Fundamental Matter and the Deconfining Phase Transition in 2+1 D

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We analyze the effect of heavy fundamentally charged particles on the finite temperature deconfining phase transition in the 2+1 dimensional Georgi-Glashow model. We show that in the presence of fundamental matter the transition turns into a crossover. The near critical theory is mapped onto the 2 dimensional Ising model in an external magnetic field. Using this mapping we determine the width of the crossover region as well as the specific heat as a function of the fundamental mass.

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Recently, significant progress has been made in understanding the deconfining phase transition in 2+1 dimensions. The transition in the $SU(2)$ Georgi-Glashow model has been analysed in detail: the order of the phase transition as well as the universality class have been established explicitly without recourse to universality arguments, and the dynamics of the phase transition was given a simple interpretation in terms of the restoration of magnetic symmetry [1]. In subsequent work, the effects of instantons at high temperature have been understood, the dynamics of the deconfining transition has been related to the properties of the confining strings, and the analysis has also been extended to $SU(N)$ gauge theories at arbitrary N [2]. The effects of the variability of the Higgs field mass were studied in [3], and some analogies between the mechanism of the deconfining transition in 2+1 dimensions and chiral symmetry restoration in QCD have been suggested [4]. These results have recently been reviewed and summarized in [5]. Also, an interesting interpretation of these results has recently been given [6] in the context of the Svetitsky-Yaffe conjecture [7] and the general role of center symmetry in abelian projection.

In this Letter we ask how the properties of the transition change in the presence of dynamical particles in the fundamental representation of the gauge group. We consider the $SU(2)$ Georgi-Glashow model in the presence of a heavy fundamental field, which we take to be a scalar (being heavy, similar results should hold for fundamental fermions [8]). The Lagrangian of the theory is

$$\mathcal{L} = -\frac{1}{2g^2}\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \frac{1}{2}(D_\mu^{ab}h^b)^2 - \frac{\lambda}{4}(h^a h^a - v^2)^2 + |D_\mu^{\alpha\beta}\Phi^\beta|^2 - M^2\Phi^*\Phi. \tag{1}$$

Here h^a is the Higgs field in the adjoint representation, and Φ is a scalar field in the fundamental representation of $SU(2)$.

We will be interested throughout this paper in the weakly coupled regime $g^2 \ll v$. In this regime, perturbatively the gauge group is broken to $U(1)$ by the large expectation value, v, of the Higgs field. The Higgs and the two gauge bosons $W^\pm$ are heavy with masses $M_H^2 = 2\lambda v^2$, and $M_W^2 = g^2v^2$. We take the fundamental field Φ to be much heavier than the charged W-bosons :

$$M^2 \gg M_W^2. \tag{2}$$

Perturbatively the theory behaves very much like 2+1 dimensional electrodynamics with spin one charged matter. However, nonperturbative effects are very important at large distances. As shown by Polyakov [9], their effect is that the photon, which is perturbatively massless, acquires a finite (but exponentially small) mass and the charged $W^\pm$ become linearly confined at large distances with nonperturbatively small string tension.

Let us first summarize what is known [1, 5] about this theory *without* the heavy fundamental matter field Φ. At zero temperature confinement is a consequence of the spontaneous breaking of the magnetic Z_2 symmetry [10, 11]. The Z_2 symmetry transformation is generated by the Wilson loop along the spatial boundary of the system

$$W(C \to \infty) = \exp\left(\frac{i}{2}\int d^2x B(x)\right), \tag{3}$$

The order parameter $V(x)$ for this Z_2 transformation is the operator that creates an elementary magnetic vortex of flux $2\pi/g$

$$V(x) = \exp\left(\frac{2\pi i}{g}\int_C dx^i\epsilon_{ij}\frac{h^a}{|h|}E_j^a(x)\right). \tag{4}$$

Here, C is a contour beginning at x and going to spatial infinity. Despite the appearance of the contour in the definition, this operator $V(x)$ is in fact local, gauge invariant and Lorentz scalar [11]. The action of the spatial Wilson loop on $V(x)$ is given by

$$W(C\to\infty)V(x)W^{\dagger}(C\to\infty) = -V(x) \tag{5}$$

which is a realization of the 't Hooft algebra [10, 11]. At low energies the theory is described by the Z_2 invariant Lagrangian of the vortex field V

$$\mathcal{L}_{\rm eff} = \partial_\mu V\,\partial^\mu V^* - \lambda\big(V\,V^* - \frac{g^2}{8\pi^2}\big)^2 + \frac{m_{ph}^2}{4}\left\{V^2+(V^*)^2\right\}. \tag{6}$$

