hep-th/0107238
DSF–22–2001

On the Phase Transition of Conformal Field Theories with Holographic Duals

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Abstract

We study the thermodynamic relations of conformal field theories (CFTs), which are holographically dual to anti-de Sitter–Schwarzschild bulk space-times. A Cardy–Verlinde formula is derived thermodynamically for CFTs living on $S^n \times \mathbb{R}$ with S^n having an arbitrary radius. The Hawking–Page phase transition of the CFT is described using Landau's theory of phase transitions, and an alternative derivation of the Cardy–Verlinde formula is presented. The condensate in the high temperature phase is identified as being composed of radiational matter.

1 Introduction

Through the holographic principle [1, 2], the thermodynamics of black holes in anti-de Sitter (AdS) space-times [3, 4] has found a new application in describing the high-temperature phase of conformal field theories (CFTs) with holographic duals [5]. Thus, the Hawking–Page phase transition has found a new interpretation as, *e.g.*, the transition between the confining and de-confining phases in $\mathcal{N}=4$ super Yang–Mills gauge theory. Recently, Verlinde [6] observed that, in these CFTs, a Cardy-type entropy formula [7] holds. Subsequently dubbed the "Cardy–Verlinde formula," its appearance in theories dual to various bulk space-times has been studied [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. Black hole phase transitions have recently been discussed in [22, 23, 24]. For a recent list of references on cosmological aspects of holography and entropy bounds in theories with holographic duals, see [20]. See also [25, 26] for related topics.

In this letter, we address three questions related to the thermodynamics of CFTs dual to AdS–Schwarzschild bulk space-times. In [6, 27], Verlinde and Savonije used Witten's result [5] for the entropy and energy of the boundary CFT and rescaled the energy and temperature according to the red-shift for an observer on a brane at a certain distance from the black hole. Although this is a valid approach, an alternative would be to derive these relations directly from the on-shell action, which we shall do in Sec. 2. Our calculation is based on Witten's [5], but we will find it crucial to express the gravitational constant in terms of the AdS length scale and a dimensionless parameter. We analyze the thermodynamics of the AdS–Schwarzschild black hole and re-derive the Cardy–Verlinde formula. In Sec. 3, we shall study the Hawking–Page phase transition of the boundary CFT using Landau's phenomenological theory of first order phase transitions [28]. The thermodynamical relations for energy and entropy are confirmed, and an alternative derivation of the Cardy–Verlinde formula is presented. Finally, in Sec. 4, we shall study from a holographic point of view the high temperature phase of the boundary CFT. The condensate, expressed phenomenologically as a non-zero order parameter, is found to consist of radiational matter. This agrees with the observation that a brane moving in the AdS–Schwarzschild background obeys the cosmological FRW equations of a radiation dominated universe [29].

2 Black Hole Thermodynamics

The metric for the (Euclidean) AdS–Schwarzschild black hole in $(n+2)$ dimensions is

$$ds^2 = f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_n^2 \ , \tag{1}$$

where

$$f(r) = 1 + \frac{r^2}{l^2} - \frac{\mu}{r^{n-1}} \ , \tag{2}$$

and $d\Omega_n^2$ is the usual metric of an unit n-sphere, S^n. l is the AdS length scale, whereas μ is a parameter controlling the mass of the black hole. The AdS–Schwarzschild space-time with metric (1) possesses an event horizon, r_+, defined by

$$f(r_+) = 0 \ . \tag{3}$$

Notice that there is only one solution to eqn. (3), because $f(r)$ is monotonous.

For large r the metric (1) behaves asymptotically as

$$ds^2 = \frac{r^2}{l^2}\left(dt^2 + l^2 d\Omega_n^2\right) \ . \tag{4}$$

Hence, asymptotically, it is conformally equivalent to $S^n \times \mathbb{R}$ (or $S^n \times S^1$, if the time dimension is compact) with S^n having radius l and with t as time parameter. More generally, let us introduce a radius ρ and rewrite eqn. (4) as

$$ds^2 = \frac{r^2}{\rho^2}\left(d\tau^2 + \rho^2 d\Omega_n^2\right) \ , \tag{5}$$

where the new time parameter is

$$\tau = \frac{\rho}{l} t \ . \tag{6}$$

According to the holographic principle, a theory living on a certain bulk space-time with a metric, whose asymptotic behaviour is given by (5), is holographically dual to a conformal field theory living on $S^n \times \mathbb{R}$ [1] with ρ being the radius of S^n and τ being the (Euclidean) time variable of the boundary theory.

