

M-Theory and de Sitter Space

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Abstract

An M-theory constructed in an eleven-dimensional supermanifold with a \star -product of field operators is shown to have a de Sitter space solution. Possible implications of this result for cosmology are mentioned.

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1 Introduction

Recently, an M-theory has been formulated in an eleven-dimensional supermanifold with coordinates [1]

$$\rho^M = x^M + \beta^M, \quad (1)$$

where $M, N = 0, 1, \dots, 10$ and x^M are commuting coordinates and β^M are Grassmann anticommuting coordinates. Both noncommutative and non-anticommutative geometries can be unified within the superspace formalism using the \star -product of two operators \hat{f} and \hat{g} [2, 3, 4]:

$$\begin{aligned} (\hat{f} \circ \hat{g})(\rho) &= \left[\exp \left(\frac{1}{2} \omega^{MN} \frac{\partial}{\partial \rho^M} \frac{\partial}{\partial \eta^N} \right) f(\rho) g(\eta) \right]_{\rho=\eta} \\ &= f(\rho) g(\rho) + \frac{1}{2} \omega^{MN} \partial_M f(\rho) \partial_N g(\rho) + O(|\omega|^2), \end{aligned} \quad (2)$$

where $\partial_M = \partial/\partial \rho^M$ and ω^{MN} is a nonsymmetric tensor

$$\omega^{MN} = \tau^{MN} + i\theta^{MN}, \quad (3)$$

with $\tau^{MN} = \tau^{NM}$ and $\theta^{MN} = -\theta^{NM}$. Moreover, ω^{MN} is Hermitian symmetric $\omega^{MN} = \omega^{\dagger MN}$, where \dagger denotes Hermitian conjugation. The familiar commutative coordinates of spacetime are replaced by the superspace operator relations

$$[\hat{\rho}^M, \hat{\rho}^N] = 2\beta^M \beta^N + i\theta^{MN}, \quad (4)$$

$$\{\hat{\rho}^M, \hat{\rho}^N\} = 2x^M x^N + 2(x^M \beta^N + x^N \beta^M) + \tau^{MN}. \quad (5)$$

In the limits that $\beta^M \rightarrow 0$ and $|\tau^{MN}| \rightarrow 0$, we get the familiar expression for noncommutative coordinate operators

$$[\hat{x}^M, \hat{x}^N] = i\theta^{MN}. \quad (6)$$

In the limits $\tau^{MN} \rightarrow 0$ and $|\theta^{MN}| \rightarrow 0$, we obtain the Clifford algebra anticommutation relation

$$\{\hat{\beta}^M, \hat{\beta}^N\} = \tau^{MN}. \quad (7)$$

We used the simpler non-anticommutative geometry obtained when $\theta^{MN} = 0$ to construct the M-theory, because it alone can lead to a finite and unitary quantum field theory and quantum gravity theory [2, 3, 4]. In the non-anticommutative field theory formalism, the product of two operators \hat{f} and \hat{g} has a corresponding \diamond -product

$$\begin{aligned} (\hat{f} \diamond \hat{g})(\rho) &= \left[\exp\left(\frac{1}{2} \tau^{MN} \frac{\partial}{\partial \rho^M} \frac{\partial}{\partial \eta^N}\right) f(\rho) g(\eta) \right]_{\rho=\eta} \\ &= f(\rho) g(\rho) + \frac{1}{2} \tau^{MN} \partial_M f(\rho) \partial_N g(\rho) + O(\tau^2). \end{aligned} \quad (8)$$

We chose as the basic action of the M-theory, the Cremmer, Julia and Scherk (CJS) [5] action for eleven-dimensional supergravity, replacing all products of field operators with the \diamond -product. This action is invariant under the generalized \diamond -product supersymmetry gauge transformations of the vielbein e_M^A , the spin 3/2 field ψ_M and the three-form field A_{MNQ} . Our M-theory has as a possible low energy limit the CJS supergravity theory when $|\tau^{MN}| \rightarrow 0$ and $\beta^M \rightarrow 0$.

