On dynamical supergravity interacting with super-p-brane sources[1]

Igor A. Bandos[1,2], José A. de Azcárraga[1], José M. Izquierdo[3] and Jerzy Lukierski[4]

[1] *Departamento de Física Teórica and IFIC, 46100-Burjassot (Valencia), Spain,*
[2] *Institute for Theoretical Physics, NSC KIPT, UA61108, Kharkov, Ukraine,*
[3] *Departamento de Física Teórica, Facultad de Ciencias, 47011-Valladolid, Spain,*
[4] *Institute for Theoretical Physics, pl. Maxa Borna 9, 50-204 Wroclaw, Poland*

Abstract

We review recent progress in a fully dynamical Lagrangian description of the supergravity–superbrane interaction. It suggests that the interacting superfield action, when it exists, is gauge equivalent to the component action of dynamical supergravity interacting with the bosonic limit of the superbrane.

1   Introduction

Superbranes [1, 2] play an important rôle in recent developments of String/M-theory [3] as well as in their applications to the study of quantum Yang–Mills theories [4] and the structure of the Universe.

A popular description of superbranes was proposed in 1989 [5, 6]. It identified them with solitonic solutions of the pure bosonic 'limit' of the supergravity equations. Although all the fermions are set equal to zero, the superbrane solutions are supersymmetric and, hence, stable. Already in [5], the superstring description as a solitonic solution of $D = 10$ dimensional supergravity was found. On the other hand, linearized supergravity was derived from the quantization of the superstring in flat superspace [7]. This allows one to identify supergravity with the low energy limit of superstring theory. As the complete supergravity action can be uniquely restored from the linearized one by requiring $D = 10$, type IIB local supersymmetry, the identification of supergravity as a low energy limit of superstring theory is quite convincing although the mechanism for the appearance of curved spacetime from the superstring is still obscure and requires further progress in constructing string field theory [8].

One may, however, consider the interaction of a superstring with a supergravity *background* *i.e.*, consider a superstring moving in a given curved superspace [9]. Then, selfconsistency requires having a smooth flat superspace limit for such a system. This implies, in particular, that the Green–Schwarz superstring has to possess local fermionic κ–symmetry [10, 7] in curved superspace, as it does in the flat one. Such a requirement immediately results in $D = 10$ superfield supergravity constraints being imposed on the background superfields [9]. The point is that these are *on-shell* constraints, *i.e.* that their selfconsistency implies equations of motion for $D = 10$ supergravity (actually, sourceless or 'free').

This could be regarded as an advantage of superstring theory, as in the bosonic string model the equations of motion for the background can be obtained only by one–loop quantum calculations (namely, from the requirement that the conformal (Weyl) symmetry is preserved in the

---

[1]Invited talk delivered at the *3rd International Sakharov Conference on Physics*, Moscow, June 24-29, 2002. To appear in the Proceedings.

quantum theory). However, on the other hand, these equations of motion are *sourceless* supergravity equations. Then their appearance from the selfconsistency conditions for the superstring might produce the impression that supersymmetry forbids the interaction of supergravity with a source.

The same situation occurs with other superbranes [1, 11]: as their worldvolume actions possess κ–symmetry when considered in the flat superspace, the selfconsistency of the model in curved superspace requires a smooth flat superspace limit and, hence, the preservation of the κ–symmetry in a curved background. Such a requirement results in the supergravity constraints, which again imply 'free' supergravity equations of motion for the physically most interesting $D = 10$ and 11 cases.

So a challenge is to see whether one can provide a full (quasi)classical Lagrangian description of the supergravity–superbrane interacting system so that supergravity is treated dynamically and not just given by a background. Such a Lagrangian should produce a complete set of supergravity equations with singular sources having support on surfaces (worldvolumes) in spacetime (superspace). In this contribution we review briefly our recent results [12, 13, 14] in this direction, *i.e.* towards a selfconsistent Lagrangian description of supergravity–superbrane coupled system.

