

More on Singularity Resolution

Amanda W. Peet

ABSTRACT. String theoretic resolution of classical spacetime singularities is discussed. Particular emphasis is on the use of brane probes, and the connection [1] of the enhançon phenomenon [2] to the $n=2*$ Pilch-Warner flow spacetime [3]. Some comments and details on the singularity of the PW spacetime are added. For the proceedings of Strings 2001, Mumbai, India.

1. Introduction

Black holes have long been sources of puzzles in theoretical physics. When treated semiclassically, black holes have two problematic aspects: their event horizons give rise to information loss, and their curvature singularities point to breakdown of the theory. A quantum theory of gravity is clearly needed to address both problems.

Classical spacetime singularities provide a valuable testing ground for a quantum gravity theory [4]. Singularities must be dealt in one of two ways: (a) *resolution* via some short-distance degrees of freedom not encoded in classical gravity, or (b) *prevention*. The existence of possibility (b) is important and oft forgotten. The essential physics behind it is that some singular classical geometries are so sick that the quantum theory should not allow them ab initio - or allow them to form starting from physical initial conditions. A simple example of a sick spacetime is Schwarzschild with mass $M < 0$. A resolution, no matter how innocuous it might look locally, would violate stability of the vacuum. Supersymmetry forbids it: GR can be embedded into $n=1$ supergravity and a Bogomolnyi bound $M \geq 0$ derived [5].

Here, quantum gravity means string theory, the low-energy limit of which is supergravity. Construction of solutions of the highly nonlinear equations of motion of supergravity, such as black holes and branes, is typically difficult. Any given search can be aided by a no-hair theorem. This guarantees uniqueness once the conserved quantum numbers associated to the spacetime are fixed, provided that cosmic censorship is not violated. If a singularity is naked, however, there is *no* guarantee that a spacetime solving the equations of motion is the correct one.

In supergravity, all types of spacetime singularities are encountered: cosmically censored and naked; spacelike, null, and timelike. It is generally hard to learn the quantum resolution mechanism for any given

classical singularity. One reason is that a spacetime which appears singular in a lower dimension may in fact become nonsingular when lifted to higher dimension [6]. For this reason it is best to analyse singularities in *ten* dimensions. Another word of warning is that whether or not a delta-function source is necessary in the right-hand side of an equation of motion can be coordinate-dependent.

String theory knows what to do in extreme regimes, and so classical spacetime singularities apparently occurring in spacetime geometries arising from fundamental objects such as strings and D-branes must have sensible resolutions. It is therefore of considerable interest to investigate geometries arising from combinations of fundamental ingredients, and much work has been done which cannot possibly be reviewed comprehensively here. The enhançon mechanism [2] was one notable example of singularity resolution; others included e.g. [7] via dielectric-brane expansion, [8] via a supergravity resolution into fluxes and [9] via geometric transitions. In the following, systems with $n=2$ supersymmetry will be the focus of attention.

The enhançon phenomenon [2] arose in the $n=2$ supersymmetric context of D-branes wrapped on K3 but it has other realisations. Consider for example N_5 D5-branes wrapped on a K3 surface. In this spacetime, the K3 volume decreases with decreasing radius. (This behaviour is to be contrasted with the case where $N_1 > N_5$ D1-branes are added, giving rise to $\text{AdS}_3 \times \text{S}^3 \times \text{K3}$ spacetime and fixed K3 volume in the interior.) At the enhançon radius, the K3 volume goes to its self-dual value, and the tension of a wrapped D-probe vanishes giving an enhanced gauge symmetry. The D-probe cannot go further in. The N_5 source-branes have an enhançon radius proportional to N_5 , and by induction it can be argued that they can never get close enough to allow a naked singularity to form. Instead, they live on a spherical shell, by Gauss's law spacetime is flat inside, and the would-be singularity is excised. In the realisation and regime of parameter space where there is a good gauge theory dual, the enhançon can be seen as a nonperturbative effect by analysing the Seiberg-Witten curve.

Although the enhançon mechanism of singularity resolution arises as an essentially stringy phenomenon, in fact supergravity already knows about it. The precise details of the excision story from the point of view of supergravity jump conditions were studied in [10]. Since there is a moduli space for motion of a probe-brane in the overall transverse directions, the shell of branes can also live at any radius greater than the enhançon radius. The jump conditions for all supergravity fields then prove to be self-consistently satisfied by D-brane sources with tension exactly as appropriate to the running K3 volume. Supergravity does not allow the branes to be packed closer than the enhançon radius: this would require unphysical negative tension.

