

GRAVITATIONAL COUPLINGS FOR yGOp-PLANES

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ABSTRACT

The Wess-Zumino action for y deformed and generalized orientifold planes (yGOp-planes) is presented and one power expansion is realized from which processes that involves yGOp-planes, RR-forms , gravitons and gaugeons , are obtained. Finally non-standard yGOp-planes are showed.

1 Introduction

The results that this paper presents is about gravitational couplings for y deformed and generalized orientifold planes (yGOp-planes) . The usual orientifold planes do not have gauge fields on their worldvolumes and do no have any kind of topological y-deformation over their worldvolumes . The y deformed and generalized orientifold planes that this paper consider have $SO(2k)$ Yang-Mills gauge fields-bundles over their corresponding worldvolumes and have topological deformations of the fields-bundles over their corresponding worldvolumes. The aim of the present paper is to display the Wess-Zumino part of the effective action for such y-deformed and generalized orientifold planes.

For the usual orientifold planes the Wess-Zumino action has the following form, which can be derived both from anomaly cancellation arguments and from direct computation on string scattering amplitudes:

$$S_{WZ(Op-plane)} = -2^{p-4} \frac{T_p}{\kappa_{p+1}} \int_{p+1} C \sqrt{\frac{L(\frac{R_T}{4})}{L(\frac{R_N}{4})}}$$

Where the Mukai vector of RR charges for the usual orientifold p-plane is given by:

$$Q(\frac{R_T}{4}, \frac{R_N}{4}) = \sqrt{\frac{L(\frac{R_T}{4})}{L(\frac{R_N}{4})}}$$

In this formula C is the vector of the RR potential forms. L is the Hirzebruch genus that generates the Hirzebruch polynomials which are given in terms of Pontryaguin classes for real bundles. The Pontryaguin classes are given in terms of the 2-form curvature of the corresponding real bundle. The formula for Q involves two real bundles over the worldvolume of the usual orientifold plane. These two bundles are the tangent bundle for the worldvolume and the normal bundle by respect to space-time for such worldvolume. Q is given then in terms of the curvatures for the tangent and normal bundles and does not have contributions from the others real bundles such as $SO(2k)$ Yang-Mills gauge bundles and does not have any kind of topological deformation.

In a recent work was presented the Mukay vector of RR charges for the generalized orientifold planes which have two $SO(2k)$ Yang-Mills gauge bundles on their worldvolumes. Such vector of RR charges is given by the following formula:

$$Q(\frac{R_T}{2}, \frac{R_N}{2}, \frac{R_E}{2}, \frac{R_F}{2}) = \sqrt{\frac{A(\frac{R_T}{2})\text{Mayer}(\frac{R_E}{2})}{A(\frac{R_N}{2})\text{Mayer}(\frac{R_F}{2})}}$$

For the generalized orientifold planes the Wess-Zumino action has the following form:

$$S_{WZ(GOp-plane)} = -2^{p-4} \frac{T_p}{\text{kappa}} \int_{p+1} C \sqrt{\frac{A(\frac{R_T}{2})\text{Mayer}(\frac{R_E}{2})}{A(\frac{R_N}{2})\text{Mayer}(\frac{R_F}{2})}}$$

The formula for the vector of RR charges corresponding to a generalized orientifold plane involves now four real bundles over the worldvolume: the tangent bundle, the normal bundle and two new SO(2k) YM gauge bundles. When one of these new SO(2k) bundles is the tangent bundle and the other is the normal bundle, one obtain the usual formula for Q corresponding to the usual orientifold planes using the following identity:

$$A(\frac{R}{2})\text{Mayer}(\frac{R}{2}) = L(\frac{R}{4})$$

Then, one has:

$$Q(\frac{R_T}{2}, \frac{R_N}{2}, \frac{R_T}{2}, \frac{R_N}{2}) = \sqrt{\frac{A(\frac{R_T}{2})\text{Mayer}(\frac{R_T}{2})}{A(\frac{R_N}{2})\text{Mayer}(\frac{R_N}{2})}}$$

$$Q(\frac{R_T}{2}, \frac{R_N}{2}, \frac{R_T}{2}, \frac{R_N}{2}) = \sqrt{\frac{L(\frac{R_T}{4})}{L(\frac{R_N}{4})}} = Q(\frac{R_T}{4}, \frac{R_N}{4})$$

In these formulas, A denotes the roof-Dirac genus and Mayer denotes the Mayer class for one SO(2k) YM gauge bundle.

In other recent work, also, was presented the Mukay vector of RR charges for the y-deformed orientifold planes which have topological y-deformations on their worldvolumes. Such vector of RR charges is given by the following formula:

$$Q(\frac{R_T}{4}, \frac{R_N}{4}, y) = \sqrt{\frac{\text{CHI}_y(\frac{R_T}{4})}{\text{CHI}_y(\frac{R_N}{4})}}$$

For the y-deformed orientifold planes the Wess-Zumino action has the following form:

$$S_{WZ(yOp-plane)} = -2^{p-4} \frac{T_p}{\text{kappa}} \int_{p+1} C \sqrt{\frac{\text{CHI}_y(\frac{R_T}{4})}{\text{CHI}_y(\frac{R_N}{4})}}$$

The formula for the vector of RR charges corresponding to a y-deformed orientifold plane involves now two real bundles over the worldvolume: the tangent bundle and the normal bundle, but in this case one has a topological y-deformation over the worldvolume. When the parameter y of the topological y-deformation is 1, then one obtain the usual formula for Q corresponding to the usual orientifold planes using the following identity:

$$\text{CHI}_1(R) = L(R)$$

Then, one has:

$$Q(\frac{R_T}{4}, \frac{R_N}{4}, 1) = \sqrt{\frac{\text{CHI}_1(\frac{R_T}{4})}{\text{CHI}_1(\frac{R_N}{4})}}$$

$$Q(\frac{R_T}{4}, \frac{R_N}{4}, 1,) = \sqrt{\frac{L(\frac{R_T}{4})}{L(\frac{R_N}{4})}} = Q(\frac{R_T}{4}, \frac{R_N}{4})$$

In these formulas, CHI sub y denotes the chi-y- genus which when y=1 is the Hirzebruch-genus and when y=0, is the Todd genus.

