London's Equation from Abelian Projection

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Confinement in non-Abelian gauge theories, such as QCD, is often explained using an analogy to type II superconductivity. In this analogy the existence of the "Meissner" effect for quarks with respect to the QCD vacuum is an important element. Here we show that using the ideas of Abelian projection it is possible to arrive at an effective London equation from a non-Abelian gauge theory. (London's equation gave a phenomenological description of the Meissner effect prior to the Ginzburg-Landau or BCS theory of superconductors). The Abelian projected gauge field acts as the E&M field in normal superconductivity, while the remaining non-Abelian components form a gluon condensate which is described via an effective scalar field. This effective scalar field plays a role similar to the scalar field in Ginzburg-Landau theory.

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I. INTRODUCTION

One of the difficult aspects of QCD is that it is a nonlinear theory. At the classical level one writes down the field equations of the nonlinear theory, and by inspiration or numerically solves the field equations. For non-Abelian gauge theories this leads to interesting classical field configurations such as monopoles, merons, instantons *etc.* In the quantized version of these theories there are suggestions of interesting configurations like those listed above. For example, the chromoelectric flux tubes that are hypothesized to exist between quarks in the dual, color superconductivity picture of confinement.

Quantizing nonlinear field theories presents additional challenges and questions. In the case of general relativity no complete quantum version of the theory exists. In the case of QCD a quantum version of the theory exists, but because the QCD coupling constant is large, the standard perturbative quantum field theory techniques do not apply in the low energy regime. This means that getting numerical predictions out of QCD is difficult. At present there is no universal analytical technique for handling detailed questions about QCD (this is in contrast with QED and the theory of the electroweak theory where the techniques of perturbative quantum field theory supply a general method for calculating results analytically). The nonperturbative aspects of QCD are dealt with using numerical techniques [1].

In the 1950's Heisenberg and co-workers proposed a scheme for investigating quantum, nonlinear spinor fields [2], which yielded interesting results concerning the spin and charge of the electron. This nonperturbative quantization technique distinguished between field operators of interacting fields and field operators of noninteracting fields (Heisenberg's ideas were developed before the widespread adoption of the path integral techniques for quantizing field theories and as such are given in terms of field operators). The algebra obeyed by the noninteracting field operators is just the standard canonical commutation relationships (see for example pg. 86 of ref. [3]). The algebra for the interacting fields, on the other hand, is determined from the Green's functions for the field operators of the nonlinear fields, $\mathcal{G}(x_1, x_2 \ldots, x_n) = \langle Q|\hat{A}(x_1), \hat{A}(x_2), \ldots \hat{A}(x_n)|Q\rangle$, where $\hat{A}(x_i)$ is the field operator at position x_i; $\mathcal{G}(x_1, x_2 \ldots, x_n)$ is the Green's function; $|Q\rangle$ is a quantum state. Conversely if the $\mathcal{G}(x_1, x_2 \ldots, x_n)$s are known then this gives the quantum states $|Q\rangle$. The Green's functions are determined from the field equations in the following way: one starts with the classical field equations and turns the classical fields into field operators. One uses these operator field equations to determine the lowest order Green's functions. Next one applies various order derivative operators on these operator field equations to obtain equations which have higher order powers of the field operators, and which yield the higher order Green's functions. In this way an infinite set of coupled, differential equations are constructed

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which connect all orders of Green's functions. In general it is not possible to solve this system of differential equations analytically, so approximation techniques must be used.