Some details on non-extremal deformations of the enhançon are also contained in [10].

2. The $n=2*$ Spacetime: Symmetries and Singularities

In the context of the original $\text{AdS}_5/\text{CFT}_4$ correspondence, many deformations were considered in which the CFT was perturbed by relevant or marginal operators, and the supergravity solution was changed correspondingly in the interior. In [3] the consistent truncation Ansatz was used to produce a proposal for a spacetime dual to the $n=2*$ flow, obtained by turning on vevs for the field theory operators $O_f = \text{Tr}(\lambda^3 \lambda^3 + \lambda^4 \lambda^4)$, \bar{O}_f and $O_b = \sum_{j=1}^4 \text{Tr}(X^j X^j) - 2 \sum_{j=5}^6 \text{Tr}(X^j X^j)$. The resulting spacetime, which is abbreviated here as PW, has a small number of integration constants. On the other hand, the moduli space for the $n=2$ gauge theory has $O(N)$ parameters, so the PW spacetime should correspond to a small subspace of that moduli space. The parameters of the PW solution are labelled γ and the $n=4$ -breaking parameter¹ $k \propto mL$, where m is the mass parameter and L the radius of curvature of the asymptotic AdS_5 and S^5 . To uncover the link to enhançon physics the parameter γ must be set to zero, the case of maximal breaking of $n=4$ in the PW solution. All expressions in the following will be written for $\gamma = 0$ only. Note that most important feature of the PW spacetime as compared to previous ‘flow’ geometries was that the $d=10$ dilaton-axion field varies with radius. All analysis will be done in $d=10$ to avoid confusion.

The $n=2*$ gauge theory has $SU(2) \times U(1)$ R-symmetry. This is easy to see by looking at the four Weyl fermions: $\lambda^{3,4}$ get mass and an $SO(2) = U(1)$ mixes them, while $\lambda^{1,2}$ are massless and mixed by an $SU(2)$. The 6 transverse scalars X^i transform as $(4 \times 4)_A$, and two of them are invariant under the R-symmetry. Since gauge theory symmetries are spacetime isometries, this condition on the X ’s gives rise to a fixed-plane in the spacetime where the radius of the transverse (squashed) sphere goes to zero. The R-symmetry does not act on the azimuthal angle φ ; the supergravity fields therefore can (and do) depend on it in complicated fashion. The spacetime possesses in addition an accidental $U(1)'$ symmetry in Einstein frame. This $U(1)'$ is a combination of a $U(1)$ subgroup of the R-symmetry group and a $U(1)$ subgroup of the supergravity S-duality group $SL(2, \mathbf{R})$.

The relation of the PW radial coordinate to the familiar $n=4$ isotropic coordinate r is given via

$$(2.1) \quad \rho^6 = c + \frac{1}{2} (c^2 - 1) \ln \left(\frac{c-1}{c+1} \right), \quad c = \cosh(kL/r).$$

Accordingly, $c \in [1, \infty)$.

¹ k is dimensionless; the proportionality constant will be fixed in section 4.

The PW spacetime has many fields turned on; the metric is in Einstein frame

$$(2.2) \quad \frac{ds_E^2}{L^2} = \frac{(cX_1X_2)^{1/4}}{\rho^3} \left\{ \frac{(k/L)^2 \rho^6}{c^2 - 1} dx_{\parallel}^2 + \frac{1}{\rho^6 (c^2 - 1)^2} dc^2 + \left[\frac{1}{c} d\vartheta^2 + \frac{\sin^2 \vartheta}{X_2} d\varphi^2 + \frac{\rho^6 \cos^2 \vartheta}{4} \left(\frac{1}{X_1} d\psi^2 + \frac{1}{cX_2} d\alpha^2 + \frac{2 \cos \psi}{cX_2} d\alpha d\beta + d\beta^2 \left(\frac{\sin^2 \vartheta}{X_1} + \frac{\cos^2 \vartheta}{cX_2} \right) \right) \right] \right\} \equiv \left[\frac{\sqrt{cX_1X_2}}{\rho^6} \right]^{1/2} \left\{ ds^2 \right\}.$$

where

$$(2.3) \quad X_1 = \cos^2 \vartheta + c\rho^6 \sin^2 \vartheta, \quad X_2 = c \cos^2 \vartheta + \rho^6 \sin^2 \vartheta.$$

