Gauge invariant formulation of systems with second class constraints.

J.Stephany and A. Restuccia

Universidad Simón Bolívar, Departamento de Física
Apartado postal 89000, Caracas 1080-A, Venezuela.

The covariant quantization of physical systems with reducible constraints is one of the unresolved problems which appear in connection with the Green-Schwarz Superstring (GSSS).The formulation of the GSSS [1] presents several advantages over the perturbative NRS formulation [2]. In particular a non perturbative second quantized Light Cone Gauge formulation was obtained in [1]and the closure of the Super-Poincare algebra [3] and multiloop analysis of the S-matrix was performed in [4]. Nevertheless the covariant first quantization of the GSSS is a problem which has not been solved. The main difficulty has been the covariant gauge fixing of the local kappa-Supersymmetry [5]. In fact, the first class constraints associated to the gauge symmetries appear mixed with second class ones, and so far no local, Lorentz covariant and finite reducible [6] [7] approach to disentangle them has been found.The zero mode structure of GSSS is described by the Brink-Schwarz Superparticle, (BSSP) [8]. The canonical structure of both theories presents a very close correspondence. In particular first and second class constraints in the BSSP also appear mixed and similar to the GSSS, problems with the infinite reducibility of the covariant generators of gauge symmetries are present.

Here we present a canonical approach which applied to the covariant quantization of the BSSP and resolves the above mentioned problems. The formulation is based upon a general canonical approach for dynamical systems restricted by reducible first and second class constraints [9]. In this approach, which is closely related to the work in [10], the phase space is extended to a larger manifold where all extended constraints are first class. By an appropriate gauge fixing one may reduce the functional integral to a functional integral on the original constrained manifold, with the correct functional measure. We show explicitly the reduction procedure. It is an off-shell approach allowing the systematic construction of the off-shell nilpotent BRST charge and of the BRST invariant effective action. For the BSSP this approach leads directly to the correct BRST charge. It gives a systematic method for the construction of the superparticle action, proposed in Ref.[11] by Kallosh which is related to this charge through the BFV formalism [12].

We start with constrained system with Hamiltonian H_0 subject to a set of reducible constraints ϕ_{a_1} $(a_1 = 1, \cdots, n)$ and a set of first class constraints φ_i $(i = 1, \cdots, k)$ which we omit in the explicit construction that follows. We limit ourselves to remark on the modifications to be done when included. So we have,

$$\phi_{a_1} = 0 \tag{1}$$

$$a_{a_2}^{a_1}\phi_{a_1} = 0 \qquad a_1 = 1\cdots n, \quad a_2 = 1\cdots m \quad . \tag{2}$$

We will not suppose ${a_{a_2}}^{a_1}$ to be of maximal rank. Instead we will impose that a (T+L) decomposition is allowed.

We have then for any objects V_{a_1} and W^{a_1}

$$V_{a_1} = V_{a_1}^{\top} + A_{a_1}^{a_2} V_{a_2}^{L}$$

$$a^{a_1}_{a_2} V^{\top}_{a_1} = 0, \qquad V^L_{a_2} = a^{a_1}_{a_2} V_{a_1} \tag{3}$$

$$W^{a_1} = W^{a_1}_{\top} + a^{a_1}_{a_2} W^{a_2}_L \tag{4}$$

$$A^{a_2}_{a_1} W^{a_1}_{\top} = 0, \qquad W^{a_2}_L = A^{a_2}_{a_1} W_{a_1} \qquad .$$

It follows that $V^L_{a_2} = a^{a_1}_{a_2} A^{b_2}_{a_1} V^L_{b_2}$, $W^{a_2}_L = A^{a_2}_{a_1} a^{a_1}_{b_2} W^{b_2}_L$ and $W^{a_1} V_{a_1} = W^{a_1}_{\top} V^{\top}_{a_1} + W^{a_2}_L V^L_{a_2}$

In the irreducible case $A^{a_2}_{a_1}$ is the inverse of $a^{a_1}_{a_2}$. In the finite reducible case this decomposition may always be done in a unique way for a given pair A, a. For infinite reducible system, we will assume that there exists such a decomposition.The constraints (2) are second class in the sense that they have an invertible Poisson Bracket matrix in the transverse sub-space.