In this paper is presented the Mukay vector of RR charges for the y- deformed and generalized orientifold planes which have two SO(2k) Yang-Mills gauge bundles on their worldvolumes and have topological y-deformations of the all bundles that are living on their worldvolumes . Such vector of RR charges is given by the following formula:

$$Q(\frac{R_T}{2}, \frac{R_N}{2}, \frac{R_E}{2}, \frac{R_F}{2}, y) = \sqrt{\frac{A(\frac{R_T}{2}, y) \text{Mayer}(\frac{R_E}{2}, y)}{A(\frac{R_N}{2}, y) \text{Mayer}(\frac{R_F}{2}, y)}}$$

For the y-deformed and generalized orientifold planes the Wess-Zumino action has the following form:

$$S_{WZ(yGOp-plane)} = -2^{p-4} \frac{T_p}{\text{kappa}} \int_{p+1} C \sqrt{\frac{A(\frac{R_T}{2}, y) \text{Mayer}(\frac{R_E}{2}, y)}{A(\frac{R_N}{2}, y) \text{Mayer}(\frac{R_F}{2}, y)}}$$

The formula for the vector of RR charges corresponding to a y-deformed and generalized orientifold plane involves now four y-deformed real bundles over the worldvolume: the tangent bundle, the normal bundle and two new SO(2k) YM gauge bundles. When one of these new SO(2k) bundles is the tangent bundle and the other is the normal bundle, one obtain the formula for Q corresponding to the y-deformed orientifold plane using the following identity:

$$A(\frac{R}{2}, y) \text{Mayer}(\frac{R}{2}, y) = CHI_y(\frac{R}{4})$$

Then, one has:

$$Q(\frac{R_T}{2}, \frac{R_N}{2}, \frac{R_T}{2}, \frac{R_N}{2}, y) = \sqrt{\frac{A(\frac{R_T}{2}, y) \text{Mayer}(\frac{R_T}{2}, y)}{A(\frac{R_N}{2}, y) \text{Mayer}(\frac{R_N}{2}, y)}}$$

$$Q(\frac{R_T}{2}, \frac{R_N}{2}, \frac{R_T}{2}, \frac{R_N}{2}, y) = \sqrt{\frac{CHI_y(\frac{R_T}{4})}{CHI_y(\frac{R_N}{4})}} = Q(\frac{R_T}{4}, \frac{R_N}{4}, y)$$

When the parameter y of the topological deformation is 1 , one obtains the formula for Q corresponding to the generalized orientifold plane using the following identity:

$$A(\frac{R_T}{2}, 1) \text{Mayer}(\frac{R_E}{2}, 1) = A(\frac{R_T}{2}) \text{Mayer}(\frac{R_E}{2})$$

Then, one has:

$$Q(\frac{R_T}{2}, \frac{R_N}{2}, \frac{R_E}{2}, \frac{R_F}{2}, 1) = \sqrt{\frac{A(\frac{R_T}{2}, 1) \text{Mayer}(\frac{R_E}{2}, 1)}{A(\frac{R_N}{2}, 1) \text{Mayer}(\frac{R_F}{2}, 1)}}$$

$$Q(\frac{R_T}{2}, \frac{R_N}{2}, \frac{R_E}{2}, \frac{R_F}{2}, 1) = \sqrt{\frac{A(\frac{R_T}{2}) \text{Mayer}(\frac{R_E}{2})}{A(\frac{R_N}{2}) \text{Mayer}(\frac{R_F}{2})}} = Q(\frac{R_T}{2}, \frac{R_N}{2}, \frac{R_E}{2}, \frac{R_F}{2})$$

When one of these new SO(2k) bundles is the tangent bundle and the other is the normal bundle, and the parameter y of the topological deformation is 1, then, one obtains the formula for Q corresponding to the usual orientifold plane using the following identity:

$$A(\frac{R}{2}, 1)Mayer(\frac{R}{2}, 1) = L(\frac{R}{4})$$

Then, one has:

$$Q(\frac{R_T}{2}, \frac{R_N}{2}, \frac{R_T}{2}, \frac{R_N}{2}, 1) = \sqrt{\frac{A(\frac{R_T}{2}, 1)Mayer(\frac{R_T}{2}, 1)}{A(\frac{R_N}{2}, 1)Mayer(\frac{R_N}{2}, 1)}}$$

$$Q(\frac{R_T}{2}, \frac{R_N}{2}, \frac{R_T}{2}, \frac{R_N}{2}, 1) = \sqrt{\frac{L(\frac{R_T}{4})}{L(\frac{R_N}{4})}} = Q(\frac{R_T}{4}, \frac{R_N}{4})$$

In these formulas, appears the y-deformed roof-Dirac genus and the y-deformed Mayer class for one SO(2k) YM gauge bundle.

In the following section the Mukay vector of RR charges for a such y-deformed and generalized orientifold p-plane (yGOp-plane) , will be given in terms of the powers of the curvatures for the four y-deformed real bundles involved over the worldvolume.

In the third section are presented the elementary processes corresponding to the power expansion for the three Q's. In the final four section some conclusions are presented about other yGOp-planes, about non-BPS yGOp-planes and non commutative yGOp-planes.

2 The Power Expantions for Q's

In this section are obtained three series power-curvature expantions corresponding to the three Q associated respectively to the GOp-planes, yOp-planes and yGOp-planes. The yGOp-planes are an unification of the GOp-planes and yOp-planes. The yGOp-planes contains the usual Op-planes, as limiting cases.