Meanwhile, the R-R self-dual 5-form field strength can be written

$$(2.4) \quad F_{(5)} = f + *f, \quad f = 4dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dw(c, \vartheta),$$

and both the R-R and NS-NS 3-form fields strengths are turned on in directions transverse to the D3-branes only. The dilaton-axion field τ runs:

$$(2.5) \quad \tau = \frac{\tau_0 - \bar{\tau}_0 B}{1 - B}, \quad \text{where} \quad B = e^{2i\varphi} \frac{\sqrt{cX_1} - \sqrt{X_2}}{\sqrt{cX_1} + \sqrt{X_2}},$$

and $\tau_0 = i/g_s + \theta_s/(2\pi)$ is the asymptotic coupling.

It is interesting to perform an analysis of the singularity of this spacetime. The above metric (2.2) is in Einstein frame. For string probe physics the string frame metric is needed, and the conversion $g_{\mu\nu}^S = e^{\Phi/2} g_{\mu\nu}^E$ involves only the $d=10$ dilaton. Study of (2.5) gives

$$(2.6) \quad e^{\Phi} = \frac{(cX_1 + X_2)}{2\sqrt{cX_1X_2}} - \cos(2\varphi) \frac{(cX_1 - X_2)}{2\sqrt{cX_1X_2}}.$$

Note that this is normalised to unity far out at $c = 1$. Large string loop corrections then occur when the dilaton is large, and it is not hard to see that this occurs on $\vartheta = \pi/2$ at large- c .

To compute curvatures in either frame it is simplest to separate off the conformal factor and use the formula for $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$

$$(2.7) \quad \tilde{R} = \frac{1}{\Omega^2} [R - 2(d-1)\nabla^2 \ln \Omega - (d-1)(d-2)(\nabla \ln \Omega)^2].$$

Now, the conformal factors entering the scalar curvature formula (2.7) are, for Einstein and string frame respectively,

$$(2.8) \quad \Omega_E^4 = \frac{\sqrt{cX_1X_2}}{\rho^6}, \quad \Omega_S^4 = \frac{1}{2\rho^6} [(cX_1 + X_2) - \cos(2\varphi)(cX_1 - X_2)].$$

The Ricci scalar curvature for the plain metric, the part inside the $\{\}$ of (2.2), can be easily found. Expressions for this and other ingredients

are cumbersome but simplify somewhat in the large- c limit². At large- c , $R \sim cf_0(\vartheta)$ generically and $R \sim 30c^3$ on $\vartheta = \pi/2$.

In the large- c limit, the Einstein conformal factor behaves as $\Omega_E^4 \sim c^2 f_1(\theta)$ generically and $\Omega_E^4 \sim c$ on $\vartheta = \pi/2$. For the string frame at large- c , $\Omega_S^4 \sim c^2 f_2(\vartheta, \varphi)$ generically and $\Omega_S^4 \sim 1$ on $(\vartheta = \pi/2, \varphi = 0)$. Thus the $1/\Omega^2$ suppression in the curvature in either Einstein or string frame, at large- c , is less powerful on the above non-generic loci, and this will result in more singular behaviour there. The other contributions to the Ricci scalar involve derivatives of the conformal factor. For the Einstein frame, the d'Alembertian term scales as $cf_3(\vartheta)$ generically and $\frac{3}{2}c^3$ on $\vartheta = \pi/2$; the gradient-squared term scales as $cf_4(\vartheta)$ for all ϑ . In string frame, the d'Alembertian term scales as $cf_5(\vartheta, \varphi)$ generically and $3c^3$ on $(\vartheta = \pi/2, \varphi = 0)$; the gradient-squared term scales as $cf_6(\vartheta, \varphi)$ generically and vanishes (!) on $(\vartheta = \pi/2, \varphi = 0)$.

Overall, for the large- c scalings, the Einstein frame results are

$$(2.9) \quad R_E \rightarrow \begin{cases} f_7(\vartheta), \vartheta \neq \pi/2 \\ 3c^{5/2}, \vartheta = \pi/2 \end{cases}$$

while in string frame the results are

$$(2.10) \quad R_S \rightarrow \begin{cases} f_8(\vartheta, \varphi), \vartheta \neq \pi/2, \varphi \neq 0 \\ c^2 f_9(\varphi), \vartheta = \pi/2, \varphi \neq 0 \\ -24c^3, \vartheta = \pi/2, \varphi = 0 \end{cases}$$

Therefore, curvature invariants are finite generically but blow up as $c \rightarrow \infty$ on the locus $\vartheta = \pi/2$, signalling the need for α' corrections.

