CONSISTENT INTERACTIONS IN A THREE-DIMENSIONAL THEORY WITH TENSOR GAUGE FIELDS OF DEGREES TWO AND THREE

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Abstract

All consistent interactions in a three-dimensional theory with tensor gauge fields of degrees two and three are obtained by means of the deformation of the solution to the master equation combined with cohomological techniques. The local BRST cohomology of this model allows the deformation of the Lagrangian action, accompanying gauge symmetries and gauge algebra. The relationship with the Chern–Simons theory is discussed.

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1 Introduction

The matter of generating consistent interactions in gauge theories [1]–[4] has been reanalyzed as a deformation problem of the master equation [5] in the framework of the antifield-BRST formalism [6]–[10]. The scope of this paper is to generate all consistent interactions that can be introduced in a free three-dimensional theory with tensor gauge fields of degrees two and three by means of the deformation of the solution to the master equation combined with cohomological techniques. The importance of the three-dimensional case is motivated by the fact that the model is irreducible, and thus its deformation displays different features as compared to the higher dimensional situations, where the theory is reducible (the reducibility order depends on the space-time dimension). The model under consideration can also be interpreted in terms of a special class of theories with mixed-symmetry type tensor gauge fields. Such theories held the attention lately on many issues,

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like, for example, the interpretation of the construction of the Pauli–Fierz theory [11], the dual formulation of linearized gravity [12]–[13], the impossibility of consistent cross-interactions in the dual formulation of linearized gravity [13], or the general scheme for dualizing higher-spin gauge fields in arbitrary irreducible representations of $GL(D,\mathbf{R})$ [14]. Meanwhile, the model under study is connected to a special class of topological BF theories [15], and also to the pure Chern–Simons theory.

Our approach is based on solving the main equations that govern the deformation of the solution to the master equation for the starting free model. Since the model is Abelian and irreducible, its antibracket-antifield BRST symmetry reduces to the sum between the Koszul–Tate differential and the exterior derivative along the gauge orbits. Initially, we compute the non-integrated density of the first-order deformation of the solution to the master equation, which lies in the local cohomological space of the BRST differential at ghost number zero. This step relies on the development of the BRST co-cycles according to the degree of the Koszul–Tate differential (antighost number), and necessitates the computation of both the cohomology of the exterior derivative along the gauge orbits and the local homology of the Koszul–Tate differential. We show that the first-order deformation stops at antighost number two and is parametrized in terms of seven independent real constants. The consistency of the first-order deformation restricts the number of independent constants to three, and consequently ends the deformation procedure at order one in the coupling constant. The resulting deformations are classified according to three complementary classes. Irrespective of the considered class, the interacting theory exhibits the following general features: (a) the interaction vertices are polynomials of order three in the undifferentiated tensor gauge fields; (b) the gauge transformations of the fields are modified; (c) the deformed gauge algebra is non-Abelian ($su(2)$), unlike that of the initial theory, which is Abelian. Finally, we discuss the link with pure three-dimensional Chern–Simons theory.

2 Free Model. Free BRST Symmetry

We begin with the Lagrangian action in three space-time dimensions

$$I_0\left[B^{\alpha\beta}{}_{(\lambda)}, A_\alpha{}^{(\lambda)}\right] = \int d^3x B^{\alpha\beta}{}_{(\lambda)}\partial_\alpha A_\beta{}^{(\lambda)}, \tag{1}$$

that involves two sorts of covariant tensor gauge fields: one of degree three, which is only antisymmetric in its first two indices, $B^{\alpha\beta}{}_{(\lambda)} = -B^{\beta\alpha}{}_{(\lambda)}$, and the other of degree two, $A_\alpha{}^{(\lambda)}$, with no further symmetries. We use the Minkowskian metric of 'mostly minus' signature, $(+,-,-)$, and work with the convention that the completely three-dimensional symbol $\varepsilon^{\alpha\beta\gamma}$ is valued like $\varepsilon^{012} = +1$. All indices (α, β, λ) are lowered and raised with the

Minkowski metric. Parentheses around an index are meant to mark that there is no symmetry between the class of indices carrying parentheses and that without. In terms of Young diagrams, $B^{\alpha\beta}{}_{(\lambda)}$ can be regarded like the tensor product between a two-cell diagram with one column and a one-cell diagram, while $A_{\alpha}{}^{(\lambda)}$ is simply the tensor product between two one-cell diagrams

$$B_{\alpha\beta(\lambda)} \simeq \begin{array}{|c|}\hline \alpha \\ \hline \beta \\ \hline \end{array} \bigotimes \boxed{\lambda},\ A_{\alpha(\lambda)} \simeq \boxed{\alpha} \bigotimes \boxed{\lambda}. \tag{2}$$

In other words, the representations of $GL\left(3, \mathbf{R}\right)$ in the corresponding spaces of covariant tensors of degree three and respectively two are reducible. The gauge symmetries of action (1) are 'spanned' by the irreducible generating set

$$\delta_{\epsilon} B^{\alpha\beta}{}_{(\lambda)} = \varepsilon^{\alpha\beta\gamma} \partial_{\gamma} \epsilon'_{(\lambda)},\ \delta_{\epsilon} A_{\alpha}{}^{(\lambda)} = \partial_{\alpha} \epsilon^{(\lambda)}, \tag{3}$$

that closes according to an Abelian algebra, with $\epsilon'_{(\lambda)}$ and $\epsilon^{(\lambda)}$ two independent real and bosonic covariant tensors of degree one. Although the parentheses might seem superfluous at degree one, we will still keep them for subsequent analysis. The above action describes a gauge theory of Cauchy order two. In the case where the covariant index of the type (λ) is replaced by one of different kind, say $a = \overline{1, N}$, the corresponding action (1) relates to a particular class of topological BF-type theories [15] involving a system of $U(1)$-Abelian vector fields A_{μ}^{a} and a set of Abelian two-form gauge fields $B_{a}^{\mu\nu}$. If, in addition, the two-form is dual to the one-form, $B_{a}^{\mu\nu} = (1/2)\, k_{ab} \varepsilon^{\mu\nu\lambda} A_{\lambda}^{b}$ (with k_{ab} the elements of a constant, symmetric and invertible matrix), then action (1) reduces to a sum of N Abelian Chern–Simons terms. In order to display the formulas in a simpler manner, it is convenient to replace $B^{\alpha\beta}{}_{(\lambda)}$ by its dual with respect to the antisymmetry indices only, $V_{\alpha(\lambda)} = \varepsilon_{\alpha\beta\gamma} B^{\beta\gamma}{}_{(\lambda)}$, in terms of which (1) and (3) become

$$I_0 \left[V_{\alpha(\lambda)}, A_{\alpha}{}^{(\lambda)}\right] = \int d^3x\, \varepsilon^{\alpha\beta\gamma} V_{\alpha(\lambda)} \partial_{\beta} A_{\gamma}{}^{(\lambda)}, \tag{4}$$

$$\delta_{\epsilon} V_{\alpha(\lambda)} = \partial_{\alpha} \epsilon'_{(\lambda)},\ \delta_{\epsilon} A_{\alpha}{}^{(\lambda)} = \partial_{\alpha} \epsilon^{(\lambda)}, \tag{5}$$

such that we deal with two types of Abelian covariant tensor gauge fields of degree two.

The main scope of this paper is to analyze all consistent interactions that can be added to action (4), namely, all interactions that preserve the field spectrum and the number of independent gauge transformations. There are three main categories of such interactions: (I) the Lagrangian action, the gauge transformations as well as their gauge algebra, are all modified; (II) the action and the gauge symmetries are changed, but not their algebra; (III) only the action is deformed, but not the gauge symmetries. We want to deform the above free action by adding to it interaction terms

$$I_0 \to I = I_0 + g I_1 + g^2 I_2 + \cdots, \tag{6}$$

and meanwhile to modify the gauge transformations

$$\bar{\delta}_\epsilon V_{\alpha(\lambda)} = \partial_\alpha \epsilon'_{(\lambda)} + g\Theta_{\alpha\lambda} + O\left(g^2\right), \ \bar{\delta}_\epsilon A_\alpha{}^{(\lambda)} = \partial_\alpha \epsilon^{(\lambda)} + g\Delta_\alpha{}^\lambda + O\left(g^2\right), \tag{7}$$

such that the deformed action I is invariant under the modified gauge transformations at each order in the coupling constant g

$$\bar{\delta}_\epsilon I = 0. \tag{8}$$

It is required that every term in the expansion (6) be a local functional in the corresponding fields. We focus on the determination of non-trivial deformations, *i.e.*, of those deformations of the action that are not due to a local redefinition of the tensor gauge fields.

