

Large N Structure of the IIB Matrix Model

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Abstract

We study large N behavior of the IIB matrix model using the equivalence between the IIB matrix model for finite N and a field theory on a non-commutative periodic lattice with $N \times N$ sites. We find that the large N dependences of correlation functions can be obtained by naively counting the number of fields in the field theory on a non-commutative periodic lattice. Furthermore, the large N scaling behavior of the coupling constant g is determined if we require that the expectation values of Wilson loops be calculable.

1 Introduction

Four years ago, large N matrix models were proposed as superstring theories [1, 2, 3, 4]. The IIB matrix model [2] is the zero volume limit [5] of a ten-dimensional super Yang-Mills theory. The gauge fields in the IIB matrix model are written as $N \times N$ hermitian matrices. The eigenvalues of these matrices are interpreted as space-time coordinates. This model represents an open space-time, because the eigenvalues of the matrices can take values in the infinite region. Some studies of a closed space-time with periodic boundary conditions have been done by changing the hermitian matrices to unitary matrices [6]. When $N=2$, we can successfully integrate the fermions by hand and analytically investigate the model with the bosonic degrees of freedom [7]. However, it is almost impossible to do something like this when $N>2$.

In the IIB matrix model, space-time coordinates are written in terms of the gauge fields and they do not commute with each other, in general. In ref.[8] toroidal compactifications of the matrix models on a non-commutative torus were investigated. Since then, non-commutative geometries in string and matrix theories have been vigorously studied.

The commutation relations of the fields in non-commutative space-time are very similar to the commutation relations of $U(N)$ algebra, which have been investigated in the context of supermembranes. [9, 10, 11, 12]. In fact, it is known that there is a one-to-one correspondence

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between the action of an $N \times N$ matrix model and that of the field theory on a non-commutative periodic lattice (NCPL) with $N \times N$ sites, which we write as $\mathbb{T}_{\frac{2}{N}}^2$ [13, 14, 15]. Since the commutation relations of coordinates of the NCPL are proportional to $\frac{1}{N}$, we expect that the matrix model in the large N limit has a one-to-one correspondence to a field theory on the continuous torus.

The naive large N limit of the IIB matrix model is the Schild action on a torus [16]. For this reason, the Schild action on a torus is a plausible candidate for the large N limit of the IIB matrix model. However, we find that the large N dependences of some expectation values in the IIB matrix model for finite N are different from those in the regularized Schild theory on a periodic $N \times N$ lattice. Since systems with infinite numbers of degrees of freedom are usually described by field theories, and the large N limit of the IIB matrix model is a system with an infinite numbers of degrees of freedom, it is natural to search for a field theory of the IIB matrix model in the large N limit. Based on the guiding principle that the N dependence for some expectation values is the same as in the IIB matrix model, we consider field theories on a torus.

We investigate the large N dependences of the correlation functions of the IIB matrix model using the field theory on an NCPL. We find that the N dependences of some correlation functions calculated in the matrix model can be straightforwardly given with the field theory on the NCPL using kinematical arguments. We also show that correlation functions such as

$$\left\langle \frac{1}{N} \text{Tr} F(A, \Psi) \right\rangle \quad (1)$$

can have non-trivial values in the large N limit if we demand $g^2 N = O(N^0)$.

This paper is organized as follows. In section 2 we review the isomorphic mapping from a $U(N)$ matrix model to a field theory on the NCPL. In section 3 we discuss the large N behavior of correlation functions using the field theory on the NCPL, which is isomorphic to the IIB matrix model. In section 4 we discuss the field theory which corresponds to the IIB matrix model in the large N limit. We show that the IIB matrix model cannot lead to the Schild action [16] straightforwardly. The final section is devoted to conclusion and discussion.

