

Accelerated Universe from Modified Chaplygin Gas and Tachyonic Fluid

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Abstract

A cosmological model with an exotic fluid is investigated. We show that the equation of state of this “modified Chaplygin” gas can describe the current accelerated expansion of the universe. We then reexpress it as FRW cosmological model containing a scalar field ϕ and find its self-interacting potential. Moreover motivated by recent works of Sen [5, 6] and Padmanbhan [13] on tachyon field theory, a map for this exotic fluid as a normal scalar field ϕ with Lagrangian $\mathcal{L}_\phi = \frac{\dot{\phi}^2}{2} - U(\phi)$ to the tachyonic field T with Lagrangian $\mathcal{L}_T = -V(T)\sqrt{1 - \dot{T}^2}$ is obtained.

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1 Introduction:

Recent measurements of redshift and luminosity-distance relations of type Ia supernovae indicate that the expansion of the universe is accelerating [1, 2]. This appears to be in strong disagreement with the standard picture of a matter dominated universe.

These observations can be accommodated theoretically by postulating that certain exotic matter with negative pressure dominates the present epoch of our universe. Such exotic matter has been called *Quintessence* and behaves like a vacuum field energy with repulsive (anti-gravitational) character arising from the negative pressure.

Negative pressure leading to an accelerating universe can also be obtained in a Chaplygin gas cosmology [3], in which the matter is taken to be a perfect fluid obeying an exotic equation of state. This cosmological model has some interesting properties. In particular, the Chaplygin gas behaves as pressureless fluid for small values of the scale factor and as a cosmological constant for large values of the scale factor which tends to accelerate the expansion.

Another interesting feature of the Chaplygin gas equation of state is its connection to string theory via a brane interpretation. It was shown in [4] that this kind of exotic equation of state can be obtained from the Nambu-Goto action for a D-brane moving in a (D+2)-dimensional spacetime in light-cone parametrization.

In this paper we consider a class of equations of state that interpolate between standard fluids at high energy densities and Chaplygin gas fluids at low energy densities. We call this type of equation of state the “modified” Chaplygin gas. In the next section, we study the behavior of this modified Chaplygin gas and show that at large cosmological scales it could account for the current observations of the acceleration of the universe. In section 3, we investigate the possibility of using Sen’s [5, 6, 7] idea of a rolling tachyon to describe the current acceleration of the present epoch where the tachyon fluid is considered as a candidate for the modified Chaplygin gas equation of state.

2 Modified Chaplygin gas:

Within the framework of FRW, we study a model based on (modified) Chaplygin gas where our principal assumption is that the energy density ρ and pressure p are related by the following equation of state :

$$p = A\rho - \frac{B}{\rho^n}, \quad n \geq 1 \quad (1)$$

where A and B positive constants.

We see that when $B=0$, it reduces to the standard equation of state of perfect fluid,

$$p = A\rho \quad (2)$$

whereas when $A = 0$, it corresponds to an exotic background fluid, Chaplygin gas, described by an equation of state :

$$p = -\frac{B}{\rho}, \quad \text{for } n = 1 \quad (3)$$

In (1), the two terms start to be of the same order when the pressure vanishes (i.e. $p = 0$). In this case, the fluid has pressureless density ρ_0 , corresponding to some cosmological scale a_0 ,

$$\rho_0 = \rho^{n+1}(a_0) = \frac{B}{A} \quad (4)$$

Now we consider a D -dimensional FRW spacetime with scale factor $a(t)$ and metric :

$$ds^2 = -dt^2 + a^2(t) d\Omega_k^2 \quad (5)$$

where $d\Omega_k^2$ is the metric of the maximally symmetric $(D-1)$ -space with curvature k , for $k = -1, 0, +1$.

If this space time is filled with a fluid of energy density ρ and pressure p , then conservation of energy momentum tensor $\nabla^\mu T_{\mu\nu} = 0$, gives,

$$\dot{\rho} + (D-1)H(\rho + p) = 0 \quad (6)$$

The Friedmann equation for the scale factor a is,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{2\rho}{(D-1)(D-2)} - \frac{k}{a^2} \quad (7)$$

These two equations imply that :

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{(D-1)p + (D-3)\rho}{(D-1)(D-2)} \quad (8)$$

A solution to this equation is obtained as follows. We define $W = a^{(D-1)(A+1)}$ and a rescaled density $\bar{\rho} = \rho W$, then (6) becomes,

$$\dot{\bar{\rho}} - \frac{B}{A+1} \frac{W^n \dot{W}}{\bar{\rho}^n} = 0 \quad (9)$$

The latter has a solution of the form :

$$\frac{\bar{\rho}^{n+1}}{n+1} = \frac{B}{A+1} \frac{W^{n+1}}{n+1} + \frac{C}{n+1} \quad (10)$$

where C is a constant of integration.

