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Scalar fluctuations in dilatonic brane–worlds

VALERIO BOZZA

Dipartimento di Fisica "E.R. Caianiello", Università di Salerno
Via S. Allende, 84081 Baronissi (SA), Italy.
INFN, Sezione di Napoli, Gruppo Collegato di Salerno, Salerno, Italy.

MAURIZIO GASPERINI

Dipartimento di Fisica, Università di Bari,
Via G. Amendola 173, 70126 Bari, Italy.
INFN, Sezione di Bari, Bari, Italy.

GABRIELE VENEZIANO

Theoretical Physics Division, CERN
CH-1211 Geneva 23, Switzerland.
Laboratoire de Physique Théorique, Université Paris Sud, 91405 Orsay, France.

We derive and solve the full set of scalar perturbation equations for a class of five-dimensional brane–world solutions, with a dilaton scalar field coupled to the bulk cosmological constant and to a 3-brane. The spectrum contains one localized massless scalar mode, to be interpreted as an effective dilaton on the brane, inducing long–range scalar interactions. Two massive scalar modes yield corrections to Newton's law at short distances, which persist even in the limit of vanishing dilaton (namely, in the standard Randall–Sundrum configuration).

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1 Introduction

In the wake of Horava–Witten heterotic M-theory [1], Randall and Sundrum (RS) have recently proposed a five dimensional model where the orbifold compactification, supplemented by a geometrical warp factor along the extra–dimension, is used to implement an effective hierarchy between the Planck mass and the electroweak scale [2]. Thanks to the warp factor, on the other hand, it is also possible to obtain consistent configurations with only one brane and an infinite extra–dimension, while preserving the long–range behaviour of the four dimensional Newton's law [3]. This is possible thanks to the dynamical localization of massless gravitons on the brane and a strong suppression of all massive, higher-dimensional modes.

As a consequence of the non-factorized structure of the metric in this type of scenarios, the decomposition of the metric fluctuations, performed according to the rotational $O(3)$ symmetry on the brane, requires a generalized gauge–invariant formalism [4]. The classical and quantum analysis of metric fluctuations is of primary importance for understanding the possible localization of massless modes on the brane, as well as the nature of the possible short-range corrections due to the continuum of massive modes, living in the bulk. Until now, the study of this problem has been mainly focused on the structure of tensor (i.e. transverse-traceless) perturbations of the bulk geometry (see [5] for a general discussion).

In all string/M-theory models, however, the graviton is accompanied by massless scalar partners (the dilaton, the compactification moduli, etc.). These typically induce long-range interactions of gravitational strength [6], not (yet) experimentally observed. The conventional way to solve this problem is to assume that the scalar partners get a non-perturbative, SUSY-breaking mass, thus suppressing the range of the associated scalar interactions. However, if scalar fluctuations would not be confined in RS–type brane–worlds, then all corresponding interactions on the brane would be suppressed, and the brane-world scenario could naturally solve a possible discrepancy between String/M-theories and experiments.

Here we present the results of Ref. [7], where we discuss the localization of scalar fluctuations in a typical brane-world scenario of the RS type, taking into account the possible existence of scalar sources and scalar fields living in the bulk. To this purpose, we shall consider a non-compact, $\mathbb{Z}_2$-symmetric, five-dimensional background, generated by a positive tension 3-brane and by a bulk dilaton field coupled to the brane and to the (negative) bulk energy density. We shall restrict ourselves to the gravi-dilaton solutions discussed in [8], which generalize the AdS_5 RS scenario in the presence of a bulk scalar field, and which are already known to guarantee the localization of tensor metric fluctuations.

