LOCALIZATION OF FIELDS ON A BRANE IN SIX DIMENSIONS

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April 25, 2020

Abstract

Universe is considered as a brane in infinite (2+4)-space. It is shown that zero modes of all kinds of matter fields and 4-gravity are localized on the brane by increasing transversal gravitational potential.

PACS: 11.10.Kk, 04.50.+h, 98.80.Cq

The papers [1, 2, 3] had excited recent interest in brane models. In present paper we want to concentrate on the localization problem in the model were our world is considered as a single shell expanding in multi-dimensions [4, 5, 6, 7, 8]. Two observed facts of modern cosmology, the isotropic runaway of galaxies and the existence of a preferred frame in the Universe where the relict background radiation is isotropic, have the obvious explanation in this picture.

We assume that trapping of physical fields on the brane has the gravitational nature, since in our world gravity is known to be the unique interaction which has universal coupling with all matter fields. To provide universal and stable trapping we assume also that on the brane (where all gravitating matter can be resided) gravitational potential should have minimal value with the respect of extra coordinates. Growing gravitational potential (warp factor) is the opposite choice compared to the one of Randall-Sundrum with the maximum on the brane [2]. However, Newton's law on the brane is the result of the cancellation mechanism introduced in [3, 6] which allows both types of gravitational potential.

To have localized multi-dimensional fields on a brane "coupling" constants appearing after integration of their Lagrangian over extra coordinates must be non-vanishing and finite. In (1+4)-dimensional models following facts were clarified: spin 0 field is localized on the brane with decreasing warp factor and spin $1/2$ field - on the brane with increasing warp factor [9]; spin 1 field is not normalizable at all [10] and spin 2 fields are localized on the brane with decreasing warp factor [2, 3]. For the case of (1+5)-dimensions it was found that spin 0, 1 and 2 fields are localized on the brane with decreasing warp factor and the spin $1/2$ field on the brane with increasing warp factor [11]. So to fulfill the localization of Standard Model particles in (1+4)-, or (1+5)-spaces it is required to introduce other interaction but gravity.

Here we want to show that zero modes of spin 0, $1/2$, 1 and 2 fields can be all localized on the brane in the (2+4)-space by increasing warp factor. Our motivation for the choice of the signature of the bulk is as follows. In the massless field case (weakest coupling with gravity) symmetries of a multi-dimensional manifold can be restored. It is well known, that in the zero-mass limit the main equations of physics are invariant under the 15-parameter nonlinear conformal transformations. A long time ago it was also discovered that the conformal group can be written as a linear Lorentz-type transformation in a (2+4)-space (for these subjects see, for example, [12]).

Action of the gravitating system in six dimensions can be written in the form

$$S = \int d^6x\sqrt{^6g}\left[-\frac{M^4}{2}(^6R + 2\Lambda) +^6 L\right], \tag{1}$$

where ^6g is the determinant, M is the fundamental scale, ^6R is the scalar curvature, Λ is the cosmological constant and ^6L is the Lagrangian of matter fields, all these values refer to six dimensions. Einstein's

6-dimensional equations can be written in the form

$$ {}^6R_{AB} = -\frac{1}{2}\Lambda g_{AB} + \frac{1}{M^4}\left(T_{AB} - \frac{1}{4}g_{AB}T\right). \tag{2} $$

Capital Latin indices run over $A, B, ... = 0, 1, 2, 3, 5, 6$.

It is convenient to introduce the new dimensionless coordinates z, v of extra (1+1)-space except of Cartesian ones

$$ x^5 = \epsilon\sqrt{z}\cosh v, \qquad x^6 = \epsilon\sqrt{z}\sinh v, \\ z = \frac{x_5^2 - x_6^2}{\epsilon^2}, \qquad \tanh v = \frac{x^6}{x^5}. \tag{3} $$

The constant ϵ which makes z, v to be dimensionless corresponds to the width of the brane.

We are looking for the solution of (2) in the form

$$ ds^2 = \phi^2(z)\eta_{\alpha\beta}(x^\nu)dx^\alpha dx^\beta + g_{ij}(z)dx^i dx^j, \tag{4} $$

where Greek indices $\alpha, \beta, ... = 0, 1, 2, 3$ numerate coordinates in 4-dimensions, while small Latin indices $i, j, ... = 5, 6$ - coordinates of the transversal space. It is assumed that in ansatz (4) the 4-dimensional conformal factor ϕ^2 and the metric tensor of transversal (1+1)-space g_{ij} depend on the extra coordinates x^i only via the coordinate z.

