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Linearized gravity, Newtonian limit and light deflection in RS1 model

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Abstract

We solve exactly the equations of motion for linearized gravity in the Randall-Sundrum model with matter on the branes and calculate the Newtonian limit in it. The result contains contributions of the radion and of the massive modes, which change considerably Newton's law at small distances. The effects of "shadow" matter, which lives on the other brane, are considered and compared with those of ordinary matter for both positive and negative tension branes. We also calculate light deflection and Newton's law in the zero mode approximation and explicitly distinguish the contribution of the radion field.

Keywords: Kaluza-Klein theories; branes; linearized gravity; Newtonian limit

1 Introduction

The present-day approach to Kaluza-Klein theories is based on the idea due to Rubakov and Shaposhnikov of localization of fields on a domain wall [1, 2]. In a first step it is natural to drop the mechanism of localization of fields and to treat the domain wall as an infinitely thin object, i.e. as a membrane, and to consider the effects due to the gravitational interaction of such 3-branes.

A particular realization of this scenario was proposed in paper [3]. In this paper an exact solution for a system of two branes interacting with gravity in a five-dimensional space-time was found. This model is called the Randall-Sundrum model (usually abbreviated as RS1 model), and it is widely discussed in the literature (see Refs. [4, 5] for reviews and references). A consistent analysis of this model without matter on the branes was made in [6]. However, the equations of motion for metric fluctuations, when there is matter located on the branes, have not been studied in detail yet. In paper [7] the Randall-Sundrum model with matter on the branes was discussed for the cases of both one and two branes (the former is usually called RS2 model). This paper utilized the Gaussian normal coordinates and the bent-brane formalism, which was argued to be inconsistent in RS2 model in paper [8]. Moreover, the use of Gaussian normal coordinates mixed the contributions of the graviton and the radion fields to the four-dimensional gravitational field. In the present paper we will decouple the equations for the graviton and the radion fields and solve exactly the equations of motion for the RS1 model with matter on the branes. Then we will calculate the Newtonian limit in this model and the light deflection by a point-like mass in the zero mode approximation.

The Randall-Sundrum model [3] describes the gravity in a five-dimensional space-time E with two branes embedded into it. We denote the coordinates by $\{x^M\} \equiv \{x^\mu, y\}$, $M = 0, 1, 2, 3, 4$, $\mu = 0, 1, 2, 3$, the coordinate $x^4 \equiv y$ parameterizing the fifth dimension. It forms the orbifold S^1/Z_2, which is realised as the circle of the circumference $2R$ with points y and $-y$ identified. Correspondingly, we have the usual periodicity condition in space-time E, which identifies points (x, y) and $(x, y + 2nR)$, and the metric g_{MN} satisfies the orbifold symmetry conditions

$$\begin{aligned} g_{\mu\nu}(x, -y) &= g_{\mu\nu}(x, y), \\ g_{\mu 4}(x, -y) &= -g_{\mu 4}(x, y), \\ g_{44}(x, -y) &= g_{44}(x, y). \end{aligned} \tag{1}$$

The branes are located at the fixed points of the orbifold, $y = 0$ and $y = R$.

The action of the model is

$$S = S_g + S_1 + S_2, \tag{2}$$

where S_g, S_1 and S_2 are given by

$$\begin{aligned} S_g &= \frac{1}{16\pi \hat{G}} \int_E (R - \Lambda) \sqrt{-g}\, d^4x dy, \\ S_1 &= V_1 \int_E \sqrt{-\tilde{g}} \delta(y) d^4x dy, \\ S_2 &= V_2 \int_E \sqrt{-\tilde{g}} \delta(y - R) d^4x dy. \end{aligned} \tag{3}$$

Here $\tilde{g}_{\mu\nu}$ is the induced metric on the branes and the subscripts 1 and 2 label the branes. We also note that the signature of the metric g_{MN} is chosen to be $(-1, 1, 1, 1, 1)$.

The Randall-Sundrum solution for the metric is given by

$$ds^2 = g_{MN} dx^M dx^N = e^{2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \tag{4}$$

where $\eta_{\mu\nu}$ is the Minkowski metric and the function $\sigma(y) = -k|y|$ in the interval $-R \le y \le R$. The parameter k is positive and has the dimension of mass, the parameters Λ and $V_{1,2}$ are related to it as follows:

$$\Lambda = -12k^2, \quad V_1 = -V_2 = -\frac{3k}{4\pi \hat{G}}.$$

We see that brane 1 has a positive energy density, whereas brane 2 has a negative one. The function σ has the properties

$$\partial_4 \sigma = -k\, sign(y), \quad \frac{\partial^2 \sigma}{\partial y^2} = -2k(\delta(y) - \delta(y - R)) \equiv -2k\tilde{\delta}. \tag{5}$$

Here and in the sequel $\partial_4 \equiv \frac{\partial}{\partial y}$.

We denote $\hat{\kappa} = \sqrt{16\pi\hat{G}}$, where $\hat{G}$ is the five-dimensional gravitational constant, and parameterize the metric g_{MN} as

$$g_{MN} = \gamma_{MN} + \hat{\kappa} h_{MN}, \tag{6}$$

h_{MN} being the metric fluctuations. Substituting this parameterization into (2) and retaining the terms of the zeroth order in $\hat{\kappa}$, we get the second variation action of this model [6]. It is invariant under the gauge transformations

$$h'_{MN}(x,y) = h_{MN}(x,y) - (\nabla_M \xi_N(x,y) + \nabla_N \xi_M(x,y)), \tag{7}$$

where ∇_M is the covariant derivative with respect to the background metric γ_{MN}, and the functions $\xi_N(x,y)$ satisfy the orbifold symmetry conditions

$$\begin{aligned} \xi^{\mu}(x,-y) &= \xi^{\mu}(x,y), \\ \xi^{4}(x,-y) &= -\xi^{4}(x,y). \end{aligned} \tag{8}$$

With the help of these gauge transformations we can impose the gauge

$$h_{\mu 4} = 0,\ h_{44} = h_{44}(x) \equiv \phi(x), \tag{9}$$

which will be called the *unitary gauge* (see [6]). We would like to emphasize once again that the branes remain straight in this gauge, i.e. we *do not* use the bent-brane formulation, which allegedly destroys the structure of the model (this problem was discussed in [8]).

