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A Formalism to Analyze the Spectrum of Brane World Scenarios

Seif Randjbar-Daemi[a1] and Mikhail Shaposhnikov[b2]

[a] *International Center for Theoretical Physics, Trieste, Italy*
[b] *Institute of Theoretical Physics, University of Lausanne, CH-1015 Lausanne, Switzerland*

Abstract

In this paper we develop a formalism to analyze the spectrum of small perturbations about arbitrary solutions of Einstein, Yang-Mills and scalar systems. We consider a general system of gravitational, gauge and scalar fields in a $D-$dimensional space-time and give the bilinear action for the fluctuations of the fields in the system around an arbitrary solution of the classical field equations. We then consider warped geometries, popular in brane world scenarios, and use the light cone gauge to separate the bilinear action into a totally decoupled spin-two, -one and -zero fluctuations. We apply our general scheme to several examples and discuss in particular localization of abelian and non-abelian gauge fields of the standard model to branes generated by scalar fields. We show in particular that the Nielsen-Olsen string solution gives rise to a normalizable localized spin-1 field in any number of dimensions.

[1]Email: seif@ictp.trieste.it
[2]Email: mikhail.shaposhnikov@ipt.unil.ch

1 Introduction

Consider a general system of fields (gravitational, gauge, scalar and fermion) in higher-dimensional space-time with the action

$$S = \int d^D x \sqrt{-G} \left(\frac{1}{\kappa^2} R - \frac{1}{4} F_{MN} F^{MN} - (D_M \Phi)^\dagger D^M \Phi - U(\Phi) + i [\bar{\Psi} \Gamma^A E_A^M (\partial_M - \Omega + ie A_M T) \Psi + \frac{1}{2} g \bar{\Psi} \Phi \Psi] \right) . \quad (1)$$

Here D is the total number of dimensions, R is the scalar curvature, A_M, Ψ and Φ are the generic (non-abelian) gauge, fermion and scalar fields, respectively. The signature of the metric is $(- + \ldots +)$, e is the gauge coupling and T is the generic notation for group generators, $D_M = \partial_M + ie T^a A^a_M$.

Suppose that the equations of motion for gravity, scalar and gauge fields resulting from the action (1) admit a solution that is consistent with four-dimensional Poincare invariance:

$$ds^2 = G_{MN} dx^M dx^N = \sigma(y) g_{\mu\nu}(x^\mu) dx^\mu dx^\nu + \gamma_{mn}(y) dy^m dy^n , \quad (2)$$

$$\Phi = \Phi(y), \quad A_\mu = 0, \quad A_a = A_a(y) , \quad (3)$$

where x^μ refer to the co-ordinates of the four-dimensional world and y^m are the extra coordinates.

We would like to know if for small energies the perturbations around a soliton solution describe four-dimensional excitations incorporating gravity and all other fields of the standard model.

It is well known that a four-dimensional theory can be constructed along these lines for Kaluza-Klein (KK) backgrounds (for a review see [1]) with $\sigma(y^m) = 1$, $\Phi(y^m) = A_a(y^m) = 0$ and a smooth compact manifold describing extra dimensions. A very attractive feature of this idea is that the gauge and scalar fields can have completely geometrical origin. The problem with this approach is that fermions are vector-like [2] and not chiral, as required by the standard model.

For KK reductions, but now with gauge and scalar backgrounds, the problem of chiral fermions can be overcome and semi-realistic models can be constructed. One of the examples is a six-dimensional model with geometry $M_4 \times S_2$, containing in addition to gravity a U(1) gauge field that has a monopole configuration on S_2. The presence of the U(1) field is essential for the chiral character of 4-dimensional fermions [3, 4]. More complicated models incorporating instanton backgrounds can also be constructed [5, 6]. The Calabi-Yau compactification in string theory or their orbifold limit [7] are more sophisticated examples of this type.

In these examples the zero modes of all the fields are separated from other excitations by a mass gap, and the resulting low energy effective theory is indeed 4-dimensional. The reason for the existence of the mass gap is the compact character of extra dimensions.

It would be interesting to understand whether compact character of extra dimensions is a necessary requirement for construction of a (semi) realistic theory, incorporating gravity and other fields. There are some partial answers to this question. In a theory without gravity a background of a topological defect (domain wall formed by a scalar field in 5d, Nielsen-Olesen string in 6d, monopole in 7d, etc.) may confine chiral fermions and scalar fields and thus give an effective theory in 4d containing fermions and scalars [8, 9]. As in KK case, the spectrum of excitations contains a mass gap that insures the effective 4d character of low-lying excitations, but contrary to the KK compactification, the spectrum of higher lying modes is continuous rather than discrete.

A combination of a topological defect and KK ideas leads to an exciting possibility of large extra dimensions [10] (see also [11]), where standard model fields are assumed to be localized to a brane [12], whereas gravity lives in a bulk.

If gravity is included, there are backgrounds that lead to an acceptable effective theory of gravity in 4 dimensions [13]. They include a non-trivial warp factor [14] $\sigma(x^a)$ and the presence of a brane, which in the field theory language is nothing but some topological defect residing in a higher-dimensional space-time. In 5 dimensions [13] or in spherical symmetric case of higher dimensions [15, 16, 17] the behavior of the warp factor is taken to be exponential, $\sigma = \exp(-cr)$, where r is a radial coordinate. A qualitative difference between KK case is that the spectrum of graviton excitations is continuous, and starts from zero without any mass gap. Still, the theory is acceptable as the interaction of bulk gravitons with other fields living on the brane is weak at small energies (see, for a review [18] and references therein).

A first step in constructing a low energy effective theory is an analysis of the spectrum of small perturbations around the solution. Consider the spectrum of small fluctuations of the metric, gauge and scalar fields and fermionic excitations near background (2,3). An effective 4-dimensional low-energy theory could arise if the following conditions are satisfied:

(i) The spectrum contains normalizable zero (or small mass) modes of graviton, gauge, scalar and fermion fields, with wave functions of the type $exp(ip_\mu x^\mu)\psi(y^m)$.

(ii) The effects of higher modes should be experimentally unobservable at low energies, i.e. there should be a mass gap between the zero modes and excited states. Another possibility is that extra, unwanted modes may be light but interact very weakly with the zero modes.

In fact, these conditions are by no means necessary for the existence of an acceptable 4-dimensional effective theory. For example, in [19], a five-dimensional theory without any normalizable modes was considered, still leading to an effective theory in 4 dimensions for metastable states localized on a domain wall. In [20, 21] it was argued that in spite of the fact that perturbative treatment of gauge field does not reveal any localized massless modes on domain wall the confinement effects may lead to the trapping of massless gauge field on a brane. In [22] it was proposed that the fermionic zero modes localized on a brane may modify through radiative corrections the effective action for the gauge fields, leading to effectively four-dimensional interactions at low energies (see, however, [23]).

The aim of the present paper is to analyze the small fluctuations of different fields for a general case of a field-theoretical brane. Only bosonic fluctuations will be considered; for some partial results concerning fermionic excitations see, e.g. [24].

To our knowledge, this problem has never been addressed in its full generality, though there are many results for a number of specific models. The most studied sector is the one related to spin-two fields (graviton). Its dynamics happens to be related to the background geometry only, as was shown already in [13]. A number of works were devoted to the problem of vector fluctuations in Randall-Sundrum (RS) model and in a model of Nielsen-Olesen string in six dimensions [25]-[34]. The problem of scalar fluctuations of field-theoretical branes, important for the analysis of their stability and of the possibility of spontaneous symmetry breaking has attracted much less attention because of its complexity (see, e.g. [35].)

The approach we choose in our paper allows to study all types of fluctuations on the same basis. The use of the light-cone gauge allows to separate easily fluctuations of different spins, the use of the action instead of equations of motion simplifies a lot of the calculations. What concerns the problem of spin-two fields, we add nothing new here and present the results just for completeness. As for the spin-one case, the main difficulty here is the mixing between the gauge fields and the metric. So, we present the general analysis of vector fluctuations for an arbitrary field theory and apply the formalism to the studied cases of the RS model and to the case of string, reproducing the known results in a much simpler and transparent way. As a new application, we consider the case of a monopole configuration. The spin-zero fluctuations represent the main difficulty because of the mixing between the components of scalar field with the components of the metric and the gauge field. The action for the scalar components, derived in this paper, has quite a formidable form presented in eqs. (43-48). In section 5 we clarify the question of scalar physical degrees of freedom in the RS modes which was obscure in the previous studies.

The paper is organized as follows. In Section 2 we consider a general decomposition of perturbations on tensors, vectors and scalars and define variables that allow independent treatment of excitations of different spins. In Section 3 we consider application of this general formalism to the metric of the type

$$ds^2 = e^{A(r)}\eta_{\mu\nu}(x^\mu)dx^\mu dx^\nu + e^{B(r)}g_{mn}(y)dy^m dy^n \ + dr^2, \tag{4}$$

where $\eta_{\mu\nu}$ is a flat metric, and warp factors A and B depend on one coordinate (r) only. In Section 4 we show how to separate fluctuations of higher spins from lower spins in this gauge with the use of Lorentz invariance on the brane. In section 5, we apply the formalism to the Randall-Sundrum model. In section 6 we consider a number of specific models of compactifications on topological defects (local string in 6d, abelian and non-abelian monopole in 7d, etc.) and address the question whether an acceptable gauge theory can arise from extra dimensions. We show that the string solution in any number of dimensions leads to normalizable localized massless vector fields while the monopole solution does so only for $D_1 \geq 4$. Section 7 contains concluding remarks. In Appendix A we give the components of the curvature tensors for the warped metric. In Appendix B we give the bilinear part of the spin-one action for the infinite tower of the Kaluza-Klein modes in a monopole background. In this appendix we show that the presence of the scalar field which generates the brane (or the branes, if we have more than one brane) does not directly affect the vector bilinears. This means that the bilinear part which is valid everywhere in the space time is formally identical to the one we would have in the bulk except that the metric functions A and B will be modified in the vicinity of the branes.

