SLIGHTLY GENERALIZED MAXWELL CLASSICAL ELECTRODYNAMICS CAN BE APPLIED TO INNERATOMIC PHENOMENA

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Abstract

In order to extend the limits of classical theory application in the microworld some weak generalization of Maxwell electrodynamics is suggested. It is shown that weakly generalized classical Maxwell electrodynamics can describe the intraatomic phenomena with the same success as relativistic quantum mechanics can do. Group-theoretical grounds for the description of fermionic states by bosonic system are presented briefly. The advantages of generalized electrodynamics in intraatomic region in comparison with standard Maxwell electrodynamics are demonstrated on testing example of hydrogen atom. We are able to obtain some results which are impossible in the framework of standard Maxwell electrodynamics. The Sommerfeld - Dirac formula for the fine structure of the hydrogen atom spectrum is obtained on the basis of such Maxwell equations without appealing to the Dirac equation. The Bohr postulates and the Lamb shift are proved to be the consequences of the equations under consideration. The relationship of the new model with the Dirac theory is investigated. Possible directions of unification of such electrodynamics with gravity are mentioned.

1 Introduction

There is no doubt that the Maxwell classical electrodynamics of macroworld (without any generalization) is sufficient for the description of electrodynamical phenomena in macro region. On the other hand it is well known that for micro phenomena (inneratomic region) the classical Maxwell electrodynamics (as well as the classical mechanics) cannot work and must be replaced by quantum theory. Trying to extend the limits of classical electrodynamics application to the intraatomic region we came to the conclusion that it is possible by means of generalization of standard Maxwell classical electrodynamics in the direction of the extencion of its symmetry. We also use the relationships between the Dirac and Maxwell equations for these purposes. Furthermore, the relationships between relativistic quantum mechanics and classical microscopical electrodynamics of media are investigated. Such relationships are considered here not only from the mathematical point of view - they are used for construction of fundamentals of a non-quantum-mechanical model of microworld.

Our non-quantum-mechanical model of microworld is a model of atom on the basis of slightly generalized Maxwell's equations, i. e. in the framework of moderately extended classical microscopical electrodynamics of media. This model is free from probability interpretation and can explain many intraatomic phenomena by means of classical physics. Despite the fact that we construct the classical model, for the purposes of such construction we use essentially the analogy with the Dirac equation and the results which were achieved on the basis of this equation. Note also that electrodynamics is considered here in the terms of field strengths without appealing to the vector potentials as the primary (input) variables of the theory.

The first step in our consideration is the unitary relationship (and wide range analogy) between the Dirac equation and slightly generalized Maxwell equations [1].

Our second step is the symmetry principle. On the basis of this principle we introduced in [2] the most symmetrical form of generalized Maxwell equations which now can describe both bosons and fermions because they have (see [2]) both spin 1 and spin 1/2 symmetries. On the other hand, namely these equations are unitarily connected with the Dirac equation. So, we have one more important argument to suggest these equations in order to describe intraatomic phenomena, i. e. to be the equations of specific intraatomic classical electrodynamics.

In our third step we refer to Sallhofer, who suggested in [3] the possibility of introduction of interaction with external field as the interaction with specific media (a new way of introduction of the interaction into the field equations). Nevertheless, our model of atom (and of electron) [1] is essentially different from the Sallhofer's one.

On the basis of these three main ideas we are able to postulate the slightly generalized Maxwell equations as the equations for intraatomic classical electrodynamics which may work in atomic, nuclear and particle physics on the same level of success as the Dirac equation can do. Below we illustrate it considering hydrogen atom within the classical model.

The interest to the problem of relationship between the Dirac and Maxwell equations dates back to the time of creation of quantum mechanics [4]. But the authors of these papers during long time considered only the most simple example of free and massless Dirac equation. The interest to this relationship has grown in recent years due to the results [3], where the investigations of the case $\mathbf{m}_0 \neq 0$ and the interaction potential $\Phi \neq 0$ were started. Another approach was developed in [5], where the quadratic relations between the fermionic and bosonic amplitudes were found and used. In our above mentioned papers [1, 2], in publications [6] and herein we consider the linear relations between the fermionic and bosonic amplitudes. In [6] we have found the relationship between the symmetry properties of the Dirac and Maxwell equations, the complete set of 8 transformations linking these equations, the relationship between the conservation laws for the electromagnetic and spinor fields, the relationship between the Lagrangians for these fields. Here we summarize our previous results and give some new details of the intraatomic electrodynamics and its application to the hydrogen atom. The possibilities of unification with gravitation are briefly discussed.

2 New classical electrodynamical hydrogen atom model

Consider the slightly generalized Maxwell equations in a medium with specific form of sources:

$$\begin{array}{c} \mathrm{curl}\overrightarrow{H} - \partial_0 \epsilon \overrightarrow{E} = \overrightarrow{j}_e, \quad \mathrm{curl}\overrightarrow{E} + \partial_0 \mu \overrightarrow{H} = \overrightarrow{j}_{mag}, \\ \mathrm{div}\epsilon\overrightarrow{E} = \rho_e, \quad \mathrm{div}\mu\overrightarrow{H} = \rho_{mag}, \end{array} \tag{1}$$

where $\overrightarrow{E}$ and $\overrightarrow{H}$ are the electromagnetic field strengths, ϵ and μ are the electric and magnetic permeabilities of the medium being the same as in the electrodynamical hydrogen atom model of H. Sallhofer [3]:

$$\epsilon\left(\overrightarrow{x}\right) = 1 - \frac{\Phi\left(\overrightarrow{x}\right) + \mathbf{m}_0}{\omega}, \quad \mu\left(\overrightarrow{x}\right) = 1 - \frac{\Phi\left(\overrightarrow{x}\right) - \mathbf{m}_0}{\omega} \tag{2}$$

where $\Phi\left(\overrightarrow{x}\right) = -Ze^2/r$ (we use the units: $\hbar = c = 1$, transition to standard system is fulfilled by the substitution $\omega \longrightarrow \hbar\omega$, $\mathbf{m}_0 \longrightarrow \mathbf{m}_0 c^2$). The current and charge densities in equations (1) have the form

$$\overrightarrow{j_e} = \text{grad} E^0, \quad \overrightarrow{j}_{mag} = -\text{grad} H^0,$$
$$\rho_e = -\epsilon\mu\partial_0 E^0 + \overrightarrow{E}\,\text{grad}\epsilon, \quad \rho_{mag} = -\epsilon\mu\partial_0 H^0 + \overrightarrow{H}\,\text{grad}\mu, \tag{3}$$

where E^0, H^0 is the pair of functions (two real scalar fields) generating the densities of gradient-like sources.

One can easily see that equations (1) are not ordinary electrodynamical equations known from the Maxwell theory. These equations have the additional terms which can be considered as the magnetic current and charge densities - in one possible interpretation, or equations (1) can be considered as the equations for compound system of electromagnetic $(\overrightarrow{E}, \overrightarrow{H})$ and scalar E^0, H^0 fields in another possible interpretation.

The reasons of our slight generalization of the classical Maxwell electrodynamics are the following.

1. The standard Maxwell electrodynamics cannot work in intraatomic region and its equations are not mathematically equivalent to any of quantum mechanical equations for electron (Schrodinger equation, Dirac equation, etc...)

2. The existence of direct relationship between the equations (1) and the Dirac equation for the massive particle in external electromagnetic field in the stationary case can be applied. Namely these equations were shown in papers [1] to be unitary equivalent with such Dirac equation (see also Sec. 3 below).

3. Equations (1) can be derived from the principle of maximally possible symmetry - these equations have both spin 1 and spin 1/2 Poincaré symmetries and in the limit of vanishing of the interaction with medium, where $\epsilon = \mu = 1$, they represent [2] the maximally symmetrical form of the Maxwell equations. This fact means first of all that from the group-theoretical point of view of Wigner, Bargmann - Wigner (and of modern field theory in general) Eqs. (1) can describe both bosons and fermions (for more details see Sec. 4. below). As a consequence of this fact one can use these equations particularly for the description of the electron. On the other hand, this fact means that intraatomic classical electrodynamics of electron needs further (relatively to that having been done by Maxwell) symmetrization of Weber - Faraday equations of classical electromagnetic theory which leads to the maximally symmetrical form (1). Below we demonstrate the possibilities of the equations (1) in the description of testing example of hydrogen atom.

