

Eight Dimensional Noncommutative Instantons and D0-D8 Bound States with B -field

Yoshiki Hiraoka

*Department of Physics, Tohoku University
Sendai 980-8578, JAPAN*

`hiraoka@tuhep.phys.tohoku.ac.jp`

Abstract

We construct some classes of instanton solutions of eight dimensional noncommutative ADHM equations generalizing the solutions of eight dimensional commutative ADHM equations found by Papadopoulos and Teschendorff, and interpret them as supersymmetric $D0$ - $D8$ bound states in a NS B -field. Especially, we consider the $D0$ - $D8$ system with anti-self-dual B -field preserving 3/16 of supercharges. This system and self-duality conditions are related with the group $Sp(2)$ which is a subgroup of the eight dimensional rotation group $SO(8)$.

1 Introduction

Recently noncommutative geometry has appeared in the context of M(atrix)-theory compactifications [1] and the D -branes in a constant NS B -field [2]. In particular, the D -branes in a constant NS B -field have attracted much interest in the understanding of non-perturbative aspects of string theory. The noncommutative Yang-Mills theory which appears as an effective world-volume field theory on the D -branes with a B -field has an interesting feature that the singularity of the instanton moduli space is naturally resolved [3].

Four dimensional $U(N)$ k -instanton is realized as k $D0$ -branes within N $D4$ -branes in type IIA string theory. When we turn on an anti-self-dual constant B -field, for preserving $1/4$ of supercharges, the instanton moduli space is resolved, and $D0$ -branes cannot escape from $D4$ -branes. From the viewpoint of $D0$ -brane theory, the moduli space of vacua of the Higgs branch coincides with the moduli space of instantons and the anti-self-dual B -field corresponds to the Fayet-Iliopoulos (FI) parameters. If the FI parameters are non-zero, the $D0$ - $D4$ system can not enter the Coulomb branch through the small instanton singularity.

On the other hand, it is also of interest to generalize the above case to higher dimensional systems in the context of both brane dynamics and the brane world-volume theories. These systems with a constant B -field are considered from various points of view in [8, 9, 10, 11, 19], and are shown equivalent by T-duality to the systems that are rotated branes at angles by several authors [18, 19, 21, 22, 23]. In particular they showed that there are three cases preserving respectively $1/16$, $1/8$ and $3/16$ of supercharges in the $D0$ - $D8$ systems with a B -field that must satisfy certain relations in each case. Concretely in the $D0$ - $D8$ system, the constant B -field satisfies the extended “self-dual” conditions given in [12, 13] which associate with the subgroup $Spin(7)$, $SU(4)$ and $Sp(2)$ of the eight dimensional rotational group $SO(8)$. These B -fields preserve $1/16$, $1/8$ and $3/16$ of supercharges respectively.

The ADHM construction is a powerful tool to construct self-dual Yang-Mills instantons [15, 16]. Especially in four dimensions we know that the instanton moduli space and the ADHM moduli space completely coincide. In eight dimensions, the ADHM construction [17] of “self-dual” instantons associated with the group $Sp(2)$ is known, but we does not know whether this ADHM construction gives all solutions of “self-dual” equations associated with the group $Sp(2)$. However, some simple solutions of eight dimensional ADHM equations are already known in [17, 21]. Eight dimensional noncommutative ADHM equations were proposed in [19], but these equations are difficult to solve even in the simple case. Some properties of moduli space of these equations were also discussed in [20].

This paper is organized as follows. In section 2, we describe eight dimensional noncom-

mutative Yang-Mills instantons associated with the group $Sp(2)$ as $D0$ - $D8$ bound states in a B -field using the noncommutative eight dimensional ADHM construction. In this case, the B -field is also necessary to satisfy the extended “(anti-)self-dual” relations. We can construct some new classes of non-trivial solutions extending the eight dimensional ADHM constructions of [17] and the solutions found by Papadopoulos and Teschendorff [21] in the commutative case, and interpret them as $D0$ - $D8$ bound states. The final section is devoted to discussions.

