Interacting mixed-symmetry type tensor gauge fields of degrees two and three: a four-dimensional cohomological approach

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Abstract

A special class of mixed-symmetry type tensor gauge fields of degrees two and three in four dimensions is investigated from the perspective of the Lagrangian deformation procedure based on cohomological BRST techniques. It is shown that the deformed solution to the master equation can be taken to be nonvanishing only at the first order in the coupling constant. As a consequence, we deduce an interacting model with deformed gauge transformations, an open gauge algebra and undeformed reducibility functions. The resulting coupled Lagrangian action contains a quartic vertex and some "mass" terms involving only the tensor of degree two. We discuss in what sense the results of the deformation procedure derived here are complementary to recent others.

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The problem of constructing consistent interactions that can be introduced among gauge fields in such a way to preserve the number of gauge symmetries [1]–[4] has been reformulated as a deformation problem of the

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master equation [5] in the framework of the antifield BRST formalism [6]–[10].

In this paper we generate all consistent interactions that can be added to a special class of mixed-symmetry type tensor gauge fields, described in the free limit by the Lagrangian action

$$S_0^L\left[A_{\alpha(\lambda)}, B^{\alpha\beta(\lambda)}\right] = \int d^4x \partial_{[\alpha} A_{\beta](\lambda)} B^{\alpha\beta(\lambda)}, \tag{1}$$

where the tensor gauge field of degree three is assumed antisymmetric in its first two indices, $B^{\alpha\beta(\lambda)} = -B^{\beta\alpha(\lambda)}$, while that of degree two displays no symmetry. We work with the conventions that the Minkowskian metric $g_{\mu\nu}$ is of 'mostly minus' signature $(+,-,-,-)$, and that the completely antisymmetric four-dimensional symbol $\varepsilon^{\alpha\beta\gamma\delta}$ is valued like $\varepsilon^{0123} = +1$. On the one hand, models with mixed-symmetry type tensor gauge fields attracted much interest lately. In this context, more problems, like, for instance, the interpretation of the construction of the Pauli-Fierz theory [11], the dual formulation of linearized gravity [12]–[13], the impossibility of consistent cross-interactions in the dual formulation of linearized gravity [13], or the general scheme for dualizing higher-spin gauge fields in arbitrary irreducible representations of $GL(D,\mathbf{R})$ [14], have been reanalyzed. On the other hand, the action (1) can be regarded in some sense as a topological Batalin-Fradkin (BF) theory. It is known that BF-like theories are deeply connected with two-dimensional gravity [15]–[21] via the so-called Poisson Sigma Models [22]–[28]. In view of its links with some important classes of gauge theories, we believe that the study of the theory under consideration might bring some contributions to the quantization of gravity without string theory.

Action (1) is found invariant under the abelian gauge transformations

$$\delta_\epsilon A_{\alpha(\lambda)} = \partial_\alpha \epsilon_{(\lambda)},\ \delta_\epsilon B^{\alpha\beta(\lambda)} = \varepsilon^{\alpha\beta\gamma\delta}\partial_\gamma {\epsilon_\delta}^{(\lambda)}, \tag{2}$$

where all gauge parameters are bosonic. The gauge generators of the tensor fields $B^{\alpha\beta(\lambda)}$ are off-shell first-stage reducible since if we make the transformation ${\epsilon_\delta}^{(\lambda)} = \partial_\delta \theta^{(\lambda)}$, with $\theta^{(\lambda)}$ arbitrary functions, then $\delta_\epsilon B^{\alpha\beta(\lambda)} = 0$. In consequence, we deal with a free normal gauge theory, of Cauchy order three.

In order to investigate the problem under consideration, we employ the antifield-BRST formalism. The BRST complex includes, besides the original tensor fields, the fermionic ghosts $\left(C_{(\lambda)}, {\eta_\alpha}^{(\lambda)}\right)$ respectively associated with the gauge parameters $\left(\epsilon_{(\lambda)}, {\epsilon_\alpha}^{(\lambda)}\right)$, the bosonic ghosts for ghosts $\eta^{(\lambda)}$ due to the first-stage reducibility relations, as well as their antifields, denoted as star variables. The BRST differential for this free model (s) decomposes as the sum between the Koszul-Tate differential and the exterior longitudinal derivative only, $s=\delta+\gamma$. The Koszul-Tate complex is graded in terms of the antighost number (agh), such that $\mathrm{agh}(\delta)=-1$, $\mathrm{agh}(\gamma)=0$, while the degree of the exterior longitudinal complex is known as the pure ghost number (pgh), with $\mathrm{pgh}(\gamma)=1$, $\mathrm{pgh}(\delta)=0$. The degrees of the BRST generators are valued like

