

Curved dilatonic brane worlds

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We construct a broad family of exact solutions to the five-dimensional Einstein equations coupled to a scalar field with an exponential potential. Embedding a three-brane in these bulk space-times in a particular way we obtain a class of self-tuned curved brane worlds in which the vacuum energy on the brane is *gravitationally idle*, the four-dimensional geometry being insensitive to the value of the brane tension. This self-tuning arises from cancellations, enforced by the junction conditions, between the scalar field potential, the brane vacuum energy and the matter on the brane. Finally, we study some physically relevant examples and their dynamics.

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I. INTRODUCTION

Brane worlds, the models in which our universe appears as some kind of domain wall or brane embedded in a higher dimensional space-time, have gained considerable attention recently. Although the idea goes back to the eighties [1], its current appeal comes from several directions. As shown by Hořava and Witten [2], the $E_8 \times E_8$ heterotic string theory at strong coupling is described in terms of M-theory in an eleven-dimensional space-time with boundaries, where the ten-dimensional gauge degrees of freedom live on the “branes at the end of the world”. This confinement of the gauge fields to a lower-dimensional submanifold, in contrast to the gravitational field that can propagate into the bulk, might help to explain the hierarchy between the electroweak and the Planck scale in four dimensions [3]. With the aim of solving the hierarchy problem, Randall and Sundrum [4] put forward a proposal, inspired by the AdS/CFT correspondence, where our four-dimensional universe is embedded in a non-factorizable way into five-dimensional anti-de Sitter (AdS) space-time. Gravity in this scenario is “trapped” on the four-dimensional brane due to the geometry of the bulk space-time [4, 5].

The phenomenological viability of brane world models is now being extensively discussed in the literature [6]. Besides the possible imprints of these models detectable in high energy experiments, cosmology naturally emerges as a very promising arena to study the possible consequences of living inside a brane (for an incomplete list of references see [7–9]). One of the important issues in cos-

mology is to explain how our homogeneous and isotropic universe could have emerged from “generic” initial conditions. A popular mechanism to address this question is to have a certain period of inflation in the early universe, during which the possible initial anisotropies and inhomogeneities would be ironed off. In the context of the brane world the issue of inflation [7, 8, 10], as well as the perturbative deviations from isotropy [11, 12] have been recently investigated.

One of the most interesting consequences of considering our universe as a brane living inside a higher-dimensional space time is that Einstein equations in four dimensions do not form a closed system [11]. As a consequence, for a four-dimensional observer it is not sufficient to know the distribution of energy-matter in her/his universe to determine its geometry, the missing element coming from the geometrical features of the space-time outside the four-dimensional universe. This “out of this world” ingredient to the right-hand side of the Einstein equations is crucial in analyzing the cosmological dynamics of the universe in four-dimensions. However, in many instances in the literature the five-dimensional solution in the bulk associated with a four-dimensional brane cosmology is not known and its effects on the brane world have to be modeled using some simplifying assumptions, or neglected altogether.

Thus, in order to gauge to what extent the five-dimensional geometry influences the cosmological dynamics on the brane, it is important to consider exact bulk solutions with various deviations from homogeneity and isotropy (see, for example, [13–15] for some studies in this direction). In this paper we will propose a systematic way of constructing five-dimensional homogeneous and inhomogeneous cosmologies coupled to a scalar field with an exponential potential. We will construct brane cosmologies using these bulk metrics and study the effect that the bulk dynamics has on the cosmological evolution

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of the brane world.

The cosmological constant problem remains a central issue to be solved in theoretical physics [16]. Brane cosmology provides new approaches that might help in the solution of this long-standing problem [17]. One of the proposals recently put forward is a self-tuning mechanism which tunes the four-dimensional cosmological constant to zero, independently of the value of the cosmological constant in the bulk [18–21]. In this paper we propose rather a new realization of this self-tuning property in which vacuum energy on the brane cancels despite the “bare” four-dimensional cosmological constant being non-zero. This is due to a non-trivial counterbalance between the non-vanishing “bare” cosmological term and the matter induced on the brane.

The paper is outlined as follows. In the next Section we will briefly describe the dynamics of dilaton brane cosmologies. After this, in Section 3, we present a solution generating technique to build five-dimensional scalar cosmologies with exponential potential starting with a vacuum solution in five dimensions. Section 4 will be devoted to the study of brane cosmologies embedded in the class of five-dimensional metrics obtained here, and how for these particular embeddings there is a self-tuning mechanism at work, the four-dimensional geometry being independent of the vacuum energy on the brane and the value of the scalar field potential. In Section 5 we illustrate our discussion with some physically interesting examples, and in Section 6 we summarize our results.

II. BRANE DYNAMICS

In the spirit of the brane world picture, we assume that the four-dimensional universe is described by a domain wall (M, g) located at some hypersurface $Y(x^A) = 0$ in the five-dimensional bulk space-time (\mathcal{M}, G) . The only matter in the bulk will be a massless scalar field with an exponential potential. Therefore, the action governing the dynamics is¹

$$S_{5D} = \int d^5x \sqrt{-G} \left[\frac{1}{2\kappa_5^2} \mathcal{R} - \frac{1}{2} \partial_A \phi \partial^A \phi - \Lambda e^{-\frac{2}{3}k\phi} \right] + \int_{Y=0} d^4x \sqrt{-g} L_{\text{brane}}, \quad (1)$$

where κ_5^2 and Λ are constants and

$$L_{\text{brane}} = -\lambda(\phi) + \frac{1}{\kappa_5^2} K^\pm + e^{4b\phi} L(e^{2b\phi} g_{\mu\nu}, \dots)_{\text{matter}}.$$

¹ In the following we will use capital Latin indices for the five dimensional coordinates, whereas the coordinates on the brane world will be denoted by Greek indices. To avoid the use of superindices to indicate the dimension, tensors in five dimensions will be indicated by capital caligraphic letters and their four-dimensional counterparts will be denoted by the corresponding roman types.