2 Mapping between the $U(N)$ Algebra and a Field on a Non-Commutative Periodic Lattice

There are some investigations of the large N limit of the $SU(N)$ algebra [9, 10, 11]. The $U(N)$ algebra¹ is generated by the following two matrices [11]:

$$U = \begin{pmatrix} 1 & & & 0 \\ & e^{i\frac{4\pi}{N}} & & \\ & & \ddots & \\ 0 & & & e^{i\frac{4\pi}{N}(N-1)} \end{pmatrix}, \quad V = \begin{pmatrix} 0 & 1 & & 0 \\ & 0 & 1 & \\ & & \ddots & \ddots \\ 0 & & & 0 & 1 \\ 1 & & & & 0 \end{pmatrix}. \quad (2)$$

¹We assume N is odd for definiteness.

These matrices satisfy the relations

$$U^N = V^N = \mathbf{1}, \quad (3)$$

$$V U = e^{i\frac{4\pi}{N}} U V. \quad (4)$$

The bases of the $U(N)$ algebra can be written

$$\mathbf{T}_{(m_1, m_2)} = e^{i\frac{2\pi}{N} m_1 m_2} U^{m_1} V^{m_2}. \quad (5)$$

The commutation relations of $\mathbf{T}_{\mathbf{m}} (\equiv \mathbf{T}_{(m_1, m_2)})$ are given by [17, 18]

$$[\mathbf{T}_{\mathbf{m}}, \mathbf{T}_{\mathbf{n}}] = -2i \sin \left(\frac{2\pi}{N} \epsilon^{ab} m_a n_b \right) \mathbf{T}_{\mathbf{m}+\mathbf{n}}. \quad (\epsilon^{12} = -\epsilon^{21} = 1, \quad a, b = 1, 2) \quad (6)$$

This type of commutation relation is satisfied by plane-wave functions on the two-dimensional non-commutative torus on which the coordinates satisfy [8]

$$[x_1, x_2] = i \frac{4\pi}{N}. \quad (7)$$

We define the operator $\Delta(\sigma)$ ($\sigma = (k^1/N, k^2/N)$, $k^a \in \{0, 1, \dots, N-1\} = \mathbb{Z}_N$), by

$$\Delta(\sigma) = \frac{1}{N} \sum_{\mathbf{m} \in \mathbb{Z}_N} \mathbf{T}_{\mathbf{m}} e^{-2\pi i \mathbf{m} \cdot \sigma}, \quad (8)$$

which maps $N \times N$ hermitian matrices to functions on the two-dimensional $N \times N$ lattice with periodic boundary conditions. The discrete space where σ takes values is written $\mathbb{T}_{\frac{2}{N}}^2$. Note that $\Delta(\sigma)$ satisfies

$$\Delta((\sigma^1 + \ell^1, \sigma^2 + \ell^2)) = \Delta((\sigma^1, \sigma^2)). \quad (\ell^a \in \mathbb{Z}) \quad (9)$$

Equation(8) represents an inverse Fourier transformation of the matrices $\mathbf{T}_{\mathbf{m}}$ which have momenta $2\pi\mathbf{m}$. Using this $\Delta(\sigma)$, a hermitian matrix A is mapped to a field on $\mathbb{T}_{\frac{2}{N}}^2$ according to

$$A(\sigma) = \text{Tr}(\Delta(\sigma) A). \quad (10)$$

Also, the product of two matrices A and B is mapped to the diamond product [13] of two fields on $\mathbb{T}_{\frac{2}{N}}^2$ according to

$$\text{Tr}(\Delta(\sigma) AB) \equiv A(\sigma) \diamond B(\sigma) = \frac{1}{N^2} \sum_{\tau, \omega \in \mathbb{T}_{\frac{2}{N}}^2} e^{2\pi i N \epsilon^{ab} (\sigma - \tau)_a (\sigma - \omega)_b} A(\tau) B(\omega). \quad (11)$$

