hep-th/0009083
September 2000

Supersymmetric Brane World Scenarios from Off-Shell Supergravity

Max Zucker[1]

Physikalisches Institut
Universität Bonn
Nussallee 12
D-53115 Bonn, Germany

Abstract

Using $N = 2$ off-shell supergravity in five dimensions, we supersymmetrize the brane world scenario of Randall and Sundrum. We extend their construction to include supersymmetric matter at the fixpoints.

[1]e-mail: zucker@th.physik.uni-bonn.de

1 Introduction

During the last months, an idea due to Randall and Sundrum [1] has gained a lot of attention. These authors consider five dimensional gravity with a cosmological constant on the orbifold $S^1/\mathbb{Z}_2$. In addition, there are cosmological constants located at the fixpoints of the $\mathbb{Z}_2$. The equations of motion of this theory are solved using a warped product ansatz which preserves four dimensional Poincaré invariance. The exponential dependence of the warp factor on the fifth direction then leads to an elegant solution of the hierarchy problem.

Clearly, it would be of great interest to supersymmetrize the Randall-Sundrum scenario. To be more precise, one may ask for five dimensional gauged supergravity [2] on $S^1/\mathbb{Z}_2$ with additional cosmological constants on the branes and supersymmetrizations thereof.

Some work has already been devoted to the topic of supersymmetric brane world scenarios. Let us mention the paper of Falkowski et al. [3], which discusses models where the supergravity in the bulk is slightly modified; namely the bulk mass term for the gravitino contains a step function. Another approach is the work of Altendorfer et al. [5], who keep the bulk theory as it stands and find the terms on the boundaries by requiring the warped product metric of Randall-Sundrum being a supersymmetric vacuum. However, their method has the disadvantage that it cannot be extended straightforwardly to include additional matter on the boundaries. Finally, we should mention a more recent preprint, which treats the subject in a more general framework [4].

In this work we present an alternative and as we think quite powerful technique, suited for the derivation of theories of the Randall-Sundrum type. This technique rests mainly on two ingredients. First, Mirabelli and Peskin [6] presented a very elegant method to couple theories, which live in the bulk of the orbifold $S^1/\mathbb{Z}_2$ to fields located at the fixpoints in a supersymmetric way. However, they apply their method to rigidly supersymmetric theories only, which is in the light of Randall-Sundrum to restrictive. We stress that the method of [6] rests upon the use of off-shell formulations. So the second ingredient is clear: An off-shell formulation of gauged supergravity in five dimensions is required in order to generalize the idea of Mirabelli and Peskin to local supersymmetry.

In two recent publications, we have worked out the $N = 2$ off-shell multiplet calculus in five dimensions. Besides the minimal multiplet and the nonlinear multiplet [7], we discussed in [8] the linear and the super Yang-Mills multiplet. The multiplet calculus is completed with the hypermultiplet which may be found in [9] and the tensor multiplet, to be discussed in the following section. In a recent publication [10] these topics have been reconsidered and extended using a different approach.

The purpose of this paper is to use our off-shell calculus in order to generalize the idea of Mirabelli and Peskin to local supersymmetry and derive in that way lagrangians of the Randall-Sundrum type.

The present paper is structured as follows. In section 2 we construct the bulk theory of our orbifold $S^1/\mathbb{Z}_2$, namely gauged $N = 2$ off-shell supergravity in five dimensions. In section 3 we develop a four dimensional $N = 1$ tensor calculus at the fixpoints. Using these results we finally study various supersymmetric theories on $S^1/\mathbb{Z}_2$ in section 4. Short conclusions and an outlook are presented in section 5.

2 Gauged off-shell supergravity

In this section we construct the gauged supergravity which will be the bulk theory for our orbifold construction.

The fundamental building block is the minimal multiplet. It has been discussed in our earlier works, [7, 8], so that we refer to these references for detailed treatments. In order to find a consistent supergravity theory, there is a compensator for the local $SU(2)_{\mathcal{R}}$ required. In principle there are three possibilities, leading to three different versions of off-shell supergravity: the nonlinear multiplet [7] (version I), the hypermultiplet [9] (version II) and the tensor multiplet (version III) which is discussed in this work and will be used for the orbifolding procedure. These are all (minimal) off-shell versions of $N=2$ supergravity which exist in five dimensions. These results are easily extended to gauged supergravity.

A detailed presentation of the three versions of off-shell supergravity, including explicit expressions for the lagrangians, may be found in [9]. An analysis of the behavior of the gauged theories under $\mathbb{Z}_2$ orbifolding is included in that work.

2.1 The tensor multiplet

We start by reminding the reader of the linear multiplet [8]. It contains a Lorentz scalar isotriplet $\vec{Y}$, a spinor ρ, a scalar N and a vector W^A. The vector W^A is constrained (for detailed formulas see [8] and for the centrally charged multiplet [9])[2]

$$\partial_A W^A + \ldots = 0. \tag{2.1}$$

As in four dimensional conformal supergravity [11], one may solve this constraint explicitly for vanishing gauge group and vanishing central charge. We can achieve that by the introduction of a 3-form tensor potential through

$$W^A = \frac{1}{12} e_M^A \varepsilon^{MNPQR} \widehat{\mathcal{D}}_N B_{PQR}. \tag{2.2}$$

This field forms together with the remaining fields of the linear multiplet the tensor multiplet. Schematically, the tensor multiplet is then given by

$$(\vec{Y},\ \rho,\ B_{MNP},\ N).$$

The introduction of the field B_{MNP} implies a new symmetry, namely tensor gauge transformations

$$\delta_\Lambda(\lambda) B_{MNP} = 3\partial_{[M}\lambda_{NP]}. \tag{2.3}$$

[2]We use the same conventions as in [7], with the exception that for five dimensional indices we use capital letters and lower case letters for four dimensional indices. Letters from the middle of the alphabet are curved, letters from the beginning of the alphabet are flat.