The density will be :

$$\rho = \left(\frac{B}{A+1} + \frac{C}{W^{n+1}} \right)^{\frac{1}{n+1}} \quad (11)$$

The constant of integration C can be expressed in terms of the cosmological scale a_0 , (i.e. $W_0 = a_0^{(D-1)(A+1)}$) where the fluid has vanishing pressure :

$$C = \frac{B}{A+1} \frac{W_0^{n+1}}{A} \quad (12)$$

The energy density ρ will be :

$$\rho = \left(\frac{B}{A+1} \right)^{\frac{1}{n+1}} \left(1 + \frac{1}{A} \left(\frac{W_0}{W} \right)^{n+1} \right)^{\frac{1}{n+1}} \quad (13)$$

We see that for a large values of the scale factor a , i.e. $W \gg W_0$, we get :

$$\begin{aligned} \rho &\simeq \left(\frac{B}{A+1} \right)^{\frac{1}{n+1}} \\ p &\simeq - \left(\frac{B}{A+1} \right)^{\frac{1}{n+1}} = -\rho \end{aligned} \quad (14)$$

which correspond to an empty universe with a cosmological constant $\left(\frac{B}{A+1} \right)^{\frac{1}{n+1}}$ (i.e. a de Sitter space).

On the other hand, for small scale factor a , i.e. $W \ll W_0$, we get :

$$\begin{aligned} \rho &\simeq \frac{\sqrt[n+1]{C}}{W} \\ p &\simeq A \frac{\sqrt[n+1]{C}}{W} = A\rho \end{aligned} \quad (15)$$

which correspond to universe dominated by an equation of state $p = A\rho$. This shows that this model interpolates between a universe dominated by matter phase with equation of state $p = A\rho$ and a de Sitter phase $p \simeq -\rho$.

Moreover, expanding equation (11) and (1) to the subleading terms at large cosmological constant, we obtain the following expressions for the energy and the pressure :

$$\begin{aligned} \rho &\simeq \left(\frac{B}{A+1} \right)^{\frac{1}{n+1}} \left(1 + \frac{1}{(n+1)A} \left(\frac{W_0}{W} \right)^{n+1} \right) \\ p &\simeq \left(\frac{B}{A+1} \right)^{\frac{1}{n+1}} \left(-1 + \frac{A+n(A+1)}{(n+1)A} \left(\frac{W_0}{W} \right)^{n+1} \right) \end{aligned} \quad (16)$$

These correspond to the mixture of a cosmological constant $\left(\frac{B}{A+1} \right)^{\frac{1}{n+1}}$ and a type of matter described by an equation of state :

$$p = (n + A(n+1)) \rho \quad (17)$$

The equation of state parameter takes the form :

$$\omega = \frac{p}{\rho} = - \frac{1 - \left(\frac{W_0}{W}\right)^{n+1}}{1 + \frac{1}{A} \left(\frac{W_0}{W}\right)^{n+1}} \quad (18)$$

which ranges over $-1 < \omega < A$, depending on the cosmological scale \mathbf{a} ,

$$\begin{aligned} \omega &= -1 \text{ for } W \gg W_0 \\ \omega &= 0 \text{ for } W = W_0 \\ \omega &= A \text{ for } W \ll W_0 \end{aligned} \quad (19)$$

The speed of sound c_s is defined as,

$$c_s^2 = \frac{\delta p}{\delta \rho} = \frac{\dot{p}}{\dot{\rho}} \quad (20)$$

Now by computing $\dot{\omega}$, we obtain :

$$\dot{\omega} = (c_s^2 - \omega) \frac{\dot{\rho}}{\rho} \quad (21)$$

It gives the following expression for the speed of sound :

$$c_s^2 = \omega + \rho \frac{d\omega}{d\rho} = \frac{n + A(n+1) + \left(\frac{W_0}{W}\right)^{n+1}}{1 + \frac{1}{A} \left(\frac{W_0}{W}\right)^{n+1}} \quad (22)$$

which implies that c_s^2 is always positive and hence there is no concern about imaginary speed of sound.

