Consequences of 't Hooft's Equivalence Class Theory and Symmetry by Coarse Graining

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Abstract

According to 't Hooft (Class.Quantum.Grav. 16 (1999), 3263), quantum gravity can be postulated as a dissipative deterministic system, where quantum states at the "atomic scale" can be understood as equivalence classes of primordial states governed by a dissipative deterministic dynamics law at the "Planck scale". In this paper, it is shown that for a quantum system to have an underlying deterministic dissipative dynamics, the time variable should be discrete if the continuity of its temporal evolution is required. Besides, the underlying deterministic theory also imposes restrictions on the energy spectrum of the quantum system. It is also found that quantum symmetry at the "atomic scale" can be induced from 't Hooft's Coarse Graining classification of primordial states at the "Planck scale".

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1 Introduction

Recently , Gerard't Hooft postulated that there should be a dissipative deterministic theory underlying quantum gravity at the so called "Planck scale"[1,2]. In his theory, the generic quantum mechanics is no longer the crucial starting point. Rather, a deterministic theory with dissipation of information at the "Planck scale" is needed to derive quantum mechanics at the "atomic scale".It seems that this viewpoint can solve problems concerning locality and causality in the so called Planck scale physics such as quantum gravity, which are quite different from those in the usual quantum field theories in some flat background space-time based on the holographic principle in quantum gravity theory [3].

In 't Hooft's opinion, at the "atomic scale" quantum states are equivalence classes of primordial states at the "Planck scale". In Ref.[4],this point of view

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was illustrated through a simple model. According to 't Hooft, if we only care the temporal evolution of equivalence classes, the information within each equivalence class could be ignored. Then from a non-time-reversible evolution, which characterizes a deterministic process with dissipation at the "Planck scale", we can obtain a time-reversible evolution of the properly defined equivalence classes of primordial states. Taking the equivalence classes to be quantum states we are then able to introduce a reversible evolution law at the "atomic scale". Apparently, here the central problem is how to classify the Planck scale states with respect to a deterministic evolution.'t Hooft's solution to this problem is as follows. He argues that two Planck scale states are equivalent at the "atomic scale" if , after some finite time interval, they evolve into the same state. This leads to a natural definition of equivalence classes: two states are in the same equivalence class if and only if they evolve into the same state after some finite time interval. Then, quantum states are identified with these equivalence classes.

Most recently we make clear the mathematical structure of 't Hooft's theory using quotient space construction and the related concepts [5]. Let the primordial states span a linear space . We find that the equivalence classes defined by 't Hooft can be identified with the cosets of the invariant subspace spanned by those primordial states annihilated by the time-evolution operator. Thus the Hilbert space of quantum states is just the corresponding quotient space and the time-reversible evolution at the "atomic scale" can naturally be induced on the quotient space by the dissipative deterministic evolution operator . Following this line, in this paper, we will make a further analysis of the mathematical aspect of 't Hooft's theory and then discuss some physical consequences implied in the theory. We will also probe the spectral structure of finite dimensional quantum system with an underlying deterministic structure and extend 't Hooft's idea to study quantum symmetry problem.

2 Some Mathematical Results

In this section we present some mathematical results closely related to the 't Hooft equivalence class theory. In the following, I, J stands for index sets not necessarily finite;if V_1 is a subspace of $V, v \in V$,the element $v + V_1$ in the quotient space V/V_1 is denoted by $\overline{v}$. All the vector spaces to be considered are over the complex number field. Physically , one should bear in mind that V will be the linear space spanned by so-called primordial states at the "Planck scale"(see below).For convenience,we list the mathematical definitions of some concepts appearing in 't Hooft's theory as follows.

Definition 1. A linear operator $T \in End(V)$ is called deterministic if there exists a basis $\{v_i|i \in I\}$ of V on which T acts in the following way:$\forall i \in I, \exists i' \in I \quad s.t.\ Tv_i = v_{i'}$. Such a basis is called $T-$deterministic basis. If, moreover, T is singular (non-invertible), then it is called dissipative deterministic.

Remark 1. In 't Hooft theory, T represents a deterministic time-evolution process (with dissipation) at the "Planck scale".

