Cosmological Anisotropy and the Cycling of Universe

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Abstract

Abstract : In this paper, we analyze the anisotropy of the scale factor in the Kantowski-Sachs spacetime. We show that the anisotropy will not increase when the expansion rate is greater than certain values while it will increase when the expansion rate is less than that value or the Universe is contracting. It is manifested that the matter dominated and radiation dominated era favor the flat spacetime if the anisotropy does not develop significantly. The relation between the cosmological anisotropy and the red-shift of the supernovae, which could be used to verify the anisotropy through the observation, has been derived.

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1. Introduction

Recent results from WMAP satellite offer a lot of precise information about the Universe[1], while not yet resolve many problems, among which is that they can not be used to discriminate between cosmological models such as cyclic model and conventional Big Bang plus inflation model[2]. The cyclic model for the Universe has been proposed by Steinhardt and Turok[3], which is based on the spin-off of ekpyrotic scenario[4]whose starting point is the unified theory known as heterotic M-theory[5]. Comparing with the consensus model of Universe, the big bang was replaced with a phase transition from contraction phase to expansion phase in the new picture. The expansion period of the cycle contains a period of radiation and matter domination ensued by an extended period of cosmic acceleration at low energy, which establishes the flat and vacuous initial condition required for the ekpyrosis and for removing the entropy, black holes and other debris produced in the preceding cycle. One of the remarkable feature of cyclic model is that it avoids the unnatural magnitude difference (100 orders) of the two period of accelerating expansion, inflation and current observed cosmic acceleration. Also, it provides a new mechanism for the generation of a scale-invariant spectrum of density perturbation and an explanation for the big bang.

In this paper, we analyze the development of cosmological anisotropy during the expansion and contraction phase in Kantowski-Sachs Universe. Different from conventional study in this field[6], we try a new approach to investigate the evolution of anisotropy and find that in a typical power-law expansion, the anisotropy will increase rapidly if the expansion rate $a(t)$ is slower than $a(t) \sim t^{\frac{1}{3}}$ for the flat Universe, or $a(t) \sim t$ for open and closed Universe, or the Universe is undergoing an contraction. If $a(t)$ expands/contracts in an exponential manner, the anisotropy will decrease/increase exponentially. The radiation dominated and matter dominated era, which correspond to $a(t) \sim t^{\frac{1}{2}}$ and $a(t) \sim t^{\frac{2}{3}}$, will not produce significant anisotropy when and only when the Universe is spatially flat, which is in agreement with the predication of inflation theory[7] as well as the very recent WMAP observations[2].

2. The Evolution of Anisotropy

The anisotropic Robertson-Walker Universe has been widely studied [6] and the corresponding metric can be written as

$$ds^2 = -dt^2 + b^2(t)dr^2 + a^2(t)(d\theta^2 + S(\theta)^2 d\phi^2) \tag{1}$$

where

$$S(\theta) = \begin{cases} \sin\theta & for \qquad k = 1, \\ \theta & for \qquad k = 0, \\ \sinh\theta & for \qquad k = -1 \end{cases} \tag{2}$$

and $k = 1, 0, -1$ corresponds to the closed, flat and open universe. Conventional discussions on the above type of spacetime focus on the quantities know as usual expansion Θ, shear σ and 3-curvature ${}^{(3)}R$, which are defined as

$$\Theta = \frac{\dot{b}}{b} + \frac{2\dot{a}}{a}, \qquad \sigma = \frac{1}{\sqrt{3}}(\frac{\dot{b}}{b} - \frac{\dot{a}}{a}), \qquad {}^{(3)}R = \frac{2k}{a^2} \tag{3}$$

