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Open String Field Theory around Universal Solutions

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Abstract

We study the physical spectrum of cubic open string field theory around universal solutions, which are constructed using the matter Virasoro operators and the ghost and anti-ghost fields. We find the cohomology of the new BRS charge around the solutions, which appear with a ghost number that differs from that of the original theory. Considering the gauge-unfixed string field theory, we conclude that open string excitations perturbatively disappear after the condensation of the string field to the solutions.

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§1. Introduction

String field theory is a powerful tool to investigate the non-perturbative physics of string theory, particularly tachyon condensation. In cubic open string field theory (CSFT),[1] there is a classical solution corresponding to the tachyon vacuum,[2] the potential height of which is equal to the D-brane tension and the physical spectrum around which contains no open string excitations.[3),4] At present, these studies have been carried out by using a level truncation approximation, and the analytic form of the tachyon vacuum solution has not yet been obtained.[2),5)–7] It is expected that more developments could be realized in string theory if such an exact solution were constructed. However, there has been little progress in the construction of the analytic solution.[8)–13]

There is a universal scalar solution in CSFT that has been found analytically and has several interesting features.[11] It has a well-defined Fock space expression that can be written in the universal basis, namely, the matter Virasoro operators and the ghost and anti-ghost fields. Furthermore, open string excitations in the original theory are not physical in the theory around the solution. These properties are necessary for the tachyon vacuum solution, since tachyon condensation should be a universal phenomenon[2),14] and the D-brane should disappear around the tachyon vacuum.[3),7),15]

In this paper, in order to explore the possibility of a universal solution describing the tachyon vacuum, we investigate the perturbative spectrum in the string field theory around it. If we expand the string field around the universal solution, we obtain the new BRS charge in the shifted theory. In order to determine the spectrum, we solve the cohomology of the new shifted BRS charge exactly.

We can construct the universal solution with one parameter, for almost all values of which the solution is pure gauge, and the new BRS charge can be expressed as the similarity transform of the original BRS charge. Therefore, the cohomology of the new BRS charge has a one-to-one correspondence with that of the original BRS charge through the similarity transformation. However, at the boundary of the parameter region, the similarity transformation has a sort of singularity, and so it is impossible to reduce the cohomology of the new BRS charge to the original one. Consequently, we find that the non-trivial parts of the new cohomology appear in the ghost number 0 and -1 sectors only. Since the string field is expanded in the ghost number one sector in the gauge-unfixed theory, we conclude that the string field theory around the solution contains no physical excitations perturbatively.

This paper is organized as follows. In §2, we review the universal solution in CSFT. In §3, we solve the cohomology of the shifted BRS charge. Then, we show that the string theory has no physical excitations perturbatively in §4. In §5, we construct other possible universal solutions and find that there is no physical spectrum around them. We give a summary and discussion in §6.

§2. Universal solution

The action in cubic open string field theory is given by

$$S[\Psi] = \frac{1}{g}\int\left(\Psi * Q_{\mathrm{B}}\Psi + \frac{2}{3}\Psi * \Psi * \Psi\right), \tag{2·1}$$

where Ψ is the string field and Q_{B} is the Kato-Ogawa BRS charge. The equation of motion is

$$Q_{\mathrm{B}}\Psi + \Psi * \Psi = 0. \tag{2·2}$$

One of the solutions of this equation has been constructed as[11)]

$$\Psi_0(h) = Q_{\mathrm{L}}\left(e^h - 1\right) I - C_{\mathrm{L}}\left((\partial h)^2 e^h\right) I, \tag{2·3}$$

where I denotes the identity string field, and Q_{L} and C_{L} are defined by

$$Q_{\mathrm{L}}(f) = \int_{C_{\mathrm{L}}}\frac{dw}{2\pi i} f(w) J_{\mathrm{B}}(w), \quad C_{\mathrm{L}}(f) = \int_{C_{\mathrm{L}}}\frac{dw}{2\pi i} f(w) c(w), \tag{2·4}$$

where $J_{\mathrm{B}}(w)$ and $c(w)$ are the BRS current and the ghost field, respectively. The contour C_{L} denotes the path along the left-half of strings. The function $h(w)$ satisfies $h(-1/w) = h(w)$ and $h(\pm i) = 0$, and as w approaches $\pm i$, $(\partial h)^2 e^h$ must approach zero sufficiently fast that $(\partial h)^2 e^h$ cancels the midpoint singularity of the ghost field on the identity and hence that the solution has a well-defined Fock space expression.

