

Fluxbranes from p-branes

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Abstract

It is shown how magnetic fluxbrane, Fp-brane, solutions are related to electric black p-branes by analytic continuation. Viewing the transverse space of branes as a warped cone, one finds that the cone base of the p-brane becomes the world-volume of the F(D-p-3)-brane and the world volume of the p-brane becomes the cone base of the F(D-p-3)-brane. An explicit example of the correspondence is given for a 2-brane and F6-brane of 11D supergravity.

1 Introduction

A long standing solution to Einstein-Maxwell theory is the Melvin Universe [1], describing how parallel lines of magnetic (electric) flux self-gravitates to form a classically stable flux-tube [2]. The solution was later extended to dilaton-Einstein-Maxwell theory in arbitrary dimensions in order to give it a place in supergravity theories [3]. It was later discovered that the dilaton-Melvin solution has a rather distinguished heritage, as it can be derived from pure gravity in one dimension higher by compactification of Minkowski space [4]. This aspect of the solution then leads it to being an exact sigma model background for string theory [5]. Further interest in the Melvin solution comes from duality arguments; it is believed that a Melvin background with field strength \mathbf{B} in type 0A and a Melvin background with \mathbf{B}' of type IIA are equivalent, for specific \mathbf{B} , \mathbf{B}' [6]. With these aspects in mind an attempt was made to generalize the Melvin solution from a fluxbrane due to a 2-form field strength to a fluxbrane for an arbitrary rank field strength [7], see also [8]. The near core and asymptotic behaviour of a class of fluxbranes was studied in [9]. The physical picture of these fluxbranes comes from giving a p-brane and anti-p-brane infinite separation, the flux from the p-brane is transmitted to the anti-p-brane along F(p+1)-branes.

There is yet another aspect to Fp-branes which warrants interest, namely the dielectric brane effect [10]. When a number of D0-branes are placed in a constant electric background 4-form field strength one finds that the 0-branes expand to form a non-commutative two-sphere. In terms of supergravity this picture cannot be entirely correct as there is no such thing as a constant form field strength, the form self-gravitates to create Fp-branes. A supergravity description of just such a system has been found in [11], [12].

Here we look at Fp-branes from a perspective prompted by [13]. In [13] it was shown how one can start with the Reissner-Nordström (RN) metric of an electrically charged black hole and, after a

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limiting procedure and analytic continuation, arrive at the Melvin solution. We aim to place this result in the more general setting of generating magnetic F(D-p-3)-branes from electrically charged p-branes. In fact such a relation was already hinted at in [3] where 0-branes (black holes) and (D-3)-fluxbranes were studied together because of the similarity of the ansätze. (See also [14] where a flux tube solution is related to the analytic continuation of a cosmological model.) Although we only explicitly work with electric p-branes and magnetic Fp-branes, electric/magnetic duality means there is a similar relation between magnetic p-branes and electric Fp-branes

The outline of the paper is to start by covering the RN-Melvin connection, to make the procedure clear, and with this in mind the general correspondence will be found. To see how this works in practise we take the non-trivial example of turning an electric 2-brane of D=11 supergravity into a magnetic F6 brane. Conclusions are then drawn at the end.

2 Melvin from Reissner-Nordström

The action that we shall be studying is that of gravity plus n-form in D dimensions,

$$S = \int \left[R * 1 - \frac{1}{2} F_{(n)} \wedge * F_{(n)} \right]. \quad (2.1)$$

which gives the following equations of motion,

$$\mathcal{R}_{MN} = \frac{1}{2(n-1)!} \left[F_{M\dots} F_N \dots - \frac{n-1}{n(D-2)} F^2 g_{MN} \right] \quad (2.2)$$

$$d * F = 0 \quad (2.3)$$

for the Ricci tensor \mathcal{R}_{MN} and $F_{(n)}$. In this paper we shall not include the dilaton as it does not add anything to the discussion.

The starting point for the derivation is the RN metric of four dimensional Einstein-Maxwell theory,

$$ds^2 = -V dt^2 + V^{-1} dr^2 + r^2 d\Omega_{(2)}^2 \quad (2.4)$$

$$V = \frac{1}{r^2} (r - r_-)(r - r_+) \quad (2.5)$$

$$F_{(2)} = \frac{2\sqrt{r_- r_+}}{r^2} dt \wedge dr \quad (2.6)$$

where $d\Omega_{(2)}^2$ is the usual spherical volume element. This is a metric describing an electrically charged black hole, and if r_{\pm} are positive it has two horizons, at $r = r_{\pm}$.

We start with a coordinate change and a redefinition of parameters,

$$r = r_- + \frac{1}{4} B^2 r_- \rho^2, \quad (2.7)$$

$$r_+ = -r_-^3 B^2, \quad (2.8)$$

noting that now there is no horizon at $r = r_+$, at the expense of an imaginary electric charge.

