PHASE TRANSITIONS, MASSIVE GRAVITONS AND EFFECTIVE ACTION IN BRANEWORLD THEORY

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We construct the holographic type nonlocal effective action in two-brane Randall-Sundrum model and show that it describes a phase transition between the local and nonlocal phases of the theory — a cumulative effect of the tower of massive Kaluza-Klein modes. We show that the corresponding renormalization group flow interpolating between the limits of short and long interbrane separations can be dynamically mediated by a repulsive interbrane potential that gives rise to braneworld cosmological scenarios with diverging branes.

1 Introduction

Recent developments in string theory [1] and the attempts to resolve the hierarchy problem [2] suggest that the observable world can be a brane embedded in a higher-dimensional spacetime with a certain number of noncompact dimensions. Moreover, string-inspired field theories imply the existence of several branes interacting and propagating in the multi-dimensional bulk. Their dynamics manifests itself for the observer as an effective four-dimensional theory that, in the cosmological context should explain the origin of structure in the Universe by means of an inflationary or some other scenario [3, 4], explain its particle phenomenology, and shed light on problems such as a possibly observable cosmological acceleration [5].

The efficient way of description for the braneworld scenario is the method of effective action. Here we calculate this action in the two-brane Randall-Sundrum model within the holographic setup characteristic of the AdS/CFT-correspondence interpretation of this model [6, 7, 8] – the braneworld action as a functional of two brane geometries effectively incorporating the dynamics of multi-dimensional gravitational field in the bulk. First we dwell on the definition of this action as an alternative to the conventional Kaluza-Klein description. In the approximation quadratic in the curvature we obtain it as a nonlocal functional of two induced metrics on branes and the corresponding radion fields. We show how conventional Kaluza-Klein bulk modes arise as the set of zeros of the nonlocal form factors of the action and analyze the first two levels of the Kaluza-Klein tower – massless and massive gravitons – within the low-derivative gradient expansion. Then we consider the reduced version of this action obtained by integrating out the fields on the negative-tension brane invisible from the viewpoint of the Planckian brane observer. We show that for small interbrane separation this action describes a Brans-Dicke type theory with the nonminimally coupled radion field and *local* Weyl-squared short distance corrections. For large interbrane distances it explicitly features the recovery of the Einstein theory with *nonlocal* short-distance corrections reflecting the well-known AdS/CFT correspondence principle. The renormalization group flow responsible for this transition between the local and nonlocal phases of the theory can be realized dynamically by means of the repulsive interbrane potential of the radion field. This generates the inflationary [9] and other cosmological scenarios alternative to models of colliding branes [3, 4].

2 Braneworld effective action – alternative to Kaluza-Klein reduction

Let us clarify the difference between the Kaluza-Klein and braneworld definitions of the effective actions. In Kaluza-Klein setting the construction of the effective action consists in the well-known procedure of decomposing the multi-dimensional field $\Phi(x,y)$, where x are the visible (four-dimensional) coordinates and y are the coordinates of extra dimensions, in a certain complete set of harmonics $Z_n(y)$ on the y-space, $\Phi(x,y)=\sum_n \phi_n(x)Z_n(y)$, and substituting the result into the fundamental action $S[\,\Phi(x,y)\,]$ of the field $\Phi(x,y)$. Subsequent integration over y gives the effective action for an infinite tower of fields $\{\phi_n(x)\}$ in x-spacetime,

$$S[\,\Phi(x,y)\,]=\int dx\,dy\,L(\Phi(x,y),\partial\Phi(x,y))=\int dx\,L_{\rm eff}(\{\phi_n(x)\},\{\partial\phi_n(x)\}). \tag{1}$$

The harmonics $Z_n(x)$ are usually taken as eigenmodes of the y-part of the full wave operator of the theory with eigenvalues m_n^2 – the masses of the Kaluza-Klein modes. This type of action was built for the two-brane Randall-

Sundrum scenario in [10]. This action, however, is very often not helpful in the braneworld context, because it does not convey a number of its important features like non-compactness of extra dimensions [11, 12], recovery of the four-dimensional Einstein gravity [13], and its interpretation in terms of the AdS/CFT-correspondence [6, 7].