Definition 2. An injective map from a set to itself is called a permutation of the set. A linear operator $T \in End(V)$ is called a permutation operator if there exists a basis of V on which T act as a permutation.

Definition 3. A linear operator on a vector space is called unitarizable if there exist an inner product on the vector space such that it is unitary relative to it.

Remark 2. Physically, time-reversible evolution is described by an unitary operator,and a reversible but not unitarizable operator usually does not correspond to any practical evolution in quantum mechanics.

Definition 4. Let V and W be two vector spaces, $T \in End(V)$ and $S \in End(W)$. If there exists an isomorphism φ between V and W such that $\varphi T = S\varphi$, T and S are called equivalent.

Having prepared the above definitions, we now state one of our central results .

Proposition 1. Let V be a vector space, $T \in End(V)$ is dissipative deterministic and V_1 is a $T-$invariant subspace such that the induced operator $\overline{T}$ on the quotient space V/V_1 is nonsingular, then $\overline{T}$ is a permutation operator; Conversely, if $S \in End(V)$ is a permutation operator, then there exists a vector space V',a dissipative deterministic operator $S' \in End(V')$ and a $S'-$invariant subspace V_1' of V' such that the induced operator $\overline{S'} \in End(V'/V_1')$ is equivalent to S.

Proof. Let $\{v_i|i \in I\}$ be a $T-$deterministic basis. Then there exists a subset $J \subset I$ such that $\{\overline{v}_i|i \in J\}$ is a basis of V/V_1.By definition

$$\overline{T}\overline{v}_i = \overline{Tv_i} = \overline{v}_{i'}\ (i, i' \in I). \tag{1}$$

As $\overline{T}$ is nonsingular,we clearly see that $\overline{T}$ acts as a permutation on the basis $\{\overline{v}_i|i \in J\}$.This proves the first half of the proposition. For the second half,let $\{v_i|i \in I\}$ be a basis of V on which S acts as a permutation, take an arbitrary element $w \notin V$ and define

$$V' = span\{v_i, w|i \in I\}$$

,$V_1' = V$.Define $S' \in End(V')$ such that $S'|_V = S$ and $S'w = 0$.It is then trivial to verify that S' is dissipative deterministic and $\overline{S'}$ is equivalent to S.The proposition is thus proved.

Remark 3. This proposition,as we will see below, tells us that 't Hooft 's underlying dissipative deterministic dynamic law at the "Planck scale" can only produce very special time-reversible evolution at the " atomic scale" .

Keep the notations in the above proposition. We have the following corollary.

Corollary. If V/V_1 is finite dimensional, then $\overline{T}$ is unitarizable.

Proof. According to the proposition, there is a basis of V/V_1 on which $\overline{T}$ acts as a permutation. If $\dim V/V_1 < \infty, \overline{T}$ is periodic,namely,there exists a positive integer n such that $\overline{T}^n = 1$.Let p be its period. Choose an arbitrary inner product $(\ ,\ )$ on V/V_1 and define a new inner product $<,>$ as follows:

$$\langle \overline{v}, \overline{w} \rangle = \sum_{j=1}^{p} (\overline{T}^j \overline{v}, \overline{T}^j \overline{w}), \forall \overline{v}, \overline{w} \in V/V_1. \tag{2}$$

It is then easy to show that $\overline{T}$ is unitary relative to the inner product $<,>$.

Proposition 1 shows us that an invertible linear operator can be induced from a deterministic operator T if and only if it is a permutation operator. The following proposition characterizes permutation operator on a finite dimensional space.

Proposition 2. Let V be a finite dimensional vector space, $T \in End(V)$. T is a permutation operator if and only if it is diagonalizable and its eigenvalues can be grouped into some classes, say, $\Delta_{n_1}, \Delta_{n_2}, \cdots, \Delta_{n_r}$,such that $\Delta_{n_j} (j = 1, 2, \cdots, r)$ exactly consists of the n_j n_jth roots of unity with the same multiplicity.

