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On the quantum matrix string*

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Abstract

We study the behavior of matrix string theory in the strong coupling region, where it is expected to reduce to discrete light-cone type IIA superstring. In the large N limit, the reduction corresponds to the double-dimensional reduction from wrapped supermembranes on $R^{10} \times S^1$ to type IIA superstrings on R^{10} in the light-cone gauge, which is shown classically, however it is not obvious quantum mechanically. We analyze the problem in matrix string theory by using the strong coupling ($1/g$) expansion. We find that the quantum corrections do not cancel out at $\mathcal{O}(1/g^2)$. Detailed calculations can be seen in Ref.[1].

1 Introduction

Supermembrane in eleven dimensions [2, 3] plays an important role to understand the fundamental degrees of freedom in M-theory. At the classical level, it was shown that the supermembrane is related to type IIA superstring in ten dimensions by the double-dimensional reduction [4]. The procedure is the following: (i) Consider the target space of $R^{10} \times S^1$. (ii) Set the compactified coordinate (with radius L) proportional to one of the spatial coordinates of the world volume, which we call ρ coordinate. (iii) Simply ignore the infinite tower of the Kaluza-Klein (non-zero) modes. However, it is not obvious whether such a reduction is justified also in quantum theory.

Sekino and Yoneya analyzed the double-dimensional reduction quantum mechanically with the light-cone supermembrane action [6]. They kept the

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Kaluza-Klein modes associated with the ρ coordinate in the wrapped supermembrane theory on the target space $R^{10} \times S^1$ and integrated them out by using the perturbative expansion with respect to the radius L. Since the gauge coupling satisfies $g \sim 1/L$ in the wrapped supermembrane theory, the expansion can be regarded as the strong coupling expansion. They calculated the effective action for the zero modes along the ρ direction to the one-loop order of $O(L^2)$ and found that the quantum corrections cancel out and the effective action agrees with the classical (free) action of type IIA superstring except at the points where the usual string interactions could occur. As is emphasized in their paper [6], however, the strong coupling expansion does not give a rigorous proof of the quantum double-dimensional reduction because the propagators are proportional to the two-dimensional δ-function, $\delta^{(2)}(\xi) \equiv \delta(\tau)\delta(\sigma)$, which will cause the ultraviolet divergences of $\delta^{(2)}(0)$ type in loops. However it is very difficult to find a suitable regularization which respects symmetries, and hence the strong coupling expansion is not yet defined rigorously. In this sense, they gave a formal argument for the vanishing of the one-loop corrections of $O(L^2)$ by demonstrating that the coefficients of $\delta^{(2)}(0)$ coming from both bosonic and fermionic degrees of freedom cancel out.

The purpose of our work is essentially to extend their analysis to the two-loop order of $O(L^2)$. However, the naive extension is not straightforward because at the two-loop level, even the coefficients of the $\delta^{(2)}(0)$ diverge due to the contribution of the infinite Kaluza-Klein towers. Thus, we need another regularization for the summation over the infinite tower of the Kaluza-Klein modes. We know the matrix regularization of the supermembrane on R^{11} in the light-cone gauge [7] and also that of the wrapped supermembrane on $R^{10} \times S^1$ in the light-cone gauge [6]. The former is called Matrix theory [8] and the latter is called matrix string theory [9, 10] which will be a non-perturbative formulation of light-cone quantized type IIA superstring theory in the large N limit. Furthermore, even at finite N, Matrix and matrix string theories are conjectured to be non-perturbative formulations of discrete light-cone quantized (DLCQ) M-theory and type IIA superstring theory, respectively [11, 12, 13]. Thus, we consider matrix string theory and study whether the reduction from matrix strings to discrete light-cone type IIA superstrings is justified quantum mechanically.[1]

[1]We use a convention of the light-cone coordinates such that $x^{\pm} = (x^0 \pm x^{10})/\sqrt{2}$. Furthermore, x^- is compactified on S^1 with radius R in DLCQ.

2 From wrapped supermembrane to matrix string

The supermembrane action on the target space R^{11} [7] in the light-cone gauge is given by

$$S = LT\int d\tau\int_0^{2\pi} d\sigma d\rho\Big[\frac{1}{2}(D_\tau X^i)^2 - \frac{1}{4L^2}\{X^i, X^j\}^2 + i\psi^T D_\tau\psi + \frac{i}{L}\psi^T\gamma^i\{X^i,\psi\}\Big], \tag{1}$$

$$D_\tau = \partial_\tau - \frac{1}{L}\{A,\ \ \},\quad \{A,B\} = \partial_\sigma A\,\partial_\rho B - \partial_\rho A\,\partial_\sigma B, \tag{2}$$

where the indices i, j run through $1, 2, \cdots, 9$, the spinor ψ has sixteen real components[2] and T is the membrane tension. At this stage, L is an arbitrary length parameter of no physical meaning. The action is invariant under the gauge transformation,

$$\delta A = \partial_\tau\Lambda + \frac{1}{L}\{\Lambda, A\},\quad \delta X^i = \frac{1}{L}\{\Lambda, X^i\},\quad \delta\psi = \frac{1}{L}\{\Lambda,\psi\}. \tag{3}$$

This gauge transformation generates the area-preserving diffeomorphism on the world volume. When the spatial surface of the supermembrane has a non-trivial topology, we have to impose further the global constraints.