2 General formalism

To analyze the spectrum of small bosonic oscillations around a given background solution of the classical field equations we insert the decomposition

$$G_{MN} \to G_{MN} + h_{MN}, \qquad A_M \to A_M + V_M, \qquad \Phi \to \Phi + \phi \tag{5}$$

in the action and expand in powers of h_{MN}, V_M and ϕ. On the right-hand side of (5) G_{MN}, A_M and Φ refer to the background solution (2,3). The zeroth order term will give us the value of the classical action evaluated at the background configuration. The first order terms will be absent because the background configuration satisfies the classical equations of motion, which are

$$D_M D^M \Phi = \frac{\partial U}{\partial \Phi^\dagger} \ , \tag{6}$$

$$D_M F^{MN} = ie\left((D^N\Phi)^\dagger T\Phi - \Phi^\dagger T D^N \Phi\right) \ , \tag{7}$$

$$\begin{aligned} R_{MN} - \frac{1}{2}G_{MN}R \ &= \ \frac{\kappa^2}{2}\Bigg(F_{MS}F_N{}^S - \frac{1}{4}G_{MN}F^2 + \\ (D_M\Phi)^\dagger D_N\Phi \ &+ \ (D_N\Phi)^\dagger D_M\Phi - G_{MN}(D_S\Phi)^\dagger D^S\Phi - G_{MN}U(\Phi)\Bigg) \ . \end{aligned} \tag{8}$$

We assume that the scalars belong to some representation of the gauge group with generators T.

These equations of motion are quite general and incorporate Kaluza-Klein compactification for Einstein-Yang-Mills systems [3]-[6], pure gravity warp-factor geometries [14, 36], thick domain walls [37, 38], global [39, 40, 41] and local [15, 17] topological defects in higher dimensions, etc.

The bilinear in perturbation parts can be written as a sum of several terms which consist of pure gravitational, gauge and scalar fluctuations plus terms which contain their mixings, viz,

$$S_2 = S_2(h,h) + S_2(V,V) + S_2(\phi,\phi) + S_2(h,V) + S_2(h,\phi) + S_2(V,\phi) \ . \tag{9}$$

Each individual term is given by

$$\begin{aligned} S_2(h,h) = \int d^DX\sqrt{-G}\Bigg\{\frac{1}{2\kappa^2}\left[\left(h^{ML}_{;M} - \frac{1}{2}h^{;L}\right)^2 - \frac{1}{2}h^{KS}_{;M}h^{;M}_{KS} + \frac{1}{4}h^{;M}h_{;M}\right] \\ -\frac{1}{4\kappa^2}R_1h^2 - \frac{1}{2}h_{KM}h^K_N\left(\frac{1}{2}F^{MS}F^N{}_S + (D^M\Phi)^\dagger D^N\Phi\right) \\ -\frac{1}{2}h^{MN}h^{KS}\left(\frac{1}{\kappa^2}R_{KMNS} - \frac{1}{2}F_{KM}F_{NS}\right)\Bigg\} \ , \end{aligned} \tag{10}$$

$$S_2(V,V) = \int d^DX\sqrt{-G}\left\{-\frac{1}{2}D_MV_ND^MV^N + \frac{1}{2}(D_MV^M)^2 - \frac{1}{2}R^{MN}V_MV_N \right. \tag{11}$$
$$\left. -eF_{MN}V^M \times V^N - \frac{e^2}{2}G^{MN}V^a_MV^b_N\Phi^\dagger\{T^a,T^b\}\Phi\right\} ,$$

$$S_2(\phi,\phi) = -\int d^DX\sqrt{-G}\left\{(D_M\phi)^\dagger D^M\phi + \frac{1}{2}\phi\frac{\partial^2U}{\partial\Phi^2}\phi + \frac{1}{2}\phi^*\frac{\partial^2U}{\partial\Phi^{*2}}\phi^* + \phi^*\frac{\partial^2U}{\partial\Phi\partial\Phi^*}\phi\right\} , \tag{12}$$

$$S_2(h,V) = -\int d^DX\sqrt{-G}\left\{\left(D^MV^N - D^NV^M\right)\left(\frac{1}{4}hF_{MN} + h_{LN}F^L{}_M\right)\right. \tag{13}$$
$$\left. +ie((D_N\Phi)^\dagger T^a\Phi - \Phi^\dagger T^aD_N\Phi)\left(\frac{1}{2}G^{MN}h - h^{MN}\right)V^a_M\right\} ,$$

$$S_2(V,\phi) = -ie\int d^DX\sqrt{-G}V^M_a\left\{(D_M\phi)^\dagger T^a\Phi + (D_M\Phi)^\dagger T^a\phi\right. \tag{14}$$
$$\left. -\phi^\dagger T^aD_M\Phi - \Phi^\dagger T^aD_M\phi\right\} ,$$

$$S_2(\phi,h) = \int d^DX\sqrt{-G}\left\{\left(h^{MN} - \frac{1}{2}G^{MN}h\right)\left((D_M\phi)^\dagger D_N\Phi + (D_M\Phi)^\dagger D_N\phi\right)\right. \tag{15}$$
$$\left. +\frac{1}{2}\phi h\frac{\partial U}{\partial\Phi} + \frac{1}{2}\phi^*h\frac{\partial U}{\partial\Phi^*}\right\} ,$$

where $h = G^{MN}h_{MN}$, and

$$\frac{2}{\kappa^2}R_1 = \frac{1}{\kappa^2}R - \frac{1}{4}F^2 - D_M\Phi^\dagger D^M\Phi - U(\Phi) \tag{16}$$

denotes the value of the classical Lagrangian evaluated at the background solution. In these expressions the covariant derivatives contain the background gravitational as well as the gauge connections. We have made use of the classical equations of motion to simplify the expressions.

The bilinear action should inherit the linearized gauge symmetries of the original Einstein, Yang-Mills, scalar system. The linearized general coordinate transformations are given by

$$\delta_\xi h_{MN} = -\xi_{N;M} - \xi_{M;N} , \tag{17}$$

$$\delta_\xi V_M = -\xi^LF_{LM} - D_M\chi , \tag{18}$$

$$\delta_\xi\phi = -\xi^MD_M\Phi + ie\chi\Phi , \tag{19}$$

where ξ is the infinitesimal parameter of the general coordinate transformations and χ is the infinitesimal parameter of an induced Yang-Mills gauge transformation defined by

$$\chi = \xi^LA_L .$$

We can write down similar transformation rules under the Yang-Mills gauge transformations. Because of such local symmetries, if we couple h_{MN}, V_M and ϕ to external sources via

$$S_{int} = \int d^D X \sqrt{-G}[\frac{1}{2} h^{MN} T_{MN} + J^M V_M + J\phi] \tag{20}$$

the source terms must satisfy the following conservation laws,

$$D_M T^{MN} = J^M F^N{}_M + J D^N \Phi \ , \tag{21}$$

$$D_M J^M = ieJ\Phi \ . \tag{22}$$

These gauge symmetries can be used to simplify the analysis of the spectrum of small oscillations. The most frequently used gauges in the literature on Kaluza Klein theory and on supergravity theories have been the covariant gauges. For the analysis of the next section we shall find it more convenient to use the light-cone gauge.

3 Warped geometries

The bilinear action of the previous section is completely general and can be used to analyze the spectrum of the small oscillations of the Einstein-Yang-Mills-scalar system around any solution of the background equations. In this section we shall specialize to a set of particular solutions, the so called warped solutions which have attracted considerable amount of interest in recent years. The space time geometry of a warped solution is characterized by the metric (4), where the coordinates x^μ, $\mu = 0, 1, ...D_1 - 1$ parameterize a flat Minkowski space with the usual metric tensor $\eta_{\mu\nu} = diag(-1, +1, ..., +1)$, while the coordinates y^m, $m = 1, ..., D_2$ cover a compact D_2 -dimensional manifold. A coordinate r can be thought of as a radial variable. The total number of the space time dimensions is thus $D = D_1 + D_2 + 1$.

The presence of the standard Poincare symmetry in the subspace spanned by the x's enable us to use the representations of this group to define the usual notions of particle physics. In addition, if the space covered by the y's also do posses isometries which leave the entire classical background invariant, there will be massless spin 1 particles in the x-subspace corresponding to the usual Kaluza Klein gauge fields. In general the final gauge fields are linear combinations of the appropriate modes contained in the fluctuations $h_{m\mu}$ and V_μ. We shall see examples of this later on.

Another advantage of the Poincare symmetry is that it allows us to use the light cone gauge to analyze the spectrum. As we shall see in this gauge the fields of different spins are readily separated from each other and, after we have eliminated the dependent and non physical degrees of freedom, we can read the spectrum almost directly.

To use the light cone gauge first we introduce the light cone coordinates $x^\pm = \frac{1}{\sqrt{2}}(x^{D_1-1} \pm x^0)$ in the x-subspace. The inner product of any two vectors then become

$$G^{MN} A_M B_N = e^{-A(r)}(A_+ B_- + A_- B_+ + A_i B_i) + e^{-B(r)} g^{mn} A_m B_n + A_r B_r \ , \tag{23}$$

where A_i and B_i are the transverse components of A_M and B_M, respectively, $A_\pm = \frac{1}{\sqrt{2}}(A_{D_1-1} \pm A_0)$. The light cone gauge is defined by

$$V_- = 0 \qquad and \qquad h_{-M} = 0, \qquad for \quad all \quad M \ . \tag{24}$$

Using these gauge conditions it turns out that the + components are not independent but can be expressed in terms of the transverse objects V_i and h_{ij}. Since the calculations leading to these results are simple but rather lengthy here we give the results only [1].

First in the Yang - Mills sector we obtain the constraint equation

$$\partial_- V_+ = -\partial_i V_i - e^A \left(\frac{D_1 - 2}{2} A' V_r + D_{\underline{m}} V^{\underline{m}} \right) - i e^A \left(\phi^\dagger T \Phi - \Phi^\dagger T \phi \right) e \ , \tag{25}$$

where the underlined index $\underline{m}$ stands for the pair of indices m , which is tangent to the y-space and r. In the Einstein sector the h_{++} field equation simply leads to

$$h = 0 \ . \tag{26}$$

This brings about considerable amount of simplification. Next after some considerable amount of calculations we obtain the constraint equations for h_{i+}, h_{m+} and h_{r+}. First consider the one for h_{i+},

$$\partial_- h_{i+} = -\partial_j h_{ij} - e^A \left(\frac{D_1 - 1}{2} A' h_{ir} + h_{i\underline{l};}{}^{\underline{l}} \right) \ . \tag{27}$$

Next consider the constraint equation for h_{m+},

$$\partial_- h_{m+} = -\partial_i h_{im} - e^A \left(\frac{D_1 - 2}{2} A' h_{rm} + h_{m\underline{l};}{}^{\underline{l}} - \kappa^2 \left(\phi^\dagger D_m \Phi + (D_m \Phi)^\dagger \phi + V_{\underline{l}} F_n{}^{\underline{l}} \right) \right) \ . \tag{28}$$

Finally we have the constraint equation for h_{r+}, which reads as follows,

$$\partial_- h_{r+} = -\partial_i h_{ir} + \frac{A'}{2} h_{ii} - e^A \left(\frac{D_1 - 2}{2} A' h_{rr} + h_{r\underline{l};}{}^{\underline{l}} - \kappa^2 \left(\phi^\dagger D_r \Phi + (D_r \Phi)^\dagger \phi + V_{\underline{l}} F_r{}^{\underline{l}} \right) \right) \ . \tag{29}$$

There is also a constraint equation for h_{++} component of the metric, which is quite complicated. We will not write it as it is not used in the analysis to follow.

