A Proposal for the Vector State in Vacuum String Field Theory

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Abstract

A previous calculation on the tachyon state arising as fluctuations of a D brane in vacuum string field theory is extended to include the vector state. We use the boundary conformal field theory approach of Rastelli, Sen and Zwiebach to construct a vector state. It is shown that the vector field satisfies the linearized equations of motion provided the two conditions $k^2 = 0$ and $k^\mu A_\mu = 0$ are satisfied. Earlier calculations using Fock space techniques by Hata and Kawano have found massless vector states that are not necessarily transverse.

1 Introduction

After the significant progress in understanding of the tachyon condensation in open string theory, Rastelli, Sen and Zwiebach proposed the so called Vacuum String Field Theory (VSFT) as a candidate for the theory of open string dynamics around the tachyon vacuum [2]. Regarded as a significant step toward solving Witten's open string field theory [1], for the last one year it has been intensively studied [3, 4, 5, 6, 8, 10]. Being constructed solely of

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world-sheet ghost fields, the kinetic operator $\mathcal{Q}$ in VSFT is non-dynamical [14]. The exact relation between Open String Field Theory (OSFT) and VSFT is not completely clear but there are significant developments in this direction [6, 7, 9, 15]. An important step toward a deeper understanding of VSFT is the study of the fluctuation modes around a classical solution to the VSFT called the sliver, and the proof that the known physical spectrum of D-branes can be reproduced. The work in this direction was initiated by Hata and Kawano who proposed a Fock space construction of the tachyon and vector states around the tachyon vacuum [12, 13]. In a very interesting paper [11] Rastelli, Sen and Zwiebach re-investigated the Hata-Kawano tachyon state using boundary conformal field theory (BCFT) and pointed out to the difficulties in the computation of the $D25$-brane tension. In our previuos paper [16] we reinvestigated the tachyon state in boundary conformal field theory language and showed that the BPZ inner product between slivers must be treated carefully. We proposed a consistent prescription for calculating both the star product of two slivers and their BPZ product and showed that the tachyon state satisfies the linearized equations of motion in the strong sense as well as in weak sense.

In [12] a proposal for vector state has been also given in framework of Fock space formalism. The authors of [12] found the correct on-shell conditions for the vector field on the brane, but they fail to find the transversality condition and the polarization vector is arbitrary rather than satisfying the transversality condition. It was pointed out in [9] that within this framework it would be a hard task to impose transversality condition on the vector or tensor massless fields. In this short note we propose a vector state in analogy to the tachyon state by making use of BCFT approach. To find the on-shell conditions for the vector field $|V\rangle$ we follow the procedure used in [11] and [16] for the tachyon field $|\chi_T\rangle$. By making use of BCFT prescription we are able to find the correct on-shell conditions for the vector field and more importantly, its transversality.

2 BCFT construction of the vector state

The open string field theory (OSFT) action proposed by Witten is given by [1]

$$S = -\kappa \left\{ \frac{1}{2} \langle \Psi | \, Q \, | \Psi \rangle + \frac{1}{3} \langle \Psi | \Psi \star \Psi \rangle \right\}, \tag{1}$$

where $|\Psi\rangle$ is the string field represented by a ghost number one state in the matter-ghost BCFT, $\langle \cdot | \cdot \rangle$ represents the BPZ inner product and $\star$ is the string star multiplication. In (1) Q is the ordinary BRST charge made up from energy-momentum tensors for matter and ghost sectors, T_{matter} and T_{ghost}, and the ghost as well. The equations of motion following from (1) are

$$Q|\Psi\rangle + |\Psi \star \Psi\rangle = 0. \tag{2}$$

After the open string tachyon condensation, the theory must represent closed string vacuum and there should be no open string perturbative excitations. VSFT is proposed to describe the dynamics around the tachyon vacuum and its kinetic operator $\mathcal{Q}$ must have trivial cohomology. The action for VSFT is given by

$$S = -\kappa \left\{ \frac{1}{2}\langle\Psi|\mathcal{Q}|\Psi\rangle + \frac{1}{3}\langle\Psi|\Psi \star \Psi\rangle \right\}, \tag{3}$$