Now we consider the wrapped supermembrane theory on the target space $R^{10}\times S^1$ and discuss the correspondence with matrix string [6]. We take the X^9 direction as S^1 and identify the radius with the above parameter L,

$$X^9 = L\rho + Y. \tag{4}$$

Thus L has the physical meaning of the radius of the X^9 direction which is regarded as the "eleventh" direction in M-theory. Substituting eq.(4) into eq.(1), we obtain the light-cone gauge supermembrane action on $R^{10}\times S^1$,

$$S = LT\int d\tau\int_0^{2\pi} d\sigma d\rho\left[\frac{1}{2}F_{\tau\sigma}^2 + \frac{1}{2}(D_\tau X^k)^2 - \frac{1}{2}(D_\sigma X^k)^2 - \frac{1}{4L^2}\{X^k, X^l\}^2 + i\psi^T D_\tau\psi - i\psi^T\gamma^9 D_\sigma\psi + \frac{i}{L}\psi^T\gamma^k\{X^k,\psi\}\right], \tag{5}$$

$$F_{\tau\sigma} = \partial_\tau Y - \partial_\sigma A - \frac{1}{L}\{A, Y\},\quad D_\sigma = \partial_\sigma - \frac{1}{L}\{Y,\ \ \}, \tag{6}$$

where the indices k, l run through $1, 2, \cdots, 8$. This is also an action of the gauge theory of the area-preserving diffeomorphism, where the gauge coupling $g \sim$

[2] We use the real and symmetric gamma matrices γ^i, which satisfy $\{\gamma^i, \gamma^j\} = 2\delta^{ij}$.

$1/L$. In Ref.[6], the area-preserving diffeomorphism in eq.(5) was regularized by the finite dimensional group $U(N)$ and it was shown that the matrix-regularized form of the action (5) agrees with that of matrix string theory,

$$S = LT \int d\tau \int_0^{2\pi} d\theta\, tr \left[\frac{1}{2}F_{\tau\theta}^2 + \frac{1}{2}(D_\tau X^k)^2 - \frac{1}{2}(D_\theta X^k)^2 + \frac{1}{4L^2}[X^k, X^l]^2 \right.$$
$$\left. + i\psi^T D_\tau \psi - i\psi^T \gamma^9 D_\theta \psi - \frac{1}{L}\psi^T \gamma^k [X^k, \psi]\right], \tag{7}$$
$$F_{\tau\theta} = \partial_\tau Y - \partial_\theta A - \frac{i}{L}[A, Y],\ D_\tau = \partial_\tau - \frac{i}{L}[A,\ \ ],\ D_\theta = \partial_\theta - \frac{i}{L}[Y,\ \ ], \tag{8}$$

where each element of the matrices is a function of (τ, θ). The action (7) is invariant under the $U(N)$ gauge transformations,

$$\delta A = \partial_\tau \Lambda + \frac{i}{L}[\Lambda, A], \quad \delta Y = \partial_\theta \Lambda + \frac{i}{L}[\Lambda, Y],$$
$$\delta X^k = \frac{i}{L}[\Lambda, X^k], \quad \delta\psi = \frac{i}{L}[\Lambda, \psi]. \tag{9}$$

The zero-modes along the ρ direction in the wrapped supermembrane are mapped to the diagonal elements of matrix string while the Kaluza-Klein modes are mapped to the off-diagonal elements [6]. Note that in the matrix regularization of the wrapped supermembrane on $R^{10} \times S^1$, there are no obvious counterparts of the global constraints, because the (matrix-regularized) Gauss law constraint, which is derived from eq.(7), cannot be manifestly interpreted as the integrability condition.

In the classical double-dimensional reduction, the Kaluza-Klein modes of every field along the ρ direction are set zero. And then in the $L \to 0$ limit, the action (5) reduces to the light-cone type IIA superstring action. As for the matrix-regularized action (7), the off-diagonal elements of every matrix are set zero in such a classical double-dimensional reduction. Then the action reduces to the DLCQ type IIA superstring action in the light-cone momentum $p^+ = N/R$ sector. It is expected that the reductions are justified also in quantum theory, however, it is not so simple [5, 6]. In particular, the quantum double-dimensional reduction of the wrapped supermembrane was analyzed for the small radius L in [6], which corresponds to the strong gauge coupling $g \sim 1/L$ in the wrapped supermembrane theory and also to the weak string coupling $g_s \sim L/\sqrt{\alpha'}$ in type IIA superstring theory. By using the perturbative expansion with respect to L, the Kaluza-Klein modes along the ρ direction were integrated out to the one-loop order of $O(L^2)$ and it was found that the effective action for the zero modes agrees with the classical (free) action of the type

IIA superstring except at the interaction points of perturbative strings. So far the result is consistent with the expectation that the wrapped supermembrane theory in the region of small radius L agrees with the perturbative type IIA superstring theory. Then we analyze the quantum reduction of the matrix string (7) to the diagonal elements for small radius L to study whether the effective action for the diagonal matrix elements agrees with the classical (free) action of the DLCQ type IIA superstring.