In previous work [2, 3, 4], we demonstrated that scalar quantum field theory and weak field quantum gravity are finite to all orders of perturbation theory, and the S-matrix for these theories is expected to obey unitarity. An analysis of the higher-dimensional field theories, including supersymmetric gauge theories generalized to the non-anticommutative formalism, shows that they will also be finite to all orders of perturbation theory. This result is mainly due to the existence of a fundamental length scale ℓ in the theory that owes its existence to a quantization of spacetime. When $\ell \rightarrow 0$ the non-anticommutative field theories reduce to the standard local point particle field theories which suffer the usual difficulties of infinities and non-renormalizable quantum gravity. The modification of standard local point field theory occurs at short distances or high energies where accelerator physics has not been probed.

There is now considerable observational evidence for the acceleration of the expansion of the universe [6]. This means that dark energy is the predominant form of matter in the present universe, which can be described either by quintessence [7] or a positive cosmological constant. The present data can be fitted with $\Omega_M \sim 0.3$ and $\Omega_\lambda \sim 0.7$, where Ω_M and Ω_λ are the ratios of the matter density ρ_M and the vacuum density ρ_{vac} to the critical density ρ_c , respectively. Hopefully, future supernovae observational data will be able to distinguish between a universe with a positive cosmological constant and quintessence [8]. In any event, this requires that our low energy M-theory has a four-dimensional de Sitter space solution.

In general, it is difficult to obtain a cosmological solution with a horizon from perturbative superstring theory and supergravity theory [9]. There exist no-go theorems preventing the existence of solutions in eleven-dimensional supergravity with

a non-vanishing cosmological constant [10]. A spontaneous compactification of this theory leads to an anti-deSitter (3+1) spacetime and a compact S^7 space with a positive cosmological constant. Demanding a supersymmetric vacuum leads to an anti-deSitter solution for spacetime, which cannot explain the current cosmological observations. In [1], we extended our generalized M-theory to the massive M-theory formulation of Chamblin and Lambert [11], which reduced to a massive type-IIA supergravity theory in ten dimensions. This scheme was able to produce a de Sitter spacetime solution in four-dimensional spacetime by a compactification of the ten-dimensional theory. However, potential problems of stability of the theory exist and there is no action for the field equations [12].

In the following, we shall investigate another approach to the problem of obtaining de Sitter space solutions of our M-theory field equations by using the Freund-Rubin [13, 14] ansatz for the four-form field F_{MNPQ} , neglecting the fermion contributions, and expanding the \diamond -product of the F-fields. We shall find that we can obtain a de Sitter (3+1) spacetime solution with a positive cosmological constant.

2 M-theory Bosonic Field Equations

The bosonic action of the M-theory takes the form

$$S = \int d^{(11)}\rho \sqrt{g^{(11)}} \diamond \left[-\frac{1}{2}R - \frac{1}{48}F_{MNPQ} \diamond F^{MNPQ} + \left[\frac{\sqrt{2}}{6 \cdot (4!)^2} \right] \left(\frac{1}{\sqrt{g^{(11)}}} \right) \right. \\ \left. \times \diamond \epsilon^{M_1 M_2 \dots M_{11}} F_{M_1 M_2 M_3 M_4} \diamond F_{M_5 M_6 M_7 M_8} \diamond A_{M_9 M_{10} M_{11}} \right]. \quad (9)$$

The metric is $(- + + + \dots +)$, $\epsilon^{0123\dots} = +1$ and $F_{MNPQ} = 4! \partial_{[M} A_{NPQ]}$ and we have set $8\pi G^{(11)} = c = 1$, where $G^{(11)}$ is the eleven-dimensional gravitational coupling constant.

