Pair Production and Vacuum Polarization of Arbitrary Spin Particles with EDM and AMM

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Abstract

The exact solutions of the wave equation for arbitrary spin particles with electric dipole and magnetic moments in the constant and uniform electromagnetic field were found. The differential probability of pair production of particles by an external electromagnetic field has been calculated on the basis of the exact solutions. We have also estimated the imaginary part of the constant and uniform electromagnetic field. The nonlinear corrections to the Maxwell Lagrangian have been calculated taking into account the vacuum polarization of arbitrary spin particles. The role of electric dipole and magnetic moments of arbitrary spin particles in instability of the vacuum is discussed.

1 Introduction

The electric dipole moment (EDM) of particles violates the CP invariance, the time-reversal (T) symmetry [1] and may be induced by the ϑ -term of the quantum chromodynamics (QCD) vacuum. In particular, the neutron and vector mesons may possess the EDM due to the ϑ-term which violates P and CP symmetries and gives CP-odd electromagnetic observable [2-4]. It should be noted that the ϑ-term is important for the solution of the $U(1)_A$ problem in strong interactions. The presence of the ϑ-term creates the "strong CP problem" which can be solved by introducing axions - scalar fields with the small mass. The experimental upper bound for the neutron leads to the experimental constraint of the EDM of the W-boson: $d_W \leq 10^{-19}e$ cm [5]. In the standard model (SM) the predicted EDM of the W-boson is $d_W \simeq 10^{-29}e$ cm [4]. The EDM of particles can be induced by the Higgs-boson exchange [6]. At the same time the EDM of the W-boson may contribute to the EDM of fermions (for example electrons). The experimental measurements of the EDM of elementary particles is important to verify the status of the SM.

The CP - violation observed in the decays of mesons is a fundamental phenomenon and remains mysterious. In the SM the CP - violating interactions can be introduced by the Kobayashi-Maskawa matrix, and the predicted EDM's of elementary particles are extremely small. Some SUSY and multi-Higgs models may predict much stronger CP - violating effects [7].

The properties of hadrons can be investigated in the framework of the renormalizable theory of strong interactions of quarks and gluons - QCD. However, the main tool in QCD is the perturbation theory in small coupling constant α_s which works only in the ultraviolet region at high energy. The characteristics of hadrons are described in the infrared region of QCD at low energy where the nonperturbative effects of chiral symmetry breaking and confinement of quarks take place. The nonperturbative methods in QCD have not developed yet, and some models of hadrons are used. It should be noted that in the framework of the QCD string theory [8] mesons and baryons possess the EDM [9]. It is very important to study various processes involving arbitrary spin particles with the EDM and anomalous magnetic moments (AMM) in view of the great interest to physics in framework and beyond the SM.

Here we proceed from the second order relativistic wave equation for arbitrary spin (s) particles with the EDM and AMM on the basis of the Lorentz representation $(0,s)\oplus(s,0)$ of the wavefunction. In the case of "normal" magnetic moment and the absence of the EDM, such a scheme was introduced in [10]. The wavefunction in this approach has the minimal number $2(2s+1)$ of components, and particles propagate causally in external electromagnetic fields.

It is interesting to study the pair production probability and the vacuum polarization of particles because there is the vacuum instability of particles in a magnetic field [11-13]. In particular, the vacuum of vector particles is non-stable in a magnetic field as there is a contribution to the negative part of the Callan-Symanzik β-function, and the vacuum is reconstructed in a magnetic field.

The problem of the pair production and vacuum polarization of vector particles with gyromagnetic ratio 2 was investigated in [14-17]. This case corresponds to the renormalizable SM where there is a certain symmetry of the vector electromagnetic vertices. The gyromagnetic ratio for W-bosons $g=1+\kappa$, and the AMM $\kappa=1$. However, for vector particles in the framework of the Proca Lagrangian, the gyromagnetic ratio $g=1$, and the AMM $\kappa=0$. The pair production probability of arbitrary spin particles with the gyromagnetic ratio g was studied in [11] on the basis of the semiclassical imaginary-time method. However, this method is valid only for weak electromagnetic fields. In [18] we found the pair production probability for higher spin particles with the gyromagnetic ratio g for arbitrary external fields. Here we generalize this result on the case of the arbitrary spin particles with the EDM, and study also the vacuum polarization.

The purpose of this paper is to find exact solutions to equations for arbitrary spin particles which possess the EDM and AMM in uniform and constant electromagnetic field, and to use these solutions for investigating the most interesting and important vacuum quantum effects - pair production and vacuum polarization. The found exact solutions of the equations can also be used for the other calculations of the electromagnetic processes with the presence of arbitrary spin particles with the EDM and AMM.

The paper is organized as follows. In section 2 we find exact solutions of the wave equation for arbitrary spin particles with the EDM and AMM in external constant and uniform electromagnetic fields. The differential pair production probability of particles and the imaginary part of the effective Lagrangian for electromagnetic fields are calculated on the basis of exact solutions in section 3. In section 4 the nonlinear corrections to Maxwell's Lagrangian caused by the vacuum polarization of arbitrary spin particles with the EDM and AMM are evaluated using the Schwinger method. Section 5 contains the conclusion.