with the photon mass [12]

$$m_{ph}^2 = \frac{16\pi^2\xi}{g^2}, \quad \xi = \text{constant}\frac{M_W^{7/2}}{g}e^{-\frac{4\pi M_W}{g^2}} \tag{7}$$

and the vortex self-coupling $\lambda = \frac{2\pi^2 M_W^2}{g^2}$. At weak coupling, $g^2 \ll v$, the radial degree of freedom of V is very heavy, and is practically frozen. Thus, the effective Lagrangian reduces to an effective Lagrangian for the phase of V:

$$Ł_{eff} = \frac{g^2}{8\pi^2}(\partial_\mu\chi)^2 + \frac{m_{ph}^2 g^2}{16\pi^2}\cos 2\chi, \tag{8}$$

where

$$V = \frac{g}{\sqrt{8\pi}}\exp i\chi. \tag{9}$$

The effective Lagrangian (8) for slowly varying fields χ is equivalent to the one derived by Polyakov [9] from the monopole (more precisely, monopole-instanton) plasma picture, with $\xi = \frac{m_{ph}^2 g^2}{16\pi^2}$ being the monopole fugacity. The only additional information contained in (6) is that the field χ should be treated as a phase. Thus rough configurations, where χ changes by 2π between adjacent points in space have finite energy. This is important, since the charged particles, $W^\pm$ in this representation show up as solitons of the field V with unit winding number, or vortices of the phase χ [11]. In these configurations χ is indeed discontinuous along some cut, but the cut itself does not cost energy.

As shown in [1], even though the $W^\pm$ bosons are heavy, they cannot be neglected at finite temperature. Their presence determines the properties of the deconfining phase transition. The physics of the phase transition is the following. At finite temperature the thermal ensemble is populated by W bosons, with density proportional to their fugacity

$$\mu \propto e^{\frac{-M_W}{T}}. \tag{10}$$

The W bosons interact via a confining potential [9]. Each W^+ boson is a source of *two* confining strings, which both end on the same nearby W^- boson, and so at low temperature the W bosons are bound in pairs.

However, this confining interaction is weak. The width of the confining string is proportional to the inverse mass $m_{\rm ph}$ of the photon, given in (7), and is therefore very large. When the average distance between the W bosons becomes the same as the width of the string, the confining interaction becomes irrelevant. The two confining strings emanating from a given W^+ do not have to end on the same W^- boson any longer, but rather the strings form a percolating network. The individual W bosons therefore have no memory of their nearest neighbours, and are free to wander independently in the thermal vacuum, thereby forming a charged plasma. Since the W bosons are vortices of the phase χ, in the plasma state the phase χ is disordered, and thus the magnetic Z_2 symmetry is restored [13]. The transition happens at the point where the fugacity (10) of the W bosons becomes equal to the fugacity (7) of monopoles,

$$T_c = \frac{g^2}{4\pi}. \tag{11}$$

To analyze the phase transition quantitatively, note that the critical temperature is much larger than the mass of the photon, and thus dimensional reduction is valid in the critical region. The dimensionally reduced theory in addition to the terms present in (8) contains contributions due to the finite density of W bosons [1]. Thus, the two dimensional Euclidean Lagrangian that describes the transition region is

$$Ł = \frac{g^2}{8\pi^2 T}(\partial_\mu\chi)^2 + \zeta\cos 2\chi + \mu\cos\tilde{\chi} \tag{12}$$

where ζ is related to the monopole fugacity by $\zeta = \xi/T$, and $\tilde{\chi}$ is the field dual to χ,

$$i\partial_\mu \tilde{\chi} = \frac{g^2}{2\pi T}\epsilon_{\mu\nu}\partial^\nu \chi. \tag{13}$$

As explained in [1], the dual field $\tilde{\chi}$ is directly related to the zeroth component of the Abelian vector potential, corresponding to the unbroken $U(1)$ gauge group in the original formulation of the Georgi-Glashow model, $\tilde{\chi} = 2g\beta A_0$. Thus the last term in eq.(12) is nothing but the potential $P^2 + h.c.$ for the fundamental Polyakov line

$$P = \exp\left(\frac{i}{2}\,\tilde{\chi}\right) \tag{14}$$

which is indeed the leading contribution to the free energy due to heavy charged particles.

