Supplementation of reducible constraints and the Green-Schwarz superstring.

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Abstract

We apply the supplementation trick [26] to the Green-Schwarz superstring. For type IIB theory both first and second class constraints are covariantly separated and then arranged into irreducible sets in the initial formulation. For $N = 1$ Green-Schwarz superstring we propose a modified action which is equivalent to the initial one. Fermionic first and second class constraints are covariantly separated, the first class constraints (1CC) turn out to be irreducible. We discuss also equations of motion in the covariant gauge for κ-symmetry. For type IIA theory the same modification leads to formulation with irreducible second class constraints.

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1 Introduction

Manifestly super Poincare invariant formulation of branes implies appearance of mixed first and second class fermionic constraints in the Hamiltonian formalism [1-10]. Typically, first and second class constraints (2CC) are treated in a rather different way in quantum theory[1]. In particular, to construct formal expression for the covariant path integral one needs to have splitted and irreducible constraints [13, 14]. So, it is necessary at first to split them, which can be achieved by using of covariant projectors of one or other kind [15-17, 12]. Details depend on the model under consideration. For example, for CBS superparticle [18, 1] one introduces two auxilliary vector variables in addition to the initial superspace coordinates

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[1] Quantization scheme for mixed constraints was developed in [11]. Application of this scheme to concrete models may conflict with manifest Poincare covariance [12].

[15]. For the Green-Schwarz (GS) superstring the projectors can be constructed in terms of the initial variables only [16]. After that the problem reduces to quantization of the covariantly separated but *infinitely* reducible constraints. Despite a lot of efforts (see [15-23] and references therein) this problem has no fully satisfactory solution up to date[2]. A revival of interest to the problem is due to recent work [24] where it was shown that scattering amplitudes for superstring can be constructed in a manifestly covariant form, as well as due to progress in the light-cone quantization of superstring on $AdS_5 \times S^5$ background [25, 24].

One possibility to avoid the problem of quantization of infinitely reducible constraints is the supplementation trick which was formulated in the Hamiltonian framework in [26]. The basic idea is to introduce finite number of an additional fermionic variables subject to their own reducible constraints (the constraints are chosen in such a way that the additional sector do not contains physical degrees of freedom). Then the original constraints can be combined with one from the additional sector into irreducible set. For the resulting 1CC one imposes a covariant and irreducible gauge. It implies, in particular, a possibility to construct correct Dirac bracket for the theory.

To apply the recipe for concrete model one needs to find a modified Lagrangian which reproduces the desired irreducible constraints. Some examples were considered [26, 27], in particular, the modified $N = 1$ GS superstring action was proposed in [28]. But it was pointed in [29] that the action is not equivalent to the initial one.

In this work we present modified action which is equivalent to $N = 1$ GS superstring and which allows one to realize the supplementation trick. Then we analyse type II GS superstring.

The work is organized as follows. To fix our notations, we review main steps of the supplementation scheme in Sec. 2. In Sec. 3 modified formulation of $N = 1$ GS superstring is presented and proved to be equivalent to the initial one. First and second class constraints are covariantly separated, 1CC form irreducible set. We discuss also equations of motion and their solution in the covariant gauge for κ-symmetry. It is shown how the usual Fock space picture can be obtained in this gauge. Type IIB theory is considered in Sec. 4. Here one has two copies of the fermionic constraints (which correspond to two $\theta^A, \ A = 1, 2$) with the same chirality. It allows one to consider their Poincare covariant combinations. In this case

[2]Infinitely reducible 1CC imply infinite tower of "ghosts for ghosts" variables [22]. For reducible 2CC the problem is that the covariant Dirac bracket obeys the Jacobi identity on the second class constraints surface only [12].

both first and second class constraints can be arranged into covariant sets in the initial formulation. For type IIA theory (Sec. 5) the two copies of constraints have an opposite chirality and can not be combined in the initial formulation. Repeating the same steps as in $N=1$ case, one finds that all the second class constraints as well as one chiral sector of the first class constraints can be combined into irreducible sets. Other sector with first class constraints remains reducible. Some technical details are omitted and can be find in [34].

