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Abstract

The physics that is traditionally formulated in one–time-physics (1T-physics) can also be formulated in two-time-physics (2T-physics). The physical phenomena in 1T or 2T physics are not different, but the spacetime formalism used to describe them is. The 2T description involves two extra dimensions (one time and one space), is more symmetric, and makes manifest many hidden features of 1T-physics. One such hidden feature is that families of apparently different 1T-dynamical systems in d dimensions holographically describe the same 2T system in $d+2$ dimensions. In 2T-physics there are two timelike dimensions, but there is also a crucial gauge symmetry that thins out spacetime, thus making 2T-physics effectively equivalent to 1T-physics. The gauge symmetry is also responsible for ensuring causality and unitarity in a spacetime with two timelike dimensions. What is gained through 2T-physics is a unification of diverse 1T dynamics by making manifest hidden symmetries and relationships among them. Such symmetries and relationships is the evidence for the presence of the underlying higher dimensional spacetime structure. 2T-physics could be viewed as a device for gaining a better understanding of 1T-physics, but beyond this, 2T-physics offers new vistas in the search of the unified theory while raising deep questions about the meaning of spacetime. In these lectures, the recent developments in the gauge field theory formulation of 2T-physics will be described after a brief review of the results obtained so far in the worldline approach.

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1 Worldline approach

A crucial element in the formulation of 2T-physics [1]-[10] is an Sp$(2,R)$ gauge symmetry in phase space. All new phenomena in 2T-physics (including the two times) can be traced to the presence of this gauge symmetry and its generalizations. In the space of all worldline theories (i.e. all possible background fields) there is an additional symmetry that corresponds to all canonical transformations in the phase space of a particle. After describing the role of these two symmetries in the worldline formalism we will discuss field theory. In field theory these two symmetries combine and get promoted to noncommutative U$_\star(1,1)$ acting on fields as functions of noncommutative phase space. U$_\star(1,1)$ symmetry provides the foundation of 2T-physics in field theory, and leads to a unification of various gauge principles in ordinary field theory, including Maxwell, Einstein and high-spin gauge principles.

1.1 Spinless particle and interactions with backgrounds

An elementary approach for understanding 2T-physics is offered by the worldline description of a spinless particle and its interactions. The action [2] has the form [6] [8]

$$I_Q=\int d\tau\left[\dot{X}^M P_M-\frac{1}{2}A^{ij}\left(\tau\right)\,Q_{ij}(X,P)\right], \tag{1}$$

where the symmetric $A_{ij}=A_{ji}$ for $i=1,2,$ denotes three Sp$(2,R)$ gauge fields, and the symmetric $Q_{ij}=Q_{ji}$ are three sp$(2,R)$ generators constructed from the phase space of the particle on the worldline $\left(X^M\left(\tau\right),P_M\left(\tau\right)\right)$. An expansion of $Q_{ij}(X,P)$ in powers of P_M in some local domain, $Q_{ij}(X,P)=\sum_s\left(f_{ij}\left(X\right)\right)^{M_1\cdots M_s}P_{M_1}\cdots P_{M_s},$ defines all the possible background fields in configuration space $\left(f_{ij}\left(X\right)\right)^{M_1\cdots M_s}$ that the particle can interact with. The local sp$(2,R)$ gauge transformations are

$$\delta X^M=-\omega^{ij}\left(\tau\right)\frac{\partial Q_{ij}}{\partial P_M},\quad \delta P_M=\omega^{ij}\left(\tau\right)\frac{\partial Q_{ij}}{\partial X^M},\quad \delta A^{ij}=\partial_\tau\omega^{ij}\left(\tau\right)+\left[A,\omega\left(\tau\right)\right]^{ij}. \tag{2}$$

The action I_Q is gauge invariant, with local parameters $\omega^{ij}\left(\tau\right),$ provided the $Q_{ij}\left(X,P\right)$ satisfy the sp$(2,R)$ Lie algebra under Poisson brackets. This is equivalent to a set of differential equations that must be satisfied by the background fields $\left(f_{ij}\left(X\right)\right)^{M_1\cdots M_s}$ [6][8]. The simplest solution is the free case denoted by $Q_{ij}=q_{ij}$ (no background fields, only the flat metric η_{MN})

$$q_{ij}=X_i^M X_j^N\eta_{MN}:\quad q_{11}=X\cdot X,\quad q_{12}=X\cdot P,\quad q_{22}=P\cdot P. \tag{3}$$

In the flat case, we have defined X_i^M with only upper spacetime indices, such that $X_1^M=X^M$ and $X_2^M=P^M$. In the curved case P_M is always defined with a lower spacetime index. The general solution with $d+2$ dimensional background fields (see Eqs.(9-12) below) describes all

[2] This action is a generalization of the familiar elementary worldline action $\int d\tau\left[\dot{X}^M P_M-\frac{1}{2}e\left(\tau\right)\,P^2\right]$ for a free massless particle, as seen by specializing to $A_{11}=A_{12}=0,$ $A_{22}=e$ and $Q_{22}=P^2$.

interactions of the spinless particle with arbitrary electromagnetic, gravitational and higher spin gauge fields in d dimensions [8].