where $\mathcal{Q}$ is a new BRST operator of ghost number one and made of ghost fields:

$$\mathcal{Q} = c_0 + \sum_{n\geq 1} f_n[c_n + (-1)^n c_n^\dagger] \tag{4}$$

where f_n are some numerical coefficients. Since $\mathcal{Q}$ has this special form, the solutions are factorized as follows

$$|\Psi\rangle = |\Psi_g\rangle \otimes |\Psi_m\rangle, \tag{5}$$

where $|\Psi_m\rangle$ and $|\Psi_g\rangle$ are the matter and ghost part respectively. The equations of motion then take the following form

$$\begin{aligned} \mathcal{Q}|\Psi_g\rangle &= |\Psi_g \star \Psi_g\rangle \\ |\Psi_m\rangle &= |\Psi_m \star \Psi_m\rangle. \end{aligned} \tag{6}$$

The ghost part is taken to be universal, but $|\Psi_m\rangle$ is a solution of projector equation and represents a particular brane. This interpretation follows from the fact that with $\mathcal{Q}$ constructed purely of ghost fields, it has trivial cohomology and hence solutions to VSFT contain no perturbative open string states, but may describe non-perturbative states such as the D-branes. The properties and the solutions to VSFT have been discussed by Rastelli, Sen and Zwiebach in several papers [2, 3, 10, 14]. The approach using the so called surface states turn out to be very useful since one can represent the BPZ inner product and the star product by simple geometric operations and using BCFT. The solution to the matter part of the equations of motion describing a $D25$-brane, called a sliver, is defined through the relation [10, 14]

$$\langle\Xi_m|\Phi\rangle = \lim_{n\to\infty} \mathcal{N}\langle f \circ \Phi(0)\rangle_{C_n}, \tag{7}$$

where C_n is a semi infinite cylinder of circumference $\frac{n\pi}{2}$ obtained by making the identification $\Re z \simeq \Re z + \frac{n\pi}{2}$ in the upper half z plane. The map f is given by $f(z) = tan^{-1}z$, $|\Phi\rangle$ is an arbitrary state in the matter Hilbert space and $\langle \ldots \rangle_{C_n}$ denotes the correlation function of the matter BCFT on the semi infinite cylinder C_n. In the limit $n \to \infty$, C_n approaches the upper half z plane. The sliver state is normalized so that

$$\langle \Xi_m | \Xi_m \rangle = K V^{(26)}, \qquad (V^{(26)} = (2\pi)^{26} \delta^{26}(0)) \tag{8}$$

where $V^{(26)}$ is the volume of 26-dimensional spacetime and K is a normalization constant that arises due to anomaly in the matter sector.

The tachyon state can be written in the form [12, 11, 16]

$$|\Psi_g\rangle \otimes |\chi_T(k)\rangle \tag{9}$$

where $|\Psi_g\rangle$ is the same state for all brane solutions and $|\chi_T(k)\rangle$ is the matter part defined as

$$\langle \chi_T(k) | \Psi \rangle = \mathcal{N} \lim_{n\to\infty} n^{2k^2} \langle e^{ik.X(\frac{n\pi}{4})} f \circ \Psi(0) \rangle_{C_n} \tag{10}$$

for any state $|\Psi\rangle$ in the Hilbert space of states of the string field[3]. It has been shown that the state $|\chi_T\rangle$ satisfies the linearized equations of motion in weak sense [11] and in strong sense as well [16].

We propose the following form of the vector state on $D25$-brane

$$\langle V(k) | \Psi \rangle = \mathcal{N} \lim_{n\to\infty} n^{2(k^2+1)} \langle A_\mu \partial_t X^\mu (\frac{n\pi}{4}) e^{ik.X(\frac{n\pi}{4})} f \circ \Psi(0) \rangle_{C_n} \tag{11}$$

i.e. we insert a vector vertex operator in the middle of the wedge state, diametrically opposite to the puncture at the origin, and in the limit $n \to \infty$ the wedge state approaches the sliver. The subscript in ∂_t is for tangential derivative to the boundary. The normalization factors are chosen in such a way that the BPZ product with Fock states is nonvanishing. As we mentioned above, BPZ inner product and $\star$-product of two slivers can be constructed by cutting and gluing cylinders C_n and $C_{n'}$ as prescribed in [10, 14]. The expression for $f \circ \Psi$ can be expanded as

$$f \circ \Psi(0) = a_\Psi \left[e^{-ik.X(0)} \right] + \left[\zeta_\nu \partial_t X^\nu (0) e^{-ik.X(0)} \right] + \cdots \tag{12}$$

where square brackets denote the conformal blocks of primary operators. The different conformal blocks in (12) are orthogonal to each other. With these definitions we can calculate (11) following the procedures outlined in [11, 16].

