NONCOMMUTATIVE PARAMETERS OF QUANTUM SYMMETRIES AND STAR PRODUCTS

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Abstract

The star product technique translates the framework of local fields on noncommutative space–time into nonlocal fields on standard space–time. We consider the example of fields on κ– deformed Minkowski space, transforming under κ–deformed Poincaré group with noncommutative parameters. By extending the star product to the tensor product of functions on κ–deformed Minkowski space and κ-deformed Poincaré group we represent the algebra of noncommutative parameters of deformed relativistic symmetries by functions on classical Poincaré group.

1 Introduction

It has been recognized recently (see. e.g. [1]–[3]) that at very short distances, comparable with Planck length $\lambda \simeq 10^{-33}$cm, the notion of classical space–time manifold should be modified. The submicroscopic quantum structure of space– time implies noncommutativity, i.e. one should replace the classical Minkowski coordinates x_μ by the generators $\widehat{x}_\mu$ of noncommutative algebra. Assuming the formula (see e.g. [4], [5])[1]

$$[\widehat{x}_\mu, \widehat{x}_\nu] = \theta_{\mu\nu}(\widehat{x}) = \theta^{(0)}_{\mu\nu} + \theta^{(1)\rho}_{\mu\nu}\widehat{x}_\rho + \theta^{(2)\rho\tau}_{\mu\nu}\widehat{x}_\rho\widehat{x}_\tau + \ldots \tag{1}$$

with $\theta_{\mu\nu}(\widehat{x})$ restricted by Jacobi identities, one arrives at different models of noncommutative space–time geometry. The simplest case is obtained if $\theta_{\mu\nu}(x)$ is a constant ($\theta_{\mu\nu}(x) \equiv \theta^{(0)}_{\mu\nu}$). Such a deformation, firstly advocated by Dopplicher, Fredenhagen and Roberts [1] has been recently extensively studied in string theory as describing world volume coordinates of D–branes (see e.g. [7]–[9]). In such a deformation the relativistic symmetries remain classical, what simplifies greatly the formalism of corresponding noncommutative theory. Indeed, if we put $\theta_{\mu\nu}(\widehat{x}) = \theta^{(0)}_{\mu\nu}$ the relations (1) remain invariant under the shifts $\widehat{x}'_\mu = \widehat{x}_\mu + a_\mu$ where a_μ are classical commutative transformations. If the rhs of (1) depends on $\widehat{x}_\mu$ the translations preserving the algebraic structure of space–time become noncommutative. The simplest framework is provided if the rhs of (1) is linear, i.e.

$$[\widehat{x}_\mu, \widehat{x}_\nu] = \theta^{(1)\rho}_{\mu\nu}\widehat{x}_\rho \tag{2}$$

In such a case the translations $\widehat{x}'_\mu = \widehat{x}_\mu + \widehat{v}_\nu$ commute with space–time algebra

$$[\widehat{x}_\mu, \widehat{v}_\nu] = 0 \tag{3}$$

and form themselves second copy of the algebra (2)

$$[\widehat{x}_\mu, \widehat{v}_\nu] = \theta^{(1)\rho}_{\mu\nu}\widehat{v}_\rho \tag{4}$$

In the general case the translations form another copy of algebra (1), but the invariance under translations implies nontrivial braiding relations between the noncommutative algebra of space– time coordinates (1) and the translations algebra (we use property $\theta_{\mu\nu} = -\theta_{\nu\mu}$):

$$[\widehat{x}_\mu, \widehat{v}_\nu] = \frac{1}{2}\left\{\theta_{\mu\nu}(\widehat{x}+\widehat{v}) - \theta_{\mu\nu}(\widehat{x}) - \theta_{\mu\nu}(\widehat{v})\right\} \tag{5}$$

[1] We shall restrict our considerations to the case when the noncommutative algebra of space–time is generated only by $\widehat{x}_\mu$. In general case, as in the first model of noncommutative space–time by Snyder [6], the operator basis of the algebra is extended by other operators (see e.g. [7])

