Ramond-Ramond Couplings of Noncommutative Branes

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ABSTRACT. We obtain the couplings of noncommutative branes of type II string theories to constant Ramond-Ramond backgrounds, for BPS as well as non-BPS branes, in the background-independent description. For the BPS branes, we also generalize these couplings to other descriptions, and thereby argue their equivalence to the known couplings in the commutative description. The first part is a review of earlier work while the second part contains some additional observations.

1. Introduction

Much insight has been gained into the dynamics of branes in string theory using noncommutativity [**2, 3**]. In the presence of a constant 2-form B-field, one finds that the world-volume action of a D-brane can be described either in terms of commutative or noncommutative variables. Using the continuous description parameter Φ, one can actually interpolate between the two types of descriptions. In Ref.[**3**] the DBI action for a D-brane in the presence of a constant B-field was proposed in a general description and was shown to be equivalent to the commutative one.

In recent times noncommutativity has also proved useful in understanding the issue of tachyon condensation in the case of unstable branes [**7**]-[**13**]. On an unstable non-BPS D-brane one gets a noncommutative field theory involving tachyonic scalars, which generically admits static solitonic solutions over which tachyon condensation can occur representing brane decay.

A distinguished feature of D-branes in Superstring theories is that they couple to the Ramond-Ramond fields given by the Chern-Simons terms. With the recent interest in the noncommutative descriptions of D-branes, a natural question to ask is: How are these couplings described in the noncommutative language? Here we try to address this question for the case of constant Ramond-Ramond fields for both BPS as well as non-BPS branes of type II string theories.

We first review the noncommutative DBI action for a single (Euclidean) Dp-brane. Thinking of a Dp-brane as a classical configuration of infinitely many D(-1)-branes, we obtain the noncommutative DBI action in $\Phi = -B$ description [**4, 5**]. We then follow the same prescription to obtain the Noncommutative Chern-Simons terms in the background-independent $\Phi = -B$ description. Next we propose analogous couplings on non-BPS branes. One key property of noncommutative solitons is that the tachyon condensation over them can produce N coincident lower dimensional D-branes starting from a single higher dimensional noncommutative brane. In recent times it has emerged that collections of N D-branes have extra commutator couplings in their world-volume theory, to Ramond-Ramond potentials, that do not exist for a single D-brane [**6, 17**]. We show that such terms can be obtained by producing the lower dimensional branes via tachyon condensation over

an appropriate noncommutative soliton starting from our proposed answer for the Ramond-Ramond couplings. Finally we propose generalisations of the noncommutative Chern-Simons terms on the BPS branes to other descriptions and show that these couplings are equivalent to the commutative ones in the DBI approximation (upto total derivatives). We work with Euclidean branes with an even number of world-volume directions. In what follows we set $2\pi\alpha' = 1$. The talk on which this article is based[1] contained a review of Ref.[19], along with some new observations about description dependence. Below, in the conclusions, we also briefly review our subsequent work, Refs.[25, 27].

In the presence of constant NS-NS B-field the dynamics of a (Euclidean) Dp-brane can be described by the following DBI action[3]:

$$\hat{S}_{DBI} \quad = \quad \hat{T}_p \int d^{p+1}x \ \sqrt{\det\big(G_{ij} + \hat{F}_{ij} + \Phi_{ij}\big)}$$

where $\hat{T}_p = (2\pi)^{\frac{1-p}{2}}/G_s$ and the field strength is

(1.2)
$$\hat{F}_{ij} = \partial_i \hat{A}_j - \partial_j \hat{A}_i - i[\hat{A}_i, \hat{A}_j]_*$$

The products of fields appearing in this equation are understood to be $*$ products given by:

(1.3)
$$f(x) * g(x) \equiv e^{\frac{i}{2}\theta^{ij}\partial_i\partial'_j} f(x)g(x')\big|_{x=x'}$$

The parameters G_{ij}, Φ_{ij}, G_s and θ^{ij} are given in terms of the commutative variables g, B, g_s by:

(1.4)
$$\frac{1}{G + \Phi} = -\theta + \frac{1}{g + B}, \qquad\qquad G_s = g_s \left(\frac{\det(G + \Phi)}{\det(g + B)}\right)^{\frac{1}{2}}$$

where Φ denotes the freedom in description. Three choices of Φ are of particular interest. They are $\Phi_{ij} = B_{ij}$, $\Phi_{ij} = 0$ and $\Phi_{ij} = -B_{ij}$. The action in terms of commutative variables with ordinary products corresponds to $\Phi_{ij} = B_{ij}$ description. $\Phi_{ij} = -B_{ij}$ is the background independent description. In this description, we have