Following Witten [5] we shall now derive the thermodynamic properties of a conformal field theory that is holographically dual to the AdS–Schwarzschild bulk. The important entity to be calculated is the value of the action of the bulk theory for the metric (1). The bulk gravity action is given by

$$\begin{aligned} I = {} & \frac{\kappa}{l^n}\int d^{n+2}x\sqrt{\tilde{g}}\left[-\tilde{R} - \frac{n(n+1)}{l^2}\right] \\ & + \frac{\kappa}{l^n}\int d^{n+1}x\sqrt{g}\left[2H + \frac{2n}{l} + \frac{l}{n-1}R + \cdots\right] \ , \end{aligned} \tag{7}$$

where bulk entities are labelled by a tilde, and those unadorned belong to the cut-off hypersurface inserted at $r = r_*$. The second line in eqn. (7) comprises the Gibbons-Hawking term with the trace of the extrinsic curvature, $H = H_i^i$, and the remaining terms are the usual holographic counter terms added in order to render I finite in the limit $r_* \to \infty$. Adding the surface terms is an alternative to Witten's subtraction of the pure AdS value of the bulk integral. Notice also that we have expressed the gravitational constant using a dimensionless parameter, κ. This is a crucial detail that will allow us later to derive the thermodynamic relations of the boundary CFT for arbitrary ρ.

For the metric (1), the curvature scalar is $\tilde{R} = -(n+1)(n+2)/l^2$, and the intrinsic and extrinsic curvatures of $r = \text{const.}$ hypersurfaces are $R = n(n-1)/r^2$ and $H = -\partial_r\sqrt{f(r)} -$

[1] The 1-sphere S^1 is used only to define the temperature of the system in thermal equilibrium.

$n\sqrt{f(r)}/r$, respectively. For finite temperature, the time variable t is compactified with periodicity β_0. Moreover, the black hole space-time is bounded by the event horizon at $r = r_+$ and by the cut-off boundary at $r = r_*$. Hence, eqn. (7) becomes

$$I = \frac{\kappa \mathcal{V}_n}{l^n}\beta_0 \left[\frac{2}{l^2}(r_*^{n+1} - r_+^{n+1}) - r_*^n \partial_{r_*} f(r_*) - 2n r_*^{n-1} f(r_*) + \frac{2n}{l} r_*^n \sqrt{f(r_*)} \left(1 + \frac{l^2}{2r_*^2}\right) + \cdots \right] . \tag{8}$$

Here, $\mathcal{V}_n$ is the volume of the unit n-sphere, and the ellipses denote further counter terms which we have not written. For large r_* we can write approximately

$$\sqrt{f(r)} \approx \frac{r_*}{l}\left(1 + \frac{l^2}{2r_*^2} - \frac{\mu l^2}{2r_*^{n+1}}\right) , \tag{9}$$

so that we obtain

$$I = \frac{\kappa \mathcal{V}_n}{l^n}\beta_0 \left(-\frac{2}{l^2} r_+^{n+1} + \mu\right) . \tag{10}$$

The terms with positive powers of r_* have exactly cancelled (using also the counter terms that are not written), the negative powers vanish in the limit $r_* \to \infty$, and the terms with $(r_*)^0$ contribute the μ term.

For non-zero μ, the period β_0 is uniquely determined by demanding completeness and smoothness of the metric at the event horizon (absence of conical singularities),

$$\beta_0 = \frac{4\pi l^2 r_+}{(n+1)r_+^2 + (n-1)l^2} . \tag{11}$$

Then, expressing μ in terms of r_+ using eqn. (3), we find from eqn. (10)

$$I = \frac{4\pi\kappa\mathcal{V}_n}{(n+1)\hat{r}^2 + (n-1)} \hat{r}^n (1 - \hat{r}^2) , \tag{12}$$

where we have introduced the dimensionless variable $\hat{r} = r_+/l$.

The on-shell action I is identified with βF, where

$$\beta = \frac{\rho}{l}\beta_0 = \frac{4\pi\rho\hat{r}}{(n+1)\hat{r}^2 + (n-1)} \tag{13}$$

is the inverse temperature measured in the dual boundary field theory, and F is the free energy. Hence,

$$F = \frac{\kappa\mathcal{V}_n}{\rho} \hat{r}^{n-1}(1 - \hat{r}^2) . \tag{14}$$

In the boundary field theory, the volume of the thermodynamic system in equilibrium is $V = \mathcal{V}_n \rho^n$. Thus, we can consider F naturally as a function of β and V and apply the whole text book apparatus of thermodynamics.