It is possible to write the above constraint equations in a more compact form. For the gauge field constraint the compact form reads as

$$D_M V^M = -i \left(\phi^\dagger T \Phi - \Phi^\dagger T \phi \right) e \ , \tag{30}$$

while for the gravitational perturbation the compact form becomes,

$$D_M h^M_N = \kappa^2 \left(\phi^\dagger D_N \Phi + (D_N \Phi)^\dagger \phi + V_{\underline{l}} F_N{}^{\underline{l}} \right) \ . \tag{31}$$

[1]This is very similar to the light cone formulation in superstring theory, except that in our field theory model the calculations leading to these results are somewhat more lengthy. For the application of the light cone gauge in the context of the higher dimensional theories of gravity see [42, 43].

Inserting our background configuration in these equations and expanding them in components reproduces the above gauge and gravitational constraint equations.

Using these constraint equations together with the light cone gauge conditions enable us to separate the bilinear Lagrangian into various spin sectors. The advantage of the light cone gauge is that the quadratic Lagrangian for the fields of different spins disentangle from each other.

3.1 Spin-2 Action

Using the constraints and the gauge conditions in the general bilinear action given in section two the spin-2 part separates from the rest and its action takes the following the form:

$$S(spin-2) = -\frac{1}{4\kappa^2}\int d^DX\sqrt{-G}\Big[\partial_\mu\tilde{h}_{ij}\partial^\mu\tilde{h}^{ij} + A'^2\tilde{h}^{ij}\tilde{h}_{ij} + G^{ik}G^{jl}\partial_r\tilde{h}_{ij}\partial_r\tilde{h}_{kl} \\ -2A'\tilde{h}^{ij}\partial_r\tilde{h}_{ij} + D_m\tilde{h}_{ij}D^m\tilde{h}^{ij}\Big] \ . \tag{32}$$

Here and in the following all the indices are raised and lowered with the r-dependent metric $G^{\mu\nu} = e^{-A(r)}\eta^{\mu\nu}$ and $G^{mn} = e^{-B(r)}g^{mn}(y)$, $\tilde{h}_{ij}$ indicates the traceless part of h_{ij}, viz,

$$h_{ij} = \tilde{h}_{ij} + \frac{1}{D_1-2}G_{ij}h^k_k \ , \tag{33}$$

where $h^k_k = G^{ik}h_{ik}$.

It is to be observed that the spin-two quadratic action is quite universal. Its dependence on the background scalar and Yang- Mills fields is only through the dependence of the space time geometry on these fields. In other words, the background scalars and gauge fields do not directly enter into the spin-2 action. This is not true for the spin-1 and spin-0 quadratic actions.

This action has more transparent form in terms of the field with one upper and one lower index:

$$S(spin-2) = -\frac{1}{4\kappa^2}\int d^DX\sqrt{-G}\left[e^{-A}\eta^{\mu\nu}\partial_\mu\tilde{h}^j_i\partial_\nu\tilde{h}^i_j + \partial_r\tilde{h}^j_i\partial_r\tilde{h}^i_j + D_m\tilde{h}^j_iD^m\tilde{h}^i_j\right] \ . \tag{34}$$

3.2 Localized spin-2 Fields

Since $\tilde{h}^j_i$ is a scalar field in the y subspace, provided this space is compact, the operator D_m will have a normalizable zero mode. We can take zero mode to be r independent, viz[2],

$$\partial_r\tilde{h}^j_i = 0 \ . \tag{35}$$

[2]There is also a r-dependent zero mode which will be discussed in section 6 in analyzing the localization of the spin-1 fields.

A little calculation will then show that the $S(spin-2)$ reduces to

$$S(spin-2) = -\frac{1}{4}\int d^D X\sqrt{-G}e^{-A}\eta^{\mu\nu}\partial_\mu \tilde{h}_i^j \partial_\nu \tilde{h}_j^i \ . \tag{36}$$

Upon integration over the D_2 dimensional y space, which is trivial as the fields are independent of y and integrating over the r subspace, which in contrast is non-trivial, we obviously obtain the light cone gauge action for a graviton in the D_1 dimensional subspace covered by the x coordinates. This action will be meaningful provided the r integral is finite. The condition for the finiteness of the r-integral is

$$\int dr e^{\frac{D_1-2}{2}A(r)+\frac{D_2}{2}B(r)} < \infty \ , \tag{37}$$

which is nothing but the condition of the finiteness of the D_1-dimensional Planck scale [13].

3.3 Spin-1 Action

The basic spin-1 fields are V_i and $h_{\underline{m}i}$. In the bilinear part of the action, in general, these fields mix with each other as well as with the derivatives of the scalar fields. However, in the light cone gauge there is a remarkable simplification due to the fact that there are no mixing with the fluctuations of spin zero fields. The bilinear action is given by,

$$S(spin-1) = \int d^D X\sqrt{-G}\left(L_v(V,V) + L_v(h,h) + L_v(h,V)\right) \ , \tag{38}$$

where the different pieces are:

$$L_v(V,V) = -\frac{1}{2}\left(\partial_\mu V_i \partial^\mu V^i + e^{-A}\partial_r V_i \partial_r V_i + D_m V_i D^m V^i + e^2 V_i^a V^{ib}\Phi^\dagger\{T^a,T^b\}\Phi\right) \ , \tag{39}$$

$$L_v(h,V) = -D^{\underline{m}}V_i h^{\underline{l}i}F_{\underline{lm}} - \frac{1}{2}A'V_i h^{\underline{l}i}F_{\underline{l}r} + ie((D_{\underline{n}}\Phi)^\dagger T\Phi - \Phi^\dagger T D_{\underline{n}}\Phi)h^{\underline{n}i}V_i \ , \tag{40}$$

$$\begin{aligned} L_v(h,h) = &-\frac{1}{2\kappa^2}\left(\partial_\mu h_{i\underline{m}}\partial^\mu h^{i\underline{m}} + \partial_r h_{i\underline{m}}\partial^r h^{i\underline{m}} + D_n h_{i\underline{m}} D^n h^{i\underline{m}}\right) \\ -\frac{1}{4\kappa^2}h_{mi}h^{mi}\left(\frac{1}{2}B'^2 + A'^2\right) &- \frac{1}{4\kappa^2}h_{ri}h^{ri}\left(D_2 A'B' - \frac{D_1-2}{2}A'^2\right) + \frac{1}{\kappa^2}A'h^{ir}D^m h_{im} \\ &-\frac{1}{2}h_{\underline{m}i}h^i_{\underline{n}}\left(\frac{1}{2}F^{\underline{m}}{}_{\underline{k}}F^{\underline{nk}} + (D^{\underline{m}}\Phi)^\dagger D^{\underline{n}}\Phi\right) \ . \end{aligned} \tag{41}$$

Note that if $F_{rn} = 0$ and Φ is r independent or does not couple to the gauge field, the only mixing is between V_i and h_{mi} and between the divergence $D^m h_{mi}$ and h_{ri}. The seemingly 3×3 system consisting of V_i, h_{mi} and h_{ri} will reduce to two 2×2 system. We shall see an example of such simplification in the next section.

3.4 Spin-0 Action

The spin zero action contains the fields $h_{\underline{mn}}$, $V_{\underline{m}}$, h^i_i and ϕ. It is the most complicated one and has the form:

$$S(spin\!-\!0) = \int d^D X \sqrt{-G} \left[L(h,h) + L(V,V) + L(\phi,\phi) + L(h,V) + L(h,\phi) + L(V,\phi) \right] \;, \tag{42}$$

where the different pieces are:

$$\begin{aligned}
L(h,h) = -\frac{1}{4\kappa^2} \Big\{ & \partial_\mu h_{\underline{mn}} \partial^\mu h^{\underline{mn}} + \partial_r h_{\underline{mn}} \partial^r h^{\underline{mn}} + D_l h_{\underline{mn}} D^l h^{\underline{mn}} \\
& + h_{mr} h^{mr} \left(\frac{4-D_1}{2} A'^2 + D_2 A'B' + 2A'' - \frac{3}{2} B'^2 - 2B'' \right) \\
& + \frac{1}{2} h_{mn} h^{mn} B'^2 + h^2_{rr} \left(\frac{4-D_1}{2} A'^2 + D_2 A'B' + 2A'' \right) \\
& - 4A' h^{r\underline{m}} D_n h^n_{\underline{m}} + 2(A'' + A'^2) h^r_r h^i_i + \frac{1}{2} A'^2 h^{i2}_i + A'B' h^i_i h^m_m \\
& + 2(B'' + B') h^m_m h^{rr} + \frac{1}{2} B'^2 h^{m2}_m - 2e^{-B} \Omega_{mn}{}^{kl} h^m_l h^n_k \\
& + 2\kappa^2 h^{\underline{l}}_{\underline{m}} h_{\underline{lm}} \left(\frac{1}{2} F^{\underline{m}}{}_{\underline{k}} F^{\underline{nk}} + (D^{\underline{m}} \Phi)^\dagger D^{\underline{n}} \Phi \right) - \kappa^2 h_{\underline{ks}} h_{\underline{mn}} F^{\underline{km}} F^{\underline{sn}} \\
& + \frac{1}{D_1 - 2} \left(\partial_\mu h^i_i \partial^\mu h^j_j + (\partial_r h^i_i)^2 + D_m h^i_i D^m h^j_j \right) \Big\} \;,
\end{aligned} \tag{43}$$

$$\begin{aligned}
L(V,V) = -\frac{1}{2} \Big(& \partial_\mu V_{\underline{m}} \partial^\mu V^{\underline{m}} + D_{\underline{m}} V_{\underline{n}} D^{\underline{m}} V^{\underline{n}} + \frac{4-3D_1}{4} A'^2 V^2_r \\
& - 2A' V_r D_{\underline{m}} V^{\underline{m}} + R_{\underline{mn}} V^{\underline{m}} V^{\underline{n}} + 2e F_{\underline{mn}} V^{\underline{m}} \times V^{\underline{n}} \\
& + e^2 V^a_{\underline{m}} V^{\underline{m}b} \Phi \{T^a T^b\} \Phi + \kappa^2 (V^{\underline{l}} F_{\underline{lm}})^2 \Big) \;,
\end{aligned} \tag{44}$$