Here $\lambda(\phi)$ is a ϕ -dependent vacuum energy (i.e. tension) on the brane, K^\pm is the extrinsic curvature on either side of the brane and L_{matter} is the Lagrangian of the matter degrees of freedom confined to the brane world minimally coupled to the metric $e^{2b\phi} g_{\mu\nu}$, $b \in \mathbf{R}$ (cf. [22]). In the following we use units in which $\kappa_5^2 = 1$. The induced metric on the brane is the projection of the five-dimensional metric onto the brane-world. If we denote by n^A the unit space-like vector normal to the brane, the four-dimensional metric will be given by

$$g_{AB} = G_{AB} - n_A n_B.$$

Because of the embedding there will be two sources of curvature for the four-dimensional universe. One will be the intrinsic curvature induced by the ambient space-time and it will be given by the projection of the five-dimensional Riemann tensor onto the brane. The second one is due to the embedding itself and it is governed by the extrinsic curvature $K_{\mu\nu} = g_\mu^C g_\nu^D \nabla_C n_D$. Using Gauss-Codazzi equations [23] we can write the Einstein equations in four dimensions as [22, 24, 25] (for a review see [26])

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{2}{3} [\mathcal{T}_{AB} g_\mu^A g_\nu^B + \left(\mathcal{T}_{AB} n^A n^B - \frac{1}{4} g^{AB} \mathcal{T}_{AB} \right) g_{\mu\nu}] + K K_{\mu\nu} - K_\mu^\sigma K_{\nu\sigma} - \frac{1}{2} g_{\mu\nu} (K^2 - K^{\alpha\beta} K_{\alpha\beta}) - E_{\mu\nu} \quad (2)$$

where we have denoted by $K \equiv K_\mu^\mu$ the trace of the second fundamental form, $E_{\mu\nu}$ is written in terms of the five-dimensional Weyl tensor \mathcal{C}_{BCD}^A as

$$E_{\mu\nu} = \mathcal{C}_{AFB}^E n^E n^F g_\mu^A g_\nu^B$$

and the five-dimensional energy-momentum tensor in the bulk derived from (1) is

$$\mathcal{T}_{AB} = \partial_A \phi \partial_B \phi - G_{AB} \left(\frac{1}{2} \partial_C \phi \partial^C \phi + \Lambda e^{-\frac{2}{3}k\phi} \right). \quad (3)$$

It is important to point out that the right-hand side of eq. (3) can be evaluated on any side of the brane, the Einstein tensor on the brane being uniquely defined. In the \mathbb{Z}_2 -symmetric case to be studied below both the extrinsic curvature and the derivatives of the dilaton field on the two sides of the brane differ just by a sign, so every term in the right hand side of (3) is well defined on the brane. If the reflection symmetry is relaxed, on the other hand, the uniqueness of the Einstein tensor is ensured in a nontrivial way [27].

In addition to this, the scalar field will satisfy the wave equation

$$\nabla^2 \phi + \frac{2k}{3} \Lambda e^{-\frac{2}{3}k\phi} = \frac{\sqrt{-g}}{\sqrt{-\tilde{g}}} [\lambda'(\phi) - b g^{\mu\nu} \tau_{\mu\nu}] \delta(Y) \quad (4)$$

where $\tau_{\mu\nu}$ is the energy momentum tensor of the matter action, as derived from $e^{4b\phi} L_{\text{matter}}$, and the prime denotes differentiation with respect to ϕ . Upon projection we obtain the four-dimensional equation for the dilaton

$$\begin{aligned} D_\mu D^\mu \phi &= a_C \partial^C \phi + K \mathcal{L}_n \phi + \mathcal{L}_n^2 \phi + \frac{2k}{3} \Lambda e^{-\frac{2}{3}k\phi} \\ &= \frac{\sqrt{-g}}{\sqrt{-\mathcal{G}}} [\lambda'(\phi) - b g^{\mu\nu} \tau_{\mu\nu}] \delta(Y) \end{aligned} \quad (5)$$

with D_μ the covariant derivative with respect to the induced metric, $a^C = n^B \nabla_B n^C$ and $\mathcal{L}_n \phi = n^A \nabla_A \phi$ the Lie derivative in the direction n^A .

As usual, we will take coordinates (χ, x^μ) in such a way that the braneworld lies on the hypersurface defined by $\chi = 0$. For later convenience we will consider that the five-dimensional metric takes the form

$$ds_{5D}^2 = N(x, \chi)^2 d\chi^2 + g_{\mu\nu}(x, \chi) dx^\mu dx^\nu$$

where the “shift” function $N(x, \chi)$ depends on all five-dimensional coordinates. Therefore $n \equiv N(x, \chi)^{-1} \partial_\chi$ and as a consequence

$$\begin{aligned} K_{\mu\nu} &= \frac{1}{2N(x, \chi)} \partial_\chi g_{\mu\nu}(x, \chi) \\ a_\mu &= -\frac{1}{N(x, \chi)} \partial_\mu N(x, \chi), \quad a_\chi = 0 \end{aligned}$$

and $E_{\mu\nu} = C_{\mu\nu}^{\chi\chi}$.