The system of units $\hbar = c = 1$, $\alpha = e^2/4\pi = 1/137$, $e > 0$ is used.

2 Arbitrary spin particles with EDM and AMM in electromagnetic fields

Let us consider the theory of arbitrary spin particles in external electromagnetic fields. We imply that the wavefunction for massive particles is transformed under the $(s,0)\oplus(0,s)$ - representation of the Lorentz group, where s is the spin of particles. The wavefunction in this representation possesses $2(2s+1)$ components. The Lorentz group representations $(s,0)$, $(0,s)$ are parity conjugated and have $(2s+1)$ components each in accordance with the number of spin projections $s_z = \pm s, \pm(s-1), \ldots$ .In the case of the spin-$1/2$ particles the $(1/2,0)\oplus(0,1/2)$ - representation corresponds to Dirac bispinors. For vector particles we arrive at the representation $(1,0)\oplus(0,1)$ of the second rank antisymmetric tensor. In this case the wavefunction has six independent components, i.e. there is doubling of the states of a vector particle with spin projections $s_z = \pm 1, 0$.

We proceed from the two (for $\varepsilon = \pm 1$) wave equations for arbitrary spin particles with the EDM and AMM in external electromagnetic fields:

$$\left[\mathcal{D}_\mu^2 - m^2 - \frac{e}{2}\left(gF_{\mu\nu} - \sigma\widetilde{F}_{\mu\nu}\right)\Sigma^{(\varepsilon)}_{\mu\nu}\right]\Psi_\varepsilon(x) = 0, \tag{1}$$

where $\mathcal{D}_\mu = \partial_\mu - ieA_\mu$ is the covariant derivative, $\partial_\mu = \partial/\partial x_\mu$, $x_\mu = (\mathbf{x}, ix_0)$ (x_0 is the time, $x_0 \equiv t$), $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor, $\widetilde{F}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the dual tensor ($\epsilon_{1234} = -i$), $\epsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita symbol. The numbers $\varepsilon = \pm 1$ correspond to the $(s,0)$ and $(0,s)$ representations with the generators of the Lorenz group $\Sigma^{(-)}_{\mu\nu}$, $\Sigma^{(+)}_{\mu\nu}$, respectively. The gyromagnetic ratio $g = 1/s + \kappa$ (κ is the AMM of a particle) and the σ give the contribution to the magnetic moment $\mu = egs/(2m)$ and the EDM of arbitrary spin particles, $d = \sigma/(2m)$. The spin matrices S_k are connected with the generators $\Sigma^{(\varepsilon)}_{\mu\nu}$ by the relationships:

$$\Sigma^{(\varepsilon)}_{ij} = \epsilon_{ijk}S_k, \qquad \Sigma^{(\varepsilon)}_{4k} = -i\varepsilon S_k \tag{2}$$

and obey the commutation relations

$$[S_i, S_j] = i\epsilon_{ijk}S_k, \qquad (S_1)^2 + (S_2)^2 + (S_3)^2 = s(s+1) \tag{3}$$

with $i,j,k = 1,2,3$ ($\epsilon_{123} = 1$).

Eq. (1) at the case $\sigma = 0$ was investigated in [18], and at the particular case of the "normal" magnetic moment of a particle, when $g = 1/s$, we arrive at the approach [10]. Here arbitrary AMM and EDM of a particle is considered. At the parity transformation, $\varepsilon \to -\varepsilon$, and the representation $(s,0)$ is converted into $(0,s)$. As a result Eqs. (1) (at $\varepsilon = \pm 1$) are invariant under the parity inversion if $\sigma = 0$. The term with the EDM in Eq. (1) violates the CP - invariance.

Now we find the solutions to Eq. (1) for a particle in the field of uniform and constant electromagnetic fields. For simplicity we choose a coordinate system in which the electric $\mathbf{E}$ and magnetic $\mathbf{H}$ fields are parallel, so that $\mathbf{E} = \mathbf{n}E$, $\mathbf{H} = \mathbf{n}H$, $\mathbf{n} = (0,0,1)$ with the 4-vector potential

$$A_\mu = (0, x_1 H, -tE, 0)\,. \tag{4}$$

The Lorentz condition $\partial_\mu A_\mu = 0$ is permitted for the potential (4). Different choices of potentials for the fixed electromagnetic fields $\mathbf{E}$, $\mathbf{H}$ are connected by the gauge transformations. When $E \neq H$, two Lorentz invariants of electromagnetic fields

$$\mathcal{F} = \frac{1}{4}F^2_{\mu\nu} = \frac{1}{2}\left(\mathbf{H}^2 - \mathbf{E}^2\right), \qquad \mathcal{G} = \frac{1}{4}F_{\mu\nu}\widetilde{F}_{\mu\nu} = \mathbf{E}\cdot\mathbf{H}, \tag{5}$$

do not vanish. This is the general case of the external uniform and constant electromagnetic fields. From Eqs. (2) we arrive at the expressions containing spin matrices and auxiliary complex vector fields:

$$\frac{1}{2}\Sigma^{(+)}_{\mu\nu}F_{\mu\nu} = \mathbf{SX}, \qquad \frac{1}{2}\Sigma^{(-)}_{\mu\nu}F_{\mu\nu} = \mathbf{SX}^*, \tag{6}$$

