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Bound States of String Networks and D-branes

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Abstract

We show the existence of non-threshold bound states of (p, q) string networks and $D3$-branes, preserving $1/4$ of the full type IIB supersymmetry, interpreted as string networks "dissolved" in $D3$-branes. We also explicitly write down the expression for the mass density of the system and discuss the extension of the construction to other Dp-branes. Differences in our construction of string networks with the ones interpreted as dyons in $N = 4$ gauge theories are also pointed out.

Non-threshold bound states of various D-branes [1, 2] have been objects of much interest due to their applications to the non-perturbative dynamics of string theory and gauge theory, including from the point of view of AdS/CFT correspondence [3, 4]. They have an interpretation as branes that are "dissolved" inside other branes, and preserve $1/2$ supersymmetry. They are also of importance in understanding the physics of black holes from a microscopic point of view [5]. Bound states of F-strings with D-branes have been analyzed as well [6, 7]. Such bound states are generally obtained by applying T-dualities [8] to delocalized brane solutions and have explicit realizations as supergravity solutions. In view of their wide applications, it is of importance to analyze these results further.

In this note, we generalize the above constructions and obtain the bound states, now interpreted as (p, q) string networks [9, 10] "dissolved" in $D3$-branes. They preserve $1/4$ of the full type IIB supersymmetry and therefore describe new non-perturbative objects in these theories. It will also be pointed out later on that, our construction of the bound states of string networks and $D3$-branes are different from the ones appearing in the context of $N = 4$ gauge theories, interpreted as dyons[11, 12, 13].

The existence of stable networks as well as web-like configurations for strings and branes is now known for several years [14] on the basis of charge conservation, tension balance and supersymmetry analysis. Although for large number of these configurations no explicit supergravity or worldvolume realizations are known, several examples in the context of string networks have been worked out from world volume point of view[15, 12]. Results in our paper give evidence for the existence of similar configurations when they are dissolved inside other D-branes.

We now start by writing down the classical supergravity solution [3] corresponding to the $D1 - D3$ bound state[2, 7], preserving $1/2$ supersymmetry :

$$\begin{aligned}
&ds^2_{str} = f^{-1/2}[-dx_0^2 + dx_1^2 + h(dx_2^2 + dx_3^2)] + f^{1/2}(dr^2 + r^2 d\Omega_5^2),\\
&f = 1 + \frac{\alpha'^2 R^4}{r^4}, \qquad h^{-1} = \sin^2\phi f^{-1} + \cos^2\phi,\\
&B_{23} = \frac{\sin\phi}{\cos\phi}\ f^{-1}h, \quad e^{2\Phi} = g^2 h,
\end{aligned}$$

$$F_{01r} = \frac{1}{g} \sin\phi \; \partial_r f^{-1},$$
$$F_{0123r} = \frac{1}{g} \cos\phi \; h \; \partial_r f^{-1}, \tag{1}$$

where $B_{\mu\nu}$ is the NS-NS antisymmetric tensor field. $F_{\mu\nu\rho}$ and $F_{\mu\nu\rho\alpha\beta}$ are respectively R-R 3-form and 5-form field strengths. The asymptotic value of the B field in eqn. (1) is $B_{23}^{\infty} = tan(\phi)$ and gives the expression for the ratio of charge densities of (smeared) $D1$ and $D3$ branes. The parameter R is defined by $cos\phi R^4 = 4\pi g n$, with n being the number of $D3$-branes. Finally $g \equiv g_\infty$ is the asymptotic value of the string coupling.

To describe explicitly the $1/2$ supersymmetry property of the $D1 - D3$ bound state, we note from their explicit solution in eqn.(1) that, they also have an alternative interpretation in terms of $D3$-branes in a constant NS-NS antisymmetric tensor background of magnetic type: $B_{23} = tan(\phi)$. The $1/2$ supersymmetry condition is then written in the following form [13]:

$$(\epsilon_L - \epsilon_R) = sin\phi \; \Gamma^{01}(\epsilon_L - \epsilon_R) + cos\phi \; \Gamma^{0123}(\epsilon_L + \epsilon_R), \tag{2}$$

where ϵ_L and ϵ_R are two positive chirality space-time spinors arising from the left and the right moving sectors of the type IIB string theory. The above condition can also be written in an alternative form:

$$\epsilon_L = -sin\phi \; \Gamma^{01}\epsilon_R + cos\phi \; \Gamma^{0123}\epsilon_R, \tag{3}$$

or equivalently,

$$\epsilon_R = -sin\phi \; \Gamma^{01}\epsilon_L - cos\phi \; \Gamma^{0123}\epsilon_L. \tag{4}$$