2 Background equations

We consider in particular a five-dimensional scalar-tensor background $\{g_{AB}, \phi\}$, possibly arising from the bosonic sector of a dimensionally reduced string/supergravity theory, and non-trivially coupled to a negative cosmological constant Λ and to a 3-brane of positive tension T_3:

$$
\begin{aligned}
S = S_{\text{bulk}} + S_{\text{brane}} &= M_5^3 \int d^5x \sqrt{|g|} \left(-R + \frac{1}{2} g^{AB} \partial_A \phi \partial_B \phi - 2\Lambda e^{\alpha_1 \phi} \right) \\
&- \frac{T_3}{2} \int d^4\xi \sqrt{|\gamma|} \left[\gamma^{\alpha\beta} \partial_\alpha X^A \partial_\beta X^B g_{AB} e^{\alpha_2 \phi} - 2 \right]. \qquad (1)
\end{aligned}
$$

Here M_5 is the fundamental mass scale of the five-dimensional bulk space-time, and the parameters α_1, α_2 control the coupling of the bulk dilaton to Λ and to the brane. We allow in general for non-minimal couplings (a single exponential potential for the dilaton can also be derived from the dimensional reduction of a suitable higher-dimensional model [8]). The brane action is parameterized by the coordinates $X^A(\xi)$ describing the embedding of the brane in the bulk manifold, and by the auxiliary metric tensor $\gamma_{\alpha\beta}(\xi)$ defined on the four-dimensional world-volume of the brane, spanned by the coordinates ξ^α.

In our conventions, greek indices run from 0 to 3, capital Latin indices from 0 to 4, lower-case Latin indices from 1 to 3. For the bulk coordinates we use the notation $x^A = (t, x^i, z)$.

The field equations are obtained by the variation of the action with respect to g_{AB}, ϕ, X^A and $\gamma_{\alpha\beta}$, respectively. These equations can be specialized to the case of a conformally flat background, with warp factor $a(z)$ and a dilaton $\phi(z)$. Also, we shall look for $\mathbb{Z}_2$–symmetric solutions, describing a flat brane rigidly located at $z = 0$, and we set

$$
g_{AB} = a^2(z)\eta_{AB}, \qquad \phi = \phi(z), \qquad X^A = \delta^A_\mu \xi^\mu. \qquad (2)
$$

where η_{AB} is the five-dimensional Minkowski metric. The induced metric thus reduces to

$$
\gamma_{\alpha\beta} = \delta^A_\alpha \delta^B_\beta g_{AB}\ e^{\alpha_2 \phi}, \qquad (3)
$$

while the brane equations are identically satisfied thanks to the $\mathbb{Z}_2$ symmetry.

The dynamical equations are obtained from the dilaton equation, which becomes

$$
3\frac{a'}{a}\phi' + \phi'' - 2\alpha_1 \Lambda a^2 e^{\alpha_1 \phi} - 2\alpha_2 T_3 a e^{2\alpha_2 \phi} \delta(z) = 0, \qquad (4)
$$

and from the (α, β) and $(4, 4)$ components of the Einstein equations, which give, respectively,

$$
-3\frac{a''}{a} = \frac{\phi'^2}{4} + \Lambda a^2 e^{\alpha_1 \phi} + \frac{T_3}{2} a e^{2\alpha_2 \phi} \delta(z), \qquad (5)
$$

$$
-6\frac{a'^2}{a^2} = -\frac{\phi'^2}{4} + \Lambda a^2 e^{\alpha_1 \phi} \qquad (6)
$$

(a prime denotes differentiation with respect to z).

If we fine-tune the parameters by choosing

$$\alpha_1 = 4\alpha_2 \ , \quad T_3 = 8\sqrt{\Lambda/\Delta} \ , \quad \alpha_1^2 = \Delta + \frac{8}{3}, \tag{7}$$

where the last equation defines Δ, we recover a one-parameter family of exact domain wall solutions (hereafter, CLP solutions) [8]. The solution corresponds to a brane of positive tension, $T_3 > 0$, provided $\Delta \leq -2$. This range of Δ guarantees a positive tension and also avoids the presence of naked singularities [8]. On the other hand, when α_1 goes to zero, i.e. when $\Delta = -8/3$, the dilaton decouples and the background reduces to Randall-Sundrum one [3]. We shall thus assume$\frac{-8}{3} \leq \Delta \leq -2$.

