Noncommutative relativistic particle on the electromagnetic background.

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Abstract

Noncommutative version of D-dimensional relativistic particle is proposed. We consider the particle interacting with the configuration space variable $\theta^{\mu\nu}(\tau)$ instead of the numerical matrix. The corresponding Poincare invariant action has a local symmetry, which allows one to impose the gauge $\theta^{0i} = 0,\ \theta^{ij} = const$. The matrix θ^{ij} turns out to be the noncommutativity parameter of the gauge fixed formulation. Poincare transformations of the gauge fixed formulation are presented in the manifest form. Consistent coupling of NC relativistic particle to the electromagnetic field is constructed and discussed.

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1 Introduction.

It is known that the noncommutative (NC) geometry [1, 2] of the position variables in some mechanical models can be obtained [3-7] as the result of direct canonical quantization [9, 10] of underlying dynamical systems with second class constraints. Nontrivial bracket for the position variables appears in this case as the Dirac bracket, after taking into account the constraints presented in the model. An apparent defect of the known NC models is lack of relativistic invariance, due to the fact that noncommutativity parameter is constant matrix.

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In this note we discuss one possibility to resolve the problem. Namely, the noncommutative version for D-dimensional relativistic particle is proposed. We show also that it is possible to write (rather exotic) interaction with an external electromagnetic field. The interaction is consistent with the Poincare invariance, as well as with local symmetries presented in the model.

The work is organized as follows. In Sec. 2 we demonstrate that a procedure used to obtain NC versions of the particular models [3, 6, 7] can be generalized to the case of an arbitrary nondegenerate mechanical system. Namely, to obtain NC version, it is sufficiently to add Chern-Simons type term to the first order Lagrangian action of the initial system. The numerical matrix $\theta^{AB} = -\theta^{BA}$, originating from the Chern-Simons term, turns out to be the NC parameter of the formulation. We point also that quantization of the NC system leads to quantum mechanics with ordinary product replaced by the Moyal product.

In Sec. 3 we show that a slight modification of the procedure allows one to obtain NC version for D-dimensional relativistic particle. Chern-Simons term can be added to the first order action of the relativistic particle, which do not spoil the reparametrization invariance. As a consequence, the model will contain the desired relativistic constraint $p^2 - m^2 = 0$. The problem is that the numerical matrix $\theta^{\mu\nu}$ do not respect the Lorentz invariance. To resolve the problem, we consider a particle interacting with a new configuration-space variable $\theta^{\mu\nu}(\tau) = -\theta^{\nu\mu}(\tau)$, instead of the constant matrix. The action constructed is manifestly Poincare invariant and has local symmetry related with the variable θ. The last one can be gauged out, an admissible gauge is $\theta^{0i} = 0$, $\theta^{ij} = const$. The noncommutativity parameter of the gauge fixed version is then the numerical matrix θ^{ij}. As it usually happens in a theory with local symmetries [11], Poincare invariance of the gauge fixed version is combination of the initial Poincare and local transformations which preserve the gauge chosen. In the case under consideration, the resulting transformations are linear and involve the constant matrix θ^{ij} (see Eqs.(29) below).

In Sec. 4 interaction with an external electromagnetic field is discussed. The standard interaction term can be added, in principle, but does not lead to an interesting situation. Consistency of the term with the local symmetries presented in the model implies it's specific dependence on the configuration space variables. As a consequence, after transition to the canonical variables, any traces of noncommutativity disappear from the formulation. As an alternative, we propose new interaction term which involve the field strength instead of the electromagnetic potential. The possibility to write the term is implied by the fact that one works with the first order Lagrangian action [1]. In the Conclusion we discuss combined interaction.