Contrary to [1], here the equations (1) are solved directly by means of separation of variables method. It is useful to rewrite these equations in the mathematically equivalent form where the sources are maximally simple:

$$\text{curl}\overrightarrow{H} - \epsilon\partial_0\overrightarrow{E} = \overrightarrow{j}_e, \quad \text{curl}\overrightarrow{E} + \mu\partial_0\overrightarrow{H} = \overrightarrow{j}_{mag},$$
$$\text{div}\overrightarrow{E} = \tilde{\rho_e}, \quad \text{div}\overrightarrow{H} = \tilde{\rho_{mag}}, \tag{4}$$

where

$$\overrightarrow{j_e} = \text{grad}E^0, \quad \overrightarrow{j}_{mag} = -\text{grad}H^0, \quad \tilde{\rho_e} = -\mu\partial_0 E^0, \quad \tilde{\rho_{mag}} = -\epsilon\partial_0 H^0. \tag{5}$$

Consider the stationary solutions of equations (4). Assuming the harmonic time dependence for the functions E^0, H^0

$$E^0(t, \overrightarrow{x}) = E^0_A(\overrightarrow{x})\cos\omega t + E^0_B(\overrightarrow{x})\sin\omega t,$$
$$H^0(t, \overrightarrow{x}) = H^0_A(\overrightarrow{x})\cos\omega t + H^0_B(\overrightarrow{x})\sin\omega t, \tag{6}$$

we are looking for the solutions of equations (4) in the form

$$\begin{aligned}\overrightarrow{E}(t,\overrightarrow{x}) &= \overrightarrow{E}_A(\overrightarrow{x})\cos\omega t + \overrightarrow{E}_B(\overrightarrow{x})\sin\omega t,\\ \overrightarrow{H}(t,\overrightarrow{x}) &= \overrightarrow{H}_A(\overrightarrow{x})\cos\omega t + \overrightarrow{H}_B(\overrightarrow{x})\sin\omega t.\end{aligned} \tag{7}$$

For the 16 time-independent amplitudes we obtain the following two nonlinked subsystems

$$\begin{gathered}\mathrm{curl}\overrightarrow{H_A} - \omega\epsilon\overrightarrow{E_B} = \mathrm{grad}E_A^0, \quad \mathrm{curl}\overrightarrow{E_B} - \omega\mu\overrightarrow{H_A} = -\mathrm{grad}H_B^0,\\ \mathrm{div}\overrightarrow{E_B} = \omega\mu E_A^0, \quad \mathrm{div}\overrightarrow{H}_A = -\omega\epsilon H_B^0,\end{gathered} \tag{8}$$

$$\begin{gathered}\mathrm{curl}\overrightarrow{H_B} + \omega\epsilon\overrightarrow{E_A} = \mathrm{grad}E_B^0, \quad \mathrm{curl}\overrightarrow{E_A} + \omega\mu\overrightarrow{H_B} = -\mathrm{grad}H_A^0,\\ \mathrm{div}\overrightarrow{E_A} = -\omega\mu E_B^0, \quad \mathrm{div}\overrightarrow{H}_B = \omega\epsilon H_A^0.\end{gathered} \tag{9}$$

Below we consider only the first subsystem (8). It is quite enough because the subsystems (8) and (9) are connected with transformations

$$\begin{gathered}E \longrightarrow H, \quad H \longrightarrow -E, \quad \epsilon E \longrightarrow \mu H, \quad \mu H \longrightarrow -\epsilon E,\\ \epsilon \longrightarrow \mu, \quad \mu \longrightarrow \epsilon,\end{gathered} \tag{10}$$

which are the generalizations of duality transformation of free electromagnetic field. Due to this fact the solutions of subsystem (9) can be easily obtained from the solutions of subsystem (8).

Furthermore, it is useful to separate equations (8) into the following subsystems:

$$\begin{gathered}\omega\epsilon E_B^3 - \partial_1 H_A^2 + \partial_2 H_A^1 + \partial_3 E_A^0 = 0,\\ \omega\epsilon H_B^0 + \partial_1 H_A^1 + \partial_2 H_A^2 + \partial_3 H_A^3 = 0,\\ -\omega\mu E_A^0 + \partial_1 E_B^1 + \partial_2 E_B^2 + \partial_3 E_B^3 = 0,\\ \omega\mu H_A^3 - \partial_1 E_B^2 + \partial_2 E_B^1 - \partial_3 H_B^0 = 0,\end{gathered} \tag{11}$$

and

$$\begin{gathered}\omega\epsilon E_B^1 - \partial_2 H_A^3 + \partial_3 H_A^2 + \partial_1 E_A^0 = 0,\\ \omega\epsilon E_B^2 - \partial_3 H_A^1 + \partial_1 H_A^3 + \partial_2 E_A^0 = 0,\\ \omega\mu H_A^1 - \partial_2 E_B^3 + \partial_3 E_B^2 - \partial_1 H_B^0 = 0,\\ \omega\mu H_A^2 - \partial_3 E_B^1 + \partial_1 E_B^3 - \partial_2 H_B^0 = 0.\end{gathered} \tag{12}$$

Assuming the spherical symmetry case, when $\Phi(\overrightarrow{x}) = \Phi(r)$, $r \equiv |\overrightarrow{x}|$, we are making the transition into the spherical coordinate system and looking for the solutions in the spherical coordinates in the form

$$(E,H)\left(\overrightarrow{r}\right) = R_{(E,H)}\left(r\right) f_{(E,H)}\left(\theta,\phi\right), \tag{13}$$

where $E \equiv \left(E^0, \overrightarrow{E}\right)$, $H \equiv \left(H^0, \overrightarrow{H}\right)$. We choose for the subsystem (11) the d'Alembert Ansatz in the form

$$\begin{gathered}\bar{E}_A^0 = \bar{C}_{E_4}\ R_{H_4} P_{l_{H_4}}^{\bar{m}_4} e^{-i\bar{m}_4\phi},\\ \bar{E}_B^k = \bar{C}_{E_k}\ R_{E_k} P_{l_{E_k}}^{\bar{m}_k} e^{-i\bar{m}_k\phi},\\ \bar{H}_B^0 = \bar{C}_{H_4}\ R_{E_4} P_{l_{E_4}}^{\bar{m}_4} e^{-i\bar{m}_4\phi}, \qquad k = 1,2,3.\\ \bar{H}_A^k = \bar{C}_{H_k}\ R_{H_k} P_{l_{H_k}}^{\bar{m}_k} e^{-i\bar{m}_k\phi},\end{gathered} \tag{14}$$

We use the following representation for $\partial_1, \partial_2, \partial_3$ operators in spherical coordinates

$$\begin{array}{c}
\partial_1 CRP_l^m e^{\mp im\phi} = \frac{e^{\mp im\phi}C}{2l+1}\cos\phi\left(R_{,l+1}P_{l-1}^{m+1} - R_{,-l}P_{l+1}^{m+1}\right) + e^{\mp i(m-1)\phi}C\frac{m}{\sin\theta}P_l^m\frac{R}{r}, \\
\partial_2 CRP_l^m e^{\mp im\phi} = \frac{e^{\mp im\phi}C}{2l+1}\sin\phi\left(R_{,l+1}P_{l-1}^{m+1} - R_{,-l}P_{l+1}^{m+1}\right) \mp e^{\mp i(m-1)\phi}C\frac{im}{\sin\theta}P_l^m\frac{R}{r}, \\
\partial_3 CRP_l^m e^{\mp im\phi} = \frac{e^{\mp im\phi}C}{2l+1}\left(R_{,l+1}(l+m)P_{l-1}^m + R_{,-l}(l-m+1)P_{l+1}^m\right).
\end{array} \tag{15}$$