3. Brane Probes and Supergravity

Generically, D-brane probes feel different physics than gravitons or other excitations of supergravity fields, because they couple differently. In the case of the original enhançon phenomenon, the physics became clearest upon probing with a brane identical to the source-branes. In particular, it became manifest that the source-branes could not get close enough together to allow formation of a naked classical singularity.

The same ideas can now be used on the PW spacetime, in order to find the connection to the original enhançon phenomenon, and to uncover the constituent structure of the source-branes giving rise to the spacetime. The natural Ansatz is to take the constituents to be D3-branes with *no* higher-brane multipoles, with a distribution to be calculated. Again, to avoid confusion, the probe analysis must be done

²In all large- c scalings presented here there appears in various contributions to the Ricci scalar a ratio of (sometimes rational powers of) polynomials of angles, which cannot be expanded here for lack of space. These functions are denoted f_i and have nothing to do with f in the R-R five-form.

in $d=10$. Details not shown explicitly here may be found in the original paper [1] (related work is in [11]).

The D p -brane probe action is, in general,

$$(3.1) \quad S_{\text{probe}} = -\frac{\mu_p}{g_s} \int d^{p+1}\xi \, e^{-\Phi} \sqrt{\det(\mathbf{P}[g+B]_{ab} + 2\pi\ell_s^2 F_{ab})} \\ + \mu_p \int \mathbf{P} \exp(2\pi\ell_s^2 F_{(2)} + B_{(2)}) \wedge \oplus_n C_{(n)},$$

where \mathbf{P} denotes pullback of bulk fields to the brane worldvolume. It is simplest to work in static gauge. Now, probe physics in the PW background is simpler than it first appears because the $\mathbf{P}B_{(2)}$, $\mathbf{P}C_{(2)}$, $F_{(2)}$ terms vanish, and the cross-terms in $\mathbf{P}F_{(5)}$ are absent.

The potential energy function vanishes on two loci. The first is the equator of the S^5 , i.e. $\vartheta = \pi/2$. The second is the special radius $\rho^6 = 0$; the coordinate $c \rightarrow \infty$ there. Two coordinate patches are needed for the fixed-plane required by the R-symmetry: the (ρ, ϑ) plane and (ϑ, φ) with identification $\vartheta \simeq \pi - \vartheta$. This gives a two-dimensional moduli space, in accordance with expectations from the $n=2$ gauge theory.

The physics becomes clearer when a change is made to a coordinate system appropriate to the $n=2$ structure. This involves writing the kinetic energy on the moduli space as

$$(3.2) \quad T(Y) = \tfrac{1}{2}\tau_3 e^{-\Phi} v^Y v^{\bar{Y}},$$

for some complex field Y . The transformation is

$$(3.3) \quad Y = \frac{kL}{2} \left(z + \frac{1}{z} \right), \quad \text{where } z = e^{-i\varphi} \sqrt{(c+1)/(c-1)}.$$

In these coordinates, the dilaton-axion field becomes

$$(3.4) \quad \tau(Y) = \frac{i}{g_s} \sqrt{\frac{Y^2}{Y^2 - k^2 L^2}} + \frac{\theta_s}{2\pi}.$$

This is, as expected, a holomorphic function of Y . The probe's kinetic energy vanishes on a particular locus, signalling the presence of an enhançon. This locus is a *line segment*, going from $Y = -kL$ to $Y = +kL$, along the branch cut³. In coordinates appropriate to the $n=2$ structure, then, the enhançon is a line segment and not a circle.

If γ had been kept as a parameter here, it would be easy to show that as $\gamma \rightarrow -\infty$, the line segment unsquashes and the brane distribution goes over to a disc, as appropriate to the $n=4$ Coulomb branch problem. Note also that this line segment is not the $n=2^*$ limit of the $n=1^*$ story of [7]; a different perturbation is considered here.

³Notice that in (3.4) it may appear at casual glance that the dilaton is zero on the enhançon locus. In fact, the Y -dependent part of the expression for $\tau(Y)$ has a real part, and once this is separated off it is easy to see that the imaginary piece does in fact exhibit the expected dilaton-blowup behaviour.