The general form of interaction with matter is standard,

$$\frac{\hat{\kappa}}{2}\int_{B_1} h^{\mu\nu}(x,0) T^1_{\mu\nu} dx + \frac{\hat{\kappa}}{2}\int_{B_2} h^{\mu\nu}(x,R) T^2_{\mu\nu}\sqrt{-\gamma_2}dx, \tag{10}$$

where $T^1_{\mu\nu}$ and $T^2_{\mu\nu}$ are energy-momentum tensors of the matter on brane 1 and brane 2 respectively:

$$T^{1,2}_{\mu\nu} = 2\frac{\delta L^{1,2}}{\delta\gamma^{\mu\nu}} - \gamma^{1,2}_{\mu\nu} L^{1,2}.$$

As follows from formula (10), $h_{\mu\nu}$ is the only physical field of the model, since only this field interacts with matter on the branes. Obviously, the unitary gauge conditions (9) do not fix the gauge of this field. In fact, after imposing these gauge conditions there remain gauge transformations of the form

$$\xi_{\mu} = e^{2\sigma}\epsilon_{\mu}(x), \tag{11}$$

which change the longitudinal components of the field $h_{\mu\nu}$. Nevertheless, it turns out that it is convenient to solve the equations of motion for linearized gravity in the unitary gauge and then to choose an appropriate gauge in our four-dimensional world on the brane. We will use the de Donder gauge for the field $h_{\mu\nu}$ on the branes, which corresponds to the choice of *harmonic coordinates*.

2 Newtonian limit

The equations of motion for different components of the metric fluctuations in the unitary gauge take the form (see [6]):

1) $\mu\nu$-component

$$
\begin{aligned}
& \frac{1}{2}\left(\partial_\rho\partial^\rho h_{\mu\nu} - \partial_\mu\partial^\rho h_{\rho\nu} - \partial_\nu\partial^\rho h_{\rho\mu} + \frac{\partial^2 h_{\mu\nu}}{\partial {x^4}^2}\right) - \\
- \quad & 2k^2 h_{\mu\nu} + \frac{1}{2}\partial_\mu\partial_\nu\tilde{h} + \frac{1}{2}\partial_\mu\partial_\nu\phi + \\
+ \quad & \frac{1}{2}\gamma_{\mu\nu}\left(\partial^\rho\partial^\sigma h_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{h} - \frac{\partial^2\tilde{h}}{\partial {x^4}^2} - 4\partial_4\sigma\partial_4\tilde{h} - \partial_\rho\partial^\rho\phi + 12k^2\phi\right) + \\
+ \quad & \left[2kh_{\mu\nu} - 3k\gamma_{\mu\nu}\phi\right]\tilde{\delta} = -\frac{\hat{\kappa}}{2}T_{\mu\nu},
\end{aligned}
\tag{12}
$$

2) $\mu 4$-component,

$$
\partial_4(\partial_\mu\tilde{h} - \partial^\nu h_{\mu\nu}) - 3\partial_4\sigma\partial_\mu\phi = 0, \tag{13}
$$

which plays the role of a constraint,

3) 44-component

$$
\frac{1}{2}(\partial^\mu\partial^\nu h_{\mu\nu} - \partial_\mu\partial^\mu\tilde{h}) - \frac{3}{2}\partial_4\sigma\partial_4\tilde{h} + 6k^2\phi = 0, \tag{14}
$$

with $T_{\mu\nu}$ being the energy-momentum tensor of the matter and $\tilde{h} = \gamma^{\mu\nu}h_{\mu\nu}$. In what follows, we will also use an auxiliary equation, which is obtained by multiplying the equation for 44-component by 2 and subtracting it from the contracted equation for $\mu\nu$-component. This equation contains $\tilde{h}$ and ϕ only and has the form:

$$
\frac{\partial^2\tilde{h}}{\partial {x^4}^2} + 2\partial_4\sigma\partial_4\tilde{h} - 8k^2\phi + 8k\phi\tilde{\delta} + \partial_\mu\partial^\mu\phi = \frac{\hat{\kappa}}{3}T^\mu_\mu. \tag{15}
$$

If $T_{\mu\nu} = 0$, the physical degrees of freedom of the model can be extracted by the substitution [6]

$$
h_{\mu\nu} = b_{\mu\nu} + \gamma_{\mu\nu}(\sigma - c)\phi + \frac{1}{2k^2}\left(\sigma - c + \frac{1}{2} + \frac{c}{2}e^{-2\sigma}\right)\partial_\mu\partial_\nu\phi. \tag{16}
$$

with $c = \frac{kR}{e^{2kR}-1}$. It turns out that the field $b_{\mu\nu}(x^\mu, y)$ describes the massless graviton [3, 9] and massive Kaluza-Klein spin-2 fields, whereas $\phi(x)$ describes a scalar field called the radion. Apparentely, the radion field was first identified in Ref. [10] (see also [11]) and discussed in [12, 13, 14, 15].

However, the situation is rather different, when there is matter on the branes. Let us first consider the case, where matter is located on the brane at 0, i.e. the energy-momentum tensor is of the form $T_{\mu\nu} = t_{\mu\nu}(x)\delta(y)$. The substitution, which allows one to decouple the equations, looks like

$$
h_{\mu\nu} = u_{\mu\nu} + \gamma_{\mu\nu}(\sigma - c)\phi + \frac{1}{2k^2}\left(\sigma - c + \frac{1}{2} + ce^{2kR}\right)\partial_\mu\partial_\nu\phi. \tag{17}
$$

After this substitution equations (13), (14) and (15), rewritten in the flat metric (i.e. $u = \eta^{\mu\nu}u_{\mu\nu}$), take the form:

$$\partial_4(e^{-2\sigma}(\partial^\nu u_{\mu\nu} - \partial_\mu u)) = 0, \tag{18}$$

$$e^{-4\sigma}(\partial^\mu\partial^\nu u_{\mu\nu} - \Box u) - 3\partial_4\sigma\partial_4(e^{-2\sigma}u) + 3ce^{2kR-2\sigma}\Box\phi = 0, \tag{19}$$

$$\partial_4(e^{2\sigma}\partial_4(e^{-2\sigma}u)) + 2\frac{c}{k}\delta(y)\left[e^{2kR} - 1\right]\Box\phi = \frac{\hat\kappa}{3}t^\mu_\mu\,\delta(y). \tag{20}$$