Two timelike dimensions is not an input, it is a result of the gauge symmetry Sp$(2,R)$ on phase space $\left(X^M, P_M\right)$. This gauge symmetry imposes the constraints $Q_{ij}(X,P)=0$ on phase space as a result of the equations of motion of the gauge field $A_{ij}(\tau)$. The meaning of the constraints is that the physical subspace of phase space should be gauge invariant under Sp$(2,R)$. There is non-trivial content in such phase space provided spacetime has $d+2$ dimensions, including two timelike dimensions [2]. The solution of the constraints is a physical phase space in two less dimensions, that is $(d-1)$ spacelike and 1 timelike dimensions. The non-trivial aspect is that there are many ways of embedding d dimensional phase space in a given $d+2$ dimensional phase space while satisfying the gauge invariance constraints. Each d dimensional solution represents a different dynamical system in 1T-physics, but all d dimensional solutions holographically represent the same higher dimensional $d+2$ theory in 2T-physics [2].

The Sp$(2,R)$ gauge symmetry is responsible for the effective holographic reduction of the $d+2$ dimensional spacetime to (a collection of) d dimensional spacetimes with one-time. Evidently, in the worldline formalism there is a single proper time τ, but the particle position $X^M(\tau)$ has two timelike dimensions $X^0(\tau), X^{0'}(\tau)$ and the particle momentum $P_M(\tau)$ has the corresponding two timelike components, $P_0(\tau), P_{0'}(\tau)$. Sp$(2,R)$ has 3 gauge parameters and 3 constraints. Two gauge parameters and two constraints can be used to eliminate one timelike and one spacelike dimensions from the coordinates and momenta. The third gauge parameter and constraint are equivalent to those associated with τ reparametrization, which is familiar in the 1T-physics worldline formulation (as in the footnote). In making the three gauge choices one must ask which combination of the two timelike dimensions is identified with τ? Evidently there are many possibilities, and once this choice is made, the 1T-Hamiltonian of the system, which will emerge from the solution of the constraints, will be the canonical momentum that is conjugate to this gauge choice of time. The many gauge choices correspond to different looking Hamiltonians with different 1T dynamical content. In this way, by various gauge fixing, the same 2T system (with the same fixed set of background fields) can be made to look like diverse 1T systems. Each 1T system in d dimensions holographically captures all the information of the 2T system in $d+2$ dimensions. Therefore there is a family of 1T systems that are in some sense dual to each other. Explicit examples of this holography/duality have been produced in the simplest 2T model [2], namely the free 2T particle in $d+2$ dimensions. The free 2T particle of Eq.(3) produces the following 1T holographic pictures in d dimensions: massless relativistic particle, massive relativistic particle, massive non-relativistic particle, particle in anti-de-Sitter space AdS_d, particle in $\mathrm{AdS}_{d-k}\times \mathrm{S}^k$ space for all $k\leq d-2$, non-relativistic Hydrogen atom ($1/r$ potential), non-relativistic harmonic oscillator in one less dimension, some examples of black holes (for d=2,3), and more ...

The higher symmetry of the $d+2$ system is present in all of the d dimensional holographic pictures. This symmetry is a global symmetry that commutes with Sp$(2,R)$, and therefore is gauge invariant. Therefore it is a symmetry of the action (not of the Hamiltonian) for any choice of gauge. Before making a gauge choice the symmetry is realized in $d+2$ dimensions. After making a gauge choice the same symmetry is non-linearly realized on the fewer d dimensions, and therefore it is harder to detect in many 1T-dynamical systems, although it is present. A special example is provided by the free 2T particle of Eq.(3) which evidently has a linearly realized SO$(d,2)$ Lorentz symmetry. This symmetry is interpreted in various ways from the point of view of 1T dynamics in the d dimensional holographic pictures: conformal symmetry for the free massless relativistic particle, dynamical SO$(d,2)$ symmetry for the H-atom, SO$(d,2)$ symmetry for the particle in AdS$_d$ (n.b. larger than SO$(d-1,2)$), etc.. The first two cases were familiar, although they were not usually thought of as having a relation to higher dimensions. All the other holographic cases mentioned above, including the non-relativistic massive particle, harmonic oscillator, AdS$_{d-k}\times$S^k, etc. all have the same hidden SO$(d,2)$ symmetry which was understood for the first time in the context of 2T-physics. The generators of the symmetry have been explicitly constructed for all the cases mentioned [2]. What is more, the symmetry is realized in the same unitary representation as characterized by the eigenvalues of the Casimir operators of SO$(d,2)$. In the classical theory, in which orders of phase space quantum operators are neglected, all the Casimir operators for SO$(d,2)$ seem to vanish (this is a non-trivial representation for the non-compact group). However, when quantum ordering is taken into account, the Casimir eigenvalues do not vanish, but take on some special values corresponding to a special representation. The ordering of phase space operators is in general difficult, but it can be implemented explicitly in a few cases (conformal, H-atom, AdS$_{d-k}\times$S^k, harmonic oscillator). For all these cases the quantum quadratic Casimir operator is the same $C_2\left(SO\left(d,2\right)\right)=1-d^2/4$, and similarly one obtains some fixed number for all higher Casimir operators. This unitary representation is the singleton/doubleton representation (name depends on d). This very specific representation, which is common to all the 1T dynamical models mentioned, corresponds to the free 2T particle. This fact is already part of the evidence of the duality that points to the existence of the unifying $d+2$ or 2T structure underlying these 1T systems.

