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Comments on the Equivalence between Chern-Simons Theory and Topological Massive Yang-Mills Theory in 3D

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Abstract

The classical formal equivalence upon a redefinition of the gauge connection between Chern-Simons theory and topological massive Yang-Mills theory in three-dimensional Euclidean space-time is analyzed at the quantum level within the BRST formulation of the Equivalence Theorem. The parameter controlling the change in the gauge connection is the inverse λ of the topological mass. The BRST differential associated with the gauge connection redefinition is derived and the corresponding Slavnov-Taylor (ST) identities are proven to be anomaly-free. Hence they can be restored order by order in the loop expansion by a recursive choice of non-invariant counterterms. The Green functions of local operators constructed only from the (λ-dependent) transformed gauge connection, as well as those of BRST invariant operators, are shown to be independent of the parameter λ, as a consequence of the validity of the ST identities. The relevance of the antighost-ghost fields, needed to take into account at the quantum level the Jacobian of the change in the gauge connection, is analyzed. Their rôle in the identification of the physical states of the model within conventional perturbative gauge theory is discussed. It is shown that they prove to be essential in keeping the correspondence between the degrees of freedom of the theory at $\lambda = 0$ (Chern-Simons theory) and at $\lambda \neq 0$.

Key-words: BRST quantization, BRST symmetry, Chern-Simons theory

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1 Introduction

It has been observed some time ago [1, 2] that it is possible to recast the action of the classical three-dimensional topological massive Yang-Mills theory in flat Euclidean space-time in the form of a pure Chern-Simons action via a non-linear redefinition of the gauge connection. One can prove [1, 2] that the transformed gauge field occurring into the Chern-Simons action can be expressed as a formal power series in the inverse $\lambda = \frac{1}{m}$ of the topological mass m:

$$\hat{A}_\mu = A_\mu + \sum_{n=1}^{\infty} \lambda^n \theta^n_\mu(A) \, , \tag{1}$$

where A_μ is the gauge connection of topological massive Yang-Mills theory and $\theta^n_\mu(A)$ is the coefficient of $\hat{A}_\mu$ of order n in the λ-expansion. $\hat{A}_\mu$ is non-local since the R.H.S. of eq.(1) does not reduce to a polynomial. However, it turns out that each coefficient $\theta^n_\mu(A)$ is local and covariant, being constructed only from the field strength of A_μ and its covariant derivatives. In particular, this implies that the transformed gauge field $\hat{A}_\mu$ is again a gauge connection.

It is worthwhile noticing that this classical formal equivalence does not hold only for the three-dimensional topological massive Yang-Mills theory. It also applies [2, 3] to any local gauge-invariant three-dimensional Yang-Mills-type action, defined as an arbitrary integrated local gauge-invariant polynomial with zero ghost number, built only with the gauge field strength and its covariant derivatives. That is, at the classical level one can obtain any local gauge-invariant three-dimensional Yang-Mills-type action (including those which are not power-counting renormalizable) by starting from the three-dimensional Chern-Simons action and then performing a gauge field redefinition within the space of gauge connections. This in turn provides a strong geometrical characterization [2] of three-dimension classical Yang-Mills-type action functionals: they can all be obtained by evaluating the Chern-Simons functional at a suitable point in the space of gauge connections which are formal power series in the parameter λ. In [3] a complete cohomological analysis of this property was given, establishing within the Batalin-Vilkovisky formalism the rigidity of (non-Abelian) Chern-Simons theories.

The problem of whether this classical equivalence can also be extended at the quantum level is yet an open issue. In this paper we will address this question within the BRST formulation of the Equivalence Theorem (ET) [4]. We will consider for the sake of definiteness the equivalence between three-dimensional Chern-Simons theory and topological massive Yang-Mills theory. Our technique can however be applied as well to the more general class of Yang-Mills-type actions.

In order to implement the classical equivalence at the quantum level one needs to take into account the contribution from the Jacobian of the field redefinition in eq.(1) [4]. This can be done by introducing a suitable set of antighost-ghost fields $(\bar{c}_\mu, c_\mu)$, carrying the same Lorentz-vector index as A_μ.

We show that the introduction of these ghost-antighost fields allows to gauge-fix the gauge symmetry of the three-dimensional Chern-Simons classical action by using a Lorentz-covariant gauge without introducing the standard Faddeev-Popov ghost and antighost scalar fields.

The resulting classical action turns out to be invariant under a new BRST differential $\tilde{s}$. $\tilde{s}$ is an extension of the gauge BRST differential s, under which the ungauged classical action of three-dimensional Chern-Simons theory is invariant, combined with the on-shell Equivalence Theorem BRST differential [4] δ, taking into account the effects of the field redefinition in eq.(1).

The absence of the conventional scalar Faddeev-Popov (FP) ghost-antighost fields constitutes an important difference with respect to the standard BRST gauge-fixing procedure. In fact it turns out that the local and covariant implementation of the Jacobian of the gauge connection redefinition in eq.(1) by means of the fields $(\bar{c}_\mu, c_\mu)$ cannot be performed if the conventional BRST gauge-fixing procedure for a Lorentz-covariant gauge has already been carried out. This is due to the fact that in this case a degeneracy arises in the two-point functions in the ghost-antighost sector of the classical action, which prevents the definition of the propagators in the sector with non-zero ghost number.

The BRST differential $\tilde{s}$ is nilpotent only modulo the equations of motion for $\bar{c}_\mu$. Hence we must resort to the full Batalin-Vilkovisky [5] formalism to obtain the Slavnov-Taylor (ST) identities associated with $\tilde{s}$. In particular, the classical action fulfilling the ST identities will acquire a non-linear dependence on the antifield $\bar{c}^{\mu *}$.

We will prove by purely algebraic arguments that these ST identities can be restored to all orders in the loop expansion. Let us remark here that the transformed action is no more of the power-counting renormalizable type. Hence we cannot apply the Quantum Action Principle (QAP) [6, 7, 8, 9, 10], which only holds for power-counting renormalizable theories, in order to guarantee that the breaking of the ST identites is equal to the insertion of the sum of a finite set of local operators of bounded dimension.

