Spontaneous Violation of Lorentz Invariance and Ultra-High Energy Cosmic Rays

J. W. Moffat[1]

Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2J 2W9, Canada

and

Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada

Abstract

We propose that local Lorentz invariance is spontaneously violated at high energies, due to a nonvanishing vacuum expectation value of a vector field ϕ^μ, as a possible explanation of the observation of ultra-high energy cosmic rays with an energy above the GZK cutoff. Certain consequences of spontaneous breaking of Lorentz invariance in cosmology are discussed.

1 Introduction

The possibility that cosmic ray protons may interact with the photons of the cosmic microwave background (CMB) with an energy larger than the Greisen-Zatsepin-Kuzmin (GZK) cutoff, 5×10^{10} GeV [1], such that the center-of-mass energy can be larger than the threshold for pion photoproduction, has been the topic of discussion recently [2, 3, 4, 5, 6, 7]. Experiments have detected cosmic rays with significantly larger energies [8, 9], although arguments have recently been made that the GZK cutoff may already have been discovered [11]. It has been proposed that the dispersion relation which connects the energy $p^0 = E/c$ and the momentum $\vec{p}$ be modified

$$F(E) \neq c[(\vec{p})^2 + m^2 c^2]^{1/2}. \tag{1}$$

The function f contains a new high-energy scale which could be associated with properties of quantum gravity [2, 6, 12, 13].

The modified dispersion relation (1) is rotationally invariant but breaks Lorentz invariance. Various proposals have been put forward to describe

---

[1]e-mail: john.moffat@utoronto.ca

Lorentz violation. A phenomenological framework for analyzing possible departures from Lorentz invariance was given by Coleman and Glashow [10], who assumed that Lorentz invariance is broken perturbatively in standard model quantum field theory. It was assumed further that the action is rotationally invariant in a preferred frame, which is the rest frame of the cosmic microwave background (CMB). In this preferred frame (1) is valid, while in other frames it is not rotationally invariant. Another possibility is to retain special relativity in a modified form i.e. the dispersion relation is valid in all inertial frames, but the linear Lorentz transformation laws are modified [5, 6]. This proposal is sometimes called "double special relativity", for the symmetry transformations possess two universal constants, the speed of light and a fundamental energy scale which is usually identified with the Planck energy $E_P \sim 10^{19}$ GeV. It has been argued that the latter proposal implies that the conservation of energy-momentum is not conserved at high-energies [7].

An alternative solution to the initial value problems of cosmology, based on a variable speed of light (VSL) [14, 15], postulated that in the very early universe at a time $t \sim t_P \sim 10^{-43}$ sec., where t_P denotes the Planck time, the local Lorentz invariance of the ground state of the universe was spontaneously broken by means of a non-zero vacuum expectation value (vev) of a vector field, $\langle \phi^\mu \rangle_0 \neq 0$. At a temperature $T < T_c$, the local Lorentz symmetry of the vacuum was restored corresponding to a "non-restoration" of the symmetry group $SO(3,1)$. Above T_c the symmetry of the ground state of the universe was broken from $SO(3,1)$ down to $SO(3)$, and the domain formed by the direction of $\langle \phi^\mu \rangle_0$ produced an arrow of time pointing in the direction of increasing entropy and the expansion of the universe [2].

The notion that as the temperature of the universe increases, a larger symmetry group $SO(3,1)$ can spontaneously break to a smaller group $SO(3)$ seems counter-intuitive. Heating a superconductor restores gauge invariance, and heating a ferromagnet restores rotational invariance. Non-restoration would appear to violate the second law of thermodynamics. This, however, is not the case, for certain ferroelectric crystals such as Rochelle or Seignette salt, possess a smaller invariance group above a critical temperature, $T = T_c$, than below it [16] [3]. Explicit models of 4-D field theories have been constructed in which the non-restoration of symmetries occurs at high temperatures [17].

In the following, we shall postulate that local Lorentz invariance of the vacuum state is spontaneously broken, so that $SO(3,1) \rightarrow SO(3)$ at some

---

[2]For alternative VSL models of cosmology, see references [19, 20, 21].