The BRS current is written in terms of the matter Virasoro operators and the ghost and anti-ghost fields, and the identity string field is represented by the total Virasoro operators.[16)] Therefore, this solution has a universal expression for arbitrary background.

If we expand the string field around the classical solution as

$$\Psi = \Psi_0 + \Psi', \tag{2·5}$$

the action becomes

$$S[\Psi_0 + \Psi'] = S[\Psi_0] + \frac{1}{g}\int\left(\Psi' * Q'_{\mathrm{B}}\Psi' + \frac{2}{3}\Psi' * \Psi' * \Psi'\right). \tag{2·6}$$

The first term on the right-hand side corresponds to the potential height of the solution, but we do not discuss it in this paper. The shifted BRS charge is given by

$$Q'_{\mathrm{B}} = Q(e^h) - C\left((\partial h)^2 e^h\right), \tag{2·7}$$

where we have defined

$$Q(f)=\oint\frac{dw}{2\pi i}f(w)J_{\rm B}(w),\quad C(f)=\oint\frac{dw}{2\pi i}f(w)c(w). \tag{2·8}$$

In general, it turns out that if the BRS charge is written in the form $Q(f)+C(g)$, it must be expressed as $\pm Q'_{\rm B}$ or $C(g)$ because of its nilpotency.*)

In the shifted theory around the solution, let us consider the redefinition of the string field as

$$\Psi''=e^{-q(h)}\Psi', \tag{2·9}$$

where, using the ghost number current $J_{\rm gh}$, $q(f)$ is defined by

$$q(f)=\oint\frac{dw}{2\pi i}f(w)J_{\rm gh}(w). \tag{2·10}$$

Through the redefinition given in Eq. (2·9), the shifted BRS charge is transformed into the original one,

$$e^{-q(h)}Q'_{\rm B}e^{q(h)}=Q_{\rm B}. \tag{2·11}$$

Then, the action in Eq. (2·6) is transformed into the action with the original BRS charge. This fact suggests that the solution might be pure gauge.

However, we can obtain non-trivial solutions if the string field redefinition itself is ill-defined. The operator $q(h)$ is separated as

$$q(h)=q_0(h)+q^{(+)}(h)+q^{(-)}(h), \tag{2·12}$$

where $q_0(h)$, $q^{(+)}(h)$ and $q^{(-)}(h)$ correspond to the zero mode part and the positive and negative frequency parts of $q(h)$, respectively. The operator $e^{\pm q(h)}$ in the redefinition can be rewritten in 'normal ordered form' as

$$e^{q(h)}=\exp\left(\frac{1}{2}[q^{(+)}(h),\,q^{(-)}(h)]\right)e^{q_0(h)}e^{q^{(-)}(h)}e^{q^{(+)}(h)}. \tag{2·13}$$

If the commutator of $q^{(+)}(h)$ and $q^{(-)}(h)$ becomes singular, the operator $e^{q(h)}$ becomes ill-defined, and so we cannot redefine the string field as in Eq. (2·9). In this case, a non-trivial solution can be found for such a function $h(w)$.

In the following, we consider the classical solution for the specific function

$$h_a(w)=\log\left(1+\frac{a}{2}\left(w+\frac{1}{w}\right)^2\right). \tag{2·14}$$

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*) This follows from the relations $\{Q(f),Q(g)\}=2\{Q_B,C(\partial f\partial g)\},\{Q(f),C(g)\}=\{Q_B,C(fg)\}$.

