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Hybrid Superstrings on Singular Calabi-Yau Fourfolds

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Abstract

Two-dimensional hybrid superstring on singular Calabi-Yau manifolds is studied by the field redefinition of the NSR formalism. The compactification on singular Calabi-Yau fourfold is described by $N = 2$ super-Liouville theory and $N = 2$ Landau-Ginzburg models. We examine the world-sheet topological $N = 4$ superconformal algebra, which is useful to identify physical states of the theory. It is shown that the space-time superconformal symmetry is not compatible with this original topological algebra. A new model is proposed by modifying the topological superconformal generators, which is consistent with space-time $N = 2$ superconformal symmetry.

Holography between type II string theory on a singular Calabi-Yau manifold and superconformal field theory provides a useful tool for studying strong coupling physics of supersymmetric gauge theories [1, 2, 3, 4]. This string theory in the decoupling limit describes a field theory on the NS fivebranes wrapped on cycles on Calabi-Yau manifolds. When a Calabi-Yau manifold becomes singular, the field theory becomes an interacting superconformal field theory. Giveon, Kutasov and Pelc [2] studied the type II superstrings on a singular hypersurface and calculated the mass spectrum in the Neveu-Schwarz-Ramond (NSR) formalism.

It is an interesting problem to add the Ramond-Ramond (RR) flux in this system. The superstring theory including the RR flux is recognized more important to understand the phase structure of related supersymmetric gauge theories[5]. Since the RR vertex operators are represented by spin fields in the NSR formalism, it is difficult to quantize the superstring theories in this formulation. Green-Schwarz formalism, on the other hand, includes the RR fields naturally. But it is difficult to quantize covariantly due to its non-linearity.

Hybrid superstrings proposed by Berkovits [6] and Berkovits and Vafa [7] is a useful approach to quantize the superstrings compactified on Calabi-Yau manifolds in a manifestly supersymmetric manner. This is defined by using only mutually local spin fields which are obtained by field redefinition of the NSR formalism. The worldsheet topological $N = 4$ superconformal algebra plays an important role for the construction of physical states. One can also introduce the RR flux in this formalism[8].

In this paper we will investigate the hybrid superstrings on singular Calabi-Yau manifolds. In particular we study the two dimensional hybrid superstrings compactified on Calabi-Yau fourfolds. This model is recently studied [9]. Singular Calabi-Yau manifolds are described by tensor products of the $N = 2$ super-Liouville theory and $N = 2$ Landau-Ginzburg models.

The space-time sector of this model includes two scalar fields (ρ, f) in addition to the superspace coordinate fields. By combining this space-time sector and the $N = 2$ Liouville theory with background charge $Q = \sqrt{\frac{2}{k}}$, we can naturally obtain the free field realization of the affine Lie superalgebra $sl(2|1)^{(1)}$ at level k. From this superalgebra, we construct a space-time $N = 2$ superconformal algebra with central charge $c = 6kp$ for integer p. For nonzero k, however, we will find generators of this space-time superconformal symmetry do not commute with the BRST operators obtained by the field redefinition of the NSR superstring. We propose a model defined by a new topological $N=4$ superconformal algebra including BRST currents. This topological algebra is consistent with space-time $N=2$ superconformal symmetry.

We begin with reviewing the hybrid superstrings on Calabi-Yau fourfolds[9]. We consider the NSR formalism of type II superstrings on $R^{1,1}\times X^8$, where $R^{1,1}$ is two-dimensional Minkowski space-time and X^8 is a Calabi-Yau fourfold. The flat space-time $R^{1,1}$ is described by worldsheet free bosons $X^m(z)$ and real free fermions $\psi^m(z)$ ($m=0,1$) with operator product expansions (OPEs)

$$X^m(z)X^n(w)\sim -\eta^{mn}\ln(z-w), \quad \psi^m(z)\psi^n(w)\sim \eta^{mn}/(z-w). \tag{1}$$

Here $\eta^{mn}=diag(-1,1)$. Introduce the light-cone basis $X^{\pm}=\frac{1}{\sqrt{2}}(\pm X^0+X^1)$, $\psi^{\pm}=\frac{1}{\sqrt{2}}(\pm\psi^0+\psi^1)$. We have

$$X^+(z)X^-(w)\sim -\ln(z-w), \quad \psi^+(z)\psi^-(w)\sim \frac{1}{z-w}. \tag{2}$$

The energy-momentum tensor T_M and $N=1$ worldsheet supercurrent G_M is given by

$$\begin{aligned} T_M &= -\partial X^+\partial X^- - \frac{1}{2}\left(\psi^+\partial\psi^- + \psi^-\partial\psi^+\right), \\ G_M &= i\psi^+\partial X^- + i\psi^-\partial X^+ \end{aligned} \tag{3}$$

The Calabi-Yau part is described by $N=2$ superconformal field theory (SCFT) with central charge $c_C=12$ ($\hat{c}_C=c_C/3=4$). Denote $J_C, G_C^{\pm}, T_C^{N=2}$ for the generators of $N=2$ superconformal algebra with OPEs

$$\begin{aligned} T_C^{N=2}(z)T_C^{N=2}(w) &\sim \frac{\frac{c_C}{2}}{(z-w)^4}+\frac{2T_C^{N=2}(w)}{(z-w)^2}+\frac{\partial T_C^{N=2}(w)}{z-w}, \\ T_C^{N=2}(z)J_C(w) &\sim \frac{J_C(w)}{(z-w)^2}+\frac{\partial J_C(w)}{z-w}, \quad T_C^{N=2}(z)G_C^{\pm}(w)\sim\frac{\frac{3}{2}G_C^{\pm}(w)}{(z-w)^2}+\frac{\partial G_C^{\pm}(w)}{z-w} \\ J_C(z)J_C(w) &\sim \frac{\hat{c}_C}{(z-w)^2}, \quad J_C(z)G_C^{\pm}(w)\sim\frac{\pm G_C^{\pm}(w)}{z-w} \\ G_C^+(z)G_C^-(w) &\sim \frac{\hat{c}_C}{(z-w)^3}+\frac{J_C}{(z-w)^2}+\frac{T_C^{N=2}(w)+\frac{1}{2}J_C(w)}{z-w} \end{aligned} \tag{4}$$