The discontinuity of the derivatives of the metric across the brane due to the energy-momentum localized on the hypersurface $Y(x^A) = 0$ is given by the Israel junction conditions [24, 28]. They relate the jump in the first derivative of the metric at $\chi = 0$ with the total energy-momentum tensor on the brane, namely

$$[K_{\mu\nu}] = - \left(S_{\mu\nu} - \frac{1}{3} g_{\mu\nu} S \right) \quad (6)$$

where we have used the usual notation $[A] \equiv A^+ - A^-$, and $S_{\mu\nu}$ is the total brane energy-momentum tensor

$$S_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} (\sqrt{-g} L_{\text{brane}}) \equiv -\lambda(\phi) g_{\mu\nu} + \tau_{\mu\nu}. \quad (7)$$

In a similar fashion we can find the jump in the derivative of the scalar field by integrating (4) across $\chi = 0$, namely

$$[\partial_\chi \phi] = N(x, 0) [\lambda'(\phi) - b g^{\mu\nu} \tau_{\mu\nu}]. \quad (8)$$

The Einstein equations in four dimensions can be now written from (2) as

$$\begin{aligned} R_{\mu\nu} &- \frac{1}{2} g_{\mu\nu} R = \frac{2}{3} \left[\partial_\mu \phi \partial_\nu \phi + \frac{5}{8} (\partial\phi)^2 g_{\mu\nu} \right] \\ &+ \frac{(\partial_\chi \phi)^2}{4N(x, 0)^2} g_{\mu\nu} - \frac{1}{2} \Lambda e^{-\frac{2}{3}k\phi} g_{\mu\nu} + K K_{\mu\nu} \\ &- K_\mu^\sigma K_{\nu\sigma} - \frac{1}{2} g_{\mu\nu} (K^2 - K^{\alpha\beta} K_{\alpha\beta}) - E_{\mu\nu}. \end{aligned} \quad (9)$$

As we discussed above, the only discontinuities on the right-hand side of (9) are contained in the extrinsic curvature $K_{\mu\nu}$, the derivative of the dilaton field “normal” to the brane $\partial_\chi \phi$ and, eventually, the potential. These terms have to be evaluated at any side of the brane, the sum of them being independent of the side chosen. All other terms, involving the scalar field and their “tangent” derivatives are continuous and can be evaluated individually without ambiguity on the hypersurface $\chi = 0$.

If we assume that our brane world is at the fixed point of a \mathbb{Z}_2 orbifold, as it is the case in the Hořava-Witten scenario, $K_{\mu\nu}^+ = -K_{\mu\nu}^-$ and the Israel junction condition (6) completely determines the extrinsic curvature in terms of the energy-momentum tensor on the brane. In the same way $(\partial_\chi \phi)^2$ can be read off eq. (8). Substitution into (9) leads then to (cf. [22])

$$\begin{aligned} R_{\mu\nu} &- \frac{1}{2} R g_{\mu\nu} = \frac{2}{3} \left[\partial_\mu \phi \partial_\nu \phi - \frac{5}{8} g_{\mu\nu} (\partial\phi)^2 \right] \\ &+ \frac{1}{6} \lambda(\phi) \tau_{\mu\nu} - \Lambda_4 g_{\mu\nu} - \frac{1}{16} [2\lambda'(\phi) - b\tau] b\tau g_{\mu\nu} \\ &+ \pi_{\mu\nu} - E_{\mu\nu}, \end{aligned} \quad (10)$$

where $\tau \equiv g^{\alpha\beta} \tau_{\alpha\beta}$ and the four-dimensional cosmological constant and the tensor $\pi_{\mu\nu}$ are given respectively by

$$\begin{aligned} \Lambda_4 &= \frac{1}{2} \left[\Lambda e^{-\frac{2k}{3}\phi} + \frac{1}{6} \lambda(\phi)^2 - \frac{1}{8} \lambda'(\phi)^2 \right] \\ \pi_{\mu\nu} &= \frac{1}{12} \tau \tau_{\mu\nu} + \frac{1}{8} g_{\mu\nu} \tau_{\alpha\beta} \tau^{\alpha\beta} - \frac{1}{4} \tau_{\mu\alpha} \tau_\nu^\alpha - \frac{1}{24} \tau^2 g_{\mu\nu}. \end{aligned} \quad (11)$$

Looking at the Einstein equations on the brane, eq. (10), we find that in general the dynamics of the four-dimensional universe is not uniquely determined by the distribution of energy inside the universe as encoded in $\tau_{\mu\nu}$ [11, 22], as it is the case with “ordinary” Einstein equations. Indeed, the only ingredient on the right-hand side of (10) which cannot be related with the matter content of the four-dimensional universe (i.e. either four-dimensional matter or the scalar field ϕ) is the tensor $E_{\mu\nu}$ which is determined by the five-dimensional Weyl tensor. In order to study the influence of the five-dimensional geometry on the cosmological evolution of the brane world via $E_{\mu\nu}$ it is necessary to consider not only the four-dimensional metric on the brane but also the higher dimensional ambient geometry. Thus, to address this problem, we will proceed to construct explicitly five-dimensional solutions to the Einstein equations in which our brane worlds will be embedded.

III. SCALAR FIELD COSMOLOGIES IN FIVE DIMENSIONS WITH AN EXPONENTIAL POTENTIAL

Since we will be interested in studying the effects of the bulk on the four-dimensional brane world our starting point will be the five-dimensional geometry in which

the brane is embedded. In particular, we want to consider solutions to the equations of motion derived from the bulk terms in eq. (1), i.e. geometries coupled to a massless scalar field with a Liouville potential.