2 Noncommutative instantons on \mathbf{R}^8 as $D0$ - $D8$ bound states with a B -field

In this section, we construct noncommutative instantons on \mathbf{R}^8 and interpret them as supersymmetric $D0$ - $D8$ bound states with a certain B -field. This noncommutativity is induced by a constant NS B -field on the $D8$ -brane. In the following, we consider the case of the gauge group $U(N)$ with the instanton number k since these states correspond to the bound states of k $D0$ -branes and N $D8$ -branes.

We also construct some solutions of the eight dimensional noncommutative ADHM equations. Eight dimensional noncommutative ADHM equations were proposed in [19], but it is difficult to solve those equations even in the $U(2)$ case. So we try to solve the equations by generalizing the solutions found by Papadopoulos and Teschendorff [21, 22] in the commutative case.

2.1 Eight dimensional ADHM construction

In this subsection, we consider the extended ADHM construction of the eight dimensional “self-dual” instantons associated with the $Sp(2)$ group given in [13, 17]. This construction of instantons is the slight extension of the four dimensional ADHM construction. When we take $\mathbf{B} = 0$, or $\mathbf{B}' = 0$ which are defined in the following, we will reproduce the four dimensional ADHM equations. Some simple solutions to these equations were constructed in [3, 4, 5, 6, 7] and others.

In order to treat the eight dimensional space, it is useful to regard eight coordinates of \mathbf{R}^8 as two quaternionic coordinates

$$\mathbf{x} = \sum_{\mu=1}^8 \tilde{\sigma}_{\mu} x^{\mu} = \begin{pmatrix} z_2 & z_1 \\ -\bar{z}_1 & \bar{z}_2 \end{pmatrix}, \quad \mathbf{x}' = \sum_{\mu=1}^8 \tilde{\sigma}'_{\mu} x^{\mu} = \begin{pmatrix} z_4 & z_3 \\ -\bar{z}_3 & \bar{z}_4 \end{pmatrix}, \quad (2.1)$$

where we defined the eight vector matrices

$$\tilde{\sigma}_\mu = (i\tau_1, 0, i\tau_2, 0, i\tau_3, 0, \mathbf{1}_2, 0), \quad (2.2)$$

$$\tilde{\sigma}'_\mu = (0, i\tau_1, 0, i\tau_2, 0, i\tau_3, 0, \mathbf{1}_2), \quad (2.3)$$

and the four complex coordinates

$$z_1 = x^3 + ix^1, \quad z_2 = x^7 + ix^5, \quad z_3 = x^4 + ix^2, \quad z_4 = x^8 + ix^6. \quad (2.4)$$

Using the $(N + 2k) \times 2k$ matrices \mathbf{A} , \mathbf{B} and \mathbf{B}' , we next define the Dirac-like operator

$$D_z = \mathbf{A} + \vec{\mathbf{B}} \cdot \vec{\mathbf{X}} \quad (2.5)$$

where $\vec{\mathbf{B}} = (\mathbf{B}, \mathbf{B}')$ and $\vec{\mathbf{X}} = (\mathbf{x}, \mathbf{x}')$.

If we solve the following Dirac-like equations

$$D_z^\dagger \psi = 0, \quad (2.6)$$

for the $(N + 2k) \times N$ matrix ψ which is normalized as $\psi^\dagger \psi = \mathbf{1}_{N \times N}$, we can construct the $U(N)$ gauge field as

$$A_\mu = \psi^\dagger \partial_\mu \psi. \quad (2.7)$$

Then using the relations

$$\Sigma_\mu \equiv \partial_\mu \vec{\mathbf{X}} = \begin{pmatrix} \tilde{\sigma}_\mu \\ \tilde{\sigma}'_\mu \end{pmatrix}, \quad (2.8)$$

and the completeness equation

$$\mathbf{1}_{N+2k} = \psi \psi^\dagger + D_z \frac{1}{D_z^\dagger D_z} D_z^\dagger, \quad (2.9)$$