In [8] the general system with background fields was studied. It was shown that all possible interactions of a point particle with background electromagnetic, gravitational and higher-spin fields in d dimensions emerges from the 2T-physics worldline theory in Eq.(3). The general Sp$(2,R)$ algebraic relations of the $Q_{ij}\left(X,P\right)$ govern the interactions, and determine equations that the background fields of any spin must obey. The constraints were solved for a certain 2T to 1T holographic image which describes a relativistic particle interacting with background fields of any spin in $(d-1)+1$ dimensions. Two disconnected branches of solutions exist, which seem to have a correspondence with massless states in string theory, one containing low spins in the zero Regge slope limit, and the other containing high spins in the infinite Regge slope limit.

The same kind of holography/duality phenomena that exist in the free case should, in principle, be expected in the presence of background fields. This includes holographic capture of the $d+2$ dimensional dynamics in various forms in d dimensions, duality relations (analogs of same Casimir, and other related (dual) quantities) among many 1T dynamical systems which have the same background fields in $d+2$ dimensions, and hidden symmetries in d dimensions which become manifest in $d+2$ dimensions. Such higher symmetries include global, local, and reparametrization symmetries inherited from $d+2$ dimensions. In principle it is possible to construct numerous duality relationships as "experimental" evidence of the underlying higher dimensional structure. There is much detail of this type yet to be explored in the worldline theory. This should be a fruitful area of investigation in 2T-physics.

1.2 Spin, supersymmetry

The worldline theory for the spinless particle has been generalized in several directions. One generalization is to spinning particles through the use of worldline supersymmetry, in which case the gauge group is OSp$(n|2)$ instead of Sp$(2,R)$ [3]. This case has also been generalized by the inclusion of some background fields [7], but the most general case analogous to Eq.(3) (including all powers of the fermion field) although straightforward, has not been investigated yet.

Another generalization involves spacetime supersymmetry, which has been obtained for the free particle (i.e. for $Q_{ij} \to q_{ij}$ as in Eq.(3)) [4][1]. In this case, in addition to the local Sp$(2,R)$, there is local kappa supersymmetry as part of a local supergroup symmetry. The action is

$$S = \int d\tau \left[\dot{X}^M P_M - \frac{1}{2} A^{ij} X_i \cdot X_j - \frac{1}{s} Str\left(L\left(\partial_\tau g g^{-1} \right) \right) \right], \tag{4}$$

where $g \in G$ is a supergroup element, and $L = L^{MN}\Gamma_{MN}$ is a coupling of the Cartan form $\partial_\tau g g^{-1}$ to the orbital SO$(d,2)$ Lorentz generators $L^{MN} = X^M P^N - X^N P^M$ via the spinor representation Γ_{MN} of SO$(d,2)$. The supergroup element g contains fermions Θ that are now coupled to phase space X^M, P_M. The constant s is fixed by the dimension of the spinor representation, and it insures a generalized form of kappa supersymmetry.

The supergroups OSp$(N|4)$, SU$(2,2|4)$, F(4), OSp$(6,2|N)$, contain SO$(d,2)$ in the spinor representation for $d = 3,4,5,6$ respectively. The 2T free superparticle of [4] based on these supergroups has a holographic reduction from $d+2$ to d dimensions which produces from Eq.(4) the superparticle in $d = 3,4,5,6$,

$$S = \int d\tau \left[\dot{x}^\mu p_\mu - \frac{1}{2} e(\tau) p^2 + \dot{\theta}\gamma^\mu \theta p_\mu \right]. \tag{5}$$

In these special dimensions the 2T approach of Eq.(4) makes manifest the hidden superconformal symmetry of the superparticle action which precisely given by the corresponding supergroup. For other supergroups that contain the bosonic subgroup SO$(d,2)$ in the *spinor* representation, the 2T supersymmetric model of Eq.(4) includes p-brane degrees of freedom. In general, the approach of [4] shows how to formulate any of these systems in terms of twistors and supertwistors (instead of particle phase space) by simply choosing gauges, and thus obtaining the spectrum of the system by using oscillator methods developed a long time ago [12] (see [11] for a related twistor approach). For example, OSp$(1|8)$, which contains SO$(4,2)$= SU$(2,2)$ (with $4+4^*$ spinors), is used to construct an action in $4+2$ dimensions as in Eq.(4); this produces a holographic picture in $3+1$ dimensions for a superparticle together with 2-brane degrees of freedom (the 4D superalgebra containing the maximal 2-brane extension). There is enough gauge symmetry in the system to remove ghosts associated with timelike dimensions of the 2-brane. The physical, and unitary, quantum states of this system correspond to a particle-brane BPS realization of the supersymmetry OSp$(1|8)$. A closely related case is the toy M-model in 11+2 dimensions based on OSp$(1|64)$ [4][1]. This produces a holographic picture that includes the 11-dimensional 2-brane and 5-brane degrees of freedom in addition to the 11-dimensional superparticle phase space (with the maximally extended superalgebra in 11D). The spectrum of the toy M-model consists of 2^8 bosons and 2^8 fermions with the quantum numbers of the 11D supergravity multiplet, but with a BPS relation among the brane charges and particle momentum. Another interesting variation of Eq.(4) is a superparticle model in 10+2 dimensions with SU$(2,2|4)$ supersymmetry, in which the coupling L lives both in SO$(4,2)$ =SU$(2,2)$ and SO(6) =SU(4) [4][1]. This produces an anti-de-Sitter holographic picture that describes the complete Kaluza-Klein towers of states that emerge in the AdS$_5\times$ S^5 compactification of IIB-supergravity. A generalization of the latter, including brane degrees of freedom, is achieved by using OSp$(8|8)$. The methods and partial details of these constructions are given in [4][1], and the full details will appear in the near future.