2.1 The Power Expantion for GOp-plane

Let E be a SO(2k)-bundle over the worldvolume of a generalized orientifold plane and consider a formal factorisation for the total Pontryaguin classs of the real bundle E, which has the following form:

$$p(E) = \prod_{i=1}^k (1 + y_i^2)$$

The total Pontryaguin classs of the real bundle E, has the following formal sumarisation in terms of the corresponding Pontryaguin classes:

$$p(E) = \sum_{j=0}^{\infty} p_j(E)$$

The total Mayer class for the real bundle E has the following formal factorisation:

$$Mayer(E) = \prod_{i=1}^k \cosh(\frac{y_i}{2})$$

The total Mayer class for the real bundle E has the following formal sumarisation in terms of the Mayer polynomials which are formed from the corresponding Pontryaguin classes :

$$Mayer(E) = \sum_{j=0}^{\infty} Mayer_j(p_1(E), ..., p_j(E))$$

The Mayer polynomials are given by:

$$Mayer_0(p_0(E)) = Mayer_0(1) = 1$$

$$Mayer_1(p_1(E)) = \frac{p_1(E)}{8}$$

$$Mayer_2(p_1(E), p_2(E)) = \frac{p_1(E)^2 + 4p_2(E)}{384}$$

$$Mayer_3(p_1(E), p_2(E), p_3(E)) = \frac{p_1(E)^3 + 12p_1(E)p_2(E) + 48p_3(E)}{46080}$$

The pontryaguin classes of the real bundle E have the following realizations in terms of the powers of the 2-form curvature for such bundle. For this curvature the y's are the eigenvalues:

$$p_1(E) = p_1(R_E) = -\frac{1}{8\pi^2} tr R_E^2$$

$$p_2(E) = p_2(R_E) = \frac{1}{16\pi^4} [\frac{1}{8}(tr R_E^2)^2 - \frac{1}{4} tr R_E^4]$$

$$p_3(E) = p_3(R_E) = \frac{1}{64\pi^6} [-\frac{1}{48}(tr R_E^2)^3 - \frac{1}{6} tr R_E^6 + \frac{1}{8} tr R_E^2 tr R_E^4]$$

Using all these expretions one can to obtain the following expansion:

$$Mayer(\frac{R_E}{2}) =$$

$$1 + \frac{p_1(R_E)}{32} + \frac{p_1(R_E)^2 + 4p_2(R_E)}{6144} + \frac{p_1(R_E)^3 + 12p_1(R_E)p_2(R_E) + 48p_3(R_E)}{2949120} + \dots$$

Now one has the following expansions:

$$A(\frac{R}{2}) = 1 - \frac{p_1(R)}{96} + \frac{7p_1(R)^2 - 4p_2(R)}{92160} + \dots$$

$$L(\frac{R}{4}) = 1 + \frac{p_1(R)}{48} + \frac{-p_1(R)^2 + 7p_2(R)}{11520} + \dots$$

Using these three expansions it is easy to obtain the following identities:

$$A(\frac{R}{2})Mayer(\frac{R}{2}) = L(\frac{R}{4})$$

$$A(R)Mayer(R) = L(\frac{R}{2})$$

$$A(2R)Mayer(2R) = L(R)$$

$$A(2^q R)Mayer(2^q R) = L(2^{q-1} R)$$

$$[A(R)2^k Mayer(R)]_{topform} = L(R)_{topform}$$

With the help from these identities one has that:

$$\sqrt{\frac{A(\frac{R_T}{2})Mayer(\frac{R_T}{2})}{A(\frac{R_N}{2})Mayer(\frac{R_N}{2})}} = \sqrt{\frac{L(\frac{R_T}{4})}{L(\frac{R_N}{4})}}$$

Using all these equations it is easy to obtain the following power expansion for Q:

$$\sqrt{\frac{A(\frac{R_T}{2})\text{Mayer}(\frac{R_F}{2})}{A(\frac{R_N}{2})\text{Mayer}(\frac{R_F}{2})}} = 1 + \frac{(4\pi^2\text{alfa})^2}{1536\pi^2}(tr R_T^2 - tr R_N^2) - \frac{(4\pi^2\text{alfa})^2}{512\pi^2}(tr R_E^2 - tr R_F^2) + \frac{(4\pi^2\text{alfa})^4}{4718592\pi^4}(tr R_T^2 - tr R_N^2)^2 + \frac{(4\pi^2\text{alfa})^4}{2949120\pi^4}(tr R_T^4 - tr R_N^4) + \frac{(4\pi^2\text{alfa})^4}{524288\pi^4}(tr R_E^2 - tr R_F^2)^2 - \frac{(4\pi^2\text{alfa})^4}{196608\pi^4}(tr R_E^4 - tr R_F^4) - \frac{(4\pi^2\text{alfa})^4}{786432\pi^4}(tr R_T^2 - tr R_N^2)(tr R_E^2 - tr R_F^2)$$

When the bundle E is the tangent bundle and the bundle F is the normal bundle one obtain the usual power expansion for Q corresponding to the usual orientifold plane:

$$\sqrt{\frac{A(\frac{R_T}{2})\text{Mayer}(\frac{R_T}{2})}{A(\frac{R_N}{2})\text{Mayer}(\frac{R_N}{2})}} = 1 + \frac{(4\pi^2\text{alfa})^2}{1536\pi^2}(tr R_T^2 - tr R_N^2) - \frac{(4\pi^2\text{alfa})^2}{512\pi^2}(tr R_T^2 - tr R_N^2) + \frac{(4\pi^2\text{alfa})^4}{4718592\pi^4}(tr R_T^2 - tr R_N^2)^2 + \frac{(4\pi^2\text{alfa})^4}{2949120\pi^4}(tr R_T^4 - tr R_N^4) + \frac{(4\pi^2\text{alfa})^4}{524288\pi^4}(tr R_T^2 - tr R_N^2)^2 - \frac{(4\pi^2\text{alfa})^4}{196608\pi^4}(tr R_T^4 - tr R_N^4) - \frac{(4\pi^2\text{alfa})^4}{786432\pi^4}(tr R_T^2 - tr R_N^2)(tr R_T^2 - tr R_N^2)$$

$$\sqrt{\frac{A(\frac{R_T}{2})\text{Mayer}(\frac{R_T}{2})}{A(\frac{R_N}{2})\text{Mayer}(\frac{R_N}{2})}} = 1 - \frac{(4\pi^2\text{alfa})^2}{768\pi^2}(tr R_T^2 - tr R_N^2) + \frac{(4\pi^2\text{alfa})^4}{1179648\pi^4}(tr R_T^2 - tr R_N^2)^2 - \frac{7(4\pi^2\text{alfa})^4}{1474560\pi^4}(tr R_T^4 - tr R_N^4)$$

$$\sqrt{\frac{A(\frac{R_T}{2})\text{Mayer}(\frac{R_T}{2})}{A(\frac{R_N}{2})\text{Mayer}(\frac{R_N}{2})}} = \sqrt{\frac{L(\frac{R_T}{4})}{L(\frac{R_N}{4})}}$$