Those with zero and negative curvature are just axisymmetric Bianchi type I and III Universe while the positive curvature model, or the closed the anisotropic Universe model, is referred to as the Kantowski-Sachs Universe. Although only the closed models fall outside of the Bianchi classification, yet one can refer to them all as Kantowski-Sachs models for convenient[8]. In this paper, we try a new approach to this models and obtain some interesting results. The Einstein equations correspond to the above setup are:

$$(\frac{\dot{a}}{a})^2 + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{k}{a^2} = \kappa\rho \tag{4}$$

$$2\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2 + \frac{k}{a^2} = -\kappa p \tag{5}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} = -\kappa p \tag{6}$$

where $\kappa = 8\pi G$, p and ρ are the pressure and energy density of the perfect fluid respectively. The energy conservation of the perfect fluid is expressed as

$$\frac{d\rho}{dt} = -\Theta(\rho + p) \tag{7}$$

Substitute Eq.(6)into Eq.(5), we have

$$\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right) + \frac{k}{a^2} = 0 \tag{8}$$

To study the evolution of the anisotropy, it is convenient to introduce a new quantity δ that is defined as $\delta \equiv \frac{b-a}{a}$. Then, the metric tensor(1) can be written as

$$ds^2 = -dt^2 + a^2(t)(1+\delta)^2 dr^2 + a^2(t)(d\theta^2 + \theta^2 d\phi^2) \tag{9}$$

where δ can also be interpreted as a perturbation of the scale factor if it is small. Now, we can rewrite Eq.(8) in terms of δ as

$$\ddot{\delta} + 3\frac{\dot{a}}{a}\dot{\delta} - \frac{k}{a^2} = 0 \tag{10}$$

It is not difficult to find the first integral of Eq.(10) as

$$\frac{\dot{\delta}(t)}{\dot{\delta}(t_0)} = \frac{1}{\dot{\delta}(t_0)a^3(t)}\int_{t_0}^{t} ka(t)dt + \left[\frac{a(t_0)}{a(t)}\right]^3 \tag{11}$$

where $\dot{\delta}(t_0)$ is the initial "speed" of the anisotropy and $a(t_0)$ is the initial value of the scale factor at $t = t_0$. In the following, we investigate the generic evolution of the anisotropy in power law expansion and exponential expansion. For the power law expansion $a(t) = a(t_0)t^q$, the solutions for Eq.(11) are:

(i) If $q \neq \pm 1$ and $q \neq \frac{1}{3}$

$$\begin{aligned}\delta(t) = \quad & \delta(t_0) - \frac{kt_0^2}{(q+1)(2-2q)a^2(t_0)} - \left[\frac{\dot{\delta}(t_0)t_0}{1-3q} - \frac{kt_0^2}{(q+1)(1-3q)a^2(t_0)}\right] \\ & + \left[\dot{\delta}(t_0)t_0^{3q} - \frac{kt_0^{3q+1}}{(q+1)a^2(t_0)}\right]\frac{t^{1-3q}}{1-3q} + \frac{kt_0^{2q}t^{2-2q}}{(q+1)(2-2q)a^2(t_0)}\end{aligned} \tag{12}$$

(ii) If $q = -1$

$$\delta(t) = \delta(t_0) + \frac{kt_0^2}{16a^2(t_0)} - \frac{\dot{\delta}(t_0)t_0}{4} + \frac{kt^4}{16a^2(t_0)t_0^2}(4\ln\frac{t}{t_0} - 1) + \frac{\dot{\delta}(t_0)t^4}{4t_0^3} \tag{13}$$

(iii) If $q = 1$

$$\delta(t) = \delta(t_0) + \frac{1}{2}[\dot{\delta}(t_0)t_0 - \frac{kt_0^2}{2a^2(t_0)}] + \frac{kt_0^2}{2a^2(t_0)}\ln(\frac{t}{t_0}) - \frac{1}{2}[\dot{\delta}(t_0)t_0^3 - \frac{kt_0^4}{2a^2(t_0)}]t^{-2} \tag{14}$$

(iv) If $q = \frac{1}{3}$

$$\delta(t) = \delta(t_0) - \frac{9kt_0^2}{16a^2(t_0)} + \frac{9kt_0^{2/3}}{16a^2(t_0)}t^{4/3} + [\dot{\delta}(t_0)t_0 - \frac{3kt_0^2}{4a^2(t_0)}]\ln\frac{t}{t_0} \tag{15}$$

Now, let's briefly discuss the above solutions.

