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Type 0 T-Duality and the Tachyon Coupling

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Abstract

We consider the T-duality relations between Type 0A and 0B theories, and show that this constraints the possible couplings of the tachyon to the RR-fields. Due to the 'doubling' of the RR sector in Type 0 theories, we are able to introduce a democratic formulation for the Type 0 effective actions, in which there is no Chern-Simons term in the effective action. Finally we discuss how to embed Type II solutions into Type 0 theories.

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Introduction

Most 10-dimensional non-supersymmetric superstring theories are plagued with tachyons, possibly endangering the consistency of the theory. This not-withstanding, they have become an active field of research mainly due to lessons learned from duality relations in the supersymmetric theories. One of the most studied examples, and subject of this paper, are the so-called Type 0 theories.

Type 0 theories can be obtained by a diagonal GSO projection on the superstring spectrum or by orbifolding the corresponding Type II theories by $(-)^{F_s}$, the total target space fermion number [1]. Note that there are two Type IIB theories, denoted IIB_+ and IIB_-,[3] which are related by spacetime parity and lead to the same Type 0B theory. From the supergravity point of view, IIB_+ differs from IIB_- in that IIB_+ has a selfdual 5-form field strength whereas IIB_- has an anti-selfdual 5-form field strength. Similar statements can be made for the Type $\mathrm{IIA}_\pm$ theories and its relation to the unique Type 0A theory.

In the notation of [2], the spectrum of the Type 0 theories are represented as

$$
\begin{aligned}
0B &: (NS_-,NS_-)\oplus(NS_+,NS_+)\oplus(R_+,R_+)\oplus(R_-,R_-)\,,\\
0A &: (NS_-,NS_-)\oplus(NS_+,NS_+)\oplus(R_+,R_-)\oplus(R_-,R_+)\,,
\end{aligned}
\tag{1}
$$

which then consists of a tachyon, the string common sector and a doubling, w.r.t. the analogous Type II theory, of the RR-fields. Since there is a doubling of the RR-fields, there is also a doubling of the D-brane content in the Type 0 theories.

In Ref. [3] the lowest order field contributions to the Type 0B string effective action were calculated from the appropriate string scattering amplitudes and, in our conventions [4], and using the diagonal basis of RR fields $\{\hat{C}^+_{(p+1)},\hat{C}^+_{(4)},\hat{C}^-_{(p+1)}\}_{p=-1,1}$, the action reads

$$
\begin{aligned}
\hat{S} \;=\; \int d^{10}\hat{x}\sqrt{|\hat{g}|}\,\Big\{ & e^{-2\hat{\varphi}}\Big[\hat{R}-4(\partial\hat{\varphi})^2+\tfrac{1}{2\cdot 3!}\hat{\mathcal{H}}^2+\tfrac{1}{2}(\partial\hat{\mathcal{T}})^2-V(\hat{\mathcal{T}})\Big]\\
&+f_+(\hat{\mathcal{T}})\Big[\tfrac{1}{2}\Big(\hat{G}^+_{(1)}\Big)^2+\tfrac{1}{2\cdot 3!}\Big(\hat{G}^+_{(3)}\Big)^2+\tfrac{1}{2\cdot 5!}\Big(\hat{G}^+_{(5)}\Big)^2\Big]\\
&+f_-(\hat{\mathcal{T}})\Big[\tfrac{1}{2}\Big(\hat{G}^-_{(1)}\Big)^2+\tfrac{1}{2\cdot 3!}\Big(\hat{G}^-_{(3)}\Big)^2\Big]\Big\}\,,
\end{aligned}
\tag{2}
$$

where the field strengths are defined by

$$
\hat{\mathcal{H}} \;=\; d\hat{\mathcal{B}} \quad , \quad \hat{G}^\pm_{(n)} \;=\; d\hat{C}^\pm_{(n-1)}\,, \tag{3}
$$

the tachyon potential is

$$
V(\hat{\mathcal{T}})=\tfrac{1}{2}m^2\hat{\mathcal{T}}^2-4c_1\hat{\mathcal{T}}^4\,, \qquad m^2=-2/\ell_s^2\,, \tag{4}
$$

[3] These are denoted IIB and IIB$'$ *resp.* in [2].

where $\ell_s = \sqrt{\alpha'}$ is the string length. It was later argued in Ref. [5] that there should be no such a potential in Type 0 superstring effective actions. However, since the tachyon is inert under T-duality, the value of the tachyon potential V will be immaterial in most of our discussion.

Finally, the functions $f_\pm(\hat{\mathcal{T}})$ are given by

$$f_\pm(\hat{\mathcal{T}}) = 1 \pm \sqrt{2}\hat{\mathcal{T}} + \hat{\mathcal{T}}^2 + \mathcal{O}\left(\hat{\mathcal{T}}^4\right) . \tag{5}$$

The RR fields are combinations of the (R_+, R_+) and (R_-, R_-) fields, denoted by C *resp.* $\overline{C}$, that diagonalize the kinetic terms:

$$\sqrt{2}\ \hat{C}^\pm_{(2n)} = \hat{C}_{(2n)} \pm \hat{\overline{C}}_{(2n)} , \tag{6}$$

and we will denote a brane charged w.r.t. $C^+_{(p+1)}$ ($C^-_{(p+1)}$) by a Dp_+-brane (Dp_--brane *resp.*).

The fields $\hat{C}_{(4)}$ and $\hat{\overline{C}}_{(4)}$ have self- and anti-selfdual field strengths and deserve further discussion. In principle, as in the Type IIB$_\pm$ cases, it is not possible to write a kinetic term for neither of them separately without the help of auxiliary fields or without breaking covariance. Combining them, however, it is possible to write a kinetic term of the form $\hat{G}_{(5)}\hat{\overline{G}}_{(5)}$. From this term one recovers a standard-looking equation of motion, but not self- or anti-self-duality which still has to be imposed by hand. It is easy to convince oneself that this cannot be done consistently in the presence of coupling to the tachyon: the equation of motion

$$d\left(f_+{}^\star\hat{G}_{(5)}\right) \ = \ 0 , \tag{7}$$

should give the Bianchi identity under the duality transformation, which has to be

$$f_+{}^\star\hat{G}_{(5)} = \hat{G}_{(5)} , \tag{8}$$

which is clearly inconsistent for non-constant f_+.

Another possibility is to combine both of them into a completely unconstrained 5-form field strength $\hat{G}^+_{(5)}$ with standard kinetic term, as it has been done here in the action Eq. (2). All we require is that it leads to the right equations of motion associated to the propagators that can be calculated from string amplitudes. Defining

$$\hat{G}^-_{(5)} \equiv f_+{}^\star\hat{G}^+_{(5)} . \tag{9}$$

we can immediately find an alternative to the action Eq. (2) in which all the kinetic terms of the fields with a minus superscript have a factor $f_-\left(\hat{\mathcal{T}}\right)$ except for $\hat{G}^-_{(5)}$ that carries a factor $f_+^{-1}\left(\hat{\mathcal{T}}\right)$.