From the constraint (2.1) it is possible to deduce the supersymmetry transformation law of the tensor field. One finds

$$\delta B_{MNP} = -i\bar{\varepsilon}\gamma_{MNP}\rho - 3\bar{\varepsilon}\vec{\tau}\gamma_{[NP}\psi_{M]}\vec{Y}.$$

It then follows that the supercovariant derivative in (2.2) is given by

$$\widehat{\mathcal{D}}_{[M}B_{NPQ]} = \partial_{[M}B_{NPQ]} + i\bar{\rho}\gamma_{[NPQ}\psi_{M]} - \frac{3}{2}\bar{\psi}_{[N}\vec{\tau}\gamma_{PQ}\psi_{M]}\vec{Y}.$$

The tensor gauge transformations (2.3) appear in the supersymmetry algebra. The additional terms are

$$[\delta_Q(\eta), \delta_Q(\varepsilon)] = \ldots + \delta_\Lambda(i\bar{\varepsilon}\gamma^P\eta B_{MNP} - \bar{\varepsilon}\vec{\tau}\gamma_{MN}\eta\vec{Y})$$

and the dots denote field dependent transformations which may be found in [7].

2.2 The action for the tensor multiplet

An action for the tensor multiplet is found as follows [11, 12]: one starts with the coupling of a Maxwell multiplet to the tensor multiplet. The tensor multiplet is inert under the gauge group. One easily finds that

$$\begin{aligned}\mathcal{L} &= 2\vec{X}\vec{Y} - 2i\bar{\Omega}\rho - 2MN - \frac{1}{12}\varepsilon^{ABCDE}G_{AB}B_{CDE} - 16\vec{Y}\vec{t}M \\ &\quad - i\bar{\rho}\gamma^M\psi_M M + \frac{1}{2}\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\vec{Y}M - \bar{\Omega}\vec{\tau}\gamma^M\psi_M\vec{Y}\end{aligned} \tag{2.4}$$

is invariant. Next, one forms a vector multiplet out of the fields of the tensor multiplet. Here one faces the first problem. The canonical embedding should start with $M \sim N + \ldots$ as it does in the case of rigid supersymmetry. However, this cannot be generalized straightforwardly to the local case [11]. Instead, one has to move on to the improved tensor multiplet [11, 13].

Since the correspondence is rather complicated, we give only the lowest component M of this vector multiplet (the full formulas may be found in [9]):

$$M = Y^{-1}N + \frac{1}{4}Y^{-3}\bar{\rho}\vec{\tau}\rho\vec{Y} + 6Y^{-1}\vec{Y}\vec{t}. \tag{2.5}$$

Here we have defined

$$Y = (\vec{Y}\vec{Y})^{1/2}$$

and let us mention that similar cohomology problems as in four dimensions [11] appear for the gauge field strength but are unimportant for our purposes. Acting repeatedly with the supersymmetry transformations on expression (2.5) gives the complete embedding.

Using this embedding of the tensor multiplet in the vector multiplet in the action formula (2.4) gives the desired action for the tensor multiplet:

$$
\begin{aligned}
\mathcal{L}_{tensor} =& -\frac{1}{4}YR(\widehat{\omega})^{AB}{}_{AB} + 4YC - \frac{1}{6}Y\widehat{F}_{AB}\widehat{F}^{AB} + Yv_{AB}v^{AB} + 20Y\vec{t}^2 - Y^{-1}N^2 \\
& - 36Y^{-1}(\vec{t}\vec{Y})^2 - \frac{1}{4}Y^{-1}\widehat{\mathcal{D}}_A\vec{Y}\widehat{\mathcal{D}}^A\vec{Y} - \frac{1}{12}Y^{-1}\varepsilon^{MNPQR}\vec{Y}\vec{V}_M\partial_N B_{PQR} \\
& + Y^{-1}W_AW^A - 4Y^{-1}\bar{\lambda}\vec{\tau}\rho\vec{Y} - 2iY\bar{\psi}_A\gamma^A\lambda - \frac{i}{4\sqrt{3}}Y^{-1}\bar{\rho}\gamma^{AB}\rho\widehat{F}_{AB} \\
& - \frac{i}{2}Y^{-1}\bar{\rho}\gamma^A\mathcal{D}_A\rho - 3Y^{-3}\bar{\rho}\vec{\tau}\rho\vec{Y}\tilde{t}\tilde{Y} - \frac{1}{2}Y^{-3}\bar{\rho}\vec{\tau}\rho\vec{Y}N - \frac{i}{4}Y^{-1}\bar{\rho}\gamma^{AB}\rho v_{AB} \\
& + \frac{1}{4}Y^{-3}\bar{\rho}\vec{\tau}\gamma^A\rho(\vec{Y}\times\widehat{\mathcal{D}}_A\vec{Y}) + \frac{1}{2}Y^{-1}\vec{Y}\bar{\psi}_A\vec{\tau}\gamma^{AB}\mathcal{D}_B\rho + iY^{-1}\bar{\rho}\psi_AW^A \\
& + \frac{1}{2}Y^{-3}\bar{\rho}\vec{\tau}\gamma^A\rho\vec{Y}W_A - \frac{1}{2}Y^{-1}\bar{\rho}\vec{\tau}\gamma^{MN}\mathcal{D}_M\psi_N\vec{Y} - \frac{i}{2}Y\bar{\psi}_P\gamma^{PMN}\mathcal{D}_M\psi_N \\
& - 12Y^{-1}N\vec{t}\vec{Y} - Y\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\vec{t} - \frac{i}{2}Y\bar{\psi}_A\psi_Bv^{AB} + 2Y^{-1}\bar{\psi}_A\vec{\tau}\gamma^A\rho(\vec{t}\times\vec{Y}) \\
& + \frac{1}{24}Y^{-3}\varepsilon^{MNPQR}\vec{Y}(\partial_M\vec{Y}\times\partial_N\vec{Y})B_{PQR} - \frac{i}{4\sqrt{3}}Y\bar{\psi}_A\gamma^{ABCD}\psi_B\widehat{F}_{CD} \\
& + Y^{-1}\bar{\rho}\vec{\tau}\gamma_B\psi_Av^{AB}\vec{Y} + 2iY^{-1}\bar{\rho}\gamma^A\psi_A\vec{t}\vec{Y} - \frac{1}{2\sqrt{3}}Y^{-1}\bar{\rho}\vec{\tau}\gamma^{ABC}\psi_A\widehat{F}_{BC}\vec{Y} \\
& - \frac{1}{4}Y^{-1}\bar{\psi}_A\vec{\tau}\gamma^{ABC}\psi_B(\vec{Y}\times\widehat{\mathcal{D}}_C\vec{Y}) - \frac{1}{2}Y^{-3}\bar{\psi}_A\vec{\tau}\gamma^{AB}\rho\vec{Y}\tilde{Y}\widehat{\mathcal{D}}_B\tilde{Y} + \frac{1}{8}Y^{-3}(\bar{\rho}\vec{\tau}\rho)^2 \\
& - \frac{i}{2}Y^{-3}\bar{\psi}_A\gamma^A\rho\bar{\rho}\vec{\tau}\rho\vec{Y} + \frac{1}{2}Y^{-1}\bar{\rho}\psi_B\bar{\rho}\gamma^{AB}\psi_A - \frac{i}{4}Y^{-1}\bar{\rho}\psi_B\bar{\psi}_A\vec{\tau}\gamma^{ABC}\psi_C\vec{Y} \\
& - \frac{3}{8}Y^{-5}(\bar{\rho}\vec{\tau}\rho\vec{Y})^2 + \frac{1}{8}Y^{-3}\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\bar{\rho}\tilde{\tau}\rho\vec{Y}\tilde{Y} + \frac{i}{4}Y^{-3}\bar{\psi}_A\vec{\tau}\tilde{\tau}\gamma^A\rho\bar{\rho}\tilde{\tau}\rho\vec{Y} \\
& - \frac{i}{4}Y^{-3}\bar{\rho}\vec{\tau}\gamma_B\rho\bar{\rho}\gamma^{AB}\psi_A\vec{Y} - \frac{1}{8}Y^{-3}\bar{\rho}\vec{\tau}\gamma_B\rho\bar{\psi}_A\tilde{\tau}\gamma^{ABC}\psi_C\vec{Y}\tilde{Y}.
\end{aligned} \tag{2.6}
$$