Following Ref. 9 and 10 let us enlarge the phase space using a set of auxiliary variables ξ^{a_1} and η_{b_1} conjugate to each other. We also introduce the combinations

$$\Phi_{a_1} = \eta_{a_1} - \frac{1}{2}\omega_{a_1 b_1}(p,q)\xi^{b_1} \overline{\Phi}_{a_1} = \eta_{a_1} + \frac{1}{2}\omega_{a_1 b_1}(p,q)\xi^{b_1} \qquad . \tag{5}$$

Here ω_{ab} is an antisymmetric matrix with vanishing Poisson Bracket with itself to be fixed by the procedure. $\overline{\Phi}$ and Φ satisfy

$$\{\Phi_{a_1}, \Phi_{b_1}\} \ = -\omega_{a_1 b_1}\{\overline{\Phi}_{a_1}, \overline{\Phi}_{b_1}\} \ = \omega_{a_1 b_1}\{\Phi_{a_1}, \overline{\Phi}_{b_1}\} \ = 0 \qquad . \tag{6}$$

In order to introduce only the complications necessary in the case of the BS superparticle we will suppose in the following that $\omega_{a_1 b_1}$ is transverse

$$a^{a_1}_{a_2} \omega_{a_1 b_1} = 0 \qquad a_1 = 1 \cdots n \qquad a_2 = 1 \cdots m \tag{7}$$

and invertible in the transverse space.

Now let us extend the constraints in the enlarged space to

$$\widetilde{\phi}_{a_1} = \phi_{a_1} + V^{c_1}_{a_1} \Phi_{c_1} = 0 \tag{8}$$

where $V^{c_1}_{a_1}(q,p)$ is also to be fixed. In general the first class constraints φ may also have to be extended in order the complete set of extended constrains be first class. In this case of the superparticle, however, the extension is not necessary. We assume $V^{b_1}_{a_1}$ to be invertible. In this case we impose the constraints (7) to be irreducible, first class and with structure functions at most linear in Φ_{a_1}. We then have

$$\{\widetilde{\phi}_{a_1}, \widetilde{\phi}_{b_1}\} = U^{c_1}_{a_1 b_1} \widetilde{\phi}_{c_1} = -2(u^{c_1}_{a_1 b_1} + v^{c_1 d_1}_{a_1 b_1} \Phi_{d_1}) \widetilde{\phi}_{c_1}. \tag{9}$$

The structure functions $U^{c_1}_{a_1 b_1}$ may depend on the phase space variables p and q. Substitution of (7) in (8) yields.

$$\begin{gathered} \{\phi_{a_1}, \phi_{b_1}\} - V^{c_1}_{a_1} V^{d_1}_{b_1} \omega_{c_1 d_1} + 2u^{c_1}_{a_1 b_1} \phi_{c_1} = 0 \\ \{\phi_{a_1}, V^{c_1}_{b_1}\} + \{V^{c_1}_{a_1}, \phi_{b_1},\} + 2v^{d_1 c_1}_{a_1 b_1} \phi_{d_1} + 2u^{d_1}_{a_1 b_1} V^{c_1}_{d_1} = 0 \\ \{V^{c_1}_{a_1}, V^{d_1}_{b_1}\} + \{V^{d_1}_{a_1}, V^{c_1}_{b_1}\} + 2V^{c_1}_{e_1} v^{e_1 d_1}_{a_1 b_1} + 2V^{d_1}_{e_1} v^{e_1 c_1}_{a_1 b_1} = 0 \qquad . \end{gathered} \tag{10}$$

We suppose here that

$$\{\phi_{a_1}, \Phi_{1 a_1}\} = 0, \quad \{V^{b_1}_{1 a_1}, \Phi_{1 c_1}\} = 0 \qquad . \tag{11}$$

