Calculation of Four Point Correlation Function of Logarithmic Conformal Field Theory Using AdS/CFT Correspondence

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Abstract

We use the correspondence between scalar field theory on AdS and induced conformal field theory on its boundary to calculate correlation functions of logarithmic conformal field theory in arbitrary dimensions.Our calculations utilize the newly proposed method of nilpotent weights.We derive expressions for the four point function assuming a generic interaction term.

keywords:conformal field theory,AdS/CFT correspondence

1 Introduction

The relationship between field theories in (d+1)-dimensional anti-desitter space and d-dimensional conformal field theories was first suggested by Maldacena and since then much work has been done on various aspects of this correspondence[2,3]. This conjecture can be stated as follows,consider the action $S[\Phi]$ defined on AdS_{d+1} and let Φ_b be the value of Φ on the boundary:

$$\Phi|_{\partial AdS} = \Phi_b \tag{1}$$

So the partition function of the AdS theory subjected to this constraint is:

$$Z_{AdS}[\Phi_b] = \int_{\Phi_b} D\Phi \exp(-S[\Phi]) \tag{2}$$

where the path integral is over configurations fulfilling (1).

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The correspondence states that the partition function of AdS theory is the generating functional of the boundary conformal field theory.

$$Z_{AdS}[\Phi_b] = \langle \exp(\int_{\partial AdS} dx \hat{O} \Phi_b) \rangle \tag{3}$$

The function Φ_b is considered as a current which couples to the scalar conformal operator $\hat{O}$ via a coupling $\int_{\partial AdS} dx \hat{O} \Phi_b$. This is an elegant and useful result, since it gives a practical way for calculation of correlation functions of conformal field theory . However since 2 and 3-point function are fixed (up to a constant) by conformal invariance , one is specially interested in $n > 3$. The conformal correlators have been studied for various cases, e.g.interacting massive scalar field theory [4] or interacting scalar spinor field theory [5].It is also interesting to find actions on AdS which induce logarithmic conformal field theory (LCFT) on its boundary. Correlation functions in a logarithmic conformal field theory exhibit logarithmic behavior and first were noted by Gurarie[6]. Logarithmic operators appear when two (or more) operators are degenerate and have the same dimension ,hence the Hamiltonian becomes non-diagonizable.In the simplest case one has a pair $\hat{A}$ and $\hat{B}$ transforming as:

$$\hat{A}(\lambda z) = \lambda^{-\Delta} \hat{A}(z) \tag{4}$$

$$\hat{B}(\lambda z) = \lambda^{-\Delta} [\hat{B}(z) - \hat{A}(z) \ln \lambda] \tag{5}$$

The bulk action which give rise to logarithmic operators on the boundary were first described in[7,8]. A new method for investigating LCFT is via nilpotent variables which was introduced in [9] and then was modified in [10,11]. In reference [11] a superfield was defined using a grassmanian variable η and different components of a logarithmic pair and fermionic fields.

$$\hat{O}(\vec{x}, \eta) = \hat{A}(\vec{x}) + \hat{\bar{\zeta}}(\vec{x})\eta + \bar{\eta}\hat{\zeta}(\vec{x}) + \bar{\eta}\eta B\hat{(}\vec{x}) \tag{6}$$

where $\hat{\zeta}(\vec{x})$and$\hat{\bar{\zeta}}(\vec{x})$ are fermionic fields with the same conformal dimension as $\hat{A}(\vec{x})$ and $\bar{\eta}\eta$ acts as the nilpotent variable . Now it is easy to see that $\hat{O}(\vec{x}, \eta)$ has the following transformation law:

$$\hat{O}(\lambda \vec{x}, \eta) = \lambda^{-(\Delta + \bar{\eta}\eta)} \hat{O}(\vec{x}, \eta) \tag{7}$$

If $\hat{O}(\vec{x}, \eta)$ were the logarithmic operator on the boundary of AdS the corresponding field $\Phi(x, \eta)$ on AdS can be extended as:

$$\Phi(x, \eta) = C(x) + \bar{\eta}\alpha(x) + \bar{\alpha}(x)\eta + \bar{\eta}\eta D(x) \tag{8}$$

where x is (d+1)-dimensional coordinate with $x^0, x^1, ..., x^d$ components and $x = (\vec{x}, x^d)$ and $x^d = 0$ corresponds to the boundary. In reference [12] the following action was introduced for free scalar massive superfield with BRST symmetry.

$$S^f = -1/2 \int d^{d+1}x \sqrt{|g|} \int d\bar{\eta} d\eta \nabla \Phi(x, \eta).\nabla \Phi(x, -\eta) + m^2(\eta)\Phi(x, \eta)\Phi(x, -\eta) \tag{9}$$

where $m^2(\eta) = m_1^2 + \bar{\eta}\eta m_2^2$ and m_1^2,m_2^2 are defined to be $\Delta(\Delta - d)$ and $(2\Delta - d)$ respectively and where g is the determinant of the metric on AdS.