The field equations are

$$R_{MN} - \frac{1}{2}g_{MN} \diamond R = -T_{F\,MN}, \quad (10)$$

$$\frac{1}{\sqrt{g^{(11)}}} \diamond \partial_M (\sqrt{g^{(11)}} \diamond F^{MNPQ}) = - \left[\frac{\sqrt{2}}{2 \cdot (4!)^2} \right] \left(\frac{1}{\sqrt{g^{(11)}}} \right) \diamond \epsilon^{M_1 \dots M_8 N P Q} \\ \times F_{M_1 M_2 M_3 M_4} \diamond F_{M_5 M_6 M_7 M_8}, \quad (11)$$

where

$$T_{F\,MN} = \frac{1}{48} (8 F_{MPQR} \diamond F_N{}^{PQR} - g_{MN} \diamond F_{SPQR} \diamond F^{SPQR}). \quad (12)$$

3 Cosmological Solutions

For cosmological purposes, we shall restrict our attention to an eleven-dimensional metric of the form

$$g_{MN} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & a_4^2(t)\tilde{g}_{ij} & 0 \\ 0 & 0 & a_7^2(t)\tilde{g}_{\Delta\Sigma} \end{pmatrix}. \quad (13)$$

Here, \tilde{g}_{ij} ($i, j = 1, 2, 3$) and $\tilde{g}_{\Delta\Sigma}$ ($\Delta\Sigma = 5, \dots, 11$) are the maximally symmetric three and seven spacelike spaces, respectively, and a_4 and a_7 are the corresponding time dependent cosmological scale factors. We have assumed for simplicity that the seven extra spacelike dimensions form a maximally symmetric space, although there is no a priori reason that this be the case. We shall assume that the Grassmann coordinates β^M are small compared to x^M , $\rho^M \approx x^M$.

The non-vanishing components of the Christoffel symbols are

$$\Gamma_{ij}^0 = \frac{\dot{a}_4}{a_4} g_{ij}, \quad \Gamma_{\Delta\Sigma}^0 = \frac{\dot{a}_7}{a_7} g_{\Delta\Sigma}, \quad \Gamma_{j0}^i = \frac{\dot{a}_4}{a_4} \delta^i_j,$$

$$\Gamma_{\Delta 0}^\Gamma = \frac{\dot{a}_7}{a_7} \delta^\Gamma_\Delta, \quad \Gamma_{jk}^i = \tilde{\Gamma}_{jk}^i, \quad \Gamma_{\Delta\Sigma}^\Gamma = \tilde{\Gamma}_{\Delta\Sigma}^\Gamma, \quad (14)$$

where $\tilde{g}_{ij} = a_4^2 \tilde{g}_{ij}$, $\tilde{g}_{\Delta\Sigma} = a_7^2 \tilde{g}_{\Delta\Sigma}$ and $\tilde{\Gamma}_{jk}^i$ and $\tilde{\Gamma}_{\Delta\Sigma}^\Gamma$ are the Christoffel symbols formed from the \tilde{g}_{ij} , $\tilde{g}_{\Delta\Sigma}$ and their derivatives.

The non-vanishing components of the Ricci tensor are

$$R_{00} = 3 \frac{\ddot{a}_4}{a_4} + 7 \frac{\ddot{a}_7}{a_7},$$

$$R_{ij} = - \left[\frac{2k_4}{a_4^2} + \frac{d}{dt} \left(\frac{\dot{a}_4}{a_4} \right) + \left(3 \frac{\dot{a}_4}{a_4} + 7 \frac{\dot{a}_7}{a_7} \right) \frac{\dot{a}_4}{a_4} \right] g_{ij},$$

$$R_{\Delta\Sigma} = - \left[\frac{2k_7}{a_7^2} + \frac{d}{dt} \left(\frac{\dot{a}_7}{a_7} \right) + \left(3 \frac{\dot{a}_4}{a_4} + 7 \frac{\dot{a}_7}{a_7} \right) \frac{\dot{a}_7}{a_7} \right] g_{\Delta\Sigma}, \quad (15)$$

where k_4 and k_7 are the curvature constants of four-dimensional and seven-dimensional space. Positive and negative values of k_4 and k_7 correspond to the sphere and the pseudosphere, respectively, while vanishing values of k_4 and k_7 correspond to flat spaces.