2.3 An action for gauged supergravity

Using the results of the preceding section we now construct an action for gauged off-shell supergravity following [11]. As compensator for the $SU(2)_{\mathcal{R}}$ $\mathcal{R}$-symmetry we use the $\vec{Y}$-field. However, before dealing with the gauge fixing, we have to consider an old problem: The appearance of a term linear in C in eq. (2.6) leads to inconsistent equations of motion. We can solve this problem in an elegant way, using the lagrangian $\mathcal{L}_{min}$ for the minimal multiplet, eq. (2.10) in [7]. The lagrangian $\mathcal{L} = \mathcal{L}_{tensor} + \mathcal{L}_{min}$ leads to consistent equations of motion. It is a well defined lagrangian for off-shell supergravity.

The gauged variant of this theory is also easily found: Using the lagrangian $\mathcal{L}_{lin}$ for the linear multiplet given in [8], eq. (3.2), the desired lagrangian is

$$
\mathcal{L}_{gauged} = \mathcal{L}_{tensor} + \mathcal{L}_{min} - \frac{\sqrt{3}}{4}g'\mathcal{L}_{lin}. \tag{2.7}
$$

Here we have introduced the cosmological constant g'. Of course, wherever W^A appears it has to be understood as supercovariant field strength for B_{MNP}, defined by (2.2).

Before we proceed, we should discuss the gauge fixing. In our formulation of off-shell supergravity, we have an $SU(2)_{\mathcal{R}}$ $\mathcal{R}$-symmetry which is gauged by an auxiliary field $\vec{V}_M$. This symmetry has to be fixed. We do that by setting [11]

$$\vec{Y} = e^u(0,1,0)^T \tag{2.8}$$

where we have introduced a new scalar u. This breaks the original $SU(2)_{\mathcal{R}}$ but leaves a residual $SO(2)$, gauged by V_M^2, intact. In addition, there is still the $U(1)$ under which only the graviphoton A_M transforms at this stage. The equations of motion for the auxiliary fields imply $V_M^2 = 2g'A_M$ so that after elimination of the auxiliary fields, we end up with the lagrangian of gauged supergravity given in [8].

Unfortunately, the lagrangian (2.7) after the gauge fixing (2.8) is quite complicated. Nevertheless it is the starting point for our orbifold construction so that we should present explicit formulas:

$$\begin{aligned}
\mathcal{L}_{gauged} =& e^u\Big(-\frac{1}{4}R(\widehat{\omega})_{AB}{}^{AB} + 4C - \frac{1}{6}\widehat{F}_{AB}\widehat{F}^{AB} + v_{AB}v^{AB} + 20\vec{t}\vec{t} - 36(t^2)^2 \\
& - \frac{1}{4}\partial^A u\partial_A u - \frac{1}{4}V_A^1V^{A1} - \frac{1}{4}V_A^3V^{A3} + 8\sqrt{3}g't^2 - \frac{i}{2}\bar{\psi}_P\gamma^{PMN}\mathcal{D}_M\psi_N \\
& - 2i\bar{\psi}_A\gamma^A\lambda - \frac{\sqrt{3}g'}{4}\bar{\psi}_A\tau^2\gamma^{AB}\psi_B - \frac{i}{2}\bar{\psi}_A\psi_B v^{AB}\Big) - 12Nt^2 + \sqrt{3}g'N \\
& - \frac{1}{\sqrt{3}}F_{AB}v^{AB} - \frac{1}{6\sqrt{3}}\varepsilon^{ABCDE}A_AF_{BC}F_{DE} - 4\bar{\lambda}\tau^2\rho - 2i\bar{\lambda}\gamma^A\psi_A \\
& + \frac{1}{2}\bar{\rho}\tau^2\psi_A\partial^A u - \frac{1}{2}\bar{\rho}\tau^1\psi^M V_M^3 + \frac{1}{2}\bar{\rho}\tau^3\psi^M V_M^1 + \frac{1}{2}\bar{\psi}_A\tau^2\gamma^{AB}\mathcal{D}_B\rho \\
& + 2i\bar{\rho}\gamma^A\psi_A t^2 - 2\bar{\psi}_A\tau^1\gamma^A\rho t^3 + 2\bar{\psi}_A\tau^3\gamma^A\rho t^1 - \frac{1}{2}\bar{\psi}_A\tau^2\gamma^{AB}\rho\partial_B u \\
& - \frac{1}{12}\varepsilon^{MNPQR}(V_M^2 - 2g'A_M)\partial_N B_{PQR} - 32\vec{t}\vec{t} - \frac{\sqrt{3}ig'}{2}\bar{\psi}_A\gamma^A\rho \\
& - \frac{1}{2}\bar{\rho}\tau^2\gamma^{MN}\mathcal{D}_M\psi_N + \bar{\rho}\tau^2\gamma_B\psi_A v^{AB} - \frac{1}{2\sqrt{3}}\bar{\rho}\tau^2\gamma^{ABC}\psi_A\widehat{F}_{BC} \\
& - \frac{i}{4\sqrt{3}}\bar{\psi}_A\gamma^{ABCD}\psi_B(e^u\widehat{F}_{CD} + \frac{1}{2}F_{CD}) + (1-e^u)\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\vec{t} \\
& + e^{-u}\Big(-\frac{i}{4}\bar{\rho}\gamma^{AB}\rho(v_{AB} + \frac{1}{\sqrt{3}}\widehat{F}_{AB}) - 3\bar{\rho}\tau^2\rho t^2 + W_AW^A - N^2 \\
& - \frac{i}{2}\bar{\rho}\gamma^A\mathcal{D}_A\rho + i\bar{\rho}\psi_A W^A\Big) - 4C - \frac{1}{2}e^{-2u}(\bar{\rho}\tau^2\rho N - \bar{\rho}\tau^2\gamma^A\rho W_A) + \mathcal{L}_{4F}.
\end{aligned} \tag{2.9}$$

