A Note on the Tachyon State in Vacuum String Field Theory

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Abstract

We re-examine the recent proposal of Rastelli, Sen and Zwiebach on the tachyon fluctuation of the vacuum string field theory representing a D25 brane, originally considered by Hata and Kawano. We show that the tachyon state satisfies the linearized equations of motion on-shell in the strong sense thereby allowing us to calculate the ratio $\frac{\mathcal{E}_c}{\mathcal{T}_{25}}$ of energy density to the tension of the D-brane to be $\frac{\mathcal{E}_c}{\mathcal{T}_{25}} \simeq \frac{\pi^2}{3}\frac{1}{16(ln2)^3} \simeq 0.62$. Our proof relies on a careful handling of the limits ($n \to \infty$) involved in the conformal theory description of the sliver and tachyon states. We conjecture that the sliver state represents a single D25 brane.

1 Introduction

In a recent paper [5] Hata and Kawano studied the fluctuation modes around a classical solution to the vacuum string field theory (VSFT) called the sliver. They identified a particular state (referred to hereafter as the HK state) as

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an off-shell tachyon on the D25 brane solution and showed that the linearized field equations around the classical background lead to the correct on-shell condition for the tachyon. Furthermore, they calculated the three tachyon amplitude using this on-shell state and found that the energy density $\mathcal{E}_c$ comes to about roughly twice the D-brane tension $\mathcal{T}_{25}$ [5]. They suggested that the sliver perhaps represents a state of two D-branes. In a very interesting paper [1] Rastelli, Sen and Zwiebach reinvestigated the HK state using boundary conformal field theory (BCFT) description as opposed to the Fock space description of Hata and Kawano. They propose a BCFT description of HK state and present numerical evidence that this description agrees with the state constructed in [5]. They further point out some problems in the computations of the D25- brane tension. Specifically, they show that a 'naive' calculation of the D brane tension yields an answer that is twice the expected value. However they show that the on-shell HK state fails to satisfy the linearized equations of motion when one takes the inner product with an HK state and hence point out that the 'naive' calculation is incorrect.

In this work we reinvestigate the properties of the state discussed by Rastelli, Sen and Zwiebach. We show that one should pay particular care to the definition of the BPZ product of the sliver state $|\chi_T(k)\rangle$ representing a tachyon state with another sliver $|\chi_T(k')\rangle$. In particular as these states are defined as $n \to \infty$ of wedge states in the CFT language, care must be exercised in taking $n, n' \to \infty$ limits. We give a consistent prescription for calculating both the star product of two slivers and their BPZ product and show that the Hata-Kawano state (or more acurately, Rastelli-Sen-Zwiebach's description in CFT language) satisfies the linearized equations of motion in the strong sense as well as in weak sense. More importantly, we calculate the ratio $\mathcal{E}_c/\mathcal{T}_{25}$ and find that it is given by $\frac{\mathcal{E}_c}{\mathcal{T}_{25}} \simeq \frac{\pi^2}{3}\frac{1}{16(ln2)^3} \simeq 0.62$. Since this is a classical estimate, we conjecture that the sliver is a single brane solution.

2 BCFT construction of the tachyon state

The Vacuum String Field theory (VSFT) action is given by

$$S = -\kappa \left\{ \frac{1}{2}\langle\Psi|Q|\Psi\rangle + \frac{1}{3}\langle\Psi|\Psi \star \Psi\rangle \right\}, \tag{1}$$

where $|\Psi\rangle$ is the string field represented by a ghost number one state in the matter-ghost BCFT, Q is a new BRST operator of ghost number one and made of ghost fields

$$Q = c_0 + \sum_{n\geq 1} f_n(c_n + (-1)^n c_n^\dagger). \tag{2}$$

$\langle\Psi|\Phi\rangle$ represents the BPZ inner product and $\star$ denotes the star product [10]. The equations of motion are