The noncommutativity of space–time translations implies necessarily the modification of space–time symmetries. In particular, one can pose the question for which functions $\theta_{\mu\nu}(\widehat{x})$ in (1) the noncommutative translations $\widehat{v}_\mu$ can be extended to the quantum Poincaré group, describing the Hopf algebra of deformed relativistic symmetries. The classification of noncommutative translations which can be extended to standard (non–braided) quantum Poincaré group was given by Podleś and Woronowicz [10]. In particular, if we wish to maintain the classical nonrelativistic $O(3)$–symmetries, the choice of the deformation is unique – one obtains the standard form of κ–deformation of relativistic symmetries ([11]–[13]).

The aim of this talk is to describe the κ–deformed field theory in the commutative framework of classical fields, with the noncommutative parameters of κ–deformed Poincaré group described by commutative parametrization. For that purpose the star product on κ–deformed Minkowski space [14], identical to the star product on the subalgebra of noncommutative translations, is extended to ten generators $(\widehat{v}_\mu, \widehat{\Lambda}^\nu{}_\kappa)$ of κ–deformed Poincaré group.

It appears that due to the fact that the cross relations between Lorentz generators $\widehat{\Lambda}_{\mu\nu}$ and translations $\widehat{v}_\mu$ are quadratic, our extended star product goes beyond the CBH formula describing star products for Lie–algebraic or Lie–superalgebraic structures.

The plan of our presentation is the following:

In the Section 2 we describe the κ–deformed Poincaré group and recall the star product for the fields defined on κ–deformed Minkowski space–time. In Sect. 3 we introduce the star product for functions on κ–deformed Poincaré group. In Sect. 4 we present final remarks and outlook.

2 κ–Deformed Poincaré Group and Fields on κ–Deformed Minkowski Space

The κ–deformed Poincaré group is described by the deformed noncommutative group parameters $(\widehat{v}_\mu, \widehat{\Lambda}^\mu{}_\nu)$ satisfying the algebraic relations [15, 16]

$$[\widehat{v}_\mu, \widehat{v}_\nu] = \frac{i}{\kappa}\left(\delta^0{}_\mu\, \widehat{v}_\nu - \delta^0{}_\nu\, \widehat{v}_\mu\right) \tag{6a}$$

$$\left[\widehat{\Lambda}^\mu{}_\nu, \widehat{v}_\rho\right] = -\frac{i}{\kappa}\left\{(\Lambda^\mu{}_0 - \delta^\mu{}_0\,)\,\widehat{\Lambda}_{\rho\nu} + \left(\widehat{\Lambda}_{0\nu} - \eta_{0\nu}\right)\delta^\mu{}_\rho\right\} \tag{6b}$$

$$\left[\widehat{\Lambda}^\mu{}_\nu, \widehat{\Lambda}^\rho{}_\tau\right] = 0 \tag{6c}$$

with the constraints $\Lambda\Lambda^T = \Lambda^T\Lambda = 1$ or

$$\widehat{\Lambda}^{\mu}{}_{\nu}\,\widehat{\Lambda}^{\tau}{}_{\nu} = \widehat{\Lambda}^{\nu}{}_{\mu}\widehat{\Lambda}^{\nu}{}_{\tau} = \eta^{\mu\tau} = \eta_{\mu\tau} \tag{7}$$

where $\mathrm{diag}\,\eta = (1,1,1,-1)$.