The generalization of the 2T superparticle with background fields is a challenging problem that remains to be investigated.

2 Field theoretic formulation of 2T-physics

The gauge symmetry Sp$(2,R)$ is at the heart of the worldline formalism and the physical results of 2T-physics. This is a phase space symmetry as seen from Eq.(2). How can one implement such a nonlocal gauge symmetry in local field theory? The answer is naturally found in noncommutative field theory [9]. In fact, a beautiful and essentially unique gauge theory formulation of 2T-physics based on noncommutative u$_\star(1,1)$ has been obtained for spinless particles [10].

Field theory emerges from the first quantization of the worldline theory. There is a phase space approach to first quantization developed in the old days by Weyl-Wigner-Moyal and others [13]-[15]. Instead of using wavefunctions in configuration space $\psi(X)$, this approach uses wavefunctions in phase space $\phi(X,P)$, which are equivalent to functions of operators X,P by the Weyl correspondence. The correct quantum results are produced provided the phase space wavefunctions are always multiplied with each other using the noncommutative Moyal star product

$$(\phi_1\star\phi_2)(X,P)=\exp\left(\frac{i\,\hbar}{2}\frac{\partial}{\partial X^M}\frac{\partial}{\partial\tilde{P}_M}-\frac{i\,\hbar}{2}\frac{\partial}{\partial P_M}\frac{\partial}{\partial\tilde{X}^M}\right)\phi_1(X,P)\,\phi_2\left(\tilde{X},\tilde{P}\right)\Big|_{X=\tilde{X},\,P=\tilde{P}}. \tag{6}$$

Then noncommutative field theory becomes a natural setting for implementing the local symmetries of 2T-physics.

2.1 Noncommutative fields from first quantization

A noncommutative field theory in phase space introduced recently [9] confirmed the worldline as well as the configuration space field theory [7] results of 2T-physics, and suggested some far reaching insights. As shown in [9], first quantization of the worldline theory is described by the noncommutative field equations

$$\begin{aligned}[Q_{ij},Q_{kl}]_\star &= i\,\hbar\left(\varepsilon_{jk}Q_{il}+\varepsilon_{ik}Q_{jl}+\varepsilon_{jl}Q_{ik}+\varepsilon_{il}Q_{jk}\right), &(7)\\ Q_{ij}\star\varphi &= 0. &(8)\end{aligned}$$

where the Moyal star product appears in all products. The first equation is the Sp$(2,R)$ commutation relations which promote the Poisson brackets relations of the worldline theory to commutators to all orders of $\hbar$. According to the Weyl correspondence, we may think of φ as an operator in Hilbert space $\varphi\sim|\psi\rangle\langle\psi|$ (the equations have a local u$_\star(1)$ symmetry applied on φ which allows it to take this special form). The φ equations are equivalent to sp$(2,R)$ singlet conditions in Hilbert space, $Q_{ij}|\psi\rangle=0$, whose solutions are physical states that are gauge invariant under sp$(2,R)$. These equations were explicitly solved in several stages in [7][8][9]. The solution space is non-empty and is unitary only when spacetime has precisely two timelike dimensions, no less and no more [9]. The solution space of these equations confirm the same physical picture conveyed by the worldline theory, including the holography/duality and hidden higher dimensional symmetries, but now in a field theoretical setting [7][9]. Up to canonical transformations of (X,P) the general solution of Eq.(7) is given by

$$\begin{aligned}Q_{11} &= X^MX^N\eta_{MN},\quad Q_{12}=X^MP_M &(9)\\ Q_{22} &= G_0(X)+G_2^{MN}(X)\,(P+A(X))_M\,(P+A(X))_N &(10)\\ &\quad+\sum_{s=3}^{\infty}G_s^{M_1\cdots M_s}(X)\;\;(P+A(X))_{M_1}\cdots(P+A(X))_{M_s} &(11)\end{aligned}$$

where η^{MN} is the flat metric in $d+2$ dimensions, $A_M(X)$ is the Maxwell gauge potential, $G_0(X)$ is a dilaton, $G_2^{MN}(X)=\eta^{MN}+h^{MN}(X)$ is the gravitational metric, and the symmetric tensors $(G_s(X))^{M_1\cdots M_s}$ for $s\geq 3$ are high spin gauge fields. The sp$(2,R)$ closure condition in Eq.(29) requires these fields to be homogeneous of degree $(s-2)$ and to be orthogonal to X^M

$$X\cdot\partial A_M = -A_M, \quad X\cdot\partial G_s = (s-2)\,G_s, \quad X^M A_M = X_{M_1} h_2^{M_1 M_2} = X_{M_1} G_{s\geq 3}^{M_1\cdots M_s} = 0, \tag{12}$$

There is remaining canonical symmetry which, when expanded in powers of $P,$ contains the gauge transformation parameters (in two less dimensions) for all of these background gauge fields [8]. In particular, the Maxwell and Einstein local symmetries in d dimensions are local symmetries of these $d+2$ dimensional equations which constrain the $d+2$ dimensional fields to be effectively d dimensional fields (modulo gauge degrees of freedom that decouple). Thus background fields $A, G_0, G_2, G_{s\geq 3}$ determine all other background fields $(f_{ij}(X))^{M_1\cdots M_s}$ up to canonical transformations. The solution of the $d+2$ dimensional equations (12) is given in [8] in terms of d dimensional background fields for Maxwell $A_\mu(x)$, dilaton $g(x)$, metric $g_{\mu\nu}(x)$ and higher spin fields $g^{\mu_1\cdots\mu_s}(x)$.