We need fermionic ghosts (b,c) and bosonic ghosts (β,γ) with conformal weights (2,-1) and $(\frac{3}{2},-\frac{1}{2})$, respectively. They satisfy

$$b(z)c(w) \sim \frac{1}{z-w}, \quad \beta(z)\gamma(w) \sim \frac{-1}{z-w}. \tag{5}$$

These fields are bosonized such as $(b,c)=(e^{i\sigma},e^{-i\sigma})$, $(\beta,\gamma)=(\partial\xi e^{-\phi},\eta e^{\phi})$ and $(\eta,\xi)=(e^{i\chi},e^{-i\chi})$ [10]. Here σ, ϕ and χ are free bosons with OPEs $\sigma(z)\sigma(w)\sim-\ln(z-w)$, $\phi(z)\phi(w)\sim-\ln(z-w)$ and $\chi(z)\chi(w)\sim-\ln(z-w)$, respectively. The ghost sector has $N=1$ superconformal symmetry. The energy-momentum tensor and the supercurrent are

$$\begin{aligned}
T^{gh}(z) &= T_{bc}+T_{\beta\gamma}, \\
T_{bc}(z) &= -2b\partial c-\partial bc, \quad T_{\beta\gamma}(z)=-\frac{3}{2}\beta\partial\gamma-\frac{1}{2}\partial\beta\gamma, \\
G^{gh}(z) &= 2c\partial\beta+3\partial c\beta+\gamma b.
\end{aligned} \tag{6}$$

We discuss the BRST structure of the superstring theory. The BRST charge Q_{BRST} is defined by

$$Q_{BRST}=\int\frac{dz}{2\pi i}J_{BRST}(z), \tag{7}$$

where J_{BRST} is the BRST current:

$$J_{BRST}=c(T_m+\frac{1}{2}T_{bc}+T_{\beta\gamma})-\gamma G_m-\gamma^2 b+\partial^2 c+\partial(c\xi\eta) \tag{8}$$

and

$$T_m=T_M+T_C^{N=2}, \qquad G_m=G_M+G_C^{+}+G_C^{-}. \tag{9}$$

Q_{BRST} is a nilpotent operator when a total central charge is zero. In fact, $T_m(z),G_m(z)$ satisfy $N=1$ superconformal algebra with $c_m=15$, while $T^{gh}(z),G^{gh}(z)$ satisfy $N=1$ superconformal algebra with $c_{gh}=-15$. The Hilbert space $\mathcal{H}_{phys}$ is characterized by the BRST cohomology; $\mathcal{H}_{phys}=\mathrm{Ker}Q_{BRST}/\mathrm{Im}Q_{BRST}$. Here Q_{BRST} acts on the small Hilbert space $\mathcal{H}_{small}$ spanned by the ghost sector (b,c,β,γ) and the matter part. When we study the BRST cohomology, it is convenient to consider the large Hilbert space $\mathcal{H}_{large}$ rather than $\mathcal{H}_{small}$. The large Hilbert space is obtained from the small Hilbert space by adding the zero mode ξ_0 of fermionic ghost ξ. In the large Hilbert space, the physical state condition becomes

$$\begin{aligned} Q_{BRST}|\psi\rangle &= 0, \quad |\psi\rangle \sim |\psi\rangle + Q_{BRST}|\Lambda\rangle, \\ \eta_0|\psi\rangle &= \eta_0|\Lambda\rangle = 0, \end{aligned} \tag{10}$$

where η_0 is the zero-mode of η which is conjugate to ξ_0.

It has been noticed in [7] that the BRST symmetry in superstring theory can be regarded as a part of topological $N=4$ superconformal symmetry. The physical state conditions are translated into the chiral state conditions in superconformal field theory. In the hybrid formalism, this world-sheet symmetry comes from a part of κ-symmetry[6, 11]. The topological $N=4$ superconformal algebra is generated by the currents

$$\begin{aligned} J(z) &= cb - \xi\eta, \quad J^{++}(z) = c\eta, \quad J^{--}(z) = \xi b, \\ G^+(z) &= J_{BRST} = c(T_m + \frac{1}{2}T_{bc} + T_{\beta\gamma}) - \gamma G_m - \gamma^2 b + \partial^2 c + \partial(c\xi\eta), \\ G^-(z) &= b, \\ \tilde{G}^+(z) &= \eta, \\ \tilde{G}^-(z) &= \xi(T_m + T_{bc} + T_{\beta\gamma}) + e^{\phi}G_m b - \eta e^{2\phi}(\partial b)b - bc\partial\xi + \partial^2\xi, \\ T(z) &= T_m + T_{bc} + T_{\beta\gamma}. \end{aligned} \tag{11}$$

The untwisted energy-momentum tensor $T^{N=2} = T - \frac{1}{2}\partial J$ has central charge $c=6$. Now the physical state conditions (10) become

$$\begin{aligned} G_0^+|\psi\rangle &= 0, \quad |\psi\rangle \sim |\psi\rangle + G_0^+|\Lambda\rangle, \\ \tilde{G}_0^+|\psi\rangle &= \tilde{G}_0^+|\Lambda\rangle = 0. \end{aligned} \tag{12}$$