3 Strong coupling expansion in matrix string theory

First every $N \times N$ hermite matrix in eq.(7) is decomposed into the diagonal and off-diagonal parts,

$$A \to a + A, \quad Y \to y + Y, \quad X^k \to x^k + X^k, \quad \psi \to \psi + \Psi, \tag{10}$$

where a, y, x^k and ψ are the diagonal and A, Y, X^k and Ψ are the off-diagonal parts of the original matrices, respectively, which are plugged into (7). The gauge transformations are also decomposed as

$$\delta a = \partial_\tau \lambda + \frac{i}{L}[\Lambda, A]_{\rm diag}, \quad \delta A = \partial_\tau \Lambda + \frac{i}{L}([\lambda, A] + [\Lambda, a] + [\Lambda, A]_{\text{off-diag}}), \ \cdots \tag{11}$$

where λ and Λ are the diagonal and off-diagonal parts of the gauge parameter, respectively. At this stage, we impose a boundary condition in θ-direction. Here, for simplicity, we choose such boundary conditions as to have the N string bits having $p^+ = 1/R$ for the diagonal matrix elements,

$$\phi_a(\theta + 2\pi) = \phi_a(\theta). \quad (\text{for } \phi = a, y, x^k, \psi) \tag{12}$$

As for the off-diagonal matrix elements, we naturally impose

$$\Phi_{ab}(\theta + 2\pi) = \Phi_{ab}(\theta). \quad (\text{for } \Phi = A, Y, X^k, \Psi) \tag{13}$$

Next [illegible] on,

$$[illegible] 0, \tag{14}$$

and we p[illegible]

$$T = \int \mathcal{D}y\mathcal{D}x^k\mathcal{D}\psi\mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}A\mathcal{D}Y\mathcal{D}X^k\mathcal{D}\Psi\mathcal{D}C\mathcal{D}\bar{C}\mathcal{D}B$$

$$\exp\Big[iLT\int d\tau\int_0^{2\pi}d\theta\,(\mathcal{L}^{string}+\mathcal{L}_0^B+L\mathcal{L}_1^B+L^2\mathcal{L}_2^B$$
$$+\mathcal{L}_0^F+L^{1/2}\mathcal{L}_{1/2}^F+L\mathcal{L}_1^F)\Big], \quad (15)$$

$$\begin{aligned}
\mathcal{L}^{string} &= \mathrm{Tr}\Big[\frac{1}{2}(\partial_\tau x^k)^2-\frac{1}{2}(\partial_\theta x^k)^2+\frac{1}{2}\{(\partial_\tau-\partial_\theta)y\}^2 \\
&\qquad +i\bar{c}(\partial_\tau-\partial_\theta)c+i\psi^T\partial_\tau\psi-i\psi^T\gamma^9\partial_\theta\psi\Big], && (16)\\
\mathcal{L}_0^B &= \mathrm{Tr}\Big[-\frac{1}{2}[x^k,A]^2+\frac{1}{2}[x^k,Y]^2+\frac{1}{2}[x^k,X^l]^2 \\
&\quad -\frac{1}{2}\Big([y,Y]+[x^k,X^k]-[y,A]\Big)^2 \\
&\quad -iB\Big([y,Y]+[x^k,X^k]-[y,A]\Big)+i[x^k,\bar{C}][x^k,C]\Big], && (17)\\
\mathcal{L}_0^F &= \mathrm{Tr}\Big[\Psi^T[y,\Psi]-\Psi^T\gamma^9[y,\Psi]-\Psi^T\gamma^k[x^k,\Psi]\Big], && (18)\\
\mathcal{L}_{1/2}^F &= \mathrm{Tr}\Big[-2\Psi^T[\psi,A]+2\Psi^T\gamma^9[\psi,Y]+2\Psi^T\gamma^k[\psi,X^k]\Big], && (19)\\
\mathcal{L}_1^B &= \mathrm{Tr}\Big[-i\partial_\tau Y[y,Y]+2i\partial_\tau Y[y,A]-i\partial_\tau A[y,Y] \\
&\qquad +2i\partial_\theta A[y,Y]-i\partial_\theta A[y,A]-i\partial_\theta Y[y,A] \\
&\qquad -i\partial_\tau X^k[y,X^k]+2i\partial_\tau X^k[x^k,A]-i\partial_\tau A[x^k,X^k] \\
&\qquad +i\partial_\theta X^k[y,X^k]-2i\partial_\theta X^k[x^k,Y]+i\partial_\theta Y[x^k,X^k] \\
&\qquad -[y,Y][A,Y]+[y,A][A,Y]-[y,X^k][A,X^k] \\
&\qquad +[x^k,A][A,X^k]+[y,X^k][Y,X^k]-[x^k,Y][Y,X^k] \\
&\qquad +[x^k,X^l][X^k,X^l]+B\partial_\theta Y-B\partial_\tau A-\partial_\theta\bar{C}[y,C] \\
&\qquad -[y,\bar{C}]\partial_\theta C+i[y,\bar{C}][Y,C]+i[x^k,\bar{C}][X^k,C] \\
&\qquad +[y,\bar{C}]\,\partial_\tau C-i[y,\bar{C}][A,C]+\partial_\tau\bar{C}[y,C]\Big], && (20)\\
\mathcal{L}_1^F &= \mathrm{Tr}\Big[i\Psi^T\partial_\tau\Psi-i\Psi^T\gamma^9\partial_\theta\Psi \\
&\qquad +\Psi^T[A,\Psi]-\Psi^T\gamma^9[Y,\Psi]-\Psi^T\gamma^k[X^k,\Psi]\Big], && (21)\\
\mathcal{L}_2^B &= \mathrm{Tr}\Big[\frac{1}{2}(\partial_\tau Y-\partial_\theta A)^2+\frac{1}{2}(\partial_\tau X^k)^2-\frac{1}{2}(\partial_\theta X^k)^2 \\
&\quad -i\partial_\tau Y[A,Y]+i\partial_\theta A[A,Y]-i\partial_\tau X^k[A,X^k] \\
&\quad +i\partial_\theta X^k[Y,X^k]-\frac{1}{2}[A,Y]^2-\frac{1}{2}[A,X^k]^2 \\
&\quad +\frac{1}{2}[Y,X^k]^2+\frac{1}{4}[X^k,X^l]^2-i\partial_\theta\bar{C}\partial_\theta C-\partial_\theta\bar{C}[Y,C]
\end{aligned}$$