$$\mathrm{pgh}\left(B^{\alpha\beta(\lambda)}\right)=\mathrm{pgh}\left(B^{*}_{\alpha\beta(\lambda)}\right)=\mathrm{pgh}\left(A_{\alpha(\lambda)}\right)=\mathrm{pgh}\left(A^{*\alpha(\lambda)}\right)=0, \tag{3}$$

$$\mathrm{pgh}\left({\eta^{*\alpha}}_{(\lambda)}\right)=\mathrm{pgh}\left(C^{*(\lambda)}\right)=\mathrm{pgh}\left(\eta^{*}_{(\lambda)}\right)=0, \tag{4}$$

$$\mathrm{pgh}\left({\eta_{\alpha}}^{(\lambda)}\right)=\mathrm{pgh}\left(C_{(\lambda)}\right)=1,\ \mathrm{pgh}\left(\eta^{(\lambda)}\right)=2, \tag{5}$$

$$\mathrm{agh}\left(B^{\alpha\beta(\lambda)}\right)=\mathrm{agh}\left(A_{\alpha(\lambda)}\right)=\mathrm{agh}\left({\eta_{\alpha}}^{(\lambda)}\right)=\mathrm{agh}\left(C_{(\lambda)}\right)=0, \tag{6}$$

$$\mathrm{agh}\left(\eta^{(\lambda)}\right)=0,\ \mathrm{agh}\left(B^{*}_{\alpha\beta(\lambda)}\right)=\mathrm{agh}\left(A^{*\alpha(\lambda)}\right)=1, \tag{7}$$

$$\mathrm{agh}\left({\eta^{*\alpha}}_{(\lambda)}\right)=\mathrm{agh}\left(C^{*(\lambda)}\right)=2,\ \mathrm{agh}\left(\eta^{*}_{(\lambda)}\right)=3, \tag{8}$$

while the operators δ and γ act on them via the definitions

$$\delta B^{\alpha\beta(\lambda)}=\delta A_{\alpha(\lambda)}=\delta{\eta_{\alpha}}^{(\lambda)}=\delta C_{(\lambda)}=\delta\eta^{(\lambda)}=0, \tag{9}$$

$$\delta B^{*}_{\alpha\beta(\lambda)}=-\partial_{[\alpha}A_{\beta](\lambda)},\ \delta A^{*\alpha(\lambda)}=2\partial_{\beta}B^{\beta\alpha(\lambda)}, \tag{10}$$

$$\delta{\eta^{*\alpha}}_{(\lambda)}=\varepsilon^{\alpha\beta\gamma\delta}\partial_{\beta}B^{*}_{\gamma\delta(\lambda)},\ \delta C^{*(\lambda)}=-\partial_{\alpha}A^{*\alpha(\lambda)},\ \delta\eta^{*}_{(\lambda)}=\partial_{\alpha}{\eta^{*\alpha}}_{(\lambda)}, \tag{11}$$

$$\gamma B^{\alpha\beta(\lambda)}=\varepsilon^{\alpha\beta\gamma\delta}\partial_{\gamma}{\eta_{\delta}}^{(\lambda)},\ \gamma A_{\alpha(\lambda)}=\partial_{\alpha}C_{(\lambda)},\ \gamma{\eta_{\alpha}}^{(\lambda)}=\partial_{\alpha}\eta^{(\lambda)}, \tag{12}$$

$$\gamma C_{(\lambda)}=\gamma\eta^{(\lambda)}=0,\ \gamma B^{*}_{\alpha\beta(\lambda)}=\gamma A^{*\alpha(\lambda)}=0, \tag{13}$$

$$\gamma{\eta^{*\alpha}}_{(\lambda)}=\gamma C^{*(\lambda)}=\gamma\eta^{*}_{(\lambda)}=0. \tag{14}$$