Proof. Let $\{v_i | i = 1, 2, \cdots, n\}$ be a basis on which T acts as a permutation. First,suppose T is a cyclic permutation the basis,namely,we have

$$Tv_1 = v_2, Tv_2 = v_3, \cdots, Tv_{n-1} = v_n, Tv_n = v_1. \tag{3}$$

Then T is a periodic operator of period n,and its minimal polynomial is $\lambda^n - 1$.Consequently, T is diagonalizable and its eigenvalues are $e^{i\frac{2k\pi}{n}}$ $(k = 1, 2, \cdots, n)$,the nth roots of unity. Now let T act as a general permutation on the basis. We notice that the basis elements can be grouped into some classes on each of which T acts as a cyclic permutation. Thus the "only if" part of the proposition easily follows.

Conversely,suppose T is diagonalizable and its eigenvalues can be grouped into some classes $\Delta_{n_1}, \Delta_{n_2}, \cdots, \Delta_{n_r}$ in such a way that $\Delta_{n_j} (j = 1, 2, \cdots, r)$ exactly consists of the n_j n_jth roots of unity with multiplicity m_j.Then there is a basis $\{v^j_{k,l} | j = 1, 2, \cdots, r; k = 1, 2, \cdots, n_j; l = 1, 2, \cdots, m_j\}$ such that

$$Tv^j_{k,l} = e^{i\frac{2k\pi}{n_j}} v^j_{k,l}. \tag{4}$$

Define the subspace $V_{j,l}$ of V as follows:

$$V_{j,l} = span\{v^j_{k,l} | k = 1, 2, \cdots, n_j\}.$$

Clearly, we have

$$V = \sum_{j=1}^{r} \sum_{l=1}^{m_j} \oplus V_{j,l}$$

and from the proof of the “only if” part we know in each subspace $V_{j,l}$ there is a basis on which T acts as a cyclic permutation of order m_j. Put together,these bases of the subspaces form a basis of V on which T acts as a permutation. This proves the “if” part of the proposition.

3 Dynamics from ’t Hooft’s Theory

In this section we focus on the physical aspect of ’t Hooft’s theory, but our analysis depends on the above mathematical results.

In ’t Hooft’s theory, primordial states at the “Planck scale” need not form a linear space. Generally they can be denoted by a set $\Sigma = \{\phi_i | i \in I\}$. *The underlying deterministic evolution is a transformation* U *(usually depending on time)of* Σ *to itself. If* U *has no inverse it is called a dissipative deterministic evolution.* Obviously, it can be represented by a matrix with the entries 0 or 1 if I is a finite set. As U is an evolution operator ,we write it as $U = U(t_f, t_{i,})$ by convention. Physically, it represents the evolution in the time interval $[t_i, t_f]$. Certainly the evolution should satisfy the so called semi-group condition

$$U(t_f, t_m)U(t_m, t_i) = U(t_f, t_i) \tag{5}$$

$$U(t, t) = 1$$

If U is singular, it describes deterministic process with dissipation. As a matter of fact, under such an evolution some states will disappear and some states will evolve into the same state,or in other words, some states with a different past may have the same deterministic fate.’t Hooft thinks that if two states evolve in such a way that their futures are identical they should represent the same state at the “atomic scale”.In this view,he divides the elements of Σ into equivalence classes, ϕ_{i_1} and ϕ_{i_2} ($i_1, i_2 \in I$) being in the same equivalence class if they are evolved into the same state after finite time interval. Denote by $\Xi = \{\bar{\phi}_j | j \in J\}$ the set of the equivalence classes. Then ’t Hooft postulates that the space of quantum states is spanned by $\{\bar{\phi}_j | j \in J\}$ and claims that the reduced evolution on the space of quantum states is reversible. We can mathematically reformulate 't Hooft's theory as follows[5].We assume that the evolution operator $U(t_2, t_1)$ only depend on the difference of $t_2$and t_1, i.e., we can write $U(t_2, t_1) = U(t_2 - t_1)$ .This is in the spirit of 't Hooft's original construction. Then the evolution at the "Planck scale" is determined by the operator $U(t, 0) \triangleq U(t)$. Let V be the vector space spanned by $\{\phi_i | i \in I\}$.Then $U(t)$ can be extended to *a deterministic operator* on V. We call V the space of primordial states in spite of the fact that generally it contains elements which are not states. Let V_1 denote the subspace of V consisting of the vectors annihilated by $U(t)$ at some t, namely, a vector v belongs to V_1if and only if there exists some $U(t)$ such that $U(t)v = 0$. Then it follows that *the space of quantum states is none other than the quotient space*