2.2 The Power Expansion for yOp-plane

For the other hand in the case of the y-Op-plane, the total Chern Class for a complex n-dimensional bundle V over the worldvolume has the following sumarization:

$$c(V) = \sum_{j=0}^{\infty} c_j(T)$$

also, the total Chern Class for the such bundle has the following factorization:

$$c(V) = \prod_{i=1}^n (1 + x_i)$$

The CHI-y- genus for the complex bundle V has the following formal factorisation:

$$CHI_y(V) = \prod_{i=1}^n \frac{(1+y\exp(-(y+1)x_i))^{x_i}}{1-\exp(-(y+1)x_i)}$$

The CHI-y- genus for the complex bundle V has the following formal summarisation in terms of the y-deformed Todd polynomials which are formed from the corresponding Chern classes and from the polynomials on y :

$$CHI_y(V) = \sum_{j=0}^{\infty} T_j(c_1(V), \dots, c_j(V), y)$$

The y-Todd polynomials are given by:

$$T_0(c_0(V), y) = T_0(1, y) = 1$$

$$T_1(c_1(V), y) = \frac{(1-y)c_1(V)}{2}$$

$$T_2(c_1(V), c_2(V), y) = \frac{(y+1)^2 c_1(V)^2 + (y^2 - 10y + 1) c_2(V)}{12}$$

$$T_3(c_1(V), c_2(V), c_3(V), y) = \frac{-(y+1)^2 (y-1) c_1(V) c_2(V) + 12y (y-1) c_3(V)}{24}$$

$$T_4(c_1(V), c_2(V), c_3(V), c_4(V), y) = \frac{(-y^4 + 474y^2 - 124y - 1 - 124y^3) c_4(V) + (y^2 - 58y + 1) (y+1)^2 c_1(V) c_3(V) + (y+1)^4 (3c_2(V)^2 + 4c_1(V)^2 c_2(V) - c_1(V)^4)}{720}$$

Now the relations between the Pontryaguin classes and the Chern Classes for the bundle V are given by the following formulas:

$$p_1(V) = -2c_2(V) + c_1(V)^2$$

$$p_2(V) = 2c_4(V) - 2c_3(V)c_1(V) + c_2(V)^2$$

Using these relations the y-deformed Todd polynomials can be written as follows:

$$T_0(c_0(V), y) = T_0(1, y) = 1$$

$$T_1(c_1(V), y) = \frac{(1-y)c_1(V)}{2}$$

$$T_2(p_1(V), c_2(V), y) = \frac{(y+1)^2 p_1(V) + 3(y-1)^2 c_2(V)}{12}$$

$$T_3(c_1(V), c_2(V), c_3(V), y) = \frac{-(y+1)^2 (y-1) c_1(V) c_2(V) + 12y (y-1) c_3(V)}{24}$$

$$T_4(c_1(V), c_3(V), c_4(V), p_1(V), p_2(V), y) = \frac{-15(y^2 + 14y + 1)(y-1)^2 c_4(V) + 15(y-1)^2 (y+1)^2 c_1(V) c_3(V) + (y+1)^4 (7p_2(V) - p_1(V)^2)}{720}$$

When y=1 the y-deformed Todd polynomials are the same Hirzebruch polynomials:

$$T_0(c_0(V), 1) = T_0(1, 1) = 1 = L_0$$

$$T_1(c_1(V), 1) = \frac{(1-1)c_1(V)}{2} = 0$$

$$T_2(p_1(V), c_2(V), 1) = \frac{(1+1)^2 p_1(V) + 3(1-1)^2 c_2(V)}{12} = \frac{p_1(V)}{3} = L_1(p_1(V))$$

$$T_3(c_1(V), c_2(V), c_3(V), 1) = \frac{-(1+1)^2 (1-1) c_1(V) c_2(V) + 12(1-1) c_3(V)}{24} = 0$$

$$T_4(c_1(V), c_3(V), c_4(V), p_1(V), p_2(V), 1) = \frac{-15(1^2 + 14 + 1)(1-1)^2 c_4(V) + 15(1-1)^2 (1+1)^2 c_1(V) c_3(V) + (1+1)^4 (7p_2(V) - p_1(V)^2)}{720} = \frac{7p_2(V) - p_1(V)^2}{45} = L_2(p_1(V), p_2(V))$$

Using all these expressions one can obtain the following expansion:

$$CHI_y\left(\frac{R_V}{4}\right) = 1 + \frac{(1-y)c_1(R_V)}{8} + \frac{(y+1)^2 p_1(R_V) + 3(y-1)^2 c_2(R_V)}{192} + \frac{-(y+1)^2 (y-1) c_1(R_V) c_2(R_V) + 12y (y-1) c_3(R_V)}{1536} + \frac{-15(y^2 + 14y + 1)(y-1)^2 c_4(R_V) + 15(y-1)^2 (y+1)^2 c_1(R_V) c_3(R_V) + (y+1)^4 (7p_2(R_V) - p_1(R_V)^2)}{184320} +$$

...

When the first chern class of V is trivial, one obtain, using again the relations between pontryaguin classes and Chern classes, the following result:

$$CHI_y(\frac{R_V}{4}) = 1 + \frac{(2(y+1)^2 - 3(y-1)^2)p_1(R_V)}{384} + \frac{12y(y-1)c_3(R_V)}{1536} +$$

$$\frac{(-60(y^2 + 14y + 1)(y-1)^2 + 56(y+1)^4)p_2(R_V) - (-15(y^2 + 14y + 1)(y-1)^2 + 8(y+1)^4)p_1(R_V)^2}{1474560} +$$

...