First, the energy is

$$E = \left.\frac{\partial(\beta F)}{\partial\beta}\right|_V = \frac{n\kappa\mathcal{V}_n}{\rho}\hat{r}^{n-1}(1+\hat{r}^2)\ . \tag{15}$$

Then, the entropy is obtained by

$$S = \beta(E-F) = 4\pi\kappa\mathcal{V}_n\hat{r}^n\ . \tag{16}$$

S is proportional to the area of the event horizon measured in units of the AdS length scale, l. Moreover, it is independent of the volume V. Next, the pressure can be found by evaluating

$$p = -\left.\frac{\partial F}{\partial V}\right|_\beta = \frac{\kappa}{\rho^{n+1}}\hat{r}^{n-1}(1+\hat{r}^2)\ , \tag{17}$$

so that one can read off the equation of state,

$$E = npV\ . \tag{18}$$

Finally, Gibbs' free energy is

$$G = F + pV = \frac{2\kappa\mathcal{V}_n}{\rho}\hat{r}^{n-1}\ . \tag{19}$$

According to Verlinde [6], one can define the Casimir energy by $E_c = nG$. Then, one easily verifies the validity of the Cardy–Verlinde formula,

$$S = \frac{2\pi\rho}{n}\sqrt{E_c(2E-E_c)}\ . \tag{20}$$

Notice that we have not used here Verlinde's argument splitting the energy into an extensive and sub-extensive part. However, expressing in eqn. (15) ρ and $\hat{r}$ by V and S, respectively, the split becomes apparent.

The interpretations arising from using the thermodynamic analogue are very suggestive. In standard thermodynamics, $\Delta G \geq 0$ holds for irreversible processes. Thus, since $G \sim E_c \sim c$, where c is a generalized central charge, $\Delta G \geq 0$ represents a "thermodynamic" Zamolodchikov theorem, and the renormalization group flow is dual to an irreversible thermodynamic process.

3 Phase Transition

The boundary field theory on $S^n\times\mathbb{R}$ exhibits the Hawking–Page phase transition [3]. This is understood as follows. On the one hand, for the pure AdS bulk geometry, the free energy is identically zero, $F_{AdS} = 0$. Moreover, the value of the inverse temperature β is not restricted by eqn. (13), so that the pure AdS bulk can exist for any temperature. On the other hand, for the AdS–Schwarzschild geometry, β obeys eqn. (13). According to eqn. (13), for fixed ρ (fixed volume), β has a maximum for $\hat{r} = \sqrt{(n-1)/(n+1)}$ with a value $\beta_{max} = 2\pi\rho/\sqrt{n^2-1}$, corresponding to Verlinde's minimum temperature of the early universe [6]. Thus, for small temperatures, $T < 1/\beta_{max}$, only the pure AdS bulk geometry is available as a dual description. For $\sqrt{(n-1)/(n+1)} \leq \hat{r} < 1$, both, pure AdS and black hole bulks are available, but the system prefers the pure AdS bulk, since $F_{BH} > F_{AdS} = 0$. For $\hat{r} > 1$, corresponding to

$$\beta < \beta_c = \frac{2\pi\rho}{n} \,, \tag{21}$$

$F_{BH} < F_{AdS} = 0$, so that the black hole configuration is preferred. Notice that for $\beta < \beta_{max}$, there are two solutions $\hat{r}$. However, the solution $\hat{r} < \sqrt{(n-1)/(n+1)}$ corresponds to the unstable black hole [3] and entails a larger free energy.

Interestingly, at the transition point, the energy and Casimir energy coincide, $E = E_c$. Moreover, eqn. (21) suggests to rewrite the Cardy–Verlinde formula (20) as

$$S = \beta_c \sqrt{E_c(2E - E_c)} \,. \tag{22}$$

We shall now try to understand the first order phase transition at $\beta = \beta_c$ using Landau's phenomenological theory [28]. We shall choose $\hat{r}$ as the order parameter, with $\hat{r} = 0$ for $T < T_c$, and $\hat{r}$ given as the larger solution of eqn. (13) for $T > T_c$. Let us forget the thermodynamic analysis of the last section and assume that we only have eqns. (13) and (14), which follow from the bulk analysis. The parameter ρ shall be kept fixed, *i.e.*, we consider a fixed volume. Eqn. (14) should be interpreted as the free energy at equilibrium, and we can try to write down a more general expression, $F(\hat{r}, T)$, describing the system also away from equilibrium. Analogously, eqn. (13) is to be interpreted as the equilibrium condition relating the temperature and the order parameter. Eqn. (13) can be rewritten in terms of the transition temperature, $T_c = n/(2\pi\rho)$, as