$$Q = V/V_1 = \{|\phi\rangle \triangleq \phi + V_1 | \phi \in V\}$$

and a non-singular evolution law of the quantum states naturally follows from $U(t)$. Let $\overline{v} \equiv |\nu\rangle$ denote the equivalence class containing v . We notice that $V_1$is invariant under $U(t)$. Thus $U(t)$ induces a natural action on the quotient space Q.We denote the induced operator by $\overline{U(t)}$, then we have

$$\overline{U(t)}\overline{v} = \overline{U(t)v}. \tag{6}$$

The following simple result is easy to prove.

Proposition 3. $\overline{U(t)}$ is non-singular.

In fact, if $\overline{U(t)}\overline{v} = \overline{0}$, then $U(t)v \in V_1$.Thus there exists some t' such that $U(t')U(t)v = 0$.It then follows that

$$U(t')U(t)v = U(t' + t)v = 0 \tag{7}$$

By definition this means $v \in V_1$,i.e.,$\overline{v} = \overline{0}$.This proves the non-singularity of $\overline{U(t)}$.

Remark 4. In Ref[1,2], 't Hooft just claims the non-singularity of the induced evolution operator. But it should be pointed out that if the condition $U(t_2, t_1) = U(t_2 - t_1)$ is not satisfied the induced evolution $\overline{U(t_2, t_1)}$ might be singular if we still use 't Hooft's principle to classify the primordial states.

We are now in a position to discuss a consequence of 't Hooft's theory. The basis consisting of the equivalence classes is called the primordial basis by 't Hooft. In our notations,$\{\overline{\phi}_j | j \in J\}$ is the primordial basis and $\overline{U(t)}$ is a (dissipative) deterministic operator on $\overline{V}$. As we have proved the non-singularity of $\overline{U(t)}$,it follows from Proposition 1 that $\overline{U(t)}$ is a permutation operator which acts as a permutation on the primordial basis. Then we easily observe that if we require $\overline{U(t)}$ to be continuous with respect to t, the time variable should be discrete. For example,if J is a finite set,or in other words,the quantum Hilbert space is finite dimensional,the induced evolution operator $U(t)$ is represented as a matrix with the entries 0 or 1 with respect to the primordial basis. Clearly,it could not be continuous if the time variable is not discrete.

4 Spectrum and Hamiltonian

Let us turn to consider restrictions on the energy spectrum of quantum system imposed by the underlying determinism. Due to the arguments in the last paragraph,we assume the time variable to be discrete. Without losing generality, let the time t take values in Z^{+},the set of non-negative integers. The deterministic evolution and the induced evolution of the quantum system is then completely determined by the operator $U(1)$. Suppose $U(1)$ is unitary. It is then can be written as $U(1) = e^{-iH}$,where H is a Hermitian operator describing the Hamiltonian of the quantum system. Now if the quantum system is finite dimensional it follows from Proposition 2 that the eigenvalues of $U(1)$ are of the form $e^{-i\frac{2k\pi}{n}}$.Thus we have the following

Proposition 4. The eigenvalues of H corresponding to the induced evolution $U(1) = e^{-iH}$ of quantum states lie in the set

$$\{\frac{2k\pi}{n} \pm 2m\pi | k, n, m \in Z^{+}\}.$$

Remark 5. We have seen that evolutions that can be induced from dissipative deterministic evolutions at the "Planck scale" belong to a special class. Firstly, there is a rather strict restriction on the corresponding Hamiltonian H . Secondly, if a quantum system with an underlying deterministic structure as is described by 't Hooft is initially in the state represented by an element of the primordial basis then the evolution will never cause coherent superposition of quantum states. As these drawbacks are inherent in the theory, to remove them we have to generalize the underlying dynamic law at the "Planck scale".