we can obtain the “self-dual” gauge field strength as

$$\begin{aligned} F_{\mu\nu} &= 2\psi^\dagger \left(\partial_{[\mu} D_z \frac{1}{D_z^\dagger D_z} \partial_{\nu]} D_z^\dagger \right) \psi \\ &= 2\psi^\dagger \vec{\mathbf{B}} \overline{N}_{\mu\nu} \frac{1}{D_z^\dagger D_z} \vec{\mathbf{B}}^\dagger \psi, \end{aligned} \quad (2.10)$$

where $\overline{N}_{\mu\nu} = \frac{1}{2}(\Sigma_\mu \Sigma_\nu^\dagger - \Sigma_\nu \Sigma_\mu^\dagger)$ is a “self-dual” tensor satisfying

$$\frac{1}{2} T_{\mu\nu\rho\sigma} \overline{N}_{\rho\sigma} = \overline{N}_{\mu\nu}. \quad (2.11)$$

Here we must require that $D_z^\dagger D_z$ commutes with Σ_μ . This is a necessary condition to obtain the “self-dual” gauge field strength on the \mathbf{R}^8 . This condition corresponds to the eight dimensional ADHM equations both for commutative and noncommutative case.

2.2 Eight dimensional noncommutative ADHM construction

We can obtain supersymmetric $D0$ - $D8$ bound states with a B -field by T-duality from intersecting $D4$ -branes at four angles. Supersymmetry condition reduces to three cases preserving $1/16$, $1/8$ and $3/16$ of supercharges respectively. In the following, we concentrate on the $3/16$ BPS states. In this case, the B -field is necessary to satisfy the generalized “(anti-)self-dual” equations [12, 13, 14] which are written by using the $Sp(2) \subset SO(8)$ invariant tensor $T_{\mu\nu\rho\sigma}$ as

$$\frac{1}{2}T_{\mu\nu\rho\sigma}B^{\rho\sigma} = \lambda B_{\mu\nu}. \quad (2.12)$$

As in the four dimensional case, it is also easy to extend the eight dimensional ADHM construction to noncommutative space because of its algebraic nature. Since we define instantons as “self-dual” configurations, the “anti-self-dual” B -field is of interest from the viewpoint of the instanton moduli space resolution. In this case the coordinates of \mathbf{R}^8 become noncommutative as

$$[z_1, \bar{z}_1] = -[z_2, \bar{z}_2] = [z_3, \bar{z}_3] = -[z_4, \bar{z}_4] = -\frac{\zeta}{2}, \quad (2.13)$$

for a positive constant parameter ζ . These commutation relations can be represented using creation and annihilation operators;

$$\begin{aligned} \sqrt{\frac{2}{\zeta}}z_1 |n_1 : n_2 : n_3 : n_4\rangle &= \sqrt{n_1 + 1} |n_1 + 1 : n_2 : n_3 : n_4\rangle, \\ \sqrt{\frac{2}{\zeta}}\bar{z}_1 |n_1 : n_2 : n_3 : n_4\rangle &= \sqrt{n_1} |n_1 - 1 : n_2 : n_3 : n_4\rangle, \\ \sqrt{\frac{2}{\zeta}}\bar{z}_2 |n_1 : n_2 : n_3 : n_4\rangle &= \sqrt{n_2 + 1} |n_1 : n_2 + 1 : n_3 : n_4\rangle, \\ \sqrt{\frac{2}{\zeta}}z_2 |n_1 : n_2 : n_3 : n_4\rangle &= \sqrt{n_2} |n_1 : n_2 - 1 : n_3 : n_4\rangle, \\ \sqrt{\frac{2}{\zeta}}z_3 |n_1 : n_2 : n_3 : n_4\rangle &= \sqrt{n_3 + 1} |n_1 : n_2 : n_3 + 1 : n_4\rangle, \\ \sqrt{\frac{2}{\zeta}}\bar{z}_3 |n_1 : n_2 : n_3 : n_4\rangle &= \sqrt{n_3} |n_1 : n_2 : n_3 - 1 : n_4\rangle, \\ \sqrt{\frac{2}{\zeta}}\bar{z}_4 |n_1 : n_2 : n_3 : n_4\rangle &= \sqrt{n_4 + 1} |n_1 : n_2 : n_3 : n_4 + 1\rangle, \\ \sqrt{\frac{2}{\zeta}}z_4 |n_1 : n_2 : n_3 : n_4\rangle &= \sqrt{n_4} |n_1 : n_2 : n_3 : n_4 - 1\rangle. \end{aligned} \quad (2.14)$$