[3]For more detais about the general construction see [10, 14].

3 The linearized equations and transversality

In this section we will show that the state defined in (11) has the correct on-shell condition for massless vector field and, in addition, it is transverse. Using the equation (12) we can write (11) as

$$\begin{aligned}\langle V(k)|\Psi\rangle = \lim_{n\to\infty} \mathcal{N} a_\Psi n^{2(k^2+1)} \big\{ A_\mu(k) \langle \partial X^\mu e^{ik.X(\frac{n\pi}{4})}\, e^{-ik.X(0)} \rangle_{C_n} \\ + \zeta_\nu(-k) A_\mu(k) \langle \partial X^\mu e^{ik.X(\frac{n\pi}{4})}\, \partial X^\nu e^{-ik.X(0)} \rangle_{C_n} \big\}\end{aligned} \tag{13}$$

where we take into account only the primary operators for the first two conformal blocks, terms containing ∂X^μ hereafter should be understood as $\partial_t X^\mu$ and the argument will always be that of the exponent following ∂X. As argued in [10], the descendents and the higher conformal blocks are more suppressed in $n \to \infty$ limit and therefore their contributions vanish.

The second term in the curly brackets in (13) is

$$\begin{aligned}&\lim_{n\to\infty} \mathcal{N} a_\Psi n^{2(k^2+1)} \zeta_\nu(-k) A_\mu(k) \langle \partial X^\mu e^{ik.X(\frac{n\pi}{4})} \partial X^\nu(0) e^{-ik.X(0)} \rangle_{C_n} \\ &= \lim_{n\to\infty} \Bigg[\mathcal{N} a_\Psi n^{2(k^2+1)} \left(\frac{4}{n}\right)^{2(k^2+1)} \zeta_\nu(-k) A_\mu(k) \\ &\quad \langle \partial X^\mu(-1) e^{ik.X(-1)} \partial X^\nu(0) e^{-ik.X(0)} \rangle_D \Big] (2\pi)^{26} \delta(0) \\ &= -\mathcal{N} a_\Psi A_\mu \zeta_\nu \eta^{\mu\nu} 2^{2k^2+3}\, V^{(26)}\end{aligned} \tag{14}$$

where the second equality is found after mapping C_n to unit disk and computing the correlations functions on the disk from the BCFT.

The first term in (11) can be evaluated in the same way and we find

$$\begin{aligned}&\lim_{n\to\infty} \mathcal{N} a_\Psi n^{2(k^2+1)} A_\mu(k) \langle \partial X^\mu e^{ik.X(\frac{n\pi}{4})} e^{-ik.X(0)} \rangle_{C_n} \\ &= \lim_{n\to\infty} \left[\mathcal{N} a_\Psi n^{2(k^2+1)} \left(\frac{4}{n}\right)^{(2k^2+1)} i A_\mu(k) \langle \partial X^\mu(-1) e^{ik.X(-1)} e^{-ik.X(0)} \rangle_D \right] \\ &= \lim_{n\to\infty} \mathcal{N} a_\Psi n A_\mu k^\mu cotan(\frac{\pi}{2}) 2^{(2k^2+1)} (2\pi)^{26} \delta(0) = 0.\end{aligned} \tag{15}$$

and therefore there is no contribution from this conformal block.