Another conclusion that can be drawn from Proposition 1 is that 't Hooft's theory is closely related to the hidden variable theory. Since $U(t)$ acts as a permutation on the primordial basis of the space of quantum states, an operator that is diagonal now with respect to this basis will continue to be diagonal in the future. Such an operator could thus be thought to represent a hidden variable. This suggests that a quantum system with an underlying dissipative deterministic mechanism might permit some kind of hidden variable theory. The corollary to Proposition 2 also shows us that if $U(t)$ is a dissipative deterministic such that the quotient space V/V_1 is finite dimensional $U(t)$ can be made unitary by properly introducing an inner product. Then $U(t)$ can be regarded as an evolution operator for a quantum system. But on the other hand, such inner product is not at all unique. Since a correct quantum theory requires a Hilbert space with properly defined inner product to define probability, this is really a problem if we wish to derive quantum dynamics from a dissipative deterministic evolution, not just to interpret a given quantum system as governed by an underlying deterministic mechanism. So a gap remains to be bridged between the so called Planck scale physics and the atomic scale physics in 't Hooft's theory.

Before passing to discuss quantum symmetry we would like to present a simple quantum system which has some characteristics of a deterministic system as shown above. We consider the following quantum system: A spinless free particle in the one dimensional region $[0, L]$ with the boundary condition $\psi(0,t) = \psi(L,t)$,where $\psi(x,t)$ is the wave function. The Hilbert space of the system is

$$\mathcal{H} =\{\psi \in \mathcal{L}^2[0,L]|\psi(0) = \psi(L)\}.$$

Clearly,

$$\triangle = \{e^{i\frac{2k\pi}{L}x}|k = 0, \pm1, \pm2, \cdots\}$$

is a basis of $\mathcal{H}$. In the case of extreme relativity, the Hamiltonian of the system is $H = -i\hbar c\frac{d}{dx}$,where c is the speed of light. Define $\overline{U(t)} = e^{-iHt}$.We have

$$\overline{U(t)}e^{i\frac{2k\pi}{L}x} = e^{-i2k\pi\frac{\hbar c}{L}t}e^{i\frac{2k\pi}{L}x} \tag{8}$$

We observe that if we take the time to be discrete, it is then possible to define a time unit such that the one step evolution acts on $\triangle$ in the following way:

$$\overline{U(1)}e^{i\frac{2k\pi}{L}x} = e^{i\frac{2k\pi}{L}x}$$

We then see that this system might be regarded as a deterministic system and $\triangle$ might serve as primordial basis for the system. If we normalize $\frac{\hbar c}{L}$ as one energy unit, then the energy spectrum of the system is$\{2k\pi|k = 0, \pm1, \pm2, \cdots\}$.This is consistent with our discussion above.

Remark 6. It should be pointed out that the above simple example is essentially the same as the example of massless neutrinos discussed in Ref.[1].

5 Quantum Symmetry by Coarse Graining

As shown above, 't Hooft's classification of primordial states implies a scheme for coarse graining. Usually, for a large close system a coarse graining process can result in quantum dissipation and decoherence in the subsystem [6]. But here the converse seems to be the case: coarse graining (or classification) can lead to a unitary dynamics for the effective system even if the evolution of primordial system is not time-reversible. Since "symmetry dominates dynamics", it is rather natural to probe the role of coarse graining in generating symmetry at the "atomic scale".

Let a deterministic system be described by an evolution operator $\overline{U(t)}$, and let $\{\overline{\phi}_j|j \in J\}$ be the primordial basis for the system. Denote by P^J the permutation group of the set J. According to Proposition 1, $\overline{U(t)}$ is a permutation operator and can be identified with an element of P^J. By definition, the group of quantum symmetry consists of those unitary operators on the state space that commute with the evolution operator. If we require that these unitary operators be induced from deterministic operators on the space of primordial states, it then follows from Proposition 1 that they belong to the centralizer of $\overline{U(t)}$ in P^J. If the space of quantum states is finite dimensional, by the trick of redefining inner product as is used in the proof of the corollary to Proposition 1,we can show that there exists an inner product such that both $\overline{U(t)}$ and the operators in its centralizer in P^J are unitary operators. Thus in this case it might be reasonable to take the group of quantum symmetry to be the centralizer of $\overline{U(t)}$ in P^J.Anyway, the symmetry group is a discrete group.

