Ghost condensates in Yang-Mills theories in nonlinear gauges

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Abstract

Ghost condensates of dimension two are analysed in a class of nonlinear gauges in pure Yang-Mills theories. These condensates are related to the breaking of the $SL(2,R)$ symmetry, present in these gauges.

1 Introduction

Recently, great attention has been devoted to pure Yang-Mills theory quantized in nonlinear gauges, the aim being that of obtaining insights about the nonperturbative infrared behavior. For instance, the so called Maximal Abelian gauge [1, 2] has been extensively studied in the context of the Abelian dominance hypothesis, according to which the relevant low energy degrees of freedom for Yang-Mills should be described by an effective abelian theory with the addition of monopoles. The condensation of the monopoles should account for the confinement of the chromoelectric charges, according to the dual superconductivity mechanism [1, 3].

The Maximal Abelian gauge, being a nonlinear gauge, requires the introduction of a four ghost interaction term, needed for the renormalizability of the model [4, 5]. As a consequence, a nontrivial vacuum state arises [6, 7, 8, 9], corresponding to a nonvanishing value for the ghost condensates $\left\langle f^{i\alpha\beta}c^{\alpha}c^{\beta}\right\rangle$, $\left\langle f^{i\alpha\beta}\overline{c}^{\alpha}\overline{c}^{\beta}\right\rangle$, $\left\langle f^{i\alpha\beta}c^{\alpha}\overline{c}^{\beta}\right\rangle$ and $\langle\overline{c}^{\alpha}c^{\alpha}\rangle$, where the index i labels the $(N-1)$ diagonal generators of the Cartan subgroup of $SU(N)$, and α,β the $N(N-1)$ off-diagonal generators. These condensates turn out to display rather interesting features. They modify the behavior of the off-diagonal ghost propagator in the infrared region [6, 7, 8] and lower the vacuum energy density, being interpreted as a low-energy manifestation of the trace anomaly $\left\langle T_{\mu}^{\mu}\right\rangle$, which is related to the gluon condensate $\langle\alpha F^{2}\rangle$.

The aim of this work is to investigate the existence of the ghost condensates $\left\langle f^{i\alpha\beta}c^{\alpha}c^{\beta}\right\rangle$, $\left\langle f^{i\alpha\beta}\overline{c}^{\alpha}\overline{c}^{\beta}\right\rangle$ in another class of nonlinear gauges [10] containing a ghost self-interaction term, usually referred as the Curci-Ferrari gauge [11, 12, 13]. This point could be of some help in order to improve our understanding of the meaning of these condensates. Our analysis shows that these condensates seem not to be related to a specific gauge, as the Maximal Abelian gauge, being present also in the Curci-Ferrari gauge. As pointed out in [14], both the Maximal Abelian and the Curci-Ferrari gauge display a $SL(2,R)$ symmetry whose generators act nontrivially on the Faddeev-Popov ghosts, while leaving the gauge fields unchanged. It turns out indeed that the ghost condensates $\left\langle f^{i\alpha\beta}c^{\alpha}c^{\beta}\right\rangle$, $\left\langle f^{i\alpha\beta}\overline{c}^{\alpha}\overline{c}^{\beta}\right\rangle$, $\left\langle f^{i\alpha\beta}c^{\alpha}\overline{c}^{\beta}\right\rangle$ are precisely related to the dynamical breaking of $SL(2,R)$. It is worth mentioning here that the breaking can occur in different channels, according to which generators are broken. More specifically, the three generators of $SL(2,R)$, namely δ, $\overline{\delta}$ and δ_{FP} are known [15] to obey the algebra $\left[\delta,\overline{\delta}\right]=\delta_{FP}$, where δ_{FP} denotes the ghost number. The condensate $\left\langle f^{i\alpha\beta}c^{\alpha}\overline{c}^{\beta}\right\rangle$ in the Curci-Ferarri gauge has been discussed by [16] and corresponds to the breaking of the generators δ, $\overline{\delta}$. In the present work we shall analyse the other condensates $\left\langle f^{i\alpha\beta}c^{\alpha}c^{\beta}\right\rangle$, $\left\langle f^{i\alpha\beta}\overline{c}^{\alpha}\overline{c}^{\beta}\right\rangle$ which are related to the breaking of (δ, δ_{FP}) and of $\left(\overline{\delta}, \delta_{FP}\right)$, respectively. We remark also that the existence of different channels for the ghost condensation has an analogy in superconductivity, known as the BCS[1] versus the Overhauser[2] effect [17]. In the present case the Faddeev-Popov charged condensates $\left\langle f^{i\alpha\beta}c^{\alpha}c^{\beta}\right\rangle$, $\left\langle f^{i\alpha\beta}\overline{c}^{\alpha}\overline{c}^{\beta}\right\rangle$ would correspond to the BCS channel, while $\left\langle f^{i\alpha\beta}c^{\alpha}\overline{c}^{\beta}\right\rangle$ to the Overhauser channel.

