Advances in String Theory in Curved Backgrounds : A Synthesis Report

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Abstract :

A synthetic report of the advances in the study of classical and quantum string dynamics in curved backgrounds is provided, namely : the new feature of Multistring solutions ; the mass spectrum of Strings in Curved backgrounds; The effect of a Cosmological Constant and of Spacial Curvature on Classical and Quantum Strings; Classical splitting of Fundamental Strings; The General String Evolution in constant Curvature Spacetimes; The Conformal Invariance Effects; Strings on plane fronted and gravitational shock waves, string falling on spacetime singularities and its spectrum.
New Developments in String Gravity and String Cosmology are reported : String driven cosmology and its Predictions; The primordial gravitational wave background ; Non-singular string cosmologies from Exact Conformal Field Theories; Quantum Field Theory, String Temperature and the String Phase of de Sitter space-time; Hawking Radiation in String Theory and the String Phase of Black Holes; New Dual Relation between Quantum Field Theory regime and String regime and the "QFT/String Tango"; New Coherent String States and Minimal Uncertainty Principle in string theory.

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In 1987, we started a programme [1] to study the string dynamics in curved spacetime and its associated physical phenomena. This study revealed new insights and new physical phenomena with respect to string propagation in flat spacetime (and with respect to quantum fields in curved spacetime). The results are relevant both for fundamental (quantum) strings and for cosmic strings, which behave essentially in a classical way. Approximative and exact solving methods have been developed. Classical and quantum string dynamics have been investigated in black hole spacetimes, cosmological backgrounds, cosmic string spacetime, gravitational wave backgrounds, supergravity backgrounds (which are necessary for fermionic strings), and near spacetime singularities. Physical phenomena like classical string instability and non oscillatory motion in time, quantum particle transmutation, string scattering, string stretching, have been found. For the results (1987-1994) see for example our Chalonge Erice Lectures 1989-1994 and references therein. See (http://www.obspm.fr/chalonge), and "String Theory in Curved Space Times", Ed. N. Sanchez, WSPC, (1998).

1 Multistring Solutions: A new feature for strings in curved backgrounds

The discovery of the multistring property [2] [3] in the propagation of strings in curved spacetimes is the consequence of several developments :

(i) The classical string equations of motion plus the string constraints were shown to be exactly integrable in D-dimensional De Sitter spacetime and equivalent to a sinh-Gordon model with a Hamiltonian unbounded from below [4]. Generalization of this result including the Cosh-Gordon and Liouville equations, for strings and multistrings in constant curvature spacetimes have been given in ref [13].

(ii) *Exact* string solutions were systematically found by soliton methods using the linear system associated to the problem (the so-called dressing method in soliton theory) [2]. In particular, exact circular string solutions were found in terms of elementary [3] and elliptic functions [5].

(iii) All these solutions describe one string, several strings or even an infinite number of different and independent strings. A single world-sheet simultaneously describes *many* different strings. This new feature appears as a consequence of the coupling of the strings with the spacetime geometry. Here, interaction among the strings (like splitting and merging) is neglected, the only interaction is with the curved background.
Different types of behaviour appear in the multistring solutions. For some of them the energy and proper size are bounded ("*stable strings*") while for many others the energy and proper size blow up for large radius of the universe ("*unstable strings*").

In all these works, strings are *test* objects propagating on the given fixed backgrounds. The string energy momentum tensor was computed and the string equation of state *derived* from the string dynamics in cosmological and black hole spacetimes. Strings obey the perfect fluid relation

$$p = (\gamma - 1)\rho$$

with three different behaviours :
(i) *Unstable* for large R, with negative pressure;
(ii) *Dual* for small R, with positive pressure, (as radiation);
(iii) *Stable* for large R, with vanishing pressure, (as cold matter).

We find the *back reaction effect* of these strings on the spacetime [6]. This is achieved by considering *selfconsistently* the strings as matter sources for the Einstein equations, as well as for the complete effective string equations, for cosmological spacetimes at the classical level. The selfconsistent solution of the Einstein-Friedman equations for string dominated universes exhibits the realistic matter dominated behaviour for large times and the radiation dominated behaviour for early times. That is, the *standard cosmological* evolution is well generated by strings. It must be noticed that there is no satisfactory derivation of inflation in the context of the effective string equations. De Sitter universe *does not* emerge as solution of the effective string equations. The effective string action (whatever be the dilaton, its potential and the central charge term) is not the appropriate framework in which to address the question of string driven inflation.