If we also require that $D_z^\dagger D_z$ commutes with Σ_μ as in the commutative case, we can obtain the “self-dual” gauge field strength on the noncommutative \mathbf{R}^8 . As in the four

dimensional case, there are also equivalence relations between different sets of matrices \mathbf{A} , \mathbf{B} and \mathbf{B}' . Using these relations, the eight dimensional noncommutative ADHM equations were proposed in [19]. However it is even difficult to find simple explicit solutions of those equations. Then instead of solving them, we solve the conditions for “self-duality” as [21], and give some simple solutions below.

$U(1)$ one-instanton solutions

In the noncommutative case, $U(1)$ instanton is already non-trivial. In this case, D_z becomes a 3×2 matrix. If we consider the following ansatz;

$$D_z = \begin{pmatrix} z_2 + z_4 & z_1 + z_3 \\ -\bar{z}_1 - \bar{z}_3 & \bar{z}_2 + \bar{z}_4 \\ A_1 & A_2 \end{pmatrix}, \quad (2.15)$$

the commuting condition of $D_z^\dagger D_z$ becomes

$$[z_1, \bar{z}_1] - [z_2, \bar{z}_2] + [z_3, \bar{z}_3] - [z_4, \bar{z}_4] + A_1^\dagger A_1 - A_2^\dagger A_2 = 0, \quad A_1^\dagger A_2 = 0. \quad (2.16)$$

When $\zeta > 0$, these equations have a non-trivial solution as

$$A_1 = \sqrt{2\zeta}, \quad A_2 = 0. \quad (2.17)$$

Then D_z becomes

$$D_z = \begin{pmatrix} z_2 + z_4 & z_1 + z_3 \\ -\bar{z}_1 - \bar{z}_3 & \bar{z}_2 + \bar{z}_4 \\ \sqrt{2\zeta} & 0 \end{pmatrix}. \quad (2.18)$$

This is the simplest extension of the four dimensional $U(1)$ one-instanton solution found by [3, 6]. Zero mode ψ of the D_z^\dagger is given by

$$\psi = \begin{pmatrix} -\sqrt{2\zeta}(z_2 + z_4) \\ \sqrt{2\zeta}(\bar{z}_1 + \bar{z}_3) \\ (z_1 + z_3)(\bar{z}_1 + \bar{z}_3) + (\bar{z}_2 + \bar{z}_4)(z_2 + z_4) \end{pmatrix}. \quad (2.19)$$

As in the four dimensional case, $\psi^\dagger \psi$ annihilates $|0 : 0 : 0 : 0\rangle \langle 0 : 0 : 0 : 0|$ so that we must normalize the zero mode in the subspace of the Fock space where $|0 : 0 : 0 : 0\rangle$ is projected out [5, 6].

We can generalize the above solution assuming the ansatz such as [21];

$$D_z = \begin{pmatrix} (\mathbf{p})^\dagger \mathbf{x} + (\mathbf{p}')^\dagger \mathbf{x}' - \mathbf{a} \\ A \end{pmatrix}. \quad (2.20)$$

Here \mathbf{p} , \mathbf{p}' and \mathbf{a} are assumed to be arbitrary quaternions, and A is a 1×2 matrix. Then the commuting condition of $D_z^\dagger D_z$ can be solved when $\zeta > 0$ as

$$A = \begin{pmatrix} \sqrt{\zeta ((\mathbf{p})^\dagger \mathbf{p} + (\mathbf{p}')^\dagger \mathbf{p}')} & 0 \end{pmatrix}. \quad (2.21)$$