The latter property is of the utmost importance in the Algebraic Renormalization of gauge theories [10]. Indeed, if the breaking of the ST identities can be characterized as the insertion of the sum of a finite set of local operators with bounded dimension, its first non-vanishing order $\Delta^{(n)}$ in the loop expansion must reduce to a local polynomial in the fields, the antifields and their derivatives. Moreover, from purely algebraic properties one can derive a set of consistency conditions for $\Delta^{(n)}$, known as the Wess-Zumino consistency conditions [10, 12], which identify $\Delta^{(n)}$ as an element of the kernel of a suitable nilpotent differential operator, given by the classical linearized

ST operator [10]. One can then characterize $\Delta^{(n)}$ by means of the powerful techniques of cohomological algebra in the space of local functionals [10, 20], to which $\Delta^{(n)}$ belongs.

The QAP cannot be extended on general grounds to non power-counting renormalizable theories [4, 11]. We will then assume that a regularization scheme has been adopted fulfilling the so-called Quasi-Classical Action Principle (QCAP) [4, 11]. The fulfillment of the QCAP only implies that the first non-vanishing order in the loop expansion of the breaking of the ST identities is a local formal power series in the fields, the antifields and their derivatives, without any reference to the all orders behaviour of the ST breaking term.

In sharp contrast with the QAP, no characterization of the full breaking of the ST identities to all orders in the loop expansion is possible by means of the QCAP. Despite this fact, in the present case the QCAP can be combined with the Wess-Zumino consistency condition [10, 12] for the ST breaking terms in order to characterize on purely algebraic grounds all possible ST breaking terms $\Delta^{(n)}$ which appear at the first non-vanishing order n in the loop expansion. One can then recursively prove that the ST identities can be restored order by order in the loop expansion by a suitable choice of non-invariant counterterms, due to the fact that the cohomology of the linearized classical ST operator $\mathcal{S}_0$ is empty in the space of local formal power series with ghost number $+1$, to which $\Delta^{(n)}$ belongs.

We will then show that the Green functions of strictly $\tilde{s}$-invariant local operators are λ-independent, as a consequence of the fulfillment of the ST identities. The ST identities also imply that the Green functions of the gauge field $\hat{A}_\mu(A, \lambda)$ and of those local composite operators only depending on $\hat{A}_\mu$ are λ-independent. We remark that these operators are $\tilde{s}$-invariant only modulo the equation of motion for $\bar{c}_\mu$.

Such a selected set of Green functions can be equivalently computed from the quantum effective action at $\lambda = 0$ and from the quantum effective action at $\lambda \neq 0$. This translates at the quantum level the classical equivalence between three-dimensional Chern-Simons theory and topological massive Yang-Mills theory in flat Euclidean space-time.

We stress that this property only holds if the antighost-ghost fields $(\bar{c}_\mu, c_\mu)$ are taken into account in the quantization of the model. They actually play an essential rôle in the identification of the asymptotic physical degrees of freedom.

We refer here to a pure perturbative analysis of the physical spectrum. As is known [21], the topological nature of three-dimensional Chern-Simons theory prevents the existence of any physical state in the sense of the conventional perturbative framework of gauge theories, with the exception of the vacuum. On the contrary, ordinary topological massive Yang-Mills theory, without the inclusion of the antighost-ghost fields $(\bar{c}_\mu, c_\mu)$, does possess local excitations [13].

In our approach the correspondence between the quantum theory at $\lambda = 0$ and at $\lambda \neq 0$ at the level of the asymptotic states is preserved by virtue of the structure of $\tilde{s}$ and the introduction of the antighost-ghost fields $(\bar{c}_\mu, c_\mu)$, trivializing the cohomology of the asymptotic BRST operator Q in the sector with zero ghost number. This could in turn shed some light on the relevance at the asymptotic level of the fields $(\bar{c}^\mu, c^\mu)$, which were originally introduced [4] as a tool to implement in the path-integral in a local and covariant way the effect of the Jacobian of the field redefinition in eq.(1).

The paper is organized as follows. In sect. 2 we discuss the classical equivalence between three-dimensional Chern-Simons theory and three-dimensional topological massive Yang-Mills theory. In sect. 3 we present the BRST formulation of the Equivalence Theorem as applied to the field redefinition in eq.(1). We discuss the gauge-fixing condition and we introduce the antighost and ghost fields $(\bar{c}^\mu, c^\mu)$ associated with the Jacobian of the field redefinition in eq.(1). We also define the BRST differentials s, δ and $\tilde{s}$ and derive the classical ST identities of the model. In sect. 4 we show that the ST identities can be recursively restored to all orders in the loop expansion by a suitable choice of non-invariant counterterms. In sect. 5 we analyze the consequences of the ST identities for the Green functions of the theory and establish the λ-independence of the connected Green functions of any BRST invariant local operators, as well as of those local operators only depending on $\hat{A}(A, \lambda)$. We notice that the latter are BRST-invariant only modulo the equation of motion for $\bar{c}^\mu$. Locality is always understood in the sense of local formal power series. In sect. 6 we analyze the physical states of the original and transformed theory within the framework of conventional perturbative field theory and prove that no physical states are present, with the exception of the vacuum. Hence the correspondence between the perturbative physical spectrum of the theory at $\lambda = 0$ (Chern-Simons theory) and at $\lambda \neq 0$ is preserved. Finally conclusions are presented in sect. 7.

2 The classical equivalence

We follow the conventions of [2]. The classical action of topological massive Yang-Mills theory is given by

$$S_{YM}(A) + S_{CS}(A) \tag{2}$$

where $S_{YM}(A)$ is the Yang-Mills action

$$S_{YM}(A) = \frac{1}{4m} \mathrm{Tr} \int d^3x \, F_{\mu\nu} F^{\mu\nu} \tag{3}$$

and $S_{CS}(A)$ denotes the Chern-Simons term

$$S_{CS}(A) = \frac{1}{2} \mathrm{Tr} \int d^3x \, \epsilon^{\mu\nu\rho} (A_\mu \partial_\nu A_\rho + \frac{2}{3} g A_\mu A_\nu A_\rho) \, . \tag{4}$$

m in eq.(3) is the topological mass [13], while g is the coupling constant of the model. A^μ takes values in the Lie algebra $\mathfrak{G}$ of a compact Lie group G:

$$A_\mu = A^a_\mu T^a \,, \tag{5}$$

where the generators T^a are assumed to be anti-hermitian. The field strength $F_{\mu\nu}$ is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu] \,. \tag{6}$$

A_μ has mass dimension 1, while the parameters g and m have mass dimension 0 and 1 respectively. We work in flat Euclidean space-time.

