Modified $N = 1$ Green-Schwarz superstring with irreducible first class constraints

A.A. Deriglazov*

Instituto de Física, Universidade Federal de Juiz de Fora, MG, Brasil.

Abstract

We propose modified action which is equivalent to $N = 1$ Green-Schwarz superstring and which allows one to realize the supplementation trick [26]. Fermionic first and second class constraints are covariantly separated, the first class constraints (1CC) turn out to be irreducible. We discuss also equations of motion in the covariant gauge for κ-symmetry. It is shown how the usual Fock space picture can be obtained in this gauge.

PAC codes: 0460D, 1130C, 1125

Keywords: Covariant quantization, Superstring

1 Introduction

Manifestly super Poincare invariant formulation of branes implies appearance of mixed first and second class fermionic constraints in the Hamiltonian formalism (equivalently one has infinitely reducible local κ-symmetry in the Lagrangian formalism) [1-10]. Typically, first and second class constraints (2CC) are treated in a rather different way in quantum theory[1]. In particular, to construct formal expression for the covariant path integral one needs to have splitted and irreducible constraints [13, 14]. So, it is necessary at first to split the constraints, which can be achieved by using of covariant projectors of one or other kind [15-17, 12]. Details depend on the model under consideration. For example, for CBS superparticle [18, 1] one introduces two auxilliary vector variables in addition to the initial superspace coordinates [15]. For the Green-Schwarz (GS) superstring the projectors can be constructed in terms of the initial

*alexei@fisica.ufjf.br On leave of absence from Dept. Math. Phys., Tomsk Polytechnical University, Tomsk, Russia

[1]Quantization scheme for mixed constraints was developed in [11]. Application of this scheme to concrete models may conflict with manifest Poincare covariance [12].

variables only [16]. After that the problem reduces to quantization of the covariantly separated but *infinitely* reducible constraints. Despite a lot of efforts (see [15-23] and references therein) this problem has no fully satisfactory solution up to date. Namely, infinitely reducible 1CC imply infinite tower of "ghost for ghosts" variables [22]. For reducible 2CC the problem is (besides the problem of quantum realization) that the covariant Dirac bracket obeys the Jacobi identity on the second class constraints surface only [12]. A revival of interest to the problem is due to recent work [24] where it was shown that scattering amplitudes for superstring can be constructed in a manifestly covariant form, as well as due to progress in the light-cone quantization of superstring on $AdS_5 \times S^5$ background [25, 24].

One possibility to avoid the problem of quantization of infinitely reducible constraints is the supplementation trick which was formulated in the Hamiltonian framework in [26]. The basic idea is to introduce additional fermionic variables subject to their own reducible constraints (the constraints are chosen in such a way that the additional sector do not contains physical degrees of freedom). Then the original constraints can be combined with one from the additional sector into irreducible set. For the latter one imposes a covariant and irreducible gauge. It implies, in particular, a possibility to construct correct Dirac bracket for the theory.

Next problem arising in this context is the problem of linearisation of the physical sector dynamics. Crucial property of the standard noncovariant gauge $\Gamma^+\theta = 0$ is that equations of motion in this case acquire linear form. Then it is possible to find their general solution. While not necessary for construction of the formal path integral, namely this fact allows one to fulfill really the canonical quantization procedure. Similarly to this, the covariant gauge will be reasonable only if it has the same property. Unfortunately, for the superstring in the covariant gauge for κ-symmetry and in the usual gauge for world sheet symmetry, equations of motion remain nonlinear. It will be shown that the problem can be avoided if one imposes "off-diagonal gauge" for $d = 2$ fields.

In symmary, the necessary steps toward to manifestly covariant formulation look as follows: mixed constraints $\mapsto$ separated but reducible constraints $\mapsto$ separeted irreducible constraints $\mapsto$ covariant irreducible gauge $\mapsto$ free dynamics of the physical sector. The final formulation admits application of the standard quantization methods in the covariant form.

To apply the recipe for concrete model one needs to find a modified Lagrangian action which reproduces the desired irreducible constraints. Some examples were considered [26, 27], in particular, the modified $N = 1$ GS superstring action was proposed in [28]. But it was pointed in [29] that the action is not equivalent to the initial one (to split the constraints, an additional bosonic variables were introduced. Zero modes of the variables survive in the physical sector).

In this work we present modified action which is equivalent to $N = 1$ GS superstring and which allows one to realize the supplementation trick. We consider $N = 1$ theory as a toy example for type II GS superstring and restrict our attention to 1CC only. The reason is that, due to special structure of type II theory, its 2CC do not represent a problem and can be combined into irreducible set.

The work is organized as follows. To fix our notations, we review main steps of the supplementation scheme in Sec. 2 (see also [26]). In Sec. 3 we consider bosonic string (in ADM representation for $d = 2$ metric) and discuss its dynamics in the off-diagonal gauge. It will be shown that the resulting Fock space picture of state spectrum is the same as in the standard gauge. We present also relation among string coordinates in these two gauges. The trick turns out to be necessary for linearisation of the superstring equations of motion in the covariant gauge for κ-symmetry. In Sec. 4 modified formulation of $N = 1$ GS superstring action is presented and proved to be equivalent to the initial one. First and second class constraints are covariantly separated, 1CC form irreducible set. Equations of motion and their solution in the covariant gauge for κ-symmetry are discussed in Sec. 5. Some technical details are arranged in the Appendixes.

2 Supplementation of the reducible constraints.

It will be convenient to work in 16-component formalism of the Lorentz group $SO(1,9)$, then θ^α, ψ_α, $\alpha = 1, \ldots, 16$, are Majorana–Weyl spinors of opposite chirality. Real, symmetric 16×16 Γ-matrices $\Gamma^\mu{}_{\alpha\beta}$, $\tilde\Gamma^{\mu\alpha\beta}$ obey the algebra $\Gamma^\mu\tilde\Gamma^\nu + \Gamma^\nu\tilde\Gamma^\mu = -2\eta^{\mu\nu}$, $\eta^{\mu\nu} = (+, -, \ldots, -)$. Momenta conjugate to configuration space variables q^i are denoted as p_{qi}.

Let us consider a dynamical system with fermionic pairs $(\theta^\alpha, p_{\theta\alpha})$ being presented among the phase space variables z^A. Typical situation for the models under consideration is that the following constraints

$$L_\alpha \equiv p_{\theta\alpha} - iB_\mu\Gamma^\mu{}_{\alpha\beta}\theta^\beta \approx 0, \qquad D^\mu D_\mu \approx 0, \tag{1}$$

are presented among others. Here, the $B^\mu(z)$ and $D^\mu(z)$ are some functions of phase variables z, so that $D^2 \approx 0$ is first class constraint. Poisson bracket of the fermionic constraints is

$$\{L_\alpha, L_\beta\} = 2iD_\mu \Gamma^\mu{}_{\alpha\beta}. \qquad (2)$$

The system $L_\alpha \approx 0$ is mixture of first and second class constraints, as it will be proved momentarily. Supplementation scheme for the mixed constraints consist of the following steps.