[3]Rochelle salt possesses a lower Curie point at $-18^o C$, below which the Rochelle crystal is orthorhombic and above which it is monoclinic.

energy E_c. The model is based on a Maxwell-Proca action for a vector field ϕ^μ, which avoids instabilities and the existence of negative energy ghost states [18, 19]. We attempt to emulate the successes of the standard model of particle physics [22], in which "soft", spontaneous breaking of the internal symmetries by a Higgs mechanism plays a crucial role. In our scenario, local Lorentz symmetry at low energies is simply an accident of nature, i.e. the ground state happens to be found at low energies in a particular local Lorentz invariant vacuum state, and transitions away from this vacuum state may well happen at high energies [4].

We shall base our formalism on four-dimensional Einstein gravity, anticipating a possible basis for a future quantum gravity theory in which the spontaneous violation of local Lorentz invariance plays a fundamental role [5]. In certain spin-foam and loop models of quantum gravity [6, 28], discrete spacetime structure at the Planck length, $L_P \sim 10^{-33}$ cm, leads to a violation of Lorentz invariance. When a cutoff is introduced into a quantum field theory or in perturbative gravitational calculations of loop graphs, Lorentz invariance is violated. Therefore, it has long been suspected that if a consistent way of breaking Lorentz invariance could be discovered, it would lead to a finite quantum field theory and quantum gravity theory.

Investigations of possible trans-Planckian energy modifications of the calculations of the primordial fluctuation spectrum have used the modified dispersion relation (1), due to the possibility that the initial tiny comoving wave modes born in the quantum ground state of the inflaton field could occur at energies above the Planck energy E_P [29].

The total action is given by

$$S = S_G + S_M + S_\phi, \tag{2}$$

where

$$S_G = \frac{1}{\kappa} \int dt d^3x \sqrt{-g}(R + 2\Lambda), \tag{3}$$

---

[4]Gravity theories with a preferred frame have been considered by Gasperini [23] and Jacobson and Mattingly [24]. In these models, the preferred frame was described by a unit timelike vector ("aether") and was a *permanent* feature of spacetime, and not related to a broken and subsequently restored symmetry phase of spacetime. A model of spontaneous breaking of Lorentz invariance inspired by string theory was constructed by Kostelecky and Samuel [25, 26, 27], and used to obtain experimental bounds on possible Lorentz violations in the standard model and in electrodynamics.

[5]In previous work [14, 15], we used a vierbein and spin-gauge connection to describe the spontaneous breaking of local Lorentz invariance. In the following, we shall use the conventional metric $g_{\mu\nu}$ and Christoffel connection $\Gamma^\lambda_{\mu\nu}$ to depict the pseudo-Riemannian geometry.

and $g = \det(g_{\mu\nu})$, $\kappa = 16\pi G/c^4$, Λ is the cosmological constant and S_M is the matter action. Moreover,

$$S_\phi = \int dt d^3x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\nu} g^{\alpha\beta} B_{\mu\alpha} B_{\nu\beta} + V(\phi) \right], \tag{4}$$

where $B_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$. The potential $V(\phi)$ is

$$V(\phi) = \frac{1}{2}\mu^2 \phi^\mu \phi_\mu + W(\phi), \tag{5}$$

where $W(\phi)$ is a function of $\phi^\mu \phi_\mu$. The total action S is diffeomorphism and locally Lorentz invariant.

The energy-momentum tensor for matter is

$$T_{M\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}}, \tag{6}$$

and we have

$$T_{\phi\mu\nu} = B_{\mu\rho} B_\nu^\rho - \frac{1}{4} g_{\mu\nu} g^{\rho\sigma} g^{\lambda\tau} B_{\rho\lambda} B_{\sigma\tau} - 2 \frac{\partial V(\phi)}{\partial g^{\mu\nu}} + g_{\mu\nu} V(\phi). \tag{7}$$

We observe that although the Maxwell-Proca action (4) is not invariant under Abelian $U(1)$ gauge transformations, *the quantized version of the action is fully consistent and free of negative energy instabilities* [18]. A Hamiltonian constraint analysis reveals that the time component ϕ^0 satisfies a constraint equation and does not physically propagate, thereby, avoiding negative energy states. This solves the problem of obtaining a physically consistent model of symmetry breaking for the local Lorentz group $SO(3,1)$, which is a non-compact group [6].