A limiting procedure is now performed to flatten the spherical volume element, using stereographic coordinates for the sphere makes this explicit,

$$d\Omega_{(2)}^2 = \frac{4}{(1 + \xi \bar{\xi})^2} d\xi d\bar{\xi} \quad (2.9)$$

$$r_- \rightarrow r_-/\lambda, \quad \xi \rightarrow \lambda(x + iz), \quad t \rightarrow \lambda t, \quad (2.10)$$

$$\lambda \rightarrow 0. \quad (2.11)$$

This is a way of getting the equivalent of the RN solution with the spherical volume element replaced by a Ricci flat metric. Such a replacement means that we find a spacetime which is not asymptotically flat, however, neither is the Melvin solution which is what we are aiming for.

Now define,

$$t = \frac{2i}{B^2 r_-} \phi, \quad x = i \frac{T}{2}, \quad z = \frac{Z}{2}, \quad (2.12)$$

With this done we find the metric and field strength,

$$ds^2 = \Lambda(\rho)^2 (-dT^2 + dZ^2) + \Lambda(\rho)^2 d\rho^2 + \rho^2 \Lambda(\rho)^{-2} d\phi^2 \quad (2.13)$$

$$\Lambda(\rho) = 1 + \frac{B^2}{4} \rho^2 \quad (2.14)$$

$$F_{(2)} = 2B\rho\Lambda(\rho)^{-2} d\rho \wedge d\phi \quad (2.15)$$

which is just the magnetic Melvin solution. We note here that the inner horizon, r_+ , of the black hole has become the origin of the Melvin solution, (2.7). The regularity of this inner black hole horizon now translates to the regularity of the core of the flux-tube, a point made in [3]. It is also worth noting that the two horizons had to have different values, (2.8), this therefore precludes the use of extremal branes. To be more explicit, suppose we started with the extremal case of $r_+ = r_- = r_H$ then

$$F_{\text{extreme}} = 2 \frac{r_H}{r^2} dt \wedge dr \quad (2.16)$$

and the analytic continuation of the world volume then takes $t \rightarrow i\tau$. To keep the form field real we then take $r_H \rightarrow iR_H$ but this makes the metric function $V(r)$ (2.5) imaginary. This is the typical situation, starting with the extreme p-branes leads to fluxbranes with imaginary coefficients, either in the metric or form field, or both.

The important observation to make is that the world-volume of the black hole, \mathcal{H} , has become the angular coordinate of the flux-tube and the angular coordinates of the black hole, $d\Omega_{(2)}$, have become the world volume of the flux-tube.

We shall see that this type of relation holds in the general case. Recalling that the metric of a cone with base metric $d\omega^2$ is $dr^2 + r^2 d\omega^2$, we can view the space transverse to the world-volume of a brane as a warped cone. The correspondence is then that the cone bases get analytically continued to become world-volumes and world-volumes are analytically continued into cone bases. As a side remark, we note that had we not flattened the $d\Omega_{(2)}^2$ we would have analytically continued this to a deSitter metric, giving a flux-tube with deSitter world-volume.

3 General correspondence

The relation between magnetic F-branes and electric p-branes is made clearer by writing down the ansatz for each of them. We start with the electric p-brane from an n -form field strength in D dimensions; $p = n - 2$, $d = n - 1$, $\tilde{d} = D - d - 2$,

$$dS^2(p\text{-brane}) = \exp(2A(r)) \bar{ds}_{(d)}^2 + \exp(2B(r)) dr^2 + \exp(2C(r)) ds_{(\tilde{d}+1)}^2 \quad (3.1)$$

$$F_{d+1} \propto \exp(dA + B - (\tilde{d} + 1)C) \eta_{(d)} \wedge dr. \quad (3.2)$$

While an n -form field strength gives rise to the following magnetic Fp-brane in D dimensions [7]; $p = D - n - 1$, $m = n - 1$, $l = D - m - 2$,

$$dS^2(Fp\text{-brane}) = \exp(2a(r)) \bar{ds}_{(l+1)}^2 + \exp(2b(r)) dr^2 + \exp(2c(r)) ds_{(m)}^2 \quad (3.3)$$

$$F_{m+1} \propto \exp(-(l + 1)a + b + mc) \eta_{(m)} \wedge dr. \quad (3.4)$$

The volume elements \bar{ds}^2 have Lorentzian signature and describe the world volumes of the branes. The transverse space is made up of a radial coordinate r and Euclidean signature metrics ds^2 , so ds^2 are the cone bases for the transverse space. The similarity of the two systems should now be clear, with us being able to move from the p-brane description to the Fp-brane by changing $d \rightarrow m$, $\bar{d} \rightarrow l$, $A(r) \rightarrow c(r)$, $B(r) \rightarrow b(r)$, $C(r) \rightarrow a(r)$. The world-volume metric, $\bar{ds}_{(d)}^2$, gets analytically continued to become the cone base metric, $ds_{(m)}^2$, and the cone base metric, $ds_{(\bar{d}+1)}^2$, gets continued to become the world-volume metric, $\bar{ds}_{(l+1)}^2$.