It has been shown in [1, 2] that the classical action in eq.(2) can be obtained from the pure Chern-Simons action in eq.(4) by a suitable field redefinition:

$$S_{CS}(\hat{A}) = S_{YM}(A) + S_{CS}(A) \,, \tag{7}$$

where

$$\hat{A}_\mu = A_\mu + \sum_{n=1}^{\infty} \frac{1}{m^n} \theta^n_\mu(D, F) \,. \tag{8}$$

In the above equation the coefficients $\theta^n_\mu(D, F)$ are local and covariant, being constructed only with the field strength $F_{\mu\nu}$ and the covariant derivative

$$D_\mu(\cdot) = \partial_\mu(\cdot) + g[A_\mu, \cdot] \,. \tag{9}$$

We find it convenient to denote by λ the inverse of m:

$$\lambda = \frac{1}{m} \,. \tag{10}$$

The field redefinition in eq.(8) is a formal power series in λ.

$S_{CS}(\hat{A})$ is invariant under

$$s\hat{A}_\mu = (D_{\hat{A}})_\mu \omega \,, \tag{11}$$

In the above equation the notation $(D_{\hat{A}})_\mu$ means that the covariant derivative is computed with respect to the gauge field $\hat{A}_\mu$. ω is the Lie-algebra valued classical ghost associated with the BRST differential s.

The requirement that s is nilpotent yields

$$s\omega = -\frac{g}{2}\{\omega, \omega\} \,. \tag{12}$$

By setting

$$sA_\mu = D_\mu \omega \tag{13}$$

we obtain

$$s\hat{A}_\mu(A,\lambda) = (D_{\hat{A}(A,\lambda)})_\mu \omega \,, \tag{14}$$

due to the fact that the coefficients θ^n_μ are covariant. In eq.(14) $\hat{A}(A,\lambda)$ is understood as the formal power series given in eq.(8). Eq.(14) means that $\hat{A}_\mu(A,\lambda)$ is a connection. As a consequence, $S_{CS}(\hat{A}(A,\lambda))$ is invariant under the BRST transformation in eq.(13). This is what one should expect, since the R.H.S. of eq.(7) is also invariant under the transformation in eq.(13).

It has been observed [2, 3] that any Yang-Mills-type action (defined as an arbitrary integrated local invariant polynomial with vanishing ghost number, built only with the field strength $F_{\mu\nu}$ and its covariant derivatives) can actually be obtained by evaluating the Chern-Simon functional in eq.(7) on a suitable gauge connection $\hat{A}_\mu$ of the form in eq.(8). This in turn provides a strong geometrical characterization [2] of three-dimension classical Yang-Mills-type action functionals: they can all be obtained by evaluating the Chern-Simons functional at a suitable point in the space of gauge connections which are formal power series in the parameter λ. In [3] a complete cohomological analysis of this property was given, establishing within the Batalin-Vilkovisky formalism the rigidity of (non-Abelian) Chern-Simons theories.

For the sake of definiteness we will analyze in the following sections the equivalence between the three-dimensional Chern-Simons action and massive topological Yang-Mills theory. We remark however that our technique can also be applied in a straightforward way to the more general situation where the equivalence between the Chern-Simons action and an arbitrary Yang-Mills-type action is considered.

3 BRST formulation of the gauge connection redefinition

The equivalence discussed in sect.2 holds at the classical level. The technique discussed in [4] provides a prescription, based on the Slavnov-Taylor (ST) identities, to implement the field redefinition in eq.(8) at the quantum level.

In particular one needs to take into account the Jacobian of the transformation in eq.(8) by introducing a suitable set of ghost and antighost fields [4]. We apply here the on-shell formalism of the Equivalence Theorem [4], thus avoiding the introduction of auxiliary Lagrange multiplier fields, which turn out to be unnecessary in the present case. We stress that from the point of view of the perturbative expansion the parameter λ is treated here as an external classical constant source. We first define

$$\begin{aligned} \Sigma &= S_{CS}(\hat{A}(A,\lambda)) \\ &= S_{YM}(A) + S_{CS}(A) \,. \end{aligned} \tag{15}$$

In order to quantize the model a gauge-fixing term must be introduced. For this purpose we consider the following action:

$$\Sigma' = \Sigma - \frac{\alpha}{2}\mathrm{Tr}\int d^3x \, \left(\partial \hat{A}(A,\lambda)\right)^2 . \tag{16}$$

Σ' is no more s-invariant, due to the Lorentz-covariant gauge-fixing in the R.H.S. of eq.(16). We will comment on the choice of the gauge-fixing condition in eq.(16) later on in this section.

By following the ET on-shell quantization prescription given in [4] we include the contribution from the Jacobian of the transformation in eq.(8) by introducing a set of antighost-ghost fields $(\bar{c}_\mu, c_\mu)$. $\bar{c}_\mu$ is s-invariant, while c_μ transforms as follows under s:

$$s c_\mu = -g\{c_\mu, \omega\} . \tag{17}$$

The BRST differential δ associated with the field redefinition in eq.(8) is given by

$$\begin{aligned} &\delta \bar{c}_\mu = - \left.\frac{\delta \Sigma'}{\delta \hat{A}^\mu}\right|_{\hat{A}^\mu = \hat{A}^\mu(A,\lambda)} , \qquad \delta A_\mu = c_\mu \, , \qquad \delta c_\mu = 0 \, , \\ &\delta \lambda = \chi \, , \qquad \delta \chi = 0 \, . \end{aligned} \tag{18}$$

