Gluon-ghost condensate of mass dimension 2 in the Curci-Ferrari gauge

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Abstract

The effective potential for an on-shell BRST invariant gluon-ghost condensate of mass dimension 2 in the Curci-Ferrari gauge in $SU(N)$ Yang-Mills is analysed by combining the local composite operator technique with the algebraic renormalization. We pay attention to the gauge parameter independence of the vacuum energy obtained in the considered framework and discuss the Landau gauge as an interesting special case.

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1 Introduction

Nowadays an increasing evidence has been reported on the relevance of the local composite operator A^2 in the Landau gauge, both from a phenomenological point of view [1, 2] as from lattice studies [3, 4, 5]. It is no coincidence that the Landau gauge is used because then A^2 equals the non-local gauge invariant operator $(VT)^{-1} \min_U \int d^4x \, (A^2)^U$ with VT the space time volume. The lattice also revealed that gluons attain a dynamical mass, see e.g. [6, 7]. Some older work already discussed the pairing of gluons in connection with a mass generation, as a result of the fact that the perturbative Yang-Mills (YM) vacuum (trivially zero) is unstable [8, 9, 10]. More recently, the connection between a condensate $\langle A^2 \rangle$ and a gluon mass has been made within the OPE framework [11, 12]. A technique to effectively calculate $\langle A^2 \rangle$ and the gluon mass was presented in [13], also in the Landau gauge.

The answer to the question how a mass is generated could be posed in a more general context than the Landau gauge. The Landau gauge is a limiting case of a class of renormalizable, generalized covariant gauges introduced in [14, 15]. We are therefore led to search for a local operator which could replace A^2. A proposal has been made in [11], where it was shown that A^2 is a special case of a more general mass dimension 2 operator, namely $\mathcal{O} = \frac{1}{2}A_\mu^a A^{\mu a} + \alpha \overline{c}^a c^a$, also involving ghosts and which is BRST invariant on-shell, however not gauge invariant (see also [16]). This should allow a BRST invariant treatment of the mass generation in those gauges. The proposed condensate is not that surprising, since it equals the operator coupled to the mass term of a massive, renormalizable $SU(N)$ model, introduced in [17, 18]. The specific form of the mass term is necessary to maintain the BRST invariance and renormalizability [17, 18, 19]. Although the Curci-Ferrari model (CF) is BRST invariant, the associated BRST operator is not nilpotent and the model is not unitary [20, 19]. Since the gauge fixing terms of the CF model and the YM theories with the gauges discussed in [11, 12, 14, 15] are the same, it seems natural to search in that direction for a suitable operator that gets a non-vanishing vacuum expectation value and invokes a dynamical mass.

The aim of this paper is to construct an effective potential for the mass dimension 2 condensate in the CF gauge. It is organized as follows. In section 2 we discuss the formalism to obtain a well-defined effective potential for the local composite operator $\mathcal{O} = \frac{1}{2}A_\mu^a A^{\mu a} + \alpha \overline{c}^a c^a$, a non-trivial task due to the compositeness of this operator [13, 21]. In section 3, we denote the Ward identities of the action, ensuring the renormalizability. A further construction of the effective action is discussed in section 4, where we also outline a subtlety on the minimization of the effective potential. In section 5, we consider the gauge parameter independence of the vacuum energy and spend some words on the BRST charge. Section 6 handles the explicit evaluation of the effective potential. We also discuss the interesting role of the Landau gauge as a limiting case of the CF gauge. We pay attention to

the similarities between CF and the Maximal Abelian gauge (MAG). A mass generating mechanism for the off-diagonal gluons in the MAG very much resembles that of the CF gauge, and could be seen as some evidence for Abelian dominance. As usual, conclusions are formulated in the last section.

2 The LCO formalism

For a more detailed introduction to the local composite operator (LCO) formalism and to the algebraic renormalization technique, the reader is referred to [13, 21], respectively [22].

Let us begin by giving the expression for the $SU(N)$ Yang-Mills action in the CF gauge

$$
\begin{aligned}
S & = S_{YM} + S_{GF+FP} = -\frac{1}{4} \int d^4x F_{\mu\nu}^a F^{a\mu\nu} + \int d^4x \left(b^a \partial_\mu A^{a\mu} + \frac{\alpha}{2} b^a b^a + \overline{c}^a \partial^\mu D_\mu^{ab} c^b \right. \\
& \left. - \frac{\alpha}{2} g f^{abc} b^a \overline{c}^b c^c - \frac{\alpha}{8} g^2 f^{abc} f^{cde} \overline{c}^a \overline{c}^b c^d c^e \right)
\end{aligned}
$$

(2.1)

where

$$
D_\mu^{ab} \equiv \partial_\mu \delta^{ab} + g f^{acb} A_\mu^c
$$

(2.2)

is the usual covariant derivative. In order to investigate if

$$
\mathcal{O} = \frac{1}{2} A_\mu^a A^{a\mu} + \alpha \overline{c}^a c^a
$$

(2.3)

gets a non-vanishing vacuum expectation value, we introduce a suitable set of LCO sources [13, 21]. In this case this task is nontrivial. It turns out that in order to introduce the local operator $\mathcal{O}$ in the starting action in a BRST invariant way, three external sources J, η^μ and τ^μ are needed, so that

$$
S_{\text{LCO}} = \int d^4x \left[J\mathcal{O} + \frac{\xi}{2} J^2 - \eta^\mu A_\mu^a c^a - \tau^\mu s(A_\mu^a c^a) \right]
$$