These features can be described in a holographic formalism in which a natural variable is the value of the field at the brane in question, $\phi(x) = \Phi(x, y_{\rm brane})$. Unlike in Kaluza-Klein description, its effective action $S_{\rm eff}[\,\phi(x)\,]$ is obtained from the fundamental action $S[\,\Phi\,]$ by a more nontrivial procedure – by substituting in $S[\,\Phi\,]$ a solution of the classical equations of motion for $\Phi(x, y)$ in the bulk, $\Phi = \Phi[\,\phi(x)\,]$, parametrized by their boundary values on the branes, that is $S_{\rm eff}[\,\phi(x)\,] = S[\,\Phi[\phi(x)]\,]$. This construction obviously generalizes to the case of several branes Σ_I enumerated by the index I and the set of brane fields $\phi^I = \Phi(\Sigma_I)$.

Such a definition corresponds to the tree-level approximation for the quantum effective action

$$\exp\Big(iS_{\rm eff}[\,\phi\,]\Big) = \int D\Phi\, \exp\Big(iS[\,\Phi\,]\Big)\,\Big|_{\,\Phi(\Sigma)\,=\,\phi}\,, \qquad (2)$$

where the functional integration over the bulk fields runs subject to these brane boundary conditions. The scope of this formula is very large, because it arises in very different contexts. In particular, its Euclidean version $(iS \to -S_{\rm Euclid})$ underlies the construction of the no-boundary wavefunction in quantum cosmology [14]. Semiclassically, in the braneworld scenario, it represents a Hamilton-Jacobi functional, and its evolutionary equations of motion in the "fifth time" y can be interpreted as renormalization-group equations [15]. It also underlies the effective action formulation of the AdS/CFT-correspondence principle between supergravity theory on an $AdS_5 \times S^5$ background and the superconformal field theory (super-Yang-Mills) on its infinitely remote boundary [16, 7, 8].

3 Two-field action in two-brane Randall-Sundrum model and massive gravitons

Here we consider the two-brane Randall-Sundrum model with Z_2 orbifold identification of points on the compactification circle of the fifth coordinate [11].

The action of this model equals

$$S[G,g]=\frac{1}{16\pi G_5}\int d^5x\, G^{1/2}\left({}^5R(G)-2\Lambda_5\right) \\ +\sum_I\int_{\Sigma_I} d^4x\, g^{1/2}\left(\frac{1}{8\pi G_5}[K]-\sigma_I\right), \qquad (3)$$

where the index $I=\pm$ enumerates two branes with two brane tensions $\sigma_\pm$ and $[K]$ is the trace of the extrinsic curvature jump on branes. These branes are located at antipoidal points of the circle labelled by the values of the y, $y=y_\pm$, $y_+=0$, $|y_-|=d$. Z_2-symmetry identifies the points on the circle y and $-y$. When the brane tensions are opposite in signs and fine tuned in magnitude to the values of the negative cosmological constant Λ_5 and the 5-dimensional gravitational constant G_5 according to the relations

$$\Lambda_5=-\frac{6}{l^2},\ \sigma_+=-\sigma_-=\frac{3}{4\pi G_5 l}, \qquad (4)$$

then in the absence of matter on branes this model admits the solution with the AdS metric in the bulk (l is its curvature radius),

$$ds^2=dy^2+e^{-2|y|/l}\eta_{\mu\nu}dx^\mu dx^\nu, \qquad (5)$$

$0=y_+\leq|y|\leq y_-=d$, and with flat induced metric $\eta_{\mu\nu}$ on both branes [11]. The metric on the negative tension brane is rescaled by the value of compactification factor $\exp(-2d/l)$ providing a possible solution for the hierarchy problem [17]. With fine tuning (4) this solution exists for arbitrary brane separation d – two flat branes stay in equilibrium.