$$
+ i\partial_\tau \bar{C}\partial_\tau C + \partial_\tau \bar{C}[A, C] + i[\bar{C}, A]_{\rm diag}[C, A]_{\rm diag}
- i[\bar{C}, Y]_{\rm diag}[C, Y]_{\rm diag} - i[\bar{C}, X^k]_{\rm diag}[C, X^k]_{\rm diag}\Big], \quad (22)
$$

where (c,$\bar{c}$) are (ghost, anti-ghost) for the first condition of (14), while (C,$\bar{C}$,B) are (ghost, anti-ghost, B-field) for the second one, respectively, a has been integrated out by using the Landau gauge condition for eq.(14) and some of the off-diagonal parts have been rescaled as [6]

$$
A \to LA, \quad Y \to LY, \quad X^k \to LX^k, \quad \Psi \to L^{1/2}\Psi, \quad C \to L^2 C. \quad (23)
$$

By using the above action, we perform the perturbative expansion with respect to L and integrate only the off-diagonal matrix elements;

$$
T = \int \mathcal{D}y\mathcal{D}x^k\mathcal{D}\psi\mathcal{D}c\mathcal{D}\bar{c}\ \exp\left(iS_{eff}[y,\, x^k,\, \bar{c},\, c,\, \psi]\right), \quad (24)
$$

$$
S_{eff}[y,\, x^k,\, \bar{c},\, c,\, \psi] = \int d\tau \int_0^{2\pi} d\theta\ \left(\mathcal{L}^{string} - i\ln Z[y,\, x^k,\, \psi]\right), \quad (25)
$$

$$
Z[y,\, x^k,\, \psi] = \int \mathcal{D}A\mathcal{D}Y\mathcal{D}X^k\mathcal{D}\Psi\mathcal{D}C\mathcal{D}\bar{C}\mathcal{D}B\ \exp\left(i\tilde{S}\right), \quad (26)
$$

$$
\tilde{S} = \int d\tau \int_0^{2\pi} d\theta \left(\mathcal{L}_0^B + L\mathcal{L}_1^B + L^2\mathcal{L}_2^B + \mathcal{L}_0^F + L^{\frac{1}{2}}\mathcal{L}_{1/2}^F + L\mathcal{L}_1^F\right), \quad (27)
$$

where we have set $LT = 1$ and $\xi = (\tau, \theta)$ for brevity. We regard (17) and (18) as the free parts and (19)–(22) the interactions. Then we read off the propagators from the free parts,

$$
\begin{aligned}
\langle \hat{X}^K_{ab}(\xi)\hat{X}^L_{ba}(\xi')\rangle &= -i\left(\delta^{KL} - \frac{(\hat{x}^K_a - \hat{x}^K_b)(\hat{x}^L_a - \hat{x}^L_b)}{(\hat{x}_a - \hat{x}_b)^2}\right)\frac{G(\xi,\xi')}{(\hat{x}_a - \hat{x}_b)^2}, && (28)\\
\langle B_{ab}(\xi)Y_{ba}(\xi')\rangle &= \langle B_{ab}(\xi)A_{ba}(\xi')\rangle = \frac{y_a - y_b}{(x_a - x_b)^2}G(\xi,\xi'), && (29)\\
\langle B_{ab}(\xi)X^k_{ba}(\xi')\rangle &= \frac{x^k_a - x^k_b}{(x_a - x_b)^2}G(\xi,\xi'), && (30)\\
\langle \bar{C}_{ab}(\xi)C_{ba}(\xi')\rangle &= \frac{G(\xi,\xi')}{(x_a - x_b)^2}, && (31)\\
\langle \Psi^\alpha_{ab}(\xi)\Psi^\beta_{ba}(\xi')\rangle &= -\frac{i}{2}\frac{(y_a - y_b)(I + \gamma^9)_{\alpha\beta} + (x^k_a - x^k_b)\gamma^k_{\alpha\beta}}{(x_a - x_b)^2}G(\xi,\xi'), && (32)
\end{aligned}
$$