(1.5)
$$\theta^{ij} = (B^{-1})^{ij}, \qquad\qquad G_{ij} = -B_{ik}g^{kl}B_{lj}, \qquad\qquad G_s = g_s\sqrt{\frac{\det B}{\det g}}$$

Working in $\Phi = -B$ description $\hat{S}_{DBI}$ can be put in the form:

(1.6)
$$\hat{S}_{DBI} = T_p \int d^{p+1}x \ \frac{\mathrm{Pf}\,Q}{\mathrm{Pf}\,\theta} \ \sqrt{\det\big(g_{ij} + (Q^{-1})_{ij}\big)}$$

Where $Q^{ij} = \theta^{ij} - \theta^{ik}\hat{F}_{kl}\theta^{lj} = -i[X^i, X^j]$ with $X^i = x^i + \theta^{ik}\hat{A}_k$ and $T_p = (2\pi)^{\frac{1-p}{2}}/g_s$

We re-express this action in terms of a trace Tr over a Hilbert space. Using

(1.7)
$$\int d^{p+1}x \to \mathrm{Tr}(2\pi)^{\frac{p+1}{2}}\mathrm{Pf}\,\theta$$

we get

(1.8)
$$\hat{S}_{DBI} = \frac{2\pi}{g_s}\mathrm{Tr}\left[\mathrm{Pf}\,Q\sqrt{\det\big(g_{ij} + (Q^{-1})_{ij}\big)}\right]$$

This action can be thought of as the action for infinitely many D(-1) - branes in a classical configuration. To see this we can start with the nonabelian DBI action for N D(-1)-branes (with N $\Rightarrow \infty$) [6]:

(1.9)
$$\hat{S}_{DBI} = \frac{2\pi}{g_s}\mathrm{Tr}\left[\sqrt{\det\big(\delta_i^j - ig_{ik}[X^k, X^j]\big)}\right]$$

Now consider the solution corresponding to a Dp-brane:

$$\begin{aligned} X^i &= x^i & \text{for} \quad i &= 1, 2, ..., p+1 \\ X^j &= 0 & \text{for} \quad j &= p+2, ..., 10. \end{aligned}$$

such that

$$(1.11) \qquad [x^i, x^j] = i\,\theta^{ij} \equiv (B^{-1})^{ij}$$

Consider fluctuations around this classical solution:

$$(1.12) \qquad \begin{aligned} X^i &= x^i + \theta^{ik}\hat{A}_k \\ X^j &= \phi^j \end{aligned}$$

Substituting these in the action for the D(-1)-branes gives the noncommutative DBI action of a Dp-brane in $\Phi = -B$ description. We follow the same prescription to obtain the Ramond-Ramond couplings on noncommutative BPS branes.

2. Chern-Simons couplings on BPS noncommutative branes

For this we start with the nonabelian Chern-Simons action for N D(-1)-branes: (N $\Longrightarrow \infty$) [6] (for earlier work on Ramond-Ramond couplings in the matrix model, see Ref.[20])

$$(2.1) \qquad S_{cs} = \frac{2\pi}{g_s}\text{Tr}\left[e^{i(\mathbf{i}_X \mathbf{i}_X)}\sum_n C^{(n)}\right]$$

For example let us take the coupling to the RR 10-form $C^{(10)}$:

$$(2.2) \qquad \begin{aligned} S_{cs} &= \frac{2\pi}{g_s}\text{Tr}\left[\frac{i^5}{5!}X^{i_{10}}X^{i_9}...X^{i_1}C^{(10)}_{i_1 i_2...i_{10}}\right] \\ &= -\frac{2\pi}{g_s}\text{Tr}\left[\frac{1}{5!2^5}(i[X^{i_1},X^{i_2}])...(i[X^{i_9},X^{i_{10}}])C^{(10)}_{i_1 i_2...i_{10}}\right] \end{aligned}$$