Here we apply a version of Heisenberg's method to non-Abelian gauge theory. In our treatment we assume that the non-Abelian gauge fields can be separated into two classes: stochastic, disordered fields and ordered fields. This is similar to Abelian Projection ideas [7] where the diagonal gauge fields associated with the Abelian subgroup are ordered (an example of this ordering would be a non-zero expectation, $\langle A_z^{diag}\rangle \neq 0$, inside a flux tube stretched between quarks) and with the off-diagonal gauge fields being disordered so that $\langle A_\mu^{off}\rangle = 0$ outside of the flux tube (but $\langle (A_\mu^{off})^2\rangle \neq 0$ outside the flux tube). This picture also has some common points with the work of Nielsen, Olesen and others [4] - [6] on the response of the QCD vacuum to a homogeneous color magnetic field, H. In ref. [4] it was shown that a homogeneous SU(2) magnetic field lowered the energy of the vacuum, but was unstable. This instability was removed by the formation of domains [5]. These domains took the form of SU(2) magnetic flux tubes which formed a random SU(2) magnetic quantum liquid [6] with the property that $\langle H\rangle = 0$ and $\langle H^2\rangle \neq 0$. The difference with our approach outlined above is that we assume the expectation values are in terms of the gauge potentials rather than the magnetic field – $\langle A_\mu^{off}\rangle = 0$ and $\langle (A_\mu^{off})^2\rangle \neq 0$. Also, in line with the ideas of Abelian Projection, the diagonal, Abelian potentials (A_z^{diag}) and the off-diagonal, non-Abelian potentials (A_μ^{off}) play different roles in the present paper.

In applying these ideas to an SU(2) gauge theory we assume that the off-diagonal components form a condensate which can be described by an effective scalar field similar to that in Ginzburg-Landau theory [9]. This is also related to work by Cornwall [10] [11] where a connection is established between a scalar field and the expectation of the Yang-Mills field strength tensor squared – $\phi(x) \leftrightarrow \langle TrG^2_{\mu\nu}\rangle$. In the present work we follow the Abelian Projection ideas and assign different roles to the Abelian and the off diagonal, non-Abelian gauge potentials. Only the off-diagonal gauge potentials are assumed be involved in the condensate. With respect to this condensate the field equation for the Abelian component then takes the form of the London equation, so that the Abelian field develops a mass as it penetrates into regions characterized by the a non-zero value for off-diagonal condensate. The mass of the Abelian field and the assumed effective mass of the condensate are not the same, which is in contrast to refs. [10] [11] where all the SU(2) gauge potentials play an equivalent role, leading to a single common effective mass for all the gluons.

II. SEPARATION OF COMPONENTS

In this section we follow the conventions of Ref. [8]. Starting with the SU(N) gauge group with generators T^B we define the SU(N) gauge fields, $\mathcal{A}_\mu = \mathcal{A}^B_\mu T^B$. Let G be a subgroup of SU(N) and $SU(N)/G$ is a coset. Then the gauge field $\mathcal{A}_\mu$ can be decomposed as

$$\mathcal{A}_\mu = \mathcal{A}^B_\mu T^B = a^a_\mu T^a + A^m_\mu T^m, \tag{1}$$
$$a^a_\mu \in G \quad \text{and} \quad A^m_\mu \in SU(N)/G \tag{2}$$

where the indices $a, b, c\dots$ belongs to the subgroup G and $m, n, \dots$to the coset SU(N)/G; B are SU(N) indices. Based on this the field strength can be decomposed as

$$\mathcal{F}^B_{\mu\nu}T^B = \mathcal{F}^a_{\mu\nu}T^a + \mathcal{F}^m_{\mu\nu}T^m \tag{3}$$

where

$$\mathcal{F}^a_{\mu\nu} = \phi^a_{\mu\nu} + \Phi^a_{\mu\nu} \quad \in G, \tag{4}$$
$$\phi^a_{\mu\nu} = \partial_\mu a^a_\nu - \partial_\nu a^a_\mu + f^{abc}a^b_\mu a^c_\nu \quad \in G, \tag{5}$$
$$\Phi^a_{\mu\nu} = f^{amn}A^m_\mu A^n_\nu \quad \in G, \tag{6}$$
$$\mathcal{F}^m_{\mu\nu} = F^m_{\mu\nu} + G^m_{\mu\nu} \quad \in SU(N)/G, \tag{7}$$
$$F^m_{\mu\nu} = \partial_\mu A^m_\nu - \partial_\nu A^m_\mu + f^{mnp}A^n_\mu A^p_\nu \quad \in SU(N)/G, \tag{8}$$
$$G^m_{\mu\nu} = f^{mnb}\left(A^n_\mu a^b_\nu - A^n_\nu a^b_\mu\right) \quad \in SU(N)/G \tag{9}$$