3 Scalar perturbations

We perturb to first order the full set of bulk equations, keeping the position of the brane fixed, $\delta X^A = 0$. We thus set

$$\delta g_{AB} = h_{AB}, \qquad \delta g^{AB} = -h^{AB}, \qquad \delta\phi = \chi, \qquad \delta X^A = 0, \tag{8}$$

where the indices of the perturbed fields are raised and lowered by the unperturbed metric, and the background fluctuations h_{AB}, χ are assumed to be inhomogeneous.

We shall expand around the CLP background in the so-called "generalized longitudinal gauge" [4], which extends the longitudinal gauge of standard cosmology [9] to the brane-world scenario. As discussed in [4], in five dimensions there are four independent degrees of freedom for the scalar metric fluctuations: in the generalized longitudinal gauge they are described by the four variables $\{\varphi, \psi, \Gamma, W\}$, defined by

$$\begin{aligned} h_{00} &= 2\varphi a^2, \qquad & h_{ij} &= 2\psi a^2 \delta_{ij}, \\ h_{44} &= 2\Gamma a^2, \qquad & h_{04} &= -W a^2. \end{aligned} \tag{9}$$

The perturbation of the backgraound equations leads then to the full set of constraints and dynamical equations governing the linearized evolution of the five scalar variables $\{\varphi, \psi, \Gamma, W, \chi\}$.

In the absence of bulk sources with anisotropic stresses we can eliminate φ from Einstein the eq. $(i \neq j)$, thus reducing the system to four scalar degrees of freedom by setting:

$$\varphi = \psi + \Gamma. \tag{10}$$

As a consequence, we find that the variable W decouples from the other fluctuations:

$$\Box_5 W = 3\left(\frac{a''}{a} - \frac{a'^2}{a^2}\right) W \tag{11}$$

but, because of the non-trivial self-interactions, W does not freely propagate in the background geometry like the graviton, which statisfies the pure five-dimensional D'Alembert equation

$$\Box_5 h_{ij} = 0, \tag{12}$$

where $\Box_5 = \nabla_M \nabla^M$.

In order to discuss the dynamics of the remaining variables ψ, Γ and χ, it is now convenient to recombine their differential equations in an explicitly covariant way, to obtain canonical evolution equation. We find :

$$\Box_5 \psi = f_\psi(\Gamma, \chi), \qquad \Box_5 \Gamma = f_\Gamma(\Gamma, \chi), \qquad \Box_5 \chi = f_\chi(\Gamma, \chi), \tag{13}$$

where the source terms depend only on Γ and χ. By introducing the fields

$$\omega_1 = 2\psi + \Gamma, \qquad \omega_2 = 6\alpha_2 \Gamma + \chi, \qquad \omega_3 = \Gamma - 2\alpha_2 \chi, \tag{14}$$

the above system of coupled equations can be diagonalized, and the perturbation equations (13) reduce to

$$\Box_5 \omega_1 = 0, \qquad \Box_5 \omega_2 = 0, \qquad \Box_5 \omega_3 = f_\omega(\omega_3). \tag{15}$$

Together with eq. (11), and the constraints between these fields coming from the Einstein equations, such decoupled equations describe the complete evolution of the scalar (metric + dilaton) fluctuations in the CLP brane-world background. Two variables (ω_1, ω_2) are (covariantly) free on the background like the graviton, while the other two variables (ω_3, W) have non-trivial self-interactions.

In all cases, it is convenient to introduce the corresponding "canonical variables" $\hat{W}$, $\hat{\omega}_i$ $(i = 1, 2, 3)$, which have canonically normalized kinetic terms [9] in the action, simply by absorbing the geometric warp factor as follows:

$$W = \hat{W} a^{-3/2}, \qquad \omega_i = \hat{\omega}_i a^{-3/2}. \tag{16}$$

When the general solution is written as a superposition of free, factorized plane-waves modes on the brane,

$$\hat{W} = \Psi_w(z) e^{-ip_\mu x^\mu}, \qquad \hat{\omega}_i = \Psi_i(z) e^{-ip_\mu x^\mu} \tag{17}$$

they define the inner product of states with measure dz, as in conventional one-dimensional quantum mechanics. Such a product is required for an appropriate definition of normalizable solutions.