The most economical and elegant way to approach the consistent deformations of a given gauge theory is to investigate the deformations of the rigid, fermionic and nilpotent symmetry that encodes, among others, the entire gauge structure of the theory— the antibracket-antifield BRST symmetry. In the case of the Abelian free model under discussion, the BRST differential s reduces to the sum between the Koszul–Tate differential δ and the exterior longitudinal derivative γ

$$s = \delta + \gamma. \tag{9}$$

While the Koszul–Tate differential is graded in terms of the antighost number (agh, $\mathrm{agh}\left(\delta\right) = -1$), the degree associated with the exterior longitudinal derivative is named pure ghost number (pgh, $\mathrm{pgh}\left(\gamma\right) = 1$). These graduations do not interfere ($\mathrm{agh}\left(\gamma\right) = 0 = \mathrm{pgh}\left(\delta\right)$). The BRST differential is controlled by an overall degree, called ghost number (gh), and defined like the difference between the pure ghost number and the antighost number, in terms of which we have that $\mathrm{gh}\left(s\right) = \mathrm{gh}\left(\gamma\right) = \mathrm{gh}\left(\delta\right) = 1$. The second-order nilpotency of s ($s^2 = 0$) is equivalent to the nilpotency and anticommutation of its components ($\delta^2 = 0$, $\gamma^2 = 0$, $\delta\gamma + \gamma\delta = 0$). The realization of these operators requires the introduction of the fermionic ghosts $\left(\eta_{(\lambda)}, C^{(\lambda)}\right)$ respectively corresponding to the gauge parameters $\left(\epsilon'_{(\lambda)}, \epsilon^{(\lambda)}\right)$, as well as of the antifields ($V^{*\alpha(\lambda)}, A^{*\alpha}{}_{(\lambda)}, \eta^{*(\lambda)}, C^*_{(\lambda)}$), with Grassmann parity opposite to that of the corresponding fields/ghosts. The pure ghost number and antighost number of the BRST generators are valued like

$$\mathrm{pgh}\left(V_{\alpha(\lambda)}\right) = \mathrm{pgh}\left(A_\alpha{}^{(\lambda)}\right) = 0, \ \mathrm{pgh}\left(\eta_{(\lambda)}\right) = \mathrm{pgh}\left(C^{(\lambda)}\right) = 1, \tag{10}$$

$$\mathrm{pgh}\left(V^{*\alpha(\lambda)}\right) = \mathrm{pgh}\left(A^{*\alpha}{}_{(\lambda)}\right) = \mathrm{pgh}\left(\eta^{*(\lambda)}\right) = \mathrm{pgh}\left(C^*_{(\lambda)}\right) = 0, \tag{11}$$

$$\mathrm{agh}\left(V_{\alpha(\lambda)}\right) = \mathrm{agh}\left(A_\alpha{}^{(\lambda)}\right) = \mathrm{agh}\left(\eta_{(\lambda)}\right) = \mathrm{agh}\left(C^{(\lambda)}\right) = 0, \tag{12}$$

$$\mathrm{agh}\left(V^{*\alpha(\lambda)}\right) = \mathrm{agh}\left(A^{*\alpha}{}_{(\lambda)}\right) = 1, \ \mathrm{agh}\left(\eta^{*(\lambda)}\right) = \mathrm{agh}\left(C^*_{(\lambda)}\right) = 2, \tag{13}$$

while the actions of δ and γ on them read as

$$\gamma\left(V_{\alpha(\lambda)}\right)=\partial_{\alpha}\eta_{(\lambda)},\ \gamma\left(A_{\alpha}{}^{(\lambda)}\right)=\partial_{\alpha}C^{(\lambda)},\ \gamma\left(\eta_{(\lambda)}\right)=0=\gamma\left(C^{(\lambda)}\right), \tag{14}$$

$$\gamma\left(V^{*\alpha(\lambda)}\right)=\gamma\left(A^{*\alpha}{}_{(\lambda)}\right)=\gamma\left(\eta^{*(\lambda)}\right)=\gamma\left(C^{*}_{(\lambda)}\right)=0, \tag{15}$$

$$\delta\left(V_{\alpha(\lambda)}\right)=\delta\left(A_{\alpha}{}^{(\lambda)}\right)=\delta\left(\eta_{(\lambda)}\right)=\delta\left(C^{(\lambda)}\right)=0, \tag{16}$$

$$\delta V^{*\alpha(\lambda)}=-\varepsilon^{\alpha\beta\gamma}\partial_{\beta}A_{\gamma}{}^{(\lambda)},\ \delta A^{*\alpha}{}_{(\lambda)}=-\varepsilon^{\alpha\beta\gamma}\partial_{\beta}V_{\gamma(\lambda)}, \tag{17}$$

$$\delta\eta^{*(\lambda)}=-\partial_{\alpha}V^{*\alpha(\lambda)},\ \delta C^{*}_{(\lambda)}=-\partial_{\alpha}A^{*\alpha}{}_{(\lambda)}. \tag{18}$$

By construction, the Koszul–Tate differential realizes an homological resolution of smooth functions defined on the stationary surface of the field equations for I_0, $\delta I_0/\delta V_{\alpha(\lambda)}\equiv\varepsilon^{\alpha\beta\gamma}\partial_{\beta}A_{\gamma}{}^{(\lambda)}=0$, $\delta I_0/\delta A_{\alpha}{}^{(\lambda)}\equiv\varepsilon^{\alpha\beta\gamma}\partial_{\beta}V_{\gamma(\lambda)}=0$, while the cohomological space of the exterior longitudinal derivative at pure ghost number zero computed in the homology of δ is nothing but the algebra of Lagrangian physical observables of the model under consideration. Moreover, the cohomological space of the BRST differential itself at ghost number zero $H^0(s)$, which contains the so-called BRST observables, is isomorphic to the same algebra of physical observables.

A remarkable feature of the BRST symmetry is that it is (anti)canonically generated in a structure named antibracket, $s\cdot=(\cdot,S)$, where the antibracket is obtained by decreeing the fields/ghosts conjugated with the corresponding antifields. Its generator is bosonic, $\varepsilon(S)=0$, has ghost number zero, $\mathrm{gh}(S)=0$, and satisfies the classical master equation

$$(S,S)=0, \tag{19}$$

that expresses the nilpotency of s at the canonical level. In our case the solution to the master equation can be written like

$$S=I_0\left[V_{\alpha(\lambda)},A_{\alpha}{}^{(\lambda)}\right]+\int d^3x\left(V^{*\alpha(\lambda)}\partial_{\alpha}\eta_{(\lambda)}+A^{*\alpha}{}_{(\lambda)}\partial_{\alpha}C^{(\lambda)}\right). \tag{20}$$

The solution to the classical master equation encodes all the information on the gauge structure of I_0. Indeed, if we expand it according to the antighost number, its antighost number zero component is precisely the Lagrangian action I_0 of the gauge theory, while the antighost number one terms describe the gauge symmetries of the action. The absence of elements with antighost number higher than one is connected with the Abelian and irreducible choice of the gauge generators.