The solution to the matter field equation is given by a Wigner distribution function constructed by Fourier transform from a wavefunctions $\psi(X_1) =< X_1|\psi >$ in configuration space

$$\begin{aligned}
\varphi(X,P) &= \int d^D Y\, \psi(X) \star e^{-iY^M P_M} \star \psi^*(X) && (13)\\
&= \int d^D Y\ \ \psi\left(X - \frac{Y}{2}\right)\ e^{-iY^M P_M}\ \ \psi^*\left(X + \frac{Y}{2}\right), && (14)
\end{aligned}$$

where the wavefunction satisfies $Q_{ij}|\psi\rangle = 0.$ The sp$(2,R)$ gauge invariant solution space [7] of this equation is non-empty and has positive norm wavefunctions only when there are two timelike dimensions. The complete set of solutions $\psi_n(X)$ in $d+2$ dimensional configuration space is holographically given explicitly in 1T spacetime in terms of a complete set of wavefunctions in d dimensional configuration space. Thus, the equations (7,8) correctly represent the 1T physics of a particle in d dimensions interacting with background gauge fields, including the Maxwell, Einstein, and high spin fields [9].

Using the complete set of physical states, $\psi_n(X)$, one may construct a complete set of physical fields $\varphi_m^n(X,P)$ in noncommutative 2T quantum phase space that correspond to $\varphi_m^n \sim |\psi_m\rangle\langle\chi_n|$. It can be shown explicitly that in noncommutative space these complete set of physical fields satisfy a closed algebra [9]

$$\varphi_{n_1}^{m_1} \star \left(\varphi^\dagger\right)_{m_2}^{n_2} \star \varphi_{n_3}^{m_3} = \delta_{m_2}^{m_1}\, \delta_{n_3}^{n_2}\, \varphi_{n_1}^{m_3},. \tag{15}$$

The positive norm (unitarity) of the physical states is captured by the δ_{nk} on the right side.

2.2 $u_\star(1,1)$ gauge principle and interactions

The goal of the field theory approach is to find a field theory, and appropriate gauge principles, from which the free Eqs.(7,8) follow as classical field equations of motion, much in the same way that the Klein-Gordon field theory arises from satisfying τ-reparametrization constraints ($p^2 = 0$), or string field theory emerges from satisfying Virasoro constraints, etc. The field theory approach, combined with gauge principles is expected to provide non-linear field interactions in 2T-physics.

The desired fundamental gauge symmetry principle that fulfill these goals is based on noncommutative u$_\star(1,1)$ in phase space [10]. There is no non-commutative su$(1,1)$ without the extra u(1) in noncommutative space, and therefore to include sp$(2,R)$=su$(1,1)$ one must take u$_\star(1,1)$ as the smallest candidate symmetry (a smaller candidate sp$_\star(2,R)$ [20] which also has a u$_\star(1)$, is eliminated on other grounds [10]). The apparently extra noncommutative u$_\star(1)$ is related to canonical transformations and plays an important role in the overall scheme.

The u$_\star(1,1)$ gauge principle completes the formalism of [9] into an elegant and concise theory which beautifully describes 2T-physics in field theory in $d+2$ dimensions. The resulting theory has deep connections to standard d dimensional gauge field theories, gravity and the theory of high spin fields. There is also a finite matrix formulation of the theory in terms of u(N,N) matrices, such that the $N\to\infty$ limit becomes the u$_\star(1,1)$ gauge theory.

The 4 noncommutative parameters of u$_\star(1,1)$ can be written in the form of a 2$\times$2 matrix, $\Omega_{ij} = \omega_{ij} + i\omega_0\varepsilon_{ij}$, whose symmetric part $\omega_{ij}(X,P)$ becomes sp$(2,R)$ when it is global, while its antisymmetric part generates the local subgroup u$_\star(1)$ with local parameter $\omega_0(X_1,X_2)$. The indices are raised with the sp$(2,R)$ metric ε^{ij}, therefore in matrix form we have

$$\Omega_k^{\ l} = \omega_k^{\ l} - i\omega_0\delta_k^{\ l} = \begin{pmatrix} \omega_{12} - i\omega_0 & \omega_{22} \\ -\omega_{11} & -\omega_{12} - i\omega_0 \end{pmatrix} \tag{16}$$

This matrix satisfies the following hermiticity conditions, $\Omega^\dagger = \varepsilon\Omega\varepsilon$. Such matrices close under matrix-star commutators to form u$_\star(1,1)$.