Such cosmologies can be constructed using a higher dimensional generalization of the theorem presented in ref. [29]. Related models were studied by Lidsey [30] in the context of heterotic M-theory; some particular examples of constant curvature dilatonic branes were also discussed in [31]. Let us consider a *vacuum* solution to the Einstein equations in five dimensions of the form

$$ds_{(\text{vac},5\text{D})}^2 = \epsilon e^{4Q(x)} d\chi^2 + e^{-2Q(x)} h_{\mu\nu}(x) dx^\mu dx^\nu \quad (12)$$

where $\epsilon = \pm 1$ depending on whether χ is a spatial direction or the time coordinate and the metric functions $Q(x)$ and $h_{\mu\nu}(x)$ are independent of χ -coordinate. The new metric

$$ds_{5\text{D}}^2 = \epsilon e^{\frac{4k}{\sqrt{k^2+6}}Q(x)+a_1\xi\chi} d\chi^2 + e^{-\frac{2k}{\sqrt{k^2+6}}Q(x)+a_2\xi\chi} h_{\mu\nu}(x) dx^\mu dx^\nu \quad (13)$$

and the scalar field

$$\phi(x, \chi) = \frac{6}{\sqrt{k^2+6}} Q(x) + \begin{cases} \frac{3k\xi}{k^2-3}\chi & k^2 \neq 3 \\ \pm\sqrt{3}\xi\chi & k^2 = 3 \end{cases} \quad (14)$$

solve Einstein equations coupled to a massless scalar field with potential $V(\phi) = \Lambda e^{-\frac{2}{\sqrt{3}}k\phi}$ where

$$\Lambda = \begin{cases} -\frac{9}{2} \frac{12-k^2}{(k^2-3)^2} \xi^2 & k^2 \neq 3 \\ -\frac{9}{2} \xi^2 & k^2 = 3 \end{cases}$$

and

$$\begin{aligned} a_1 &= \frac{k^2}{3} a_2 \equiv \frac{2k^2}{k^2-3} & \text{if } k^2 \neq 3, \\ a_1 &= a_2 \equiv 2 & \text{if } k^2 = 3. \end{aligned} \quad (15)$$

Finally, ξ fixes the scale of the cosmological constant Λ .

Though some algebra is involved, the result can be easily proven following the same steps as in the four-dimensional case [29], we therefore leave this to the reader. Using the theorem we can construct five-dimensional dilaton gravity solutions with a cosmological constant (negative when $k^2 < 12$). At a glance, there are several interesting values of k for which the metric (13) admits different physical interpretations. The first one is $k^2 = 12$ when the potential for the scalar field vanishes. The second, and more interesting one, is $k^2 = 0$ at which the coupling between the dilaton field and the cosmological constant is zero and we are left with a negative cosmological constant $\Lambda = -6\xi^2$. In this case the geometries are characterized by the line element

$$ds_{k=0}^2 = \epsilon d\chi^2 + e^{-2\xi\chi} h_{\mu\nu}(x) dx^\mu dx^\nu \quad (16)$$

which solve the Einstein equations with a negative cosmological constant and a massless scalar field $\phi(x) =$

$\sqrt{6}Q(x)$. Taking $\epsilon = 1$ and χ as the bulk coordinate, we are provided with generalizations of the Randall-Sundrum model with a generic four-dimensional metric and a massless scalar field. If $k^2 = 18$ and $\epsilon = 1$ we obtain solutions to the low-energy field equations of the $E_8 \times E_8$ heterotic string at strong coupling compactified on a three-fold Calabi-Yau, with the scalar field ϕ representing the breathing mode of the internal manifold [30, 32].

Whenever $\epsilon = 1$ it can be easily realized that the vacuum five-dimensional metric (12) can be thought of as the “oxidation” of a four-dimensional massless scalar field cosmology with a scalar field $\psi(x)_{4\text{D}} = \sqrt{6}Q(x)$ (cf. for example [33]). Therefore in order to get (12) one can start with *any* four-dimensional solution to the Einstein-scalar equations. If $k^2 = 0$ the previous results tell us that *any* four-dimensional metric coupled to a massless scalar field can be embedded into a five-dimensional bulk space-time with negative cosmological constant by the ansatz (16) with $\epsilon = 1$. Finally, in the case $Q(x) = 0$ the Einstein vacuum equations for (12) imply that $h_{\mu\nu}$ has to be a Ricci flat four-dimensional metric. For the particular choice $h_{\mu\nu} = \eta_{\mu\nu}$ we recover the family of metrics considered in [18] after the obvious change of coordinates $dx_5 = \exp(\frac{1}{2}a_1\xi\chi) d\chi$ and a rescaling of the dilaton field. On the other hand, taking $h_{\mu\nu}$ to be the Schwarzschild metric in four-dimensions we can construct embeddings of four-dimensional black holes in five dimensions of the form

$$ds^2 = e^{a_1\xi\chi} d\chi^2 + e^{a_2\xi\chi} \left[-\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega_2^2 \right],$$

which, for $k^2 = 0$, corresponds to the AdS black string considered by Chamblin, Hawking and Reall [34]. When $k^2 \neq 0$ the four-dimensional black hole is embedded into a five-dimensional bulk space-time with a nontrivial profile for the dilaton field, which on the other hand is constant on the brane.

IV. BRANE COSMOLOGIES WITH IDLE VACUUM ENERGY

In the previous Section we have constructed a generic procedure to obtain five-dimensional cosmologies coupled to a scalar field with an exponential potential (or a negative cosmological constant). The final aim is to use these solutions as bulk geometries of four-dimensional brane worlds. As we have reviewed in Section 2, given the solution in the bulk, the matter/energy content of the brane world is strongly constrained by the junction conditions for both the metric and the scalar field.

Among the different possibilities to embed a four-dimensional brane world in the five-dimensional solution (13) the simplest ones correspond to take the codimen-

sion of the brane along one of the four space-like coordinates.

Let us consider the solutions (13) and (14) with $\epsilon = 1$ so that the brane world lies on the hypersurface defined by the equation $\chi = 0$. This choice leads naturally to warped geometries that generalize the Randall-Sundrum construction to include a scalar field with a Liouville potential.

