

hep-th/0011057
HU-EP-00/51

T-duality and Actions for Non-Commutative D-Branes

Radu Tatar¹

*Humboldt-Universität zu Berlin, Institut für Physik,
Invalidenstrasse 110, 10115 Berlin, Germany*

Abstract

We show how the T-duality is realized for D-branes with noncommutative world-volume coordinates. We discuss D-branes wrapped on tori and the result is that the recently found noncommutative actions form a consistent collection due to the T-duality mapping between noncommutative D-branes and rotated commutative D-branes on deformed tori.

¹tatar@qft.physik.hu-berlin.de

1 Introduction

Recently, the actions for the BPS and non-BPS D-branes have been discussed extensively in order to study their dynamics. By turning on constant NS-NS 2-form B -field along the world-volume of either type of D-branes, the world-volume action becomes a noncommutative field theory [1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12]. The product of the fields of the non-commutative field theories is a \star product which involves a θ parameter related to the value of the B field. Besides the B field, we can turn on different magnetic fluxes F on the D-branes and then the non-commutative field theories on the D-branes have multiple θ parameters related to the different values of $F+B$ [13, 14, 15].

In [16], the Chern-Simons couplings for D-branes were generalized to the case of branes with noncommutative world-volume coordinate in a manifestly background independent way. For D-branes with B and F fields, this includes a factor of $\frac{B}{F+B}$ in front of the usual coupling between the B and F fields and the RR-forms in type II theories.

One important check of the results of [16] is to study the transformation of the noncommutative Chern-Simons under T-duality. This is the subject of the present paper. We consider non-commutative D-branes wrapped on tori with different dimensions and we check the transformation of their actions under T-dualities with respect to different directions on the tori. We use the results of [17, 18, 19, 20, 21, 22] concerning the effect of T-duality on the noncommutative D-branes. The results is that, after T-duality, a noncommutative D-brane becomes a commutative D-brane rotated with respect to the original coordinates of the torus and the torus itself becomes deformed. The rotation of the brane is related to the value of $B+F$ and the deformation of the torus is related to the value of B .

By starting with a non-commutative D9-brane, we map the Chern-Simons terms obtained via double and direct dimensional reduction as done previously for commutative BPS and non-BPS D-branes in [23, 24, 25, 26, 27].

2 T-duality and Chern-Simons terms For Non-commutative D-branes

We check the consistency of the Chern-Simons terms for non-commutative D-branes discussed on [16]. We start with a D9 brane with two compact directions on a T^2 torus, with NS 2-form B and magnetic flux F turned on the directions of the torus. Therefore the field theory on the brane becomes non-commutative.

As described in [16], one possible choice for the values of the fields is to

take $F + B = Q^{-1}$ constant and to allow a variable value for $B = \theta^{-1}$. In this case the Born-Infeld and Chern-Simons actions could be written as:

$$S_{DBI} = T_9 \int d^{p+1}x \frac{\text{Pf } Q}{\text{Pf } \theta} \sqrt{\det(g_{ij} + (F + B)_{ij})} \quad (1)$$

and

$$S_{CS} = \mu_9 \int_x \frac{\text{Pf } Q}{\text{Pf } \theta} \sum_n C^{(n)} e^{F+B} \quad (2)$$

where T_9 and μ_9 are the tension and charge of the D9 brane. In the case of only 2 directions of B and F fields, we can write $\text{Pf } Q = Q$ and $\text{Pf } \theta = \theta$ so the above formulas become:

$$S_{DBI} = T_9 \int d^2x \frac{Q}{\theta} \sqrt{\det(g_{ij} + (F + B)_{ij})} \quad (3)$$

and

$$S_{CS} = \mu_9 \int_x \frac{Q}{\theta} \sum_n C^{(n)} e^{F+B} \quad (4)$$

Let us consider that the directions of the two-torus T^2 and of the B and F fields are on the (x^1, x^2) plane. Therefore, if we want to discuss the action of T-duality, we need to distinguish between T-duality on the compact (x^1, x^2) directions and T-dualities with respect to any other possible compact direction.