2 Noncommutative version of an arbitrary nondegenerate mechanics.

Our starting point is some nondegenerate mechanical system with the configuration space variables $q^A(t), \ A = 1, 2, \ldots, n$, and the Lagrangian action

$$S = \int dt L(q^A, \ \dot{q}^A). \tag{1}$$

Due to nondegenerate character of the system, there are no constraints in the Hamiltonian formulation. Let p_A are conjugated momentum for q^A, one can write the Hamiltonian action

$$S_H = \int dt \left[p_A \dot{q}^A - H_0(q^A, \ p_A) \right]. \tag{2}$$

Equations of motion which follow from Eq.(1) and (2) are equivalent (they remain equivalent for any degenerated system also [10, 12], in this case the Hamiltonian includes the Lagrangian multipliers). Equivalently, one can describe the initial system (1) by means of the first order *Lagrangian action*

$$S_1 = \int dt \left[v_A \dot{q}^A - H_0(q^A, \ v_A) \right]. \tag{3}$$

Here $q^A(t), \ v_A(t)$ are the configuration space variables of the formulation [2]. The noncommutative version of the system (1) is described

[1] Let us note that the same interaction term can be written for ordinary relativistic particle (in the first order formulation) as well.

[2] The Lagrangian formulations (1), (3) are equivalent. Actually, denoting the conjugated momentum for the variables $q^A, \ v_A$ as $p_A, \ \pi^A$ one finds, in the Hamiltonian formulation for the action (3), the second class constraints $p_A - v_A = 0, \ \pi^A = 0$. Introducing the corresponding Dirac bracket, one can treat the constraints as the strong equations. Then the Hamiltonian formulation for (3) is the same as for (1), namely Eq.(2).

by the following Lagrangian action

$$S_N = \int dt \left[v_A \dot{q}^A - H_0(q^A,\ v_A) + \dot{v}_A \theta^{AB} v_B \right], \tag{4}$$

where θ^{AB} is some constant matrix. It turns out to be the noncommutativity parameter for the variables q^A.

Let us analyse the model (4) in the Hamiltonian framework (see [8] for details). All the expressions for determining of the momentum turn out to be the primary constraints of the model $(p_A, \pi^A$ are conjugated momentum for the variables $q^A, v_A)$

$$G_A \equiv p_A - v_A = 0, \qquad T^A \equiv \pi^A - \theta^{AB} v_B = 0, \tag{5}$$

with the Poisson bracket algebra being of second class

$$\{G_A, G_B\} = 0, \qquad \{T^A, T^B\} = -2\theta^{AB}, \qquad \{G_A, T^B\} = -\delta_A^B. \tag{6}$$

The constraints can be taken into account by transition to the Dirac bracket. After that, one can take the variables $(q^A,\ p_A)$ as the physical one, while $(v_A,\ \pi^A)$ can be omitted from consideration using Eq.(5). The resulting noncommutative system has the following properties.

1) It has the same number of physical degrees of freedom as the initial system S, namely $q^A,\ p_A$.

2) Equations of motion of the system are the same as for the initial system S, modulo the term which is proportional to the parameter θ^{AB}

$$\dot{q}^A = \frac{\partial H_0}{\partial p_A} - 2\theta^{AB} \frac{\partial H_0}{\partial q^B}, \qquad \dot{p}_A = -\frac{\partial H_0}{\partial q^A}, \tag{7}$$

where $H_0(q,\ p) = H_0(q,\ v)|_{v \to p}$.

3) The physical variables have the brackets

$$\{q^A, q^B\} = -2\theta^{AB}, \qquad \{q^A, p_B\} = \delta_B^A, \qquad \{p_A, p_B\} = 0. \tag{8}$$

In particular, brackets of the configuration space variables are noncommutative. One can show that other possibilities to choose the physical variables: $q^A,\ v_A$, or $q^A,\ \pi_A$ lead to an equivalent description.

To quantize the resulting system, one possibility is to find variables which have the canonical brackets. For the case under consideration they are

$$\tilde{q}^A = q^A - \theta^{AB} p_B, \qquad \tilde{p}_A = p_A, \tag{9}$$

and obey $\{\tilde{q},\tilde{q}\} = \{\tilde{p},\tilde{p}\} = 0,\ \{\tilde{q},\tilde{p}\} = 1$. Equations of motion in terms of these variables acquire the standard form

$$\dot{\tilde{q}}^A = \{\tilde{q}^A, \tilde{H}_0\}, \qquad \dot{\tilde{p}}_A = \{\tilde{p}_A, \tilde{H}_0\}, \tag{10}$$

where $\tilde{H}_0 = H_0(\tilde{q} + \theta\tilde{p},\ \tilde{p})$. It leads to quantum mechanics with the Moyal product (see [7] and references therein)

$$H_0(\tilde{q}^A + \theta^{AB}\tilde{p}_B,\ \tilde{p}_B)\Psi(\tilde{q}^C) = H_0(\tilde{q}^A,\ \tilde{p}_B) * \Psi(\tilde{q}^C). \tag{11}$$