The overall degree from the BRST complex is the ghost number (gh), defined like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}(s)=1$. The BRST symmetry admits a canonical action in the antibracket $(,)$, $sF=(F,S)$, where the canonical generator S is bosonic, of ghost number zero, and satisfies the classical master equation $(S,S)=0$, which is equivalent to the second-order nilpotency of s, $s^{2}=0$. In the case of the free model under study, since both the gauge generators and reducibility functions are field-independent, it follows that the solution to the master equation is given by

$$S=S_{0}^{L}+\int d^{4}x\left(A^{*\alpha(\lambda)}\partial_{\alpha}C_{(\lambda)}+\varepsilon^{\alpha\beta\gamma\delta}B^{*}_{\alpha\beta(\lambda)}\partial_{\gamma}{\eta_{\delta}}^{(\lambda)}+{\eta^{*\alpha}}_{(\lambda)}\partial_{\alpha}\eta^{(\lambda)}\right). \tag{15}$$

A consistent deformation of the free action (1) and of its gauge invariances (2) defines a deformation of the corresponding solution to the master equation that preserves both the master equation and the field-ghost/antifield spectra. So, if $S_0^L + g\int d^4x\alpha_0 + O(g^2)$ stands for a consistent deformation of the free action, with deformed gauge transformations $\bar{\delta}_\epsilon A_{\alpha(\lambda)} = \partial_\alpha \epsilon_{(\lambda)} + g\beta_{\alpha\lambda} + O(g^2)$, $\bar{\delta}_\epsilon B^{\alpha\beta(\lambda)} = \varepsilon^{\alpha\beta\gamma\delta}\partial_\gamma {\epsilon_\delta}^{(\lambda)} + g\beta^{\alpha\beta\lambda} + O(g^2)$, then the deformed solution to the master equation

$$\bar{S} = S + g\int d^4x\alpha + O\left(g^2\right), \tag{16}$$

satisfies $\left(\bar{S},\bar{S}\right) = 0$, where the first-order deformation α begins like $\alpha = \alpha_0 + A^{*\alpha(\lambda)}\bar{\beta}_{\alpha\lambda} + B^*_{\alpha\beta(\lambda)}\bar{\beta}^{\alpha\beta\lambda} +$ 'more' (g is the so-called deformation parameter or coupling constant). The terms $\bar{\beta}_{\alpha\lambda}$and $\bar{\beta}^{\alpha\beta\lambda}$ are obtained by replacing the gauge parameters $\left(\epsilon_{(\lambda)}, {\epsilon_\alpha}^{(\lambda)}\right)$ respectively with the fermionic ghosts $\left(C_{(\lambda)}, {\eta_\alpha}^{(\lambda)}\right)$ in the functions $\beta_{\alpha\lambda}$ and $\beta^{\alpha\beta\lambda}$.

The master equation $\left(\bar{S},\bar{S}\right) = 0$ holds to order g if and only if

$$s\alpha = \partial_\mu j^\mu, \tag{17}$$

for some local j^μ. In order to solve this equation, we develop α according to the antighost number

$$\alpha = \alpha_0 + \alpha_1 + \cdots + \alpha_I,\ \mathrm{agh}\left(\alpha_K\right) = K,\ \mathrm{gh}\left(\alpha_K\right) = 0,\ \varepsilon\left(\alpha_K\right) = 0. \tag{18}$$

The number of terms in the expansion (18) is finite and it can be shown that we can take last term in α to be annihilated by γ, $\gamma\alpha_I = 0$. Consequently, we need to compute the cohomology of γ, $H(\gamma)$, in order to determine the component of highest antighost number in α. From (12–14) it is simple to see that $H(\gamma)$ is spanned by $F_{\alpha\beta(\lambda)} \equiv \partial_{[\alpha}A_{\beta](\lambda)}$, $\partial_\beta B^{\beta\alpha(\lambda)}$ and $\chi^* = \left(B^*_{\alpha\beta(\lambda)}, A^{*\alpha(\lambda)}, {\eta^{*\alpha}}_{(\lambda)}, C^{*(\lambda)}, \eta^*_{(\lambda)}\right)$, by their spacetime derivatives, as well as by the undifferentiated ghosts $\left(C_{(\lambda)}, \eta^{(\lambda)}\right)$. (The derivatives of these ghosts are removed from $H(\gamma)$ since they are γ-exact, in agreement with the second and third relations in (12).) If we denote by $e^M\left(C_{(\lambda)}, \eta^{(\lambda)}\right)$ the elements with pure ghost number M of a basis in the space of the polynomials in the corresponding ghosts, it follows that the general solution to the equation $\gamma\alpha_I = 0$ takes the form