If we consider x^3 as a compact direction and take a T-duality with respect to it, then the D9 brane will become a D8 brane wrapped on (x^1, x^2, x^3) , and the action will be:

$$S_{DBI} = T_8 \int d^2x \frac{Q}{\theta} \sqrt{\det(\hat{g}_{ij} + (\hat{F} + \hat{B})_{ij})} \quad (5)$$

and

$$S_{CS} = \mu_8 \int_x \frac{Q}{\theta} \sum_n \hat{C}^{(n)} e^{\hat{F} + \hat{B}} \quad (6)$$

where T_8 and μ_8 are the tension and charge of the D8 brane. In this case, the 9-th component of the BI vector A_3 gets mapped into the transverse dimension ϕ_3 to the D8 brane in the usual sense.

What happens now if we consider a T-duality with respect to x^1 or x^2 directions? As discussed in [18, 19, 20, 21, 22], there are two things which happen: the T^2 torus gets deformed and the D8 branes is rotated in the (x^1, x^2) plane. If we consider a T-duality along the x^2 direction, we obtain a torus with a rotation of the x^2 axis into an x'^2 axis by an angle $\pi/2 - \alpha$ given by:

$$\cot \alpha = B = \theta^{-1} \quad (7)$$

(so it makes an angle α with the x^1 axis) and the D8 brane has a direction on the deformed torus (x^1, x'^2) at an angle β with respect to the direction x^1 where β is given by:

$$\cot \beta = B + F = Q^{-1} \quad (8)$$

We want to see the consistency of T-duality at the level of Chern-Simons action. From the formula (6), we see that for the D9 brane we have the couplings $C^{(10)} + C^{(8)} \wedge (F + B)$ when the F, B are in the (x^1, x^2) directions and $C^{(8)}$ is in the (x^0, x^3, \dots, x^9) directions. Together with the factor $\frac{Q}{\theta}$ this will give

$$\int \frac{Q}{\theta} (C^{(10)} + C^{(8)} \wedge (F + B)) \quad (9)$$

A double dimensional reduction of the D9 brane on the x^2 direction gives a commutative D8 brane in the x^1, \dots, x^{10} directions with Chern-Simons terms containing the term

$$\int \frac{\tan \alpha}{\tan \beta} (C^9 + C^8 d\chi) \quad (10)$$

where the nine-dimensional scalar field comes from the component of the BI vector in the direction x^2 over which we reduce.

We then consider a direct dimensional reduction of a commutative D8 brane on the (x'^2, \dots, x^{10}) directions whose Chern-Simons term in 10 dimensions does not contain the $\frac{Q}{\theta}$ term in the action. After reduction, it has a Chern-Simons terms containing the sum

$$\int (C^9 + C^8 d\chi') \quad (11)$$

where $\chi' = \Phi^2$ comes from the reduction on the transverse direction.

The integral in (10) is taken over dx^1 and in (11) is taken over dx'^2 . But $\frac{dx'^2}{dx^1}$ is just $\tan \alpha$ so we see that we see the appearance of $\tan \alpha$ in (11). Moreover, the directly dimensional reduced D8 brane is at an angle β with respect to x^1 so there is a factor $\cot \beta$ also appearing in the action. Therefore we have a mapping between the 2-nd component of the BI vector and the extra transverse direction.

What happens if we take two T-dualities in the x^1, x^2 directions? The D9 brane becomes a commutative D7 branes. In [17, 20] it has been argued that the coordinates x^1, x^2 of the D7 brane do not commute in this case. By turning on B_{12} field and keeping the other 8 directions non-compact, this induces D7 - branes on the world-volume of the D9-branes, the D7-branes must couple to the RR 10-form potential as discussed by Myers [28] and this induces a term like

$$[\phi^1, \phi^2] \wedge C_{12i_1 \dots i_8} \quad (12)$$

The D7 branes obtained by T-duality were discussed to be generated by applying an asymmetric rotation to an ordinary D7-brane with pure Neumann or Dirichlet boundary conditions [17, 20]. The commutator of the transverse coordinates becomes in the large F, B limit as $[x_1, x_2] = 1/(F + B) = Q$ so if we identify $\phi^1 = x^1, \phi^2 = x^2$, the Myers coupling becomes $Q_{12} C_{12i_1 \dots i_8}$. There is also a term $1/\theta$ coming into the action because of the deformation of the torus. So the Chern-Simons term for the D7 branes will contain a part of the form

$$\int \frac{Q}{\theta} C_{12i_1 \dots i_8}. \quad (13)$$

and terms with derivatives of the transverse directions. If we now consider the term $\frac{Q}{\theta} C^{(10)}$ in the D9 brane action, this will be mapped into the Chern-Simons term for the D7 brane after the two T-dualities.