The field equations obtained from the action principle are given by

$$G_{\mu\nu} = \frac{\kappa}{2} T_{\mu\nu} + g_{\mu\nu}\Lambda, \tag{9}$$

$$-\nabla_\nu B^{\mu\nu} + \frac{\partial V(\phi)}{\partial \phi_\mu} = 0. \tag{10}$$

---

[6]For example, if we had chosen the kinetic part of the action for ϕ^μ to be

$$S_{\phi\text{kin}} = \frac{1}{2} \int dt d^3x \sqrt{-g} \partial_\mu \phi^\alpha \partial^\mu \phi_\alpha, \tag{8}$$

then for the compact group $SO(4)$ there will not be any negative energy (ghost) modes, for $SO(4)$ is a compact group, whereas for the noncompact group $SO(3,1)$ negative energy modes will cause the kinetic energy contribution to be unstable and the Hamiltonian will not be bounded from below.

We have $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and

$$T_{\mu\nu} = T_{M\mu\nu} + T_{\phi\mu\nu}. \tag{11}$$

The Bianchi identities $\nabla_\nu G^{\mu\nu} = 0$ lead to the conservation law: $\nabla_\nu T^{\mu\nu} = 0$.

Let us now choose $V(\phi)$ to be of the form of a "Mexican hat" potential

$$V(\phi) = -\frac{1}{2}\mu^2\phi_\mu\phi^\mu + \frac{1}{4}\lambda(\phi_\mu\phi^\mu)^2, \tag{12}$$

where the coupling constant $\lambda > 0$, $\mu^2 > 0$, and the potential is bounded from below. If V has a minimum at

$$\phi_\mu \equiv v_\mu = \langle\phi_\mu\rangle_0, \tag{13}$$

then the spontaneously broken solution is given by

$$\phi^\mu\phi_\mu \equiv v^2 = \mu^2/\lambda, \tag{14}$$

where $v^2 = (v_\mu)^2$. This manifold of points of the minima of $V(\phi)$ is invariant under local Lorentz transformations

$$\phi'^\mu = \Lambda^\mu_\nu\phi^\nu. \tag{15}$$

We can choose the ground state to be described by the timelike vector

$$\phi^{(0)}_\mu = \delta_{\mu 0}v = \delta_{\mu 0}(\mu^2/\lambda)^{1/2}. \tag{16}$$

The homogeneous Lorentz group $SO(3,1)$ is broken down to the spatial rotation group $SO(3)$. The three rotation generators J_i ($i = 1, 2, 3$) leave the vacuum invariant, $J_i v_i = 0$, while the three Lorentz-boost generators K_i break the vacuum symmetry, $K_i v_i \neq 0$.

We shall consider the perturbations about the stable vacuum state, $\chi(x), \theta^i(x)$ (i=1,2,3):

$$\phi^0 = v + \chi, \quad \phi^i = \theta^i. \tag{17}$$

The potential term in the action for the perturbations in a local, flat patch of spacetime, for which $g_{\mu\nu} \approx \eta_{\mu\nu}$ (where $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is the Minkowski spacetime metric) has the form

$$V(\phi) = -\frac{\mu^2}{2}[(v + \chi)^2 - (\theta^i)^2] + \lambda[(v + \chi)^2 - (\theta^i)^2]^2, \tag{18}$$

and the action becomes

$$S_\phi = \int dt d^3x \Big\{\frac{1}{4}[-(\partial_i\theta_j - \partial_j\theta_i)^2 + 2(\partial_0\theta_i - \partial_i\chi)^2] + \mu^2\chi^2$$

$$+\text{cubic and quartic terms in } \chi, \theta + \text{const.} \Bigg\}. \tag{19}$$

We see that the action of the perturbations is invariant under $SO(3)$ rotations of the fields θ^i, since it only contains the combination $(\theta^i)^2$, but it does not possess the full Lorentz $SO(3,1)$ symmetry invariance. It follows that θ^i are three massless Nambu-Goldstone fields.