The full BRST differential $\tilde{s}$ is

$$\tilde{s} = s + \delta \, . \tag{19}$$

For convenience we gather here the $\tilde{s}$-transformations of the fields and the external sources λ, χ, ω:

$$\begin{aligned} &\tilde{s} A_\mu = D_\mu \omega + c_\mu \, , \qquad \tilde{s}\omega = -\frac{g}{2}\{\omega,\omega\} \, , \qquad \tilde{s}\lambda = \chi \, , \qquad \tilde{s}\chi = 0 \, , \\ &\tilde{s} c_\mu = -g\{c_\mu, \omega\} \, , \qquad \tilde{s}\bar{c}_\mu = - \left.\frac{\delta \Sigma'}{\delta \hat{A}_\mu}\right|_{\hat{A}_\mu = \hat{A}_\mu(A,\lambda)} . \end{aligned} \tag{20}$$

Then the classical action

$$\begin{aligned} \Sigma_0 &= \Sigma' + \mathrm{Tr}\int d^3x \, \bar{c}^\mu \tilde{s} \hat{A}^\mu \\ &= \Sigma' + \mathrm{Tr}\int d^3x \, \bar{c}^\mu \left(\partial_\mu \omega + g[\hat{A}_\mu, \omega] \right) \\ &\quad + \mathrm{Tr}\int d^3x \, \bar{c}^\mu \left(\frac{\delta \hat{A}^\mu}{\delta A^\rho} c^\rho + \frac{\delta \hat{A}^\mu}{\delta \lambda}\chi \right) \end{aligned} \tag{21}$$

is $\tilde{s}$-invariant. The term in the last line of eq.(21) takes into account the Jacobian of the field redefinition. We remark that ω is an external anti-commuting classical source. It does not need to be promoted to a ghost field, since Σ' in eq.(16) is already gauge-fixed. This constitutes an important difference with the standard Faddeev-Popov BRST gauge-fixing procedure for gauge theories [14]. In fact the addition of a conventional Lorentz-covariant gauge-fixing term

$$\mathrm{Tr}\int d^3x\, s\left(\bar{c}(\frac{\alpha}{2}B - \partial A)\right) = \mathrm{Tr}\int d^3x\, \left(\frac{\alpha}{2}B^2 - B\partial A - \partial^\mu \bar{c} D_\mu c\right) \tag{22}$$

by means of the set of scalar Faddeev-Popov fields $\bar{c}, c$ and the Nakanishi-Lautrup multiplier field B, transforming as follows under s

$$\begin{aligned} & sA_\mu = D_\mu c\,, \qquad sc = -\frac{g}{2}\{c, c\}\,, \\ & s\bar{c} = B\,, \qquad sB = 0\,, \end{aligned} \tag{23}$$

would lead to a degeneracy in the two-point functions in the ghost-antighost sector of the classical action, hence preventing the construction of the ghost propagators.

The rôle of the antighost-ghost fields $(\bar{c}_\mu, c_\mu)$ in the definition of the asymptotic physical states of the model within the conventional framework of perturbative field theory will be analyzed in sect. 6.

λ and χ are constant external classical sources, with χ anti-commuting.

We can assign the ghost number as follows: A_μ and λ have ghost number zero, $\bar{c}^\mu$ has ghost number -1, c^μ has ghost number $+1$. ω and χ have ghost number $+1$. With these assignments Σ_0 in eq.(21) has ghost number zero.

The mass dimension of $\bar{c}^\mu$ is 2, the mass dimension of c^μ is 1. λ, χ and ω have mass dimension zero.

We remark that $\tilde{s}$ is nilpotent on $\bar{c}^\mu$ only modulo the equation of motion for $\bar{c}_\mu$:

$$\tilde{s}^2 \bar{c}^\mu = -\frac{\delta^2 \Sigma'}{\delta \hat{A}_\mu \delta \hat{A}_\nu} \tilde{s} \hat{A}_\nu = -\frac{\delta^2 \Sigma'}{\delta \hat{A}_\mu \delta \hat{A}_\nu} \frac{\delta \Sigma_0}{\delta \bar{c}^\nu}\,. \tag{24}$$

Therefore the full Batalin-Vilkovisky formalism is needed [5] in order to write the ST identities associated with $\tilde{s}$. The classical action will acquire a non-linear dependence on the antifield $\bar{c}^*_\mu$.

We consider the following classical action, constructed according to the prescription given in [4] for the on-shell version of the Equivalence Theorem:

$$\begin{aligned} \Gamma^{(0)} \quad = \quad & \Sigma_0 \\ & + \mathrm{Tr}\int d^3x \sum_{j=1}^\infty (-1)^j \frac{1}{j!} (\bar{c}^{\mu_1})^* \dots (\bar{c}^{\mu_j})^* \frac{\delta^{(j)}\Sigma'}{\delta \hat{A}_{\mu_1} \dots \delta \hat{A}_{\mu_j}}\,. \end{aligned} \tag{25}$$

$\Gamma^{(0)}$ is a local formal power series in the antifield $(\bar{c}^\mu)^*$. This follows as a consequence of the fact that Σ_0 in eq.(21) is a local formal power series in the field A_μ, due to the gauge-fixing term entering into eq.(16).

$\Gamma^{(0)}$ satisfies the following ST identities:

$$
\begin{aligned}
\mathcal{S}(\Gamma^{(0)}) &= \mathrm{Tr} \int d^3x \, \Bigg((\partial_\mu \omega + g[A_\mu, \omega] + c_\mu) \frac{\delta \Gamma^{(0)}}{\delta A_\mu} + \frac{\delta \Gamma^{(0)}}{\delta (\bar{c}^\mu)^*} \frac{\delta \Gamma^{(0)}}{\delta \bar{c}_\mu} \\
&\qquad - g\{c_\mu, \omega\} \frac{\delta \Gamma^{(0)}}{\delta c_\mu} - \frac{g}{2}\{\omega, \omega\} \frac{\delta \Gamma^{(0)}}{\delta \omega} \Bigg) + \chi \frac{\partial \Gamma^{(0)}}{\partial \lambda} = 0 \, . \qquad (26)
\end{aligned}
$$