To obtain the noncommutative Chern-Simons terms for a Dp-brane, we substitute the solutions corresponding to a Dp-brane along with the fluctuations into the action above. In the case of a D9-brane this gives rise to:

$$(2.3) \qquad \begin{aligned} \hat{S}^{(D9)}_{cs} &= \frac{2\pi}{g_s}\text{Tr}\left[\frac{1}{5!2^5}\epsilon_{i_1 i_2...i_{10}}Q^{i_1 i_2}...Q^{i_9 i_{10}}\frac{1}{10!}\epsilon^{j_1 j_2...j_{10}}C^{(10)}_{j_1 j_2...j_{10}}\right] \\ &= \frac{2\pi}{g_s}\text{Tr}\left[\text{Pf}Q\,C^{(10)}\right] \end{aligned}$$

Converting into an integral:

$$(2.4) \qquad \hat{S}^{(D9)}_{cs} = T_9\int d^{10}x\frac{\text{Pf}Q}{\text{Pf}\theta}\left[\frac{1}{10!}\epsilon^{j_1 j_2...j_{10}}C^{(10)}_{j_1 j_2...j_{10}}\right]$$

where again $Q = \theta - \theta\hat{F}\theta$. For obtaining lower form couplings, we repeat the above exercise starting with that form coupling to infinite D(-1)-branes. Using the identity:

$$(2.5) \qquad \begin{aligned} &\frac{\text{Pf}Q}{2^r r!(2n-2r)!}\epsilon^{i_1 i_2\cdots i_{2n}}Q^{-1}_{i_1 i_2}\cdots Q^{-1}_{i_{2r-1}i_{2r}}C^{(2n-2r)}_{i_{2r+1}\cdots i_{2n}} \\ &= \frac{(-1)^r}{2^{n-r}(n-r)!}Q^{i_{2r+1}i_{2r+2}}\cdots Q^{i_{2n-1}i_{2n}}C^{(2n-2r)}_{i_{2r+1}\cdots i_{2n}} \end{aligned}$$

the result can be put in the following form:

$$(2.6) \qquad \hat{S}^{(D9)}_{cs} = T_9\int_x \frac{\text{Pf}Q}{\text{Pf}\theta}\sum_n C^{(n)}\,e^{Q^{-1}}$$

For lower dimensional branes one can consider coupling to higher RR-forms as well. In this case one has to again start with the nonabelian Chern-Simons terms on N D(-1)-branes and substitute the appropriate brane solution along with the fluctuations. For illustration let us start with the case of a Euclidean D1-brane coupling to the RR 4-form in type IIB. The coupling of N D-instantons to the RR 4-form is

$$(2.7) \qquad \text{Tr}\left(\frac{1}{2!2^2}\left(-i[\phi^{i_1},\phi^{i_2}]\right)\left(-i[\phi^{i_3},\phi^{i_4}]\right)C^{(4)}_{i_1i_2i_3i_4}\right)$$

Here ϕ^i represent all 10 transverse scalars to the D-instantons. Now insert $\phi^1 = X^1$, $\phi^2 = X^2$. The remaining ϕ^i are renamed as $\hat{\Phi}^a$, they represent the scalars transverse to the noncommutative D1-brane. Thus we find the coupling:

$$(2.8) \qquad \frac{1}{2!2}\,\epsilon_{ij}\,\text{tr}\left(\left(-i[X^i,X^j]\right)\left(-i[\hat{\Phi}^a,\hat{\Phi}^b]\right)-\left(-i[X^i,\hat{\Phi}^a]\right)\left(-i[X^j,\hat{\Phi}^b]\right)\right)C^{(4)}_{12ab}$$

Making the replacements

$$\begin{aligned} -i[X^1,X^2] &= Q^{12} = \theta^{12}(1+\theta^{12}\hat{F}_{12}) \\ [X^i,\hat{\Phi}^a] &= i\theta^{ij}D_j\hat{\Phi}^a \end{aligned}$$

the operator turns into:

$$(2.10) \qquad \theta^{12}\left((1+\theta^{12}\hat{F}_{12})\left(-i[\hat{\Phi}^a,\hat{\Phi}^b]\right)+\theta^{ij}D_j\hat{\Phi}^aD_i\hat{\Phi}^b\right)$$

Therefore we seem to have a non-vanishing coupling of a single noncommutative D1-brane to a 4-form RR field. However it is easy to see from the last expression that this coupling is zero in the DBI approximation (neglecting $\mathcal{O}(\partial\hat{F})$ and $\mathcal{O}(\partial^2\hat{\Phi})$ terms). Similarly one can find the coupling of a noncommutative brane of any dimension to any RR-form.

