Remarks on the BRST Cohomology of Supersymmetric Gauge Theories

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Abstract

The supersymmetric version of the descent equations following from the Wess-Zumino consistency condition is discussed. A systematic framework in order to solve them is proposed.

1 Introduction

One of the most attractive properties of supersymmetric quantum field theories is their softer ultraviolet behavior. Supersymmetry has allowed to establish several nonrenormalization theorems [1], which have provided examples of gauge theories with vanishing beta function to all orders of perturbation theory.

Recently, a criterion of general applicability for the ultraviolet finiteness has been proven [2]. The result allows to give a purely cohomological algebraic characterization of the ultraviolet behavior of gauge field theories, including the supersymmetric models as well. Moreover, it also covers the case of theories whose beta function receives only one-loop contribution, as it happens for the $N = 2$ supersymmetric gauge theories in four dimensions.

The aforementioned criterion makes use of the set of descent equations stemming from the Wess-Zumino consistency condition. In the case of supersymmetric theories it turns out that these equations take a peculiar form, leading to a system of nonstandard equations which highly constrains the possible invariant counterterms and anomalies allowed by the gauge invariance and by the global supersymmetry. These equations have been proven to be very useful in the algebraic proof of the ultraviolet finiteness properties of both $N = 2$ [3] and $N = 4$ supersymmetric gauge theories [4, 5].

It is worth mentioning that, unlike the nonsupersymmetric case, where systematic procedures are available in order to solve the descent equations [6, 7], in the supersymmetric case the task is considerably more difficult, even when a superspace formulation is available [8].

The aim of this letter is to pursue the investigation of the structure of the descent equations for supersymmetric gauge theories, by providing a systematic framework to solve them.

The paper is organized as follows. In Sect.2 a short review of the quantization of the supersymmetric gauge theories is given. In Sect.3 the supersymmetric descent equations are discussed and a way to solve them is presented. In Sect.4 the example of the $N = 1$ supersymmetric Yang-Mills in four dimensions is worked out.

2 Algebraic structure of supersymmetric gauge theories

A brief account of the quantization of the supersymmetric gauge theories in the Wess-Zumino gauge is given here, following the procedure outlined in refs.[4, 9, 10]. Let us start by considering a supersymmetric gauge theory described by the classical action $\Sigma_{\rm inv}(\Phi)$, where Φ denotes collectively the gauge and matter fields. In the following we shall refer to renormalizable gauge theories in four dimensions, the generalization to other dimensions being straightforward.

In the absence of central charges and adopting the Wess-Zumino gauge, the supersymmetry algebra has the typical form

$$
\begin{aligned}
\left\{ Q^i_\alpha , \overline{Q}^j_{\dot\alpha} \right\} &= -2i\delta^{ij}\, \sigma^\mu_{\alpha\dot\alpha} \partial_\mu + (\text{gauge transf.}) + (\text{eqs. of motion}) , \\
\left\{ Q^i_\alpha , Q^j_\beta \right\} &= \left\{ \overline{Q}^i_{\dot\alpha} , \overline{Q}^j_{\dot\beta} \right\} = (\text{gauge transf.}) + (\text{eqs. of motion})
\end{aligned}
\tag{1}
$$

where $Q^i_\alpha , \overline{Q}^j_{\dot\alpha}$ are the supersymmetric charges, with $\alpha, \dot\alpha = 1, 2$ being the spinor indices and $i\,,j = 1, ..., N$ labelling the number of supersymmetries.

The action $\Sigma_{\rm inv}(\Phi)$ is left invariant by the charges $Q^i_\alpha , \overline{Q}^j_{\dot\alpha}$. It is also required to be invariant under gauge transformations, which give rise to the nilpotent BRST operator s when the local gauge parameter is replaced by the Faddeev-Popov ghost. In order to properly quantize the theory, one has to introduce the gauge-fixing and the antifield terms in the action. The standard procedure in order to take into account both BRST and supersymmetry invariance, is to collect them into a unique generalized BRST operator $\mathcal{Q}$ [4, 9, 10]. In addition to the Faddeev-Popov ghost, the introduction of constant ghosts $\varepsilon^\alpha_i , \bar\varepsilon^{\dot\alpha}_j$ corresponding to global supersymmetry is required. The resulting generalized operator $\mathcal{Q}$ is found to be

$$
\mathcal{Q} = s + \varepsilon^\alpha_i Q^i_\alpha + \bar\varepsilon^{\dot\alpha}_j\, \overline{Q}^j_{\dot\alpha} \ . \tag{2}
$$