$$\alpha_I = a_I\left(\left[F_{\alpha\beta(\lambda)}\right], \left[\partial_\beta B^{\beta\alpha(\lambda)}\right], \left[\chi^*\right]\right) e^I\left(C_{(\lambda)}, \eta^{(\lambda)}\right), \tag{19}$$

where $\mathrm{agh}\left(a_I\right) = I$. The notation $f([q])$ means that f depends on q and its spacetime derivatives up to a finite order. The equation (17) projected on antighost number $(I-1)$ becomes

$$\delta\alpha_I + \gamma\alpha_{I-1} = \partial^\mu \overset{(I-1)}{m}_\mu. \tag{20}$$

Replacing (19) in (20), it follows that the last equation possesses solutions with respect to α_{I-1} if the coefficients a_I pertain to the homological space of the Koszul-Tate differential modulo the exterior spacetime differential at antighost number I, $H_I(\delta|d)$, i.e., $\delta a_I = \partial_\mu l^\mu_{I-1}$. In the meantime, since our free model is linear and of Cauchy order equal to three, according to the results from [29] we have that $H_J(\delta|d)$ vanishes for $J > 3$, so we can assume that the first-order deformation stops at antighost number three ($I = 3$)

$$\alpha = \alpha_0 + \alpha_1 + \alpha_2 + \alpha_3, \tag{21}$$

where α_3 is of the form (19), with a_3 from $H_3(\delta|d)$. On the one hand, the most general representatives of $H_3(\delta|d)$ can be taken of the type $\eta^*_{(\lambda)}$. On the other hand, the elements $e^3\left(C_{(\lambda)}, \eta^{(\lambda)}\right)$ are precisely $\left(\eta^{(\mu)} C_{(\nu)}, C_{(\mu)} C_{(\nu)} C_{(\rho)}\right)$. Then, α_3 is of the form $\eta^*_{(\lambda)}\left(f^{\lambda\nu}_\mu \eta^{(\mu)} C_{(\nu)} + f^{\lambda\mu\nu\rho} C_{(\mu)} C_{(\nu)} C_{(\rho)}\right)$, with $f^{\lambda\nu}_\mu$ and $f^{\lambda\mu\nu\rho}$ some constants. By covariance arguments, $f^{\lambda\nu}_\mu$ must contain at least one spacetime derivative, which, if applied on the basis $\eta^{(\mu)} C_{(\nu)}$, leads to trivial (γ-exact) terms, such that we can set $f^{\lambda\nu}_\mu = 0$. In the meantime, the only manifestly covariant constants $f^{\lambda\mu\nu\rho}$ in four spacetime dimensions that do not involve spacetime derivatives can only be proportional with the completely antisymmetric symbol $\varepsilon^{\lambda\mu\nu\rho}$. In consequence, the last representative from the first-order deformation (21) reads as

$$\alpha_3 = \frac{1}{3} \varepsilon^{\lambda\mu\nu\rho} \eta^*_{(\lambda)} C_{(\mu)} C_{(\nu)} C_{(\rho)}. \tag{22}$$

By taking into account the relations (9–14), it follows that the solution to the equation (20) for $I = 3$ is precisely given by

$$\alpha_2 = -\varepsilon^{\lambda\mu\nu\rho}\left(\eta^{*\alpha}{}_{(\lambda)} A_{\alpha(\mu)} + \varepsilon^{\alpha\beta\gamma\delta} \frac{1}{4} B^*_{\alpha\beta(\lambda)} B^*_{\gamma\delta(\mu)}\right) C_{(\nu)} C_{(\rho)} + \xi C^*_{(\lambda)} \eta^{(\lambda)}, \tag{23}$$

with ξ a numerical constant. Further, we compute the component α_1 as solution to the equation $\delta\alpha_2 + \gamma\alpha_1 = \partial^\mu \overset{(1)}{m}_\mu$, and find that

$$\begin{aligned} \alpha_1 = {} & B^*_{\alpha\beta(\lambda)}\left(\varepsilon^{\alpha\beta\lambda\rho} + \varepsilon^{\lambda\mu\nu\rho} \varepsilon^{\alpha\beta\gamma\delta} A_{\gamma(\mu)} A_{\delta(\nu)}\right) C_{(\rho)} + \\ & B^{*\mu\nu}{}_{(\nu)} C_{(\mu)} - \xi A^{*\alpha}{}_{(\lambda)} \eta_\alpha{}^{(\lambda)}. \end{aligned} \tag{24}$$