Moreover it has the following asymptotic limit,

$$\begin{aligned} c_s^2 &= n + A(n+1) \quad \text{for } W \gg W_0 \\ c_s^2 &= A(n+1) \quad \text{for } W = W_0 \\ c_s^2 &= A \quad \text{for } W \ll W_0 \end{aligned} \quad (23)$$

The speed of sound never exceeds that of light for smaller scale \mathbf{a} (i.e. W) or of the order of the scale $\mathbf{a_0}$ where the pressure vanishes, provided that $A < \frac{1}{n+1}$ and will exceed it for large scale compared to $\mathbf{a_0}$.

A way to describe this cosmological model from field theoretical point of view is to introduce a scalar field ϕ and self-interacting potential $U(\phi)$ with the following Lagrangian [8] :

$$\mathcal{L}_\phi = \frac{\dot{\phi}^2}{2} - U(\phi). \quad (24)$$

The energy momentum tensor contributed by the scalar field ϕ is identical to a fluid with energy density ρ_ϕ and pressure p_ϕ given by :

$$\begin{aligned}\rho_\phi &= \frac{\dot{\phi}^2}{2} + U(\phi) = \rho \\ p_\phi &= \frac{\dot{\phi}^2}{2} - U(\phi) = A\rho - \frac{B}{\rho^n}.\end{aligned}\tag{25}$$

It follows that :

$$\begin{aligned}\dot{\phi}^2 &= (1 + \omega_\phi) \rho_\phi \\ U(\phi) &= \frac{1}{2} (1 - \omega_\phi) \rho_\phi\end{aligned}\tag{26}$$

Now since $\dot{\phi} = \phi' \dot{W}$ where the prime denotes derivation with respect to W and $\dot{W} = (D-1)(A+1)HW$, we get :

$$\phi'^2 = \frac{D-2}{2(D-1)(A+1)^2} \frac{1+\omega}{W^2}\tag{27}$$

Here we have used (7) for the Hubble constant and guided by the cosmic microwave background CMB data which is strongly consistent with a flat universe, we have restricted ourselves to the flat case $k=0$.

A use of (18) for $n=1$, gives :

$$\begin{aligned}\phi' &= \sqrt{\frac{D-2}{2(D-1)A(A+1)}} \frac{\frac{W_0}{W^2}}{\sqrt{1 + \frac{1}{A} \left(\frac{W_0}{W}\right)^2}} \\ U(\phi) &= \frac{1}{2} \left(\frac{B}{A+1}\right)^{\frac{1}{2}} \frac{2 + \frac{1-A}{A} \left(\frac{W_0}{W}\right)^2}{\sqrt{1 + \frac{1}{A} \left(\frac{W_0}{W}\right)^2}}\end{aligned}\tag{28}$$

The first equation can be integrated easily which gives :

$$A \left(\frac{W}{W_0}\right)^2 = \frac{4e^{2\alpha\phi}}{(1 - e^{2\alpha\phi})^2} = \frac{1}{sh^2(\alpha\phi)}\tag{29}$$

where $\alpha = \sqrt{\frac{2(A+1)(D-1)}{D-2}}$.

We note that for larger scales (i.e. $W \gg W_0$), the scalar vanishes (i.e. $\phi = 0$) and becomes infinite (i.e. $\phi \rightarrow +\infty$) for small scales ($W \ll W_0$).

Finally, by substituting the latter expression in our previous equations, we can write all our physical quantities ρ_ϕ, p_ϕ and ω_ϕ in terms of the scalar field ϕ as,

$$\begin{aligned}\rho_\phi &= \left(\frac{B}{A+1}\right)^{\frac{1}{2}} \cosh(\alpha\phi) \\ p_\phi &= \left(\frac{B}{A+1}\right)^{\frac{1}{2}} \left(A \cosh(\alpha\phi) - \frac{A+1}{\cosh(\alpha\phi)}\right) \\ \omega_\phi &= -\frac{1 - A \sinh^2(\alpha\phi)}{\cosh^2(\alpha\phi)}\end{aligned}\tag{30}$$

Notice that these physical quantities do not depend on the intermediate constant W_0 (i.e. constant of integration C).