$$\frac{1}{2}\Sigma^{(+)}_{\mu\nu}\widetilde{F}_{\mu\nu}=\mathbf{S}\widetilde{\mathbf{X}}, \qquad \frac{1}{2}\Sigma^{(-)}_{\mu\nu}\widetilde{F}_{\mu\nu}=\mathbf{S}\widetilde{\mathbf{X}}^*,$$

where $\mathbf{X}=\mathbf{H}+i\mathbf{E}$, $\mathbf{X}^*=\mathbf{H}-i\mathbf{E}$, $\widetilde{\mathbf{X}}=-\mathbf{E}+i\mathbf{H}$, $\widetilde{\mathbf{X}}^*=-\mathbf{E}-i\mathbf{H}$. The auxiliary complex vector fields $\mathbf{X}$, $\widetilde{\mathbf{X}}$ (and $\mathbf{X}^*$, $\widetilde{\mathbf{X}}^*$) are transformed as $\mathbf{X}\to\widetilde{\mathbf{X}}$ under the dual transformations of the electromagnetic fields $\mathbf{E}\to\mathbf{H}$, $\mathbf{H}\to-\mathbf{E}$.

Let the wavefunction $\Psi_\varepsilon(x)$ be the eigenfunction of the operators $\mathbf{SX}$ and $\mathbf{S}\widetilde{\mathbf{X}}$. Then we have the diagonal representation of matrices (6), and equations for eigenvalues are given by

$$\begin{aligned}
&\mathbf{SX}\Psi^{(s_z)}_+(x)=s_zX\Psi^{(s_z)}_+(x), &&\mathbf{SX}^*\Psi^{(s_z)}_-(x)=s_zX^*\Psi^{(s_z)}_-(x),\\
&\mathbf{S}\widetilde{\mathbf{X}}\Psi^{(s_z)}_+(x)=s_z\widetilde{X}\Psi^{(s_z)}_+(x), &&\mathbf{S}\widetilde{\mathbf{X}}^*\Psi^{(s_z)}_-(x)=s_z\widetilde{X}^*\Psi^{(s_z)}_-(x),
\end{aligned} \tag{7}$$

where $X=H+iE$, $\widetilde{X}=-E+iH$, and the spin projections s_z take the values

$$s_z=\begin{cases} \pm s, \pm(s-1), \cdots 0 & \text{for bosons,}\\ \pm s, \pm(s-1), \cdots \pm\frac{1}{2} & \text{for fermions.}\end{cases} \tag{8}$$

Using Eqs. (6), (7), we can represent Eq. (1) (for $\varepsilon=\pm1$) as

$$\begin{aligned}
&\left[\mathcal{D}^2_\mu-m^2-es_z\left(gX-\sigma\widetilde{X}\right)\right]\Psi^{(s_z)}_+(x)=0,\\
&\left[\mathcal{D}^2_\mu-m^2-es_z\left(gX^*-\sigma\widetilde{X}^*\right)\right]\Psi^{(s_z)}_-(x)=0,
\end{aligned} \tag{9}$$

where s_z is given by Eq. (8). For every projection s_z equations (9) are the Klein-Gordon type equations with the complex "effective" masses:

$$\begin{aligned}
&m^2_{eff}=m^2+es_z\left(gX-\sigma\widetilde{X}\right),\\
&(m^2_{eff})^*=m^2+es_z\left(gX^*-\sigma\widetilde{X}^*\right).
\end{aligned} \tag{10}$$

Introducing the variables [19] (see also [20])

$$\begin{aligned}
&\eta=\frac{p_2-eHx_1}{\sqrt{eH}}, \qquad \tau=\sqrt{eE}\left(t+\frac{p_3}{eE}\right),\\
&\Psi^{(s_z)}_+(x)\equiv\Psi^{(s_z)}(x)=\exp\left[i\left(p_2x_2+p_3x_3\right)\right]\Phi^{(s_z)}(\eta,\tau),
\end{aligned} \tag{11}$$

Eq. (9) becomes

$$\left[eH\left(\partial^2_\eta-\eta^2\right)-eE\left(\partial^2_\tau+\tau^2\right)-m^2-es_z\left(gX-\sigma\widetilde{X}\right)\right]\Phi^{(s_z)}(\eta,\tau)=0, \tag{12}$$

plus complex conjugated equation where $\partial_\eta = \partial/\partial\eta$, $\partial_\tau = \partial/\partial\tau$. The variables η, τ are separated in this equation and the solution to Eq. (12) can be written in the form

$$\Phi^{(s_z)}(\eta,\tau) = \phi^{(s_z)}(\eta)\chi^{(s_z)}(\tau), \tag{13}$$

with the eigenfunctions $\phi^{(\lambda)}(\eta)$, $\chi(\tau)$ obeying the following equations

$$\left[eH\left(\partial_\eta^2 - \eta^2\right) - m^2 - es_z\left(gX - \sigma\widetilde{X}\right) + k_{s_z}^2\right]\phi^{(s_z)}(\eta) = 0, \tag{14}$$

$$\left[eE\left(\partial_\tau^2 + \tau^2\right) + k_{s_z}^2\right]\chi^{(s_z)}(\tau) = 0, \tag{15}$$

