

Time-dependent cosmological constant in the Jackiw-Teitelboim cosmology

M. Alves* and J. Barcelos-Neto†
Instituto de Física
Universidade Federal do Rio de Janeiro,
RJ 21945-970 – Brazil

M. Novello‡ and J.M. Salim§
Centro Brasileiro de Pesquisas Físicas,
Rua Dr. Xavier Sigaud 150,
Urca 22290-180 Rio de Janeiro, RJ – Brazil
(Dated: April 25, 2020)

We study the obtainment of a time-dependent cosmological constant at $D=2$ in a model based on the Jackiw-Teitelboim cosmology. We show that the cosmological term goes to zero asymptotically and can induce a nonsingular behavior at the origin.

PACS numbers: 98.80.Bp, 98.80.Cq

I. INTRODUCTION

The presence of cosmological constant is an interesting and intriguing subject in many cosmological models [1]. If it actually exists in the present stage of the universe, it has to be very small. But the point is that it could have not been small since the beginning. Consequently, realistic models to describe the evolution of the universe should include a time-dependent of the cosmological *constant* [2].

In a previous paper [3], three of us have presented a model where the origin of this time dependence was related to a possible quantum scenario of the initial evolution of the universe. This was achieved by showing that the chiral gauge anomaly could be conveniently adapted in order to generate a time-dependent cosmological *constant*. In this way, the presence of this term today would be a reminiscence of that initial quantum behavior. Later on, this model was applied [4] in a particular scenario where the geometry was initially Bianchi-like (spatially homogeneous but anisotropic) [5]. We have shown that the Einstein equations in the presence of the cosmological term (as well as the Maxwell one) leads to an asymptotic solution that is compatible with the Friedmann universe [1, 6].

In these two papers, the time-dependence we have mentioned was implicit into the possible quantum behavior of the initial evolution of the universe, but we have not found this time dependence explicitly. It could have been achieved if the equations of motion of the theory were completely solved. However, since we are involved with nonlinear equations, this is not an easy task.

We address to this problem in the present paper. We try to circumvent the algebraic difficulties by working in a gravitational theory in spacetime dimension $D=2$, in the formulation given by Jackiw and Teitelboim [7]. We deal with two distinct situations in order to implement the time cosmological constant in the model. The first one is considered by following the same steps of reference [4] adapted to $D=2$. We find that the only possible solution is an actually cosmological constant. Even though there is no time dependence for the cosmological term at $D=2$, the development of this part will be useful to envisage the asymptotic solution of the next case. Secondly, we use a Chern-Simons [8] term at $D=2$ [9] as the starting point to generate the cosmological term. Even though the equations of motion we obtain are much simpler than the four dimensional case, they are still very complicated to be completely solved. However, with the help of the solution we have obtained into the first case, we show that it is possible to have a cosmological term which tends to zero as the time goes to infinity. This is the behavior we would expect for a time-dependent cosmological constant. We also consider its behavior closed to the origin, where the presence of the cosmological *constant* permit us to have both singular and nonsingular solutions, depending on the boundary conditions we use.

Our paper is organized as follow. In Sec. II we review the general features of the model adapted to the two-dimensional case. The applications mentioned above are displayed in Sects. III. and IV respectively. We left Sec. V for some concluding remarks.

II. TIME-DEPENDENT COSMOLOGICAL CONSTANT IN THE JACKIW-TEITELBOIM COSMOLOGY

It is well-known that the Einstein-Hilbert action in $D=2$ is a topological invariant quantity, in a sense that its corresponding Lagrangian is a total derivative. So, there is no way to extract any dynamics from it. The

*Electronic address: msalves@if.ufrj.br

†Electronic address: barcelos@if.ufrj.br

‡Electronic address: novello@cbpf.br

§Electronic address: jsalim@cbpf.br

formalism due to Jackiw and Teitelboim [7] starts from a kind of two-dimensional version of the Einstein equation (but with no action as origin), i.e.

$$R - \Lambda = 8\pi G T \quad (2.1)$$

where R is the Ricci scalar, Λ is a cosmological constant, G is the gravitational constant and T is the trace of the energy-momentum tensor.

It is opportune to mention that in the original Jackiw-Teitelboim model the right-hand side of (2.1) was zero. The introduction of the trace of the energy-momentum tensor into the equation was first done by Mann et al. [10].