Supersymmetry conditions of eqns. (3) and (4) reduce to that of a standard $D3$-brane for $\phi = 0$. From eqn. (1), we also notice that D-string in the above bound state lies along the x^1 axis and is smeared in the remaining spatial directions x^2 and x^3, giving it the interpretation of a D-string being "dissolved" in a $D3$-brane. A generalization of the supergravity solution in eq. (1), representing $((F, D1), D3)$ bound state, is known and corresponds to the case when both electric and magnetic type $B_{\mu\nu}$ fields are turned on [3, 16]. These solutions also preserve $1/2$ supersymmetry, thereby ensuring their stability.

The mass-density of these $((F1, D1), D3)$ bound states can be expressed (in string-frame) as [2, 17, 18]:

$$m^2 = T_0^2 \left[\frac{n^2}{g^2} + |p + q\tau|^2\right], \tag{5}$$

where we have one (p, q)-string along, say x^1 direction per $(2\pi)^2\alpha'$ area [19] over the $x^2 - x^3$ plane, and n is the $D3$-brane charge. Also, $T_0 = \frac{1}{(2\pi)^3\alpha'^2}$ and axion-dilaton moduli are given as: $\tau \equiv \chi + \frac{i}{g}$. We also notice that mass-density (5), is a sum of distinct energy densities, associated with a (p, q)-string and that of a $D3$-brane. Moreover the contributions of (p, q)-string for different (p, q)'s remain identical to the one, when $D3$-brane is absent.

We now discuss the construction of the bound state of string networks and $D3$-brane from supersymmetry point of view. Following above reasoning, these objects can also be viewed as (p, q) string networks dissolved in a $D3$-brane. To discuss the network construction we now complexify eqns.(3) and (4):

$$(\epsilon_L - i\epsilon_R) = isin\phi\ \Gamma^{01}(\epsilon_L + i\epsilon_R) + icos\phi\ \Gamma^{0123}(\epsilon_L - i\epsilon_R), \tag{6}$$

giving the $1/2$ supersymmetry projection of a $(D1 - D3)$-bound state. Then, to write down the supersymmetry projection of a bound state $((F1, D1), D3)$ of a (p, q)-string and $D3$-brane, we use the fact that they can be generated by applying $SL(2, Z)$ duality [20] on the $D1 - D3$ bound state discussed above. This procedure also gives the $1/2$ supersymmetry condition for the $((F1, D1), D3)$ bound state, by using that the spinors, $(\epsilon_L \pm i\epsilon_R)$, transform covariantly under the maximal compact subgroup, $SO(2) \in SL(2, R)$, with $SL(2, R)/SO(2)$ parametrizing the moduli space represented by axion-dilaton fields. The transformation properties of spinors are given as:

$$(\epsilon_L \pm i\epsilon_R) \to e^{i\frac{\alpha}{2}}(\epsilon_L \pm i\epsilon_R). \tag{7}$$

To obtain the phase α for a given $SL(2, Z)$ transformation, one notes that using the vielbein E, corresponding to the axion-dilaton moduli $\mathcal{M} \equiv EE^T$, any $SL(2, R)$ vector can be turned into an $SO(2)$ vector. As a result, the phase transformation parameter α can be read off from the corresponding $SL(2,Z)$ parameters. The supersymmetry condition for a (p,q)-string dissolved in a $D3$ brane can then be generated from the one in (6) and has a form:

$$(\epsilon_L - i\epsilon_R) = e^{i\Theta(p,q,\tau)} sin\phi\ \Gamma^{01}(\epsilon_L + i\epsilon_R) + icos\phi\ \Gamma^{0123}(\epsilon_L - i\epsilon_R). \tag{8}$$

We notice that a phase factor Θ, dependent on axion- dilaton moduli (τ) as well as $SL(2,Z)$ quantum numbers (p,q): $e^{i\Theta(p,q,\tau)} = p + q\tau/|p+q\tau|$, appears in the first term in the R.H.S. representing the supersymmetry condition of a (p,q) string. Since $D3$-branes are $SL(2,Z)$ invariant objects, the second term in the R.H.S. of eqn. (8) remains unchanged with respect to the one for $D1 - D3$ case.