Finally, we get the following potential which has a simple form :

$$U(\phi) = \frac{1}{2} \left(\frac{B}{A+1} \right)^{1/2} \left(\frac{1+A}{\cosh(\alpha\phi)} + (1-A) \cosh(\alpha\phi) \right) \quad (31)$$

3 Rolling Tachyon :

Recently, some works have appeared in the literature which study the tachyon field cosmology [9]–[17]. These follow from Sen's [5, 6] idea which suggests that the tachyon condensate of string theory in a gravitational background may be described by an effective field theory with an action of the form :

$$S = \int d^D x \sqrt{-g} \left(R - V(T) \sqrt{1 + g^{\mu\nu} \nabla_\mu T \nabla_\nu T} \right) \quad (32)$$

where T is the tachyon field, $V(T)$ is the tachyon potential and $g_{\mu\nu}$ is the FRW metric spacetime. The tachyon potential $V(T)$ has two extremal points. The extremal point $T=0$ is a maximum with $V(T=0) = V_0$ as the tension of some unstable bosonic D -brane and the extremal point $T \rightarrow T_0$ is a minimum where the potential vanishes. The energy momentum tensor is :

$$\begin{aligned} T_{\mu\nu} &= - \frac{2\delta S}{\sqrt{-g} \delta g^{\mu\nu}} = - V(T) \sqrt{1 + g^{\mu\nu} \nabla_\mu T \nabla_\nu T} g_{\mu\nu} + V(T) \frac{\nabla_\mu T \nabla_\nu T}{\sqrt{1 + g^{\mu\nu} \nabla_\mu T \nabla_\nu T}} \\ &= p_T g_{\mu\nu} + (p_T + \rho_T) u_\mu u_\nu \end{aligned} \quad (33)$$

where the 4-velocity u_μ is :

$$u_\mu = \frac{-\nabla_\mu T}{\sqrt{-g^{\mu\nu} \nabla_\mu T \nabla_\nu T}} \quad (34)$$

with $u^\mu u_\mu = -1$.

Then the energy density ρ_T and the pressure p_T of the tachyon field which we consider homogeneous but time dependent is (i.e. $\nabla_i T = 0$),

$$\begin{aligned} \rho_T &= \frac{V(T)}{\sqrt{1 - \dot{T}^2}} \\ p_T &= -V(T) \sqrt{1 - \dot{T}^2} \end{aligned} \quad (35)$$

The condition of accelerating universe (i.e. $\frac{\ddot{a}}{a} > 0$) requires that :

$$\dot{T}^2 < \frac{2}{D-1} \quad (36)$$

Then the equation of state parameter is :

$$\omega_T = \frac{p_T}{\rho_T} = - \left(1 - \dot{T}^2 \right) \quad (37)$$

Now consider the pressure $p_\phi = \frac{\dot{\phi}^2}{2} - U(\phi)$ and the energy density $\rho_\phi = \frac{\dot{\phi}^2}{2} + U(\phi)$ for the scalar field that describes the modified Chaplygin gas for the case $n = 1$ and rewrite them as :

$$\begin{aligned} p_\phi &= -U(\phi) \left(1 - \frac{\dot{\phi}^2}{2U(\phi)} \right) \\ \rho_\phi &= U(\phi) \left(1 + \frac{\dot{\phi}^2}{2U(\phi)} \right) \end{aligned} \quad (38)$$

In the approximation $\frac{\dot{\phi}^2}{U(\phi)} \ll 1$, the above pressure and energy density can be expressed in the following form :

$$\begin{aligned} p_\phi &\simeq -U(\phi) \sqrt{1 - \frac{\dot{\phi}^2}{U(\phi)}} \\ \rho_\phi &\simeq \frac{U(\phi)}{\sqrt{1 - \frac{\dot{\phi}^2}{U(\phi)}}} \end{aligned} \quad (39)$$

Mapping the pressure p_ϕ and energy density ρ_ϕ for the scalar field in this approximation with the corresponding tachyon field p_T and ρ_T (35), we obtain that :

$$\begin{aligned} \dot{T}^2 &= \frac{\dot{\phi}^2}{U(\phi)} = 2 \left(\frac{p_\phi + \rho_\phi}{\rho_\phi - p_\phi} \right) \\ V(T) &= U(\phi) = \frac{1}{2} (\rho_\phi - p_\phi) \end{aligned} \quad (40)$$

It follows then that :

$$\dot{T} = \sqrt{2 \left(\frac{p_\phi + \rho_\phi}{\rho_\phi - p_\phi} \right)} = \sqrt{2 \left(\frac{1 + \omega_\phi}{1 - \omega_\phi} \right)} \quad (41)$$

We see that the above approximation $\frac{\dot{\phi}^2}{U(\phi)} \ll 1$ is valid only when $\omega_\phi \ll -\frac{1}{3}$. It implies that the fluid with pressure p_ϕ and energy density ρ_ϕ has an equation of state $p_\phi \ll -\frac{1}{3}\rho_\phi$ which can describe the current observations of accelerating universe. Moreover for large values of the scale factor $W \gg W_0$ (i.e. $\phi \rightarrow 0$), $T = 0$ which corresponds to $\omega_T = -1$.

