

Acceleration of the Universe, String Theory and a Varying Speed of Light

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Abstract

The existence of future horizons in spacetime geometries poses serious problems for string theory and quantum field theories. The observation that the expansion of the universe is accelerating has recently been shown to lead to a crisis for the mathematical formalism of string and M-theories, since the existence of a future horizon for an eternally accelerating universe does not allow the formulation of physical S-matrix observables. Postulating that the speed of light varies in an expanding universe in the future as well as in the past can eliminate future horizons, allowing for a consistent definition of S-matrix observables.

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1 Introduction

Recently, it has been shown that a critical situation arises in string and M-theories due to the existence of future horizons in spacetimes [1, 2, 3], and for models of the universe exhibiting an accelerating expansion [4]. String theories and M-theories, in their present mathematical frameworks, are critically dependent on their background geometries. The observables of string theory are determined for asymptotically free particle states in asymptotically flat Minkowski spacetime. The existence of an S-matrix is crucially dependent on having a large enough space at infinity in which particles are separated into a system of non-interacting objects. The description of observables in AdS spacetimes by boundary correlators of bulk fields is analogous to S-matrix elements, and is supported by an infinite asymptotic space with non-interacting particles at infinity.

The standard quantum field theory formalism, which provides the basis for the remarkable standard model agreement with observational data, is a perturbative theory based largely on the notion of asymptotically free

particles and fields. In dS spacetime with a positive cosmological constant problems arise, for there is no unique Fock space vacuum and it is difficult to define momentum space due to the ill-defined nature of particle annihilation and creation operators. The indication that observational data tell us that the universe is accelerating produces a crisis for our conventional quantum field theory formalism. The perturbative and non-perturbative string theories, formulated for strictly on-shell S-matrix elements, do not fare any better and are possibly in worse shape due to their on-shell definitions. This crisis in our interpretation of modern particle theories has been waiting to happen; the advent of observational data supporting an accelerated expansion of the universe, forces us to confront particle theory with the complications of spacetimes with future horizons.

Realistic cosmology models are described by neither flat spacetime nor AdS spacetime. Spatially flat FRW models can be described by S-vectors as suggested by Witten [3], by assuming that the initial state of the universe is unique and that the final state is described by asymptotically free particles with a Fock space of asymptotic out-fields. In standard FRW models there is no future particle horizon, so that particles can communicate with particles at infinity and an S-vector can be meaningfully constructed.

It is possible that string theory can be completely reformulated, so that it can cope with spacetimes with future horizons [5]. On the other hand, it is possible that if the data supporting an accelerating universe are confirmed, then we may not be able to reformulate the language of quantum field theory and string theories in a satisfactory way, so that we are forced to consider new ideas that can resolve the crisis. In the following, we shall explore the idea that a varying light speed in the *future* universe can remove future horizons from all spacetime geometries and rid us of the challenging (and maybe impossible) problem of making sense of present theories in cosmological backgrounds with future horizons.

The idea that the speed of light varies was proposed as an alternative to standard inflation theory [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. The idea [6] originated with the hypothesis that there is a phase of spontaneously broken, local Lorentz invariance in the early universe, due to a non-vanishing vacuum expectation value of a field. The speed of light underwent an abrupt phase transition as the universe expanded decreasing to its presently observed value. This idea was reformulated as a bimetric theory based on vector-tensor and scalar-tensor structures [15, 16, 17]. The notion of a varying speed of light (VSL) has also been formulated in theories with extra-dimensions and in 5-dimensional brane-bulk models with violations of Lorentz invariance [18]. There are observational indications that the fine-structure constant $\alpha = e^2/\hbar c$ varies with time consistent with an increasing

speed of light [19].

In the following, we shall show that VSL theories can remove the problem of future horizons, allowing for a physical framework to define a consistent S-matrix as a basis for quantum field theory and string/M-theories.

2 Varying Speed of Light Model

We shall use a minimal scheme proposed in refs. [8, 9, 10] to illustrate the resolution of the future horizon problem, and defer the application of a more geometrically rigorous theory of VSL, such as the bimetric theory [15, 16, 17] to a later publication. In a minimally coupled VSL theory, one replaces c by a field in a preferred frame of reference, $\chi(x^\mu) = c^4$. The dynamical variables in the Lagrangian \mathcal{L} are the metric $g_{\mu\nu}$, matter variables contained in the matter Lagrangian \mathcal{L}_M , and the scalar field χ which is assumed not to couple to the metric explicitly. In the preferred frame the curvature tensor is to be calculated from $g_{\mu\nu}$ at constant χ in the normal manner. Varying the action with respect to the metric gives the field equations

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{\chi}T_{\mu\nu}, \quad (1)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, Λ is the cosmological constant and $T_{\mu\nu}$ denotes the stress energy-momentum tensor. This theory is not locally Lorentz invariant. Choosing a specific time to be the comoving proper time, and assuming that the universe is spatially homogeneous and isotropic, so that c only depends on time $c = c(t)$, then the FRW metric can still be written as

$$ds^2 = c^2 dt^2 - a^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (2)$$

where $k = 0, +1, -1$ for spatially flat, closed and open universes, respectively. The Einstein equations are still of the form

