IFUP-TH 2002/41

Finite-temperature properties of the supersymmetric generalization of 3D compact QED

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Abstract

The finite-temperature properties of supersymmetric version of (2+1)D compact QED are explored. Integration over the dual photino yields an action of the dual photon in the form of a certain series. Only the first term of this series is of the potential form, while all the others are some unimportant corrections to the kinetic term. Disregarding these, we arrive at the action where besides the purely bosonic sector with monopoles of charge 2, monopoles of charge 1 and 3 are also present yielding certain corrections to the Debye mass of the dual photon. The finite-temperature analysis of this action is performed, and the RG trajectories of all the fugacities are found in the vicinity of the Berezinsky-Kosterlitz-Thouless (BKT) critical point. It is found that in this critical region, the fugacities ξ_n's of monopoles of charge $n = 1, 2, 3$ are scaled with the correlation radius, a, as $\xi_n \sim a^{-2n^2}$.

PACS: 11.10.Wx, 14.80.Hv, 11.10.Kk

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1 Introduction. The model.

Finite-temperature behavior of (2+1)D compact QED has for the first time been analysed in ref. [1]. In that paper, it has been demonstrated that the monopole plasma in the respective dimensionally-reduced theory undergoes the BKT phase transition [2] to the molecular phase, where the confining properties of the model, discovered in ref. [3], disappear. It is also interesting to study the finite-temperature behavior of the more general theory, namely of the (2+1)D Georgi-Glashow model. This theory becomes reduced to compact QED in the limit of infinitely large Higgs-boson mass, but in addition contains W-bosons which play an important role for the dynamics of the phase transition [4]. From the finite-temperature analysis of the (2+1)D Georgi-Glashow model (reviewed in ref. [5]), it in particular becomes clear that a lot of new effects open up when one includes into the theory some matter fields [6]. This is the main motivation for the present research. It is devoted to the investigation of finite-temperature properties of the supersymmetric generalization of (2+1)D compact QED [7] which contains besides the dual photon also its superpartner, photino. This theory is described by the supersymmetric sine-Gordon model, whose action in the superfield notation reads [1]

$$S = \int d^3x d\bar{\theta} d\theta \left[\frac{1}{2}(\bar{D}_\alpha \Phi)(D_\alpha \Phi) - \zeta \cos(g_m \Phi)\right]. \tag{1}$$

In this equation, the scalar supermultiplet and the supercovariant derivative have the form [2]

$$\Phi(\mathbf{x}, \theta) = \chi + 2\bar{\theta}\lambda + 2\bar{\theta}\theta F, \quad D_\alpha = \frac{1}{2}\frac{\partial}{\partial\bar{\theta}^\alpha} + (\gamma_i \theta)_\alpha \partial_i.$$

Here, χ is the dual-photon field (real scalar), λ is the photino field, which is the two-component real (Majorana) spinor ($\bar{\lambda} = i\lambda^T \sigma_2$), F is an auxiliary scalar field, and the Euclidean γ-matrices are expressed in terms of the Pauli matrices as $\gamma_1 = i\sigma_3$, $\gamma_2 = i\sigma_1$, $\gamma_3 = -i\sigma_2$. Next, g_m is the magnetic coupling of dimensionality $[\text{mass}]^{-1/2}$, and ζ is the monopole fugacity of dimensionality $[\text{mass}]^2$ which is exponentially small with respect to g_m^{-4} [3, 7].

Performing the Taylor expansion of $\cos(g_m \Phi)$ in the Grassmann variables and integrating them out, one readily rewrites the action (1) in the component notations as

$$S = \int d^3x \left[\frac{1}{2}(\partial_i \chi)^2 + \frac{i}{2}\bar{\lambda}\hat{\partial}\lambda - \frac{1}{2}F^2 + 2g_m \zeta F \sin(g_m \chi) + V(\mathbf{x})\bar{\lambda}\lambda\right] \xrightarrow{\int \mathcal{D}F}$$

$$\xrightarrow{\int \mathcal{D}F} \int d^3x \left[\frac{1}{2}(\partial_i \chi)^2 + \frac{i}{2}\bar{\lambda}\hat{\partial}\lambda + V(\mathbf{x})\bar{\lambda}\lambda - (g_m\zeta)^2 \cos(2g_m \chi)\right],$$