Yet another possibility, which we will use later on, is to write an almost standard kinetic terms for $\hat{C}^+_{(4)}$ and $\hat{C}^-_{(4)}$ with the understanding that self- and anti-self-duality have to be imposed on the subsequent equations of motion. In this case, the kinetic term would be

$$\int d^{10}\hat{x}\,\sqrt{|\hat{\jmath}|}\,\left\{f_{+}\tfrac{1}{4\cdot 5!}\left(\hat{G}^{+}_{(5)}\right)^{2}+f_{+}^{-1}\tfrac{1}{4\cdot 5!}\left(\hat{G}^{-}_{(5)}\right)^{2}\right\}\,, \tag{10}$$

and we would impose Eq. (59) as a constraint. This *non-selfdual* (NSD) action would be a generalization of the Type IIB one [6, 4]. Eliminating the $\hat{G}^{-}_{(5)}$ combination with the above constraint would take us to Eq. (2). Eliminating $\hat{G}^{+}_{(5)}$ would give us the alternative action in terms of $\hat{G}^{-}_{(5)}$.

A further remark must be made: the Type 0 theories have to be invariant under a $\mathbb{Z}_2$ group associated to $(-)^{f_L}$, the worldsheet fermion number. This implies that the effective action should be invariant under the transformation $\hat{\mathcal{T}}\to-\hat{\mathcal{T}}$ combined with an interchange of the $^{+}$- and $^{-}$ fields. A quick look at Eq. (2) then reveals that this can only be true if

$$f_{+}\left(\hat{\mathcal{T}}\right)\;=\;f_{-}\left(-\hat{\mathcal{T}}\right)\,. \tag{11}$$

Furthermore, since in the interchange of the $^{+}$- and $^{-}$ fields $\hat{G}^{+}_{(5)}$ is transformed into $\hat{G}^{-}_{(5)}$, it is clear that the action is not strictly invariant under this transformation. Actually, it takes us to the alternative action[4] but with the 5-form kinetic term carrying a tachyon factor $f_{+}\left(-\hat{\mathcal{T}}\right)$ instead of $f_{+}^{-1}\left(\hat{\mathcal{T}}\right)$. This implies that

$$f_{+}\left(-\hat{\mathcal{T}}\right)\;=\;f_{+}^{-1}\left(\hat{\mathcal{T}}\right)\;=\;f_{-}\left(\hat{\mathcal{T}}\right)\,. \tag{12}$$

These constraints, which will also coincide with the constraints coming from T-duality, determine to some extent the form of the functions f, as we will discuss later.

Due to the similarity of Type 0B with Type IIB we expect more terms in the RR field strengths and a Chern-Simons term in the action. These terms could in principle be determined from more complicated string amplitudes, but we are going to try to determine as many as we can of these additional terms by imposing T-duality between the Type 0B and Type 0A string effective actions using dimensional reduction as in Refs. [7, 8, 4]. This is our main goal. The Type 0A string effective action has not been calculated from first principles as yet. However, it is clear that the tachyon-independent part of the NSNS sector effective action (the so-called "common sector") is identical to the Type 0B one. Furthermore, it is also clear that T-duality acts on this sector according to the usual Buscher rules[5] [9] which also implies that the tachyon is invariant under T-duality. We

[4]We could also say that the transformation has to be supplemented by a dualization of $\hat{G}^{-}_{(5)}$ to be a symmetry of the action.

[5]In closed string theory, T-duality is always a symmetry that interchanges momentum and winding modes associated to a given compact direction whose radius is simultaneously inverted. It is worth remarking that the string effective action does not contain any field associated to these modes (they are massive). One could take into account massive Kaluza-Klein (momentum) modes arising in the compactification of the massless fields contained in the effective action, but there is no known way to take into account winding modes, which are stringy (not field-theoretical) objects. Given this fact, one may wonder how, if at all, can the string effective action give a description of T-duality. The answer lies in the observation that all Kaluza-Klein modes are charged with respect to the massless Kaluza-Klein vector coming

are going to show that these facts, plus our knowledge of the field content of the Type 0A theory and its T-duality relation to the Type 0B theory is enough to determine the effective action of both the Type 0A and Type 0B theories to identical orders in the fields.

In the next section we are going to reduce first the Type 0B action Eq. (2) to nine dimensions. From the form of the 9-dimensional action plus the T-duality invariance of the tachyon field we will immediately be able to derive an effective action for the Type 0A theory, including tachyon couplings about which we will obtain more information. Next, we will notice that we need additional terms in the RR field strengths to establish T-duality with the Type 0A effective action. This will follow from our knowledge of Buscher's rules in the NSNS sector. The introduction of the new terms on the RR field strengths will also force us to introduce a Chern-Simons term.

1 The Type 0A action and T-duality

As was said above, the Type 0A action has not been calculated from first principles, although such an action was proposed in Ref. [10]. In this section we will use T-duality as a guideline for the construction of the Type 0A effective action. We will leave the construction of the massive theory for section (2).

1.1 Reduction to $d = 9$ of the Type 0B Effective Action

Our Kaluza-Klein Ansatz to reduce the the Type 0B action Eq. (2) in the direction of the coordinate $y = x^9$ will be similar to the one used in establishing Type IIA/B T-duality in Ref. [4] (identical in the NSNS sector, actually). The relation between the 10-dimensional fields

$$\{\hat{\jmath}, \hat{\mathcal{B}}, \hat{\varphi}, \hat{\mathcal{T}}, \hat{C}^{+}_{(0)}, \hat{C}^{-}_{(0)}, \hat{C}^{+}_{(2)}, \hat{C}^{-}_{(2)}, \hat{C}^{+}_{(4)}\}\,, \tag{13}$$

and the 9-dimensional fields

$$\{g, B, A^{(1)}, A^{(2)}, k, \phi, T, C^{+}_{(0)}, C^{+}_{(1)}, C^{+}_{(2)}, C^{+}_{(3)}, C^{+}_{(4)}, C^{-}_{(0)}, C^{-}_{(1)}, C^{-}_{(2)}, \}\,, \tag{14}$$

is, in the NSNS sector

from the metric while all winding modes are charged with respect to the *winding* vector coming from the Kalb-Ramond 2-form. The interchange of momentum and winding modes implies the interchange of the Kaluza-Klein and winding vectors of the string effective action. Furthermore, the inversion of the radius is expressed in the effective action as the inversion of the Kaluza-Klein scalar that measures that radius. The transformation of the remaining massless fields follows from these and from covariance. This is the content of the Buscher rules and this is why they have to take the same form in the NSNS sector of any closed string theory effective action.

$$
\begin{aligned}
\hat{\jmath}_{\mu\nu} &= g_{\mu\nu} - k^{-2} A^{(2)}{}_{\mu} A^{(2)}{}_{\nu}\,, & \hat{\mathcal{B}}_{\mu\nu} &= B_{\mu\nu} + A^{(1)}{}_{[\mu} A^{(2)}{}_{\nu]}\,,\\
\hat{\jmath}_{\mu\underline{y}} &= -k^{-2} A^{(2)}{}_{\mu}\,, & \hat{\mathcal{B}}_{\mu\underline{y}} &= A^{(1)}{}_{\mu}\,,\\
\hat{\jmath}_{\underline{yy}} &= -k^{-2}\,, & \hat{\varphi} &= \phi - \tfrac{1}{2}\log k\,,\\
\hat{\mathcal{T}} &= T\,,
\end{aligned}
\tag{15}
$$

and, in the RR sector

$$
\begin{aligned}
\hat{C}^{\pm}_{(2n)\,\mu_1\cdots\mu_{2n}} &= C^{\pm}_{(2n)\,\mu_1\cdots\mu_{2n}} - 2n A^{(2)}{}_{[\mu_1} C^{\pm}_{(2n-1)\,\mu_2\cdots\mu_{2n}]}\,,\\
\hat{C}^{\pm}_{(2n)\,\mu_1\cdots\mu_{2n-1}\underline{y}} &= -C^{\pm}_{(2n-1)\,\mu_1\cdots\mu_{2n-1}}\,.
\end{aligned}
\tag{16}
$$