The relations (6) were firstly obtained [15] by the quantization of Poisson–Lie bracket for the functions on Poincaré group with the following classical r–matrix

$$r = \frac{1}{\kappa} N_i \,\Lambda\, P_i\,, \tag{8}$$

where P_i are three–momenta and $N_i \equiv M_{i0}$ are Lorentz boost generators. Another way to obtain the relations (6a)–(6c) was to construct the dual Hopf algebra to the κ–deformed Poincaré algebra $\mathcal{U}_\kappa(\mathcal{P}_4)$ written in bicrossproduct basis ([13],[16]). The coproduct for $\widehat{v}_\kappa, \widehat{\Lambda}^{\kappa}{}_{\nu})$ remain undeformed

$$\Delta(\widehat{v}_\mu) \;=\; \widehat{v}_\nu \otimes \widehat{\Lambda}_{\mu}{}^{\nu} + \mathbf{1}\otimes \widehat{v}_\mu \tag{9a}$$

$$\Delta(\widehat{\Lambda}^{\mu}{}_{\nu}) \;=\; \widehat{\Lambda}^{\mu}{}_{\rho}\otimes\widehat{\Lambda}^{\rho}{}_{\nu} \tag{9b}$$

i.e. the composition of two quantum Poincaré group transformations is described by standard classical formulae.

The κ–deformed Minkowski space described in the formula (9a) by $\widehat{x} = \widehat{v}_\mu \otimes \mathbf{1}$ satisfies the relations (6a), or, more explicitly,

$$[\widehat{x}_0, \widehat{x}_i] = \frac{i}{\kappa}\widehat{x}_i \qquad [\widehat{x}_i, \widehat{x}_j] = 0 \tag{10}$$

The κ–deformed field theory is described by the operator functions $\Phi_A(\widehat{x})$. Following the arguments given in [17],[14] we shall use for the fields $\Phi_A(\widehat{x})$ the κ–deformed Fourier transform

$$\Phi_A(\widehat{x}) = \frac{1}{(2\pi)^4}\int d^4p\,\widetilde{\Phi}_\kappa(p) : e^{ip\widehat{x}} : \tag{11}$$

where

$$: e^{ip\widehat{x}} : \equiv e^{-ip_0\widehat{x}_0} e^{i\vec{p}\vec{\widehat{x}}} \tag{12}$$

and

$$\widetilde{\Phi}_\kappa(p) = e^{\frac{3p_0}{\kappa}}\,\widetilde{\Phi}\left(e^{\frac{p_0}{\kappa}}\,\vec{p}, p_0\right) \tag{13}$$

We have

$$: e^{ip\widehat{x}} :: e^{ip'\widehat{x}} :=: e^{i\Delta^{(2)}(p,p')\widehat{x}} : \tag{14}$$

where $\Delta^{(2)}_\mu = (\Delta^{(2)}_0 = p_0 + p'_0, \Delta^{(2)}_i = p_i e^{\frac{p'_0}{\kappa}} + p'_i)$.

The algebraic relation (14) is translated into star product framework by the replacement $\widetilde{x}_\mu \to x_\mu$ where x_μ are classical space–time coordinates and the ordering in eq. (12) is reflected in explicite choice of the star multiplication:

$$e^{ipx} \star e^{ip'x} = e^{i\Delta^{(2)}(p,p')x} \tag{15}$$

i.e. after replacement $\Phi(\widehat{x}) \to \phi(x)$ one gets

$$\phi(x) \star \chi(x) = \frac{1}{(2\pi)^4} \int d^4p \; d^4p' \widetilde{\phi}_\kappa(p) \widetilde{\chi}_\kappa(p') e^{i\Delta^{(2)}(p,p')x} \tag{16}$$

In the following section we shall extend the star product (15–16), valid for the noncommutative translations, to the whole quantum κ–deformed Poincaré group.

3 The Star Product for κ–Deformed Poincaré Group

In order to extend the action of Poincaré group on Minkowski space to the noncommutative case we have to replace the classical Poincaré group by its κ–deformed counterpart

$$(a_\mu, \Lambda^\mu{}_\nu) \Longrightarrow \left(\widehat{a}_\mu, \widehat{\Lambda}^\mu{}_\nu\right) \tag{17}$$

The noncommutativity of symmetry group parameters raises the question of the physical interpretation of deformed symmetries. In this chapter we shall show how one can replace the operator algebra of functions on κ–deformed Poincaré group (6a–6c) by the functions on classical Poincaré group, with suitably chosen star product multiplication.