Since the $\tilde{G}_0^+$-cohomology is trivial, *i.e.* for a state $|\psi\rangle$ satisfying $\tilde{G}_0^+|\psi\rangle = 0$ there is a state $|V\rangle \in \mathcal{H}_{large}$ such that $|\psi\rangle = \tilde{G}_0^+|V\rangle$. The physical state conditions for $|V\rangle$ become

$$G_0^+\tilde{G}_0^+|V\rangle = 0, \quad |V\rangle \sim |V\rangle + G_0^+|\Lambda\rangle + \tilde{G}_0^+|\tilde{\Lambda}\rangle. \tag{13}$$

We now investigate the space-time supersymmetry. In order to introduce space-time spinor, we bosonize the worldsheet fermions $\psi^+ = e^{iH_1}$, $\psi^- = e^{-iH_1}$, where $H_1(z)H_1(w) \sim -\ln(z-w)$. The space-time supercharges are given by $Q_\alpha^\pm = \int \frac{dz}{2\pi i} q_\alpha^\pm(z)$ ($\alpha = 1, 2$) where $q_\alpha^\pm(z)$ are the supercurrents in the $-\frac{1}{2}$ picture:

$$q_\alpha^\pm = e^{-\frac{\phi}{2} + \frac{i\epsilon_\alpha H_1}{2} \pm \frac{i}{2}H_C}, \tag{14}$$

where H_C is defined by $J_C = i\partial H_C$ and $\epsilon_1 = 1$, $\epsilon_2 = -1$. From the GSO projection, we may take $q_1^{\pm}(z)$ for mutually local operators. The supersymmetry algebra is

$$\{Q_1^+, Q_1^-\} = \int \frac{dz}{2\pi i} e^{-\phi}\psi^+(z). \tag{15}$$

Due to the picture number, supersymmery is not manifest in the NSR formalism.

In order to realize the space-time supersymmetry manifestly, we change the picture number of q_1^+ from $-\frac{1}{2}$ to $+\frac{1}{2}$ by applying the picture changing operator $Z = Q_{BRST}\xi$;

$$Z = c\partial\xi - e^{\phi}G_m - \partial\eta e^{2\phi}b + \partial(\eta e^{2\phi}b). \tag{16}$$

Then q_1^+ becomes

$$q_1^+ = \eta b e^{\frac{3\phi}{2}+\frac{iH_1}{2}+\frac{iH_C}{2}} + i\partial X^+ e^{\frac{\phi}{2}-\frac{iH_1}{2}+\frac{iH_C}{2}} - G_C^- e^{\frac{\phi}{2}+\frac{iH_1}{2}+\frac{iH_C}{2}}. \tag{17}$$

The new space-time supercharges

$$Q_1^{\pm} = \int \frac{dz}{2\pi i} q_1^{\pm}(z) \tag{18}$$

obey the anti-commutation relation

$$\{Q_1^+, Q_1^-\} = \int \frac{dz}{2\pi i} i\partial X^+. \tag{19}$$

In this realization, certain linear combinations of the fields ϕ, H_1 and H_C are important. We define the new variables

$$p_1 = e^{-\frac{1}{2}\phi+\frac{i}{2}H_1-\frac{i}{2}H_C}, \quad p_2 = \eta b e^{\frac{3}{2}\phi+\frac{i}{2}H_1+\frac{i}{2}H_C}, \tag{20}$$

and their conjugate fields

$$\theta^1 = e^{\frac{1}{2}\phi-\frac{i}{2}H_1+\frac{i}{2}H_C}, \quad \theta^2 = c\xi e^{-\frac{3}{2}\phi-\frac{i}{2}H_1-\frac{i}{2}H_C}, \tag{21}$$

satisfying $p_a(z)\theta^b(w) \sim \frac{\delta_a^b}{z-w}$. Then the supercurrents become

$$\begin{aligned} q_1^- &= p_1, \\ q_1^+ &= p_2 + i\partial X^+\theta^1 - G_C^- e^{\frac{\phi}{2}+\frac{iH_1}{2}+\frac{iH_C}{2}}. \end{aligned} \tag{22}$$

The fermionic system (p_a, θ^a) has conformal weights $(1,0)$ and regarded as fundamental fields in the hybrid superstrings. The fermions θ^a are fermionic coordinates of target superspace. Let us express them in the bosonized form $p_a = e^{i\phi_a}$ and $\theta^a = e^{-i\phi_a}$ ($a = 1, 2$), where

$$\begin{aligned} i\phi_1 &= -\frac{1}{2}\phi + \frac{i}{2}H_1 - \frac{i}{2}H_C, \\ i\phi_2 &= i\sigma + i\chi + \frac{3}{2}\phi + \frac{i}{2}H_1 + \frac{i}{2}H_C. \end{aligned} \tag{23}$$

Since (p_a, θ^a) are not orthogonal to the $U(1)$ scalar field H_C and the ghost fields, we define the new $U(1)$ field $\widehat{H}_C$ and the free bosons(ρ, f).

$$\begin{aligned} i\widehat{H}_C &= iH_C + 4(\phi + i\chi), \\ \rho &= 2\sqrt{2}\phi + \frac{3}{\sqrt{2}}i\chi + \frac{1}{\sqrt{2}}iH_C + \frac{1}{\sqrt{2}}i\sigma, \\ if &= \frac{1}{\sqrt{2}}(-\phi - i\chi + iH_1 - i\sigma). \end{aligned} \tag{24}$$

These are orthogonal to (p_a, θ^a). Due to the change of the $U(1)$ current J_C, the generators of $N = 2$ superconformal algebra are modified as

$$\begin{aligned} \widehat{J}_C &= i\partial\widehat{H}_C, \quad \widehat{T}_C^{N=2} = T_C^{N=2} + \frac{1}{2}i\partial H_C\partial(\phi + i\chi) + 2(\partial\phi + i\partial\chi)^2, \\ \widehat{G}_C^{\pm} &= e^{\pm(\phi + i\chi)}G_C^{\pm}. \end{aligned} \tag{25}$$