$\mathcal{L}_{4F}$ contains four fermion terms which play no rôle for us. They may be found in [9]. The covariant derivatives which appear here are covariant w.r.t. local Lorentz and local $SO(2)$ transformations, e.g.

$$\mathcal{D}_M\rho = \partial_M\rho + \frac{1}{4}\widehat{\omega}_{MAB}\gamma^{AB}\rho - \frac{i}{2}\tau^2\rho V_M^2.$$

Field	e_m^a	$e_m^{\dot 5}$	e_5^a	$e_{\dot 5}^5$	ψ_m	ψ_5	A_m	A_5	V_m^1	V_m^2	
Parity $\mathcal{P}$	$+1$	-1	-1	$+1$	$+1$	-1	-1	$+1$	-1	-1	
Field	V_m^3	V_5^1	V_5^2	V_5^3	v_{ab}	$v_{a\dot 5}$	λ	C	t^1	t^2	t^3
Parity $\mathcal{P}$	$+1$	$+1$	$+1$	-1	-1	$+1$	$+1$	$+1$	$+1$	$+1$	-1

Table 1: Parities of the minimal multiplet.

Let us stress, that we work in what follows till the end with the gauged $SU(2)_{\mathcal{R}}$. The very last step is to impose the condition (2.8).

3 Supergravity on $S^1/\mathbb{Z}_2$

We now consider gauged supergravity on $S^1/\mathbb{Z}_2$. The $\mathbb{Z}_2$ acts on the fifth coordinate, $x^5 \to -x^5$. A generic bosonic field transforms like

$$\varphi(x^m, x^5) \to \mathcal{P}\varphi(x^m, -x^5), \qquad \text{with} \qquad \mathcal{P}^2 = 1.$$

Fermionic fields transform like

$$\psi(x^5) \to \mathcal{P} i\tau^3\gamma^{\dot 5}\psi(-x^5). \tag{3.1}$$

An extended discussion of $\mathbb{Z}_2$ assignments to symplectic Majorana spinors can be found in [4]. We use the notation that a dot on an index 5 denotes a Lorentz index.

3.1 The minimal multiplet

Starting from $\mathcal{P}(e_m^a) = +1$, the parity assignments of all fields belonging to the minimal multiplet are dictated by supersymmetry. We have collected these assignments in table 1. Note that the orbifold condition breaks the $SU(2)_{\mathcal{R}}$ at the fixpoints to a residual $U(1)$.

Let us define

$$\begin{aligned} b_a &= v_{a\dot 5} && (3.2a) \\ a_m &= -\frac{1}{2}(V_m^3 - \frac{2}{\sqrt{3}}\widehat{F}_{m5}e_{\dot 5}^5 + 4e_m^a v_{a\dot 5}) && (3.2b) \\ S &= C - \frac{1}{2}e_{\dot 5}^5(\partial_5 t^3 - \bar\lambda\tau^3\psi_5 + V_5^1 t^2 - V_5^2 t^1) && (3.2c) \end{aligned}$$

at the fixpoints. The important observation is then that the fields

$$(e_m^a,\ \psi_m,\ b_a,\ a_m,\ \lambda,\ S,\ t^1,\ t^2),$$

taken at the boundaries, form a non-minimal $N = 1$ supergravity multiplet in four dimensions with $(16 + 16)$ components. This multiplet, sometimes called the intermediate multiplet has been studied in detail by Sohnius and West [14]. These authors also constructed the corresponding multiplet calculus.

The $U(1)$ which survives the orbifold projection is to be identified with the chiral $U(1)$ gauged by the auxiliary field a_m. The fields transform as follows under this symmetry (with parameter α):

$$\begin{gathered}\delta\psi_m = \gamma^{\dot{5}}\psi_m\alpha, \qquad \delta\lambda = \gamma^{\dot{5}}\lambda\alpha, \\ \delta a_m = \partial_m\alpha, \qquad \delta t^1 = -2t^2\alpha, \qquad \delta t^2 = 2t^1\alpha.\end{gathered}$$