$$\begin{aligned}
L(\phi,\phi) = & -(D_M \phi)^\dagger D^M \phi - \frac{1}{2} \phi \frac{\partial^2 U}{\partial \Phi^2} \phi - \frac{1}{2} \phi^* \frac{\partial^2 U}{\partial \Phi^{*2}} \phi^* - \phi^* \frac{\partial^2 U}{\partial \Phi \partial \Phi^*} \phi \\
& + \frac{e^2}{2} \left(\phi^\dagger T \Phi - \Phi^\dagger T \phi \right)^2 - \frac{\kappa^2}{2} \left(\phi^\dagger D_{\underline{m}} \Phi + (D_{\underline{m}} \Phi)^\dagger \phi \right)^2 \;,
\end{aligned} \tag{45}$$

$$L(h,V) = V^{\underline{n}} \left(D_{\underline{m}} h_{\underline{ln}} F^{\underline{lm}} - h^{\underline{m}}_{\underline{l}} D_{\underline{m}} F^{\underline{l}}{}_{\underline{n}} \right) - \frac{1}{2} A' F^{r\underline{m}} V_{\underline{m}} h^i_i + \frac{D_1 - 2}{2} A' F^{\underline{lm}} V_{\underline{m}} h_{r\underline{l}} \;, \tag{46}$$

$$
\begin{aligned}
L(h,\phi) = h^{\underline{lm}}{}_{;\underline{l}} \left(\phi^\dagger D_{\underline{m}}\Phi + (D_{\underline{m}}\Phi)^\dagger\phi\right) - \frac{1}{2}h^i_i A' \left(\phi^\dagger D_r\Phi + (D_r\Phi)^\dagger\phi\right) \\
+\frac{D_1-2}{2}h^{r\underline{m}}A' \left(\phi^\dagger D_{\underline{m}}\Phi + (D_{\underline{m}}\Phi)^\dagger\phi\right) \\
+h^{\underline{mn}}(D_{\underline{m}}\phi^\dagger D_{\underline{n}}\Phi + D_{\underline{m}}\Phi^\dagger D_{\underline{n}}\phi) \ ,
\end{aligned} \tag{47}
$$

$$
\begin{aligned}
L(V,\phi) = 2iV^{\underline{m}} \left(\phi^\dagger T D_{\underline{m}}\Phi - (D_{\underline{m}}\Phi)^\dagger T\phi\right) - i\frac{D_1-2}{2}A'V_r \left(\phi^\dagger T\Phi - \Phi^\dagger T\phi\right) \\
-\kappa^2 F^{\underline{lm}}V_{\underline{m}} \left(\phi^\dagger D_{\underline{l}}\Phi + (D_{\underline{l}}\Phi)^\dagger\phi\right) \ .
\end{aligned} \tag{48}
$$

We recall that the three fields h^m_m, h^r_r and h^i_i are not independent as their sum vanishes due to the constraint $h=0$. The spin-zero action can be used for an analysis of stability of different brane solutions, see, e.g. [44].

4 Lorentz invariance and the modes decomposition

The light cone gauge is ideally suited for the study of massless excitations because it leaves only physical degrees of freedom for D_1-dimensional graviton and for the vector fields. As for the massive modes, it distributes the members of the same spin multiplet over tensor, vector and scalar equations. A spin two massive particle has $D_1(D_1-1)/2-1$ degrees of freedom, from which $D_1(D_1-3)/2$ components are in the gravity multiplet (33), D_1-2 components in the vector sector (38), and one component in the scalar sector (42).

By the same reasoning, for massive vector excitations D_1-2 components are present in the vector part (38) while one is hidden in the scalar sector (42). This means that one combination of vector fields and one combination of scalar fields must obey equations which give the same spectrum of fluctuation as the tensor part. If the total number of physical vector fields is N_v, then N_v combinations of scalar fields can be decoupled from scalar equations and must have the same massive spectrum, as follows from the vector equations. In this section we will show how to single out these combinations.

To study a spectrum of D_1-dimensional perturbations it is convenient to work in Fourier space, with

$$
h_{MN} \propto V_M \propto \exp\left(ip_\mu x^\mu\right) \ . \tag{49}
$$

For the massive modes one can always choose a coordinate (rest frame of the massive particle) system in which $p_\mu = (m,\ldots,0)$, where m is a mass of an excitation. We shall be using this frame in this Section.

We start from the massive spin-two multiplet. From the D_1-dimensional point of view it is described by a tensor field $H_{\mu\nu}$ which is transverse and traceless,

$$
\partial_\mu H^\mu_\nu = 0, \quad H^\mu_\mu = 0 \ . \tag{50}
$$

This tensor is invariant under D_1-dimensional general-coordinate transformations.

In the rest frame the conditions (50) give $H_{0\mu} = 0$ and $\sum_{i=1}^{D_1-1} H_{ii} = 0$. The components of the tensor $H_{\mu\nu}$ automatically satisfying the conditions (50) can be expressed through the arbitrary metric as $\tilde{h}_{ij}$, then $h_{i(D_1-1)}$, and, finally $\sum_{i=1}^{D_1-2} h_{ii} - \frac{D_1-2}{D_1-1}\sum_{i=1}^{D_1-1} h_{ii}$. The first term is contained in the spin two part of the action (33), the second term is counted as a vector in the light-cone gauge, and the last one as a scalar. The expression in terms of the fields in this gauge are just h_{i+} for the vectors, which is converted to

$$\frac{D_1-1}{2} A' h_{ir} + h_{i\underline{l};}{}^{\underline{l}} \ . \tag{51}$$

with the use of constraint (27). It is this combination of the vector fields that may be decoupled from the vector equations (38) and should have the same spectrum as the tensor part. The scalar field that can be decoupled from (42) and has the same sect rum as (33) is given by

$$h_i^i + (D_1-2) h_+^+ \ , \tag{52}$$

where h_{++} has to be expressed through physical scalar components via constraint equation, which we do not write for a general case because of its complexity (see, however, below).

A similar line of reasoning can be applied for the massive vector fields. From the four-dimensional point of view they are described by the vector fields $h_{\underline{m}\nu}$ and V_μ which are transverse,

$$\partial_\mu h^\mu_{\underline{n}} = 0, \quad \partial_\mu V^\mu \ = 0. \tag{53}$$

In the rest frame we have $h^0_{\underline{n}} = 0$ and $V^0 = 0$. Thus, the components $h_{+\underline{n}}$ and V_+, expressed through the constraints (25), (28) and (29) should give the desired combinations of scalar fields, that have the same spectrum as the corresponding vector modes:

$$\frac{D_1-2}{2} A' V_r + D_{\underline{m}} V^{\underline{m}} - i\left(\phi^\dagger T \Phi - \Phi^\dagger T \phi\right) e \ , \tag{54}$$

$$\frac{D_1-2}{2} A' h_{rm} + h_{m\underline{l};}{}^{\underline{l}} - \kappa^2\left(\phi^\dagger D_m \Phi + (D_m\Phi)^\dagger \phi + V_{\underline{l}} F_n{}^{\ \underline{l}}\right) \ , \tag{55}$$

$$\frac{A'}{2} h_i^i - \left(\frac{D_1-2}{2} A' h_{rr} + h_{r\underline{l};}{}^{\underline{l}} - \kappa^2\left(\phi^\dagger D_r \Phi + (D_r\Phi)^\dagger \phi + V_{\underline{l}} F_r{}^{\ \underline{l}}\right)\right) \ . \tag{56}$$

5 The Randall-Sundrum model

The simplest model where the general equations presented above can be applied is the Randall-Sundrum model in which a thick domain wall is formed by a real scalar field Φ with potential $U(\Phi)$. This model has been studied in many papers. Here we just present some peculiarities that are present in the high-cone gauge. For this model $D_1 = 4$, $D_2 = 0$. However, we shall write the equations in some generality which will also be useful for the study of the monopole background in the next section. For arbitrary D_1 and D_2 we choose the following anzatz for the gauge and the real scalar field,

$$\Phi = \Phi(r), \quad A_\mu = A_r = 0, \quad A_m = A_m(y) \ , \tag{57}$$

where x_μ, $\mu = 0, 1, ..., D_1 - 1$ refer to the co-ordinates of the D_1-dimensional world and y^m, $m = 1, ..., D_2$ and r are the extra coordinates.

Inserting our anzatz into the bosonic field equations we obtain,

$$\begin{aligned}\frac{D_1}{4}(D_1-1)A'^2 + (D_1-1)A'' + \frac{D_2}{2}(D_1-1)A'B' \\ +\frac{1}{4}D_2(D_2+1)B'^2 + D_2B'' - e^{-B}\Omega = \kappa^2 L \ ,\end{aligned} \tag{58}$$

$$\frac{D_1}{4}(D_1-1)A'^2 + \frac{1}{2}D_1D_2A'B' + \frac{D_2}{4}(D_2-1)B'^2 - e^{-B}\Omega = \kappa^2(\Phi'^2 + L) \ , \tag{59}$$

$$\begin{aligned}\frac{D_2}{4}(D_2-1)B'^2 + (D_2-1)B'' + \frac{D_1}{2}(D_2-1)A'B' + \frac{1}{4}D_1(D_1+1)A'^2 + D_1A'' \\ +(\frac{1}{D_2} - \frac{1}{2})e^{-B}\Omega = \kappa^2\left(\frac{e^{-2B}}{e^2}f^2 + L\right) \ ,\end{aligned} \tag{60}$$

where f^2 in eq.(60) is defined by $g^{mn}F_{mk}F_{nl} = f^2 g_{mk}$, $L = -\frac{1}{4}e^{-2B}f^2 - \frac{1}{2}\Phi'^2 - U(\Phi)$. With this assumption we are anticipating that our background solution is going to be of the same type as the ones already analyzed in [45].

The Klein-Gordon equation for Φ can be written as

$$\partial_r(\frac{1}{2}\Phi'^2 - U) = -(\frac{D_1}{2}A' + \frac{D_2}{2}B')\Phi'^2 \ . \tag{61}$$

We have four equations for three functions A, B and Φ. One can however, show that the equations are not over determined and they are consistent. These general equations can be employed to analyze the spectrum of many models in detail. We shall use them in the next section for the case of a monopole background.

Here we specialize to the simple case of the Randall Sundrum model for which $D_1 = 4$ and $D_2 = 0$. The gauge field is also set to zero. The equation of motion for the scalar field then becomes:

$$\Phi'' + 2A'\Phi' = \frac{\partial U}{\partial \Phi} \ , \tag{62}$$

whereas Einstein equations can be written in the form

$$A'' + A'^2 = -\frac{1}{3}\kappa^2\left(\frac{1}{2}\Phi'^2 + U\right) \ , \tag{63}$$

$$A'^2 = \frac{1}{3}\kappa^2\left(\frac{1}{2}\Phi'^2 - U\right) \ . \tag{64}$$

The physical fields are h_{12} and $h_{11}-h_{22}$ (gravity multiplet), h_{ri} (vector multiplet or graviphoton) and two scalar fields h_i^i and ϕ.