We introduce a 2$\times$2 matrix $\mathcal{J}_{ij} = J_{ij} + iJ_0\varepsilon_{ij}$ that parallels the form of the parameters Ω_{ij}. There will be a close relation between the fields $J_{ij}(X,P)$ and Q_{ij} as we will see soon. When one of the indices is raised, the matrix $\mathcal{J}$ takes the form

$$\mathcal{J}_i^{\ j} = \begin{pmatrix} J_{12} - iJ_0 & J_{22} \\ -J_{11} & -J_{12} - iJ_0 \end{pmatrix} \tag{17}$$

Next we consider matter fields that transform under the noncommutative group U$_\star^L(1,1)\times$U$_\star^R(1,1)$. In this notation $\mathcal{J}$ transforms as the adjoint under U$_\star^L(1,1)$ and is a singlet under U$_\star^R(1,1)$, thus it is in the $(1,0)$ representation, which means its gauge transformations are defined by the matrix-star products in the form $\delta\mathcal{J} = \mathcal{J}\star\Omega^L - \Omega^L\star\mathcal{J}$. For the matter field we take the $\left(\frac{1}{2},\frac{1}{2}\right)$ representation given by a 2×2 complex matrix $\Phi_i^\alpha(X_1,X_2)$. This field is equivalent to a complex symmetric tensor Z_{ij} and a complex scalar φ. We define $\bar{\Phi} = \varepsilon\Phi^\dagger\varepsilon$. The U$_\star^L(1,1)\times$U$_\star^R(1,1)$ transformation rules for this field are $\delta\Phi = -\Omega^L\star\Phi + \Phi\star\Omega^R$, where Ω^L,Ω^R are the infinitesimal parameters for U$_\star^L(1,1)\times$U$_\star^R(1,1)$.

We now construct an action that will give the noncommutative field theory equations (7,8) in a linearized approximation and provide unique interactions in its full form. The action has a resemblance to the Chern-Simons type action introduced in [9], but now there is one more field, J_0, and the couplings among the fields obey a higher gauge symmetry

$$S_{J,\Phi} = \int d^{2D}X\, Tr\left(-\frac{i}{3}\mathcal{J}\star\mathcal{J}\star\mathcal{J} - \mathcal{J}\star\mathcal{J} + i\mathcal{J}\star\Phi\star\bar{\Phi} - V_\star\left(\Phi\star\bar{\Phi}\right)\right). \tag{18}$$

The invariance under the local $\mathrm{U}_\star^L(1,1)\times\mathrm{U}_\star^R(1,1)$ transformations is evident. $V(u)$ is a potential function with argument $u=\Phi\star\bar{\Phi}$.

The form of this action is unique as long as the maximum power of $\mathcal{J}$ is 3. We have not imposed any conditions on powers of Φ or interactions between $\mathcal{J},\Phi,$ other than obeying the symmetries. A possible linear term in $\mathcal{J}$ can be eliminated by shifting $\mathcal{J}$ by a constant, while the relative coefficients in the action are all absorbed into a renormalization of $\mathcal{J},\Phi$. A term of the form $Tr\left(\mathcal{J}\star\mathcal{J}\star f\left(\Phi\star\bar{\Phi}\right)\right)$ that is allowed by the gauge symmetries can be eliminated by shifting $\mathcal{J}\rightarrow\left(\mathcal{J}-\frac{1}{3}f\left(\Phi\star\bar{\Phi}\right)\right)$. This changes the term $i\bar{\Phi}\star\mathcal{J}\star\Phi$ by replacing it with interactions of $\mathcal{J}$ with any function of $\bar{\Phi},\Phi$ that preserves the gauge symmetries. However, one can do field redefinitions to define a new Φ so that the interaction with the linear $\mathcal{J}$ is rewritten as given, thus shifting all complications to the function $V_\star\left(\bar{\Phi}\star\Phi\right)$. When the maximum power of $\mathcal{J}$ is cubic we have the correct link to the first quantized worldline theory. Therefore, with the only assumption being the cubic restriction on $\mathcal{J}$, this action explains the first quantized worldline theory, and generalizes it to an interacting theory based purely on a gauge principle.

The equations of motion are

$$\mathcal{J}\star\mathcal{J}-2i\mathcal{J}-\Phi\star\bar{\Phi}=0,\quad \left(\mathcal{J}+iV'\right)\star\Phi=0. \tag{19}$$

where $V'(u)=\partial V/\partial u$. It is shown in [10] that one can choose gauges for $\Phi_i^\alpha=\varphi\delta_i^\alpha$ to simplify these equations, such that J_0 is fully solved

$$J_0=-1+\frac{V'\left(-|\lambda|^2\right)}{|\lambda|^2}\varphi\star\varphi^\dagger+\left(1-\frac{1}{2}J_{ij}\star J^{ij}\right)^{1/2}. \tag{20}$$

while the remaining fields satisfy

$$J_{ij}\star\varphi=0,\quad \varphi\varphi^\dagger\varphi=|\lambda|\,\varphi,\quad 0=\left(1+V'\left(-|\lambda|^2\right)\right)^2-1-|\lambda|^2. \tag{21}$$

and

$$[J_{11},J_{12}]_\star = i\left\{J_{11},\sqrt{1-C_2(J)}\right\}_\star \tag{22}$$

$$[J_{11},J_{22}]_\star = 2i\left\{J_{12},\sqrt{1-C_2(J)}\right\}_\star \tag{23}$$

$$[J_{11},J_{22}]_\star = i\left\{J_{22},\sqrt{1-C_2(J)}\right\}_\star \tag{24}$$

where the expression

$$C_2(J) = \frac{1}{2} J_{kl} \star J^{kl} = \frac{1}{2} J_{11} \star J_{22} + \frac{1}{2} J_{22} \star J_{11} - J_{12} \star J_{12} \tag{25}$$

looks like a Casimir operator. But this algebra is not a Lie algebra, and in general one cannot show that $C_2(J)$ commutes with J_{ij}. However, assuming no anomalies in the associativity of the star product, the Jacobi identities $[J_{11}, [J_{12}, J_{22}]_\star]_\star + cyclic = 0$ require

$$\left[J_{ij}, \left[J^{ij}, \sqrt{1 - C_2(J)} \right]_\star \right]_\star = 0, \tag{26}$$

but generally this is a weaker condition than the vanishing of $[J_{ij}, C_2(J)]_\star$.