The allowed mass spectrum of $m^2 = \eta^{\mu\nu} p_\mu p_\nu$, for the scalar fluctuations on the brane, can then be obtained by solving an eigenvalue problem in the Hilbert space $L^2(R)$ for the canonical variables Ψ_w, Ψ_i, satisfying a Schrödinger-like equation in z, which is obtained from the equations (11), (15) for W and ω_i, and which can be written in the conventional form as:

$$\Psi_w'' + \left(m^2 - \frac{\xi_w''}{\xi_w} \right) \Psi_w = 0, \qquad \Psi_i'' + \left(m^2 - \frac{\xi_i''}{\xi_i} \right) \Psi_i = 0. \tag{18}$$

Here, by analogy with cosmological perturbation theory [9], we have introduced four "pump fields" ξ_w, ξ_i, defined as follows:

$$\begin{gathered} \xi_w = a^{\beta_w}, \qquad\qquad \xi_i = a^{\beta_i}, \\ \beta_w = -\tfrac{3}{2}, \quad \beta_1 = \beta_2 = \tfrac{3}{2}, \quad \beta_3 = -\tfrac{1}{2}(1+3\alpha_1^2) = -\tfrac{3}{2}(\Delta+3). \end{gathered} \tag{19}$$

The effective potential generated by the derivatives of the pump fields depends on β_w, β_i, and contains in general a smooth part, peaked at $z = 0$, plus a positive or negative δ-function contribution at the origin. We may have, in principle, not only volcano-like potentials, which correspond to the free covariant d'Alembert equation with $\beta = 3/2$ [8] (and which are known to localize gravity [3, 5]), but also potentials that are positive everywhere and admit no bound states.

4 Localization of the massless modes

The general solutions of the canonical perturbation equations (18) are labelled by the mass eigenvalue m, by their parity with respect to z-reflections, and by the parameters β_w, β_i, which depend on the type of perturbation. To describe a bound state, we shall restrict such solutions to those with a normalizable canonical variable with respect to the measure dz, namely to $\Psi(z) \in L^2(R)$. Among the acceptable solutions, we shall finally select those satisfying all the constraints coming from Einstein equations. The above set of conditions will determine the class of brane-world backgrounds allowing the four-dimensional localization of long-range scalar interactions.

Analyzing Eqs. (18) for $m = 0$, we find that W and ω_3 are not normalizable. The even solutions of the free d'Alembert equation are instead normalizable, so we have acceptable solutions for ω_1 and ω_2. However, because of the constraints, ω_1 is forced to vanish when $W = 0$, unless the fluctuations are static, $\dot\omega_1 = 0$. It follows that there are two independent massless modes localized on the brane: one, ω_2, is propagating and the other, ω_1, is static.

We may thus conclude that all CLP backgrounds with $-8/3 \le \Delta \le -2$ localize on the brane not only the massless spin-2 degrees of freedom [8], but also one propagating massless scalar degree of freedom (ω_2), corresponding to a long-range scalar interaction generated by the dilaton field. The second independent massless degree of freedom localized on the brane (ω_1) is not propagating ($\dot\omega_1 = 0$), but is essential to reproduce the standard long-range gravitational interaction in the static limit, as we shall discuss later. In the limiting case of a pure AdS$_5$ solution ($\Delta = -8/3$) the dilaton disappears from the background, and the dilaton fluctuation $\chi = \omega_2$ decouples from the others. The only (static) contribution to the scalar sector of metric fluctuations comes from ω_1, which generates the long-range Newton potential $\varphi = \psi$ on the brane.

5 The massive mode spectrum

The massive part of the spectrum of the canonical equations (18) is not localized on the brane; it may induce higher-dimensional, short-range corrections to the long-range scalar forces, which represent physical effects from the fifth dimension on our brane.