Let us now review how the time-dependent Λ can be generated [3, 4] at $D=2$. We consider an action S_Λ as

$$S_\Lambda = \int d^2x \sqrt{-g} Y(\mathcal{G}) \quad (2.2)$$

where g is the determinant of the metric tensor and \mathcal{G} is given by [3]

$$\mathcal{G} = -\frac{1}{2\sqrt{-g}} \epsilon^{\mu\nu} F_{\mu\nu} \quad (2.3)$$

$F_{\mu\nu}$ is a gauge field strength (which for simplicity we are considering to be Abelian) and $\epsilon^{\mu\nu}$ is the usual Levi-Civita tensor density (we shall adopt the convention $\epsilon^{01} = 1$).

The energy-momentum tensor related to the action (2.2) is [11]

$$\begin{aligned} T_{\mu\nu}^\Lambda &= \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} (\sqrt{-g} Y) \\ &= \frac{2}{\sqrt{-g}} \frac{\delta \sqrt{-g}}{\delta g^{\mu\nu}} Y + 2 \frac{dY}{d\mathcal{G}} \frac{\delta \mathcal{G}}{\delta g^{\mu\nu}} \\ &= \left(\mathcal{G} \frac{dY}{d\mathcal{G}} - Y \right) g_{\mu\nu} \end{aligned} \quad (2.4)$$

Considering that [4]

$$Y = \frac{k}{4} \mathcal{G}^{p+1} \quad (2.5)$$

where k is a constant and p is some rational number, we have

$$T_{\mu\nu}^\Lambda = \frac{kp}{4} \mathcal{G}^{p+1} g_{\mu\nu} \quad (2.6)$$

This expression permit us to identify the cosmological constant

$$\Lambda = \frac{kp}{4} \mathcal{G}^{p+1} \quad (2.7)$$

which can also be related to the trace of $T_{\mu\nu}^\Lambda$ as

$$\Lambda = g^{\mu\nu} T_{\mu\nu}^\Lambda = T^\Lambda \quad (2.8)$$

Notice that for $p=0$, there is no cosmological constant and for $p=-1$, it is actually a constant.

Introducing the cosmological constant generated in this way into the Jackiw-Teitelboim equation, we get

$$R - T^\Lambda = 8\pi G \tilde{T} \quad (2.9)$$

where we have absorbed a factor $8\pi G$ into the constant \tilde{T} of the Lagrangian density \mathcal{L} . \tilde{T} is the trace of the remaining part of the energy-momentum tensor of the theory (including other terms besides S_Λ). We emphasize that in our formalism the cosmological term was generated by the dynamics.

In the next section we use this model to an specific example involving a general electrodynamic theory at $D=2$.

III. COSMOLOGICAL TERM WITH ELECTRODYNAMICS AT D=2

Let us consider the general action at involving fermions and Abelian gauge fields interacting at $D=2$,

$$S = \int d^2x \sqrt{-g} \left[Y(\mathcal{G}) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \gamma^\mu (\nabla_\mu - ie A_\mu) \psi \right] \quad (3.1)$$

where ∇_μ is the covariant derivative.

Even though the Jackiw-Teitelboim equation (2.9) does not come from (3.1), we have used it as a source for the energy-momentum tensor and, particularly, for the cosmological constant. Considering that $T^\Lambda = \Lambda$ is given by the combination of (2.3) and (2.7), and that $\tilde{T} = -\frac{1}{2} F_{\mu\nu} F^{\mu\nu}$ is related to the electromagnetic energy-momentum tensor (whose trace is not zero at $D=2$), we have for the Jackiw-Teitelboim equation

$$R - \frac{kp}{4} \left(-\frac{1}{2\sqrt{-g}} \epsilon^{\mu\nu} F_{\mu\nu} \right)^{p+1} = -4\pi G F_{\mu\nu} F^{\mu\nu} \quad (3.2)$$

Now, it is necessary to combine (3.2) with the other equations obtained from (3.1), which are

$$\gamma^\mu (\nabla_\mu - ie A_\mu) \psi = 0 \quad (3.3)$$

$$\begin{aligned} \sqrt{-g} F^{\mu\nu}{}_{;\nu} + \frac{k}{4} (p+1) \epsilon^{\mu\nu} \left[\left(-\frac{1}{2\sqrt{-g}} \epsilon^{\sigma\rho} F_{\sigma\rho} \right)^p \right]_{,\nu} \\ - e \sqrt{-g} \bar{\psi} \gamma^\mu \psi = 0 \end{aligned} \quad (3.4)$$

where the notation ∇_μ and ∂_μ means covariant and usual derivatives respectively.