We are now in a position to prove that the vector state $|V\rangle$ (11) satisfies the linearized equations of motion. To do so we have to calculate the quantity $\langle V \star \Xi + \Xi \star V|\Psi\rangle$ for arbitrary $|\Psi\rangle$. According to [10, 11, 16] we have

$$\begin{aligned}\langle \Xi \star V + V \star \Xi|\Psi\rangle = \lim_{n\to\infty} \mathcal{N} a_\Psi n^{2(k^2+1)} \big\{ A_\mu \big[&\langle \partial X^\mu e^{ik.X(\frac{n\pi}{4})}\, e^{-ik.X(0)} \rangle_{C_{2n-1}} \\ &\langle \partial X^\mu e^{ik.X(\frac{(3n-2)\pi}{4})}\, e^{-ik.X(0)} \rangle_{C_{2n-1}} \big] \\ &+ A_\mu \zeta_\nu \big[\langle \partial X^\mu e^{ik.X(\frac{n\pi}{4})}\, \partial X^\nu e^{-ik.X(0)} \rangle_{C_{2n-1}} \\ &+ \langle \partial X^\mu e^{ik.X(\frac{(3n-2)\pi}{4})}\, \partial X^\nu e^{-ik.X(0)} \rangle_{C_{2n-1}} \big] \big\} \end{aligned} \tag{16}$$

where, according to the rules outlined in [16], in the star product we take the semi infinite cylinders representing $|V\rangle$ and $|\Xi\rangle$ to be of equal circumference $n\pi/2$. After mapping to unit disk D we find

$$\begin{aligned}\langle \Xi \star V + V \star \Xi|\Psi\rangle = \lim_{n\to\infty} \mathcal{N} a_\Psi n^{2(k^2+1)} \big\{ A_\mu \big[&\langle \partial X^\mu e^{ik.X(i)} e^{-ik.X(0)} \rangle_D \\ &+ \langle \partial X^\mu e^{ik.X(-i)} e^{-ik.X(0)} \rangle_D \big] \\ &+ A_\mu \zeta_\nu \big[\langle \partial X^\mu e^{ik.X(i)}\, \partial X^\nu e^{-ik.X(0)} \rangle_D \\ &+ \langle \partial X^\mu e^{ik.X(-i)}\, \partial X^\nu e^{-ik.X(0)} \rangle_D \big] \big\}\, V^{(26)} \\ = &- 2\mathcal{N} a_\Psi \left[\eta^{\mu\nu} + k^\mu k^\nu\right] \zeta_\mu A_\mu 2^{k^2+2}\, V^{(26)} \\ = &- \mathcal{N} a_\Psi \left[\eta^{\mu\nu} + k^\mu k^\nu\right] \zeta_\mu A_\mu 2^{k^2+3}\, V^{(26)}. \end{aligned} \tag{17}$$

where the correlation functions in the first square brackets cancel each other and the evaluation of the correlators in the second brackets gives the final result. But from (14) and (15) we have

$$\langle V|\Psi\rangle = -\mathcal{N} a_\Psi \zeta_\nu A_\mu \eta^{\mu\nu} 2^{2k^2+3} V^{(26)}. \tag{18}$$

The expression (18) is equal to (17) if

$$k^2 = 0 \tag{19}$$

and

$$k^\mu A_\mu = 0 \tag{20}$$

for any Fock state $|\Psi\rangle$ and thus the equations of motion give the correct on-shell conditions for the massless vector field A_μ. We point out that along with $k^2 = 0$ we found that the vector field A_μ is transverse which the authors of [12] failed to find using oscilator approach.

To obtain the kinetic term for A_μ in the action we must first check that the vector state $|V\rangle$ satisfies the linearized equations of motion against another sliver (which was a problem in [10] but a resolution was proposed in [16]). According to the general procedure developed in [10], we have

$$\begin{aligned}\langle V \star \Xi|\chi\rangle = \lim_{n_i\to\infty} n_1^{2k'^2} n_2^{2(k^2+1)} A_\mu \times \\ \langle \partial X^\mu e^{ik.X[(2n_2+n_3+n_1-4)\pi/4]} e^{ik'.X(0)} \rangle_{C_{n_1+n_2+n_3-3}}. \end{aligned} \tag{21}$$