It is important to note that modes with negative squared mass (tachyons) are not included in the spectrum, as they would not correspond to a normalizable canonical variable (Ψ would blow up in z). Another consequence of the normalization condition is the mass gap between the localized massless mode and the massive corrections, in the limiting background with $\Delta = -2$ (already noticed in [8] for the case of pure tensor interactions).

Imposing all the constraints from Einstein equations, we find that, in contrast with the massless case, none of the four scalar fluctuations is forced to vanish. However, only two amplitudes are independent. By taking, for instance, ω_2 and ω_3 as independent variables, we can indeed express W and ω_1 in terms of the other fields, for all values of Δ.

For such backgrounds we thus have four types of higher-dimensional contributions to the scalar interactions on the brane, arising from the massive spectrum of ω_i and W. The massive ω_2 modes can be considered as the KK excitations of the effective dilaton zero–mode localized on the brane. The massive ω_3 are still present even if we turn off the dilaton and are therefore a product of the extra–dimension itself. In the limiting RS case, $\omega_3 \equiv \Gamma$ is the breathing mode of the fifth dimension, which induces short range corrections to the gravitational force.

6 Static limit and leading-order corrections

We can now compute the effective scalar-tensor interaction induced on the brane, in the weak field limit, by a static and point-like source of mass M and dilatonic charge Q.

In our longitudinal gauge (9), in which the decomposition of the metric fluctuations is based on the $O(3)$ symmetry of the spatial hypersurfaces of the brane, the energy density of a point-like particle only contributes to the scalar part of the perturbed matter stress tensor (with T_{00} as the only non-vanishing component), and provides a δ-function source to the $(0,0)$ scalar perturbation equation. Similarly, the charge Q acts as a point-like source in the dilaton perturbation equation.

As a consequence, we obtain three δ-function sources in the equations for the three ω_i fluctuations, $S_i\delta^3(x-x')\delta(z)$, with three scalar charges S_i, which are "mixtures" of M and Q, while no source term is obtained in the static limit for the W fluctuation.

The exact static solutions of eqs. (15) can be easily obtained using the static limit of the retarded Green function evaluated on the brane ($z=0$), i.e.

$$\omega_i(x,x') = -S_i G_i(\nu,x,x'), \tag{20}$$

where

$$G_i(\nu, \vec{x}, \vec{x}', z = z' = 0) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{x}-\vec{x}')} \left\{ \frac{\left[\psi_0^+(0)\right]^2}{p^2} + \int_{m_0}^{\infty} dm \frac{\left[\psi_m^+(0)\right]^2}{p^2+m^2} \right\}, \tag{21}$$

is the static Green function, constructed by the exact massless and massive eigenfunctions $\psi_0^+(z), \psi_m^+(z)$ of Eqs. (18) (respectively). The first term in the integrand corresponds to the long-range forces generated by the massless modes, the second term to the "short-range" corrections due to the massive modes, and m_0 is the lower bound for the massive spectrum ($m_0 = k/2$ if $\Delta = -2$, while $m_0 = 0$ if $\Delta \neq -2$).

We should note that in the ω_1, ω_2 case we have to include both the massless and massive contributions, while in the ω_3 case only the massive ones survive. Here we report the ω_i solutions in the case $\Delta < -2$:

$$\begin{aligned}
\omega_1 &= -\frac{S_1 A_{\nu_0}}{r}\left[1 + B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right], \\
\omega_2 &= -\frac{S_2 A_{\nu_0}}{r}\left[1 + B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right], \\
\omega_3 &= -\frac{S_3 A_{\nu_0}}{r} B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2},
\end{aligned} \tag{22}$$

where $\nu_0 = \frac{\Delta}{2(\Delta+2)}$ and the constants A_{ν_0} and B_{ν_0} depend, through ν_0, on the particular values of the dilaton coupling parameters, and are fixed by the correct normalization of the canonical "wave function" [7].