A). Manifestly covariant separation of the constraints.
Let us extend the initial phase space by a pair of vectors $(\Lambda^\mu, p_{\Lambda\mu})$ subject to constraints

$$\Lambda^2 \approx 0, \qquad p^\mu_\Lambda \approx 0. \qquad (3)$$

Supposing that $\Lambda D \neq 0$ (which will be true for the models under consideration), one can extract two 2CC: $\Lambda^2 \approx 0$, $p_\Lambda D \approx 0$ and nine 1CC: $\tilde{p}^\mu_\Lambda \equiv p^\mu_\Lambda - \frac{p_\Lambda D}{\Lambda D}\Lambda^\mu \approx 0$ (there is identity $D\tilde{p}_\Lambda \equiv 0$). Thus the variables introduced are non physical. Eq.(3) has first stage of reducibility and can be quantized by standard methods [34, 35].

Below the following two facts will be used systematically (proof is presented in the Appendix 2).
1). Let $\Psi^\alpha = 0$ are 16 equations. Then
a). The system

$$D^\mu \Gamma^\mu \Psi = 0, \qquad (4)$$

$$\Lambda^\mu \Gamma^\mu \Psi = 0, \qquad (5)$$

is equivalent to $\Psi^\alpha = 0$.
b) The quantities (4), (5) belong to P_-, P_+ subspaces correspondingly.
c) Let $\Psi^\alpha = 0$ represent 16 independent equations. Then Eq.(4) contains 8 independent equations. In $SO(8)$ notations they mean that 8_s part $\bar{\Psi}_{\dot{a}}$ of Ψ^α can be presented through 8_c part Ψ_a (or vice-versa). The same is true for Eq.(5).
2). Let $\Psi^\alpha = 0$, $\Phi^\alpha = 0$ are $16 + 16$ independent equations. Consider the system

$$D^\mu \Gamma^\mu \Psi = 0, \qquad \Lambda^\mu \Gamma^\mu \Phi = 0. \qquad (6)$$

Then a) Eq.(6) contains 16 independent equations according to 1).
b) The equations

$$D^\mu \Gamma^\mu \Psi + \Lambda^\mu \Gamma^\mu \Phi = 0, \qquad (7)$$

are equivalent to the system (6). Thus the system (7) consist of 16 independent equations (i.e. it is irreducible).

The system (1) can be rewritten now in the equivalent form

$$L^{(1)\alpha} \equiv D_\mu \tilde{\Gamma}^{\mu\alpha\beta} L_\beta \approx 0, \tag{8}$$

$$L^{(2)\alpha} \equiv \Lambda_\mu \tilde{\Gamma}^{\mu\alpha\beta} L_\beta \approx 0. \tag{9}$$

The equivalence follows from the statement 1), or directly from invertibility of the matrix[2] $(\Lambda_\mu + D_\mu)\Gamma^\mu{}_{\alpha\beta}$: $(\Lambda_\mu + D_\mu)\Gamma^\mu(L^{(1)} + L^{(2)}) \approx -2(D\Lambda)L$. Among 1CC $L^{(1)} \approx 0$ and 2CC $L^{(2)} \approx 0$ there are in eight linearly independent (see Appendix 1).

B). Auxiliary sector subject to reducible constraints.
Let us further introduce a pair of spinors $(\eta^\alpha, p_{\eta\alpha})$ subject to the constraints

$$p_{\eta\alpha} \approx 0, \qquad T_\alpha \equiv \Lambda_\mu \Gamma^\mu{}_{\alpha\beta}\eta^\beta \approx 0. \tag{10}$$

These equations contain 8 independent 1CC among $p_\eta^{(1)} \equiv \Lambda_\mu \tilde{\Gamma}^\mu p_\eta \approx 0$ and 8+8 independent 2CC among $p_\eta^{(2)} \equiv D_\mu \tilde{\Gamma}^\mu p_\eta \approx 0$, $\Lambda_\mu \Gamma^\mu \eta \approx 0$. Note that the covariant gauge $D_\mu \Gamma^\mu \eta = 0$ may be imposed. After that the complete system (constraints + gauges) is equivalent to $p_\eta \approx 0$, $\eta \approx 0$.

C). Supplementation up to irreducible constraints.
Now part of the constraints can be combined into covariantly separated and irreducible sets. According to the statement 2), the system (8)-(10) is equivalent to

$$\Phi^{(1)\alpha)} \equiv L^{(1)\alpha} + p_\eta^{(1)\alpha} = D_\mu \tilde{\Gamma}^{\mu\alpha\beta} L_\beta + \Lambda_\mu \tilde{\Gamma}^{\mu\alpha\beta} p_{\eta\beta} \approx 0, \tag{11}$$

$$\Phi^{(2)\alpha} \equiv L^{(2)\alpha} + p_\eta^{(2)\alpha} = \Lambda_\mu \tilde{\Gamma}^{\mu\alpha\beta} L_\beta + D_\mu \tilde{\Gamma}^{\mu\alpha\beta} p_{\eta\beta} \approx 0, \tag{12}$$

$$T_\alpha \equiv \Lambda_\mu \Gamma^\mu{}_{\alpha\beta}\eta^\beta \approx 0, \tag{13}$$

where $\Phi^{(1)\alpha} \approx 0$ ($\Phi^{(2)\alpha} \approx 0$) are 16 irreducible 1CC (2CC) and $T_\alpha \approx 0$ include 8 linearly independent 2CC. In the result first class constraints of the extended formulation form irreducible set (11). As it was mentioned above the type II superstring presents an example of more attractive situation as compare to the general case (11)-(13). Due to special structure of the theory the 2CC can also be combined into irreducible set.

[2] To split the constraints one can also use true projectors instead of the matrices $D_\mu \Gamma^\mu$, $\Lambda_\mu \Gamma^\mu$, see Appendix 2. It allows one to avoid possible "second class patalogy" [30] in the 1CC algebra [16].

3 Bosonic string in "off-diagonal gauge".

In this section, on example of the bosonic string, we discuss two additional tools which will be used below. First, the modified superstring action acquires more elegant form in ADM representation for $d = 2$ metric. Second, to analyze equations of motion in the covariant gauge for κ-symmetry it will be convenient to use a trick which we refer here as off-diagonal gauge for $d = 2$ fields.

Starting from the bosonic string

$$S = -\frac{T}{2}\int d^2\sigma \frac{1}{\sqrt{-g}} g^{ab}\partial_a x^\mu \partial_b x^\mu, \tag{14}$$

let us consider ADM representation

$$g^{00} = \frac{1}{\gamma N^2}, \quad g^{01} = \frac{N_1}{\gamma N^2}, \quad g^{11} = \frac{N_1^2 - N^2}{\gamma N^2}, \quad \sqrt{-g} = \frac{1}{\gamma N}, \tag{15}$$

then

$$N = \frac{\sqrt{-\det g^{ab}}}{g^{00}}, \qquad N_1 = \frac{g^{01}}{g^{00}}, \qquad \gamma = \frac{-g^{00}}{\det g^{ab}}. \tag{16}$$

The action (14) acquires now the form

$$\begin{aligned} S &= -\frac{T}{2}\int d^2\sigma \frac{1}{N} D_+ x^\mu D_- x^\mu, \\ D_\pm x^\mu &\equiv \partial_0 x^\mu + N_\pm \partial_1 x^\mu, \qquad N_\pm \equiv N_1 \pm N, \end{aligned} \tag{17}$$

while the world-sheet reparametrisations in this representation look as

$$\delta\sigma^a = \xi^a, \qquad \delta N_\pm = \partial_0\xi^1 + (\partial_1\xi^1 - \partial_0\xi^0)N_\pm - \partial_1\xi^0 N_\pm^2. \tag{18}$$