We now adopt the Freund-Rubin ansatz for which all components of the four-form field F_{MNPQ} vanish except [13, 14]:

$$F_{\mu\nu\rho\sigma} = mf(t) \frac{1}{\sqrt{-g^{(4)}}} \epsilon_{\mu\nu\rho\sigma}, \quad (16)$$

where $\mu, \nu = 0, 1, 2, 3$ and m is a constant. With this ansatz, the trilinear contributions in A_{MNP} and its derivatives in the action vanish. We shall use (8) to expand

the products of the F-tensors in small values of τ^{MN} . We shall neglect the contributions from the \diamond -products of the metric g_{MN} and its derivatives compared to the \diamond -products of the F-tensors. Using the results that $\epsilon_{\mu\alpha\rho\sigma}\epsilon_\nu^{\alpha\rho\sigma} = 6m^2 f^2(t) g_{\mu\nu}(-g^{(4)})$ and $\epsilon_{\mu\nu\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = 24m^2 f^2(t)(-g^{(4)})$, we find to first order in $|\tau|$:

$$T_{FMN} = \frac{1}{2}\epsilon m^2 \left(f^2 - \frac{\dot{f}^2}{2\Lambda^2} \right) g_{MN}, \quad (17)$$

where we have chosen

$$\tau^{00} = -\frac{1}{\Lambda^2}, \quad (18)$$

and Λ is a constant with the dimensions of energy. Moreover, $\epsilon = +1$ for $M, N = \mu, \nu$, and -1 for $M, N = \Delta, \Sigma$. Eq.(11) becomes

$$\frac{d}{dt} \left[(a_7(t))^7 f(t) \right] = 0. \quad (19)$$

We have

$$R_{MN} - \frac{1}{2}g_{MN}R = \lambda_{11}g_{MN}, \quad (20)$$

where λ_{11} is the eleven-dimensional cosmological constant:

$$\lambda_{11} = -\frac{1}{2}\epsilon m^2 \left(f^2 - \frac{\dot{f}^2}{2\Lambda^2} \right). \quad (21)$$

This leads to

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \lambda_4 g_{\mu\nu}, \quad (22)$$

and

$$R_{\Delta\Sigma} - \frac{1}{2}g_{\Delta\Sigma}R = \lambda_7 g_{\Delta\Sigma}, \quad (23)$$

where

$$\lambda_4 = -\frac{1}{2}m^2 \left(f^2 - \frac{\dot{f}^2}{2\Lambda^2} \right), \quad \lambda_7 = \frac{1}{2}m^2 \left(f^2 - \frac{\dot{f}^2}{2\Lambda^2} \right). \quad (24)$$

Let us assume that

$$C \approx f^2 - \frac{\dot{f}^2}{2\Lambda^2}, \quad (25)$$

where C is a constant. In the limit, $\Lambda \rightarrow \infty$, we obtain the standard supersymmetric vacuum result

$$\lambda_4 = -\frac{1}{2}m^2 f^2, \quad \lambda_7 = \frac{1}{2}m^2 f^2. \quad (26)$$

The eleven-dimensional space becomes a product of a four-dimensional Einstein anti-de Sitter space with negative cosmological constant and a seven-dimensional Einstein space with positive cosmological constant [15]. If we require the vacuum to be supersymmetric by demanding covariantly constant spinors ψ for which

$$\delta\psi = \bar{D}_M \theta = 0, \quad (27)$$

where

$$\bar{D}_M = D_M + \frac{i\sqrt{2}}{288}(\Gamma_M^{NPQR} - 8\Gamma^{PQR}\delta_M^N)F_{NPQR}, \quad (28)$$

then for $m \neq 0$, $N=8$ supersymmetry uniquely chooses $AdS \times S^7$ with an $SO(8)$ -invariant metric on S^7 [15, 16].