The spin-two Lagrangian has a simple form

$$L(spin-2) = -\frac{1}{4\kappa^2}\int d^DX\sqrt{-G}\left(e^{-A}\eta^{\mu\nu}\partial_\mu\tilde{h}_i^j\partial_\nu\tilde{h}_j^i + \partial_r\tilde{h}_i^j\partial_r\tilde{h}_j^i\right) \ . \tag{65}$$

The equation for determination of four-dimensional spectrum is

$$-e^{-2A}\partial_r\left(e^{2A}\partial_r\Psi\right) = m^2e^{-A}\Psi \tag{66}$$

and allows a normalizable zero mode (graviton) provided $A\to\infty$ for $r\to\pm\infty$ which is achieved if the fine-tuning condition of [13] is satisfied.

The spin-one action is

$$L(spin-1) = -\frac{1}{2\kappa^2}\left(\partial_\mu h_{ir}\partial^\mu h^{ir} + \partial_r h_{ir}\partial^r h^{ir}\right) + \frac{1}{4\kappa^2}h_{ri}h^{ri}A'^2 - \frac{1}{4}h_{ri}h^{ri}\Phi'^2 \tag{67}$$

and can be expressed entirely through the metric with the use of equation $\frac{\kappa^2}{3}(\Phi')^2 + A'' = 0$ following from (64). The corresponding equation for the determination of the spectrum

$$-e^{-A}\partial_r\left(e^A\partial_r\Psi\right) - \frac{1}{2}\left(2A'' + A'^2\right) = m^2e^{-A}\Psi \tag{68}$$

is a partner equation of (66) from the point of view of supersymmetric quantum mechanics [46]. It has exactly the same spectrum as for spin-two excitations with removed zero mode – a result which one would expect from four-dimensional Poincare invariance on the brane (five components of a four-dimensional massive spin two field are distributed among the multiplets in the light-cone gauge as $\tilde{h}_i^j$ (two degrees of freedom), h_{ri} (two degrees of freedom), with fifth degree of freedom hiding in some combination of the two scalar fields ϕ and $h_{rr} = -h_i^i$.

Finally, the spin-zero action is

$$\begin{aligned} S(spin-0) = \int d^4xdre^{2A}\Bigg[&-\frac{3}{8}\left(e^{-A}\eta^{\mu\nu}\partial_\mu h_{rr}\partial_\nu h_{rr} + (h'_{rr})^2 - h_{rr}^2(2A'' + A'^2)\right) \\ &-\frac{1}{2}\left(e^{-A}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + (\phi')^2 + \frac{\partial^2U}{\partial^2\Phi^2}\phi^2 + \phi^2(\Phi')^2\right) \\ &-\phi h_{rr}\left(\Phi'' + \frac{1}{2}A'\Phi\right)\Bigg] \ . \end{aligned} \tag{69}$$

It contains fluctuations belonging to the four dimensional massive spin-2 multiplet and a genuine scalar field. A component corresponding to the spin-2 field can be found in a way discussed in the previous section, see eq.(52). In the particle rest-frame it is

$$-h_{rr} + 2h_+^+ \ , \tag{70}$$

where

$$h_+^+ = -h_{rr} + \frac{e^A}{2m^2}\left[3h_{rr}(A'^2 - A'') + 3A'h'rr + 2\phi(\Phi'' - A'\Phi') - 2\phi'\Phi'\right] \ , \tag{71}$$

and m is the particle mass.

Another independent component, corresponding to a genuine scalar field in four dimensions can be found from the requirement of gauge invariance with respect to general-coordinate transformations as in ref. [35] and is equal to

$$\phi + \frac{\Phi'}{3A'}(h_{rr} + h_+^+) \ . \tag{72}$$

According to [35], there are no normalizable zero modes in the scalar sector.

6 Vector fluctuations in specific models

The question of gauge field localization by gravity has been addressed in a number of papers, see, e.g. [25]-[34]. It has been shown that gauge fields cannot be localized in five-dimensional space time, but a normalizable zero mode can exist for $D_1 = 4$, $D_2 \geq 1$. The spectrum of vector field excitation does not have a mass gap, but this may not be dangerous from phenomenological point of view because of the screening phenomenon discussed in [28].

In this section we shall consider some specific models in which a defect is formed by interacting gauge and scalar field. Here analysis of gauge field localization happens to be quite complicated because of the mixing of the gauge fields coming from the metric with the gauge fields. Nevertheless due to the complete separation of fields of different spins in the light cone gauge the analyzes of the physical spectrum in our formalism becomes considerably simpler than in other gauges.

6.1 Nielsen-Olesen string in six dimensions

As we have seen above the minimal Randall-Sundrum model containing gravity and scalar field does not have any four-dimensional vector fields. Models in higher dimensions contain more degrees of freedom and the appearance of localized vector fields is more probable. Consider as an example a Nielsen-Olesen string in six dimensions. It is known that this theory can localize gravity as in the thin string limit [15] as well as for a thick string [17].

The model contains a U(1) gauge field, a complex scalar field and gravity with $D_2 = 1$. We keep D_1 arbitrary. The background metric has the general form (4) with $dy = a d\theta$, $0 \leq \theta < 2\pi$ being an angular coordinate and $0 \leq r < \infty$ a radial coordinate, a is a parameter of the dimension of a length. For a gravity localizing solution A and B go like $-cr$ for $r \to \infty$, where c is some positive constant. The background vector field $A_\theta(r)$ and scalar field $\Phi(r)$ are non-zero and are given by a standard Nielsen-Olesen configuration,

$$\begin{aligned} \Phi(r,\theta) &= v f(r) e^{in\theta}, \\ A_\theta(r) &= \frac{1}{ae}[P(r) - n] \ , \end{aligned} \tag{73}$$

where v is the vev of the scalar field, and the functions $f(r)$ and $P(r)$ satisfy the following boundary conditions:

$$\begin{aligned} f(0) &= 0, & \lim_{r\to\infty} f(r) &= 1, \\ P(0) &= n, & \lim_{r\to\infty} P(r) &= 0. \end{aligned} \tag{74}$$

Their dependence on r can be found by numerical integration of equations of motion and is presented in [17].

In the light-cone gauge one gets three four-dimensional vector fields, namely h_{ri}, $h_{\theta i}$ and V_i. One of the vector degrees of freedom, defined by (51), is in fact a part of four-dimensional massive spin two field and thus is not interesting for us. A combination of V_i and $h_{\theta i}$, to be identified below will represent a localized spin one zero mode, found recently by a direct computation in [47, 48], while a second combination will be a massive spin one field. In the absence of the U(1) gauge field the massless zero mode corresponds to a graviphoton of U(1) isometry group [31].

To determine the structure of zero mode it is helpful [3] to consider a combined action of linearized U(1) gauge transformation with the gauge function $\alpha(x^\mu)$ and a general coordinate transformation of the U(1) isometry group $\delta(a\theta) = \xi^\theta(x^\mu)$ on the $(\theta\mu)$ components of the metric and μ component of the vector field. Using the transformation rules given in equations (17) to (19) we find

$$\delta h_{\theta\mu} = -e^B \partial_\mu \xi^\theta \ , \tag{75}$$

$$\delta V_\mu = -\partial_\mu \xi^\theta A_\theta - \frac{1}{e}\partial_\mu \alpha \ . \tag{76}$$

Under this transformation different scalar fields transform as

$$\delta h_{r\theta} = \delta V_r = 0 \ , \tag{77}$$

$$\delta\phi = (-\xi^\theta \partial_{a\theta} + i\alpha)\Phi \ . \tag{78}$$

From eq. (78) one can easily see that the group $U(1)_{gauge}\times U(1)_{isometry}$ is spontaneously broken down to $U(1)_\gamma$ that corresponds to a gauge transformation with $\alpha = \frac{n}{a}\xi^\theta$. Under this transformation the change of the vector field is

$$\delta V_\mu = -\partial_\mu \xi^\theta (A_\theta + \frac{n}{ae}) = -\frac{1}{ae}\partial_\mu \xi^\theta P(r) \ , \tag{79}$$

while the field h_{ir} is invariant. These transformation rules indicate that the field $\gamma_\mu(x^\nu)$ defined by

$$V_\mu = \frac{1}{ae} P(r)\gamma_\mu, \quad h_{\mu\theta} = e^B \gamma_\mu \tag{80}$$

should correspond to a four-dimensional massless vector field. Indeed, a direct computation gives for the action of γ_i:

$$S = -\frac{b}{2}\int d^{D_1}x \eta^{\mu\nu}\partial_\mu\gamma_i\partial_\nu\gamma_i \tag{81}$$

with a finite normalization factor

$$b = 2\pi a \int_0^\infty dr e^{(\frac{D_1}{2}-2)A+\frac{1}{2}B} \left(\frac{P^2(r)}{a^2 e^2} + \frac{e^B}{\kappa^2} \right) \ . \tag{82}$$

In deriving of eq. (81) one also need to use the difference between the 00 and $\theta\theta$ components of the background equation which takes the form

$$e^B \left(\frac{1}{2}A'' - \frac{1}{2}B'' + \frac{D_1}{4}A'^2 - \frac{1}{4}B'^2 - \frac{1}{4}(D_1-1)A'B' \right) = \frac{\kappa^2}{2a^2}\left(\frac{1}{e^2}P'^2 + 2v^2 f^2 P^2\right) \ . \tag{83}$$

We note that all these integrals are finite even for $D_1 = 4$ for which the warp factor e^A disappears from the exponential of the integrand defining b.

6.2 t'Hooft-Polyakov monopole in seven dimensions

As was shown in [16] (see also [49]), localization of gravity is possible on a t'Hooft-Polyakov monopole configuration provided the model parameters are tuned is a specific way. For this theory $D_1 = 4,\ D_2 = 2$, and $g_{mn}(y)dy^m dy^n = \sin^2\theta d\varphi^2 + d\theta^2$, with $0 \le \theta \le \pi$ and $0 \le \varphi \le 2\pi$ being the angular variables, whereas $0 \le r < \infty$ is a radial coordinate. Asymptotics of the functions A and B for $r \to \infty$ are $A \propto -cr$, $B \to const$. We do not attempt to make an analysis of the vector field zero mode problem here, because of the complexity of the monopole solution (see below for the discussion of an abelian monopole). However, even if the zero modes exist, they are not normalizable because of the behavior of B at infinity.