By direct application of the Dirac procedure one obtains the Hamiltonian

$$H = \int d\sigma \left[-\frac{N}{2}\left(\frac{1}{T}p^2 + T(\partial_1 x)^2\right) - N_1(p\partial_1 x) + \lambda_N p_N + \lambda_{N1} p_{N1}\right], \tag{19}$$

where λ_q are the Lagrangian multipliers for the corresponding primary constraints. Dynamics is governed by the equations of motion

$$\partial_0 x^\mu = -\frac{N}{T}p^\mu - N_1\partial_1 x^\mu, \quad \partial_0 p^\mu = -\partial_1[TN\partial_1 x^\mu + N_1 p^\mu], \tag{20}$$

which are accompanied by the first class constraints

$$p_N = 0, \qquad p_{N1} = 0, \tag{21}$$

$$(\frac{1}{T}p^\mu \pm \partial_1 x^\mu)^2 = 0. \tag{22}$$

In the standard gauge

$$N = 1, \qquad N_1 = 0, \tag{23}$$

one has

$$\partial_0 x^\mu = -\frac{1}{T}p^\mu, \qquad \partial_0 p^\mu = -T\partial_1\partial_1 x^\mu, \tag{24}$$

which implies $(\partial_0^2 - \partial_1^2)x^\mu = 0$, $(\partial_0^2 - \partial_1^2)p^\mu = 0$. Now one can look for solution in terms of oscillators. Let us show that, instead of this, one can equivalently to start from the system $\partial_-\tilde{x}^\mu = 0$, $\partial_-\tilde{p}^\mu = 0$ and look for its general solution[3]. From the latter one restores the general solution of Eq.(24). Actually, Eq.(24) can be rewritten as

$$\partial_+ x^\mu = -\frac{1}{T}\tilde{p}^\mu, \tag{25}$$

$$\partial_-\tilde{p}^\mu = 0, \tag{26}$$

where $\tilde{p}^\mu \equiv p^\mu - T\partial_1 x^\mu$. Eq.(26) has the solution $\tilde{p}^\mu(\tau,\sigma) = \tilde{p}^\mu(\sigma^+)$. Then (25) is ordinary differential equation with the solution $x^\mu = z^\mu - \frac{1}{T}\int_0^{\sigma^+} dl\tilde{p}^\mu(l)$, where z^μ is general solution for $\partial_+ z^\mu = 0$. Equivalently, one solves $\partial_-\tilde{x}^\mu = 0$, then $z^\mu = \tilde{x}^\mu(\sigma^+ \mapsto \sigma^-)$. Collecting all this one has the following result:
Let $\tilde{x}^\mu$, $\tilde{p}^\mu$ represent general solution of the system

$$\partial_-\tilde{x}^\mu = 0, \qquad \partial_-\tilde{p}^\mu = 0. \tag{27}$$

Then the quantities

$$\begin{aligned} x^\mu(\tau,\sigma) &= \tilde{x}^\mu(\sigma^+ \mapsto \sigma^-) - \frac{1}{T}\int_0^{\sigma^+} dl\tilde{p}^\mu(l), \\ p^\mu(\tau,\sigma) &= \frac{1}{2}[\tilde{p}^\mu - T\partial_-\tilde{x}^\mu(\sigma^+ \mapsto \sigma^-)], \end{aligned} \tag{28}$$

give general solution of the system (24).

Note that Eq.(27) can be obtained from Eq.(20) if one takes[4]

$$N = 0, \qquad N_1 = -1, \tag{29}$$

instead of the gauge (23). Note also that while the action (17) is not well defined for the value $N = 0$, the Hamiltonian formulation

[3] It follows immediately from the wave equation for x^μ. We prefer to work with the Hamiltonian equations of motion since it gives automatically brackets for oscillators.

[4] Curious fact is that the membrane equations of motion turn out to be free also for similar choice.

(19)-(22) admits formally Eq.(29) as the gauge fixing conditions for the constraints (21).

One more observation is that the transition (28) to the initial variables is not necessary in the canonical quantization framework. Namely, solution of Eq.(27) in terms of oscillators leads to the same description of state space as those of (24). Actually, solution of Eq.(27) is ($0 \leq \sigma \leq \pi$, closed string)

$$
\begin{aligned}
x^\mu(\tau,\sigma) &= X^\mu + \frac{i}{\sqrt{\pi T}}\sum_{n\neq 0}\frac{1}{n}\beta^\mu_n e^{2in(\tau+\sigma)}, \\
p^\mu(\tau,\sigma) &= \frac{1}{\pi}P^\mu - 2\sqrt{\frac{T}{\pi}}\sum_{n\neq 0}\gamma^\mu_n e^{2in(\tau+\sigma)}.
\end{aligned} \tag{30}
$$

From these expressions one extracts the Poisson brackets for coefficients. For the variables

$$
\bar{\alpha}^\mu_n \equiv \beta^\mu_n + \gamma^\mu_n, \quad \alpha^\mu_n \equiv \beta^\mu_{-n} - \gamma^\mu_{-n}, \quad \alpha^\mu_0 = -\bar{\alpha}^\mu_0 = \frac{1}{2\sqrt{\pi T}}P^\mu, \tag{31}
$$

one obtains the properties

$$
\begin{aligned}
\{\alpha^\mu_n, \alpha^\nu_k\} = \{\bar{\alpha}^\mu_n, \bar{\alpha}^\nu_k\} = in\eta^{\mu\nu}\delta_{n+k,0}, &\quad \{X^\mu, P^\nu\} = \eta^{\mu\nu}, \\
(\alpha^\mu_n)^* = \alpha^\mu_{-n}, &\qquad (\bar{\alpha}^\mu_n)^* = \bar{\alpha}^\mu_{-n}.
\end{aligned} \tag{32}
$$

In terms of these variables the Virasoro constraints (22) acquire the standard form

$$
L_n = \frac{1}{2}\sum_{\forall k}\alpha_{n-k}\alpha_k = 0, \quad \bar{L}_n = \frac{1}{2}\sum_{\forall k}\bar{\alpha}_{n-k}\bar{\alpha}_k. \tag{33}
$$

Eqs.(32), (33) have the same form as those of string in the gauge (23) [31-33]. Thus, instead of the standard gauge one can equivalently use the conditions (29), which gives the same structure of state space. This fact will be used in Sec. 5 for the superstring. If it is necessary, the initial string coordinate x^μ can be restored by means of Eq.(28).

4 $N = 1$ Green-Schwarz superstring with irreducible first class constraints.