where $\hat{\partial} \equiv \gamma_i \partial_i$ and $V(\mathbf{x}) \equiv -g_m^2 \zeta \cos(g_m \chi)$. The obtained action coincides with the one presented in ref. [7]. Integrating further over photino we obtain

$$\int \mathcal{D}\bar{\lambda}\mathcal{D}\lambda \exp\left\{-\int d^3x \left[\frac{i}{2}\bar{\lambda}\hat{\partial}\lambda + V(\mathbf{x})\bar{\lambda}\lambda\right]\right\} = \exp\left\{\frac{1}{2}\text{tr}\ \ln\left[\left(\frac{i}{2}\hat{\partial} + V\right)\left(-\frac{i}{2}\hat{\partial} + V\right)\right]\right\} \equiv \mathrm{e}^{-\frac{1}{2}\mathcal{S}},$$

[1] For simplicity, we consider the case of vanishing Θ-parameter, although the inclusion of a nonvanishing Θ is straightforward.

[2] We use the convention $\int d\bar{\theta} d\theta \bar{\theta}\theta = 1$.

where $\mathcal{S} = \operatorname{tr} \int d^3x \int\limits_0^\infty \frac{d\tau}{\tau} \langle \mathbf{x} | \, \mathrm{e}^{-\mathcal{E}\tau\left(-\frac{1}{4}\partial^2 + \frac{\sigma}{4}\right)} | \mathbf{x} \rangle$. In this formula, $\frac{\sigma}{4} \equiv \frac{i}{2}\hat{\partial} V + V^2$, and $\mathcal{E}$ (the so-called einbein) is an arbitrary constant which we shall set for convenience to be equal 4. Next, to evaluate $\mathcal{S}$, it is natural to apply the heat-kernel method (see e.g. ref. [8]) that leads to the following expression:

$$\mathcal{S} = \operatorname{tr} \int d^3x \int\limits_0^\infty \frac{d\tau}{\tau} \int \frac{d^3p}{(2\pi)^3} \mathrm{e}^{-\tau p^2} \mathrm{e}^{\tau\left(\partial^2 + 2ip_i\partial_i\right)} \mathrm{e}^{-\tau\sigma}.$$

The second exponent in this formula results into the factor 1, since all its other terms yield full derivatives. We obtain

$$\mathcal{S} = \operatorname{tr} \int d^3x \int\limits_0^\infty \frac{d\tau}{\tau} \frac{\mathrm{e}^{-4\tau V^2}}{\tau^{3/2}} \mathrm{e}^{-2i\tau\hat{\partial} V} = 8 \operatorname{tr} \int d^3x \sum_{n=0}^{\infty} \left(-\frac{i}{2}\right)^n \frac{\Gamma\left(n - \frac{3}{2}\right)}{n!} V^{3-2n} \left(\hat{\partial} V\right)^n$$

with "Γ" standing for the Gamma-function. The parameter of this expansion can be estimated as $\frac{|\partial_i V|}{V^2} \sim \tan^2(g_m \chi)$, i.e., the expansion is well convergent in the weak-field limit $g_m |\chi| \ll 1$. However, obviously only the zeroth term of the expansion does not contain the derivatives of χ. Other terms, containing such derivatives, are proportional to some powers of $g_m^4 \zeta$. [3] Owing to the above-mentioned fact that $g_m^4 \zeta$ is an exponentially small quantity, all these terms can be disregarded with respect to the leading kinetic term of the field χ. Within this approximation, the action of the model takes the form

$$S \simeq \int d^3x \left[\frac{1}{2}(\partial_i \chi)^2 - (g_m \zeta)^2 \cos(2g_m\chi) - \frac{32\sqrt{\pi}}{3} (g_m^2 \zeta)^3 \cos^3(g_m \chi)\right]. \tag{2}$$

The Debye mass of the dual photon stemming from this action reads $m_D \simeq 2g_m^2 \zeta \left(1 + 4\sqrt{\pi} g_m^4 \zeta\right)$. Here the second term in the brackets, which is due to the $\cos^3$-term in the action, represents the leading correction to the purely bosonic expression $m_D \simeq 2g_m^2 \zeta$. Besides that, since $\cos^3(g_m\chi) = \frac{1}{4}\left[\cos(3g_m\chi) + 3\cos(g_m\chi)\right]$, the appearance of the $\cos^3$-term in the action means that monopoles of charge 1 and 3 (in the units of g_m) become present in the plasma of monopoles of charge 2. However, due to the extra factor ζ at the $\cos^3$-term, the densities of these monopoles are exponentially small with respect to the density of monopoles of charge 2. Clearly, this situation is different from the standard case of (2+1)D compact QED, where the monopoles of charge 1 have the highest density among monopoles of all possible charges.