$$T = \frac{T_c}{2n\hat{r}} \left[(n+1)\hat{r}^2 + (n-1) \right] \,. \tag{23}$$

Let us make the ansatz

$$F(\hat{r}, T) = \frac{\kappa \mathcal{V}_n}{\rho} \left(a\hat{r}^{n-1} - bT\hat{r}^n + c\hat{r}^{n+1} \right) \,, \tag{24}$$

where a, b and c are three constants to be determined. Substituting into eqn. (24) the equilibrium temperature, eqn. (23), and comparing with the equilibrium expression for F, eqn. (14), we find

$$\begin{aligned} a &= \frac{(n-1)bT_c}{2n} + 1 \,, \\ c &= \frac{(n+1)bT_c}{2n} - 1 \,. \end{aligned} \tag{25}$$

In addition, we demand that the equilibrium condition derived from eqn. (24),

$$\frac{\partial F(\hat{r}, T)}{\partial \hat{r}} = 0 \,, \tag{26}$$

coincide with eqn. (23). This yields

$$T = \frac{1}{nb\hat{r}} \left[c(n+1)\hat{r}^2 + a(n-1) \right] \ , \tag{27}$$

which, together with eqn. (25), leads to

$$a = c = n \ , \quad b = 2n\beta_c \ . \tag{28}$$

Thus, we have the free energy from eqn. (24),

$$F(\hat{r}, T) = \frac{\kappa \mathcal{V}_n n}{\rho} \left(\hat{r}^{n-1} - 2\beta_c T \hat{r}^n + \hat{r}^{n+1} \right) \ . \tag{29}$$

The free energy (29) correctly describes the first order phase transition at $T = T_c$ in terms of the order parameter $\hat{r}$. In fact, for fixed $T < T_c$, the absolute minimum of F is at $\hat{r} = 0$, whereas for $T > T_c$, F assumes its absolute minimum for $\hat{r}$ given by the larger solution of eqn. (13). The smaller solution corresponds to the local maximum of F.

In equilibrium, we have the entropy

$$S = -\frac{dF(\hat{r}, T)}{dT} = -\frac{\partial F(\hat{r}, T)}{\partial T} = \frac{2n\kappa \mathcal{V}_n}{\rho} \beta_c \hat{r}^n = 4\pi\kappa \mathcal{V}_n \hat{r}^n \ , \tag{30}$$

where we have made use of eqn. (26). Eqn. (30) is in agreement with eqn. (16). We also confirm the expression (15) for the energy at equilibrium,

$$E = F + TS = \frac{n\kappa \mathcal{V}_n}{\rho} \hat{r}^{n-1} (1 + \hat{r}^2) \ . \tag{31}$$

We shall now give another, more general, derivation of the Cardy–Verlinde formula. Let us rewrite eqn. (29) generically as

$$F(\hat{r}, T) = \frac{1}{2} E_c(\hat{r}) \left(1 - 2\beta_c T \hat{r} + \hat{r}^2 \right) \ , \tag{32}$$

where we have introduced an energy $E_c(\hat{r})$ that depends only on the order parameter. In eqn. (29), E_c coincides with the thermodynamic Casimir energy of the CFT dual to the AdS–Schwarzschild bulk. In general, we shall assume that $E_c(\hat{r})$ grows monotonously, and that $E_c(0) = 0$. Then, $F(\hat{r}, T)$ describes a system with a first order phase transition from $\hat{r} = 0$ for $T < T_c$ to some $\hat{r}(T) > 0$ for $T > T_c$. We find the equilibrium entropy and energy,

$$S = \beta_c E_c(\hat{r}) \hat{r} \ , \tag{33}$$

$$E = \frac{1}{2} E_c(\hat{r}) (1 + \hat{r}^2) \ . \tag{34}$$

Obviously,

$$E_c(2E - E_c) = (E_c \hat{r})^2 = (ST_c)^2 \ , \tag{35}$$

which is the Cardy–Verlinde formula in the form (22). Thus, the Cardy–Verlinde formula arises generically in theories with first order phase transitions, which are described by a free energy of the form (32).