This function $h_a(w)$ can be written in a Laurent expansion as

$$h_a(w) = -\log(1-Z(a))^2 - \sum_{n=1}^{\infty} \frac{(-1)^n}{n} Z(a)^n \left(w^{2n} + w^{-2n}\right),$$
$$Z(a) = \frac{1+a-\sqrt{1+2a}}{a}, \tag{2·15}$$

where $-1 \leq Z(a) < 1$ ($-1/2 \leq a < \infty$). This series is convergent within the annulus $|Z(a)|^{1/2} < |w| < 1/|Z(a)|^{1/2}$. For this $h_a(w)$, the commutator of $q^{(\pm)}$ can be evaluated as

$$[q^{(+)}(h_a),\, q^{(-)}(h_a)] = 2\sum_{n=1}^{\infty} \frac{1}{n} Z(a)^{2n} = -2\log(1-Z(a)^2). \tag{2·16}$$

In the case $Z(a=-1/2)=-1$, the commutator becomes divergent, and the string field redefinition is ill-defined. Then, we can obtain a non-trivial universal solution. It should be noted that for $a=-1/2$, the Laurent expansion of Eq. (2·15) is ill-defined, because the annulus $|Z(a)|^{1/2} < |w| < 1/|Z(a)|^{1/2}$ shrinks to a unit circle. However, the expansion becomes well-defined on the unit circle as a Fourier expansion if we redefine w as $w = \exp(i\sigma)$. It is sufficient for our argument to expand $h(w)$ in a Fourier series.

For $a=-1/2$ case, the non-trivial universal solution is given by[11)]

$$\Psi_0 = Q_{\mathrm{L}}\left(-\frac{1}{4}\left(w+\frac{1}{w}\right)^2\right) I + C_{\mathrm{L}}\left(w^{-2}\left(w+\frac{1}{w}\right)^2\right) I, \tag{2·17}$$

and the shifted BRS charge becomes

$$Q'_{\mathrm{B}} = Q\left(-\frac{1}{4}\left(w-\frac{1}{w}\right)^2\right) + C\left(w^{-2}\left(w+\frac{1}{w}\right)^2\right). \tag{2·18}$$

§3. Cohomology of the new BRS charge

The shifted BRS charge contains several level operators, and therefore it is difficult to solve its cohomology in the usual manner. We consider rewriting the shifted BRS charge as a fixed level expression through some transformation. From the operator product expansions of the ghost number current with the BRS current and the ghost field, we can derive the following commutation relations:[11)]

$$[q(f),\, Q(g)] = Q(fg) - 2C(\partial f \partial g), \tag{3·1}$$
$$[q(f),\, C(g)] = C(fg). \tag{3·2}$$

Using the commutation relations in Eqs. (3·1) and (3·2), we find that

$$e^{q(f)} Q(g) e^{-q(f)} = Q(g\, e^f) - C\left(\{(\partial f)^2 g + 2\partial f \partial g\} e^f\right), \tag{3·3}$$
$$e^{q(f)} C(g) e^{-q(f)} = C(g\, e^f). \tag{3·4}$$

Then, the shifted BRS charge of Eq. (2·7) is transformed as

$$e^{q(f)}Q'_{\mathrm{B}}e^{-q(f)} = Q(e^{h+f}) - C(\{\partial(h+f)\}^2 e^{h+f}). \tag{3·5}$$

Applying Eq. (3·5) to Eq. (2·18), we find that the shifted BRS charge for $a=-1/2$ can be transformed into a fixed level operator as

$$e^{q(\lambda)}\,Q'_{\mathrm{B}}e^{-q(\lambda)} = -\frac{1}{4}Q(w^2) + C(1), \tag{3·6}$$

where $\lambda(w)$ is given by

$$\lambda(w) = 2\sum_{n=1}^{\infty}\frac{1}{n}w^{-2n} = -2\log(1-w^{-2}). \tag{3·7}$$

Using the oscillator expressions, these equations can be written as

$$e^{q(\lambda)}Q'_{\mathrm{B}}e^{-q(\lambda)} = -\frac{1}{4}Q_2 + c_2, \tag{3·8}$$

$$Q'_{\mathrm{B}} = \frac{1}{2}Q_{\mathrm{B}} - \frac{1}{4}(Q_2+Q_{-2}) + 2c_0 + c_2 + c_{-2}, \tag{3·9}$$

$$q(\lambda) = 2\sum_{n=1}^{\infty}\frac{1}{n}q_{-2n}, \tag{3·10}$$

where we have expanded the BRS current J_{B}, the ghost and anti-ghost fields c and b, and the ghost number current J_{gh} as

$$J_{\mathrm{B}}(w) = \sum_{n=-\infty}^{\infty} Q_n w^{-n-1}, \tag{3·11}$$

$$c(w) = \sum_{n=-\infty}^{\infty} c_n w^{-n+1}, \quad b(w) = \sum_{n=-\infty}^{\infty} b_n w^{-n-2}, \tag{3·12}$$

$$J_{\mathrm{gh}}(w) = c\,b(w) = \sum_{n=-\infty}^{\infty} q_n w^{-n-1}. \tag{3·13}$$