Finally, the antighost number zero piece in (21) is subject to the equation $\delta\alpha_1 + \gamma\alpha_0 = \partial^\mu \overset{(0)}{m}_\mu$, whose solution can be written like

$$\begin{aligned} \alpha_0 = {} & \varepsilon^{\lambda\mu\nu\rho}\left(A_{\lambda(\mu)} A_{\nu(\rho)} - \frac{1}{3!} \varepsilon^{\alpha\beta\gamma\delta} A_{\alpha(\lambda)} A_{\beta(\mu)} A_{\gamma(\nu)} A_{\delta(\rho)}\right) + \\ & \frac{1}{2}\left(A^\mu{}_{(\mu)} A^\nu{}_{(\nu)} - A^{\alpha(\mu)} A_{\mu(\alpha)}\right) - \frac{\xi}{4} \varepsilon_{\alpha\beta\gamma\delta} B^{\alpha\beta(\lambda)} B^{\gamma\delta}{}_{(\lambda)}. \end{aligned} \tag{25}$$

So far, we have completely generated the first-order deformation of the solution to the master equation in the case of the analysed model, (21), where the concrete form of the terms $(\alpha_a)_{a=0,1,2,3}$ can be found in the right hand-side of formulas (22–25).

Next, we investigate the equations that control the higher-order deformations. If we denote by $S_2 = \int d^4x\beta$ the second-order deformation, the master equation $\left(\bar{S}, \bar{S}\right) = 0$ holds to order g^2 if and only if

$$\frac{1}{2}\Delta = -s\beta + \partial_\mu t^\mu, \tag{26}$$

where $(S_1, S_1) = \int d^4x\Delta$. Making use of (21) and (22–25), we deduce that

$$\begin{aligned}
&\frac{1}{2}\Delta = \partial_\mu t^\mu + \xi \left(\varepsilon^{\lambda\mu\nu\rho}\left(\frac{1}{3}C^*_{(\lambda)}C_{(\rho)} + \eta^*_{(\lambda)}\eta_{(\rho)} + B^*_{\gamma\delta(\lambda)}B^{\gamma\delta}{}_{(\rho)}\right)C_{(\mu)}C_{(\nu)} + \right.\\
&2\left(\varepsilon^{\lambda\mu\nu\rho}B^{\gamma\delta}{}_{(\lambda)}A_{\gamma(\mu)}A_{\delta(\nu)} + \frac{1}{4}\varepsilon^{\gamma\delta\lambda\rho}B_{\gamma\delta(\lambda)} - B^{\rho\lambda}{}_{(\lambda)}\right)C_{(\rho)} - \\
&2\varepsilon^{\lambda\mu\nu\rho}\left(\eta^{*\alpha}{}_{(\lambda)}A_{\alpha(\mu)} + \frac{1}{4}\varepsilon^{\alpha\beta\gamma\delta}B^*_{\alpha\beta(\lambda)}B^*_{\gamma\delta(\mu)}\right)C_{(\nu)}\eta_{(\rho)} + A^{\alpha(\mu)}\eta_{\mu(\alpha)} \\
&\left(B^*_{\alpha\beta(\lambda)}\left(\varepsilon^{\alpha\beta\lambda\rho} + \varepsilon^{\lambda\mu\nu\rho}\varepsilon^{\alpha\beta\gamma\delta}A_{\gamma(\mu)}A_{\delta(\nu)}\right) + B^{*\rho\nu}{}_{(\nu)}\right)\eta_{(\rho)} + \\
&\varepsilon^{\lambda\mu\nu\rho}\left(\left(A^{*\alpha}{}_{(\lambda)}A_{\alpha(\mu)} + \eta^{*\alpha}{}_{(\lambda)}\eta_{\alpha(\mu)}\right)C_{(\nu)} + 2\varepsilon^{\alpha\beta\gamma\delta}B^*_{\alpha\beta(\lambda)}A_{\gamma(\mu)}\eta_{\delta(\nu)}\right)C_{(\rho)} + \\
&\left. 2\left(\frac{1}{3}\varepsilon^{\lambda\mu\nu\rho}\varepsilon^{\alpha\beta\gamma\delta}A_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)} - \varepsilon^{\alpha\mu\delta\rho}A_{\alpha(\mu)}\right)\eta_{\delta(\rho)} - A^{\mu}{}_{(\mu)}\eta^{\nu}{}_{(\nu)}\right).
\end{aligned} \tag{27}$$