2 Supplementation of the reducible constraints.

It will be convenient to work in 16-component formalism of the Lorentz group $SO(1,9)$, then θ^α, ψ_α, $\alpha=1,\ldots,16$, are Majorana–Weyl spinors of opposite chirality. Real, symmetric 16×16 Γ-matrices $\Gamma^\mu{}_{\alpha\beta}$, $\tilde\Gamma^{\mu\alpha\beta}$ obey the algebra $\Gamma^\mu\tilde\Gamma^\nu+\Gamma^\nu\tilde\Gamma^\mu=-2\eta^{\mu\nu}$, $\eta^{\mu\nu}=(+,-,\ldots,-)$. Momenta conjugate to configuration space variables q^i are denoted as p_{qi}.

Let us consider a dynamical system with fermionic pairs $(\theta^\alpha,p_{\theta\alpha})$ being presented among the phase space variables z^A. Typical situation for the models under consideration is that the following constraints

$$L_\alpha\equiv p_{\theta\alpha}-iB_\mu\Gamma^\mu{}_{\alpha\beta}\theta^\beta\approx 0, \qquad (1)$$

$$D^\mu D_\mu\approx 0, \qquad \Lambda^\mu\Lambda_\mu\approx 0, \qquad (2)$$

are presented among others. Here, the $B^\mu(z),\ D^\mu(z),\ \Lambda^\mu(z)$ are some functions of phase variables z, so that $D^2\approx 0$, $\Lambda^2\approx 0$ are first class constraints[3]. It is supposed also $(D\Lambda)\neq 0$ which is true for the models considered below. Poisson bracket of the fermionic constraints is

$$\{L_\alpha,L_\beta\}=2iD_\mu\Gamma^\mu{}_{\alpha\beta}. \qquad (3)$$

The system $L_\alpha\approx 0$ is mixture of first and second class constraints, as it will be proved momentarily.

Below the following two simple facts will be used systematically (see [34] for the proof).

1). Let $\Psi^\alpha=0$ are 16 equations. Then: a) The system

$$D^\mu\Gamma^\mu\Psi=0, \qquad (4)$$

[3]In some cases (for example, for the superparticle) the quantity Λ^μ is absent in the initial formulation. Then one needs to introduce an additional vector variable [26].

$$\Lambda^\mu \Gamma^\mu \Psi = 0, \tag{5}$$

is equivalent to $\Psi^\alpha = 0$.
b) Let $\Psi^\alpha = 0$ represent 16 independent equations. Then Eq.(4) contains 8 independent equations. In $SO(8)$ notations they mean that 8_s part $\bar{\Psi}_{\dot{a}}$ of Ψ^α can be presented through 8_c part Ψ_a (or vice-versa). The same is true for Eq.(5).
2). Let $\Psi^\alpha = 0, \;\; \Phi^\alpha = 0$ are $16+16$ independent equations. Consider the system

$$D^\mu \Gamma^\mu \Psi = 0, \qquad \Lambda^\mu \Gamma^\mu \Phi = 0. \tag{6}$$

Then: a) Eq.(6) contains 16 independent equations according to 1).
b) The equations

$$D^\mu \Gamma^\mu \Psi + \Lambda^\mu \Gamma^\mu \Phi = 0, \tag{7}$$

are equivalent to the system (6). Thus the system (7) consist of 16 independent equations (i.e. it is irreducible).

Supplementation scheme for the mixed constraints consist of the following steps.

A). Manifestly covariant separation of the constraints.
By virtue of the statement 1), the system (1) can be rewritten in the equivalent form[4]

$$L^{(1)\alpha} \equiv D_\mu \tilde{\Gamma}^{\mu\alpha\beta} L_\beta \approx 0, \tag{8}$$

$$L^{(2)\alpha} \equiv \Lambda_\mu \tilde{\Gamma}^{\mu\alpha\beta} L_\beta \approx 0. \tag{9}$$

Eq.(8) contains 8 independent constraints which turn out to be 1CC as a consequence of Eqs.(2), (3). Similarly, Eq.(9) contains 8 independent constraints. One can rewrite them in $SO(8)$ notations and verify that they form 2CC system.