Expanding the integrand of (8) in powers of η and $\bar{\eta}$,integrating over η and $\bar{\eta}$ and writing it in terms of four components one finds:

$$S^f = -\frac{1}{2}\int d^{d+1}x\sqrt{|g|}[2\nabla C(x).\nabla D(x) + 2m_1^2 C(x)D(x) + m_2^2 C^2(x) + 2\nabla\bar{\alpha}(x).\nabla\alpha(x) + 2m_1^2\bar{\alpha}(x)\alpha(x)] \qquad (10)$$

We see that the bosonic part of this action is the same as the one proposed in [7].Also note that in the free theory, as we expect,the bosonic and fermionic part are decoupled. In the language of superfield the AdS/CFT correspondence becomes

$$\langle \exp\int d\bar{\eta}d\eta \int_{\partial AdS} d^d\vec{x}\hat{O}(\vec{x},\eta)\Phi_b(\vec{x},\eta)\rangle = \exp(-S_{cl}[\Phi_b(\vec{x},\eta)]) \qquad (11)$$

where $\Phi_b(\vec{x},\eta)$ is the value of $\Phi(x,\eta)$ on the boundary. In reference [12] the two-point correlation functions were calculated using equation (11),now our aim is to calculate n-point functions with this method.However the method used to calculate two-point functions will not give non-trivial 4-point functions unless interactions are also added.

2 Interactions and n-point functions

In order to find n-point functions[4,5,7,8,13] ($n \geq 3$)one should consider interaction terms in $\Phi(x,\eta)$ in addition to the free theory. Furthermore we wish the action to have BRST symmetry so that the correlation functions remain invariant under BRST transformations.In the language of superfield the infinitesimal BRST transformation is of the form:

$$\delta\Phi(x,\eta) = (\bar{\epsilon}\eta + \bar{\eta}\epsilon)\Phi(x,\eta) \qquad (12)$$

where $\bar{\epsilon}$ and ϵ are infinitesimal anticommuting parameters. If we consider the interaction term of degree n in $\Phi(x,\eta)$the general form is:

$$S^I = \lambda\int d^{d+1}x\sqrt{|g|}\int d\bar{\eta}d\eta \qquad (13)$$

$$V = \Phi(x,\eta_1)...\Phi(x,\eta_i)...\Phi(x,\eta_n) \qquad (14)$$

with $\eta_i = n_i\eta$ and $n_i \in Z$.

Then change of V under BRST transformation is given by:

$$\delta V = \Phi(x,\eta_1)...\Phi(x,\eta_i)...\Phi(x,\eta_n)[\bar{\epsilon}\sum_{i=1}^{n}\eta_i + \sum_{i=1}^{n}\bar{\eta}_i\epsilon] \qquad (15)$$

BRST invariance leads to:

$$\sum_{i=1}^{n} n_i = 0 \qquad (16)$$

which clearly does not have a unique solution. For n even the most symmetric choice is:

$$V = \Phi^{\frac{n}{2}}(x,\eta)\Phi^{\frac{n}{2}}(x,-\eta) \tag{17}$$

But for n odd we must choose an antisymmetric division such as:

$$V = \Phi^{n-1}(x,\eta)\Phi(x,-(n-1)\eta) \tag{18}$$

which is true for any n regardless of being odd or even.

Let us first consider interaction terms of type (17) for n even.

$$S^I = \int d^{d+1}x\sqrt{|g|}\int d\bar{\eta}d\eta\frac{\lambda_n(\eta)}{n!}\Phi^{\frac{n}{2}}(x,\eta)\Phi^{\frac{n}{2}}(x,-\eta) \tag{19}$$

$$\lambda_n(\eta) = \lambda_n + \lambda'_n\bar{\eta}\eta \tag{20}$$

To write S^I in terms of its four components,we expand (19) in powers of η's,then integrate it over them and we find:

$$\int d^{d+1}x\sqrt{|g|}[\frac{C^{n-1}(x)}{n!}(\lambda'_n C(x)+n\lambda_n D(x))\frac{\lambda_n}{(n-1)!}C^{n-2}(x)\bar{\alpha}(x)\alpha(x)] \tag{21}$$

It is observed that the pure bosonic part of $S^I$is the same as the on proposed in [7]and the fermionic part despite the free action is coupled with bosonic field $C(x)$,this means that the exact solution of equation of motion for this action and the one in [7] will be different,but we will show that in tree level to first order in λ_n the correlation functions are the same. Now the total action is:

$$S = S^f + S^I \tag{22}$$

The equation of motion for the the field $\Phi(x,\eta)$ is:

$$(\nabla^2 - m^2)\Phi(x,\eta) + \frac{\lambda_n(\eta)}{(n-1)!}\Phi^{\frac{n}{2}}(x,\eta)\Phi^{\frac{n}{2}-1}(x,-\eta) = 0 \tag{23}$$