The field strengths are related, in flat indices, by

$$
\left\{
\begin{aligned}
\hat{\mathcal{H}}_{abc} &= H_{abc}\,,\\
\hat{\mathcal{H}}_{aby} &= k F^{(1)}{}_{ab}\,,
\end{aligned}
\right.
\tag{17}
$$

where

$$
\left\{
\begin{aligned}
H &= dB - \tfrac{1}{2} A^{(1)} F^{(2)} - \tfrac{1}{2} A^{(2)} F^{(1)}\,,\\
F^{(1,2)} &= dA^{(1,2)}\,,
\end{aligned}
\right.
\tag{18}
$$

in the NSNS sector and by

$$
\left\{
\begin{aligned}
\hat{G}^{\pm}_{(2n+1)\,a_1\cdots a_{2n+1}} &= G^{\pm}_{(2n+1)\,a_1\cdots a_{2n+1}}\,,\\
\hat{G}^{\pm}_{(2n+1)\,a_1\cdots a_{2n} y} &= -k G^{\pm}_{(2n)\,a_1\cdots a_{2n}}\,,
\end{aligned}
\right.
\tag{19}
$$

where

$$
\left\{
\begin{aligned}
G^{\pm}_{(2n+1)} &= dC^{\pm}_{(2n)} + F^{(2)} C^{\pm}_{(2n-1)}\,,\\
G^{\pm}_{(2n)} &= dC^{\pm}_{(2n-1)}\,,
\end{aligned}
\right.
\tag{20}
$$

in the RR sector. The reduced action is

$$
\begin{aligned}
S \;=\; \int d^9x\,\sqrt{|g|}\;\Big\{ e^{-2\phi}\big[& R - 4(\partial\phi)^2 + \tfrac{1}{2\cdot 3!}H^2 + (\partial \log k)^2 \\
& -\tfrac{1}{4}k^2\left(F^{(1)}\right)^2 - \tfrac{1}{4}k^{-2}\left(F^{(2)}\right)^2 + \tfrac{1}{2}\left(\partial T\right)^2 - V(T)\big] \\
& + f_+(T)\left[\tfrac{1}{2}k^{-1}\left(G^+_{(1)}\right)^2 - \tfrac{1}{4}k\left(G^+_{(2)}\right)^2 + \tfrac{1}{2\cdot 3!}k^{-1}\left(G^+_{(3)}\right)^2 \right. \\
& \left. -\tfrac{1}{2\cdot 4!}k\left(G^+_{(4)}\right)^2 + \tfrac{1}{2\cdot 5!}k^{-1}\left(G^+_{(5)}\right)^2\right] \\
& + f_-(T)\left[\tfrac{1}{2}k^{-1}\left(G^-_{(1)}\right)^2 - \tfrac{1}{4}k\left(G^-_{(2)}\right)^2 + \tfrac{1}{2\cdot 3!}k^{-1}\left(G^-_{(3)}\right)^2\right]\Big\}\,.
\end{aligned}
\tag{21}
$$

1.2 The Type 0A Effective Action and its Reduction to $d = 9$

We should compare the above action with the dimensionally reduced Type 0A effective action which we do not know in detail. Let us summarize our knowledge of this action: first of all, it contains the same 10-dimensional NSNS fields as the Type 0B action and all of them (except, possibly, the tachyon) appear in it in identical fashion. This implies that the T-duality rules in this sector will be Buscher's and also implies that the tachyon will appear also in the same form and will be invariant under T-duality (its reduction is trivial).

As for the RR fields, the Type 0A string effective action contains 2 1-forms, and 2 3-forms:[6] $\hat{C}_{(1)}, \hat{C}_{(3)}, \hat{\hat{C}}_{(1)}, \hat{\hat{C}}_{(3)}$ that may couple to the tachyon as in the Type 0B case. Whatever the couplings to the tachyon are, we can always diagonalize the kinetic terms. We denote the potentials in the diagonal basis by $\hat{C}^+_{(1)}, \hat{C}^+_{(3)}, \hat{C}^-_{(1)}, \hat{C}^-_{(3)}$ but we will not make any assumption about the relation with the original potentials. It is now evident that the fields with index $+$ (resp. $-$) will couple to the tachyon through $f_+(\hat{T})$ (resp. $f_-(\hat{T})$), since otherwise it would be impossible to get the reduced action Eq. (21).

Thus, to the order considered, the Type 0A string effective action must be of the form

$$
\begin{aligned}
\hat{S}_{0A} \;=\; \int d^{10}\hat{x}\sqrt{|\hat{g}|}\;\Big\{ e^{-2\hat{\phi}} & \left[\hat{R} - 4(\partial\hat{\phi})^2 + \tfrac{1}{2\cdot 3!}\hat{H}^2 + \tfrac{1}{2}(\partial\hat{T})^2 - V(\hat{T})\right] \\
& + \textstyle\sum_{\alpha=+,-} f_\alpha(\hat{T})\left[-\tfrac{1}{4}\left(\hat{G}^\alpha_{(2)}\right)^2 - \tfrac{1}{2\cdot 4!}\left(\hat{G}^\alpha_{(4)}\right)^2\right]\Big\}\,,
\end{aligned}
\tag{22}
$$

where the field strengths are defined as in Eq. (3) and the tachyon potential and coupling functions are identical to those of the Type 0B theory.

[6]There must be a massive extension of the Type 0A theory, with two constant field strengths $\hat{G}_{(0)}$ and $\hat{\hat{G}}_{(0)}$. We will consider it later.

Let us now reduce this action to 9-dimensions in the direction of the coordinate x. The relation between the 10-dimensional fields

$$\{\hat{g}, \hat{B}, \hat{\phi}, \hat{T}, \hat{C}^{+}_{(1)}, \hat{C}^{+}_{(3)}, \hat{C}^{-}_{(1)}, \hat{C}^{-}_{(3)}\}\,, \tag{23}$$

and the 9-dimensional fields

$$\{g, B, A^{(1)}, A^{(2)}, k, \phi, T, C^{+}_{(0)}, C^{+}_{(1)}, C^{+}_{(2)}, C^{+}_{(3)}, C^{-}_{(0)}, C^{-}_{(1)}, C^{-}_{(2)}, C^{-}_{(3)}\}\,, \tag{24}$$

is, in the NSNS sector[7]

$$\begin{array}{rclcrcl}
\hat{g}_{\mu\nu} & = & g_{\mu\nu} - k^2 A^{(1)}{}_{\mu} A^{(1)}{}_{\nu}\,, & \qquad & \hat{B}_{\mu\nu} & = & B_{\mu\nu} - A^{(1)}{}_{[\mu} A^{(2)}{}_{\nu]}\,, \\
\hat{g}_{\mu\underline{x}} & = & -k^2 A^{(1)}{}_{\mu}\,, & & \hat{B}_{\mu\underline{x}} & = & A^{(2)}{}_{\mu}\,, \\
\hat{g}_{\underline{xx}} & = & -k^2\,, & & \hat{\phi} & = & \phi + \frac{1}{2}\log k\,, \\
\hat{T} & = & T\,. & & & &
\end{array} \tag{25}$$

