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String representation of the SU(N)-inspired dual Abelian-Higgs–type theory with the Θ-term

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Abstract

String representation of the $[U(1)]^{N-1}$ gauge-invariant dual Abelian-Higgs–type theory, which is relevant to the $SU(N)$-QCD with the Θ-term and provides confinement of quarks, is derived. The N-dependence of the Higgs vacuum expectation value is found, at which the tension of the string joining quarks becomes N-independent, similarly to the real QCD. Contrary to that, the inverse coupling constant of the rigidity term of this string always behaves approximately as $1/N$. A long-range Aharonov-Bohm–type interaction of a dyon (i.e., a quark which acquired a magnetic charge due to the Θ-term) with a closed electric string becomes nontrivial at $\Theta \neq N\pi \times$ integer. On the contrary, at these critical values of Θ, the scattering of dyons over strings is absent.

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1 Introduction. The model.

During the last years, the method of Abelian projections [1] has been extensively used both analytically and numerically to describe confinement in QCD by the monopole mechanism (for recent reviews see [2] and refs. therein). In particular, several attempts have been done to address the case of arbitrary number of colors [3, 4]. On the way of using the method of Abelian projections, it is reasonable to base the respective 3D *continuum* models on the assumption that monopoles form a dilute plasma (see e.g. ref. [5] for the $SU(2)$-case). This is because such a monopole configuration is an approximate stationary point of the action of the $SU(N)$ 3D Georgi-Glashow model, and the confining mechanism of the latter is supposed to be similar to that of Abelian-projected theories [1]. In the present letter, we shall work in 4D and explore another $SU(N)$-inspired theory describing Abelian-projected monopoles, which provides confinement of quarks. It is based on the alternative assumption [6] that monopoles form magnetic Higgs condensate, rather than the plasma. This assumption looks more appropriate in 4D, where Abelian-projected monopoles are known to be proliferating [7], and therefore cannot be treated in the approximation of a dilute plasma. The model we are going to deal with is a straightforward generalization of the respective $SU(3)$-one [8], whose string representation has been explored in refs. [9, 10] (see also [11] where the collective effects of vortex loops in this model have been studied). Similarly to ref. [10], we shall consider the general case of a theory extended by the Θ-term, owing to which quarks acquire a nonvanishing magnetic charge (i.e., become dyons) and scatter over the dual electric Abrikosov-Nielsen-Olesen strings [12]. Note that the simplest model of this type, corresponding to the Abelian-projected $SU(2)$-QCD with the Θ-term, has for the first time been considered in ref. [13]. As one of the results of the present letter, we shall get the critical values of Θ in the $SU(N)$-case, at which the long-range topological interaction of dual strings with dyons disappears. These values in particular reproduce the respective $SU(2)$- and $SU(3)$-ones, obtained in the above-mentioned papers.

The partition function of the effective $[U(1)]^{N-1}$ gauge-invariant Abelian-projected theory we are going to explore [1] reads

$$\mathcal{Z}_\alpha = \int \left(\prod_i |\Phi_i| \mathcal{D} |\Phi_i| \mathcal{D}\theta_i\right) \mathcal{D}\mathbf{B}_\mu \delta\left(\sum_i \theta_i\right) \exp\left\{-\int d^4x \left[\frac{1}{4}\left(\mathbf{F}_{\mu\nu} + \mathbf{F}^{(\alpha)}_{\mu\nu}\right)^2 + \right.\right.$$

$$\left.\left. + \sum_i \left[\left|\left(\partial_\mu - i g_m \mathbf{q}_i \mathbf{B}_\mu\right)\Phi_i\right|^2 + \lambda\left(|\Phi_i|^2 - \eta^2\right)^2\right] - \frac{i\Theta g_m^2}{16\pi^2}\left(\mathbf{F}_{\mu\nu} + \mathbf{F}^{(\alpha)}_{\mu\nu}\right)\left(\tilde{\mathbf{F}}_{\mu\nu} + \tilde{\mathbf{F}}^{(\alpha)}_{\mu\nu}\right)\right]\right\}. \qquad (1)$$