(2.4)

where ξ is the LCO parameter and s denotes the BRST operator acting as

$$
\begin{aligned}
s A_\mu^a & = -D_\mu^{ab} c^b \\
s c^a & = \frac{g}{2} f^{abc} c^b c^c \\
s \overline{c}^a & = b^a \\
s b^a & = -J c^a \\
s J & = 0 \\
s \eta^\mu & = \partial^\mu J \\
s \tau^\mu & = \eta^\mu
\end{aligned}
$$

(2.5)

The parameter ξ has to be introduced since the introduction of the source term $J\mathcal{O}$ gives rise to novel vacuum energy divergences proportional to J^2. These new divergences, related to those of the connected Green's function $\langle \mathcal{O}(x)\mathcal{O}(y) \rangle_c$ for $x \to y$, are cancelled by a counterterm $\delta\xi\frac{J^2}{2}$.

After introduction of the sources, we still have a BRST invariant action

$$s\left(S_{YM} + S_{GF+FP} + S_{\text{LCO}}\right) = 0 \tag{2.6}$$

but it should be observed that, due to the presence of the sources (J, η^μ, τ^μ), the BRST operator is no more nilpotent, namely

$$
\begin{aligned}
s^2\Phi &= 0, \quad \Phi = (A, c, J, \eta^\mu) \\
s^2\overline{c}^a &= -Jc^a \\
s^2 b &= -J\frac{g}{2}f^{abc}c^b c^c \\
s^2\tau^\mu &= \partial^\mu J
\end{aligned}
\tag{2.7}
$$

As a consequence, setting

$$s^2 = \delta_J \tag{2.8}$$

we have

$$\delta_J\left(S_{YM} + S_{GF+FP} + S_{\text{LCO}}\right) = 0 \tag{2.9}$$

The operator δ_J is related to the $SL(2,R)$ symmetry [14, 15, 23] exhibited by the Curci-Ferrari action. The generators of this $SL(2,R)$ symmetry are, next to the Faddeev-Popov ghost number δ_{FP}, given by

$$
\begin{aligned}
\delta\overline{c}^a &= c^a \\
\delta b^a &= \frac{g}{2}f^{abc}c^b c^c \\
\delta A_\mu^a &= \delta c^a = 0
\end{aligned}
\tag{2.10}
$$

and

$$
\begin{aligned}
\overline{\delta}c^a &= \overline{c}^a \\
\overline{\delta}b^a &= \frac{g}{2}f^{abc}\overline{c}^b \overline{c}^c \\
\overline{\delta}A_\mu^a &= \overline{\delta}\overline{c}^a = 0
\end{aligned}
\tag{2.11}
$$

The action of the δ symmetry can be enlarged to the sources as $\delta J = 0$, $\delta\eta_\mu = 0$ and $\delta\tau_\mu = 0$. Then it is obvious from (2.7) that

$$\delta_J = s^2\big|_{J=0} - J\delta = -J\delta \tag{2.12}$$

Also, expression (2.8) shows that, in the massive case, the δ_J-invariance is a consequence of the modified BRST transformations. The lack of nilpotency of the BRST operator together with (2.8) are well known features of the CF gauge in the presence of a mass term

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[24]. Next to the δ_J invariance, the action $S_{YM} + S_{GF+FP} + S_{LCO}$ is still invariant under the NO algebra[1] [23], meaning that irrespective of the fact that $\langle \mathcal{O} \rangle$ gains a non-trivial value, the NO (and thus the $SL(2, R)$) symmetry is unaffected.

Notice that in the present case the operator s^2 always contains the source J which will be set to zero at the end of the computation.

3 Ward identities

Let us now translate the previous invariances into Ward identities. To this purpose, we introduce external sources Ω_μ^a and L^a coupled to the BRST variation of A_μ^a and c^a

$$S_{ext} = \int d^4x \left[-\Omega^{a\mu} D_\mu^{ab} c^b + L^a \frac{g}{2} f^{abc} c^b c^c \right] \tag{3.1}$$

with

$$s\Omega_\mu^a = sL^a = 0$$

The complete action

$$\Sigma = S_{YM} + S_{GF+FP} + S_{LCO} + S_{ext} \tag{3.2}$$

turns out to obey the following identities:

- The Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) = 0 \tag{3.3}$$

  with

$$\mathcal{S}(\Sigma) = \int d^4x \left(\frac{\delta\Sigma}{\delta A_\mu^a} \frac{\delta\Sigma}{\delta \Omega^{a\mu}} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta\Sigma}{\delta c^a} + b^a \frac{\delta\Sigma}{\delta \overline{c}^a} + \partial_\mu J \frac{\delta\Sigma}{\delta \eta_\mu} + \eta^\mu \frac{\delta\Sigma}{\delta \tau^\mu} - Jc^a \frac{\delta\Sigma}{\delta b^a} \right) \tag{3.4}$$

  The δ_J Ward identity

$$\mathcal{W}(\Sigma) = 0 \tag{3.5}$$

  with

$$\mathcal{W}(\Sigma) = \int d^4x \left(Jc^a \frac{\delta\Sigma}{\delta \overline{c}^a} + J \frac{\delta\Sigma}{\delta L^a} \frac{\delta\Sigma}{\delta b^a} - \partial_\mu J \frac{\delta\Sigma}{\delta \tau_\mu} \right) \tag{3.6}$$

Proceeding as in [25], these identities imply the renormalizability of the model and, in particular, the multiplicative renormalizability of the local operator $\mathcal{O}$.