Consider GS superstring action with $N = 1$ space time supersymmetry

$$
S = -\frac{T}{2}\int d^2\sigma\left[\frac{1}{\sqrt{-g}}g^{ab}\Pi^\mu_a\Pi^\mu_b + 2i\varepsilon^{ab}\partial_a x^\mu\theta\Gamma^\mu\partial_b\theta\right], \tag{34}
$$

where $\sqrt{-g}=\sqrt{-\det g^{ab}}$, $\Pi^\mu_a\equiv\partial_a x^\mu-i\theta\Gamma^\mu\partial_a\theta$, $\varepsilon^{01}=-1$. Let us denote

$$\begin{aligned}B^\mu\equiv p^\mu+T\Pi^\mu_1,\qquad \hat{p}^\mu\equiv p^\mu-iT\theta\Gamma^\mu\partial_1\theta,\\ D^\mu\equiv\hat{p}^\mu+T\Pi^\mu_1=p^\mu+T\partial_1 x^\mu-2iT\theta\Gamma^\mu\partial_1\theta,\\ \Lambda^\mu\equiv\hat{p}^\mu-T\Pi^\mu_1=p^\mu-T\partial_1 x^\mu.\end{aligned}\tag{35}$$

Then constraints under the interest for Eq.(34) can be written as

$$L_\alpha\equiv p_{\theta\alpha}-iB_\mu\Gamma^\mu{}_{\alpha\beta}\theta^\beta\approx 0,\qquad D^2\approx 0,\tag{36}$$

$$\Lambda^2\approx 0.\tag{37}$$

From Eqs.(35)-(36) and from the standard requirement that the induced metric is non degenerated it follows

$$D\Lambda=\hat{p}^2-T\Pi^2_1\neq 0.\tag{38}$$

The constraints obey the algebra

$$\{L_\alpha,L_\beta\}=2iD_\mu\Gamma^\mu{}_{\alpha\beta}\delta(\sigma-\sigma');\tag{39}$$

$$\begin{aligned}\{D^2,D^2\}=4T[D^2(\sigma)+D^2(\sigma')]\partial_\sigma\delta(\sigma-\sigma'),\\ \{\Lambda^2,\Lambda^2\}=-4T[\Lambda^2(\sigma)+\Lambda^2(\sigma')]\partial_\sigma\delta(\sigma-\sigma'),\\ \{D^2,\Lambda^2\}=0;\end{aligned}\tag{40}$$

$$\{L_\alpha,D^2\}=8iTD^\mu(\Gamma^\mu\partial_1\theta)_\alpha\delta(\sigma-\sigma')\tag{41}$$

From comparison of Eqs(36)-(39) with Eqs.(1)-(3) one concludes that the first step of supplementation scheme is *not necessary* here, since the quantity Λ^μ with the properties $\Lambda^2=0$, $(D\Lambda)\neq 0$ is constructed from the variables in our disposal. Thus one needs to find only a modification which will lead to Eq.(10).

To achieve this it will be convenient to work in the ADM representation (15). The action (34) acquires then the following form:

$$S=-\frac{T}{2}\int d^2\sigma\left[\frac{1}{N}\Pi^\mu_+\Pi^\mu_-+2i\varepsilon^{ab}\partial_a x^\mu\theta\Gamma^\mu\partial_b\theta\right],\tag{42}$$

where it was denoted

$$\Pi^\mu_\pm\equiv\Pi^\mu_0+N_\pm\Pi^\mu_1,\qquad N_\pm\equiv N_1\pm N.\tag{43}$$

Modified action to be examined is

$$S=-\frac{T}{2}\int d^2\sigma\left[\frac{1}{N}\Pi^\mu_+(\Pi^\mu_-+i\eta\Gamma^\mu\chi)+2i\varepsilon^{ab}\partial_a x^\mu\theta\Gamma^\mu\partial_b\theta-\frac{1}{4N}(\eta\Gamma^\mu\chi)^2\right],\tag{44}$$

where two additional Majorana-Weyl fermions $\eta^\alpha(\tau,\sigma)$, $\chi^\alpha(\tau,\sigma)$ were introduced. Our aim now will be to show canonical equivalence of this action and the initial one. Then the additional sector will be used to arrange 1CC of the theory into irreducible set.

Direct application of the Dirac algorithm gives us the Hamiltonian

$$H = \int d\sigma \left[-\frac{N}{2}\left(\frac{1}{T}\hat{p}^2 + T\Pi_1^2\right) - N_1(\hat{p}\Pi_1) - \frac{i}{2}(\hat{p}^\mu - T\Pi_1^\mu)\eta\Gamma^\mu\chi + \right. \\ \left. \lambda_N p_N + \lambda_{N1}p_{N1} + \lambda_\eta p_\eta + \lambda_\chi p_\chi + L\lambda_\theta \right], \quad (45)$$

where λ_q are the Lagrangian multipliers for the corresponding primary constraints. After determining of secondary constraints, complete constraint system can be written in the following form (in the notations (35))

$$p_N = 0, \qquad p_{N1} = 0, \quad (46)$$

$$H_+ \equiv D^2 - 4TL\partial_1\theta = 0, \quad H_- \equiv \Lambda^2 - 4T\partial_1\eta p_\eta - 4T\partial_1\chi p_\chi = 0, \quad (47)$$

$$L_\alpha \equiv p_{\theta\alpha} - iB_\mu\Gamma^\mu{}_{\alpha\beta}\theta^\beta = 0, \quad (48)$$

$$G_\alpha \equiv \Lambda^\mu(\Gamma^\mu\eta)_\alpha = 0, \qquad p_{\eta\alpha} = 0. \quad (49)$$

$$S_\alpha \equiv \Lambda^\mu(\Gamma^\mu\chi)_\alpha = 0, \qquad p_{\chi\alpha} = 0. \quad (50)$$

Note that the desired constraints (10) appear in duplicate form (49), (50). Combinations of the constraints in Eq.(47) are chosen in such a way that all mixed brackets (i.e. those among Eq.(47) and Eqs.(48)-(50)) vanish. The constraints $H_\pm$, L_α obey the Poisson bracket algebra (39), (40), while the remaining non zero brackets are written in the Appendix 3. The constraints (46), (47) are first class. Important moment is that there are no of tertiary constraints in the problem. Actually, from the condition that the constraints (48)-(50) are conserved in time, one obtains

$$D^\mu\Gamma^\mu(\lambda_\theta - N_+\partial_1\theta) = 0, \quad (51)$$

$$Z^\alpha \equiv \Lambda^\mu(\Gamma^\mu\lambda_\eta)_\alpha + iT[\partial_1(\eta\Gamma^\mu\chi)](\Gamma^\mu\eta)_\alpha = 0, \quad (52)$$

and the same as (52) for λ_χ. Eq.(51) allows one to determine half of the multipliers λ_θ. According to statement 1), Eq.(52) is equivalent to

$$\Lambda^\mu\tilde{\Gamma}^\mu Z = 0, \quad (53)$$

$$D^\mu \tilde{\Gamma}^\mu Z = 0. \tag{54}$$

One finds that (53) vanishes on the constraint surface, while manifest form of (54) is

$$2(D\Lambda)\tilde{P}_{-\beta}^{\alpha}\lambda_\eta^\beta = iTD^\mu[\partial_1(\eta\Gamma^\nu\chi)](\tilde{\Gamma}^\mu\Gamma^\nu\eta)^\alpha. \tag{55}$$

Here $\tilde{P}_\pm$ are covariant projectors (A.12) on eight-dimensional subspaces. From Eqs.(54), (55), (A.17) it follows that both sides of Eq.(55) belong to the same subspace $\tilde{P}_-$. So, Eq.(55) do not contains of new constraints and allows one to determine half of the multipliers λ_η.