Here, the index i runs from 1 to the number of positive roots $\mathbf{q}_i$'s of the $SU(N)$-group, that is $N(N-1)/2$. Next, g_m is the magnetic coupling constant related to the electric one, g, by means of the topological quantization condition $g_m g = 4\pi n$. In what follows, we shall for simplicity restrict ourselves to the monopoles possessing the minimal charge only, i.e., set $n = 1$, although the generalization to an arbitrary n is straightforward. Note that the origin of root vectors in eq. (1) is the fact that monopole charges are distributed along them. Further, $\Phi_i = |\Phi_i| e^{i\theta_i}$ are the dual Higgs fields, which describe the condensates of monopoles, and $\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu$ is the field-strength tensor of the $(N-1)$-component "magnetic" potential $\mathbf{B}_\mu$. The latter is dual to the "electric" potential, whose components are diagonal gluons. Since the $SU(N)$-group is special,

[1]Throughout the present letter, all the investigations will be performed in the Euclidean space-time.

the phases θ_i's of the dual Higgs fields are related to each other by the constraint $\sum\limits_i \theta_i = 0$, which is imposed by introducing the corresponding δ-function into the r.h.s. of eq. (1). Next, the index α runs from 1 to N and denotes a certain quark color. Finally, $\tilde{\mathcal{O}}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}\mathcal{O}_{\lambda\rho}$, and $\mathbf{F}^{(\alpha)}_{\mu\nu}$ is the field-strength tensor of a test quark of the color α, which moves along a certain contour C. This tensor obeys the equation $\partial_\mu \tilde{\mathbf{F}}^{(\alpha)}_{\mu\nu} = g\mathbf{m}_\alpha j_\nu$, where $j_\mu(x) = \oint\limits_C dx_\mu(\tau)\delta(x - x(\tau))$, and $\mathbf{m}_\alpha$ is a weight vector of the group $SU(N)$. One thus has $\mathbf{F}^{(\alpha)}_{\mu\nu} = g\mathbf{m}_\alpha \tilde{\mathcal{F}}_{\mu\nu}$, where $\mathcal{F}_{\mu\nu}$ can be chosen e.g. in the form $\mathcal{F}_{\mu\nu} = -\Sigma_{\mu\nu}$. Here, $\Sigma_{\mu\nu}(x) = \int\limits_\Sigma d\sigma_{\mu\nu}(x(\xi))\delta(x - x(\xi))$ is the vorticity tensor current associated with the world sheet Σ of the open electric string, bounded by the contour C [2]. From now on, we shall omit the normalization constant in front of all the functional integrals implying for every color α the normalization condition $\mathcal{Z}_\alpha\left[C = 0\right] = 1$.

Note that the Θ-term can be rewritten as

$$-\frac{i\Theta g_m^2}{16\pi^2}\left(\mathbf{F}_{\mu\nu} + \mathbf{F}^{(\alpha)}_{\mu\nu}\right)\left(\tilde{\mathbf{F}}_{\mu\nu} + \tilde{\mathbf{F}}^{(\alpha)}_{\mu\nu}\right) = \frac{i\Theta g_m}{\pi}\mathbf{m}_\alpha \int d^4x \mathbf{B}_\mu j_\mu, \tag{2}$$

which means that by virtue of this term quarks start interacting with the magnetic gauge field $\mathbf{B}_\mu$ [14]. This is only possible provided they acquire some magnetic charge, i.e., become dyons. According to eq. (2), this charge is indeed nonvanishing and equals to $\Theta g_m/\pi$.

Expanding for a while $|\Phi_i|$ around the Higgs v.e.v. η, one gets the mass of the dual vector boson, $m = g_m\eta\sqrt{N}$. In what follows, we shall work in the London limit of the model (1), which admits a construction of the string representation. This is the limit when m is much smaller than the mass of any of the Higgs fields, $m_H = \eta\sqrt{2\lambda}$. Since we would like the model under study be consistent with QCD, we must have $g \sim \sqrt{\bar{\lambda}/N}$, where $\bar{\lambda}$ is the 't Hooft coupling constant, which remains finite in the large-N limit. Therefore, in the London limit, the Higgs coupling λ should grow with N faster than $\mathcal{O}\left(N^2\right)$, namely it should obey the inequality $\lambda \gg 8\pi^2 N^2/\bar{\lambda}$.