Equation (41) can be rewritten as :

$$\dot{T} = -\sqrt{\frac{D-2}{(D-1)(1-\omega_\phi^2)}} \frac{\dot{\rho}_\phi}{\rho_\phi^{3/2}} \quad (42)$$

where we have used equation (6) and (7).

Then the tachyon field T can be found in terms of the energy density ρ_ϕ for the scalar field as :

$$T = -\sqrt{\frac{D-2}{D-1}} \int \frac{d\rho_\phi}{\sqrt{1-\omega_\phi^2} \rho_\phi^{3/2}} \quad (43)$$

An approximate solution to this integral can be found by replacing ω_ϕ with its mean value $\langle \omega_\phi \rangle$,

$$\begin{aligned} T &\simeq -\sqrt{\frac{D-2}{(D-1)(1-\langle \omega_\phi \rangle^2)}} \int \frac{d\rho_\phi}{\rho_\phi^{3/2}} \\ &= 2\sqrt{\frac{D-2}{(D-1)(1-\langle \omega_\phi \rangle^2)}} \frac{1}{\sqrt{\rho_\phi}} \end{aligned} \quad (44)$$

This means that :

$$\rho_\phi \simeq 4 \left(\frac{D-2}{(D-1)(1-\langle \omega_\phi \rangle^2)} \right) \frac{1}{T^2} \quad (45)$$

and the potential $V(T)$ for the tachyon field can be expressed as,

$$\begin{aligned} V(T) &= \frac{1}{2}(\rho_\phi - p_\phi) = \frac{1}{2} \left((1-A)\rho_\phi + \frac{B}{\rho_\phi} \right) \\ &\simeq \frac{1}{2} \left(\frac{4(1-A)(D-2)}{(D-1)(1-\langle \omega_\phi \rangle^2)} \frac{1}{T^2} + \frac{B(D-1)(1-\langle \omega_\phi \rangle^2)}{4(D-2)} T^2 \right) \end{aligned} \quad (46)$$

Even though the tachyonic potential $V(T)$ has different form than the scalar potential $U(\phi)$. They lead to the same cosmological evolution that is : an accelerated expansion for the universe.

This was made possible because of the mapping that exists between normal scalar field ϕ with potential $U(\phi)$ and tachyonic field T with potential $V(T)$.

Before we close, we remark that the mapping between scalar field ϕ and tachyon field T which is valid for small gradient $\dot{T}^2 = \frac{\dot{\phi}^2}{U(\phi)} \ll 1$, suggests that the tachyonic Lagrangian (which is the pressure in this case),

$$\begin{aligned} p_T &= -V(T) \sqrt{1 - \dot{T}^2} \\ &\simeq -V(T) \left(1 - \frac{\dot{T}^2}{2} \right) \end{aligned} \quad (47)$$

is identical to the k -essence Lagrangian [18], $p_k(T, \frac{\dot{T}^2}{2}) = -V(T) \tilde{p}(\frac{\dot{T}^2}{2})$.

This implies that the mapping between ϕ and T can be reinterpreted now as a mapping between normal scalar field ϕ and k -essence field.

4 Conclusions :

We have considered an exotic fluid and studied its cosmological implications. Such an exotic fluid obeys an equation of state, we called it " modified Chaplygin " equation of state, which has some interesting properties.

It has been shown here that this " modified Chaplygin " gas of state describes the evolution of a universe from a phase dominated by an equation of state $p = A\rho$ for small values of the scale cosmological factor to a phase dominated by a cosmological constant $\left(\frac{B}{A+1}\right)^{\frac{1}{n+1}}$ for large values of the scale factor.

We have then described this " modified Chaplygin " gas as FRW cosmological model having a scalar field and found its self-interacting potential.

Furthermore we have discussed the cosmological evolution of the tachyon field in gravitational background to drive the accelerated expansion of the universe and obtained a mapping between a normal scalar field with its potential and a tachyonic field with its corresponding potential.

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