It is easy to see that none of the terms proportional with ξ in the right hand-side of (27) can be written like an s-exact modulo d quantity. In conclusion, the consistency of the first-order deformation requires that $\xi = 0$. With this value at hand, the equation (26) is satisfied with the choice $\beta = 0$, which further induces that the second-order deformation of the solution to the master equation can be taken to vanish, $S_2 = 0$. Further, all the higher-order equations are satisfied if we set $S_3 = S_4 = \cdots = 0$. Consequently, the complete deformed solution to the master equation, consistent to all orders in the coupling constant, reduces in this situation to the sum between the free solution (15) and the first-order deformation in which we replace ξ by zero, and hence is expressed by

$$\begin{aligned}
&\bar{S} = S_0^L + \int d^4x\left(g\varepsilon^{\lambda\mu\nu\rho}\left(A_{\lambda(\mu)}A_{\nu(\rho)} - \frac{1}{3!}\varepsilon^{\alpha\beta\gamma\delta}A_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)}A_{\delta(\rho)}\right) + \right.\\
&\frac{g}{2}\left(A^{\mu}{}_{(\mu)}A^{\nu}{}_{(\nu)} - A^{\alpha(\mu)}A_{\mu(\alpha)}\right) + A^{*\alpha(\lambda)}\partial_\alpha C_{(\lambda)} + \varepsilon^{\alpha\beta\gamma\delta}B^*_{\alpha\beta(\lambda)}\partial_\gamma\eta_\delta{}^{(\lambda)} + \\
&gB^*_{\alpha\beta(\lambda)}\left(\frac{1}{2}\left(g^{\alpha\rho}g^{\beta\lambda} - g^{\alpha\lambda}g^{\beta\rho}\right) + \varepsilon^{\alpha\beta\lambda\rho} + \varepsilon^{\lambda\mu\nu\rho}\varepsilon^{\alpha\beta\gamma\delta}A_{\gamma(\mu)}A_{\delta(\nu)}\right)C_{(\rho)} +
\end{aligned}$$

$$\eta^{*\alpha}{}_{(\lambda)}\partial_\alpha \eta^{(\lambda)} - g\varepsilon^{\lambda\mu\nu\rho}\left(\eta^{*\alpha}{}_{(\lambda)}A_{\alpha(\mu)} + \varepsilon^{\alpha\beta\gamma\delta}\frac{1}{4}B^*_{\alpha\beta(\lambda)}B^*_{\gamma\delta(\mu)}\right)C_{(\nu)}C_{(\rho)} +$$
$$\left.\frac{g}{3}\varepsilon^{\lambda\mu\nu\rho}\eta^*_{(\lambda)}C_{(\mu)}C_{(\nu)}C_{(\rho)}\right). \tag{28}$$

With the help of the last formula, we are able to identify the interacting tensor gauge field theory behind the deformation procedure. For instance, the antighost number zero component in (28) is nothing but the Lagrangian action of the resulting coupled model

$$\bar{S}_0^L\left[A_{\alpha(\lambda)}, B^{\alpha\beta(\lambda)}\right] = \int d^4x \left(\partial_{[\alpha}A_{\beta](\lambda)}B^{\alpha\beta(\lambda)} + g\varepsilon^{\lambda\mu\nu\rho}A_{\lambda(\mu)}A_{\nu(\rho)} + \right.$$
$$\left.\frac{g}{2}\left(A^\mu{}_{(\mu)}A^\nu{}_{(\nu)} - A^{\alpha(\mu)}A_{\mu(\alpha)}\right) - \frac{g}{3!}\varepsilon^{\lambda\mu\nu\rho}\varepsilon^{\alpha\beta\gamma\delta}A_{\alpha(\lambda)}A_{\beta(\mu)}A_{\gamma(\nu)}A_{\delta(\rho)}\right) \tag{29}$$