To understand the content of the nonlinear gauge field equations (22- 24) we setup a perturbative expansion around a background solution

$$J_{ij} = J_{ij}^{(0)} + g J_{ij}^{(1)} + g^2 J_{ij}^{(2)} + \cdots \tag{27}$$

such that $J_{ij}^{(0)}$ is an exact solution, and then analyze the full equation perturbatively in powers of g. For the exact background solution we assume that $\frac{1}{2} J_{kl}^{(0)} \star J^{(0)kl}$ commutes with $J_{ij}^{(0)}$, therefore the background solution satisfies a Lie algebra. Then we can write the exact background solution to Eqs.(22-24) in the form

$$J_{ij}^{(0)} = Q_{ij} \star \frac{1}{\sqrt{1 + \frac{1}{2} Q_{kl} \star Q^{kl}}} \tag{28}$$

where Q_{ij} satisfies the sp$(2, R)$ algebra with the normalization of Eq.(7)

$$[Q_{11}, Q_{12}]_\star = 2i Q_{11}, \quad [Q_{11}, Q_{22}]_\star = 4i Q_{12}, \quad [Q_{12}, Q_{22}]_\star = 2i Q_{22}, \tag{29}$$

and $\frac{1}{2} Q_{kl} \star Q^{kl}$ is a Casimir operator that commutes with all Q_{ij} that satisfies the sp$(2, R)$ algebra. Then the background $Q_{ij}(X_1, X_2)$ has the form of Eqs.(9 -12) up to a u$_\star(1)$ subgroup gauge symmetry. The square root is understood as a power series involving the star products and can be multiplied on either side of Q_{ij} since it commutes with the Casimir operator. For such a background, the matter field equations reduce to

$$Q_{ij} \star \varphi = 0. \tag{30}$$

Summarizing, we have shown that our action $S_{J,\Phi}$ has yielded precisely what we had hoped for. The linearized equations of motion (0^{th} power in g) in Eqs.(29,30) are exactly those required by the first quantization of the worldline theory as given by Eqs.(7, 8). There remains to understand the propagation and self interactions of the fluctuations of the gauge fields $g J_{ij}^{(1)} + g^2 J_{ij}^{(2)} + \cdots$, which are not included in Eqs.(29,30). However, the full field theory includes all the information uniquely, in particular the expansion of Eqs.(22-24) around the background solution $J_{ij}^{(0)}$ of Eq.(28) should determine both the propagation and the interactions of the fluctuations involving photons, gravitons, and high spin fields. At the linearized level in lowest order, it is shown in [10] that these fields satisfy the Klein-Gordon type equation.

3 Remarks and Projects

We have learned that we can consistently formulate a worldline theory as well as a field theory of 2T-physics in $d+2$ dimensions based on basic gauge principles. The equations, compactly written in $d+2$ dimensional phase space in the form of Eq.(19), yield a unified description of various gauge fields in configuration space, including Maxwell, Einstein, and high spin gauge fields interacting with matter and among themselves in d dimensions.

All results follow from the field theory action in Eq.(18), which is essentially unique save for the assumption of maximum cubic power of $\mathcal{J}$. At this time it is not known what would be the consequences of relaxing the maximum cubic power of $\mathcal{J}$.

It appears that our approach provides for the first time an action principle that should contribute to the resolution of the long studied but unfinished problem of high spin fields [17][18][8][19]. The nature and detail of the interactions can in principle be extracted from our $d+2$ dimensional theory. The details of the interactions remain to be worked out, and a comparison to the perturbative equations in [17] is desirable.

For spinning particles the worldline theory introduced osp$(n|2)$ in place of sp$(2,R)$ [3]. For the field theory counterpart, we may guess that the appropriate gauge group for the supersymmetric noncommutative field theory would be u$_\star(n|1,1)$.

In the case of spacetime supersymmetry, the worldline theory with background fields remains to be constructed. We expect this to be a rather interesting and rewarding exercise, because kappa supersymmetry is bound to require the background fields to satisfy dynamical equations of motion, as it does in 1T physics [22]. The supersymmetric field equations thus obtained in $d+2$ dimensions should be rather interesting as they would include some long sought field theories in $d+2$ dimensions, among them super Yang-Mills and supergravity theories. Perhaps one may also attempt directly the spacetime supersymmetrization of the field theory approach, bypassing the background field formulation of the worldline theory. Based on the arguments given in [4][1] and [23] we expect the supersymmetry osp$(1|64)$ to play a crucial role in the relation of this work to M-theory (osp$(1|64)$ has also appeared later in [24][25]).

The noncommutative field theory can be reformulated as a matrix theory in a large N limit [10]. It is conceivable that these methods would lead to a formulation of a covariant version of M(atrix) theory [21]. Using matrix methods one could relate to the 2T-physics formulation of strings and branes.

A different formulation of 2T-physics for strings (or p-branes) on the worldsheet (or worldvolume) was initiated in [5]. Tensionless strings (or p-branes) were described in the 2T approach. However, tensionful strings did not emerge yet in the formulation. The systematics of the 2T formulation for strings or branes is not as well understood as the particle case. The suspicion is that either the action in [5] was incomplete or the correct gauge choice (analogous to the massive particle) remains to be found. This is still a challenge.