B). Auxiliary sector subject to reducible constraints.
Let us introduce a pair of spinors $(\eta^\alpha, p_{\eta\alpha})$ subject to the constraints

$$p_{\eta\alpha} \approx 0, \qquad T_\alpha \equiv \Lambda_\mu \Gamma^\mu{}_{\alpha\beta} \eta^\beta \approx 0. \tag{10}$$

These equations contain 8 independent 1CC among $p^{(1)}_\eta \equiv \Lambda_\mu \tilde{\Gamma}^\mu p_\eta \approx 0$ and 8+8 independent 2CC among $p^{(2)}_\eta \equiv D_\mu \tilde{\Gamma}^\mu p_\eta \approx 0$, $\Lambda_\mu \Gamma^\mu \eta \approx 0$. Note that the covariant gauge $D_\mu \Gamma^\mu \eta = 0$ may be imposed. After that the complete system (constraints + gauges) is equivalent to $p_\eta \approx 0$, $\eta \approx 0$.

[4] To split the constraints one can also use true projectors instead of the matrices $D_\mu \Gamma^\mu, \; \Lambda_\mu \Gamma^\mu$. It allows one to avoid possible "second class patalogy" [30] in the 1CC algebra [16].

C). Supplementation up to irreducible constraints.
Now part of the constraints can be combined into covariantly separated and irreducible sets. According to the statement 2), the system (8)-(10) is equivalent to

$$\Phi^{(1)\alpha)} \equiv L^{(1)\alpha} + p_\eta^{(1)\alpha} = D_\mu \tilde{\Gamma}^{\mu\alpha\beta} L_\beta + \Lambda_\mu \tilde{\Gamma}^{\mu\alpha\beta} p_{\eta\beta} \approx 0, \tag{11}$$

$$\Phi^{(2)\alpha} \equiv L^{(2)\alpha} + p_\eta^{(2)\alpha} = \Lambda_\mu \tilde{\Gamma}^{\mu\alpha\beta} L_\beta + D_\mu \tilde{\Gamma}^{\mu\alpha\beta} p_{\eta\beta} \approx 0, \tag{12}$$

$$T_\alpha \equiv \Lambda_\mu \Gamma^\mu{}_{\alpha\beta} \eta^\beta \approx 0, \tag{13}$$

where $\Phi^{(1)\alpha} \approx 0$ ($\Phi^{(2)\alpha} \approx 0$) are 16 irreducible 1CC (2CC) and $T_\alpha \approx 0$ contains 8 linearly independent 2CC. In the result first class constraints of the extended formulation form irreducible set (11). As it was mentioned above, the type IIB superstring presents an example of more attractive situation as compare to the general case (11)-(13). Due to special structure of the theory the 2CC can also be combined into irreducible set.

3 $N = 1$ Green-Schwarz superstring with irreducible first class constraints.

Consider GS superstring action with $N = 1$ space-time supersymmetry

$$S = -\frac{T}{2} \int d^2\sigma \left[\frac{1}{\sqrt{-g}} g^{ab} \Pi_a^\mu \Pi_b^\mu + 2i\varepsilon^{ab} \partial_a x^\mu \theta \Gamma^\mu \partial_b \theta \right], \tag{14}$$

where $\sqrt{-g} = \sqrt{-\det g^{ab}}$, $\Pi_a^\mu \equiv \partial_a x^\mu - i\theta\Gamma^\mu \partial_a \theta$, $\varepsilon^{01} = -1$. Let us denote

$$\begin{array}{ll} B^\mu \equiv p^\mu + T\Pi_1^\mu, & \hat{p}^\mu \equiv p^\mu - iT\theta\Gamma^\mu \partial_1 \theta, \\ D^\mu \equiv \hat{p}^\mu + T\Pi_1^\mu, & \Lambda^\mu \equiv \hat{p}^\mu - T\Pi_1^\mu. \end{array} \tag{15}$$

Then constraints under the interest for Eq.(14) are those of Eqs.(1), (2). From Eqs.(15), (2) and from the standard requirement that the induced metric is non degenerated it follows $D\Lambda = \hat{p}^2 - T\Pi_1^2 \neq 0$. Using the quantities D^μ, Λ^μ, the fermionic constraints L_α can be decomposed now on first and second class subsets as in Eq.(8), (9). To proceed further, one needs to find a modification which will lead to Eq.(10).