3. Noncommutative Chern-Simons terms on non-BPS branes

Type II theories have unstable Dp-branes, where p is odd for type IIA and even for type IIB. Like the BPS branes, these branes also couple to Ramond-Ramond forms [14, 15, 16]. These terms on a single unstable brane in commutative description are given by:

$$(3.1) \qquad S_{cs} = \frac{T_{p-2}}{2T_{min}}\int_x dT\,\sum_n C^{(n)}e^{F+B}$$

Where T_{min} is the value of the tachyon at the minimum of the tachyon potential $V(T)$. We propose that the Chern-Simons action on the Euclidean D9 brane of type IIA in the noncommutative description is:

$$(3.2) \qquad \hat{S}_{cs} = \frac{T_8}{2T_{min}}\int_x \frac{\text{Pf}Q}{\text{Pf}\theta}\,\mathcal{D}T\,\sum_n C^{(n)}\,e^{Q^{-1}}$$

where $\mathcal{D}_iT = -i(Q^{-1})_{ij}[X^j,T]$. A non-trivial check of this action is the following. Consider a noncommutative soliton which represents the decay of a (Euclidean) D9 brane into N coincident D(-1)-branes. Condensing the tachyon over that soliton we will get N coincident non-BPS D(-1)-branes, which carry Myers type couplings to the RR-forms [17]. Substituting the solitonic solution along with fluctuations around it should give us the nonabelian Chern-Simons action on these branes. It is easy to verify that the proposed action does pass this check. The action for other unstable branes can be obtained by taking appropriate noncommutative soliton representing lower branes and substituting the solution along with fluctuations.

For illustration let us do the tachyon condensation over the following soliton, which is supposed to represent the decay of a D9-brane into N D7-branes.

$$
\begin{aligned}
T_{cl} &= T_{max}P_N + T_{min}(1 - P_N) \\
X_{cl}^i &= P_N x^i \quad \text{for} \quad i = 1, 2, \ldots, 8 \\
X_{cl}^i &= 0 \quad \text{for} \quad i = 9, 10
\end{aligned}
$$

This solution has the property that $[T_{cl}, X_{cl}^i] = 0$. Substituting this solution along with the fluctuations into the $C^{(9)}$ coupling in the proposed action for the unstable D9-brane, we get

(3.4)
$$
\begin{aligned}
\hat{S}_{CS}^{UD7} &= \frac{T_6}{2T_{min}} \mathrm{tr}_N \int_x (-i) \, [\delta X^9, \delta X^{10}] \\
&\quad (-i)(Q_{cl}^{-1})_{12} \, [X_{cl}^2, \delta T] \, C_{23\ldots10}^{(9)}
\end{aligned}
$$

And also

$$
\hat{S}_{CS}^{UD7} = \frac{T_6}{2T_{min}} \mathrm{tr}_N \int_x (-i)[\delta X^{10}, \delta T] \, C_{12\ldots8,10}^{(9)}
$$

where tr_N denotes the trace over the $N \times N$ Chan-Paton matrices. These are actually the two kinds of couplings that exist on a stack unstable branes [17].

4. General Φ

Here we propose the noncommutative Chern-Simons couplings on a BPS brane in other descriptions and use them to argue the equivalence of the actions in various descriptions in the DBI approximation, on the lines of Ref.[3]. As we have seen the RR couplings to a noncommutative Dp-brane are given by:

(4.1)
$$
\hat{S}_{CS} = T_p \int_x \sqrt{\det(1 - \theta\hat{F})} \sum_n C^{(n)} e^{Q^{-1}}
$$

We propose that the answer in all other descriptions is given by (see also [26]):

(4.2)
$$
\hat{S}(\Phi)_{CS} = T_p \int_x \sqrt{\det(1 - \theta\hat{F})} \sum_n C^{(n)} e^{B + \hat{F}(1 - \theta\hat{F})^{-1}}
$$

where θ is the noncommutativity parameter for the corresponding description, as given in Eq.(1.4). To verify this we consider the variations of the action with respect to θ and show that these variations vanish up to total derivatives and $\mathcal{O}(\partial\hat{F})$ terms. First we illustrate this for the case of the top form coupling. The variation of $\hat{F}$ is [3]