where f^{ABC} are the structural constants of SU(N). The SU(N) Yang-Mills field equations can be decomposed as

$$d_\nu\left(\phi^{a\mu\nu} + \Phi^{a\mu\nu}\right) = -f^{amn}A^m_\nu\left(F^{n\mu\nu} + G^{n\mu\nu}\right), \tag{10}$$
$$D_\nu\left(F^{m\mu\nu} + G^{m\mu\nu}\right) = -f^{mnb}\left[A^n_\nu\left(\phi^{b\mu\nu} + \Phi^{b\mu\nu}\right) - a^b_\nu\left(F^{n\mu\nu} + G^{n\mu\nu}\right)\right] \tag{11}$$

where $d_\nu[\cdots]^a = \partial_\nu[\cdots]^a + f^{abc}a^b_\nu[\cdots]^c$ is the covariant derivative on the subgroup G and $D_\nu[\cdots]^m = \partial_\nu[\cdots]^m + f^{mnp}A^n_\nu[\cdots]^p$

Specializing to the SU(2) case we let $SU(N) \to SU(2)$, $G \to U(1)$, and $f^{ABC} \to \epsilon^{ABC}$. Setting the indices as $a = 3$ and consequently $m, n = 1, 2$, our classical equations become

$$\partial_\nu \left(\phi^{\mu\nu} + \Phi^{\mu\nu}\right) = -\epsilon^{3mn} A^m_\nu \left(F^{n\mu\nu} + G^{n\mu\nu}\right), \tag{12}$$

$$D_\nu \left(F^{m\mu\nu} + G^{m\mu\nu}\right) = -\epsilon^{3mn} \left[A^n_\nu \left(\phi^{\mu\nu} + \Phi^{\mu\nu}\right) - a_\nu \left(F^{n\mu\nu} + G^{n\mu\nu}\right)\right] \tag{13}$$

Since $G = U(1)$ we have $d_\nu = \partial_\nu$.

III. HEISENBERG QUANTIZATION

In quantizing the classical system given in Eqs. (12) - (13) via Heisenberg's method one first replaces the classical fields by field operators $a_\mu \to \hat{a}_\mu$ and $A^m_\mu \to \hat{A}^m_\mu$. This yields the following differential equations for the operators

$$\partial_\nu \left(\hat{\phi}^{\mu\nu} + \hat{\Phi}^{\mu\nu}\right) = -\epsilon^{3mn} \hat{A}^m_\nu \left(\hat{F}^{n\mu\nu} + \hat{G}^{n\mu\nu}\right), \tag{14}$$

$$D_\nu \left(\hat{F}^{m\mu\nu} + \hat{G}^{m\mu\nu}\right) = -\epsilon^{3mn} \left[\hat{A}^n_\nu \left(\hat{\phi}^{\mu\nu} + \hat{\Phi}^{\mu\nu}\right) - \hat{a}_\nu \left(\hat{F}^{n\mu\nu} + \hat{G}^{n\mu\nu}\right)\right] \tag{15}$$

These nonlinear equations for the field operators of the nonlinear quantum fields can be used to determine expectation values for the field operators $\hat{a}_\mu$ and $\hat{A}^m_\mu$ (*e.g.* $\langle \hat{a}_\mu \rangle$, where $\langle \cdots \rangle = \langle Q | \cdots | Q \rangle$ and $|Q\rangle$ is some quantum state). One can also use these equations to determine the expectation values of operators that are built up from the fundamental operators $\hat{a}_\mu$ and $\hat{A}^m_\mu$. For example, the "electric" field operator, $\hat{E}_z = \partial_0 \hat{a}_z - \partial_z \hat{a}_0$ giving the expectation $\langle \hat{E}_z \rangle$. The simple gauge field expectation values, $\langle \mathcal{A}_\mu(x) \rangle$, are obtained by average Eqs. (14) (15) over some quantum state $|Q\rangle$