6.3 $SU(2) \times U(1)$ Gauge Fields

In this subsection we will study the case of an abelian monopole configuration and show that it allows the existence of vector zero modes living in the x^μ subspace with the symmetry group SU(2)×U(1). These modes are localized on a brane for $D_1 \ge 5$. For the physically interesting case of $D_1 = 4$ they are not normalizable. Perhaps, the localization of these modes may be achieved by the radiative corrections to the action of the gauge fields as in [22], or by some non-perturbative mechanism [20, 23].

Let the gauge group $G = U(1)$ and assume that $D_2 = 2$ and that the y coordinates cover a S^2 with the metric

$$ds^2 = a^2(d\theta^2 + \sin\theta^2 d\varphi^2) \ , \tag{84}$$

where a is the radius of the sphere. We will assume that there is a single real scalar field in the system which generates a domain wall and eventually through their Yukawa coupling to the fermions produce chiral fermions localized to the wall as we have demonstrated in [24]. The SU(2) gauge fields will have their origin in the isometries of S^2. The gauge field configuration should thus preserve this symmetry. There is a unique choice with this symmetry, namely, the magnetic monopole configuration on S^2. This configuration will also be responsible for the chirality of fermionic fields localized to the brane.

The U(1) monopole configuration is given by

$$A = \frac{n}{2}(cos\theta - 1)d\varphi \ , \tag{85}$$

where n is an integer.

This solution is one of the class of solutions considered in [45]. There it was shown that the bulk Einstein and gauge field equation in the absence of the scalar field are solved by

$$A(r) = -cr, \qquad B(r) = const, \qquad F_{nk}F_m{}^{\ k} = f^2 g_{mn}, \quad and \quad F_{rm=0} \ , \tag{86}$$

where $c > 0$ and f are constants. Without any loss of generality we can take $B = 0$. The strength of the magnetic field which is given by the constant f ($f^2 = \frac{n^2}{2}$) and which should be quantized for topological reasons.

It is easy to verify that under the assumption that the real scalar field approaches the minima of its potential for $r \to \pm\infty$ the bulk equations will still be solve with the above anzatz. In this section we shall assume that, with the same anzatz for the gauge field, there exists a core solution for the scalar field which joins smoothly the above bulk solution. Near the core the functions A and B will of course be more complicated and will not be given by simple expressions given above. The details of the scalar potential or the solution for the functions Φ, A or B will turn out to be irrelevant to the existence of the massless modes.

Assume that the Φ field configuration creates a $D_1 + 2$ dimensional defects located at some points along the r direction. The D_1 dimensional subspace is conformal to flat Minkowski subspace, while the other 2 directions cover a S^2 of radius a. To study the possibility of localizing gauge fields on the $\mathbb{R}^{D_1}$ part we need to look at the fluctuations around the background solution. Our interest in this section is only on a sub-sector of the vector part comprising V_i and h_{mi} subject to the condition $\nabla_m h^{mi} = (\partial_m + \Gamma^l_{lm})h^m_i = 0$. This condition makes the only other vector field in our problem, namely, h_{ri} to decouple from this sub-sector. To obtain the spectrum in $\mathbb{R}^{D_1}$ we perform harmonic expansion on S^2. The fields $V_i(x, \theta, \phi, r)$ and $h_{mi}(x, \theta, \phi, r)$ can be expanded on spherical modes on S^2 according to the prescription given in [3, 4]. Each expansion mode will carry a pair of $SU(2)$ labels l (specifying the $SU(2)$ irreducible representation) and λ taking on $2l + 1$ values. For the field V_i the values of l start from zero. The expansion is given by,

$$V_i = \sum_{\geq 0} \sqrt{\frac{2l+1}{4\pi}} \sum_{|\lambda| \leq l} V_i^{l\lambda}(x, r) D_0^{l,\lambda}(\theta, \varphi) \ . \tag{87}$$

The $l = 0$ mode is a $SU(2)$ singlet and it decouples from the rest. After integrating over θ and ϕ its action becomes

$$S^{l=0} = a^2 \int d^{D_1}x \int dr e^{(\frac{D_1}{2}-1)A+B}[-\frac{1}{2e^2}e^{-A}\eta^{\mu\nu}\partial_\mu V_i(x,r)\partial_\nu V_i(x,r) \ + \tag{88}$$

$$\frac{1}{2e^2}V_i(\partial_r^2 + ((\frac{D_1}{2} - 1)A' + B')\partial_r)V_i] \ . \tag{89}$$

The operator $e^{(\frac{D_1}{2}-1)A+B}\left(\partial_r^2+\left((\frac{D_1}{2}-1)A'+B'\right)\partial_r\right)$ has the general structure of [3] $e^N(\partial_r^2+N'\partial_r)$ with $N=(\frac{D_1}{2}-1)A+B$. This operator is a negative Hermitian operator. It has two zero modes. The first one is the trivial one, namely, $\partial_r V_i=0$ This mode obviously produces a massless gauge field in $\mathbb{R}^{D_1}$ with an effective gauge coupling constant given by

$$\frac{1}{g^2}=\frac{a^2}{e^2}\int dre^{(\frac{D_1}{2}-2)A+B}\ . \tag{90}$$

Provided that this gauge field has the correct coupling to the chiral fermions localized on $\mathbb{R}^{D_1}$ we can identify it with the $U(1)_Y$ gauge field of the standard $SU(2)_L\times U(1)_Y$ electroweak theory.

The second zero mode has a non trivial dependence on r and is given by $V_i(x,y,r)=\int^r dr'e^{-N(r')}\tilde{V}_i(x,y)$. Using this mode the above coupling should be replaced by

$$\frac{1}{g^2}=\frac{a^2}{e^2}\int dre^{(\frac{D_1}{2}-2)A+B}\psi^2\ , \tag{91}$$

where $\psi(r)=\int^r dr'e^{-N(r')}$.

The value of the kinetic term for the massless vector or the effective gauge coupling after r integration will depend on the details of the background solution. However, there are some general statements which we can make about the convergence of the integral, which will also be valid for the $SU(2)$ coupling to be discussed below. As we are assuming that the defect is created by a scalar field configuration of a kink type we shall assume that the coordinate r ranges from $-\infty$ to $+\infty$. The kink will be assumed to be localized at $r=0$. Away from $r=0$ the scalar field will approach one of the minima of its potential $U(\Phi)$, and it will be very slowly varying so that in the background field equations we can set $\Phi'=0$. As stated above it is easily verified that in this bulk region the background equations admit a solution of the form $B=0$ and $A=cr$, where the constant c is related to the radius of S^2 and the magnetic field f through

$$c^2=\frac{4}{D_1}\left(\frac{\kappa^2 f^2}{2e^2}-\frac{1}{a^2}\right)\ . \tag{92}$$

We thus have two solutions for c differing in sign. Both signs lead to regular bulk geometries. In the vicinity of the brane the shape of the functions $A(r)$ and $B(r)$ will depend on the details of the scalar potential and the detailed form of the scalar field configuration in that region. Unfortunately, neither the trivial r-zero mode nor the non trivial one given by $\int^r dr'e^{-N(r')}$ produce a convergent integral for the gauge coupling for $D_1=4$. On the other hand the trivial zero mode in the gravitational sector produces a finite $D_1=4$ effective Newton's constant[4],

[3]A similar operator exists also for the string solution discussed in section (6.1).

[4]We can perform similar analysis for the localization of graviton. The resulting effective D_1 dimensional gravitational constant will be give by $\frac{1}{\kappa_{D_1}^2}=\frac{a^2}{\kappa^2}\int dre^{(\frac{D_1}{2}-1)A+B}\Psi^2(r)$ where this time Ψ is the gravitational r zero mode which is 1 in the trivial sector and $\Psi(r)=\int^r dr'exp\left(-\frac{D_1}{2}A-B\right)$ in the non-trivial sector. It is easy to see that the trivial r zero mode will give a convergent effective D_1 dimensional gravitational coupling constant, even for $D_1=4$ provided we select the exponentially decaying asymptotic solution for the warp factor.

provided we select the exponentially decreasing warp factor, i.e in the asymptotic region we set $c = -|c|\varepsilon(r)$. This feature will persist also for the non-Abelian effective gauge couplings. We shall offer some speculative ideas which may help to overcome this important difficulty.

Now consider the $l = 1$ sector. This comprises the $SU(2)$ triplets $h_{i\lambda}$ and $V_{i\lambda}$ contained in the harmonic expansion of $h_{mi}(x, \theta, \phi, r)$ and $V_i(x, \theta, \phi, r)$.

According to [3] in order to expand h_{im} into harmonic modes on S^2 we need to use the isohelicity $\pm$ basis. This is constructed from the components of a vector relative to some orthonormal frame at any point in the tangent space to S^2. Let h_{1i}, h_{2i} denote the components of h_{mi} relative to some orthonormal frame. The complex fields $h_{\pm i}$ are defined by

$$h_{\pm i} = \frac{1}{\sqrt{2}}(h_{1i} \mp h_{2i}) \; . \tag{93}$$

Using the formalism of [3, 4] we can now write down the following mode expansion for the fields $h_{\pm i}$

$$h_{i+} = \bar{h}_{-i} = \sum_{l \geq 1} \sqrt{\frac{2l+1}{4\pi}} \sum_{|\lambda| \leq l} h^{l\lambda}_{i+}(x, r) D^{l,\lambda}_{1}(\theta, \varphi) \; . \tag{94}$$

It is not so hard to obtain the bilinear action for the infinite tower of the Kaluza Klein modes. We give the general result in Appendix B. Here we shall restrict our attention to the $l = 1$ sector only. The corresponding action in terms of $h_{i\lambda}$ and $V_{i\lambda}$, where λ is the $SU(2)$ triplet index, is rather complicated. However, if we perform a change of variables from $h_{i\lambda}$ to $U_{i\lambda} = e^{-B} h_{i\lambda}$ and make use of the background equations in an efficient way then we obtain considerable simplification. Again we omit the tedious calculations. The result is

$$\begin{aligned}
S_{l=1}(U, V) \;\; = \;\; & a^2 \int d^{D_1}x \int dr e^{(\frac{D_1}{2}-1)A+B} [-\frac{e^{-A}}{2e^2}\eta^{\mu\nu}\partial_\mu V_i \partial_\nu V_i + \\
& + \frac{1}{2e^2}\bar{V}_i\{\partial_r^2 + ((\frac{D_1}{2}-1)A' + B')\partial_r - \frac{2}{a^2}e^{-B}\}V_i + \\
& + \frac{\kappa}{e^2}\frac{f}{a}e^{-B}(\bar{V}_i U_i + V_i \bar{U}_i) - \\
& - e^{-A+B}\eta^{\mu\nu}\partial_\mu \bar{U}_i \partial_\nu U_i + e^B \bar{U}_i(\partial_r^2 + ((\frac{D_1}{2}-1)A' + 2B')\partial_r - \frac{\kappa^2}{e^2}f^2 e^{-2B})U_i] \; .
\end{aligned}$$