$$Q|\Psi\rangle + |\Psi\star\Psi\rangle = 0. \tag{3}$$

Because of the special form of Q, one looks for a factorized solution

$$|\Psi\rangle = |\Psi_g\rangle \otimes |\Psi_m\rangle, \tag{4}$$

where $|\Psi_g\rangle$ denotes the ghost state and $|\Psi_m\rangle$ the matter state. The equations of motion then read

$$\begin{aligned} Q|\Psi_g\rangle &= |\Psi_g\star\Psi_g\rangle \\ |\Psi_m\rangle &= |\Psi_m\star\Psi_m\rangle. \end{aligned} \tag{5}$$

The ghost solution $|\Psi_g\rangle$ is taken to be universal and $|\Psi_m\rangle$ obtained as a solution to the projector equation, corresponds to different D-brane solutions. This interpretation follows from the fact that with Q constructed purely of ghost fields, it has trivial cohomology and hence solutions to VSFT contain no perturbative open string states, but may describe non-perturbative states such as the D-branes. In several exciting papers [2, 3, 4] Rastelli, Sen and Zwiebach discuss the properties and solutions to VSFT[3]. They construct a solution to the matter part of the equations of motion, describing a D25-brane solution, called a sliver state $|\Xi_m\rangle$ defined through the relation [2, 3]

$$\langle\Xi_m|\Phi\rangle = \lim_{n\to\infty}\mathcal{N}\langle f\circ\Phi(0)\rangle_{C_n}, \tag{6}$$

where $f(z) = tan^{-1}z$, $|\Phi\rangle$ is an arbitrary state in the matter Hilbert space, $\mathcal{N}$ is a normalization constant and $\langle\ldots\rangle_{C_n}$ denotes the correlation function of the matter BCFT on a semi infinite cylinder C_n of circumference $\frac{n\pi}{2}$ obtained by making identification $\mathcal{R}e\,z \simeq \mathcal{R}e\,z + \frac{n\pi}{2}$ in the upper half z plane. In the $n\to\infty$ limit C_n approaches the upper half plane and

$$\langle\Xi_m|\Xi_m\rangle = KV^{(26)}, \qquad (V^{(26)} = (2\pi)^{26}\delta^{26}(0)) \tag{7}$$

where $V^{(26)}$ is the volume of 26-dimensional spacetime and K is a normalization constant that arises due to anomaly in the matter sector.

[3]For a recent nice review see [7]

Rastelli, Sen and Zwiebach propose that the HK tachyon state can be written in the form

$$|\Psi_g\rangle \otimes |\chi_T(k)\rangle$$

where $|\Psi_g\rangle$ is the same state as in eq. (4) and $|\chi_T(k)\rangle$ is the matter part defined through the relation

$$\langle \chi_T(k)|\Psi\rangle = \mathcal{N} \lim_{n\to\infty} n^{2k^2} \langle e^{ik.X(\frac{n\pi}{4})} f \circ \Psi(0)\rangle_{C_n}. \tag{8}$$

Here $|\Psi\rangle$ is any state in the Hilbert space of states of the string field. This relation tells us to insert a tachyon vertex in the middle of the sliver at $n\pi/4$ which is diametrically opposite the puncture at the origin. BPZ and star products of two sliver states can be constructed by cutting and pasting of the cylinders C_n and $C_{n'}$ according to the prescription in [2, 7].

For arbitrary $|\Psi\rangle$ (8) can be calculated following [1] by writting

$$f \circ \Psi(0) = a_\Psi e^{-ik.X(0)} + \left[descendents\ of\ e^{-ik.X(0)}\right] + \cdots \tag{9}$$