Take now the case when induced metrics on branes differ from the background values by perturbations

$$g^\pm_{\mu\nu}(x)=a^2_\pm\,\eta_{\mu\nu}+h^\pm_{\mu\nu}(x) \qquad (6)$$

(with $a_\pm=a(y_\pm)$ given in terms of the interbrane distance $a_+=1$, $a_-=e^{-2d/l}\equiv a$), which induce the perturbed solution of Einstein equations in the bulk

$$ds^2=dy^2+e^{-2|y|/l}\eta_{\mu\nu}dx^\mu dx^\nu+h_{AB}(x,y)\,dx^A dx^B, \qquad (7)$$

and consider the calculation of the braneworld action of the above type in the approximation quadratic in $h^\pm_{\mu\nu}(x)$. In the wording of Sect.2 the braneworld action of $\phi(x)=(g^\pm_{\mu\nu}(x),\psi^\pm(x)$, $S_{\rm eff}[\,\phi(x)\,]=S_4[\,g^\pm_{\mu\nu}(x),\psi^\pm(x)\,]$, is invariant under the two four-dimensional diffeomorphisms acting on the branes. In the linearized approximation they reduce to the transformations of metric perturbations, $h^{\pm}_{\mu\nu} \to h^{\pm}_{\mu\nu} + f^{\pm}_{\mu,\nu} + f^{\pm}_{\nu,\mu}$, with two *independent* local vector field parameters $f^{\pm}_{\mu} = f^{\pm}_{\mu}(x)$. Therefore the action is expressible in terms of the tensor invariants of these transformations — linearized Ricci tensors of $h_{\mu\nu} = h^{\pm}_{\mu\nu}(x)$,

$$R^{\pm}_{\mu\nu} = \frac{1}{2}\left(-\Box h_{\mu\nu} + h^{\lambda}_{\nu,\lambda\mu} + h^{\lambda}_{\mu,\lambda\nu} - h_{,\mu\nu}\right)^{\pm}, \tag{8}$$

on *flat* four-dimensional backgrounds of both branes.

Due to metric perturbations the branes no longer stay at fixed values of the fifth coordinate. Up to four-dimensional diffeomorphisms, their embedding variables consist of two four-dimensional scalars — the radions $\psi^{\pm}(x)$ — which also enter as arguments of the braneworld action. In the Randall-Sundrum gauge, $h_{A5} = 0$, $h_{\mu\nu}{}^{,\nu} = h^{\mu}_{\mu} = 0$, these embeddings are defined by the equations

$$\Sigma_{\pm}: \quad y = y_{\pm} + \frac{l}{a^2_{\pm}}\psi^{\pm}(x), \quad y_{+} = 0,\ y_{-} = d. \tag{9}$$

The answer for the braneworld action, which we advocate here, and which can be derived either by the direct substitution of the linearized solution (7) of bulk Einstein equations in (3) [18] or by functionally integrating effective 4-dimensional equations of motion [19], is given in terms of the invariant fields of the above type, $(R^{\pm}_{\mu\nu}(x), \psi^{\pm}(x))$, by the following spacetime integral of a 2×2 quadratic form,

$$\begin{aligned} S_4\left[\,g^{\pm}_{\mu\nu}, \psi^{\pm}\right] = \frac{1}{16\pi G_4}\int d^4x \left[\mathbf{R}^T_{\mu\nu}\,\frac{2\mathbf{F}(\Box)}{l^2\Box^2}\,\mathbf{R}^{\mu\nu} + \frac{1}{6}\,\mathbf{R}^T\,\frac{\mathbf{K}(\Box)-6\mathbf{F}(\Box)}{l^2\Box^2}\,\mathbf{R}\right.\\ \left. -3\Big(\Box\mathbf{\Psi}+\frac{1}{6}\mathbf{R}\Big)^T\frac{\mathbf{K}(\Box)}{l^2\Box^2}\,\Big(\Box\mathbf{\Psi}+\frac{1}{6}\mathbf{R}\Big)\right]. \end{aligned} \tag{10}$$

Here G_4 is an effective four-dimensional gravitational coupling constant, $G_4 = G_5/l$, $(\mathbf{R}^{\mu\nu}, \mathbf{\Psi})$ and $(\mathbf{R}^T_{\mu\nu}, \mathbf{\Psi}^T)$ are the two-dimensional columns

$$\mathbf{R}_{\mu\nu} = \begin{bmatrix} R^{+}_{\mu\nu}(x) \\ R^{-}_{\mu\nu}(x) \end{bmatrix}, \quad \mathbf{\Psi} = \begin{bmatrix} \psi^{+}(x) \\ \psi^{-}(x) \end{bmatrix} \tag{11}$$

and rows $\mathbf{R}^T_{\mu\nu} = \begin{bmatrix} R^{+}_{\mu\nu}(x) & R^{-}_{\mu\nu}(x) \end{bmatrix}, \quad \mathbf{\Psi}^T = \begin{bmatrix} \psi^{+}(x) & \psi^{-}(x) \end{bmatrix}$ of the Ricci curvatures and radions associated with two branes. The nonlocal form factors in