More recently, [7], new classes of exact **multistring** solutions were found. The multistring solutions were classified and their physical properties described.
In Anti de Sitter spacetime, the solutions describe an **infinity** number of infinitely long stationary strings of equal energy but different pressures. In De Sitter spacetime, outside the horizon, they describe infinitely many **dynamical** strings, infalling non radially, scattering at the horizon and going back to spatial infinity in different directions. For special values of the constant of motion, there are families of solutions with **selected finite** numbers of different and independent strings. The strings appear **distributed** in **packets**, the number of strings in each packet, the number of "turns" or "festoons" in each string is precisely determined and solely dictated by the dynamics, exactly solved in terms of elliptic functions.

In Black hole spacetimes, (without cosmological constant) no multistring solutions are found [8]. In the Schwarzschild black hole, inside the horizon, the string infalls, with **indefinitely** growing size and energy, into the r=0 singularity and the string motion stops there.

In the (2+1)- black hole anti-de Sitter background, the string stops at r=0 with **finite** length [9]; the reason being that the point r=0 is not a strong curvature singularity in the (2+1)-black hole anti- de Sitter spacetime. Outside the horizon, in this spacetime, the multistring solution describes infinitely many, infinitely long open strings.

2 The String Mass Spectrum in the presence of Cosmological Constant

The string mass spectrum in the presence of a cosmological constant (for both de Sitter and Anti de Sitter spacetimes) was found in ref.[9], [11]. New features as compared to the string spectrum in flat spacetime appear, as a **fine structure effect** (splitting of levels) at all the states beyond the graviton, (in both de Sitter (dS) and AntideSitter (AdS) spacetimes), and the **absence** of a critical Hagedorn temperature in AdS spacetime (the partition function for a gas of strings in AdS spacetime is well defined for all temperature).

The presence of a cosmological constant reduces (although do not totally removes) the degeneracy of states as compared with flat Minkowski spacetime. In AdS spacetime, the density of states ρ(m) grows like $\exp[(\Lambda\ m)^{\frac{1}{2}}]$, (while, as is known, in Minkowski spacetime, ρ(m) grows like the Exponential of the mass m).

The high mass spectrum changes drastically with respect to flat Minkowski spacetime. The level spacing **grows** with the eigenvalue of the number operator, N, in AdS spacetime, while is approximatively constant (although smaller than in Minkowski spacetime and slightly decreasing) in dS spacetime.

There is an infinite number of states with arbitrarily high mass in AdS space time, while in dS there is a **finite** number of oscillating states only.
The string mass has been expressed in terms of the Casimir operator C = $L_{\mu\nu}\ L^{\mu\nu}$ of the O(3,1) De Sitter group [O(2,2) group in Anti deSitter [11]. See also section 3 below.

3 Spatial Curvature Effects

The effects of the spatial curvature on the classical and quantum string dynamics is studied in ref.[10]. The general solution of the circular string motion in static Robertson-Walker spacetimes with closed or open sections has been found [10]. This is given closely and completely in terms of elliptic functions.
The **back reaction effect** of these strings on the spacetime is found : the self-consistent solution to the Einstein equations is a spatially closed ($\mathbf{K}>0$) spacetime with a selected value of the curvature index **K**, $\mathbf{K} = (\mathbf{G}/\alpha')^{\frac{2}{3}}$ (the scale factor is normalized to unity). No self-consistent solutions with $\mathbf{K}<0$ exist.

We semiclassically quantize the circular strings and find the mass m in each case. For $\mathbf{K}>0$, the very massive strings, oscillating on the full hypersphere, have

$$\mathbf{m}^2 \sim \mathbf{K}\mathbf{N}^\mathbf{2}, (\mathbf{N}\epsilon\mathbf{N}_0)$$

independent of α' and the level spacing **grows** with n, while the strings oscillating on one hemisphere (without crossing the equator) have

$$\mathbf{m}^2 \sim \alpha'\mathbf{N}$$

and a **finite** number of states N $\sim$ 1/(Kα').