$$\left\langle Q \middle| \partial_\nu \left(\hat{\phi}^{\mu\nu} + \hat{\Phi}^{\mu\nu}\right) + \epsilon^{3mn} \hat{A}^m_\nu \left(\hat{F}^{n\mu\nu} + \hat{G}^{n\mu\nu}\right) \middle| Q \right\rangle = 0, \tag{16}$$

$$\left\langle Q \middle| D_\nu \left(\hat{F}^{m\mu\nu} + \hat{G}^{m\mu\nu}\right) + \epsilon^{3mn} \left[\hat{A}^n_\nu \left(\hat{\phi}^{\mu\nu} + \hat{\Phi}^{\mu\nu}\right) - \hat{a}_\nu \left(\hat{F}^{n\mu\nu} + \hat{G}^{n\mu\nu}\right)\right] \middle| Q \right\rangle = 0 \tag{17}$$

One problem in using these equations to obtain expectation values like $\langle A^m_\mu \rangle$, is that these equations involve not only powers or derivatives of $\langle A^m_\mu \rangle$ (*i.e.* terms like $\partial_\alpha \langle A^m_\mu \rangle$ or $\partial_\alpha \partial_\beta \langle A^m_\mu \rangle$) but also contain terms like $\mathcal{G}^{mn}_{\mu\nu} = \langle A^m_\mu A^n_\nu \rangle$. Starting with Eqs. (16)– (17) one can generate an operator differential equation for the product $\hat{A}^m_\mu \hat{A}^n_\nu$ thus allowing the determination of the Green's function $\mathcal{G}^{mn}_{\mu\nu}$. However this equation will in turn contain other, higher order Green's functions. Repeating these steps leads to an infinite set of equations connecting Green's functions of ever increasing order. This construction, leading to an infinite set of coupled, differential equations, does not have an exact, analytical solution and so must be handled using some approximation.

Operators are only well determined if there is a Hilbert space of quantum states. Thus we need to ask about the definition of the quantum states $|Q\rangle$ in the above construction. The resolution to this problem is as follows: There is an one-to-one correspondence between a given quantum state $|Q\rangle$ and the infinite set of quantum expectation values over any product of field operators, $\mathcal{G}^{mn\cdots}_{\mu\nu\cdots}(x_1, x_2 \ldots) = \langle Q | A^m_\mu(x_1) A^n_\nu(x_2) \ldots | Q \rangle$. So if all the Green's functions – $\mathcal{G}^{mn\cdots}_{\mu\nu\cdots}(x_1, x_2 \ldots)$ – are known then the quantum states, $|Q\rangle$ are known, *i.e.* the action of $|Q\rangle$ on any product of field operators $\hat{A}^m_\mu(x_1) \hat{A}^n_\nu(x_2) \ldots$ is known. The Green's functions are determined from the above, infinite set of equations (following Heisenberg's idea).

Another problem associated with products of field operators like $\hat{A}^m_\mu(x) \hat{A}^n_\nu(x)$ which occur in Eq. (15) is that the two operators occur at the same point. For *non-interacting* field it is well known that such products have a singularity. In this paper we are considering *interacting* fields so it is not known if a singularity would arise for such products of operators evaluated at the same point. Physically it is hypothesized that there are situations in interacting field theories where these singularities do not occur (*e.g.* for flux tubes in Abelian or non-Abelian theory quantities like the "electric" field inside the tube, $\langle E^a_z \rangle < \infty$, and energy density $\varepsilon(x) = \langle (E^a_z)^2 \rangle < \infty$ are nonsingular). Here we take as an assumption that such singularities do not occur.