The hybrid superstrings are described by the free fields $X^{\pm}$, (p_a, θ^a) ($a = 1, 2$), ρ, f and $N = 2$ SCFT $(\widehat{J}_C, \widehat{G}_C^{\pm}, \widehat{T}_C^{N=2})$. In terms of these new variables, the space-time supercurrents take the form

$$q_1^- = p_1, \quad q_1^+ = p_2 + i\partial X^+\theta^1 - e^{\frac{1}{\sqrt{2}}(\rho + if)}\widehat{G}_C^-. \tag{26}$$

The last term in q_1^+ is removed by the similarity transformation $\mathcal{O} \to (\mathcal{O})^R \equiv e^R\mathcal{O}e^{-R}$ with the operator $R = \int \frac{dz}{2\pi i}c\widehat{G}_C^-$:

$$\begin{aligned} (q_1^-)^R &= p_1, \\ (q_1^+)^R &= p_2 + i\partial X^+\theta^1. \end{aligned} \tag{27}$$

Further similarity transformation by the operator $U = \int \frac{dz}{2\pi i}\frac{-i}{2}\theta^1\theta^2\partial X^+$ leads to the supercurrennts realized in a more symmetric way:

$$\begin{aligned} (q_1^-)^{R+U} &= p_1 + \frac{i}{2}\partial X^+\theta^2, \\ (q_1^+)^{R+U} &= p_2 + \frac{i}{2}\partial X^+\theta^1. \end{aligned} \tag{28}$$

It is easy to see that the supercharges $Q_1^{\pm} = \int \frac{dz}{2\pi i}(q_1^{\pm})^{R+U}$ satisfy the anticommutation relation (19). Combining these with the anti-holomorphic part, we get $N = (2,2)$ supersymmetry for type IIA or $N = (4,0)$ supersymmetry for type IIB.

We study the topological $N = 4$ structure in the hybrid superstrings. When we perform the similarity transformation by $R+U$, we may check that the energy-momentum tensor T and $SU(2)$ current J, $J^{\pm\pm}$ are invariant. In the currents $G^{\pm}$, the Calabi-Yau part and the flat space-time part decouple after the transformation. In terms of hybrid variables $(X^m, p^a, \theta_a, \rho, f, \widehat{J}_C, \widehat{G}_C^{\pm}, \widehat{T}_C^{N=2})$ the generators (11) are shown to be expressed as follows:

$$\begin{aligned}
(J)^{R+U} &= -\sqrt{2}\partial\rho + i\partial\widehat{H}_C, \quad (J^{\pm\pm})^{R+U} = e^{\pm(-\sqrt{2}\rho + i\widehat{H}_C)}, \\
(T)^{R+U} &= -\partial X^+\partial X^- - p_1\partial\theta^1 - p_2\partial\theta^2 - \frac{1}{2}(\partial\rho)^2 - \frac{1}{2}(\partial f)^2 - \frac{1}{\sqrt{2}}\partial^2(\rho - if) \\
&\quad + \widehat{T}_C^{N=2} + \frac{1}{2}\partial\widehat{J}_C^{N=2}, \\
(G^+)^{R+U} &= e^{\frac{1}{\sqrt{2}}(\rho+if)}\Big[\Big(-i\partial X^- + \frac{1}{2}(\theta^1\partial\theta^2 - \partial\theta^1\theta^2)\Big)\Big(p_1 - \frac{i}{2}\theta^2\partial X^+\Big) - i\sqrt{2}\partial f\partial\theta^2 - \frac{3}{4}\partial^2\theta^2\Big] \\
&\quad + e^{\frac{1}{\sqrt{2}}(\rho-if)}(p_1 - \frac{i}{2}\theta^2\partial X^+) + \widehat{G}_C^+, \\
(G^-)^{R+U} &= e^{-\frac{i}{\sqrt{2}}(\rho+if)}(p_2 - \frac{i}{2}\theta^1\partial X^+) + \widehat{G}_C^-.
\end{aligned} \tag{29}$$

In the supercurrents $\tilde{G}^{\pm}$, the flat space-time part and the Calabi-Yau part do not decouple. One finds that

$$\begin{aligned}
(\tilde{G}^+)^{R+U} &= e^{-\frac{1}{\sqrt{2}}(3\rho+if)+i\widehat{H}_C}(p_2 - \frac{i}{2}\theta^1\partial X^+) + e^{-\sqrt{2}\rho+i\widehat{H}_C}\widehat{G}_C^-, \\
(\tilde{G}^-)^{R+U} &= e^{\frac{1}{\sqrt{2}}(3\rho+if)-i\widehat{H}_C}\Big[\Big(-i\partial X^- + \frac{1}{2}(\theta^1\partial\theta^2 - \partial\theta^1\theta^2)\Big)\Big(p_1 - \frac{i}{2}\theta^2\partial X^+\Big) - i\sqrt{2}\partial f\partial\theta^2 - \frac{3}{4}\partial^2\theta^2\Big] \\
&\quad + e^{\frac{1}{\sqrt{2}}(3\rho-if)-i\widehat{H}_C}(p_1 - \frac{i}{2}\theta^2\partial X^+) + e^{\sqrt{2}\rho-i\widehat{H}_C}\widehat{G}_C^+.
\end{aligned} \tag{30}$$

In the following we shall use the variables after the similarity transformations. We use $\mathcal{O}$ instead of $\mathcal{O}^{R+U}$ for simplicity.