The rest of the paper will be devoted to the finite-temperature analysis of the obtained action (2).

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[3] For instance, the first nonvanishing term of this kind reads

$$-\sqrt{\pi} \operatorname{tr} \int d^3x \frac{(\hat{\partial} V)^2}{V} = 2\sqrt{\pi} \int d^3x \frac{\left(\partial_i V\right)^2}{V} = 2\sqrt{\pi} g_m^4 \zeta \int d^3x \tan(g_m\chi) \sin(g_m\chi) (\partial_i \chi)^2.$$

2 RG analysis at finite temperature.

At finite temperature $T \equiv 1/\beta$, one should supply the field χ with the periodic boundary conditions in the temporal direction, with the period equal to β. Because of that, the lines of magnetic field emitted by a monopole cannot cross the boundary of the one-period region and, at the distances larger than β, should approach this boundary, going almost parallel to it. Therefore, monopoles separated by such distances interact via the 2D Coulomb potential, rather than the 3D one. The minimal average distance between monopoles in the plasma is that of the monopoles of charge 2. It is of the order of $(g_m\zeta)^{-2/3}$, and therefore at $T > (g_m\zeta)^{2/3}$, the monopole ensemble becomes two-dimensional. On the other hand, the critical temperature of the BKT phase transition reads $T_c = 8\pi/g_m^2$, [4] which is exponentially larger than $(g_m\zeta)^{2/3}$. Therefore, the idea of dimensional reduction is perfectly applicable at the temperatures of the order of T_c.

The factor β, appearing in front of the action (2) upon the dimensional reduction, can be removed [and the action can be cast to the original form (2) with the substitution $d^3x \to d^2x$] by the obvious rescaling $\chi_{\rm new} = \sqrt{\beta}\chi$. We thus arrive at the following dimensionally-reduced theory: $S_{\rm d.-r.}[\chi] = \int d^2x \left[\frac12(\partial_i\chi)^2 - \sum\limits_{n=1}^{3}\xi_n\cos\left(n\sqrt{K}\chi\right)\right]$, where $K \equiv g_m^2 T$, $\xi_1 \equiv 8\sqrt{\pi}\beta(g_m^2\zeta)^3$, $\xi_2 \equiv \beta(g_m\zeta)^2$, and $\xi_3 \equiv \frac{8\sqrt{\pi}}{3}\beta(g_m^2\zeta)^3$. Note once more that the BKT critical temperature, T_c, is determined by the term $\cos\left(\sqrt{K}\chi\right)$. This temperature, at which monopoles of charge 1 bind into molecules, is larger than such temperatures for monopoles of charges $n = 2, 3$, equal to $8\pi/(ng_m)^2$. [5]