where $G(\xi,\xi') \equiv \delta^{(2)}(\xi - \xi')$, $(x_a - x_b)^2 \equiv (x^k_a - x^k_b)(x^k_a - x^k_b)$, the spinor indices α, β run through $1, 2, \cdots, 16$ and we have introduced the hatted variables $\hat{X}^K$ and $\hat{x}^K$ $(K = k, 9, 10\ \ (k = 1, 2, \cdots, 8))$,

$$
\hat{X}^k = X^k, \quad \hat{X}^9 = Y, \quad \hat{X}^{10} = iA, \quad \hat{x}^k = x^k, \quad \hat{x}^9 = y, \quad \hat{x}^{10} = iy, \quad (33)
$$

and $(\hat{x}_a - \hat{x}_b)^2 \equiv (\hat{x}_a^K - \hat{x}_b^K)(\hat{x}_a^K - \hat{x}_b^K) = (x_a - x_b)^2$. Notice that $(x_a - x_b)^2$ (for $a \neq b$, $1 \leq a, b \leq N$) must be non-zero in order that the perturbative expansion makes sense since the propagators are singular at $(x_a - x_b)^2 = 0$. We recall that in matrix string theory, the usual string interactions are described by the exchange of coincident diagonal matrix elements and hence the perturbative expansion does not make sense even for small radius L at the interaction points. Thus, henceforth we ignore the interaction points and integrate out the off-diagonal matrix elements to get the effective action for the diagonal matrix elements, which is expected to agree with the classical (free) action of DLCQ type IIA superstring.

The perturbative calculation is, however, formal as in Ref.[6]: The propagators (28)-(32) are proportional to the δ-function $G(\xi, \xi') = \delta^{(2)}(\xi - \xi')$ and the loops suffer from the ultraviolet divergences like $\delta^{(2)}(0)$. However, it is very difficult to find a suitable regularization which respects symmetries. If we adopt a certain regularization, e.g. cutoff regularization for large momenta, the regularized δ-function $G_{(r)}(\xi, \xi')$ would not satisfy $f(\xi)G_{(r)}(\xi, \xi') = f(\xi')G_{(r)}(\xi, \xi')$, which causes an ambiguity of how we choose the arguments of $(x_a^k - x_b^k)$ and $(y_a - y_b)$, which appear in the propagators (28)-(32). To avoid the ambiguity, henceforth we consider only the configurations of the diagonal matrix elements in which the differences of arbitrary two elements $(x_a^k - x_b^k)$, $(y_a - y_b)$ and $(\psi_a - \psi_b)$ are independent of ξ, although x_a^k, x_b^k, y_a, y_b, ψ_a and ψ_b themselves depend on ξ, in general. We have not yet found such a suitable regularization, however, we give a formal argument about the quantum corrections order by order in the strong-coupling expansion.

(i) $O(L^0)$: The lowest order contribution in eq.(26) is the one-loop determinant of the free action. Actually, the determinant is unity due to the coincidence between bosonic and fermionic degrees of freedoms.

(ii) $O(L^{1/2})$: The next contribution in eq.(26) comes from $\tilde{S}^F_{1/2} = \int \mathcal{L}^F_{1/2}$. $\langle i\tilde{S}^F_{1/2} \rangle$ vanishes because there is no way to self-contract in $i\tilde{S}^F_{1/2}$.

(iii) $O(L^1)$: The $O(L^1)$ contributions in eq.(26) come from $\tilde{S}^B_1$, $\tilde{S}^F_{1/2}$ and $\tilde{S}^F_1$. There are tree kinds of contributions, $\langle i\tilde{S}^B_1 \rangle$, $(1/2!)\langle i\tilde{S}^F_{1/2} i\tilde{S}^F_{1/2} \rangle$ and $\langle i\tilde{S}^F_1 \rangle$. The first one is given by

$$\begin{aligned}
\langle i\tilde{S}^B_1 \rangle \;=\; iL \int d^2\xi d^2\xi' \sum_{a,b=1}^{N} \Big\{ & i(y_a - y_b)\partial_\tau \langle Y_{ab}(\xi) Y_{ba}(\xi') \rangle \\
& - i(y_a - y_b)\partial_\tau \langle Y_{ab}(\xi) A_{ba}(\xi') \rangle - i(y_a - y_b)\partial_\theta \langle A_{ab}(\xi) Y_{ba}(\xi') \rangle \\
& + i(y_a - y_b)\partial_\theta \langle A_{ab}(\xi) A_{ba}(\xi') \rangle + i(y_a - y_b)\partial_\tau \langle X^k_{ab}(\xi) X^k_{ba}(\xi') \rangle \\
& - i(x^k_a - x^k_b)\partial_\tau \langle X^k_{ab}(\xi) A_{ba}(\xi') \rangle - i(y_a - y_b)\partial_\theta \langle X^k_{ab}(\xi) X^k_{ba}(\xi') \rangle
\end{aligned}$$