The vector field ϕ^μ perturbations have three massless Nambu-Goldstone modes that could produce long-range fifth force effects, when ϕ^μ interacts with matter. Fifth force experiments have not detected any observable effects, so this will put a bound on the strength of the coupling of ϕ^μ with matter [30].

2 Spontaneously Broken Lorentz Invariance and Cosmology

In the spontaneously broken phase of the evolution of the universe, the space-time manifold has been broken down to $R \times SO(3)$. The three-dimensional space with $SO(3)$ symmetry is assumed to be the homogeneous and isotropic FRW solution:

$$d\sigma^2 = R^2(t)\Big[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big], \tag{20}$$

where $k = 0, +1, -1$ corresponding to a flat, closed and open universe, respectively, and t is the external time variable. This describes the space of our ground state in the symmetry broken phase and it has the correct subspace structure for our FRW universe with the metric

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = dt^2 c^2 - R^2(t)\Big[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big]. \tag{21}$$

The Newtonian "time" t is the *cosmological time* measured by standard clocks.

The Friedmann equations are

$$H^2 - \frac{kc^2}{R^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3}, \tag{22}$$

$$\ddot{R} = \frac{4\pi G}{3}\Big(\rho + 3\frac{p}{c^2}\Big)R + \frac{\Lambda c^2}{3}R, \tag{23}$$

where $H = \dot{R}/R$ and $\rho = \rho_M + \rho_\phi$ and $p = p_M + p_\phi$.

In our homogeneous and isotropic cosmology, we have $B_{\mu\nu} = 0$ and $\phi_i = 0$ (i=1,2,3). We have from Eq.(10) that $dV(\phi)/d\phi^0 = 0$ and from (12), we find that at the minimum of the potential: $\phi_0 \equiv \langle\phi\rangle_0 = (\mu^2/\lambda)^{1/2} = v$, which is the result (16) obtained previously for our choice of gauge and spontaneous symmetry breaking. The energy momentum tensor for the ϕ_μ field is now given by

$$T_\mu^\nu = \delta_\mu^\nu V(\phi), \qquad (24)$$

where

$$V(\phi) = \mu^4/2\lambda. \qquad (25)$$

For the fluid in comoving coordinates, we have

$$T_\mu^\nu = \mathrm{diag}(\rho c^2, -p, -p, -p). \qquad (26)$$

We obtain from (24):

$$T_{\phi 0}^0 \equiv \rho_\phi c^2 = V(\phi), \quad T_{\phi i}^j \equiv -p_\phi \delta_i^j = \delta_i^j V(\phi), \qquad (27)$$

which gives the equation of state, $p_\phi = -\rho_\phi c^2$, associated with the effective cosmological constant

$$\Lambda_{\mathrm{eff}} = \Lambda + \Lambda_\phi. \qquad (28)$$

If the vacuum density ρ_ϕ dominates over the matter density ρ_M, then in the spontaneously broken phase the universe will be inflationary with $\ddot{R} > 0$ and be dominated by a de Sitter solution [31]. If the period of spontaneous symmetry breaking lasts long enough to generate $\sim$ 60-efolds of inflation, then it solves the horizon and flatness problems and can generate a scale invariant fluctuation spectrum. The inflationary period ends when the local $SO(3,1)$ symmetry is restored.

3   The GZK Cutoff and Conclusions

We have shown that by beginning with a diffeomorphism invariant action that includes the Einstein-Hilbert action, due to a nonvanishing vev $\langle\phi^\mu\rangle_0 \neq 0$, the local Lorentz invariance of the vacuum state is spontaneously broken at high energies. In a small patch of flat spacetime the homogeneous Lorentz group $SO(3,1)$ is broken down to the homogeneous rotation group $SO(3)$. In the spontaneously broken phase at high energies, the modified dispersion relation:

$$E^2 - c^2(\vec{p})^2 - m^2 c^4 = f(E^2), \qquad (29)$$

is invariant under the subgroup of rotations $SO(3)$, but is not locally Lorentz invariant. In this scenario, a possible modified dispersion relation would correspond to choosing