We now enumerate our basic assumptions:

1. After quantization the components $\hat{A}^m_\mu(x)$ become stochastic. In mathematical terms we write this assumption as

$$\left\langle A^m_\mu(x) \right\rangle = 0 \qquad \text{and} \qquad \left\langle A^m_\mu(x) A^n_\nu(x) \right\rangle = -\varphi(x) \delta^{mn} \eta_{\mu\nu} \tag{18}$$

   where $\varphi(x)$ is some scalar field, $\eta = \{+1, -1, -1, -1\}$. This would give a problem with the time components in that $\langle A^m_0 A^m_0 \rangle < 0$. Thus to deal with this we also assume that the fields are static and have no time component, *i.e.* $A^m_0 = 0$.

2. The components a^a_μ of the subgroup G can have some order so that certain expectation values can have non-zero values, for example

$$\langle H^a_z \rangle = \langle (\nabla \times \vec{a})_z \rangle \neq 0. \tag{19}$$

Such conditions are meant to imply that a_μ (or certain quantities derived from it) develops a non-zero expectation value for some non-trivial, non-vacuum boundary conditions (*e.g.* the presence of external quarks). Such conditions are not connected with vacuum states since this would imply a violation of the Lorentz symmetry of the QCD vacuum.

3. The gauge potentials a^a_μ and A^m_μ are not correlated. Mathematically this means that

$$\left\langle f(a^a_\mu) g(A^m_\nu) \right\rangle = \left\langle f(a^a_\mu) \right\rangle \left\langle g(A^m_\mu) \right\rangle \tag{20}$$

where f, g are any functions

These assumptions are a variation of the Abelian Projection ideas, since there the SU(N)/G components of the gauge fields are suppressed. The characterization of the off-diagonal fields as stochastic is a result of the first part of Eq. (18), $\left\langle A^m_\mu(x) \right\rangle = 0$. The second part of Eq. (18) is related to some recent work [12] [13] which demonstrates the physical importance of the expectation value of the square of the non-Abelian gauge potential to the dynamics of non-Abelian field theory. The surprising thing about this is that the non-Abelian gauge potential (and its square) is gauge *variant*, and one would think that physical quantities should only be constructed from gauge *invariant* quantities. In previous work [6] [10] [11] one had conditions similar to the first assumption above, but in terms of the expectation values of the Yang-Mills field strength tensor and its square – $\langle G_{\mu\nu} \rangle = 0$ and $\langle G^2_{\mu\nu} \rangle \neq 0$. One way of looking at the condition, $\left\langle A^m_\mu(x) A^n_\nu(x) \right\rangle = -\varphi(x) \delta^{mn} \eta_{\mu\nu}$, is that it represents the condensation of the off-diagonal SU(2) gluons into effective scalar fields, $\varphi(x)$. This provides a physical motivation for a connection of the present work to the Ginzburg-Landau model of superconductivity. In Ginzburg-Landau theory the scalar field represents a condensation of electrons *i.e.* the Cooper pairs. This association between the expectation of the square of the off-diagonal gauge potentials with a scalar field is also similar to ref. [10] except there the association was between $\langle G^2_{\mu\nu} \rangle \neq 0$ and the scalar field.

IV. LONDON'S EQUATION

In this section we want to show how London's equation emerges from Eqs. (14)-(15) under the setup outlined above. London's equation describes the Meissner effect in ordinary superconductivity. Showing that the same equation emerges from a quantized non-Abelian gauge theory gives support to the dual superconducting picture of the QCD vacuum. Because of the stochastic assumption above we will not be interested in the off-diagonal components of the gauge fields, $\left\langle A^m_\mu \right\rangle$. Thus we will not worry about Eq. (15) which is the equation that determines these off-diagonal components. The Abelian field a_μ is determined from Eq. (14) which is linear in a_μ. Because of this we take the Abelian gauge field as classical [14]. This leads to the following equation

$$\partial_\nu \left(\phi^{\mu\nu} + \langle \Phi^{\mu\nu} \rangle \right) = -\epsilon^{3mn} \left(\langle A^m_\nu F^{n\mu\nu} \rangle + \langle A^m_\nu G^{n\mu\nu} \rangle \right). \tag{21}$$