This will give a 9-dimensional NSNS sector identical to that of the action Eq. (21). The only possible relation between the 10- and 9-dimensional RR fields is

$$\left\{\begin{array}{rcl}
\hat{C}^{\pm}_{(2n-1)\,\mu_1\cdots\mu_{2n-1}} & = & C^{\pm}_{(2n-1)\,\mu_1\cdots\mu_{2n-1}} + (2n-1)A^{(1)}{}_{[\mu_1} C^{\pm}_{(2n-2)\,\mu_2\cdots\mu_{2n-1}]}\,, \\
\hat{C}^{\pm}_{(2n-1)\,\mu_1\cdots\mu_{2n-2}\underline{x}} & = & C^{\pm}_{(2n-2)\,\mu_1\cdots\mu_{2n-2}}\,,
\end{array}\right. \tag{26}$$

and we remark that it involves $A^{(1)}$ and not $A^{(2)}$, as in the Type 0B case. We cannot change this without spoiling T-duality in the NSNS sector.

The field strengths are related in flat indices by

$$\left\{\begin{array}{rcl}
\hat{H}_{abc} & = & H_{abc}\,, \\
\hat{H}_{abx} & = & k^{-1} F^{(2)}{}_{ab}\,,
\end{array}\right. \tag{27}$$

in the NSNS sector where the 9-dimensional field strengths are also given by Eq. (18).

The RR field strengths are related by

$$\left\{\begin{array}{rcl}
\hat{G}^{\pm}_{(2n)\,a_1\cdots a_{2n}} & = & G^{\pm}_{(2n)\,a_1\cdots a_{2n}}\,, \\
\hat{G}^{\pm}_{(2n)\,a_1\cdots a_{2n-1}x} & = & k^{-1} G^{\pm}_{(2n-1)\,a_1\cdots a_{2n-1}}\,,
\end{array}\right. \tag{28}$$

where the 9-dimensional ones are defined as follows

[7] We use the T-dual KK Ansatz. This ensures that the resulting 9-dimensional actions are the same in stead of being related by $k \to k^{-1}$ and $A^{(1)} \leftrightarrow A^{(2)}$.

$$
\left\{
\begin{array}{lcl}
G^{\pm}_{(2n+1)} & = & dC^{\pm}_{(2n)}\,, \\
 & & \\
G^{\pm}_{(2n)} & = & dC^{\pm}_{(2n-1)} + F^{(1)}C^{\pm}_{(2n-2)}\,.
\end{array}
\right.
\tag{29}
$$

The even ones involve $F^{(1)}$ while in the Type 0B the odd ones involve $F^{(2)}$.

Summarizing, we have obtained the action

$$
\begin{array}{rcl}
S & = & \int d^9x\,\sqrt{|g|}\ \left\{e^{-2\phi}\left[R - 4(\partial\phi)^2 + \frac{1}{2\cdot 3!}H^2 + (\partial\log k)^2\right.\right. \\
 & & \\
 & & \left. -\frac{1}{4}k^2\left(F^{(1)}\right)^2 - \frac{1}{4}k^{-2}\left(F^{(2)}\right)^2 + \frac{1}{2}\left(\partial T\right)^2 - V(T)\right] \\
 & & \\
 & & +\sum_{\alpha=+,-} f_\alpha(T)\left[\frac{1}{2}k^{-1}\left(G^{\alpha}_{(1)}\right)^2 - \frac{1}{4}k\left(G^{\alpha}_{(2)}\right)^2\right. \\
 & & \\
 & & \left.\left. +\frac{1}{2\cdot 3!}k^{-1}\left(G^{\alpha}_{(3)}\right)^2 - \frac{1}{2\cdot 4!}k\left(G^{\alpha}_{(4)}\right)^2\right]\right\}\,.
\end{array}
\tag{30}
$$

This action is different from the one we obtained from the Type 0B theory Eq. (21) in two points: the definition of the 9-dimensional field strengths involves only one of the two 9-dimensional vectors: the Kaluza-Klein one. Since they are interchanged by T-duality, we need both vectors to appear in the field strengths. On the other hand, in the Type 0B case we have obtained one RR field strength which is not present in the reduced Type 0A theory: $G^{+}_{(5)}$ and in the Type 0A case we have obtained another RR field strength absent in the reduced Type 0B action: $G^{-}_{(4)}$.

The first problem can only be solved by making the winding vector appear in the reduced RR field strengths, which implies that $\hat{B}$ must appear in the 10-dimensional RR field strengths. Up to possible field redefinitions, there is only one way of doing this: precisely defining the RR field strengths as in the Type II theories, i.e.

$$
\hat{G}^{\pm}_{(n)} = d\hat{C}^{\pm}_{(n-1)} - \hat{\mathcal{H}}\hat{C}^{\pm}_{(n-3)}\,,
\tag{31}
$$

in both the Type 0B and 0A theories. This is consistent with the fact that the amplitudes involving two RR fields of the same sector and a NSNS field (different fro the tachyon) are identical to those of the Type $\mathrm{IIB}_{\pm}$ theories. The only difference could be the sign of the second term. We can set it to minus, as above, for the Type 0B $\hat{G}^{+}_{(2n+1)}$ field strengths by fixing the relative sign between $\hat{\mathcal{B}}$ and the $\hat{C}^{+}_{(2n)}$ potentials. In principle the sign of the second term in the Type 0B field strengths $\hat{G}^{-}_{(2n+1)}$ could still be arbitrarily chosen by changing the sign of all the RR potentials $\hat{C}^{-}_{2n}$ because the $+$ and $-$ RR potentials are decoupled in the action Eq. (21). However, as we are going to argue next, we are going to have to introduce a Chern-Simons term that may couple them and we have to be open to the two possible signs.

The second problem can only be solved by Hodge-dualizing $G^{+}_{(5)}$ and identifying the dual field with $-G^{-}_{(4)}$. This is somewhat reminiscent of the procedure followed in Type II theories [4]. For this dualization to give the right form of $G^{-}_{(4)}$ it will be necessary to add to the 10-dimensional Type 0B action a Chern-Simons term and this will force us to introduce another one in the 10-dimensional Type 0A action. A subtle point arises here: when one dualizes a field strength whose kinetic term comes multiplied by a function, the kinetic term of the dual field comes multiplied by the *inverse* function. Thus, the $G^{-}_{(4)}$ kinetic term will carry an $f_{+}^{-1}(T)k$ factor. We expected the k factor, but we also expected an $f_{-}(T)$ factor. To establish T-duality, then, we must have $f_{+}^{-1} = f_{-}$ which together with Eq. (11) implies that

$$f_{\pm}\left(\hat{\mathcal{T}}\right) = \exp\left(\pm h(\hat{\mathcal{T}})\right) , \tag{32}$$

where h is an odd function of $\hat{\mathcal{T}}$. Note that this result was anticipated in Ref. [11] by means of tadpole considerations; here it arises as a necessity for T-duality to work.