4. Gauge Theory Connection and Brane Distribution

One advantage of working in coordinates appropriate to the $n=2$ structure is that it becomes easy to extract the distribution of branes using

$$(4.1) \quad \tau_{\text{SUGRA}} = \tau_{\text{SYM}}.$$

In computing the coupling function τ_{SYM} for a Coulomb branch configuration, it is best to begin by recalling from the Seiberg-Witten story that quantum mechanically $SU(N)$ is always broken down to $U(1)^{N-1}$. The vevs in energy units, $a \equiv Y/(2\pi\ell_s^2)$, can then be parametrised as $\text{diag}(\{a_i\})$, where $\sum_i a_i = 0$. The next step is to compute the prepotential F for the $N-1$ abelian vector multiplets,

$$(4.2) \quad F = F_{\text{classical}} + F_{\text{pert}} + F_{\text{nonpert}}.$$

The perturbative correction to the classical prepotential is 1-loop exact, and is computed by integrating out the charged fields (“W”-bosons). The nonperturbative part is generated by instantons and is difficult to calculate for large- N , but it is important only when there are light BPS states, i.e. when the eigenvalue spacings are smaller than order $1/N$. Such effects are exponentially suppressed in the supergravity regime where N is large. Nonperturbative corrections turn on sharply at the enhançon locus, where the spacing between the eigenvalue representing the probe and those representing the source-branes can in fact become close.

To find the perturbative part of the prepotential, place a probe at distance u . Then the vevs are changed to $\text{diag}(u, \{a_i - u/N\})$, giving F_{pert} and thence

$$(4.3) \quad \tau_{\text{SYM}}(u) = \frac{i}{g_s} + \frac{\theta_s}{2\pi} + \frac{i}{2\pi} \sum_i \ln \left[\frac{(u - a_i - u/N)^2}{(u - a_i - u/N)^2 - m^2} \right].$$

The $n=4$ breaking is small in the $n=2^*$ solution of PW. This means that the logarithm can be Taylor expanded. In the large- N limit, the sum can be approximated by an integral, and the normalised brane distribution extracted by matching to supergravity:

$$(4.4) \quad \rho(u) = \frac{2}{m^2 g_s} \sqrt{a_0^2 - u^2},$$

where the size of the enhançon (in energy units) is

$$(4.5) \quad a_0 = kL = mL^2 = m\sqrt{\frac{g_s N}{\pi}}.$$

In the supergravity approximation, $g_s N$ is large, and so the size of the enhançon is much greater than the breaking parameter m .

As emphasised in [1], this general method of analysis, matching gravity and gauge theory τ functions, may provide clues to the supergravity solution representing a more general gauge theory flow. Work along these lines was undertaken in [12]; some subtleties remain to be understood.

5. Outlook

In both the enhançon and the PW spacetimes, the supergravity description turns out to be strongly coupled at the enhançon locus; it is not clear whether there is any weakly coupled description for the physics there. The original enhançon spacetime can also be studied without taking the decoupling limit, and in that case the brane expansion mechanism of singularity resolution can occur in a regime where the supergravity description is everywhere weakly coupled. Some progress was made in [10] for finite temperature in that $n=2$ system, and more work is still needed.

Going to finite temperature in systems exhibiting gravity/gauge duality can be simpler than in the general case, because the gauge theory may guide expectations on the gravity side. Some progress has been made in this way on gravity duals at finite temperature without naked singularities, but only at high temperature [13]-[17]. It will be interesting to discover the mechanisms for singularity resolution in those systems at low temperature, and indeed for the more general cases.

To our knowledge, the only examples to date where singularity resolutions are understood explicitly involve timelike and null singularities. The case of spacelike singularities is harder to understand, and may be intertwined with the black hole information problem.

In the Lorentzian AdS/CFT correspondence, the CFT at finite temperature has been argued to correspond to an AdS-Schwarzschild black hole. Wick rotation in the bulk, something which does not appear to make sense in general in quantum gravity, can in AdS/CFT be defined via Wick rotation in the CFT [18]. It can then be argued that the region behind the horizon of an AdS-Schwarzschild black hole containing the spacelike singularity may not be represented in the CFT, because that region is absent in the Euclidean continuation. On the other hand, in [19] the opposite point of view was taken. There, for the (unexcited) AdS bulk it was pointed out that the conformal isometry of the bulk spacetime dictates working with global AdS and that this must include regions behind horizons. The CFT then lives on S^3 not \mathbf{R}^3 . This requirement changes the global structure of the spacetime, which then cannot be obtained as the near-horizon geometry of a bunch of D3-branes. Questions about brane renditions of bulk locality and diffeomorphism invariance remain.