(4.3)
$$
\delta\hat{F}_{ij}(\theta) = \delta\theta^{kl} \left[\hat{F}_{ik}\hat{F}_{jl} - \frac{1}{2}\hat{A}_k(\partial_l\hat{F}_{ij} + \hat{D}_l\hat{F}_{ij}) \right] + \mathcal{O}(\partial\hat{F})
$$

Since we are going to ignore derivatives of $\hat{F}$, we drop the $*$ products between $\hat{F}$'s but leave them in the definitions of $\hat{F}$ and in $\hat{D}_l\hat{F}_{ij}$. Keeping the closed string RR field constant the variation of the action is given by:

$$
\begin{aligned}
\delta\left[\sqrt{\det(1 - \theta\hat{F})} \right] &= -\frac{1}{2}\sqrt{\det(1 - \theta\hat{F})} \, \mathrm{Tr}\left[\frac{1}{1 - \theta\hat{F}}\delta\theta\hat{F} + \frac{1}{1 - \theta\hat{F}}\theta\delta\hat{F} \right] \\
&= -\frac{1}{2}\sqrt{\det(1 - \theta\hat{F})} \, \mathrm{Tr}\left[\frac{1}{1 - \theta\hat{F}}\delta\theta\hat{F} + \left(\frac{1}{1 - \theta\hat{F}} \right)^j{}_m \right. \\
&\quad \left. \theta^{mi}\delta\theta^{kl}\left(\hat{F}_{ik}\hat{F}_{jl} - \frac{1}{2}\hat{A}_k(\partial_l\hat{F}_{ij} + \hat{D}_l\hat{F}_{ij}) \right) \right] + \mathcal{O}(\partial\hat{F}) \\
&= -\frac{1}{2}\sqrt{\det(1 - \theta\hat{F})} \, \mathrm{Tr}\left[\frac{1}{1 - \theta\hat{F}}\delta\theta\hat{F} - \frac{1}{1 - \theta\hat{F}}\theta\hat{F}\delta\theta\hat{F} \right]
\end{aligned}
$$

$$
\begin{aligned}
&\left. - \delta\theta\hat{F}\right] + \ \mathcal{O}(\partial\hat{F}) + \text{total derivatives} \\
&= \quad 0 + \mathcal{O}(\partial\hat{F}) + \text{total derivatives}
\end{aligned}
$$

(4.4)

In the last step we have used the following identities:

$$
\begin{aligned}
\partial_l \ \sqrt{\det(1-\theta\hat{F})} &= -\frac{1}{2}\sqrt{\det(1-\theta\hat{F})}\left(\frac{1}{1-\theta\hat{F}}\right)^j{}_m \theta^{mi}\partial_l\hat{F}_{ij} \\
\hat{D}_l \ \sqrt{\det(1-\theta\hat{F})} &= -\frac{1}{2}\sqrt{\det(1-\theta\hat{F})}\left(\frac{1}{1-\theta\hat{F}}\right)^j{}_m \theta^{mi}\hat{D}_l\hat{F}_{ij} \\
&\quad + \mathcal{O}(\partial_l\hat{F}\hat{D}_l\hat{F})
\end{aligned}
$$

(4.5)

and

$$
\delta\theta^{kl}(\partial_l\hat{A}_k + \hat{D}_l\hat{A}_k) = \delta\theta^{kl}\hat{F}_{lk}
$$

Now let us turn to the next lower form coupling.

$$
\begin{aligned}
&\delta\left[\sqrt{\det(1-\theta\hat{F})}\ \hat{F}\frac{1}{1-\theta\hat{F}}\right] \\
&= \ -\frac{1}{2}\sqrt{\det(1-\theta\hat{F})}\text{Tr}\left[\frac{1}{1-\theta\hat{F}}\delta\theta\hat{F} + \frac{1}{1-\theta\hat{F}}\theta\delta\hat{F}\right]\hat{F}\frac{1}{1-\theta\hat{F}} \\
&+ \ \sqrt{\det(1-\theta\hat{F})}\left[\delta\hat{F}\ \frac{1}{1-\theta\hat{F}} + \hat{F}\frac{1}{1-\theta\hat{F}}\delta\theta\hat{F}\frac{1}{1-\theta\hat{F}}\right. \\
&+ \ \left.\hat{F}\frac{1}{1-\theta\hat{F}}\theta\delta\hat{F}\frac{1}{1-\theta\hat{F}}\right]
\end{aligned}
$$