The supersymmetry transformation laws on the boundary are in our notations

$$\begin{aligned}
\delta e_m^a &= -i\bar{\varepsilon}\gamma^a\psi_m \\
\delta\psi_m &= \mathcal{D}_m\varepsilon - 3\gamma^{\dot{5}}\varepsilon b_m + \gamma_m(\gamma^a\gamma^{\dot{5}}b_a + 2i\tau^1t^1 + 2i\tau^2t^2)\varepsilon \\
\delta b_a &= -\frac{i}{4}\bar{\varepsilon}\gamma_a{}^{bc}\gamma^{\dot{5}}\widehat{\mathcal{R}}_{bc} - 2i\bar{\varepsilon}\gamma_a\gamma^{\dot{5}}\lambda \\
\delta a_m &= -\frac{i}{2}\bar{\varepsilon}\gamma^{\dot{5}}\gamma_m\lambda' \\
\delta\lambda &= -\frac{1}{4}\gamma^{\dot{5}}\varepsilon\widehat{\mathcal{D}}_a b^a + \varepsilon S - \frac{i}{2}\tau^1\gamma^a\varepsilon\widehat{\mathcal{D}}_a t^1 - \frac{i}{2}\tau^2\gamma^a\varepsilon\widehat{\mathcal{D}}_a t^2 \\
\delta S &= -\frac{i}{2}\bar{\varepsilon}(\gamma^m\widehat{\mathcal{D}}_m + \gamma^a\gamma^{\dot{5}}b_a)\lambda - \bar{\varepsilon}(\tau^1t^1 + \tau^2t^2)(\frac{1}{2}\gamma^{ab}\widehat{\mathcal{R}}_{ab} + 8\lambda) \\
\delta t^1 &= \bar{\varepsilon}\tau^1\lambda \\
\delta t^2 &= \bar{\varepsilon}\tau^2\lambda.
\end{aligned} \tag{3.3}$$

Note that fields like ψ_5 and A_5 have dropped out. The covariant derivatives which appear here are covariant w.r.t. four dimensional local Lorentz transformations and chiral $U(1)$ transformations. Further, we have introduced the useful definition [14]

$$\lambda' = 12\lambda + \gamma^{ab}\widehat{\mathcal{R}}_{ab}.$$

λ' transforms quite simple under supersymmetry:

$$\delta\lambda' = 12\varepsilon S - \frac{1}{4}\varepsilon\widehat{R}(\widehat{\omega})^{ab}{}_{ab} + 48\varepsilon((t^1)^2 + (t^2)^2) - 6\varepsilon b_a b^a - \frac{1}{2}\gamma^{ab}\gamma^{\dot{5}}\varepsilon\widehat{f}_{ab}$$

and $\widehat{f}_{mn}$ is the supercovariant field strength of a_m. We remind the reader that in these equations, the fermions ψ_m and λ satisfy the chirality constraints

$$\lambda = i\tau^3\gamma^{\dot{5}}\lambda \qquad \text{and} \qquad \psi_m = i\tau^3\gamma^{\dot{5}}\psi_m.$$

We can compute the gauge algebra for the multiplet (3.3). It is the projection of the five dimensional gauge algebra:

$$\begin{aligned}[\delta_Q(\varepsilon), \delta_Q(\eta)] =& \delta_{g.c.}(i\bar{\eta}\gamma^m\varepsilon) + \delta_Q(i\bar{\varepsilon}\gamma^m\eta\psi_m) + \delta_{ch}(i\bar{\varepsilon}\gamma^m\eta a_m) \\ &+ \delta_{Lt}(i\bar{\eta}\gamma^m\varepsilon\widehat{\omega}_{mab} - 2i\bar{\varepsilon}\gamma_{abc}\gamma^{\dot{5}}\eta b^c + 4\bar{\varepsilon}\tau^1\gamma_{ab}\eta t^1 + 4\bar{\varepsilon}\tau^2\gamma_{ab}\eta t^2).\end{aligned} \tag{3.4}$$

$\delta_{ch}(\alpha)$ denotes a chiral transformation with parameter α and all transformation have to be understood in the four dimensional sense.

We are now in a position to elaborate on the matter multiplets which may be coupled to the supergravity multiplet (3.3). This has been done in great detail in [14]. We restrict ourselves to the presentation of formulas which are required for the remaining sections. A more extended discussion may be found in the original work, for details using our notations, [9] should be consulted.

3.2 The chiral multiplet

This multiplet has also been given in [14]. Since it is very well known we restrict ourselves to the transformation laws using our conventions. The field content of the chiral multiplet is

$$\mathbb{A} = (A,\ B,\ \psi,\ F,\ G).$$

It exists for arbitrary chiral weight w. The transformation laws are:

$$\begin{aligned}
\delta A &= \bar{\varepsilon}\tau^2\psi + wB\alpha \\
\delta B &= \bar{\varepsilon}\tau^2\gamma^{\dot{5}}\psi - wA\alpha \\
\delta\psi &= -\frac{i}{2}\gamma^a\tau^2\varepsilon\widehat{\mathcal{D}}_a A - \frac{1}{2}\varepsilon F - \frac{i}{2}\gamma^a\gamma^{\dot{5}}\tau^2\varepsilon\widehat{\mathcal{D}}_a B - \frac{1}{2}\gamma^{\dot{5}}\varepsilon G + (w-1)\gamma^{\dot{5}}\psi\alpha \\
\delta F &= i\bar{\varepsilon}\gamma^a(\widehat{\mathcal{D}}_a\psi + \gamma^{\dot{5}}\psi b_a) + 4\bar{\varepsilon}(\tau^1\psi t^1 - \tau^1\gamma^{\dot{5}}\psi t^2) \\
&+ \frac{w}{2}\bar{\varepsilon}\tau^2(A + \gamma^{\dot{5}}B)\lambda' + (2-w)G\alpha \\
\delta G &= i\bar{\varepsilon}\gamma^{\dot{5}}\gamma^a(\widehat{\mathcal{D}}_a\psi + \gamma^{\dot{5}}\psi b_a) - 4\bar{\varepsilon}\gamma^{\dot{5}}(\tau^1\psi t^1 - \tau^1\gamma^{\dot{5}}\psi t^2) \\
&+ \frac{w}{2}\bar{\varepsilon}\tau^2\gamma^{\dot{5}}(A + \gamma^{\dot{5}}B)\lambda' + (w-2)F\alpha.
\end{aligned}$$

We have also indicated the transformation properties under chiral rotations.