Only the first term carrying momentum $-k$ contributes to (8) in the $n \to \infty$ limit as $e^{-ik.X(0)}$ is the only primary operator non orthogonal to the insertion $e^{ik.X}$. One finds readily upon evaluating the correlation function on right side of (8) that

$$\langle \chi_T(k)|\Psi\rangle = \mathcal{N} 2^{2k^2} a_\Psi V^{(26)} \tag{10}$$

by using the procedure for evaluating star and BPZ products of two sliver states. It is established in [1] that the state $|\chi_T(k)\rangle$ satisfies the linearized equations of motion on-shell (i.e. $k^2 = 1$) in the weak form

$$\langle \chi_T(k)|\Psi\rangle = \langle \Xi_m \star \chi_T(k) + \chi_T(k) \star \Xi_m|\Psi\rangle. \tag{11}$$

However they show that eq. (11) ceases to be valid if the Hilbert space state $|\Psi\rangle$ is replaced by another sliver $|\chi_T(k')\rangle$ (i.e. Fock space state). Since this product arises in the action for the tachyon they conclude that the result for the ratio $\mathcal{E}_c/\mathcal{T}_{25}$ is incorrect. We now show that the linearized equations in the form

$$\langle \chi_T(k)|\chi_T(k')\rangle = \langle \Xi \star \chi_T(k) + \chi_T(k) \star \Xi|\chi_T(k')\rangle \tag{12}$$

is actually valid on-shell if we use proper caution in taking $n, n' \to \infty$ limits. According to the rules in [2, 7]

$$\langle \chi_T(k)|\chi_T(k')\rangle = \lim_{n\to\infty, n'\to\infty} n^{2k^2} n'^{2k'^2} \langle e^{ik'.X\left((n+n'-2)\frac{\pi}{4}\right)} e^{ik.X(0)}\rangle_{C_{n+n'-2}}. \tag{13}$$

At first glance the definition of BPZ product in (8) and (13) appear to involve different rules. But careful look shows that they are in fact consistent. Let us demonstrate the equivalence of (13) with (8). The normalization constants in (13) are of the form $(\frac{length}{\pi/2})^{2k^2}$ where the length refers to the circumference of the cylinders defining the states $|\Psi\rangle$ as surface states in a boundary conformal field theory. In (13) the two cylinders defining $|\chi_T(k)\rangle$ and $|\chi_T(k')\rangle$ have circumferences $(n-1)\frac{\pi}{2}$ and $(n'-1)\frac{\pi}{2}$ respectively. Hence take for the moment these normalization factors to be $l_n = n-1$ and $l_{n'} = n'-1$. We now make a change of variables to $\tilde{z} = z/(n'-1)$. Then the length of the first strip representing $|\chi_T(k')\rangle$ in the BPZ product becomes $\pi/2$ while the length of the second strip is $(\tilde{n} - \varepsilon_{n'})\frac{\pi}{2}$, where $\tilde{n} = n/(n'-1)$ and $\varepsilon_{n'} = 1/(n'-1)$. Then, a simple computation yields

$$\langle\chi_T(k)|\chi_T(k')\rangle = K l_{n'}^{2k'^2} \tilde{l}_{\tilde{n}}^{2k^2} \left(\frac{1}{n'-1}\right)^{k^2+k'^2} \langle e^{ik'.X\left((\tilde{n}+1-\varepsilon_{n'})\frac{\pi}{4}\right)} e^{ik.X(0)}\rangle_{C_{\tilde{n}+1-\varepsilon_{n'}}}. \tag{14}$$

Now in the limit $\tilde{n}, n' \to \infty$ this become

$$\langle\chi_T(k)|\chi_T(k')\rangle = \lim_{n\to\infty} n^{2k^2} \langle e^{ik'.X(n\frac{\pi}{4})} e^{ik.X(0)}\rangle_{C_n} \tag{15}$$

which is a special case of (13). The above correlation function can be evaluated after mapping C_n into unit disk D and we find

$$\langle\chi_T(k)|\chi_T(k')\rangle = K 2^{2k^2} (2\pi)^{26} \delta(k+k'). \tag{16}$$