Here, we have defined the ghost number current using $SL(2,R)$ normal ordering.

It is thus found that we can reduce the cohomology of Q'_{B} to that of the fixed level operator given in Eq. (3·8). In order to determine the cohomology of the fixed level operator, let us consider the replacement of the ghost and anti-ghost oscillators c_n and b_n by $c_n^{(k)} = c_{n+k}$ and $b_n^{(k)} = b_{n-k}$ in some operator ϕ. We express the operator in which this replacement has been made by $\phi^{(k)}$. For example, we apply this bc-shift operation to the modes of the ghost number current q_n, the ghost Virasoro operators $L_n^{(bc)}$, and the BRS charge Q_{B}, and then we rewrite them in terms of the unshifted operators:

$$q_n^{(k)} = q_n + k\delta_{n,0}, \tag{3·14}$$

$$L_n^{(bc)\,(k)} = L_n^{(bc)} + kq_n + \frac{1}{2}(k^2 - 3k)\delta_{n,0}, \tag{3·15}$$

$$Q_{\rm B}^{(k)} = Q_k - k^2 c_k. \tag{3·16}$$

The algebra of these bc-shifted operators can be calculated using the following commutation relations:

$$[q_m,\, q_n] = m\delta_{m+n,0}, \tag{3·17}$$

$$[L_m^{(bc)},\, L_n^{(bc)}] = (m-n)L_{m+n}^{(bc)} - \frac{13}{6}(m^3 - m)\delta_{m+n,0}, \tag{3·18}$$

$$[L_m^{(bc)},\, q_n] = -nq_{m+n} - \frac{3}{2}m(m+1)\delta_{m+n,0}, \tag{3·19}$$

$$\{Q_m,\, Q_n\} = 2mn\{Q_m,\, c_n\}, \tag{3·20}$$

$$\{Q_m,\, c_n\} = \{Q_{\rm B},\, c_{m+n}\}, \tag{3·21}$$

$$\{Q_m,\, b_n\} = L_{m+n} + mq_{m+n} + \frac{3}{2}m(m-1)\delta_{m+n,0}, \tag{3·22}$$

$$[Q_m,\, q_n] = -Q_{m+n} + 2mnc_{m+n}. \tag{3·23}$$

It turns out that the algebra of $q_n^{(k)}$, $L_n^{(bc)\,(k)}$ and $Q_{\rm B}^{(k)}$ is the same as that of the unshifted operators q_n, $L_n^{(bc)}$ and $Q_{\rm B}$. In general, the algebra of $\phi^{(k)}$ is invariant through the bc-shift operation, because $c_m^{(k)}$ and $b_n^{(k)}$ satisfy the anti-commutation relations $\{c_m^{(k)},\, b_n^{(k)}\} = \delta_{m+n,0}$, and these relations are the same as those before the replacement of the ghost modes.

The $SL(2,\, R)$ invariant vacuum $|0\rangle$ has the following properties for the bc-shifted modes:

$$\begin{aligned} c_n^{(k)}\,|0\rangle &= 0, \quad (n \geq 2-k) \\ b_n^{(k)}\,|0\rangle &= 0. \quad (n \geq -1+k) \end{aligned} \tag{3·24}$$

We define the bc-shifted vacuum $|0\rangle^{(k)}$ as

$$c_n^{(k)}\,|0\rangle^{(k)} = 0, \quad (n \geq 2) \tag{3·25}$$

$$b_n^{(k)}\,|0\rangle^{(k)} = 0, \quad (n \geq -1) \tag{3·26}$$

From Eq. (3·24), we find that the bc-shifted vacuum can be expressed in terms of the $SL(2,\, R)$ invariant vacuum:

$$|0\rangle^{(k)} = \begin{cases} b_{-k-1}b_{-k}\cdots b_{-2}\,|0\rangle\,, & (k \geq 1) \\ c_{k+2}c_{k+1}\cdots c_1\,|0\rangle\,. & (k \leq -1) \end{cases} \tag{3·27}$$