3 Deformation of the Free Model

The reformulation of the problem of consistent deformations of action (4) and of its gauge symmetries (5) in terms of the BRST setting is based on the observation that if a deformation (6–7) of the classical theory can be consistently constructed, then the solution (20) to the master equation (19) for the free theory can be deformed into

$$\bar{S} = S + gS_1 + g^2 S_2 + O\left(g^3\right), \; \varepsilon\left(\bar{S}\right) = 0, \; \mathrm{gh}\left(\bar{S}\right) = 0, \tag{21}$$

such that

$$(\bar{S}, \bar{S}) = 0. \tag{22}$$

The projection of (22) along the various powers in the coupling constant induces the following tower of equations:

$$g^0 : (S, S) = 0, \tag{23}$$

$$g^1 : (S_1, S) = 0, \tag{24}$$

$$g^2 : \frac{1}{2}(S_1, S_1) + (S_2, S) = 0, \tag{25}$$

$$\vdots$$

The first equation is satisfied by hypothesis. The second one governs the first-order deformation of the solution to the master equation, S_1, and it expresses the fact that S_1 is a BRST co-cycle, $sS_1 = 0$, and hence it exists and is local. The remaining equations are responsible for the higher-order deformations of the solution to the master equation. No obstructions arise in finding solutions to them as long as no further restrictions, such as space-time locality, are imposed. Taking into account (6–7), the first two pieces of S_1 regarded as an expansion according to the antighost number

$$S_1 = \overset{[0]}{S}_1 + \overset{[1]}{S}_1 + \cdots \overset{[j]}{S}_1, \; \varepsilon\left(\overset{[k]}{S}_1\right) = 0, \; \mathrm{gh}\left(\overset{[k]}{S}_1\right) = 0, \; \mathrm{agh}\left(\overset{[k]}{S}_1\right) = k, \tag{26}$$

are exactly

$$\overset{[0]}{S}_1 = I_1, \; \overset{[1]}{S}_1 = \int d^3x \left(V^{*\alpha(\lambda)}\bar{\Theta}_{\alpha\lambda} + A^{*\alpha}{}_{(\lambda)}\bar{\Delta}_\alpha{}^\lambda\right), \tag{27}$$

where $\bar{\Theta}_{\alpha\lambda}$ and $\bar{\Delta}_\alpha{}^\lambda$ are obtained from $\Theta_{\alpha\lambda}$ and $\Delta_\alpha{}^\lambda$ in (7) where we replace the gauge parameters by the ghosts. Obviously, only non-trivial first-order deformations should be considered, since trivial ones ($S_1 = sB$) lead to trivial deformations of the initial theory, and can be eliminated by convenient redefinitions of the fields. Ignoring the trivial deformations, it follows that

S_1 is a non-trivial BRST-observable, $S_1 \in H^0(s)$. Once that the deformation Eqs. (24–25), etc., have been solved by means of specific cohomological techniques, from the consistent non-trivial deformed solution to the master equation we can extract all the information on the gauge structure of the accompanying interacting theory. Eventually, we can make a selection of the coupled models with special physical characteristics, like space-time locality, PT-invariance, etc.

Although the cohomological approach to the deformation of gauge theories seems to deal with the same equations as the standard one, it is far more effective as it organizes the recursive construction in a systematic manner, precisely relates the consistent interactions to co-cycles of the BRST differential, reduces the trivial deformations to trivial classes of the BRST cohomology, and brings in the powerful tools of homological algebra.

3.1 First-order deformation

Initially, we approach the Eq. (24), responsible for the first-order deformation of the solution to the master equation. By making the notation $S_1 = \int d^3x\, a_1$, this equation takes the local form

$$sa_1 = \partial_\mu j^\mu, \tag{28}$$

for some local current j^μ. The established fact that S_1 belongs to $H^0(s)$ implies that a_1 pertains to the zeroth order local cohomological space of the BRST differential, $H^0(s|d)$, where d is the exterior space-time differential. Then, the trivial solutions to (28), namely, $a_1 = sb + \partial_\mu k^\mu$, can be discarded. In order to solve the equation (28), we recall the expansion (26), where we make the notations $\overset{[k]}{S}_1 = \int d^3x\, \overset{[k]}{a}_1$, which provides the development of a_1 along the antighost number

$$a_1 = \overset{[0]}{a}_1 + \overset{[1]}{a}_1 + \cdots + \overset{[j]}{a}_1,\ \varepsilon\left(\overset{[k]}{a}_1\right) = 0,\ \mathrm{gh}\left(\overset{[k]}{a}_1\right) = 0,\ \mathrm{agh}\left(\overset{[k]}{a}_1\right) = k. \tag{29}$$

The number of terms in (29) is finite and it can be proven that the last representative can be taken to be annihilated by the exterior longitudinal derivative, $\gamma\, \overset{[j]}{a}_1 = 0$, with the precaution that it should not be trivial (γ-exact), $\overset{[j]}{a}_1 \neq \gamma c$. Then, as $\overset{[j]}{a}_1$ belongs to $H(\gamma)$, we have to compute the cohomology of γ in order to generate the component of highest antighost number from the first-order deformation. Looking at the definitions (14–15), we read that $H(\gamma)$ is spanned by the polynomials in the (undifferentiated) ghosts with coefficients that are functions of the 'field strengths' $\partial_{[\alpha} V_{\beta](\lambda)}$, $\partial_{[\alpha} A_{\beta]}^{(\lambda)}$, of the antifields $\chi^* \equiv \left(V^{*\alpha(\lambda)}, A^{*\alpha}_{(\lambda)}, \eta^{*(\lambda)}, C^*_{(\lambda)}\right)$, as well as of their space-time derivatives up to a finite order. The space-time derivatives of the ghosts are removed from $H(\gamma)$ since their first-order derivatives are already γ-exact, as can be observed from the first and second definitions in (14). In consequence, we have that

$$\overset{[j]}{a}_1 = \alpha_j\left(\left[\partial_{[\alpha}V_{\beta](\lambda)}\right],\left[\partial_{[\alpha}A_{\beta]}^{(\lambda)}\right],[\chi^*]\right)e^j\left(\eta_{(\lambda)},C^{(\lambda)}\right), \tag{30}$$

where e^j are the elements of pure ghost number j of a basis in the ghosts, the notation $f([q])$ signifies that f depends on q and its space-time derivatives up to a finite order, and the coefficients have $\mathrm{agh}(\alpha_j) = j$. Inserting (30) into the Eq. (28) projected on antighost number $(j-1)$

$$\delta\overset{[j]}{a}_1 + \gamma\overset{[j-1]}{a}_1 = \partial_\mu m^\mu, \tag{31}$$

we obtain that a necessary condition for the existence of $\overset{[j-1]}{a}_1$ is that the coefficients α_j belong to the local homology of the Koszul–Tate differential at antighost number j, $\alpha_j \in H_j(\delta|d)$, *i.e.*, $\delta\alpha_j = \partial_\mu d^\mu$. The free model under investigation is described by a normal gauge theory of Cauchy order two, and thus we have that the local homology of the Koszul–Tate differential vanishes [16] at antighost numbers strictly greater than two, $H_i(\delta|d) = 0$ for $i > 2$. This forces the development (29) to stop at antighost number two

$$a_1 = \overset{[0]}{a}_1 + \overset{[1]}{a}_1 + \overset{[2]}{a}_1, \tag{32}$$

where $\gamma\overset{[2]}{a}_1 = 0$, such that $\overset{[2]}{a}_1$ is of the form (30), with $j = 2$ and α_2 from $H_2(\delta|d)$. On the one hand, the most general representatives of $H_2(\delta|d)$ are

$$k_1\eta^{*(\lambda)} + k_2 C^*_{(\lambda)}, \tag{33}$$

with $k_{1,2}$ real constants, and, on the other hand, the elements of pure ghost number two of a basis in the ghosts are

$$\eta_{(\mu)}\eta_{(\nu)},\ \eta_{(\mu)}C^{(\nu)},\ C^{(\mu)}C^{(\nu)}, \tag{34}$$

such that $\overset{[2]}{a}_1$ is of the form

$$\begin{aligned}\overset{[2]}{a}_1 = {} & \eta^*_{(\lambda)}\left(f_1^{\lambda\mu\nu}\eta_{(\mu)}\eta_{(\nu)} + f_2^{\lambda\mu\nu}\eta_{(\mu)}C_{(\nu)} + f_3^{\lambda\mu\nu}C_{(\mu)}C_{(\nu)}\right)\\ & + C^*_{(\lambda)}\left(f_4^{\lambda\mu\nu}\eta_{(\mu)}\eta_{(\nu)} + f_5^{\lambda\mu\nu}\eta_{(\mu)}C_{(\nu)} + f_6^{\lambda\mu\nu}C_{(\mu)}C_{(\nu)}\right),\end{aligned} \tag{35}$$

where $\left(f_a^{\lambda\mu\nu}\right)_{a=\overline{1,6}}$ must be non-derivative constants. (If any of these constants contains at least one space-time derivative, then the corresponding term in $\overset{[2]}{a}_1$ is γ-exact, and thus trivial in $H(\gamma)$.) By covariance arguments, all these constants can only be proportional with the completely antisymmetric symbol in three dimensions, such that the last term in the first-order deformation (32) becomes (up to trivial γ-exact solutions)

$$\overset{[2]}{a}_1 = \varepsilon^{\lambda\mu\nu}\left(\eta^*_{(\lambda)}\left(c_1\eta_{(\mu)}\eta_{(\nu)} + c_2\eta_{(\mu)}C_{(\nu)} + c_3C_{(\mu)}C_{(\nu)}\right)\right.$$
$$\left.+C^*_{(\lambda)}\left(c_4\eta_{(\mu)}\eta_{(\nu)} + c_5\eta_{(\mu)}C_{(\nu)} + c_6C_{(\mu)}C_{(\nu)}\right)\right), \tag{36}$$

with c_1, etc., arbitrary real constants.