We begin by assuming Z_2 -symmetry around the location of the brane at $\chi = 0$. In this case we are led to the following non-factorizable geometry in five dimensions

$$ds_{5D}^2 = e^{\frac{4k}{\sqrt{k^2+6}}Q(x)+a_1\xi|\chi|} d\chi^2 + e^{-\frac{2k}{\sqrt{k^2+6}}Q(x)+a_2\xi|\chi|} h_{\mu\nu}(x) dx^\mu dx^\nu \quad (17)$$

with a_1, a_2 given by eq. (15). The dilaton, on the other hand, is given by

$$\phi(x, \chi) = \varphi(x) + \begin{cases} \frac{3k\xi}{k^2-3}|\chi| & k^2 \neq 3 \\ \pm\sqrt{3}\xi|\chi| & k^2 = 3 \end{cases} \quad (18)$$

where $\varphi(x) = \frac{6}{\sqrt{k^2+6}}Q(x)$.

In principle, for generic $k \neq 0$, the metric (17) can have a curvature singularity located at $\chi = \pm\infty$ (cf. [18, 35]), in addition to the possible singularities of the metric $h_{\mu\nu}$. Depending on the value of k and on the particular solution considered this singularity can be harmless provided it is located at an infinite proper distance from the brane location. It can be easily seen that this is the case whenever $k^2 > 3$. However, in this case the four-dimensional Planck scale obtained by integrating out the coordinate χ in the five-dimensional action (1) diverges. On the other hand, when $k^2 < 3$ the singularity is located at finite proper distance from the brane but the four-dimensional Planck scale is finite as well. Whenever needed, a second brane can be located at $\chi_0 > 0$, imposing reflection symmetry around this point, so the bulk coordinate is restricted to the interval $[0, \chi_0]$ and the singularity is screened.

Since we have completely determined the bulk geometry and assumed reflection symmetry around $\chi = 0$, the junction conditions (6) will determine the energy-momentum tensor of the matter fields on the brane. Imposing the matching condition on the scalar field (8) at the brane location for the dilaton (18) we find the following relation

$$\lambda'(\varphi) - b g^{\mu\nu} \tau_{\mu\nu} = \begin{cases} \frac{6k\xi}{k^2-3} e^{-\frac{k}{3}\varphi} & k^2 \neq 3 \\ \pm 2\sqrt{3}\xi e^{\mp\frac{1}{\sqrt{3}}\varphi} & k^2 = 3 \end{cases} \quad (19)$$

where we have used the fact that in our case the shift function $N(x, \chi)$ can be written in terms of the scalar field as

$$N(x, \chi) = e^{\frac{k}{3}\phi(x, \chi)} \quad (20)$$

Moreover, using (19) and the expression of the extrinsic curvature of the hypersurface $\chi = 0$ embedded in the metric (17), $K_{\mu\nu} = \frac{1}{2} a_2 \xi e^{-\frac{k}{3}\varphi} g_{\mu\nu}$, we can write a differential equation for $\lambda(\varphi)$

$$\lambda'(\varphi) - 4b\lambda(\varphi) = \begin{cases} 6\xi \frac{k+12b}{k^2-3} e^{-\frac{k}{3}\varphi} & k^2 \neq 3 \\ 2\xi (12b \pm \sqrt{3}) e^{\mp\frac{1}{\sqrt{3}}\varphi} & k^2 = 3 \end{cases} \quad (21)$$

This equation can be solved to get the functional dependence of the vacuum energy on the brane with the dilaton field

$$\lambda(\varphi) = -\rho_0 e^{4b\varphi} - \begin{cases} \frac{18\xi}{k^2-3} e^{-\frac{k}{3}\varphi} & k^2 \neq 3 \\ 6\xi e^{\mp\frac{1}{\sqrt{3}}\varphi} & k^2 = 3 \end{cases} \quad (22)$$

where ρ_0 is an integration constant. We can now plug (22) into eqs. (7) and (6) to obtain the form of the energy-momentum tensor $T_{\mu\nu}$. Proceeding in this way we find

$$\tau_{\nu}^{\mu} = -\rho_0 e^{4b\varphi} \delta_{\nu}^{\mu}. \quad (23)$$

Therefore the matter content of the brane described by L_{matter} has as its effective equation of state $p = -\rho_0$. Notice that the form of the energy-momentum tensor is independent of the value of k and therefore valid for *all* brane models constructed using the solution generating technique of Section 3, and that the constant energy density and the equation of state do not depend on the value of the dilaton coupling b . We now evaluate the tensor $\pi_{\mu\nu}$ which appears on the right-hand side of (10). The result is

$$\pi_{\mu\nu} = -\frac{1}{12} \rho_0^2 e^{8b\varphi} g_{\mu\nu}.$$

With these expressions for the tensors appearing on the right-hand side of (10) we can study the source of the brane gravitational field. Remarkably, we find that

$$\frac{1}{6} \lambda(\varphi) \tau_{\mu\nu} - \Lambda_4 g_{\mu\nu} - \frac{1}{16} [2\lambda'(\varphi) - b\tau] b \tau g_{\mu\nu} + \pi_{\mu\nu} = 0 \quad (24)$$

so the “effective” energy-momentum tensor in (10) only receives contributions from the dilaton field and the tidal bulk effects

$$T_{\mu\nu}^{\text{eff}} = \frac{2}{3} \left[\partial_{\mu} \varphi \partial_{\nu} \varphi - \frac{5}{8} g_{\mu\nu} (\partial \varphi)^2 \right] - E_{\mu\nu}.$$

An interesting thing to notice about the cancellation (24) is that the four-dimensional cosmological constant, as defined by eq. (11), is different from zero. Only when $\rho_0 = 0$ and $Q(x) = 0$ the four-dimensional cosmological constant Λ_4 vanishes exactly independently of the value of the cosmological constant in the bulk. A particular case is the Randall-Sundrum model [4], where $h_{\mu\nu} = \eta_{\mu\nu}$, $k = 0$ and $\Lambda^2 = -6\Lambda$. In the generic case, however, Λ_4 is canceled by the matter contributions encoded in the

energy-momentum tensor and the dilaton-induced terms on the right-hand side of (10). This counterbalance of the four-dimensional vacuum energy by matter cosmological terms is imposed by the junction conditions representing a different realization of the self-tuning mechanism of [18, 19].