Equation(10) can be rewritten as

$$A(\sigma) = \sum_{\mathbf{m}} \left\{ \frac{1}{N} \text{Tr}(\mathbf{T}_{\mathbf{N}-\mathbf{m}} A) \right\} e^{-2\pi i (\mathbf{N}-\mathbf{m}) \cdot \sigma} \equiv \sum_{\mathbf{m}} A_{\mathbf{m}} e^{2\pi i \mathbf{m} \cdot \sigma}, \quad (12)$$

and using these Fourier transformed modes, $A(\sigma) \diamond B(\sigma)$ in eq.(11) is given by,

$$\begin{aligned} A(\sigma) \diamond B(\sigma) &= \sum_{\mathbf{m}, \mathbf{n}} e^{-\frac{2\pi i}{N} \epsilon^{ab} m_a n_b} (A_{\mathbf{m}} e^{2\pi i \mathbf{m} \cdot \sigma}) (B_{\mathbf{n}} e^{2\pi i \mathbf{n} \cdot \sigma}) \\ &\equiv A(\sigma) e^{\frac{i}{2\pi N} \epsilon^{ab} \overleftarrow{\partial}_a \overrightarrow{\partial}_b} B(\sigma), \end{aligned} \quad (13)$$

where the partial derivative ∂_a is used symbolically, but it becomes a normal derivative in the large N limit. Then the diamond product is a discrete-space version of the star product [13].

Let us consider the action of matrices A^i ,

$$S = \text{Tr} F(A^i). \quad (14)$$

The Δ mapping (10) maps this action (14) to

$$S = \frac{1}{N} \sum_{\sigma \in \mathbb{T}_{\frac{2}{N}}^2} F_{\diamond}(A^i(\sigma)), \quad (15)$$

where $F_{\diamond}(\cdot)$ denotes the quantity whose functional form is the same as that of $F(\cdot)$ but in which all the products of matrices are replaced by the diamond products of the corresponding fields on $\mathbb{T}_{\frac{2}{N}}^2$. The measure of the path integral is given by

$$\mathcal{D}A^i(\sigma) = \prod_{0 \leq m_a < N} dA_{\mathbf{m}}^i. \quad (16)$$

To this point we have considered a $U(N)$ matrix model, but it is obvious that we can follow the same procedures for a $SU(N)$ matrix model. Since matrices are traceless, the resultant fields on $\mathbb{T}_{\frac{2}{N}}^2$ have no zero mode.

3 Field Theory on a Non-commutative Periodic Lattice as a IIB Matrix Model

We now apply the mapping in the previous subsection to the IIB matrix model [2]. The action of the IIB matrix model [2] is given by

$$S_{\text{IIB}}^M = -\frac{1}{g^2} \text{Tr} \left(\frac{1}{4} [A^\mu, A^\nu]^2 + \frac{1}{2} \overline{\Psi} \Gamma_\mu [A^\mu, \Psi] \right), \quad (17)$$

where A_μ ($\mu = 1, 2, \dots, 10$) and Ψ are $N \times N$ hermitian matrices and Ψ are ten-dimensional Majorana-Weyl spinors. Then the corresponding action of the field theory on $\mathbb{T}_{\frac{2}{N}}^2$ is given by (cf. eqs.(14) and (15)) [14, 15]

$$S_{\text{IIB}_N} = -\frac{1}{g^2 N} \sum_{\sigma \in \mathbb{T}_{\frac{2}{N}}^2} \left(\frac{1}{4} [A^\mu(\sigma), A^\nu(\sigma)]_{\diamond}^2 + \frac{1}{2} \overline{\Psi}(\sigma) \diamond \Gamma_\mu [A^\mu(\sigma), \Psi(\sigma)]_{\diamond} \right). \quad (18)$$

We now consider the N dependence of correlation functions with the field theory on the NCPL (18) and we show that the results obtained from Monte Carlo simulations [19, 24, 20] and perturbative investigations [19] are straightforwardly derived. The starting point is the fact that the expectation values of the bosonic and the fermionic parts of the action are proportional to N^2 , respectively [19]. We explain this point using the bosonic part of the action here. Let us first define the partition function with a parameter κ ,

$$\begin{aligned} Z[\kappa] &= \int \mathcal{D}A \mathcal{D}\Psi e^{\frac{1}{g^2 N} \sum_{\sigma} \left(\frac{\kappa}{4} [A^{\mu}(\sigma), A^{\nu}(\sigma)]_{\diamond}^2 + \frac{1}{2} \bar{\Psi}(\sigma) \diamond \Gamma_{\mu} [A^{\mu}(\sigma), \Psi(\sigma)]_{\diamond} \right)} \\ &= \kappa^{-(\frac{D}{4} + 2^{\lfloor \frac{D}{2} \rfloor - 4})(N^2 - 1)} Z[\kappa = 1]. \end{aligned} \quad (19)$$

