Tachyons in a slice of AdS

A. Delgado* **and M. Redi†**
Department of Physics and Astronomy
Johns Hopkins University
3400 North Charles St.
Baltimore, MD 21218-2686

Abstract

Stability of AdS space allows scalar fields to have negative mass squared as long as the Breitenlohner-Freedman bound is satisfied. In a compactification of AdS instead, to avoid instabilities, a tachyonic bulk mass must be supplemented by appropriate brane actions. In this paper we clarify the meaning of the lower bound in the Randall-Sundrum scenario with two branes and explain how the instability disappears in the infinite space limit. A CFT interpretation is also given as radiative symmetry breaking.

*email: adelgado@pha.jhu.edu
†email: redi@pha.jhu.edu

In the old days of gauged supergravity, it was typically found that the scalar fields required by supersymmetry have a potential which is unbounded from below. Anti de Sitter space (AdS) is a solution of the supergravity equations of motion when the scalars are at a local maximum of the potential. In flat space this is clearly unacceptable because the theory would be unstable under small fluctuations around the vacuum but it was found that this conclusion does not hold in curved space. In fact, a scalar field can be stable in AdS space even though the potential is unbounded from below [1]. In particular, a tachyonic mass does not lead to inconsistencies if the mass satisfies the Breitenlohner-Freedman (BF) bound which, for the five dimensional case we will be interested in, is $M^2 \geq -4/L^2$, where L is the radius of curvature of AdS_5. This happens because AdS space has a boundary where conditions on the fields need to be specified such that the energy functional is bounded from below even though the potential is not. In the light of the AdS/CFT correspondence negative scalars have particular interest because they correspond to deformations of the CFT by relevant operators (see [2]). In the CFT language the lower bound on the mass is understood as a unitary bound on the dimensions of the gauge invariant operators.

In this brief paper we will study the rôle of negative mass squared scalars in the Randall-Sundrum compactification of AdS and propose a new holographic interpretation. We became aware that this subject has been previously investigated in [3] with which our work has some overlapping. In our case we consider in detail the two brane scenario and show that a tachyonic bulk scalar naturally leads to instabilities (at least as long as gravity is non dynamical) for any distance between the two branes. By studying the limit where one of the branes is sent to the boundary of AdS we explain how the BF bound arises. Also, our AdS/CFT interpretation is completely different.

In order to be specific, we consider the compactification of five dimensional AdS space which has drawn a lot of attention for its relevance for phenomenological applications, but we believe that our results hold in any number of dimension with minor modification of the formulas. As shown in [4], for the particular case of five dimensions, it is possible to compactify AdS space onto an orbifold $S^1/\mathbb{Z}_2$. This geometry is obtained by adding two 3-branes parallel to the boundary of AdS to reproduce the singularities of the metric at the fixed points of the orbifold. Once the tensions of the branes are tuned with the cosmological constant of the bulk, this configuration is an exact solution of Einstein's equation for any distance between the two branes. This space has a four dimensional ground state which is ordinary flat space. Our strategy will therefore be to perform a Kaluza-Klein (KK) decomposition of the fields and use ordinary flat space arguments to discuss stability.

We consider the AdS metric in Poincaré coordinates:

$$ds^2 = \frac{L^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) \tag{1}$$

where L is the radius of curvature of AdS. The space has a boundary at $z = 0$ while $z = \infty$ is the horizon. The physical space of the orbifold is an interval with metric given by (1) bounded by two branes located at z_1 (UV brane) and at z_2 (IR brane).

The action of a free scalar propagating in this geometry, including brane localized mass terms is:

$$S = S_{bulk} + S_{UV} + S_{IR} \tag{2}$$

where:

$$\begin{aligned} S_{bulk} &= \int d^4x \int dz \sqrt{G}(\frac{1}{2}g^{MN}\partial_M\phi\partial_N\phi - \frac{1}{2}m^2\phi^2) \\ S_{UV} &= -\int d^4x \sqrt{g_{UV}}\frac{\lambda_1}{L}\phi^2 \\ S_{IR} &= -\int d^4x \sqrt{g_{IR}}\frac{\lambda_2}{L}\phi^2 \end{aligned} \tag{3}$$

g_{UV} and g_{IR} are the induced metrics on the branes and $\lambda_{1,2}$ are dimensionless couplings. In general, brane localized kinetic terms can also be included and they naturally arise from the dynamic of the system in the interacting theory, but they will not be relevant for the present discussion. In the infinite space limit, boundary conditions on the fields have to be imposed at $z = 0$. In the finite slice, the boundary conditions at z_1 and z_2 are implied by the brane actions.