$$
\begin{aligned}
&+ i(x_a^k - x_b^k)\partial_\theta \langle X_{ab}^k(\xi) Y_{ba}(\xi')\rangle - \partial_\theta \langle B_{ab}(\xi) Y_{ba}(\xi')\rangle \\
&+ \partial_\tau \langle B_{ab}(\xi) A_{ba}(\xi')\rangle + 2(y_a - y_b)\,\partial_\theta \langle \bar{C}_{ab}(\xi) C_{ba}(\xi')\rangle \\
&\qquad - 2(y_a - y_b)\,\partial_\tau \langle \bar{C}_{ab}(\xi) C_{ba}(\xi')\rangle \Big\}\, \delta^{(2)}(\xi - \xi'). \qquad (34)
\end{aligned}
$$

From eqs.(28)-(32), we see that the quantity in the braces is antisymmetric in a and b and hence $\langle i\tilde{S}_1^B \rangle$ is zero. Similarly we find that both $\langle i\tilde{S}_{1/2}^F\, i\tilde{S}_{1/2}^F \rangle$ and $\langle i\tilde{S}_1^F \rangle$ are zero. Note that in order to show that the quantum correction of $O(L)$ is zero, we have never used the fact that $G(\xi, \xi')$ is a δ-function. Hence it would hold even if $G(\xi, \xi')$ is some regularized δ-function.

(iv) $O(L^{3/2})$: There are tree kinds of contributions,

$$
\frac{1}{2!}\langle i\tilde{S}_1^B\, i\tilde{S}_{1/2}^F \rangle, \quad \frac{1}{3!}\langle i\tilde{S}_{1/2}^F\, i\tilde{S}_{1/2}^F\, i\tilde{S}_{1/2}^F \rangle, \quad \frac{1}{2!}\langle i\tilde{S}_1^F\, i\tilde{S}_{1/2}^F \rangle.
$$

Each of them is zero because there is no way of contraction, respectively.

(v) $O(L^2)$: The contributions are

$$
\begin{aligned}
&\langle i\tilde{S}_2^B \rangle, \quad \frac{1}{2!}\langle i\tilde{S}_1^B i\tilde{S}_1^B \rangle, \quad \frac{1}{2!}\langle i\tilde{S}_1^F\, i\tilde{S}_1^F \rangle, \quad \frac{1}{2!}\langle i\tilde{S}_1^B i\tilde{S}_1^F \rangle, \\
&\frac{1}{3!}\langle i\tilde{S}_{1/2}^F i\tilde{S}_{1/2}^F i\tilde{S}_1^B \rangle, \quad \frac{1}{3!}\langle i\tilde{S}_{1/2}^F i\tilde{S}_{1/2}^F i\tilde{S}_1^F \rangle, \quad \frac{1}{4!}\langle i\tilde{S}_{1/2}^F i\tilde{S}_{1/2}^F i\tilde{S}_{1/2}^F i\tilde{S}_{1/2}^F \rangle.
\end{aligned}
$$

The last three terms contain fermionic diagonal elements ψ_a and they each vanish due to the anti-commutativity of the Grassmann variables ψ_a. Also it is easy to see that there is no contribution from $(1/2!)\langle i\tilde{S}_1^B i\tilde{S}_1^F \rangle$. Then $(1/2!)\langle i\tilde{S}_1^B i\tilde{S}_1^B \rangle$, $\langle i\tilde{S}_2^B \rangle$ and $(1/2!)\langle i\tilde{S}_1^F\, i\tilde{S}_1^F \rangle$ are to be considered below.

(v-1) One-loop: The one-loop contributions coming from $(1/2!)\langle i\tilde{S}_1^B i\tilde{S}_1^B \rangle$, $\langle i\tilde{S}_2^B \rangle$ and $(1/2!)\langle i\tilde{S}_1^F\, i\tilde{S}_1^F \rangle$ are referred to as $(1/2!)\langle i\tilde{S}_1^B i\tilde{S}_1^B \rangle^{(1)}$, $\langle i\tilde{S}_2^B \rangle^{(1)}$ and $(1/2!)\langle i\tilde{S}_1^F\, i\tilde{S}_1^F \rangle^{(1)}$, respectively. They are given by