In what follows, we shall adapt the usual RG strategy [9] based on the integration over the high-frequency modes. Splitting the momenta into two ranges, $0 < p < \Lambda'$ and $\Lambda' < p < \Lambda$, one can then define these modes as $h = \chi_\Lambda - \chi_{\Lambda'}$, where $\chi_{\Lambda'}(\mathbf{x}) = \int\limits_{0<p<\Lambda'} \frac{d^2p}{(2\pi)^2} {\rm e}^{i\mathbf{px}}\chi(\mathbf{p})$ and consequently, $h(\mathbf{x}) = \int\limits_{\Lambda'<p<\Lambda} \frac{d^2p}{(2\pi)^2} {\rm e}^{i\mathbf{px}}\chi(\mathbf{p})$. The partition function, $\mathcal{Z}_\Lambda = \int\limits_{0<p<\Lambda} \mathcal{D}\chi(\mathbf{p}){\rm e}^{-S_{\rm d.-r.}[\chi_\Lambda]}$, can be rewritten as $\mathcal{Z}_\Lambda = \int\limits_{0<p<\Lambda'} \mathcal{D}\chi(\mathbf{p}){\rm e}^{-\frac12\int d^2x(\partial_i\chi_{\Lambda'})^2}\mathcal{Z}'$, where $\mathcal{Z}' = \left\langle \exp\left\{\int d^2x\left[\sum\limits_{n=1}^{3}\xi_n\cos\left(n\sqrt{K}\chi_\Lambda\right)\right]\right\}\right\rangle_h$ and $\langle\mathcal{O}\rangle_h \equiv \frac{\int\limits_{\Lambda'<p<\Lambda}\mathcal{D}\chi(\mathbf{p})\exp\left[-\frac12\int d^2x(\partial_i h)^2\right]\mathcal{O}}{\int\limits_{\Lambda'<p<\Lambda}\mathcal{D}\chi(\mathbf{p})\exp\left[-\frac12\int d^2x(\partial_i h)^2\right]}$. Next, because of the $\cos\left(3\sqrt{K}\chi_\Lambda\right)$-term, in the course of average over h, one should keep the third irreducible average (also called cumulant) of $\cos\left(\sqrt{K}\chi_\Lambda\right)$. It is defined by the formula

$$\langle\langle f_1f_2f_3\rangle\rangle = \langle f_1f_2f_3\rangle - \langle\langle f_1f_2\rangle\rangle\langle f_3\rangle - \langle\langle f_1f_3\rangle\rangle\langle f_2\rangle - \langle\langle f_2f_3\rangle\rangle\langle f_1\rangle - \langle f_1\rangle\langle f_2\rangle\langle f_3\rangle, \quad (3)$$

[4] In the (2+1)D Georgi-Glashow model, T_c can be rewritten in terms of the electric coupling, g, as $T_c = \frac{g^2}{2\pi}$ [1], where g is related to g_m as $gg_m = 4\pi$.

[5] Note that if our theory were considered as being originated from some extension of the (2+1)D Georgi-Glashow model, possessing charged matter fields, the true phase transition would be determined by the equality of density of these fields to that of monopoles [4]. Since ξ_1 is exponentially smaller than ξ_2 (i.e., the density of monopoles of charge 2 is the highest one), such a phase transition would then be determined by the monopoles of charge 2, rather than 1.

where $\langle\langle f_1 f_2\rangle\rangle = \langle f_1 f_2\rangle - \langle f_1\rangle\langle f_2\rangle$ and in our case $f_i \equiv \cos\left(\sqrt{K}\chi_\Lambda(\mathbf{x}_i)\right)$, $\langle\ldots\rangle \equiv \langle\ldots\rangle_h$. Denoting for brevity $\sqrt{K}\chi_{\Lambda'}(\mathbf{x})$ by $\phi_{\mathbf{x}}$ we first obtain for a single average and for the second cumulant (cf. ref. [9]):

$$\left\langle\cos\left(\sqrt{K}\chi_\Lambda(\mathbf{x})\right)\right\rangle = D(0)\cos\phi_{\mathbf{x}}, \tag{4}$$

$$\left\langle\left\langle\cos\left(\sqrt{K}\chi_\Lambda(\mathbf{x})\right)\cos\left(\sqrt{K}\chi_\Lambda(\mathbf{y})\right)\right\rangle\right\rangle =$$

$$= \frac{D^2(0)}{2}\left\{\left[D^2(\mathbf{x}-\mathbf{y})-1\right]\cos\left(\phi_{\mathbf{x}}+\phi_{\mathbf{y}}\right)+\left[D^{-2}(\mathbf{x}-\mathbf{y})-1\right]\cos\left(\phi_{\mathbf{x}}-\phi_{\mathbf{y}}\right)\right\}, \tag{5}$$

where $D(\mathbf{x}) \equiv \exp\left[-\frac{K}{2}\int\limits_{\Lambda'<p<\Lambda}\frac{d^2p}{(2\pi)^2}\frac{\mathrm{e}^{i\mathbf{px}}}{p^2}\right]$. In the same way, we obtain the following expression for the three-local average:

$$\left\langle\cos\left(\sqrt{K}\chi_\Lambda(\mathbf{x})\right)\cos\left(\sqrt{K}\chi_\Lambda(\mathbf{y})\right)\cos\left(\sqrt{K}\chi_\Lambda(\mathbf{z})\right)\right\rangle = \frac{D^3(0)}{4}\Big[D^2(\mathbf{x}-\mathbf{y})D^2(\mathbf{x}-\mathbf{z})D^2(\mathbf{y}-\mathbf{z})\times$$

$$\times\cos\left(\phi_{\mathbf{x}}+\phi_{\mathbf{y}}+\phi_{\mathbf{z}}\right)+D^{-2}(\mathbf{x}-\mathbf{y})D^{-2}(\mathbf{x}-\mathbf{z})D^2(\mathbf{y}-\mathbf{z})\cos\left(\phi_{\mathbf{x}}-\phi_{\mathbf{y}}-\phi_{\mathbf{z}}\right)+D^{-2}(\mathbf{x}-\mathbf{y})D^2(\mathbf{x}-\mathbf{z})\times$$

$$\times D^{-2}(\mathbf{y}-\mathbf{z})\cos\left(\phi_{\mathbf{x}}-\phi_{\mathbf{y}}+\phi_{\mathbf{z}}\right)+D^2(\mathbf{x}-\mathbf{y})D^{-2}(\mathbf{x}-\mathbf{z})D^{-2}(\mathbf{y}-\mathbf{z})\cos\left(\phi_{\mathbf{x}}+\phi_{\mathbf{y}}-\phi_{\mathbf{z}}\right)\Big]. \tag{6}$$

The third cumulant can now be readily evaluated by virtue of eqs. (3)-(6). The expression for it is, however, rather lengthy and for the sake of shortness we shall not present it here. It is only important that similarly to eq. (5), the third cumulant vanishes when the distance between any two points in it grows. This fact enables one to approximate, for example, $\cos\left(\phi_{\mathbf{x}}+\phi_{\mathbf{y}}-\phi_{\mathbf{z}}\right)$ by $\cos\phi_{\mathbf{u}}$ [6] and $\cos\left(\phi_{\mathbf{x}}+\phi_{\mathbf{y}}+\phi_{\mathbf{z}}\right)$ by $\cos\left(3\phi_{\mathbf{u}}\right)$, where $\mathbf{u}$ is the center of mass of the triangle with the vertices $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$. The partition function $\mathcal{Z}'$ then takes the form

$$\mathcal{Z}' \simeq \exp\Bigg\{-\int d^2x\Bigg[\xi_1 D(0)\left(1+\frac{\xi_1^2\alpha_1}{24}D^2(0)\right)\cos\phi_{\mathbf{x}}+D^2(0)\left(\xi_2 D^2(0)-\frac{\xi_1^2\alpha_2}{4}\right)\cos(2\phi_{\mathbf{x}})+$$

$$+D^3(0)\left(\xi_3 D^6(0)+\frac{\xi_1^3\alpha_3}{24}\right)\cos(3\phi_{\mathbf{x}})-\int d^2y\left(D^{-2}(\mathbf{y})-1\right)\left[1-\frac{K^2}{2}\left(y_i\partial_i\phi_{\mathbf{x}}\right)^2\right]\Bigg]\Bigg\},$$

where

$$\alpha_1 = \int d^2y d^2z\Big\{D^2(\mathbf{y})\Big[D^{-2}(\mathbf{z})D^{-2}(\mathbf{y}-\mathbf{z})-1\Big]+D^{-2}(\mathbf{y})\Big[D^2(\mathbf{z})D^{-2}(\mathbf{y}-\mathbf{z})+$$

[6] The next term, $-\frac{1}{2}\left[(\mathbf{x}-\mathbf{z})_i\partial_i\phi_{\mathbf{u}}\right]^2\cos\phi_{\mathbf{u}}$, may be omitted, since it results into the exponentially small correction to the kinetic term, analogous to the one discussed in footnote 3.

$$+ D^{-2}(\mathbf{z})D^2(\mathbf{y}-\mathbf{z}) - 2\Big] + 7 - D^2(\mathbf{z}) - 2D^{-2}(\mathbf{z}) - 2D^2(\mathbf{y}-\mathbf{z}) - 2D^{-2}(\mathbf{y}-\mathbf{z})\Big\},$$

$$\alpha_2 = \int d^2y \left[D^2(\mathbf{y}) - 1\right], \quad \alpha_3 = \int d^2y d^2z \Big\{D^2(\mathbf{y})\left[D^2(\mathbf{z})D^2(\mathbf{y}-\mathbf{z}) - 1\right] + 1 - D^2(\mathbf{z})\Big\}.$$