For K $<$0, there are infinitely many strings states with masses

$$\mathbf{m}\log\mathbf{m} \sim \mathbf{N},$$

that is the level spacing grows **slower** than **N**.
The stationary string solutions as well as the generic string fluctuations around the center of mass are also found and analyzed in closed form.

4 Classical String Splitting

We find exact solutions of the string equations of motion and constraints describing the classical splitting of a string into two [12]. For the same Cauchy data, the strings which split have smaller action that the string without splitting. This phenomenon is already present in flat space- time. The splitting process takes place in real (lorentzian signature spacetime).

The solutions in which the string splits are perfectly natural within the classical theory of strings. There is no need of extra interactions, (nor extra terms in the action to produce splitting). The difference with the non splitting solutions is on the boundary conditions.

The mass, energy and momentum carried out by the strings are computed. We show that the splitting solution describes a natural decay process of one string of mass **M** into two strings with a smaller total mass and some kinetic energy. The standard non-splitting solution is contained as a particular case.

We also described the splitting of a closed string in the background of a singular gravitational plane wave, and showed how the presence of the strong gravitational field increases (and amplifies by an overall factor) the negative difference between the action of the splitting and non-splitting solutions.

5 General String Evolution in Constant Curvature Space-Times

In ref. [13], we have found that the fundamental quadratic form of the classical string propagation in (2+1)-dimensional constant curvature spacetimes, solves the sinh-Gordon equation, the cosh-Gordon equation, or the Liouville equation. In both de Sitter and anti-de Sitter spacetimes, (as well as in the 2+1 black hole anti-de Sitter spacetime), *all* three equations must be included to cover the generic string dynamics. This is particularly enlightening since *generic* properties of the string evolution can be thus *directly* extracted from the properties of these three equations and their associated Hamiltonians or potentials, *irrespective of any solution.*
These results complete and generalize our previous results on this topic since (until now, only the sinh-Gordon sector in de Sitter spacetime was known).

We also construct new classes of multistring solutions, in terms of elliptic functions, to all three equations in both de Sitter and anti de Sitter spacetimes, which generalize our previous ones.
These results can be straighforwardly generalized to constant curvature spacetimes of arbitrary dimension, by replacing the sinh-Gordon equation, the cosh-Gordon equation, and the Liouville equation by their higher dimensional generalizations.

Our results indicate the existence of various kinds of dualities relating the different sectors and their solutions in de Sitter and anti-de Sitter spacetimes : in the sinh-Gordon sector of de Sitter spacetime, small strings are dual (that is, under S $\Rightarrow$ 1/S, S being the proper string size, they are mapped to large strings). And, similarly, in the sinh-Gordon sector of anti-de Sitter spacetime. Furthermore, in the cosh-Gordon sector, small (large) strings in de Sitter spacetime are dual to large (small) strings in the anti-de Sitter spacetime.

6 Conformal Invariance Effects

Classical and quantum strings in the conformally invariant background corresponding to the $SL(2R)$ WZWN model has been studied in ref [14]. This background is locally anti-de Sitter spacetime with non-vanishing torsion. Conformal invariance is expressed as the torsion being parallelizing; and the precise effect of the conformal invariance on the dynamics of both circular and generic classical strings has been extracted [14].
In particular, the conformal invariance gives rise to a repulsive interaction of the string with the background which precisely cancels the dominant attractive term arising from gravity.

We perform both semi-classical and canonical string quantization, in order to see the effect of the conformal invariance of the background on the string mass spectrum. Both approaches yield that the high-mass states are governed by

$$m \sim HN(N \in N_0, N\text{“large”}),$$

where m is the string mass and H is the Hubble constant.
It follows that the level spacing grows proportionally to N :

$$\frac{d(m^2\alpha')}{dN} \sim N,$$

while the string entropy goes like

$$S \sim \sqrt{m}.$$

Moreover, it follows that there is no Hagedorn temperature, so that the partition function is well defined for any positive temperature.

All results are compared with the analogue results in anti-de Sitter spacetime, which is a nonconformal invariant background.

It appears that conformal invariance *simplifies* the mathematics of the problem but the physics remains mainly *unchanged*. Differences between conformal and non-conformal backgrounds only appear in the intermediate region of the string mass spectrum, but these differences are minor. For low and high masses, the string mass spectra in conformal and non-conformal backgrounds are identical.
Interestingly enough, conformal invariance fixes the value of the spacetime curvature to be $-69/(26\alpha')$.