Substitutions (14) and (15) together with the assumptions

$$\begin{array}{c}
R_{E_\alpha} = R_E, \quad l_{E_\alpha} = l_E, \quad R_{H_\alpha} = R_H, \quad l_{H_\alpha} = l_H, \\
\bar{m}_1 = \bar{m}_2 = \bar{m}_3 - 1 = \bar{m}_4 - 1 = m, \\
\bar{C}_{H_1} = i\,\bar{C}_{H_2}, \quad \bar{C}_{E_2} = -i\,\bar{C}_{E_1}, \quad \bar{C}_{H_4} = -i\,\bar{C}_{E_3}, \quad \bar{C}_{H_3} = -i\,\bar{C}_{E_4}, \\
\bar{C}^I_{H_2} = \bar{C}^I_{E_4}\,(l^I_H + m + 1), \quad \bar{C}^I_{E_3} = -\,\bar{C}^I_{E_4} \equiv \bar{C}^I, \\
\bar{C}^I_{E_1} = \bar{C}^I_{E_3}\,(l^I_E - m), \quad l^I_H = l^I_E - 1 \equiv l^I, \\
\bar{C}^{II}_{H_2} = -\,\bar{C}^{II}_{E_4}\,(l^{II}_H - m), \quad \bar{C}^{II}_{E_3} = -\,\bar{C}^{II}_{E_4} \equiv \bar{C}^{II}, \\
\bar{C}^{II}_{E_1} = -\bar{C}^{II}_{E_3}\,(l^{II}_E + m + 1), \quad l^{II}_H = l^{II}_E + 1 \equiv l^{II}
\end{array} \tag{16}$$

into the subsystem (11) guarantee the separation of variables in these equations and lead to the pair of equations for two radial functions R_E, R_H (for the subsystem (12) the procedure is similar):

$$\epsilon\omega R^I_E - R^I_{H,-l} = 0, \quad \mu\omega R^I_H + R^I_{E,l+2} = 0, \tag{17}$$

$$\epsilon\omega R^{II}_E - R^{II}_{H,l+1} = 0, \quad \mu\omega R^{II}_H + R^{II}_{E,-l+1} = 0; \qquad R_{,a} \equiv \left(\frac{d}{dr} + \frac{a}{r}\right)R. \tag{18}$$

For the case $\Phi = -ze^2/r$ the equations (17), (18) coincide exactly with the radial equations for the hydrogen atom of the Dirac theory and, therefore, the procedure of their solution is the same as in well-known monographs on relativistic quantum mechanics. It leads to the well-known Sommerfeld - Dirac formula for the fine structure of the hydrogen spectrum. We note only that here the discrete picture of energetic spectrum in the domain $0 < \omega < \mathbf{m}_0 c^2$ is guaranteed by the demand for the solutions of the radial equations (17), (18) to decrease on infinity (when $r \to \infty$). From the equations (17), (18) and this condition the Sommerfeld - Dirac formula

$$\omega = \omega^{hyd}_{nj} = \frac{\mathbf{m}_0 c^2}{\hbar\sqrt{1 + \frac{\alpha^2}{\left(n_r + \sqrt{k^2 - \alpha^2}\right)^2}}} \tag{19}$$

follows, where the notations of the Dirac theory (see, e. g., [7]) are used: $n_r = n - k$, $k = j + 1/2$, $\alpha = e^2/\hbar c$. Let us note once more that the result (19) is obtained here not from the Dirac equation, but from the Maxwell equations (1) with sources (3) in the medium (2).

Substituting (16) into (14) one can easy obtain the angular part of the hydrogen solutions for the $(\vec{E}, \vec{H}, E^0, H^0)$ field and calculate according to (3) the corresponding currents and charges. Let us write down the explicit form for the set of electromagnetic field strengths $(\vec{E}, \vec{H})$, which are the hydrogen solutions of equations (1), and also for the currents and charges generating these field strengths (the complete set of solutions is represented in [1]:

$$\begin{gathered}\overrightarrow{E^I} = R_E^I \begin{vmatrix} (-l+m-1)\, P_{l+1}^m \cos m\phi \\ (l-m+1)\, P_{l+1}^m \sin m\phi \\ -P_{l+1}^{m+1} \cos{(m+1)}\,\phi \end{vmatrix}, \quad \overrightarrow{H^I} = R_H^I \begin{vmatrix} (l+m+1)\, P_l^m \sin m\phi \\ (l+m+1)\, P_l^m \cos m\phi \\ -P_l^{m+1} \sin{(m+1)}\,\phi \end{vmatrix}, \\ \overrightarrow{j_e^I} = gradR_H^I P_l^{m+1} \cos{(m+1)}\,\phi, \quad \overrightarrow{j_{mag}^I} = -gradR_e^I P_{l+1}^{m+1} \sin{(m+1)}\,\phi, \\ \rho_e^I = -\left(\varepsilon R_E^I\right)_{,l+2} P_l^{m+1} \cos{(m+1)}\,\phi, \quad \rho_{mag}^I = -\left(\mu R_H^I\right)_{,-l} P_{l+1}^{m+1} \sin{(m+1)}\,\phi, \end{gathered} \tag{20}$$

$$\begin{gathered}\overrightarrow{E^{II}} = R_E^{II} \begin{vmatrix} (l+m)\, P_{l-1}^m \cos m\phi \\ (-l-m)\, P_{l-1}^m \sin m\phi \\ P_{l-1}^{m+1} \cos(m+1)\phi \end{vmatrix}, \quad \overrightarrow{H^{II}} = R_H^{II} \begin{vmatrix} (-l+m)\, P_l^m \sin m\phi \\ (-l+m)\, P_l^m \cos m\phi \\ -P_l^{m+1} \sin{(m+1)}\,\phi \end{vmatrix} \\ \overrightarrow{j_e^{II}} = gradR_H^{II} P_l^{m+1} \cos{(m+1)}\,\phi, \quad \overrightarrow{j_{mag}^{II}} = -gradR_E^{II} P_{l-1}^{m+1} \sin{(m+1)}\,\phi, \\ \rho_e^{II} = -\left(\varepsilon R_E^{II}\right)_{,-l+1} P_l^{m+1} \cos{(m+1)}\,\phi, \quad \rho_{mag}^{II} = -\left(\mu R_H^{II}\right)_{,l+1} P_{l-1}^{m+1} \sin{(m+1)}\,\phi. \end{gathered} \tag{21}$$

In one of the possible interpretations the states of the hydrogen atom are described by these field strength functions $\vec{E}, \vec{H}$ generated by the corresponding currents and charge densities.

It is evident from (1) that currents and charges in (20), (21) are generated by scalar fields (E^0, H^0). Corresponding to (20), (21) (E^0, H^0) solutions of equations (1) are the following:

$$\begin{gathered} E^{I0} = R_H^I P_l^{m+1} \cos{(m+1)}\,\phi, \quad H^{I0} = R_E^I P_{l+1}^{m+1} \sin{(m+1)}\,\phi, \\ E^{II0} = R_H^{II} P_l^{m+1} \cos{(m+1)}\,\phi, \quad H^{II0} = R_E^{II} P_{l-1}^{m+1} \sin{(m+1)}\,\phi. \end{gathered} \tag{22}$$

As in quantum theory, the numbers $n = 0, 1, 2, ...;$ $j = k - \frac{1}{2} = l \mp \frac{1}{2}$ $(k = 1, 2, ..., n)$ and $m = -l, -l+1, ..., l$ mark both the terms (19) and the corresponding exponentially decreasing field functions $\vec{E}, \vec{H}$ (and E^0, H^0) in (20)-(22), i. e. they mark the different discrete states of the classical electrodynamical field (and the densities of the currents and charges) which by definitions describes the corresponding states of hydrogen atom in the model under consideration.

Note that the radial equations (17), (18) cannot be obtained if one neglects the sources in equations (1), or one (electric or magnetic) of these sources. Moreover, in this case there is no solution effectively concentrated in atomic region.