Finally using this last expansion and the relations between the Pontryaguin classes and the 2-form curvature, one can to obtain the following development for the Q of the yOp-planes:

$$\sqrt{\frac{CHI_y(\frac{R_T}{4})}{CHI_y(\frac{R_N}{4})}} = 1 + \frac{(y^2 - 10y + 1)(4\pi^2 \text{alfa})^2}{6144\pi^2} (tr R_T^2 - tr R_N^2) +$$

$$\frac{(4\pi^2 \text{alfa})^3}{256} y(y-1)(c_3(R_T) - c_3(R_N)) +$$

$$\frac{(4\pi^2 \text{alfa})^4}{75497472\pi^4} (y^2 - 10y + 1)^2 (tr R_T^2 - tr R_N^2)^2 -$$

$$\frac{(4\pi^2 \text{alfa})^4}{188743680\pi^4} (-4y^4 - 496y^3 + 1896y^2 - 496y - 4)(tr R_T^4 - tr R_N^4)$$

When y=1, then one obtain the development for the Q of the usual Op-plane:

$$\sqrt{\frac{CHI_1(\frac{R_T}{4})}{CHI_1(\frac{R_N}{4})}} = 1 + \frac{(1^2 - 10 + 1)(4\pi^2 \text{alfa})^2}{6144\pi^2} (tr R_T^2 - tr R_N^2) +$$

$$\frac{(4\pi^2 \text{alfa})^3}{256} 1(1-1)(c_3(R_T) - c_3(R_N)) +$$

$$\frac{(4\pi^2 \text{alfa})^4}{75497472\pi^4} (1^2 - 10 + 1)^2 (tr R_T^2 - tr R_N^2)^2 -$$

$$\frac{(4\pi^2 \text{alfa})^4}{188743680\pi^4} (-4 - 496 + 1896 - 496 - 4)(tr R_T^4 - tr R_N^4)$$

$$\sqrt{\frac{CHI_1(\frac{R_T}{4})}{CHI_1(\frac{R_N}{4})}} = 1 - \frac{(4\pi^2 \text{alfa})^2}{768\pi^2} (tr R_T^2 - tr R_N^2) +$$

$$\frac{(4\pi^2 \text{alfa})^4}{1179648\pi^4} (tr R_T^2 - tr R_N^2)^2 - \frac{7(4\pi^2 \text{alfa})^4}{1474560\pi^4} (tr R_T^4 - tr R_N^4)$$

$$\sqrt{\frac{CHI_1(\frac{R_T}{4})}{CHI_1(\frac{R_N}{4})}} = \sqrt{\frac{L(\frac{R_T}{4})}{L(\frac{R_N}{4})}}$$

2.3 The Power Expantion for yGOp-plane

Let E be a y-deformed SO(2k)-bundle over the worldvolume of a y-deformed and generalized orientifold plane and consider a formal factorisation for the total Pontryaguin class of the y-deformed real bundle E, which has the following form:

$$p(E) = \prod_{i=1}^k (1 + y_i^2)$$

The total Pontryaguin class of the real SO(2k)- bundle E, has the following formal sumarisation in terms of the corresponding Pontryaguin classes:

$$p(E) = \sum_{j=0}^{\infty} p_j(E)$$

From the other hand the total Chern class of the complex $SU(k)$ -bundle E , has the following formal factorisation:

$$c(E) = \prod_{i=1}^k (1 + y_i)$$

also, the total Chern Class for the such bundle has the following summarisation:

$$c(E) = \sum_{j=0}^{\infty} c_j(E)$$

The total y -deformed Mayer class for the real-complex bundle E has the following formal factorisation:

$$Mayer(E, y) = \prod_{i=1}^k \frac{\exp(\frac{y_i(y+1)}{4}) + y \exp(-\frac{y_i(y+1)}{4})}{2}$$

The total y -deformed Mayer class for the real-complex bundle E has the following formal summarisation in terms of the y -deformed Mayer polynomials which are formed from the corresponding Pontryaguin classes, from the corresponding Chern classes and from polynomials for the parameter y :

$$Mayer(E, y) = \sum_{j=0}^{\infty} Mayer_j(p_1(E), \dots, c_1(E), \dots, y)$$

The y -deformed Mayer polynomials are given by:

$$Mayer_0(p_0(E), y) = Mayer_0(1, y) = \frac{(y+1)^4}{16}$$

$$Mayer_1(p_1(E), y) = \frac{y(y+1)^4 p_1(E)}{128}$$

$$Mayer_{\frac{3}{2}}(c_3(E), y) = \frac{y(y-1)(y+1)^4 c_3(E)}{256}$$

$$Mayer_2(p_1(E), p_2(E), y) = \frac{y(y+1)^4 ((1-y+y^2)p_1(E)^2 - 2(-4y+1+y^2)p_2(E))}{6144}$$

The total Pontryaguin classs of the real tangent bundle T of the worldvolume of the y -GOP-plane, has the following formal summarisation in terms of the corresponding Pontryaguin classes:

$$p(T) = \sum_{j=0}^{\infty} p_j(T)$$

also, the formal factorisation for the total Pontryaguin classs of the y -deformed real tangent bundle T , has the following form:

$$p(T) = \prod_{i=1}^{\frac{p+1}{2}} (1 + x_i^2)$$

The total y -deformed Dirac-roof genus for the real bundle T has the following formal factorisation:

$$A(T, y) = \prod_{i=1}^{\frac{p+1}{2}} \frac{\frac{x_i}{2}}{\sinh(\frac{(y+1)x_i}{4})}$$

The total y -deformed Dirac-roof genus for the real bundle T has the following formal summarisation in terms of the y -deformed Dirac polynomials which are formed from the corresponding Pontryaguin classes and from polynomials for the parameter y :

$$A(T, y) = \sum_{j=0}^{\infty} A_j(p_1(T), \dots, p_j(T), y)$$

The y -deformed Dirac polynomials are given by:

$$A_0(p_0(T), y) = A_0(1, y) = \frac{16}{(y+1)^4}$$

$$A_1(p_1(T), y) = -\frac{p_1(T)}{6(y+1)^2}$$

$$A_2(p_1(T), p_2(T), y) = \frac{7p_1(T)^2 - 4p_2(T)}{5760}$$

It is easy to check that when $y=1$ the y -deformed Mayer polynomials and the y -deformed Dirac polynomials are the same usuals Mayer polynomials and Dirac polynomials.