Suppose also that the extra coordinates enter the stress-energy T_{AB} from the metric (4) only. This means that strength of a gauge fields A_B towards the extra directions and covariant derivatives of scalar Φ and spinor Ψ fields with respect to the extra coordinates are zero [5]

$$ F_{iB} = 0, \qquad D_i\Phi = 0, \qquad D_i\Psi = 0. \tag{5} $$

Then ansatz for multi-dimensional matter energy-momentum tensor can be written in the form

$$ T_{\alpha\beta} = \frac{\tau_{\alpha\beta}(x^\nu)}{\epsilon^2\phi^2(z)}, \qquad T_{ij} = -g_{ij}(z)\frac{L(x^\nu)}{\epsilon^2\phi^4(z)}. \tag{6} $$

Because of conformal mapping in the space (4) the 4-dimensional Lagrangian of matter fields $L(x^\nu)$ and the 4-dimensional stress-energy $\tau_{\alpha\beta}(x^\nu)$ automatically appear to be independent from z (see for example [12]).

So we are looking for the solution of 6-dimensional Einstein's and matter fields equations for the case of brane-Universe when the metric and matter energy-momentum tensor have the general structures (4) and (6) respectively. As it was shown in [5] this configuration corresponds to the solution with the minimal energy and thus is stable.

On the brane we require to have 4-dimensional Einstein's equations without a cosmological term

$$ R_{\alpha\beta} = \frac{1}{\epsilon^2 M^4\phi^2}\left(\tau_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta}\tau\right). \tag{7} $$

The Ricci tensor in four dimensions $R_{\alpha\beta}$ is constructed from the 4-dimensional metric tensor $\eta_{\alpha\beta}(x^\nu)$ in the standard way. Then remaining in (2) equations reduce to [8]

$$ g_{ij} = c\eta_{ij}\phi', \qquad z\phi^3\phi' + A\phi^5 + B\phi + C = 0, \tag{8} $$

were prime denotes a derivative with respect to z. Here η_{ij} is the metric tensor of flat extra (1+1)-space, c and C are the integration constants and

$$ A = -\frac{\Lambda\epsilon^2 c}{40}, \qquad B = \frac{c(\tau + 2L)}{16M^4} \tag{9} $$

are the dimensionless parameters. In general B depends on the 4-coordinates x^ν.

To localize matter on the brane without extra sources the factor $1/\phi^2(z)$ in (6) and (7) should have δ-like behavior. It means that $\phi^2(z)$ (and transversal gravitational potential) must be a growing function starting from the brane location. On the brane we assume $\phi(0) = 1$, any other constant will correspond to an overall re-scaling of the coordinates. To have convergent transversal volume when z runs from 0 to ∞ the needed solution $\phi(z)$ of (8) must approach some finite value $a > 1$ at the infinity.

Boundary conditions are taken in the form

$$\phi(z \to 0) \approx 1 + z/|c|, \qquad \phi(z \to \infty) \approx a - 1/b|c|z^b, \tag{10}$$

where $b > 0$ is some parameter. This choice corresponds to the following geometries on the brane and in the transversal infinity:

$$\begin{aligned} ds^2(z \to 0) &\approx \eta_{\alpha\beta}(x^\nu)dx^\alpha dx^\beta + \eta_{ij}dx^i dx^j, \\ ds^2(z \to \infty) &\approx a^2\eta_{\alpha\beta}(x^\nu)dx^\alpha dx^\beta + \frac{1}{z^{b+1}}\eta_{ij}dx^i dx^j. \end{aligned} \tag{11}$$

Substitution of the conditions (10) to (8) impose certain relations

$$\begin{aligned} Aa^5 + Ba + C \simeq 0, &\qquad A + B + C \simeq 0, \\ ba^3 - 5Aa^4 - B \simeq 0, &\qquad 1 + 5A + B \simeq 0. \end{aligned} \tag{12}$$

From these relations one can find [8]

$$\begin{aligned} b &\approx \frac{4a^3 + 3a^2 + 2a + 1}{a^3(a^3 + 2a^2 + 3a + 4)}, \\ A = -\frac{\Lambda\epsilon^2 c}{40} &\approx \frac{1}{a^4 + a^3 + a^2 + a - 4}, \\ B = \frac{c(\tau + 2L)}{16M^4} &\approx -\frac{a^4 + a^3 + a^2 + a + 1}{a^4 + a^3 + a^2 + a - 4}. \end{aligned} \tag{13}$$

Using the relations (12) it can be shown also that the solution of (8) has an inflection point on the brane $z = 0$ (at the inflection point second derivative of a function is zero, while the first is not). It means that on the brane, at the minimum of the transversal gravitational potential and of the total energy of gravitating system, transversal curvature R_{zz} is zero. The function ϕ has no other inflection points outside the brane and smoothly grows from 1 to its maximal value a.