The action $\Sigma_{\rm inv}(\Phi)$ is invariant under the $\mathcal{Q}$-transformations which, due to the algebra (1), turn out to be nilpotent only up equations of motion and space-time translations, namely

$$
\mathcal{Q}^2 = \varepsilon^\mu \partial_\mu + (\text{eqs. of motion}) \ , \tag{3}
$$

with $\varepsilon^\mu = -2i\varepsilon^\alpha_i \sigma^\mu_{\alpha\dot\alpha} \bar\varepsilon^{\dot\alpha}_i$.

Hence, the complete classical action Σ is given by

$$
\Sigma = \Sigma_{\rm inv}(\Phi) + \Sigma_{\rm gf}(\Phi, \pi, c, \bar c) + \Sigma_{\rm ext}(\Phi, \Phi^*, c, c^*) \ , \tag{4}
$$

where $\Sigma_{\rm gf}(\Phi, \pi, c, \bar c)$ is the gauge-fixing action depending on the gauge and matter fields Φ, the Lagrange multiplier π, the Faddeev-Popov ghost c and antighost $\bar c$. The term $\Sigma_{\rm ext}(\Phi, \Phi^*, c, c^*)$ denotes the antifields action, which is constructed by coupling the nonlinear $\mathcal{Q}$-transformations to external fields

Φ^* and c^* associated respectively to Φ and c, *i.e.*

$$\Sigma_{\rm ext}(\Phi,\Phi^*,c,c^*) = \int d^4x \left(\sum_\Phi \Phi^*\, Q\Phi + c^* Qc + (\text{terms quadratic in } \ \Phi^* , c^*) \right) . \tag{5}$$

As is well known, the terms quadratic in the external fields (Φ^*, c^*) are needed in order to account for the on-shell nilpotency of the generalized operator Q [4, 9, 10]. The invariance of the action $\Sigma_{\rm inv}(\Phi)$ under Q may be now translated into the classical Slavnov-Taylor identity, whose typical form is [11, 3, 5]

$$\mathcal{S}(\Sigma) = \varepsilon^\mu \Delta^{\rm cl}_\mu \ , \tag{6}$$

with

$$\mathcal{S}(\Sigma) = \int d^4x \left(\sum_\Phi \frac{\delta\Sigma}{\delta\Phi^*}\frac{\delta\Sigma}{\delta\Phi} + \frac{\delta\Sigma}{\delta c^*}\frac{\delta\Sigma}{\delta c} + Q\overline{c}\frac{\delta\Sigma}{\delta\overline{c}} + Q\pi\frac{\delta\Sigma}{\delta\pi} \right) \ . \tag{7}$$

It is worth underlining that the breaking term $\Delta^{\rm cl}_\mu$ is a classical breaking, as it turns out to be linear in the quantum fields. As such, it will not be affected at the quantum level [6].

Introducing the so called linearized Slavnov-Taylor operator $\mathcal{B}_\Sigma$

$$\mathcal{B}_\Sigma = \int d^4x \left(\sum_\Phi \left(\frac{\delta\Sigma}{\delta\Phi^*}\frac{\delta}{\delta\Phi} + \frac{\delta\Sigma}{\delta\Phi}\frac{\delta}{\delta\Phi^*} \right) + \frac{\delta\Sigma}{\delta c^*}\frac{\delta}{\delta c} + \frac{\delta\Sigma}{\delta c}\frac{\delta}{\delta c^*} + Q\overline{c}\frac{\delta}{\delta\overline{c}} + Q\pi\frac{\delta}{\delta\pi} \right) \ , \tag{8}$$

it follows that

$$\mathcal{B}_\Sigma \mathcal{B}_\Sigma = \varepsilon^\mu \partial_\mu \ , \tag{9}$$

meaning that $\mathcal{B}_\Sigma$ is nilpotent only modulo a total derivative. Of course, this property follows from the supersymmetric structure of the theory. Moreover, the operator $\mathcal{B}_\Sigma$ is strictly nilpotent when acting on the space of the integrated local functionals of the fields, antifields and their derivatives. This is precisely the functional space to which all invariant counterterms and anomalies belong.