For a chiral multiplet with weight $w = 2$ we can write down an invariant F-term density:

$$[\mathbb{A}]_F = F + i\bar{\psi}_m\gamma^m\psi + \frac{1}{2}\bar{\psi}_m\tau^2\gamma^{mn}(A + \gamma^{\dot{5}}B)\psi_n - 12t^2A - 12t^1B. \tag{3.5}$$

3.3 The vector multiplet

The field content of the super Yang-Mills multiplet is

$$\mathbb{Y} = (\ u_m,\ \chi,\ D).$$

The abelian vector multiplet can be derived straightforwardly from the general multiplet. We skip this multiplet, refer the interested reader for technical details to [9] and confine

Field	Y^1	Y^2	Y^3	ρ	N	B_{mnp}	B_{mn5}
Parity $\mathcal{P}$	$+1$	$+1$	-1	$+1$	$+1$	$+1$	-1

Table 2: Parities of the tensor multiplet.

ourselves to the presentation of the transformation laws of the vector multiplet:

$$\begin{aligned}
\delta u_m &= i\bar{\varepsilon}\gamma_m\chi \\
\delta\chi &= -\gamma^{\dot{5}}\varepsilon D + \frac{1}{4}\gamma^{ab}\varepsilon\widehat{u}_{ab} \\
\delta D &= \frac{i}{2}\bar{\varepsilon}\gamma^{\dot{5}}\gamma^m\widehat{\mathcal{D}}_m\chi - \frac{3i}{2}\bar{\varepsilon}\gamma^a\chi b_a,
\end{aligned}$$

where the fields take their values in the Lie algebra of the gauge group. We have defined the supercovariant field strength $\widehat{u}_{mn}$ of u_m. Of course, the gauge algebra (3.4) is modified by the appearance of a field dependent gauge transformation.

An action can be derived by forming a chiral multiplet from fields of the Yang-Mills multiplet. One finds [14, 9]

$$\begin{aligned}
\mathcal{L}_{sym} = Tr\, [DD &- \frac{1}{4}\widehat{u}_{ab}\widehat{u}^{ab} - 3i\bar{\chi}\gamma^a\gamma^{\dot{5}}\chi b_a \\
&+ i\bar{\chi}\gamma^a\widehat{\mathcal{D}}_a\chi - \bar{\psi}_m\gamma^m(i\gamma^{\dot{5}}\chi D + \frac{i}{4}\gamma^{ab}\chi\widehat{u}_{ab}) \\
&+ \frac{1}{4}\bar{\psi}_m\tau^2\gamma^{mn}\psi_n\bar{\chi}\tau^2\chi + \frac{1}{4}\bar{\psi}_m\tau^2\gamma^{mn}\gamma^{\dot{5}}\psi_n\bar{\chi}\tau^2\gamma^{\dot{5}}\chi].
\end{aligned} \tag{3.6}$$

4 Lagrangians at the fixpoints

In this section we construct lagrangians of the Randall-Sundrum type, i.e. theories which live on an orbifold $S^1/\mathbb{Z}_2$. In the bulk there is only gauged supergravity, but this can be extended easily to more complicated configurations using the results in [8, 9]. On the boundaries, the original Randall-Sundrum scenario [1] requires a cosmological constant. We present some generalizations of this model.

4.1 A cosmological constant – The Randall-Sundrum scenario

To start with the supersymmetrization of the Randall-Sundrum scenario we analyze the parity assignments of the tensor multiplet from section 2. The results are shown in table 2. Inspection of the transformation laws shows, that on the boundaries, they form chiral multiplets with chiral weight $w = 2$. The precise correspondence is

$$\big(A,\ B,\ \psi,\ F,\ G\big) = \big(\ Y^2,\ Y^1,\ \rho,\ \ -2N + \widehat{\mathcal{D}}_{\dot{5}}Y^3,\ \ +2W^{\dot{5}} + 12(t^1Y^2 - Y^1t^2)\big), \tag{4.1}$$

on the boundary.

Using the result for the F-term density of a chiral multiplet, eq. (3.5), we are then in a position to write down the desired theory. Before doing that, however, we have to fix the $\mathcal{R}$-symmetry. This is achieved by imposing (2.8) which breaks the $SU(2)_{\mathcal{R}}$ in the bulk to a residual $U(1)$, gauged by V_M^2, and breaks the chiral $U(1)$ on the boundaries completely. The minimal multiplet on the boundary is then extended to an $N=1$, $D=4$ supergravity multiplet with $(20+20)$ components. One then uses the expressions (4.1) in (3.5) to obtain

$$\mathcal{L}_{cc} = -2N + e^u V_{\dot{5}}^1 - \bar{\rho}\tau^3\psi_{\dot{5}} + i\bar{\psi}_m\gamma^m\rho + \frac{1}{2}e^u\bar{\psi}_a\tau^2\gamma^{ab}\psi_b - 12e^u t^2. \tag{4.2}$$

Our complete action is

$$S = \int d^5x\; e\left(\mathcal{L}_{gauged} + \Lambda_1\delta(x^5)\mathcal{L}_{cc} + \Lambda_2\delta(x^5-\ell)\mathcal{L}_{cc}\right) \tag{4.3}$$

where Λ_1 and Λ_2 are real constants. We use the unusual convention that $\delta(x^5)$ is a scalar, not a density. If one eliminates the auxiliary fields, terms of the form $\delta(x^5)^2$ appear but drop out at the end. One finds a result similar to the one of Altendorfer et al. [5],

$$\mathcal{L} = \widetilde{\mathcal{L}}_{gauged} - \big(\Lambda_1\delta(x^5) + \Lambda_2\delta(x^5-\ell)\big)\big(\sqrt{3}g' - \frac{1}{2}\bar{\psi}_a\tau^2\gamma^{ab}\psi_b\big), \tag{4.4}$$

with the on-shell lagrangian $\widetilde{\mathcal{L}}_{gauged}$ of gauged supergravity given in [8]. g' is the cosmological constant. The on-shell transformation laws are

$$\begin{aligned}
\delta e_M^A =& -i\bar{\varepsilon}\gamma^A\psi_M \\
\delta\psi_M =& \mathcal{D}_M\varepsilon + \frac{1}{4\sqrt{3}}\gamma_{MAB}\varepsilon\widehat{F}^{AB} - \frac{1}{\sqrt{3}}\gamma^N\varepsilon\widehat{F}_{MN} + \frac{ig'}{2\sqrt{3}}\gamma_M\tau^2\varepsilon \\
&+ i\Lambda_1\delta_M^5\delta(x^5)\tau^2\gamma^{\dot{5}}\varepsilon e_5^{\dot{5}} + i\Lambda_2\delta_M^5\delta(x^5-\ell)\tau^2\gamma^{\dot{5}}\varepsilon e_5^{\dot{5}} \\
\delta A_M =& -\frac{\sqrt{3}i}{2}\bar{\varepsilon}\psi_M.
\end{aligned} \tag{4.5}$$

As a consistency check, we have verified the invariance of the lagrangian (4.4) under the on-shell transformations (4.5).