The critical temperature $T_c = g^2/4\pi$ is special for three reasons. First, this is the point at which the operators $\cos 2\chi$ and $\cos\tilde{\chi}$ have the same scaling dimension, equal to one. Second, at this point the fields $\chi \pm \frac{\tilde{\chi}}{2}$ become chiral (antichiral), as can be seen from (13) :

$$(\partial_1 \pm i\partial_2)\left(\chi \pm \frac{\tilde{\chi}}{2}\right) = 0. \tag{15}$$

Third, at T_c the coefficients of the two "interaction terms" in (12) become equal, $\zeta = \mu$. These facts all imply that the theory can be conveniently fermionized by using the standard bosonization/fermionization techniques [16]. Defining the chiral and antichiral fermionic fields

$$\psi_R = a^{-1/2}\frac{1}{\sqrt{2}}\exp\left[i\left(\chi + \frac{\tilde{\chi}}{2}\right)\right] \quad , \quad \psi_L = a^{-1/2}\frac{1}{\sqrt{2}}\exp\left[-i\left(\chi - \frac{\tilde{\chi}}{2}\right)\right] \tag{16}$$

the potential terms in (12) become

$$a^{-1}\cos 2\chi = \psi_R^\dagger\psi_L - \psi_L^\dagger\psi_R \quad , \quad a^{-1}\cos\tilde{\chi} = \psi_R^\dagger\psi_L^\dagger - \psi_L\psi_R. \tag{17}$$

The dimensional constant a plays the role of the UV cutoff in the effective theory, and is of the order of T [14]. Defining the Majorana fermions In terms of the Majorana fermions

$$\rho = \frac{\psi + \psi^\dagger}{\sqrt{2}}, \quad \sigma = \frac{\psi - \psi^\dagger}{i\sqrt{2}} \tag{18}$$

the effective Lagrangian (12) becomes

$$L = \frac{1}{2}\bar{\rho}\gamma_\mu\partial_\mu\rho + \frac{1}{2}\bar{\sigma}\gamma_\mu\partial_\mu\sigma + i\frac{\zeta + \mu}{2}\rho^T\gamma_2\rho + i\frac{\zeta - \mu}{2}\sigma^T\gamma_2\sigma \tag{19}$$

where the gamma matrices are taken as the Pauli matrices : $\gamma_\mu = \tau_\mu$, and $\bar{\psi} = \psi^\dagger\gamma_1$. Note that the two Majorana fermions ρ and σ in (19) do not interact with one another. The fermion ρ has Majorana mass of the order of the photon mass, while the fermion σ is massless at criticality. At large distances ($d \gg \zeta^{-1}$) the massive fermion decouples, and so the long distance physics at criticality is governed by the theory of one massless Majorana fermion, which describes the critical point of a single 2D Ising model. The implications of this for the deconfining phase transition are analyzed in [1].

We now ask how this picture changes in the presence of the heavy fundamentally charged matter field Φ. First, we note that when M^2 is finite, the nature of the magnetic symmetry changes. As discussed in [15] it becomes a local rather than a global symmetry. The operator V in (4) is no longer a local operator. The operator V^2 is still local, but it is not an order parameter for Z_2. Thus there is no local order parameter that can distinguish between broken and unbroken magnetic symmetry. In this situation we do not expect the deconfining transition to remain second order. It should either become first order with finite latent heat, or disappear altogether into a sharp but analytic crossover.

The way the fundamental matter affects the physics of the transition can be understood qualitatively from the following simple consideration. For large M^2 the fugacity of the fundamental Φ particles is very small, and thus they are present in the thermal ensemble with very low density, proportional to

$$h = e^{-\frac{M}{T}}. \tag{20}$$

As discussed earlier, below $T = T_c$, the ensemble of W bosons consists of dipoles bound together by a *pair* of confining strings. A single Φ particle is a source (or a sink) of only one confining string. Any extra Φ particles in this ensemble therefore have to bind in pairs between themselves. Thus, below the transition the fundamental particles form an extra component of the "dipole plasma", which does not mix with the dominant (higher density) component consisting of W's. On the other hand, above the transition the strings emanating from W bosons percolate through the whole ensemble, rather than ending on a nearest particle. In this situation, a Φ particle loses all memory of any other Φ particles in its neighbourhood, since its own confining string can easily end on a neighbouring W boson, whose density is much higher. This change of the distribution of Φ particles clearly leads to an increase of the entropy in the system. Below the transition the contribution of the Φ particles to the entropy can be estimated from considering the thermal ensemble as an ensemble of "dipoles" with fugacity h^2