We have seen that if we adhere to the principle that things happening in the space of primordial states bear the mark of determinism, then logically,things happening in the space of quantum states bear the mark of discreteness. To change the situation we need to loosen the restriction of determinism in the strict sense of this word used by 't Hooft. Let us conclude this paper by a short discussion of quantum symmetry derived from a not necessarily deterministic operator on the space of primordial states. Let V be the space of primordial states and $S \in End(V)$ satisfies $SV_1 \subset V_1$.

Proposition 5. $\overline{U(t)}\overline{S} - \overline{S}\overline{U(t)} = 0$ if and only if there exists some t' such that

$$U(t')(U(t)S - SU(t)) = 0 \tag{9}$$

The proof of this result is immediate. It directly follows from eq.(9) that $(U(t)S - SU(t))V \subset V_1$(cf. Section 3).In other words,we have

$$\overline{U(t)}\overline{S} - \overline{S}\overline{U(t)} = 0 \tag{10}$$

This proves the "if" part. The "only if" part can be proved by reversing the deduction.

If the time is discrete and takes values in Z^+,then the evolution at the "Planck scale" is determined by $U(1) \triangleq U$.Notice that $U^n = U(n)$ in this case.It follows that eq.(9) is equivalent to the following equation

$$U^n(US - SU) = 0 \tag{11}$$

for some positive integer n.Let us take 't Hooft's example in Ref.[1] to illustrate the above idea. We have

$$U = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{12}$$

Let $e_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}^T, e_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}^T, e_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}^T,$ $e_4 = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}^T$.Then

$$\overline{U} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with respect to the basis $\{\ \overline{e_i} | i = 1, 2, 3\}$.The general matrix T that commutes with $\overline{U}$ is of the form

$$T = \begin{pmatrix} a & b & c \\ b & a & c \\ j & j & k \end{pmatrix}.$$

Suppose $SV_1 \subset V_1$. Then the general solution of eq.(11) is

$$S = \begin{pmatrix} a & b & c & b \\ b-m & f & g & f \\ j & j & k & j \\ m & a-f & c-g & a-f \end{pmatrix}$$

It is clear that for each T commuting with $\overline{U}$ there exists S such that $\overline{S} = T$.In fact, as $\overline{e_2 = e_4}$, the above S has the representation

$$\overline{S} = \begin{pmatrix} a & b & c \\ b & a & c \\ j & j & k \end{pmatrix} \tag{13}$$

with respect to the basis $\{\ \overline{e_i} | i = 1, 2, 3\}$. Mathematically,this is a trivial fact. On the other hand,we have

$$US - SU = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a-f & m & c-g & m \\ 0 & 0 & 0 & 0 \\ -a+f & -m & -c+g & -m \end{pmatrix}.$$

This simply means that for $\overline{S}$ to commute with $\overline{U}$, S does not necessarily commute with U.

In the representation where $\overline{U}$ is diagonal we have

$$\overline{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Thus the matrix T that commutes with $\overline{U}$ takes a block diagonal form.It then follows that the symmetry group of the system is $U(2) \times U(1)$.But if we impose the restriction that S is a deterministic operator,as is required by determinism,it then turns out that the set of nonsingular $\overline{S}$ commuting with $\overline{U}$ is $\{1, \overline{U}\}$,the centralizer of $\overline{U}$ in P^3.

To sum up,if we loosen the restriction of determinism it is possible to induce quantum symmetry from transformations on the space of primordial states through a procedure of coarse graining as shown above. On the other hand,quantum symmetry at the "atomic scale" does not necessitate symmetry at the "Planck scale".

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References

[1] G 't Hooft, Class.Quant.Grav. 16 (1999) 3263-3279, also see LANL e-print gr-qc/9903084

[2] G 't Hooft, Determinism and Dissipation in Quantum Gravity, Erice lecture in LANL e-print hep-th/0003004

[3] G 't Hooft, The Holographic Principle, LANL e-print hep-th/0003005

[4] G 't Hooft, Quantummechanical behaviour in a deterministic model, LANL e-print,quant-ph/9612018

[5] C.P.Sun X.F.Liu and S.X.Yu, Quotient Construction of 't Hooft's Quantum Equivalence Classes, LANL e-print, hep-th/0006105

[6] M.Gell-Mann, J.B.Hartle, Phys.Rev. D, 47, 3345(1993).