The constants $\alpha_{1,3}$ here stem from the third cumulant. Taking $\Lambda' = \Lambda - d\Lambda$, we readily obtain $\alpha_1 = -\alpha_3 = \mathcal{A}K\frac{d\Lambda}{\Lambda^3}\left[\alpha + \mathcal{O}\left(\frac{d\Lambda}{\Lambda}\right)\right]$. Here, $\mathcal{A}$ is the (fixed) area of the system, and α stands for some momentum-space-slicing dependent positive constant, whose concrete value will turn out to be unimportant for the final expressions describing the RG flow. In the same way, we have $\alpha_2 = -\alpha K d\Lambda/\Lambda^3$. Taking further into account that to the leading order in $d\Lambda/\Lambda$, $D(0) \simeq 1 - \frac{K}{4\pi}\frac{d\Lambda}{\Lambda}$, we obtain the following renormalizations of fugacities:

$$d\xi_1 = \frac{\xi_1 K}{4}\left(-\frac{1}{\pi} + \frac{\alpha\mathcal{A}\xi_1^2}{6\Lambda^2}\right)\frac{d\Lambda}{\Lambda}, \quad d\xi_2 = K\left(-\frac{\xi_2}{\pi} + \frac{\alpha\xi_1^2}{4\Lambda^2}\right)\frac{d\Lambda}{\Lambda}, \quad d\xi_3 = -\frac{K}{4}\left(\frac{9\xi_3}{\pi} + \frac{\alpha\mathcal{A}\xi_1^3}{6\Lambda^2}\right)\frac{d\Lambda}{\Lambda},$$

where the terms higher in $1/\Lambda$ than those presented in the brackets have been disregarded. The field itself becomes renormalized as [7]

$$\chi_{\Lambda'}^{\text{new}} = \sqrt{1 + \beta\left(K\xi_1 D(0)\right)^2\frac{d\Lambda}{\Lambda^5}}\chi_{\Lambda'} \simeq \left(1 + \frac{\beta}{2}(K\xi_1)^2\frac{d\Lambda}{\Lambda^5}\right)\chi_{\Lambda'},$$

where β is another (analogous to α) momentum-space-slicing dependent positive constant. Finally, the renormalizations of the coupling K and of the free-energy density $F \equiv -\frac{\ln\mathcal{Z}'}{\mathcal{A}}$ have the form:

$$dK = -\beta K^3\xi_1^2\frac{d\Lambda}{\Lambda^5}\left[1 + \mathcal{O}\left(\frac{d\Lambda}{\Lambda}\right)\right], \quad dF = -\frac{\alpha}{4}K\xi_1^2\frac{d\Lambda}{\Lambda^3}\left[1 + \mathcal{O}\left(\frac{d\Lambda}{\Lambda}\right)\right],$$

where the last terms in the brackets will further be disregarded.

Following the notations of ref. [9], we shall further make the change of variables from the momentum scale to the real-space one: $\Lambda \to a \equiv 1/\Lambda$, $d\Lambda \to -d\Lambda$, that obviously modifies the above RG equations as

$$d\xi_1 = -\frac{\xi_1 K}{4\pi}\frac{da}{a}\left(1 - \frac{\pi\alpha\mathcal{A}(a\xi_1)^2}{6}\right), \quad d\xi_2 = -\frac{K}{\pi}\frac{da}{a}\left(\xi_2 - \frac{\pi\alpha(a\xi_1)^2}{4}\right),$$

$$d\xi_3 = -\frac{9K}{4\pi}\frac{da}{a}\left(\xi_3 + \frac{\pi\alpha\mathcal{A}a^2\xi_1^3}{54}\right), \quad dK = -\beta\xi_1^2(aK)^3 da, \quad dF = \frac{\alpha}{4}K\xi_1^2 a da.$$

Our aim below is to obtain the RG flow in the vicinity of the BKT critical point [2, 9, 1] $K_c = 8\pi$, $z_{1\,c} = 0$, where $z_1 \equiv \xi_1 a^2$. [The value of K_c here stems, in particular, from the value of T_c, at which monopoles of charge 1 bind into molecules (cf. the first two paragraphs of this

[7] The respective RG equation obviously reads $d\chi = \frac{\beta}{2}(K\xi_1)^2\chi d\Lambda/\Lambda^5$.