In case (i), one can easily find that when $k = 0$, the anisotropy δ will increase with time if $q < \frac{1}{3}$ while it will decrease if $q > \frac{1}{3}$. However, when $k = \pm 1$, the anisotropy δ will increase with time if $q < 1$ while it will decrease if $q > 1$. Clearly, the evolution of δ is different for closed Universe ($k = 1$) and open Universe ($k = -1$), and the difference is dependent on the choice of the initial $a(t_0)$, t_0, k and $\dot{\delta}(t_0)$.

In case (ii), one can find that δ will always increase with time but obviously in different manners, which are also dependent on the choice of the initial $a(t_0)$, t_0, k and $\dot{\delta}(t_0)$.

In case (iii), one can find that when $k = 0$, δ will always decrease with time. But when $k = \pm 1$, δ will always increase in different manners dependent on $a(t_0)$, t_0, k and $\dot{\delta}(t_0)$.

In case (iv), one can find that δ will always increase with time but in different manners for different $a(t_0)$, t_0, k and $\dot{\delta}(t_0)$.

It is also interesting to consider the anisotropy in the exponential expansion $a(t) = a(t_0)\exp[H(t - t_0)]$. It is not difficult to find that the δ evolves as

$$\delta(t) = \delta(t_0) + \frac{k}{6H^2a^2(t_0)} + \frac{\dot{\delta}(t_0)}{3H} - \frac{k}{2H^2a^2(t_0)} e^{-2H(t-t_0)} + \left[\frac{k}{3H^2a^2(t_0)} - \frac{\dot{\delta}(t_0)}{3H}\right] e^{-3H(t-t_0)} \tag{16}$$

It is quite clear that $\delta(t)$ will decrease with t if $a(t)$ expand exponentially, and it will increase if $a(t)$ contracts exponentially. So far, we can conclude that for the standard Big Bang cosmology, the cosmological anisotropy will increase during the radiation dominated and matter dominated era if the Universe is open or close. It can only survive for the spatially flat cosmology, which is an important prediction of inflation and confirmed by the recent WMAP data. In the subsequent section, we will devote our discussions mainly to the case in a spatially flat spacetime.

3. the Evolution of Anisotropy in Flat Spacetime and Cyclic Model

From Eq.(11), by setting $k = 0$, we have the first integral of Eq.(10) in flat Universe as

$$\frac{\dot{\delta}(t)}{\dot{\delta}(t_0)} = \left[\frac{a(t_0)}{a(t)}\right]^3 \tag{17}$$

From Eq.(17), we have $\frac{\dot{\delta}(t)}{\dot{\delta}(t_0)} > 0$, therefore $\delta(t)$ is a monotonic function. When $\dot{\delta}(t_0) > 0$, $\delta(t)$ increases monotonically; when $\dot{\delta}(t_0) < 0$, $\delta(t)$ decreases monotonically.

It does not lose generality that we assume $b(t_0) > a(t_0)$, i.e., $\delta(t_0) \geq 0$ for simplicity. The generalized scale factor $a(t)$ is determined by the pressure and energy density of the perfect fluid in the Einstein equations (4)-(6). For example, we can apply these equations specifically to the initial stage of cosmological evolution which is assumed to be governed by the ordinary scalar field or the rolling tachyon filed. In this paper, these equations are applicable to matter with energy momentum tensor of arbitrary form so that we should discuss the varied form of $a(t)$. From Eq.(17), we have

$$\delta(t) \leq \delta(t_0) + |\dot{\delta}(t_0)| \cdot | \int_{t_0}^{t} \left[\frac{a(t_0)}{a(t)}\right]^3 dt| \tag{18}$$