Finally, let us try to give generic interpretations of $E_c(\hat{r})$. First, from eqn. (34) follows that $E = E_c$ for $T \leq T_c$ ($E = E_c = 0$ for $T < T_c$), whereas $E > E_c$ for $T > T_c$. Second, in a microcanonical ensemble, we should consider S and E_c as a function of the energy and possibly other parameters, call them x. Then, from eqn. (35) follows

$$T_c^2 S \left.\frac{\partial S}{\partial x}\right|_E = (E - E_c) \left.\frac{\partial E_c}{\partial x}\right|_E , \tag{36}$$

so that E_c is the energy of the microcanonical ensemble at which the entropy of the system is stationary under changes of the other system parameters.

4 Holographic Description of the Condensate

In the previous section, we have discussed from a thermodynamic point of view the phase transition of the boundary CFT. The theory acquires a non-zero order parameter in the high temperature phase. We would like to understand this process also from a holographic point of view. The non-zero order parameter should manifest itself as a physical condensate of some quantum field. For this purpose, let us consider the one-point function of the energy momentum tensor of the boundary CFT. It is obtained from the first order variation of the action I, eqn. (7), under the variation of the metric, $g_{ij} \to g_{ik}(\delta^k_j + h^k_j)$. One finds

$$\delta I = \frac{\kappa}{l^n} \int d^{n+1}x \sqrt{g} \left[-H^i_j + H\delta^i_j + \frac{n}{l}\delta^i_j + \frac{l}{n-1}\left(-R^i_j + \frac{1}{2}R\delta^i_j \right) + \cdots \right] h^j_i , \tag{37}$$

where the bulk term vanishes, because we are on-shell. The integral is evaluated at the cut-off, $r = r_*$, and the counter terms ensure that δI is finite in the limit $r_* \to \infty$. In the course of the calculation one realizes that the counter terms, which are not written explicitly, do not contribute to the finite result, but serve only to remove the divergent terms.

With

$$\begin{aligned} H^t_t &= -\partial_r \sqrt{f(r)} , & H^\alpha_\beta &= -\frac{1}{r}\sqrt{f(r)}\delta^\alpha_\beta , \\ R_{tt} &= R_{t\alpha} = H_{t\alpha} = 0 , & R^\alpha_\beta &= \frac{n-1}{r}\delta^\alpha_\beta , \end{aligned} \tag{38}$$

($\alpha, \beta = 1, \ldots, n$; t as index denotes the $\mathbf{e}_t$-components of the tensors) and using eqns. (2) and (9), we find

$$\delta I = \frac{\kappa}{2l^n} \int d^{n+1}x \left(n\mu h^t_t - \mu \delta^\alpha_\beta h^\beta_\alpha \right) + \text{anomaly terms} . \tag{39}$$

The anomaly terms arise from additional counter terms to be added when n is odd [30]. They contribute a gravitational anomaly to the energy momentum tensor one-point function, but we shall not consider them explicitly. Thus, from

$$\left\langle \Theta^i_j \right\rangle = -2 \frac{\delta I}{\delta h^j_i} \tag{40}$$

and using eqn. (3) to eliminate μ, we find the energy momentum tensor one-point functions,

$$\left\langle \Theta^t_t \right\rangle = -\frac{n\kappa}{l} \hat{r}^{n-1}(1+\hat{r}^2) \ , \quad \left\langle \Theta^\alpha_\beta \right\rangle = \frac{\kappa}{l} \hat{r}^{n-1}(1+\hat{r}^2)\delta^\alpha_\beta \ , \tag{41}$$

apart from the possible gravitational anomaly. Θ^i_j is the energy momentum tensor of the boundary CFT living on $S^n \times \mathbb{R}$, with t as time parameter and l being the radius of S^n. In order to obtain the energy momentum tensor for an arbitrary radius ρ, one simply multiplies the expressions in eqn. (41) by l/ρ. This yields

$$\left\langle \Theta^\tau_\tau \right\rangle = -\frac{E}{\mathcal{V}_n} \ , \quad \left\langle \Theta^\alpha_\beta \right\rangle = \rho^n p \delta^\alpha_\beta \ , \tag{42}$$

with E and p given by eqns. (15) and (17), respectively. Obviously, $\Theta^i_i = 0$ (up to the gravitational anomaly), which corresponds to radiation-type matter. Thus, we conclude that the boundary CFT contains a (temperature dependent) condensate of radiational matter in the high temperature phase. From a bulk point of view, this radiation consists of the Hawking radiation emitted from the black hole and traversing the brane of the observer.