Given this correspondence we should ask whether the well known p-brane solutions can uncover new fluxbrane solutions. The Fp-branes of most interest would be those where the cone base was a sphere with its round metric. The correspondence says that these are equivalent to p-branes with deSitter world-volume, a system which has not yet been solved.

We can however use the correspondence to understand some features of [7]. That paper described Fp-branes where the world-volume and cone base metrics were Einstein metrics, independent of the radial coordinate. These then should match with black p-branes whose world volume and cone base are also Einstein and independent of the radial distance. Given this it is clear that the ‘standard’ black branes of Duff *et al* [15] will not analytically continue to these Fp-branes as there the world volume has a radial dependence. There is a class of black brane solutions however where the world volume is flat and has a boost symmetry [16], these should therefore be continued to give Fp branes where the cone base is flat. These black brane solutions then explain some properties of the Fp-branes found in [7]. The Fp-branes with deSitter world volume and flat cone base were found to be asymptotically flat, this now translates to the asymptotic flatness of the black brane solutions with Minkowski world volume. Similarly, the non-asymptotic flatness of Fp-branes with Ricci flat world volume gets translated to the same property of black branes with a Ricci flat cone base.

4 Explicit example; $D=11$ F6-brane from the 2-brane.

We now give an explicit, non trivial example of how this correspondence relates the black branes of [16] to the Fp-branes of [7]. The example we choose is motivated by 11D supergravity, starting with an electric 2-brane we generate a magnetic F6-brane as follows.

Taking note of the different convention for the form normalization we start with the electric 2-brane solution of [16],

$$ds^2 = \Lambda^{-\frac{2}{3}} d\underline{x}_{(3)}^2 \quad (4.1)$$

$$+ \Lambda^{\frac{1}{3}} \left(\frac{r_+^6 - r_-^6}{r_+^6 - r_-^6} \right)^{-\frac{5}{6}} (r_+^6 - r_-^6)^{-\frac{5}{3}} r^{10} dr^2$$

$$+ \Lambda^{\frac{1}{3}} \left(\frac{r_+^6 - r_-^6}{r_+^6 - r_-^6} \right)^{\frac{1}{6}} (r_+^6 - r_-^6)^{\frac{1}{3}} d\Omega_{(7)}^2,$$

$$F_{(4)} = 2Q * \eta_{(7)}, \quad (4.2)$$

$$Q^2 = 9r_-^6 \left[\sqrt{\frac{7}{3}} (r_+^6 - r_-^6) + r_-^6 \right], \quad (4.3)$$

$$\Lambda = \left\{ \left(1 + \frac{r_-^6}{\sqrt{\frac{7}{3}}(r_+^6 - r_-^6)} \right) \left(\frac{r_+^6 - r_-^6}{r_+^6 - r_-^6} \right)^{-\frac{1}{2}\sqrt{\frac{7}{3}}} - \frac{r_-^6}{\sqrt{\frac{7}{3}}(r_+^6 - r_-^6)} \left(\frac{r_+^6 - r_-^6}{r_+^6 - r_-^6} \right)^{\frac{1}{2}\sqrt{\frac{7}{3}}} \right\} \quad (4.4)$$

With $\eta_{(7)}$ being the volume form for the round 7-sphere. Following in analogy with the RN-Melvin

case we take the parameter describing the outer horizon, r_+ , to be negative and we define a new radial coordinate, R ,

$$r_+^6 = -B^{2c_0}, \quad \frac{r^6 - r_+^6}{r^6 - r_-^6} = \left(\frac{\Gamma R}{B}\right)^{2c_0}. \quad (4.5)$$

Two radial scales, R_0 and R_1 , are introduced by the relations

$$\left(\frac{B}{\Gamma R_1}\right)^{2c_0\sqrt{\frac{7}{3}}} = \frac{\sqrt{\frac{7}{3}}(r_+^6 - r_-^6) + r_-^6}{r_-^6} \quad (4.6)$$

$$R_0 = \frac{B}{\Gamma} \quad (4.7)$$

and we introduce the constant κ by,

$$(B^{2c_0} + r_-^6) \left[\left(\frac{R_0}{R_1}\right)^{c_0\sqrt{\frac{7}{3}}} + \left(\frac{R_0}{R_1}\right)^{-c_0\sqrt{\frac{7}{3}}} \right]^{-1} = \frac{\kappa}{6\sqrt{\frac{7}{3}}} \quad (4.8)$$