Taking into account that the equation for $\overset{[1]}{a}_1$ is obtained from (31) where we set $j = 2$

$$\delta \overset{[2]}{a}_1 + \gamma \overset{[1]}{a}_1 = \partial_\mu m^\mu, \tag{37}$$

with the help of (36) and (14–18) we arrive at

$$\overset{[1]}{a}_1 = \varepsilon^{\lambda\mu\nu}\left(V^{*\alpha}{}_{(\lambda)}\left(\left(2c_1V_{\alpha(\mu)} + c_2A_{\alpha(\mu)}\right)\eta_{(\nu)} + \left(2c_3A_{\alpha(\mu)} + c_2V_{\alpha(\mu)}\right)C_{(\nu)}\right) +\right.$$
$$\left.A^{*\alpha}{}_{(\lambda)}\left(\left(2c_6A_{\alpha(\mu)} + c_5V_{\alpha(\mu)}\right)C_{(\nu)} + \left(2c_4V_{\alpha(\mu)} + c_5A_{\alpha(\mu)}\right)\eta_{(\nu)}\right)\right) + \overset{[1]}{a'}_1, \tag{38}$$

where $\overset{[1]}{a'}_1$ denotes the general non-trivial solution to the 'homogeneous' equation $\gamma \overset{[1]}{a'}_1 = \partial_\mu m'^\mu$. Such solutions stem from $\overset{[2]}{a'}_1 = 0$, such that they do not modify the gauge algebra, but deform the gauge transformations (type (II) interactions). Following a standard reasoning, it can be shown that the above 'homogeneous' equation is equivalent, up to trivial redefinitions, to the equation with vanishing m'^μ

$$\gamma \overset{[1]}{a'}_1 = 0. \tag{39}$$

According to (30), the general solution to the Eq. (39) is of the type

$$\overset{[1]}{a'}_1 = \alpha_1^\lambda\left(\left[\partial_{[\alpha}V_{\beta](\lambda)}\right], \left[\partial_{[\alpha}A_{\beta]}^{\ \ (\lambda)}\right], \left[V^{*\alpha(\lambda)}\right], \left[A^{*\alpha}{}_{(\lambda)}\right]\right)\eta_{(\lambda)}$$
$$+\alpha_1'^\lambda\left(\left[\partial_{[\alpha}V_{\beta](\lambda)}\right], \left[\partial_{[\alpha}A_{\beta]}^{\ \ (\lambda)}\right], \left[V^{*\alpha(\lambda)}\right], \left[A^{*\alpha}{}_{(\lambda)}\right]\right)C_{(\lambda)}, \tag{40}$$

since the elements with pure ghost number equal to one of a basis in the ghosts are precisely $\eta_{(\lambda)}$ and $C_{(\lambda)}$. (The antifields $\eta^{*(\lambda)}$, $C^*_{(\lambda)}$ and their space-time derivatives are not allowed to enter $\overset{[1]}{a'}_1$ as they have the antighost number equal to two.) Meanwhile, the quantities $\partial_{[\alpha}V_{\beta](\lambda)}$, $\partial_{[\alpha}A_{\beta]}^{\ \ (\lambda)}$ and their space-time derivatives are already δ-exact (see (17)), such that the dependence of α_1^λ and $\alpha_1'^\lambda$ on them leads to trivial (δ-exact and γ-closed, hence s-exact) terms, which further implies that

$$\alpha_1^\lambda = \alpha_1^\lambda\left(\left[V^{*\alpha(\lambda)}\right], \left[A^{*\alpha}{}_{(\lambda)}\right]\right), \ \alpha_1'^\lambda = \alpha_1'^\lambda\left(\left[V^{*\alpha(\lambda)}\right], \left[A^{*\alpha}{}_{(\lambda)}\right]\right). \tag{41}$$

On the other hand, α_1^{λ} and $\alpha_1^{\prime\lambda}$ can only be linear in the undifferentiated antifields $V^{*\alpha(\lambda)}$ and $A^{*\alpha}{}_{(\lambda)}$. This is because the appearance of at least one space-time derivative in these antifields leads, via an integration by parts in the corresponding functional $\overset{[1]}{S'}_1 = \int d^3\, \overset{[1]}{a'}_1$, to a γ-exact (and hence trivial) solution to (39), which can always be taken to vanish. In consequence, the most general solution to the 'homogeneous' Eq. (39) is expressed (up to trivial γ-exact solutions) by

$$\overset{[1]}{a'}_1 = \varepsilon^{\alpha\lambda\mu}\left(V^*_{\alpha(\lambda)}\left(c'_1 C_{(\mu)} + c'_2 \eta_{(\mu)}\right) + A^*_{\alpha(\lambda)}\left(c'_3 C_{(\mu)} + c'_4 \eta_{(\mu)}\right)\right), \tag{42}$$

with c'_1 and so on arbitrary real constants.

By projecting (28) on antighost number zero, we deduce the equation verified by the antighost number zero component of the first-order deformation

$$\delta \overset{[1]}{a}_1 + \gamma \overset{[0]}{a}_1 = \partial_\mu n^\mu. \tag{43}$$

Using the expression (38) of $\overset{[1]}{a}_1$ with $\overset{[1]}{a'}_1$ given by (42), by direct computation we infer that

$$\begin{aligned}
\delta \overset{[1]}{a}_1 = {} & \partial_\mu n'^\mu + \varepsilon^{\lambda\mu\nu}\varepsilon^{\alpha\beta\gamma}\left(\left(2c_1\left(\partial_\gamma A_{\alpha(\lambda)}\right)V_{\beta(\mu)} + c_5 A_{\alpha(\lambda)}\partial_\gamma V_{\beta(\mu)}\right)\eta_{(\nu)}\right.\\
& + \left(2c_6\left(\partial_\gamma V_{\alpha(\lambda)}\right)A_{\beta(\mu)} + c_2 V_{\alpha(\lambda)}\partial_\gamma A_{\beta(\mu)}\right)C_{(\nu)}\\
& \left.-\frac{1}{2}\left(c_5 V_{\beta(\mu)}V_{\gamma(\nu)}\gamma\left(A_{\alpha(\lambda)}\right) + c_2 A_{\beta(\mu)}A_{\gamma(\nu)}\gamma\left(V_{\alpha(\lambda)}\right)\right)\right)\\
& + c'_2\left(A^\beta{}_{(\beta)}\gamma\left(V^\gamma{}_{(\gamma)}\right) - A^\gamma{}_{(\beta)}\gamma\left(V^\beta{}_{(\gamma)}\right)\right)\\
& + c'_3\left(V^\gamma{}_{(\gamma)}\gamma\left(A^\beta{}_{(\beta)}\right) - V^\beta{}_{(\gamma)}\gamma\left(A^\gamma{}_{(\beta)}\right)\right)\\
& - \gamma\left(\frac{1}{3}\varepsilon^{\lambda\mu\nu}\varepsilon^{\alpha\beta\gamma}\left(c_3 A_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)} + c_4 V_{\alpha(\lambda)}V_{\beta(\mu)}V_{\gamma(\nu)}\right)\right.\\
& + \frac{c'_1}{2}\left(A^\gamma{}_{(\beta)}A^\beta{}_{(\gamma)} - A^\beta{}_{(\beta)}A^\gamma{}_{(\gamma)}\right)\\
& \left.+\frac{c'_4}{2}\left(V^\gamma{}_{(\beta)}V^\beta{}_{(\gamma)} - V^\beta{}_{(\beta)}V^\gamma{}_{(\gamma)}\right)\right),
\end{aligned} \tag{44}$$