As in the case of four or six dimensional superstrings[6], these currents are written in terms of supercovariant derivatives. Let d_a ($a = 1, 2$) be supercovariant derivatives which anticommute with $q_1^{\pm}(z)$

$$d_1 = p_1 - \frac{i}{2}\theta^2\partial X^+,$$

$$d_2 = p_2 - \frac{i}{2}\theta^1\partial X^+. \tag{31}$$

We introduce also

$$\begin{aligned}
\pi^+ &= i\partial X^+, \\
\pi^- &= i\partial X^- + \frac{1}{2}(\partial\theta^1\theta^2 - \theta^1\partial\theta^2), \\
\omega^1 &= \partial\theta^1, \quad \omega^2 = \partial\theta^2.
\end{aligned} \tag{32}$$

They satisfy the following algebra:

$$\begin{aligned}
d_1(z)d_2(w) &\sim \frac{-\pi^+(w)}{(z-w)^2}, \quad \pi^+(z)\pi^-(w) \sim \frac{1}{(z-w)^2}, \\
d_1(z)\pi^-(w) &\sim \frac{-\omega^2(w)}{z-w}, \quad d_2(z)\pi^-(w) \sim \frac{-\omega^1(w)}{z-w}, \\
d_1(z)\omega^1(w) &\sim \frac{1}{(z-w)^2}, \quad d_2(z)\omega^2(w) \sim \frac{1}{(z-w)^2}.
\end{aligned} \tag{33}$$

Then the generators of the topological $N=4$ algebra become

$$\begin{aligned}
J &= -\sqrt{2}\partial\rho + i\partial\widehat{H}_C, \quad J^{\pm\pm} = e^{\pm(-\sqrt{2}\rho + i\widehat{H}_C)}, \\
T &= \pi^+\pi^- - d_1\partial\theta^1 - d_2\partial\theta^2 - \frac{1}{2}(\partial\rho)^2 - \frac{1}{2}(\partial f)^2 - \frac{1}{\sqrt{2}}\partial^2(\rho - if) \\
&\quad + \widehat{T}_C^{N=2} + \frac{1}{2}\partial\widehat{J}_C, \\
G^+ &= e^{\frac{1}{\sqrt{2}}(\rho+if)}\left(-\pi^- d_1 - i\sqrt{2}\partial f\partial\theta^2 - \frac{3}{4}\partial^2\theta^2\right) + e^{\frac{1}{\sqrt{2}}(\rho-if)}d_1 + \widehat{G}_C^+, \\
G^- &= e^{-\frac{1}{\sqrt{2}}(\rho+if)}d_2 + \widehat{G}_C^-, \\
\tilde{G}^+ &= e^{-\frac{1}{\sqrt{2}}(3\rho+if)+i\widehat{H}_C}d_2 + e^{-\sqrt{2}\rho+i\widehat{H}_C}\widehat{G}_C^-, \\
\tilde{G}^- &= e^{\frac{1}{\sqrt{2}}(3\rho+if)-i\widehat{H}_C}\left(-\pi^- d_1 - i\sqrt{2}\partial f\partial\theta^2 - \frac{3}{4}\partial^2\theta^2\right) + e^{\frac{1}{\sqrt{2}}(3\rho-if)-i\widehat{H}_C}d_1 + e^{\sqrt{2}\rho-i\widehat{H}_C}\widehat{G}_C^+.
\end{aligned} \tag{34}$$

The space-time supersymmetry generators (28) commute with G_0^+ and $\tilde{G}_0^+$. The space-time supermultiplet of a observable belongs to the BRST cohomology. We note that G^+ can be decomposed in three mutually anticommuting parts:

$$G^+ = G_{(1)}^+ + G_{(2)}^+ + G_C^+, \tag{35}$$

where $G_{(1)}^+ = e^{\frac{1}{\sqrt{2}}(\rho+if)}\left(-(\pi^- d_1) - i\sqrt{2}\partial f\partial\theta^2 - \frac{3}{4}\partial^2\theta^2\right)$ and $G_{(2)}^+ = e^{\frac{1}{\sqrt{2}}(\rho-if)}d_1$.

Now we will study the hybrid superstrings on singular Calabi-Yau fourfolds. A singular Calabi-Yau manifold X^8 is described by $R_\varphi \times S^1 \times LG$[2], where $R_\varphi \times S^1$ denotes the $N=2$ supersymmetric Liouville theory with the central charge $c_L = 3 + 3Q^2$. LG is an $N=2$ Laundau-Ginzburg model associated with ADE type singularity and has the central charge $c_{LG} = 12 - c_L = 9 - 3Q^2$.

The $N=2$ Liouville part is realized by two complex boson $\Phi = \frac{1}{\sqrt{2}}(\varphi + iY)$, $\bar{\Phi} = \frac{1}{\sqrt{2}}(\varphi - iY)$ and complex fermions $\Psi = \frac{1}{\sqrt{2}}(\Psi^1 + i\Psi^2)$, $\bar{\Psi} = \frac{1}{\sqrt{2}}(\Psi^1 - i\Psi^2)$, where φ is a Liouville field with background charge Q and Y is a free boson compactified on S^1. The OPEs are given by $\varphi(z)\varphi(w) \sim -\ln(z-w)$, $Y(z)Y(w) \sim -\ln(z-w)$ and $\Psi^a(z)\Psi^b(w) \sim \delta^{ab}/(z-w)$ $(a,b=1,2)$. The worldsheet $N=2$ algebra with the central charge $c_L = 3+3Q^2$ can be obtained from the generators:

$$
\begin{aligned}
T_L &= -\partial\Phi\partial\bar{\Phi} - \frac{Q}{2\sqrt{2}}\partial^2(\Phi + \bar{\Phi}) - \frac{1}{2}\bar{\Psi}\partial\Psi - \frac{1}{2}\Psi\partial\bar{\Psi}, \\
G_L^+ &= i\Psi\partial\bar{\Phi} + \frac{iQ}{\sqrt{2}}\partial\Psi, \quad G_L^- = i\bar{\Psi}\partial\Phi + \frac{iQ}{\sqrt{2}}\partial\bar{\Psi}, \\
J_L &= \Psi\bar{\Psi} + iQ\partial Y.
\end{aligned} \tag{36}
$$