Section).] Substituting this value of K_c into the equation for ξ_1, we see that in the critical region this fugacity scales as $\xi_1 \sim a^{-2}$, [8] and the term $\sim (a\xi_1)^2$ in the brackets is of the order of a^{-2}. At the same time, in the molecular phase, the correlation radius a becomes infinite due to the short-rangeness of the molecular fields, and it diverges at $T \to T_c - 0$ [9]. Therefore, the above-mentioned term $\sim (a\xi_1)^2$ in the RG equation for ξ_1 may be disregarded in the critical region, as well as the analogous term in the equation for ξ_2 and the term $\sim a^2\xi_1^3$ in the equation for ξ_3. In what follows, we shall adapt this approximation, that is equivalent to the neglection of the above-described contribution of the third cumulant to ξ_n's. The scaling behaviors of ξ_2 and ξ_3 in the critical region then read $\xi_2 \sim a^{-8}$ and $\xi_3 \sim a^{-18}$. One can write down the following general RG equation $d\ln\xi_n = -n^2\frac{K}{4\pi}d\ln a$, that in the vicinity of the critical point yields the scaling law $\xi_n \sim a^{-2n^2}$. We thus see that the absolute value of power of the correlation radius, with which the fugacity of monopoles of a certain charge vanishes towards the critical point, grows as the squared charge of respective monopoles. This is a formal expression for a heuristic fact that at a given temperature, the fugacity of monopoles of charge n is smaller for a larger n, since the phase transitions occur in the sequence $n=3$, $n=2$, and $n=1$.

It is further convenient to introduce instead of K the new coupling $x = 2 - \frac{K}{4\pi}$, which is positive at $T < T_c$ and vanishes at $K = K_c$. Performing then rescalings $a_{\rm new} = 2(8\pi^2\beta)^{1/4}a$, $z_{1\,{\rm new}} = \xi_1 a_{\rm new}^2$, we obtain equations $dx = z_1^2 da/a$, $dz_1^2 = 2z_1^2 x da/a$. They yield the BKT-type RG flow $z_1^2 - x^2 = \tau$, where $\tau \propto (T_c - T)/T_c$ is some constant. In particular, $x \simeq z_1$ at $T \to T_c - 0$. Owing to the above equation for z_1^2, this relation yields $z_{1\,({\rm in})}^{-1} - z_1^{-1} = \ln\left(a/a_{({\rm in})}\right)$, where the subscript "$_{({\rm in})}$" means the initial value. Taking into account that $z_{1\,({\rm in})}$ is exponentially small, while $z_1 \sim 1$ [the value at which the growth of $z_1(a)$ stops], we obtain in the case $x_{({\rm in})} \le \sqrt{\tau}$: $\ln\left(a/a_{({\rm in})}\right) \sim z_{1\,({\rm in})}^{-1} \sim \tau^{-1/2}$. According to this relation, at $T \to T_c - 0$, the correlation radius diverges with an essential singularity as $a(\tau) \sim \exp\left({\rm const}/\sqrt{\tau}\right)$ with ${\rm const} > 0$.

In order to describe the RG flows of ξ_n', $n = 2, 3$, it is natural to introduce the dimensionless quantities analogous to z_1, $z_n \equiv \xi_n a^2$, which obey the equations $d\ln z_n^2 = 4(1-n^2)d\ln a$. Taking into account that $dx \simeq \tau d\ln a$ at $x \ll 1$, we obtain in this region the following equation: $\frac{dz_n^2}{dx} = \frac{4(1-n^2)}{\tau}z_n^2$. The respective RG flow has the form $z_n = C_n {\rm e}^{-\frac{2(n^2-1)}{\tau}x}$ with C_n's being the positive integration constants. We see that in any of the planes (x, z_2) and (x, z_3), the line $\tau = 0$ plays the same role as it plays in the (x, z_1)-plane. Namely, this line [coinciding in the $(x, z_{2,3})$-planes with the line $x = 0$] is the separatrix between two distinct classes of RG trajectories. First of them are those with $\tau < 0$, which decrease towards C_n with the decrease of x, while the second ones, corresponding to $\tau > 0$, increase from zero to C_n. At $x = 0$, both $z_{2,3}$ have nonvanishing values $C_{2,3}$, that is similar to z_1, $z_1(x=0) = \sqrt{\tau}$. As it has been discussed, these values vanish at $K \to K_c$ according to the law $C_n \sim a^{2(1-n^2)}$.