$$f(E^2) = \eta E^2 \left(\frac{E}{E_P}\right)^\alpha \sim \eta c^2 (\vec{p})^2 \left(\frac{E}{E_P}\right)^\alpha, \tag{30}$$

where η is a dimensionless parameter of order 1. This modified dispersion relation [2, 3] has been related to theories of quantum gravity [2, 6, 12]. For $\alpha \sim 1 - 2$, this deformed dispersion relation will show a significant upward shift above the GZK cutoff determined by

$$E\epsilon = \frac{c^4[(m_{\text{prot}} + m_\pi)^2 - m_{\text{prot}}^2]}{4}, \tag{31}$$

where E and ϵ denote the energy of the proton and the photon, respectively, in the photoproduction process $p + \gamma \to p + \pi$.

In the Lorentz invariant deformed models of dispersion relations, or the "doubly special relativity" models [5], it is not clear how one incorporates gravity theory, and, in particular, Einstein's theory of gravity. In contrast, in our model the initial action contains the Einstein-Hilbert gravity theory. An important feature of the Maxwell-Proca action S_ϕ is that the vector field ϕ^μ can be consistently quantized, even though the action is not gauge invariant. It is hoped that future experiments will confirm whether there is an ultra-high energy cosmic ray "threshold anomaly" associated with the GZK cutoff.

The spontaneously broken symmetry model described here could be important for early universe inflationary and VSL cosmology [31, 14, 15].

Acknowledgments

References

[1] G. T. Zatsepin and V. A. Kuzmin, JETP Lett. **41**, 78 (1966); K. Greisen, Phys. Rev. Lett. **16**, 748 (1966).

[2] G. Amelino-Camelia, J. Ellis, N. E. Mavromatos, D. V. Nanoloulous and S. Sarkar, Nature **393**, 763 (1998), astro-ph/0712103.

[3] G. Amelino-Camelia and T. Piran, Phys. Rev. **D64**, 036005 (2001), astro-ph/0008107.

[4] F. W. Stecker and S. L. Glashow, Astropart. Phys. **16**, 97 (2001), astro-ph/0102226.

[5] G. Amelino-Camelia, Nature **418**, 34 (2002), gr-qc/0207049; J. Kowaleski-Glikman and S. Nowak, Phys. Lett. **B539**, 126 (2002), hep-th0203040.; J. Lukierski and A. Nowicki, hep-th/0203065; S. Judes and M. Visser, gr-qc/0205067.J. Rembielinski and K. A. Smolinski, hep-th/0207031; A. Granik, hep-th/0207113.

[6] J. Magueijo and L. Smolin, Phys. Rev. Lett. **88**, 190403 (2002), 0112090; gr-qc/0207085.

[7] M. Toller, hep-ph/0211094.

[8] M. Takeda et al., Phys. Rev. Lett. **81**, 1163 (1998); astro-ph/0209422.

[9] F. W. Stecker, astro-ph/0208507.

[10] S. Coleman and S. L. Glashow, Phys. Lett. **405**, 249 (1997), hep-ph/9703240; Phys. Rev. **D59**, 116008 (1999), hep-ph/9812418.

[11] J. N. Bahcall and E. Waxman, hep-ph/0206217.

[12] T. Kifune, Astroph. J. **518**, L 21(1999), astro-ph/9904164.

[13] S. Majid and H. Ruegg, Phys. Lett. **B334**, 348 (1994).

[14] J. W. Moffat, Int. J. Mod. Phys. **D2**, 351 (1993), gr-qc/9211020; J. W. Moffat, Found. of Phys. **23**, 411 (1993), gr-qc/9209001; J. W. Moffat, *Fluctuating Paths and Fields*, Festschrift dedicated to Hagen Kleinert, edited by H. Jancke, A. Pelster, H.-J. Schmidt, and M. Bachmann, World Scientific, Singapore, p. 741, 2001, astro-ph/9811390.

[15] J. W. Moffat, hep-th/0208122; astro-ph/0210042.