Let us consider Fourier expansion of all terms of equation (20) with respect to coordinate y. Since the term with the derivative ∂_4 has no zero mode, this equation implies that

$$\Box\phi = \frac{\hat\kappa}{6R}t, \quad t \equiv t^\mu_\mu, \tag{21}$$

$$\partial_4(e^{-2\sigma}u) = 0. \tag{22}$$

From the last equation and equation (18) it follows that

$$\partial^\nu u_{\mu\nu} = e^{2\sigma}A_\mu(x), \tag{23}$$

$$\partial_\mu u^\nu_\nu = e^{2\sigma}B_\mu(x), \tag{24}$$

where $A_\mu(x)$ and $B_\mu(x)$ depend on four-dimensional coordinates only. It is easy to see that the remaining gauge transformations (11) allow us to impose the de Donder gauge condition on the field $u_{\mu\nu}$

$$\partial^\nu\left(u_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}u\right) = 0. \tag{25}$$

Having imposed this gauge, we are still left with residual gauge transformation

$$\xi_\mu = e^{2\sigma}\epsilon_\mu(x), \quad \Box\epsilon_\mu = 0. \tag{26}$$

The gauge transformations with ξ_μ satisfying these conditions are important for determining the number of degrees of freedom of the massless graviton. It follows from equation (19) that

$$\Box u = e^{2\sigma}\frac{\hat\kappa k}{1 - e^{-2kR}}t. \tag{27}$$

Now let us consider $\mu\nu$-equation. After substitution (17) and in the de Donder gauge it takes the form

$$\begin{aligned}&\frac{1}{2}e^{-2\sigma}\Box u_{\mu\nu} + \frac{1}{2}\partial_4\partial_4 u_{\mu\nu} - 2k^2u_{\mu\nu} - \partial_4\partial_4\sigma u_{\mu\nu} = \\ &= -\frac{\hat\kappa}{2}f_{\mu\nu}\delta(y) + \frac{\hat\kappa k}{12(1 - e^{-2kR})}\left(\eta_{\mu\nu} + 2\frac{\partial_\mu\partial_\nu}{\Box}\right)t,\end{aligned} \tag{28}$$

where

$$f_{\mu\nu} = t_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)t. \tag{29}$$

One can see that $f_{\mu\nu}$ is transverse-traceless. Here and below the inverse d'Alembertian is an integral operator uniquely defined by the radiation conditions.

To solve this equation, we make the following substitution

$$u_{\mu\nu} = v_{\mu\nu} + \frac{\hat{\kappa}k}{6(1-e^{-2kR})} e^{2\sigma} \Box^{-1} \left(\eta_{\mu\nu} + 2\frac{\partial_\mu \partial_\nu}{\Box} \right) t, \tag{30}$$

which does not violate (25) that takes the form

$$\begin{aligned} \partial^\nu v_{\mu\nu} &= 0, \\ v^\mu_\mu &= 0. \end{aligned} \tag{31}$$

Substituting (30) into (28), we get

$$\frac{1}{2} e^{-2\sigma} \Box v_{\mu\nu} + \frac{1}{2} \frac{\partial^2}{\partial y^2} v_{\mu\nu} - 2k^2 v_{\mu\nu} - \frac{\partial^2}{\partial y^2} \sigma v_{\mu\nu} = -\frac{\hat{\kappa}}{2} f_{\mu\nu} \delta(y). \tag{32}$$

This equation can be solved exactly. To this end, let us first perform Fourier transform of the field $v_{\mu\nu}$ with respect to the x-coordinates

$$\frac{1}{2} e^{-2\sigma} (-p^2) \tilde{v}_{\mu\nu} + \frac{1}{2} \frac{\partial^2}{\partial y^2} \tilde{v}_{\mu\nu} - 2k^2 \tilde{v}_{\mu\nu} - \frac{\partial^2}{\partial y^2} \sigma \tilde{v}_{\mu\nu} = -\frac{\hat{\kappa}}{2} \tilde{f}_{\mu\nu} \delta(y), \tag{33}$$

where $p^2 = -p_0^2 + \vec{p}^2$.

First let us solve this equation in the bulk (see [6], [9]). Here the solution of equation (33) looks like

$$\tilde{v}_{\mu\nu}(p,y) = C_{\mu\nu} J_2\left(\frac{\sqrt{|p^2|}}{k} e^{k|y|} \right) + D_{\mu\nu} N_2\left(\frac{\sqrt{|p^2|}}{k} e^{k|y|} \right), p^2 < 0 \tag{34}$$

$$\tilde{v}_{\mu\nu}(p,y) = C_{\mu\nu} I_2\left(\frac{\sqrt{|p^2|}}{k} e^{k|y|} \right) + D_{\mu\nu} K_2\left(\frac{\sqrt{|p^2|}}{k} e^{k|y|} \right), p^2 > 0 \tag{35}$$

Substituting it into equation (33) and comparing the terms at the boundaries, we get the values of the constant tensors $C_{\mu\nu}$ and $D_{\mu\nu}$. Having got them, we obtain the solutions of the $\mu\nu$-equation with matter on the positive tension brane:

1) $p^2 < 0$

$$\begin{aligned} \tilde{v}_{\mu\nu}(p,y) = \left[\tilde{t}_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \tilde{t} \right] \frac{\hat{\kappa}}{2\sqrt{-p^2}} \times \\ \times \frac{N_2\left(\frac{\sqrt{-p^2}}{k} e^{k|y|}\right) J_1\left(\frac{\sqrt{-p^2}}{k} e^{kR}\right) - J_2\left(\frac{\sqrt{-p^2}}{k} e^{k|y|}\right) N_1\left(\frac{\sqrt{-p^2}}{k} e^{kR}\right)}{J_1\left(\frac{\sqrt{-p^2}}{k}\right) N_1\left(\frac{\sqrt{-p^2}}{k} e^{kR}\right) - N_1\left(\frac{\sqrt{-p^2}}{k}\right) J_1\left(\frac{\sqrt{-p^2}}{k} e^{kR}\right)}, \end{aligned} \tag{36}$$

2) $p^2 > 0$

$$\tilde{v}_{\mu\nu}(p,y) = -\left[\tilde{t}_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\tilde{t}\right]\frac{\hat{\kappa}}{2\sqrt{p^2}} \times$$
$$\times \frac{K_2\left(\frac{\sqrt{p^2}}{k}e^{k|y|}\right) I_1\left(\frac{\sqrt{p^2}}{k}e^{kR}\right) + I_2\left(\frac{\sqrt{p^2}}{k}e^{k|y|}\right) K_1\left(\frac{\sqrt{p^2}}{k}e^{kR}\right)}{I_1\left(\frac{\sqrt{p^2}}{k}\right) K_1\left(\frac{\sqrt{p^2}}{k}e^{kR}\right) - I_1\left(\frac{\sqrt{p^2}}{k}e^{kR}\right) K_1\left(\frac{\sqrt{p^2}}{k}\right)}. \tag{37}$$

Since we want to calculate the Newtonian limit, $\tilde{t}_{\mu\nu}$ is proportional to $\delta(p_0)$ in this case. It means, that we need the solution for $p^2 > 0$.