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}, \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + 3\frac{p}{c^2} \right). \quad (4)$$

We have set $\Lambda = 0$, since we shall be primarily concerned with quintessence models in which it is assumed that the cosmological constant is zero. The conservation equations are modified due to the time dependence of c :

$$\dot{\rho} + 3\frac{\dot{a}}{a} \left(\rho + \frac{p}{c^2} \right) = \frac{3kc^2}{4\pi Ga^2} \frac{\dot{c}}{c}. \quad (5)$$

Let us assume that matter obeys the equation of state

$$p = w\rho c^2, \quad (6)$$

where w is a constant. We shall assume that c changes at a rate proportional to the expansion of the universe

$$c = \bar{c}a^n, \quad (7)$$

where \bar{c} and n are constants. The present observational value of c is defined to be

$$c(t_0) \equiv c_0 = \bar{c}a^n(t_0), \quad (8)$$

where t_0 denotes the present time. Barrow [9] has found an exact solution of (5) of the form

$$\rho = \frac{B}{a^{3(1+w)}} + \frac{3k\bar{c}^2na^{2(n-1)}}{4\pi G(2n-2+3(1+w))}, \quad (9)$$

where $B \geq 0$ is constant if $2n-2+3(1+w) \neq 0$. For $n = 0$ the speed of light is constant and the equations reduce to the usual adiabatic expansion laws of FRW cosmology. As first shown in ref. [6] and subsequently in refs. [8, 9, 15] VSL theories can solve the horizon, flatness and particle relic problems of early universe cosmology.

3 Future Horizons, Quintessence and Varying Speed of Light

We shall adopt the theory of quintessence [20] as an alternative to a positive cosmological constant. According to this theory, the dark energy of the universe is dominated by the potential $V(\phi)$ of a scalar field ϕ , which rolls down to its minimum at $V = 0$. We recall that a cosmological constant corresponds to $w = -1$, radiation domination to $w = \frac{1}{3}$ and matter domination to $w = 0$. On the other hand, quintessence gives an equation of state with

$$-1 < w < -\frac{1}{3}, \quad (10)$$

while the observational evidence for a cosmological constant is given by the bound:

$$-1 < w_{\text{observed}} \leq -\frac{2}{3}. \quad (11)$$

We shall now analyze the causal structure of the universe with w in the range (10) with a varying light speed $c = c(t)$. Our results can be straightforwardly extended to higher-dimensional theories such as brane-bulk models.

We obtain from (4) the condition for an accelerating expansion of the universe, $p < -\rho/3$. The proper horizon distance is given by

$$\delta_H(t) = a(t)I, \quad (12)$$

where

$$I = \int_{t_0}^{\infty} \frac{dt' c(t')}{a(t')}. \quad (13)$$

Whenever I diverges there exist no future event horizons in the spacetime geometry. On the other hand, when I converges, the spacetime geometry exhibits a future horizon, and events whose coordinates at time \bar{t} are located beyond δ_H can never communicate with the observer at $r = 0$.

The variation of the expansion scale factor at large $a(t)$, when the curvature becomes negligible, approaches

$$a(t) \sim t^{2/3(1+w)}. \quad (14)$$

We now have

$$I = \bar{c} \int_{t_0}^{\infty} dt' t'^{[2(n-1)/3(1+w)]}. \quad (15)$$

We see that for the quintessence w range (10), we can choose n so that I diverges and the future horizon has been eliminated. For $n = 0$, I will converge for the quintessence w range and will generate a future horizon, which prohibits an S-matrix description of particles. Consider, as an example, the choice $w = -\frac{2}{3}$, then (15) diverges for $n \geq \frac{1}{2}$ and the future horizon is eliminated.

Consider finally a dS universe with $\Lambda > 0$ and the asymptotic scale factor behavior

$$a(t) \sim \exp\left(\sqrt{\frac{\Lambda}{3}}t\right). \quad (16)$$

We now get

$$I = \bar{c} \int_{t_0}^{\infty} dt' \exp\left[(n-1)\sqrt{\frac{\Lambda}{3}}t'\right] \quad (17)$$

and the dS event horizon is removed for $n \geq 1$.

4 Conclusions

We have concerned ourselves in this note with the serious difficulty in defining a consistent S-matrix description of quantum field theory and String/M-theories in spaces associated with an eternal accelerated expansion of the universe. We have shown that if we postulate that the speed of light varies in the future as well as in the past universe, then we can solve the initial value problems of cosmology (horizon and flatness problems), and remove asymptotically all future horizons associated with quintessence models and an accelerating universe. If future data confirm the accelerated expansion of the universe, and it proves impossible to formulate a consistent theory of quantum fields, strings and M-theory, based on physically meaningful quantum observables, then we may be forced to seriously consider a scenario such as that provided by VSL theories to preserve our present understanding of particle physics and future theories of quantum gravity.

Hopefully, new supernovae red-shift data will be able to distinguish between quintessence models and a non-vanishing cosmological constant Λ . Whatever the outcome of these observations, the VSL theories can eliminate the problem of asymptotic states in string theory and quantum field theory, whether or not quintessence models can be found that lead in the future to a decelerating universe or whether the cosmological constant is non-zero, resulting in an eternal accelerating universe.

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