(10) express in terms of the nonlocal operator $\mathbf{F}(\Box)$ the following 2×2-matrix valued function of the $\Box$

$$\mathbf{F}(\Box) = -\frac{1}{J_2^+ Y_2^- - J_2^- Y_2^+}\left[\begin{array}{cc} \sqrt{\Box} z_+ u_+(z_-) & -2/\pi \\ -2/\pi & \sqrt{\Box} z_- u_-(z_+) \end{array}\right], \tag{12}$$

$$u_\pm(z) = Y_1^\pm J_2(z\sqrt{\Box}) - J_1^\pm Y_2(z\sqrt{\Box}), \tag{13}$$

$$J_\nu^\pm \equiv J_\nu(z_\pm\sqrt{\Box}), \;\; Y_\nu^\pm \equiv Y_\nu(z_\pm\sqrt{\Box}), \tag{14}$$

expressible in terms of the Bessel (J_ν) and Neumann (Y_ν) cylindrical functions. The second form factor $\mathbf{K}(\Box)$ equals $\mathbf{K}(\Box) = \mathbf{F}(\Box) + l^2\Box\,\mathrm{diag}[-1,\,1/a^2]$.

The zero-eigenvalue eigenvectors of these nonlocal operators correspond to propagating modes of the theory. In the sector of transverse-traceless metric perturbations (graviton sector), for example, these modes $\mathbf{v}_n(x)$ are defined by the equation $\mathbf{F}(m_n^2)\mathbf{v}_n = 0$, where m_n are the masses of these excitations, $(\Box - m_n^2)\mathbf{v}_n(x) = 0$, given by the roots $\Box = m_n^2$ of the equation $\det\mathbf{F}(\Box) = 0$. For the operator (12) these equation reduces to

$$\det \mathbf{F}(\Box) \sim \Box\left(Y_1^- J_1^+ - Y_1^+ J_1^-\right) = 0 \tag{15}$$

and thus gives a well-known mass spectrum of Kaluza-Klein modes in the Randall-Sundrum model, because these combination of Bessel functions is exactly the left-hand side of the eigenvalue problem for the harmonics $Z_n(y)$ satisfying the relevant Neumann-type boundary conditions on the two branes.

Eq. (15) shows that the massless graviton is guaranteed to exist in the spectrum, its localization on the positive-tension (Planckian) brane reflecting the recovery of Einstein theory on this brane for low energies satisfying the bounds

$$l\sqrt{\Box} \ll 1, \;\; \frac{l\sqrt{\Box}}{a} \ll 1. \tag{16}$$

In this domain the nonlocal form factor can be expanded in powers of derivatives to have the structure

$$\frac{\mathbf{F}(\Box)}{l^2} = -\mathbf{M} + \mathbf{D}\Box + O(\Box^2), \tag{17}$$

$$\mathbf{M} = \frac{1}{l^2}\,\frac{4}{1-a^4}\left[\begin{array}{cc} a^4 & -a^2 \\ -a^2 & 1 \end{array}\right], \; \mathbf{D} = \frac{1-a^2}{6(1+a^2)^2}\left[\begin{array}{cc} a^2+3 & 2 \\ 2 & 3+a^{-2} \end{array}\right], \tag{18}$$

which contains both the mass matrix $\mathbf{M}$ and the matrix of the kinetic term $\mathbf{D}$. The mass matrix is degenerate in accordance with the presence of the massless mode, $\det\mathbf{M} = 0$, $\mathrm{rank}\,\mathbf{M} = 1$, while the kinetic matrix is positive definite. This allows one to simultaneously diagonalize the both matrices in the basis of new fields $h^0_{\mu\nu}$ and $h^M_{\mu\nu}$, so that the low-energy action in the graviton sector describes two fields — the massless graviton and the massive transverse-traceless tensor of the mass M,