When matter is on brane 2 (at $y = R$), all the reasonings are the same, as presented above. The full substitution looks like

$$h_{\mu\nu} = u_{\mu\nu} + e^{2\sigma}\eta_{\mu\nu}(\sigma - c)\phi + \frac{1}{2k^2}\left(\sigma + \frac{1}{2}\right)\partial_\mu\partial_\nu\phi,$$
$$u_{\mu\nu} = v_{\mu\nu} + \frac{\hat{\kappa}k}{6(e^{2kR}-1)}e^{2\sigma}\Box^{-1}\left(\eta_{\mu\nu} + 2\frac{\partial_\mu\partial_\nu}{\Box}\right)t. \tag{38}$$

The gauge conditions are the same too. The equations of motion for the field ϕ and solution for the field $v_{\mu\nu}$ with matter on brane 2 can be derived analogously and read

$$\Box\phi = \frac{\hat{\kappa}}{6R}t, \tag{39}$$

1) $p^2 < 0$

$$\tilde{v}_{\mu\nu}(p,y) = \left[\tilde{t}_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\tilde{t}\right]\frac{\hat{\kappa}}{2\sqrt{-p^2}}\frac{1}{e^{kR}} \times$$
$$\times \frac{N_2\left(\frac{\sqrt{-p^2}}{k}e^{k|y|}\right) J_1\left(\frac{\sqrt{-p^2}}{k}\right) - J_2\left(\frac{\sqrt{-p^2}}{k}e^{k|y|}\right) N_1\left(\frac{\sqrt{-p^2}}{k}\right)}{J_1\left(\frac{\sqrt{-p^2}}{k}\right) N_1\left(\frac{\sqrt{-p^2}}{k}e^{kR}\right) - N_1\left(\frac{\sqrt{-p^2}}{k}\right) J_1\left(\frac{\sqrt{-p^2}}{k}e^{kR}\right)}, \tag{40}$$

2) $p^2 > 0$

$$\tilde{v}_{\mu\nu}(p,y) = -\left[\tilde{t}_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\tilde{t}\right]\frac{\hat{\kappa}}{2\sqrt{p^2}}\frac{1}{e^{kR}} \times$$
$$\times \frac{K_2\left(\frac{\sqrt{p^2}}{k}e^{k|y|}\right) I_1\left(\frac{\sqrt{p^2}}{k}\right) + I_2\left(\frac{\sqrt{p^2}}{k}e^{k|y|}\right) K_1\left(\frac{\sqrt{p^2}}{k}\right)}{I_1\left(\frac{\sqrt{p^2}}{k}\right) K_1\left(\frac{\sqrt{p^2}}{k}e^{kR}\right) - I_1\left(\frac{\sqrt{p^2}}{k}e^{kR}\right) K_1\left(\frac{\sqrt{p^2}}{k}\right)}. \tag{41}$$

An important point is that these equations are written in the coordinates $\{x^\mu\}$, which are *Galilean on brane 1* (not on brane 2) and are inappropriate for studying physical effects on brane 2 (we recall that coordinates are called Galilean, if $g_{\mu\nu} = diag(-1,1,1,1)$ [16]).

Obviously, when there is matter on both branes, the solution for the metric fluctuation is just the sum of solutions for each brane separately, which follows from the linearity of the equations.

Now let us examine the four-dimensional theory on the branes. First we consider the case when matter and observer are located on brane 1. It is easy to see that $h_{\mu\nu}$ (17), (30) does not satisfy the de Donder gauge condition. The residual gauge transformations (26) are not sufficient to pass to this gauge. But since we consider only the effective theory on brane 1, we can drop the $\Box\epsilon_\mu = 0$ condition, which fixes the gauge for the field $v_{\mu\nu}$. Then we can pass to the de Donder gauge condition for the field $h_{\mu\nu}$ *on brane 1* with the help of the gauge functions of the form

$$\epsilon_\mu = \frac{1}{2}\left(\frac{R}{2k} + \frac{1}{4k^2}\right)\partial_\mu\phi + \frac{kR}{e^{2kR}-1}\frac{\partial_\mu}{\Box}\phi. \tag{42}$$

Having been made, these transformations result in

$$h_{\mu\nu}|_{y=0} = v_{\mu\nu}|_{y=0} + \eta_{\mu\nu}kR\phi + 2kR\frac{\partial_\mu\partial_\nu}{\Box}\phi. \tag{43}$$

This formula gives the four-dimensional gravitational field on brane 1, and it takes into account the contributions of the massless and of the massive gravitons and the contribution of the radion.

Now let us calculate the Newtonian limit in this case. Let us consider a static point mass with energy-momentum tensor $t_{00} = M\delta(\vec{x}), t_{0k} = t_{ij} = 0$ (and $\tilde{t}_{00} = 2\pi M\delta(p_0)$), M denoting the inertial mass. We need to calculate only h_{00}-component of the metric fluctuations (see (37))

$$\tilde{v}_{00}(p,0) = -\frac{\hat{\kappa}}{2\sqrt{p^2}}\frac{K_2\left(\frac{\sqrt{p^2}}{k}\right)I_1\left(\frac{\sqrt{p^2}}{k}e^{kR}\right) + I_2\left(\frac{\sqrt{p^2}}{k}\right)K_1\left(\frac{\sqrt{p^2}}{k}e^{kR}\right)}{I_1\left(\frac{\sqrt{p^2}}{k}\right)K_1\left(\frac{\sqrt{p^2}}{k}e^{kR}\right) - I_1\left(\frac{\sqrt{p^2}}{k}e^{kR}\right)K_1\left(\frac{\sqrt{p^2}}{k}\right)} \times \\ \times \frac{4\pi}{3}M\delta(p_0) \tag{44}$$

and

$$h_{00}(x,0) = v_{00}(x,0) - kR\frac{1}{4\pi}\frac{\hat{\kappa}}{6R}\frac{M}{r}, \tag{45}$$