3 The supersymmetric descent equations

In order to discuss the structure of the supersymmetric descent equations, let us begin by considering the Wess-Zumino consistency condition for the invariant counterterms which can be freely added to any order of the perturbation theory, namely

$$\mathcal{B}_\Sigma \int d^4x\, \Omega^0 = 0 \ , \tag{10}$$

where Ω^0 has the same quantum numbers of the classical Lagrangian, *i.e.* it is a local polynomial of dimension four and vanishing Faddeev-Popov charge. The integrated consistency condition (10) can be translated at the local level as

$$\mathcal{B}_\Sigma \Omega^0 = \partial^\mu \Omega^1_\mu \,, \tag{11}$$

where Ω^1_μ is a local polynomial of Faddeev-Popov charge 1 and dimension 3. Applying now the operator $\mathcal{B}_\Sigma$ to both sides of (11) and making use of eq.(9), one obtains the condition

$$\partial^\mu \left(\mathcal{B}_\Sigma \Omega^1_\mu - \varepsilon_\mu \Omega^0 \right) = 0 \,, \tag{12}$$

which, due to the algebraic Poincaré Lemma [6], implies

$$\mathcal{B}_\Sigma \Omega^1_\mu = \varepsilon_\mu \Omega^0 + \partial^\nu \Omega^2_{[\nu\mu]} \,, \tag{13}$$

for some local polynomial $\Omega^2_{[\nu\mu]}$ antisymmetric in the Lorentz indices μ, ν and with Faddeev-Popov charge 2. This procedure can be easily iterated, yielding the following set of descent equations

$$\begin{aligned}
\mathcal{B}_\Sigma \Omega^0 &= \partial^\mu \Omega^1_\mu \,, \\
\mathcal{B}_\Sigma \Omega^1_\mu &= \partial^\nu \Omega^2_{[\nu\mu]} + \varepsilon_\mu \Omega^0 \,, \\
\mathcal{B}_\Sigma \Omega^2_{[\mu\nu]} &= \partial^\rho \Omega^3_{[\rho\mu\nu]} + \varepsilon_\mu \Omega^1_\nu - \varepsilon_\nu \Omega^1_\mu \,, \\
\mathcal{B}_\Sigma \Omega^3_{[\mu\nu\rho]} &= \partial^\sigma \Omega^4_{[\sigma\mu\nu\rho]} + \varepsilon_\mu \Omega^2_{[\nu\rho]} + \varepsilon_\rho \Omega^2_{[\mu\nu]} + \varepsilon_\nu \Omega^2_{[\rho\mu]} \,, \\
\mathcal{B}_\Sigma \Omega^4_{[\mu\nu\rho\sigma]} &= \varepsilon_\mu \Omega^3_{[\nu\rho\sigma]} - \varepsilon_\sigma \Omega^3_{[\mu\nu\rho]} + \varepsilon_\rho \Omega^3_{[\sigma\mu\nu]} - \varepsilon_\nu \Omega^3_{[\rho\sigma\mu]} \,.
\end{aligned} \tag{14}$$

It should be observed that these equations are of an unusual type, as the cocycles with lower Faddeev-Popov charge appear in the equations of those with higher Faddeev-Popov charge, turning the system (14) highly nontrivial. We also remark that the last equation for $\Omega^4_{[\mu\nu\rho\sigma]}$ is not homogeneous, a property which strongly constrains the possible solutions. Eqs.(14) immediately generalize to possible anomalies and to cocycles with arbitrary Faddeev-Popov charge. To some extent, the system (14) displays a certain similarity with the descent equations in $N=1$ superspace [8]. Actually, it is possible to solve the eqs.(14) in a rather direct way by making use of the supersymmetric structure of the theory. This goal is achieved by introducing an operator $\mathcal{W}_\mu$ which, together with the linearized Slavnov-Taylor operator $\mathcal{B}_\Sigma$, gives rise to the algebra

$$\{ \mathcal{W}_\mu, \mathcal{B}_\Sigma \} = \partial_\mu \quad , \quad \{ \mathcal{W}_\mu, \mathcal{W}_\nu \} = 0 \,.$$