Let us now discuss the case when we have four compact directions which are seen as $T^2 \times T^2$ on the x^1, x^2 and x^3, x^4 directions respectively. We turn on B and F fields such that $B_{12} = B_1, F_{12} = F_1$ and $B_{34} = B_2, F_{34} = F_2$. Then we define

$$Q_i^{-1} = B_i + F_i, i = 1, 2 \quad (14)$$

and

$$\theta_i^{-1} = B_i, i = 1, 2 \quad (15)$$

and we insert in formula (2) $\text{Pf } Q = Q_1 Q_2$ and $\text{Pf } \theta = \theta_1 \theta_2$.

We then consider two T-dualities in the x_3 and x_4 directions. Under T-duality, the D9 brane goes into a D7 brane rotated in the x^1, x^2 plane at an angle β_1 with respect to the x^1 direction and rotated in the x^3, x^4 plane at an angle β_2 with respect to the x^3 direction. The angles are given by

$$\cot \beta_i = B_i + F_i = Q_i^{-1}, i = 1, 2. \quad (16)$$

In the same time the direction x^2 is rotated by an angle $\pi/2 - \alpha_1$ where $\cot \alpha_1 = Q_1$ and the direction x^4 gets rotated by an angle $\pi/2 - \alpha_2$ where $\cot \alpha_2 = Q_2$.

For the D9 brane we have the coupling

$$\sum_{i=1}^2 C^{(10)} + C^{(8)} \wedge (F_i + B_i) \quad (17)$$

so the Chern-Simons term is:

$$\int \prod_{i=1}^2 \frac{Q_i}{\theta_i} (C^{(10)} + \sum_{i=1}^2 C^{(8)} \wedge (F_i + B_i)) \quad (18)$$

or

$$\int \left(\prod_{i=1}^2 \frac{Q_i}{\theta_i} C^{(10)} + \sum_{i=1}^2 \frac{Q_i}{\theta_1 \theta_2} C^{(8)} \right) \quad (19)$$

which is

$$\int \left(\prod_{i=1}^2 \frac{Q_i}{\theta_i} C^{(10)} + \sum_{i=1}^2 \tan \alpha_1 \tan \alpha_2 Q_i C^{(8)} \right) \quad (20)$$

A double reduction of the D9 brane in the x^2 direction gives a noncommutative D8 brane with a Chern-Simons coupling:

$$\int \left(\prod_{i=1}^2 \frac{Q_i}{\theta_i} (C^{(9)} + C^{(8)} d\chi) + \tan \alpha_1 \tan \alpha_2 Q_1 (C^{(7)} + C^{(6)} d\chi) \right) \quad (21)$$

We consider also a direct dimensional reduction of a noncommutative D8 brane on (x^2, \dots, x^{10}) whose noncommutative Chern-Simons action in 10 dimensions is:

$$\sum_{i=1}^2 \frac{Q_2}{\theta_2} (C^{(9)} + C^{(7)} \wedge (F_i + B_i)) \quad (22)$$

After reduction the Chern-Simons action becomes:

$$\int Q_2 \tan \alpha_2 (C^{(9)} + C^{(8)} d\chi' + (C^{(7)} + C^{(6)} d\chi') \wedge (F_i + B_i)) \quad (23)$$

We now compare equations (21) and (23). We see that the difference is a factor $\frac{\tan \alpha_1}{\tan \beta}$. But this is exactly coming from the deformation of the torus and from the rotation of the D-brane in the (x^1, x^2) plane as explained above. So again there is a mapping between the 2-nd component of the BI vector and the extra transverse direction. The same discussion applies if we consider a T-duality on the x^4 direction.

