$D_N(\omega)\psi_M = \partial_N\psi_M + \frac{1}{4}\omega_{NAB}\diamondsuit\Gamma^{AB}\psi_M. \tag{13}$$

The spin connection ω_{MAB} is defined by

$$\omega_{MAB} = \omega^0_{MAB}(e) + T_{MAB}, \tag{14}$$

where T_{MAB} is the spin torsion tensor.

The transformation laws are

$$\delta e_M^A = -i\kappa\bar{\epsilon}\diamondsuit\Gamma^A\psi_M, \tag{15}$$

$$\delta\psi_M = \frac{1}{\kappa}D_M(\hat{\omega})\diamondsuit\epsilon + \frac{i}{144}(\Gamma^{OPQR}{}_M - 8\Gamma^{PQR}\delta_M^O)\epsilon\diamondsuit\hat{F}_{OPQR} \equiv \frac{1}{\kappa}\hat{D}_M\epsilon, \tag{16}$$

$$\delta A_{MNP} = \frac{3}{2}\bar{\epsilon}\diamondsuit\Gamma_{[MN}\psi_{P]}, \tag{17}$$

where

$$\hat{F}_{MNPQ} = F_{MNPQ} - 3\kappa\bar{\psi}_{[M}\diamondsuit\Gamma_{NP}\psi_{Q]}. \tag{18}$$

We also have

$$\hat{\omega}_{MAB} = \omega_{MAB} + \frac{i\kappa^2}{4}\bar{\psi}_O\diamondsuit\Gamma_{MAB}{}^{OP}\psi_P. \tag{19}$$

In the limit $|\tau^{MN}| \to 0$ and $\beta^M \to 0$, (10) reduces to the Cremmer, Julia and Scherk (CJS) eleven-dimensional supergravity Lagrangian [1], which should be the correct low energy limit of an M-theory. The finiteness and gauge invariance of the M-theory should be guaranteed by the non-anticommutative field theory. However, this finiteness was proved for scalar field theory and weak field quantum gravity [4, 5, 6], but is expected to hold also for our non-perturbative M-theory, due to the existence of a finite, fundamental length ℓ. The symmetric tensor τ^{MN} can be written as

$$\tau^{MN} = \ell^2 s^{MN} = \frac{1}{\Lambda^2}s^{MN}, \tag{20}$$

where Λ is a fundamental energy scale that can be chosen to be the Planck energy $\Lambda = M_{PL}$.

The focus of experimental cosmology has been on the now significant data that indicates that the universe is undergoing an accelerated expansion [7]. It has been know for a long time [14] that spontaneous compactification of the eleven-dimensional supergravity field equations leads to solutions with a vacuum state corresponding to the product of a four-dimensional anti-de Sitter space with negative cosmological constant and a seven-dimensional Einstein space with positive cosmological constant. Unfortunately, this direct compactification leads us to an unacceptable four-dimensional cosmology. There exist no-go theorems which forbid attempts to realize de Sitter space within string theory and M-theory [15].

Chamblin and Lambert (CL) have extended standard eleven-dimensional supergravity theory to a massive supergravity theory, in which de Sitter space is a natural ground state [8]. The CL scenario may not have anything directly to do with standard string theory, but within our construction of M-theory this is not important. The important issue is that it may provide a natural embedding of de Sitter space into our modified eleven-dimensional supergravity or M-theory.

The CL modification of supergravity theory is based on a conformal spin $CSpin(1,10)$ connection, defined in our superspace by

$$\Upsilon_A = \frac{1}{4}\Omega_A{}^{BC}\Gamma_{BC} + 2k_A, \tag{21}$$

where

$$\Omega_{AB}{}^{C} = \omega_{AB}{}^{C} + 2(e^C_A \diamondsuit k_B - e_{AB} \diamondsuit k^C), \tag{22}$$

and for which the conformal part of the curvature dk vanishes. If $k = d\chi$, then the redefinition that returns the equations of motion back to their usual ones is

$$e^A_M \to \exp(-2\chi) \diamondsuit e^A_M, \quad \psi_M \to \exp(-2\chi) \diamondsuit \psi_M. \tag{23}$$

This modification is non-trivial for a non-simply connected space.