One of the motivations for the present work where the hope to remove the fine tuning of the bulk and boundary cosmological constants, inherent to the original work of Randall and Sundrum, by supersymmetry. It should be clear that the method of Altendorfer et al. [5] is not suited to answer the question whether this fine tuning is removed by supersymmetry. The alternative work [3] implies that this fine tuning is removed by supersymmetry. This is not true for our result (4.4), at least on the lagrangian level, due to the appearance of the parameters Λ_1 and Λ_2.

Before proceeding, let us shortly comment on the solutions to the equation of motion and the Killing spinor equations. The Einstein equations are solved by the Randall-Sundrum metric

$$ds^2 = e^{-2\sigma}\eta_{mn}dx^m dx^n - (dx^5)^2 \qquad \text{with} \qquad \sigma \equiv \frac{g'}{\sqrt{3}}|x^5|, \tag{4.6}$$

provided that

$$\Lambda_1 = -\Lambda_2 = 1. \tag{4.7}$$

Note that we have used according to common practice, that[3] $\theta(x^5)^2 = 1$, where θ is the standard step function.

By decomposing the spinors in two-component objects

$$\varepsilon^i = \begin{pmatrix} \varepsilon^i_+ \\ \varepsilon^i_- \end{pmatrix}. \tag{4.8}$$

and using the representation of gamma matrices given in [6], one may try to solve the Killing spinor equations $\delta\psi_M = 0$ using the Ansatz of Altendorfer et al. [5]

$$\varepsilon^1_+ = e^{-\frac{1}{2}\sigma}\eta, \qquad \varepsilon^2_+ = -i\theta(x^5)e^{-\frac{1}{2}\sigma}\eta, \tag{4.9}$$

with a constant spinor η. However, in this case one finds the constraint

$$\Lambda_1 = -\Lambda_2 = 2. \tag{4.10}$$

Note that (4.10) is in contradiction with (4.7) and thus the spinors (4.9) represent no Killing spinors of the Randall-Sundrum solution.[4] However, since the Randall-Sundrum metric (4.6) is a solution to the Einstein equations we have to expect that there exist consistent solutions to the Killing spinor equations. Otherwise we would have constructed a theory where the geometry of spacetime breaks supersymmetry spontaneously.

4.2 Matter at the fixpoints

Our next models contain as bulk theory gauged supergravity and cosmological constants on the boundaries. In addition, we allow for chiral multiplets or super Yang-Mills multiplets on the boundaries. Let us start with the chiral multiplet.

For notational convenience we consider only one brane located at $x^5 = 0$. It is completely trivial to add the second one.

The systematic construction of a kinetic term for the chiral multiplet as it is needed here requires the general multiplet which we do not discuss. For technical details we refer to [14, 9]. The bosonic part of the lagrangian for a chiral multiplet with chiral weight w is

$$\begin{aligned} \mathcal{L}_{kin} = &- \mathcal{D}_a B \mathcal{D}^a B - \mathcal{D}_a A \mathcal{D}^a A - 8t^2 GB - F^2 - G^2 \\ &+ 8t^2 FA + 8t^1 FB + 8t^1 GA - 4\mathcal{D}_a BAb^a + 4\mathcal{D}_a ABb^a \\ &+ (A^2 + B^2)(12wS - \frac{w}{4}R_{ab}{}^{ab} + 48(w-1)((t^1)^2 + (t^2)^2) - 6wb_a b^a - 8S). \end{aligned} \tag{4.11}$$

[3] A counterexample is found as follows: Taking the derivative of $\theta(x^5)^2 = 1$ leads to $\delta(x^5)\theta(x^5) = 0$ and multiplying this equation with $\theta(x^5)$ leads, using $\theta(x^5)^2 = 1$ to $\delta(x^5) = 0$. More compactly stated: $(\theta(x^5)\theta(x^5))\delta(x^5) \neq \theta(x^5)(\theta(x^5)\delta(x^5))$. See also [15]

[4] In equation (13) of [5], the transformation law for the gravitino ψ^+_m contains a term $\sim i\Lambda e_m{}^a \sigma_a \bar{\eta}_+$ which is missing in eq. (20). If this term is included, eqs. (21) and (22) have to be changed. After performing these corrections, one recovers the $\Lambda_1 = -\Lambda_2 = 2$ case and our calculation is in agreement with [5]. We would like to thank Jan Conrad for extended discussions on this point.

Note the appearance of the curvature scalar. After inserting the definitions (3.2) and fixing the gauge via (2.8), we take our action to be

$$S = \int d^5x\, e\left(\mathcal{L}_{gauged} + \Lambda_1\delta(x^5)\mathcal{L}_{cc} + \tilde{\Lambda}_1\delta(x^5)\mathcal{L}_{kin}\right),$$

with $\tilde{\Lambda}_1$ a constant. After gauge fixing, the final step would be the elimination of the auxiliary fields. This turns out to be a remarkably complicated exercise which we have not completed.

The other possibility is to consider a super Yang-Mills multiplet on the boundary. This is conceptually similar to the model of Hořava and Witten [16, 17].