---

[1]This algebra is generated by the $SL(2, R)$ and (anti-)BRST transformations s and $\overline{s}$. It is a trivial task to check that the action is also anti-BRST invariant, and relations similar to (2.8), (2.9) and (2.12) arise for the anti-BRST transformation $\overline{s}$.

4 Renormalizability of $\mathcal{O}$ and the effective action

As established explicitly in [12, 26], the operator $\mathcal{O}$ is indeed multiplicative renormalizable in the CF gauge. Denoting the bare operator by $\mathcal{O}_B$, one has

$$\mathcal{O}_B = Z_{\mathcal{O}}\mathcal{O}_R \tag{4.1}$$

with[2] [12, 26]

$$Z_{\mathcal{O}} = 1 + \left[\frac{35}{6} - \frac{\alpha}{2}\right]\frac{g^2 N}{16\pi^2}\frac{1}{\varepsilon} + \left[\left(\frac{2765}{72} - \frac{11\alpha}{3}\right)\frac{1}{\varepsilon^2} + \left(\frac{\alpha^2}{16} + \frac{11\alpha}{16} - \frac{449}{48}\right)\frac{1}{\varepsilon}\right]\left(\frac{g^2 N}{16\pi^2}\right)^2 + \dots \tag{4.2}$$

For the anomalous dimension $\gamma_{\mathcal{O}}$ of $\mathcal{O}$, one has [12, 26]

$$\gamma_{\mathcal{O}}(g^2, \alpha) = -\mu\frac{\partial \ln Z_{\mathcal{O}}}{\partial\mu} = \left(\frac{35}{6} - \frac{\alpha}{2}\right)\frac{g^2 N}{16\pi^2} + \left(\frac{449}{24} - \frac{\alpha^2}{8} - \frac{11\alpha}{8}\right)\left(\frac{g^2 N}{16\pi^2}\right)^2 + \dots \tag{4.3}$$

Notice that $\gamma_{\mathcal{O}}$ depends on the gauge parameter α. This is due to the explicit dependence from α of the operator $\mathcal{O}$. Moreover, in the limit $\alpha \to 0$, expression (4.3) reduces to the anomalous dimension of the Landau gauge [13]. Let us also give, for further use, the β-function of the gauge parameter α in the CF gauge [12, 26].

$$\beta_{\alpha}(g^2, \alpha) = \frac{\mu}{\alpha}\frac{\partial\alpha}{\partial\mu} = \left(\frac{13}{3} - \frac{\alpha}{2}\right)\frac{g^2 N}{16\pi^2} - \frac{\alpha^2 + 17\alpha - 118}{16}\left(\frac{g^2 N}{16\pi^2}\right)^2 + \dots \tag{4.4}$$

In order to obtain the effective potential for the operator $\mathcal{O}$, we set to zero the sources Ω_{μ}^a, L^a, η^{μ} and τ^{μ}, obtaining for the generating functional the following expression

$$\exp -i\mathcal{W}(J) = \int [D\phi] \exp iS(J) \tag{4.5}$$

with

$$S(J) = S_{YM} + S_{GF+FP} + \int d^4x \left[J\mathcal{O} + \frac{\xi}{2}J^2\right] \tag{4.6}$$

and ϕ denoting the relevant fields.

From the bare Lagrangian associated to (4.6), one obtains that the quantity $\xi(\mu)$ obeys the following renormalization group equation (RGE)

$$\mu\frac{d\xi}{d\mu} = 2\gamma_{\mathcal{O}}(g^2, \alpha)\xi + \delta(g^2, \alpha) \tag{4.7}$$

where

$$\delta(g^2, \alpha) = \left(\varepsilon + 2\gamma_{\mathcal{O}}(g^2, \alpha) - \beta(g^2)\frac{\partial}{\partial g^2} - \alpha\beta_{\alpha}(g^2, \alpha)\frac{\partial}{\partial\alpha}\right)\delta\xi \tag{4.8}$$

---

[2]We use dimensional regularization in $d = 4-\varepsilon$ dimensions and employ the $\overline{MS}$ renormalization scheme.