Choosing to compactify on the non-simply connected supermanifold, $SM_{10} \times S^1$, results in the non-anticommutative ten-dimensional supergravity equations of motion [8]

$$\begin{aligned} R_{ab} - \frac{1}{2} g_{ab} \diamondsuit R = & -2[D_a \diamondsuit D_b \phi - g_{ab} \diamondsuit D_c \diamondsuit D^c \phi + g_{ab} \diamondsuit D_c \phi \diamondsuit D^c \phi] \\ & + \frac{1}{2}(F_{ac} \diamondsuit F_b{}^c - \frac{1}{4} g_{ab} \diamondsuit F^{cd} \diamondsuit F_{cd}) \diamondsuit \exp(2\phi) \\ & -18m(D_{(a} A_{b)} - g_{ab} \diamondsuit D^c A_c) - 36m^2(A_a \diamondsuit A_b + 4 g_{ab} \diamondsuit A^c \diamondsuit A_c) \\ & -12m A_{(a} \diamondsuit \partial_{b)} \phi - 30m g_{ab} \diamondsuit A^c \diamondsuit \partial_c \phi - 144 m^2 g_{ab} \diamondsuit \exp(-2\phi), \end{aligned} \tag{24}$$

$$D^b F_{ab} = 18m A_b \diamondsuit F_a{}^b + 72m^2 \exp(-2\phi) \diamondsuit A_a - 24m \exp(-2\phi) \diamondsuit \partial_a \phi, \tag{25}$$

$$\begin{aligned} 6D^a \diamond D_a \phi - 8 D_a \phi \diamondsuit D^a \phi = & -R + \frac{3}{4} \exp(2\phi) \diamondsuit F^{ab} \diamondsuit F_{ab} \\ & +360 m^2 \exp(-2\phi) + 288 m^2 A^a \diamondsuit A_a + 96 m A^b \diamondsuit \partial_b \phi - 36 m D^b A_b, \end{aligned} \tag{26}$$

where $a, b = 0, 1, ...9$ and $F_{ab} = \partial_a A_b - \partial_b A_a$.

These equations of motion were obtained by turning off the four-form field strength and fermions. They correspond to the eleven-dimensional equations of motion with no dependence on the y coordinate with $k = m dy$, where dy is the tangent vector to the circle. Thus, solutions of these equations of motion are solutions to our finite M-theory. In the limits that $|\tau^{ab}| \to 0$ ($\ell \to 0$) and $m \to 0$, we recover the standard massless type IIA ten-dimensional supergravity and the relation of M-theory to perturbative string theory. A dimensional reduction of eleven-dimensional supergravity with our $\diamondsuit$-product modification over a *noncompact dimension*, yields the same equations of motion [16].

The compactification of the eleven-dimensional M-theory on a circle has built into it a mass generating mechanism, such that the two-form eats the scalar, the three-form eats the vector and the four-form eats the three-form. However, the $U(1)$ symmetry of the vector A_a is violated, so that the vector is tachyonic and the solutions will be unstable. Even though the equations of motion are supersymmetric, there is no Noether theorem conserved supercharge associated with the Hamiltonian of the massive theory. Thus, massive M-theory compactified on S^1 with a topologically non-trivial conformal connection does not possess a globally conserved supercharge.

The massive M-theory does not in its present form possess an action. The equations of motion come from gauging the scale symmetry of modified eleven-dimensional supergravity, and whereas the equations of motion of CL theory obey this gauge symmetry, the action does not. This is an unusual situation, since normally after a spontaneous symmetry breaking the modified action exists, given a suitable spontaneous symmetry breaking potential. Clearly this issue requires further study.

The fact that the massive M-theory is embedded in a de Sitter space can be seen by turning off all the gauge potentials to give

$$R_{ab} = 36m^2 \exp(-2\phi)\Diamond g_{ab}, \tag{27}$$

which corresponds to a constant dilaton field ϕ. In the limit $\ell \to 0$ we recover ten-dimensional de Sitter space with an effective cosmological constant

$$\lambda = 576m^2 \exp(-2\phi). \tag{28}$$

As pointed out in [8], there is no need to consider other fields to induce a cosmological constant, for a positive cosmological constant asserts itself in ten dimensions. In de Sitter space we have $\sum\{Q, Q^*\} = 0$ and there is no positive energy theorem.

The question remains to be asked: Do there exist any stable solutions to the massive M-theory with a natural embedding in de Sitter space?