If we choose $f^2 = \hat{f}^2/2\Lambda^2$, we get $\lambda_4 = \lambda_7 = 0$ and flat $(3+1)$ and seven-dimensional spaces. On the other hand, if we choose $f^2 < \hat{f}^2/2\Lambda^2$, then $\lambda_4 > 0$ and $\lambda_7 < 0$ and we obtain a positive cosmological constant in $(3+1)$ spacetime, corresponding to a de Sitter universe, and a seven-dimensional anti-de Sitter space.

From (22) and (23), we obtain

$$3\ddot{a}_4 + 7\frac{\ddot{a}_7 a_4^2}{a_7} = \lambda_4 a_4^2, \quad (29)$$

$$2k_4 + \ddot{a}_4 a_4 + 2\dot{a}_4^2 + 7\frac{\dot{a}_7 \dot{a}_4 a_4}{a_7} = \lambda_4 a_4^2, \quad (30)$$

$$2k_7 + \ddot{a}_7 a_7 + 6\dot{a}_7^2 + 3\frac{\dot{a}_4 \dot{a}_7 a_7}{a_4} = \lambda_7 a_7^2. \quad (31)$$

Combining (29) and (30) gives

$$\left(\frac{\dot{a}_4}{a_4}\right)^2 + \frac{k_4}{a_4^2} - \frac{7}{6}\frac{\ddot{a}_7}{a_7} + \frac{7}{2}\frac{\dot{a}_7 \dot{a}_4}{a_4 a_7} = \frac{1}{3}\lambda_4. \quad (32)$$

For solutions in which the scale factor a_4 expands faster than a_7 , we get the standard four-dimensional Friedmann equation:

$$\left(\frac{\dot{a}_4}{a_4}\right)^2 + \frac{k_4}{a_4^2} = \frac{1}{3}\lambda_4. \quad (33)$$

This has the de Sitter inflationary solution for $k_4 = 0$ and $\lambda_4 > 0$:

$$a_4 = B \exp\left(\sqrt{\frac{\lambda_4}{3}}t\right), \quad (34)$$

where B is a constant.

4 Conclusions

Our M-theory describes a finite quantum field theory in an eleven-dimensional super manifold in which the action is constructed from a \star -product of field operators based on eleven-dimensional CJS supergravity theory. The quantum gravity and quantum gauge field parts of the action will be finite for $\Lambda < \infty$ due to the finiteness of the non-anticommutative quantum field theory. In the limits $\Lambda \rightarrow \infty$ and $\beta^M \rightarrow 0$, we obtain the low energy limit of eleven-dimensional CJS supergravity. The

compactified version of this theory has the same massless ten-dimensional particle spectrum as type-IIA superstring theory and is connected to the latter theory by a duality transformation.

The M-theory eleven-dimensional field equations are invariant under generalized κ -product supersymmetric gauge transformations, which can be thought of as classical deformations of the standard supersymmetric gauge transformations of CJS supergravity. The generalized κ -product supersymmetric vacuum leads to de Sitter space solutions and thus provides an empirical basis for a realistic cosmology. The no-go theorems in [10] do not apply in this case, because they are derived from standard supersymmetric theories. Our M-theory with generalized supersymmetric equations predicts an inflationary period in the early universe. As the universe expands, f^2 can tend towards $f^2/2\Lambda^2$ and produce a small four-dimensional cosmological constant with $\lambda_4 > 0$ and an accelerating universe at present. However, we have neglected the fermion contributions to the generalized M-theory field equations, so we have to include them and matter and radiation contributions to the equations of motion to describe a fit to observational data. This will be the subject of a future investigation.

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