By using the consensus Cauchy criterion, we discuss the convergence of definite integral in Eq.(18). At $t \gg t_0$, we rewrite $a(t)$ as $\frac{f(t)}{t^q}$, if $q > \frac{1}{3}$ and $f(t) \leq constant \leq +\infty$, then the integral is convergent; If $q \leq \frac{1}{3}$ and $f(t) > constant \geq 0$, then the integral is divergent. This argument can be extended to the following form: if $\left[\frac{a(t_0)}{a(t)}\right]^3 = f(t) \cdot g(t)$ for $t \gg t_0$, $\int_{t_0}^{\infty} g(t)dt$ is a convergent integral and $f(t)$ is a monotonic and boundary function, then $I \equiv \int \left[\frac{a(t_0)}{a(t)}\right]^3 dt$ is a convergent integral. The observed expansion of the universe and the observed cosmic black body radiation provide the empirical basis for a Friedmann-Lemaitre-Robertson-Walker model of the universe, sometimes called the isotropic model. Therefore, we assume that the initial values of anisotropy must be tiny as compared with the scale factor, i.e., $\delta(t_0) \ll 1$ and $|\frac{\dot{b}}{b} - \frac{\dot{a}}{a}|_{t=t_0} \ll 1$. Therefore, the anisotropy degree $\delta(t)$ is still tiny at the late time when I is a convergent integral. We now have proved a lemma that is stated as follows:

Lemma 1. There is an anisotropic perturbation at $t = t_0$, which is described by $\delta(t_0)$ and $\dot{\delta}(t_0)$ in the isotropic universe with the scale factor $a(t)$. The anisotropy degree $\delta(t)$ is still tiny at the late time evolution when the expansion rate is greater than certain value while it will become large when the expansion rate is equal or smaller than that value or the universe is undergoing an contraction stage appeared in the cyclic model.

Now, let's consider the power law expansion of the scale factor $a(t) = a(t_0)\frac{t}{t_0}^q$. The cases $q = 1/2$, $q = 2/3$ and $q > 1$ represent the radiation dominated era, matter dominated era and accelerating expansion era respectively. The $q < 0$ case corresponds to the contraction phase in the cyclic universe model. Using $a(t) = a(t_0)\frac{t}{t_0}^q$, one can easily obtain

$$\delta(t) = \begin{cases} \delta(t_0) - \frac{\dot{\delta}(t_0)t_0}{1-3q} + \frac{\dot{\delta}(t_0)t_0^{3q}}{1-3q}t^{1-3q}, & q \neq \frac{1}{3} \\ \delta(t_0) + \dot{\delta}(t_0)t_0 \ln(\frac{t}{t_0}), & q = \frac{1}{3} \end{cases} \tag{19}$$

If the condition $q > 1/3$ is satisfied, we have

$$\delta(t_0) - \frac{|\dot{\delta(t_0)}|}{3q-1}t_0 \leq \delta(t) \leq \delta(t_0) + \frac{|\dot{\delta(t_0)}|}{3q-1}t_0 \tag{20}$$

so that the anisotropy degree $\delta(t)$ is small forever, and is specified by its initial value. In the radiation and matter dominated era as well as the accelerating expansion era, the expansion satisfy the condition $q > 1/3$. Therefore, the evolution of anisotropy degree has been so significantly damped that it could not be detected by current observation. When the expansion is slower than $t^{1/3}$, or the universe is contracting, the anisotropy degree will increase rapidly with time.

When the scale factor is in the exponential expansion or contraction $a(t) = a(t_0)e^{\pm H(t-t_0)}$, we will have

$$\delta(t) = \delta(t_0) \pm \frac{\dot{\delta}(t_0)}{3H}[1 - e^{\pm 3H(t_0-t)}] \tag{21}$$

where H is the Hubble constant and the plus and minus signs represent the expansion and contraction respectively. It is obvious that the anisotropy degree $\delta(t)$ satisfies

$$\delta(t_0) - \frac{|\dot{\delta(t_0)}|}{3H} \leq \delta(t) \leq \delta(t_0) + \frac{|\dot{\delta(t_0)}|}{3H} \tag{22}$$

when the scale factor is expanding exponentially, while the anisotropy degree will increase rapidly with time when the scale factor contracts exponentially.