If we take two T-dualities in the x^2, x^4 directions, we need to compare a twice doubly dimensional reduced noncommutative D9 brane and a twice directly dimensional reduced D7 brane. We can start directly in 9 dimensions with a non-commutative D8 brane and make only one reduction. We can start with both formulas (21) or (23). If we start with (21) and make a double dimensional reduction on the x^4 direction, we obtain a commutative D7 brane with a Chern-Simons action:

$$\int \left(\prod_{i=1}^2 \frac{Q_i}{\theta_i} (C^{(8)} + C^{(7)} d\tau + C^{(7)} d\chi + C^{(6)} d\tau d\chi) \right) \quad (24)$$

where τ is the eight-dimensional scalar field which comes from the component of the nine-dimensional BI vector in the direction x^4 over which we reduce.

We then consider a direct dimensional reduction of a commutative D7 brane on the $(x^2, x^4, \dots, x^{10})$ directions from ten to eight dimensions, this has a Chern-Simons term:

$$\int (C^{(8)} + C^{(7)} d\tau' + C^{(7)} d\chi' + C^{(6)} d\tau' d\chi') \quad (25)$$

where $\tau' = \Phi^4$ comes from the reduction on the transverse direction. The supplementary factor $\prod_{i=1}^2 \frac{Q_i}{\theta_i}$ in (24) comes from the rotation of the two tori T^2 and from the rotation of the directions of the D7 brane in the (x^1, x^2) plane and in the (x^3, x^4) plane, therefore we see that the two components χ, τ of the BI vector are mapped into the extra transverse directions χ', τ' , as required by T duality.

We would like now to discuss the non-abelian case. In [27, 28], the action for the case of multi-branes on top of each other is given and it involves the replacement of partial derivatives for the transverse fields by covariant derivatives. By starting from the D9 branes, after reduction to 9 dimensions, there is no other scalar field except the 10-th component of the gauge field so we still have partial derivatives and the above discussion is the same, the only difference being that the Chern-Simons action (2) now writes:

$$S_{CS} = \mu_9 \int_x \frac{\text{Pf } Q}{\text{Pf } \theta} \text{Tr} \sum_n C^{(n)} e^{F+B} \quad (26)$$

where by Tr we mean the symmetric trace description.

If we consider the case of N D9 branes on T^2 in the (x^1, x^2) directions with magnetic fluxes $F_i, i = 1, \dots, N$, a T-duality in the x^4 direction would give a deformed T^2 torus by an angle $\cot \alpha = B$ and N D8 branes rotated by angles $\cot \beta_i = B + F_i, i = 1, \dots, N$. In this case, we need to consider the spectrum of the open strings between the different D8 branes which could contain tachyons so the system might be unstable. As discussed in [18, 20], D-branes at angles could become unstable and behave as D $-D$ systems. It would be very interesting to have a complete discussion of these phenomena.

We can continue the above discussion for the case of six compact directions which seen as a product of three tori $T^2 \times T^2 \times T^2$ in the (x^1, x^2) , (x^3, x^4) and (x^5, x^6) directions respectively. We turn on B and F fields such that $B_{12} = B_1, B_{34} = B_2, B_{56} = B_3$ and $F_{12} = F_1, F_{34} = F_2, F_{56} = F_3$. We then have three values for Q_i and three values for θ_i and $\text{Pf } Q = Q_1 Q_2 Q_3$ and $\text{Pf } \theta = \theta_1 \theta_2 \theta_3$.

By applying three T-dualities with respect to (x^2, x^4, x^6) , the factor $\frac{\text{Pf } Q}{\text{Pf } \theta}$ appears naturally when comparing the double and direct direction and the formula (2) stands also for this case.

Another check for the formulas of [16] is made when we start with a D7 brane on (x^1, \dots, x^8) directions with B and F fields on the (x^1, x^2) directions. As discussed in [16], we have the transverse coordinates Φ^8, Φ^9 which are functions of the noncommuting brane coordinates so they do not commute. The Chern-Simons term will now look like:

$$S_{CS} = \mu_7 \int_x \frac{Q}{\theta} P[e^{i(i_\Phi * i_\Phi)} \sum_n C^{(n)}] e^{Q^{-1}} \quad (27)$$

where P represents the pullback of the transverse brane coordinates and $Q^{12} = (F^{12} + B^{12})^{-1}$. The exponential term $e^{i(i_\Phi * i_\Phi)}$ just implies the appearance of a Myers-type term like $Q^{9,10} = -i [\Phi^9, \Phi^{10}]$.