Now we can show on the basis of this model that the assertions known as *Bohr's postulates are the consequences of equations (1) and of their classical interpretation*, i. e. these assertions can be derived from the model, there is no necessity to postulate them from beyond the framework of classical physics as it was in Bohr's theory. To derive the first Bohr's postulate one can calculate the generalized Pointing vector for the hydrogen solutions (20)-(22), i. e. for the compound system of stationary electromagnetic and scalar fields $(\vec{E}, \vec{H}, E^0, H^0)$

$$\vec{P}_{gen} = \int d^3x (\vec{E} \times \vec{H} - \vec{E} E^0 - \vec{H} H^0). \tag{23}$$

The straightforward calculations show that not only vector (23) is identically equal to zero but the Pointing vector itself and the term with scalar fields (E^0, H^0) are also identically equal to zero:

$$\vec{P} = \int d^3x (\vec{E} \times \vec{H}) \equiv 0, \quad \int d^3x (\vec{E} E^0 + \vec{H} H^0) \equiv 0. \tag{24}$$

This means that in stationary states hydrogen atom does not emit any Pointing radiation neither due to the electromagnetic $(\overrightarrow{E}, \overrightarrow{H})$ field, nor to the scalar (E^0, H^0) field. That is the mathematical proof of the first Bohr postulate.

The similar calculations of the energy for the same system (in formulae (23)-(25) the functions $(\overrightarrow{E}, \overrightarrow{H}, E^0, H^0)$ are taken in appropriate physical dimension which is given by the formula (49) below)

$$P^0 = \frac{1}{2}\int d^3x \mathcal{E}^\dagger \mathcal{E} = \frac{1}{2}\int d^3x(\overrightarrow{E}^2 + \overrightarrow{H}^2 + E_0^2 + H_0^2) = \omega_{nj}^{hyd} \tag{25}$$

give a constant $\mathbf{W}_{nl}$, depending on n, l (or n, j) and independent of m. In our model this constant is to be identified with the parameter ω in equations (1) which in the stationary states of $(\overrightarrow{E}, \overrightarrow{H}, E^0, H^0)$ field appears to be equal to the Sommerfeld - Dirac value ω_{nj}^{hyd} (19). By abandoning the $\hbar = c = 1$ system and putting arbitrary $"A"$ in equations (1) instead of $\hbar$ we obtain final ω_{nj}^{hyd} with $"A"$ instead of $\hbar$. Then the numerical value of $\hbar$can be obtained by comparison of ω_{nj}^{hyd} containing $"A"$ with the experiment. These facts complete the proof of the second Bohr postulate.

This result means that in this model the Bohr postulates are no longer postulates, but the direct consequences of the classical electrodynamical equation (1). Moreover, this means that together with Dirac or Schrodinger equations we have now the new equation which can be used for finding the solutions of atomic spectroscopy problems. In contradiction to the well-known equations of quantum mechanics our equation is the classical one.

Being aware that few interpretations of quantum mechanics (e.g.: Copenhagen, statistical, Feynman's, Everett's, transactional, see e. g. [8]) exist, we are far from thinking that here the interpretation can be the only one. But the main point is that now the classical interpretation (without probabilities) is possible.

Today we prefer the following interpretation of hydrogen atom in the approach, when one considers only the motion of electron in the external field of the nucleon. In our model the interacting field of the nucleon and electron is represented by the medium with permeabilities ϵ, μ given by formulae (2). The atomic electron is interpreted as the stationary electromagnetic-scalar wave $(\overrightarrow{E}, \overrightarrow{H}, E^0, H^0)$ in medium (2), i.e. as the stationary electromagnetic wave interacting with massless scalar fields (E^0, H^0), or with complex massless scalar field $\mathcal{E}^0 = E^0 - iH^0$ with spin $s = 0$. In other words, the electron can be interpreted as an object having the structure consisting of a photon and a massless meson with zero spin connected, probably, with leptonic charge. The role of the massless scalar field is the following: it generates the densities of electric and magnetic currents and charges $(\rho, \overrightarrow{j})$, which are the secondary objects in such model. The mass is the secondary parameter too. There is no electron as an input charged massive corpuscle in this model! The mass and the charge of electron appear only outside such atom according to the law of electromagnetic induction and its gravitational analogy. That is why no difficulties of Rutherford - Bohr's model (about different models of atom see, e. g., [9]) of atom are present here! The Bohr postulates are shown to be the consequences of the model. This interpretation is based on the hypothesis of bosonic nature of matter (on the speculation of the bosonic structure of fermions) according to which all the fermions can be constructed from different bosons (something like new SUSY theory). Of course, before the experiment intended to observe the structure of electron and before the registration of massless spinless meson it is only the hypothesis but based on the mathematics presented here. We note that such massless spinless boson has many similar features with the Higgs boson and the transition here from intraatomic (with high symmetry properties) to macroelectrodynamics (with loss of many symmetries) looks similarly to the symmetry breakdown mechanism.

The successors of magnetic monopole can try to develop here the monopole interpretation (see [10] for the review and some new ideas about monopole) - we note that there are few interesting possibilities of interpretation but we want to mark first of all the mathematical facts which are more important than different ways of interpretation.

3 The unitary relationship between the relativistic quantum mechanics and classical electrodynamics in medium

Let us consider the connection between the stationary Maxwell equations

$$\begin{array}{c}\mathrm{curl}\overrightarrow{H}-\omega\epsilon\overrightarrow{E}= \ \mathrm{grad}E^0, \quad \mathrm{curl}\overrightarrow{E}-\omega\mu\overrightarrow{H}=-\mathrm{grad}H^0,\\ \mathrm{div}\overrightarrow{E}=\omega\mu E^0, \quad \mathrm{div}\overrightarrow{H}=-\omega\epsilon H^0,\end{array} \tag{26}$$

which follow from the system (8) after ommitting indices A, B, and the stationary Dirac equation following from the ordinary Dirac equation

$$\left(i\gamma^\mu\partial_\mu-\mathbf{m}_0+\gamma^0\Phi\right)\Psi=0, \quad \Psi\equiv(\Psi^\alpha), \tag{27}$$

with $m\neq 0$ and the interaction potential $\Phi\neq 0$. Assuming the ordinary time dependence

$$\Psi(x)=\Psi(\overrightarrow{x})e^{-i\omega t}\Longrightarrow\partial_0\Psi(x)=-i\omega\Psi(x), \tag{28}$$

for the stationary states and using the standard Pauli - Dirac representation for the γ matrices, one obtains the following system of equations for the components Ψ^α of the spinor Ψ:

$$\begin{array}{c}-i\omega\epsilon\Psi^1+(\partial_1-i\partial_2)\Psi^4+\partial_3\Psi^3=0,\\ -i\omega\epsilon\Psi^2+(\partial_1+i\partial_2)\Psi^3-\partial_3\Psi^4=0,\\ -i\omega\mu\Psi^3+(\partial_1-i\partial_2)\Psi^2+\partial_3\Psi^1=0,\\ -i\omega\mu\Psi^4+(\partial_1+i\partial_2)\Psi^1-\partial_3\Psi^2=0,\end{array} \tag{29}$$

where ϵ and μ are the same as in (2). After substitution in Eqs. (29) instead of Ψ the following column

$$\Psi=\mathrm{column}\left|-H^0+iE^3,-E^2+iE^1,E^0+iH^3,-H^2+iH^1\right|. \tag{30}$$

one obtains Eqs. (26). A complete set of 8 such transformations can be obtained with the help of the Pauli - Gursey symmetry operators [11] similarly to [6].