2   Statement of the problem

It is natural to assume that such a Lagrangian description is based on the sum of *some* supergravity action S_{SG} and the super–p–brane action S_p,

$$S = S_{SG} + S_p \ . \tag{1}$$

A superbrane is a brane moving in superspace. This means that the super–p–brane worldvolume $\mathcal{W}^{p+1}$ is a hypersurface $Z^M = \hat{Z}^M(\xi)$ in superspace $\Sigma^{(D|n)}$ with D bosonic coordinates x^μ ($\mu = 0, 1, \ldots, D-1$) and $n = N\dim(Spin(1, D-1))$ fermionic coordinates $\theta^{\check{\alpha}}$ ($\check{\alpha} = 1, \ldots, n$; below we assume N=1 for simplicity),

$$\mathcal{W}^{p+1} \ \subset \ \Sigma^{(D|n)} \ : \ Z^M = \hat{Z}^M(\xi) \ \leftrightarrow \ \begin{cases} x^\mu = \hat{x}^\mu(\xi) \\ \theta^{\check{\alpha}} = \hat{\theta}^{\check{\alpha}}(\xi) \end{cases} ; \tag{2}$$

the $\xi^m = (\tau, \vec{\sigma})$ are $\mathcal{W}^{p+1}$ local coordinates, $m = 0, \ldots, p$, and the hat indicates dependence on ξ. The action S_p is formulated in terms of supervielbein and gauge field superforms on $\Sigma^{(D|n)}$, characteristic of the superfield description of supergravity,

$$E^A(Z) = dZ^M E_M^A(Z) = (E^a, E^\alpha) \ , \tag{3}$$
$$C_q(Z) = \tfrac{1}{q!} dZ^{M_q} \wedge \ldots \wedge dZ^{M_1} C_{M_1 \ldots M_q}(Z) \ ,$$

pulled back to $\mathcal{W}^{p+1}$

$$S_p = \int_{\mathcal{W}^{p+1}} \mathcal{L}_{p+1}(\hat{E}^a, \hat{C}_q) \equiv S_p[\hat{E}^a, \hat{C}_q], \tag{4}$$

$$\hat{E}^A = E^A(\hat{Z}(\xi)) := d\hat{Z}^M(\xi) E_M^A(\hat{Z}(\xi)) \ , \quad \hat{C}_q = C_q(\hat{Z}(\xi)) \tag{5}$$

and, perhaps, a set of worldvolume gauge fields (one–form $A(\xi) = d\xi^m A_m(\xi)$ for Dirichlet superbranes). For instance, for the 'standard' branes which do not carry additional worldvolume gauge fields, the action S_p reads [2]

$$S_p = S_p[\hat{E}^a, \hat{C}_{p+1}] = \int_{\mathcal{W}^{p+1}} \left(\tfrac{1}{2(p+1)} \ * \hat{E}_a \wedge \hat{E}^a - \hat{C}_{p+1} \right) , \tag{6}$$

2

where $*$ is the Hodge star operator for the induced metric, $* \frac{1}{(p+1)} \hat{E}_a \wedge \hat{E}^a = d^{p+1}\xi \sqrt{|det(\hat{E}_{ma}\hat{E}_n^a)|}$.

Hence, to have a well posed variational problem for the interacting system action (1), one has to assume that the supergravity action is also formulated in terms of superfields $E_M^A(Z)$, $C_{M_1...M_q}(Z)$,

$$S = S_{SG}[E^A, C_q] + S_p[\hat{E}^a, \hat{C}_q] . \tag{7}$$

The problem, however, is that for the most interesting cases ($D = 10$ and $D = 11$) no *superfield* action for supergravity is known.

As, in contrast, the *component* action $S_{SG}[e^a, e^\alpha = dx^\mu \psi_\mu^\alpha, \ldots]$ for all $D \leq 11$ supergravity theories is now known, one might think of using it in (7) instead of the superfield supergravity action, and decompose the superfields in $S_p[\hat{E}^a, \hat{C}_q]$ in terms of component fields, *e.g.* $E_\mu^a = e_\mu^a(x) + \mathcal{O}(\theta)$, $E_\mu^\alpha = \psi_\mu^\alpha(x) + \mathcal{O}(\theta)$. However, the problem is that to find the decompositions of these superfields one has to use the superspace constraints. For $D = 10, 11$ these are the *on–shell* supergravity constraints, which imply that the fields $e_\mu^a(x)$, $\psi_\mu^\alpha(x)$ obey the 'free' supergravity equations (without any source term). Moreover, as these fields are solutions of equations of motion, one is not free to take their arbitrary variations. Thus, despite the first impression, the variational problem for the action $S_{SG}[e^a, dx^\mu \psi_\mu^\alpha, \ldots] + S_p[\hat{E}^a, \hat{C}_q]$ is not well posed.