$$
\begin{aligned}
\frac{1}{2!}\,\langle i\tilde{S}_1^B i\tilde{S}_1^B \rangle^{(1)} &= L^2 \int d^2\xi d^2\xi' \sum_{a\neq b} \Bigg[-\frac{17(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \partial_\tau \partial_{\theta'} G(\xi, \xi')\, G(\xi, \xi') \\
&+ \left(\frac{1}{(x_a - x_b)^2} + \frac{17}{2}\, \frac{(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \right) \partial_\tau \partial_{\tau'} G(\xi, \xi')\, G(\xi, \xi') \\
&- \left(\frac{1}{(x_a - x_b)^2} - \frac{17}{2}\, \frac{(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \right) \partial_\theta \partial_{\theta'} G(\xi, \xi')\, G(\xi, \xi') \Bigg], \qquad (35) \\
\langle i\tilde{S}_2^B \rangle^{(1)} &= L^2 \int d^2\xi d^2\xi' \sum_{a\neq b} \Bigg[\frac{(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2}\, \partial_\tau \partial_{\theta'} G(\xi, \xi')\, \delta^{(2)}(\xi - \xi') \\
&\left(\frac{3}{(x_a - x_b)^2} - \frac{1}{2}\, \frac{(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \right) \partial_\tau \partial_{\tau'} G(\xi, \xi')\, \delta^{(2)}(\xi - \xi')
\end{aligned}
$$

$$
- \left(\frac{3}{(x_a - x_b)^2} + \frac{1}{2} \frac{(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \right) \partial_\theta \partial_{\theta'} G(\xi, \xi') \, \delta^{(2)}(\xi - \xi') \Bigg], \quad (36)
$$

$$
\frac{1}{2!} \langle i\tilde{S}_1^F i\tilde{S}_1^F \rangle^{(1)} = L^2 \int d^2\xi d^2\xi' \sum_{a \neq b} \Bigg[\frac{16(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \partial_\tau \partial_{\theta'} G(\xi, \xi') \, G(\xi, \xi')
$$

$$
+ \left(\frac{-4}{(x_a - x_b)^2} - \frac{8(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \right) \partial_\tau \partial_{\tau'} G(\xi, \xi') \, G(\xi, \xi')
$$

$$
+ \left(\frac{4}{(x_a - x_b)^2} - \frac{8(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \right) \partial_\theta \partial_{\theta'} G(\xi, \xi') \, G(\xi, \xi') \Bigg]. \quad (37)
$$

Note that we have never used the fact that $G(\xi, \xi')$ is the δ-function in deriving eqs.(35)-(37). Thus they are expected to be unaltered even if we adopt a certain regularization. At this stage we first use the fact that $G(\xi, \xi')$ is the δ-function and it is shown that they are canceled, [3]

$$
\frac{1}{2!} \langle i\tilde{S}_1^B i\tilde{S}_1^B \rangle^{(1)} + \langle i\tilde{S}_2^B \rangle^{(1)} + \frac{1}{2!} \langle i\tilde{S}_1^F \, i\tilde{S}_1^F \rangle^{(1)} = 0. \quad (38)
$$

(v-2) Two-loop: The two-loop contributions, which are not calculated in Ref.[6], are referred to as $(1/2!) \langle i\tilde{S}_1^B i\tilde{S}_1^B \rangle^{(2)}$, $\langle i\tilde{S}_2^B \rangle^{(2)}$ and $(1/2!) \langle i\tilde{S}_1^F \, i\tilde{S}_1^F \rangle^{(2)}$, respectively. We obtain

$$
\frac{1}{2!} \langle i\tilde{S}_1^B i\tilde{S}_1^B \rangle^{(2)} = -iL^2 \int d^2\xi d^2\xi' \sum_{a \neq b, \, b \neq c, \, c \neq a} \Bigg\{ \frac{33}{2} \frac{1}{(x_a - x_b)^2 (x_b - x_c)^2}
$$

$$
- 16 \frac{\{(x_a^k - x_b^k)(x_c^k - x_a^k)\}^2}{(x_a - x_b)^2 (x_b - x_c)^2 \{(x_c - x_a)^2\}^2}
$$

$$
- \frac{1}{2} \frac{\{(x_a^k - x_b^k)(x_b^k - x_c^k)\}^2}{\{(x_a - x_b)^2 (x_b - x_c)^2\}^2} \Bigg\} (G(\xi, \xi'))^3, \quad (39)
$$

$$
\langle i\tilde{S}_2^B \rangle^{(2)} = iL^2 \int d^2\xi d^2\xi' \Bigg[\sum_{a \neq b, \, b \neq c, \, c \neq a} \Bigg\{ \frac{73}{2} \frac{1}{(x_a - x_b)^2 (x_b - x_c)^2}
$$

$$
- \frac{1}{2} \frac{\{(x_a^k - x_b^k)(x_b^k - x_c^k)\}^2}{\{(x_a - x_b)^2 (x_b - x_c)^2\}^2} \Bigg\} + 54 \sum_{a \neq b} \frac{1}{\{(x_a - x_b)^2\}^2} \Bigg]
$$

$$
\times (G(\xi, \xi'))^2 \, \delta^{(2)}(\xi - \xi'), \quad (40)
$$

---

[3] This result in matrix string theory is essentially the same as the one in the wrapped supermembrane theory [6]. In Ref.[6], however, the zero-mode gauge field a is restricted to be zero by hand, while we have just fixed the gauge ($a = y$) and added the corresponding FP-ghost part following the standard procedure [14]. In this sense the configuration of a is not restricted in our calculations.