We shall consider the algebra of the following ordered exponentials

$$: e^{i(\alpha_\mu \widehat{v}^\mu + b^\nu_\mu \widehat{\Lambda}^\mu{}_\nu)} := e^{-i\alpha_0 \widehat{v}_0} e^{i\vec{\alpha}\vec{v}} e^{ib^\mu_\nu \widehat{\Lambda}_\mu{}^\nu} \tag{18}$$

The product of two ordered exponentials (18) is given by the formula:

$$\begin{aligned} &: e^{i\alpha_\mu \widehat{v}^\mu + ib^\nu{}_\mu \widehat{\Lambda}^\mu{}_\nu} :: e^{i\alpha'_\mu \widehat{v}^\mu + ib'^\nu_\mu \widehat{\Lambda}^\mu{}_\nu} : \\ &=: e^{i\Delta^{(2)}_\mu(\alpha,\alpha')\widehat{v}^\mu} : e^{i(b^\nu{}_\mu f^\mu_\nu(g^\rho_\sigma(\widehat{\Lambda},\vec{\alpha}'),\alpha) + b^\nu{}_\mu \widehat{\Lambda}^\mu{}_\nu)} \end{aligned} \tag{19}$$

where

$$e^{-i\lambda\alpha^0} \widehat{\Lambda}^\kappa{}_\nu e^{i\lambda\alpha^0} \;\; = \;\; f^\kappa_\nu\left(\widehat{\Lambda}, \lambda\right) \tag{20a}$$

$$e^{-i\vec{\lambda}\vec{\alpha}}\widehat{\Lambda}^{\kappa}{}_{\nu}e^{i\vec{\lambda}\vec{\alpha}} \quad = \quad g^{\kappa}{}_{\nu}\left(\widehat{\Lambda},\vec{\lambda}\right) \tag{20b}$$

The functions $f^{\kappa}{}_{\nu}$ and $g^{\kappa}{}_{\nu}$ can be calculated explicitly. The functions defined by (20a) read:

$$\begin{aligned}
f^{0}{}_{0}(\widehat{\Lambda},\lambda) &= \tanh\frac{\lambda}{\kappa}\left(\frac{1+\coth\frac{\lambda}{\kappa}\widehat{\Lambda}^{0}{}_{0}}{1+\tanh\frac{\lambda}{\kappa}\widehat{\Lambda}^{0}{}_{0}}\right)\\
f^{0}{}_{k}(\widehat{\Lambda},\lambda) &= \left(\cosh\frac{\lambda}{\kappa}\right)^{-1}\frac{\widehat{\Lambda}^{0}{}_{k}}{1+\tanh\frac{\lambda}{\kappa}\widehat{\Lambda}^{0}{}_{0}}\\
f^{k}{}_{0}(\widehat{\Lambda},\lambda) &= \left(\cosh\frac{\lambda}{\kappa}\right)^{-1}\frac{\Lambda^{k}{}_{0}}{1+\tanh\frac{\lambda}{\kappa}\widehat{\Lambda}^{0}{}_{0}}\\
f^{i}{}_{k}(\widehat{\Lambda},\lambda) &= \frac{\widehat{\Lambda}^{i}{}_{k}+\tanh\frac{\lambda}{\kappa}\left(\widehat{\Lambda}^{0}{}_{0}\widehat{\Lambda}^{i}{}_{k}-\widehat{\Lambda}^{i}{}_{0}\widehat{\Lambda}^{0}{}_{k}\right)}{1+\tanh\frac{\lambda}{\kappa}\widehat{\Lambda}^{0}{}_{0}}
\end{aligned} \tag{21}$$