3   Group manifold approach to supergravity–superbrane interaction

A solution to the above problem (for any D) was proposed in [12] using in (7) the *group manifold* or rheonomic action for supergravity [15],

$$S_{gmSG} = S_{gmSG}[\tilde{E}^A, \tilde{w}^{ab}, \tilde{C}_q] , \tag{8}$$

which is written in terms of superfields, but pulled back to a surface $\mathcal{M}^D$ of maximal bosonic dimension in superspace,

$$\mathcal{M}^D \subset \Sigma^{(D|n)} : Z^M = \tilde{Z}^M(x) \Leftrightarrow \begin{cases} x^\mu - arbitrary \\ \theta^{\check{\alpha}} = \tilde{\theta}^{\check{\alpha}}(x) \end{cases} , \tag{9}$$

$$\tilde{E}^A := E^A(\tilde{Z}(x)) = d\tilde{Z}^M(x)E_M^A(\tilde{Z}(x)) ,$$

$$\tilde{w}^{ab} := w^{ab}(\tilde{Z}(x)) , \qquad \tilde{C}_q := C_q(\tilde{Z}(x)) , \tag{10}$$

where the tilde indicates x–dependence.

As both S_{gmSG} and S_p are formulated in terms of pull–backs of the same superfields, the variational problem for the coupled action

$$S = S_{gmSG}[\tilde{E}^A, \tilde{w}^{ab}, \tilde{C}_q] + S_p[\hat{E}^a, \hat{C}_q] \tag{11}$$

is now well posed.

A salient feature of (11) is that the interacting dynamical system involves two types of fermionic coordinate functions, $\tilde{\theta}^{\check{\alpha}}(x)$ and $\hat{\theta}^{\check{\alpha}}(\xi)$, which are apparently independent. However, it may be seen [12] that the variation with respect to the bosonic supervielbein produces an equation the consequences of which include not only the superfield (superform) generalization of the Einstein equation (with a source) pulled back to the surface $\mathcal{M}^D$,

$$\tilde{M}_{(D-1)a} := \tilde{R}^{bc} \wedge \tilde{E}_{abc}^{\wedge(D-3)} + \ldots = J_{(D-1)a} , \tag{12}$$

but also the identification of the supergravity and superbrane fermionic coordinate functions on $\mathcal{W}^{p+1}$ [12],

$$\hat{\theta}^{\check{\alpha}}(\xi) = \tilde{\theta}^{\check{\alpha}}(\hat{x}(\xi)) . \tag{13}$$

This implies that, on the mass shell, the worldvolume $\mathcal{W}^{p+1}$ lays on the surface $\mathcal{M}^D$, $\mathcal{W}^{p+1} \subset \mathcal{M}^D$. The $(D-1)$ current density distribution J_{D-1} in Eq. (12) has the form

$$J^a_{(D-1)} = dx^{\wedge(D-1)}_\mu \int_{\mathcal{W}^{p+1}} *\hat{E}^a \wedge d\hat{x}^\mu(\tau)\delta^D(x - \hat{x}(\tau)) \, , \tag{14}$$

and the exterior power is defined by *e.g.*, $E^{\wedge(D-q)}_{a_1...a_q} = \frac{1}{q!}\epsilon_{a_1...a_q b_1...b_{D-q}} E^{b_1} \wedge ... \wedge E^{b_{D-q}}$ for the bosonic supervilebein.

More details on the group–manifold based approach to the supergravity–superbrane interacting system can be found in ref. [12]. Here we would only like to mention some features of this approach which, although reasonable (as seen below and in [13, 14]), were perhaps unexpected.
i) As the superbrane action (4) does not contain the pull–back $\hat{E}^\alpha$ of the fermionic supervielbein E^α, the variation of the action (11) with respect to E^α produces the pull–back of the superfield (superform) generalization of the Rarita–Schwinger equation *without* a source term,

$$\tilde{\Psi}_{(D-1)\alpha} := \frac{4i}{3}\mathcal{D}\tilde{E}^\beta \wedge \tilde{E}^{\wedge(D-3)}_{abc}\Gamma^{abc}_{\beta\alpha} + \ldots = 0 \, . \tag{15}$$