The need to introduce a 10-dimensional Chern-Simons term can also be seen directly in 10-dimensions starting with the NSD action with the kinetic terms Eq. (10). It is instructive to derive the Chern-Simons term using an argument different from T-duality. Let us for the moment set to zero the tachyon field, in order to simplify the calculations (in any case, it does not play any role in the determination of the Chern-Simons term). The kinetic terms are just

$$\int d^{10}\hat{x}\sqrt{|\hat{\jmath}|}\left[\tfrac{1}{2\cdot 5!}\left(\hat{G}_{(5)}\right)^{2} + \tfrac{1}{2\cdot 5!}\left(\hat{\overline{G}}_{(5)}\right)^{2}\right] . \tag{33}$$

The Chern-Simons term has to be such that selfduality of $\hat{G}_{(5)}$ and the anti-selfduality of $\hat{\overline{G}}_{(5)}$ can be consistently imposed, i.e. such that the equations of motion are identical to the Bianchi identities,

$$\begin{aligned} d\left(\hat{G}_{(5)} + \hat{H}\hat{C}_{(2)}\right) &= 0\,, \\ d\left(\hat{\overline{G}}_{(5)} + \hat{H}\hat{\overline{C}}_{(2)}\right) &= 0\,, \end{aligned} \tag{34}$$

using the (anti-) selfduality constraints. The Chern-Simons is, therefore, given by the addition of the Type IIB$_{+}$ and Type IIB$_{-}$ Chern-Simons terms, namely

$$\int d^{10}\hat{x}\sqrt{|\hat{\jmath}|}\left\{\tfrac{1}{2\cdot 5!}\left(\hat{G}_{(5)}\right)^{2} + \tfrac{1}{2\cdot 5!}\left(\hat{\overline{G}}_{(5)}\right)^{2} + \tfrac{10}{(5!)^{2}}\hat{\epsilon}\left[\hat{G}_{(5)}\hat{\mathcal{H}}\hat{C}_{(2)} - \hat{\overline{G}}_{(5)}\hat{\mathcal{H}}\hat{\overline{C}}_{(2)}\right]\right\} , \tag{35}$$

which, written in terms of the diagonal fields is

$$\int d^{10}\hat{x}\sqrt{|\hat{\jmath}|}\left\{\tfrac{1}{4\cdot 5!}\left(\hat{G}^{+}_{(5)}\right)^{2} + \tfrac{1}{4\cdot 5!}\left(\hat{G}^{-}_{(5)}\right)^{2} + \tfrac{1}{4!\cdot 5!}\hat{\epsilon}\left[\hat{G}^{+}_{(5)}\hat{\mathcal{H}}\hat{C}^{-}_{(2)} + \hat{G}^{-}_{(5)}\hat{\mathcal{H}}\hat{C}^{+}_{(2)}\right]\right\} . \tag{36}$$

We can now Poincaré-dualize $\hat{G}^{-}_{(5)}$, adding to the above action a Lagrange-multiplier term to enforce its Bianchi identity

$$\int d^{10}\hat{x}\sqrt{|\hat{\jmath}|}\,\tfrac{1}{4!\cdot 5!}\hat{\epsilon}\partial\tilde{\hat{C}}^{-}_{(4)}\left(\hat{G}_{(5)}+10\hat{H}\hat{C}_{(2)}\right)\,, \tag{37}$$

then, solving for $\hat{G}^{-}_{(5)}$

$$\hat{G}^{-}_{(5)} = {}^{\star}\tilde{\hat{G}}^{-}_{(5)}\,, \qquad \tilde{\hat{G}}^{-}_{(5)} = 5\partial\tilde{\hat{C}}^{-}_{(4)} - 10\hat{\mathcal{H}}\hat{C}^{-}_{(2)}\,, \tag{38}$$

and substituting this solution into the action and, finally, identifying $\tilde{\hat{G}}^{-}_{(5)} = \hat{G}^{+}_{(5)}$, we find

$$\int d^{10}\hat{x}\sqrt{|\hat{\jmath}|}\left[\tfrac{1}{2\cdot 5!}\left(\hat{G}^{+}_{(5)}\right)^{2} - \tfrac{1}{4\cdot 5!}\frac{\hat{\epsilon}}{\sqrt{|\hat{\jmath}|}}\hat{G}^{+}_{(5)}\partial\hat{C}^{-}_{(2)}\mathcal{B}\right]\,, \tag{39}$$

which contains the actual kinetic term that we have and the Chern-Simons term that we should expect.

1.3 Corrected 10-dimensional Type 0A/B Effective Actions and T-Duality Rules

It is quite straightforward to carry on with the program. First, we consider again the action Eq. (2) but with RR field strengths given by Eq. (31) in both, + and −, sectors and repeat the dimensional reduction. The Kaluza-Klein Ansatz is the same for all the fields, and the 10-dimensional field strengths decompose in 9-dimensional field strengths in the same form and so, we get an action of the form Eq. (21), but with lower-dimensional RR-field strengths defined by

$$\left\{\begin{array}{rcl} G^{\pm}_{(2n+1)} & = & dC^{\pm}_{(2n)} - HC^{\pm}_{(2n-2)} + F^{(2)}C^{\pm}_{(2n-1)}\,, \\ G^{\pm}_{(2n)} & = & dC^{\pm}_{(2n-1)} - HC^{\pm}_{(2n-3)} + F^{(1)}C^{\pm}_{(2n-2)}\,. \end{array}\right. \tag{40}$$

Now, both the Kaluza-Klein and winding vector field are present in the 9-dimensional RR field strengths.

The next step is to Poincaré-dualize $G^{+}_{(5)}$ into $G^{-}_{(4)}$: we add to the 9-dimensional action a Lagrange-multiplier term to enforce the Bianchi identity

$$d\left[G^{+}_{(5)} + HC^{+}_{(2)} - F^{(2)}C^{+}_{(3)}\right] = 0\,. \tag{41}$$

The Lagrange multiplier has to be a 3-form that will become the dual potential $C^{-}_{(3)}$. Then, the Lagrange multiplier term will take the form

$$\alpha\int d^{9}x\,\epsilon\,\partial C^{-}_{(3)}\left[G^{+}_{(5)} + 10HC^{+}_{(2)} - 10F^{(2)}C^{+}_{(3)}\right]\,, \tag{42}$$

where α is a constant whose value has to be chosen so as to get the right normalization for the kinetic term of $C^{-}_{(3)}$. In the action Eq. (21) with the above Lagrange-multiplier term, $C^{+}_{(4)}$ only appears through $G^{+}_{(5)}$. We can consider it as a functional of $G^{+}_{(5)}$ since we can always recover the expression of $G^{+}_{(5)}$ in terms of $C^{+}_{(4)}$ through equation of motion of the Lagrange-multiplier. Now, the $G^{+}_{(5)}$ equation of motion is

$$G^{+}_{(5)} = -\alpha f_{+}^{-1}(T) k \frac{\epsilon}{\sqrt{|g|}} \partial C^{-}_{(3)} \,. \tag{43}$$