Now that the relations between the bc-shifted and unshifted operators and their vacua have been established, we can determine the cohomology of the shifted BRS charge. First, let us recall the cohomology of the Kato-Ogawa BRS charge.[17)–19)]

Proposition 1. *Any state* $|\psi\rangle$ *satisfying* $Q_{\rm B}\,|\psi\rangle = 0$ *can be written*

$$|\psi\rangle = |P\rangle \otimes c_1\,|0\rangle\ + |P'\rangle \otimes c_0 c_1\,|0\rangle\ + Q_{\rm B}\,|\phi\rangle\,. \qquad (3\cdot28)$$

Here $|P\rangle$ *and* $|P'\rangle$ *are DDF states.*[*)]

The first term on the right-hand side of Eq. (3·28) corresponds to the cohomology with the additional subsidiary condition $b_0\,|\psi\rangle = 0$,[17)] which is the Siegel gauge condition in string field theory. The second term is obtained as the additional term in case that we remove the Siegel gauge condition.[18),19)]

Since the algebra of the bc-shifted operators is the same as that of the unshifted one, a similar proposition also holds if we apply the bc-replacement in Proposition 1. The BRS charge with bc-replacement is given by Eq. (3·16). Considering the correspondence between ($Q_{\rm B}^{(k)}$, $|0\rangle^{(k)}$) and ($Q_{\rm B}$, $|0\rangle$), we can obtain the cohomology of $Q_{\rm B}^{(k)}$.

Proposition 2. *Any state* $|\psi\rangle$ *satisfying* $Q_{\rm B}^{(k)}\,|\psi\rangle = 0$ *can be written*

$$|\psi\rangle = |P\rangle \otimes c_1^{(k)}\,|0\rangle^{(k)}\ + |P'\rangle \otimes c_0^{(k)} c_1^{(k)}\,|0\rangle^{(k)}\ + Q_{\rm B}^{(k)}\,|\phi\rangle\,. \qquad (3\cdot29)$$

From Eqs. (3·8) and (3·16), the new BRS charge $Q'_{\rm B}$ can be written as a similarity transformation of the bc-shifted BRS charge:

$$Q'_{\rm B} = -\frac{1}{4} e^{-q(\lambda)} Q_{\rm B}^{(2)} e^{q(\lambda)}. \qquad (3\cdot30)$$

Therefore, with the help of Proposition 2, any state $|\psi\rangle$ satisfying $Q'_{\rm B}\,|\psi\rangle = 0$ can be written

$$|\psi\rangle = |P\rangle \otimes U b_{-2}\,|0\rangle\ + |P'\rangle \otimes U\,|0\rangle\ + Q'_{\rm B}\,|\phi\rangle\,, \qquad (3\cdot31)$$

where U is given by

$$U = \exp\left(-\sum_{n=1}^{\infty} \frac{2}{n} q_{-2n}\right). \qquad (3\cdot32)$$

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*) Here, we have not discussed the infrared states with zero momenta.

Thus, we have obtained the cohomology of the shifted BRS charge $Q'_{\rm B}$.

It should be noted that, for the derivation of the cohomology, it is important that the operator $e^{q(\lambda)}$ be invertible. Instead of Eq. (3·30), the new BRS charge can be written

$$Q'_{\rm B}=-\frac{1}{4}e^{-q(\lambda')}Q_{\rm B}^{(-2)}e^{q(\lambda')}, \tag{3·33}$$

where $\lambda'(w)=-\log(1-w^2)$ and

$$q(\lambda')=2\sum_{n=1}^{\infty}\frac{1}{n}q_{2n}. \tag{3·34}$$

However, we cannot reduce the cohomology of $Q'_{\rm B}$ to that of $Q_{\rm B}^{(-2)}$, because the operator $e^{q(\lambda')}$ has 'zero modes'. Indeed, as an example of 'zero modes', we find that

$$e^{q(\lambda')}\times e^{-q(\lambda)}\left|0\right\rangle\;=\;\exp\left(-4\sum_{n=1}^{\infty}\frac{2}{n}\right)e^{-q(\lambda)}\left|0\right\rangle=0. \tag{3·35}$$