It is useful to represent the right-hand side of (30) in terms of components of the following complex function

$$\mathcal{E}\equiv\left|\begin{array}{c}\overrightarrow{\mathcal{E}}\\ \mathcal{E}^0\end{array}\right|=\mathrm{column}\left|E^1-iH^1,E^2-iH^2,E^3-iH^3,E^0-iH^0\right|, \tag{31}$$

where $\overrightarrow{\mathcal{E}}=\overrightarrow{E}-i\overrightarrow{H}$ is the well-known form for the electromagnetic field used by Majorana as far back as near 1930 (see, e.g., [4]), and $\mathcal{E}^0=E^0-iH^0$ is a complex scalar field. In these terms the connection between the spinor and electromagnetic (together with the scalar) fields has the form

$$\mathcal{E}=W\Psi, \quad \Psi=W^\dagger\mathcal{E}, \tag{32}$$

where the unitary operator W is the following:

$$W = \left| \begin{array}{cccc} 0 & iC_- & 0 & C_- \\ 0 & -C_+ & 0 & iC_+ \\ iC_- & 0 & C_- & 0 \\ iC_+ & 0 & C_+ & 0 \end{array} \right|; \quad C_\mp \equiv \frac{1}{2}(C \mp 1), \quad C\Psi \equiv \Psi^*, \quad C\mathcal{E} \equiv \mathcal{E}^*. \tag{33}$$

The unitarity of the operator (33) can be verified easily by taking into account that the equations

$$(AC)^\dagger = CA^\dagger, \quad aC = Ca^*, \quad (aC)^* = Ca \tag{34}$$

hold for an arbitrary matrix A and a complex number a. We note that in the real algebra (i. e. the algebra over the field of real numbers) and in the Hilbert space of quantum mechanical amplitudes this operator has all properties of unitarity: $WW^{-1} = W^{-1}W = 1$, $W^{-1} = W^\dagger$, plus linearity.

The operator (33) transforms the stationary Dirac equation

$$\left[(\omega - \Phi)\,\gamma^0 + i\gamma^k\partial_k - \mathbf{m}_0\right]\Psi\left(\overrightarrow{x}\right) = 0 \tag{35}$$

from the standard representation (the Pauli - Dirac representation) into the bosonic representation

$$\left[(\omega - \Phi)\,\tilde{\gamma}^0 + \tilde{i}\tilde{\gamma}^k\partial_k - \mathbf{m}_0\right]\mathcal{E}\left(\overrightarrow{x}\right) = 0. \tag{36}$$

Here the $\tilde{\gamma}^\mu$ matrices have the following unusual explicit form

$$\begin{array}{c} \tilde{\gamma}^0 = \left| \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right| C, \quad \tilde{\gamma}^1 = \left| \begin{array}{cccc} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -1 \\ i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right|, \\ \tilde{\gamma}^2 = \left| \begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -1 & 0 & 0 & 0 \end{array} \right|, \quad \tilde{\gamma}^3 = \left| \begin{array}{cccc} -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{array} \right| \end{array} \tag{37}$$

in which $\tilde{\gamma}^0$ matrix explicitly contains operator C of complex conjugation. We call the representation (37) the bosonic representation of the γ matrices. In this representation the imaginary unit i is represented by the 4×4 matrix operator:

$$\tilde{i} = \left| \begin{array}{cccc} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{array} \right|. \tag{38}$$

Due to the unitarity of the operator (33) the $\tilde{\gamma}^\mu$ matrices still obey the Clifford-Dirac algebra

$$\tilde{\gamma}^\mu\tilde{\gamma}^\nu + \tilde{\gamma}^\nu\tilde{\gamma}^\mu = 2g^{\mu\nu} \tag{39}$$

and have the same Hermitian properties as the Pauli - Dirac γ^μ matrices:

$$\tilde{\gamma}^{0\dagger} = \tilde{\gamma}^0, \quad \tilde{\gamma}^{k\dagger} = -\tilde{\gamma}^k. \tag{40}$$

Thus, the formulae (37) give indeed an exotic representation of γ^μ matrices.

In the vector-scalar form the equation (36) is as follows

$$-i\,\mathrm{curl}\,\overrightarrow{\mathcal{E}} + \left[(\omega - \Phi)\,C - \mathbf{m}_0\right]\overrightarrow{\mathcal{E}} = -\mathrm{grad}\mathcal{E}^0, \quad \mathrm{div}\,\overrightarrow{\mathcal{E}} = \left[(\omega - \Phi)\,C + \mathbf{m}_0\right]\mathcal{E}^0. \tag{41}$$

Fulfilling the transition to the common real field strengths according to the formula $\mathcal{E} = E - iH$ and separating the real and imaginary parts we obtain equations (26) which are mathematically equivalent to the equations (1) in stationary case.

We emphasize that the only difference between the equation (36) in the case of description of fermions and in the case of bosons is the possibility of choosing γ^μ matrices: for the case of fermions these matrices may be chosen in arbitrary form (in each of representations of Pauli - Dirac, Majorana, Weyl, ...), in the case of the description of bosons the representation of γ^μ matrices and their explicit form *must be fixed* in the form (37). In the case of bosonic interpretation of Eq. (35) one must fixes the explicit form of γ^μ matrices and of Ψ (30).

The mathematical facts considered here prove the one-to-one correspondence between the solutions of the stationary Dirac and the stationary Maxwell equations with 4-currents of gradient-like type. Hence, one can, using (30), write down the hydrogen solutions of the Maxwell equations (1) (or (4)) starting from the well-known hydrogen solutions of the Dirac equation (27), i. e. without special procedure of finding the solutions of the Maxwell equations, see [1].

4 Some group-theoretical grounds of the model

Consider briefly the case of absence of interaction of the compound field $(\overrightarrow{E}, \overrightarrow{H}, E^0, H^0)$ with media, i. e. the case $\epsilon = \mu = 1$, and the symmetry properties of the corresponding equations. In this case equations (1) for the system of electromagnetic and scalar fields $(\overrightarrow{E}, \overrightarrow{H}, E^0, H^0)$ have the form:

$$\begin{gathered} \partial_0 \overrightarrow{E} = \mathrm{curl}\overrightarrow{H} - gradE^0, \quad \partial_0 \overrightarrow{H} = -\mathrm{curl}\overrightarrow{E} - gradH^0, \\ \mathrm{div}\overrightarrow{E} = -\partial_0 E^0, \quad \mathrm{div}\overrightarrow{H} = -\partial_0 H^0. \end{gathered} \tag{42}$$

The Eqs. (42) are nothing more than the weakly generalized Maxwell equations ($\epsilon = \mu = 1$) with gradient-like electric and magnetic sources $j^e_\mu = -\partial_\mu E^0$, $j^{mag}_\mu = -\partial_\mu H^0$, i. e.

$$\overrightarrow{j}_e = -\mathrm{grad}E^0, \quad \overrightarrow{j}_{mag} = -\mathrm{grad}H^0, \quad \rho_e = -\partial_0 E^0, \quad \rho_{mag} = -\partial_0 H^0. \tag{43}$$

In terms of complex 4-component object $\mathcal{E} = E - iH$ from formula (31) (and in terms of following complex tensor

$$\mathsf{E} = (\mathsf{E}^{\mu\nu}) \equiv \left|\begin{array}{cccc} 0 & \mathcal{E}^1 & \mathcal{E}^2 & \mathcal{E}^3 \\ -\mathcal{E}^1 & 0 & i\mathcal{E}^3 & -i\mathcal{E}^2 \\ -\mathcal{E}^2 & -i\mathcal{E}^3 & 0 & i\mathcal{E}^1 \\ -\mathcal{E}^3 & i\mathcal{E}^2 & -i\mathcal{E}^1 & 0 \end{array}\right|) \tag{44}$$

Eqs. (42) can be rewritten in the manifestly covariant forms

$$\partial_\mu \mathcal{E}_\nu - \partial_\nu \mathcal{E}_\mu + i\varepsilon_{\mu\nu\rho\sigma}\partial^\rho \mathcal{E}^\sigma = 0, \quad \partial_\mu \mathcal{E}^\mu = 0 \tag{45}$$

(vector form) and

$$\partial_\nu \mathsf{E}^{\mu\nu} = \partial^\mu \mathcal{E}^0 \tag{46}$$

- tensor-scalar form. It is useful also to consider the following form of Eqs. (42)= (45)=(46):

$$(i\partial_0 - \overrightarrow{S} \cdot \overrightarrow{p})\overrightarrow{\mathcal{E}} - i\mathrm{grad}\mathcal{E}^0 = 0, \quad \partial_\mu \mathcal{E}^\mu = 0, \tag{47}$$

where $\overrightarrow{S} \equiv (S^j)$ are the generators of irreducible representation $D(1)$ of the group $SU(2)$:

$$S^1 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{vmatrix}, \quad S^2 = \begin{vmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{vmatrix}, \quad S^3 = \begin{vmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad \overrightarrow{S}^2 = 1(1+1)I. \tag{48}$$