ii) The field equations (Eqs. (12), (15), *etc.*) are defined on an *arbitrary* surface $\mathcal{M}^D$ in superspace (the fermionic function is not restricted by any equation, except for (13) which could rather be treated as a relation for $\hat{\theta}^{\check{\alpha}}(\xi)$). Nevertheless, in contrast with the case of 'free' supergravity, these equations cannot be extended ('lifted') to the whole superspace.
iii) The local supersymmetry

$$\delta_{ls}\tilde{E}^a = -2i\tilde{E}^\alpha\Gamma^a_{\alpha\beta}\epsilon^\beta(x, \tilde{\theta}(x)) \, , \quad \delta_{ls}\tilde{E}^\alpha = \mathcal{D}\epsilon^\alpha(x, \tilde{\theta}(x)) + \ldots \quad \ldots \tag{16}$$

(pull–back of the variational version of the superspace general coordinate symmetry on $\mathcal{M}^D$; $\delta_{ls}x^\mu = 0 = \delta_{ls}\tilde{\theta}^{\check{\alpha}}(x)$, *cf.* $\tilde{\delta}_{gc}$ in [13]), which is a gauge symmetry of the group manifold action (8), is partially (1/2) broken on the worldvolume $\mathcal{W}^{p+1}$ for the coupled system (11). The 1/2 of the supersymmetry preserved on $\mathcal{W}^{p+1}$ has the form of a κ–symmetry transformation. To formulate the statement in a more precise manner, note that the superfield $\epsilon^\alpha(x, \theta)$ can be easily restored from its pull–back on $\mathcal{M}^D$, $\tilde{\epsilon}^\alpha := \epsilon^\alpha(x, \tilde{\theta}(x))$. Then this superfield parameter of local supersymmetry (16) is arbitrary in all 'points' of superspace, except for the worldvolume $\mathcal{W}^{p+1}$, where it is restricted by the condition

$$\hat{\epsilon}^\alpha := \epsilon^\alpha(\hat{x}(\xi), \hat{\theta}(\xi)) = (1 - \bar{\gamma})^\alpha{}_\beta\kappa^\beta(\xi) \, . \tag{17}$$

In (17) $\bar{\gamma}^\alpha{}_\beta$ is the kappa–symmetry projector (see [2]) which satisfies $(1 - \bar{\gamma})^2 = 2(1 - \bar{\gamma})$ (as a result, only 1/2 of the components of spinor $\kappa^\beta(\tau)$ enter effectively in (17) and, hence, in field transformations). On the other side, κ–symmetry does not exist as a separate symmetry of the action (11).
iv) There exists another fermionic gauge symmetry (the pull–back of superspace superdiffeomorphism symmetry [13]) which transforms the fermionic coordinate function by

$$\delta_{sdiff}\tilde{\theta}^{\check{\alpha}}(x) = \varepsilon^{\check{\alpha}}(x, \tilde{\theta}(x)) \, . \tag{18}$$

This clearly allows one to fix the gauge

$$\tilde{\theta}^{\check{\alpha}}(x) = 0 \, . \tag{19}$$

In the light of the identification (13), Eq. (19) implies also

$$\hat{\theta}^{\check{\alpha}}(\xi) = 0 \, . \tag{20}$$

4

As setting $\tilde{\theta}(x) = 0$ in the group manifold action (8) one arrives at a first order component action for supergravity (written in terms of spacetime fields),

$$S_{gmSG}[\tilde{E}^A, \tilde{w}^{ab}, \tilde{C}_q]|_{\tilde{\theta}(x)=0} \equiv S_{SG}[e^a(x), dx^\mu \psi_\mu^\alpha(x), \omega^{ab}(x)] , \qquad (21)$$

one concludes that in the gauge (19) the group–manifold based action for interacting supergravity–superbrane system, Eq. (11), becomes the action for (component) supergravity interacting with a purely *bosonic* brane.