from which we find,

$$ds^2 = \left(\frac{c_0}{3}\right)^{\frac{2}{3}} \left(\frac{\kappa}{6\sqrt{\frac{7}{3}}}\right)^{\frac{2}{3}} \tilde{\Lambda}^{-\frac{2}{3}} \left(\frac{3}{c_0}\right)^{\frac{2}{3}} (B^{2c_0} + r_-^6)^{\frac{2}{3}} d\underline{x}_{(3)}^2 \quad (4.9)$$

$$+ \frac{c_0^2}{9} \left(\frac{\kappa}{6\sqrt{\frac{7}{3}}}\right)^{\frac{1}{3}} \tilde{\Lambda}^{\frac{1}{3}} \left(\left(\frac{R}{R_0}\right)^{-c_0} - \left(\frac{R}{R_0}\right)^{c_0} \right)^{-\frac{7}{3}} \frac{dR^2}{R^2}$$

$$+ \left(\frac{\kappa}{6\sqrt{\frac{7}{3}}}\right)^{\frac{1}{3}} \tilde{\Lambda}^{\frac{1}{3}} \left(\left(\frac{R}{R_0}\right)^{-c_0} - \left(\frac{R}{R_0}\right)^{c_0} \right)^{-\frac{1}{3}} d\Omega_{(7)}^2$$

$$F = 2\kappa \left(\frac{c_0}{3}\right)^2 \left(\frac{\kappa}{6\sqrt{\frac{7}{3}}}\right)^{-2} \tilde{\Lambda}^{-2} \left(\frac{3}{c_0}\right) (B^{2c_0} + r_-^6) i dx^0 \wedge dx^1 \wedge dx^2 \wedge \frac{dR}{R} \quad (4.10)$$

$$\tilde{\Lambda} = \left\{ \left(\frac{R}{R_1}\right)^{c_0\sqrt{\frac{7}{3}}} + \left(\frac{R}{R_1}\right)^{-c_0\sqrt{\frac{7}{3}}} \right\} \quad (4.11)$$

The final step is a rescaling of the world-volume coordinates and the analytic continuation,

$$\left(\frac{3}{c_0}\right)^{\frac{1}{3}} (B^{2c_0} + r_-^6)^{\frac{1}{3}} \underline{x} = \underline{Y} \quad (4.12)$$

$$(iY^0, Y^1, Y^2) \rightarrow (X^9, X^{10}, X^{11}), \quad (4.13)$$

$$d\Omega_{(7)} \rightarrow d\omega_{(7)}, \quad (4.14)$$

where $d\omega_{(7)}$ is the deSitter volume element, the analytic continuation of the round sphere metric.

What we have ended up with is an F6-brane with deSitter world volume and a transverse space which has a flat cone base, as such we may compare it to the metric and field strength found in [7], section 6 with $m=3$, $l=6$, $\Lambda_{(L)}=1$, which should describe such a fluxbrane,

$$ds^2 = \exp(2a(\xi)) d\omega_{(7)}^2 + \exp(2(A(\xi) + a(\xi)) + \exp(2c(\xi)) dX_{(3)}^2 \quad (4.15)$$

$$F = \kappa \exp(6c(\xi)) dx^9 \wedge dx^{10} \wedge dx^{11} \wedge d\xi \quad (4.16)$$

$$A(\xi) = -\ln \left\{ -\frac{6}{c_0} \sinh[c_0(\xi - \xi_0)] \right\} \quad (4.17)$$

$$c(\xi) = -\frac{1}{3} \ln \left\{ -\frac{\kappa}{c_0 \sqrt{\frac{7}{3}}} \cosh \left[c_0 \sqrt{\frac{7}{3}} (\xi - \xi_1) \right] \right\} \quad (4.18)$$

$$a(\xi) = \frac{1}{6}(A(\xi) - 3c(\xi)), \quad b(\xi) = A(\xi) + c(\xi). \quad (4.19)$$

After a coordinate change, $\xi = \ln(R)$ and redefinition of constants, $\xi_0 = \ln(R_0)$, $\xi_1 = \ln(R_1)$ we find precise agreement between the metric and form field derived from the black p-brane.

5 Conclusions.

The well known objects of supergravity theory, p-branes, have been related to their less studied cousins, Fp-branes. We found that the argument of Gibbons and Herdeiro [13], which relates the Reissner-Nordström black hole to the Melvin flux-tube, can be extended to a more general relation between electric black p-branes and magnetic Fp-branes. The end result being that the transverse space cone base (world volume) of a p-brane gets analytically continued to the world volume (transverse space cone base) of an F(D-p-3) brane. In particular, the interesting case of an Fp-brane with a round sphere for the cone base relates to the problem of a black brane with deSitter world volume.

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