$$\frac{1}{2!}\langle i\tilde{S}_1^F\, i\tilde{S}_1^F\rangle^{(2)} = -iL^2\int d^2\xi d^2\xi' \sum_{a\neq b,\, b\neq c,\, c\neq a}\left\{20\,\frac{1}{(x_a-x_b)^2(x_b-x_c)^2}\right.$$
$$\left. +\,16\,\frac{\{(x_a^k-x_b^k)(x_c^k-x_a^k)\}^2}{(x_a-x_b)^2(x_b-x_c)^2\{(x_c-x_a)^2\}^2}\right\}(G(\xi,\xi'))^3. \quad (41)$$

Note that in calculating eqs.(39)–(41) we have never used the fact that $G(\xi,\xi')$ is the δ-function and hence eqs.(39)–(41) are expected to be unaltered even if we adopt a certain regularization. At this stage we first use the fact that $G(\xi,\xi')$ is the δ-function and sum up eqs.(39)–(41) to get

$$\frac{1}{2!}\langle i\tilde{S}_1^B i\tilde{S}_1^B\rangle^{(2)} + \langle i\tilde{S}_2^B\rangle^{(2)} + \frac{1}{2!}\langle i\tilde{S}_1^F\, i\tilde{S}_1^F\rangle^{(2)}$$
$$= iL^2\int d^2\xi d^2\xi' \sum_{a\neq b}\frac{54}{\{(x_a-x_b)^2\}^2}\,(G(\xi,\xi'))^3. \quad (42)$$

The two-loop quantum corrections at $O(L^2)$ do not cancel out! One comment is in order: The remaining term is exactly that of the second summation in eq.(40). If we assume that the differences of the diagonal elements can be estimated as $(x_a^k - x_b^k) \sim O(N^\alpha)$[4] with some common constant α for large N, we will see that the terms canceled in eq.(42) behave as $\sum_{a\neq b,\, b\neq c,\, c\neq a}(x_a^k - x_b^k)^{-2}(x_b^k - x_c^k)^{-2} \sim O(N^{3-4\alpha})$, while the remaining term behaves as $\sum_{a\neq b}(x_a^k - x_b^k)^{-4} \sim O(N^{2-4\alpha})$. In this sense, we could say that only the leading terms in the large N are canceled in the two-loop quantum corrections to the classical string action at $O(L^2)$.

4 Conclusion

We have studied in matrix string theory whether the reduction to the diagonal elements of the matrices is justified quantum mechanically. We have seen that the quantum corrections do not cancel out at $O(L^2)$. We should note, however, that no suitable regularization for the divergences of $\delta^{(2)}(0)$ type is found so far, and hence we have only studied a mechanism of cancellation of quantum corrections. In fact, we have found that at the two-loop level of $O(L^2)$, the sub-leading term in the large N comes only from the bosonic degrees of freedom and cannot be canceled out. Even if we find a suitable regularization, such a structure seems to be unaltered and hence our result will be unchanged.

[4] According to the correspondence of a long string in matrix string theory with the wrapped supermembrane, $\alpha = -1$ for $|a-b| \ll N$ [6].

Finally, we comment on the global constraints in the wrapped supermembrane theory. Such constraints should be taken into account in the calculations of the quantum double-dimensional reduction.[5] In matrix string theory, however, there are no counterparts of such constraints. In particular, in the standard derivation of matrix string theory, they do not appear naturally. However we can show that the global constraint does not alter our result [15].

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References

[1] S. Uehara and S. Yamada, JHEP **0209**, 019 (2002) [arXiv:hep-th/0207209].

[2] J. Hughes, J. Liu and J. Polchinski, Phys. Lett. B **180**, 370 (1986).

[3] E. Bergshoeff, E. Sezgin and P. K. Townsend, Phys. Lett. B **189**, 75 (1987).

[4] M. J. Duff, P. S. Howe, T. Inami and K. S. Stelle, Phys. Lett. B **191**, 70 (1987).

[5] J. G. Russo, Nucl. Phys. B **492**, 205 (1997) [arXiv:hep-th/9610018].

[6] Y. Sekino and T. Yoneya, Nucl. Phys. B **619**, 22 (2001) [arXiv:hep-th/0108176].

[7] B. de Wit, J. Hoppe and H. Nicolai, Nucl. Phys. B **305**, 545 (1988).

[8] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, Phys. Rev. D **55**, 5112 (1997) [arXiv:hep-th/9610043].

[9] L. Motl, arXiv:hep-th/9701025.

[10] R. Dijkgraaf, E. Verlinde and H. Verlinde, Nucl. Phys. B **500**, 43 (1997) [arXiv:hep-th/9703030].

[11] L. Susskind, arXiv:hep-th/9704080.

[5] The global constraints are not considered in the calculations [6].

[12] N. Seiberg, Phys. Rev. Lett. **79**, 3577 (1997) [arXiv:hep-th/9710009].

[13] A. Sen, Adv. Theor. Math. Phys. **2**, 51 (1998) [arXiv:hep-th/9709220].

[14] T. Kugo and S. Uehara, Nucl. Phys. B **197**, 378 (1982).

[15] S. Uehara and S. Yamada, in preparation.