Note however, that in the case at hand we are not making any *a priori* assumption on the form of the energy-momentum tensor on the brane $E_{\mu\nu}$. Israel junction condition, together with the jump condition for the dilaton across $\chi = 0$, force it to be of vacuum type, the net cosmological term (24) being zero independently of the value of ρ_0 , k and ξ .

For the five-dimensional metrics of the form (17) we can evaluate the tensor $E_{\mu\nu}$ in terms of the curvature and the scalar field. The result is

$$\begin{aligned} E_{\mu\nu} = & -\frac{1}{3} \left(R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R \right) \\ & - \frac{2k^2}{27} \left[\partial_\mu \varphi \partial_\nu \varphi - \frac{1}{4} (\partial\varphi)^2 g_{\mu\nu} \right] \\ & - \frac{2k}{9} \left(D_\mu D_\nu \varphi - \frac{1}{4} D^2 \varphi g_{\mu\nu} \right). \end{aligned} \quad (25)$$

Using this expression the effective energy-momentum tensor driving the four-dimensional geometry can be written solely in terms of the four-dimensional projection of the scalar field $\varphi(x)$. The four-dimensional equations are given by

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = & \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} g_{\mu\nu} (\partial\varphi)^2 \\ & + \frac{k}{3} \left(D_\mu D_\nu \varphi - \frac{1}{4} g_{\mu\nu} D^2 \varphi \right) \\ & + \frac{k^2}{9} \left[\partial_\mu \varphi \partial_\nu \varphi - \frac{1}{4} g_{\mu\nu} (\partial\varphi)^2 \right] \end{aligned} \quad (26)$$

The first important thing to notice is that when $k=0$ we recover an ordinary four-dimensional scalar cosmology with $\varphi(x) = \sqrt{6} Q(x)$. Actually, by looking at the original ansatz for the vacuum five-dimensional metric in eq. (12) we realize that this is exactly the four-dimensional geometry whose “oxidation” produces (12). This is yet another way to see our previous conclusion, that any four-dimensional cosmology coupled to a massless scalar field can be trivially embedded into a five-dimensional cosmology with negative cosmological constant by the ansatz (16). In this case $E_{\mu\nu}$ has a purely “geometrical” origin, being determined just by the four-dimensional curvature, as can be seen from (25).

If $k \neq 0$ things get more involved, however, since now the effective energy-momentum tensor in (26) cannot be interpreted as that of a minimally coupled massless scalar field. The nontrivial cosmological dynamics in the x -coordinate translates into Brans-Dicke-like terms in the energy-momentum tensor (cf. [36]). It is interesting to notice that in the large- $|k|$ limit even if the dilaton $\varphi(x)$ is switched off, there is a residual contribution coming

from the last two terms in (26). Since $\varphi(x) \sim 1/|k|$ for large $|k|$ the weakness of the scalar field is compensated by the extra powers of k in (26) so a source for the gravitational field is left in that limit, given by the terms in the “effective” energy-momentum tensor with explicit k -dependence.

As it turns out the four-dimensional cosmologies can be conformally related to the equations of low energy string cosmology. In order to see this note that the vacuum Einstein equations for (12) imply that $D_h^2 \varphi = 0$, where the covariant derivative is defined with respect to the metric $h_{\mu\nu}$; when written in terms of the four-dimensional metric $g_{\mu\nu} = e^{-\frac{2}{3}\varphi} h_{\mu\nu}$ the condition translates into $D^2 \varphi = -\frac{k}{3} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$. In this way, $E_{\mu\nu}$ is given by

$$E_{\mu\nu} = - \left(\frac{1}{3} + \frac{k^2}{9} \right) \partial_\mu \varphi \partial_\nu \varphi + \frac{1}{12} g_{\mu\nu} (\partial\varphi)^2 - \frac{k}{3} D_\mu D_\nu \varphi,$$

whereas the four-dimensional Einstein equations can be written as

$$R_{\mu\nu} = \partial_\mu \varphi \partial_\nu \varphi + \frac{k}{3} D_\mu D_\nu \varphi + \frac{k^2}{9} \partial_\mu \varphi \partial_\nu \varphi.$$

Dilaton gravity is then recovered through the field redefinitions (cf. [30])

$$\begin{aligned} \bar{g}_{\mu\nu} &= \exp \left[\left(\frac{k}{3} \pm \sqrt{\frac{k^2+6}{3}} \right) \varphi \right] g_{\mu\nu} \\ \Phi &= \pm \sqrt{\frac{k^2+6}{3}} \varphi \end{aligned} \quad (27)$$

so we arrive to the usual equations for dilaton gravity in the string frame (with the normalization $\mathcal{L}_{\text{grav}} \sim e^{-\Phi} \bar{R}$)

$$\begin{aligned} \bar{R}_{\mu\nu} &= -\bar{D}_\mu \bar{D}_\nu \Phi, \\ \bar{D}^2 \Phi &= \bar{g}^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi. \end{aligned} \quad (28)$$

An important stage in the evolution of the early universe is inflation. As shown above the solutions on the brane can be conformally related to low energy string cosmology in the string frame. With one further conformal transformation they, of course, can be related to the usual Einstein frame in which the metric is just $h_{\mu\nu}$ and the physics is described by Einstein relativity coupled to a massless scalar field $\psi(x)_{4D} = \sqrt{6} Q(x)$. In this frame standard inflation does not take place. However, in the string frame there is the possibility of implementing inflation in the framework of the pre-big-bang scenario [37]. In particular, if the four-dimensional metric has a space-like Killing vector, scale factor duality along that direction will be a symmetry of the equations (28).