such that the existence of $\overset{[0]}{a}_1$ implies the following conditions on the various constants

$$2c_1 = c_5,\ 2c_6 = c_2,\ c'_2 = c'_3. \tag{45}$$

Replacing (45) in (44), the solution to the Eq. (43), expressed in terms of seven independent real constants, reads as

$$\overset{[0]}{a}_1 = \varepsilon^{\lambda\mu\nu}\varepsilon^{\alpha\beta\gamma}\left(\frac{1}{2}\left(c_2 V_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)} + c_5 A_{\alpha(\lambda)}V_{\beta(\mu)}V_{\gamma(\nu)}\right)\right.$$

$$+\frac{1}{3}\left(c_3 A_{\alpha(\lambda)} A_{\beta(\mu)} A_{\gamma(\nu)} + c_4 V_{\alpha(\lambda)} V_{\beta(\mu)} V_{\gamma(\nu)}\right)\right)$$
$$+\frac{1}{2}\left(c_1'\left(A^{\gamma}{}_{(\beta)} A^{\beta}{}_{(\gamma)} - A^{\beta}{}_{(\beta)} A^{\gamma}{}_{(\gamma)}\right) + c_4'\left(V^{\gamma}{}_{(\beta)} V^{\beta}{}_{(\gamma)} - V^{\beta}{}_{(\beta)} V^{\gamma}{}_{(\gamma)}\right)\right)$$
$$+ c_2'\left(A^{\gamma}{}_{(\beta)} V^{\beta}{}_{(\gamma)} - A^{\beta}{}_{(\beta)} V^{\gamma}{}_{(\gamma)}\right) + \overset{[0]}{a''}_1 \equiv \overset{[0]}{\bar{a}}_1 + \overset{[0]}{a''}_1 \, . \tag{46}$$

In the above, $\overset{[0]}{a''}_1$ represents the general non-trivial solution to the 'homogeneous' equation

$$\gamma \overset{[0]}{a''}_1 = \partial_\mu n''^{\mu}. \tag{47}$$

Such solutions come from $\overset{[1]}{a''}_1 = 0$, such that they do not deform either the gauge algebra or the gauge transformations (type (III) interactions), but simply add to the original action γ-invariant modulo d terms. There are two types of solutions to (47). The first one corresponds to $n''^{\mu} = 0$ and is given by arbitrary polynomials in $\partial_{[\alpha} V_{\beta](\lambda)}$, $\partial_{[\alpha} A_{\beta]}^{\ \ (\lambda)}$ and their space-time derivatives. However, this type of solutions is easily seen to be trivial (s-exact) due to the simultaneous δ-exactness and γ-closedness of $\partial_{[\alpha} V_{\beta](\lambda)}$ and $\partial_{[\alpha} A_{\beta]}^{\ \ (\lambda)}$, and hence they can be removed by taking the associated $\overset{[0]}{a''}_1$ to vanish. The second one leads to non-vanishing local currents n''^{μ} and is expressed by generalized Chern–Simons terms

$$\overset{[0]}{a''}_1 = \varepsilon^{\alpha\beta\gamma}\left(\sum_{k=0}^{m} d_k \left(\partial_{\mu_1\cdots\mu_k} A_{\alpha(\lambda)}\right)\left(\partial^{\mu_1\cdots\mu_k}\partial_\beta A_\gamma{}^{(\lambda)}\right)\right.$$
$$+\sum_{k=0}^{m'} d_k' \left(\partial_{\mu_1\cdots\mu_k} V_{\alpha(\lambda)}\right)\left(\partial^{\mu_1\cdots\mu_k}\partial_\beta V_\gamma{}^{(\lambda)}\right)$$
$$\left.+\sum_{k=0}^{m''} d_k'' \left(\partial_{\mu_1\cdots\mu_k} V_{\alpha(\lambda)}\right)\left(\partial^{\mu_1\cdots\mu_k}\partial_\beta A_\gamma{}^{(\lambda)}\right)\right), \tag{48}$$

where all d_k, d_k' and d_k'' are constants. The solution (48) leads to a deformed Lagrangian action with fields equations involving more than one derivative in the fields, unlike the original action, which has first-order derivative field equations. In order to preserve the first-order character at the level of the deformed field equations, we are tempted to forbid this discontinuous behavior by dropping out all the terms from (48) with $k > 0$. We show that this is not necessary as $\overset{[0]}{a''}_1$ like in (48) can be removed from a_1 by adding to it trivial terms. Indeed, it is easy to see that $\overset{[0]}{a''}_1$ is δ-exact

$$\overset{[0]}{a''}_1 = -\delta\left(\sum_{k=0}^{m} d_k \left(\partial^{\mu_1\cdots\mu_k} V^{*\alpha(\lambda)}\right)\left(\partial_{\mu_1\cdots\mu_k} A_{\alpha(\lambda)}\right)\right.$$

$$+\sum_{k=0}^{m'} d'_k \left(\partial^{\mu_1\cdots\mu_k} A^{*\alpha(\lambda)}\right)\left(\partial_{\mu_1\cdots\mu_k} V_{\alpha(\lambda)}\right)$$
$$+\sum_{k=0}^{m''} d''_k \left(\partial^{\mu_1\cdots\mu_k} V^{*\alpha(\lambda)}\right)\left(\partial_{\mu_1\cdots\mu_k} V_{\alpha(\lambda)}\right)\Bigg) \equiv \delta T. \tag{49}$$

In the meantime, the solution (42) to the 'homogeneous' Eq. (39) is unique up to a trivial (γ-exact) quantity, so we can replace it by

$$\overset{[1]}{a'}_1 \rightarrow \overset{[1]}{\bar{a}'}_1 = \overset{[1]}{a'}_1 + \gamma T. \tag{50}$$

In agreement with (47) and (49), this trivial term simply adds a divergence to the δ-variation of the corresponding $\overset{[1]}{a}_1$

$$\overset{[1]}{a}_1 \rightarrow \overset{[1]}{\bar{a}}_1 = \overset{[1]}{a}_1 + \gamma T,\ \delta \overset{[1]}{a}_1 \rightarrow \delta \overset{[1]}{\bar{a}}_1 = \delta \overset{[1]}{a}_1 + \partial_\mu n''^\mu, \tag{51}$$

such that (46) remains the general solution also to the Eq. (43) with $\overset{[1]}{\bar{a}}_1$ instead of $\overset{[1]}{a}_1$. Then, according to (9), (46), (49) and (51), we have that the introduction of this term amounts in the first-order deformation to

$$a_1 \rightarrow \bar{a}_1 = \overset{[2]}{a}_1 + \overset{[1]}{\bar{a}}_1 + \overset{[0]}{a}_1 = \overset{[2]}{a}_1 + \overset{[1]}{a}_1 + \overset{[0]}{\bar{a}}_1 + sT. \tag{52}$$

As the first-order deformation is unique up to trivial (s-exact modulo d) elements, we can safely discard sT from $\bar{a}_1$, which is equivalent to setting

$$\overset{[0]}{a''}_1 = 0, \tag{53}$$

in (46). In consequence, we will work with the non-trivial first-order deformation of the solution to the master equation

$$a_1 = \overset{[2]}{a}_1 + \overset{[1]}{a}_1 + \overset{[0]}{\bar{a}}_1. \tag{54}$$

We cannot stress enough that no assumption on the number of derivatives in a_1 is necessary, since all higher-order derivative terms can be completely eliminated from $\overset{[0]}{a}_1$ by means of trivial quantities.