It follows from Eqs. (14), (15) that a magnetic field H forms a motion of a particle depending on the variables x_1, x_2 and an electric field determines a motion depending on x_3, t. In accordance with Eqs. (13)-(15) these two motions are described by the functions $\phi^{(s_z)}(\eta)$, $\chi^{(s_z)}(\tau)$ and they are independent. The solution to Eq. (14) is given by the Hermite functions [21]

$$\phi^{(s_z)}(\eta) = N_0 \exp\left(-\frac{\eta^2}{2}\right) H_n(\eta)), \tag{16}$$

which is finite at $\eta \to \infty$. Here N_0 is the normalization constant, and $H_n(\eta)$ are the Hermite polynomials. The requirement that the solution to Eq. (14) be finite leads to the equation for eigenvalues $k_{s_z}^2$:

$$k_{s_z}^2 - m^2 - es_z\left(gX - \sigma\widetilde{X}\right) = eH(2n+1), \tag{17}$$

where $n = 1, 2, ...$ is the principal quantum number. So, the spectral parameter $k_{s_z}^2$ is a quantized but not an arbitrary value.

Solutions to Eq. (15) can be expressed through the parabolic-cylinder functions $D_\nu(x)$ [21] as

$$\begin{gathered} {}_+\chi^{(s_z)}(\tau) = D_\nu[-(1-i)\tau], \qquad {}^-\chi^{(s_z)}(\tau) = D_\nu[(1-i)\tau], \\ {}^+\chi^{(s_z)}(\tau) = D_{-(\nu+1)}[(1+i)\tau], \qquad {}_-\chi^{(s_z)}(\tau) = D_{-(\nu+1)}[-(1+i)\tau], \end{gathered} \tag{18}$$

Four solutions (18) possess different asymptotic at $t \to \pm\infty$ [19], and ν reads

$$\nu = \frac{ik_{s_z}^2}{2eE} - \frac{1}{2}.$$

Using Eqs. (13), (16) and (18), we arrive at the solutions of Eqs. (9):

$$ {}^\pm_\pm\Psi^{(s_z)}(x) = N_0 \exp\left\{i(p_2x_2 + p_3x_3) - \frac{\eta^2}{2}\right\} H_n(\eta) {}^\pm_\pm\chi^{(s_z)}(\tau). \tag{19}$$

The solutions ${}^{\pm}_{\pm}\Psi^{(s_z)}(x)$ are labelled by four conserved numbers: two momentum projections p_2, p_3, the spin projection s_z and the spectral parameter k_{s_z}. Wavefunctions (19) are the exact solutions of Eq. (9) for a particle in external constant and uniform electromagnetic fields corresponding to the principal quantum number n and the spin projection s_z. The exact solutions (19) can be used for different electrodynamic calculations of the quantum processes with the presence of arbitrary spin particles. In such approach the interaction with an external electromagnetic field is taken into consideration with its exact value, and therefore the calculations may be used without the perturbative theory in a small parameter. It allows us to investigate some nonperturbative and nonlinear effects.

3 Pair production of arbitrary spin particles by electromagnetic fields

According to the well known Schwinger result [22] a uniform and constant electric field produces pairs of particles. The probabilities for the production of scalar and spin 1/2 pairs of particles have been evaluated. Now we consider more general case of pair production of arbitrary spin particles with the EDM and AMM by electric and magnetic fields. The probability of pair production of particles by constant and uniform electromagnetic fields may be obtained using the asymptotic form of the solutions [19]. In this approach one avoids the Klein "paradox" when the total scattering probability of a wave packet of a particle is smaller than unity.

The functions ${}^{+}_{+}\chi^{(s_z)}(\tau)$ (19) have positive frequency and ${}^{-}_{-}\chi^{(s_z)}(\tau)$ have negative frequency when the time approaches $\pm\infty$. The set of wavefunctions ${}^{\pm}\chi^{(s_z)}(\tau)$ and ${}_{\pm}\chi^{(s_z)}(\tau)$ (19) are equivalent and satisfy the relationships [19]:

$$
\begin{aligned}
{}_{+}\Psi^{(s_z)}(x) &= c_{1ns_z}\,{}^{+}\Psi^{(s_z)}(x) + c_{2ns_z}\,{}^{-}\Psi^{(s_x)}(x),\\
{}^{+}\Psi^{(s_z)}(x) &= c^{*}_{1ns_z}\,{}_{+}\Psi^{(s_z)}(x) - c_{2ns_z}\,{}_{-}\Psi^{(s_z)}(x),\\
{}_{-}\Psi^{(s_z)}(x) &= c^{*}_{2ns_z}\,{}^{+}\Psi^{(s_z)}(x) + c^{*}_{1ns_z}\,{}^{-}\Psi^{(s_z)}(x),\\
{}^{-}\Psi^{(s_z)}(x) &= -c^{*}_{2ns_z}\,{}_{+}\Psi^{(s_z)}(x) + c_{1ns_z}\,{}_{-}\Psi^{(s_z)}(x),
\end{aligned}
\tag{20}
$$

where variables c_{1ns_z}, c_{2ns_z} are

$$
c_{2ns_z} = \exp\left[-\frac{\pi}{2}(\lambda + i)\right], \qquad \lambda = \frac{m^2 + es_z\left(gX - \sigma\widetilde{X}\right) + eH(2n+1)}{eE},
$$

$$
\mid c_{1ns_z}\mid^2 - \mid c_{2ns_z}\mid^2 = 1 \qquad \text{for bosons,} \tag{21}
$$

$$| c_{1ns_z} |^2 + | c_{2ns_z} |^2 = 1 \qquad\qquad\qquad \text{for fermions}.$$