Many more general questions may be asked. One question is whether maximal analytic extensions of supergravity geometries are physical. It is sometimes argued that such extensions are necessary in order to avoid conical singularities, but these are rather mild singularities in string theory. Another question is whether eternal black holes exist. (White holes probably do not exist because they are unstable to collapsing on very short timescales.) Another problem, related to black hole complementarity, is to understand the physics of an observer who has crossed the horizon and is falling toward a spacelike singularity. Cosmological singularities are more difficult yet as the spacetimes are time-dependent. Cosmological horizons also present present basic difficulties [20, 21, 22]. According to [23] the initial “big bang” singularity has a quite different character to a black hole interior even though their Carter-Penrose diagrams may look similar.

A proposal has been made very recently [24] that eternal black holes in AdS may be understood via a direct product of two entangled CFT’s. It will be interesting to understand whether this can be generalised.

Acknowledgements

Research of the author is supported by NSERC of Canada and the University of Toronto. The author thanks participants of the ITP conference “Avatars of M Theory”, Tom Banks, Raphael Bousso, Lenny Susskind and Arkady Tseytlin for useful interactions.

References

- [1] A. Buchel, A. W. Peet and J. Polchinski, Phys. Rev. D **63** (2001) 044009 [hep-th/0008076].
- [2] C. V. Johnson, A. W. Peet and J. Polchinski, Phys. Rev. D **61** (2000) 086001 [hep-th/9911161].
- [3] K. Pilch and N. P. Warner, Nucl. Phys. B **594** (2001) 209 [hep-th/0004063].
- [4] G. T. Horowitz and R. Myers, Gen. Rel. Grav. **27** (1995) 915 [gr-qc/9503062].
- [5] E. Witten, Commun. Math. Phys. **80** (1981) 381.
- [6] G. W. Gibbons, G. T. Horowitz and P. K. Townsend, Class. Quant. Grav. **12** (1995) 297 [hep-th/9410073].
- [7] J. Polchinski and M. J. Strassler, hep-th/0003136.
- [8] I. R. Klebanov and M. J. Strassler, JHEP **0008** (2000) 052 [hep-th/0007191].
- [9] F. Cachazo, K. Intriligator and C. Vafa, [hep-th/0103067].
- [10] C. V. Johnson, R. C. Myers, A. W. Peet and S. F. Ross, hep-th/0105077, accepted for publication in Phys. Rev. D.
- [11] N. Evans, C. V. Johnson and M. Petrini, JHEP **0010** (2000) 022 [hep-th/0008081].
- [12] J. Babington, N. Evans and J. Hockings, hep-th/0105235.
- [13] A. Buchel, Nucl. Phys. B **600** (2001) 219 [hep-th/0011146].
- [14] A. Buchel, C. P. Herzog, I. R. Klebanov, L. Pando Zayas and A. A. Tseytlin, JHEP **0104** (2001) 033 [hep-th/0102105].
- [15] S. S. Gubser, C. P. Herzog, I. R. Klebanov and A. A. Tseytlin, JHEP **0105** (2001) 028 [hep-th/0102172].
- [16] A. Buchel and A. Frey, hep-th/0103022.

- [17] C. P. Herzog and P. Ouyang, hep-th/0104069.
- [18] V. Balasubramanian, P. Kraus and A. E. Lawrence, Phys. Rev. D **59** (1999) 046003 [hep-th/9805171].
- [19] G. T. Horowitz and H. Ooguri, Phys. Rev. Lett. **80** (1998) 4116 [hep-th/9802116].
- [20] R. Bousso, JHEP **0011** (2000) 038 [hep-th/0010252].
- [21] W. Fischler, A. Kashani-Poor, R. McNees and S. Paban, hep-th/0104181.
- [22] S. Hellerman, N. Kaloper and L. Susskind, JHEP **0106** (2001) 003 [hep-th/0104180].
- [23] T. Banks and W. Fischler, hep-th/0102077; T. Banks, talk at ITP “Avatars of M Theory” conference, 5-8 June 2001, Santa Barbara, USA, http://online.itp.ucsb.edu/online/mtheory_c01/banks/
- [24] J. M. Maldacena, hep-th/0106112.

AUTHOR ADDRESS: UNIVERSITY OF TORONTO, DEPARTMENT OF PHYSICS,
60 ST. GEORGE ST, TORONTO, ONTARIO, CANADA M5S 1A7.