$$S_{\rm graviton}[\,h^0_{\mu\nu},h^M_{\mu\nu}] = \frac{1}{32\pi G_4}\int d^4x\Big(\,h_0\Box\,h_0 + h_M(\Box - M^2)\,h_M\Big), \quad (19)$$

$$M^2 = \frac{24}{l^2}\,\frac{a^2(1+a^2)}{(1-a^2)^2}. \quad (20)$$

In view of standard arguments of gauge invariance under the two diffeomorphism transformations above, the massless graviton has two dynamical degrees of freedom, while the massive tensor field has all five polarizations of a generic transverse-traceless tensor field.

Strictly speaking, even the lowest-lying massive mode cannot be consistently described within the low-derivative expansion (because the mass (20) as a candidate for m_1^2 violates the second of the low-energy restrictions (16)). However, numerical calculation of the first nontrivial root, $m_1 = \sqrt{\Box}$, of (15) shows that for a wide range of values of a it coincides with good precision with (20). Therefore, the massive graviton mode with the mass (20) seems to provide a good description of the first non-zero level. Its application in the form of the phenomenon of radion induced graviton oscillations, potentially interesting in the gravitational wave astronomy, is considered in [20].

4 Local to nonlocal phase transitions and AdS/CFT correspondence

The difficulties with a consistent description of massive modes originate from the fact that the low-energy approximation on the Planckian brane turns out to belong the physical high-energy limit on the negative-tension brane (remember that the physical energy there is determined by the scale of $\eta^{\mu\nu}\partial_\mu\partial_\nu/a^2$). This situation becomes even worse for large brane separation $a = \exp(-d/l) \to 0$. To avoid this difficulty let us consider the reduced version of the braneworld action corresponding to tracing (integrating) out the fields on the negative-tension brane. This procedure can be motivated by a simple physical fact that this brane is invisible from the viewpoint of the observer sitting on the Planckian brane. In the tree-level approximation this reduction, $S_{\rm eff}[\,\phi^+,\phi^-] \Rightarrow S_{\rm red}[\,\phi^+]$, is equivalent to the exclusion of the fields on the negative-tension brane in terms of those on the positive-tension one, $S_{\rm red}[\,\phi^+] = S_{\rm eff}[\,\phi^+,\phi^-[\,\phi^+]]$, as solutions of their respective equations of motion, $\delta S_{\rm eff}[\,\phi^{\pm}]/\delta\phi^{-}=0$, $\phi^{-}=\phi^{-}[\,\phi^{+}]$.

Below we present the result of this reduction in two energy domains — one corresponding to (16) and another for the case of large brane separation when the second of inequalities (16) is violated. For small or finite interbrane distance in the range of (16) the low-energy reduced action has the following form

$$S_{\rm red}[\,g_{\mu\nu},\varphi\,]=\int d^4x\sqrt{g}\,\Big[\Big(\frac{m_P^2}{16\pi}-\frac{1}{12}\varphi^2\Big)\,R+\frac{1}{2}\varphi\,\Box\varphi + \frac{l^2m_P^2}{32\pi}\,\kappa(\varphi^2)\,C^2_{\mu\nu\alpha\beta}\Big]\,, \qquad (21)$$

$$\kappa(\varphi^2)=\frac{1}{4}\,\Big[\,\ln\frac{1}{a^2}-(1-a^2)-\frac{1}{2}(1-a^2)^2\,\Big]_{\,a^2=4\pi\varphi^2/3m_P^2}\,. \qquad (22)$$

in terms of the new scalar field $\varphi(x)$ — the reparametrization of the original radion field,

$$\varphi(x)=\sqrt{\frac{3}{4\pi}}m_P\,e^{-d/l-\psi(x)}, \qquad (23)$$

and the four-dimensional Planck mass, $m_P=\sqrt{l/G_5}$. This action represents the Eistein gravity nonminimally coupled to the Brans-Dicke type scalar φ describing the local, depending on x, distance between the branes and having the short distance corrections in terms of the squared Weyl tensor $C^2_{\mu\nu\alpha\beta}$ with local (but φ-dependent) coeffficient (22).