If we use the above discussion, a T-duality on the x^1 direction would give a rotated D6 brane in a tilted torus. The D6 brane is now commutative so the transverse directions are functions of commuting coordinate and they commute. This would mean that in this case we do not have Myers-type term because $[\Phi^9, \Phi^{10}] = 0$, and this is expected because the Chern-Simons term for the D6 brane involves only angles of rotation which are related to Q^{12} and θ^{12} and not to $Q^{9,10}$.

The non-BPS D-branes could be treated in a similar fashion by using the results of [25, 26] concerning the T-duality for commutative non-BPS D-branes. A formula has been proposed in [16] for a non-BPS D8 brane as

$$S_{CS} = \frac{\mu_8}{2T_{min}} \int \frac{\text{Pf } Q}{\text{Pf } \theta} \mathcal{D}T C^{(n)} e^{Q^{-1}} \quad (28)$$

where $\mathcal{D}_i T = -i (Q^{-1})_{ij} [X^j, T]$ is a covariant derivative which is background independent and linear in $[X^j, T]$.

Another important example of non-commutative field theories appear when the D-branes are not wrapped on tori but on S^2 cycles. When studying D3 branes orthogonal to orbifolds or conifolds, the D5 branes wrapped on the resolution vanishing 2-cycles have naturally B and F fields on their world-volume and this implies Chern-Simons couplings which induce fractional D3 brane charges [29, 30, 31, 32, 33, 34, 35, 36, 37]. In the case of non-BPS systems of branes, by turning on different F fluxes on different branes we can make the system stable as discussed in [38, 13, 39, 40].

Acknowledgements

We would like to thank Eric Bergshoeff and Ralph Blumenhagen for discussions and to Dieter Lust and Keshav Dasgupta for comments on the manuscript. The work was supported by DFG.

References

- [1] V. Schomerus, *D-branes and Deformation Quantization*, hep-th/9903205, JHEP **06** (1999) 030.
- [2] N. Seiberg, E. Witten, *String Theory and Noncommutative Geometry*, hep-th/9908142, JHEP **09** (1999) 032.
- [3] R. Gopakumar, S. Minwalla, A. Strominger, *Noncommutative Solitons*, hep-th/0003160, JHEP **05** (2000) 020.
- [4] K. Dasgupta, S. Mukhi and G. Rajesh, *Noncommutative Tachyons*, hep-th/0005006, JHEP **06** (2000) 022.
- [5] J. A. Harvey, P. Kraus, F. Larsen, E. J. Martinec, *Strings and Branes as Noncommutative Solitons*, hep-th/0005031, JHEP **07** (2000) 042.
- [6] E. Witten, *Noncommutative Tachyons And String Field Theory*, hep-th/0006071.
- [7] R. Gopakumar, S. Minwalla, A. Strominger, *Symmetry Restoration and Tachyon Condensation in Open String Theory*, hep-th/0007226.
- [8] N. Seiberg, *Note on Background Independence in Noncommutative Gauge Theories, Matrix Model and Tachyon Condensation*, hep-th/0008013, JHEP **09** (2000) 003.
- [9] J. A. Harvey, P. Kraus, F. Larsen, *Tensionless Branes and Discrete Gauge Symmetry*, hep-th/0008064.
- [10] A. Sen, *Some Issues in Non-commutative Tachyon Condensation*, hep-th/0009038.
- [11] M. Aganagic, R. Gopakumar, S. Minwalla, A. Strominger, *Unstable Solitons in Noncommutative Gauge Theory*, hep-th/0009142.
- [12] J. A. Harvey, P. Kraus, F. Larsen, *Exact Noncommutative Solitons*, hep-th/0010060.
- [13] R. Tatar, *A Note on Non-Commutative Field Theory and Stability of Brane-Antibrane Systems*, hep-th/0009213.
- [14] L. Dolan, C. R. Nappi, *A Scaling Limit With Many Noncommutativity Parameters*, hep-th/0009225.
- [15] K. Dasgupta, Z. Yin, *Non-Abelian Geometry*, hep-th/0011034.