The problem of obtaining fermions possessing chirality can be resolved in two ways: (a) by performing a compactification on a *noncompact* direction, as shown by Wetterich [17], or, (b) by performing an orbifold compactification with the resulting periodic boundary conditions on the fermion fields leading to chirality of the fermions [18]. We have aleady observed that the massive eleven-dimensional supergravity theories give massive type IIA ten-dimensional supergravity theories when they are compactified along noncompact directions [16]. The Witten chirality index theorem for Kaluza-Klein theories only holds for *compact* Riemannian manifolds [19]. Our finite M-theory can reduce to a standard model with chiral fermions in four dimensions when either of (a) or (b) compactifications are performed. The eleven-dimensional manifold of M-theory can contain the $SU(3) \times SU(2) \times U(1)$ standard model [20].

The finite M-theory we have constructed is a *nonlocal* quantum field theory in eleven-dimensions. The nonlocal nature of the theory will persist after compactification for $\Lambda < \infty$ ($\ell \neq 0$), and will guarantee that the quantum gravity sector of the four-dimensional theory is finite to all orders of perturbation theory [6]. That M-theory is nonlocal should not come as a surprise, for it is now generally accepted that string and D-brane theories are intrinsically nonlocal theories. This is the case for noncommutative string and D-brane theories on a B-field background. The nonlocality of the fields will only occur at short distances and will vanish as $\Lambda \to \infty$.

If we have succeeded in finding a finite M-theory with our non-anticommutative geometry and can guarantee that it is naturally embedded in de Sitter space with stable solutions, then we are faced with the problem of a future horizon, either through a positive cosmological constant or through a quintessence-like equation of state [9]. For a positive cosmological constant, the universe will undergo eternal acceleration, whereas for a quintessence dark energy, it may be possible to construct a quintessence model of cosmology in which the universe will begin to decelerate in the future, thereby avoiding the existence of a future horizon [12]. The existence of a future horizon would make our formulation of finite M-theory incompatible with the existence of an S-matrix and consistent physical observables. Particles would be immersed in a heat bath with a finite entropy and there would not exist asymptotic in and out states at infinity. The number of degrees of freedom would be finite and the dimensions of our Hilbert superspace would also be finite, which would invalidate the basic notions of our non-anticommutative and noncommutative quantum field theory.

One way to resolve the problem of future horizons is to postulate that the speed of light varies in the future as well as in the past, for with a suitable varying speed of light as the universe expands, any future horizon can be eliminated, allowing for an infinite spacetime with appropriate in and out states at infinity and a consistent S-matrix [11]. A varying speed of light is a natural outcome of higher-dimensional theories as is the existence of multiple light cones, which can undergo expansion or contraction and remove future horizons [21, 22, 23].

4 Conclusions

We have formulated an eleven-dimensional superspace with the algebra of functions on a noncommutative and non-anticommutative space isomorphic to the algebra of functions with commutative x^M and anticommutative β^M coordinates, with the general $\hat{f} \circ \hat{g}$-product of operators $\hat{f}$ and $\hat{g}$. We constructed a conjectured finite M-theory, using the simpler non-anticommutative geometry with the $\hat{f}\diamondsuit\hat{g}$-product and eleven-dimensional supergravity theory. This M-theory should produce a finite quantum gravity theory and Yang-Mills gauge theory coupled to the Majorana spin $\frac{3}{2}$ fermion field. In the limit $|\tau^{MN}| \rightarrow 0$, this finite M-theory reduces to eleven-dimensional CJS supergravity theory [1], which is the correct low energy effective theory of M-theory, related by duality to type IIA superstring theory.

Demanding that the finite M-theory be naturally embedded in a de Sitter space with positive cosmological constant $\lambda > 0$, led us to formulate a non-anticommutative geometrical, massive M-theory based on the CL [8] massive supergravity theory. The field equations of this theory reduce in the limits $\beta^M \rightarrow 0$, $|\tau^{MN}| \rightarrow 0$ and $m \rightarrow 0$ to massless type IIA supergravity theory, formulated in flat Minkowski space. However, this theory may not possess stable solutions, although it is possible that it could relax to a true ground state which is stable and is still embedded in de Sitter space. These issues require further investigation.

The problem of the existence of a future horizon in a de Sitter space can be removed by postulating that the speed of light varies in the future universe as well as in the past universe [11]. Multiple expanding or contracting light cones, existing in the higher-dimensional theory as well as in the compactified theory, can remove a future horizon and allow for a consistent S-matrix formulation of our finite M-theory [21, 22, 23].

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