The procedure described above can be applied to some degenerated systems as well. The necessary condition is that a part of variables enter into the initial action without the time derivatives, and such that they can be identified with the Lagrangian multipliers of the Hamiltonian formulation. Then the system admits the first order *Lagrangian formulation* (3). The relativistic particle and the string are examples of such a system (see [13] for the first order formulation of the string). We suppose that the procedure can be applied to the spinning particle [14] and to the superparticle [15]. It may be interesting [16] since both models are supersymmetric. If the relativistic invariance is presented in the initial formulation, a slight modification of the procedure is required to keep the symmetry in the NC version. The modification is presented in the next section.

3 Noncommutative relativistic particle.

The configuration space variables of the model are $x^\mu(\tau),\ v^\mu(\tau),\ e(\tau),\ \theta^{\mu\nu}(\tau)$, with the Lagrangian action being

$$S = \int d\tau \left[\dot{x}^\mu v_\mu - \frac{e}{2}(v^2 - m^2) + \frac{1}{\theta^2}\dot{v}_\mu\theta^{\mu\nu}v_\nu\right]. \tag{12}$$

Here $\theta^2 \equiv \theta^{\mu\nu}\theta_{\mu\nu},\ \eta^{\mu\nu} = (+,-,\ldots,-)$. Insertion of the term θ^2 in the denominator has the same meaning as for the eibein in the action of massless particle: $L = \frac{1}{2e}\dot{x}^2$. Technically, it rules out the degenerated gauge $e = 0$. The action is manifestly invariant under the Poincare transformations

$$x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu, \quad v'^\mu = \Lambda^\mu{}_\nu v^\nu, \quad \theta'^{\mu\nu} = \Lambda^\mu{}_\rho\Lambda^\nu{}_\sigma\theta^{\rho\sigma}. \tag{13}$$

Local symmetries of the model are reparametrizations (with $\theta^{\mu\nu}$ being the scalar variable), and the following transformations with the parameter $\epsilon_{\mu\nu}(\tau)=-\epsilon_{\nu\mu}(\tau)$

$$\delta x^\mu=-\epsilon^{\mu\nu}v_\nu, \qquad \delta\theta_{\mu\nu}=-\theta^2\epsilon_{\mu\nu}+2\theta_{\mu\nu}(\theta\epsilon). \tag{14}$$

To analyse physical sector of this constrained system, we rewrite it in the Hamiltonian form. Starting from the action (12), one finds in the Hamiltonian formalism the primary constraints

$$\begin{aligned} G^\mu\equiv p^\mu-v^\mu=0, \qquad & T^\mu\equiv\pi^\mu-\frac{1}{\theta^2}\theta^{\mu\nu}v_\nu=0, \\ p_\theta^{\mu\nu}=0, \qquad & p_e=0, \end{aligned} \tag{15}$$

and the Hamiltonian

$$H=\frac{e}{2}(v^2-m^2)+\lambda_{1\mu}G^\mu+\lambda_{2\mu}T^\mu+\lambda_e p_e+\lambda_{\theta\mu\nu}p_\theta^{\mu\nu}. \tag{16}$$

Here $p,\ \pi$ are conjugated momentum for $x,\ v$ and λ are the Lagrangian multipliers for the constraints. On the next step there is appear the secondary constraint

$$v^2-m^2=0, \tag{17}$$

as well as equations for determining the Lagrangian multipliers

$$\lambda_2^\mu=0, \qquad \lambda_1^\mu=ev^\mu+\frac{2}{\theta^2}(\lambda_\theta v)^\mu-\frac{4}{\theta^4}(\theta\lambda_\theta)(\theta v)^\mu. \tag{18}$$