Now, following [13], it is possible to set the hitherto *free* parameter ξ such a function of g^2 and α, so that if g^2 runs according to $\beta(g^2)$ and α to $\beta_\alpha(g^2)$, $\xi(g^2, \alpha)$ will run according to its RGE (4.7). Specifying, $\xi(g^2, \alpha)$ is the particular solution of

$$\left(\beta(g^2)\frac{\partial}{\partial g^2} + \alpha\beta_\alpha(g^2, \alpha)\frac{\partial}{\partial\alpha}\right)\xi(g^2, \alpha) = 2\gamma_\mathcal{O}(g^2, \alpha)\xi(g^2, \alpha) + \delta(g^2, \alpha) \qquad (4.9)$$

Furthermore[3], $\xi(g^2, \alpha)$ is multiplicatively renormalizable ($\xi + \delta\xi = Z_\xi\xi$). It is easy to see that $\xi(g^2, \alpha)$ will be of the form

$$\xi(g^2, \alpha) = \frac{\xi_0(\alpha)}{g^2} + \xi_1(\alpha) + \xi_2(\alpha)g^2 + \dots \qquad (4.10)$$

Performing the calculation at 1-loop, we find that

$$\delta\xi = -\frac{(N^2 - 1)}{16\pi^2}\frac{(3 - \alpha^2)}{\varepsilon} \qquad (4.11)$$

Consequently, solving (4.9) for ξ_0 as a function of the gauge parameter α, one finds

$$\xi_0(\alpha) = \frac{9}{13}\frac{N^2 - 1}{N}s_0(\alpha) \qquad (4.12)$$

$$s_0(\alpha) = 1 + \frac{311}{117}\alpha + 6\alpha\left(1 - \frac{3\alpha}{26}\right)\ln\left|-\frac{26}{\alpha} + 3\right| + c\alpha(-26 + 3\alpha) \qquad (4.13)$$

with c an integration constant. Notice that $s_0(0) = 1$, so that we recover the result of [13] in the case of the Landau gauge. In the next section, we will show that the vacuum energy is gauge parameter independent. Henceforth, we can forget about the integration constant and set $c = 0$.

Taking now the functional derivative of $\mathcal{W}(J)$ with respect to J, we obtain

$$\left.\frac{\delta\mathcal{W}(J)}{\delta J}\right|_{J=0} = -\langle\mathcal{O}\rangle \qquad (4.14)$$

The presence of the J^2 term in $\mathcal{W}(J)$ seems to spoil an energy interpretation. However, this can be dealt with by introducing a Hubbard-Stratonovich field σ so that

$$J\mathcal{O} + \frac{\xi}{2}J^2 \Rightarrow -\frac{\sigma^2}{2\xi g^2} + \frac{\sigma}{g\xi}\mathcal{O} + \frac{\sigma}{g}J - \frac{1}{2\xi}\mathcal{O}^2 \qquad (4.15)$$

Therefore

$$\exp -i\mathcal{W}(J) = \int [D\phi]\exp i\left(S_\sigma + \int d^4 x\frac{\sigma}{g}J\right) \qquad (4.16)$$

---

[3]The integration constant showing up when (4.9) is solved, has been put to zero according to [13].

where

$$S_\sigma = S_{YM} + S_{GF+FP} + \int d^4x \left(-\frac{\sigma^2}{2\xi g^2} + \frac{\sigma}{g\xi}\mathcal{O} - \frac{1}{2\xi}\mathcal{O}^2 \right) \tag{4.17}$$

J now appears as a linear source. Hence, we have back an energy interpretation and the $1PI$ machinery applies.

Differentiating the functional generator with respect to J, one gets the relationship

$$\langle\sigma\rangle_{S_\sigma} = g\langle\mathcal{O}\rangle \tag{4.18}$$

Recapitulating, we have constructed a multiplicatively renormalizable action S_σ incorporating the effects of a possible non-vanishing vacuum expectation value for $\mathcal{O}$. The corresponding effective action Γ obeys a linear, homogeneous RGE. Notice that to get actual knowledge of the n-loop effective action, one needs the values of $\xi_0, \ldots, \xi_n$. This means, recalling (4.9), that we need the $(n+1)$-loop values of the renormalization group functions. In [27], a slightly different Hubbard-Stratonovich transformation was used, so that

$$JO + \frac{\xi}{2}J^2 \Rightarrow -\frac{\sigma^2}{2g^2} + \frac{\sigma}{g\sqrt{\xi}}\mathcal{O} + \frac{\sqrt{\xi}\sigma}{g}J - \frac{1}{2\xi}\mathcal{O}^2 \tag{4.19}$$

resulting in

$$\exp -i\mathcal{W}(J) = \int [D\phi]\exp i\left(S_\sigma + \int d^4x \frac{\sqrt{\xi}\sigma}{g}J \right) \tag{4.20}$$

where

$$S_\sigma = S_{YM} + S_{GF+FP} + \int d^4x \left(-\frac{\sigma^2}{2g^2} + \frac{\sigma}{g\sqrt{\xi}}\mathcal{O} - \frac{1}{2\xi}\mathcal{O}^2 \right) \tag{4.21}$$

With this action, it seems that it suffices to know $\xi_0, \ldots, \xi_{n-1}$ to construct the n-loop effective potential. However, some attention should be paid here. It is indeed so that with (4.21), we do not need ξ_n for $\Gamma_{n-\text{loop}}$, but since the source J is now coupled to the operator $\frac{\sqrt{\xi}\sigma}{g}$, we formally have for the effective action Γ, being the Legendre transform of $\mathcal{W}(J)$

$$\Gamma\left(\frac{\sqrt{\xi}\sigma}{g}\right) = -\mathcal{W}(J) - \int d^4y J(y)\frac{\sqrt{\xi}\sigma(y)}{g} \tag{4.22}$$