It has been known for some time that the SL(2,R) WZWN model reduces to Liouville theory. In ref [15] we give a direct and physical derivation of this result based on the classical string equations of motion and the proper string size. This allows us to extract precisely the physical effects of the metric and antisymmetric tensor, respectively, on the *exact* string dynamics in the SL(2,R) background. Also the general solution to the proper string size has been found [15].

We show that the antisymmetric tensor (corresponding to conformal invariance) generally gives rise to repulsion, and it precisely cancels the dominant attractive term arising from the metric. Both the sinh-Gordon and the cosh-Gordon sectors of the string dynamics in non-conformally invariant AdS spacetime reduce here to the Liouville equation (with different signs of the potential), while the original Liouville sector reduces to the free wave equation.
Only the very large classical string size is affected by the torsion. Medium and small size string behaviors are unchanged.
We also find illustrative classes of string solutions in the SL(2,R) background: dynamical closed as well as stationary open spiralling strings, for which the effect of torsion is somewhat like the effect of rotation in the metric. Similarly, the string solutions in the 2+1 BH-AdS background with torsion and angular momentum are fully analyzed [15].

7 Strings on plane waves and shock waves. The falling of strings on space-time singularities and its spectrum

In ref. [24], we studied the dynamics of strings near spacetime singularities. We considered plane fronted gravitational-wave backgrounds with a singularity of the type $|U|^{-\beta}$, U being a null coordinate. The case with a $\delta(U)$

shock-wave singularity turns out to be similar to the $\beta = 1$ case.
New features in the string behavior appear : when $\beta \geq 2$, the string does not propagate through the gravitational wave and it escapes to infinity grazing the singularity plane $U = 0$; one tranverse coordinate does not oscillate in time (neither classically nor quantum mechanically) and the tunnel effect does not take place.

The expectation value of the mass squared $\langle M^2_> \rangle$ and mode number $\langle N_> \rangle$ operators and of the energy-momentum tensor are computed. When the transverse size (ρ_0) of the gravitational- wave front is infinite, divergences in $\langle M^2_> \rangle$ and $\langle N_> \rangle$ appear for $1 \leq \beta < 2$ and $3/2 \leq \beta < 2$, respectively.
The short-distance spacetime singularity at $U = 0$ is not responsible for these divergences, but the infinite amount of energy carried by the gravitational wave when $\rho_0 = \infty$.

In summary, the propagation of strings through these singular space-times is proven to be physically meaningful for $\beta \geq 2$ and $\beta < 1$. And this is also the case for $1 \leq \beta < 2$.

In conclusion, test strings do propagate consistently in singular plane wave space-times and in shock-wave space-times. We recall that the Klein-Gordon equation (for a point particle) is ill defined in this geometry, whereas the string equations are well behaved [25], [26]. Analogous conclusions hold for quantum strings in the Schwarzschild geometry where a regular behavior was found at the horizon and at the $r = 0$ singularity [27]. That is, strings feel the space-time singularities much less than point particles.
Furthermore, we would not be surprised by the presence of space-time singularities in string theory as long as one sticks to a geometry description using a metric tensor $G_{AB}(X)$ (in spite of the fact that it fulfills the string-corrected Einstein equations). We do not expect that a space-time description in terms of a Riemannian manifold with local coordinates X^A will be meaningful at the Planck scale.

In ref. [28] we fully investigated at the quantum level the nonlinear transformation relating the string operators (zero modes and oscillators) and Fock space states before and after the collision with gravitational shock waves. This throws light on the rôle of the space-time geometry in this problem. The treatement was done for a general shock wave space-time of any localized source. We computed the *exact* expectation values of the total number (N) and mass (M^2) operators and show that they are <u>finite</u>.

We study the energy-momentum tensor of the string and compute the exact expectation values of all its components. We analyze vacuum polarization and quadratic fluctuations. All these physical magnitudes are *finite*.

We express all these quantities in terms of exact integral representations in which the role of the real pole singularities characteristic of the tree level string spectrum (real mass resonances) are clearly exhibited. The presence of such poles is not at all related to the structure of the space-time geometry (which may or may not be singular). Claims on the divergences of the mass $\langle M^2 \rangle$ and total number $\langle N \rangle$ on these backgrounds(G.T.Horowitz and A.R. Steiff, Phys. Rev. Lett. 64, 260 (1990), Phys. Rev. D42, 1950 (1990)) totally overlocked this problem.