There is no of tertiary constraints in the problem. Equations of motion follow from (16)-(18), in particular, for the variables $x,\ p$ one has

$$\dot{x}^\mu=ep^\mu+\frac{2}{\theta^2}(\lambda_\theta v)^\mu-\frac{4}{\theta^4}(\theta\lambda_\theta)(\theta v)^\mu, \qquad \dot{p}^\mu=0 \tag{19}$$

Poisson brackets of the constraints are

$$\begin{aligned} \{G^\mu,G^\nu\}=0, \qquad & \{T^\mu,T^\nu\}=-\frac{2}{\theta^2}\theta^{\mu\nu}, \\ \{G_\mu,T^\nu\}=-\delta_\mu^\nu, \qquad & \{T_\mu,p_\theta^{\rho\sigma}\}=-\frac{1}{\theta^2}\delta_\mu^{[\rho}v^{\sigma]}+\frac{4}{\theta^4}(\theta v)_\mu\theta^{\rho\sigma}. \end{aligned} \tag{20}$$

The constraints $G^\mu,\ T^\mu$ form the second class subsystem and can be taken into account by transition to the Dirac bracket. Then the remaining constraints can be classified in accordance with their properties relatively to the Dirac bracket. Consistency of the procedure is guaranteed by the known theorems [10]. Introducing the Dirac bracket

$$\{A,B\}_D=\{A,B\}+\{A,G_\mu\}\frac{2}{\theta^2}\theta^{\mu\nu}\{G_\nu,B\}-\\ \{A,G^\mu\}\{T_\mu,B\}+\{A,T_\mu\}\{G^\mu,B\}, \tag{21}$$

one finds, in particular, the following brackets for the fundamental variables (all the nonzero brackets are presented)

$$\{x^\mu,x^\nu\}=-\frac{2}{\theta^2}\theta^{\mu\nu}, \quad \{x^\mu,p_\nu\}=\delta^\mu_\nu, \quad \{p_\mu,p_\nu\}=0; \tag{22}$$

$$\{x^\mu,v_\nu\}=\delta^\mu_\nu, \quad \{x^\mu,\pi^\nu\}=-\frac{1}{\theta^2}\theta^{\mu\nu}, \quad \{\theta_{\mu\nu},p_\theta^{\rho\sigma}\}=-\delta^{[\rho}_\mu\delta^{\sigma]}_\nu,\\ \{x^\mu,p_\theta^{\rho\sigma}\}=-\{\pi^\mu,p_\theta^{\rho\sigma}\}=\frac{1}{\theta^2}\eta^{\mu[\rho}v^{\sigma]}-\frac{4}{\theta^4}(\theta v)^\mu\theta^{\rho\sigma}. \tag{23}$$

Let us choose x^μ, p^μ as the physical sector variables (one can equivalently take (x,v) or (x,π), which leads to the same final results, similarly to the non relativistic case [7]). The variables v,π can be omitted now from the consideration.

Up to now the procedure preserves the manifest Poincare invariance of the model. Let us discuss the first class constraints $p_\theta^{\rho\sigma}=0$. As the gauge fixing conditions one takes

$$\theta^{0i}=0, \qquad \theta^{ij}=const. \tag{24}$$

Then $\theta^{\mu\nu}\theta_{\mu\nu}=-\theta_{ij}\theta_{ji}$, and the gauge is admissible if $\theta_{ij}\theta_{ji}\neq 0$, see Eq.(22), (19). From the equation of motion $\dot\theta=\lambda_\theta$ one determines the remaining Lagrangian multipliers: $\lambda_\theta=0$. Using this result in Eq.(19), the final form of the equations of motion is

$$\dot x^\mu=ep^\mu, \qquad \dot p^\mu=0. \tag{25}$$

They are supplemented by the remaining first class constraints $p^2-m^2=0$, $p_e=0$. Brackets for the physical variables are given by Eqs.(22).