4 Dynamical supergravity interacting with a bosonic brane

The supergravity interaction with a bosonic brane described by the sum of the component action for supergravity and the action for the bosonic brane,

$$S = S_{SG}[e^a(x), dx^\mu \psi_\mu^\alpha(x), \omega^{ab}(x)] + S_p[\hat{e}^a, \hat{C}_q] , \qquad (22)$$

could be expected to be not supersymmetric and even inconsistent. However, as shown in [13], *the dynamical system (22) is selfconsistent when the bosonic brane is the pure bosonic 'limit' of some superbrane* (its 'associated superbrane') due to the fact that in this case *a one–half of the local supersymmetry is preserved* on the worldvolume of the bosonic brane. To be more precise, when the bosonic brane is the limit of a superbrane, the dynamical system (22) possesses a local supersymmetry with parameter $\epsilon^\alpha(x)$ arbitrary out of the worldvolume but restricted on the worldvolume by the conditions

$$\hat{\epsilon}^\alpha := \epsilon^\alpha(\hat{x}(\xi)) = (1 - \bar{\gamma}_0)^\alpha{}_\beta \kappa^\beta(\xi) \qquad (23)$$

(*cf.* Eq. (17)), where $\bar{\gamma}_0{}^\alpha{}_\beta = \bar{\gamma}^\alpha{}_\beta|_{\hat{\theta}=0}$ is the pure bosonic 'limit' of the κ–symmetry projector of the associated superbrane.

An inconsistency could now be expected as a result of this loss of supersymmetry. Indeed, local supersymmetry is used to gauge away the spin $1/2$ irreducible representation included in the gravitino field $\psi_\mu^\alpha(x)$. But if local supersymmetry is broken, this spin $1/2$ part could remain 'dynamical' and produce a spin $1/2$ ghost. Thus, it would appear that there are still problems even if the bosonic brane is the bosonic limit of a superbrane, because in this case $1/2$ of the supersymmetry is broken on the worldline due to (23).

However, it was shown in [13] that the presence of a source in the Einstein equation (the $\tilde{\theta}(x) = \hat{\theta}(\xi) = 0$ 'limit' of Eq. (12)) and the absence of a source in the gravitino equation ($\tilde{\theta}(x) = \hat{\theta}(\xi) = 0$ 'limit' of (15)) results in a condition on the supercurrent which, in turn, is equivalent to a *fermionic equation for the bosonic brane* variables which include the pull–back $\hat{e}^\alpha$ of the fermionic form $e^\alpha = dx^\mu \psi_\mu^\alpha$ (specifically, $\hat{e}^\beta \Gamma_{\alpha\beta}^a \hat{e}_{\tau a} = 0$ for a bosonic particle; this is the bosonic 'limit' of the superparticle equation $\hat{E}^\beta \Gamma_{\alpha\beta}^a \hat{E}_{\tau a} = 0$). As shown in [13], precisely these *fermionic equations for the bosonic brane* take up the rôle of gauge fixing conditions for the part of local supersymmetry that is lost on the bosonic brane worldvolume due to (23). In other words, $\hat{e}^\beta \Gamma_{\alpha\beta}^a \hat{e}_{\tau a} = 0$ or its higher p counterpart, together with the remaining $1/2$ of the local supersymmetry, remove the spin $1/2$ ghost part of the gravitino field on the worldvolume of the bosonic brane.

To clarify further the reasons for such selfconsistency of the supergravity–bosonic brane system, we have investigated in [14] a full superfield description of the supergravity–superparticle interacting system in a simple case where the superfield supergravity action exists, namely in $D = 4$ $N = 1$ superspace, to which we now turn.

5 Superfield description of the $D = 4$, $N = 1$ supergravity–superparticle interacting system

The superfield action for supergravity–superparticle interacting system is

$$S = S_{SG\,4D}[E^A] + S_0[\hat{E}^a] , \qquad (24)$$

where

$$S_{SG\,4D}[E^A] = \int d^4x d^4\theta \, sdet(E_M^A) \equiv \int d^8Z \, E \qquad (25)$$

is the Wess–Zumino superfield action for $D = 4$, $N = 1$ supergravity [16], and

$$S_0 := S_0[\hat{E}^a] = \frac{1}{2} \int_{\mathcal{W}^1} l(\tau) \hat{E}_{\tau a} \hat{E}^a \qquad (26)$$

is the Brink–Schwarz superparticle action ($\hat{E}^a = d\hat{Z}^M(\tau) E_M^a(\hat{Z}) = d\tau \hat{E}_\tau^a$ and $l(\tau)$ is a Lagrange multiplier which can be treated as a worldline einbein).