$$\exp\left(TS_\Phi\right) = \sum_{n=0}^{\infty}(xh^2Va^{-2})^n\frac{1}{n!} = \exp\left(xh^2a^{-2}V\right) \tag{21}$$

where the factor of volume arises from an independent integration over the center of mass coordinates of the dipoles. Also x is a number of order unity, encoding the fact that the fugacity of the dipole is not exactly h^2 due to the interaction between the two particles forming the dipole, and that there is an extra "renormalization" due to the integration over the internal states of the dipole. Thus parametrically below the transition

$$S_< \propto h^2. \tag{22}$$

On the other hand, above the transition one can consider the Φ particles as noninteracting and randomly distributed. The result for the entropy is then

$$S_> \propto h. \tag{23}$$

Since h is small, this means that the entropy rises strongly in the transition region. These estimates (22) and (23) of the entropy are only valid far enough from the transition, where S can be expanded in powers of h. Thus, this simple consideration is not sufficient to determine whether the change from $S_<$ to $S_>$ takes place abruptly at some value of temperature (a first order transition), or smoothly over a finite range of temperatures ΔT (a smooth crossover). To probe this question more precisely we must determine how the presence of the heavy fundamental Φ particles modifies the dimensionally reduced Lagrangian (12) close to criticality.

First, it is clear that the presence of the heavy Φ particles does not change the self-interaction of the light photon, just as the presence of heavy W's does not. However, in the presence of the fundamentally charged field Φ, the low energy theory (8) must now admit solitons with half the topological charge. Thus the field χ now has period π rather than 2π. In addition, at finite temperature the presence of the Φ particles induces a new term, similar to the last term in (12), but with twice the periodicity and with a coefficient proportional to the fugacity, h, of the Φ particles. This contribution is proportional to the first power of the Polyakov line in (14). The derivation is completely analogous to that presented in [1] and we will not discuss it in detail. The dimensionally reduced Lagrangian therefore is

$$Ł = \frac{g^2}{8\pi^2T}(\partial_\mu\chi)^2 + \zeta\cos 2\chi + \mu\cos\tilde{\chi} + h\cos\frac{\tilde{\chi}}{2}. \tag{24}$$

Close to the transition temperature we can again fermionize the theory. Expanding to leading order in $T - T_c$, and keeping only relevant terms we find

$$L = \frac{1}{2}\bar{\rho}\gamma_\mu\partial_\mu\rho + i\zeta\rho^T\gamma_2\rho + \frac{1}{2}\bar{\sigma}\gamma_\mu\partial_\mu\sigma - i\frac{\tau}{2}\sigma^T\gamma_2\sigma + h\Sigma \tag{25}$$

with

$$\tau = \left(T - \frac{g^2}{4\pi}\right)\frac{16\pi^2M_W}{g^4}\zeta. \tag{26}$$

Since the fugacity h of the heavy fundamental fields is small, we can treat the term $h\Sigma$ as a perturbation. Recall from (19) that the system *without* this perturbation is that of two decoupled 2D Ising models, one of them close to criticality and another far away from criticality. The term $h\cos\frac{\tilde{\chi}}{2}$ makes the two Ising models coupled, resulting in the so-called Baxter–Ashkin-Teller model [17]. In his translation table between the sine-Gordon operators and the Baxter operators, Ogilvie [17] has identified the operator $\cos\frac{\tilde{\chi}}{2}$ (mass dimension $\frac{1}{4}$) with a product of a spin operator of one Ising model (A) and a disorder operator of another Ising model (B) : $\cos\frac{\tilde{\chi}}{2} \leftrightarrow \hat{\sigma}^{(A)}\hat{\mu}^{(B)}$. The conformal dimensions of both spin and disorder operators are each $\frac{1}{16}$. The operators $\cos 2\chi \pm \cos\tilde{\chi}$ (mass dimension 1) are identified with the energy (mass) operators $\hat{\varepsilon}^{(A,B)}$, each having conformal dimension=$\frac{1}{2}$, of the Ising model A and B, respectively.

In the regime we are interested in, the Ising model A is deeply into the ordered phase, so the operator $\hat{\sigma}^{(A)}\hat{\mu}^{(B)}$ can be substituted by $\hat{\mu}^{(B)}$ with mass dimension $\frac{1}{8}$. An alternative interpretation of this result is to integrate out the heavy Majorana fermion. Its effect in the re-bosonized theory is the placement of a background charge at infinity that enforces $c = \frac{1}{2}$, and a multiplicative renormalization of the sine-Gordon field. Then the vertex operators $e^{i\tilde{\chi}}$ and $e^{i\tilde{\chi}/2}$ with the rescaled $\tilde{\chi}$ have conformal dimensions $\frac{1}{2}$ and $\frac{1}{16}$ in the presence of the background charge, respectively.