The relations (20) represent the Bogolubov transformations with the coefficients c_{1ns_z}, c_{2ns_z} which contain the information about producing pairs of particles in the state with the principal quantum number n and the spin projection s_z. So, the value $| c_{2ns_z} |^2$ is the probability of pair production of arbitrary spin particles in the state with quantum numbers n, s_z throughout all space and during all time, and according to Eq. (21) is given by

$$| c_{2ns_z} |^2 = \exp\left\{-\pi\left[\frac{m^2}{eE} + s_z\left(\sigma + g\frac{H}{E}\right) + \frac{H}{E}(2n+1)\right]\right\}. \tag{22}$$

The expression (22) represents also the probability of the pair annihilation with quantum numbers n, s_z. It follows from Eq. (22) that the maximum of pair creation probability at $H \gg E$ occurs in the state with the smallest energy when $n = 0$, $s_z = -s$. According to the approach [19], the average number of particle pairs produced from a vacuum is given by

$$\overline{N} = \sum_{n,s_z} | c_{2ns_z} |^2 \frac{e^2 EHVT}{(2\pi)^2}, \tag{23}$$

where V is the normalization volume, T is the time of observation.

Inserting expression (22) into Eq. (23) and calculating the sum over the principal quantum number n and spin projections s_z (see [18]), we find the pair production probability per unit volume and per unit time

$$I(E,H) = \frac{\overline{N}}{VT} = \frac{e^2 EH}{8\pi^2}\frac{\exp\left[-\pi m^2/(eE)\right]}{\sinh\left(\pi H/E\right)}\frac{\sinh\left[(s+1/2)\pi\left(\sigma + gH/E\right)\right]}{\sinh\left[(\pi/2)\left(\sigma + gH/E\right)\right]}. \tag{24}$$

The expression (24) is non-analytic in E and it is impossible to obtain it with the help of a perturbative theory. We find from Eq. (24) that in the case $\sigma = g = 0$, the pair production of arbitrary spin particles is $(2s+1)$ times that for the scalar particle pair production due to the $(2s+1)$ physical degrees of freedom of the arbitrary spin field. At $\sigma = 0$ Eq. (24) converts into the expression derived in [18]. The intensity of the pair creation (24) is the generalization of the results [11,18] on the case of arbitrary spin particles with the EDM and AMM. In the general case $\sigma \neq 0$, $g \neq 0$ there is also a pair production of arbitrary spin particles if $E = 0$, $H > m^2/e$ at $gs > 1$ (see [11]), i.e. there is instability of the vacuum in a magnetic field. This occurs because the ground state with quantum numbers $n = 0$, $s_z = -s$ becomes a tachyon state when the magnetic field H is greater than the critical value $H_0 = m^2/e$, i.e. $H > H_0$, and the vacuum is reconstructed.

When the magnetic field vanishes, $H=0$, equation (24) transforms into

$$I(E)=\frac{e^2E^2}{8\pi^3}\frac{\exp\left(-\pi E_0/E\right)}{\sinh\left[\pi\sigma/2\right]}\sinh\left[(2s+1)\pi\sigma/2\right], \tag{25}$$

where $E_0=m^2/e$ is the critical value of the electric field. At $E<E_0$ the intensity of pair production is exponentially small. However at the critical value of the electric field $E=E_0$ particles are created rapidly. So, the intensity for the electron-positron pair production ($s=1/2$, $\sigma=0$) at $E=m_e^2/e$, where m_e is the mass of the electron, is given by

$$I(E_0)=\frac{m_e^4}{4\pi^3}\exp(-\pi)\simeq 4\times 10^{48}\ cm^{-3}sec^{-1}.$$

The pair production probability (25) does not depend on the gyromagnetic ratio g and depends only on the EDM. It follows from Eq. (24) that the magnetic field increases the producing of higher spin particles and decreases the pair production of scalar particles.