Next, we differentiate $Z[\kappa]$ with respect to κ and take the $\kappa \rightarrow 1$ limit. Then the expectation value of the bosonic part of the action is given by

$$\left\langle -\frac{1}{4g^2 N} \sum_{\sigma \in \mathbb{T}_{\frac{2}{N}}^2} [A^{\mu}(\sigma), A^{\nu}(\sigma)]_{\diamond}^2 \right\rangle = \left(\frac{D}{4} + 2^{\lfloor \frac{D}{2} \rfloor - 4} \right) (N^2 - 1). \quad (20)$$

If we assume that the order of N is not affected by the commutator and that the summation over σ gives $\sum_{\sigma} = O(N^2)$ [21], we have

$$A^{\mu}(\sigma) = O((g^2 N)^{\frac{1}{4}}). \quad (21)$$

We can make a similar argument to the fermionic part of the action, and we obtain

$$\Psi(\sigma) = O((g^2 N)^{\frac{3}{8}}). \quad (22)$$

From eq.(21), we can estimate the N dependence of correlation functions as

$$\left\langle \frac{1}{N} \text{Tr}(A^{\mu_1} \dots A^{\mu_{2k}}) \right\rangle = \left\langle \frac{1}{N^2} \sum_{\sigma \in \mathbb{T}_{\frac{2}{N}}^2} \left(A^{\mu_1}(\sigma) \diamond \dots \diamond A^{\mu_{2k}}(\sigma) \right) \right\rangle = O((g^2 N)^{\frac{k}{2}}). \quad (23)$$

As a special case of (23), the distribution of the eigenvalues of the bosonic matrices is estimated as

$$\left\langle \frac{1}{N} \text{Tr}(A^2) \right\rangle = O(g N^{\frac{1}{2}}). \quad (24)$$

Furthermore, we can also estimate the N dependences for any correlation functions including fermions. Let $F_{(m,n)}[A, \Psi]$ be a homogeneous polynomial of terms of order m in A^{μ} and order n in Ψ . For example, $F_{(1,2)}[A^{\mu}, \Psi] = \bar{\Psi} A^{\mu} \Psi$. Then we obtain

$$\left\langle \frac{1}{N} \text{Tr} (F_{(m,n)}[A^{\mu}, \Psi]) \right\rangle = O((g^2 N)^{\frac{1}{4}m + \frac{8}{3}n}). \quad (25)$$

We consider Wilson loops,

$$\left\langle \left[\frac{1}{N} \text{Tr} \left(P e^{i \int d\sigma (k(\sigma) \cdot A + \bar{\lambda}(\sigma) \Psi)} \right) \right] \dots \right\rangle, \quad (26)$$

which are not homogeneous polynomials. However, we will see that they are finite if $g^2 N$ is fixed in the large N limit. From refs. [19, 20] and [24] it is known that any Wilson loops are finite if $g^2 N$ is fixed in the large N limit.² We have studied this from the kinematical point of view, and it is interesting that our naive argument reproduce the results obtained using Monte Carlo simulations. This suggests that investigating the IIB matrix model from the NCPL point of view will be useful.

It is obvious that we can follow the same arguments with the bosonic model, for which

$$S_b = -\frac{1}{4g^2} \text{Tr}([A^\mu, A^\nu]^2), \quad (27)$$

whose large N limit is shown to exist in ref.[22], and it gives the same results for the N dependences as the supersymmetric model. (17) Hence, for example, eq. (24) holds. In refs. [19] and [20], some correlation functions were calculated with Monte Carlo simulations and the results there agree with ours.