When $\mathbf{p} = \mathbf{p}' = \mathbf{1}_2$ and $\mathbf{a} = \mathbf{0}_2$ the above solution reduces to (2.18), and when $\mathbf{p}' = \mathbf{a} = \mathbf{0}$ and $\mathbf{p} = \mathbf{1}_2$ to the four dimensional noncommutative $U(1)$ one-instanton solution found by [5, 6] corresponding to $D4$ - $D8$ bound states in this case. If instead $\mathbf{p} = \mathbf{a} = \mathbf{0}$ and $\mathbf{p}' = \mathbf{1}_2$, the solution reduces to the other four dimensional noncommutative $U(1)$ one-instanton solution. Therefore our solutions are natural to extend the four dimensional solutions.

$U(1)$ two-instanton solutions

In this case D_z becomes a 5×4 matrix. We assume the ansatz such as [21];

$$D_z = \begin{pmatrix} (\mathbf{p}_1)^\dagger \mathbf{x}^1 + (\mathbf{p}'_1)^\dagger \mathbf{x}^2 - \mathbf{a}_1 & 0 \\ 0 & (\mathbf{p}_2)^\dagger \mathbf{x}^1 + (\mathbf{p}'_2)^\dagger \mathbf{x}^2 - \mathbf{a}_2 \\ -\lambda_1 A_1 & -\lambda_2 A_2 \end{pmatrix}, \quad (2.22)$$

where $\mathbf{p}_{1,2}$, $\mathbf{p}'_{1,2}$ and $\mathbf{a}_{1,2}$ are assumed to be arbitrary quaternions, $A_{1,2}$ are 1×2 matrices, and $\lambda_{1,2}$ are constants. Then if we take $\lambda_{1,2}$ as

$$\lambda_1 = \sqrt{\zeta ((\mathbf{p}_1)^\dagger \mathbf{p}_1 + (\mathbf{p}'_1)^\dagger \mathbf{p}'_1)}, \quad \lambda_2 = \sqrt{\zeta ((\mathbf{p}_2)^\dagger \mathbf{p}_2 + (\mathbf{p}'_2)^\dagger \mathbf{p}'_2)}, \quad (2.23)$$

the commuting condition of $D_z^\dagger D_z$ becomes the following equations;

$$(A_1^\dagger A_1)_{11} - (A_1^\dagger A_1)_{22} = 1, \quad (A_2^\dagger A_2)_{11} - (A_2^\dagger A_2)_{22} = 1, \quad (2.24)$$

$$A_1^\dagger A_2 \propto 1_{2 \times 2}, \quad A_2^\dagger A_1 \propto 1_{2 \times 2}, \quad (A_1^\dagger A_1)_{12} = (A_2^\dagger A_2)_{12} = 0. \quad (2.25)$$

Now A_1 and A_2 are 1×2 matrices, these equations seem to have no non-trivial solutions. However if we set \mathbf{p}_1 , \mathbf{p}_2 or \mathbf{p}'_1 , \mathbf{p}'_2 to zero, these equations reduce to the four dimensional $U(1)$ two-instanton ADHM equations, then can have non-trivial solutions. As we will see below, in the $U(2)$ case we can construct non-trivial eight dimensional two-instanton solutions using the above ansatz.

$U(2)$ one-instanton solutions

In this case, D_z becomes a 4×4 matrix. Here also we assume the ansatz such as

$$D_z = \begin{pmatrix} (\mathbf{p})^\dagger \mathbf{x}^1 + (\mathbf{p}')^\dagger \mathbf{x}^2 - \mathbf{a} \\ -\lambda A \end{pmatrix}, \quad (2.26)$$

where \mathbf{p} , \mathbf{p}' and \mathbf{a} are assumed to be arbitrary quaternions, A is a 2×2 matrix which is not necessary to be quaternion, and λ is a constant. Then the commuting condition of $D_z^\dagger D_z$ becomes the following equations;

$$\lambda^2 \left((A^\dagger A)_{11} - (A^\dagger A)_{22} \right) = \zeta \left((\mathbf{p})^\dagger \mathbf{p} + (\mathbf{p}')^\dagger \mathbf{p}' \right), \quad (A^\dagger A)_{12} = 0. \quad (2.27)$$

If we take λ as

$$\lambda = \sqrt{\zeta \left((\mathbf{p})^\dagger \mathbf{p} + (\mathbf{p}')^\dagger \mathbf{p}' \right)}, \quad (2.28)$$

then a matrix A is for example given by

$$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \quad (2.29)$$

This gives an eight dimensional noncommutative $U(2)$ one-instanton solution.