The modified action which we propose below acquires more elegant form in ADM representation for the world-sheet metric: $g^{00} =$

$\gamma^{-1}N^{-2}$, $g^{01}=\gamma^{-1}N^{-2}N_1$, $g^{11}=\gamma^{-1}N^{-2}(N_1^2-N^2)$. The GS action (14) acquires then the following form:

$$S=-\frac{T}{2}\int d^2\sigma\left[\frac{1}{N}\Pi_+^\mu\Pi_-^\mu+2i\varepsilon^{ab}\partial_a x^\mu\theta\Gamma^\mu\partial_b\theta\right], \qquad (16)$$

where it was denoted $\Pi_\pm^\mu\equiv\Pi_0^\mu+N_\pm\Pi_1^\mu,\ N_\pm\equiv N_1\pm N$. The world-sheet reparametrisations in this representation look as $\delta\sigma^a=\xi^a,\ \delta N_\pm=\partial_0\xi^1+(\partial_1\xi^1-\partial_0\xi^0)N_\pm-\partial_1\xi^0N_\pm^2$. Modified action to be examined is

$$S=-\frac{T}{2}\int d^2\sigma\left[\frac{1}{N}\Pi_+^\mu(\Pi_-^\mu+i\eta\Gamma^\mu\chi)+2i\varepsilon^{ab}\partial_a x^\mu\theta\Gamma^\mu\partial_b\theta-\frac{1}{4N}(\eta\Gamma^\mu\chi)^2\right], \qquad (17)$$

where two additional Majorana-Weyl fermions $\eta^\alpha(\tau,\sigma),\ \chi^\alpha(\tau,\sigma)$ were introduced. Our aim now will be to show canonical equivalence of this action and the initial one. Then the additional sector will be used to arrange 1CC of the theory into irreducible set. Direct application of the Dirac algorithm gives us the Hamiltonian

$$H=\int d\sigma\left[-\frac{N}{2}\left(\frac{1}{T}\hat{p}^2+T\Pi_1^2\right)-N_1(\hat{p}\Pi_1)-\frac{i}{2}(\hat{p}^\mu-T\Pi_1^\mu)\eta\Gamma^\mu\chi+\lambda_Np_N+\lambda_{N1}p_{N1}+\lambda_\eta p_\eta+\lambda_\chi p_\chi+L\lambda_\theta\right], \qquad (18)$$

where λ_q are the Lagrangian multipliers for the corresponding primary constraints. After determining of secondary constraints, complete constraint system can be presented as (in the notations (15))

$$p_N=0, \qquad p_{N1}=0, \qquad (19)$$

$$D^2-4TL\partial_1\theta=0, \quad \Lambda^2-4T\partial_1\eta p_\eta-4T\partial_1\chi p_\chi=0, \qquad (20)$$

$$L_\alpha\equiv p_{\theta\alpha}-iB_\mu\Gamma^\mu{}_{\alpha\beta}\theta^\beta=0, \qquad (21)$$

$$\Lambda^\mu(\Gamma^\mu\eta)_\alpha=0, \qquad p_{\eta\alpha}=0, \qquad (22)$$

$$\Lambda^\mu(\Gamma^\mu\chi)_\alpha=0, \qquad p_{\chi\alpha}=0. \qquad (23)$$

Note that the desired constraints (10) appear in duplicate form (22), (23). Combinations of constraints in Eq.(20) are chosen in such a way that all mixed brackets (i.e. those among Eq.(20) and Eqs.(21)-(23)) vanish. The fermionic constraints L_α obey the Poisson bracket algebra (2). The constraints (19), (20) are first class.

To proceed further, let us make partial fixation of gauge. One imposes $N=1,\ N_1=0$ for Eq.(19) and $D^\mu\Gamma^\mu\chi=0$ for 1CC $\Lambda^\mu\Gamma^\mu p_\chi=$

0 contained in Eq.(23). After that, the pairs $(N, p_N), (N_1, p_{N1}), (\chi, p_\chi)$ can be omitted from consideration. The Dirac bracket for the remaining variables coincides with the Poisson one. In the same fashion, the pair η, p_η can be omitted also. Then the remaining constraints (as well as equations of motion) coincide with those of the GS superstring, which proves equivalence of the actions (14) and (17).

On other hand, retaining the variables η, p_η and the corresponding constraints (22), the system (21), (22) can be rewritten equivalently as in (11)-(13), with the irreducible 1CC (11), which are separated from the 2CC (12), (13).