The corresponding linearized ST operator $\mathcal{S}_0$ is given by

$$
\begin{aligned}
\mathcal{S}_0 &= \mathrm{Tr} \int d^3x \, \Bigg((\partial_\mu \omega + g[A_\mu, \omega] + c_\mu) \frac{\delta}{\delta A_\mu} + \frac{\delta \Gamma^{(0)}}{\delta (\bar{c}^\mu)^*} \frac{\delta}{\delta \bar{c}^\mu} + \frac{\delta \Gamma^{(0)}}{\delta \bar{c}_\mu} \frac{\delta}{\delta (\bar{c}^{\mu *})} \\
&\qquad - g\{c_\mu, \omega\} \frac{\delta}{\delta c_\mu} - \frac{g}{2}\{\omega, \omega\} \frac{\delta}{\delta \omega} \Bigg) + \chi \frac{\partial}{\partial \lambda} \, . \qquad (27)
\end{aligned}
$$

We notice that λ forms together with its partner χ a $\mathcal{S}_0$-doublet. This in turn will allow to control the dependence of the Green functions of the model on λ. The propagators for A_μ and $(\bar{c}^\mu, c^\mu)$, computed from $\Gamma^{(0)}$, both exist, as it can be easily checked. The classical action $\Gamma^{(0)}$ together with the ST identities in eq.(26) is the starting point for the quantization of the model.

4 Quantization

In this section we show that the ST identities in eq.(26) can be restored to all orders in the loop expansion. A formal proof of the fact that the ST identities in eq.(26) are anomaly-free at the quantum level is needed because the inclusion of the gauge BRST differential s into $\tilde{s}$ prevents a direct application of the results given in [4] on the recursive fulfillment of the Equivalence Theorem ST identities, associated with the pure Equivalence Theorem BRST differential δ.

We first remark that the Quantum Action Principle [6, 7, 8, 9, 10] does not apply, since we are dealing with a non-power counting renormalizable theory. Let us comment on this point further. For power-counting renormalizable theories the QAP characterizes the possible breaking of the ST identities, to all orders in perturbation theory, as the insertion of a finite sum of local operator with bounded dimension. Consequently, if such an insertion were zero up to order $n-1$, at the n-th order it must reduce to a local polynomial in the fields and the external sources and their derivatives with bounded dimension. This follows from the locality of the insertion and the topological interpretation of the $\hbar$-expansion as a loop-wise expansion.

One might notice that all what is needed to carry out the recursive analysis of the ST breaking terms by using cohomological and algebraic methods is that part of the QAP, saying that at the first non-vanishing order in the loop expansion the ST breaking term is a local polynomial in the fields and the external sources and their derivatives.

It has then been proposed [11] that this consequence of the QAP could be extended to non power-counting renormalizable theories in the form of the so-called Quasi-Classical Action Principle, stating that

> *In the loop-wise perturbative expansion the first non-vanishing order of ST identities is a classical local integrated formal power series in the fields and external sources and their derivatives.*

We remark that the QCAP does not allow to characterize the possible breaking of ST identities to all orders in the loop expansion, independently of the non-invariant counterterms and the normalization conditions chosen, as the QAP does.

Although its plausibility, no satisfactory proof is available on general grounds for the QCAP. Thus from now on we will assume that it is fulfilled by the regularization used to construct the vertex functional Γ. Moreover, we point out that we do not require that the breaking term is polynomial: loosing the power-counting entails that no bounds on the dimension can in general be expected.

Since we assume the QCAP, we can now use the power of cohomological algebra to constrain the possible ET ST breaking terms and show that the ET ST identities can always be restored by a suitable order by order choice of non-invariant counterterms.

We suppose that the ST identities have been fulfilled up to order $n-1$ in the loop expansion:

$$\mathcal{S}(\Gamma)^{(j)} = 0\,, \qquad j = 0, 1, \ldots, n-1\,. \tag{28}$$

By virtue of the Quasi-Classical Action Principle the n-th order ST breaking term $\Delta^{(n)}$, given by

$$\Delta^{(n)} \equiv \mathcal{S}(\Gamma)^{(n)}\,, \tag{29}$$

is a local formal power series in the fields, the external sources and their derivatives with ghost number $+1$. Moreover it satisfies the Wess-Zumino consistency condition [10, 12]

$$\mathcal{S}_0(\Delta^{(n)}) = 0\,, \tag{30}$$

where $\mathcal{S}_0$ is the linearized classical ST operator given in eq.(27).

We are going to show by purely algebraic methods that eq.(30) implies

$$\Delta^{(n)} = \mathcal{S}_0(-\Xi^{(n)}) \tag{31}$$

for a local formal power series $\Xi^{(n)}$ with ghost number zero.

Then the n-th order ST identities can be restored by adding the non-symmetric counterterm functional $-\Xi^{(n)}$ to the n-th order vertex functional. In order to prove eq.(31) we will first derive some general results on the cohomology $H(\mathcal{S}_0, \mathcal{F})$ of the linearized ST operator $\mathcal{S}_0$ in the space of Lorentz-invariant integrated local formal power series spanned by $A_\mu, \bar{c}^\mu, \bar{c}^{\mu *}, c^\mu, \omega$ and λ, χ and their derivatives. At the first stages of this analysis we do not impose any restrictions on the ghost number of the functionals belonging to $\mathcal{F}$. In the end we will specialize to the sector with ghost number $+1$, to which $\Delta^{(n)}$ in eq.(29) belongs.

Let us remark first that (λ, χ) form a set of doublets under $\mathcal{S}_0$, since

$$\mathcal{S}_0(\lambda) = \chi\,, \qquad \mathcal{S}_0(\chi) = 0\,. \tag{32}$$

(λ, χ) is a coupled doublet, in the sense that the counting operator

$$\mathcal{N} = \lambda \frac{\partial}{\partial \lambda} + \chi \frac{\partial}{\partial \chi} \tag{33}$$

does not commute with $\mathcal{S}_0$. Doublets can be handled by the powerful methods of homological perturbation theory [15, 16, 17, 20]. It can be proven [16, 17, 18] that the cohomology of any nilpotent differential ρ in the space of local formal integrated power series $\mathcal{P}$ spanned by a set of variables $\{\varphi, z, w\}$ and their derivatives is isomorphic to the cohomology of the restriction ρ' of ρ to the space $\mathcal{P}' \subset \mathcal{P}$ of local formal integrated power series independent of (z, w), whenever (z, w) form a set of coupled doublets under ρ, i.e.

$$\rho z = w\,, \qquad \rho w = 0 \tag{34}$$

and

$$[\mathcal{N}, \rho] \neq 0\,, \tag{35}$$

where $\mathcal{N} = z\frac{\delta}{\delta z} + w\frac{\delta}{\delta w}$ is the counting operator for (z, w). We notice that no restriction on the ghost number of the functionals in $\mathcal{P}$ and $\mathcal{P}'$ is made. It might also happen that $\mathcal{P}$ and $\mathcal{P}'$ split into a sum of subspaces with different ghost number.