In Eq. (25) the supervielbein $E_M^A(Z)$ is assumed to obey the supergravity constraints $T_{\alpha\dot\beta}{}^a = -2i\sigma_{\alpha\dot\beta}^a$ as well as $T_{\alpha\beta}{}^A = 0 = T_{\dot\alpha\dot\beta}{}^A$, $T_{\alpha\dot\beta}{}^{\dot\gamma} = 0$, $T_{\alpha b}{}^c = 0$, and $R_{\alpha\dot\beta}{}^{ab} = 0$ (or $T_{ab}{}^c = 0$ as, $e.g.$, in [17]). This makes the variational problem slightly subtle (see [16, 14]), but nevertheless well posed.

The superfield equations of motion which result from the variation of the coupled action (24) with respect to the superfield variables turn out to be (see [14])

$$\bar{R} := -\frac{1}{12} R_{\alpha\beta}{}^{ab}(\sigma_a \tilde\sigma_b)^{\alpha\beta} = 0 , \qquad (27)$$

$$G_a := 2i(T_{a\beta}{}^\beta - T_{a\dot\beta}{}^{\dot\beta}) = \mathcal{J}_a , \qquad (28)$$

which imply the following superfield generalization of the Rarita–Schwinger and Einstein equations for the coupled system

$$\Psi_{\dot\alpha}^a \equiv \epsilon^{abcd} T_{bc}{}^\alpha \sigma_{d\alpha\dot\alpha} = \tfrac{i}{4}\bar{\mathcal{D}}_{\dot\alpha}\mathcal{J}^a , \qquad (29)$$

$$R_{bc}{}^{ac} = \tfrac{1}{16}\tilde\sigma_b^{\dot\beta\beta}[\mathcal{D}_\beta, \bar{\mathcal{D}}_{\dot\beta}]\mathcal{J}^a . \qquad (30)$$

The current potential $\mathcal{J}_a$ entering Eq. (28) (and Eqs. (29), (30)) is expressed through the set of current prepotentials

$$\mathcal{K}_a^B(Z) := \int_{\mathcal{W}^1} \frac{l(\tau)}{\hat{E}} \hat{E}_{a\tau} \hat{E}^B \delta^8(Z - \hat{Z}), \quad \delta^8(Z - \hat{Z}) := \delta^4(x - \hat{x})(\theta - \hat{\theta})^4 , \qquad (31)$$

by

$$\tfrac{1}{6}\mathcal{J}_a = -i\mathcal{D}_\alpha \mathcal{K}_a^\alpha + i\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{K}}_a^{\dot\alpha} + \tfrac{1}{4}\tilde\sigma_b^{\dot\alpha\alpha}[\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}]\mathcal{K}_a{}^b . \qquad (32)$$

and, *due to the structure of the supergravity equations*, satisfies the identities

$$\mathcal{D}^\alpha \mathcal{J}_{\alpha\dot\alpha} = 0 , \qquad \bar{\mathcal{D}}^{\dot\alpha}\mathcal{J}_{\alpha\dot\alpha} = 0 , \qquad \mathcal{D}^a \mathcal{J}_a = 0 \qquad (33)$$

($\sigma_{\alpha\dot\alpha}^a \mathcal{J}_a \equiv \mathcal{J}_{\alpha\dot\alpha}$) *which are equivalent to the superparticle equations of motion,*

$$\hat{E}^\alpha \sigma_{\alpha\dot\alpha}^a \hat{E}_{a\tau} = 0 , \qquad \hat{E}_{a\tau}\sigma_{\alpha\dot\alpha}^a \hat{\bar{E}}^{\dot\alpha} = 0 , \qquad \mathcal{D}(l\,\hat{E}_{a\tau}) = 0 . \qquad (34)$$

(only the superparticle Lagrange multiplier $l(\tau)$ produces an independent equation, $\hat{E}^a_\tau \hat{E}_{a\tau} = 0$). Note that the bosonic counterpart of the above statement is well known in general relativity (see, *e.g.* pp. 19, 44-48 and Eq. (1.6.13) in [19] and p. 240 in [20]).

The above mentioned dependence of the superparticle equations of motion is the content of a Noether identity which, in the language of the second Noether theorem (see [13, 18]), reflects the fact that the action (24) possesses a gauge symmetry (superspace superdiffeomorphisms) the transformations of which act additively on the superparticle coordinate functions. In other words, the superparticle coordinate functions behave like *compensators* (pure gauge fields) with respect to superspace diffeomorphism symmetry.