Either way, we interpret the effective Lagrangian (24) near criticality, and with heavy fundamental matter fields, as that of the $c = \frac{1}{2}$ conformal field theory of a Majorana fermion with two perturbations, one of conformal dimension $\frac{1}{2}$ and another one of conformal dimension $\frac{1}{16}$. This perturbed conformal theory describes a single Ising model in an external magnetic field h close to, but away from, the critical temperature [18–20]. The coefficient τ (see (26)) of the perturbation of conformal dimension $\frac{1}{2}$ is proportional to the deviation from the critical temperature, while the coefficient h of the perturbation of conformal dimension $\frac{1}{16}$ is proportional to the external magnetic field. The fact that our Georgi-Glashow model, with heavy fundamental matter, maps onto this dimensionally reduced Ising system means that we can use the known Ising results to study the nature of the phase transition in the presence of the heavy fundamental matter field Φ, whose fugacity h plays the role of the external magnetic field in the Ising language.

The Ising model with these two perturbations has been studied extensively [18–20]. It is believed not to be exactly soluble, although many exact results are known both at $h = 0$ for all τ [21], and at $\tau = 0$ for all h [22–24]. Nevertheless, much is known about the system (25) with both perturbations. For example, it has been shown in [20] that for the Ising system perturbed by the operators of conformal dimension $\frac{1}{2}$ and $\frac{1}{16}$, the free energy can be written as

$$F(\tau, h) = \frac{2\tau^2}{15\pi} \log h + f\left(\frac{\tau}{|h|^{8/15}}\right) \tag{27}$$

where the function $f(x)$ on the RHS is an *analytic* function for all real x, including $x = 0$. This is a highly nontrivial result, which has not yet been proved rigorously, but which is strongly supported by numerical results [20], as well as by the exact results available from the $\tau = 0$ and the $h = 0$ limits. The analyticity relation (27) is a very significant and powerful result. The analyticity of the function $f(x)$ means that the theory has no phase transitions. This implies that the second order Ising transition at $M^2 \to \infty$ (i.e, in the theory *without* fundamental matter fields) becomes a crossover at finite M^2. Thus, the second order deconfining phase transition found in [1] for the finite temperature 2+1 dimensional Georgi-Glashow model changes into a crossover with the inclusion of fundamental matter fields that have a heavy but finite mass.

It is also known [20] that away from $\tau = 0$, the free energy has an expansion in powers of $\alpha = \frac{h}{\tau^{15/8}}$. (This unusual power $15/8$ is a simple consequence of the fact that the field h has mass dimension $15/8$, while τ has mass dimension 1). Below the transition, where $\tau < 0$, this expansion contains only even powers of α, while above the transition, where $\tau > 0$, it contains both odd and even powers of α. This result is consistent with our earlier physical estimates, in (22) and (23), of the behaviour of the entropy on the fugacity h, based on the dipole picture. The width of the crossover region is determined by the temperature for which the expansion parameter α is small, and therefore

$$T - \frac{g^2}{4\pi} \propto \zeta^{-1} h^{\frac{8}{15}} = \exp\left(-\frac{32\pi M}{15 g^2} + \frac{4\pi M_W}{g^2}\right). \tag{28}$$

The increase in entropy which we estimated before in (22) and (23) happens within this range of temperatures. In particular this tells us that the dependence of the specific heat on h is

$$C_V = T \frac{\partial S}{\partial T}\bigg|_V \propto \zeta h^{\frac{7}{15}} = \exp\left(-\frac{28\pi M}{15 g^2} - \frac{4\pi M_W}{g^2}\right). \tag{29}$$

To conclude, we note that it would be interesting to connect our results with those of [25] which considers massless fundamental fermions. We also note that our results can also be interpreted in the framework of the Z_2 gauge theory. As shown in [15], the dual description of the Georgi-Glashow model with heavy fundamental matter is a local Z_2 gauge theory with matter fields at weak coupling. The Z_2 gauge coupling constant is related to the mass of the fundamental fields as $e^2 \propto M^{-1}$. Thus our results predict that the hot Z_2 theory with matter does not undergo a phase transition but rather a sharp crossover, with the width of the crossover region nonperturbatively small at weak coupling $\Delta T \propto \exp(-\frac{aT_c}{e^2})$.

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