Integrating $|\Phi_i|$'s out, we arrive at the following expression for the partition function (1) in the London limit:

$$\mathcal{Z}_\alpha = \int \left(\prod_i \mathcal{D}\theta_i^{\rm sing}\mathcal{D}\theta_i^{\rm reg}\right)\mathcal{D}\mathbf{B}_\mu \mathcal{D}k\,\delta\left(\sum_i \theta_i^{\rm sing}\right)\exp\Biggl\{-\int d^4x\Biggl[\frac{1}{4}\left(\mathbf{F}_{\mu\nu} + \mathbf{F}^{(\alpha)}_{\mu\nu}\right)^2 +$$

$$+\eta^2\sum_i\left(\partial_\mu\theta_i - g_m\mathbf{q}_i\mathbf{B}_\mu\right)^2 - ik\sum_i \theta_i^{\rm reg} - \frac{i\Theta g_m^2}{16\pi^2}\left(\mathbf{F}_{\mu\nu} + \mathbf{F}^{(\alpha)}_{\mu\nu}\right)\left(\tilde{\mathbf{F}}_{\mu\nu} + \tilde{\mathbf{F}}^{(\alpha)}_{\mu\nu}\right)\Biggr]\Biggr\}. \tag{3}$$

Here, we have decomposed the total phases of the dual Higgs fields into multivalued and single-valued (else oftenly called singular and regular, respectively) parts, $\theta_i = \theta_i^{\rm sing} + \theta_i^{\rm reg}$, and imposed the constraint of vanishing of the sum of regular parts by introducing the integration over the Lagrange multiplier $k(x)$. The fields $\theta_i^{\rm sing.}$'s describing a certain configuration of closed dual strings are related to the world sheets Σ_i's of these strings by means of the equation

$$\varepsilon_{\mu\nu\lambda\rho}\partial_\lambda\partial_\rho\theta_i^{\rm sing}(x) = 2\pi\Sigma^i_{\mu\nu}(x) \equiv 2\pi\int\limits_{\Sigma_i} d\sigma_{\mu\nu}\left(x^{(i)}(\xi)\right)\delta\left(x - x^{(i)}(\xi)\right). \tag{4}$$

[2]Another possible choice of $\mathcal{F}_{\mu\nu}$ is $\mathcal{F}_{\mu\nu}(x) = \partial^x_\nu \int d^4y D_0(x-y)j_\mu(y) - (\mu \leftrightarrow \nu)$, where $D_0(x) = 1/(4\pi^2x^2)$ is the massless propagator. The obvious difference between these two choices is the dimensionality of the support of $\mathcal{F}_{\mu\nu}$ – either it is a 2D Dirac sheet Σ, or the whole 4D space-time. It is known, however, that this ambiguity in the choice of the solution to the equation $\partial_\mu\mathcal{F}_{\mu\nu} = j_\nu$ does not affect physical results.

This equation is the covariant formulation of the 4D analogue of the Stokes' theorem for the gradient of the field θ_i, written in the local form. In eq. (4), $x^{(i)}(\xi) \equiv x_\mu^{(i)}(\xi)$ is a vector, which parametrizes the world sheet Σ_i with $\xi = (\xi^1, \xi^2)$ standing for the 2D coordinate. As far as the regular parts of the phases, $\theta_i^{\rm reg}$'s, are concerned, those describe single-valued fluctuations around the string configuration described by $\theta_i^{\rm sing}$'s. Note that owing to the one-to-one correspondence between $\theta_i^{\rm sing}$'s and Σ_i's, established by eq. (4), the integration over $\theta_i^{\rm sing}$'s is implied in the sense of a certain prescription of the summation over string world sheets. For the $SU(3)$-inspired model, one of the possible concrete forms of such a prescription, corresponding to the summation over the grand canonical ensemble of virtual pairs of strings with opposite winding numbers, has been considered in ref. [11]. It is also worth noting that by virtue of eq. (4) it is possible to demonstrate that the integration measure $\mathcal{D}\theta_i$ becomes factorized into the product $\mathcal{D}\theta_i^{\rm sing}\mathcal{D}\theta_i^{\rm reg}$.