The initial Poincare transformations (13) do not preserve the gauge (24) and must be accompanied by compensating local transformation, with the parameter $\epsilon^{\mu\nu}$ chosen in appropriate way. It gives the Poincare symmetry of the gauge fixed version. To find it, one has the conditions ($\Lambda^\mu{}_\nu = \delta^\mu_\nu + \omega^\mu{}_\nu$)

$$(\delta_\omega + \delta_\epsilon)\theta^{0i} = \omega^0{}_j\theta^{ji} + \theta^2\epsilon^{0i} = 0,$$
$$(\delta_\omega + \delta_\epsilon)\theta^{ij} = \omega^{[i}{}_k\theta^{kj]}\theta^2\epsilon^{ij} + 2\theta^{ij}(\theta^{kp}\epsilon_{kp}) = 0 \tag{26}$$

The solution is

$$\epsilon^{0i}(\omega) = \frac{1}{\theta^2}\omega^{0j}\theta^{ji}, \quad \epsilon^{ij}(\omega) = \frac{1}{\theta^2}\omega^{[i}{}_k\theta_k{}^{j]}, \tag{27}$$

or, equivalently

$$\epsilon^{\mu\nu}(\omega) = -\frac{1}{\theta^2}\omega^{[\mu}{}_\rho\theta^{\rho\nu]}, \tag{28}$$

where Eq.(24) is implied. Then the Poincare transformations of the gauge fixed version are

$$\delta x^\mu = \omega^\mu{}_\nu x^\nu + \frac{1}{\theta^2}p_\nu\omega^{[\nu}{}_\rho\theta^{\rho\mu]}, \quad \delta p^\mu = \omega^\mu{}_\nu p^\nu. \tag{29}$$

4 Interaction with an external electromagnetic field.

The standard interaction term $A_\mu(x)\dot{x}^\mu$ can not be added to the NC action (12), since it will break the local symmetry (14). To preserve the symmetry, one needs to take the electromagnetic field depending on the gauge-invariant combination: $A_\mu\left(x - \frac{\theta v}{\theta^2}\right)\dot{x}^\mu$. Then, in terms of the canonical variables (9), any traces of noncommutativity disappear in the final formulation, similarly to the free particle case.

Other natural possibility, which is implied by the first order formulation, is coupling to the field strenght of the form $\dot{v}_\mu F^{\mu\nu}v_\nu$, where $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}(x)$. So, let us consider the action

$$S = \int d\tau\left[\dot{x}^\mu v_\mu - \frac{e}{2}(v^2 - m^2) + \frac{1}{\theta^2}\dot{v}_\mu\theta^{\mu\nu}v_\nu + \dot{v}_\mu F^{\mu\nu}v_\nu\right]. \tag{30}$$

Note that the interaction term can not be removed by shift of the θ-variable, due to presence of θ^2 in the denominator. Local symmetries of the model are reparametrizations, $U(1)$ gauge transformations, and the modified ϵ transformations which look now as follow:

$$\delta x^\mu = -\epsilon^{\mu\nu}v_\nu, \qquad \delta\theta_{\mu\nu} = -\theta^2(\epsilon_{\mu\nu} + \delta_\epsilon F_{\mu\nu}) +$$

$$2\theta_{\mu\nu}\left[(\theta\epsilon)+(\theta\delta_\epsilon F)\right]. \tag{31}$$

Hamiltonian analysis of the model is similar to the free particle case discussed above. The interaction term leads to deformation of the constraint structure as compare with (15)-(23). The constraints of the model are

$$G^\mu \equiv p^\mu - v^\mu = 0, \qquad T^\mu \equiv \pi^\mu - \frac{1}{\theta^2}\theta^{\mu\nu}v_\nu - F^{\mu\nu}v_\nu = 0,$$
$$p_\theta^{\mu\nu} = 0, \qquad p_e = 0, \qquad v^2 - m^2 = 0. \tag{32}$$

The Poisson bracket algebra is deformed also and acquires the form

$$\{G^\mu, G^\nu\} = 0, \qquad \{T^\mu, T^\nu\} = \Delta^{\mu\nu},$$
$$\{T^\mu, G^\nu\} = \Delta_1{}^{\mu\nu}, \qquad \{G^\mu, T^\nu\} = -(\Delta_1^T)^{\mu\nu}, \tag{33}$$

where it was denoted

$$\Delta^{\mu\nu} = -2\left(\frac{\theta^{\mu\nu}}{\theta^2} + F^{\mu\nu}\right), \qquad \Delta_1{}^{\mu\nu} = \eta^{\mu\nu} - \partial^\nu F^{\mu\rho}v_\rho. \tag{34}$$