Writing the equation of motion (23) in terms of the four components we have:

$$\nabla^2 C(x) - (m_1)^2 C(x) + \frac{\lambda_n}{(n-1)!}C^{n-1}(x) = 0 \tag{24}$$

$$\nabla^2\alpha(x) - (m_1)^2\alpha(x) + \frac{\lambda_n}{(n-1)!}C^{n-2}(x)\alpha(x) = 0 \tag{25}$$

$$\nabla^2\bar{\alpha}(x) - (m_1)^2\bar{\alpha}(x) + \frac{\lambda_n}{(n-1)!}C^{n-2}(x)\bar{\alpha}(x) = 0 \tag{26}$$

$$\nabla^2 D(x) - (m_1)^2 D(x) - (m_2)^2 C(x) + \frac{C^{n-2}(x)}{(n-1)!}((n-1)\lambda_n D(x) + \lambda'_n C(x)) + \frac{\lambda_n}{(n-2)!}C^{n-3}(x)\bar{\alpha}(x)\alpha(x) = 0 \tag{27}$$

We see that as pointed out earlier the equation of motion for D(x) is different with the corresponding one in [7],also as we expected the fermionic fields to behave like ordinary field with dimension Δ interacting with C(x). The Dirichlet Green function for this system satisfies the equation:

$$(\nabla^2 - m^2)G(x, y, \eta) = \delta(x - y) \tag{28}$$

together with the boundary condition:

$$G(x, y, \eta)|_{x\epsilon\partial AdS} = 0 \tag{29}$$

The classical field $\Phi(x, \eta)$ satisfying equation of motion (24)with Dirichlet boundary condition on ∂AdS satisfies the integral equation:

$$\begin{aligned}\Phi(x, \eta) = \int_{\partial AdS} d^d y \sqrt{|h|} n^\mu \frac{\partial}{\partial y^\mu} G(x, y, \eta)\Phi_b(y, \eta) - \\ \int_{AdS} d^{d+1}y\sqrt{|g|}G(x, y, \eta))\frac{\lambda_n(\eta)}{(n-1)!}\Phi^{\frac{n}{2}}(x, \eta)\Phi^{\frac{n}{2}-1}(x, -\eta) = 0\end{aligned} \tag{30}$$

where h is the determinant of the induced metric on ∂AdS and n^μ the unit vector normal to ∂AdS and pointing outwards. We shall denote the surface term in (25)by $\Phi^0(x, \eta)$ and the remainder by $\Phi^1(x, \eta)$. Then substituting the classical solution (28)into (23) integrating by part and using the properties of Green function we obtain to tree level:

$$S_{cl} = -\frac{1}{2}\int d^d\vec{x}\sqrt{|h|}\int d\bar{\eta}d\eta n^\mu \Phi^0(x, \eta)\partial_\mu\Phi^0(x, \eta) + \int d^{d+1}x\sqrt{|g|}\int d\eta d\bar{\eta}\frac{\lambda_n(\eta)}{n!}(\Phi^0(x, \eta))^n \tag{31}$$

The Green function for this problem is calculated in reference [12] substituting it for $\Phi^0(x, \eta)$ one obtains:

$$\Phi^0(x, \eta) = a(\eta)\int d^d\vec{y}(\frac{x^d}{(x^d)^2 + |\vec{x} - \vec{y}|^2})^{\Delta+\bar{\eta}\eta}\Phi_b(\vec{y}, \eta) \tag{32}$$

with

$$a(\eta) = \frac{\Gamma(\Delta + \bar(\eta)\eta)}{2\pi^{d/2}\Gamma(\alpha + 1)} = a + \bar{\eta}\eta a' \tag{33}$$

Using the solution (32),the classical action to first order in λ_n becomes:

$$\begin{aligned}S^I_{cl} = \int d^{d+1}x \int d^d\vec{y}_1...d^d\vec{y}_n \int d\bar{\eta}d\eta a^n(\eta) \times \frac{(x^d)^{-(d+1)+n(\Delta+\bar{\eta}\eta)}}{\prod_{i=1}^n[(x^d)^2 + |\vec{x} - \vec{y}_i|^2]^{\Delta+\bar{\eta}\eta}} \times \\ \Phi_b(\vec{y}_1, \eta)...\Phi_b(\vec{y}_{n/2}, \eta)\Phi_b(\vec{y}_{n/2+1}, -\eta)...\Phi_b(\vec{y}_n, -\eta)\end{aligned} \tag{34}$$