This extends the well-known result [10, 20] stating that the cohomology of an arbitrary nilpotent differential ρ is independent of the set of variables (z, w), provided that they form a set of decoupled doublets, i.e. provided that, in addition to eq.(34), the following commutation relation holds:

$$[\mathcal{N}, \rho] = 0\,. \tag{36}$$

By making use of the results of [16, 17, 18] we can hence restrict ourselves to the analysis of the cohomology of the restriction $\mathcal{S}_0'$ of $\mathcal{S}_0$ to the space of Lorentz-invariant local formal power series independent of (λ, χ). We denote this space by $\mathcal{F}'$.

$\mathcal{S}_0'$ is explicitly given by

$$\mathcal{S}_0' = \mathrm{Tr}\int d^3x \Bigg((\partial_\mu\omega + g[A_\mu,\omega] + c_\mu)\frac{\delta}{\delta A_\mu} + \frac{\delta\Gamma^{(0)}}{\delta\bar{c}^{\mu *}}\Bigg|_{\lambda=\chi=0}\frac{\delta}{\delta\bar{c}^\mu} \\ +(\partial_\mu\omega + g[A_\mu,\omega] + c_\mu)\frac{\delta}{\delta\bar{c}^{\mu *}} - g\{c_\mu,\omega\}\frac{\delta}{\delta c_\mu} - \frac{g}{2}\{\omega,\omega\}\frac{\delta}{\delta\omega}\Bigg) . \quad (37)$$

It is convenient to change generators according to

$$c_\mu' = c_\mu + \partial_\mu\omega + g[A_\mu,\omega]\,, \qquad A_\mu' = A_\mu - \bar{c}^*_\mu\,. \quad (38)$$

$\mathcal{S}_0'$ reads in the new variables:

$$\mathcal{S}_0' = \mathrm{Tr}\int d^3x \left(c_\mu'\frac{\delta}{\delta\bar{c}^*_\mu} + \frac{\delta\Gamma^{(0)}}{\delta\bar{c}^{\mu *}}\Bigg|_{\lambda=\chi=0}\frac{\delta}{\delta\bar{c}^\mu} - \frac{g}{2}\{\omega,\omega\}\frac{\delta}{\delta\omega}\right) . \quad (39)$$

$(\bar{c}^*_\mu, c_\mu')$ are again a set of coupled doublets under $\mathcal{S}_0'$ and by the same arguments used before the cohomology $H(\mathcal{S}_0',\mathcal{F}')$ turns out to be isomorphic to the cohomology of the restriction $\mathcal{S}_0''$ of $\mathcal{S}_0'$ to the subspace $\mathcal{F}''$ of Lorentz-invariant integrated local formal power series independent of $\bar{c}^*_\mu, c_\mu'$. Notice that in this subspace $A_\mu' = A_\mu$. $\mathcal{S}_0''$ has the following form:

$$\mathcal{S}_0'' = \mathrm{Tr}\int d^3x\Bigg(- \frac{\delta\Sigma'}{\delta\hat{A}^\mu}\Bigg|_{\begin{pmatrix}\lambda=\chi=0,\\ \hat{A}_\mu = A_\mu,\\ c_\mu' = \bar{c}^*_\mu = 0\end{pmatrix}}\frac{\delta}{\delta\bar{c}^\mu} - \frac{g}{2}\{\omega,\omega\}\frac{\delta}{\delta\omega}\Bigg) . \quad (40)$$

We have now to study the cohomology $H(\mathcal{S}_0'',\mathcal{F}'')$ of $\mathcal{S}_0''$ in the subspace $\mathcal{F}''$ spanned by Lorentz-invariant monomials generated by $A_\mu,\omega,\bar{c}_\mu$ and their derivatives. For that purpose we use a very general result in cohomological algebra [10, 20] stating that the cohomology $H(\mathcal{S}_0'',\mathcal{F}'')$ is isomorphic to a subset of the cohomology $H(\mathcal{L},\mathcal{F}'')$ of the Abelian approximation $\mathcal{L}$ to $\mathcal{S}_0''$, which is given by

$$\mathcal{L} = \int d^3x \left(-\Gamma^{ab}_{\mu\nu}A^b_\nu\frac{\delta}{\delta\bar{c}^{\mu a}}\right) . \quad (41)$$

In the above equation $\Gamma^{ab}_{\mu\nu}$ is the 1-PI classical two point-function, defined by

$$\Gamma^{ab}_{\mu\nu}(x-y) = \frac{\delta^2\Sigma'}{\delta A^a_\mu(x)\delta A^b_\nu(y)}\Bigg|_{\lambda=0,A_\mu=0} . \quad (42)$$

In order to study the cohomology of $\mathcal{L}$ we make use of the relationship between $\mathcal{L}$ and the Koszul-Tate differential δ_{KT} [20] associated with the classical action

$$S = -\mathrm{Tr}\int d^3x \left(+\frac{1}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - \frac{\alpha}{2}(\partial A)^2\right) . \quad (43)$$

The Koszul-Tate differential δ_{KT} acts in the space $\mathcal{F}_{KT}$ of local formal power series in A_μ, ω, their antifields $\bar{c}_\mu, \omega^*$ (with $\bar{c}_\mu$ playing the rôle of the antifield for A_μ) and their derivatives. The action of δ_{KT} on the generators of $\mathcal{F}_{KT}$ is the following:

$$
\begin{aligned}
&\delta_{KT} A^a_\mu = \mathcal{L} A^a_\mu = 0\,, \qquad \delta_{KT}\omega^a = \mathcal{L}\omega^a = 0\,, \\
&\delta_{KT} \bar{c}^a_\mu = \mathcal{L}\bar{c}^a_\mu = \frac{\delta S}{\delta A^a_\mu}\,, \qquad \delta_{KT}\omega^{*a} = -\partial^\mu \bar{c}^a_\mu\,.
\end{aligned} \tag{44}
$$

From the above equation we see that $\mathcal{L}$ coincides with the restriction of δ_{KT} to the subspace $\mathcal{F}'' \subset \mathcal{F}_{KT}$ of ω^*-independent elements.