Let us consider the important cases of particles with spins $1/2$ and 1. For fermions with spin $1/2$ the pair production probability by the electromagnetic field takes the form

$$I_{1/2}(E,H)=\frac{e^2EH}{4\pi^2}\frac{\exp\left[-\pi m^2/(eE)\right]}{\sinh\left(\pi H/E\right)}\cosh\left[(\pi/2)\left(\sigma+gH/E\right)\right]. \tag{26}$$

For the probability of electron-positron production we have $g=2$, $\sigma=0$ and Eq. (26) converts into those obtained in [19]. At the particular case of an external electric field we arrive at the Schwinger formula (for $g=2$, $\sigma=0$) [22]. The presence of the EDM and AMM of a spinor particle increases the pair production. From the general expression (24) we find after some transformations the intensity of producing of the vector particles ($s=1$)

$$I_1(E,H)=\frac{e^2EH}{8\pi^2}\frac{\exp\left[-\pi m^2/(eE)\right]}{\sinh\left(\pi H/E\right)}\left[2\cosh\left[\pi\left(\sigma+gH/E\right)\right]+1\right]. \tag{27}$$

The pair production probability (27) is greater than those for spinor particles (26) due to the greater number of physical degrees of freedom (spin projections) of the vector field. At $\sigma=0$ and $H=0$ expression (27) converts into the intensity of pair production of vector particles obtained in [14] for the case $g=2$.

The imaginary part of the density of the electromagnetic field Lagrangian can be obtained with the help of [19]:

$$\mathrm{Im}\mathcal{L} = \frac{1}{2}\int \sum_{n,s_z} \ln \mid c_{1ns_z} \mid^2 \frac{e^2EH}{(2\pi)^2}. \tag{28}$$

Using Eqs. (21) we find

$$\mathrm{Im}\mathcal{L} = \frac{e^2EH}{16\pi^2}\sum_{n=1}^{\infty}\frac{\beta_n}{n}\exp\left(-\frac{\pi m^2 n}{eE}\right)\frac{\sinh\left[n(s+1/2)\pi\left(\sigma+gH/E\right)\right]}{\sinh\left(n\pi H/E\right)\sinh\left[n\left(\pi/2\right)\left(\sigma+gH/E\right)\right]}, \tag{29}$$

$$\beta_n = \begin{cases} (-1)^{n-1} & \text{for integer spins,} \\ 1 & \text{for half-integer spins.} \end{cases}$$

In accordance with [22,19] the first term ($n=1$) in (29) gives the half of the pair production probability per unit volume and per unit time. It should be noted that Im$\mathcal{L}$ (29) and the pair production probabilities (24)-(27) do not depend on the scheme of renormalization as all divergences and the renormalizability are contained in Re$\mathcal{L}$ [22].

4 Polarization of arbitrary spin particle vacuum

Let us consider the problem of one-loop corrections to the Lagrange function of a constant and uniform electromagnetic field due to the field interaction with a vacuum of arbitrary spin particles with the EDM and AMM. This problem has been solved for fields of spins 0, $1/2$ and 1 (at $\sigma=0$, $g=2$) in [22,14,15]. The nonlinear corrections to the Maxwell Lagrangian due to vacuum polarization are connected with the cross-section of scattering photons by photons. Using the Schwinger method [22], we obtain the one-loop corrections to Lagrangian of a constant and uniform electromagnetic field [1]

$$\mathcal{L}^{(1)} = \frac{\epsilon}{32\pi^2}\int_0^\infty d\tau\tau^{-3}\exp\left(-m^2\tau - l(\tau)\right)\mathrm{tr}\exp\left[\frac{e_0\tau}{2}\Sigma_{\mu\nu}\left(gF_{\mu\nu}-\sigma\widetilde{F}_{\mu\nu}\right)\right], \tag{30}$$

$$\Sigma_{\mu\nu} = \Sigma^{(+)}_{\mu\nu}\oplus\Sigma^{(-)}_{\mu\nu}, \qquad l(\tau) = \frac{1}{2}\mathrm{tr}\ln\left[(e_0F\tau)^{-1}\sin(e_0F\tau)\right],$$
$$\exp\left[-l(\tau)\right] = \frac{(e_0\tau)^2\mathcal{G}_0}{\mathrm{Im}\cosh(e_0\tau X_0)}, \tag{31}$$

where $\oplus$ is the direct sum, $\epsilon=1$ for bosons and $\epsilon=-1$ for fermions due to different signs of loop integrals for bosons and fermions. The index

[1] The factor ϵ was omitted in [18].

0 in Eqs. (30), (31) refers to the unrenormalized variables, so that e_0 is the bare electric charge, $\mathbf{X}_0 = \mathbf{H}_0 + i\mathbf{E}_0$, $\mathcal{G}_0 = \mathbf{E}_0 \cdot \mathbf{H}_0$; $\mathbf{E}_0$, $\mathbf{H}_0$ are bare electric and magnetic fields, respectively. The Lagrangian $\mathcal{L}^{(1)}$ (30) is the effective nonlinear Lagrangian which is an integral over the proper time τ. Calculating with the help of Eqs. (6), (7) the trace (tr) of the matrices we arrive at

$$\mathrm{tr}\exp\left[\frac{e_0\tau}{2}\Sigma_{\mu\nu}\left(gF_{\mu\nu}-\sigma\widetilde{F}_{\mu\nu}\right)\right]=2\mathrm{Re}\frac{\sinh\left[(s+1/2)e_0\tau\left(gX_0-\sigma\widetilde{X}_0\right)\right]}{\sinh\left[(e_0\tau/2)\left(gX_0-\sigma\widetilde{X}_0\right)\right]}. \tag{32}$$