4 Large N Limit of the IIB Matrix Model

It would be quite difficult to write down the action of the IIB matrix model in the large N limit directly from the original action (17). However, eq. (18), the field theory action on the NCPL, resembles the Schild action [16], and hence one may think that the Schild action would be the correct action in this limit. In fact the following arguments support such expectations. First, eq.(7), the commutator between two coordinates of the NCPL, vanishes in the $N \rightarrow 0$ limit. This means that a field theory on the NCPL (18) becomes a field theory on an ordinary commutative continuous torus. Next, the commutation relations of fields with respect to the diamond product are given by

$$\begin{aligned} [A(\sigma), B(\sigma)]_\diamond &= 2i A(\sigma) \sin \left(\frac{1}{2\pi N} \epsilon^{ab} \overleftarrow{\partial}_a \overrightarrow{\partial}_b \right) B(\sigma) \\ &= \frac{i}{\pi N} \epsilon^{ab} \partial_a A(\sigma) \partial_b B(\sigma) + O \left(\frac{1}{N^2} \right), \end{aligned} \quad (28)$$

where the partial derivative in the first term on the r.h.s. may be regarded as the difference on a periodic lattice. In the large N limit we could neglect $O(\frac{1}{N^2})$ terms in (28), and then the action of the IIB matrix model (18) would become

$$S_{\text{IIB}} = \frac{1}{g^2 N^3} \sum_{\sigma \in \mathbb{Z}_N^2} \left[\frac{1}{4} (\epsilon^{ab} \partial_a a^\mu(\sigma) \partial_b a^\nu(\sigma))^2 - \frac{i}{2} \bar{\psi}(\sigma) \epsilon^{ab} \partial_a \not{a}(\sigma) \partial_b \psi(\sigma) \right], \quad (29)$$

$$= \frac{1}{g^2 N} \int_{T^2} d^2 \sigma \left[\frac{1}{4} (\epsilon^{ab} \partial_a a^\mu(\sigma) \partial_b a^\nu(\sigma))^2 - \frac{i}{2} \bar{\psi}(\sigma) \epsilon^{ab} \partial_a \not{a}(\sigma) \partial_b \psi(\sigma) \right], \quad (30)$$

²The correlation functions of the Wilson loops are not the connected parts in our case. It is shown in ref.[24] that connected n point Wilson loops are $O(N^{-(2n-1)})$ for the bosonic model and $O(N^{-n})$ for the supersymmetric model. These orders are due to the factorization property and our result does not contradict these results.

where

$$\begin{aligned} a^\mu(\sigma) &= \pi^{-\frac{1}{2}} A^\mu(\sigma) = \pi^{-\frac{1}{2}} \text{Tr}(\Delta(\sigma) A^\mu), \\ \psi(\sigma) &= \pi^{-\frac{1}{4}} \Psi(\sigma) = \pi^{-\frac{1}{4}} \text{Tr}(\Delta(\sigma) \Psi). \end{aligned} \quad (31)$$

This means that the IIB matrix model of finite N could be approximated by the lattice-regularized Schild action. The size of the matrix, N , is the same number as the number of sites for each direction. This argument is the inverse of that for the matrix regularization of the Schild action [16] using the NCPL.

Despite the above arguments, we show that the Schild theory (30) is not the large N limit of the IIB matrix model. First, we have seen that the large N behavior of the fields (21) and (22) depends on the N dependence of the action. However, the N dependence of the IIB matrix model (18) is different from that of the lattice-regularized Schild action (29). In fact, following arguments similar to there given in the previous section, we find that the action (29) gives

$$\left\langle \frac{1}{N} \text{Tr}(A^2) \right\rangle_{\text{Schild}} = O(gN^{-\frac{1}{2}}), \quad (32)$$

Therefore we must conclude that (30) itself, obtained in the naive large N limit, is not the large N limit of the IIB matrix model.

What is wrong with the argument that the Schild action (30) can be regarded as the large N limit of the IIB matrix model? The crucial oversight of such arguments is that we have neglected $O(\frac{1}{N^2})$ terms in (28). This is justified only if the Kaluza-Klein momenta on the torus, \mathbf{m} , is sufficiently small compared to N . The assumption in the previous section, that the order of N is not affected by the commutation relations when estimating the large N dependence [21], implies that the configurations of large momenta of order N play a crucial role in calculating correlation functions.