We define the antifield number as follows: A^a_μ and ω^a carry zero antifield number, $\bar{c}^a_\mu$ carries antifield number $+1$ and ω^{*a} carries antifield number $+2$.[2]

The general results on the homology groups of Koszul-Tate differentials [20] allow us to conclude that the homology groups $H_m(\delta_{KT}, \mathcal{F}_{KT})$ are zero in strictly positive antifield number m. In particular $H(\delta_{KT}|_{\mathcal{F}''}, \mathcal{F}'') = H(\mathcal{L}, \mathcal{F}'')$ must be $\bar{c}^\mu_a$-independent. We remark that in general $H(\mathcal{L}, \mathcal{F}'')$ is however A^a_μ-dependent [20].

By using the fact that $H(\mathcal{L}, \mathcal{F}'')$ must be $\bar{c}^\mu_a$-independent we can then further restrict the analysis to the space $\mathcal{F}'''$ of Lorentz-invariant integrated local formal power series spanned by (ω, A_μ) and their derivatives. That is, the cohomology $H(\mathcal{S}_0, \mathcal{F})$ is isomorphic to a subset of the cohomology $H(\mathcal{S}'''_0, \mathcal{F}''')$ of the restriction $\mathcal{S}'''_0$ of $\mathcal{S}''_0$ to the space $\mathcal{F}'''$:

$$
\mathcal{S}'''_0 = \mathrm{Tr} \int d^3x \Big(-\frac{g}{2}\{\omega, \omega\}\frac{\delta}{\delta\omega}\Big)\,. \tag{45}
$$

We now specialize to the sector $\mathcal{F}'''_{+1}$ of $\mathcal{F}'''$ with ghost number $+1$. Any element $Y \in H(\mathcal{S}'''_0, \mathcal{F}'''_{+1})$ must contain just one ω and must be $\mathcal{S}'''_0$-invariant. It follows that it is zero, and then we can conclude that $H(\mathcal{S}'''_0, \mathcal{F}'''_{+1})$ is empty. Going back to the cohomology of the full linearized classical ST operator $\mathcal{S}_0$ we obtain that $H(\mathcal{S}_0, \mathcal{F}_{+1})$ is also empty. Equivalently, eq.(31) holds. This ends the proof that the ST identities can be restored also at the n-th order in the loop expansion.

5 Consequences of the ST identities for the Green functions

In this section we analyze the consequences of the ST identities

$$
\mathcal{S}(\Gamma) = 0 \tag{46}
$$

[2] Since with these assignments δ_{KT} acts as a boundary operator (it has degree -1 with respect to the antifield degree) the groups $H_m(\delta_{KT}, \mathcal{F}_{KT})$ in antifield degree m are said “homology groups” rather than “cohomology groups” of the differential δ_{KT}.

for the Green functions of the model. We remark that we can introduce in Σ' (and correspondingly in the classical action in eq.(25)) a set of external sources $\zeta_i(x)$, coupled to BRST-invariant local composite operators $\mathcal{Q}_i(x)$, without spoiling the validity of the ST identities in eq.(46). The ST identities are also not broken if we add a set of external sources $\tau_j(x)$ coupled to local composite operators $\mathcal{O}_j(\hat{A}(A,\lambda))$, which depend only on $\hat{A}(A,\lambda)$. Notice that the operators $\mathcal{O}_j$ are BRST-invariant only modulo the equation of motion for $\bar{c}_\mu$.

We use the collective notation $\beta = \{\zeta_i, \tau_j\}$ to denote all the sources ζ_i, τ_j. The connected generating functional W is defined by the Legendre transform of Γ with respect to the quantum fields of the model:

$$W[J_\mu, J_{\bar{c}_\mu}, J_{c_\mu}; \omega, \lambda, \chi, \bar{c}^{*\mu}, \beta] = \Gamma + \mathrm{Tr} \int d^3x \, \left(J_\mu A^\mu + J_{\bar{c}_\mu} \bar{c}^\mu + J_{c_\mu} c^\mu \right) . \tag{47}$$

The ST identities in eq.(46) read for W

$$\begin{aligned} \mathcal{S}(W) = -\mathrm{Tr} \int d^3x \, \Bigg\{ \left((\partial_\mu \omega + g[\frac{\delta W}{\delta J_\mu}, \omega] + \frac{\delta W}{\delta J_{c_\mu}} \right) J^\mu + \frac{\delta W}{\delta \bar{c}^{\mu *}} J_{\bar{c}_\mu} \\ + \frac{g}{2}\{\omega, \omega\} \frac{\delta W}{\delta \omega} - g\{\frac{\delta W}{\delta J_{c_\mu}}, \omega\} J_{c_\mu} \Bigg\} + \chi \frac{\delta W}{\delta \lambda} = 0 \end{aligned} \tag{48}$$