Now, to show the possibility of a string network construction, we consider a (p,q) string lying in $x^1 - x^2$ plane at an angle θ with the x^1 axis. Then equation (8) is replaced by:

$$(\epsilon_L - i\epsilon_R) = e^{i\Theta(p,q,\tau)} sin\phi\ \Gamma^0(\Gamma^1\ cos\theta + \Gamma^2\ sin\theta)(\epsilon_L + i\epsilon_R) + icos\phi\ \Gamma^{0123}(\epsilon_L - i\epsilon_R). \tag{9}$$

It can be seen that, as in the case of string networks in the absence of $D3$-brane, if one identifies the orientation of the (p,q)- string inside $D3$-brane, with its phase in the internal space: $\theta = \Theta(p,q,\tau)$, then the above supersymmetry condition is solved by the following projections:

$$(\epsilon_L - i\epsilon_R) = sin\phi\ \Gamma^{01}(\epsilon_L + i\epsilon_R) + icos\phi\ \Gamma^{0123}(\epsilon_L - i\epsilon_R), \tag{10}$$

and

$$(\epsilon_L - i\epsilon_R) = isin\phi\ \Gamma^{02}(\epsilon_L + i\epsilon_R) + icos\phi\ \Gamma^{0123}(\epsilon_L - i\epsilon_R). \tag{11}$$

We notice that the projection condition (10) corresponds to that of an F-sting along x^1 axis dissolved in a $D3$-brane. Similarly, the projection condition (11) corresponds to that of a D-string along x^2 axis, dissolved in the same $D3$-brane. These together imply that supersymmetry is broken to $1/4$ of the original one. Interestingly, the supersymmetry condition, eqn.(9), is satisfied for arbitrary (p, q) with only finite number of projections, provided the above identification of the phases, $\theta = \Theta(p, q, \tau)$ holds. The projection conditions, eqns. (10) and (11), also reduce to the ones in [9], for $\phi = \frac{\pi}{2}$, which corresponds to the case when there is no $D3$-brane. We have therefore shown the existence of a $((p, q) string\ network, D3)$, bound state preserving $1/4$ supersymmetry.

To confirm the $1/4$ supersymmetry property of our configuration further, we now show that simultaneous solutions for ϵ_L and ϵ_R, of appropriate type, do exist for eqns. (10) and (11). In this connection, we note that eqn. (10), representing the supersymmetry of F-string dissolved in $D3$- brane, can be written as

$$\epsilon_L = sin\phi\ \Gamma^{01}\epsilon_L + cos\phi\ \Gamma^{0123}\epsilon_R, \tag{12}$$

or equivalently as

$$\epsilon_R = -sin\phi\ \Gamma^{01}\epsilon_R - cos\phi\ \Gamma^{0123}\epsilon_L. \tag{13}$$

The supersymmetry conditions of dissolved D-strings, along x^1, were already written in eqn. (3), or (equivalently in) (4). From eqn. (11), we get conditions that are identical to the ones in eqns. (3) and (4), when we replace Γ^{01} by Γ^{02}. In particular, for our argument we use:

$$\epsilon_R = -sin\phi\ \Gamma^{02}\epsilon_L - cos\phi\ \Gamma^{0123}\epsilon_L, \tag{14}$$

as well as eqn. (12) as independent conditions following from (10) and (11). Now, substituting ϵ_R from eqn. (14) into (12), one gets:

$$\epsilon_L = \Gamma^0(\Gamma^1\ sin\phi - \Gamma^3\ cos\phi)\epsilon_L. \tag{15}$$

The $1/4$ supersymmetry now directly follows from eqns.(14) and (15).

We now obtain the mass density of the $(string\ network, D3)$ bound state that we have constructed. For this purpose, one can start with the expression of the mass density for a bound state of (p, q)-string with $D3$ branes as in eqn.(5). As already emphasized, (p, q)-strings inside $D3$-branes have distinct contribution to the total mass formula. Now, for the case of $(string\ network, D3)$ bound state the contribution to the total mass, coming from the network, can be written as sum of contributions from different strings in that network[9]:

$$m^2_{network} = (\Sigma_i l_i T_i)^2, \tag{16}$$

where l_i's are the the lenghts of various links and T_i's are the corresponding tensions. Final expression for mass density is then obtained by adding contribution from the $D3$-branes as well. In other words, the modification to the mass formula in the string network case is essentially due to the replacement of the (p,q)-string tension, by the corresponding network mass formula in eqn. (5).