So far the field theory has been analyzed at the classical level. The action can now be taken as the starting point for a second quantized approach to 2T-physics. The technical aspects of this are open.

It would be interesting to consider phenomenological applications of the noncommutative field theory approach of 2T-physics, including spinning particles, and non-Abelian gauge groups.

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References

[1] I. Bars, "Survey of Two-Time Physics" , hep-th/0008164.

[2] I. Bars, C. Deliduman and O. Andreev, Phys. Rev. **D58** (1998) 066004, hep-th/9803188; I. Bars, Phys. Rev. **D58** (1998) 066006, hep-th/9804028; hep-th/9809034; Phys. Rev. **D59** (1999) 045019, hep-th/9810025; ; S. Vongehr, hep-th/9907077.

[3] I. Bars and C. Deliduman, Phys. Rev. **D58** (1998) 106004, hep-th/9806085;

[4] I. Bars, C. Deliduman and D. Minic, "Supersymmetric Two-Time Physics", Phys. Rev. **D59** (1999) 125004, hep-th/9812161; "Lifting M-theory to Two-Time Physics", Phys. Lett. **B457** (1999) 275, hep-th/9904063.
I. Bars, "*2*T formulation of superconformal dynamics relating to twistors and supertwistors", Phys. Lett. **B483** (2000) 248, hep-th/0004090; "$AdS_5 \times S^5$ supersymmetric Kaluza-Klein towers as a 12-dimensional theory in 2T-Physics", in preparation (see also [1]); "A Toy M-model", in preparation (partial results in hep-th/9904063 and [1]).

[5] I. Bars, C. Deliduman and D. Minic, "String, Branes and Two-Time Physics", Phys. Lett. **B466** (1999) 135, hep-th/9906223.

[6] I. Bars, "Two-Time Physics with Gravitational and Gauge Field Backgrounds", Phys. Rev. **D62** (2000) 085015, hep-th/0002140.

[7] I. Bars, "Two-Time Physics in Field Theory", Phys. Rev. **D62** (2000) 046007, hep-th/0003100.

[8] I. Bars and C. Deliduman, "High spin gauge fields and Two-Time Physics As The Foundation of 2T-Physics in Field Theory", hep-th/0103042.

[9] I. Bars and S.J. Rey, "Noncommutative Sp(2,R) Gauge Theories - A Field Theory Approach to Two-Time Physics", hep-th/0104135.

[10] I. Bars, "$U_\star(1,1)$ Noncommutative Gauge Theory of 2T-Physics ", hep-th/0105013.

[11] I. Bandos, J. A. de Azcarraga, J. M. Izquierdo, J. Lukierski, "BPS states in M theory and twistorial constituents", hep-th/0101113.

[12] I. Bars and M. Günaydin, Comm. Math. Phys. **91** (1983) 31. M. Günaydin and N. Marcus, Class. and Quant. Grav. **2** (1985) L11; M. Günaydin, D. Minic and M. Zagermann, Nucl. Phys. **B534** (1998) 96; Nucl. Phys. **B544** (1999) 737. M. Günaydin and D. Minic, Nucl. Phys. **B523** (1998) 145.

[13] H. Weyl, Zeit. für Phys. **46** (1927) 1.

[14] E. Wigner, Phys. Rev. **40** (1932) 749; in *Perspectives in Quantum Theory*, Eds. W. Yourgrau and A. van de Merwe (MIT Press, Cambridge, 1971).

[15] J. Moyal, Proc. Camb. Phil. Soc. **45** (1949) 99.

[16] P.A.M Dirac, Ann. Math. 37 (1936) 429; H. A. Kastrup, Phys. Rev. 150 (1966) 1183; G. Mack and A. Salam, Ann. Phys. 53 (1969) 174; C. R. Preitschopf and M. A. Vasiliev, hep-th/9812113.

[17] M. A. Vasiliev, Int. J. Mod. Phys. **D5** (1996) 763; "Higher spin gauge theories, star products and AdS space", hep-th/9910096.

[18] A. Yu. Segal, "Point particle in general background fields and generalized equivalence principle", hep-th/0008105; "A Generating formulation for free higher spin massless fields", hep-th/0103028.

[19] E. Sezgin and P. Sundell, "Doubletons and higher spin gauge theories", hep-th/0105001.

[20] I. Bars, M.M. Sheikh-Jabbari, M.A. Vasiliev, "Noncommutative $o_\star(N)$ and $usp_\star(2N)$ algebras and the corresponding gauge field theories", hep-th/0103209.

[21] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, Phys. Rev. **D55** (1997) 5112, hep-th/9610043. N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. **B498** (1997) 467, hep-th/9612115.

[22] E. Witten, "Twistor-like transform in ten-dimensions", Nucl.Phys.B266:245,1986.

[23] I. Bars, “S-theory”, Phys. Rev. **D55** (1997) 2373, [hep-th/9607112]; “Algebraic structure of S theory”, hep-th/9608061.

[24] P. West, “Hidden superconformal symmetry in M theory”, **JHEP** (2000) 0008:007, [hep-th/0005270].

[25] R. D’Auria, S. Ferrara and M.A. Lledo, “On the embedding of space-time symmetries into simple superalgebras”, hep-th/0102060.