So far, we obtained the most general expression of the non-trivial first-order deformation of the solution to the master equation for the model under study like in (54), where its various components are pictured by (36), (38), (42), (46) and (53), and the constants involved are subject to the conditions (45). Putting together all these results, we arrive at

$$S_1 = \int d^3x \left(\varepsilon^{\lambda\mu\nu} \left(\eta^*_{(\lambda)} \left(\frac{c_5}{2}\eta_{(\mu)}\eta_{(\nu)} + c_2\eta_{(\mu)}C_{(\nu)} + c_3 C_{(\mu)}C_{(\nu)}\right)\right.\right.$$

$$
\begin{aligned}
&+C^{*}_{(\lambda)}\left(\frac{c_2}{2}C_{(\mu)}C_{(\nu)}+c_5C_{(\mu)}\eta_{(\nu)}+c_4\eta_{(\mu)}\eta_{(\nu)}\right)\\
&+V^{*\alpha}{}_{(\lambda)}\left(\left(c_5V_{\alpha(\mu)}+c_2A_{\alpha(\mu)}\right)\eta_{(\nu)}+\left(2c_3A_{\alpha(\mu)}+c_2V_{\alpha(\mu)}\right)C_{(\nu)}\right)\\
&+A^{*\alpha}{}_{(\lambda)}\left(\left(c_2A_{\alpha(\mu)}+c_5V_{\alpha(\mu)}\right)C_{(\nu)}+\left(2c_4V_{\alpha(\mu)}+c_5A_{\alpha(\mu)}\right)\eta_{(\nu)}\right)\\
&+\varepsilon^{\alpha\beta\gamma}\left(\frac{1}{2}\left(c_2V_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)}+c_5A_{\alpha(\lambda)}V_{\beta(\mu)}V_{\gamma(\nu)}\right)\right.\\
&\left.\left.+\frac{1}{3}\left(c_3A_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)}+c_4V_{\alpha(\lambda)}V_{\beta(\mu)}V_{\gamma(\nu)}\right)\right)\right)\\
&+\varepsilon^{\alpha\lambda\mu}\left(V^{*}_{\alpha(\lambda)}\left(c'_1C_{(\mu)}+c'_2\eta_{(\mu)}\right)+A^{*}_{\alpha(\lambda)}\left(c'_4\eta_{(\mu)}+c'_2C_{(\mu)}\right)\right)\\
&+\frac{c'_1}{2}\left(A^{\gamma}{}_{(\beta)}A^{\beta}{}_{(\gamma)}-A^{\beta}{}_{(\beta)}A^{\gamma}{}_{(\gamma)}\right)+\frac{c'_4}{2}\left(V^{\gamma}{}_{(\beta)}V^{\beta}{}_{(\gamma)}-V^{\beta}{}_{(\beta)}V^{\gamma}{}_{(\gamma)}\right)\\
&\left.\left.+c'_2\left(A^{\gamma}{}_{(\beta)}V^{\beta}{}_{(\gamma)}-A^{\beta}{}_{(\beta)}V^{\gamma}{}_{(\gamma)}\right)\right)\right). \qquad (55)
\end{aligned}
$$

It has a beautiful symmetric form with respect to the permutation of the two tensors, accompanying BRST generators and arbitrary constants, being invariant under the transformation

$$V_{\alpha(\lambda)}\longleftrightarrow A_{\alpha(\lambda)},\ V^{*\alpha(\lambda)}\longleftrightarrow A^{*\alpha(\lambda)},\ \eta_{(\nu)}\longleftrightarrow C_{(\nu)},\ \eta^{*(\nu)}\longleftrightarrow C^{*(\nu)}, \qquad (56)$$

$$c_2\longleftrightarrow c_5,\ c_3\longleftrightarrow c_4,\ c'_1\longleftrightarrow c'_4,\ c'_2\longleftrightarrow c'_2. \qquad (57)$$

3.2 Higher-order deformations

The next step of the deformation procedure consists in the determination of the second- and higher-order deformations of the solution to the master equation. The second-order deformation is subject to the Eq. (25), and it shows that the existence of local solutions S_2 requires that the antibracket (S_1, S_1) is the s-variation of a local functional. By direct computation, we find that

$$
\begin{aligned}
&\frac{1}{2}(S_1,S_1)=\int d^3x\left((4c_3c_4-c_2c_5)\left(\eta^{*(\lambda)}C^{(\mu)}\eta_{(\lambda)}\eta_{(\mu)}+C^{*(\lambda)}\eta^{(\mu)}C_{(\lambda)}C_{(\mu)}\right.\right.\\
&+V^{*\alpha(\lambda)}\left(\left(V_{\alpha(\lambda)}\eta^{(\mu)}-V_{\alpha}{}^{(\mu)}\eta_{(\lambda)}\right)C_{(\mu)}+A_{\alpha}{}^{(\mu)}\eta_{(\lambda)}\eta_{(\mu)}\right)\\
&+A^{*\alpha(\lambda)}\left(\left(A_{\alpha(\lambda)}C^{(\mu)}-A_{\alpha}{}^{(\mu)}C_{(\lambda)}\right)\eta_{(\mu)}+V_{\alpha}{}^{(\mu)}C_{(\lambda)}C_{(\mu)}\right)\\
&\left.+\varepsilon^{\alpha\beta\gamma}\left(A_{\alpha}{}^{(\lambda)}A_{\beta}{}^{(\mu)}V_{\gamma(\lambda)}\eta_{(\mu)}+V_{\alpha}{}^{(\lambda)}V_{\beta}{}^{(\mu)}A_{\gamma(\lambda)}C_{(\mu)}\right)\right)\\
&+(2c_4c'_1-c_2c'_4)\left(\left(V^{*\alpha(\lambda)}\eta_{(\alpha)}+A^{*\lambda(\mu)}C_{(\mu)}-A^{*\mu}{}_{(\mu)}C^{(\lambda)}\right)\eta_{(\lambda)}\right.\\
&\left.+\varepsilon^{\alpha\lambda\mu}\left(\left(A_{\alpha}{}^{(\beta)}V_{\beta(\lambda)}-A_{\beta}{}^{(\beta)}V_{\alpha(\lambda)}\right)\eta_{(\mu)}+V_{\alpha(\lambda)}V_{\beta(\mu)}C^{(\beta)}\right)\right)\\
&+(2c_3c'_4-c_5c'_1)\left(\left(A^{*\alpha(\lambda)}C_{(\alpha)}+V^{*\lambda(\mu)}\eta_{(\mu)}-V^{*\mu}{}_{(\mu)}\eta^{(\lambda)}\right)C_{(\lambda)}\right.\\
&\left.+\varepsilon^{\alpha\lambda\mu}\left(\left(V_{\alpha}{}^{(\beta)}A_{\beta(\lambda)}-V_{\beta}{}^{(\beta)}A_{\alpha(\lambda)}\right)C_{(\mu)}+A_{\alpha(\lambda)}A_{\beta(\mu)}\eta^{(\beta)}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
&-\varepsilon^{\alpha\lambda\mu}\left(c_2'\left(c_1' A_{\alpha(\lambda)}C_{(\mu)} + c_4' V_{\alpha(\lambda)}\eta_{(\mu)}\right)\right.\\
&\left.+\left(\left(c_2'\right)^2 + c_1'c_4'\right)\left(A_{\alpha(\lambda)}\eta_{(\mu)} + V_{\alpha(\lambda)}C_{(\mu)}\right)\right)\bigg) .
\end{aligned} \tag{58}
$$

We observe that none of the terms in the right hand-side of (58) can be written like an s-exact quantity, such that the consistency of the first-order deformation requires that these terms must vanish. This takes place if and only if the constants satisfy the equations

$$4c_3c_4 - c_2c_5 = 0, \tag{59}$$

$$2c_4c_1' - c_2c_4' = 0,\ 2c_3c_4' - c_5c_1' = 0, \tag{60}$$

$$c_2'c_1' = 0,\ c_2'c_4' = 0,\ \left(c_2'\right)^2 + c_1'c_4' = 0. \tag{61}$$

Consequently, we can take the second-order deformation to vanish, $S_2 = 0$, and, moreover, we further find that all the higher-order deformations can also be set equal to zero, $S_3 = \cdots = 0$. In consequence, the deformation procedure stops at the first order and grants the consistency of the solution to the master equation at all orders in the coupling constant. In order to classify the resulting deformations, we solve the Eqs. (59–61) starting with (61), that yields

$$c_2' = 0,\ c_1'c_4' = 0, \tag{62}$$

so we obtain three distinct possibilities.