It is interesting to note that from the point of view of a five-dimensional observer there are inflating solutions on the brane. Using the form of the energy momentum tensor in 5 dimensions the strong energy condition reads, for any timelike vector t^A ($t_A t^A = -1$),

$$0 < \mathcal{R}_{AB} t^A t^B = (t^A \partial_A \phi)^2 - \frac{2}{3} \Lambda e^{-\frac{2}{3} k \phi}.$$

This condition can in principle be violated for $\Lambda > 0$, i.e. if $k^2 > 12$.

V. EXAMPLES

A. Bianchi I brane cosmologies

As an illustrating example we can construct Bianchi I brane cosmologies generalizing the analysis of [14] to nonzero profiles for a scalar field with a nontrivial potential. We start with a vacuum Kasner-like line element in five dimensions and apply the algorithm described to find, after a suitable redefinition of the constants, the following solution (we will consider that $k^2 \neq 3$. The case $k^2 = 3$ can be easily obtained)

$$ds^2 = t^{\frac{2k}{3}\beta} e^{\frac{2k^2-6}{k^2-3}|\chi|} d\chi^2 + e^{\frac{6\epsilon}{k^2-3}|\chi|} (-dt^2 + t^{2\alpha_1} dx^2 + t^{2\alpha_2} dy^2 + t^{2\alpha_3} dz^2)$$

$$\phi(t, \chi) = \beta \log t + \frac{3k\epsilon}{k^2-3}|\chi|$$

where the constants α_i , β satisfy the following relations

$$\sum_{i=1}^3 \alpha_i = 1 - \frac{k\beta}{3}, \quad \sum_{i=1}^3 \alpha_i^2 = 1 - \beta^2 - \left(\frac{k\beta}{3}\right)^2. \quad (29)$$

We notice that for $k=0$ the usual condition for a Kasner metric coupled to a massless scalar field are retrieved. As we saw in the general analysis of Section 4, for vanishing ϵ the four-dimensional effective energy-momentum tensor is that of a massless scalar field. When $k \neq 0$ the usual Kasner conditions get modified. Looking at the scaling of powers of $k\beta$ in (29) we see that the first condition gets modified by the linear terms in the dilaton field in (26) whereas the modification of the second condition comes from the quadratic ones. In the Hořava-Witten case the conditions on the Kasner exponents reduce to the ones found in [38].

Looking at the constraints (29) it can be easily seen that there is no volume inflation for any value of the parameter β and ϵ , since the average scale factor scales as t^n with $0 < n < \frac{2}{3}$.

The case when $|k| \rightarrow \infty$ is of some interest. In this limit, regularity of the five-dimensional metric requires that $\beta k \sim \text{constant}$ so the homogeneous part of the scalar field vanishes. The result is a Bianchi I metric in four dimensions where the Kasner exponents α_i satisfy the constraints

$$\sum_{i=1}^3 \alpha_i = 1 - C, \quad \sum_{i=1}^3 \alpha_i^2 = 1 - C^2$$

with $|C| \leq 1$ a numerical constant. In this case the (traceless) effective energy-momentum tensor is that of an anisotropic fluid with energy density and pressure given by

$$\rho = \frac{C(1-C)}{t^2}, \quad p_i = \frac{C\alpha_i}{t^2}.$$

B. Friedmann-Robertson-Walker models

Another physically interesting example are Friedmann-Robertson-Walker (FRW) cosmologies. These can be easily constructed by considering the five-dimensional vacuum line element [33]

$$ds_{\text{vac}}^2 = \frac{t^2}{1 - \kappa t^2} d\chi^2 - dt^2 + (1 - \kappa t^2) \left[\frac{dr^2}{1 - \kappa r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

where $\kappa = 0, \pm 1$. Taking χ as the bulk coordinate and applying the algorithm of Section 3 we get, after changing into conformal time, the following four-dimensional metrics

$$ds_{4\text{D}}^2 = a^2(\eta) \left[-d\eta^2 + \frac{dr^2}{1 - \kappa r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

where

$$a(\eta) = \begin{cases} (\cosh \eta)^{\frac{\sqrt{k^2+6}+k}{2\sqrt{k^2+6}}} (\sinh \eta)^{\frac{\sqrt{k^2+6}-k}{2\sqrt{k^2+6}}} & \kappa = -1 \\ \eta^{\frac{\sqrt{k^2+6}-k}{2\sqrt{k^2+6}}} & \kappa = 0 \\ (\cos \eta)^{\frac{\sqrt{k^2+6}+k}{2\sqrt{k^2+6}}} (\sin \eta)^{\frac{\sqrt{k^2+6}-k}{2\sqrt{k^2+6}}} & \kappa = 1 \end{cases} \quad (30)$$

while the dilaton field is

$$\varphi(\eta) = \begin{cases} \frac{3}{\sqrt{k^2+6}} \log \tanh \eta & \kappa = -1 \\ \frac{3}{\sqrt{k^2+6}} \log \eta & \kappa = 0 \\ \frac{3}{\sqrt{k^2+6}} \log \tan \eta & \kappa = 1 \end{cases}$$

In the flat ($\kappa=0$) case we recover the solution discussed in [35]. There is no inflation for any value of k , since the exponent of the scale factor in (30) is always positive. It is interesting to notice, however, that when $k \rightarrow -\infty$ we recover a radiation-dominated universe. Looking at the effective energy-momentum tensor on the brane, eq. (26), we see that this corresponds to the dynamics of the universe being dominated by $E_{\mu\nu}$. The opposite limit $k \rightarrow \infty$ gives Minkowski space-time as a result.