For the Landau-Ginzburg part, we write the generators of $N=2$ algebra as $J_{LG}, G_{LG}^{\pm}, T_{LG}^{N=2}$ Then we have

$$
\begin{aligned}
\hat{J}_C &= J_L + J_{LG}, \quad \hat{T}_C^{N=2} = T_L^{N=2} + T_{LG}^{N=2} \\
\hat{G}_C^{\pm} &= G_L^{\pm} + G_{LG}^{\pm}.
\end{aligned} \tag{37}
$$

In [2], it is argued that the superstrings on singular Calabi-Yau manifolds correspond to the space-time $N=2$ superconformal field theory with central charge $c = 6kp$ in the decoupling limit. Here k is related to the Liouville background charge Q by the formula $k = 2/Q^2$ and p is an integer.

We expect that the space-time $N=2$ supersymmetry (28) is enhanced to $N=2$ superconformal symmetry generated by $\mathcal{L}_n$, $\mathcal{I}_n$ $(n \in \mathbf{Z})$ and $\mathcal{G}_r^{\pm}$ $(r \in \mathbf{Z} + \frac{1}{2})$. Their algebra is

$$
\begin{aligned}
[\mathcal{L}_n, \mathcal{L}_m] &= (n-m)\mathcal{L}_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}, \\
\left[\mathcal{L}_n, \mathcal{G}_r^{\pm}\right] &= \left(\frac{n}{2} - r\right)\mathcal{G}_{n+r}^{\pm},
\end{aligned}
$$

$$
\begin{aligned}
[\mathcal{L}_n, \mathcal{I}_m] &= -m\mathcal{I}_{n+m}, \\
\left\{\mathcal{G}_r^+, \mathcal{G}_s^-\right\} &= \mathcal{L}_{r+s} + \frac{1}{2}(r-s)\mathcal{I}_{r+s} + \frac{c}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}, \\
\left[\mathcal{I}_n, \mathcal{G}_r^\pm\right] &= \pm\mathcal{G}_{n+r}^\pm, \\
[\mathcal{I}_n, \mathcal{I}_m] &= \frac{c}{3}n\delta_{n+m,0}.
\end{aligned} \tag{38}
$$

$\mathcal{G}^\pm_{-\frac{1}{2}}$ generate the $N=2$ supersymmetry and $\mathcal{L}_{-1}$ corresponds to the translation: $\mathcal{G}^\pm_{-\frac{1}{2}} = Q_1^\pm$, $\mathcal{L}_{-1} = \int \frac{dz}{2\pi i} i\partial X^+$.

We discuss a finite dimensional subalgebra of $N=2$ superconformal symmetry. In terms of zero-modes of superspace coordinates (X^-, θ^a), the $N=2$ space-time superconformal generators $\mathcal{L}_0, \mathcal{L}_{\pm 1}, \mathcal{G}_{\pm\frac{1}{2}}, \bar{\mathcal{G}}_{\pm\frac{1}{2}}, \mathcal{I}_0$ are given by

$$
\begin{aligned}
\mathcal{L}_{-1} &= -i\frac{\partial}{\partial X^-}, \\
\mathcal{L}_0 &= -X^-\frac{\partial}{\partial X^-} - \frac{1}{2}\left(\theta^1\frac{\partial}{\partial\theta^1} + \theta^2\frac{\partial}{\partial\theta^2}\right), \\
\mathcal{L}_1 &= i(X^-)^2\frac{\partial}{\partial X^-} - iX^-\left(\theta^1\frac{\partial}{\partial\theta^1} + \theta^2\frac{\partial}{\partial\theta^2}\right), \\
\mathcal{G}^+_{\frac{1}{2}} &= -iX^-\left(\frac{\partial}{\partial\theta^2} - \frac{i}{2}\theta^1\frac{\partial}{\partial X^-}\right) - \frac{1}{2}\theta^1\theta^2\frac{\partial}{\partial\theta^2}, \\
\mathcal{G}^+_{-\frac{1}{2}} &= \frac{\partial}{\partial\theta^2} - \frac{i}{2}\theta^1\frac{\partial}{\partial X^-}, \\
\mathcal{G}^-_{\frac{1}{2}} &= -iX^-\left(\frac{\partial}{\partial\theta^1} - \frac{i}{2}\theta^2\frac{\partial}{\partial X^-}\right) - \frac{1}{2}\theta^1\theta^2\frac{\partial}{\partial\theta^1}, \\
\mathcal{G}^-_{-\frac{1}{2}} &= \frac{\partial}{\partial\theta^1} - \frac{i}{2}\theta^2\frac{\partial}{\partial X^-}, \\
\mathcal{I}_0 &= \theta^1\frac{\partial}{\partial\theta^1} - \theta^2\frac{\partial}{\partial\theta^2}.
\end{aligned} \tag{39}
$$