From the elements of antighost number one in (28), we read the deformed gauge transformations of the tensor fields

$$\bar{\delta}_\epsilon A_{\alpha(\lambda)} = \partial_\alpha \epsilon_{(\lambda)} \equiv \left(Z^{(A)}_{\alpha\lambda}\right)^\rho \epsilon_{(\rho)},\ \bar{\delta}_\epsilon B^{\alpha\beta(\lambda)} = \varepsilon^{\alpha\beta\gamma\delta}\partial_\gamma \epsilon_\delta{}^{(\lambda)} + \frac{g}{2}\epsilon^{[(\alpha)}g^{\beta]\lambda} +$$
$$g\left(\varepsilon^{\alpha\beta\lambda\rho} + \varepsilon^{\lambda\mu\nu\rho}\varepsilon^{\alpha\beta\gamma\delta}A_{\gamma(\mu)}A_{\delta(\nu)}\right)\epsilon_{(\rho)} \equiv$$
$$\left(Z^{(B)\alpha\beta\lambda}\right)^\delta{}_\tau \epsilon_\delta{}^{(\tau)} + \left(Z^{(B)\alpha\beta\lambda}\right)^\rho \epsilon_{(\rho)}, \tag{30}$$

where the nonvanishing gauge generators involved with the coupled theory are given by

$$\left(Z^{(A)}_{\alpha\lambda}\right)^\rho = \delta_\lambda{}^\rho \partial_\alpha,\ \left(Z^{(B)\alpha\beta\lambda}\right)^\delta_\tau = \varepsilon^{\alpha\beta\gamma\delta}\delta^\lambda{}_\tau \partial_\gamma, \tag{31}$$

$$\left(Z^{(B)\alpha\beta\lambda}\right)^\rho = \frac{g}{2}\left(g^{\alpha\rho}g^{\beta\lambda} - g^{\alpha\lambda}g^{\beta\rho}\right) + g\left(\varepsilon^{\alpha\beta\lambda\rho} + \varepsilon^{\lambda\mu\nu\rho}\varepsilon^{\alpha\beta\gamma\delta}A_{\gamma(\mu)}A_{\delta(\nu)}\right). \tag{32}$$

Related to the antighost number two contribution of (28), we remark that the reducibility functions and relations are not modified during the deformation mechanism with respect to the initial free model. In change, the original abelian gauge algebra is deformed into an open one, where the associated non-abelian commutators among the gauge generators are provided by

$$\left(Z^{(A)}_{\delta\tau}\right)^\sigma \frac{\delta\left(Z^{(B)\alpha\beta\lambda}\right)^\rho}{\delta A_{\delta(\tau)}} - \left(Z^{(A)}_{\delta\tau}\right)^\rho \frac{\delta\left(Z^{(B)\alpha\beta\lambda}\right)^\sigma}{\delta A_{\delta(\tau)}} =$$
$$-2g\varepsilon^{\sigma\rho\tau\mu}A_{\delta(\mu)}\left(Z^{(B)\alpha\beta\lambda}\right)^\delta_\tau + g\varepsilon^{\sigma\rho\lambda\mu}\varepsilon^{\alpha\beta\gamma\delta}\frac{\delta\bar{S}_0^L}{\delta B^{\gamma\delta(\mu)}}. \tag{33}$$

The pieces of antighost number three in (28) offer information on the second-order structure functions due to the open character of the deformed gauge algebra.

In conclusion, in this paper we have investigated a special class of mixed-symmetry type tensor gauge fields of degrees two and three in four dimensions from the perspective of the Lagrangian deformation procedure based on cohomological BRST techniques. We have shown that the deformed solution to the master equation can be taken to be nonvanishing only at the first order in the coupling constant. Thus, we reveal an interacting model with: (i) deformed gauge transformations; (ii) an open gauge algebra; (iii) a quartic vertex and "mass" terms involving only the tensors of degree two; (iv) undeformed reducibility functions. It is interesting to mention that, in spite of the fact that the tensor gauge fields involved here are of the same type with those studied in [30] (up to the notational replacement $A \longleftrightarrow B$), the resulting interacting theories are in a way complementary. More precisely, while here we deform the gauge algebra, but do not affect the reducibility, in [30] the reducibility is essentially changed, although the gauge algebra is not modified. Moreover, the first-order deformation of the solution to the master equation derived here contains no spacetime derivatives, while that from [30] involves only derivative terms. Finally, we note that those interactions from [30] that actually deform the gauge symmetry only exist in four spacetime dimensions, by contrast to the model discussed here, which is suited to further generalization. In a future paper we hope to solve this problem.

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