We expected

$$G^{+}_{(5)} = -\sqrt{f_{-}/f_{+}}\, k\, {}^{\star}G^{-}_{(4)} \,, \tag{44}$$

with

$$G^{-}_{(4)} = dC^{-}_{(3)} - HC^{-}_{(1)} + F^{(1)} C^{-}_{(2)} \,. \tag{45}$$

This fixes the normalization constant $\alpha = +\frac{1}{3!\cdot 5!}$, implies $f_{+}^{-1}(T) = f_{-}(T)$ and also tells us that there should be a 9-dimensional Chern-Simons term in the 9-dimensional action Eq. (21) of the form

$$-\tfrac{1}{2\cdot 3!\cdot 5!} \int d^9x\, \epsilon\, G^{+}_{(5)} \left[2HC^{-}_{(1)} - 3F^{(1)} C^{-}_{(2)}\right] \,, \tag{46}$$

to get Eq. (44). This term can only come from the 10-dimensional Chern-Simons term in Eq. (39) that we can also write, up to total derivatives, in the form

$$-\tfrac{1}{96} \int d^{10}\hat{x}\, \hat{\epsilon}\, \partial \hat{C}^{+}_{(4)} \partial \hat{C}^{-}_{(2)} \hat{\mathcal{B}} \,, \tag{47}$$

which gives rise to the term we wanted and another term not involving $C^{+}_{(4)}$ in any way:

$$-\tfrac{1}{2\cdot 3!\cdot 5!} \int d^9x\, \epsilon \left[G^{+}_{(5)} \left(2HC^{-}_{(1)} - 3F^{(1)} C^{-}_{(2)}\right) - 5G^{+}_{(4)} HC^{-}_{(2)}\right] \,. \tag{48}$$

Observe that the Chern-Simons term Eq. (39,39) is very similar to the Chern-Simons term in the Type IIB NSD string effective action [6, 4]. Here, however, it mixes non-trivially the two RR sectors.

Adding to this 9-dimensional Chern-Simons the Lagrange-multiplier term Eq. (42) with the value of α that we have calculated, we find the equation of motion Eq. (44), and, using it to eliminate $G^{+}_{(5)}$, we finally get the 9-dimensional Type 0 string effective action

$$
\begin{aligned}
S \;=\; \int d^9x \, \sqrt{|g|} \; \Big\{ e^{-2\phi} \Big[& R - 4(\partial\phi)^2 + \tfrac{1}{2\cdot 3!} H^2 + (\partial \log k)^2 \\
& -\tfrac{1}{4} k^2 \left(F^{(1)}\right)^2 - \tfrac{1}{4} k^{-2} \left(F^{(2)}\right)^2 + \tfrac{1}{2} \left(\partial T\right)^2 - V(T) \Big] \\
& + \textstyle\sum_{\alpha=+,-} f_\alpha(T) \Big[\tfrac{1}{2} k^{-1} \left(G^{\alpha}_{(1)}\right)^2 - \tfrac{1}{4} k \left(G^{\alpha}_{(2)}\right)^2 \\
& \qquad + \tfrac{1}{2\cdot 3!} k^{-1} \left(G^{\alpha}_{(3)}\right)^2 - \tfrac{1}{2\cdot 4!} k \left(G^{\alpha}_{(4)}\right)^2 \Big] \\
& -\tfrac{1}{36} \tfrac{\epsilon}{\sqrt{|g|}} \Big\{ \partial C^{+}_{(3)} \partial C^{-}_{(3)} A^{(2)} - \tfrac{9}{2} C^{+}_{(2)} C^{-}_{(2)} \partial A^{(1)} \left(\partial B - A^{(1)} \partial A^{(2)} - A^{(2)} \partial A^{(1)}\right) \\
& + \tfrac{3}{2} \Big[\partial C^{+}_{(3)} \partial C^{-}_{(2)} \left(B + A^{(1)} A^{(2)}\right) + 2 \partial C^{+}_{(3)} C^{-}_{(2)} A^{(2)} \partial A^{(1)} A^{(2)} \\
& + \partial C^{-}_{(3)} \partial C^{+}_{(2)} \left(B + A^{(1)} A^{(2)}\right) + 2 \partial C^{-}_{(3)} C^{+}_{(2)} A^{(2)} \partial A^{(1)} A^{(2)} \Big] \Big\} \Big\} \,.
\end{aligned}
\tag{49}
$$

In order to establish T-duality then, we have to find a 10-dimensional Chern-Simons term to add to the Type 0A string effective action Eq. (30) leading to the above 9-dimensional Type 0 string effective action using the same Ansatz as before. This is a very non-trivial check of our construction. It takes little time to see that the sought for Chern-Simons term is

$$
-\tfrac{1}{72} \int d^{10}\hat{x} \, \hat{\epsilon} \, \partial \hat{C}^{+}_{(3)} \partial \hat{C}^{-}_{(3)} \hat{B} \,. \tag{50}
$$

Again, this Chern-Simons term looks very similar to the one in the Type IIA string effective action. In fact, we could rewrite it in the form

$$
-\tfrac{1}{144} \int d^{10}\hat{x} \, \hat{\epsilon} \, \left[\partial \hat{C}_{(3)} \partial \hat{C}_{(3)} \hat{B} - \partial \hat{\overline{C}}_{(3)} \partial \hat{\overline{C}}_{(3)} \hat{B} \right] \,. \tag{51}
$$

which would be the sum of the Chern-Simons terms of the Type IIA_{+} and Type IIA_{-} theories (which are related by target space parity).

The resulting Type 0A string effective action (Eq. (22) plus the Chern-Simons term Eq. (50)) is left-right invariant (i.e. invariant under the interchange of the two RR sectors $\hat{C}^{\pm} \rightarrow \pm \hat{C}^{\pm}$ and sign reversal of the Kalb-Ramond form $\hat{B} \rightarrow -\hat{B}$), as it should.

In the same way, the complete Type 0B action (Eq. (2) plus Eq. (47)) is invariant under the transformation that changes the sign of the tachyon and interchanges the $+$ and $-$ RR field strengths if we dualize $\hat{G}^{-}_{(5)}$ into $\hat{G}^{+}_{(5)}$.

We have just established T-duality between the Type 0A and 0B string effective actions, as we intended to do. The T-duality rules are identical to those of the Type II theories [4], but now working inside each of the $^+$ and $^-$ diagonal RR sectors.

2 Democratic Type 0 Actions and Massive 0A

In Ref. [12] a "democratic" pseudo-action for Type II theories was proposed in which all RR potentials appear on the same footing. The pseudo-action has to be supplemented by duality constraints relating "electric" and "magnetic" RR fields (hence the "pseudo") and one of its properties is that it has no Chern-Simons term and only the kinetic terms for all the field strengths appear in it. In the Type 0 case, it is a simple exercise to get an action in which RR field strengths of all orders appear in the same footing: in the 0B action we can dualize $\hat{G}^-_{(3)}$ and $\hat{G}^-_{(1)}$ into $\hat{G}^+_{(7)}$ and $\hat{G}^+_{(9)}$ respectively, and in this order[8] by the standard Poincaré-dualization procedure. There is no need to impose any duality constraint as the resulting $\hat{G}^+_{(2n+1)}\,,\quad n = 0, 1, 2, 3, 4$ field strengths are independent. Actually, not all "electric" and "magnetic" field strengths appear, but only some electric and some magnetic. In any case, the action obtained in this way is really much simpler than the one we arrived at in the previous section given by Eq. (2) plus Eq. (39) or Eq. (47) with RR field strengths given by Eq. (31). In particular, there is no Chern-Simons term and only the kinetic terms of all field strengths $\hat{G}^+_{(2n+1)}$ $(n = 0, 1, 2, 3, 4)$ appear:

$$
\begin{aligned}
\hat{S}_{0B} \;=\; & \int d^{10}\hat{x}\sqrt{|\hat{\jmath}|}\,\Big\{e^{-2\hat{\varphi}}\Big[\hat{R} - 4(\partial\hat{\varphi})^2 + \tfrac{1}{2\cdot 3!}\hat{\mathcal{H}}^2 + \tfrac{1}{2}(\partial\hat{\mathcal{T}})^2 - V(\hat{\mathcal{T}})\Big] \\
& \qquad\qquad + f_+(\hat{\mathcal{T}})\textstyle\sum_{n=1}^{n=4}\tfrac{1}{2\cdot(2n+1)!}\Big(\hat{G}^+_{(2n+1)}\Big)^2\Big\}\,.
\end{aligned} \tag{52}
$$

We could have dualized instead the $\hat{G}^+_{(5)}$, $\hat{G}^+_{(3)}$ and $\hat{G}^+_{(1)}$ field strengths, in this order, and we would have obtained the above action with the $+$ indices replaced by $-$ indices. The transformation that changes the sign of the tachyon and interchanges the two RR sectors would take us back to the above action which is, thus, invariant under a combination of that transformation and the Poincaré-dualization of all the field strengths.