$U(2)$ two-instanton solutions

In this case, D_z is a 6×4 matrix. Here also we assume the ansatz such as

$$D_z = \begin{pmatrix} (\mathbf{p}_1)^\dagger \mathbf{x}^1 + (\mathbf{p}'_1)^\dagger \mathbf{x}^2 - (\mathbf{a}_1) & 0 \\ 0 & (\mathbf{p}_2)^\dagger \mathbf{x}^1 + (\mathbf{p}'_2)^\dagger \mathbf{x}^2 - (\mathbf{a}_2) \\ -\lambda_1 A_1 & -\lambda_2 A_2 \end{pmatrix}, \quad (2.30)$$

where $\mathbf{p}_{1,2}$, $\mathbf{p}'_{1,2}$ and $\mathbf{a}_{1,2}$ are assumed to be arbitrary quaternions, $A_{1,2}$ are 2×2 matrices which are not necessary to be quaternions, and $\lambda_{1,2}$ are constants. Then if we take $\lambda_{1,2}$ as

$$\lambda_1 = \sqrt{\zeta \left((\mathbf{p}_1)^\dagger \mathbf{p}_1 + (\mathbf{p}'_1)^\dagger \mathbf{p}'_1 \right)}, \quad (2.31)$$

$$\lambda_2 = \sqrt{\zeta \left((\mathbf{p}_2)^\dagger \mathbf{p}_2 + (\mathbf{p}'_2)^\dagger \mathbf{p}'_2 \right)}, \quad (2.32)$$

the commuting condition of $D_z^\dagger D_z$ becomes the following equations;

$$(A_1^\dagger A_1)_{11} - (A_1^\dagger A_1)_{22} = 1, \quad (A_2^\dagger A_2)_{11} - (A_2^\dagger A_2)_{22} = 1, \quad (2.33)$$

$$A_1^\dagger A_2 \propto 1_{2 \times 2}, \quad A_2^\dagger A_1 \propto 1_{2 \times 2}, \quad (A_1^\dagger A_1)_{12} = (A_2^\dagger A_2)_{12} = 0. \quad (2.34)$$

In contrast with the $U(1)$ two-instanton case, these equations have for example a non-trivial solution;

$$A_1 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \quad (2.35)$$

This gives an eight dimensional noncommutative $U(2)$ two-instanton solution.

$U(2)$ k -instanton solutions

The above case can be easily generalized to multi-instanton solution. In this case, D_z becomes a $(2 + 2k) \times (2k)$ matrix. We consider the following ansatz;

$$D_z = \begin{pmatrix} (\mathbf{p}_1)^\dagger \mathbf{x}^1 + (\mathbf{p}'_1)^\dagger \mathbf{x}^2 - (\mathbf{a}_1) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & (\mathbf{p}_k)^\dagger \mathbf{x}^1 + (\mathbf{p}'_k)^\dagger \mathbf{x}^2 - (\mathbf{a}_k) \\ -\lambda_1 A_1 & \dots & -\lambda_k A_k \end{pmatrix}, \quad (2.36)$$

where $\mathbf{p}_{1,\dots,k}$, $\mathbf{p}'_{1,\dots,k}$ and $\mathbf{a}_{1,\dots,k}$ are assumed to be arbitrary quaternions, $A_{1,\dots,k}$ are 2×2 matrices which are not necessary to be quaternions, and $\lambda_{1,\dots,k}$ are constants. Then if we take $\lambda_{1,\dots,k}$ as