In evaluating $\langle\Xi_m \star \chi_T(k) + \chi_T(k) \star \Xi_m|\chi_T(k')\rangle$ we first express $\langle\Xi_m \star \chi_T(k)\rangle$ (and $|\chi_T(k) \star \Xi_m\rangle$) as a sliver (i.e we let $n_2 = n_3 = n$ in these states and $\tilde{n}$ as in the above) we then find

$$\begin{aligned}
&\langle\Xi_m \star \chi_T(k)|\chi_T(k')\rangle \\
&\quad = K \lim_{n_1\to\infty} n_1^{2k'^2} \left[\lim_{n_2,n_3\to\infty} n_2^{2k^2} \langle e^{ik.X\left((2n_2+n_3+n'-4)\frac{\pi}{4}\right)} e^{ik'.X(0)}\rangle_{C_{n'+n_2+n_3-3}}\right] \\
&\quad = K \lim_{\tilde{n}\to\infty} \tilde{n}^{2k^2} \langle e^{ik.X\left((3\tilde{n}+1-3\varepsilon_{n_1})\frac{\pi}{4}\right)} e^{ik'.X(0)}\rangle_{C_{2\tilde{n}+1-2\varepsilon_{n_1}}}.
\end{aligned} \tag{17}$$

We change to a new variable $w = exp(\frac{4iz}{2\tilde{n}+1-2\varepsilon_{n_1}})$ so that $C_{2\tilde{n}+1-2\varepsilon_{n_1}}$ is mapped to a unit disk D. We let $\tilde{n} \to \infty$ and take also $n_1 \to \infty$. We obtain

$$\begin{aligned}
&\langle\Xi_m \star \chi_T(k)|\chi_T(k')\rangle \\
&\quad = K \lim_{n_1\to\infty} \lim_{\tilde{n}\to\infty} \left\{ n^{2k^2} \left(\frac{4}{2\tilde{n}+1-2\varepsilon_{n_1}}\right)^{k^2+k'^2} \langle e^{ik.X\left(e^{\frac{3i\pi}{2}}\right)} e^{ik'.X(1)}\rangle_D \right\} \\
&\quad = K\, 2^{k^2} (2\pi)^{26} \delta(k+k').
\end{aligned} \tag{18}$$

$\langle \chi_T(k) \star \Xi_m | \chi_T(k') \rangle$ is calculated similarly and is equal to (18). We find

$$\langle \Xi_m \star \chi_T(k) + \chi_T(k) \star \Xi_m | \chi_T(k') \rangle = K\, 2^{k^2+1} (2\pi)^{26} \delta(k+k'). \tag{19}$$

Comparing (19) with equation (16) we see that

$$2^{1-k^2} \langle \chi_T(k) | \chi_T(k') \rangle = \langle \Xi_m \star \chi_T(k) + \chi_T(k) \star \Xi_m | \chi_T(k') \rangle \tag{20}$$

which for $k^2 = 1$ reduces to eq. (11), thus giving the correct on-shell condition for the tachyon on the D-brane. Thus the tachyon state $|\chi_T(k)\rangle$ satisfies the linearized equations of motion in the strong sense. As we notice from the above steps, it is crucial to take limits $n, n' \to \infty$ separately and not let $n = n' \to \infty$. The above calculations lead us to the following formal rules in manipulating with sliver states:

a) When we take a star product of two slivers, it is permissible to let $n_1 = n_2 = n \to \infty$ and express the starproduct state as a sliver.

b) When we take the BPZ product of two slivers one of the limits $n \to \infty$ (keeping the other n' large but fixed) first and express the result as a limit over the other n'.

These rules correspond to the procedure explained above and used in calculating (16) and (19).

3 The D25-brane tension and energy density

In this section we compute the D25-brane tension by using its relation to the three tachyon coupling g_T [8, 9].

If the classical solution $|\Psi_m\rangle$ to VSFT were to represent a D25-brane, then the ratio of the energy density $\mathcal{E}_c$ to the brane tension $\mathcal{T}_{25}$ is expected to be unity. Previous estimates [1] and [5, 6] give roughly 2, giving rise to the speculation that $|\Xi_m\rangle$ in fact is a solution representing two D25 branes.