Needless to say one can also write down a democratic formulation of the Type 0A action created in the foregoing section and it has the same features as the democratic 0B action, namely only kinetic terms appear. T-duality is then established by extending the decomposition rules (16,26) to include the higher ("magnetic") RR forms.

In Ref. [8] it was shown that in order to establish T-duality between Type IIB and massive Type IIA, Romans' theory for short [13], one has to apply *Generalized Dimensional Reduction* on the Type IIB side and standard dimensional reduction on Romans' side. The

[8] We cannot directly dualize $\hat{G}^-_{(1)}$ because there are explicit $\hat{C}^-_{(0)}$ potentials in $\hat{G}^-_{(3)}$. We could absorb them into a redefinition of $\hat{C}^-_{(2)}$, but this would introduce unnecessary complications.

symmetry abused to perform the GDR is the invariance under the addition of constants to the Type IIB RR scalar, *i.e.* $\delta\hat{C}_{(0)} = m = cte$.

In the democratic formulation, the symmetry under constant shifts of the two RR zero forms also acts on the higher RR forms and can be written as

$$\hat{C}^{\pm} \;=\; \hat{C}^{\pm} \;+\; a^{\pm}e^{\hat{B}} \;, \tag{53}$$

so we can apply GDR in much the same way as in Ref. [8] and oxidize the 9-dimensional theory to the massive 0A action. In the democratic 0B action there is however only one RR scalar present so that it might seem that we would end up with only one mass parameter, whereas Type 0A can support 2 mass parameters associated to the 2 D8-branes present in its spectrum. This is however only an illusion: the 9-form field strength in Type 0B will induce a 9-form field strength in d=9, which in its turn can only be related to a 10-form field strength in Type 0A. It is this 10-form field strength that couples to the second D8-brane.

Generalized Dimensional Reduction, then, boils down to using the decomposition[9]

$$\hat{C}^{+}_{(2n)} \;=\; C^{+}_{(2n)} \;-\; C^{+}_{(2n-1)}\left(d\underline{y} + A^{(2)}\right) \;+\; \underline{y}\; G^{+}_{(0)}\tfrac{1}{n!}\hat{B}^{n} \;, \tag{54}$$

in stead of (16), in the reduction carried out in Sec. (1). The resulting 9-dimensional action can be obtained by standard dimensional reduction from the action

$$\begin{aligned}\hat{S}_{0A} \;=\; & \int d^{10}\hat{x}\sqrt{|\hat{g}|}\;\Big\{e^{-2\hat{\phi}}\left[\hat{R} - 4(\partial\hat{\phi})^2 + \tfrac{1}{2\cdot 3!}\hat{H}^2 + \tfrac{1}{2}(\partial\hat{\mathcal{T}})^2 - V(\hat{\mathcal{T}})\right] \\ & -f_{+}(\hat{\mathcal{T}})\textstyle\sum_{n=0}^{n=5}\tfrac{1}{2\cdot(2n)!}\left(\hat{G}^{+}_{(2n)}\right)^{2}\Big\} \;.\end{aligned} \tag{55}$$

where the RR field strengths read

$$\hat{G}^{\pm} \;=\; d\hat{C}^{\pm} \;-\; \hat{H}\hat{C}^{\pm} \;+\; G^{\pm}_{(0)}e^{B} \;. \tag{56}$$

The non-democratic formulation of massive 0A can be obtained by dualizing the 10-, 8- and 6-form field strengths, resulting in

$$\begin{aligned}\hat{S}_{0A} \;=\; & \int d^{10}\hat{x}\sqrt{|\hat{g}|}\;\Big\{e^{-2\hat{\phi}}\left[\hat{R} - 4(\partial\hat{\phi})^2 + \tfrac{1}{2\cdot 3!}\hat{H}^2 + \tfrac{1}{2}(\partial\hat{\mathcal{T}})^2 - V(\hat{\mathcal{T}})\right] \\ & -\textstyle\sum_{\alpha=\pm} f_{\alpha}\sum_{n=0}^{2}\tfrac{1}{2\cdot(2n)!}G^{\alpha\;2}_{(2n)} \\ & -\tfrac{1}{72\sqrt{\hat{g}}}\hat{\epsilon}\;\left[\partial\hat{C}^{+}_{(3)}\partial\hat{C}^{-}_{(3)}\hat{B} + \tfrac{1}{4}\hat{G}^{-}_{(0)}\partial\hat{C}^{+}_{(3)}\hat{B}^{3} + \tfrac{1}{4}\hat{G}^{+}_{(0)}\partial\hat{C}^{-}_{(3)}\hat{B}^{3} + \tfrac{9}{80}\hat{G}^{+}_{(0)}\hat{G}^{-}_{(0)}\hat{B}^{5}\right]\Big\} \;,\end{aligned} \tag{57}$$

which is just what one would expect.

[9]Please note the ˆ on the B.

3 Type 0 D-brane solutions from Type II

In this section we are going to see how to adapt Type II BPS solutions to the Type 0 setting. This will be done under the assumption of a constant tachyon field $\hat{\mathcal{T}}_0$, and we will absorb any tachyon dependence in the equations of motion into the RR-fields. In order to do this consistently however, we must investigate the tachyon equation of motion, *i.e.*

$$\nabla^\mu \left(e^{-2\phi}\partial_\mu \mathcal{T}\right) + V'(\hat{\mathcal{T}}) - h'(\mathcal{T}) \left[\sum_n \tfrac{(-)^n}{2\cdot n!} F^{+}_{(n)}{}^2 - \textstyle\sum_n \tfrac{(-)^n}{2\cdot n!} F^{-}_{(n)}{}^2\right] = 0\,, \tag{58}$$

where we made use of Eq. (32) and $F^{\pm}_{(n)}$ denotes the rescaled field strength. It is natural to choose a constant value of the tachyon $\hat{\mathcal{T}}_0$ that minimizes the potential $V'(\hat{\mathcal{T}}_0) = 0$. We will assume that a minimum exists and, further, that $h'(\hat{\mathcal{T}}_0)$ is finite[10]. The tachyon equation of motion, then, leads to the constraint

$$\sum_n \tfrac{(-)^n}{2\cdot n!} F^{+}_{(n)}{}^2 - \textstyle\sum_n \tfrac{(-)^n}{2\cdot n!} F^{-}_{(n)}{}^2 = 0\,. \tag{59}$$