2 String representation.

Let us now construct the string representation of the model (3). First, similarly to the $SU(3)$-case [9, 10], one can show that due to the equality $\sum\limits_i \mathbf{q}_i = 0$, the integration over k yields only an inessential constant factor, and we get

$$\int\left(\prod\limits_i \mathcal{D}\theta_i^{\rm sing}\mathcal{D}\theta_i^{\rm reg}\right)\mathcal{D}k\delta\left(\sum\limits_i\theta_i^{\rm sing}\right)\exp\left\{-\int d^4x\left[\eta^2\sum\limits_i\left(\partial_\mu\theta_i - g_m\mathbf{q}_i\mathbf{B}_\mu\right)^2 - ik\sum\limits_i\theta_i^{\rm reg}\right]\right\} =$$

$$= \int\left(\prod\limits_i\mathcal{D}x^{(i)}(\xi)\mathcal{D}h_{\mu\nu}^i\right)\delta\left(\sum\limits_i\Sigma_{\mu\nu}^i\right)\exp\left\{-\int d^4x\left[\frac{1}{24\eta^2}\left(H_{\mu\nu\lambda}^i\right)^2 - i\pi h_{\mu\nu}^i\Sigma_{\mu\nu}^i + ig_m\mathbf{q}_i\mathbf{B}_\mu\partial_\nu\tilde{h}_{\mu\nu}^i\right]\right\}.$$

Here, the Kalb-Ramond field $h_{\mu\nu}^i$ is dual to $\theta_i^{\rm reg}$, and $H_{\mu\nu\lambda}^i = \partial_\mu h_{\nu\lambda}^i + \partial_\lambda h_{\mu\nu}^i + \partial_\nu h_{\lambda\mu}^i$ stands for the strength tensor of this field. We have also used the relation (4) and referred the Jacobians [15] emerging in course of the change of variables $\theta_i^{\rm sing} \to x^{(i)}$ to the integration measures $\mathcal{D}x^{(i)}(\xi)$'s.

The action of the dual-gauge-field sector of the model can then be written as follows:

$$\int d^4x\left[\frac{1}{4}\mathbf{F}_{\mu\nu}^2 + \frac{1}{4}\left(\mathbf{F}_{\mu\nu}^{(\alpha)}\right)^2 + \mathbf{B}_\mu\partial_\nu\left(ig_m\mathbf{q}_i\tilde{h}_{\mu\nu}^i - g\mathbf{m}_\alpha\tilde{\Sigma}_{\mu\nu} - \frac{i\Theta g_m^2}{4\pi^2}\tilde{\mathbf{F}}_{\mu\nu}^{(\alpha)}\right)\right].$$

The $\mathbf{B}_\mu$-fields can then be integrated out as Lagrange multipliers by passing to the new fields $B_\mu^i = \mathbf{q}_i\mathbf{B}_\mu$, using the formula [16] [3] $\left(B_\mu^i\right)^2 = \frac{N}{2}\mathbf{B}_\mu^2$, and introducing the numbers $s_i^{(\alpha)}$'s according to the definition $\mathbf{m}_\alpha = \mathbf{q}_i s_i^{(\alpha)}$. The resulting partition function reads as follows:

$$\mathcal{Z}_\alpha = \int\left(\prod\limits_i\mathcal{D}x^{(i)}(\xi)\mathcal{D}h_{\mu\nu}^i\right)\delta\left(\sum\limits_i\Sigma_{\mu\nu}^i\right)\exp\left\{-\int d^4x\left[\frac{1}{24\eta^2}\left(H_{\mu\nu\lambda}^i\right)^2 - i\pi h_{\mu\nu}^i\Sigma_{\mu\nu}^i + \right.\right.$$

$$\left.\left. + \frac{N}{8}\left(g_m h_{\mu\nu}^i + igs_i^{(\alpha)}\Sigma_{\mu\nu} - \frac{\Theta g_m}{\pi}s_i^{(\alpha)}\tilde{\mathcal{F}}_{\mu\nu}\right)^2 + \frac{1}{4}\left(\mathbf{F}_{\mu\nu}^{(\alpha)}\right)^2\right]\right\}. \qquad (5)$$

[3] See also the last paper in ref. [4] for the discussion of this formula.

To proceed with the analysis of this expression, we obviously need to know possible values of $s_i^{(\alpha)}$'s, as well as $\left(s_i^{(\alpha)}\right)^2$ for a fixed α. First of all, it is straightforward to see that for a given α, only $(N-1)$ numbers $s_i^{(\alpha)}$'s are different from zero. This is simply because only $(N-1)$ $\mathbf{q}_i$'s out of $N(N-1)/2$ positive roots are so that $\mathbf{m}_\alpha \mathbf{q}_i = 1/2$, while the others are orthogonal to $\mathbf{m}_\alpha$. Next, by noting that every root vector can be represented as a difference of two weight vectors and by using the normalization condition $\mathbf{m}_\alpha \mathbf{m}_\beta = \left(\delta_{\alpha\beta} - N^{-1}\right)/2$, these nonvanishing $s_i^{(\alpha)}$'s can be found to be $\pm N^{-1}$ (with $\sum\limits_i s_i^{(\alpha)} = 0$), so that $\left(s_i^{(\alpha)}\right)^2 = (N-1)/N^2$. Owing to this result, the singular term $\frac{1}{4}\left(\mathbf{F}_{\mu\nu}^{(\alpha)}\right)^2$ in eq. (5) cancels out, and we get the following intermediate expression for the partition function:

$$\mathcal{Z}_\alpha = \exp\left[-\frac{N-1}{8N}\left(\frac{\Theta g_m}{\pi}\right)^2 \int d^4x \mathcal{F}_{\mu\nu}^2 - \frac{2i\Theta(N-1)}{N}\hat{L}(\Sigma, C)\right] \times$$

$$\times \int \left(\prod_i \mathcal{D}x^{(i)}(\xi)\mathcal{D}h_{\mu\nu}^i\right) \delta\left(\sum_i \Sigma_{\mu\nu}^i\right) \exp\left\{-\int d^4x \left[\frac{1}{24\eta^2}\left(H_{\mu\nu\lambda}^i\right)^2 + \frac{Ng_m^2}{8}\left(h_{\mu\nu}^i\right)^2 - i\pi h_{\mu\nu}^i \Sigma_{\mu\nu}^{i\,(\alpha)}\right]\right\}. \tag{6}$$

Here, $\hat{L}(\Sigma, C) \equiv \int d^4x d^4y \tilde{\Sigma}_{\mu\nu}(x) j_\nu(y) \partial_\mu^x D_0(x-y)$ is the (formal expression for the) 4D Gauss' linking number of the surface Σ with its boundary C, which eventually becomes cancelled from the final expression for $\mathcal{Z}_\alpha$, and $\Sigma_{\mu\nu}^{i\,(\alpha)} \equiv \Sigma_{\mu\nu}^i - Ns_i^{(\alpha)}\Sigma_{\mu\nu} - \frac{i\Theta N g_m^2}{4\pi^2} s_i^{(\alpha)} \tilde{\mathcal{F}}_{\mu\nu}$, so that $\partial_\mu \Sigma_{\mu\nu}^{i\,(\alpha)} = Ns_i^{(\alpha)} j_\nu$.

Further integration over the Kalb-Ramond fields is straightforward and yields

$$\int \left(\prod_i \mathcal{D}h_{\mu\nu}^i\right) \exp\left\{-\int d^4x \left[\frac{1}{24\eta^2}\left(H_{\mu\nu\lambda}^i\right)^2 + \frac{Ng_m^2}{8}\left(h_{\mu\nu}^i\right)^2 - i\pi h_{\mu\nu}^i \Sigma_{\mu\nu}^{i\,(\alpha)}\right]\right\} =$$

$$= \exp\left\{-2\pi^2 \int d^4x d^4y D_m(x-y) \left[\eta^2 \Sigma_{\mu\nu}^{i\,(\alpha)}(x) \Sigma_{\mu\nu}^{i\,(\alpha)}(y) + \frac{2}{g_m^2}\frac{N-1}{N} j_\mu(x) j_\mu(y)\right]\right\},$$

where $D_m(x) = mK_1(m|x|)/(4\pi^2|x|)$ is the massive propagator with K_1 standing for the modified Bessel function. Simplifying the integral $\int d^4x d^4y \Sigma_{\mu\nu}^{i\,(\alpha)}(x) D_m(x-y) \Sigma_{\mu\nu}^{i\,(\alpha)}(y)$ (see ref. [10] for the analogous transformations in the $SU(3)$-case) we eventually arrive at the following final expression for the partition function:

$$\mathcal{Z}_\alpha = \exp\left\{-\frac{N-1}{4N}\left[g^2 + \left(\frac{\Theta g_m}{\pi}\right)^2\right] \int d^4x d^4y j_\mu(x) D_m(x-y) j_\mu(y)\right\} \int \left(\prod_i \mathcal{D}x^{(i)}(\xi)\right) \times$$