Using all these y -deformed polynomials and the relations between the Pontryagin classes and the 2-form curvatures, one can obtain the following expansion for the Q of the y GOp-planes:

$$\begin{aligned} \sqrt{\frac{A(\frac{R_T}{2}, y) \text{Mayer}(\frac{R_E}{2}, y)}{A(\frac{R_N}{2}, y) \text{Mayer}(\frac{R_F}{2}, y)}} &= 1 + \frac{(4\pi^2 \text{alfa})^2}{6144\pi^2} (y+1)^2 (tr R_T^2 - tr R_N^2) - \\ &\frac{(4\pi^2 \text{alfa})^2}{512\pi^2} y (tr R_E^2 - tr R_F^2) + \frac{(4\pi^2 \text{alfa})^3}{256} y(y-1) (c_3(R_E) - c_3(R_F)) + \\ &\frac{(4\pi^2 \text{alfa})^4}{75497472\pi^4} (y+1)^4 (tr R_T^2 - tr R_N^2)^2 + \\ &\frac{(4\pi^2 \text{alfa})^4}{47185920\pi^4} (y+1)^4 (tr R_T^4 - tr R_N^4) + \frac{(4\pi^2 \text{alfa})^4}{524288\pi^4} y^2 (tr R_E^2 - tr R_F^2)^2 + \\ &\frac{(4\pi^2 \text{alfa})^4}{393216\pi^4} y(-4y+1+y^2) (tr R_E^4 - tr R_F^4) - \\ &\frac{(4\pi^2 \text{alfa})^4}{3145728\pi^4} y(y+1)^2 (tr R_T^2 - tr R_N^2) (tr R_E^2 - tr R_F^2) \end{aligned}$$

When $y=1$ the y GOp-plane is reduced to the GOp-plane. When $E=T$ and $F=N$, the y GOp-plane is reduced to the y Op-plane. When $y=1$ and $E=T$ and $F=N$, then the y GOp-plane is reduced to the usual Op-plane.

3 The Elementary Processes

In this section are presented the elementary gravitational processes for Op-planes, GOp-planes, y Op-planes and y GOp-planes corresponding to the series power-curvature expansions of the three Q 's obtained in the section two.

3.1 The Elementary Processes for Op-planes

The WZ action for the usual orientifold p -plane can be written as a sum of the WZ actions for three elementary processes:

$$S_{WZ(Op-plane)} = \sum_{j=1}^3 S_{WZ(Op-plane),j}$$

The WZ actions for the three elementary processes are given by the following expressions:

$$\begin{aligned} S_{WZ(Op-plane),1} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p+1} \\ S_{WZ(Op-plane),2} &= \\ -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-3} &[-(\frac{(4\pi^2 \text{alfa})^2}{768\pi^2} (tr R_T^2 - tr R_N^2))] \end{aligned}$$

$$S_{WZ(Op-plane),3} = -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-7} \left(\frac{(4\pi^2 \alpha)^4}{1179648\pi^4} (tr R_T^2 - tr R_N^2)^2 - \frac{7(4\pi^2 \alpha)^4}{1474560\pi^4} (tr R_T^4 - tr R_N^4) \right)$$

The first WZ action describes an elementary process for which the usual orientifold p-plane emits one (p+1)-form RR potential. The second WZ action describes an elementary process for which the usual Op-plane absorbs two gravitons and emits one (p-3)-form RR potential. The third WZ action describes an elementary process for which the Op-plane absorbs four gravitons and emits one (p-7)-form RR potential.

3.2 The Elementary Processes for GOp-planes

From the result of the section two, the WZ action for a generalized orientifold p-plane can be written as a sum of the WZ actions for some elementary processes:

$$S_{WZ(GOp-plane)} = \sum_{j=1}^6 S_{WZ(GOp-plane),j}$$

The WZ actions for the six elementary processes are given by the following expressions:

$$\begin{aligned} S_{WZ(GOp-plane),1} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p+1} \\ S_{WZ(GOp-plane),2} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-3} \frac{(4\pi^2 \alpha)^2}{1536\pi^2} (tr R_T^2 - tr R_N^2) \\ S_{WZ(GOp-plane),3} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-3} \left(-\frac{(4\pi^2 \alpha)^2}{512\pi^2} (tr R_E^2 - tr R_F^2) \right) \\ S_{WZ(GOp-plane),4} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-7} \left(\frac{(4\pi^2 \alpha)^4}{4718592\pi^4} (tr R_T^2 - tr R_N^2)^2 + \frac{(4\pi^2 \alpha)^4}{2949120\pi^4} (tr R_T^4 - tr R_N^4) \right) \\ S_{WZ(GOp-plane),5} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-7} \left(\frac{(4\pi^2 \alpha)^4}{524288\pi^4} (tr R_E^2 - tr R_F^2)^2 - \frac{(4\pi^2 \alpha)^4}{196608\pi^4} (tr R_E^4 - tr R_F^4) \right) \\ S_{WZ(GOp-plane),6} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-7} \left(-\frac{(4\pi^2 \alpha)^4}{786432\pi^4} (tr R_T^2 - tr R_N^2) (tr R_E^2 - tr R_F^2) \right) \end{aligned}$$

The first WZ action describes an elementary process for which the generalized orientifold p-plane emits one (p+1)-form RR potential. The second WZ action describes an elementary process for which the generalized Op-plane absorbs two gravitons and emits one (p-3)-form RR potential. The third WZ action describes an elementary process for which the generalized Op-plane absorbs two gaugeons and emits one (p-3)-form RR potential. The fourth WZ action describes an elementary process for which the GOp-plane absorbs four gravitons and emits one (p-7)-form RR potential. The fifth WZ action describes an elementary process for which the GOp-plane absorbs four gaugeons and emits one (p-7)-form RR potential. The sixth WZ action describes an elementary process for which the GOp-planes absorbs two gravitons and two gaugeons and emits one (p-7)-form RR potential.