(see (43)) where $r = \sqrt{\vec{x}^2}$ and

$$v_{00}(x,0) = \frac{1}{(2\pi)^4}\int e^{-i\eta_{\mu\nu}p^\mu x^\nu}\tilde{v}_{00}(p,0)d^4p. \tag{46}$$

Using the last relation and equation (44) one can easy find that

$$\begin{aligned} v_{00}(x,0) &= \frac{1}{(2\pi)^4}\frac{4\pi}{r}\frac{4\pi}{3}M\int_0^\infty g(p)sin(pr)pdp, \\ g(p) &= \frac{\hat{\kappa}}{2p}\frac{K_2\left(\frac{p}{k}\right)I_1\left(\frac{p}{k}e^{kR}\right) + I_2\left(\frac{p}{k}\right)K_1\left(\frac{p}{k}e^{kR}\right)}{I_1\left(\frac{p}{k}\right)K_1\left(\frac{p}{k}e^{kR}\right) - I_1\left(\frac{p}{k}e^{kR}\right)K_1\left(\frac{p}{k}\right)}, \end{aligned} \tag{47}$$

where $p = \sqrt{\vec{p}^2}$. It is impossible to evaluate the integral in (47) analytically. But we can estimate the integrand in the following cases: 1) $p << e^{-kR}k$, 2) $e^{-kR}k << p << k$ and 3) $k << p$. Using these estimates in the intervals $0 \leq p \leq e^{-kR}k$, $e^{-kR}k \leq p \leq k$ and $k < p < \infty$ respectively, we can estimate the integral in equation (47).

First, utilizing the recurrent relations for the McDonald functions, we represent the factor $g(p)$ in the integrand in the following form

$$g(p) = \frac{\hat{\kappa}k}{p^2} - \frac{\hat{\kappa}}{2p} \frac{K_0\left(\frac{p}{k}\right) I_1\left(\frac{p}{k}e^{kR}\right) + I_0\left(\frac{p}{k}\right) K_1\left(\frac{p}{k}e^{kR}\right)}{I_1\left(\frac{p}{k}\right) K_1\left(\frac{p}{k}e^{kR}\right) - I_1\left(\frac{p}{k}e^{kR}\right) K_1\left(\frac{p}{k}\right)}. \tag{48}$$

With the help of this transformation we have picked out the contribution of the zero mode to v_{00}. Substituting asymptotic formulas for McDonald functions into (48), we get

$$g(p) \approx \frac{\hat{\kappa}k}{p^2} + \left[\begin{array}{lll} \frac{\hat{\kappa}k}{e^{2kR}-1}\frac{1}{p^2} - \frac{\hat{\kappa}}{2k\left(1-e^{-2kR}\right)} ln\left(\frac{p}{k}\right) & , & p << e^{-kR}k \\ -\frac{\hat{\kappa}}{2k} ln\left(\frac{p}{k}\right) & , & e^{-kR}k << p << k \\ \frac{\hat{\kappa}}{2p} & , & k << p. \end{array} \right. \tag{49}$$

All the integrals arising after substituting (49) into (47) can be calculated analytically. The integrals of the form $\int_a^\infty sin(pr)dp$ are calculated with the use of the regularization

$$\int_a^\infty sin(pr)dp = \lim_{\epsilon \to 0} \left(\int_a^\infty e^{-\epsilon p} \sin(pr)dp \right) = \frac{\cos(ar)}{r}. \tag{50}$$

It is well known that the gravitational potential is expressed through the g_{00}-component of the metric as $g_{00} = -1 - 2V$ [17]. Since $g_{00} = -1 + \hat{\kappa}h_{00}$, we get

$$V = -\frac{\hat{\kappa}}{2}h_{00}. \tag{51}$$

Thus, using equations (43), (47) and (49), we get

$$V \approx -\frac{M}{r}G_1 \left(1 + \frac{4\cos(kr)}{3\pi kr} - \frac{4}{3\pi k^2r^2}\left[sin(kr) - si(kr)\right]\right), \tag{52}$$

where $G_1 = \frac{\hat{G}k}{1-e^{-2kR}}$ and

$$si(b) = \int_0^b \frac{\sin t}{t} dt.$$

Thus, we have examined the case of the mass and the observer being located on brane 1. But there are three more possible cases to be examined. It is the case of "shadow" matter, when the mass is located on brane 2 and the observer is located on brane 1, the case of the mass and the observer being located on brane 2, and the case of the mass being located on brane 1, whereas the observer being located on brane 2 ("shadow" matter effect as well). All the calculations in these cases are the same, as described above. But if the observer is located on brane 2, it is necessary to pass to Galilean coordinates on brane 2 to get a correct result. This problem was discussed in detail in paper [6]. The energy-momentum tensor $t_{\mu\nu}$ takes the form $t_{00} = M\delta(\vec{x}), t_{0k} = t_{ij} = 0$ in the Galilean coordinates on the brane the mass is located on. In the formulas presented below (in the Galilean coordinates on the observer's brane) the energy-momentum tensor always has this canonical form, and passing to the Galilean coordinates is taken into account by a factor in front of the brackets.

Now let us discuss, how to pass to Galilean coordinates on brane 2 by the example of the case when the mass is located on brane 1 and the observer is located on brane 2 ("shadow" matter effect). We consider the following form of the Fourier transform with the integrand appropriate for this case

$$A_{\mu\nu}(x_0, \vec{x}) = \frac{1}{(2\pi)^4} B_{\mu\nu} \int e^{i\eta_{\mu\nu}p^\mu x^\nu} g(p)\delta(p_0) d^4p, \tag{53}$$

with $B_{\mu\nu}$ being a constant. One can easily find, that in Galilean coordinates on brane 2 this expression looks like

$$A'_{\mu\nu}(z_0, \vec{z}) = \frac{1}{(2\pi)^4} B_{\mu\nu}\, e^{-kR} \int e^{i\eta_{\mu\nu}q^\mu z^\nu} g(e^{-kR}q)\delta(q_0) d^4q, \tag{54}$$

where $\{z^\mu\}$ are Galilean coordinates on brane 2. In particular, this means, that the intervals, in which we have to estimate the integrand, change. Now we have to choose the following domains of estimate: $q << k$, $k << q << e^{kR}k$ and $e^{kR}k << q$.

Another difficulty, which arises in this case, is that one of the integrals cannot be evaluated analytically. It has the following form

$$\int_k^\infty e^{-\frac{q}{k}} \sqrt{q} \sin(qr)\, dq = k^{3/2} \int_1^\infty \sqrt{t} \sin(tkr) e^{-t}\, dt, \tag{55}$$

where $r = \sqrt{\vec{z}^2}$. But we can estimate it in the limiting cases $1 << kr$ and $kr << 1$.