$$\times \delta\left(\sum_i \Sigma_{\mu\nu}^i\right) \exp\left[-2(\pi\eta)^2 \int d^4x d^4y \hat{\Sigma}_{\mu\nu}^i(x) D_m(x-y) \hat{\Sigma}_{\mu\nu}^i(y) - 2i\Theta s_i^{(\alpha)} \hat{L}\left(\Sigma_i, C\right) + \right.$$

$$\left. + 2i\Theta \int d^4x d^4y \left(\frac{N-1}{N}\tilde{\Sigma}_{\mu\nu}(x) - s_i^{(\alpha)} \tilde{\Sigma}_{\mu\nu}^i(x)\right) j_\mu(y) \partial_\nu^x D_m(x-y)\right], \tag{7}$$

where $\hat{\Sigma}_{\mu\nu}^i \equiv \Sigma_{\mu\nu}^i - Ns_i^{(\alpha)}\Sigma_{\mu\nu}$. This formula is the main result of the present letter. Note that for every color α, it is straightforward to integrate out one of the world sheets Σ_i's by resolving the constraint imposed by the δ-function.

The first exponent on the r.h.s. of eq. (7) represents the short-ranged interaction of quarks via dual vector bosons. Noting that for any α, $\mathbf{m}_\alpha^2 = (N-1)/(2N)$, we immediately read from this term the total charge of the quark, $\sqrt{g^2 + (\Theta g_m/\pi)^2}$. The magnetic part of this charge coincides with the one following from eq. (2). Further, the first term in the second exponent on the r.h.s. of eq. (7) is the short-ranged (self-)interaction of closed world sheets Σ_i's and an open one Σ. In particular, by virtue of the general formulae obtained in ref. [17], one can get from the $\Sigma \times \Sigma$-interaction the following values of the string tension and of the inverse coupling constant of the rigidity term, corresponding to the confining-string world sheet Σ:

$$\sigma = 2\pi(N-1)\eta^2 \ln \frac{m_H}{m}, \quad \alpha^{-1} = -\frac{\pi(N-1)}{4g_m^2 N} = \mathcal{O}\left(\frac{1}{N}\right).$$

Here, in the derivation of σ, we have in the standard way [12] set for a characteristic small dimensionless quantity in the model under study the ratio m/m_H and adapted the logarithmic accuracy, i.e., assumed that not only $\frac{m_H}{m} \gg 1$, but also $\ln \frac{m_H}{m} \gg 1$. While the $1/N$ behavior of α^{-1} is fixed by the requirement that $g_m^2 \sim N$, the N-dependence of σ is subject to such a dependence of η. In QCD, to the leading order in the parameter of the strong-coupling expansion, $\beta = 2N/g^2$, the string tension for the rectangular loop is known to be N-independent: $\sigma_{\rm QCD} = \frac{1}{a^2} \ln \frac{2N^2}{\beta} = \frac{1}{a^2} \ln \bar{\lambda}$, where a is the lattice spacing [4]. Thus, if we adjust the N-dependence of η as $\eta \sim \left[(N-1) \ln \frac{\sqrt{\lambda}}{N}\right]^{-1/2}$, where the N-dependence of λ was discussed in the paragraph following after eq. (2), then the resulting string tension will be as N-independent, as it is in QCD.

Next, the last term on the r.h.s. of eq. (7) describes the short-range interactions of dyons with both closed and open strings (obviously, the latter confine these very dyons themselves). Finally, the term $-2i\Theta s_i^{(\alpha)} \hat{L}(\Sigma_i, C)$ in eq. (7) describes the long-range interaction of dyons with closed world sheets, that is the 4D analogue of the Aharonov-Bohm effect [18]. Since nonvanishing values of $s_i^{(\alpha)}$'s were found to be $\pm N^{-1}$, at $\Theta \neq N\pi \times$ integer, dyons (due to their magnetic charge) do interact by means of this term with the closed dual strings. On the contrary, these critical values of Θ correspond to such a relation between the magnetic charge of a dyon and the electric flux inside the string when the scattering of dyons over strings is absent. Note finally once more that these critical values of Θ generalize the $SU(2)$- and $SU(3)$-ones obtained in refs. [13] and [10], respectively.

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[4]This fact stems also from the natural conjecture that the linear term in the quark-antiquark potential should have the same N-dependence as the Coulomb term, that is $V_{\rm Coul}(R) = -\frac{g_{\rm QCD}^2}{4\pi R} \frac{N^2-1}{2N} = \mathcal{O}\left(N^0\right)$.

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