The calculation of (20b) is more complicated, but also possible. They are described by the following set of relations

$$e^{-i\lambda\, a^{k}}\,\Lambda^{0}{}_{0}\,e^{i\lambda\, a^{k}} \quad = \quad \frac{\left(\frac{\Lambda^{0}{}_{0}-1}{2\kappa^{2}}\right)\lambda^{2}-\frac{\Lambda^{k}{}_{0}}{\kappa}\lambda+\Lambda^{0}{}_{0}}{\frac{\left(\Lambda^{0}{}_{0}-1\right)}{2\kappa^{2}}\lambda^{2}-\frac{\Lambda^{k}{}_{0}}{\kappa}\lambda+1} \tag{22a}$$

$$e^{-i\lambda\, a^{k}}\,\Lambda^{k}{}_{0}\,e^{i\lambda\, a^{k}} \quad = \quad \frac{-\left(\Lambda^{0}{}_{0}-1\right)\frac{\lambda}{\kappa}+\Lambda^{k}{}_{0}}{\frac{\left(\Lambda^{0}{}_{0}-1\right)}{2\kappa^{2}}\lambda^{2}-\frac{\Lambda^{k}{}_{0}}{\kappa}\lambda+1} \tag{22b}$$

$$e^{-i\lambda\, a^{k}}\,\Lambda^{i}{}_{0}\,e^{i\lambda\, a^{k}} \quad = \quad \frac{\Lambda^{i}{}_{0}}{\frac{\left(\Lambda^{0}{}_{0}-1\right)}{2\kappa^{2}}\lambda^{2}-\frac{\Lambda^{k}{}_{0}}{\kappa}\lambda+1} \tag{22c}$$

$$e^{-i\lambda\, a^{k}}\,\Lambda^{0}{}_{k}\,e^{i\lambda\, a^{k}} \quad = \quad \frac{\Lambda^{0}{}_{k}+\frac{\Lambda^{k}{}_{k}\left(\Lambda^{0}{}_{0}-1\right)+\Lambda^{0}{}_{k}\Lambda^{k}{}_{0}}{\kappa}\lambda}{\frac{\left(\Lambda^{0}{}_{0}-1\right)}{2\kappa^{2}}\lambda^{2}-\frac{\Lambda^{k}{}_{0}}{\kappa}\lambda+1} \tag{22d}$$

$$e^{-i\lambda\, a^{k}}\,\Lambda^{k}{}_{k}\,e^{i\lambda\, a^{k}} \quad = \quad \frac{-\frac{\left(\Lambda^{k}{}_{k}\left(\Lambda^{0}{}_{0}-1\right)+\Lambda^{0}{}_{k}\Lambda^{k}{}_{0}\right)}{2\kappa^{2}}\lambda^{2}-\frac{\Lambda^{0}{}_{k}\lambda}{\kappa}+\Lambda^{k}{}_{k}}{\frac{\left(\Lambda^{0}{}_{0}-1\right)}{2\kappa^{2}}\lambda^{2}-\frac{\Lambda^{k}{}_{0}}{\kappa}\lambda+1} \tag{22e}$$

$$e^{-i\lambda\, a^k}\, \Lambda^0{}_i\, e^{i\lambda\, a^k} \quad = \quad \frac{\Lambda^0{}_i + \frac{\left(\Lambda^0{}_0 - 1\right) + \Lambda^k{}_i + \Lambda^k{}_0 \Lambda^0{}_i}{\kappa}\, \lambda}{\frac{\left(\Lambda^0{}_0 - 1\right)}{2\kappa^2}\, \lambda^2 - \frac{\Lambda^k{}_0}{\kappa}\, \lambda + 1} \tag{22f}$$

$$e^{-i\lambda\, a^k}\, \Lambda^k{}_i\, e^{i\lambda\, a^k} \quad = \quad \frac{\Lambda^k{}_i - \frac{\Lambda^0{}_i}{\kappa}\, \lambda + \frac{\left(\Lambda^0{}_0 - 1\right) + \Lambda^k{}_i + \Lambda^k{}_0 \Lambda^0{}_i}{2\kappa^2}\, \lambda^2}{\frac{\left(\Lambda^0{}_0 - 1\right)}{2\kappa^2}\, \lambda^2 - \frac{\Lambda^k{}_0}{\kappa}\, \lambda + 1} \tag{22g}$$