The general solution of Eqs. (42)= (45)=(46)=(47) was found in the last references within [6], their symmetry properties were considered in [2]. This solution was found in the manifold $(S(R^4) \otimes C^4)^*$ of Schwartz's generalized functions directly by application of Fourier method. In terms of helicity amplitudes $c^\mu(\overrightarrow{k})$ this solution has the form

$$\mathcal{E}(x) = \int \mathrm{d}^3k \sqrt{\frac{2\omega}{(2\pi)^3}} \left\{ \begin{array}{c} \left[c^1 e_1 + c^3 (e_3 + e_4)\right] \mathrm{e}^{-ikx} + \\ \left[c^{*2} e_1 + c^{*4} (e_3 + e_4)\right] \mathrm{e}^{ikx} \end{array} \right\}, \quad \omega \equiv \sqrt{\overrightarrow{k}^2}, \tag{49}$$

where 4-component basis vectors e_α are taken in the form

$$e_1 = \begin{vmatrix} \overrightarrow{e_1} \\ 0 \end{vmatrix}, \quad e_2 = \begin{vmatrix} \overrightarrow{e_2} \\ 0 \end{vmatrix}, \quad e_3 = \begin{vmatrix} \overrightarrow{e_3} \\ 0 \end{vmatrix}, \quad e_4 = \begin{vmatrix} 0 \\ 1 \end{vmatrix}. \tag{50}$$

Here the 3-component basis vectors which, without any loss of generality, can be taken as

$$\overrightarrow{e_1} = \frac{1}{\omega\sqrt{2\left(k^1k^1 + k^2k^2\right)}} \begin{vmatrix} \omega k^2 - ik^1k^3 \\ -\omega k^1 - ik^2k^3 \\ i\left(k^1k^1 + k^2k^2\right) \end{vmatrix}, \quad \overrightarrow{e_2} = \overrightarrow{e_1}^*, \quad \overrightarrow{e_3} = \frac{\overrightarrow{k}}{\omega}, \tag{51}$$

are the eigenvectors for the quantummechanical helicity operator for the spin $s = 1$.

Note that if the quantities E^0, H^0 in Eqs. (42) are some given functions for which the representation

$$E^0 - iH^0 = \int \mathrm{d}^3k \sqrt{\frac{2\omega}{(2\pi)^3}} \left(c^3 \mathrm{e}^{-\mathrm{i}kx} + c^4 \mathrm{e}^{\mathrm{i}kx}\right) \tag{52}$$

is valid, then Eqs. (42) are the Maxwell equations with the given sources, $j^e_\mu = -\partial_\mu E^0, j^{mag}_\mu = -\partial_\mu H^0$ (namely these 4 currents we call the gradient-like sources). In this case the general solution of the Maxwell equations (42)= (45)=(46)=(47) with the given sources, as follows from (49), has the form

$$\begin{array}{c} \overrightarrow{E}(x) = \int \mathrm{d}^3k \sqrt{\frac{\omega}{2(2\pi)^3}} \left(c^1 \overrightarrow{e}_1 + c^2 \overrightarrow{e}_2 + \alpha \overrightarrow{e}_3\right) \mathrm{e}^{-\mathrm{i}kx} + c.c \\ \overrightarrow{H}(x) = i \int \mathrm{d}^3k \sqrt{\frac{\omega}{2(2\pi)^3}} \left(c^1 \overrightarrow{e}_1 - c^2 \overrightarrow{e}_2 + \beta \overrightarrow{e}_3\right) \mathrm{e}^{-\mathrm{i}kx} + c.c \end{array} \tag{53}$$

where the amplitudes of longitudinal waves $\overrightarrow{e}_3 \exp(-\mathrm{i}kx)$ are $\alpha = c^3 + c^4$, $\beta = c^3 - c^4$ and c^3, c^4 are determined by the functions E^0, H^0 according to the formula (52).

Equations (42)= (45)=(46)=(47) are directly connected with the free massless Dirac equation

$$i\gamma^\mu \partial_\mu \Psi(x) = 0. \tag{54}$$

There is no reason to appeal here to the stationary case as it was done in Sec. 3, where the case with nonzero interaction and mass was considered. The substitution of

$$\psi = \left| \begin{array}{c} E^3 + iH^0 \\ E^1 + iE^2 \\ iH^3 + E^0 \\ -H^2 + iH^1 \end{array} \right| = U\mathcal{E}, \quad U = \left| \begin{array}{cccc} 0 & 0 & C_+ & C_- \\ C_+ & iC_+ & 0 & 0 \\ 0 & 0 & C_- & C_+ \\ C_- & iC_- & 0 & 0 \end{array} \right|, \quad C_\mp \equiv \frac{1}{2}(C \mp 1), \tag{55}$$

into Dirac equation (54) with γ matrices in standard Pauli - Dirac representation guarantees its transformation into the generalized Maxwell equations (42)= (45)=(46)=(47). The complete set of 8 transformations like (55), which relate generalized Maxwell equations (42) and massless Dirac equation (54), was found in [6]. Unitary relationship between the generalized Maxwell equations (45) and massless Dirac equation (54) was considered in the way similar to the Sec. 3 and can be found in some our papers from among the references within [6].

Equations (45) (or their another representations (42)= (45)=(46)=(47)) are the maximally symmetrical form of the generalized Maxwell equations. We consider here representation (45) as an example. The following theorem is valid.

Theorem. *The generalized Maxwell equations (45) are invariant with respect to the three different transformations, which are generated by three different representations* P^V, P^{TS}, P^S *of the Poincaré group* $P(1,3)$ *given by the formulae*

$$\begin{array}{c} \mathcal{E}(x) \rightarrow \mathcal{E}^V(x) = \Lambda\mathcal{E}(\Lambda^{-1}(x-a)), \\ \mathcal{E}(x) \rightarrow \mathcal{E}^{TS}(x) = F(\Lambda)\mathcal{E}(\Lambda^{-1}(x-a)), \\ \mathcal{E}(x) \rightarrow \mathcal{E}^S(x) = S(\Lambda)\mathcal{E}(\Lambda^{-1}(x-a)), \end{array} \tag{56}$$

where Λ *is a vector (i. e.* $(\frac{1}{2},\frac{1}{2})$*),* $F(\Lambda)$ *is a tensor-scalar (*$(0,1)\otimes(0,0)$*) and* $S(\Lambda)$ *is a spinor representation (*$(0,\frac{1}{2})\otimes(\frac{1}{2},0)$*) of* $SL(2,C)$ *group. This means that the equations (45) have both spin 1 and spin 1/2 symmetries.*

Proof. Let us write the infinitesimal transformations, following from (56), in the form

$$\mathcal{E}^{V,TS,S}(x) = (1 - a^\rho\partial_\rho - \frac{1}{2}\omega^{\rho\sigma}j^{V,TS,S}_{\rho\sigma})\mathcal{E}(x). \tag{57}$$

Then the generators of the transformations (57) have the form

$$\partial_\rho = \frac{\partial}{\partial x^\rho}, \quad j^{V,TS,S}_{\rho\sigma} = x_\rho\partial_\sigma - x_\sigma\partial_\rho + s^{V,TS,S}_{\rho\sigma}, \tag{58}$$

where

$$(s^V_{\rho\sigma})^\mu_\nu = \delta^\mu_\rho g_{\sigma\nu} - \delta^\mu_\sigma g_{\rho\nu}, \quad s^V_{\rho\sigma} \in \left(\frac{1}{2},\frac{1}{2}\right), \tag{59}$$

$$s^{TS}_{\rho\sigma} = \left| \begin{array}{cc} s^T_{\rho\sigma} & 0 \\ 0 & 0 \end{array} \right| \in (1,0)\oplus(0,0), \quad s^T_{\rho\sigma} = -s^T_{\sigma\rho}: \quad s^T_{mn} = -i\varepsilon^{mnj}S^j, \quad s^T_{0j} = S^j, \tag{60}$$