1) $1 << kr$. In this case (55) can be estimated by integration by parts

$$\int_1^\infty \sqrt{t} \sin(tkr) e^{-t}\, dt \approx e^{-1} \frac{\cos(kr)}{kr}. \tag{56}$$

2) $kr << 1$. In this case we can make the expansion of $sin(tkr)$ for the small argument

$$\begin{aligned} \int_1^\infty \sqrt{t} \sin(tkr) e^{-t}\, dt &\approx \int_0^\infty \sqrt{t} \sin(tkr) e^{-t}\, dt - \\ &- \int_0^1 \sqrt{t} \left(krt - \frac{(kr)^3 t^3}{6} + \ldots \right) e^{-t}\, dt. \end{aligned} \tag{57}$$

The first integral was taken from the table of integrals [18] and the second one was calculated numerically. The result reads as follows

$$\int_1^\infty \sqrt{t} \sin(tkr) e^{-t}\, dt \approx \left(\frac{3}{4}\sqrt{\pi} - 0.2 \right) kr + 0.099 \frac{1}{6} (kr)^3. \tag{58}$$

And the last very interesting point of "shadow" matter cases is passing to the de Donder gauge. Let us show it in the case of the field $h_{\mu\nu}|_{y=R}$. It does not satisfy the de Donder gauge condition on brane 2 (see (38)), analogously to the case of matter located on brane 1. With the help of the gauge functions of the following form

$$\epsilon_\mu = \frac{e^{2kR}}{8k^2}\partial_\mu\phi + \frac{kRe^{2kR}}{e^{2kR}-1}\frac{\partial_\mu}{\Box}\phi \tag{59}$$

we can pass to the de Donder gauge. But after this transformation $h_{\mu\nu}|_{y=R}$ appears to have the following form

$$h_{\mu\nu}|_{y=R} = v_{\mu\nu}|_{y=R}\,. \tag{60}$$

This means, that $h_{\mu\nu}|_{y=R}$ satisfies transverse-traceless gauge conditon

$$\begin{aligned} \partial^\nu h_{\mu\nu}|_{y=R} &= 0, \\ h^\nu_\nu|_{y=R} &= 0, \end{aligned} \tag{61}$$

which is quite natural, because there is no matter on brane 2.

Thus, the Newtonian limit in this case has the following form

1) $1 << kr$

$$V \approx -\frac{4}{3}G_1 e^{-kR}\frac{M}{r}\left(1+\sqrt{\frac{2}{\pi}}e^{-1}\frac{\cos(kr)}{kr}\right), \tag{62}$$

2) $kr << 1$

$$\begin{aligned} V \approx & -G_1 e^{-kR}Mk\frac{8}{3\pi}\left(1+\frac{3\pi}{4\sqrt{2}} - 0.2\sqrt{\frac{\pi}{2}} + 0.099\sqrt{\frac{\pi}{2}}\frac{1}{6}(kr)^2\right) \approx \\ & \approx -G_1 e^{-kR}Mk\left(2.05 + 0.02(kr)^2\right). \end{aligned} \tag{63}$$

Below the results for the last two cases are presented. All calculations were made similarly to the presented above.

I) The mass and the observer are located on brane 2:

$$V \approx -G_2\frac{M}{r}\left(1 - e^{2kR} + \frac{8e^{2kR}}{3\pi}\,si(kr) + \frac{2e^{2kR}}{3\pi}\left[\frac{\cos(kr)}{kr} + \frac{\sin(kr)}{(kr)^2}\right]\right), \tag{64}$$

where $G_2 = \frac{\hat{G}k}{e^{2kR}-1}$ is the effective gravitational constant on brane 2.

II) The mass is located on brane 2 and the observer is located on brane 1 ("shadow" matter also):

1) $e^{kR} << kr$

$$V \approx -\frac{4}{3}e^{kR}G_2\frac{M}{r}\left(1+\sqrt{\frac{2}{\pi}}e^{kR}e^{-1}\frac{\cos(kre^{-kR})}{kr}\right), \tag{65}$$

2) $kr << e^{kR}$

$$\begin{aligned} V &\approx -G_2 Mk\frac{8}{3\pi}\left(1+\frac{3\pi}{4\sqrt{2}} - 0.2\sqrt{\frac{\pi}{2}} + 0.099\,e^{-2kR}\sqrt{\frac{\pi}{2}}\frac{1}{6}(kr)^2\right) \approx \\ &\approx -G_2 Mk\left(2.05 + 0.02\,e^{-2kR}(kr)^2\right). \end{aligned} \tag{66}$$

We would like to note that in all formulas for the Newtonian limit, presented above, parameter M denotes the physical mass in Galilean coordinates on the brane the mass is located on.

In paper [6] possible values of parameter k were discussed. It was shown that the value $k \sim 1Tev$ is the only one admissible for a correct physical interpretation of the theory on brane 2. This means, that for $r \sim 1cm$ the value $kr \sim 10^{17}$ and the terms proportional to $\sin(kr)$ and $\cos(kr)$ can be dropped as negligible. For example, since $si(t \to \infty) \to \frac{\pi}{2}$, equation (64) takes the form

$$V \approx -G_2 \frac{M}{r} \left(1 + \frac{1}{3} e^{2kR}\right). \tag{67}$$

The same arguments can be applied to other cases of matter and observer disposition. Since the contribution of massive modes are negligible for $r >> 10^{-17}cm$, only the zero modes constitute the Newtonian limit. Unfortunately, in formulas (52), (62), (64) and (65) the contributions of the massless graviton and of the radion are "mixed". This problem can be solved with the help of the zero mode approximation.