Note the Abelian term, $\phi^{\mu\nu}$, is treated classically while the remaining terms which involve combinations of the off-diagonal fields are treated as quantum degrees of freedom via the expectation values. To calculate these expectation values we take, as a first approximation, the scalar function of Eq. (18) as a constant, *i.e.* $\varphi(x) = \varphi_0$

$$\left\langle A^m_\mu(x) A^n_\nu(x) \right\rangle = -\varphi_0 \delta^{mn} \eta_{\mu\nu} \tag{22}$$

Then this gives

$$\langle \Phi_{\mu\nu} \rangle = \epsilon^{3mn} \left\langle A^m_\mu A^n_\nu \right\rangle = 0, \tag{23}$$

$$\langle A^m_\nu G^{n\mu\nu} \rangle = \epsilon^{np3} \left(\langle A^m_\nu A^{p\mu} \rangle a^\nu - \langle A^m_\nu A^{p\nu} \rangle a^\mu \right) = -3\varphi_0 \epsilon^{3mn} a^\mu. \tag{24}$$

The next term is

$$\langle A^m_\nu F^{n\mu\nu} \rangle = \langle A^m_\nu \partial^\mu A^{n\nu} \rangle - \langle A^m_\nu \partial^\nu A^{n\mu} \rangle + \epsilon^{npq} \langle A^m_\nu A^{p\mu} A^{q\nu} \rangle. \tag{25}$$

For the disordered, non-diagonal components we will set $\langle A^{m_1}_{\mu_1}(x) A^{m_2}_{\mu_2}(x) \ldots A^{m_n}_{\mu_n}(x) \rangle \equiv 0$ if n is odd. For the other terms in the right-hand-side of Eq. (25) we note that

$$\partial_\alpha \left\langle A^m_\mu A^n_\nu \right\rangle = \left\langle \partial_\alpha A^m_\mu A^n_\nu \right\rangle + \left\langle A^m_\mu \partial_\alpha A^n_\nu \right\rangle = 0, \tag{26}$$
$$\left\langle \partial_\alpha A^m_\mu A^n_\nu \right\rangle = - \left\langle A^m_\mu \partial_\alpha A^n_\nu \right\rangle.$$

For these stochastic, non-diagonal components the last expression should not depend on the order of the indices (m, n) and (μ, ν) *i.e.* $\langle \partial_\alpha A^m_\mu A^n_\nu \rangle = \langle \partial_\alpha A^n_\nu A^m_\mu \rangle = \langle A^m_\mu \partial_\alpha A^n_\nu \rangle$. Using this with Eq. (26) gives

$$\langle \partial_\alpha A^m_\mu A^n_\nu \rangle = \langle A^m_\mu \partial_\alpha A^n_\nu \rangle = 0. \tag{27}$$

Putting all this together gives from Eq. (21)

$$\partial_\nu \phi^{\mu\nu} = 6\varphi_0 a^\mu. \tag{28}$$

applying the Lorentz gauge condition, $\partial_\nu a^\nu = 0$, then yields

$$\partial_\nu \partial^\nu a^\mu = -6\varphi_0 a^\mu. \tag{29}$$

This is London's equation for the U(1) ordered phase in the presence of disordered SU(2)/U(1) phase. To demonstrate how this leads to a Meissner-like effect for the U(1) gauge field, a_μ, we take half of all space as being filled by the stochastic phase (*e.g.* $\varphi(x) = \varphi_0 \neq 0$ for $y > 0$ and $\varphi(x) = 0$ for $y < 0$). In this case the Abelian gauge field has only a dependence on y, $a_\mu(y)$, and Eq. (29) becomes

$$\frac{d^2 a_\mu}{dy^2} = 6\varphi_0 a_\mu \tag{30}$$

which has the solution

$$a_\mu = a_{0\mu} e^{-\sqrt{6\varphi_0} y}. \tag{31}$$

Thus the magnetic field $H_z = H_{0z} e^{-\sqrt{6\varphi_0} y}$ is exponentially damped as it penetrates the region with the stochastic phase.