To compute the tachyon coupling we must determine the quadratic and cubic terms in the tachyon field arising in the action (1).

We follow [1] in using the general expansion

$$|\Psi\rangle = |\Psi_g\rangle \otimes \left\{ |\Xi_m\rangle + \int d^{26}k\, n^{-k^2} T(k) |\chi_T(k)\rangle + \cdots \right\}. \tag{21}$$

Here $T(k)$ is the tachyonic field amplitude and ellipses denote of higher order excitations. Subsituting (21) into (1) we get [1]

$$
\begin{aligned}
S =& S(|\Psi_g\rangle \otimes |\Xi_m\rangle) - \langle\Psi_g|Q|\Psi\rangle \times \\
& \Big\{\frac{1}{2}\int d^{26}k\, d^{26}k' T(k)T(k')\langle\chi_T(k')|\big[|\chi_T(k)\rangle - |\Xi_m \star \chi_T(k)\rangle - |\chi_T(k)\star\Xi_m\rangle\big] \\
& + \frac{1}{3}\int d^{26}k_1\, d^{26}k_2\, d^{26}\, k_3 T(k_1)T(k_2)T(k_3)\langle\chi_T(k_1)|\chi_T(k_2)\star\chi_T(k_3)\rangle\Big\}.
\end{aligned}
\tag{22}
$$

From equations (16) and (19) we have for off-shell the relation

$$
2^{1-k^2}|\chi_T(k_3)\rangle = |\Xi\star\chi_T(k)\rangle + |\chi_T(k)\star\Xi\rangle. \tag{23}
$$

The quadratic term in the action can now be written as

$$
\begin{aligned}
S^{(2)} =& -\frac{1}{2}\langle\Psi_g|Q|\Psi\rangle\int d^{26}k\, d^{26}k'\, T(k)T(k')\left[1-2^{1-k^2}\right]\langle\chi_T(k')|\chi_T(k)\rangle \\
\simeq& \frac{1}{2}\langle\Psi_g|Q|\Psi\rangle\, ln2\int d^{26}k\, d^{26}k'\, T(k)T(k')[k^2-1]\langle\chi_T(k')|\chi_T(k)\rangle,
\end{aligned}
\tag{24}
$$

where in the second step we have taken the tachyon field $T(k)$ near on-shell $k^2 \simeq 1$.

Subsituting for the inner product from (16) we find for the quadratic term

$$
S^{(2)} \simeq \frac{K}{2}\, 4ln2\,\langle\Psi_g|Q|\Psi\rangle(2\pi)^{26}\int d^{26}k\, d^{26}k'\, T(-k)T(k)[k^2-1]. \tag{25}
$$

By a redefinition of $T(k)$

$$
\hat{T}(k) = 2\sqrt{K\, ln2\,\langle\Psi_g|Q|\Psi_g\rangle}T(k) \tag{26}
$$

we can write $S^{(2)}$ as

$$
S^{(2)} \simeq \frac{1}{2}(2\pi)^{26}\int d^{26}k\,(k^2-1)\hat{T}(-k)\hat{T}(k). \tag{27}
$$

Consider next the cubic term

$$
S^{(3)} = \frac{1}{3}\int d^{26}k_1\, d^{26}k_2\, d^{26}k_3\, T(k_1)T(k_2)T(k_3)\langle\chi_T(k_1)|\chi_T(k_2)\star\chi_T(k_3)\rangle. \tag{28}
$$

The BPZ product in (28) can be readily calculated following [1] and by using the rules outlined in the previous section.