Introducing explicitly the four-dimensional gravitational constant G, and going back through the transformation (14), the scalar and dilaton fluctuations can finally be written in the form

$$\begin{aligned}
\varphi &= -\frac{GM}{r}\left[1 + \frac{2\alpha_2}{1+12\alpha_2^2}\left(\frac{Q}{M} + 2\alpha_2\right) + \frac{4}{3}B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right], \\
\psi &= -\frac{GM}{r}\left[1 - \frac{2\alpha_2}{1+12\alpha_2^2}\left(\frac{Q}{M} + 2\alpha_2\right) + \frac{2}{3}B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right], \\
\Gamma &= -\frac{GM}{r}\left[\frac{4\alpha_2}{1+12\alpha_2^2}\left(\frac{Q}{M} + 2\alpha_2\right) + \frac{2}{3}B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right], \\
\chi &= -\frac{GQ}{r}\left[\frac{2}{1+12\alpha_2^2}\left(1 + 2\alpha_2\frac{M}{Q}\right) + 2B_{\nu_0}\left(\frac{1}{kr}\right)^{2\nu_0-2}\right],
\end{aligned} \tag{23}$$

where G is related to the five-dimensional mass scale M_5 through the appropriate integral over the "warped" volume external to the brane. It should be noted that the short-range corrections induced by the massive scalar modes have the same qualitative behaviour as in the tensor case, discussed in [8], in spite of the fact that the massive scalar modes have different spectra.

The limiting case $\Delta = -8/3$ corresponds to a pure AdS$_5$ background, if there are no scalar charges on the brane. In that case ω_2 exactly corresponds to the dilaton fluctuation χ (see eq. (14)), and can be consistently set to zero (toghether with the dilaton background) if we want to match, in particular, the "standard" brane-world configuration originally considered by Randall and Sundrum [3]. In this limit, $B_2 = 1/2$, and we exactly recover previous results for the effective gravitational interaction on the brane [10], i.e.

$$\varphi = -\frac{GM}{r}\left(1+\frac{2}{3k^2r^2}\right), \qquad \psi = -\frac{GM}{r}\left(1+\frac{1}{3k^2r^2}\right). \tag{24}$$

The massless-mode truncation reproduces in this case the static, weak field limit of linearized general relativity. The massive tower of scalar fluctuations, however, induces deviations from Einstein gravity already in the static limit (as noted in [10]), and is the source of a short-range force due to the "breathing" of the fifth dimension,

$$\Gamma = -\frac{GM}{3r}\frac{1}{(kr)^2}, \tag{25}$$

even in the absence of bulk scalar fields, and of scalar charges for the matter on the brane.

In a more general gravi-dilaton background ($\Delta \neq -8/3$), the static expansion (23) describes an effective scalar-tensor interaction on the brane, which is potentially dangerous for the brane-world scenario, as it contains not only short-range corrections, but also long-range scalar deviations from general relativity (and, possibly, violations of the Einstein equivalence principle), even in the interaction of ordinary masses, i.e. for $Q = 0$. This seems to offer an interesting window to investigate the effects of the bulk geometry on the four-dimensional physics of the brane.

7 Conclusions

We have analyzed the full set of coupled equations governing the evolution of scalar fluctuations in a dilatonic brane-world background, supporting a flat 3-brane rigidly located at the fixed point of Z_2 symmetry. We have diagonalized the system of dynamical equations, and found four decoupled but self-interacting variables representing, in a five-dimensional bulk, the four independent degrees of freedom of scalar excitations of the gravi-dilaton background.

We have presented the exact solutions of the canonical perturbation equation for all the scalar degrees of freedom, and we have discussed, the effects of their massless and massive spectrum for the scalar interactions on the brane.

For all dilaton couplings, there is one propagating massless mode localized on the brane, associated with a long-range dilatonic interaction in four dimensions. In addition, KK modes in the bulk yield short–range scalar corrections to the Newton's law. These corrections are present even in the RS limit where the dilaton vanishes, because of the breathing mode of the fifth dimension.

Finally, we note that our results are different from those obtained in the case of thick branes with a confining scalar potential [11].

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