The operator $\mathcal{W}_\mu$ has been introduced first in the case of topological field theories [12, 13], and subsequently in the case of extended supersymmetry

[2, 5]. In the next section the explicit form of $\mathcal{W}_\mu$ for the case of $N=1$ gauge theories will be given.

Once the operator $\mathcal{W}_\mu$ has been introduced, it can be used as a climbing operator for the descent equations (14). It turns out in fact that, provided an explicit form for $\Omega^4_{[\mu\nu\rho\sigma]}$ is available, a solution of the system is obtained by repeated applications of $\mathcal{W}_\mu$ on $\Omega^4_{[\mu\nu\rho\sigma]}$, according to

$$
\begin{aligned}
\Omega^0 &= \frac{1}{4!}\mathcal{W}^\mu\mathcal{W}^\nu\mathcal{W}^\rho\mathcal{W}^\sigma\Omega^4_{[\sigma\rho\nu\mu]} \ , \\
\Omega^1_\mu &= \frac{1}{3!}\mathcal{W}^\nu\mathcal{W}^\rho\mathcal{W}^\sigma\Omega^4_{[\sigma\rho\nu\mu]} \ , \\
\Omega^2_{[\mu\nu]} &= \frac{1}{2!}\mathcal{W}^\rho\mathcal{W}^\sigma\Omega^4_{[\sigma\rho\mu\nu]} \ , \\
\Omega^3_{[\mu\nu\rho]} &= \mathcal{W}^\sigma\Omega^4_{[\sigma\mu\nu\rho]} \ .
\end{aligned}
\tag{15}
$$

We are left thus with the characterization of $\Omega^4_{[\mu\nu\rho\sigma]}$. This point can be faced by introducing a new operator $\mathcal{F}_\Sigma$ defined as

$$
\mathcal{F}_\Sigma = \mathcal{B}_\Sigma - \varepsilon^\mu\mathcal{W}_\mu \ . \tag{16}
$$

Unlike $\mathcal{B}_\Sigma$, the new operator $\mathcal{F}_\Sigma$ has the remarkable property of being strictly nilpotent, *i.e.*

$$
\mathcal{F}_\Sigma\mathcal{F}_\Sigma = 0 \quad , \quad \{\mathcal{W}_\mu, \mathcal{F}_\Sigma\} = \partial_\mu \ .
$$

In particular, thanks to (15), the last equation for $\Omega^4_{[\mu\nu\rho\sigma]}$ in (14) can be cast in the form of a homogeneous equation

$$
\mathcal{F}_\Sigma\Omega^4_{[\mu\nu\rho\sigma]} = 0 \ . \tag{17}
$$

This means that $\Omega^4_{[\mu\nu\rho\sigma]}$ can be obtained from the knowledge of the cohomology of the nilpotent operator $\mathcal{F}_\Sigma$, for which standard techniques are available [7]. This gives us a systematic framework for solving the descent equations in the supersymmetric case.

4 The example of $N=1$ super Yang-Mills theory

The $N=1$ super Yang-Mills action $S^{\mathrm{N=1}}$ in the Wess-Zumino gauge is given by

$$
S^{\mathrm{N=1}} = \frac{1}{g^2}\mathrm{Tr}\int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\,\lambda^\alpha\sigma^\mu_{\alpha\dot\beta}D_\mu\bar\lambda^{\dot\beta} + \frac{1}{2}\mathcal{D}^2 \right) \ , \tag{18}
$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$ is the field strength, λ^α and $\overline{\lambda}^{\dot\beta}$ are two-component spinors and $\mathcal{D}$ is an auxiliary scalar field, introduced for the off-shellness closure of the supersymmetric algebra.