Hence

$$\frac{\delta}{\delta\left(\frac{\sqrt{\xi}\sigma(y)}{g}\right)}\Gamma\left(\frac{\sqrt{\xi}\sigma(x)}{g}\right) = -J(y) \tag{4.23}$$

Since

$$\Gamma = \frac{\Gamma_0}{g^2} + \Gamma_1 + \ldots \tag{4.24}$$

$$\frac{\sqrt{\xi}}{g} = \sqrt{\xi_0}\left(\frac{1}{g^2} + \frac{\xi_1}{\xi_0} + \ldots\right) \tag{4.25}$$

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it becomes clear that, in order to have $J = 0$ up to the considered order in a g^2 expansion (i.e. to end up in the vacuum state), one must solve (for constant configurations)

$$\frac{d}{d\left(\frac{\sqrt{\xi}\sigma}{g}\right)}V = 0 \tag{4.26}$$

which will not[4] produce the same (correct) σ_{min} as by solving

$$\frac{dV}{d\sigma} = 0 \tag{4.27}$$

as it was done in [27]. The most efficient way to solve (4.26) is by performing the transformation

$$\sigma \to \frac{\sigma}{\sqrt{\xi}} \tag{4.28}$$

and this exactly transforms the action (4.21) into the one of (4.17). Notice that the action (4.21) is not incorrect, one should only be careful how the vacuum configuration is constructed. The conclusion is that one cannot escape the job of doing $(n+1)$-loop calculations for n-loop results.

We draw attention to the fact that the action S_σ is BRST invariant[5], while this BRST transformation is nilpotent for $J = 0$. This means that the action, evaluated in its minimum, i.e. the vacuum energy, should be independent of the gauge parameter α order by order. In the next section, we pay some more attention to this α independence.

5   Gauge parameter independence of the vacuum energy

We begin our argumentation from the generating functional (4.16). It will be useful to consider also the 'original' action $\widetilde{S}(J)$ (i.e. before the Hubbard-Stratonovich transformation) defined in (4.6). To avoid confusion with (4.16)-(4.17), we added a $\sim$ to the notation. The relation between $\mathcal{W}(J)$ and $\widetilde{S}(J)$ is obtained via the insertion of a unity

$$1 = \frac{1}{N}\int[D\sigma]\exp\left[i\int d^4x\left(-\frac{1}{2\xi}\left(\frac{\sigma}{g} - \mathcal{O} - \xi J\right)^2\right)\right] \tag{5.1}$$

with N an appropriate normalization factor. Explicitly, we have

$$\exp(-i\mathcal{W}(J)) = \int[D\phi][D\sigma]\exp i\left[\widetilde{S}(J) + \int d^4x\left(-\frac{1}{2\xi}\left(\frac{\sigma}{g} - \mathcal{O} - \xi J\right)^2\right)\right] \tag{5.2}$$

---

[4]Because ξ itself is a series in g^2.

[5]It is obvious that $s\sigma = gs\mathcal{O}$.

Since evidently

$$\frac{d}{d\alpha}\frac{1}{N}\int [D\sigma]\exp\left[i\int d^4x\left(-\frac{1}{2\xi}\left(\frac{\sigma}{g}-\mathcal{O}-\xi J\right)^2\right)\right]=0 \tag{5.3}$$

we find

$$-\frac{d\mathcal{W}(J)}{d\alpha}=\left\langle s\left(\frac{\bar{c}b}{2}-\frac{g^2}{4}f^{abc}\bar{c}^a\bar{c}^bc^c\right)\right\rangle_{J=0}+\text{terms proportional to }J \tag{5.4}$$

The effective action Γ is related to $\mathcal{W}(J)$ through a Legendre transformation

$$\Gamma\left(\frac{\sigma}{g}\right)=-\mathcal{W}(J)-\int d^4y J(y)\frac{\sigma(y)}{g} \tag{5.5}$$

The effective potential $V(\sigma)$ is then defined as

$$-V(\sigma)\int d^4x=\Gamma\left(\frac{\sigma}{g}\right) \tag{5.6}$$

Let σ_{min} be the solution of

$$\left.\frac{dV(\sigma)}{d\sigma}\right|_{\sigma=\sigma_{min}}=0 \tag{5.7}$$

Hence, we have that[6]

$$\sigma=\sigma_{min}\Rightarrow J=0 \tag{5.8}$$

Invoking (5.8), we derive from (5.5)-(5.6)

$$\left.\frac{d}{d\alpha}V(\sigma)\right|_{\sigma=\sigma_{min}}\int d^4x=\left.\frac{d}{d\alpha}\mathcal{W}(J)\right|_{J=0} \tag{5.9}$$

Finally, combining (5.4) and (5.9), we conclude that

$$\left.\frac{d}{d\alpha}V(\sigma)\right|_{\sigma=\sigma_{min}}=0 \tag{5.10}$$

Some extra words concerning (5.8) and its consequences (5.9)-(5.10) are in order. Obviously, this is based on the relation

$$\frac{\delta}{\delta\left(\frac{\sigma}{g}\right)}\Gamma=-J \tag{5.11}$$

---

[6]To have (5.8) correct at any order in g^2, the minima should be computed correctly, as explained in the previous section.