$$\lambda_1 = \sqrt{\zeta ((\mathbf{p}_1)^\dagger \mathbf{p}_1 + (\mathbf{p}'_1)^\dagger \mathbf{p}'_1)}, \quad (2.37)$$

$$\vdots$$

$$\lambda_k = \sqrt{\zeta ((\mathbf{p}_k)^\dagger \mathbf{p}_k + (\mathbf{p}'_k)^\dagger \mathbf{p}'_k)}, \quad (2.38)$$

the commuting condition of $D_z^\dagger D_z$ becomes the following equations;

$$(A_1^\dagger A_1)_{11} - (A_1^\dagger A_1)_{22} = 1, \dots, (A_k^\dagger A_k)_{11} - (A_k^\dagger A_k)_{22} = 1, \quad (2.39)$$

$$A_1^\dagger A_k \propto 1_{2 \times 2}, \quad A_2^\dagger A_k \propto 1_{2 \times 2}, \dots, A_{k-1}^\dagger A_k \propto 1_{2 \times 2}, \quad (2.40)$$

$$(A_1^\dagger A_1)_{12} = \dots = (A_k^\dagger A_k)_{12} = 0. \quad (2.41)$$

These equations have for example a non-trivial solution;

$$A_1 = A_2 = \dots = A_{k-1} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad A_k = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \quad (2.42)$$

This gives an eight dimensional noncommutative $U(2)$ k -instanton solution.

3 Discussions

In this paper, we described eight dimensional noncommutative Yang-Mills instantons associated with the group $Sp(2)$ as $D0$ - $D8$ bound states in a B -field using the noncommutative eight dimensional ADHM construction. In this case, the B -field was necessary to satisfy the extended “(anti-)self-dual” relations for preserving some proportions of supercharges. We constructed some new classes of non-trivial solutions of the eight dimensional

noncommutative ADHM equations generalizing the solutions found by Papadopoulos and Teschendorff [21] in the commutative case, and interpreted them as $D0$ - $D8$ bound states.

However in the noncommutative case it is non-trivial whether the instanton number is actually the expected one, therefore it would be interesting to calculate explicitly the instanton numbers.

In the commutative case, (2.30) is a solution of eight dimensional ADHM equations when $A_1(=A_2)$ is arbitrary quaternion and $\lambda_{1,2}$ are arbitrary constants. Then there exists following equivalence relations ;

$$\{\mathbf{p}_1, \mathbf{p}'_1, \mathbf{p}_2, \mathbf{p}'_2, \mathbf{a}_1^\dagger, \mathbf{a}_2^\dagger, A_1^\dagger\} \sim \{\mathbf{p}_1\mathbf{s}, \mathbf{p}'_1\mathbf{s}, \mathbf{p}_2\mathbf{s}, \mathbf{p}'_2\mathbf{s}, \mathbf{a}_1^\dagger\mathbf{s}, \mathbf{a}_2^\dagger\mathbf{s}, A_1^\dagger\mathbf{s}\},$$

where $\mathbf{s} \in \mathbf{H}$. Therefore the moduli space structure of the above solution may be related with the Grassmannian ;

$$\frac{Sp(2)}{Sp(1) \times Sp(1)},$$

and this may suggest that the moduli space of the eight dimensional instanton will have an $Sp(2)$ holonomy [19]. It is not yet made clear how the noncommutativity deforms the moduli space structure of the eight dimensional instantons, but we expect that our solutions will shed light on this problem.

It is also of interest to search for the ADHM constructions associated with the group $Spin(7)$ and $SU(4)$. In the $Spin(7)$ case, some solutions are known as octonionic Yang-Mills instantons in [24] which are constructed on the $Spin(7)$ holonomy manifold (gravitational instanton) by the standard embedding of the spin connection into the Yang-Mills connection. In four dimensional case, the ADHM constructions of instantons on an ALE space (gravitational instanton) are well-known [25]. Eight dimensional ALE spaces are considered in [26, 27]. Therefore we expect that similar constructions are possible in the eight-dimensional case, and our solutions give some insight into the moduli space structures of eight dimensional instantons.

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