In cyclic model, the cyclic scenario can be effectively described in terms of the evolution of a scalar field in a specific potential $V(\phi)$. The essential difference is in the form of the potential and the couplings between the scalar field, matter and radiation[3]. In the anisotropic R-W metric(9), the action for the model can be expressed as

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - V(\phi)) + \beta^4(\phi)(\rho_M + \rho_R)\right] \tag{23}$$

where g is the determinant of the metric $g_{\mu\nu}$ and $\mathcal{R}$ is the Ricci scalar. The coupling $\beta(\phi)$ between ϕ and the matter (ρ_M) and radiation (ρ_R) densities is crucial because it allows the densities to remain finite at the big crunch/big bang transition. Varying the action(23), one can obtain the Einstein equation up to the linear order of the anisotropic perturbation as

$$3H^2 + 2H\dot{\delta} = \kappa\left(\frac{1}{2}\dot{\phi}^2 + V + \beta^4\rho_R + \beta^4\rho_M\right) \tag{24}$$

$$2\frac{\ddot{a}}{a} + H^2 = -\kappa\left(\dot{\phi}^2 - V + \beta^4\rho_R + \frac{1}{2}\beta^4\rho_M\right) \tag{25}$$

$$2\frac{\ddot{a}}{a}+3H\dot{\delta}+H^2+\ddot{\delta}=-\kappa\left(\dot{\phi}^2-V+\beta^4\rho_R+\frac{1}{2}\beta^4\rho_M\right) \tag{26}$$

The fluid equation of motion for matter or fluid is

$$\frac{d\rho_i}{dt}=\left[-3\frac{\dot{\hat{a}}}{\hat{a}}-\dot{\delta}\right](\rho_i+p_i) \tag{27}$$

where $\hat{a}=\beta(\phi)a$ and $\beta(\phi)$ is such chosen that the quantity $\hat{a}$ is finite when a approaches zero.

Now, we can consider the cosmological anisotropy in a similar fashion as that in the previous section of this paper. It is clear that the evolution equation for the anisotropy can also be expressed as Eqs.(19) and (21) for the power law and exponential expansion/contraction respectively. Therefore the conclusions in the former part of this section are still held true. When the universe is in the contraction phase, the anisotropy will develop to be very significant. It must be pointed out that the increase of anisotropy does not necessarily exclude the cycling of Universe as described in the cyclic model if the initial "speed" of anisotropy $\dot{\delta}(t_0)$ is extremely small so that the anisotropy will not develop to be very significant even after a long period increase as shown in Eq.(19). This is physically possible because the lasting accelerated expansion before the contraction phase will reduce the initial "speed" of anisotropy. On the other hand, one can also consider a different coupling between the scalar field and the matter as well as the radiation, for example, by taking into account of the higher order anisotropic effects and the corresponding coupling that dependent on direction, to avoid the the increase of anisotropy. The later possibility will be attempted in a preparing work.

In the following section, we will correlate the anisotropy with the red-shift of the supernovae, which might be a way to identify the anisotropy not only when it increases, but also when it damps.

4. Relate the Anisotropy with the Red-shift

The red-shift data of supernovae has been used to reconstruct the potential[10–12]or the equation of state[13] of the field that drive the expansion of the universe. Now, we generalize the red-shift relations in isotropic flat R-W spacetime by introducing the different red-shift in different directions. In our set-up here, we consider the simple case that the red-shift is different in the x direction and the y, z plane. Accordingly, we introduce the following definitions:

$$x(Z_x) = \int_{t(Z_x)}^{0} \frac{du}{b(u)} = \int_{0}^{Z_x} \frac{d\xi}{H_x(\xi)} \tag{28}$$

$$y(Z_y) = \int_{t(Z_y)}^{0} \frac{du}{a(u)} = \int_{0}^{Z_y} \frac{d\xi}{H_y(\xi)} \tag{29}$$

where $x(Z_x)$ and $y(Z_y)$ are the coordinate distance in x and y directions respectively. Z_x and Z_y are the red-shift in x and y direction respectively. $H_x(Z)$ and $H_y(Z)$ are defined as

$$(\frac{\dot{b}}{b})^2 = H_x(Z_x)^2 = \frac{1}{(dx/dZ_x)^2} \tag{30}$$

$$(\frac{\dot{a}}{a})^2 = H_y(Z_y)^2 = \frac{1}{(dy/dZ_y)^2} \tag{31}$$

$H_x(Z) \equiv \frac{\dot{b}}{b}$ and $H_y(Z) \equiv \frac{\dot{a}}{a}$. Since the z direction has the same scale factor as that in y direction, we consider the corresponding coordinate distance in z direction is the same as in y direction. Obviously, the above definitions is a direct generalization of the red-shift in the isotropic spacetime

$$r(Z) = \int_{t(Z)}^{t_0} \frac{du}{a(u)} = \int_{0}^{Z} \frac{d\xi}{H(\xi)} \tag{32}$$