Covariant and irreducible gauge for Eq. (11) can be chosen as

$$R_\alpha \equiv \Lambda^\mu(\Gamma^\mu\theta)_\alpha + D^\mu(\Gamma^\mu\eta)_\alpha = 0, \tag{24}$$

or, equivalently

$$\Lambda^\mu(\Gamma^\mu\theta)_\alpha = 0, \qquad D^\mu(\Gamma^\mu\eta)_\alpha = 0. \tag{25}$$

Matrix of the Poisson brackets

$$\{\Phi^{(1)\alpha}, R_\beta\} = [2(D\Lambda)\delta^\alpha_\beta + 4iTD^\mu(\tilde{\Gamma}^\mu\Gamma^\nu\partial_1\theta)^\alpha(\Gamma^\nu\eta)_\beta]\delta(\sigma-\sigma'), \tag{26}$$

has a body on its diagonal and is invertible. It means that Eqs.(11), (24) allows one to construct the Dirac bracket without fermionic inconsistencies.

To conclude this section, let us discuss dynamics of the physical sector variables in the covariant gauge for κ-symmetry. Crucial property of the standard noncovariant gauge $\Gamma^+\theta = 0$ is that equations of motion in this case acquire linear form. Then it is possible to find their general solution. While not necessary for construction of the formal path integral, namely this fact allows one to fulfill really the canonical quantization procedure. Similarly to this, the covariant gauge will be reasonable only if it has the same property. To study equations of motion for physical variables, let us consider the gauge $D^\mu\Gamma^\mu\chi = 0$ and Eq.(25). Then the variables $(\chi, p_\chi), (\eta, p_\eta)$ can be omitted. The remaining constraints are (19)-(21), which are accompanied by the covariant gauge condition

$$\Lambda^\mu\Gamma^\mu\theta = 0. \tag{27}$$

Equations of motion for the theory look now as follows

$$\begin{aligned}
\partial_0 x^\mu &= -\frac{N}{T}p^\mu - N_1\partial_1 x^\mu + 2iN\theta\Gamma^\mu\tilde{P}_-\partial_1\theta, \\
\partial_0 p^\mu &= \partial_1[-TN\partial_1 x^\mu - N_1 p^\mu + 2iNT\theta\Gamma^\mu\tilde{P}_-\partial_1\theta],
\end{aligned}$$

$$\partial_0\theta^\alpha = -N_1\partial_1\theta^\alpha - \tilde{K}^\alpha{}_\beta\partial_1\theta^\beta, \tag{28}$$

where $\tilde{P}^\alpha_{\pm\beta}$ are covariant projectors on eight-dimensional subspaces [34]. In the standard gauge

$$N = 1, \qquad N_1 = 0, \tag{29}$$

for the constraints (19) the equations of motion remain non linear. Nevertheless, usual picture of the Fock space can be obtained in the covariant gauge. To resolve the problem one can use a trick which we refer here as "off-diagonal gauge" for $d = 2$ fields. Namely, let us consider

$$N = 0, \qquad N_1 = -1, \tag{30}$$

instead of (29). Then Eq.(28) acquires the linear form[5]

$$\partial_- x^\mu = 0, \qquad \partial_- p^\mu = 0, \qquad \partial_-\theta_a = 0, \tag{31}$$

where $\theta_a,\ a = 1,\ldots,8$ is 8_c part of θ^α, while 8_s part $\theta_{\dot{a}}$ is determined by the covariant gauge condition $\Lambda^\mu\Gamma^\mu\theta = 0$.

Fermionic dynamics is the same as in the light-cone gauge. Let us demonstrate that bosonic sector of (31) leads also to the same description of state space as those of string in the usual gauge (29). Actually, solution of Eq.(31) is ($0 \le \sigma \le \pi$, closed string)

$$\begin{aligned} x^\mu(\tau,\sigma) &= X^\mu + \frac{i}{\sqrt{\pi T}}\sum_{n\neq 0}\frac{1}{n}\beta^\mu_n e^{2in(\tau+\sigma)}, \\ p^\mu(\tau,\sigma) &= \frac{1}{\pi}P^\mu - 2\sqrt{\frac{T}{\pi}}\sum_{n\neq 0}\gamma^\mu_n e^{2in(\tau+\sigma)}. \end{aligned} \tag{32}$$