To proceed further, let us make partial fixation of gauge. One imposes $N = 1,\ N_1 = 0$ for Eq.(46) and $D^\mu\Gamma^\mu\chi = 0$ for 1CC $\Lambda^\mu\Gamma^\mu p_\chi = 0$ contained in Eq.(50). After that, the pairs $(N, p_N), (N_1, p_{N1}), (\chi, p_\chi)$ can be omitted from consideration. The Dirac bracket for the remaining variables coincides with the Poisson one. In the same fashion, the pair η, p_η can be omitted also. Then the remaining constraints (as well as equations of motion) coincide with those of the GS superstring, which proves equivalence of the actions (34) and (44).

On other hand, retaining the constraints (49), the system (48), (49) can be rewritten equivalently as in (11)-(13), or, in the manifest form

$$\Phi^\alpha \equiv (\hat{p}^\mu + T\Pi_1^\mu)(\tilde{\Gamma}^\mu L)^\alpha + (\hat{p}^\mu - T\Pi_1^\mu)(\tilde{\Gamma}^\mu p_\eta)^\alpha = 0, \tag{56}$$

$$\begin{aligned} (\hat{p}^\mu - T\Pi_1^\mu)(\tilde{\Gamma}^\mu L)^\alpha + (\hat{p}^\mu + T\Pi_1^\mu)(\tilde{\Gamma}^\mu p_\eta)^\alpha = 0, \\ (\hat{p}^\mu - T\Pi_1^\mu)(\Gamma^\mu\eta)_\alpha = 0, \end{aligned} \tag{57}$$

with the irreducible 1CC (56) which are separated from the 2CC (57).

Covariant and irreducible gauge for Eq. (56) can be chosen as

$$R_\alpha \equiv \Lambda^\mu(\Gamma^\mu\theta)_\alpha + D^\mu(\Gamma^\mu\eta)_\alpha = 0, \tag{58}$$

or, equivalently

$$\Lambda^\mu(\Gamma^\mu\theta)_\alpha = 0, \qquad D^\mu(\Gamma^\mu\eta)_\alpha = 0. \tag{59}$$

Matrix of the Poisson brackets

$$\{\Phi^\alpha, R_\beta\} = [2(D\Lambda)\delta_\beta^\alpha + 4iTD^\mu(\tilde{\Gamma}^\mu\Gamma^\nu\partial_1\theta)^\alpha(\Gamma^\nu\eta)_\beta]\delta(\sigma - \sigma'), \tag{60}$$

has a body on its diagonal and is invertible. It means that Eqs.(56), (58) allows one to construct the Dirac bracket without fermionic patalogies.

5 Superstring dynamics in the covariant gauge for κ-symmetry.

To study equations of motion for physical variables let us consider the gauge $D^\mu\Gamma^\mu\chi=0$ and Eq.(59). Then the variables $(\chi,p_\chi),(\eta,p_\eta)$ can be omitted. Equations of motion for the theory look now as follows

$$\begin{aligned}\partial_0 x^\mu &= -\frac{N}{T}p^\mu - N_1\partial_1 x^\mu - i\theta\Gamma^\mu(\lambda_\theta - N_+\partial_1\theta),\\ \partial_0 p^\mu &= \partial_1[-TN\partial_1 x^\mu - N_1 p^\mu - iT\theta\Gamma^\mu(\lambda_\theta - N_+\partial_1\theta)],\\ \partial_0\theta &= -\lambda^\alpha_\theta.\end{aligned} \tag{61}$$

The remaining constraints are (46)-(48), which are acompanied by the covariant gauge condition

$$\Lambda^\mu\Gamma^\mu\theta=0. \tag{62}$$

Its conservation in time gives the condition

$$\Lambda^\mu\Gamma^\mu(\lambda_\theta - N_-\partial_1\theta)=0. \tag{63}$$

The Lagrangian multipliers λ_θ can be determined now from Eqs.(51), (63)

$$\lambda^\alpha_\theta = N_1\partial_1\theta^\alpha + N\tilde{K}^\alpha{}_\beta\partial_1\theta^\beta. \tag{64}$$

By using of this result in Eq.(61) one has

$$\begin{aligned}\partial_0 x^\mu &= -\frac{N}{T}p^\mu - N_1\partial_1 x^\mu + 2iN\theta\Gamma^\mu\tilde{P}_-\partial_1\theta,\\ \partial_0 p^\mu &= \partial_1[-TN\partial_1 x^\mu - N_1 p^\mu + 2iNT\theta\Gamma^\mu\tilde{P}_-\partial_1\theta],\\ \partial_0\theta^\alpha &= -N_1\partial_1\theta^\alpha - \tilde{K}^\alpha{}_\beta\partial_1\theta^\beta.\end{aligned} \tag{65}$$

In the standard gauge $N=1,\ N_1=0$ for the constraints (46) one obtains equations of motion in the following form: $\partial_0\theta^\alpha=-\tilde{K}^\alpha{}_\beta\partial_1\theta^\beta$, $\partial_0 x^\mu = -\frac{1}{T}p^\mu + i\theta\Gamma^\mu\partial_+\theta$, $\quad \partial_0 p^\mu = -T\partial_1[\partial_1 x^\mu - i\theta\Gamma^\mu\partial_+\theta]$. Thus, they remain nonlinear. Moreover, in this gauge we failed to find an appropriate set of variables that would obey to the free equations (the only quantity with desired property is Λ^μ: $\quad \partial_-\Lambda^\mu=0$).

Nevertheless, usual picture of the Fock space can be obtained in the covariant gauge. To resolve the problem one can use the same trick which was considered above for the bosonic case. Namely, after substitution $N=0,\ N_1=-1$ (equivalent choice is $N_1=1$) into Eq.(65), one obtains free equations of motion

$$\partial_- x^\mu=0, \qquad \partial_- p^\mu=0, \qquad \partial_-\theta_a=0, \tag{66}$$

where $\theta_a,\ a=1,\ldots,8$ is 8_c part of θ^α, while 8_s part $\theta_{\dot a}$ is determined by the covariant gauge condition $\Lambda^\mu\Gamma^\mu\theta=0$. Fermionic dynamics is the same as in the light-cone gauge $\Gamma^+\theta=0$. Bosonic sector leads to correct description as it was proved in Sec. 3. Thus the covariant gauge (62) allows one to obtain the same structure of state space as those in the light-cone gauge.

6 Conclusion

In this work we have proposed modified action (44) for $N=1$ GS superstring. In addition to usual superspace coordinates it involves a pair of the Majorana-Weyl spinors. The additional variables are subject to reducible constraints (49), (50) which supply their non-physical character (see discussion after Eq.(10)). Equivalence of the modified action and the initial one was proved in the canonical quantization framework (see discussion after Eq.(55)). We have demonstrated also how the state spectrum can be studied in the covariant gauge for κ-symmetry.