Let us consider the metric

$$ds^2 = dt^2 - a^2(t) dx^2 \quad (3.5)$$

So, the Jackiw-Teitelboim equation (3.2) leads to

$$\frac{\ddot{a}}{a} - \frac{kp}{4} \left(\frac{F_{10}}{a} \right)^{p+1} = 8\pi G \left(\frac{F_{01}}{a} \right)^2 \quad (3.6)$$

For the kind of metric given by (3.5), we have that all fields are just function of t . Thus, if one takes $\nu = 0$ into (3.3), and considering that derivatives of fields with respect to μ are zero, we get

$$\begin{aligned} \psi^\dagger \psi &= 0 \\ \Rightarrow \psi &= 0 \end{aligned} \quad (3.7)$$

Now, taking $\nu = 0$ into (3.3) and using (3.7), we also obtain

$$\frac{F_{10}}{a} + \frac{k}{4}(p+1) \left(\frac{F_{10}}{a} \right)^p = E_0 \quad (3.8)$$

where E_0 is a constant. We observe that the value of F_{10}/a depends on a , but, for any a , it is always a constant. Consequently, this model has generated an actually cosmological constant and E_0 is also a constant. In this case, the Jackiw- Teitelboim equation (3.6) leads to well-known results [10].

IV. USING ANOTHER CHERN-SIMONS TERM

The quantity we have used in the previous section is considered to be the Chern-Simons term at $D=2$. However, it is possible to have another sequence of Chern-Simons terms at any spacetime dimensions [9], where for $D=2$ this term is (in curved space)

$$\mathcal{G} = -\frac{1}{2\sqrt{-g}} \epsilon^{\mu\nu} F_{\mu\nu} \phi \quad (4.1)$$

where ϕ is a scalar field. We observe that $T_{\mu\nu}^\Lambda$ is still given by (2.7), but with \mathcal{G} replaced by the new one. So, the cosmological *constant* is

$$\Lambda = \frac{kp}{4} \left(\frac{F_{10}}{a} \phi \right)^{p+1} \quad (4.2)$$

Also here we have that for $p=0$ there is no cosmological *constant* and for $p=-1$ the cosmological term is actually a constant. The Jackiw-Teitelboim equation turns to be

$$\frac{\ddot{a}}{a} - \frac{kp}{4} \left(\frac{F_{10}}{a} \phi \right)^{p+1} = 8\pi G \left(\frac{F_{01}}{a} \right)^2 \quad (4.3)$$

Now, the action we have to use is

$$S = \int d^2x \sqrt{-g} \left[Y(\mathcal{G}) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} \right] \quad (4.4)$$

We have not included the fermionic field because it will not contribute according the metric convention we are using.

As in the previous case, the Jackiw-Teitelboim equation has to be combine with the other ones obtained from the action. These equations are now given by

$$\frac{F_{10}}{a} + \frac{k}{4}(p+1) \phi \left(\frac{F_{10}}{a} \phi \right)^p = E_0 \quad (4.5)$$

$$\dot{a}\dot{\phi} + a\ddot{\phi} - \frac{k}{4}(p+1)F_{10} \left(\frac{F_{10}}{a} \phi \right)^p = 0 \quad (4.6)$$

The set we have to use is given by Eqs. (4.3), (4.5), and (4.6). The complete solution of this set of equations is not an easy task. However, one can infer the asymptotic behavior of the cosmological constant, and consequently the solution, as $a \rightarrow \infty$ and closed to the origin.

In the previous section, we just had solutions where F_{10}/a was a constant, independently of the region of the space. This was given by solving Eq. (3.8). The corresponding relation in the present section is given by Eq. (4.5). Now, F_{10}/a is not necessarily a constant for all values of a by virtue of the presence of ϕ into the second term. One observes in (4.5) that there is a possibility of asymptotic solution $F_{10}/a = E_0$ if $\phi \rightarrow 0$ (for $p > -1$). We also observe that this solution is consistent with Eq. (4.6) and leads to a nice behavior for the cosmological *constant* that appear in the dynamical Jackiw-Teitelboim equation (4.3), where it goes to zero asymptotically.