To write the expression for the mass density in a concrete form, we consider the case when the string network as well as $D3$-brane are wrapped on a T^2. In this context, for the wrapping of the string network, one defines lattice vectors $\vec{a}, \vec{b}$, constructed out of the link vectors $\vec{l}_i$,$(i = 1, 2, 3)$ of a 3-prong string junction in a periodic string network of strings with quantum numbers (p_i, q_i), $(i = 1, 2, 3)$ [9, 21], obeying charge conservation on the junction. The T^2 is parametrized by moduli: $\lambda_1 = \vec{a}.\vec{b}/\vec{a}^2$, $\lambda_2 = |\vec{a} \times \vec{b}|/\vec{a}^2$. Total mass density (per unit length) then turns out to be of the form (now in Einstein-frame):

$$m^2 = T_0^2 n^2 A^2 + m^2_{network}, \tag{17}$$

where $A = |\vec{a} \times \vec{b}|$ is the area of T^2. Explicit expression for network contribution to this mass-density: $m^2_{network}$ is identical to the one in [9], with appropriate replacements of charges by charge-densities, coming from the smearing of the resulting (delocalized) particle-like state in the unwrapped direction of the $D3$ brane. Putting the factors of α' etc. appropriately:

$$m^2_{network} = \frac{1}{(2\pi)^4 \alpha'^3} A \begin{pmatrix} p_1 & q_1 & p_2 & q_2 \end{pmatrix} (M \pm L) \begin{pmatrix} p_1 \\ q_1 \\ p_2 \\ q_2 \end{pmatrix}, \tag{18}$$

where we have one wrapped string network per $2\pi\sqrt{\alpha'}$ length along the unwrapped direction of $D3$-brane. Also,

$$M = \frac{1}{\lambda_2}\begin{pmatrix} \mathcal{M} & \lambda_1\mathcal{M} \\ \lambda_1\mathcal{M} & |\lambda|^2\mathcal{M} \end{pmatrix}, \qquad L = \begin{pmatrix} 0 & \mathcal{L} \\ -\mathcal{L} & 0 \end{pmatrix}, \tag{19}$$

$$\mathcal{M} = \frac{1}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}, \qquad \mathcal{L} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{20}$$

We have therefore given the expression for the mass density of the bound states discussed above. We also notice that the above mass formula reduces to the one for the conventional string networks [9] in the absence of $D3$-branes, $(n = 0)$. Moreover, by setting charges $(p_2, q_2) = (0, 0)$, which implies the reduction to the case of a straight string with (p_1, q_1) charges, one can obtain the energy spectrum of $((F, D1), D3)$ bound state with $1/2$ supersymmetry. In the compactified theory, the mass formula (17) also corresponds to that of a bound state of a particle with U-duality charges, dissolved in a string with, $(0, 0, 1)$ charge. Our result therefore turns out to be consistent with a general supersymmetry analysis of BPS objects in eight dimensions [22] implying $1/4$ supersymmetry for particle-like objects and $1/2$ for string-like ones in $D = 8$. Apart from the supersymmetry analysis that we have presented in the paper, we also note that the non-threshold BPS mass formula, written in eqn. (17), implies that such bound states of string-networks and D3-branes are also energetically favorable, and therefore likely to be formed, when several (p, q)-strings are dissolved inside these branes. However, a more detailed analysis is needed in this context.

We emphasize that BPS configuration obtained above are different from the ones obtained in [13] in the context of noncommutative gauge theory. First of all, the string networks of [13] preserve $1/4$ [11] of the $D3$ brane supersymmetry, with an interpretation as a dyon in these theories, whereas in our case we have $1/4$ of the full type II supersymmetry. Also, our string network lies completely inside the $D3$ brane, compared to the ones representing dyons which connect different branes. It will certainly be interesting to examine our string network configurations from the point of view of noncommutative worldvolume theory, appearing in this context.

These results can also be generalized to other bound states of fundamental and D-

objects discussed in the literature [23, 19] and will give rise to lower supersymemtries than the known ones. For example, one can generalize the construction of the $D1-D5$ system[3] to string networks "dissolved" in (p,q)-webs of 5-branes. It will then be of interest to analyze implications of these results to black hole physics.

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