(4.6)

From here it is easy to show, after substituting the expression for $\delta\hat{F}$ and doing similar manipulations as for the top form case, that $\delta\left[\sqrt{\det(1-\theta\hat{F})}\ \hat{F}\frac{1}{1-\theta\hat{F}}\right]$ is also zero up to total derivatives and terms which are beyond the DBI approximation. Similarly, one can show that

$$
\begin{aligned}
&\delta\left[\sqrt{\det(1-\theta\hat{F})}\ \hat{F}\frac{1}{1-\theta\hat{F}} \wedge \hat{F}\frac{1}{1-\theta\hat{F}} \wedge \cdots \wedge \hat{F}\frac{1}{1-\theta\hat{F}}\right] \\
&= 0 + \mathcal{O}(\partial\hat{F}) + \text{total derivatives}
\end{aligned}
$$

(4.7)

Using these and keeping the B-field fixed under the variation, we can show that the proposed noncommutative Chern-Simons terms are equivalent to the commutative ones in the DBI approximation.

5. Conclusions

We have obtained the couplings of the noncommutative branes to constant Ramond-Ramond fields for both BPS and non-BPS branes of type II superstring theories. For the case of BPS branes we have used the fact that a noncommutative brane can be obtained as a classical configuration of infinitely many lower dimensional branes. We have also proposed these couplings for a generic value of the description parameter and then used them to show that these couplings are equivalent to the ones in the commutative $\Phi = B$ description in the DBI approximation.

For non-BPS branes we have proposed the couplings guided by the requirement of background independence in $\Phi = -B$ description. We showed how to obtain the couplings of a bunch of non-BPS branes to Ramond-Ramond forms by condensing the noncommutative tachyon over level N noncommutative soliton. The couplings obtained this way exactly match with the ones found in the literature. The couplings presented here have been verified to be consistent with T-duality in [**21**].

Finally we would like to mention how these couplings can be generalized to the case of non-constant Ramond-Ramond fields [**24, 25, 26**]. For this one must

take the RR-fields to be a functional of transverse coordinates in the action of N instantons. Then to obtain the coupling on a Dp-brane one can follow the same procedure as for the BPS branes in the case of constant RR-fields. Expanding the RR-field in a nonabelian taylor series in the momentum space, one would get an open Wilson line [**22, 23**]. The symmetrized trace prescription for the matrices in the action of instantons would lead to the smearing of the operators found in the case of constant RR fields over this open Wilson line. Comparing these couplings with their commutative counterparts one gets a bunch of interesting identities relating commutative and noncommutative variables including a closed form for the Seiberg-Witten map. An application of these results in finding an infinite subset of derivative corrections to both commutative DBI and Chern-Simons actions in the Seiberg-Witten limit can be found in [**27**].

References

[1] N.V. Suryanarayana, *"Ramond-Ramond Couplings of Noncommutative Branes"*, http://theory.tifr.res.in/strings/Proceedings/nemani/.

[2] A. Connes, M. R. Douglas and A. Schwarz, *"Noncommutative Geometry and Matrix Theory: Compactification on Tori"*, hep-th/9711162, JHEP **9802**, 003 (1998)
M. Douglas and C. Hull, *"D-branes and the Noncommutative Torus"*, hep-th/9711165, JHEP **9802**, 008 (1998)
F. Ardalan, H. Arfaei and M. M. Sheikh-Jabbari, *"Noncommutative Geometry from Strings and Branes"*, hep-th/9810072, JHEP **9902**, 016 (1999)
C. Chu and P. Ho, *"Noncommutative Open String and D-brane"*, hep-th/9812219, Nucl. Phys. B **550**, 151 (1999)
V. Schomerus, *"D-branes and Deformation Quantization"*, hep-th/9903205, JHEP **9906**, 030 (1999).

[3] N. Seiberg and E. Witten, *"String theory and noncommutative geometry"*, JHEP**9909**, 032 (1999) [hep-th/9908142].

[4] L. Cornalba and R. Schiappa, *"Matrix Theory Star Products from the Born-Infeld Action"*, hep-th/9907211,
L. Cornalba, *"D-brane Physics and Noncommutative Yang-Mills Theory"*, hep-th/9909081,
N. Ishibashi, *"A Relation Between Commutative and Noncommutative Descriptions of D-branes"*, hep-th/9909176.