[16] F. Jona and G. Shirane, *Ferroelectric Crystals*, Pergamon, Oxford, 1962.

[17] S. Weinberg, Phys. Rev. **D19**, 3357 (1974); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **42**, 1651 (1979); Phys. Rev. **D20**, 3390 (1979); P. Langacker and S. Pi, Phys. Rev. Lett. **45**, 1 (1980); P. Salomonson and B. K. Skagerstam, Phys. Lett. **B155**, 100 (1985); S. Dodelson and L. M. Widrow, Phys. Rev. **D42**, 326 (1990).

[18] J. A. Isenberg and J. M. Nester, Ann. Phys. **107**, 56 (1977).

[19] M. A. Clayton and J. W. Moffat, Phys. Lett. **B460**, 263 (1999), arXive:astro-ph/9812481.

[20] M. A. Clayton and J. W. Moffat, Phys. Lett. **B477**, 269 (2000), gr-qc/9910112; M. A. Clayton and J. W. Moffat, Phys. Lett. **B506**, 177 (2001), gr-qc/0101126 v2; M. A. Clayton and J. W. Moffat, Int. J. of Mod. Phys. **D11**, 187 (2002), gr-qc/0003070; J. W. Moffat, to be published in Int. J. Mod. Phys. (2002), gr-qc/0202012; M. A. Clayton and J. W. Moffat, astro-ph/0203164; I. T. Drummond, Phys. Rev. **D63** 043503 (2001), astro-ph/0008234; B. A. Bassett, S. Liberati, C. Molinari-Paris, and M. Visser, Phys. Rev. **D62** 103518 (2000), astro-ph/0001441 v2.

[21] A. Albrecht and J. Magueijo, Phys. Rev. **D59**, 043516, (1999), astro-ph/9811018; J. D. Barrow, Phys. Rev. **D59**, 043515 (1999); J. D. Barrow and J. Magueijo, Phys. Lett. **B477**, 246 (1999), astro-ph/9811073 v2; J. Magueijo, Phys. Rev. **D62**, 103521 (2000), gr-qc/0007036.

[22] See e.g.: Ta-Pei Cheng and Ling-Fong Li, *Gauge Theory of Elementary Particle Physics*, Clarendon Press, Oxford, 1984.

[23] M. Gasperini, Class. Quantum Grav. **4**, 485 (1987); Gen. Rel. Grav. **30**, 1703 (1998).

[24] T. Jacobson and D. Mattingly, gr-qc/0007031; Phys. Rev. **D63**, 041502 (2001).

[25] V. A. Kostelecky and S. Samuel, Phys. Rev. **D40**, 1886 (1989).

[26] D. Colladay and V. A. Kostelecky, Phys. Rev. **D58**, 116002 (1998), hep-th/9809521.

[27] V. A. Kostelecky and R. Lehnert, Phys. Rev. **D63**, 065008 (2001), hep-th/0012060.

[28] For a recent review of spin-foam models, see: D. Oriti, Rept. Prog. Phys. **64**, 1489 (2001), gr-qc/0106091.

[29] See, for example: J. Martin and R. H. Brandenberger, Phys. Rev. **D63**, 123501 (2001), hep-th/0005209; J. C. Niemeyer and R. Parentani, Phys. Rev. **D64**, 101301, astro-ph/0101451; A. Kempf and J. C. Niemeyer, Phys. Rev. **D64**, 103501 (2001), astro-ph/0103225; R. Easther, B. R. Greene, W. H. Kinney and G. Shiu, Phys. Rev. **D64**, 103502 (2001), hep-th/01044102.

[30] E. Fischbach, G. T. Gillies, D. E. Krause, J. G. Schwan, and C. Talmadge, Metrologia **29**, 213 (1992).

[31] A. H. Guth, Phys. Rev. **D23**, 347 (1981); A. Albrecht and P. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982); A. D. Linde, Phys. Lett. **B129**, 177 (1983); A. D. Linde, *Particle Physics and Inflationary Cosmology*, (Harwood, Chur, Switzerland, 1990); A. Albrecht, astro-ph/0210527.