Now it is easy to show that 4-dimensional gravity is localized on the brane in spite of growing character of transversal potential. Using formulae for decomposition of the scalar curvature and determinant

$$\begin{aligned} {}^6R &= \frac{R}{\phi^2} - 3\Lambda + \frac{\tau + 2L}{2\epsilon^2 M^4 \phi^4}, \\ \sqrt{{}^6g} &= |c\phi^{'}|\phi^4\sqrt{-\eta}, \end{aligned} \tag{14}$$

integral of gravitational part of the action (1) can be written in the form:

$$\begin{aligned} S_g &= -\int d^6x\sqrt{{}^6g}\frac{M^4}{2}\left({}^6R + 2\Lambda\right) = \\ &= -|c|\epsilon^2\int_{-1}^{1} dv \int d^4x \int_0^\infty dz\phi^4\phi^{'}\sqrt{-\eta}\frac{M^4}{2}\left(\frac{R}{\phi^2} - \Lambda + \frac{\tau + 2L}{2\epsilon^2 M^4\phi^4}\right). \end{aligned} \tag{15}$$

Placing of the minimum of the warp factor at $z = 0$ means that in the frame of the center of the expanding shell-Universe ($x^5 = x^6 = 0$) its walls move towards the transversal (1+1)-space with a velocity close to the speed of light. In the considered space (4) physical fields are independent from v. Integration of the action over v gives large but finite universal factor for all kinds of fields (corresponding to the transversal velocity of the brane) and can be ignored in calculations. So we must show that physical fields are localized on the brane only with respect to the coordinate z.

Using the relations (12) and

$$\int \sqrt{{}^6g}dz = |c|\int_0^\infty \phi^4\phi^{'}\sqrt{-\eta} = |c|\int_1^a \phi^4\sqrt{-\eta}d\phi \tag{16}$$

one can find that after integration over z last two terms in (15) exactly cancel each other. Also we see that integral over z of remained term is finite in spite of growing character of $\phi(z)$, since $\phi(z)$ varies in the finite range $(1 \div a)$. So 4-dimensional gravity is localized on the brane and the total action (1) reduces to

$$
\begin{aligned}
S = \int d^6x\sqrt{^6g}\left[-\frac{M^4}{2}(^6R + 2\Lambda) + {^6L}\right] \simeq \\
\simeq \int d^4x \int_0^\infty dz\phi^4\phi^{'}\sqrt{-\eta}\left(-\frac{M^4}{2}\frac{R}{\phi^2} + \frac{L}{\epsilon^2\phi^4}\right) \simeq \int d^4x\sqrt{-\eta}\left(-\frac{m_P^2}{2}R + L\right).
\end{aligned}
\tag{17}
$$

Appearing in (17) effective 4-dimensional scale (Planck's scale)

$$
m_P^2 \sim M^4\epsilon^2a^2 \tag{18}
$$

is constructed from the fundamental scale M, the width of our world ϵ and the value of the transversal gravitational potential at the infinity a.

For the realistic values (similar to [1]) of our physical parameters

$$
m_P^2 >> M^4\epsilon^2, \qquad (\tau + 2L) \sim M^4 > 0, \tag{19}
$$

from the relations (13) follows

$$
a >> 1, \qquad c \sim -10, \qquad \Lambda > 0, \qquad b \sim 1/a^3, \qquad \epsilon^2 \sim 1/\Lambda a^4. \tag{20}
$$

Smallness of Newton's constant $\sim 1/m_P^2$ and of the width of our world $\sim \epsilon$ can be the result of the large values of the transversal gravitational potential a and of bulk cosmological constant Λ.

It must be noted that, since c is negative and ϕ' is Positive, as it is seen from the first equation of (8) a suitable solution of our model does not exist in the case of space-like transversal 2-space of (1+5)-models studied in [13, 14, 15, 16, 17, 18, 19].

Now we want to check that in (2+4)-space zero-modes of matter fields also are localized on the brane with increasing warp factor. To have self consistent theory we must follow the assumptions (5) we had used to show localization of 4-dimensional gravity on the brane.

Equation of a massless scalar field in six dimension coupled to gravity has the form $\partial_A(\sqrt{^6g}g^{AB}\partial_B\Phi) = 0$. If we take that Φ is independent from the extra coordinates we shall receive ordinary 4-dimensional Klein-Gordon equation and the action of spin 0 field can be cast to

$$
S_\Phi = -\frac{1}{2}\int d^6x\sqrt{^6g}g^{AB}\partial_A\Phi\partial_B\Phi \simeq -\frac{1}{2}\epsilon^2\int_1^a\phi^2d\phi\int d^4x\sqrt{-\eta}\eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi. \tag{21}
$$

The localization condition requires the integral over ϕ in (21) to be finite, as it is actually.