Although the analysis of the condensate $\langle\overline{c}^{\alpha}c^{\alpha}\rangle$ is out of the aim of the present work, we underline that, in the Maximal Abelian gauge, it is believed to be part of a more general condensate, namely $\left(\frac{1}{2}\left\langle A^{\alpha}_{\mu}A^{\mu\alpha}\right\rangle - \xi\left\langle\overline{c}^{\alpha}c^{\alpha}\right\rangle\right)$, where ξ denotes the gauge parameter. This condensate has been proposed in [18] due to its BRST invariance. It is expected to provide effective masses for both off-diagonal gauge and ghost fields [9, 18], thus playing a very important role for the Abelian dominance. It is useful to note here that the operator $\left(\frac{1}{2}A^2 - \xi\overline{c}c\right)$ generalizes to the Curci-Ferrari gauge, displaying the important property of being multiplicatively renormalizable [19, 20].

The paper is organized as follows. In Sect.2 we present the model and its BRST quantization. Sect.3 is devoted to the evaluation of the one-loop effective potential for the ghost condensates and to the study of the vacuum configurations.

2 Yang-Mills in nonlinear gauges

Let us begin by reviewing the quantization of pure $SU(N)$ Yang-Mills in the Curci-Ferrari gauge. The gauge fixed action turns out to be

$$S = S_{\mathrm{YM}} + S_{\mathrm{gf}} \;, \tag{2.1}$$

where S_{YM} is the Yang-Mills action

$$S_{\mathrm{YM}} = -\frac{1}{4}\int d^4x F^{a\mu\nu}F^{a}_{\mu\nu} \;,$$

and S_{gf} denotes the nonlinear gauge fixing term with the quartic ghost interaction

$$S_{\mathrm{gf}} \;\; = \;\; s\int d^4x\left(\overline{c}^a\partial A^a + \frac{\xi}{2}\overline{c}^a b^a - \frac{\xi}{4}gf^{abc}\overline{c}^a\overline{c}^b c^c\right) \tag{2.2}$$

[1] Particle-particle and hole-hole pairing.

[2] Particle-hole pairing.

$$= \int d^4x \left(b^a \partial_\mu A^{\mu a} + \frac{\xi}{2} b^a b^a + \overline{c}^a \partial^\mu (D_\mu c)^a - \frac{\xi}{2} g f^{abc} b^a \overline{c}^b c^c - \frac{\xi}{8} g^2 f^{abc} \overline{c}^a \overline{c}^b f^{cmn} c^m c^n \right) .$$

The color indices run here over all generators of $SU(N)$, *i.e.* a,b,$c = 1, ..., N^2 - 1$. The operator s is the nilpotent BRST operator acting on the fields as

$$\begin{aligned} sA^a_\mu &= -(D_\mu c)^a \ , \qquad sc^a = \frac{g}{2} f^{abc} c^b c^c \ , \\ s\overline{c}^a &= b^a \qquad , \qquad sb^a = 0 \ , \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \ , \\ (D_\mu c)^a &= \partial_\mu c^a + g f^{abc} A^b_\mu c^c \ . \end{aligned} \tag{2.4}$$

Notice that expression (2.2) contains a unique parameter ξ. Moreover, as already mentioned, in addition to the BRST invariance, the model displays a further global $SL(2, R)$ symmetry [15]. The corresponding generators δ, $\overline{\delta}$ and δ_{FP} are given by

$$\begin{aligned} \delta\overline{c}^a &= c^a \ , \qquad \delta b^a = \frac{g}{2} f^{abc} c^b c^c \ , \\ \delta A^a_\mu &= 0 \ , \qquad \delta c^a = 0 \ , \end{aligned} \tag{2.5}$$

$$\begin{aligned} \overline{\delta} c^a &= \overline{c}^a \ , \qquad \overline{\delta} b^a = \frac{g}{2} f^{abc} \overline{c}^b \overline{c}^c \ , \\ \overline{\delta} A^a_\mu &= 0 \ , \qquad \overline{\delta}\overline{c}^a = 0 \ , \end{aligned} \tag{2.6}$$

and

$$\begin{aligned} \delta_{FP} c^a &= c^a \ , \qquad \delta_{FP} \overline{c}^a = -\overline{c}^a \ , \\ \delta_{FP} A^a_\mu &= 0 \ , \qquad \delta_{FP} b^a = 0 \ , \end{aligned} \tag{2.7}$$

with

$$\left[\delta, \overline{\delta} \right] = \delta_{FP} \ . \tag{2.8}$$

As proven in [12], the BRST symmetry together with the δ invariance are sufficient to ensure the perturbative renormalizability of the model, meaning that the gauge-fixing (2.2) is stable under radiative corrections.