In the modified formulation first class constraints form irreducible set (56) and are separated from the second class one (57). The corresponding covariant gauge (58) is irreducible also, which garantees applicability of the usual path integral methods for the 1CC sector of the theory. In the considered theory with one supersymmetry, reducibility of the second class constraints can not be avoided. But it turns out to be possible for type IIB GS superstring. For this case the theory has two copies of the fermionic constraints (which correspond to two $\theta^A,\ A=1,2$) with the same chirality. It allows one to consider their Poincare covariant combinations. In this case both first and second class constraints can be arranged into covariant sets in the initial formulation. For the type IIA theory the two copies of constraints have an opposite chirality and can not be combined in the initial formulation. Repeating the same procedure as in $N=1$ case, one finds that all the second class constraints can be combined into irreducible sets. These results will be presented in a forthcoming work.

Acknowledgments.

The work has been supported partially by Project GRACENAS No 97-6.2-34.

Appendix 1. $SO(1,9)$ and $SO(8)$ notations

Manifest expression of $SO(1,9)$, Γ-matrices 16×16 through $SO(8)$ γ-matrices is

$$\Gamma^0=\begin{pmatrix}\mathbf{1}_8 & 0\\ 0 & \mathbf{1}_8\end{pmatrix},\quad \tilde{\Gamma}^0=\begin{pmatrix}-\mathbf{1}_8 & 0\\ 0 & -\mathbf{1}_8\end{pmatrix},$$
$$\Gamma^i=\begin{pmatrix}0 & \gamma^i{}_{a\dot{a}}\\ \bar{\gamma}^i{}_{\dot{a}a} & 0\end{pmatrix},\quad \tilde{\Gamma}^i=\begin{pmatrix}0 & \gamma^i{}_{a\dot{a}}\\ \bar{\gamma}^i{}_{\dot{a}a} & 0\end{pmatrix},$$
$$\Gamma^9=\begin{pmatrix}\mathbf{1}_8 & 0\\ 0 & -\mathbf{1}_8\end{pmatrix},\quad \tilde{\Gamma}^9=\begin{pmatrix}\mathbf{1}_8 & 0\\ 0 & -\mathbf{1}_8\end{pmatrix}. \tag{A.1}$$

Here $\gamma^i{}_{a\dot{a}}$, $\bar{\gamma}^i{}_{\dot{a}a}\equiv(\gamma^i{}_{a\dot{a}})^{\mathrm{T}}$ are real $SO(8)$ γ-matrices which obey [31]

$$\gamma^i\bar{\gamma}^j+\gamma^j\bar{\gamma}^i=2\delta^{ij}\mathbf{1}_8, \tag{A.2}$$

and $i,a,\dot{a}=1,\ldots,8$. Majorana-Weyl spinors of $SO(1,9)$ group Ψ^α, Φ_α can be decomposed in terms of their $SO(8)$ components. Namely, from Eq.(A.1) it follows that in the decomposition

$$\Psi^\alpha=(\Psi_a,\bar{\Psi}_{\dot{a}}),\qquad \Phi^\alpha=(\Phi_a,\bar{\Phi}_{\dot{a}}), \tag{A.3}$$

the part Ψ_a ($\bar{\Psi}_{\dot{a}}$) is 8_c (8_s) representation of $SO(8)$ group. The matrices $\Gamma^\pm=\frac{1}{2}(\Gamma^0\pm\Gamma^9)$ can be used to extract these components: $\Gamma^+\Psi\subset 8_c$, $\Gamma_-\Psi\subset 8_s$. It breaks $SO(1,9)$ symmetry up to $SO(8)$ subgroup. To keep $SO(1,9)$ group one needs to use covariant projectors described in Appendix 2.

Fermionic constraints (1) in $SO(8)$ notations are

$$\begin{aligned}L_a&=p_{\theta a}-i(\sqrt{2}B^-\theta_a-B^i\gamma^i_{a\dot{a}}\bar{\theta}_{\dot{a}})=0,\\ \bar{L}_{\dot{a}}&=\bar{p}_{\theta\dot{a}}-i(\sqrt{2}B^+\bar{\theta}_{\dot{a}}-B^i\bar{\gamma}^i_{\dot{a}a}\theta_a)=0,\end{aligned} \tag{A.4}$$

and obey the algebra

$$\{L_a,L_b\}=2\sqrt{2}iD^-\delta_{ab},\qquad \{\bar{L}_{\dot{a}},\bar{L}_{\dot{b}}\}=2\sqrt{2}iD^+\delta_{\dot{a}\dot{b}},$$
$$\{L_a,\bar{L}_{\dot{b}}\}=-2iD^i\gamma^i_{a\dot{b}}, \tag{A.5}$$

as a consequence of Eq.(2). Thus they have no of definite class. In the separated form (8), (9) one has

$$\begin{aligned}\{L^{(1)\alpha},L^{(1)\beta}\}&=\{L^{(1)\alpha},L^{(2)\beta}\}\approx 0,\\ \{L^{(2)\alpha},L^{(2)\beta}\}&\approx -4i(D\Lambda)(\Lambda\tilde{\Gamma})^{\alpha\beta},\end{aligned} \tag{A.6}$$

on the constraint surface. Eqs.(8), (9) contains in eight linearly independent equations according to the statement 1). In $SO(8)$ notations one has

$$L^{(1)}_a=-\sqrt{2}D^+L_a-D^i\gamma^i_{a\dot{a}}\bar{L}_{\dot{a}},\quad \bar{L}^{(1)}_{\dot{a}}=-\sqrt{2}D^-\bar{L}_{\dot{a}}-D^i\bar{\gamma}^i_{\dot{a}a}L_a,$$

$$L^{(2)}_a = -\sqrt{2}\Lambda^+ L_a - \Lambda^i \gamma^i_{a\dot a}\bar L_{\dot a}, \quad \bar L^{(2)}_{\dot a} = -\sqrt{2}\Lambda^- \bar L_{\dot a} - \Lambda^i \tilde\gamma^i_{\dot a a} L_a. \quad (A.7)$$

One can take $L^{(1)}_a = 0,\ L^{(2)})_a = 0$ as linearly independent sets of first and second class constraints. Then the corresponding non zero bracket

$$\{L^{(2)}_a, L^{(2)}_b\} \approx -4\sqrt{2}i(D\Lambda)\Lambda^- \delta_{ab}, \quad (A.8)$$

is manifestly invertible.

Appendix 2. Covariant projectors and their properties.

To extract 8_c or 8_s part of any quantity Ψ^α (Φ_α) one can use the matrices $\Gamma^\pm$ ($\tilde\Gamma^\pm$). Covariant generalisation of the latter is given by the projectors $\tilde P_\pm$ ($P_\pm$) defined below.