When the gaugeons corresponding to the bundles E and F are the same gravitons corresponding to the bundles T and N respectively, then the six elementary process for the GOp-plane are reduced to the usuals three elementary process for the usual Op-plane: Op-plane emits one (p+1)-form RR potential, Op-plane absorbs two gravitons and emits one (p-3)-form RR potential; and, Op-plane absorbs four gravitons and emits one (p-7)-form RR potential.

3.3 The Elementary Processes for yOp-planes

Of other hand, from the result of the section two, the WZ action for a y-deformed orientifold p-plane can be written as a sum of the WZ actions for some elementary processes:

$$S_{WZ(yOp-plane)} = \sum_{j=1}^4 S_{WZ(yOp-plane),j}$$

The WZ actions for the four elementary processes are given by the following expressions:

$$\begin{aligned} S_{WZ(yOp-plane),1} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p+1} \\ S_{WZ(yOp-plane),2} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-3} \left[- \left(\frac{(y^2-10y+1)(4\pi^2\alpha)^2}{6144\pi^2} (tr R_T^2 - tr R_N^2) \right) \right] \\ S_{WZ(yOp-plane),3} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-5} \left(\frac{(4\pi^2\alpha)^3}{256} y(y-1) (c_3(R_T) - c_3(R_N)) \right) \\ S_{WZ(yOp-plane),4} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-7} \left(\frac{(4\pi^2\alpha)^4}{75497472\pi^4} (y^2 - 10y + 1)^2 (tr R_T^2 - tr R_N^2)^2 - \frac{(4\pi^2\alpha)^4}{188743680\pi^4} (-4y^4 - 496y^3 + 1896y^2 - 496y - 4) (tr R_T^4 - tr R_N^4) \right) \end{aligned}$$

The first WZ action describes an elementary process on which the yOp-plane emits one (p+1)-form RR potential. The second WZ action describes an elementary process for which the y-deformed Op-plane absorbs two gravitons and emits one (p-3)-form RR potential. The third WZ action describes an elementary process for which the y-deformed Op-plane absorbs three gravitons and emits one (p-5)-form RR potential. The fourth WZ action describes an elementary process for which the yOp-plane absorbs four gravitons and emits one (p-7)-form RR potential. When y=1, then the four elementary process for the yOp-plane are reduced to the usuals three elementary process for the usual Op-plane: Op-plane emits one (p+1)-form RR potential, Op-plane absorbs two gravitons and emits one (p-3)-form RR potential; and, Op-plane absorbs four gravitons and emits one (p-7)-form RR potential.

3.4 The Elementary Processes for yGOp-planes

From the result of the section two, the WZ action for a y-deformed and generalized orientifold p-plane can be written as a sum of the WZ actions for some elementary processes:

$$S_{WZ(yGOp-plane)} = \sum_{j=1}^7 S_{WZ(yGOp-plane),j}$$

The WZ actions for the seven elementary processes are given by the following expressions:

$$\begin{aligned}
S_{WZ(yGOp-plane),1} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p+1} \\
S_{WZ(yGOp-plane),2} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-3} \frac{(4\pi^2 \alpha)^2}{6144\pi^2} (y+1)^2 (tr R_T^2 - tr R_N^2) \\
S_{WZ(yGOp-plane),3} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-3} \left(-\frac{(4\pi^2 \alpha)^2}{512\pi^2} y (tr R_E^2 - tr R_F^2) \right) \\
S_{WZ(yGOp-plane),4} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-5} \left(\frac{(4\pi^2 \alpha)^3}{256} y(y-1) (c_3(R_E) - c_3(R_F)) \right) \\
S_{WZ(yGOp-plane),5} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-7} \left(\frac{(4\pi^2 \alpha)^4}{75497472\pi^4} (y+1)^4 (tr R_T^2 - tr R_N^2)^2 + \frac{(4\pi^2 \alpha)^4}{47185920\pi^4} (y+1)^4 (tr R_T^4 - tr R_N^4) \right) \\
S_{WZ(yGOp-plane),6} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-7} \left(\frac{(4\pi^2 \alpha)^4}{524288\pi^4} y^2 (tr R_E^2 - tr R_F^2)^2 + \frac{(4\pi^2 \alpha)^4}{393216\pi^4} y(-4y+1+y^2) (tr R_E^4 - tr R_F^4) \right) \\
S_{WZ(yGOp-plane),7} &= -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C_{p-7} \left(-\frac{(4\pi^2 \alpha)^4}{3145728\pi^4} y(y+1)^2 (tr R_T^2 - tr R_N^2) (tr R_E^2 - tr R_F^2) \right)
\end{aligned}$$

The first WZ action describes an elementary process for which the y -deformed and generalized orientifold p -plane emits one $(p+1)$ -form RR potential. The second WZ action describes an elementary process for which the y -deformed and generalized Op-plane absorbs two gravitons and emits one $(p-3)$ -form RR potential. The third WZ action describes an elementary process for which the y -deformed and generalized Op-plane absorbs two gaugeons and emits one $(p-3)$ -form RR potential. The fourth WZ action describes an elementary process for which the y GOp-plane absorbs three gaugeons and emits one $(p-5)$ -form RR potential. The fifth WZ action describes an elementary process for which the y GOp-plane absorbs four gravitons and emits one $(p-7)$ -form RR potential. The sixth WZ action describes an elementary process for which the y GOp-plane absorbs four gaugeons and emits one $(p-7)$ -form RR potential. The seventh WZ action describes an elementary process for which the y GOp-plane absorbs two gravitons and two gaugeons and emits one $(p-7)$ -form RR potential. When $y=1$ the elementary processes for the y GOp-plane are reduced to the elementary processes for the GOp-plane. When $E=T$ and $F=N$ the elementary processes for the y GOp-plane are reduced to the elementary processes for the y Op-plane. When $y=1$ and $E=T$ and $F=N$ the elementary processes for the y GOp-plane are reduced to the elementary processes for the usual Op-plane.