This is tantamount to saying that in the supergravity–superparticle interacting system described by the action (24) the superparticle coordinate functions $\hat{x}^\mu(\tau)$, $\hat{\theta}^{\check{\alpha}}(\tau)$ are the Goldstone fields for a gauge symmetry which is the superdiffeomorphism symmetry. The fact that the Goldstone fields are defined on the worldline or the worldvolume rather than on the whole superspace makes the super–Higgs effect [21] rather subtle in the presence of superbranes (we plan to discuss this separately). Here we wish to note only that the Goldstone nature of the fermionic coordinate functions *i.e.*, its additive transformation law under superdiffeomorphisms, allows one to fix the fermionic 'unitary' gauge

$$\hat{\theta}^{\underline{\alpha}}(\tau) = 0 \; . \tag{35}$$

It was shown in [14] that in this gauge the current potential is proportional to the superspace Grassmann coordinate θ (not to be confused with $\hat{\theta}(\tau)$):

$$\hat{\theta}(\tau) = 0 \qquad \Rightarrow \qquad \mathcal{J}_a \propto (\theta)^2 \; . \tag{36}$$

As a result $\mathcal{D}\mathcal{J}_a|_{\theta=0} = 0$ and the spacetime gravitino field equation for the supergravity–superparticle interacting system (given by the leading component of the superfield equation (29)) becomes sourceless in the gauge (35),

$$\hat{\theta}(\tau) = 0 \quad \Rightarrow \quad \Psi^a_\alpha|_{\theta=0} \equiv 2e e^a_\mu \epsilon^{\mu\nu\rho\sigma} \mathcal{D}_{[\nu}\psi^\alpha_{\rho]}(x) e^b_\sigma \sigma_{b\alpha\dot{\alpha}} = 0 \; . \tag{37}$$

In contrast, the component Einstein equation retains the source term in the gauge (35),

$$e(x)R_{bc}{}^{ac}|_{\theta=0} = c\int_{\mathcal{W}^1} l(\tau)[\hat{e}_{b\tau}\hat{e}^a]\delta^4(x-\hat{x}) \; . \tag{38}$$

The explicit form of the *r.h.s*'s of Eqs. (37), (38) uses the Wess–Zumino gauge, which can be fixed simultaneously with the gauge (35) [14].

Eqs. (37), (38) coincide with the graviton and gravitino equations for the supergravity–bosonic particle interacting system. Moreover, one can show (see [12]) that the supergravity auxiliary fields vanish (on shell) in this gauge and that, in it, the superfield action (24) reduces to the action of the supergravity–bosonic particle interacting system (22) upon integration over the Grassmann coordinates and elimination of the supergravity auxiliary fields by using their purely algebraic equations of motion.

Thus, the superfield description of $D = 4$ $N = 1$ supergravity–superparticle interacting system is gauge equivalent to the dynamical system of component supergravity interacting with the massless bosonic particle.

6   Conclusions

The example of the above $D = 4$, $N = 1$ interacting system shows that the complete coupled superfield action possesses all the gauge symmetries characteristic of the 'free' superfield supergravity and of the superparticle, including worldvolume κ–symmetry. On the other hand,

the gauge fixed version of the superfield coupled action describes the component supergravity interacting with a bosonic particle; moreover, such a gauge fixed action produces the gauge fixed version of *all* dynamical equations of the superfield interacting system. This suggests that the supergravity–superbrane interacting system described by a complete *superfield* action (still unknown for the interesting cases of $D = 10$ and 11 supergravity) is gauge equivalent to the system of dynamical supergravity interacting with the bosonic brane obtained by taking the bosonic 'limit' of the superbrane.

As the *component* supergravity actions are known for all $D \leq 11$ supergravities, the above mentioned gauge equivalence would allow one to study any supergravity–superbrane interacting system, including systems of $D = 10, 11$ supergravity interacting with super–D–branes and super–M–branes. The bosonic brane action appearing as a gauge fixed version of the superbrane action keeps trace of its origin: when it is added to the (component) action for supergravity, the resulting coupled action still possesses on the worldvolume half of the 'free' supergravity local supersymmetry. This 'preserved' part of the local supersymmetry is defined through the (pure bosonic limit of the) κ–symmetry projector and is a remnant of the original superbrane κ–symmetry.