3 The ghost condensates

In order to study the ghost condensates $\left\langle f^{abc}\overline{c}^a\overline{c}^b\right\rangle$, $\left\langle f^{abc}c^bc^c\right\rangle$ we first eliminate the Lagrange multiplier field b^a. From

$$\frac{\delta S}{\delta b^a} = \partial_\mu A^{\mu a} + \xi b^a - \frac{\xi}{2} g f^{abc}\overline{c}^b c^c \ , \tag{3.9}$$

we get

$$\begin{aligned} S &= S_{\mathrm{YM}} + \int d^4x \left(-\frac{1}{2\xi}\left(\partial A^a\right)^2 + \overline{c}^a \partial^\mu \left(D_\mu c\right)^a + \frac{g}{2} f^{abc}\partial A^a \overline{c}^b c^c \right) \\ &\quad + \int d^4x \left(-\frac{\xi}{8} g^2 \left(f^{abc}\overline{c}^b c^c\right)\left(f^{amn}\overline{c}^m c^n\right) - \frac{\xi}{8} g^2 \left(f^{abc}\overline{c}^a\overline{c}^b\right)\left(f^{cmn}c^m c^n\right) \right) \ . \end{aligned} \tag{3.10}$$

Using the Jacobi identity

$$\left(f^{abc}\overline{c}^b c^c\right)\left(f^{amn}\overline{c}^m c^n\right) = -\frac{1}{2}\left(f^{abc}\overline{c}^a\overline{c}^b\right)\left(f^{cmn}c^m c^n\right) \ , \tag{3.11}$$

it follows

$$\begin{aligned} S &= S_{\mathrm{YM}} + \int d^4x \left(-\frac{1}{2\xi}\left(\partial A^a\right)^2 + \overline{c}^a \partial^\mu \left(D_\mu c\right)^a + \frac{g}{2} f^{abc}\partial A^a \overline{c}^b c^c \right) \\ &\quad - \int d^4x \left(\frac{\xi}{16} g^2 \left(f^{abc}\overline{c}^a\overline{c}^b\right)\left(f^{cmn}c^m c^n\right) \right) \ . \end{aligned} \tag{3.12}$$

To evaluate the one-loop effective potential for the ghost condensation we introduce auxiliary fields σ^a, $\overline{\sigma}^a$ with ghost number 2 and -2, so that

$$-\frac{\xi}{16} g^2 \left(f^{abc}\overline{c}^a\overline{c}^b\right)\left(f^{cmn}c^m c^n\right) \Rightarrow \mathcal{L}_{\sigma\overline{\sigma}} \tag{3.13}$$

where

$$\mathcal{L}_{\sigma\overline{\sigma}} = -\frac{1}{\xi g^2}\sigma^a\overline{\sigma}^a + \frac{\overline{\sigma}^a}{4} f^{abc}c^b c^c - \frac{\sigma^a}{4} f^{abc}\overline{c}^b\overline{c}^c \ . \tag{3.14}$$

The relevant part of the action for the evaluation of the one-loop effective potential is given by

$$\begin{aligned} S_{c\overline{c}}^{\mathrm{quad}} &= \int d^4x \left(-\frac{1}{\xi g^2}\sigma^a\overline{\sigma}^a + \overline{c}^a\partial^2 c^a + \frac{\overline{\sigma}^a}{4} f^{abc}c^b c^c - \frac{\sigma^a}{4} f^{abc}\overline{c}^b\overline{c}^c \right) \\ &= \int d^4x \left[-\frac{1}{\xi g^2}\sigma^a\overline{\sigma}^a + \frac{1}{2}\begin{pmatrix} \overline{c}^a & c^a \end{pmatrix} \mathcal{M}^{ab} \begin{pmatrix} \overline{c}^b \\ c^b \end{pmatrix} \right] \ , \end{aligned} \tag{3.15}$$

where $\mathcal{M}^{ab}$ denotes the $(N^2-1)\times(N^2-1)$ matrix

$$\mathcal{M}^{ab} = \left(\begin{array}{cc} -\frac{1}{2}\sigma^c f^{cab} & \partial^2\delta^{ab} \\ -\partial^2\delta^{ab} & \frac{1}{2}\overline{\sigma}^c f^{cab} \end{array} \right) . \tag{3.16}$$

For the one-loop effective potential we get

$$V^{\rm eff}(\sigma,\overline{\sigma}) = \frac{1}{\xi g^2}\sigma^a\overline{\sigma}^a + \frac{i}{2}{\rm tr}\log\det\mathcal{M}^{ab}. \tag{3.17}$$

where $\sigma^a, \overline{\sigma}^a$ have to be considered constant fields and where we have factorized the space-time volume.