An explicit evaluation of the effective potential results in a series for $V(\sigma)$, and consequently in a gap equation via (5.7). Said otherwise, $J = 0$ means in practice that J equals zero up to a certain order in g^2 as a consequence of the solved gap equation, which is of the form

$$V_0(\sigma) + V_1(\sigma)g^2 + \ldots + V_{n-1}(\sigma)\left(g^2\right)^{n-1} = 0 \tag{5.12}$$

Returning to (5.4), the terms proportional to J are themselves some series in g^2. This means that the product of such a term with J is again a series, which has to be cut off at the considered order; thus some terms are dropped. When (5.11)-(5.12) are used, it turns out that the product of such a term with J is also zero, but up to terms of higher order. Henceforth, the gauge parameter independence is *not exact*, but holds up to terms of higher order. The same holds true for the BRST charge Q_{BRST}, which will not be exactly nilpotent, but again up to higher order terms. As it is well known, Q_{BRST} is used to define physical states as those annihilated by Q_{BRST} and which are not exact (i.e. $\neq Q_{BRST}\,|\text{something}\rangle$). The nilpotency of Q_{BRST} is needed to move freely in the space of gauge parameter choices. With all this in mind, the α derivative of the action is reduced to an exact BRST variation. This is the usual argument used to show that physical operators, including the vacuum energy, are independent of the choice for the gauge parameter α [22]. We underline again that here, all this is not exact, but only valid up to terms of higher order.

Concluding this section, we have shown that the effective potential, evaluated at its minimum (i.e. the vacuum energy), is gauge parameter independent at any order in a loop (g^2) expansion, at least up to terms that are of higher order.

6 Evaluation of the 1-loop effective potential

In order to evaluate the 1-loop effective potential, it is sufficient to consider only the quadratic terms of S_σ, namely

$$S_\sigma^{quad} = \int d^4x \left(-\frac{\sigma^2}{2\xi g^2} + \overline{c}^a \Sigma^{ab} c^b + \frac{1}{2} A^{a\mu} \Omega_{\mu\nu}^{ab} A^{b\nu} \right) \tag{6.1}$$

where

$$\Sigma^{ab} = \delta^{ab} \left(\partial^2 + \frac{\sigma\alpha}{g\xi} \right) \tag{6.2}$$

and

$$\Omega_{\mu\nu}^{ab} = \delta^{ab} \left[\left(\partial^2 + \frac{\sigma}{g\xi} \right) g_{\mu\nu} - \left(1 - \frac{1}{\alpha} \right) \partial_\mu \partial_\nu \right] \tag{6.3}$$

To calculate V, we use the background formalism with the trivial background $A_\mu = 0$. This means that we restrict ourselves to the pure short-range contributions to $\langle \mathcal{O} \rangle$. If we would like to include long-range effects, we could for example use an instanton background [3]. An asset of considering only short-range contributions is that one does not have to

worry about Gribov ambiguities, since these short-range contributions are calculated with a purely perturbative expansion, and perturbation theory is not affected by Gribov copies, since the considered distances are "too short" to make different gauge copies aware of each other [28, 29, 30, 31].

For the 1-loop effective potential we get

$$V_1(\sigma) = \frac{\sigma^2}{2\xi_0}\left(1 - \frac{\xi_1}{\xi_0}g^2\right) + i\ln\det\Sigma^{ab} - \frac{i}{2}\ln\det\Omega^{ab}_{\mu\nu} \tag{6.4}$$

In d dimensions, it holds that

$$\ln\det\delta^{ab}\left[g_{\mu\nu}\left(\partial^2 + m^2\right) - \left(1 - \frac{1}{\alpha}\right)\partial_\mu\partial_\nu\right]$$
$$= \left(N^2 - 1\right)\left[(d-1)\mathrm{tr}\ln\left(\partial^2 + m^2\right) + \mathrm{tr}\ln\left(\frac{\partial^2}{\alpha} + m^2\right)\right] \tag{6.5}$$

Working up to order ε^0 and order g^2, we find

$$i\ln\det\Sigma^{ab} = i\left(N^2 - 1\right)\int\frac{d^dk}{(2\pi)^d}\ln\left(-k^2 + \frac{\sigma\alpha}{g\xi}\right)$$
$$= -\frac{(N^2-1)}{32\pi^2}\left(\frac{g^2\sigma^2\alpha^2}{\xi_0^2}\right)\left(\ln\frac{g\sigma\alpha}{\xi_0\overline{\mu}^2} - \frac{3}{2} - \frac{2}{\varepsilon}\right) \tag{6.6}$$

$$-\frac{i}{2}\ln\det\Omega^{ab}_{\mu\nu} = -\frac{i}{2}\left(N^2 - 1\right)\int\frac{d^dk}{(2\pi)^d}\left[(d-1)\ln\left(-k^2 + \frac{\sigma}{g\xi}\right)\right.$$
$$+ \left.\ln\left(-\frac{k^2}{\alpha} + \frac{\sigma}{g\xi}\right)\right]$$
$$= \frac{3\left(N^2-1\right)}{64\pi^2}\left(\frac{g^2\sigma^2}{\xi_0^2}\right)\left(\ln\frac{g\sigma}{\xi_0\overline{\mu}^2} - \frac{5}{6} - \frac{2}{\varepsilon}\right)$$
$$+ \frac{(N^2-1)}{64\pi^2}\left(\frac{g^2\alpha^2\sigma^2}{\xi_0^2}\right)\left(\ln\frac{g\alpha\sigma}{\xi_0\overline{\mu}^2} - \frac{3}{2} - \frac{2}{\varepsilon}\right) \tag{6.7}$$