From these expressions one extracts the Poisson brackets for coefficients. For the variables

$$\bar{\alpha}^\mu_n \equiv \beta^\mu_n + \gamma^\mu_n, \quad \alpha^\mu_n \equiv \beta^\mu_{-n} - \gamma^\mu_{-n}, \quad \alpha^\mu_0 = -\bar{\alpha}^\mu_0 = \frac{1}{2\sqrt{\pi T}}P^\mu, \tag{33}$$

one obtains the properties

$$\begin{aligned} \{\alpha^\mu_n, \alpha^\nu_k\} = \{\bar{\alpha}^\mu_n, \bar{\alpha}^\nu_k\} = in\eta^{\mu\nu}\delta_{n+k,0}, \quad \{X^\mu, P^\nu\} = \eta^{\mu\nu}, \\ (\alpha^\mu_n)^* = \alpha^\mu_{-n}, \qquad (\bar{\alpha}^\mu_n)^* = \bar{\alpha}^\mu_{-n}. \end{aligned} \tag{34}$$

In terms of these variables bosonic part of the Virasoro constraints $D^2 = \Lambda^2 = 0$ acquire the standard form

$$L_n = \frac{1}{2}\sum_{\forall k}\alpha_{n-k}\alpha_k = 0, \quad \bar{L}_n = \frac{1}{2}\sum_{\forall k}\bar{\alpha}_{n-k}\bar{\alpha}_k = 0. \tag{35}$$

[5] While the action (17) is not well defined for the value $N = 0$, the Hamiltonian formulation (18)-(23), (28) admits formally Eq.(30) as the gauge fixing conditions for the constraints (19).

Eqs.(34), (35) have the same form as those of string in the gauge (29) and contain all the necessary information for canonical quantization [31-33]. Thus, instead of the standard gauge one can equivalently use the conditions (30) and the covariant gauge for κ-symmetry (27), which gives the same structure of state space.

From Eq.(32) it follows that $x^\mu(\tau,\sigma)$ is not true string coordinate. If it is necessary, the latter can be restored by using of the following statement (see [34]):

Let $\tilde{x}^\mu,\ \tilde{p}^\mu$ represent general solution of the system $\partial_-\tilde{x}^\mu = 0$, $\partial_-\tilde{p}^\mu = 0$. Then the quantities

$$\begin{aligned} x^\mu(\tau,\sigma) &= \tilde{x}^\mu(\sigma^+ \mapsto \sigma^-) - \int_0^{\sigma^+} dl\tilde{p}^\mu(l), \\ p^\mu(\tau,\sigma) &= \frac{1}{2}[\tilde{p}^\mu - \partial_-\tilde{x}^\mu(\sigma^+ \mapsto \sigma^-)], \end{aligned} \tag{36}$$

give general solution of the string equations of motion in the standard gauge (29) $\partial_0 x^\mu = -p^\mu$, $\partial_0 p^\mu = -\partial_1\partial_1 x^\mu$.

4 Type IIB Green-Schwarz superstring with irreducible first and second class constraints.

Here we show that type IIB theory is essentially different from other models with κ-symmetry. Namely, in this case *both* first and second class constraints can be arranged into irreducible sets *in the initial formulation*, without introducing of an additional variables.

Denoting two Majorana-Weyl spinors of the same chirality as $\theta^{1\alpha},\ \theta^{2\alpha},\ \alpha = 1,\ldots,16$, the type IIB GS superstring action is

$$\begin{aligned} S = T\int d^2\sigma \left[\frac{1}{2\sqrt{-g}}g^{ab}\Pi^\mu_a\Pi^\mu_b - i\varepsilon^{ab}\partial_a x^\mu(\theta^1\Gamma^\mu\partial_b\theta^1 - \theta^2\Gamma^\mu\partial_b\theta^2) + \right. \\ \left. \varepsilon^{ab}(\theta^1\Gamma^\mu\partial_a\theta^1)(\theta^2\Gamma^\mu\partial_b\theta^2)\right], \end{aligned} \tag{37}$$

where $\Pi^\mu_a \equiv \partial_a x^\mu - i\theta^A\Gamma^\mu\partial_a\theta^A$. Let us denote