Thus the work outlined here may provide a convenient framework for a further study of various aspects of brane physics in string/M-theory and its applications. In particular, it might be useful in the search for new solutions of the supergravity equations with nonvanishing fermionic fields and in the analysis of anomalies in M-theory (see [22]).

References

[1] E. Bergshoeff, E. Sezgin and P.K. Townsend, *Phys. Lett.* **B189**, 75-78 (1987); *Ann. Phys. (NY)* **185**, 330 (1988).

[2] A. Achúcarro, J.M. Evans, P.K. Townsend and D.L. Wiltshire, *Phys. Lett.* **B198**, 441 (1987).

[3] J.H. Schwarz, *Nucl. Phys. Proc. Suppl.* **B55**, 1-32 (1997) (hep-th/9607067); P.K. Townsend, *Four Lectures on M–theory*, hep-th/9612121.

[4] O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Phys. Rep.* **323**, 183-386, (2000) (hep-th/9905111) and refs. therein.

[5] A. Dabholkar, G.W. Gibbons, J.A. Harvey and F. Ruiz Ruiz, *Nucl. Phys.* **B340**, 33-55 (1990).

[6] M.J. Duff, R.R. Khuri and J.X. Lu, *Phys. Rep.* **259**, 213-326 (1995); K.S. Stelle, *BPS Branes in Supergravity*, hep-th/9803116.

[7] M. Green, J. Schwarz and E. Witten, *Superstring Theory*, I, II, CUP, 1987.

[8] E. Witten, *Nucl. Phys.* **B268**, 253 (1986); W. Siegel, *Introduction to String Field Theory*, World Scientific, Singapore, 1988 (hep-th/0107094); I.Ya. Aref'eva, D.M. Belov, A.S. Koshelev and P.B. Medvedev, *Nucl. Phys.* **B638**, 21-40 (2002) (hep-th/0107197); I. Bars and Y. Matsuo, *Phys. Rev.* **D66**, 066003 (2002) (hep-th/0204260), and refs. therein.

[9] M.T. Grisaru, P.S. Howe, L. Mezincescu, B. Nilsson and P.K. Townsend, *Phys. Lett.* **B162**, 116 (1985).

[10] J. A. de Azcárraga and J. Lukierski, *Phys. Lett.* **B113**, 170 (1982); *Phys. Rev.* **D28**, 1337 (1983); W. Siegel, *Phys. Lett.* **B128**, 397 (1983).

[11] J.A. Shapiro and C.C. Taylor, *Phys. Rep.* **191**, 221-287 (1990) and refs. therein.

[12] I.A. Bandos, J. A. de Azcárraga, J. M. Izquierdo and J. Lukierski, *Phys. Rev.* **D65**, 021901 (2002) (hep-th/0104209).

[13] I.A. Bandos, J.A. de Azcárraga and J.M. Izquierdo, *Phys. Rev.* **D65**, 105010 (2002) (hep-th/0112207).

[14] I.A. Bandos, J. A. de Azcárraga, J. M. Izquierdo and J. Lukierski, *D = 4 supergravity dynamically coupled to a massless superparticle in a superfield Lagrangian approach*, hep-th/0207139.

[15] Y. Neeman and T. Regge, *Riv. Nuovo Cim.* **1**, 1 (1978); L. Castellani, R. D'Auria and P. Fré, *Supergravity and superstrings, a geometric perspective*, vol. 2, World Scientific, 1991, and references therein.

[16] J. Wess and B. Zumino, *Phys. Lett.* **B66**, 361-364 (1977)

[17] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton University Press, 1992.

[18] I.A. Bandos, J.A. de Azcárraga and J.M. Izquierdo, *On the local symmetries of gravity and supergravity models*, hep-th/0201067.

[19] L. Infeld and J. Plebanski, *Motion and Relativity*, Pergamon Press, 1960.

[20] V. Fock, *The theory of space, time and relativity*, Pergamon Press, 1966.

[21] D.V. Volkov and V.A. Soroka, *JETP Lett.* **18**, 312 (1973)

[22] E. Witten, *J. Geom. Phys.* **22**, 103 (1997) (hep-th/961023); K. Lechner, P.A. Marchetti and M. Tonin, *Phys. Lett.* **B524**, 199-207 (2002) (hep-th/0107061); M. Cariglia and K. Lechner, *Phys. Rev.* **D66**, 045003 (2002) (hep-th/0203238) and refs. therein.