§4. String field theory around the universal solution

The universal classical solution given in Eq. (2·3) has been found from the equation of motion without gauge fixing. In fact, this solution contains the ghost zero mode c_0, and therefore it is outside the Siegel gauge. Therefore, in the context of the gauge-unfixed theory, a simpler argument can be given to understand the spectrum around the solution.

If we consider the equation $(\Box\eta_{\mu\nu}-\partial_\mu\partial_\nu)A^\nu=0$ in an abelian gauge theory, the solution is given by $A_\mu=A_\mu^{(+)}+A_\mu^{(-)}+\partial_\mu\theta$, where $A_\mu^{(\pm)}$ correspond to the transverse modes and θ is the gauge freedom. Then, we can identify the transverse modes as the physical degrees of freedom. In the following, we proceed similarly. We consider the gauge-unfixed theory from beginning to end, where the kinetic operator $\Box\eta_{\mu\nu}-\partial_\mu\partial_\nu$ is replaced by $Q'_{\rm B}$; that is, in the gauge-unfixed theory, we solve the equation of motion and expand the string field around it. Then, we investigate the fluctuations in the shifted theory without gauge-fixing and identify their physical modes up to the gauge symmetry, not the BRS symmetry.

In open string field theory, the string field Ψ is assigned the ghost number $N_{\rm FP}=1$.[1),20)] Here, we have defined the ghost number $N_{\rm FP}$ of the $SL(2,R)$ invariant vacuum $\left|0\right\rangle$ as zero. Using the oscillator representation, the string field Ψ is expanded as

$$\left|\Psi\right\rangle=\phi(x)\,c_1\left|0\right\rangle+A_\mu(x)\,\alpha_{-1}^\mu c_1\left|0\right\rangle\,+B(x)\,c_{-1}c_1\left|0\right\rangle+C(x)\left|0\right\rangle+\cdots. \tag{4·1}$$

The assignment $N_{\rm FP}=1$ implies that the component fields ϕ, A_μ, B and C are assigned the ghost numbers 0, 0, -1 and 1, respectively. In the gauge invariant theory without gauge fixing, the ghost number of the component fields should be fixed to zero. In other words, the fields $\mathcal{B}$, $\mathcal{C}$, and so on, are forbidden.

Let us consider the perturbative spectrum in the gauge-unfixed theory around the universal solution. The linearized equation of motion is given by

$$Q'_{\rm B}\Psi = 0. \tag{4·2}$$

Using Eq. (3·31), it is possible to find the solution of this equation. Since the string field does not contain a component field carrying nonzero ghost number, the solution of Eq. (4·2) is given by

$$\Psi = Q'_{\rm B}\phi. \tag{4·3}$$

In the linearized approximation, the string field theory around the solution is invariant under the gauge transformation

$$\delta\Psi = Q'_{\rm B}\Lambda, \tag{4·4}$$

where Λ is a gauge transformation parameter. This symmetry is not BRS but gauge symmetry, and so both the parameter Λ and its component fields are assigned ghost number zero.

From Eqs. (4·3) and (4·4), it follows that all of the on-shell modes can be eliminated by the gauge transformation in the gauge-unfixed theory. Therefore, we conclude that the open string excitations in the original theory disappear perturbatively after the condensation of the string field to the universal solution.

§5. Other universal solutions

We can construct other universal solutions, because the universal solution in Eq. (2·3) can be constructed for any function $h(w)$ that makes the operator $e^{q(h)}$ singular. Let us consider the solution for the function

$$h_a^l(w) = \log\left(1 - \frac{a}{2}(-1)^l\left(w^l - (-1)^l\frac{1}{w^l}\right)^2\right), \tag{5·1}$$

where $l = 1, 2, \cdots$. The $l = 1$ case corresponds to the solution given previously. This function satisfies all of the conditions discussed in §2, and so it yields a universal solution. The function h_a^l is expanded in a Laurent series as

$$h_a^l(w) = -\log(1 - Z(a))^2 - \sum_{n=1}^{\infty}\frac{(-1)^{nl}}{n}Z(a)^n\left(w^{2ln} + w^{-2ln}\right). \tag{5·2}$$

From this expansion, it follows that if we take $a \to -1/2$ limit, the operator $e^{q(h_a^l)}$ becomes singular. Therefore, we can obtain a non-trivial universal solution for $h_{-1/2}^l$.