Subsequently, we obtain for the one-loop effective potential in the $\overline{MS}$ scheme[7]

$$V_1(\sigma) = \frac{\sigma^2}{2\xi_0}\left(1 - \frac{\xi_1}{\xi_0}g^2\right) + \frac{3\left(N^2-1\right)}{64\pi^2}\left(\frac{g^2\sigma^2}{\xi_0^2}\right)\left(\ln\frac{g\sigma}{\xi_0\overline{\mu}^2} - \frac{5}{6}\right)$$
$$- \frac{(N^2-1)}{64\pi^2}\left(\frac{g^2\sigma^2\alpha^2}{\xi_0^2}\right)\left(\ln\frac{g\alpha\sigma}{\xi_0\overline{\mu}^2} - \frac{3}{2}\right) \tag{6.8}$$

with ξ_0 given by (4.12). In principle, as soon one knows the value of ξ_1, one can set $\overline{\mu}^2 = \frac{\sigma}{\sqrt{\xi_0}}$ and use the renormalization group equation for $V(\sigma)$ to sum leading logarithms

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[7]It is easily checked that using the renormalized version of the Hubbard-Stratonovich transformation (4.15), the counterterm proportional to $\delta\xi$ removes the infinities coming from (6.6) and (6.7).

and solve the gap equation. This leads to a value for the vacuum energy E, gluon mass m_{gluon}, and through the trace anomaly, one also finds an estimation for $\left\langle \frac{\alpha_s}{\pi} F^2 \right\rangle = -\frac{32}{11} E$. Since the aim of this paper is merely to describe the mass generation *mechanism* in the CF gauge, we do not perform the 2-loop calculation leading to ξ_1 and corresponding numerical values. Moreover, since the vacuum energy is gauge parameter independent, we may choose a specific α. Therefore, we restrict ourselves to the case $\alpha = 0$, for which ξ_1 has already been determined [13].

The Landau gauge is by far the most interesting choice. It is a fixed point of the renormalization group for the gauge parameter at any order. Due to the transversality condition $\partial_\mu A^\mu = 0$, it is a quite physical gauge. It has some interesting non-renormalization properties [22]. Even more interesting is the already mentioned fact that $\mathcal{O}$ reduces to A^2, which has a gauge-invariant meaning in the Landau gauge, since it equals $(VT)^{-1} \min_U \int d^4x \, (A^2)^U$, a gauge-invariant (however in general non-local) operator[8]. As a consequence, the gauge invariance[9] of the formalism is more obvious in the Landau gauge [13]. The relevance of the Landau gauge has also been pointed out from a more topological point of view [2]. In case of compact 3-dimensional QED, A^2 was shown to be an order parameter for the monopole condensation [1, 2]. If monopole condensation has something to do with confinement, there might exist a relation between A^2 and confinement in case of QCD too. All these things are less clear in the case of the $\mathcal{O}$ operator in the CF gauge.

Having said all this, it might look like that our efforts are not that important for $\alpha \neq 0$. This is however not the case. We have given a consistent framework to *calculate* the dynamically generated gluon mass for the CF gauge. Notice that the obtained Lagrangian in the condensed vacuum is however not the one of the Curci-Ferrari model [17, 18]. The question, also posed in [13], is if the dynamically massive YM action (4.17) breaks unitarity? From a pragmatic point of view, a possible lack of unitarity in the gluon sector should not be considered very problematic. After all, since gluons are not observables due to confinement, massive gluons are a fortiori unphysical. In fact, a deep connection might exist between massive gluons and confinement, as it was explored in [33]. See [34] for an attempt to construct a string theory incorporating a $\langle A^2 \rangle$ condensate.

We notice that the action (2.1) can be rewritten as

$$S = S_{YM} + s\overline{s} \int d^4x \left(\frac{1}{2} A_\mu^a A^{\mu a} - \frac{\alpha}{2} c^a \overline{c}^a \right) \tag{6.9}$$

with[10]

$$\overline{s} A_\mu^a = -D_\mu^{ab} \overline{c}^b$$

---

[8]Although this correspondence is somewhat troubled by Gribov copies [32], but this is of no relevance in the presented approach.

[9]Which is in fact a stronger statement than gauge parameter independence.