5 Conclusions

In this paper we have studied the thermodynamics of CFTs which are holographically dual to AdS-Schwarzschild black holes. We have analyzed the Hawking-Page phase transition using Landau's theory of first order phase transitions. This has led us to an alternative derivation of the Cardy-Verlinde formula. In the course of our analysis we have reformulated some relations for the thermodynamical quantities in a way which is less dependent on the specific form of the AdS black hole solution, suggesting a broader range of validity of the Cardy-Verlinde formula. This is especially true in the expression (32) for the free energy, which allows to derive the Cardy-Verlinde formula under the simple hypothesis of monotonicity of the function $E_c(\hat{r})$, interpreted as the order parameter dependent Casimir energy. It might be interesting to find statistical systems which show the generic behaviour leading to the Cardy-Verlinde formula, providing microscopic interpretations of the Casimir energy.

We have also used the bulk action to show that, at the classical level, the high temperature phase of the holographically dual matter satisfies the equation of state of radiation, in agreement with the recent construction of FRW space-times on branes moving in a AdS-Schwarzschild background.

Acknowledgments

W. M. gratefully acknowledges financial support by the European Commission RTN programme HPRN-CT-2000-00131, in which he is associated with INFN, Sezione di Frascati.

References

[1] G. 't Hooft, `gr-qc/9310026`.

[2] L. Susskind, *J. Math. Phys.* **36**, 6377 (1995), `hep-th/9409089`.

[3] S. W. Hawking and D. Page, *Commun. Math. Phys.* **87**, 577 (1983).

[4] J. D. Brown, J. Creighton and R. B. Mann, *Phys. Rev. D* **50**, 6394 (1994), `gr-qc/9405007`.

[5] E. Witten, *Adv. Theor. Math. Phys.* **2**, 505 (1998), `hep-th/9803131`.

[6] E. Verlinde, `hep-th/0008140`.

[7] J. L. Cardy, *Nucl. Phys. B* **270**, 317 (1986).

[8] F.-L. Lin, *Phys. Rev. D* **63**, 064026 (2001), `hep-th/0010127`.

[9] S. Nojiri and S. D. Odintsov, `hep-th/0011115`.

[10] R.-G. Cai, *Phys. Rev. D* **63**, 124018 (2001), `hep-th/0102113`.

[11] A. K. Biswas and S. Mukherji, *J. High Energy Phys.* **103**, 046 (2001), `hep-th/0102138`.

[12] S. Nojiri and S. D. Odintsov, `hep-th/0103078`.

[13] D. Birmingham and S. Mokhtari, *Phys. Lett. B* **508**, 365 (2001), `hep-th/0103108`.

[14] D. Klemm, A. C. Petkou, G. Siopsis and D. Zanon, `hep-th/0104141`.

[15] D. Youm, `hep-th/0105036`.

[16] S. Nojiri, O. Obregon, S. D. Odintsov, H. Quevedo and M. P. Ryan, `hep-th/0105052`.

[17] D. Youm, `hep-th/0105093`.

[18] S. Nojiri, S. D. Odintsov and S. Ogushi, `hep-th/0105117`.

[19] R.-G. Cai and Y.-Z. Zhang, `hep-th/0105214`.

[20] D. Youm, `hep-th/0105249`.

[21] B. Wang, E. Abdalla and R.-K. Su, `hep-th/0106086`.

[22] S. Surya, K. Schleich and D. M. Witt, *Phys. Rev. Lett.* **86**, 5231 (2001), `hep-th/0101134`.

[23] G. J. Stephens and B. L. Hu, `gr-qc/0102052`.

[24] P. Creminelli, A. Nicolis and R. Rattazzi, `hep-th/0107141`.

[25] T. Shiromizu and D. Ida, *Phys. Rev. D* **64**, 044015 (2001) `hep-th/0102035`.

[26] T. Shiromizu, T. Torii and D. Ida, `hep-th/0105256`.

[27] I. Savonije and E. Verlinde, *Phys. Lett. B* **507**, 305 (2001), `hep-th/0102042`.

[28] L. D. Landau and E. M. Lifshitz, *Statistical Physics*, Pergamon Press, Oxford (1963).

[29] S. S. Gubser, *Phys. Rev. D* **63**, 084017 (2001), `hep-th/9912001`.

[30] M. Henningson and K. Skenderis, *J. High Energy Phys.* **07**, 023 (1998), `hep-th/9806087`.