Starting from $SO(1,9)$ vectors $D^\mu,\ \Lambda^\mu$ and antisymmetric product of Γ-matrices

$$\tilde\Gamma^{\mu\nu} \equiv \tilde\Gamma^\mu\Gamma^\nu - \tilde\Gamma^\nu\Gamma^\mu = 2(\tilde\Gamma^\mu\Gamma^\nu + \eta^{\mu\nu}) = -2(\eta^{\mu\nu} + \tilde\Gamma^\nu\Gamma^\mu), \quad (A.9)$$

one has

$$D^\mu\Lambda^\nu(\tilde\Gamma^{\mu\nu})^\alpha{}_\gamma D^\rho\Lambda^\delta(\tilde\Gamma^{\rho\delta})^\gamma{}_\beta = 4[(D\Lambda) - D^2\Lambda^2]\delta^\alpha_\beta. \quad (A.10)$$

Let $D^2 = \Lambda^2 = 0,\ (D\Lambda) \neq 0$. Then the matrix

$$\tilde K^\alpha{}_\beta \equiv \frac{1}{2(D\Lambda)} D^\mu\Lambda^\nu(\tilde\Gamma^{\mu\nu})^\alpha{}_\beta, \quad (A.11)$$

obeys $\tilde K^\alpha{}_\gamma \tilde K^\gamma{}_\beta = \delta^\alpha_\beta$. It allows one to define the projectors ($\tilde P_+ + \tilde P_- = 1,\ \tilde P^2_- = \tilde P_-,\ \tilde P^2_+ = \tilde P_+,\ \tilde P_+\tilde P_- = 0$)

$$\tilde P^\alpha_{\pm\beta} = \frac{1}{2}(\delta^\alpha_\beta \pm \tilde K^\alpha{}_\beta). \quad (A.12)$$

It is convenient to introduce also the "untilded" projectors

$$P_{\pm\alpha}{}^\beta = \frac{1}{2}(\delta^\beta_\alpha \pm K_\alpha{}^\beta),$$
$$K_\alpha{}^\beta \equiv \frac{1}{2(D\Lambda)} D^\mu\Lambda^\nu(\Gamma^{\mu\nu})_\alpha{}^\beta, \qquad K^2 = 1. \quad (A.13)$$

Their properties are as follows:

$$(P_\pm)_\alpha{}^\beta = (\tilde P_\mp)^\beta{}_\alpha,$$
$$(D\tilde\Gamma)(\Lambda\Gamma) = -2(D\Lambda)\tilde P_-, \qquad (\Lambda\tilde\Gamma)(D\Gamma) = -2(D\Lambda)\tilde P_+. \quad (A.14)$$

Commutation rules for the matrices $\tilde{K},\ K$

$$
\begin{aligned}
\tilde{K}^{\alpha}{}_{\gamma}(\tilde{\Gamma}^{\mu})^{\gamma\beta} = \frac{2}{(D\Lambda)}[D^{\mu}(\Lambda\tilde{\Gamma}) - \Lambda^{\mu}(D\tilde{\Gamma})]^{\alpha\beta} + (\tilde{\Gamma}^{\mu})^{\alpha\gamma}K_{\gamma}{}^{\beta}, \\
\tilde{K}(D\tilde{\Gamma}) = -\frac{1}{(D\Lambda)}[(D\Lambda)D\tilde{\Gamma} - D^2\Lambda\tilde{\Gamma}], \\
\tilde{K}(\Lambda\tilde{\Gamma}) = \frac{1}{(D\Lambda)}[(D\Lambda)\Lambda\tilde{\Gamma} - \Lambda^2 D\tilde{\Gamma}], \\
(\Lambda\Gamma)\tilde{K} = -\frac{1}{(D\Lambda)}[(D\Lambda)\Lambda\Gamma - \Lambda^2 D\Gamma], \\
(D\Gamma)\tilde{K} = \frac{1}{(D\Lambda)}[(D\Lambda)D\Gamma - D^2\Lambda\Gamma],
\end{aligned}
\tag{A.15}
$$

imply

$$
\tilde{P}_{\pm}\tilde{\Gamma}^{\mu} = \tilde{\Gamma}^{\mu}P_{\pm} \pm \frac{1}{(D\Lambda)}D^{\mu}(\Lambda\tilde{\Gamma}) \mp \frac{1}{(D\Lambda)}\Lambda^{\mu}(D\tilde{\Gamma}), \tag{A.16}
$$

$$
\begin{aligned}
\tilde{P}_{+}(D\tilde{\Gamma}) &= \frac{D^2}{2(D\Lambda)}\Lambda\tilde{\Gamma} \approx 0, \\
\tilde{P}_{+}(\Lambda\tilde{\Gamma}) &= \Lambda\tilde{\Gamma} - \frac{\Lambda^2}{2(D\Lambda)}D\tilde{\Gamma} \approx \Lambda\tilde{\Gamma}, \\
\tilde{P}_{-}(D\tilde{\Gamma}) &= D\tilde{\Gamma} - \frac{D^2}{2(D\Lambda)}\Lambda\tilde{\Gamma} \approx D\tilde{\Gamma}, \\
\tilde{P}_{-}(\Lambda\tilde{\Gamma}) &= \frac{\Lambda^2}{2(D\Lambda)}D\tilde{\Gamma} \approx 0,
\end{aligned}
\tag{A.17}
$$

$$
\begin{aligned}
(D\Gamma)\tilde{P}_{+} &= D\Gamma - \frac{D^2}{2(D\Lambda)}\Lambda\Gamma \approx D\Gamma, \\
(\Lambda\Gamma)\tilde{P}_{+} &= \frac{\Lambda^2}{2(D\Lambda)}D\Gamma \approx 0, \\
(D\Gamma)\tilde{P}_{-} &= \frac{D^2}{2(D\Lambda)}\Lambda\Gamma \approx 0, \\
(\Lambda\Gamma)\tilde{P}_{-} &= \Lambda\Gamma - \frac{\Lambda^2}{2(D\Lambda)}D\Gamma \approx \Lambda\Gamma.
\end{aligned}
\tag{A.18}
$$

Properties for the untilded quantities are obtained from Eqs.(A.16)-(A.18) by substitution $\tilde{P}_{\pm} \mapsto P_{\pm},\ \tilde{\Gamma} \leftrightarrow \Gamma$. Eqs.(A.17), (A.18) mean that the matrix $(D\tilde{\Gamma})_{\alpha\beta}$ belong to $\tilde{P}_{-}$ subspace for the first index and to P_{+} subspace for the second index. The matrix $(\Lambda\tilde{\Gamma})$ has an opposite properties.

These properties allows one to prove two statements formulated in Sec.2. 1a) follows from invertibility of the matrix $(\Lambda_\mu + D_\mu)\tilde{\Gamma}^\mu$: $(\Lambda_\mu+D_\mu)\tilde{\Gamma}^\mu[D\Gamma\Psi+\Lambda\Gamma\Psi] \approx -2(D\Lambda)\Psi$. 1b) follows from Eqs.(A.17), (A.18). 1c) follows from manifest form of Eq.(4) in $SO(8)$ notations

$$\sqrt{2}D^-\Psi_a - D^i\gamma^i_{a\dot{a}}\bar{\Psi}_{\dot{a}} = 0, \tag{A.19}$$

$$\sqrt{2}D^+\bar{\Psi}_{\dot{a}} - D^i\bar{\gamma}^i_{\dot{a}a}\Psi_a = 0. \tag{A.20}$$

From Eq.(A.20) one has

$$\bar{\Psi}_{\dot{a}} = \frac{1}{\sqrt{2}D^+}D^i\bar{\gamma}^i_{\dot{a}a}\Psi_a. \tag{A.21}$$

Substitution of Eq.(A.21) into Eq.(A.19) gives identity on the surface $\Lambda^2 = D^2 = 0$.

The statement 2) is immediate consequence of the statement 1) and Eqs.(A.17), (A.18).