(i) The first case corresponds to the vanishing of the solution to the 'homogeneous' equation (39), $\overset{[1]}{a'}_1 = 0$

$$c_1' = c_2' = c_4' = 0, \tag{63}$$

such that (60) is automatically satisfied. Then, the overall deformed solution to the master equation, consistent to all orders in the coupling constant, reads as

$$
\begin{aligned}
\bar{S}_{(i)} = S + g\varepsilon^{\lambda\mu\nu}\int d^3x\, &\left(\eta^*_{(\lambda)}\left(\frac{c_5}{2}\eta_{(\mu)}\eta_{(\nu)} + c_2\eta_{(\mu)}C_{(\nu)} + c_3C_{(\mu)}C_{(\nu)}\right)\right.\\
&+ C^*_{(\lambda)}\left(\frac{c_2}{2}C_{(\mu)}C_{(\nu)} + c_5C_{(\mu)}\eta_{(\nu)} + c_4\eta_{(\mu)}\eta_{(\nu)}\right)\\
&+ V^{*\alpha}_{(\lambda)}\left(\left(c_5V_{\alpha(\mu)} + c_2A_{\alpha(\mu)}\right)\eta_{(\nu)} + \left(2c_3A_{\alpha(\mu)} + c_2V_{\alpha(\mu)}\right)C_{(\nu)}\right)\\
&+ A^{*\alpha}_{(\lambda)}\left(\left(c_2A_{\alpha(\mu)} + c_5V_{\alpha(\mu)}\right)C_{(\nu)} + \left(2c_4V_{\alpha(\mu)} + c_5A_{\alpha(\mu)}\right)\eta_{(\nu)}\right)\\
&+ \varepsilon^{\alpha\beta\gamma}\left(\frac{1}{2}\left(c_2V_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)} + c_5A_{\alpha(\lambda)}V_{\beta(\mu)}V_{\gamma(\nu)}\right)\right.\\
&\left.\left.+\frac{1}{3}\left(c_3A_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)} + c_4V_{\alpha(\lambda)}V_{\beta(\mu)}V_{\gamma(\nu)}\right)\right)\right),
\end{aligned} \tag{64}
$$

where the four real constants are subject to the equation (59), and S is the free solution (20).

(ii) The second situation is described by

$$c_1' = c_2' = 0,\ c_4' \neq 0, \tag{65}$$

and induces the solution to (59–60) like

$$c_2 = c_3 = 0, \tag{66}$$

with c_4 and c_5 arbitrary real constants. In this case, the full deformed solution of the master equation is

$$\begin{aligned}
\bar{S}_{(ii)} &= S + g \int d^3x \left(\varepsilon^{\lambda\mu\nu} \left(\frac{c_5}{2} \eta^*_{(\lambda)} \eta_{(\mu)} \eta_{(\nu)} \right.\right. \\
&+ C^*_{(\lambda)} \left(c_5 C_{(\mu)} \eta_{(\nu)} + c_4 \eta_{(\mu)} \eta_{(\nu)} \right) + c_5 V^{*\alpha}{}_{(\lambda)} V_{\alpha(\mu)} \eta_{(\nu)} \\
&+ A^{*\alpha}{}_{(\lambda)} \left(c_5 V_{\alpha(\mu)} C_{(\nu)} + \left(2c_4 V_{\alpha(\mu)} + c_5 A_{\alpha(\mu)} \right) \eta_{(\nu)} \right) \\
&\left. + \varepsilon^{\alpha\beta\gamma} \left(\frac{c_5}{2} A_{\alpha(\lambda)} V_{\beta(\mu)} V_{\gamma(\nu)} + \frac{c_4}{3} V_{\alpha(\lambda)} V_{\beta(\mu)} V_{\gamma(\nu)} \right) \right) \\
&\left. + c_4' \left(\varepsilon^{\alpha\lambda\mu} A^*_{\alpha(\lambda)} \eta_{(\mu)} + \frac{1}{2} \left(V^{\gamma}{}_{(\beta)} V^{\beta}{}_{(\gamma)} - V^{\beta}{}_{(\beta)} V^{\gamma}{}_{(\gamma)} \right) \right) \right).
\end{aligned} \tag{67}$$

(iii) Finally, in the third case we have that

$$c_2' = c_4' = 0,\ c_1' \neq 0, \tag{68}$$

which further leads to the solution of (59–60) under the form

$$c_4 = c_5 = 0, \tag{69}$$

with c_2 and c_3 arbitrary real constants. In this situation, the consistent deformed solution of the master equation is expressed by

$$\begin{aligned}
\bar{S}_{(iii)} &= S + g \int d^3x \left(\varepsilon^{\lambda\mu\nu} \left(\frac{c_2}{2} C^*_{(\lambda)} C_{(\mu)} C_{(\nu)} \right.\right. \\
&+ \eta^*_{(\lambda)} \left(c_2 \eta_{(\mu)} C_{(\nu)} + c_3 C_{(\mu)} C_{(\nu)} \right) + c_2 A^{*\alpha}{}_{(\lambda)} A_{\alpha(\mu)} C_{(\nu)} \\
&+ V^{*\alpha}{}_{(\lambda)} \left(c_2 A_{\alpha(\mu)} \eta_{(\nu)} + \left(2c_3 A_{\alpha(\mu)} + c_2 V_{\alpha(\mu)} \right) C_{(\nu)} \right) \\
&\left. + \varepsilon^{\alpha\beta\gamma} \left(\frac{c_2}{2} V_{\alpha(\lambda)} A_{\beta(\mu)} A_{\gamma(\nu)} + \frac{c_3}{3} A_{\alpha(\lambda)} A_{\beta(\mu)} A_{\gamma(\nu)} \right) \right) \\
&\left. + c_1' \left(\varepsilon^{\alpha\lambda\mu} V^*_{\alpha(\lambda)} C_{(\mu)} + \frac{1}{2} \left(A^{\gamma}{}_{(\beta)} A^{\beta}{}_{(\gamma)} - A^{\beta}{}_{(\beta)} A^{\gamma}{}_{(\gamma)} \right) \right) \right).
\end{aligned} \tag{70}$$

It is interesting to notice that in each situation the deformed solution to the master equation is parametrized in terms of three independent real constants.

4 Identification of the Interacting Model

We are now in the position to identify the resulting interaction models. As we have previously mentioned, the antighost number zero component in the deformed solution to the master equation determines the Lagrangian action of the coupled model. From the antighost number one elements we read the deformed gauge transformations. The pieces with higher antighost number offer information on the nature of the deformed gauge algebra. In the first of the cases discussed in the above, we infer the deformed Lagrangian action and accompanying gauge transformations under the form

$$
\begin{aligned}
&I_{(i)}\left[V_{\alpha(\lambda)}, A_{\alpha}{}^{(\lambda)}\right] = \varepsilon^{\alpha\beta\gamma}\int d^3x \left(V_{\alpha(\lambda)}\partial_\beta A_\gamma{}^{(\lambda)}\right. \\
&+ g\varepsilon^{\lambda\mu\nu}\left(\frac{1}{2}\left(c_2 V_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)} + c_5 A_{\alpha(\lambda)}V_{\beta(\mu)}V_{\gamma(\nu)}\right)\right. \\
&\left.\left.+\frac{1}{3}\left(c_3 A_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)} + c_4 V_{\alpha(\lambda)}V_{\beta(\mu)}V_{\gamma(\nu)}\right)\right)\right), \qquad (71)
\end{aligned}
$$

$$
\begin{aligned}
&\bar{\delta}_\epsilon^{(i)} V^{\alpha(\lambda)} = \partial^\alpha \epsilon'^{(\lambda)} + g\varepsilon^{\lambda\mu\nu}\left(\left(c_5 V^\alpha{}_{(\mu)} + c_2 A^\alpha{}_{(\mu)}\right)\epsilon'_{(\nu)}\right. \\
&\left.+\left(2c_3 A^\alpha{}_{(\mu)} + c_2 V^\alpha{}_{(\mu)}\right)\epsilon_{(\nu)}\right), \qquad (72)
\end{aligned}
$$

$$
\begin{aligned}
&\bar{\delta}_\epsilon^{(i)} A^{\alpha(\lambda)} = \partial^\alpha \epsilon^{(\lambda)} + g\varepsilon^{\lambda\mu\nu}\left(\left(c_2 A^\alpha{}_{(\mu)} + c_5 V^\alpha{}_{(\mu)}\right)\epsilon_{(\nu)}\right. \\
&\left.+\left(2c_4 V^\alpha{}_{(\mu)} + c_5 A^\alpha{}_{(\mu)}\right)\epsilon'_{(\nu)}\right). \qquad (73)
\end{aligned}
$$

We remark that the above deformed action, as well as its gauge transformations, preserve the symmetry under the transformations (56–57).