The action $S^{\rm N=1}$ is invariant under both BRST and supersymmetry transformations. Following the general procedure, we shall collect the BRST differential s and the supersymmetry generators $\left(Q_\alpha, \overline{Q}_{\dot\alpha}\right)$ into an extended operator $\mathcal{Q}$

$$\mathcal{Q} = s + \varepsilon^\alpha Q_\alpha + \overline{\varepsilon}^{\dot\alpha}\overline{Q}_{\dot\alpha} \ , \tag{19}$$

where ε^α and $\overline{\varepsilon}^{\dot\alpha}$ are global ghosts. The operator $\mathcal{Q}$ acts on the fields as

$$\begin{aligned}
\mathcal{Q}A_\mu &= -D_\mu c + \varepsilon^\alpha \sigma_{\mu\,\alpha\dot\beta}\overline{\lambda}^{\dot\beta} + \lambda^\alpha \sigma_{\mu\,\alpha\dot\beta}\,\overline{\varepsilon}^{\dot\beta}, \\
\mathcal{Q}\lambda^\beta &= \left\{c, \lambda^\beta\right\} - \frac{1}{2}\varepsilon^\alpha \left(\sigma^{\mu\nu}\right)_\alpha^{\ \beta} F_{\mu\nu} - \varepsilon^\beta \mathcal{D}, \\
\mathcal{Q}\overline{\lambda}^{\dot\beta} &= \left\{c, \overline{\lambda}^{\dot\beta}\right\} + \frac{1}{2}\left(\overline{\sigma}^{\mu\nu}\right)^{\dot\beta}_{\ \dot\alpha}\,\overline{\varepsilon}^{\dot\alpha} F_{\mu\nu} + \overline{\varepsilon}^{\dot\beta}\mathcal{D}, \\
\mathcal{Q}\mathcal{D} &= [c, \mathcal{D}] - i\,\varepsilon^\alpha \sigma^\mu_{\alpha\dot\beta} D_\mu \overline{\lambda}^{\dot\beta} + i\,D_\mu \lambda^\alpha \sigma^\mu_{\alpha\dot\beta}\overline{\varepsilon}^{\dot\beta}, \\
\mathcal{Q}c &= c^2 + 2i\varepsilon^\alpha \sigma_{\mu\,\alpha\dot\beta}\,\overline{\varepsilon}^{\dot\beta} A^\mu.
\end{aligned} \tag{20}$$

For the complete gauge-fixed action Σ we have

$$\Sigma = S^{\rm N=1} + S_{\rm gf} + S_{\rm ext} \ , \tag{21}$$

where $S_{\rm gf}$ is the gauge-fixing term in the Landau gauge and $S_{\rm ext}$ contains the coupling of the non-linear transformations $\mathcal{Q}\Phi_i$ to the antifields $\Phi_i^* = (A_\mu^*$, c^*, $\lambda^{\alpha\,*}$, $\overline{\lambda}^*_{\dot\alpha}$, $\mathcal{D}^*)$. They are given by

$$\begin{aligned}
S_{\rm gf} &= {\rm Tr}\int d^4x\, \mathcal{Q}\left(\bar{c}\partial A\right) \ , \\
S_{\rm ext} &= {\rm Tr}\int d^4x \left(A_\mu^* \mathcal{Q}A^\mu + c^*\mathcal{Q}c + \lambda^{\alpha\,*}\mathcal{Q}\lambda_\alpha + \overline{\lambda}^*_{\dot\alpha}\mathcal{Q}\overline{\lambda}^{\dot\alpha} + \mathcal{D}^*\mathcal{Q}\mathcal{D}\right),
\end{aligned} \tag{22}$$

with $\mathcal{Q}\bar{c} = b$ and $\mathcal{Q}b = -2i\varepsilon^\alpha \sigma^\mu_{\alpha\dot\beta}\,\overline{\varepsilon}^{\dot\beta}\partial_\mu \bar{c}$.