- [16] S Mukhi, V. Suryanarayana, *Chern-Simons Terms on Noncommutative Branes*, hep-th/0009101.
- [17] C. S. Chu and P. M. Ho, *Noncommutative Open String and D-brane*, hep-th/9812219, Nucl.Phys. **B550** (1999) 151.
- [18] B. Chen, H. Itoyama, T. Matsuo, K. Murakami, *p-p' System with B-field, Branes at Angles and Noncommutative Geometry*, hep-th/9910263, Nucl.Phys. **B576** (2000) 177.
- [19] Y. Imamura, *T-duality of non-commutative gauge theories*, hep-th/0001105, JHEP **01** (2000) 039.
- [20] R. Blumenhagen, L. Goerlich, B. Kors, D. Lust, *Asymmetric Orbifolds, Noncommutative Geometry and Type I String Vacua*, hep-th/0003024, Nucl.Phys. **B582** (2000) 44.
- [21] R. Blumenhagen, L. Goerlich, B. Kors, D. Lust, *Noncommutative Compactifications of Type I Strings on Tori with Magnetic Background Flux*, hep-th/0007024, JHEP **10** (2000) 006.
- [22] R. Blumenhagen, L. Goerlich, B. Kors, D. Lust, *Magnetic Flux in Toroidal Type I Compactification*, hep-th/0010198.
- [23] Enrique Alvarez, J.L.F. Barbon, J. Borlaf, *T duality for open Strings*, hep-th/9603089, Nucl.Phys. **B479** (1996) 218.
- [24] E. Bergshoeff, M. de Roo, *D-branes and T-duality*, hep-th/9603123, Phys.Lett. **B380** (1996) 265.
- [25] M. R. Garoussi, *Tachyon couplings on non-BPS D-branes and Dirac-Born-Infeld action*, hep-th/0003122, Nucl.Phys. **B584** (2000) 28.
- [26] E.A. Bergshoeff, M. de Roo, T.C. de Wit, E. Eyras, S. Panda, *T-duality and Actions for Non-BPS D-branes*, hep-th/0003221, JHEP **05** (2000) 009.
- [27] B. Janssen and P. Meessen, *A Nonabelian Chern-Simons term for Non-BPS Branes*, hep-th/0009025.
- [28] R. C. Myers, *Dielectric-Branes*, hep-th/9910053, JHEP **12** (1999) 022.
- [29] M. Douglas, *Enhanced Gauge Symmetry in M(atric) Theory*, hep-th/9612126, JHEP **07** (1997) 004

- [30] D.-E. Diaconescu, M. Douglas and J. Gomis, *Fractional Branes and Wrapped Branes*, hep-th/9712230, JHEP **02** (1998) 013.
- [31] A. Karch, D. Lüster and D. Smith, *Equivalence of Geometric Engineering and Hanany-Witten via Fractional Branes*, hep-th/9803232, Nucl. Phys. **B533** (1998) 348.
- [32] S. S. Gubser and I. R. Klebanov, *Baryons and Domain Walls in an $N=1$ Superconformal Gauge Theory*, Phys. Rev. **D58** (1998) 125025; hep-th/9808075.
- [33] K. Dasgupta and S. Mukhi, *Brane Constructions, Fractional Branes and Anti-de Sitter Domain Walls*, hep-th/9904131, JHEP **07** (1999) 008.
- [34] I. R. Klebanov, N. A. Nekrasov, *Gravity Duals of Fractional Branes and Logarithmic RG Flow*, hep-th/9911096.
- [35] K. Oh and R. Tatar, *Renormalization Group Flows on D3 branes at an Orbifolded Conifold*, hep-th/0003183, JHEP **05** (2000) 030.
- [36] D. Berenstein, V. Jejjala, R. G. Leigh, *Marginal and Relevant Deformations of $N=4$ Field Theories and Non-Commutative Moduli Spaces of Vacua*, hep-th/0005087.
- [37] K. Dasgupta, S. Hyun, K. Oh, R. Tatar, *Conifolds with Discrete Torsion and Noncommutativity*, hep-th/0008091, JHEP.
- [38] Y. Oz, T. Pantev, D. Waldram, *Brane-Antibrane Systems on Calabi-Yau Spaces*, hep-th/0009112
- [39] P. Kraus, A. Rajaraman, S. Shenker, *Tachyon Condensation in Non-commutative Gauge Theory*, hep-th/0010016
- [40] M. Li, *Note on Noncommutative Tachyon in Matrix Models*, hep-th/0010058