To realize the infinite dimensional algebra, it is convenient to introduce (β,γ) system instead of (X^+, X^-) by

$$
\beta = i\partial X^+, \quad \gamma = -iX^-. \tag{40}
$$

which has the OPE $\beta(z)\gamma(w) \sim (-1)/(z-w)$. The space-time $N=2$ superconformal algebra is now realized in terms of hybrid variables (β,γ), (p_a, θ^a) and (ρ, f). In addition, $\mathcal{I}_0$ corresponds to the $\mathcal{R}$-charge of the theory and is characterized by the $U(1)$ current $i\partial Y$. In the AdS_3 case[12], we need the Liouville field to construct the space-time Virasoro algebra. In the present case, φ plays a similar role. These field contents lead to the free field realization of currents of affine Lie superalgebra $sl(2|1)^{(1)}$ at level k[13]:

$$\begin{aligned}
\hat{J}^{++} &= \gamma^2\beta - \gamma\left(\theta^1 p_1 + \theta^2 p_2 - \partial(R - a\Phi) + \partial(\bar{R} + a\bar{\Phi})\right) \\
&\quad + (k-1)\frac{1}{2}(\theta^1\partial\theta^2 - \partial\theta^1\theta^2) - \frac{1}{2}\theta^1\theta^2\left(\partial(R - a\Phi) + \partial(\bar{R} + a\bar{\Phi})\right) - k\partial\gamma, \\
\hat{J}^3 &= -\gamma\beta + \frac{1}{2}\left(\theta^1 p_1 + \theta^2 p_2 - \partial(R - a\Phi) + \partial(\bar{R} + a\bar{\Phi})\right), \\
\hat{J}^{--} &= \beta, \\
\hat{I} &= \frac{1}{2}\left(\theta^1 p_1 - \theta^2 p_2 - \partial(R - a\Phi) - \partial(\bar{R} + a\bar{\Phi})\right), \\
\hat{j}^{+(+)} &= \gamma(p_2 + \frac{1}{2}\theta^1\beta) - (k - \frac{1}{2})\partial\theta^1 - \theta^1\partial(\bar{R} + a\bar{\Phi}) - \frac{1}{2}\theta^1\theta^2 p_2, \\
\hat{j}^{+(-)} &= \gamma(p_1 + \frac{1}{2}\theta^2\beta) - (k - \frac{1}{2})\partial\theta^2 + \theta^2\partial(R - a\Phi) + \frac{1}{2}\theta^1\theta^2 p_1, \\
\hat{j}^{-(+)} &= p_2 + \frac{1}{2}\theta^1\beta, \\
\hat{j}^{-(-)} &= p_1 + \frac{1}{2}\theta^2\beta.
\end{aligned} \tag{41}$$

where $a = \sqrt{k} = \frac{\sqrt{2}}{Q}$ and

$$R = \frac{1}{\sqrt{2}}(\rho + if), \qquad \bar{R} = \frac{1}{\sqrt{2}}(\rho - if). \tag{42}$$

From the affine Lie superalgebra, one may construct the space-time $N = 2$ superconformal algebra as discussed in the AdS_3 case[14]:

$$\begin{aligned}
\mathcal{L}_n &= \oint \frac{dz}{2\pi i}\left(\frac{1}{2}n(n+1)\gamma^{n-1}\left(\hat{J}^{++} + \frac{k}{2}\partial(\theta^1\theta^2)\right) + (n^2-1)\gamma^n\hat{J}^3 + \frac{1}{2}n(n-1)\gamma^{n+1}\hat{J}^{--}\right), \\
\mathcal{G}_r^+ &= \oint \frac{dz}{2\pi i}\left((r + \frac{1}{2})\gamma^{r-\frac{1}{2}}(\hat{j}^{+(+)} + k\partial\theta^1) - (r - \frac{1}{2})\gamma^{r+\frac{1}{2}}\hat{j}^{-(+)}\right), \\
\mathcal{G}_r^- &= \oint \frac{dz}{2\pi i}\left((r + \frac{1}{2})\gamma^{r-\frac{1}{2}}\hat{j}^{+(-)} - (r - \frac{1}{2})\gamma^{r+\frac{1}{2}}\hat{j}^{-(-)} - (r^2 - \frac{1}{4})\gamma^{r-\frac{3}{2}}\frac{k - \frac{1}{2}}{2}\theta^1\theta^2\partial\theta^2\right), \\
\mathcal{I}_n &= \oint \frac{dz}{2\pi i}\left(2\gamma^n\hat{I} - n\gamma^{n-1}\left((k - \frac{1}{2})\theta^1\partial\theta^2 - \frac{1}{2}\partial\theta^1\theta^2 - \frac{1}{2}\theta^1\theta^2(\partial(R - a\Phi) - \partial(\bar{R} + a\bar{\Phi})\right)\right).
\end{aligned} \tag{43}$$

These generators satisfy $N = 2$ space-time superconformal algebra with the central charge $c = 6kp$, where

$$p = \int \frac{dz}{2\pi i}\frac{\partial\gamma}{\gamma}. \tag{44}$$

We must, however, examine consistency of this space-time $N=2$ superconformal symmetry with the topological $N=4$ structure of the theory. Since the physical symmetry have to (anti-)commute with the BRST charge, the space-time charges (43) must (anti-) commute with operators $G_0^+ = \int \frac{dz}{2\pi i} G^+(z)$ and $\tilde{G}_0^+ = \int \frac{dz}{2\pi i}\tilde{G}^+(z)$. For $sl(2|1)$ currents, however, it is shown that the currents $\hat{J}^{++}$ and $\hat{j}^{+(+)}$ are not G_0^+-invariant. Moreover, $\hat{J}^{++}$ and $\hat{j}^{+(-)}$ are not $\tilde{G}_0^+$-invariant. This implies that the original model proposed in [2] does not have the space-time superconformal symmetry in their physical spectrum. Here we modify the model in order to allow the space-time superconformal symmetry.