As usual, $\bar{c}, b$ denote the antighost and the Lagrange multiplier. The operator $\mathcal{Q}$ turns out to be nilpotent only up to space-time translations

$$\mathcal{Q}^2 = -2i\varepsilon^\alpha \sigma^\mu_{\alpha\dot\beta}\,\overline{\varepsilon}^{\dot\beta}\partial_\mu \ . \tag{23}$$

The complete action Σ satisfies the following Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) = -2i\varepsilon^\alpha \sigma^\mu_{\alpha\dot\beta}\,\overline{\varepsilon}^{\dot\beta}\Delta^{\rm cl}_\mu \ , \tag{24}$$

where

$$\mathcal{S}(\Sigma) = \mathrm{Tr}\int d^4x \left(\frac{\delta\Sigma}{\delta\Phi_i^*}\frac{\delta\Sigma}{\delta\Phi_i} + b\frac{\delta\Sigma}{\delta\bar{c}} - 2i\varepsilon^{\alpha}\sigma^{\mu}_{\alpha\dot{\beta}}\,\overline{\varepsilon}^{\dot{\beta}}\partial_{\mu}\bar{c}\frac{\delta\Sigma}{\delta b} \right) \tag{25}$$

and the classical breaking $\Delta^{\mathrm{cl}}_{\mu}$ is

$$\Delta^{\mathrm{cl}}_{\mu} = \mathrm{Tr}\int d^4x \left(-A^{*\nu}\partial_{\mu}A_{\nu} + c^*\partial_{\mu}c + \lambda^{\alpha\,*}\partial_{\mu}\lambda_{\alpha} + \overline{\lambda}^*_{\dot{\alpha}}\partial_{\mu}\overline{\lambda}^{\dot{\alpha}} - \mathcal{D}^*\partial_{\mu}\mathcal{D} \right) . \tag{26}$$

From eq.(24) and (25) it follows that the linearized operator $\mathcal{B}_{\Sigma}$ defined as

$$\mathcal{B}_{\Sigma} = \mathrm{Tr}\int d^4x \left(\frac{\delta\Sigma}{\delta\Phi_i^*}\frac{\delta}{\delta\Phi_i} + \frac{\delta\Sigma}{\delta\Phi_i}\frac{\delta}{\delta\Phi_i^*} + b\frac{\delta}{\delta\bar{c}} - 2i\varepsilon^{\alpha}\sigma^{\mu}_{\alpha\dot{\beta}}\,\overline{\varepsilon}^{\dot{\beta}}\partial_{\mu}\bar{c}\frac{\delta}{\delta b} \right) \tag{27}$$

is nilpotent modulo a total space-time derivative, namely

$$\mathcal{B}_{\Sigma}\mathcal{B}_{\Sigma} = \varepsilon^{\alpha}d_{\alpha} \ , \tag{28}$$

with the operator d_{α} given by

$$d_{\alpha} = -2i\sigma^{\mu}_{\alpha\dot{\beta}}\,\overline{\varepsilon}^{\dot{\beta}}\partial_{\mu} \, . \tag{29}$$

The integrated cohomology of $\mathcal{B}_{\Sigma}$ is characterized by a consistency condition of the kind (10) which, in the present case, can be written as [14]

$$\begin{array}{rcl} \mathcal{B}_{\Sigma}\Omega^{0} & = & d_{\alpha}\Omega^{1\alpha} \ , \\ \mathcal{B}_{\Sigma}\Omega^{1\alpha} & = & d_{\beta}\Omega^{2[\beta\alpha]} + \varepsilon^{\alpha}\Omega^{0} \ , \\ \mathcal{B}_{\Sigma}\Omega^{2[\beta\alpha]} & = & \varepsilon^{\beta}\Omega^{1\alpha} - \varepsilon^{\alpha}\Omega^{1\beta} \ . \end{array} \tag{30}$$

It should be noted that the presence of the operator d_{α} in the first equation of (30) is due to the supersymmetric character of the theory, following by observing that in the nonsupersymmetric case the pure Yang-Mills Lagrangian $\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ is pointwise invariant.