Let us proceed by working out in detail the example of $SU(2)$. In this case, we have $f^{abc} = \varepsilon^{abc}$ ($\varepsilon^{123}=1$), and $\mathcal{M}^{ab}$ is a 6×6 matrix. After a simple computation one has

$$V^{\rm eff}(\sigma,\overline{\sigma}) = \frac{1}{\xi g^2}\sigma^a\overline{\sigma}^a + i\int\frac{d^4k}{(2\pi)^4}\log\left(\left(k^2\right)^2 + \frac{\sigma^a\overline{\sigma}^a}{4}\right) . \tag{3.18}$$

From

$$\int\frac{d^4k}{(2\pi)^4}\log\left(\left(k^2\right)^2+\varphi^2\right) = \frac{i}{32\pi^2}\varphi^2\left(-\frac{1}{\varepsilon} - 2\gamma + 2\log 4\pi - \log\frac{\varphi^2}{\mu^4} + 3\right) , \tag{3.19}$$

and making use of the minimal subtraction scheme, the effective potential $V^{\rm eff}(\sigma,\overline{\sigma})$ is found to be

$$V^{\rm eff}(\sigma,\overline{\sigma}) = \frac{1}{\xi g^2}\sigma^a\overline{\sigma}^a + \frac{1}{32\pi^2}\frac{\sigma^a\overline{\sigma}^a}{4}\left(\log\frac{\sigma^a\overline{\sigma}^a}{4\left(4\pi\right)^2\mu^4} - 3 + 2\gamma\right) . \tag{3.20}$$

Let us look now at the minimum of the potential (3.20). It is given by the condition

$$\sigma^a_{\rm min}\overline{\sigma}^a_{\rm min} = 64\pi^2\mu^4 e^{(2-2\gamma)}\exp\left(-\frac{128}{\xi}\frac{\pi^2}{g^2}\right) , \tag{3.21}$$

which is physically consistent for any $\xi > 0$. Setting now

$$\sigma^a = \sigma\delta^{a3} \; , \quad \overline{\sigma}^a = \overline{\sigma}\delta^{a3} \; , \tag{3.22}$$

for the vacuum configuration we obtain

$$\sigma_{\rm min}\overline{\sigma}_{\rm min} = 64\pi^2\mu^4 e^{(2-2\gamma)}\exp\left(-\frac{128}{\xi}\frac{\pi^2}{g^2}\right) . \tag{3.23}$$

The nontrivial minimum configuration (3.23) means that the local operators $\varepsilon^{3bc}c^bc^c$, $\varepsilon^{3bc}\overline{c}^b\overline{c}^c$ acquire a nonvanishing vacuum expectation value, implying a dynamical breaking of $SL(2,R)$. Indeed

$$\begin{aligned}
\left\langle \varepsilon^{3bc}c^bc^c \right\rangle &= \frac{1}{2}\left\langle \delta_{FP}\left(\varepsilon^{3bc}c^bc^c\right)\right\rangle = \left\langle \delta\left(\varepsilon^{3bc}\overline{c}^bc^c\right)\right\rangle \ , \\
\left\langle \varepsilon^{3bc}\overline{c}^b\overline{c}^c \right\rangle &= -\frac{1}{2}\left\langle \delta_{FP}\left(\varepsilon^{3bc}\overline{c}^b\overline{c}^c\right)\right\rangle = \left\langle \overline{\delta}\left(\varepsilon^{3bc}c^b\overline{c}^c\right)\right\rangle \ .
\end{aligned} \tag{3.24}$$

Observe also that the vacuum configuration (3.22), (3.23) leaves invariant the Cartan subgroup $U(1)$ of $SU(2)$, in analogy with the Maximal Abelian gauge.

It remains now to face the important question of the stability of the nontrivial vacuum (3.23), whose physical meaning relies on the existence of a positive value for the parameter ξ, which plays the role of a coupling constant for the quartic ghost self-interaction. A natural choice for the parameter ξ would be given by the fixed point of the corresponding renormalization group function β_ξ. This would require the knowledge of the existence of a nonperturbative fixed point for ξ, which is beyond our present possibilities. However, it worth to remind that a detailed analysis of the renormalization of the Curci-Ferrari gauge has been performed at one-loop level by [19]. Recently, the authors [20] have worked out the two and three loops contributions. In particular, according to [19, 20], the running of ξ at one-loop order is found to be

$$\beta_\xi = \frac{\mu}{\xi}\frac{\partial\xi}{\partial\mu} = \left(\frac{13}{3} - \frac{\xi}{2}\right)\frac{g^2N}{16\pi^2} \ , \tag{3.25}$$

showing that the value $\xi = 26/3$ is a fixed point, matching the requirements for a nontrivial vacuum. Although one cannot provide a definitive answer, these results give evidences for the existence of the ghost condensation in the Curci-Ferrari gauge.

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