$$
\begin{aligned}
&\langle\chi_T(k_1)|\chi_T(k_2)\star\chi_T(k_3)\rangle = \\
&= K n_1^{2k_1^2} n_2^{2k_2^2} n_3^{2k_3^2}\langle e^{ik_1.X(0)} e^{ik_2.X\left((\tilde{n}+1-\varepsilon_{n_1})\frac{\pi}{4}\right)} e^{ik_2.X\left((3\,\tilde{n}+1-3\varepsilon_{n_1})\frac{\pi}{4}\right)}\rangle_{C_{2\tilde{n}+1-2\varepsilon_{n_1}}} \\
&= K\,\tilde{n}^{2k_2^2+2k_3^2}\left(\frac{4}{2\,\tilde{n}+1-2\varepsilon_{n_1}}\right)^{k_1^2+k_2^2+k_3^2}\langle e^{k_1.X(1)} e^{ik_2.X(e^{i\pi/2})} e^{ik_2.X(e^{i3\pi/2})}\rangle_D,
\end{aligned}
\tag{29}
$$

where in the last step we used a change of variable $w = e^{i4z/(2\tilde{n}+\varepsilon_{n_1}-3)}$ to map the cylinder $C_{2\tilde{n}+\varepsilon_{n_1}-3}$ into a unit disk D. Evaluating the correlation function on D we find that

$$\begin{aligned}\langle \chi_T(k_1)|\chi_T(k_2)\star\chi_T(k_3)\rangle \simeq &\tilde{n}^{k_2^2+k_3^2-k_1^2}2^{k_1^2}(2\pi)^{26}\delta(k_1+k_2+k_3)\\ \simeq &\tilde{n}^{k_2^2+k_3^2-k_1^2}2(2\pi)^{26}\delta(k_1+k_2+k_3).\end{aligned} \tag{30}$$

Thus the cubic term in the action near on-shell becomes

$$\begin{aligned}S^{(3)} \simeq &-2\frac{K}{3}(2\pi)^{26}\langle\Psi_g|Q|\Psi_g\rangle\int d^{26}k_1\, d^{26}k_2\, d^{26}k_3\,\delta(k_1+k_2+k_3)\,T(k_1)T(k_2)T(k_3)\\ = &-\frac{1}{3}\frac{2(2\pi)^{26}}{(2ln2)^3\sqrt{K\langle\Psi_g|Q|\Psi_g\rangle}}\int d^{26}k_1\, d^{26}k_2\, d^{26}k_3\,\hat{T}(k_1)\hat{T}(k_2)\hat{T}(k_3)\,\delta(k_1+k_2+k_3).\end{aligned} \tag{31}$$

The on-shell three tachyon coupling is therefore given by

$$g_T = \frac{2}{(2\sqrt{ln2})^3}\frac{1}{\sqrt{K\langle\Psi_g|Q|\Psi_g\rangle}}\,. \tag{32}$$

According to [8, 9] the D-brane tension is related to the above coupling g_T in the following way

$$\mathcal{T}_{25} = \frac{1}{2\pi^2 g_T^2} = \frac{1}{2\pi^2}K\langle\Psi_g|Q|\Psi_g\rangle\frac{(2\sqrt{ln2})^6}{4} \tag{33}$$

we note that our expression for the brane tension $\mathcal{T}_{25}$ differs from that in [1]. We recall also the expression for the energy density $\mathcal{E}_c$ corresponding to the sliver

$$\mathcal{E}_c = \frac{K}{6}\langle\Psi_g|Q|\Psi_g\rangle. \tag{34}$$

From (33) and (34) we get for the ratio of the energy density to the brane tension

$$\frac{\mathcal{E}_c}{\mathcal{T}_{25}} = \frac{\pi^2}{3}\frac{1}{16(ln2)^3} \simeq 0.62. \tag{35}$$

4 Conclusions

In view of the fact that the calculated value of $\mathcal{E}_c/\mathcal{T}_{25}$ is less than unity we are inclined to conjecture that the sliver state does in fact represent a single

D25 brane. We can attribute the deviation from unity to the fact that the result (35) is a perturbative estimate.

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