Replacing (32) into (30) and subtracting the constant to have vanishing $\mathcal{L}^{(1)}$ when the electromagnetic fields approach zero, (see [22]) we arrive at

$$\mathcal{L}^{(1)}=\frac{\epsilon}{16\pi^2}\int_0^\infty d\tau\tau^{-3}\exp\left(-m^2\tau\right)$$
$$\times\left\{\frac{(e_0\tau)^2\mathcal{G}_0}{\mathrm{Im}\cosh(e_0\tau X_0)}\mathrm{Re}\frac{\sinh\left[(s+1/2)e_0\tau\left(gX_0-\sigma\widetilde{X}_0\right)\right]}{\sinh\left[(e_0\tau/2)\left(gX_0-\sigma\widetilde{X}_0\right)\right]}-(2s+1)\right\}. \tag{33}$$

Setting $\sigma = 0$, $g = 2$, $\epsilon = -1$ in Eq. (33) we come to the Schwinger Lagrangian [22]. The integral (30) is the nonlinear correction to Maxwell's Lagrangian due to the vacuum polarization of arbitrary spin particles with the EDM and AMM. The Lagrangian (33) has the quadratic term in the electromagnetic fields, and that renormalizes the Lagrangian of the free electromagnetic fields

$$\mathcal{L}^{(0)}=-\mathcal{F}_0=\frac{1}{2}\left(\mathbf{E}_0^2-\mathbf{H}_0^2\right). \tag{34}$$

Expanding Lagrangian (33) in weak electromagnetic fields and adding the Lagrangian of the free electromagnetic fields (34) we obtain the renormalized Lagrangian of electromagnetic fields that takes into account the vacuum polarization of arbitrary spin particles with the EDM and AMM:

$$\mathcal{L}=\mathcal{L}^{(0)}+\mathcal{L}^{(1)}=-\mathcal{F}+\frac{\epsilon}{16\pi^2}\int_0^\infty d\tau\tau^{-3}\exp\left(-m^2\tau\right)$$
$$\times\left\{\frac{(e\tau)^2\mathcal{G}}{\mathrm{Im}\cosh(e\tau X)}\mathrm{Re}\frac{\sinh\left[(s+1/2)e\tau\left(gX-\sigma\widetilde{X}\right)\right]}{\sinh\left[(e\tau/2)\left(gX-\sigma\widetilde{X}\right)\right]}\right. \tag{35}$$
$$\left.-(2s+1)-\frac{(2s+1)(e\tau)^2\mathcal{F}}{3}\left[s(s+1)\left(g^2-\sigma^2\right)-1\right]\right\},$$

where we renormalize fields and charges:

$$\mathcal{F}=Z_3^{-1}\mathcal{F}_0,\qquad e=Z_3^{1/2}e_0.$$

The renormalization constant is given by

$$Z_3^{-1} = 1 - \frac{\epsilon e_0^2 (2s+1) [s(s+1)(g^2-\sigma^2)-1]}{48\pi^2} \int_0^\infty d\tau \tau^{-1} \exp\left(-m^2\tau\right). \tag{36}$$

If the electromagnetic fields $\mathbf{E}$, $\mathbf{H}$ are absent, Lagrangian (35) vanishes. The renormalization constant Z_3^{-1} diverges logarithmically when the cutoff factor τ_0 at the lower limit in the integral (36) approaches zero ($\tau_0 \to 0$). It follows from Eq. (36) that if the inequality

$$\epsilon \left[s(s+1)\left(g^2-\sigma^2\right)-1\right] > 0 \tag{37}$$

is valid, the renormalization constant of the charge $Z_3^{1/2} > 1$. This case indicates asymptotic freedom in the field [23,24], and is not realized in QED because $g = 2$, $\sigma = 0$ and $\epsilon = -1$. However, for boson fields, when $\epsilon = 1$, asymptotic freedom occurs for the small value of the EDM σ. In accordance with Eq. (37) the asymptotically free behavior in the boson fields is due to the AMM, and the EDM of particles suppresses the phenomena of asymptotic freedom. However this discussion concerns only the renormalizable theories. We can argue that the formal counting of the divergences corresponding to Eq. (1) leads to a renormalizable theory due to the form of the field propagator that proportional to $1/p^2$, and the smallness of the field self-interaction constant. If $g = 2$, $\sigma = 0$, we have the linear approximation to the renormalizable gage theory for the vector field when the mass of the field is acquired by the Higgs mechanism. Eq. (36) allows us to obtain the Callan-Zymanzik β function that corresponds to the renormalizable theory:

$$\beta = -\frac{\epsilon e_0^2 (2s+1) [s(s+1)(g^2-\sigma^2)-1]}{48\pi^2}. \tag{38}$$

Asymptotic freedom occurs if the β function is negative ($\beta < 0$) that is equivalent to Eq. (37). The AMM and spin (s) of bosons assures asymptotic freedom and instability of the vacuum in a magnetic field.