To compute (21) we follow the procedure outlined in [16] and first map the first strip to a strip of length $\pi/2$. The length of the other two strips become $\tilde{n}_2 = \frac{n_2}{n_1-1}$ and $\tilde{n}_3 = \frac{n_3}{n_1-1}$, and we define also $\epsilon_1 = \frac{1}{n_1-1}$. Since we have the star product $\langle V \star \Xi|$, we take $\tilde{n}_2 = \tilde{n}_3 = \tilde{n}$. A simple computation yields

$$\begin{aligned}\langle V \star \Xi|\chi\rangle &= \lim_{n_1,\tilde{n}\to\infty} \tilde{n} i A_\mu 2^{k'^2+k^2+1} \langle \partial X^\mu e^{ik.X(3\pi/2)} e^{ik'.X(0)}\rangle_D \\ &= -\lim_{\tilde{n}\to\infty} \tilde{n} i k^\mu A_\mu 2^{k^2+1} V^{(26)} \end{aligned} \tag{22}$$

and taking into account that the result for $\langle \Xi \star V|\chi\rangle$ is the same but with opposite sign, we find

$$\langle \Xi \star V + V \star \Xi|\chi\rangle = 0. \tag{23}$$

Analogous calculations yield

$$\langle V \star \Xi + \Xi \star V|V\rangle = -(\eta^{\mu\nu} + k^\mu k^\nu) A_\mu A_\nu 2^{k^2+3} V^{(26)} \tag{24}$$

and

$$\langle V|\chi\rangle = \lim n k^\mu A_\mu 2^{2(k^2+1)} cotan(\frac{\pi}{2}) V^{(26)} = 0, \tag{25}$$

$$\langle V|V\rangle = -\eta^{\mu\nu} A_\mu A_\nu 2^{2k^2+3} V^{(26)}. \tag{26}$$

Comparing (23-26) we conclude that the linearized equations of motion are satisfied also against slivers under conditions (19) and (20).

To compute the kinetic term in the action (3), we define the off-shell tachyon field $T(k)$ and vector field $A_\mu(k)$ in the momentum space in the following way

$$\begin{aligned}|\Psi\rangle = |\Psi_g\rangle \otimes \Big[&|\Xi_m\rangle \\ &+ \int d^{26}k \left(n^{-k^2} T(k) |\chi_T(k)\rangle + n^{-k^2-1} A_\mu(k) |V^\mu(k)\rangle + \ldots \right) \Big] \end{aligned} \tag{27}$$

where ellipses denote of higher order excitations. Substitution of (27) into (3) gives for the quadratic part in A_μ

$$\begin{aligned} S^{(2)} = &-\frac{1}{2} \langle \Psi_g|\mathcal{Q}|\Psi_g\rangle \int d^{26}k_1 d^{26}k_2\, A_\mu(k_1) A_\nu(k_2)\, n_1^{-k_1^2-1} n_2^{-k_1^2-1} \\ &\langle V^\mu(k_1)| \left[V^\nu(k_2)\rangle - |\Xi_m \star V^\nu(k_2)\rangle - |V^\nu(k_2) \star \Xi\rangle \right]. \end{aligned} \tag{28}$$

The BPZ products in the above expression are readily computed in (21- 26) and we have simply to substitute their values into (28). Then for the kinetic term we get

$$S^{(2)} = -\frac{1}{2}\langle\Psi_g|\mathcal{Q}|\Psi_g\rangle \int d^{26}k_1\, d^{26}k_1\, n_1^{-k_1^2-1}\, n_2^{-k_2^2-1} A_\mu(k_1)\, A_\nu(k_2)$$
$$[\langle V^\mu(k_1)|V^\nu(k_2)\rangle - \langle V^\mu(k_1)|\Xi_m \star V^\nu(k_2)\rangle - \langle V^\mu(k_1)|V^\nu(k_2)\star\Xi\rangle]$$
$$\simeq -\frac{1}{2}\langle\Psi_g|\mathcal{Q}|\Psi_g\rangle \int d^{26}k_1\, d^{26}k_2\, n_1^{-k_1^2-1}\, n_2^{-k_2^2-1}\, k_1^2\, A_\mu(k_1)\, A_\nu(k_2)$$
$$\left\{\eta^{\mu\nu}\, 2^{2k_1^2+3} - (\eta^{\mu\nu} - k_1^\mu k_2^\mu)2^{k_1^2+3}\right\}\delta(k_1+k_2)$$
$$\simeq -\frac{\langle\Psi_g|\mathcal{Q}|\Psi_g\rangle}{2} 2^3\, ln 2 \int d^{26}k(k^2\eta^{\mu\nu} - \frac{k^\mu k^\nu}{ln\, 2}) A_\mu(k)\, A_\nu(-k) \qquad (29)$$