[5] N. Seiberg, *"A Note on Background Independence in Noncommutative Gauge Theories, Matrix Model and Tachyon Condensation"*, hep-th/0008013.

[6] M. Van Raamsdonk and W. Taylor, *"Multiple Dp-branes in Weak Background Fields"*, hep-th/9910052; Nucl.Phys. **B573** (2000) 703,
R. C. Myers, *"Dielectric-Branes"*, hep-th/9910053, JHEP **9912**, 022 (1999),
W. Taylor, *"The M(atrix) Model of M Theory"*, hep-th/0002016.

[7] R. Gopakumar, S. Minwalla and A. Strominger, *"Noncommutative solitons"*, hep-th/0003160; JHEP **05** (2000) 020.

[8] K. Dasgupta, S. Mukhi and G. Rajesh, *"Noncommutative Tachyons"*, hep-th/0005006; JHEP **06** (2000) 022.

[9] J. Harvey, P. Kraus, F. Larsen and E. Martinec, *"D-Branes and Strings as Noncommutative Solitons"*, hep-th/0005031, JHEP **07** (2000) 042.

[10] J. Harvey, P. Kraus and F. Larsen, *"Tensionless Branes and Discrete Gauge Symmetry"*, hep-th/0008064.

[11] C. Sochichiu, *"Noncommutative Tachyonic Solitons. Interaction with Gauge Field"*, hep-th/0007217; JHEP **08** (2000) 026.

[12] R. Gopakumar, S. Minwalla and A. Strominger, *"Symmetry Restoration and Tachyon Condensation in Open String Theory"*, hep-th/0007226.

[13] A. Sen, *"Some Issues in Noncommutative Tachyon Condensation"*, hep-th/0009038.

[14] A. Sen, *"Supersymmetric World-Volume Action For Non-BPS D-Branes"*, hep-th/9909062; JHEP **10** (1999) 008.

[15] M. Billò, B. Craps and F. Roose, *"Ramond-Ramond Coupling of Non-BPS D-Branes"*, hep-th/9905157; JHEP **06** (1999) 033.

[16] C. Kennedy and Wilkins, *"Ramond-Ramond Couplings on Brane-Anti-Brane Systems"*, hep-th/9905195; Phys. Lett. **B464** (1999) 206.

[17] B. Janssen and P. Meessen, *"A Nonabelian Chern-Simons Term for Non-BPS D-Branes"*, hep-th/0009025.

[18] M.R. Garousi, *"Tachyon Couplings on Non-BPS D-Branes and Dirac-Born-Infeld Action"*, hep-th/0003122; Nucl.Phys. **B584** (2000) 284.

[19] S. Mukhi and N.V. Suryanarayana, *"Chern-Simons Terms on Noncommutative Branes"*, hep-th/0009101, JHEP **0011**, 006 (2000).

[20] D. Brace, B. Morariu and B. Zumino, *"T-Duality and Ramond-Ramond Backgrounds in the Matrix Model"*, hep-th/9811213, Nucl.Phys. **B549** 181 (1999).

[21] R. Tatar, *"T-Duality and Actions for Noncommutative D-branes"*, hep-th/0011057.

[22] H. Liu, *" ✴-Trek II: ✴ₙ Operations, Open Wilson Lines and the Seiberg-Witten Map"*, hep-th/0011125.

[23] S.R. Das and S. Trivedi, *"Supergravity Couplings to Noncommutative Branes, Open Wilson Lines and Generalized Star Products"*, hep-th/0011131, JHEP **0102**, 046 (2001).

[24] Y. Okawa and H. Ooguri, *"An Exact Solution to Seiberg-Witten Equation of Noncommutative Gauge Theory"*, hep-th/0104036.

[25] S. Mukhi and N.V. Suryanarayana, *"Gauge-invariant Couplings of Noncommutative Branes to Ramond-Ramond Backgrounds"*, hep-th/0104045, JHEP **0105**, 023 (2001).

[26] H. Liu and J. Michelson, *"Ramond-Ramond Couplings of Noncommutative D-Branes"*, hep-th/0104139.

[27] S.R. Das, S. Mukhi and N.V. Suryanarayana, *Derivative Corrections from Noncommutativity*, hep-th/0106024.

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