Let us suppose that we are able to find a solution to (20) with all the required conditions. In order to demonstrate the equivalence of our system in the enlarged phase space to the original system we have to impose additional restriction besides (7). A counting of the degrees of freedom suggests which ones should be chosen in this generalized situation. The original model has $2N$ phase space variables p, q restricted by $(n - m_L)$ transverse constraints with m_L the rank of $a_{a_1}^{a_2}$. The enlarged model has $2N$ variables p, q and $2n$ variables ξ, η restricted by n constraints $\widetilde{\phi}_{a_1}$ and n gauge fixing conditions $\widetilde{\chi}_{a_1}$. To match we need $(n - m_L)$ additional constraints. We take them to be

$$\overline{\Phi}_{a_1}^{\top} = 0 \quad . \tag{12}$$

Since

$$[\overline{\Phi}_{a_1}^{\top}, \overline{\Phi}_{a_2}^{\top}] = \omega_{a_1 a_2}^{\top} \tag{13}$$

the constraints (11) are in our hypothesis second class.The advantage of this formulation is that he field dependence in $\omega_{a_1 a_2}^{\top}$ may be simpler than in $\{\phi_{a_1}, \phi_{b_1}\}$ since $V_{a_1}^{a_2}$ may be also a field dependent object.

A gauge invariant extension of the hamiltonian H_0 may be written in the form [9]

$$\widetilde{H} = H_0 + h^{a_1} \Phi_{a_1} \quad . \tag{14}$$

h^{a_1} is fixed imposing

$$\{\widetilde{H}, \widetilde{\phi}_{a_1}\} = W_{a_1}^{b_1} \widetilde{\phi}_{b_1} \quad . \tag{15}$$

Introducing the ghost variables C^{a_1} and μ_{a_1} the BRST operator and the extended hamiltonian are obtained in the standard way [6]

The BRST invariant effective action in a phase space representation is given by [6] [18]

$$S_{eff} =< p\dot{q} + \mu_{a_1} \dot{C}^{a_1} + \eta_{a_1} \dot{\xi}^{a_1} - \widehat{H} + \widehat{\delta}(\lambda^{a_1} \mu_{a_1}) + \widehat{\delta}(\overline{C}_{a_1} \chi^{a_1}) > \tag{16}$$

where χ^{a_1} are the gauge fixing conditions and $\widehat{\delta}$ is defined by $\widehat{\delta} F = [\Omega, F]$ for any function F of the canonical variables of the enlarged superphase-space. For the non-canonical sector we have

$$\widehat{\delta}\lambda^{a_1} = \theta^{a_1} \quad , \ \widehat{\delta}\theta^{a_1} = 0 \widehat{\delta}\overline{C}_{a_1} = B_{a_1} \quad , \ \widehat{\delta} B_{a_1} = 0 \delta\mu_{a_1} = \widetilde{\phi}_{a_1} \quad . \tag{17}$$

We will show now that with an adequate gauge fixing condition one can reduce the path integral corresponding to the enlarged system to the Senjanovic-Fradkin expression for the original system.

We may choose the gauge conditions

$$\chi^{a_1} = \xi^{a_1} \quad . \tag{18}$$

The functional integral is

$$I(\chi) = \int Dz \delta(\overline{\Phi}^{\top}) (det \omega^{\top})^{1/2} e^{-Seff} \tag{19}$$

where Dz is the Liouville measure

Integrating in θ one gets $\delta(\mu)$.Integrating in B_{a_1}, $\overline{C}_{a_1}$,and $\lambda_L^{a_2}$ and using Eq.(9) the factor in the measure of (18) becomes

$$(det\omega^{\top})^{1/2}\delta(\eta)\delta(\xi)\delta(C)detV_{\top}^{\top} \qquad . \tag{20}$$

Now we note from (9) that

$$(det\omega^{\top})^{1/2}detV_{\top}^{\top} = (det\{\phi_{\top},\phi_{\top}\}) \qquad . \tag{21}$$

Doing the trivial integrals in η, ξ and C, we finally obtain

$$I = \int DqDpD\lambda^{\top}(det\{\phi_{\top},\phi_{\top}\})exp- < p\dot{q} - H + \lambda_{\top}\phi^{\top} > \tag{22}$$

which is the correct Senjanovic-Fradkin expression of the functional integral of this system.

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