The only nonzero bracket of the constraint $p_\theta^{\mu\nu} = 0$ with others is the same as in (20). The Dirac bracket which corresponds to the constraints $G^\mu = 0,\ T^\mu = 0$ is

$$\{A,B\}_D = \{A,B\} - \{A,G_\mu\}\left(\Delta_1^{-1}\Delta\Delta_1^{-1T}\right)^{\mu\nu}\{G_\nu,B\} -$$
$$\{A,G_\mu\}(\Delta_1^{-1})^{\mu\nu}\{T_\nu,B\} + \{A,T_\mu\}(\Delta_1^{-1T})^{\mu\nu}\{G_\nu,B\}. \tag{35}$$

One takes the same gauge as in (24) for the first class constraints $p_\theta^{\mu\nu} = 0$, and the gauge $e = 1$ for $p_e = 0$. The resulting system can be taken into account by transition to the corresponding Dirac bracket. Then the remaining variables of the theory are x^μ, p_μ. Classical dynamics of these variables is described by the Hamiltonian equations

$$\dot{x}^\mu = (\Delta_1^{-1})^{\mu\nu}p_\nu, \qquad \dot{p}^\mu = 0, \tag{36}$$

which are accompanied by the constraint $p^2 - m^2 = 0$. Brackets for the variables are

$$\{x^\mu, x^\nu\} = \left(\Delta_1^{-1}\Delta\Delta_1^{-1T}\right)^{\mu\nu},$$
$$\{x^\mu, p_\nu\} = \delta^\mu{}_\nu + \left(\Delta_1^{-1}\right)^{\mu\rho}\partial_\nu F^{\rho\sigma}p_\sigma, \qquad \{p_\mu, p_\nu\} = 0. \tag{37}$$

One notes that in terms of the variables x, p the noncommutativity parameter θ does not enter into the equations of motion. From Eq.(36) one finds the second order equation for the position variable

$$\ddot{x}^\mu = (\Delta_1^{-1})^{\mu\sigma}\partial_\alpha\partial_\beta F_{\sigma\rho}\Delta_1{}^{\rho\nu}\dot{x}^\nu\dot{x}^\alpha\dot{x}^\beta. \tag{38}$$

Interesting property of the interaction is that dynamics of NC variables in the constant electromagnetic field is governed by free equation. All the information on dynamics is encoded in this case in the noncommutative brackets (37). For the non relativistic systems the same property was discussed in [7]. Let us point also that it may be mechanical analogy of duality relations [17].

5 Conclusion.

In this work we have presented noncommutative version (ref12)of D-dimensional relativistic particle. It couples to the electromagnetic background through the field strength, see (30). The interaction introduced is consistent with the Poincare invariance as well as with local symmetries presented in the model. Some relevant comments are in order.

1) The same interaction term can be added to the first order action of usual (commutative) relativistic particle as well. It may be interesting to study this trick in the context of higher spin particle models [18] (it is well known that the standard coupling is not consistent with symmetries of higher spin actions [19-21]).

2) Let us point that in the second order formulation, a similar interaction term could be $F_{\mu\nu}\dot{x}^\mu\ddot{x}^\nu$. One expects that it will lead to different physical picture as compare with 1). The term involve the higher derivative, which indicates on appearance of extra physical degrees of freedom.

3) At last, we point that the standard coupling can be combined with the one considered in Sec. 4, one takes

$$S_{int} = \int d\tau \left[A_\mu\left(x - \frac{\theta v}{\theta^2}\right)\dot{x}^\mu + \dot{v}_\mu F^{\mu\nu}(x)v_\nu\right]. \tag{39}$$

Since the bracket algebra (37) is deformed as compare with the free case (due to presence of the field strength term), this interacting system will be different from the corresponding commutative one.

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