3 Zero mode approximation

Let us consider equations (12), (13) and (13) with the energy-momentum tensor of the form $T_{\mu\nu} = t_{\mu\nu}(x)\delta(y)$ (in the unitary gauge). As described above, with the help of (17) these equation (except for the $\mu\nu$-equation) take the form (18), (19) and (20). Then we can get equations (21) and (22) as it was made above. It is well known that the field $u_{\mu\nu}$ in the presence of matter is a combination of zero and massive modes [7], whose eigenfunctions are orthogonal [6]. In particular, the zero mode can be represented as $u^0_{\mu\nu} = e^{2\sigma}\alpha_{\mu\nu}$, where $\alpha_{\mu\nu}$ depends on x only. It also means that with the help of the residual gauge transformations $\xi_\mu = e^{2\sigma}\epsilon_\mu(x)$ it is possible to impose the gauge condition

$$\begin{aligned} \partial^\nu u^m_{\mu\nu} &= 0, \\ u^{m\mu}_\mu &= 0, \end{aligned} \tag{68}$$

on the massive modes $u^m_{\mu\nu}$ and the de Donder gauge condition on the zero mode

$$\partial^\nu \left(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha\right) = 0. \tag{69}$$

After imposing this gauge, we are still left with residual gauge transformation (26). It follows from equations (19), (21),(68), (25) that

$$\Box\alpha = \frac{\hat{\kappa} c e^{2kR}}{R} t. \tag{70}$$

Now let us consider equation (12). Making substitution (17) with condition (21) and passing to the gauge (68), (69) we get

$$\begin{aligned} &\frac{1}{2}\Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) + \frac{1}{2}e^{-2\sigma}\Box u^m_{\mu\nu} + \frac{1}{2}\partial_4\partial_4 u^m_{\mu\nu} - 2k^2 u^m_{\mu\nu} - \partial_4\partial_4\sigma u^m_{\mu\nu} = \\ &= -\frac{\hat{\kappa}}{2} t_{\mu\nu}\delta(y) + \frac{\hat{\kappa} k e^{2kR}}{6(e^{2kR}-1)} \left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right) t - \frac{\hat{\kappa}}{6}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right) t\,\delta(y). \end{aligned} \tag{71}$$

Since we are going to calculate the Newtonian limit and light deflection in the zero mode approximation, we have to find an equation for the field $\alpha_{\mu\nu}$. If we multiply equation (71) by $e^{2\sigma}$, integrate it over y and take into account the orthonormality condition for the wave functions of the modes, we get

$$\Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) = -\frac{\hat{\kappa}k}{(1 - e^{-2kR})}t_{\mu\nu}. \tag{72}$$

The equation for massive modes takes the form

$$\begin{aligned} &\frac{1}{2}e^{-2\sigma}\Box u^m_{\mu\nu} + \frac{1}{2}\partial_4\partial_4 u^m_{\mu\nu} - 2k^2 u^m_{\mu\nu} - \partial_4\partial_4\sigma u^m_{\mu\nu} = -\frac{\hat{\kappa}}{2}t_{\mu\nu}\delta(y) + \\ &+ \frac{\hat{\kappa}ke^{2kR}}{2(e^{2kR}-1)}t_{\mu\nu} + \frac{\hat{\kappa}ke^{2kR}}{6(e^{2kR}-1)}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right)t - \frac{\hat{\kappa}}{6}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right)t\,\delta(y). \end{aligned} \tag{73}$$

We note that equation (71) has been solved exactly in Section 2. But it turned out to be impossible to evaluate analytically all the arising integrals even for a simple form of $t_{\mu\nu}$ (for example, for a static point mass).

When matter is on brane 2 (at $y = R$), all the reasonings are the same, as presented above. The substitution has the form (38). The gauge conditions are the same as in case of matter on brane 1 too. The equations of motion for the fields $\alpha_{\mu\nu}$, ϕ in the presence of matter on brane 2 can be derived analogously and read

$$\begin{aligned} \frac{1}{2}\Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) \quad &+ \quad \frac{1}{2}e^{-2\sigma}\Box u^m_{\mu\nu} + \frac{1}{2}\partial_4\partial_4 u^m_{\mu\nu} - 2k^2 u^m_{\mu\nu} - \partial_4\partial_4\sigma u^m_{\mu\nu} = \\ = -\frac{\hat{\kappa}}{2}t_{\mu\nu}\delta(y-R) \quad &+ \quad \frac{\hat{\kappa}k}{6(e^{2kR}-1)}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right)t^\rho_\rho - \\ &- \quad \frac{\hat{\kappa}}{6}\left(\frac{\partial_\mu\partial_\nu}{\Box} - \eta_{\mu\nu}\right)t^\rho_\rho\,\delta(y-R), \end{aligned} \tag{74}$$

$$\Box(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha) = -\frac{\hat{\kappa}k}{(e^{2kR}-1)}t_{\mu\nu}, \tag{75}$$

$$\Box\phi = \frac{\hat{\kappa}}{6R}t^\mu_\mu. \tag{76}$$

We would like to note once again that these equations are written in the coordinates $\{x^\mu\}$, which are *Galilean on brane 1* and are inappropriate for studying physical effects on brane 2.

Now we are ready to find the Newtonian limit and light deflection by a point-like static mass. Let us first make it for brane 1. The substitution (17) in the zero mode approximation is

$$h^0_{\mu\nu} = e^{2\sigma}\alpha_{\mu\nu} - \frac{kR}{e^{2kR}-1}\eta_{\mu\nu}\phi + \frac{R}{2k}\partial_\mu\partial_\nu\phi + \frac{1}{4k^2}\partial_\mu\partial_\nu\phi. \tag{77}$$

The equation for $h_{\mu\nu}$ on brane 1 in the zero mode approximation looks like

$$\begin{aligned} \Box h^0_{\mu\nu} \quad &= \quad -\frac{\hat{\kappa}ke^{2kR}}{e^{2kR}-1}\left(t_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}t^\rho_\rho\right) - \\ - \frac{\hat{\kappa}k}{6(e^{2kR}-1)}\eta_{\mu\nu}t \quad &+ \quad \frac{\hat{\kappa}}{12k^2R}\left(kR + \frac{1}{2}\right)\partial_\mu\partial_\nu t. \end{aligned} \tag{78}$$

Let us consider a static point mass with energy-momentum tensor $t_{00} = M\delta(\vec{x}), t_{0k} = t_{ij} = 0$, M denoting the inertial mass again. Then we can find a solution of equation (78) for h_{00}-component

$$h_{00} = \frac{\hat{\kappa}k(e^{2kR} + \frac{1}{3})}{8\pi(e^{2kR} - 1)} \frac{M}{r}, \tag{79}$$

where $r = \sqrt{\vec{x}^2}$. Thus, we get

$$V = -\hat{G}k\frac{(e^{2kR} + \frac{1}{3})}{(e^{2kR} - 1)} \frac{M}{r} = -G_1 \left(1 + \frac{1}{3}e^{-2kR}\right) \frac{M}{r}. \tag{80}$$

Now let us show that the Randall-Sundrum model in the zero mode approximation is equivalent to the linearized Brans-Dicke theory (apparently, it was first noted in [7]). Appropriate formulas for the light deflection in the Brans-Dicke theory are well known and can be found, for example, in [17].