It is not difficult to prove that the relations between t and Z_x and Z_y are

$$\frac{dZ_x}{dt} = -(1+Z_x)H_x(Z_x) = -(1+Z_x)\frac{dx}{dZ_x} \tag{33}$$

$$\frac{dZ_y}{dt} = -(1+Z_y)H_y(Z_y) = -(1+Z_y)\frac{dy}{dZ_y} \tag{34}$$

To correlate the red-shift Z_x and Z_y with the anisotropy δ, we can consider Eqs.(11) and (30). From Eq.(31), one have

$$d\ln a = \frac{dt}{dy/dZ_y} \tag{35}$$

Together with Eq.(11), we have

$$\begin{aligned} \dot{\delta}(Z_x, Z_y) &\equiv f(Z_x, Z_y) \\ &= \dot{\delta}_0 \exp[\int \frac{3(dx/dZ_x)}{(dy/dZ_y)(1+Z_x)} dZ_x + 3\int \frac{dZ_y}{(1+Z_y)}] \end{aligned} \tag{36}$$

Therefore, the correlation equations could be expressed as:

$$\frac{\partial\delta}{\partial Z_x}+\frac{\partial\delta}{\partial Z_y}\frac{(1+Z_y)(dy/dZ_y)}{(1+Z_x)(dx/dZ_x)}=-\frac{f(Z_x,Z_y)}{(1+Z_x)(dx/dZ_x)} \tag{37}$$

$$\delta(Z_x,Z_y)=-\int[\frac{f(Z_x,Z_y)dZ_x}{(1+Z_x)(dx/dZ_x)}+\frac{f(Z_x,Z_y)dZ_y}{(1+Z_y)(dy/dZ_y)}] \tag{38}$$

It is obvious that once we know the coordinate distance x and y as the function of red-shift Z_x and Z_y, we will be able to obtain the anisotropy δ through the correlation relations Eqs.(37) and (38).

4. Discussion and Conclusion

In this paper, we study the evolution of cosmological anisotropy in Kantowski-Sachs spacetime. We show that the anisotropy will increase significantly if the expansion rate is slower than a critical value, which is different for different type Universe(Closed, open or flat). We also show that the expansion in the conventional big bang cosmology, the radiation dominated era, matter dominated era and the current accelerating expansion era, is fast enough to damp the anisotropy if the Universe is spatially flat, which is in agreement with the theoretical predication of inflation theory as well as current observations. But if the Universe is not flat, the anisotropy will develop to be significant during the radiation and matter dominated era. This itself could be considered as a important theoretical support to the inflation theory. In the cyclic Universe model, the anisotropy will increase when the Universe expands slower than the critical value or contracts. But, as we explained in section 3, this could be alleviated by the tuning of the condition before Universe expands slower than the critical value, or by considering different and direction-dependent coupling between the scalar field and radiation as well as matter. Especially, it needs to point out that in the cyclic model, q does not remain small all the way to the bounce. Rather, q is greater than $\frac{1}{3}$ once the branes get to within a few Planck lengths[9]. So, the above increase of anisotropy could also be considered as a mechanism employed by the cyclic model to seed acceptable anisotropy observed in the present Universe.

On the other hand, observation of the red-shift of SNeIa has been proved to be an important way to probe the Universe[14]. We derive the relation between the cosmological anisotropy and the red-shift, which indicates that one can gain some information about the anisotropy by fitting the red-shift data. When the Universe slows down or contracts, we may reconstruct the anisotropy through the red-shift data.

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