Note that we have suppressed the SU(2) triplet index λ which is summed over the values $\pm 1, 0$. The r derivative operator acting on V_i, U_i have the general structure of $e^N(\partial_r^2 + N'\partial_r)$. This is an Hermitian and negative operator, as we mentioned above. It has again two zero modes, one with trivial dependence on r and the other with a non trivial r dependence of the type we discussed above. In both of the zero mode sectors the action can be diagonalized. For example in the trivial sector the simple transformation $V_i = \alpha V_i' + \beta U_i'$ and $U_i = \frac{1}{2a\kappa f}\alpha V'_i - \frac{1}{2}\frac{a\kappa f}{e^2}\beta U'_i$ where α and β are arbitrary numbers diagonalizes the action. The diagonal action for both the trivial and non-trivial zero mode sectors take the form of

$$S_{l=1}(V', U') = \int d^{D_1}x \left(-\frac{1}{2g_2^2}\eta^{\mu\nu}\partial_\mu \bar{V}_i' \partial_\nu V_i' + \bar{U}'_i(\frac{c}{2}\eta^{\mu\nu}\partial_\mu\partial_\nu - m^2)U_i' \right) \; . \tag{95}$$

Thus the field $V'_{i\lambda}(x_)$ is a triplet of massless vectors to be identified with the gauge fields of $SU(2)_L$ with a coupling constant g_2 given by

$$\frac{1}{g_2^2} = |\alpha|^2 \frac{a^2}{e^2} \int dr e^{(\frac{D_1}{2}-2)A+B} \Psi^2(r) \left(1 + \frac{2e^2}{(a\kappa f)^2} e^B\right) \ , \tag{96}$$

where $\Psi(r) = 1$ for the trivial r-zero mode and $\Psi(r) = \int^r dr' exp\left(-(\frac{D_1}{2}-1)A - B\right)$ for the non-trivial zero mode. It is seen that neither the trivial nor the non trivial r zero modes will cut the r integral when $|r|$ goes to infinity, for the case of $D_1 = 4$. The trivial r zero mode together with asymptotically decaying warp factor, on the other hand produces convergent integrals for both the Abelian as well as the non Abelian gauge couplings for any $D_1 > 4$ and a finite effective gravitational constant for any $D_1 \geq 4$.

The other constants in S above are defined through,

$$c = \frac{a^2|\beta|^2}{e^2} \int dr e^{(\frac{D_1}{2}-2)A+B} \Psi^2(r) \left(1 + \frac{(a\kappa f)^2}{2e^2} e^{-B}\right) \ , \tag{97}$$

$$m^2 = \frac{|\beta|^2}{e^2} \int dr e^{(\frac{D_1}{2}-1)A} \Psi^2(r) \left(1 + \frac{(a\kappa f)^2}{2e^2} e^{-B} + \frac{(a\kappa f)^4}{4e^4} e^{-2B}\right) \ . \tag{98}$$

Note that the complex numbers α and β are not fixed. The normalization of the non linear terms in the Yang-Mills action and the fermion SU(2) coupling constant should fix them.

Thus, together with the singlet massless vector we obtain the four massless vectors of the standard S(2)×U(1) gauge theory. We also obtain a SU(2) triplet of massive vectors U'_i. It is hoped that this vector and other massive bulk modes can be made arbitrarily heavy or weakly coupled to the standard model sector.

6.4 SU(3)×SU(2)×U(1) Gauge Fields

To obtain color SU(3) gauge fields using the above Kaluza-Klein mechanism in the context of the brane world scenarios, we add an extra component to the compact internal space, namely, we take the y subspace to be $S^2 \times CP^2$. Thus $D_2 = 6$. It is sufficient to start from a D-dimensional U(1) gauge theory with a single real scalar field which will produce a defect as in the monopole case. The Einstein, Yang-Mills system will be solved again with the S^2 metric as given in the previous section and with the standard Fubini-Study metric on CP^2. Using a pair of complex coordinates $\zeta^a, a = 1, 2$ on CP^2 this metric is given by

$$ds^2 = \frac{4b^2}{1+\zeta^\dagger\zeta} d\bar{\zeta}^a \left(\delta^{ab} - \frac{\zeta^a\bar{\zeta}^b}{1+\zeta^\dagger\zeta}\right) d\zeta^b, \tag{99}$$

where b is the radius of CP^2. The anzatz for the U(1) gauge field is a magnetic monopole on S^2 and a self -dual instanton on CP^2, which is given by,

$$A = iq\frac{1}{2\left(1+\zeta^\dagger\zeta\right)} \left(\zeta^\dagger d\zeta - d\zeta^\dagger\zeta\right) \ , \tag{100}$$

where q is a real number. The field strength of this vector potential is self-dual. For this reason we call it a U(1) instanton. It is well known that in the absence of a U(1) gauge field of the above type there is a global obstruction to defining a spinor field on CP^2. With the background gauge field as given above this obstruction is removed provided q is taken to be one half of an odd integer[5].

The configuration of the gauge and the metric fields given above solve the coupled bulk Einstein, Yang-Mills equations in the region where the scalar field sits at the minima of its potential. We assume that there is a solution in the core of the defect which smoothly joins the bulk solution as in the monopole case. The isometry group of the Fubini-Study metric is SU(3). Using the techniques similar to those outlined for the monopole case above we will obtain SU(3)× SU(2)× U(1) gauge fields in the subspace spanned by x^μ.

In ref [50] it has been shown that the $S^2 \times CP^2$ together with the magnetic monopole on S^2 and the self dual instanton on CP^2 gives rise to a rich spectrum of chiral fermions in the x-subspace. One can choose the magnetic charge and the instanton number such that the chiral fermions belong to the doublet representation of SU(2) and be at the same time triplets of SU(3).

As in the monopole case in the previous section, these zero modes are localized on the defect for $D_1 \geq 5$ and not normalizable for $D_1 = 4$.

If we wish to obtain all the gauge fields of the standard model from a geometrical origin the minimum number of dimensions needed in our scheme turns out to be $4 + 2 + 4 + 1 = 11$. At the end of the next section we speculate on a possible origin for these gauge fields in the 11-dimensional M-theory.

7 Conclusions

In this paper we gave a detailed analysis of the bosonic fluctuations of field-theoretical branes – topological defects. The general equations we obtained can be used for constructing a low-energy effective theory coming from the fluctuations around classical solutions residing in higher-dimensional space-time.

Whereas localization of gravity is possible provided certain fine-tuning conditions are satisfied, existence of non Abelian normalizable zero modes for the gauge fields is a serious problem in the case of $D_1 \leq 4$. This is not so for the $U(1)$ gauge fields as shown in section 6.1. In this section we demonstrated that in the background of a Nielsen - Olesen string in any $D_1 + 2$ dimensional theory there is a normalizable $U(1)$ gauge field localized to the D_1 dimensional space-time transverse to the string for any D_1. In the monopole background where non Abelian D_1 dimensional gauge fields can be obtained, the spin-1 kinetic energy of the effective D_1 dimensional theory is finite only for $D_1 > 4$. Perhaps, higher dimensional field theories are

[5]This background is one of the configurations studied in detail in reference [50] in the context of a new proposal to solve the gauge hierarchy problem in theories with large extra dimensions.

not relevant for four-dimensional non Abelian physics and a resolution of the problem of the gauge-field localization should come from string theory and D-branes.

A way out at the field theory level would be to add, say, to U(1) Higgs model in six dimensions, forming a string, an extra gauge group that does not couple to the gauge and Higgs field forming a topological defect. However, this does not look particularly natural from model building point of view. Another possibility would be to modify the action of gravity and/or of the gauge field by writing, say $F_{AB}^2 Z(\Phi)$ where Z is some function with the property that it goes to zero when Φ reaches its asymptotic value. This does not look very appealing either. So, it remains to be seen if a natural realistic four-dimensional theory can be constructed along these lines.

It is possible of course that part or all of the standard model gauge group emerges as an unbroken subgroup of a big non-abelian gauge group in higher dimensions. This is what happens in the Calabi-Yau compactification of gauge theories. Being Ricci flat the Calabi Yau spaces do not admit continuous isometries. The 4-dimensional gauge symmetries of particle physics emerge as unbroken subgroups of the $E_8 \times E_8$ group of the heterotic string theory in ten dimensions. If we insist on a geometrical origin for the gauge symmetries the minimum number of dimensions in our scheme is 4+ 2+4+1=11. This is identical to the number of the dimensions of the M-theory which is believed to have the 11-dimensional supergravity as its low energy limit. The 11-dimensional supergravity has neither elementary scalar fields nor a $U(1)$ gauge field in its spectrum. Its bosonic spectrum consists of a third rank antisymmetric potential A_{MNP} and of course the gravity G_{MN}. The component $A_{m\mu\nu}$, where, m is tangent to S^2 or to CP^2 will look like a vector field on S^2 and a vector field on CP^2. It will be like a scalar field from the point of view of the 4-dimensional space time covered by the x^μ coordinates. It can play the role of the $U(1)$ gauge field in our construction above. There are also several candidates for real scalar fields which can depend on the r coordinate only, for example, $A_{\mu\nu\lambda}$ is one of them. Unfortunately the 11-dimensional supergravity, at least at the classical level, does not have a non zero potential for these scalars, as the only non linear term involving the tensor field A_{MNP} is the Chern-Simons term. It remains to be seen, nevertheless, if we can generate a solution of the type we need, at the quantum level.