$$\begin{aligned} B^\mu_1 &\equiv p^\mu + T\partial_1 x^\mu - iT\theta^1\Gamma^\mu\partial_1\theta^1, \\ B^\mu_2 &\equiv p^\mu - T\partial_1 x^\mu + iT\theta^2\Gamma^\mu\partial_1\theta^2, \\ \hat{p}^\mu &\equiv p^\mu - iT\theta^1\Gamma^\mu\partial_1\theta^1 + iT\theta^2\Gamma^\mu\partial_1\theta^2, \end{aligned} \tag{38}$$

$$\begin{aligned} D^\mu &\equiv \hat{p}^\mu + T\Pi^\mu_1 = p^\mu + T\partial_1 x^\mu - 2iT\theta^1\Gamma^\mu\partial_1\theta^1, \\ \Lambda^\mu &\equiv \hat{p}^\mu - T\Pi^\mu_1 = p^\mu - T\partial_1 x^\mu + 2iT\theta^2\Gamma^\mu\partial_1\theta^2, \end{aligned} \tag{39}$$

Then constraints under the interest for Eq.(37) can be written as

$$H_+ \equiv D^2 - 4TL^1_\beta\partial_1\theta^{1\beta} = 0, \quad H_- \equiv \Lambda^2 + 4TL^2_\beta\partial_1\theta^{2\beta} = 0, \tag{40}$$

$$\begin{array}{c} L_\alpha^1 \equiv p_{\theta 1\alpha} - iB_1^\mu(\Gamma^\mu\theta^1)_\alpha = 0, \\ L_\alpha^2 \equiv p_{\theta 2\alpha} - iB_2^\mu(\Gamma^\mu\theta^2)_\alpha = 0, \end{array} \tag{41}$$

Poisson brackets for the fermionic constraints are

$$\begin{array}{c} \{L_\alpha^1, L_\beta^1\} = 2iD_\mu{\Gamma^\mu}_{\alpha\beta}\delta(\sigma-\sigma'), \\ \{L_\alpha^2, L_\beta^2\} = 2i\Lambda_\mu{\Gamma^\mu}_{\alpha\beta}\delta(\sigma-\sigma'). \end{array} \tag{42}$$

As for $N=1$ case, the constraints $H_\pm$ were chosen such that they commutes with (41). In contrast to $N=1$ case, one has now the fermionic constraints in duplicate form. On this reason the second step of supplementation scheme is *not necessary* here. Eq.(41) can be rewritten equivalently as

$$D^\mu(\tilde\Gamma^\mu L^1)^\alpha = 0, \qquad \Lambda^\mu(\tilde\Gamma^\mu L^2)^\alpha = 0; \tag{43}$$

$$\Lambda^\mu(\tilde\Gamma^\mu L^1)^\alpha = 0, \qquad D^\mu(\tilde\Gamma^\mu L^2)^\alpha = 0, \tag{44}$$

where Eq.(43) (Eq.(44)) contains 1CC (2CC) correspondingly. According to the statement 2), the equivalent system is

$$\begin{array}{c} \Phi^{(1)\alpha} \equiv D^\mu(\tilde\Gamma^\mu L^1)^\alpha + \Lambda^\mu(\tilde\Gamma^\mu L^2)^\alpha = 0 \\ \Phi^{(2)\alpha} \equiv \Lambda^\mu(\tilde\Gamma^\mu L^1)^\alpha + D^\mu(\tilde\Gamma^\mu L^2)^\alpha = 0. \end{array} \tag{45}$$

Here $\Phi^{(1)\alpha}$ consist of 16 irreducible 1CC and $\Phi^{(2)\alpha}$ represents 16 irreducible 2CC. Non zero Poisson bracket is

$$\{\Phi^{(2)\alpha}, \Phi^{(2)\beta}\} = -4i(D\Lambda)(D^\mu + \Lambda^\mu)\tilde\Gamma^{\mu\alpha\beta}\delta(\sigma-\sigma'), \tag{46}$$

and is manifestly non degenerated.

Covariant and irreducible gauge for 1CC $\Phi^{(1)\alpha}=0$ can be chosen as

$$R_\alpha \equiv \Lambda^\mu(\Gamma^\mu\theta^1)_\alpha + D^\mu(\Gamma^\mu\theta^2)_\alpha = 0. \tag{47}$$

The corresponding bracket

$$\begin{array}{r} \{\Phi^{(1)\alpha}, R_\beta\} = [2(D\Lambda)\delta_\beta^\alpha + 4iT(D^\mu(\tilde\Gamma^\mu\Gamma^\nu\partial_1\theta^1)^\alpha(\Gamma^\nu\theta^2)_\beta - \\ \Lambda^\mu(\tilde\Gamma^\mu\Gamma^\nu\partial_1\theta^2)^\alpha(\Gamma^\nu\theta^1)_\beta)]\delta(\sigma-\sigma'), \end{array} \tag{48}$$

is manifestly non degenerated also.