$i \neq k$

$$e^{-i\lambda\, a^k}\, \Lambda^i{}_k\, e^{i\lambda\, a^k} =$$

$$= \Lambda^i{}_k - \frac{1}{\kappa} \int\limits_0^\lambda d\mu\, \frac{\Lambda^i{}_0 \left[-\frac{\Lambda^k{}_k \left(\Lambda^0{}_0 - 1\right) + \Lambda^0{}_k \Lambda^k{}_0}{2\kappa^2}\, \mu^2 - \frac{\Lambda^0{}_k}{\kappa}\, \mu + \Lambda^k{}_k \right]}{\left[\frac{\left(\Lambda^0{}_0 - 1\right)}{2\kappa^2}\, \mu^2 - \frac{\Lambda^k{}_0}{\kappa}\, \mu + 1 \right]^2} \tag{22h}$$

$i \neq k, j \neq k$

$$e^{-i\lambda\, a^k}\, \Lambda^i{}_j\, e^{i\lambda\, a^k} =$$

$$= \Lambda^i{}_j - \frac{1}{\kappa} \int\limits_0^\lambda d\mu\, \frac{\Lambda^i{}_0 \left[-\frac{\left(\Lambda^0{}_0 - 1\right) \Lambda^k{}_j + \Lambda^k{}_0\, \Lambda^0{}_j}{2\kappa^2}\, \mu^2 - \frac{\Lambda^0{}_k}{\kappa}\, \mu + \Lambda^k{}_j \right]}{\left[\frac{\left(\Lambda^0{}_0 - 1\right)}{2\kappa^2}\, \mu^2 - \frac{\Lambda^k{}_0}{\kappa}\, \mu + 1 \right]^2} \tag{22i}$$

In order to represent the relation (19) in star product framework we reproduce the multiplication (19) by a new star product of the basic functions on classical Poincaré group parameters:

$$e^{i(\alpha_\kappa v^\kappa + b^\nu_\mu \Lambda^\mu{}_\nu)} \circledast e^{i(\alpha'_\kappa v^\kappa + b^{\nu\prime}_\mu \Lambda^\mu{}_\nu)}$$

$$= e^{i(\Delta_\mu(\alpha,\alpha') v^\kappa + b^\nu_\mu f^\kappa_\nu (g^\rho_\sigma(1,\vec{\alpha}'),\alpha_0) + b^{\nu\prime}_\mu \Lambda^\kappa{}_\nu)} \tag{23}$$

As it is seen from (22a –22i) the function g^ρ_σ are not linear in $\widehat{\Lambda}^\kappa{}_\nu$, due to the quadratic commutator (6b). On the other hand, due to the commutativity (6c) the formulae for f^κ_ν and g^κ_ν can be obtained in explicite form.

4 Final Remarks

We would like to make the following comments:

i) It should be observed that the nice coproduct formula (15) for noncommutative translations can not be extended to the Lorentz sector. One can pose the question whether by a suitable choice of noncommutative $b^{\mu}{}_{\nu}$ such extension can be achieved.

ii) The relations (6a–6c) describe a quadratic algebra, which implies that in the exponential on rhs of (23) there are arbitrary powers of $\Lambda^{\kappa}{}_{\nu}$. It should be observed, however, that the multiplication formula (23) has an explicite form.

iii) Using the star product (23) one can discuss the covariance of κ–deformed local field theory under the κ–deformed relativistic transformations. At present it is only clear how to show in the star–product framework the covariance under the subgroup of noncommutative translations (see also [14]).

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