We will solve the above problem by modifying both the worldsheet topological currents and the space-time currents. After some calculations, we are able to find the following solution. Firstly, the $sl(2|1)$ currents are modified as

$$\hat{J}^{(new)++} = \hat{J}^{++} + aie^{-R}\theta^1\Psi, \quad \hat{j}^{(new)+(-)} = \hat{j}^{+(-)} - aie^{-R}\Psi, \tag{45}$$

and other currents are invariant. The space-time $N=2$ superconformal generators are obtained by replacing the affine currents to the new ones in (43). But $\mathcal{G}_r^-$ and $\mathcal{I}_n$ receive further corrections:

$$\begin{aligned}
\mathcal{G}_r^{new-} &= \mathcal{G}_r^{old-} - \int \frac{dz}{2\pi i}\left((r^2 - \frac{1}{4})\gamma^{r-\frac{3}{2}}\frac{ai}{2}\theta^1\theta^2 e^{-R}\Psi\right), \\
\mathcal{I}_n^{new} &= \mathcal{I}_n^{old} - \int \frac{dz}{2\pi i} ain\gamma^{n-1}e^{-R}\theta^1\Psi.
\end{aligned} \tag{46}$$

The BRST currents which is consistent with these space-time currents take the form

$$\begin{aligned}
G^{new+} &= G^+_{(1)} + \Delta G^+_{(1)} + G^+_L + G^+_{LG}, \\
\tilde{G}^{new+} &= \tilde{G}^{old+} + \Delta\tilde{G}^+,
\end{aligned} \tag{47}$$

where

$$\begin{aligned}
\Delta G^+_{(1)} &= e^R\left\{(\partial R + a\partial\bar{\Phi})\partial\theta^2 + \partial^2\theta^2\right\}, \\
\Delta\tilde{G}^+ &= -aie^{-\sqrt{2}\rho + i\hat{H}_C}(\bar{\Psi}\partial R + \partial\bar{\Psi}).
\end{aligned} \tag{48}$$

In $G^+(z)$, the term $G^+_{(2)}$ is eliminated in order to keep the OPE with $G^-(z)$. The energy-momentum tensor T and $SU(2)$ currents $J^{\pm\pm}, J^3$ do not change. The supercurrents G^-

and $\tilde{G}^-$ are modified due to the change of G^+ and $\tilde{G}^+$. To summarize, the worldsheet $N=4$ currents are

$$
\begin{aligned}
T &= -\beta\partial\gamma - p_1\partial\theta^1 - p_2\partial\theta^2 - \partial\bar{R}\partial R - \partial^2\bar{R} \\
&\qquad - \partial\bar{\Phi}\partial\Phi - \frac{Q}{\sqrt{2}}\partial^2\bar{\Phi} - \bar{\Psi}\partial\Psi + T_{LG} + \frac{1}{2}\partial J_{LG}, \\
G^+ &= e^R\left(-\pi^- d_1 + \partial(\bar{R}+a\bar{\Phi})\partial\theta^2 + \frac{1}{4}\partial^2\theta^2\right) + i\left(\Psi\partial\bar{\Phi} + \frac{1}{a}\partial\Psi\right) + G^+_{LG}, \\
G^- &= e^{-R}d_2 + i\left(\bar{\Psi}\partial(\Phi - aR) - \left(a - \frac{1}{a}\right)\partial\bar{\Psi}\right) + G^-_{LG}, \\
\tilde{G}^+ &= e^{-2R-\bar{R}+iH_L+iH_{LG}}d_2 + e^{-R-\bar{R}+iH_L+iH_{LG}}\left(\bar{\Psi}\partial(\Phi - aR) + \left(a - \frac{1}{a}\right)\partial\bar{\Psi}\right) \\
&\qquad - e^{-R-\bar{R}+iH_L+iH_{LG}}G^-_{LG}, \\
\tilde{G}^- &= e^{2R+\bar{R}-iH_L-iH_{LG}}(-\pi^- d_1 + \partial(\bar{R}+a\bar{\Phi})\partial\theta^2 + \frac{1}{4}\partial^2\theta^2) \\
&\qquad + e^{R+\bar{R}-iH_L-iH_{LG}}\left(\Psi\partial\bar{\Phi} + \frac{1}{a}\partial\Psi\right) - e^{R+\bar{R}-iH_L-iH_{LG}}G^+_{LG}, \\
J &= -\partial(R+\bar{R}) + i\partial H_L + i\partial H_{LG}, \\
J^{++} &= e^{-R-\bar{R}+iH_L+iH_{LG}}, \\
J^{--} &= e^{R+\bar{R}-iH_L-iH_{LG}},
\end{aligned}
\tag{49}
$$

where $J_L = i\partial H_L$ and $J_{LG} = i\partial H_{LG}$. New $N=2$ superconformal generators (43), (45) and (46) are consistent with this $N=4$ topological structure. The new model allows the space-time superconformal symmetry in the physical spectrum. Based on these currents, we may study the BRST cohomology since it is not clear that the new BRST current reproduces the consistent physical spectrum with that in [2]. In the following paper[15], we will investigate the BRST cohomology and physical spectrum in detail.

In this paper, we have constructed the space-time $N=2$ superconformal algebra in the superstrings compactified on singular Calabi-Yau fourfolds. We find that the original BRST operator (7) does not (anti-)commute with the superconformal generators. The space-time supersymmetry of the original model [2], therefore, does not extend to the superconformal. We construct a new topological $N=4$ superconformal algebra including the new BRST operator which commute with the space-time superconformal generators. These generators nontrivialy mix the two-dimensional and Liouville parts as expected.

It is an interesting problem to generalize the present approach to the case of 4 or 6 dimensional hybrid superstrings on a singular Calabi-Yau three or two-fold, respectively. We would be able to study higher dimensional superconformal field theory in terms of hybrid superstring formalism. It is also interesting to apply the present approach to the case of the superstrings with RR backgrounds [8] and the theory compactified on G_2 and $Spin(7)$ holonomy manifolds[16]. These subjects will be discussed elsewhere.

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