In conclusion of this letter, the dual photino present in the supersymmetric version of (2+1)D compact QED leads to the increase of the Debye mass of the dual photon by means of the appearance of the admixture of monopoles of charge 1 and 3 in the plasma of monopoles of charge 2 (the latter correspond to the purely bosonic sector of the model). At finite temperature, the BKT phase transitions of monopoles of charge n occur in the sequence $n = 3$, $n = 2$,

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[8] In particular, substituting the behavior $\xi_1 \sim a^{-2}$ into the equation for the free-energy density, we see that the latter scales in the vicinity of the critical point as $F \sim a^{-2}$, i.e, remains continuous.

[9] The functional form of this divergence will be discussed below.

$n=1$. In the critical region [corresponding to the ($n=1$)-phase transition], the fugacities ξ_n's are found to vanish as $\xi_n \sim a^{-2n^2}$, where the correlation radius a diverges with an essential singularity, $a(\tau) \sim \exp\left(\mathrm{const}/\sqrt{\tau}\right)$. This scaling of ξ_n's is in accordance with the above sequence of the phase transitions. When the parameter x (measuring the distance to the critical point) is much smaller than unity, the dimensionless fugacity $z_1 = \xi_1 a^2$ scales according to the standard BKT RG flow as $z_1^2 - x^2 = \tau$. At the same time, the analogous fugacities for monopoles of higher charges, $z_n = \xi_n a^2$, $n = 2, 3$, behave as $z_n = C_n \mathrm{e}^{-\frac{2(n^2-1)}{\tau}x}$ with $C_n \sim a^{2(1-n^2)}$. For these monopoles (similarly to the case $n=1$), the line $\tau = 0$ is the separatrix between the RG trajectories corresponding to $\tau < 0$ (which decrease towards C_n with the decrease of x) and those corresponding to $\tau > 0$ (which increase from zero to C_n with the decrease of x).

3 Acknowledgments.

The author is grateful to Prof. A. Di Giacomo for useful discussions and to Dr. A. Kovner for informing him about the existence of ref. [7]. He is also grateful to Prof. A. Di Giacomo and to the whole staff of the Physics Department of the University of Pisa for cordial hospitality. The work has been supported by INFN and partially by the INTAS grant Open Call 2000, Project No. 110.

References

[1] N.O. Agasyan and K. Zarembo, Phys. Rev. **D 57** (1998) 2475.

[2] V.L. Berezinsky, Sov. Phys.- JETP **32** (1971) 493; J.M. Kosterlitz and D.J. Thouless, J. Phys. **C 6** (1973) 1181; J.M. Kosterlitz, J. Phys. **C 7** (1974) 1046.

[3] A.M. Polyakov, Nucl. Phys. **B 120** (1977) 429.

[4] G. Dunne, I.I. Kogan, A. Kovner, and B. Tekin, JHEP **01** (2001) 032.

[5] I.I. Kogan and A. Kovner, hep-th/0205026; D. Antonov, hep-th/0207224.

[6] N.O. Agasian and D. Antonov, Phys. Lett. **B 530** (2002) 153; G.V. Dunne, A. Kovner, and Sh.M. Nishigaki, Phys. Lett. **B 544** (2002) 215.

[7] I. Affleck, J. Harvey, and E. Witten, Nucl. Phys. **B 206** (1982) 413.

[8] J.F. Donoghue, E. Golowich, and B.R. Holstein, *Dynamics of the Standard Model* (Cambridge Univ. Press, Cambridge, 1992).

[9] J.B. Kogut, Rev. Mod. Phys. **51** (1979) 659; B. Svetitsky and L.G. Yaffe, Nucl. Phys. **B 210 [FS6]** (1982) 423.