(S^j are given by the formula (48)) and

$$s^S_{\rho\sigma} = \frac{1}{4}[\hat{\gamma}_\rho, \hat{\gamma}_\sigma], \quad \hat{\gamma} = U^\dagger\gamma U, \tag{61}$$

(the unitary operator U is given by the formula (55), the explicit form of $\hat{\gamma}$ matrices here is essentially different from the explicit form of the matrices (37) and may be easily found from the definition in (61)). Now the proof of the theorem is reduced to the verification that all the generators (58) obey the commutation relations of the $P(1,3)$ group and commute with the operator of the generalized Maxwell equations (42)= (45)=(46)=(47), which can be rewritten in the Dirac form

$$\hat{\gamma}^{\mu}\partial_{\mu}\mathcal{E}(x) = 0 \tag{62}$$

(for some details see Ref. [2]). **QED.**

This result about the generalized Maxwell equations (42)= (45)=(46)=(47) means the following. From group theoretical point of view these equations (coinciding with Eqs. (1) in the case $\epsilon = \mu = 1$) can describe both bosons and fermions. This means that one has direct group-theoretical grounds to apply these equations for the description of electron, as it is presented above in Sec. 2.

A distinctive feature of the equation (45) for the system $\mathcal{E} = (\overrightarrow{\mathcal{E}}, \mathcal{E}^0)$ (i.e. for the system of interacting irreducible $(0,1)$ and $(0,0)$ fields) is the following. It is the manifestly covariant equation with minimal number of components, i. e. the equation without redundant components for this system.

Note that each of the three representations (56) of the $P(1,3)$ group is a local one, because each matrix part of transformations (56) (matrices Λ, $F(\Lambda)$ and $S(\Lambda)$) does not depend on coordinates $x \in R^4$, and, consequently, the generators of (56) belong to the Lie class of operators. Each of the transformations in (56) may be understood as connected with special relativity transformations in the space-time $R^4 = \{x\}$, i. e. with transformations in the manifold of inertial frame of references.

It follows from the Eqs. (45) that the field $\mathcal{E} = (\overrightarrow{\mathcal{E}}, \mathcal{E}^0)$ is massless, i. e. $\partial^{\nu}\partial_{\nu}\mathcal{E}^{\mu} = 0$. Therefore it is interesting to note that neither P^V, nor P^{TS} symmetries cannot be extended to the local conformal $C(1,3)$ symmetry. Only the known spinor C^S representation of $C(1,3)$ group obtained from the local P^S representation is the symmetry group for the generalized Maxwell equations (45). This fact is understandable: the electromagnetic field $\overrightarrow{\mathcal{E}} = \overrightarrow{E} - i\overrightarrow{H}$ obeying Eqs. (45) is not free, it interacts with the scalar field $\mathcal{E}^0$.

Consider the particular case of standard (non-generalized) Maxwell equations, namely, the case of equations (45) without magnetic charge and current densities, i. e. the case when $H^0 = 0$ but $E^0 \neq 0$. The symmetry properties of such standard equations are strongly restricted in comparison with the generalized Eqs. (45): they are invariant only with respect to tensor-scalar (spins 1 and 0) representation of Poincaré group defined by the corresponding representation $(0,1) \otimes (0,0)$ of proper ortochronous Lorentz group $SL(2,C)$. Another symmetries mentioned in the theorem are lost for this case. The proof of this assertion follows from the fact that the vector $\left(\left(\frac{1}{2},\frac{1}{2}\right)\right)$ and the spinor $\left(\left(0,\frac{1}{2}\right) \oplus \left(\frac{1}{2},0\right)\right)$ transformations of $\mathcal{E} = (\overrightarrow{\mathcal{E}}, \mathcal{E}^0)$ mix the $\mathcal{E}^0$ and $\overrightarrow{\mathcal{E}}$ components of the field $\mathcal{E}$, and only the tensor-scalar $(0,1) \oplus (0,0)$ transformations do not mix them.

For the free Maxwell equation in vacuum without sources (the case $E^0 = H^0 = 0$) the losing of above mentioned symmetries is evident from the same reasons. Moreover, it is well known that such equations are invariant only with respect to tensor (spin 1) representations of Poincaré and conformal groups and with respect to dual transformation: $\overrightarrow{E} \rightarrow \overrightarrow{H}, \overrightarrow{H} \rightarrow -\overrightarrow{E}$. We have obtained the extended 32-dimensional Lie algebra [12] (and the corresponding group) of invariance of free Maxwell equations, which is isomorphic to $C(1,3) \oplus C(1,3) \oplus dual$ algebra. We were successful to prove it appealing not to Lie class of symmetry operators but to a more general, namely, to the simplest Lie - Backlund class of operators. The corresponding generalization of symmetries of Eqs. (45) presented in the above theorem leads to a wide 246-dimensional Lie algebra in the class of first order Lie - Backlund operators. Thus, the Maxwell equations

(45) with electric and magnetic gradient-like sources have the maximally possible symmetry properties among the standard and generalized equations of classical electrodynamics.

Finally, knowing the operator U (55), it is easy to obtain the relationship between the amplitudes $a^r(\overrightarrow{k})$, $b^r(\overrightarrow{k})$ determining the well known fermionic solution of the massless Dirac equation (in Pauli - Dirac representation), and the amplitudes $c^\alpha(\overrightarrow{k})$, determining the bosonic solution (49). Corresponding formulae (direct and inverse) related fermionic and bosonic amplitudes were found in [6]

$$
\begin{aligned}
a^1 &= \frac{1}{2\omega}\left[i\sqrt{(\omega-k^3)(\omega+k^3)}(c^1-c^2)-(\omega-k^3)c^3+(\omega+k^3)c^4\right], \quad \omega \equiv \sqrt{\overrightarrow{k}^2}, \\
a^2 &= \frac{1}{2\omega}\left[-i(k^1+ik^2)\left(\sqrt{\frac{\omega+k^3}{\omega-k^3}}c^1+\sqrt{\frac{\omega-k^3}{\omega+k^3}}c^2\right)+(k^1+ik^2)(c^3+c^4)\right], \\
b^1 &= \frac{1}{2\omega}\left[i\sqrt{(\omega-k^3)(\omega+k^3)}(c^1+c^2)+(\omega+k^3)c^3+(\omega-k^3)c^4\right], \\
b^2 &= \frac{1}{2\omega}\left[i(k^1+ik^2)\left(\sqrt{\frac{\omega-k^3}{\omega+k^3}}c^1-\sqrt{\frac{\omega+k^3}{\omega-k^3}}c^2\right)+(k^1+ik^2)(c^3-c^4)\right].
\end{aligned}
\tag{63}
$$

In terms of unitary operator V this formulae have the form:

$$
\widehat{a} \equiv \begin{vmatrix} a^1 \\ a^2 \\ b^1 \\ b^2 \end{vmatrix} = \frac{1}{2\omega}\begin{vmatrix} i\sqrt{pq} & -p & -i\sqrt{pq} & q \\ -iz^*\sqrt{\frac{q}{p}} & z^* & -iz^*\sqrt{\frac{p}{q}} & z^* \\ i\sqrt{pq} & q & i\sqrt{pq} & p \\ iz\sqrt{\frac{p}{q}} & z & -iz\sqrt{\frac{q}{p}} & -z \end{vmatrix} \cdot \begin{vmatrix} c^1 \\ c^3 \\ c^2 \\ c^4 \end{vmatrix} = V \cdot \widehat{c}, \tag{64}
$$

where $p = \omega - k^3$, $q = \omega + k^3$, $z = k^1 - ik^2$, $z^* = k^1 + ik^2$, $\omega \equiv \sqrt{\overrightarrow{k}^2}$. The operator V (the image of operator U (55) in the space of quantum-mechanical amplitudes $\widehat{c}$ and $\widehat{a}$, i. e. in the rigged Hilbert space $S_3^4 \subset H \subset S_3^{*4}$, where $S_3^{*4} \equiv (S(R^3) \otimes C^4)^*$ is the space of 4-component generalized Schwartz functions) is unitary one: $VV^{-1} = V^{-1}V = 1, \quad V^{-1} = V^\dagger$, plus linearity.