The ST identities in eq.(48) allow to control the dependence of the connected Green functions on λ and in particular to prove that the Green functions of BRST-invariant operators are independent of λ [10, 19]. Moreover, they also allow to prove that the Green functions of the operators $\mathcal{O}_j(\hat{A}(A,\lambda))$, which are $\tilde{s}$-invariant only modulo the classical equation of motion of $\bar{c}_\mu$, are also independent of λ. The proof is standard [10, 19, 4] and is reported here for completeness. We first take the derivative of eq.(48) with respect to χ and to $\beta_{j_1}(x_1), \ldots, \beta_{j_n}(x_n)$ and finally go on-shell by setting $J_\mu = J_{\bar{c}_\mu} = J_{c_\mu} = \bar{c}^{*\mu} = \beta = \omega = \chi = 0$. This yields

$$\left. \frac{\delta^{(n+1)} W}{\delta \lambda \delta \beta_{j_1}(x_1) \ldots \delta \beta_{j_n}(x_n)} \right|_{on-shell} = 0 \, , \tag{49}$$

stating the independence of the Green functions

$$G^{(n)}_{j_1,\ldots,j_n}(x_1, \ldots, x_n) \equiv \left. \frac{\delta^{(n)} W}{\delta \beta_{j_1}(x_1) \ldots \delta \beta_{j_n}(x_n)} \right|_{on-shell} \tag{50}$$

of λ. Hence $G^{(n)}_{j_1,\ldots,j_n}$ can be equivalently computed at $\lambda \neq 0$ and in the limit $\lambda = 0$. Since $\lambda = \frac{1}{m}$, eq.(49) entails that the Green functions in eq.(50) are independent of the topological mass parameter m.

6 Physical states

Due to the validity of the ST identities in eq.(46), the asymptotic BRST operator Q [24, 25, 26], acting on the asymptotic states of the theory, is a conserved charge. The action of Q on the asymptotic fields of the model is given in the present case by the following commutation relations:

$$
\begin{aligned}
&[Q, A_\mu^{(as)}(x)] = c_\mu^{(as)}\,, \qquad && \{Q, c_\mu^{(as)}\} = 0\,, \\
&\{Q, \bar{c}_\mu^{(as)}\} = 0\,.
\end{aligned}
\tag{51}
$$

From the above equation we see that, unlike $\tilde{s}$, Q is nilpotent. This follows from the fact that Q acts on on-shell fields.

The existence of a nilpotent conserved asymptotic BRST charge Q allows us to apply the conventional BRST quantization of gauge theories [14, 26] to the present model. The physical Hilbert subspace is defined as usual by $\mathcal{H}_{phys} = \mathrm{Ker}\, Q/\mathrm{Im}\, Q$. The vacuum is Q-invariant. Moreover, we are only interested in cohomology classes with zero ghost number. From eq.(51) we see that the pair $(A_\mu^{(as)}, c_\mu^{(as)})$ forms a Q-doublet and hence it does belong to $\mathcal{H}_{phys}$. The only states belonging to $\mathcal{H}_{phys}$ are those generated by $\bar{c}^\mu$, which, however, correspond to cohomology classes with negative ghost number.

We conclude that no physical states with zero ghost number exist in this theory, apart from the vacuum. This is a well-known result for three-dimensional Chern-Simons theory: with the exception of the vacuum, there are no physical states in the usual sense of perturbative field theory [21]. The non-trivial physical Hilbert space of Chern-Simons theory, as described for instance in [22, 23] for the case of the three-manifold $\Sigma \otimes R$ with Σ a Riemann surface of non-trivial genus, is beyond the purely perturbative framework of the present analysis and thus cannot be dealt with by the cohomological arguments of this section.

We point out that ordinary topological massive Yang-Mills theory, without the inclusion of the antighost-ghost fields $(\bar{c}_\mu, c_\mu)$, does possess local excitations. The correspondence at the asymptotic level between the quantum theory at $\lambda = 0$ and at $\lambda \neq 0$ is actually preserved only thanks to the presence of the antighost-ghost fields $(\bar{c}_\mu, c_\mu)$.

We remark that our approach has the virtue to explicitly show by purely cohomological arguments the absence in this model of asymptotic states different from the vacuum, which are physical in the sense of conventional perturbative quantum field theory. It is the structure of the BRST asymptotic charge Q that enforces the triviality of $\mathcal{H}_{phys}$ in the sector with zero ghost number. In this construction the antighost-ghost fields, needed to take into account the Jacobian of the gauge field redefinition in eq.(8), play an essential rôle. This in turn shows their relevance also at the asymptotic level for a proper quantum implementation of the classical equivalence between three-dimensional Chern-Simons theory and topological massive Yang-Mills theory.

7 Conclusions

In this paper we have analyzed the extension to the quantum level of the classical equivalence upon a redefinition of the gauge connection between Chern-Simons theory and topological massive Yang-Mills theory in three-dimensional Euclidean space-time, within the BRST formulation of the Equivalence Theorem. The parameter controlling the change in the gauge connection is the inverse λ of the topological mass m. At the quantum level one needs to take into account the Jacobian of the gauge field redefinition in eq.(8) in order to properly extend the classical equivalence to the full λ-dependent quantum effective action. This can be done in a local and covariant way by introducing a set of antighost and ghost fields $\bar{c}^\mu, c^\mu$ with the same Lorentz-vector index as A_μ. The BRST differential $\tilde{s}$ associated with the gauge field redefinition has then been derived and the corresponding Slavnov-Taylor (ST) identities have been proven to be anomaly-free. $\tilde{s}$ incorporates both the BRST differential s, issued from the gauge invariance of the ungauged topological Chern-Simons theory, and the BRST differential δ, associated with the field redefinition in eq.(8). As a consequence of the ST identities, the Green functions of local operators depending only on the transformed gauge connection $\hat{A}(A, \lambda)$, as well as those of BRST invariant operators, are shown to be λ-independent. Hence they can be equivalently computed at $\lambda \neq 0$ and in the limit $\lambda = 0$. Since $\lambda = \frac{1}{m}$, this entails that such a selected set of Green functions is independent of the topological mass m. The identification of the physical states of the model within the conventional perturbative approach to gauge theories can be carried out by following the standard BRST cohomological construction. It turns out that the antighost and ghost fields $\bar{c}^\mu, c^\mu$ play an essential rôle in this procedure, trivializing the cohomology of the asymptotic BRST charge in the sector with zero ghost number. This guarantees that the correspondence at the level of asymptotic states between the quantum theory at $\lambda = 0$ and at $\lambda \neq 0$ is actually preserved. This in turn provides an additional insight into the relevance of the antighost and ghost fields, associated with the gauge field redefinition, for a proper quantum implementation of the classical equivalence between Chern-Simons theory and massive topological Yang-Mills theory in three-dimensional flat Euclidean space-time.

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