4 Conclutions

The WZ action for the y GOp-planes can be modified or extended by various ways. When the bundles have non-trivial second Stiefel-Whitney classes one can

to write the following WZ action which incorporates an effect of the magnetic monopoles:

$$S_{WZ} = -2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C \sqrt{\frac{A(\frac{R_T}{2}, y) \text{Mayer}(\frac{R_E}{2}, y) e^{\frac{d_1}{2}}}{A(\frac{R_N}{2}, y) \text{Mayer}(\frac{R_F}{2}, y) e^{\frac{d_2}{2}}}}$$

where:

$$d_1 = \text{reduction.mod.2}(w_2(T) + w_2(E))$$

$$d_2 = \text{reduction.mod.2}(w_2(N) + w_2(F))$$

This action describes processes on which the yGOp-plane emits RR-forms and absorbs gravitons, gaugeons and magnetic monopoles.

From the other side one can to write the following actions for GOp-planes non standard:

$$S_{WZ} = 2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C (2 \sqrt{\frac{A(R_T)}{A(R_N)}} - \sqrt{\frac{A(\frac{R_T}{2}) \text{Mayer}(\frac{R_E}{2})}{A(\frac{R_N}{2}) \text{Mayer}(\frac{R_F}{2})})$$

$$S_{WZ} = \frac{T_p}{\kappa} \int_{p+1} C (\sqrt{\frac{A(R_T)}{A(R_N)}} - 2^{p-4} \sqrt{\frac{A(\frac{R_T}{2}) \text{Mayer}(\frac{R_E}{2})}{A(\frac{R_N}{2}) \text{Mayer}(\frac{R_F}{2})})$$

In the same way, one can to write the following actions for yOp-planes non standard:

$$S_{WZ} = 2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C (2 \sqrt{\frac{A(R_T)}{A(R_N)}} - \sqrt{\frac{\text{CHI}_y(\frac{R_T}{4})}{\text{CHI}_y(\frac{R_N}{4})})$$

$$S_{WZ} = \frac{T_p}{\kappa} \int_{p+1} C (\sqrt{\frac{A(R_T)}{A(R_N)}} - 2^{p-4} \sqrt{\frac{\text{CHI}_y(\frac{R_T}{4})}{\text{CHI}_y(\frac{R_N}{4})})$$

These actions correspond respectively to the Sp-type yOp-planes and the yOp-planes that give rise to gauge symmetries of type SO(2n+1). Such non-standard yOp-planes are building from combinations of the D-p-branes and standard yOp-planes.

In the same way, one can to write the following actions for yGOp-planes non standart:

$$S_{WZ} = 2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C (2 \sqrt{\frac{A(R_T)}{A(R_N)}} - \sqrt{\frac{A(\frac{R_T}{2}, y) \text{Mayer}(\frac{R_E}{2}, y)}{A(\frac{R_N}{2}, y) \text{Mayer}(\frac{R_F}{2}, y)}})$$

$$S_{WZ} = \frac{T_p}{\kappa} \int_{p+1} C (\sqrt{\frac{A(R_T)}{A(R_N)}} - 2^{p-4} \sqrt{\frac{A(\frac{R_T}{2}, y) \text{Mayer}(\frac{R_E}{2}, y)}{A(\frac{R_N}{2}, y) \text{Mayer}(\frac{R_F}{2}, y)}})$$

These actions correspond respectively to the Sp-type yGOp-planes and the yGOp-planes that give rise to gauge symmetries of type SO(2n+1). Such non-standard yGOp-planes are building from combinations of the D-p-branes and standard yGOp-planes.

By combination of Dp-branes, yDp-branes, Op-planes, GOp-planes, yOp-planes and yGOp-planes one can to have gauge teories with symmetries Sp and SO-odd whose WZ actions are give respectively by:

$$\begin{aligned}
S_{WZ} &= 2^{p-4} \frac{T_p}{\kappa} \int_{p+1} C \left(\sqrt{\frac{A(R_T)}{A(R_N)}} + \sqrt{\frac{A(R_{T,y})}{A(R_{N,y})}} - \frac{1}{4} \left(\sqrt{\frac{CHI_y(\frac{R_T}{4})}{CHI_y(\frac{R_N}{4})}} + \right. \right. \\
&\quad \left. \left. \sqrt{\frac{L(\frac{R_T}{4})}{L(\frac{R_N}{4})}} + \sqrt{\frac{A(\frac{R_T}{2})Mayer(\frac{R_E}{2})}{A(\frac{R_N}{2})Mayer(\frac{R_F}{2})}} + \sqrt{\frac{A(\frac{R_T}{2},y)Mayer(\frac{R_E}{2},y)}{A(\frac{R_N}{2},y)Mayer(\frac{R_F}{2},y)}} \right) \right) \\
S_{WZ} &= \frac{T_p}{\kappa} \int_{p+1} C \left(\frac{1}{2} \left(\sqrt{\frac{A(R_T)}{A(R_N)}} + \sqrt{\frac{A(R_{T,y})}{A(R_{N,y})}} \right) - 2^{p-4} \frac{1}{4} \left(\sqrt{\frac{CHI_y(\frac{R_T}{4})}{CHI_y(\frac{R_N}{4})}} + \right. \right. \\
&\quad \left. \left. \sqrt{\frac{L(\frac{R_T}{4})}{L(\frac{R_N}{4})}} + \sqrt{\frac{A(\frac{R_T}{2})Mayer(\frac{R_E}{2})}{A(\frac{R_N}{2})Mayer(\frac{R_F}{2})}} + \sqrt{\frac{A(\frac{R_T}{2},y)Mayer(\frac{R_E}{2},y)}{A(\frac{R_N}{2},y)Mayer(\frac{R_F}{2},y)}} \right) \right)
\end{aligned}$$

Finally one can think about non-BPS GOp-planes, non-BPS yOp-planes and non-BPS yGOp-planes with the tachyon effect. One can also think about noncommutative Op-planes, GOp-planes, yOp-planes and yGOp-planes

In conclusion gauge theories with symmetries SO-even, Sp and SO-odd can be obtained from the combination of the Dp-branes, yDp-branes, Op-planes, GOp-planes, yOp-planes and yGOp-planes of the string theory.

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