After expanding in powers of η and $\bar{\eta}$ and integrating over η's , one obtains for the classical solution:

$$S^I_{cl} = \frac{a^n}{n!}\int d^{d+1}x\int d^d\vec{y}_1...d^d\vec{y}_n[\lambda'_n\Psi_1 + \lambda_n(\Psi_2 + n\frac{a'}{a} + \ln\frac{(x^d)^n}{\prod_{i=1}^n[(x^d)^2 + |\vec{x} - \vec{y}_i|^2]})] \tag{35}$$

with

$$J_n(\vec{y_1}....\vec{y_n},x) = \frac{(x^d)^{-(d+1)+n\Delta}}{\prod_{i=1}^n [(x^d)^2 + |\vec{x}-\vec{y_i}|^2]^\Delta} \tag{36}$$

and

$$\Phi_b(\vec{y}_1,\eta)...\Phi_b(\vec{y}_{n/2},\eta)\Phi_b(\vec{y}_{n/2+1},-\eta)...\Phi_b(\vec{y}_n,\eta) = \Psi_1 + \bar{\eta}\Psi + \bar{\Psi}\eta + \bar{\eta}\eta\Psi \tag{37}$$

Now we can derive the correlation of operator fields on boundary by using equation (11). Expanding both sides of equation (11) in powers of $\Phi_b(\vec{x},\eta)$ and integrating over η's the n-point functions of different components of $\hat{O}(\vec{x},\eta)$ can be found.So the connected part of the tree level n-point function to order λ_n for components of $\hat{O}(\vec{x},\eta)$ are:

$$\langle\hat{B}(y_1....\hat{B}(y_n)\rangle_{conn} = a^n \int d^{d+1}x J_n(\vec{y_1}\cdots\vec{y_n},x)\lambda'_n + \lambda_n(n\frac{a'}{a} + \ln\frac{(x^d)^n}{\prod_{i=1}^n[(x^d)^2+|\vec{x}-\vec{y_i}|^2]})] \tag{38}$$

$$\langle\hat{B}(y_1)....\hat{B}(y_{i-1})\hat{A}(y_i)\hat{B}(y_{i+1})....\hat{B}(y_n)\rangle_{conn} = -\lambda_n a^n I_n(\vec{y_1}....\vec{y_n}) \tag{39}$$

$$\langle\hat{B}(y_1)....\hat{B}(y_{i-1})\hat{\bar{\zeta}}(y_i)\hat{B}(y_{i+1})...\hat{B}(y_j)\hat{\zeta}(y_j)\hat{B}(y_{j+1})...\hat{B}(y_n)\rangle_{conn} = \pm\lambda_n a^n I_n(\vec{y_1}....\vec{y_n}) \tag{40}$$

with

$$I_n(\vec{y_1}....\vec{y_n}) = \int d^{d+1}x J_n(\vec{y_1}...\vec{y_n},x) \tag{41}$$

where the minus sign in (35-3) refers to cases $(i<j<n/2),(n/2<i<j)$ and $(i>n/2, j<n/2)$and all the other correlation functions being zero. As we expected logarithmic terms appear in the correlation functions of $\hat{B}$'s with themselves and fermionic fields behave just like ordinary fields of dimension Δ,but their fermionic nature inhibits appearing of more than two fermionic fields in nonzero correlation functions.The integral I_n can be made simpler after integrating over x^d and using Feynman parameterization [4]. The result for $(n=4)$ is:

$$I_4 = \frac{2\Delta - \frac{d}{2}}{\Gamma(2\Delta)} \times \frac{2\pi^{\frac{d}{2}}}{(\eta\zeta\prod_{i<j} y_{ij})^{\frac{2}{3}\Delta}} \int_0^\infty F(\Delta,\Delta,2\Delta;1-\frac{(\eta+\zeta)^2}{(\eta\zeta)^2} - \frac{4}{\eta\zeta}\sinh^2(z))dz \tag{42}$$

with

$$y_{ij} = |\vec{y_i}-\vec{y_j}|\eta = \frac{y_{12}y_{34}}{y_{14}y_{23}}\zeta = \frac{y_{12}y_{34}}{y_{13}y_{24}} \tag{43}$$

Coming back to odd powers of $\Phi(x,\eta)$, choosing it of the form (19) we observe that it is not possible to write a consistent equation of motion,for $\Phi(x,\eta)$ as a whole. However integrating over η's ,one can derive a consistent set of equations for the components. Our method does have the weakness that choosing either of the forms (18) and (19) is arbitrary. Even requiring BRST symmetry does not fix the choice. It is not clear to the authors what extra requirement is necessary to fix this choice. When the power is even, of course an extra symmetry under reflection $\Phi(x,\eta) \to -\Phi(x,\eta)$ exists! Therefore only in the case of even powers, a choice can be fixed.

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