The equation of motion following from (2) is:

$$\frac{L^3}{z^3}(-\partial_z^2 + \frac{3}{z}\partial_z + \eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{m^2L^2}{z^2} + \frac{2\lambda_1}{z}\delta(z-z_1) + \frac{2\lambda_2}{z}\delta(z-z_2))\phi(q,z) = 0 \tag{4}$$

The solutions of the bulk equation are linear combinations of Bessel functions of order $\alpha = \sqrt{4 + M^2L^2}$:

$$\phi(q,z) = z^2(A\ J_\alpha(q\ z) + B\ Y_\alpha(q\ z)) \tag{5}$$

where q is the four dimensional momentum. The delta Dirac impose the following boundary conditions on the fields:

$$\begin{aligned} z\partial_z\phi(q,z_1) &= \lambda_1\phi(q,z_1) \\ z\partial_z\phi(q,z_2) &= -\lambda_2\phi(q,z_2) \end{aligned} \tag{6}$$

We have here assumed that ϕ is even under the Z_2 projection. For an odd scalar, we should impose that the wave function vanishes at the orbifold fixed points. This can be considered as a special case of Dirichlet boundary conditions which can always be imposed. Substituting (5) into (6) and using the properties of the Bessel functions, we obtain an equation for the $4D$ KK masses m, namely:

$$\frac{(2-\alpha-\lambda_1)J_\alpha(mz_1)+mz_1J_{\alpha-1}(mz_1)}{(2-\alpha-\lambda_1)Y_\alpha(mz_1)+mz_1Y_{\alpha-1}(mz_1)}=\frac{(2-\alpha+\lambda_2)J_\alpha(mz_2)+mz_2J_{\alpha-1}(mz_2)}{(2-\alpha+\lambda_2)Y_\alpha(mz_2)+mz_2Y_{\alpha-1}(mz_2)} \tag{7}$$

Due to the reality of the action, solutions to this equation can either be purely real or purely imaginary. For $\lambda_{1,2}=0$, corresponding to ordinary Neumann boundary conditions, we find that the spectrum of the KK masses is non tachyonic if and only if $M^2\geq 0$. In particular, for $M^2\geq -4/L^2$ (corresponding to the BF bound in $5D$), there is no massless mode in the spectrum unless the bulk mass is zero. Therefore, a scalar propagating in the RS background is unstable whenever $M^2<0$ as in flat space. Of course, it is always possible to tune the brane masses to compensate for the negative bulk mass but this can be done independently of the BF bound, so it is not different from a tachyon in flat space. In [5] it was considered the case of supersymmetric theories in a slice of AdS$_5$. SUSY in AdS background requires some of the scalars to have negative mass satisfying the bound. By ordinary $4D$ flat space supersymmetry all the KK masses have to be positive so it was found that invariance of the action under supersymmetry transformation requires $\lambda_{1,2}\neq 0$. In general the brane terms will depend on the position of the two branes. This is clearly unsatisfactory, especially in a theory where gravity is dynamical since in this case the distance between the two branes is free to fluctuate.

It is natural to ask what is meaning of the lower bound in a slice of AdS. For zero brane actions we have checked numerically that a negative mass scalar with $M^2\geq -4/L^2$ always generates a single $4D$ tachyon plus positive excitations, independently of the location of the two branes. If we now send the UV brane toward the boundary, the mass of the tachyon grows to infinity so effectively this mode decouples from the theory. It will be convenient for us to keep the UV brane fixed and move the IR brane to the horizon of the space at $z=\infty$. Appropriately Weyl rescaling the four dimensional metric will be physically equivalent to moving the UV brane while keeping the IR brane fixed. In the limit $z_2\to\infty$, the positive modes become continuous while the tachyon has a finite mass squared of the order of $-1/z_1^2$. Technically this happens because, for imaginary values of m and $|mz_2|>>1$, equation (7) becomes:

$$(2-\alpha)H^1_\alpha(mz_1)+mz_1H^1_{\alpha-1}(mz_1)=0 \tag{8}$$

where $H^1_\alpha(mz)=J_\alpha(mz)+iY_\alpha(mz)$. From (8) we see that the properties of the tachyon are associated with the UV brane. This type of equation has been widely studied in the contest of general properties of Bessel functions [6] and it can be proved analytically that equation (8) has indeed zero imaginary solutions when $M^2 \geq 0$ and two conjugate when $-4/L^2 \leq M^2 < 0$ which fully agrees with our previous numerical checks.

We can Weyl rescale the four dimensional metric according to:

$$g_{\mu\nu} \to (z_2/L)^2 g_{\mu\nu} \tag{9}$$

to obtain the configuration where we keep the IR brane fixed (where the $5D$ metric has canonical normalization) and the UV brane is located at $z_1 = L^2/z_2$ (assuming that the UV brane was originally at $z_1 = L$). It follows from (9) that the mass of the tachyon is proportional to $1/z_1$ for small z_1.

In the limit where the UV brane has been moved all the way to the boundary of AdS, the equation (7) for the positive modes also simplifies:

$$(2-\alpha)J_\alpha(mz_2) + mz_2 J_{\alpha-1}(mz_2) = 0 \tag{10}$$

The mass gap between the positive KK modes remains finite and the mass scale is set by $1/z_2$. Basically, the different behavior of the tachyon stems from the fact that its wave function is localized on the UV brane while all the other excitations are peaked on the IR brane. As a consequence, the two types of modes decouple from each other when $z_1 \to 0$ and the tachyon is projected out of the physical spectrum.