For large interbrane distance corresponding to high-energy domain on the invisible brane,

$$l\sqrt{\Box}\ll 1,\ \frac{\sqrt{\Box}}{a}\gg 1, \qquad (24)$$

this action has a different behavior

$$S_{\rm red}[\,g_{\mu\nu}\,]=\frac{m_P^2}{16\pi}\int d^4x\,\sqrt{g}\,\Big[\,R+\frac{l^2}{2}\,C_{\mu\nu\alpha\beta}\,k(\Box)C^{\mu\nu\alpha\beta}\,\Big]\,, \qquad (25)$$

$$k(\Box)=\frac{1}{4}\Big(\,\ln\frac{4}{l^2(-\Box)}-\mathbf{C}\,\Big)\,. \qquad (26)$$

Radion field essentially decouples from gravity and Weyl-squared term becomes nonlocal with the logarithmic form factor characteristic of the AdS/CFT-correspondence phenomenon — immitation of quantum logarithms of conformal field theory on the brane by the tree-level (super)gravitational action calculated in the bulk [6, 7, 8].

Transition from the local phase (21) to the nonlocal phase (25) of the theory represents the renormalization group flow (AdS flow) interpolating between the limits of small and large interbrane distances. The scalar field φ starting at $\varphi = \sqrt{3/4\pi} m_P$, ($a = 1$ – the point of coinciding branes) tends to zero, ($a \to 0$ – the limit of large interbrane separation). The condensate of this field in the form of the Coleman-Weinberg type of potential logarithmic in φ^2/m_P^2, $\kappa(\varphi^2)$, (22), then delocalizes into the logarithmic form factor $k(\Box)$, (26), of the Weyl squared term. The dominant logarithmic term of $\kappa(\varphi^2) \sim (1/4)\ln(m_P^2/\varphi^2)$ at $\varphi \to 0$ instead of an infinite growth saturates at the logarithmic scale of the graviton radiation characterized by spacetime inhomogeneity of the Weyl tensor – the logarithmic nonlocality of $k(\Box) \sim (1/4)\ln(4/l^2\Box)$. The physics of this transition is transparent: the tower of massive Kaluza-Klein modes which are infinitely heavy at the initial point of merging branes (see (20) at $a \to 1$), for $a \to 0$ becomes very light — its spectrum getting practically continuous and resulting in a cumulative effect in the form of the logarithmic nonlocality characteristic of the AdS/CFT-correspondence.

5 Conclusions. Radion induced inflation and dynamical realization of the AdS flow

Thus, we have constructed the holographic type braneworld action in the two-brane Randall-Sundrum model, demonstrated the existence of massive graviton modes in its spectrum and showed that it features the renormalization group flow from small to large interbrane distances associated with different, respectively, local and nonlocal phases of the theory.

From physics viewpoint it is certainly interesting to see whether this flow can be realized at the dynamical level, that is enforced as a solution of effective equations of motion. In [9] it was suggested that a small detuning of brane tensions from their Randall-Sundrum values (4) leads to the origin of the four-dimensional cosmological term in the action (21). Then after the reparametrization of this action to the Einstein frame of new fields $(\bar g_{\mu\nu}, \phi)$ (with ϕ minimally coupled to $\bar g_{\mu\nu}$ and related to the old field φ by $\varphi = \sqrt{3/4\pi}\, m_P \tanh(\sqrt{4\pi/3}\,\phi/m_P)$) one finds that the new field ϕ acquires the potential, $V(\phi) \sim \cosh^4(\sqrt{4\pi/3}\,\phi/m_P)$, capable to maintane a slow-roll inflation. It corresponds to two branes diverging under the repulsive force of this interbrane potential (remember, that the limit $\phi \to 0$ corresponds to $a \to 0$) and, thus, dynamically realizes the renormalization flow of the above type. This model qualitatively differs from the inflation and other scenarios of colliding branes [3, 4]. Unfortunately, this model suffers from an essential drawback. This is the necessity to introduce by hand a four-dimensional cosmological constant — the brane tension detuning of the above type.