8 Minimal String Driven Cosmology and its Predictions

In refs. [16], [17] we constructed a minimal model for the Universe evolution fully extracted from effective String Theory.

By linking this model to a minimal but well established observational information, we proved that it gives realistic predictions on early and current energy density and its results are compatible with General Relativity.

Interestingly enough, this model predicts the current energy density Omega=1 and a lower limit Omega larger or equal 4/9. On the other hand, the energy density at the exit of inflationary stage is also predicted $\mathrm{Omega}_{\mathrm{infl}}$ = 1.
This result shows agreement with General Relativity (spatially flat metric gives critical energy density) within an unequivalent Non-Eistenian context (string low energy effective equations).

The order of magnitude of the energy density-dilaton coupled term at the beginning of radiation dominated stage agrees with GUT scale.

Whithout solving the known problems about higher order corrections and graceful exit of inflation, we find this model closer to the observational Universe properties than the current available string cosmology scenarii.

At a more fundamental level, this model is by its construction close to the standard cosmological evolution, and it is driven selfconsistently by the evolution of the string equation of state itself.

The inflationary String Driven stage is able to reach an enough amount of inflation, describing a Big Bang like evolution for the metric.

9 The Primordial Gravitational Wave Background in String Cosmology

In ref.[16] we found the spectrum P(w)dw of the gravitational wave background produced in the early universe in string theory.

We work in the framework of String Driven Cosmology, whose scale factors are computed with the low-energy effective string equations as well as selfconsistent solutions of General Relativity with a gas of strings as source.

The scale factor evolution is described by an early string driven inflationary stage with an instantaneous transition to a radiation dominated stage and successive matter dominated stage. This is an expanding string cosmology always running on positive proper cosmic time.

A careful treatment of the scale factor evolution and involved transitions is made. A full prediction of the power spectrum of gravitational waves without any free-parameters is given.

We study and show explicitly the effect of the dilaton field, characteristic to this kind of cosmologies.

We compute the spectrum for the same evolution description with three differents approachs.

Some features of gravitational wave spectra, as peaks and asymptotic behaviours, are found direct consequences of the dilaton involved and not only of the scale factor evolution.

10 Non-Singular String-Cosmologies From Exact Conformal Field Theories

In ref. [18] we constructed non-singular two and three dimensional string cosmologies using the exact conformal field theories corresponding to SO(2,1)/SO(1,1) and SO(2,2)/SO(2,1) coset models.

All semi-classical curvature singularities are canceled in the exact theories for both of these cosets, but some new curvature singularities emerge in the quantum models.
However, considering different patches of the global manifolds, allows the construction of non-singular spacetimes with cosmological interpretation.

In both, two and three dimensions, we constructed non-singular oscillating cosmologies, non-singular expanding and inflationary cosmologies including a de Sitter (exponential) stage with positive scalar curvature. Non-singular contracting and deflationary cosmologies were also constructed.

We analyse these cosmologies in detail with respect to the behaviour of the scale factors, the scalar curvature and the string-coupling.

The sign of the scalar curvature turns out to be changed by the quantum corrections in oscillating cosmologies and evolves with time in the non-oscillating cases.

Similarities between the two and three dimensional cases suggest a general picture for higher dimensional coset cosmologies :
(i) Anisotropy seems to be a generic unavoidable feature,
(ii) cosmological singularities are generically avoided and
(iii) it is possible to construct non-singular cosmologies where some spatial dimensions are experiencing inflation while the others experience deflation.
De Sitter stage can be achieved asymptotically at early times or late times, but there is not a conformal coset model of this type describing a de Sitter background globally.

11 Quantum Field Theory, String Temperature and the String Phase of De Sitter Spacetime

The density of mass levels ρ(m) and the critical temperature for strings in de Sitter space-time were found in ref. [20].

Quantum Field Theory (QFT) and string theory in de Sitter space have been compared in refs. [20] and [22].

A 'Dual'-transform is introduced which relates classical to quantum string lengths, and more generally, QFT and string domains.

Interestingly, the string temperature in De Sitter space turns out to be the Dual transform of the QFT-Hawking-Gibbons temperature.