We are now ready to show the inconsistency that arise when $M^2 < -4/L^2$. Moving the IR brane to infinity one finds that additional tachyons are generated. Actually, equation (8) has now an infinite number of imaginary solutions approaching zero. This clearly leads to an instability. After Weyl rescaling according to (9), each mode has a mass which grows as $1/z_1$ but, due to the new tachyons that are generated, there are always states with $m^2 = -O(1/L^2)$. Even if we start with a localized mass term on the UV brane such that the spectrum is tachyon free, by moving the UV brane toward the boundary we will eventually generate tachyons since the required mass grows as $1/z_1$.

One of the features of these scenarios is that there is an AdS/CFT duality that relates the $5D$ warped theory to a $4D$ CFT (see [7]). The standard correspondence is that a theory on a slice of AdS$_5$ is dual to a $4D$ CFT with a cut-off given by the position of the UV brane and in which conformal invariance is spontaneously broken at a scale set by the IR brane. This last fact explains the existence of a mass gap between the KK modes in the case where the IR brane is kept at a finite distance and the UV brane is removed. Fields living in the bulk of the extra dimension correspond to fundamental $4D$ fields living outside the CFT and coupled to gauge invariant operators of the CFT. In the case of scalars the conformal dimension of the operator is related to the bulk mass of the scalar by:

$$\Delta_\pm = 2 \pm \sqrt{4 + M^2 L^2} \tag{11}$$

where the minus sign (necessary to saturate the unitarity bound $\Delta \geq 1$ of $4D$ CFT) is allowed only in the region $-4/L^2 < M^2 < -3/L^2$. [1] With our boundary conditions the positive modes grow as z^{Δ_+} for small z, in the limit where the UV brane is removed, which corresponds to the choice of the plus sign in the formula (11) [8]. The correspondence also asserts that classical computations on the AdS side correspond to large-N contributions in loops of CFT.

The interpretation of the appearance of a tachyon mode when this kind of scalars is compactified is as follows. In the CFT dual a mass term localized on the UV brane corresponds to adding a mass term for this fundamental scalar as well as a deformation of the CFT, while a mass term on the IR brane is interpreted as a threshold effect when the CFT is broken. In the case when no brane term is added there is a massless scalar at the UV scale which is coupled to a CFT through a relevant operator (i.e. $\Delta < 4$), and the large-N loop contributions drive that scalar to have a negative mass in the IR. If some brane term is added on the AdS side, to cancel the negative mass as explained before, then the holographic interpretation is that there is a massive scalar in the UV and the CFT does not have enough room to make it tachyonic. Since the value of the brane term, or UV mass, depends on the size of the bulk there is a way to go from a healthy scalar to a tachyon just by changing the position of the UV brane, that is by changing the scale of the cut-off. This means that depending on the scale where the CFT is spontaneously broken the scalar will have positive or negative m^2. In other words, when this scalar is charged under some gauge group, this resembles the case of radiative symmetry breaking, i.e., a perfectly massive scalar in the UV whose mass is driven negative in the IR by CFT loops. In order for this scalar to have phenomenological interest for electroweak symmetry breaking the UV scale should not be much separated from the IR scale, because as has been shown, the negative mass of the tachyon is of the order of the UV cut-off, so unfortunately this mechanism can not be used in the usual RS1 set-up with the Higgs field in the bulk. One can also understand why in the case when the UV brane is sent to the boundary the tachyonic mode decouples. The dual interpretation of that is simply that the UV brane sets the cut-off of the CFT, so when there is no UV brane there is no cut-off and the external fields added became non-dynamical so this potential tachyon decouples from the rest of the fields.

To conclude we can summarize the results of this paper. We have studied the behavior of tachyonic scalars in the Randall-Sundrum scenario with two branes (UV/IR), the reason for that being that full AdS allows that kind of scalars as long as the mass satisfies the Breitenlohner-Freedman bound. It has been shown that, in general, those scalars will generate instabilities in a slice of AdS, unless suitable brane actions are included. It has also been pointed out that, as long as the bound is satisfied, when the UV brane is sent to the boundary then the instabilities disappear. This is very well understood from the KK decomposition because there

[1] We would like to thank Prof. Igor Klebanov for clarifying this point to us.

is a single tachyonic mode whose wave function is localized near the UV brane while the positive modes are peaked on the IR brane. Finally a CFT interpretation has been proposed relating the generation of this tachyon mode with large-N loops of the CFT. Any possible phenomenological application or the extension of this analysis to other geometries is left for future investigation.

Acknowledgments

We thank Jon Bagger and Raman Sundrum for useful discussions and for reading the manuscript. A. D. is supported by NSF Grants P420D3620414350 and P420D3620434350.

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