Interestingly, the present results suggest a natural mechanism of interbrane repulsion based on the Weyl-squared term of (21) and (25). When the brane Universe is filled with the graviton radiation $C^2_{\mu\nu\alpha\beta} > 0$, and for small brane separation this term forms the interbrane potential $-(l^2/2)\kappa(\varphi^2)C^2_{\mu\nu\alpha\beta}$. It has a maximum at the point of coinciding branes $a = 1$ because the coefficient $\kappa(\varphi^2)$, Eq. (22), is strictly positive. The repelling force is very small, though, and vanishes at $a = 1$, because of the behaviour of $\kappa(\varphi^2) \sim (1-a^2)^3/12$, $a^2 = 4\pi\varphi^2/3m_P^2$ (note that the expression (22) represents the logarithm $\ln(1/a^2)$ with exactly the first two terms of its Taylor series in $(1-a^2)$ subtracted). Unfortunately, this potential is strictly negative, because $\kappa(\varphi^2) \geq 0$, and, therefore, cannot maintain inflation. Rather, it can serve as a basis of brane models with AdS_4 geometry embedded in AdS_5 [21]. It can also be useful in the model of "thick" branes in the Big Crunch/Big Bang transitions of ekpyrotic and cyclic cosmologies [4].

Thus, the two-brane model is likely to have a natural mechanism to realize the AdS flow dynamically. Concrete implications of this flow and the corresponding transitions between the local and nonlocal phases of the theory still have to be worked out. In particular, they might be important in view of the growing interest in nonlocal modifications of gravity within the cosmological constant problem [22]. They are currently under study.

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[1] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995); P. Hořava and E. Witten, Nucl. Phys. B **460**, 506 (1996).

[2] I. Antoniadis, Phys. Lett. B **246**, 377 (1990); N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429**, 263 (1998).

[3] G. Dvali and S.-H. H. Tye, Phys. Lett. **B450** (1999) 72.

[4] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D **64**, 123522 (2001); R. Kallosh, L. Kofman and A. D. Linde, Phys. Rev. D **64**, 123523 (2001); P. J. Steinhardt and N. Turok, Phys. Rev. D **65**, 126003 (2002).

[5] N. A. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, Science **284**, 1481 (1999).

[6] S. S. Gubser, Phys. Rev. D **63**, 084017 (2001).

[7] S. B. Giddings, E. Katz and L. Randall, JHEP **0003**, 023 (2000).

[8] S. W. Hawking, T. Hertog and H. S. Reall, Phys. Rev. D **62**, 043501 (2000).

[9] A. O. Barvinsky, Phys. Rev. D **65**, 062003 (2002).

[10] E. E. Boos, Y. A. Kubyshin, M. N. Smolyakov and I. P. Volobuev, "Effective Lagrangians for linearized gravity in Randall-Sundrum model", hep-th/0105304.

[11] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).

[12] V. A. Rubakov, Phys. Usp. **44**, 871 (2001).

[13] J. Garriga and T. Tanaka, Phys. Rev. Lett. **84**, 2778 (2000).

[14] J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983); S. W. Hawking, Nucl. Phys. B **239**, 257 (1984); A. Vilenkin, Phys. Lett. B **117**, 25 (1982).

[15] E. Verlinde and H. Verlinde, JHEP **0005**, 034 (2000), hep-th/9912018.

[16] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)].

[17] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[18] A. O. Barvinsky and D. V. Nesterov, "Duality Of Boundary Value Problems and Braneworld Action In Curved Brane Models", hep-th/0210005.

[19] A. O. Barvinsky, A. Yu. Kamenshchik, C. Kiefer and A. Rathke, "Non-local Braneworld Action: an Alternative to Kaluza-Klein Description", hep-th/0206188.

[20] A. O. Barvinsky, A. Yu. Kamenshchik, C. Kiefer and A. Rathke, "Radion-Induced Graviton Oscillations in the Two-Brane World, talk given by A. Yu. Kamenshchik at the 3rd Sakharov Conference on Physics, Moscow, June 2002.

[21] A. Karch and L. Randall, JHEP **0105**, 008 (2001); Phys. Rev. Lett. **87**, 061601 (2001).

[22] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and G. Gabadadze, "Non-Local Modification of Gravity and the Cosmological Constant Problem", hep-th/0209227.