Hence, the fermionic states may be constructed as linear combinations of bosonic states, namely, of the states of the coupled electromagnetic $\overrightarrow{\mathcal{E}} = \overrightarrow{E} - i\overrightarrow{H}$ and scalar $\mathcal{E}^0 = E^0 - iH^0$ fields. The inverse relationship between bosonic and fermionic states is also valid. We prefer the first possibility which is new (bosonic) realization of the old idea (Thomson, Abraham, etc) of electromagnetic nature of mass and of material world. Thus, today on the basis of (25), (55), (63), (64) we may speak about more general conception of the bosonic field nature of material world.

On the basis of this relationship, in [6] the relationship between quantized electromagnetic-scalar and massless spinor field was obtained. The possibility of both Bose and Fermi quantization types for electromagnrtic-scalar field (and, inversly, for the Dirac spinor field) was proved. We will not tauch here the problems of quantization because we are trying here to demonstrate new possibilities of classical theory.

5 A brief remark about gravity

The unified theory of electromagnetic and gravitational phenomena may be constructed in the approach under consideration in the following way. The main primary equations again are written as (1) and gravity is considered as a medium in these equations, i. e. the electric ϵ and magnetic μ permeabilities of the medium are some functions of the gravitational potential Φ_{grav}:

$$\epsilon = \epsilon(\Phi_{grav}), \quad \mu = \mu(\Phi_{grav}). \tag{65}$$

Gravity as a medium may generate all the phenomena which in standard Einstein's gravity are generated by Riemann geometry. For example, the refraction of the light beam near a big mass star is a typical medium effect in such a unified model of electromagnetic and gravitationaal phenomena. The idea of such consideration consists in the following. The gravitational interaction between massive objects can be represented as the interaction with some medium, similarly as here (in Eqs. (1)) the electromagnetic interaction between charged particles is considered.

6 Brief conclusions

One of the conclusions of our investigation presented here and in [2, 6] is that a field equation itself does not answer the question what kind of particles (Bose or Fermi) is described by this equation. To answer this question one needs to find all the representations of the Poincaré group under which the equation is invariant. If more than one such Poincaré representations are found [2], including the representations with integer and half-integer spins, then the given equation describes both Bose and Fermi particles, and both quantization types (Bose and Fermi) [6] of the field function, obeying this equation, satisfy the microcausality condition. The strict group-theoretical ground of this assertion is the following [2]: both slightly generalized Maxwell equations (1) (with $\epsilon = \mu = 1$) and Dirac equation (54) (with $\mathbf{m}_0 = 0$, $\Phi = 0$) are invariant with respect to three different local representations of Poincaré group, namely the standard spinor, vector and tensor-scalar representations generating by the $(0, \frac{1}{2}) \otimes (\frac{1}{2}, 0)$, $(\frac{1}{2}, \frac{1}{2})$, $(0, 1) \otimes (0, 0)$ representations of the Lorentz SL(2,C) group, respectively.

Now it is clear that only the pair of notions "equation" plus "fixed Bose or Fermi representation of Poincare group" answers the question what kind of particle, boson or fermion, is describing.

So if one fixes the pair "Dirac equation plus reducible, spins 1 and 0, representation" he may describe bosonic system (photon plus boson).

If one fixes another pair "Dirac equation plus spin 1/2 representation" one may describe fermions (electron, neutrino, etc.).

If one fixes the pair "generalized Maxwell equation plus spins 1 and 0 representation" he may describe bosonic system (photon plus boson).

Finally, if one fixes the pair "generalized Maxwell equation plus spin 1/2 representation" one may describe fermions. Namely this last possibility is under main consideration in this paper.

The simple case $\mathbf{m}_0 = 0$, $\Phi = 0$ is considered in details in formulae (64), where it is shown that amplitudes of fermionic states (or their creation - annihilation operators) are the linear combinations of amplitudes (or of creation - annihilation operators) of bosonic states. In this sense our model, where the electron is considered as a compound system of photon plus massless spinless boson, i. e. the states of electron are the linear combinations of the states of electromagnetic-scalar field, has the analogy with modern quark models of hadrons.

In the model of atom under consideration based on the equations of Maxwell's electrodynamics, not of quantum mechanics, the atomic electron is interpreted as a classical stationary electromagnetic-scalar wave (the details of the interpretation see in Sec. 2). That is why this model is essentially distinguished from the first electrodynamical hydrogen atom model suggested by Sallhofer, see, e.g., [3, 9].

A few words can be said about the interpretation of the Dirac Ψ function. As follows from the consideration presented here, e.g., from the relationship (30), the new interpretation of the Dirac Ψ function can be suggested too: Ψ function is the combination of the electromagnetic field strengths $\left(\overrightarrow{E}, \overrightarrow{H}\right)$ and two scalar fields (E^0, H^0) generating the electromagnetic sources, i.e. in this case the probability or Copenhagen interpretation of the function Ψ is not necessary.

In the approach based on the equations (1), it is possible to solve another stationary problems of atomic physics without any appealing to the Dirac equation and the probability or Copenhagen interpretation.

Some nonstationary problems, e.g., the problem of transitions between the stationary states caused by the external perturbation, can be, probably, solved in terms of the electrodynamical model under consideration similarly to the solution of this problem on the basis of the stationary Schrödinger equation with the corresponding perturbation.

It is evident from the hydrogen example presented in Sec. 2 that discretness of the physical system states (and its characteristics such as energy, etc) may be a consequence not only of quantum systems (Schrödinger, Dirac), but also of the classical (Maxwell) equations for the given system. In the case under consideration this discretness is caused by the properties of medium, which are given by the electric and magnetic permeabilities (2).

It is very useful to consider the case of Lamb shift in the approach presented here. This specific quantum electrodynamical effect (as modern theory asserts) can be described here in the framework of classical electrodynamics of media. In order to obtain Lamb shift one must add to $\Phi\left(\overrightarrow{x}\right) = -Ze^2/r$ in (2) the quasipotential (known, e. g., from [13], which follows, of course, from quantum electrodynamics) and solve the equations (1) for such medium similarly to the procedure of Sec. 2. Finally one obtains the Lamb shift correction to the Sommerfeld - Dirac formula (19). Such Lamb shift can be interpreted as a pure classical electrodynamical effect. It can be considered here as a consequence of polarization of medium (2) and not of polarization of such abstract concept as vacuum in quantum electrodynamics. This brief example demonstrates that our consideration can essentially extend the limits of classical theory application in microworld, which was the main purpose of our investigations.

The main conclusion from Sec. 3 is the following. The unitary equivalence between the stationary Dirac equation and the stationary Maxwell equations with gradient-like currents and charges in medium (2) gives the possibility to reformulate all the problems of atomic and nuclear physics (not only the problem of hydrogen atom description, which here is only an example of possibilities), which can be solved on the basis of the stationary Dirac equation, in the language of classical electrodynamical stationary Maxwell equations. It means that our model in stationary case is equally successful as the conventional relativistic quantum mechanics.

Thus, the new features which follow from our approach are: (i) the classical interpretation, (ii) new equation and method in atomic and nuclear physics based on classical electrodynamics in inneratomic medium like (2), (iii) the hypothesis of bosonic nature of matter (bosonic structure of fermions), (iv) extension of the limits of classical theory application in the microworld, (v) foundations of a unified model of electromagnetic and gravitational phenomena, in which gravitation is considered as a medium in generalized equations.

Acknowledgement

The authors are much grateful to Prof. Hans Sallhofer for multiple discussions of the details and essential support of the main idea, and to Prof. Boris Struminsky for useful discussions of alternatives in gravity.

This work is supported by the National Fund of Fundamental Researches of Ukraine, grant #F7/458-2001.

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