From eqs. (30)-(31) the effective mass of the Abelian field is $m_{eff} = \sqrt{6\varphi_0}$. On the other hand eq. (22) (up to a group factor of 2/3) is similar to the relationship given in ref. [11] (see eq. (32) of that reference) between the effective gluon mass and the expectation of the square of *all* the gauge potentials. Thus from eq. (22) we find that in our model the effective mass of the SU(2) gluons associated with the off-diagonal gauge potentials is different from the effective mass of the gauge boson associated with the Abelian gauge potential. The difference in masses between the condensate represented by the scalar field and the gauge boson is also found in spontaneous symmetry breaking of a gauge symmetry. For example, consider the Ginzburg-Landau Lagrangian with an Abelian gauge field, A_μ, and a complex scalar field, φ, describing the condensate

$$\mathcal{L}_{GL} = (D_\mu \varphi)(D_\mu \varphi)^* - m^2 |\varphi|^2 - \lambda |\varphi|^4 \tag{32}$$

where $D_\mu = \partial_\mu + ieA_\mu$. The condensate has a mass of m while the gauge boson A_μ will acquire a mass of $\sqrt{\frac{e^2 m^2}{2\lambda}}$. In our case the condensate comes from the same set of SU(2) gauge fields as the Abelian gauge field. The different behavior/roles of the Abelian and off-diagonal, non-Abelian gauge fields results from using ideas similar to Abelian Projection through our first assumption given in eq. (18) above.

V. CONCLUSIONS

In this paper we have shown how the London equation emerges from a non-Abelian gauge theory by combining ideas of a nonperturbative quantization technique pioneered by Heisenberg and co-workers, with ideas similar to Abelian Projection. The importance of this is that the London equation gives a phenomenological description of the Meissner effect in superconductors, and the vacuum of some non-Abelian gauge theories (*e.g.* QCD) is often modeled as a dual superconductor in order to explain confinement. In our approach we split the gauge group (SU(2) in our case) into a subgroup (U(1) in our case) and the coset space (SU(2)/U(1) in our case). The gauge bosons associated with the coset SU(2)/U(1) were taken to be in an disordered, stochastic phase, $\langle A^m_\mu(x) \rangle = 0$. Mathematically this statement was contained in Eq. (18) where the scalar field can be compared to the scalar field in the Ginzburg-Landau treatment of superconductivity. In the Ginzburg-Landau model the scalar field represents a condensation of electrons into Cooper pairs. In our work the scalar field can be thought of as a condensation of gluons. Just as the E&M field is excluded from the superconductor, so in our example the diagonal, Abelian gauge field is excluded from the disordered phase.

There is a difference between Abelian Projection and the treatment in the present paper. In Abelian Projection the off-diagonal components are constructed by applying gauge fixing, but in our case they emerge from applying the three assumptions given in section III to the dynamical equations. In the first approximation we have neglected the dynamical behavior of the stochastic phase, by setting $\varphi(x) = \varphi_0$. In this way we obtained an equation for the Abelian components of the gauge field which was similar to London's equation for the vector potential in superconductivity theory. Higher order approximations in the above procedure would result in higher order powers and derivatives of φ. This would hopefully lead to dynamical equations for $\varphi(x)$ similar to the field equations which result from the Ginzburg-Landau Lagrangian given in eq. (32). (Note in this regard that there are two scalar fields in eq. (32), since there φ is complex, and in eq. (18) there are also effectively two scalar fields coming from $m = n = 1$ and $m = n = 2$). In this case one would be able to construct Nielsen-Olesen flux tube solutions [15], which would be very suggestive toward making a firm connection with the dual superconducting model of QCD. Such a construction of an effective Ginzburg-Landau equation for φ would be important in bolstering the claim of a connection between our approach and the dual superconducting model of the QCD vacuum.

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