The quantum back reaction problem for strings in de Sitter space is addressed selfconsistently in the framework of the 'string analogue' model (or thermodynamical approach), which is well suited to combine QFT and string studies [20], [21], [22].

We find de Sitter space-time is a self-consistent solution of the semi-classical Einstein equations in this framework. Two branches for the scalar curvature $R_{(\pm)}$ show up : a classical, low curvature solution (-), and a quantum high curvature solution (+), entirely sustained by the strings.

There is a maximal value for the curvature R_{max} due to the string back reaction.

Interestingly, our Dual relation manifests itself in the back reaction solutions : the (-) branch is a classical phase for the geometry with intrinsic temperature given by the QFT-Hawking-Gibbons temperature.

The (+) is a stringy phase for the geometry with temperature given by the intrinsic string de Sitter temperature.

2 + 1 dimensions are considered, but conclusions hold generically in D dimensions.

12 Hawking Radiation in String Theory and the String Phase of Black Holes

The quantum string emission by Black Holes is computed in the framework of the 'string analogue model' (or thermodynamical approach), which is well suited to combine QFT and string theory in curved backgrounds (particulary here, as black holes and strings posses intrinsic thermal features and temperatures).

The QFT-Hawking temperature T_H is upper bounded by the string temperature T_S in the black hole background.

The black hole emission spectrum is an incomplete gamma function of (T_H - T_S).

For $T_H \ll T_S$, the spectrum yields the QFT-Hawking emission.

For T_H near to T_S, it shows highly massive string states dominate the emission and undergo a typical string phase transition to a microscopic 'minimal' black hole of mass M_{min} or radius r_{min} (inversely proportional to T_S) and string temperature T_S.

The semiclassical QFT black hole (of mass M and temperature T_H) and the string black hole (of mass M_{min} and temperature T_S) are mapped one into another by a 'Dual' transform which links semi classical-QFT and quantum string regimes.

The string back reaction effect (selfconsistent black hole solution of the semiclassical Einstein equations with mass M_+ (radius r_+) and temperature T_+) is computed.

Both, the QFT and string black hole regimes are well defined and bounded: $r_{min} \leq r_+ \leq r_S$, $M_{min} \leq M_+ \leq M_S$, $T_{min} \leq T_+ \leq T_S$.

The string 'minimal' black hole has a life time τ_{min} = (K/Gh) T_S $^{-3}$.

13 New Dual Relation between Quantum Field Theory Regime and String Regime in Curved Backgrounds

In ref.[22] we introduced a R "Dual" transform which relates Quantum Field Theory and Quantum String regimes, both in a curved background.

This operation maps the characteristic length of one regime into the other and, as a consequence, maps mass domains as well.

The Hawking-Gibbons temperature and the string maximal or critical temperature are dual of each other.

If back reaction of quantum matter is included, Quantum Field and Quantum String phases appear, and duality relations between them manifest as well.

This Duality is shown in two relevant examples : Black Hole and de Sitter space times, and appears to be a generic feature, analogous to the "wave-particle" duality.

14 New Coherent String States and Minimal Uncertainty Principle in String Theory.

We study the properties of **exact** (all level k) quantum coherent states in the context of string theory on a group manifold (WZWN models).

Coherent states of WZWN models may help to solve the unitary problem : Having positive norm, they consistently describe the very massive string states (otherwise excluded by the spin-level condition).

These states can be constructed by (at least) two alternative procedures : (i) as the exponential of the creation operator on the ground state, and (ii) as eigenstates of the annhilation operator. In the $k \to \infty$ limit, all the known properties of ordinary coherent states of Quantum Mechanics are recovered.

States (i) and (ii) (which are equivalent in the context of ordinary quantum mechanics and string theory in flat spacetime) are not equivalent in the context of WZWN models.

The set (i) was constructed by Larsen and Sanchez in ref. [19]. The construction of states (ii) was provided in ref. [8] by the same authors. We compare the two sets and discuss their properties.

We analyze the uncertainty relation, and show that states (ii) satisfy automatically the *minimal uncertainty* condition for any k; they are thus *quasiclassical*, in some sense more classical than states (i) which only satisfy it in the $k \to \infty$ limit.
The modification to the Heisenberg relation is given by $2\mathcal{H}/k$, where $\mathcal{H}$ is connected to the string energy.

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