Substituting $h_{-1/2}^l$ into Eq. (2·3), the solution is obtained as

$$\Psi_0^{(l)} = Q_{\mathrm{L}}\left(-\frac{1}{2}+\frac{(-1)^l}{4}(w^{2l}+w^{-2l})\right)I + C_{\mathrm{L}}\left(l^2 w^{-2}\left(2-(-1)^l(w^{2l}+w^{-2l})\right)\right)I. \tag{5·3}$$

We can obtain the Fock space expression of the universal solution as

$$\left|\Psi_0^{(l)}\right\rangle = \Bigg[\sum_{m=1}^{\infty}\frac{(-1)^m}{2\pi}\left(\frac{2}{2m+1}-\frac{1}{2m+1-2l}-\frac{1}{2m+1+2l}\right)\left(-Q_{-(2m+1)}+4l^2 c_{-(2m+1)}\right) - \frac{1}{2\pi}\frac{8l^2}{4l^2-1}Q_{-1} + \frac{1}{2\pi}\sum_{k=1}^{l}\frac{4l^2}{2k-1}c_1 + \frac{1}{2\pi}\left(-\sum_{k=1}^{l}\frac{4l^2}{2k-1}+\frac{32l^4}{4l^2-1}\right)c_{-1}\Bigg]|I\rangle, \tag{5·4}$$

where use has been made of the equations[11)]

$$\begin{aligned} J_{\mathrm{B}}(w)\,|I\rangle &= \sum_{n=1}^{\infty} Q_{-n}(w^n-(-1)^n w^{-n})w^{-1}\,|I\rangle\,, \\ c(w)\,|I\rangle &= \Bigg[-c_0\frac{w-w^3}{1+w^2}+c_1\frac{1}{1+w^2}+c_{-1}\frac{1+w^2+w^4}{1+w^2} \\ &\qquad +\sum_{n=2}^{\infty}c_{-n}(w^n-(-1)^n w^{-n})w\Bigg]\,|I\rangle\,. \end{aligned} \tag{5·5}$$

If we expand the string field around this solution, the new BRS charge in the shifted theory is given by

$$\tilde{Q}_{\mathrm{B}} = Q\left(e^{h_{-1/2}^l}\right) - C\left((\partial h_{-1/2}^l)^2 e^{h_{-1/2}^l}\right) \tag{5·6}$$

$$= \frac{1}{2}Q_{\mathrm{B}} + \frac{(-1)^l}{4}(Q_{2l}+Q_{-2l}) + 2l^2\left(c_0 - \frac{(-1)^l}{2}(c_{2l}+c_{-2l})\right). \tag{5·7}$$

Let us consider the spectrum in the shifted theory. As in the case $l=1$, the new BRS charge can be written as a similarity transformation of the bc-shifted BRS charge. We have

$$\tilde{Q}_{\mathrm{B}} = \frac{(-1)^l}{4}e^{-q(\lambda^l)}\,Q_{\mathrm{B}}^{(2l)}\,e^{q(\lambda^l)}, \tag{5·8}$$

where $\lambda^l(w) = -2\log(1+(-1)^l w^{-2l})$, and $q(\lambda^l)$ is given by

$$q(\lambda^l) = 2\sum_{n=1}^{\infty}\frac{(-1)^{n(l+1)}}{n}q_{-2nl}. \tag{5·9}$$