5 Type IIA Green-Schwarz superstring with irreducible second class constraints.

The type IIA formulation can be obtained from Eqs.(37)- (40) by substitution $\theta^{2\alpha} \mapsto \theta_\alpha^2$ as well as $\Gamma^\mu_{\alpha\beta} \mapsto \tilde\Gamma^{\mu\alpha\beta}$ in each place where

Γ-matrix is associated with the spinor θ^2. The fermionic constraints are

$$\begin{aligned} L^1_\alpha &\equiv p_{\theta 1\alpha} - iB_1^\mu(\Gamma^\mu\theta^1)_\alpha = 0, \\ L^{2\alpha} &\equiv p^\alpha_{\theta 2} - iB_2^\mu(\tilde\Gamma^\mu\theta^2)^\alpha = 0. \end{aligned} \tag{49}$$

They belong to two inequivalent representations of $SO(1,9)$ group and thus can not be recombined in the Poincare covariant way. Moreover, there is no of natural object in the problem which can be used for raising (lowering) of spinor index. Nevertheless, after separation of the constraints, part of them can be arranged into irreducible subsets. Repeating the same procedure as in $N=1$ case, one obtains the additional sector

$$\Lambda^\mu(\Gamma^\mu\eta)_\alpha = 0, \qquad p_{\eta\alpha} = 0. \tag{50}$$

Then Eqs.(49), (50) can be rewritten in the equivalent form

$$\Phi^{(1)\alpha} \equiv D^\mu(\tilde\Gamma^\mu L^1)^\alpha + \Lambda^\mu(\tilde\Gamma^\mu p_\eta)^\alpha = 0, \tag{51}$$

$$\Lambda^\mu(\Gamma^\mu L^2)_\alpha = 0, \tag{52}$$

$$\begin{aligned} \Phi^{(2)\alpha} &\equiv \Lambda^\mu(\tilde\Gamma^\mu L^1)^\alpha + D^\mu(\tilde\Gamma^\mu p_\eta)^\alpha = 0, \\ G^{(2)\alpha} &\equiv D^\mu(\Gamma^\mu L^2)_\alpha + \Lambda^\mu(\Gamma^\mu\eta)_\alpha = 0, \end{aligned} \tag{53}$$

Here $\Phi^{(1)\alpha}$ are irreducible 1CC while $\Phi^{(2)\alpha}$, $G^{(2)\alpha}$ form irreducible set of 2CC. Thus, only 1CC in Eq.(52) remain reducible.

6 Conclusion

In this work we have analysed a possibility to apply the supplementation scheme [26] to the Green-Schwarz superstring. It was shown that type IIB theory represents an exceptional case, namely, both first and second class constraints can be arranged into irreducible sets (45) in the initial formulation. It implies that the standard path integral quantization methods can be applied to the resulting system (45) in a manifestly covariant form.

For the type IIA theory only 2CC can be arranged into irreducible set (53).

For both cases consistent Dirac bracket can be constructed and then the corresponding 2CC can be eliminated in a covariant way. It leads to the intriguing suggestion that it may exist manifestly $D10$ supersymmetric action which is equivalent to Type II GS superstring and which contains 1CC only.

For $N = 1$ case we have proposed the modified action (17). In addition to usual superspace coordinates it involves a pair of the Majorana-Weyl spinors. The additional variables are subject to reducible constraints (22), (23), which supply their nonphysical character (see discussion after Eq.(10)). Equivalence of the modified action and the initial one was proved in the canonical quantization framework (see discussion after Eq.(23)). We have demonstrated also how the state spectrum can be studied in the covariant gauge for κ-symmetry. In the modified formulation first class constraints form irreducible set (11) and are separated from the second class one (12), (13). The corresponding covariant gauge (24) is irreducible also, which garantees applicability of the usual path integral methods for the 1CC sector of the theory.

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