5 Conclusion and Discussion

We have taken notice of the equivalence of the IIB matrix model (17) and the field theory on an NCPL (18). Specifically, a matrix A is given by $\frac{1}{N} \sum_{\sigma} A(\sigma) \Delta(\sigma)$ in the coordinate representation and $\sum_{\mathbf{m}} A_{\mathbf{m}} \mathbf{T}_{\mathbf{m}}$ in the momentum representation, and eq. (18) is the action in the coordinate representation. We found that we can easily determine the large N dependences for correlation functions using the action in the coordinate representation. We have shown that any correlation function $\langle \frac{1}{N} \text{Tr} F(A, \Psi) \rangle$ can have finite and nontrivial values if we take $g^2 N \sim O(N^0)$. It is interesting that the connected parts of the Wilson loops are renormalizable in the supersymmetric model when the same limit is taken [24]. We need an effective theory of the Wilson loops to obtain the string field theory from the IIB matrix model [3]. It is natural to demand that no correlation functions like (25) diverge from the field theoretical point of view. This leads to the following double scaling limit:

$$\begin{aligned} N &\rightarrow \infty, \\ g &\rightarrow 0. \quad (g^2 N : \text{fixed}) \end{aligned} \quad (33)$$

IIB matrix model (S_{IIB}^M)	Schild action (S_I)
$SU(N)$	area preserving diffeomorphism
$\delta_\Lambda A^\mu = i[\Lambda, A^\mu]$ $\delta_\Lambda \Psi = i[\Lambda, \Psi]$	$\delta_{\lambda(\sigma)} A^\mu(\sigma) = \epsilon^{ab} \partial_a \lambda(\sigma) \partial_b A^\mu(\sigma)$ $\delta_{\lambda(\sigma)} \Psi(\sigma) = \epsilon^{ab} \partial_a \lambda(\sigma) \partial_b \Psi(\sigma)$
supersymmetry	supersymmetry
$\delta_\varepsilon A^\mu = i\bar{\varepsilon} \Gamma^\mu \Psi$ $\delta_\varepsilon \Psi = \frac{i}{2} [A^\mu, A^\nu] \Gamma_{\mu\nu} \varepsilon$	$\delta_\varepsilon A^\mu(\sigma) = i\bar{\varepsilon} \Gamma^\mu \Psi(\sigma)$ $\delta_\varepsilon \Psi(\sigma) = \frac{i}{2} \epsilon^{ab} \partial_a A^\mu(\sigma) \partial_b A^\nu(\sigma) \Gamma_{\mu\nu} \varepsilon$
rotational invariance	rotational invariance
$\delta_\omega A^\mu = \omega^\mu{}_\nu A^\nu$ $\delta_\omega \Psi = 0$ $(\omega_{\nu\mu} = -\omega_{\mu\nu})$	$\delta_\omega A^\mu(\sigma) = \omega^\mu{}_\nu A^\nu(\sigma)$ $\delta_\omega \Psi(\sigma) = 0$ $(\omega_{\nu\mu} = -\omega_{\mu\nu})$
shift of the bosonic matrices	shift of the bosonic fields
$\delta_\alpha A^\mu = \alpha^\mu \mathbf{1}$ $\delta_\alpha \Psi = 0$	$\delta_\alpha A^\mu(\sigma) = \alpha^\mu$ $\delta_\alpha \Psi(\sigma) = 0$
shift of the fermionic matrices	shift of the fermionic fields
$\delta_\chi A^\mu = 0$ $\delta_\chi \Psi = \chi \mathbf{1}$	$\delta_\chi A^\mu(\sigma) = 0$ $\delta_\chi \Psi(\sigma) = \chi$

Table 1: Correspondence between symmetries of the IIB matrix model and S_I

We should note, however, that Eq. (33) raises a question about scales. If we fix $g^2 N$, then $\alpha'^2 \sim g^2 N$ [3]. On the other hand, $\langle \frac{1}{N} \text{Tr} A^2 \rangle$, which is often interpreted as the extent of spacetime, also behaves like $g^2 N$. Then we must know the origin of this huge hierarchical difference between scales of string and spacetime. We may take another limit. Since $\langle \frac{1}{N} \text{Tr} A^2 \rangle \sim g^2 N$ is the extent of spacetime, it may be infinite in the large N limit, and we can fix $\alpha'^2 \sim g^2 N^{1-\kappa}$ ($\kappa > 0$) [25]. In this case, however, we should impose the condition that at least connected parts of the correlation functions of the (renormalized) Wilson loops are finite in the large N limit while various correlation functions go to infinity.