From Proposition 2, the cohomology of the new BRS charge $\tilde{Q}_{\mathrm{B}}$ $(l \geq 2)$ can be written

$$|\psi\rangle = |P\rangle \otimes U_l\, b_{-2l}b_{-2l+1}\cdots b_{-2}\,|0\rangle \tag{5·10}$$

$$+\,|P'\rangle \otimes U_l\, b_{-2l+1}\cdots b_{-2}\,|0\rangle + \tilde{Q}_{\mathrm{B}}\,|\phi\rangle\,, \tag{5·11}$$

where $|P\rangle$ and $|P'\rangle$ denote DDF states, and U_l is given by

$$U_l = \exp\left(-2\sum_{n=1}^{\infty}\frac{(-1)^{n(l+1)}}{n}q_{-2nl}\right). \tag{5·12}$$

Thus, the non-trivial cohomology is assigned to the ghost numbers $-2l+1$ and $-2l+2$. Therefore, again applying the argument used in the case $l=1$, we conclude that there is no physical spectrum perturbatively in the string field theory around the solution.

§6. Summary and discussion

We have obtained the cohomology for the new BRS charge around the universal solution in CSFT. The non-trivial states belong to the ghost number 0 and -1 sectors. Consequently, the shifted theory around the solution contains no physical spectrum perturbatively. We have found other possible solutions in CSFT. Around these solutions too, the shifted theories have no physical spectrum.

We believe that the universal solutions correspond to the tachyon vacuum solution, because they have universal expressions and there is no physical spectrum around them. However, the universal solutions have several possible expressions. If these solutions represent the tachyon condensation, they must be equivalent up to gauge transformations, and the shifted theories must be related by string field redefinitions.

Let us consider the string field redefinition generated by the operator $K_n = L_n - (-1)^n L_{-n}\ (n \geq 1)$. This field redefinition preserves the three string vertex, and it only changes the kinetic operator $\tilde{Q}_{\rm B}$ to $e^K \tilde{Q}_{\rm B} e^{-K}$.[21),22)] Since the operators Q_n and c_n have the algebra

$$[K_m,\, Q_n] = -nQ_{n+m} + (-1)^m nQ_{n-m}, \tag{6·1}$$
$$[K_m,\, c_n] = -(2m+n)c_{n+m} - (-1)^m(2m-n)c_{n-m}, \tag{6·2}$$

the transform of the new BRS charge remains of the form $Q(f)+C(g)$, for some functions $f(w)$ and $g(w)$. As pointed out in §2, the BRS charge with this form must be written as $Q(e^h)-C((\partial h)^2 e^h)$, because of its nilpotency. Therefore, it is sufficient to consider the ghost kinetic parts in order to determine the number of inequivalent classes of the new BRS charge through the string field redefinition $\Psi' = e^K \Psi$.

According to Ref. 22), with this field redefinition, the ghost kinetic terms $a_0 c_0 + \sum_{n\geq 1} a_n (c_n + (-1)^n c_{-n})$ can be classified into at least two classes: those which annihilate the identity string field and those which do not. For all our solutions, the ghost parts annihilate the identity string field.[10),23)] Therefore, there is no contradiction at present if the shifted theories can be transformed into each other through field redefinitions. The feasibility of such a procedure requires further investigation.

At present, there are many important issues that remain incompletely understood. In this paper, we did not discuss the potential height at our solutions, but determining this is an important problem in order to conclude whether or not they correspond to the tachyon vacuum. Unfortunately, it is difficult to evaluate the action at the solution that is constructed using the identity string field, because we often encounter disastrous divergences in the computation of quantities of the form $\langle I|\ldots|I\rangle$. We should find a consistent regularization in order to treat the identity string field correctly in these calculations.

We should comment on the relation between our result and that in Ref. 24). In that work, it is shown in terms of the level truncation scheme that the cohomology of the new BRS charge around the tachyon vacuum is trivial. This would seem to contradict our result if the universal solutions represent the tachyon condensation, because our solutions yield a non-trivial cohomology. We have not yet understood this apparent contradiction.

As is well known, vacuum string field theory (VSFT)[25),26)] is another approach to investigate the tachyon condensation, and there is some evidence that VSFT represents the theory around the tachyon vacuum.[27)] It would be interesting to examine the relations between our solutions in the context of CSFT and VSFT.

Around the tachyon vacuum, there would be closed string excitations instead of open ones. It is a future problem to investigate how to treat closed string around our solutions in the context of CSFT. It might be necessary to fix the gauge appropriately and perform a second quantization.

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