where in the second equality we take the vector field near on-shell and in the third step we used the procedure outlined in [16] to calculate the correlation function keeping terms quadratic in k_μ. It is clear from (29) that the expected action for the vector field on $D25$-brane reproduced from VSFT. From (29) we conclude that the proposed vector state gives the correct on-shell conditions and, though A_μ is transverse, the correct action for a massless vector field.

Due to the twist symmetry of the VSFT action, the interaction part will contain only terms linear in χ_T and quadratic in V and they can be easily calculated using the above technique. For this terms we find in coordinate space

$$S^{(2)} \sim 3\int d^{26}x T A_\mu A^\mu + 2\int d^{26}x\, [\partial_\mu\partial_\nu T A^\mu A^\nu - \partial_\mu T\partial_\nu A^\mu A^\nu + T\partial_\nu A^\mu \partial_\mu A^\nu] \qquad (30)$$

where, as in the case of tachyon interaction term, the multiplicative constant is infinite.

4 Conclusions

In this note we have proposed a vector state around the classical solution of VSFT representing a $D25$-brane. A careful analysis shows that the proposed state satisfies the linearized equations of motion with the correct on-shell conditions for a massless vector field A_μ. We also find the transversality condition on A_μ, a property that seems difficult to demonstrate in the oscilator approach (see [12, 13]). Due to the twist symmetry, the interaction term must contain two vector field and one tachyon and we found it to be of the form (30). Terms with two tachyons and one vector field do not appear. This is because the twist operation Ω, i.e a combined world-sheet parity reversal plus $SL(2,R)$ transformations, is a symmetry of the OSFT and VSFT as well [4]. Under the action of Ω the massless vector field A_μ is odd and changes sign while the tachyon field, being even, remains unchanged. For instance, one can easily compute $T(k_1)\,T(k_1)\,A_\mu(k_3)$ term:

$$\langle\chi|\chi\star V\rangle = \lim_{n_i\to\infty} N\, n_1^{2k_1^2}\, n_2^{2k_2^2}\, n_3^{2(k_3^2+1)} A_\mu(k_3)$$
$$\langle e^{ik_1.X(0)} e^{ik_2.X[(n_1+n_2-2)\pi/4]} \partial X^\mu e^{ik_3.X[(n_1+2n_2+n_3-4)\pi/4]}\rangle_{C_{n_1+n_2+n_3-3}}. \quad (31)$$

To compute (31), we perform the following steps

a) map the first strip to $[-\pi/4, \pi/4]$.

b) change the variable $w = e^{i4\pi/(2n+\epsilon_1-3)}$ to map to unit disk [5].

c) take n and n_1 large and compute the correlator.

We end up with:

$$\begin{aligned}\langle\chi|\chi\star V\rangle &= \lim_{n,n_1\to\infty} N\, n^{k_2^2+k_3^2-k_1^2}\, 2^{k_1^2+k_2^2+k_3^2} A_\mu(k_3) \\ &\quad \langle e^{ik_1.X(e^0)}\, e^{ik_2.X(e^{2i\pi/2})}\, \partial X^\mu e^{ik_3.X(e^{3i\pi/2})}\rangle_D \\ &\sim i\,(k_2-k_1)^\mu A_\mu(k_3) 2^{k_1^2-1}(2\pi)^{26}\delta(k_1+k_2+k_3) \end{aligned} \quad (32)$$

which is what is expected for this coupling.[6] It would be interseting to investigate the fluctuation modes around the other D-brane solutions of VSFT and coincident D-branes as well. We hope to report on these issues in the near future.

Acknowledgements: R.R. would like to thank Simon Fraser University for warm hospitality. This work has been supported by an operating grant from the Natural Sciences and Engineering Research Council of Canada.

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