Our analysis is applicable to the bosonic model (27) since supersymmetry does not play an important role in counting the powers of N in our arguments. One might think that the N dependences of correlation functions would be different in supersymmetric and bosonic models. However, the distributions of the eigenvalues of the bosonic matrices in these models agree with each other [4, 19]. This supports our analysis at least in the leading order of N . Other correlation functions, such as $\langle \frac{1}{g^2} \text{Tr} A^4 \rangle$ and the Wilson loops, were previously calculated in the bosonic matrix model using Monte Carlo simulations [19, 20] and our results agree with those. It is surprising that our two equations (21) and (22) lead to the correct large N behavior for the correlation functions, such as $\langle \frac{1}{N} \text{Tr} A^2 \rangle$, $\langle \frac{1}{g^2} \text{Tr} A^4 \rangle$ and the Wilson loops, when $g^2 N$ is fixed. However, an analytical proof for validity of the assumptions used to obtain the equations is not given here. These results suggest that all the momentum modes of the fields, or the components of the matrices, contribute significantly at least in the bosonic model.

If we regard the IIB matrix model for finite N (17) as a regularized action of some field theory, the Schild action [16] is a plausible candidate for such a field theory. Actually, the a naive large N limit of the action of the field theory on the NCPL (18) is the Schild action

(30). We have seen that the N dependence of the action is crucial for calculating the N dependences of correlation functions. The lattice-regularized Schild action (29), however, has a different N dependence from that of the IIB matrix model (18). This difference comes from the fact that we have neglected the $O(\frac{1}{N^2})$ terms in the commutation relations in (28). This is justified only if large momentum modes do not contribute to the correlation functions. Thus the large momentum modes $\mathbf{m} = O(N)$ contribute equally or significantly to correlation functions, since our results agree with those obtained by the matrix model [19, 20].

What sort of action can we have for the IIB matrix model in the large N limit? Noting that $\partial_a = O(N)$ [21], we may consider the action

$$S_1 = \frac{1}{g^2 N^3} \int_{T^2} d^2 \sigma \left[\frac{1}{4} (\epsilon^{ab} \partial_a A^\mu(\sigma) \partial_b A^\nu(\sigma))^2 - \frac{i}{2} \bar{\Psi}(\sigma) \epsilon^{ab} \partial_a \mathbf{A}(\sigma) \partial_b \Psi(\sigma) \right]. \quad (34)$$

This action is different from the Schild action (30) by the factor N^{-2} . Although we can write down S_1 by rescaling the fields in the Schild action (30), we cannot obtain S_1 directly from the action of the IIB matrix model (17). S_1 , however, is one of the most likely candidates for the action of the IIB matrix model in the large N limit because each symmetry of the IIB matrix model corresponds to one of the symmetries of S_1 (see Table 1). On the other hand, we found that we cannot neglect $O(\frac{1}{N^2})$ terms of the commutation relations (28) in the large N limit. This implies that the large N limit of a field theory on an NCPL, or the matrix model, will be a theory on a non-commutative torus or a non-local theory on a torus. Non-commutative field theories have the special feature that there exist stringy modes as well as usual particle modes [23]. This suggests that the large N field theory on an NCPL describes the Planck scale physics in which the coordinates do not commute with each other, and hence it supports the conjecture that the IIB matrix model describes the theory at the Planck scale.

There may be other field theories which have the same N dependence as the IIB matrix model, and one of them must be the true action of the IIB matrix model in the large N limit. If we could find such an action, we would have much more information, such as that regarding the true vacuum. Searching for such an action is one of the most important subjects in studying the IIB matrix model.

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