Instead of the weak projectors (A.12), (A.13) one can define the strong one, starting from the matrix

$$\tilde{K}^\alpha{}_\beta \equiv \frac{1}{2b}D^\mu\Lambda^\nu(\tilde{\Gamma}^{\mu\nu})^\alpha{}_\beta, \qquad b \equiv \sqrt{(D\Lambda) - D^2\Lambda^2}, \tag{A.22}$$

instead of Eq.(A.11).

Appendix 3. Constraint algebra

Some useful Poisson brackets are

$$\begin{aligned} \{D^\mu, D^\nu\} = 2T\eta^{\mu\nu}\partial_\sigma\delta, \qquad \{D^\mu, \Lambda^\nu\} = 0,& \\ \{\Lambda^\mu, \Lambda^\nu\} = -2T\eta^{\mu\nu}\partial_\sigma\delta,& \\ \{L_\alpha, \Lambda^\mu\} = 0, \qquad \{L_\alpha, D^\mu\} = 4iT(\Gamma^\mu\partial_1\theta)_\alpha\delta,& \end{aligned} \tag{A.23}$$

where $\delta \equiv \delta(\sigma - \sigma')$. Non zero brackets of the constraints (47)-(50) consist of Eqs.(39), (40) as well as the following one:

$$\begin{aligned} \{G_\alpha, p_{\eta\beta}\} = -\Lambda^\mu\Gamma^\mu_{\alpha\beta}\delta, \qquad \{S_\alpha, p_{\chi\beta}\} = -\Lambda^\mu\Gamma^\mu_{\alpha\beta}\delta,& \\ \{G_\alpha, G_\beta\} = -T[(\eta\Gamma^\mu)_\alpha(\eta\Gamma^\mu)_\beta(\sigma)+& \\ (\eta\Gamma^\mu)_\alpha(\eta\Gamma^\mu)_\beta(\sigma')]\partial_\sigma\delta + T(\eta\Gamma^\mu\partial_1\eta)\Gamma^\mu_{\alpha\beta}\delta,& \\ \{S_\alpha, S_\beta\} = -T[(\chi\Gamma^\mu)_\alpha(\chi\Gamma^\mu)_\beta(\sigma)+& \\ (\chi\Gamma^\mu)_\alpha(\chi\Gamma^\mu)_\beta(\sigma')]\partial_\sigma\delta + T(\chi\Gamma^\mu\partial_1\chi)\Gamma^\mu_{\alpha\beta}\delta,& \\ \{G_\alpha, S_\beta\} = -2T[(\eta\Gamma^\mu)_\alpha(\chi\Gamma^\mu)_\beta(\sigma)\partial_\sigma\delta-& \\ 2T(\eta\Gamma^\mu)_\alpha(\partial_1\chi\Gamma^\mu)_\beta\delta.& \end{aligned} \tag{A.24}$$

Note that to check Eq.(39) one needs to use $D = 10$ Γ-matrix identity

$$\Gamma^\mu_{\alpha(\beta}\Gamma^\mu{}_{\gamma\sigma)} = 0. \tag{A.25}$$

References

[1] L. Brink and J. H. Schwarz, Phys. Lett. B **100** (1981) 310.

[2] M.B. Green and J.H. Schwarz, Phys. Lett. B **136** (1984) 367.

[3] W. Siegel, Phys. Lett. B **128** (1983) 397.

[4] E. Bergshoeff, E. Sezgin, and P.K. Townsend, Ann. of Physics, **185** (1988) 330.

[5] P.K. Townsend, Phys. Lett B **373** (1996) 68.

[6] M. Cederwall, A. von Gussich, B.E.W. Nilsson, and A. Westenberg, Nucl. Phys. B **490** (1997) 163; Nucl. Phys. B **490** (1997) 179.

[7] M. Aganagic, C. Popesku, and J.H. Schwarz, Nucl. Phys. B **495** (1997) 99.

[8] A. Achucarro, J. Evans, P. Townsend and D. Wiltshire, Phys. Lett. B **198** (1987) 441.

[9] M.J. Duff, Int. J. Mod. Phys. A **11** (1996) 5623.

[10] A.A. Deriglazov and D.M. Gitman, Int. J. Mod. Phys. A **14** (1999) 2769;

[11] I.A. Batalin and I.V. Tyutin, Nucl. Phys. B **381** (1992) 619; Phys. Lett. B **317** (1993) 354; Mod. Phys. Lett. A **8** (1993) 3757.

[12] A.A. Deriglazov, A.V. Galajinsky and S.L. Lyakhovich, Nucl. Phys. B **473** (1996) 245.

[13] D. M. Gitman and I. V. Tyutin, *Quantization of Fields with Constraints*, Springer-Verlag, 1990.

[14] I. V. Tyutin, Soviet J. Nucl. Phys. **51** (1990) 1495.

[15] L. Brink, M. Henneaux, and C. Teitelboim, Nucl. Phys. B **293** (1987) 505.

[16] J.M. Evans, Phys. Lett. B **233** (1989) 307.

[17] J.M. Evans, Nucl. Phys. B **331** (1990) 711.

[18] R. Casalbuoni, Phys. Lett B **62** (1976) 49.

[19] D.P. Sorokin, V.I. Tkach and D.V. Volkov, Mod. Phys. Lett. A **4** (1989) 901.

[20] N. Berkovits, Nucl. Phys. B **350** (1991) 193.

[21] Y. Eisenberg, Phys. Lett B **276** (1992) 325.

[22] F. Ebler, M. Hatsuda, E. Laenen, W. Siegel, J. Yamron, T. Kimura, and A. Micovic, Nucl. Phys. B **364** (1991) 67.

[23] M.B. Green and C.M. Hull, Nucl. Phys. B **344** (1990) 115; Mod. Phys. Lett. A **5** (1990) 1399.

[24] N. Berkovits, JHEP 0004 (2000) 018.

[25] R. Metsaev and A. Tseytlin, Nucl. Phys. B **533** (1998) 109; *Supertstring action in* $AdS_5 \times S^5$*:* κ*-symmetry light-cone gauge*, hep-th/0007036.

[26] A.A. Deriglazov and A.V. Galajinsky, Phys. Lett. B **381** (1996) 105.

[27] S. Belucci and A.V. Galajinsky, Phys. Lett. B **432** (1998) 103; B **460** (1999) 288.

[28] A.A. Deriglazov and A.V. Galajinsky, Phys. Rev. D **54** (1996) 5195.

[29] N. Berkovits, Phys. Rev. D **59** (1999) 048901.

[30] A. Dresse, J. Fisch, M. Henneaux and C. Schomblond, Phys. Lett B **210** (1988) 141.

[31] M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory*, v.1,2, Cambridge Univ. Press, Cambridge, 1987.

[32] L. Brink and M. Henneaux, *Principles of String Theory*, Plenum Press, NY and London, 1988.

[33] J. Polchinski, *String Theory*, v.1,2, Cambridge University Press, 1998.

[34] I.A. Batalin and G.A. Vilkovisky, Phys. Lett. B **120** (1983) 166.

[35] I.A. Batalin and E.S. Fradkin, Phys. Lett. B **122** (1983) 157; Lett. Nuovo Cim. **38** (1983) 393.