The equation for the fluctuations of metric in the linearized Brans-Dicke theory looks like

$$\Box \left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) = -16\pi G \left(t_{\mu\nu} - \frac{1}{2\omega + 3} \left(\eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box}\right) t\right), \tag{81}$$

where ω is the BD-parameter. It is easy to see that $h_{\mu\nu}$ satisfies the de Donder gauge condition $\partial^\nu \left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) = 0$. The light deflection angle is given by [17]

$$\Delta\varphi \approx \frac{4MG}{r_0} \left(\frac{2\omega + 3}{2\omega + 4}\right), \tag{82}$$

with r_0 being the impact parameter and M being the mass of the static point-like source.

Now let us examine the four-dimensional effective theory on brane 1. It is easy to see that $h^0_{\mu\nu}$ (77) does not satisfy the de Donder gauge condition. We can impose it with the help of the gauge functions (42).

In this gauge equation (78) takes the form

$$\Box \left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) = -16\pi G_1 \left(t_{\mu\nu} - \frac{e^{-2kR}}{3} \left(\eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box}\right) t\right). \tag{83}$$

Here and in what follows we drop the superscript 0 of the zero mode approximation. By comparing (83) with (81), we get $\omega = \frac{3}{2}\left(e^{2kR} - 1\right)$. Substituting this into (82), we find the angle of light deflection by a static point-like source on brane 1

$$\Delta\varphi \approx \frac{4MG_1}{r_0} \left(\frac{1}{1 + \frac{1}{3}e^{-2kR}}\right). \tag{84}$$

The second term in the denominator of formula (84) and the second term in formula (80) correspond to the contribution of the radion. One can see that the contribution of the radion is e^{2kR} times smaller than the contribution of the massless graviton.

Direct calculations lead us to the following results for the remaining cases. All the reasonings concerning Galilean coordinates are the same as in section 2.

1) We live on brane 1, the mass is located on brane 2 ("shadow" matter)

$$\Box\left(h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h\right)=-16\pi G_1 e^{-kR}\left(t_{\mu\nu}-\frac{1}{3}\left(\eta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box}\right)t\right). \tag{85}$$

The light deflection angle is

$$\Delta\varphi\approx\frac{3MG_1e^{-kR}}{r_0}=4MG_1e^{-kR}\left(\frac{1}{1+\frac{1}{3}}\right)\frac{1}{r_0}. \tag{86}$$

Newton's Law looks like

$$V=-\frac{4}{3}G_1e^{-kR}\frac{M}{r}=-\left(1+\frac{1}{3}\right)G_1e^{-kR}\frac{M}{r}. \tag{87}$$

The second term in the denominator of formula (86) and the second term in the brackets in formula (87) correspond to the contribution of the radion. One can see that the contribution of the radion is of the same order, as the contribution of the massless graviton, but the effect of the "shadow" matter is e^{kR} times smaller, than the effect of the ordinary matter (compare with (84) and (80)). It means that the effective theory on brane 1 is phenomenologically acceptable, but not interesting (see [6]).

2) We live on brane 2, and the mass is located on the same brane

$$\Box\left(h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h\right)=-16\pi G_2\left(t_{\mu\nu}-\frac{e^{2kR}}{3}\left(\eta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box}\right)t\right) \tag{88}$$

with $G_2=\hat{G}k\frac{1}{e^{2kR}-1}$.

The light deflection angle is

$$\Delta\varphi\approx\frac{4MG_2}{r_0}\left(\frac{1}{1+\frac{e^{2kR}}{3}}\right). \tag{89}$$

Newton's Law looks like

$$V=-G_2\left(1+\frac{e^{2kR}}{3}\right)\frac{M}{r}. \tag{90}$$

The second term in the denominator of formula (89) and the second term in the brackets in formula (90) correspond to the contribution of the radion. Now the contribution of the radion is e^{2kR} times stronger than the contribution of the massless graviton. It means that, in the case of the massless radion, scalar gravity is realized on brane 2. However, if one assumes some mechanism for generating the radion mass, for example, the Goldberger-Wise mechanism [19], gravity in the zero mode approximation becomes tensor and the hierarchy problem solves.

3) We live on brane 2, the mass is located on brane 1 ("shadow" matter also)

$$\Box\left(h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h\right)=-16\pi G_2 e^{kR}\left(t_{\mu\nu}-\frac{1}{3}\left(\eta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box}\right)t\right). \tag{91}$$

The light deflection angle is

$$\Delta\varphi \approx \frac{4MG_2 e^{kR}}{r_0}\left(\frac{1}{1+\frac{1}{3}}\right) = \frac{3MG_1 e^{-kR}}{r_0}. \tag{92}$$

Newton's Law looks like

$$V = -\left(1+\frac{1}{3}\right) G_2 e^{kR}\frac{M}{r} = -\frac{4}{3}G_1 e^{-kR}\frac{M}{r}. \tag{93}$$

As before, the second term in the denominator of formula (92) and the second term in the brackets in formula (93) correspond to the contribution of the radion. We can see that the contribution of the radion is of the same order, as contribution of the massless graviton, but the contribution of the "shadow" matter to the Newtonian limit is e^{kR} times smaller, than the contribution of the ordinary matter, because of the interaction with the massless radion (compare with (90)). But if the radion is massive (otherwise we have scalar gravity in "our" world on brane 2), the contribution of the "shadow" matter to the Newtonian limit is e^{kR} times stronger, than the contribution of the ordinary matter. It could lead to some observable effects, resulting from the distribution of matter in the "mirror" world (brane 1).

One can shaw that equations (80), (87), (90) and (93) coincide with the zero-mode parts of equations (52), (65), (64) and (62) respectively.

We would like to note that in paper [7] the effects of the "shadow" matter were also considered. In this paper the deflection angles with the same impact parameters and by the same Newtonian masses were compared (Newtonian mass is the coefficient in front of the $\frac{1}{r}$ term in Newton's Law). We have compared the effects of the "shadow" and the ordinary matter, as it was made in [7], and we have got the following results: 44% instead of 25% in [7] for brane 1 and the difference of the order of e^{4kR} instead of 25% in [7] for brane 2. This discrepancy appears because we use the Galilean coordinates on brane 2. In the case of the massive radion there is no difference in the zero mode approximation at all for both branes.

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