Our model is the following. In the bulk, we have gauged supergravity. On the boundary there is a cosmological constant plus a super Yang-Mills multiplet. Our action is

$$S = \int d^5x\, e\left(\mathcal{L}_{gauged} + \Lambda_1\delta(x^5)\mathcal{L}_{cc} + \tilde{\Lambda}_1\delta(x^5)\mathcal{L}_{sym}\right)$$

with $\tilde{\Lambda}_1$ a constant and $\mathcal{L}_{sym}$ is given in (3.6). We next use the definitions (3.2) and fix the gauge using (2.8). In this case it is easy to eliminate the auxiliary fields from the lagrangian. The result is

$$\begin{aligned}
&\mathcal{L} = \widetilde{\mathcal{L}}_{gauged} - \Lambda_1\delta(x^5)(\sqrt{3}g' - \frac{1}{2}\bar{\psi}_a\tau^2\gamma^{ab}\psi_b)\\
&+\tilde{\Lambda}_1\delta(x^5)\big(- \frac{1}{4}\widehat{u}_{ab}\widehat{u}^{ab} - \frac{i\sqrt{3}}{4}\bar{\chi}\gamma^a\gamma^{\dot{5}}\chi\widehat{F}_{a\dot{5}} + i\bar{\chi}\gamma^m\widehat{D}_m\chi - \frac{i}{2}\bar{\chi}\gamma^{mab}\psi_m\widehat{u}_{ab}\\
&+\frac{1}{4}\bar{\psi}_m\tau^2\gamma^{mn}\psi_n\bar{\chi}\tau^2\chi + \frac{1}{4}\bar{\psi}_m\tau^2\gamma^{mn}\gamma^{\dot{5}}\psi_n\bar{\chi}\tau^2\gamma^{\dot{5}}\chi\big).
\end{aligned} \tag{4.12}$$

5 Conclusions and Outlook

Using off-shell supergravity, we have presented a technique which allows the systematic construction of models of the Randall-Sundrum type. We assign consistently $\mathbb{Z}_2$ transformation laws to the fields belonging to the supergravity multiplet in the bulk. The fields with positive parity form a non-minimal $N = 1$, $D = 4$ supergravity multiplet at the fixpoints. Using this multiplet, we develop parts of an $N = 1$ tensor calculus located at the branes. We apply this calculus to the explicit construction of lagrangians of the Randall-Sundrum type.

The configurations presented here are easily extended to more complicated ones, using the results in [7, 8, 9]. That is, one could introduce additional matter to the bulk which would then extend the possible couplings to matter at the fixpoints of the orbifold.

Of particular interest would be the study of supersymmetry breaking, since the brane world scenario represents a geometrization of hidden sector supergravity models (for a review see [18]).

First steps in this direction were undertaken in [6] and in [19]. These authors considered rigidly supersymmetric theories on $S^1/\mathbb{Z}_2$. On one brane a supersymmetry breaking D- or F-term was placed, respectively, and the transmission of the supersymmetry breaking to the other wall studied. Using the techniques developed in the present work, it should be quite simple to extend these investigations to local supersymmetry. In this context we should also mention [3], where gaugino condensation as supersymmetry breaking mechanism has been considered.

Acknowledgments: We would like to thank Jonathan Bagger, Jan Conrad, Stefan Förste, Zygmunt Lalak and Hans-Peter Nilles for helpful discussions. This work was partially supported by the European Commission programs ERBFMRX-CT96-0045 and CT96-0090.

References

[1] L. Randall and R. Sundrum, "A large mass hierarchy from a small extra dimension," Phys. Rev. Lett. **83** (1999) 3370 [hep-ph/9905221].

[2] R. D'Auria, E. Maina, T. Regge and P. Fre, "Geometrical First Order Supergravity In Five Space-Time Dimensions," Annals Phys. **135** (1981) 237.

[3] A. Falkowski, Z. Lalak and S. Pokorski, "Supersymmetrizing branes with bulk in five-dimensional supergravity," hep-th/0004093.

[4] E. Bergshoeff, R. Kallosh and A. V. Proeyen, "Supersymmetry in Singular Spaces," hep-th/0007044.

[5] R. Altendorfer, J. Bagger and D. Nemeschansky, "Supersymmetric Randall-Sundrum scenario," hep-th/0003117v2.

[6] E. A. Mirabelli and M. E. Peskin, "Transmission of supersymmetry breaking from a 4-dimensional boundary," Phys. Rev. **D58** (1998) 065002 [hep-th/9712214].

[7] M. Zucker, "Minimal off-shell supergravity in five dimensions," Nucl. Phys. **B570** (2000) 267 [hep-th/9907082].

[8] M. Zucker, "Gauged N = 2 off-shell supergravity in five dimensions," JHEP **08** (2000) 016 [hep-th/9909144].

[9] M. Zucker, "Off-shell supergravity in five dimensions and supersymmetric brane world scenarios," thesis, Physikalisches Institut, Universität Bonn, August 2000, BONN-IR-2000-10.

[10] T. Kugo and K. Ohashi, "Supergravity Tensor Calculus in 5D from 6D," hep-ph/0006231.

[11] B. de Wit, R. Philippe and A. Van Proeyen, "The Improved Tensor Multiplet In N=2 Supergravity," Nucl. Phys. **B219** (1983) 143.

[12] E. Bergshoeff, E. Sezgin and A. Van Proeyen, "Superconformal Tensor Calculus And Matter Couplings In Six-Dimensions," Nucl. Phys. **B264** (1986) 653.

[13] B. de Wit and M. Rocek, "Improved Tensor Multiplets," Phys. Lett. **B109**, 439 (1982).

[14] M. F. Sohnius and P. C. West, "Supergravity With One Auxiliary Spinor," Nucl. Phys. **B216** (1983) 100.

[15] J. O. Conrad, "Brane tensions and coupling constants from within M-theory," Phys. Lett. **B421** (1998) 119 [hep-th/9708031].

[16] P. Horava and E. Witten, "Eleven-Dimensional Supergravity on a Manifold with Boundary," Nucl. Phys. **B475** (1996) 94 [hep-th/9603142].

[17] P. Horava and E. Witten, "Heterotic and type I string dynamics from eleven dimensions," Nucl. Phys. **B460** (1996) 506 [hep-th/9510209].

[18] H. P. Nilles, "Supersymmetry, Supergravity And Particle Physics," Phys. Rept. **110** (1984) 1.

[19] G. Honecker, "Übertragung der Supersymmetrie-Brechung durch ein fünf-dimensionales Hypermultiplett," Diploma Thesis; Bonn, January 1999, (in german)