The behavior closed to the origin can present two typical situations: a singularity when $a(0) = 0$ or a minimum when $a(0) = a_0$. The singular case is more restrictive. Eq. (4.5) imposes that this situation is only possible if $p=1$ and $k < 0$. If $a(0) = a_0$ then equation (4.5) only determines the value of E_0 in terms of F_{10}/a_0 and ϕ_0 . The condition that a_0 is a minimum impose that $\dot{a}_0 = 0$, then Eq. (4.6) only determines $\dot{\phi}$ in terms of ϕ_0 , F_{10}/a_0 and the parameters k and p . In addition, the condition for the existence of a minimum at $t=0$, implies that the model exhibits an accelerated expansion. So, considering Eq. (4.3), we have

$$\frac{\ddot{a}_0}{a_0} = \frac{kp}{4} \left(\frac{F_{10}}{a_0} \phi_0 \right)^{p+1} + 8\pi G \left(\frac{F_{10}}{a_0} \right)^2 > 0. \quad (4.7)$$

which does not fix the sign of k .

V. CONCLUSION

In this paper we have studied the behavior of a time-dependent cosmological constant in $D=2$ by using the Jackiw-Teitelboim cosmology. We have considered two starting Lagrangians to generate the cosmological term. The first one was based on the topological quantity $\mathcal{G} = \epsilon^{\mu\nu} F_{\mu\nu} / \sqrt{-g}$, which led to a cosmological term that was actually a constant. This part was mimicking what we have done in previous work for $D=4$. In the second case we have considered another topological quantity, given by $\mathcal{G} = \phi \epsilon^{\mu\nu} F_{\mu\nu} / \sqrt{-g}$. The cosmological term so obtained had a nice behavior, which we would like to have in a four-dimensional spacetime. It went to zero asymptotically (with the possibility of having a positive acceleration) and with singular or non singular solution closed to the origin.

This behavior we have obtained into the second case, where the topological term involves two fields, suggests us to use a similar procedure in $D=4$. Here, the topological would be $\mathcal{G} = \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} B_{\rho\lambda} / 4\sqrt{-g}$, where $B_{\mu\nu}$ is the Kalb-Ramon field [12]. This work is presently under study and possible results shall be reported elsewhere.

Acknowledgments

This work is supported in part by Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq (Brazilian Research Agency). One of us, J.B.-N. has also the support of PRONEX 66.2002/1998-9.

-
- [1] For a general review of the cosmological constant problem, as well as a complete list of references, see S. Weinberg, *Rev. Mod. Phys.* **61** (1989) 1.
 - [2] Concerning the time-dependent cosmological constant, we mention, among others, M. Özer and M.O. Taha, *Nucl. Phys.* **B287** (1987) 776; J.M. Overduin and F.I. Cooperstock, *Phys. Rev.* **D58** (1998) 043506 and L.M. Diaz-Rivera and L.O. Pimentel, *Phys. Rev.* **D60** (1999) 123501, where can be found a large list of references.
 - [3] M. Novello, J. Barcelos-Neto and J.M. Salim, *Class. Quantum Grav.* **18** (2001) 1261.
 - [4] M. Novello, J. Barcelos-Neto and J.M. Salim, *Class. Quantum Grav.* **19** (2002) 3107.
 - [5] See, for example, V.A. Belinsky, E.M. Lifshitz, and I.M. Khalatnikov, *Adv. Phys.* **19** (1970) 525, and references therein.
 - [6] For other isotropization mechanisms, see M. Novello and S.L.S. Duque, *Physica* **A168** (1990) 1073, and references therein.
 - [7] R. Jackiw, *Quantum theory of gravity*, Ed. S. Christensen (Adam Hilger, Bristol, 1984), p.403; C. Teitelboim, *ibid*, p.327.
 - [8] S. Deser, R. Jackiw, and S. Templeton, *Ann. Phys.* **140** (1982) 372.
 - [9] J. Barcelos-Neto and E.C. Marino, *Europhys. Lett.* **57** (2002) 473.
 - [10] R.B. Mann, A. Shiekh and L. Tarasov, *Nucl. Phys.* **B341** (1990) 134; R.B. Mann, S.M. Morsink, A.E. Sikkema and T.G. Stelle, *Phys. Rev.* **D43** (1991) 3948.
 - [11] See, for example, N.D. Birrell and P.C.W. Davies, *Quantum fields in curved space*, Cambridge (1982).
 - [12] M. Kalb and P. Ramond, *Phys. Rev.* **D9** (1974) 2273.