[10]We disregard S_{LCO} here.

$$\overline{s}c^a = \frac{g}{2}f^{abc}\overline{c}^b\overline{c}^c$$
$$\overline{s}c^a = -b^a + gf^{abc}c^b\overline{c}^c$$
$$\overline{s}b^a = -gf^{abc}b^b\overline{c}^c \qquad (6.10)$$

Another very interesting renormalizable gauge is the modified Maximal Abelian gauge (MAG) [35], particularly useful in the context of the dual superconductivity mechanism for confinement. This gauge partially fixes the local $SU(N)$ freedom, i.e. up to the Abelian degrees of freedom. The MAG shares a close similarity with the CF gauge, since its gauge fixing is given by

$$S = S_{YM} + s\overline{s}\int d^4x \left(\frac{1}{2}A_\mu^{a'}A^{\mu a'} - \frac{\alpha}{2}c^{a'}\overline{c}^{a'}\right) \qquad (6.11)$$

where the accent means that the color index runs strictly over the non-Abelian degrees of freedom. In particular, in [23] it has been shown that the remaining Abelian degrees of freedom can be fixed so that the resulting theory displays a global $SL(2, R)$ symmetry, in complete analogy with the CF gauge. Furthermore, due to the similarity (6.9)-(6.11), it is not difficult to show that a quite analogous treatment with a source J coupled to the $U(1)^{N-1}$ invariant operator

$$\mathcal{O}' = \frac{1}{2}A_\mu^{a'}A^{\mu a'} + \alpha\overline{c}^{a'}c^{a'} \qquad (6.12)$$

will provide us with a dynamical mass for the off-diagonal gluons and ghosts [11, 23, 36, 37], a hint for some kind of Abelian dominance [38]. This strategy for the MAG was already put forward in [11]. Just as the operator $\mathcal{O}$ is multiplicatively renormalizable in the CF gauge, the operator $\mathcal{O}'$ will be multiplicatively renormalizable in the MAG [37]. So far for the similarities between CF and MAG. Although it would be nice to stretch the similarity further and simply put $\alpha = 0$ from the beginning, in which case the MAG reads in differential form $D_\mu^{a'b'}A^{\mu b'} = 0$ with $D_\mu^{a'b'}$ the $U(1)^{N-1}$ Abelian covariant derivative. As such, we have some kind of $U(1)^{N-1}$ invariant version of the Landau gauge. Unfortunately, the limit $\alpha \to 0$ is now far from being trivial [39]. Moreover, $\alpha = 0$ is not a fixed point of the renormalization group [39, 40]. Also, although for $\alpha = 0$ the tree level action (6.11) does not contain a 4-ghost interaction, radiative corrections will reintroduce this interaction [35], unlike the Landau gauge. Making a long story short, we are forced to let the gauge parameter α free and perform a similar analysis as done in the previous sections. At the end of such a more general analysis, one could investigate if the limit $\alpha \to 0$ can be taken.

Before we formulate our conclusion, we quote the results obtained for the Landau gauge in [13]

$$\xi_1 = \frac{161}{52}\frac{N^2 - 1}{16\pi^2}$$
$$\left.\frac{g^2N}{16\pi^2}\right|_{1\text{-loop}} = \frac{36}{187}$$
$$m_{\text{gluon}} \approx 485\text{MeV for } N = 3$$

14

$$
\begin{aligned}
E &\approx -0.001 \mathrm{GeV}^4 \text{ for } N = 3 \\
\left\langle \frac{\alpha_s}{\pi} F^2 \right\rangle &\approx 0.003 \mathrm{GeV}^4 \text{ for } N = 3
\end{aligned}
\tag{6.13}
$$

As the relevant expansion parameter, i.e. $g^2 N / 16 \pi^2$, is relatively small and results do not change much if the second loop correction to $V(\sigma)$ is included [13], qualitatively acceptable results are achieved. The value for the 1-loop dynamical gluon mass m_{gluon} is also in qualitative agreement with lattice values [6, 7], reporting something like $m_{\mathrm{gluon}} \sim 600$ MeV.

7 Conclusion

In this paper, we have constructed a renormalizable effective potential for the on-shell BRST invariant local composite operator of mass dimension 2 in the Curci-Ferrari gauge, namely $\mathcal{O} = \frac{1}{2} A^a_\mu A^{\mu a} + \alpha \overline{c}^a c^a$. This gauge reduces to the Landau gauge in the limit $\alpha = 0$. It is worth underlining that, in the Landau gauge, the operator $\mathcal{O}$ equals the gauge invariant operator A^2. Much attention has been paid recently to the condensate $\langle A^2 \rangle$. The generalization to $\alpha \neq 0$ has also its importance due to the close analogy with the Maximal Abelian gauge, where the $\alpha \to 0$ limit is not as obvious as in case of the CF gauge. In particular, we have shown that the vacuum energy obtained in the presented formalism for the CF gauge is independent from the gauge parameter α. As already underlined the α-independence has to be understood in a g^2 expansion and up to terms of higher order.

We restricted ourselves in this paper to the on-shell BRST invariant condensate resulting in a mass for the particles. A gluon mass modifies the behaviour of the gluon propagator in the infrared (see e.g. [6]) and might be relevant for the confinement problem. A more intensive study would also include the pure ghost condensates, also of mass dimension 2, discussed in [23, 27, 36, 38, 39, 41, 42]. These are not directly related to the mass generation for the gluons [23, 36], but are relevant for the $SL(2, R)$ symmetry and can modify the ghost propagator.

Acknowledgments

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