In the second and third cases, we infer that

$$
\begin{aligned}
&I_{(ii)}\left[V_{\alpha(\lambda)}, A_{\alpha}{}^{(\lambda)}\right] = \int d^3x \left(\varepsilon^{\alpha\beta\gamma}\left(V_{\alpha(\lambda)}\partial_\beta A_\gamma{}^{(\lambda)}\right.\right. \\
&\left.\left.+g\varepsilon^{\lambda\mu\nu}\left(\frac{c_5}{2}A_{\alpha(\lambda)}V_{\beta(\mu)}V_{\gamma(\nu)} + \frac{c_4}{3}V_{\alpha(\lambda)}V_{\beta(\mu)}V_{\gamma(\nu)}\right)\right)\right. \\
&\left.+g\frac{c_4'}{2}\left(V^\gamma{}_{(\beta)}V^\beta{}_{(\gamma)} - V^\beta{}_{(\beta)}V^\gamma{}_{(\gamma)}\right)\right), \qquad (74)
\end{aligned}
$$

$$
\bar{\delta}_\epsilon^{(ii)} V^{\alpha(\lambda)} = \partial^\alpha \epsilon'^{(\lambda)} + gc_5\varepsilon^{\lambda\mu\nu}V^\alpha{}_{(\mu)}\epsilon'_{(\nu)}, \qquad (75)
$$

$$
\begin{aligned}
&\bar{\delta}_\epsilon^{(ii)} A^{\alpha(\lambda)} = \partial^\alpha \epsilon^{(\lambda)} + gc_4'\varepsilon^{\alpha\lambda\mu}\epsilon'_{(\mu)} \\
&+ g\varepsilon^{\lambda\mu\nu}\left(c_5 V^\alpha{}_{(\mu)}\epsilon_{(\nu)} + \left(2c_4 V^\alpha{}_{(\mu)} + c_5 A^\alpha{}_{(\mu)}\right)\epsilon'_{(\nu)}\right), \qquad (76)
\end{aligned}
$$

respectively,

$$I_{(iii)}\left[V_{\alpha(\lambda)},A_{\alpha}{}^{(\lambda)}\right]=\int d^3x\left(\varepsilon^{\alpha\beta\gamma}\left(V_{\alpha(\lambda)}\partial_\beta A_\gamma{}^{(\lambda)}\right.\right.$$
$$+g\varepsilon^{\lambda\mu\nu}\left.\left(\frac{c_2}{2}V_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)}+\frac{c_3}{3}A_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)}\right)\right)$$
$$\left.+g\frac{c_1'}{2}\left(A^\gamma{}_{(\beta)}A^\beta{}_{(\gamma)}-A^\beta{}_{(\beta)}A^\gamma{}_{(\gamma)}\right)\right), \tag{77}$$

$$\bar\delta_\epsilon^{(iii)}V^{\alpha(\lambda)}=\partial^\alpha\epsilon'^{(\lambda)}+gc_1'\varepsilon^{\alpha\lambda\mu}\epsilon_{(\mu)}$$
$$+g\varepsilon^{\lambda\mu\nu}\left(c_2A^\alpha{}_{(\mu)}\epsilon'_{(\nu)}+\left(2c_3A^\alpha{}_{(\mu)}+c_2V^\alpha_{(\mu)}\right)\epsilon_{(\nu)}\right), \tag{78}$$

$$\bar\delta_\epsilon^{(iii)}A^{\alpha(\lambda)}=\partial^\alpha\epsilon^{(\lambda)}+gc_2\varepsilon^{\lambda\mu\nu}A^\alpha{}_{(\mu)}\epsilon_{(\nu)}. \tag{79}$$

Related to the last two situations, we notice that they can be obtained one from the other by performing the transformations (56–57). In this sense, the cases (ii) and (iii) are complementary.

By inspecting the structure of the terms with antighost number two appearing in each of the solutions (64), (67) and (70), we observe that in all cases the gauge algebra is deformed and closes according to a $su(2)$ Lie algebra, with the structure constants proportional with the components of the completely antisymmetric symbol in three dimensions, $\varepsilon^{\lambda\mu\nu}$.

Finally, we investigate the particular case where the degree three tensor gauge field $B^{\alpha\beta}{}_{(\lambda)}$ is dual to that of degree two with respect to the antisymmetry indices only, $A_{\alpha(\lambda)}=\varepsilon_{\alpha\beta\gamma}B^{\beta\gamma}{}_{(\lambda)}$, which is translated into $A_{\alpha(\lambda)}=V_{\alpha(\lambda)}$ at the level of action (4) and of its gauge symmetries (5). In this situation, we start from the Lagrangian action and associated gauge transformations

$$I_0\left[A_\alpha{}^{(\lambda)}\right]=\int d^3x\varepsilon^{\alpha\beta\gamma}A_{\alpha(\lambda)}\partial_\beta A_\gamma{}^{(\lambda)},\ \delta_\epsilon A_\alpha{}^{(\lambda)}=\partial_\alpha\epsilon^{(\lambda)}, \tag{80}$$

such that the free solution to the master equation (20) becomes

$$S=I_0\left[A_\alpha{}^{(\lambda)}\right]+\int d^3x\,A^{*\alpha}{}_{(\lambda)}\partial_\alpha C^{(\lambda)}. \tag{81}$$

The analogue of the first-order deformation (55) is found of the type

$$S_1=\int d^3x\left(c\varepsilon^{\lambda\mu\nu}\left(\frac{1}{2}C^*_{(\lambda)}C_{(\mu)}C_{(\nu)}+A^{*\alpha}{}_{(\lambda)}A_{\alpha(\mu)}C_{(\nu)}\right.\right.$$
$$\left.+\frac{1}{3}\varepsilon^{\alpha\beta\gamma}A_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)}\right)$$
$$\left.+c'\left(\varepsilon^{\alpha\lambda\mu}A^*_{\alpha(\lambda)}C_{(\mu)}+\left(A^\gamma{}_{(\beta)}A^\beta{}_{(\gamma)}-A^\beta{}_{(\beta)}A^\gamma{}_{(\gamma)}\right)\right)\right), \tag{82}$$

while its consistency, (25), holds if and only if

$$c'=0, \tag{83}$$

while c remains arbitrary, and can be fixed equal to the unity, $c=1$. The deformed Lagrangian action and gauge symmetries will accordingly be

$$I\left[A_{\alpha}{}^{(\lambda)}\right]=\varepsilon^{\alpha\beta\gamma}\int d^{3}xA_{\alpha(\lambda)}\left(\partial_{\beta}A_{\gamma}{}^{(\lambda)}+\frac{g}{3}\varepsilon^{\lambda\mu\nu}A_{\beta(\mu)}A_{\gamma(\nu)}\right), \tag{84}$$

$$\bar{\delta}_{\epsilon}A_{\alpha}{}^{(\lambda)}=\partial_{\alpha}\epsilon^{(\lambda)}+g\varepsilon^{\lambda\mu\nu}A_{\alpha(\mu)}\epsilon_{(\nu)}, \tag{85}$$

such that the deformed gauge algebra is again $su(2)$. This particular case is similar to what happens during the deformation of pure three-dimensional Abelian Chern–Simons theory into the non-Abelian version.

5 Conclusion

In conclusion, in this work we have constructed all consistent Lagrangian interactions that can be added to a free theory involving tensor gauge fields of degrees two and three in three space-time dimensions with the help of the free local BRST cohomology. We have proven that the deformed solution to the master equation can be taken to be non-vanishing only at the first order in the coupling constant. As a consequence, we determine three types of interacting models (among which two are complementary in the sense that they can be obtained one from the other by performing a certain transformation) with deformed gauge transformations, a non-Abelian ($su(2)$) gauge algebra and polynomial vertices of order three in the undifferentiated fields. In the special case where the tensor gauge fields are dual to each other, we obtain an analogue of pure three-dimensional Chern–Simons theory.

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