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Appendix A: The curvature tensor for a warped metric

Our convention for the Ricci tensor is $R\mu\nu = R^{\lambda}_{\mu\lambda\nu}$, where the Riemann tensor is defined by

$$R^{\mu}_{\gamma\alpha\beta} = \partial_\alpha \Gamma^{\mu}_{\beta\gamma} - \partial_\beta \Gamma^{\mu}_{\alpha\gamma} + \Gamma^{\mu}_{\alpha\lambda}\Gamma^{\lambda}_{\beta\gamma} - \Gamma^{\mu}_{\beta\lambda}\Gamma^{\lambda}_{\alpha\gamma} \tag{A.1}$$

For the warped metric (4) the different non-zero components of the connections with mixed indicies are:

$$\Gamma^{r}_{\mu\nu} = -\frac{1}{2}\eta_{\mu\nu}A'e^{A} \ , \Gamma^{\mu}_{\nu r} = \frac{1}{2}\delta^{\mu}_{\nu}A' \ , \tag{A.2}$$

$$\Gamma^{r}_{mn} = -\frac{1}{2}g_{mn}B'e^{B} \ , \Gamma^{m}_{nr} = \frac{1}{2}\delta^{m}_{n}B' \ . \tag{A.3}$$

The curvature and Ricci tensors are:

$$R_{\mu\nu\rho\sigma} = \frac{1}{4}\left(A'e^{A}\right)^2 \left(\eta_{\nu\rho}\eta_{\mu\sigma} - \eta_{\nu\sigma}\eta_{\mu\rho}\right) \ , \tag{A.4}$$

$$R_{r\mu r\nu} = -\frac{1}{2}e^{A}\eta_{\mu\nu}\left(A'' + \frac{1}{2}A'^2\right) \ , \tag{A.5}$$

$$R_{m\mu n\nu} = -\frac{1}{4}A'B'e^{A+B}\eta_{\mu\nu}g_{mn} \ , \tag{A.6}$$

$$R_{rmrn} = -\frac{1}{2}e^{B}g_{mn}\left(B'' + \frac{1}{2}B'^2\right) \ , \tag{A.7}$$

$$R_{mnkl} = e^{B}\Omega_{mnkl} + \frac{1}{4}\left(B'e^{B}\right)^2 \left(g_{nk}g_{ml} - g_{nl}g_{mk}\right) \ , \tag{A.8}$$

$$R_{\mu\nu} = -\frac{1}{2}e^{A(r)}\eta_{\mu\nu}\left(A'' + \frac{A'}{2}(D_1A' + D_2B')\right) \ , \tag{A.9}$$

$$R_{rr} = -\frac{D_1}{2}\left(A'' + \frac{1}{2}A'^2\right) - \frac{D_2}{2}\left(B'' + \frac{1}{2}B'^2\right) \ , \tag{A.10}$$

$$R_{mn} = \Omega_{mn} - \frac{1}{2}e^{B(r)}g_{mn}\left(B'' + \frac{B'}{2}(D_1A' + D_2B')\right) \ , \tag{A.11}$$

and the scalar curvature is

$$R = e^{-B(r)}\Omega - D_1A'' - D_2B'' - \frac{1}{4}(D_1A' + D_2B')^2 - \frac{1}{4}(D_1A'^2 + D_2B'^2) \ , \tag{A.12}$$

where Ω_{mnkl}, Ω_{mn} and Ω are the curvature, Ricci and the scalar curvatures constructed from the metric $g_{mn}(y)$.

Appendix B: General Vector Bilinears

The general vector bilinear in the presence of a neutral scalar can be shown by some calculation to be given by (we make judicious use of the background field equations and rescale $h_{mi} \to e^B h_{mi}$ and assume that $F_{mr} = 0$),

$$
\begin{aligned}
S = \quad & \int d^D x \; e^{\frac{D_1}{2}A + \frac{D_2}{2}B} \left[\frac{1}{2e^2} \; e^{-2A} \; V_i \eta^{\mu\nu} \; \partial_\mu \partial_\nu \; V_i \right. \\
& + \frac{1}{2e^2} \; e^{-A} V_i \left\{ \partial_r^2 + \left(\left(\frac{D_1}{2} - 1 \right) A' + \frac{D_2}{2} B' \right) \; \partial_r \; - e^{-B} g^{mn} D_m D_n \right\} \; V_i \\
& - \frac{\kappa}{e^2} \; e^{-A-B} g^{mn} \; g^{\ell k} D_n V_i \; h_{ki} \; F_{m\ell} \\
& + \frac{1}{2} \; g^{mn} \; h_{mi} \left\{ \; e^{-2A-B} \; \eta^{\mu\nu} \; \partial_\mu \; \partial_\nu \right. \\
& + e^{-A+B} \left\{ \partial_r^2 + \left(\left(\frac{D_1}{2} - 1 \right) \; A' + \left(\frac{D_2}{2} + 1 \right) \; B' \right) \; \partial_r \right. \\
& \left. \left. \left. + e^{-B} \left(\frac{1}{D_2} \Omega + g^{k\ell} \nabla_k \; \nabla_\ell \right) \; - \frac{\kappa^2 f^2}{e^2} \; e^{-2B} \right\} \; h_{ni} \right] \; .
\end{aligned}
$$

In principle we can expand V_i and h_{mi} on the basis of the eigenfunctions of hermitian differential operators, $e^{\left(\frac{D_1}{2} - 1\right) \; A + \frac{D_2}{2} \; B} \left\{ \partial_r^2 + \left(\left(\frac{D_1}{2} - 1 \right) A' + \frac{D_2}{2} B' \right) \partial_r \right\}$ acting on V_i and

$$
e^{\left(\frac{D_1}{2} - 1\right) A + \left(\frac{D_2}{2} + 1\right) B} \left\{ \partial_r^2 + \left(\left(\frac{D_1}{2} - 1 \right) A' + \left(\frac{D_1}{2} + 1 \right) B' \right) \partial_r \right\} \tag{B.1}
$$

acting on h_{mi}. Both of these operators are of the form $e^N (\partial_r^2 + N' \partial_r)$. Obvious zero modes of these operators are given by,

$$
\partial_r \; V_i = \partial_r \; h_{im} = 0 \; . \tag{B.2}
$$

If r ranges from 0, to ∞ or $-\infty$ to ∞ these modes are not normalizable in the inner product defined by $(\psi, \chi) = \int_0^\infty \; dr \; \psi^* \chi$. This may not matter for the construction of a viable effective theory localized to the defect.

There is another non trivial zero mode defined by

$$
\left(\partial_r^2 + N' \partial_r \right) V_i = 0 \; , \tag{B.3}
$$

$$
V_i(x, y, r) = \tilde{V}_i(x, y) \int^r dr' e^{-N(r')} \; . \tag{B.4}
$$

Then

$$
(V_i, V_i) = \tilde{V}_i(x, y) \; \tilde{V}_i(x, y) \int^\infty dr \; \int^r \; dr' \; e^{-N(r')} \int^r \; dr'' e^{-N(r'')} \; . \tag{B.5}
$$

For us $N = \left(\frac{D_1}{2} - 1\right) A + \alpha B$, where $\alpha = \frac{D_2}{2}$ or $\frac{D_2}{2} + 1$.

It is conceivable that there are solutions for which the above integrals are convergent.

Let us keep the discussion general and write

$$V_i(x, y, r) = e^{\psi(r)}\ \tilde{V}_i(r, y)\ , \tag{B.6}$$

$$h_{im}(x, y, r) = e^{\chi(r)}\ \tilde{h}_{im}(x, y)\ , \tag{B.7}$$

such that V_i and h_{im} are annihilated by their respective operators $e^N(\partial_r^2 + N'\partial_r)$.

For these zero modes the bilinear action becomes,

$$\begin{aligned} S = \quad & \int d^3x\ e^{\frac{D_1}{2}A + \frac{D_2}{2}B} \Bigg[\ \tilde{V}_i \left\{ \frac{e^{-A+2\psi}}{2e^2} \left(e^{-A}\eta^{\mu\nu}\partial_\mu\partial_\nu + e^{-B}g^{mn}D_m D_n\right) \right\} \tilde{V}_i \\ & - \frac{\kappa}{e^2} e^{-A-2B+\psi+\chi} g^{mn} g^{k\ell} D_m \tilde{V}_i \tilde{h}_{ni} F_{m\ell} \\ & + \frac{1}{2} g^{mn} \tilde{h}_{mi} \left\{ e^{-A+2\chi} \left(e^{-A+B}\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{1}{D_2}\Omega + g^{k\ell}\nabla_k\nabla_\ell - \frac{n^2 f^2}{e^2} e^{-B} \right) \right\}\ h_{mi} \Bigg]\ . \end{aligned}$$

This bilinear action is quite general, applicable to the Abelian and non-Abelian backgrounds. Now we specialize to the monopole background.

Take $D_2 = 2$ and $G = U(1)$ with $F_{mn} = f\varepsilon_{mn}$. We need to make the following substitutions [3]:

$$g^{mn} D_m D_n \tilde{V}_i \to -\frac{1}{a^2}\ell(\ell+1)\tilde{V}_i \quad \ell \geq 0\ , \tag{B.8}$$

$$g^{mn} D_m D_n \tilde{h}_{\pm i} = -\frac{1}{a^2}\left\{\ell(\ell+1) - 1\right\}\ \tilde{h}_{\pm i} \quad \ell \geq 1\ . \tag{B.9}$$

Upon integrating over S^2 we obtain

$$
\begin{aligned}
S = \quad & a^2 \int d^{D_1}x \int dr\ e^{\left(\frac{D_1}{2}-2\right)A+B+2\psi} \frac{1}{2e^2} \tilde{V}_i \eta^{\mu\nu} \partial_\mu \partial_\nu \tilde{V}_i \\
& + a^2 \int d^{D_1}x \int dr e^{\frac{D_1}{2}A+B} \sum_{\ell \geq 1} \sum_{-\ell \leq \lambda \leq \ell} \\
& \overline{V}_i^{\ell,\lambda} \left\{ \frac{e^{-A+2\psi}}{2e^2} \left(e^{-A} \eta^{\mu\nu} \partial_\mu \partial_\nu - \frac{1}{a^2} e^{-B} \ell(\ell+1) \right) V_i^{\ell,\lambda} \right. \\
& + \frac{\kappa f}{e^2 a} \sqrt{\frac{\ell(\ell+1)}{2}}\ e^{-A-B+\psi+\chi} \left(\overline{V}_i^{\ell,\lambda}\ U_i^{\ell,\lambda} + V_i^{\ell,\lambda} \overline{U}_i^{\ell,\lambda} \right) \\
& + \overline{U}_i^{\ell,\lambda} \left\{ e^{-A+2\chi} \left(e^{-A+B} \eta^{A+B} \eta^{\mu\nu} \partial_\mu \partial_\nu + \right. \right. \\
& \left. \left. \left. \frac{1}{a^2} - \frac{1}{a^2} \left(\ell(\ell+1) - 1 \right) - \frac{\kappa^2 f^2}{e^2} e^{-B} \right) \right\} U_i^{\ell,\lambda} \right\} \ ,
\end{aligned}
$$

where $U_i^{\ell,\lambda}$ are the $SU(2)$ modes of the rescaled h_{mi}. The first term is the $l = 0$ contribution given in section 6.3. It is easy to check that the $l = 1$ term also coincides with the one given in section 6.3.

Although the general result has been obtained with no assumption about the behavior of the solution near the core of the defect or defects, it in fact coincides with the result which we would have obtained in the absence of the scalar field and in the bulk region. In this case of course the function B has to be set equal to a constant. In other words the effect of the scalar field which generates the defects is felt in the general bilinear equation above only through its effect on the functions A, B and the zero mode functions ψ and χ.

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