For the non-flat FRW models we find again that inflation does not occur for any value of k . In the case of the negatively curved model ($\kappa=-1$) the universe approaches a radiation dominated regime at late times ($\eta \rightarrow \infty$) in which the dilaton field is frozen, independently of k . As in the flat case, the limit $k \rightarrow -\infty$ retrieves a radiation dominated open FRW model. When $k \rightarrow \infty$, on the other hand, we get a regular vacuum FRW model with scale factor $a(\eta) = \cosh \eta$. The conclusions are similar in the case of the models with $\kappa=1$: no value of k renders an inflationary universe.

We will not present further explicit examples, but just mention again that the ansatz (12) is quite generic. One may start directly with any vacuum five-dimensional solution and put it into (12) form, or rather start with a four-dimensional dilaton solution lifting it to a five-dimensional vacuum geometry. Thus, for example, the solutions of [13] and their generalizations may be easily obtained by using the dilatonic plane wave solutions given in [39].

VI. CONCLUSIONS AND OUTLOOK

Brane cosmology is special, as compared to standard Einstein gravity, in that the four-dimensional world is not dynamically self-contained, in the sense that the matter/energy content of the universe encoded in the energy-momentum tensor does not determine the gravitational field. The “missing” part on the right-hand side of the Einstein equations comes from gravitational effects in the bulk which are not sourced by four-dimensional matter. It is important to notice, however, that the term containing the tidal bulk effects is the only one not suppressed by powers of the five-dimensional Newton’s constant.

Here we have studied the physical and cosmological relevance of bulk effects by looking at five-dimensional cosmologies coupled to a Liouville scalar field and embedding the brane world into them. One of the remarkable properties of the class of brane cosmologies under study is that there is a natural self-tuning of the vacuum energy on the brane. This insensitivity of the brane solutions to the value of the brane tension happens because the “bare” cosmological constant on the brane is dynamically counterbalanced by the brane matter, which the junction conditions force to be of vacuum energy type. So, the dynamics of the four-dimensional universe is driven just by the dilaton field and the non-local bulk effects contained in the tensor $E_{\mu\nu}$.

The five-dimensional bulk cosmologies we constructed using the theorem stated in Section 3 generalize, and include, those studied previously in the literature, (see, for example, [13, 18, 30, 31, 35, 40]). In addition, the brane cosmologies obtained by warped embeddings extend and complement the self-tuning mechanism of [18, 20] to generic non Ricci-flat branes. Incidentally, the technique proposed can be also used to construct static five-dimensional solutions which on the brane reduce to four-dimensional black holes.

The authors of [41, 42] have argued that the self-tuning mechanism proposed in [18] is actually a fine tuning in disguise due to the presence of the singularity in the bulk. For the brane worlds studied in this paper, however, we find that whenever $k^2 < 3$ the four-dimensional effective theory is well defined, in the sense that the four-dimensional Planck scale is finite and the consistency condition of ref. [42] is satisfied without adding extra

sources due to the “on-shell” identity

$$\frac{2}{3}\Lambda \int_{-\infty}^{\infty} d\chi \sqrt{-\mathcal{G}} e^{-\frac{2}{3}k\phi(\chi,x)} + \frac{1}{3}\sqrt{-g} \left[\lambda(\varphi) + \rho_0 e^{4b\varphi(x)} \right] = 0, \quad (31)$$

where we have used the fact that for our family of solutions $\mathcal{L}_{\text{matter}} = -\rho_0$. Actually eq. (31) is automatically enforced by Israel junction conditions and the matching condition for the dilaton, which in turn determine both the energy-momentum tensor on the brane and $\lambda(\varphi)$. If $k^2 > 3$ the divergence in the four-dimensional Planck scale can be cut off, provided a second “hidden” vacuum brane is located at some $\chi_0 > 0$. The coordinate χ is then restricted to the interval $[0, \chi_0]$ by assuming \mathbb{Z}_2 reflection symmetry around the location of the second brane. Imposing the junction conditions at χ_0 we find $\lambda(\phi)_{\text{hidden}} = -\lambda(\phi)$, where $\lambda(\phi)$ is given by eq. (22) with $\rho_0 = 0$. This value of the vacuum energy for the second brane implies again that the model satisfies the consistency condition of [42] for any value of k .

In order to illustrate the physics of the family of brane worlds considered, we have analyzed a number of explicit examples of physical relevance. For Bianchi-I and all the FRW models we find that the time-dependence of the scale factor(s) is only controlled by the constant k measuring the slope of the potential of the scalar field in the bulk. The result is that no inflation occurs on the brane for any value of k , whereas there is a region in parameter space for which inflation takes place in the bulk. The difference between the behavior on the brane and in the bulk is mainly due to difference in the character of the velocity of the fluid flow as derived from the scalar field on the brane and in the bulk. While the five-velocity in the bulk is not orthogonal to the hypersurfaces of constant time its projection onto the brane is.

In the analysis presented here we have assumed that the brane world is trapped at an orbifold point and it does not move in the bulk. It can be easily seen that this is a consistent assumption for the self-tuning brane cosmologies studied in Section 4. Once \mathbb{Z}_2 symmetry is relaxed in general the brane will move and the problem of the dynamical stability of the brane world arises. A specially interesting way to relax reflection symmetry in our case is to consider bulk space-times in which k takes different values at the two sides of the brane. If we regard the dilaton potential as arising from some kind of phase transition, the brane world plays the role of a (infinitely thin) domain wall separating two regions of space with different values for the cosmological constant, in the spirit of the scenarios discussed in [43]. In this case the possible motion of the brane in the bulk will affect the gravitational dynamics on the brane world [44, 45]. A detailed study of these non-symmetric brane worlds and their stability will be presented elsewhere.

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