In terms of the rescaled RR field strengths, denoted by F, the equations of motions can be written as,[11]

$$\begin{aligned}
0 &= d^\star F^{\pm} + H \wedge F^{\pm}\,,\\
0 &= d\left(e^{-2\phi\star}H\right) + \frac{1}{2}\sum_\alpha {}^\star F^{\alpha} \wedge F^{\alpha}\,,\\
0 &= R + 4\left(\partial\phi\right)^2 - 4\nabla^2\phi + \tfrac{1}{2\cdot 3!}H^2\,,\\
R_{\mu\nu} &= 2\nabla_\mu\nabla_\nu\phi - \tfrac{1}{4}H_\mu{}^{\rho\sigma}H_{\nu\rho\sigma} + \tfrac{1}{4}e^{2\phi}\sum_{n,\alpha}\tfrac{(-1)^n}{n!}T^{\alpha(n)}{}_{\mu\nu}\,,
\end{aligned} \tag{60}$$

where $T^{\pm(n)}{}_{\mu\nu}$ are the energy-momentum tensor of the RR fields

$$T^{\pm(n)}{}_{\mu\nu} = n\; F^{\pm}{}_{(n)\mu}{}^{\rho_1\cdots\rho_{n-1}} F^{\pm}_{(n)\nu\rho_1\cdots\rho_{n-1}} - \tfrac{1}{2}g_{\mu\nu}F^{\pm}_{(n)}{}^2\,. \tag{61}$$

The Type II equations of motion can be obtained from these by taking for example all the $^-$ RR-fields to vanish.

Now, a typical Type II brane solution cannot, except for the D3-brane, be a solution of the constraint Eq. (59), and so the best thing we can do is to distribute each Type II D-brane charge evenly over the $^+$ and $^-$ $(p+2)$-form field strength[12]. The constraint is,

---

[10]If $h'(\hat{\mathcal{T}}_0) = 0$ we end up with no extra constraint and we can embed every Type II solution in Type 0. Therefore, from now on we will consider the $h' \neq 0$ case only.

[11]We will adapt the same philosophy as in [14] meaning that in the equations of motion also the dual fields are given. This for instance means that not only $F^{+}_{(5)}$, but also $F^{-}_{(5)}$ will contribute.

[12]A similar idea was proposed in Ref. [15].

then, automatically satisfied and the solution reads

$$
\begin{aligned}
ds^2 &= H^{-1/2}\left(dt^2 - d\vec{y}^{\,2}_{(p)}\right) - H^{1/2} d\vec{x}^{\,2}_{(9-p)} \;, \\
e^{2\phi} &= H^{\frac{p-3}{4}} \;, \\
C^{+}_{ty_1\ldots y_p} &= \pm C^{-}_{ty_1\ldots y_p} \;=\; \tfrac{1}{\sqrt{2}} H^{-1} \;,
\end{aligned}
\tag{62}
$$

where H only depends on the transverse coordinates, $\vec{x}_{(9-p)}$, and is harmonic. Since this solution bears $^+$ and $^-$ charge, but has the form of only one object we are destined to interpret these solutions as the $Dp_{\pm}$-brane [16], a bound state of a Dp_+- and a Dp_--brane[13]. Observe that these solutions are simpler in terms of the original (but rescaled), non-diagonal $C_{(p+1)}, \overline{C}_{(p+1)}$ RR potentials because they are charged only with respect to one of them. They are also trivially related by T-duality as Type II Dp-branes are.

The fact that this pairing occurs follows naturally from the Type IIB $D3$-brane solution: Since it is selfdual it automatically satisfies the condition (59), but as before the $D3$-brane charge must be divided by $\sqrt{2}$ in order to satisfy the equations of motion. Consider then T-duality in a worldvolume direction; in the democratic formulation, the electric component of the 5-form field strength gives rise to the electric component of $G^{+}_{(4)}$, whereas its magnetic part gives rise to a magnetic $G^{+}_{(6)}$ and thus leads to an electric $G^{-}_{(4)}$. Needless to say it works also in the other direction, implying that all the $Dp_{\pm}$ branes are connected by T-duality.

In Ref. [16] it was shown that the potential between a $D(p+r)_{\pm}$-brane and a $D(p+s)_{\pm}$ vanishes if $r+s=4$. This means that we can expect the, adapting the notation of [18] to the case at hand, $(p \mid D(p+r)_{\pm}, D(p+4-r)_{\pm})$ intersection to be described by the *harmonic superposition rule* [17]. In Type II the $r+s=4$ class can be generated by T-duality from the $(1 \mid D3, D3)$, which in the Type 0 setting has to be interpreted as a $(1 \mid D3_{\pm}, D3_{\pm})$ intersection. Since we embed a solution which is based on a selfdual 5-form, Eq. (58) is identically satisfied, and as before will give rise to a solution once we divide the Type II RR-field Ansatz by $\sqrt{2}$. Applying T-duality to this Type 0 intersection, we can generate the whole class of $(p \mid D(p+r)_{\pm}, D(p+4-r)_{\pm})$ intersections.[14] Please observe that Eq. (58) is automatically obeyed: by using the $Dp_{\pm}$-branes in stead of a Dp_+-brane, say, we are effectively identifying the $^+$ and $^-$ sector, automatically satisfying Eq. (59).

It should be clear that we can embed every Type II solution into Type 0: just distribute the Type II RR-charge over the $^+$ and $^-$ sector in the appropriate way. For Type II brane solutions this means changing the Dp-brane for a $Dp_{\pm}$-brane. In particular this means that the Type II BPS intersections will give rise to Type 0 intersections of various $Dp_{\pm}$-branes. It would be nice to study the stability of these Type 0 intersections.

[13]Please note that in this notation the system of a coincident electric and magnetic $D3$ brane [11] is denoted by $D3_{\pm}$.

[14]Note that this will not reproduce [16]'s $D5_{\pm}D1_{\pm}$ solution as we took the tachyon potential to be zero in contrast to [16].

4 Conclusion

In this paper we have, starting from the Type 0B effective action, constructed the Type 0A effective action by means of T-duality. Although there is a doubling of RR-fields in the Type 0 theories w.r.t. the Type II theories, T-duality does not mix RR-fields from the different sectors. Due to this doubling of RR-fields, one can write down a democratic formulation of the action, in which we dualize the fields of one sector giving rise to an action with only kinetic terms for the RR-fields, *i.e.* in the democratic formulation there is no Chern-Simons term in the action. Using this democratic formulation we applied generalized dimensional reduction based on the translational symmetry of the RR-scalar(s), in order to find the Type 0 analogue of Romans' theory, massive Type 0A.

Type 0 inherits a $\mathbb{Z}_2$ symmetry from the left worldsheet fermion number operator, which takes the tachyon to its negative and interchanges the electric ($^+$) and magnetic ($^-$) RR-sector. This discrete symmetry together with the consistency of T-duality between the Type 0 effective actions, then constrains the possible couplings of the tachyon to the RR-fields.

Finally we have shown how to create Type 0 solutions starting from Type II solutions, assuming a constant tachyon. In short, it all boils down to change a Type II Dp-brane to a Type 0 $Dp_\pm$- brane, which is nothing but a bound state of a Dp_+- and a Dp_--brane. In particular we can embed the Type II BPS intersections. For these intersections the stability is guaranteed by supersymmetry which to a large extent also dictates the form of the solution. Now the form of the Type 0 intersections is the same, making the question of stability in this class of solution an interesting one.

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