The equation and the action of $U(1)$ vector field in the case of constant extra components $A_i = const$ also reduce to the 4-dimensional Maxwell equations and to the action which is multiplied by finite integral over extra coordinates

$$
S_A = -\frac{1}{4}\int d^6x\sqrt{^6g}g^{AB}g^{MN}F_{AM}F_{BN} \simeq -\frac{1}{4}\epsilon^2\int_1^a d\phi\int d^4x\sqrt{-\eta}\eta^{\mu\nu}\eta^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}. \tag{22}
$$

From the convergent character of the volume element (16) and formulae (21) and (22) it easy to see that localization of the Abelian-Higgs model (investigated in the papers [18, 19] for the signature $(1+5)$) is the particular example of our model. In addition here we have localization of zero modes of spinor fields also.

In the case of spinor fields we shall introduce the vierbien $h^{\bar{M}}_M$, where $\bar{M}, \bar{N}, ...$ denote local Lorentz indices. Relation between the curved gamma matrices Γ^M and the flat gamma ones $\gamma^{\bar{M}}$ is given by the formula $\Gamma^M = h^M_{\bar{M}}\gamma^{\bar{M}}$, so

$$
\Gamma_\mu = \phi\gamma_\mu, \qquad \Gamma_i = \sqrt{|c|\phi'}\gamma_i. \tag{23}
$$

The spin connection is defined as

$$
\omega^{\bar{M}\bar{N}}_M = \frac{1}{2}h^{N\bar{M}}(\partial_M h^{\bar{N}}_N - \partial_N h^{\bar{N}}_M) - \frac{1}{2}h^{N\bar{N}}(\partial_M h^{\bar{M}}_N - \partial_N h^{\bar{M}}_M) - \frac{1}{2}h^{P\bar{M}}h^{Q\bar{N}}(\partial_P h_{Q\bar{R}} - \partial_Q h_{P\bar{R}})h^{\bar{R}}_M. \tag{24}
$$

The non-vanishing components of the spin-connection for the background metric (4) are

$$\omega_\nu^{\bar M \bar N} = (\delta^{i\bar N}\delta_\nu^{\bar M} - \delta^{i\bar M}\delta_\nu^{\bar N})\partial_i\phi/\sqrt{|c|\phi'}, \qquad \omega_j^{\bar M \bar N} = (\delta^{i\bar N}\delta_j^{\bar M} - \delta^{i\bar M}\delta_j^{\bar N})\partial_i\sqrt{\phi'}/\sqrt{\phi'}. \tag{25}$$

Therefore, the covariant derivatives have the form

$$D_\mu\Psi = (\partial_\mu + \Gamma^j\Gamma_\mu\partial_j\phi/2\phi)\Psi, \qquad D_i\Psi = (\partial_i + \Gamma^j\Gamma_i\partial_j\sqrt{\phi'}/2\sqrt{\phi'})\Psi. \tag{26}$$

We are looking for the solution in the form $\Psi(x^A) = \psi(x^\nu)H(x^j)$, where ψ satisfies the massless 4-dimensional Dirac equation $\gamma^\nu\partial_\nu\psi = 0$. Then 6-dimensional Dirac equation reduces to

$$\left(\partial_j - 2\partial_j\phi/\phi - \partial_j\sqrt{\phi'}/2\sqrt{\phi'}\right)H(x^j) = 0 \tag{27}$$

The solution of this equation with unit integration constant is

$$H(x^j) = \phi^2(\phi')^{1/4}. \tag{28}$$

Then the action of spin $1/2$ field takes the form

$$S_\Psi = \int d^6x\sqrt{^6g}\bar\Psi i\Gamma^A D_A\Psi \simeq \epsilon^2\int_0^\infty dz\phi^7(\phi')^{3/2}\int d^4x\sqrt{-\eta}\bar\psi i\gamma^\nu\partial_\nu\psi. \tag{29}$$

Since ϕ and ϕ' are monotone and finite functions of z the integral over z in (29) is finite. So massless Dirac fermions are also localized on the brane.

When we consider interaction of scalars, or fermions with the electromagnetic field we must make usual replacements

$$\partial_i \to \partial_i - iA_i, \qquad \Psi \to e^{iA_ix^i}\Psi \tag{30}$$

in above formulae for localization. Here x^i are coordinates of transversal (1+1)-space and A_i are constant extra components of electromagnetic field.

To summarize, in this paper it is shown that for the realistic values of the fundamental scale and the brane stress-energy, there exists a non-singular static solution of (2+4)-dimensional Einstein equations. This solution provides gravitational trapping of the 4-dimensional gravity and the matter on the brane without extra δ-like sources. In contrast to Randall-Sundrum's case, the factor responsible for this trapping is the growing away from the brane gravitational potential, but has a convergent volume integral, although the transversal 2-space is infinite. Study of the fluctuation of the metric, which is crucial for the stability of the model, will be the subject of future investigations.

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