Defining now the climbing operator $\mathcal{W}_{\alpha}$

$$\mathcal{W}_{\alpha} = \left[\frac{\partial}{\partial\varepsilon^{\alpha}}, B_{\Sigma} \right] \ , \tag{31}$$

it is easily verified that

$$\{\mathcal{W}_{\alpha}, B_{\Sigma}\} = d_{\alpha} \ , \quad \{\mathcal{W}_{\alpha}, \mathcal{W}_{\beta}\} = 0 \ . \tag{32}$$

As discussed in the previous section, the next step is the introduction of the operator $\mathcal{F}_{\Sigma}$

$$\mathcal{F}_{\Sigma} = \mathcal{B}_{\Sigma} - \varepsilon^{\alpha}\mathcal{W}_{\alpha} \, . \tag{33}$$

Accordingly, the last equation of (30) reads

$$\mathcal{F}_\Sigma \Omega^{2[\beta\alpha]} = 0 \ . \tag{34}$$

Furthermore, up to trivial exact cocycles, $\Omega^{2[\beta\alpha]}$ is found to be

$$\Omega^{2[\beta\alpha]} = \varepsilon^{\beta\alpha} \ \mathrm{Tr} \ \lambda^\gamma \lambda_\gamma \ . \tag{35}$$

The higher cocycles are obtained by applying repeatedly the operator $\mathcal{W}_\alpha$ on $\Omega^{2[\beta\alpha]}$

$$\begin{aligned} \Omega^0 &= \frac{1}{2} \mathcal{W}_\alpha \mathcal{W}_\beta \Omega^{2[\beta\alpha]} \ , \\ \Omega^{1\alpha} &= \mathcal{W}_\beta \Omega^{2[\beta\alpha]} \ . \end{aligned} \tag{36}$$

Acting now with $\partial/\partial g$ on both sides of the Slavnov-Taylor identity (24) and observing that the linear breaking term Δ^{cl}_μ does not depend on the coupling constant g, we get the condition

$$\mathcal{B}_\Sigma \frac{\partial \Sigma}{\partial g} = 0 \ , \tag{37}$$

which shows that $\partial\Sigma/\partial g$ is invariant under the action of $\mathcal{B}_\Sigma$. In fact $\partial\Sigma/\partial g$ identifies the cohomology of $\mathcal{B}_\Sigma$ in sector of the integrated polynomials with dimension four and ghost number zero, belonging to the same cohomology class of $\int d^4x \, \Omega^0$.

From eqs.(36) , the usefulness of the operator $\mathcal{W}_\mu$ becomes now apparent. In particular, it allows to establish the following relation

$$\frac{\partial \Sigma}{\partial g} = \frac{1}{4g^3} \varepsilon^{\alpha\beta} \ \mathcal{W}_\alpha \mathcal{W}_\beta \ \mathrm{Tr} \ \int d^4x \lambda^\gamma \lambda_\gamma \ . \tag{38}$$

Equation (38) implies that the origin of the action of $N = 1$ super Yang-Mills can be traced back to the gauge invariant local polynomial $\mathrm{Tr} \ \int d^4x \lambda^\gamma \lambda_\gamma$. This relationship has recently been pointed out in [14].

The construction of the nilpotent operator $\mathcal{F}_\Sigma$ of equation (16) is easily generalized to the cases of $N = 2$ and $N = 4$ gauge theories, so that the analogue of the equation (38) can be worked out from the knowledge of its cohomology, a representative of which has been given in [3, 5].

5 Conclusion

The structure of the descent equations for supersymmetric gauge theories has been discussed. Due to the supersymmetry algebra (1) , these equations are of an unusual type, a property which makes their analysis rather cumbersome. However, it has been shown that a suitable climbing operator $\mathcal{W}_\mu$ can be introduced by making use of the proper supersymmetric algebra. Provided the solution $\Omega^4_{[\mu\nu\rho\sigma]}$ of the last equation of the system (14) is available, a solution of the whole system is obtained by repeated applications of $\mathcal{W}_\mu$ on $\Omega^4_{[\mu\nu\rho\sigma]}$. Concerning the characterization of $\Omega^4_{[\mu\nu\rho\sigma]}$, we have been able to prove that it belongs to the cohomology of the nilpotent operator to $\mathcal{F}_\Sigma$ of eq.(16) . As a consequence, it can be determined by standard cohomolgy arguments [6, 7], providing thus a systematic framework for analysing the supersymmetric version of the descent equations.

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