Expanding Eq. (35) in the weak electromagnetic fields, and using the equality

$$\int_0^\infty d\tau \tau \exp\left(-m^2\tau\right) = \frac{1}{m^4}$$

we obtain after renormalization the Maxwell Lagrangian with the nonlinear corrections:

$$\mathcal{L} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{H}^2\right) - \frac{2s(s+1)\sigma g}{s(s+1)(g^2-\sigma^2)-1}\left(\mathcal{G}-\mathcal{G}_0\right) + \frac{\epsilon\alpha^2(2s+1)}{90m^4}$$

$$\times\left\{\left[s(s+1)(3s^2+3s-1)\left(g^4-6g^2\sigma^2+\sigma^4\right)-10s(s+1)(g^2-\sigma^2)+7\right]\mathcal{F}^2\right.$$
$$+\left[s(s+1)(3s^2+3s-1)\left(6g^2\sigma^2-g^4-\sigma^4\right)+1\right]\mathcal{G}^2$$
$$\left.+4s(s+1)\sigma g\left[2(3s^2+3s-1)\left(g^2-\sigma^2\right)-5\right]\mathcal{G}\mathcal{F}\right\}, \quad (39)$$

where $\alpha = e^2/(4\pi)$. The second and last terms in Eq. (39) indicate parity violation due to the EDM of a particle. We can consider the second term in Eq. (39) as anomaly for a particle with the EDM because such a quadratic in fields term does not present in the bare Lagrangian (34). Lagrangian (39) is the Heisenberg-Euler type Lagrangian [25,26] for the case of arbitrary spin particles with the EDM and AMM. It can be verified that for the case $\sigma = g = 0$, we arrive from Eq. (39) at

$$\mathcal{L}(\sigma = g = 0) = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{H}^2\right) + \epsilon(2s+1)\mathcal{L}_{\text{spin }0}, \quad (40)$$

where $\mathcal{L}_{\text{spin }0}$ is the nonlinear correction to the Lagrangian of the electromagnetic fields due to the vacuum polarization of scalar particles [22]:

$$\mathcal{L}_{\text{spin }0} = \frac{\alpha^2}{360m^4}\left[7\left(\mathbf{E}^2 - \mathbf{H}^2\right)^2 + 4(\mathbf{EH})^2\right] \quad (41)$$

Eq. (40) tells us that there is an equal contribution of $(2s+1)$ degrees of freedom (spin projections) of arbitrary spin fields at $\sigma = g = 0$ when Eq. (1) is converted into a Klein-Gordon equation. The factor ϵ corresponds to different statistics for bosons and fermions. For the case of QED at $s = 1/2$, $g = 2$, $\epsilon = -1$ Eq. (39) becomes the Schwinger Lagrangian [22].

It is interesting to consider the particular case of spin 1 fields with gyromagnetic ratio $g = 2$ and $\sigma = 0$ which corresponds to the linear approximation to the renormalizable SM. However, it should be noted that Eq. (1) at $s = 1$ is based on the $(1,0)\oplus(0,1)$ representation of the Lorenz group, and is not equivalent to the Proca equation [18]. Setting $g = 2$, $\sigma = 0$, $s = 1$, $\epsilon = 1$ in Eq. (39) we arrive at the Lagrangian of the electromagnetic field that takes into account the vacuum polarization of a charged vector particles

$$\mathcal{L}_{\text{spin }1} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{H}^2\right) + \frac{\alpha^2}{10m^4}\left[\frac{29}{4}\left(\mathbf{E}^2 - \mathbf{H}^2\right)^2 - 53(\mathbf{EH})^2\right] \quad (42)$$

which is a little different from those obtained in [14] on the basis of the Proca equation.

Eq. (35) allows us to find the limit at $eE/m^2 \to \infty$ and $eH/m^2 \to \infty$ if we ignore the dependence of the AMM and EDM on the strong external electromagnetic fields that is some approximation.

5 Conclusion

The pair-production probability (24), and the effective Lagrangian for electromagnetic fields (35) which takes into account the polarization of the vacuum, are the generalization of the Schwinger result on the case of the theory of particles with arbitrary spins, EDM and AMM in the external electric and magnetic fields. It follows from Eq. (24) that there is a pair production of particles by a purely magnetic field ($H > H_0$) in the case of $gs > 1$ assuring asymptotic freedom and instability of the vacuum in a magnetic field. The presence of the EDM of a particle does not lead to instability of the vacuum in a magnetic field but it suppresses the phenomena of asymptotic freedom. In the presence of the magnetic field the probability decreases for scalar particles and increases for higher spin particles.

The intensity of pair production of arbitrary spin particles (24) does not depend on the renormalization scheme as all divergences and the renormalizability are contained in Re$\mathcal{L}$. Therefore the formula (24) is reliable. The procedure of obtaining the vacuum polarization corrections for electromagnetic fields uses the renormalization, and we imply that the scheme considered is the linearized version of renormalizable gauge theory. This point of view is justified if we imply the smallness of the arbitrary spin field self-interaction constant.

We have just studied the vacuum quantum effects of pair production and vacuum polarization of arbitrary spin particles with the EDM which violates CP - symmetry. The investigation of quantum processes with CP - violation is important because they may give a sensitive probe for New Physics. The value of the gyromagnetic ratio $g \neq 2$ for vector particles, and the EDM $d \neq 0$ leads to physics beyond the SM. Now this is of interest because experimental muon AMM data [27] have challenged the SM (there is a discrepancy of 2.6σ deviation between the theory and the averaged experimental value).

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