Gauging dual symmetry

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(Dated: April 25, 2020)

The idea of gauging (*i.e.* making local) symmetries is a central feature of many modern field theories. Usually, one starts with a Lagrangian for some scalar or spinor fields, with the Lagrangian being invariant under some global symmetry transformation of these fields. Making this global symmetry local requires the introduction of vector fields. Therefore the vector field is a consequence or result of the gauge principle. Here we show that for some symmetries the process of transforming from a global to a local symmetry can be achieved by introducing a scalar rather than a vector field. The symmetry that we study is electric-magnetic dual symmetry which "rotates" electric and magnetic quantities into one another. Starting from an initial Lagrangian which contains vector fields and satisfies a global electric-magnetic duality, we show that it is possible to make the symmetry local by introducing a scalar field.

I. INTRODUCTION

Local or gauge symmetry is deeply ingrained in modern physics. The strong and electroweak interactions of particle physics are formulated as gauge interactions. General relativity can be viewed as taking the global spacetime symmetries of special relativity and making them local. In the case of the strong and electroweak interactions the vector fields can be said to be derived from the gauge principle, in contrast to the matter fields which are "put in by hand". Here we show that it is possible to gauge certain symmetries using scalar rather than vector fields. Starting with a Lagrangian with a global electric-magnetic dual symmetry and vector fields, we find that making this dual symmetry local requires the introduction of a complex scalar field. The final Lagrangian contains both the original vector fields as well as the scalar field which arises from the alternative gauge principle. This Lagrangian is different from the standard scalar electrodynamics Lagrangian in that the coupling between the vector and scalar field is a derivative coupling as opposed to a polynomial coupling.

For definiteness and in order to make comparisons, we briefly review the textbook example [1] of scalar electrodynamics where the ordinary gauge principle is applied to a complex scalar field. Starting with the Lagrange density

$$\mathcal{L}_{scalar} = (\partial_\mu \phi)^* (\partial^\mu \phi) - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \tag{1}$$

one finds that it is possible to allow the scalar fields to have the following, local symmetry

$$\phi(x) \to e^{-ie\Lambda(x)} \phi(x) \qquad \phi^*(x) \to e^{ie\Lambda(x)} \phi^*(x) \tag{2}$$

if one introduces a four-vector, gauge field A_μ which promotes the ordinary derivative to a covariant derivative – $\partial_\mu \to \partial_\mu - ieA_\mu \equiv D_\mu$. In conjunction with the transformation in Eqs. (2) A_μ transforms as

$$A_\mu \to A_\mu - \partial_\mu \Lambda(x) \tag{3}$$

One also introduces a term which contains only the gauge fields and is invariant under Eq. (3) namely $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The new Lagrange density which is invariant under Eqs.(2) (3) is

$$\mathcal{L'}_{scalar} = (D_\mu \phi)^* (D^\mu \phi) - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{4}$$

Starting with a Lagrange density with scalar fields it is necessary to introduce a vector field in order to allow the global phase symmetry of the scalar fields to become local.

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II. DUAL SYMMETRY

Source-free electromagnetism possesses a dual symmetry between electric and magnetic fields which can be written in terms of $F_{\mu\nu}$ and its dual $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ [2]

$$F_{\mu\nu} \to \cos(\Lambda)F_{\mu\nu} + \sin(\Lambda)\tilde{F}_{\mu\nu}$$
$$\tilde{F}_{\mu\nu} \to -\sin(\Lambda)F_{\mu\nu} + \cos(\Lambda)\tilde{F}_{\mu\nu} \tag{5}$$

The standard form of this dual symmetry in terms of the $\mathbf{E}$ and $\mathbf{B}$ fields is obtained by making the replacements $F_{\mu\nu} \to \mathbf{E}$ and $\tilde{F}_{\mu\nu} \to \mathbf{B}$ [3]. This dual symmetry of Maxwell's equations can be extended to the case with sources if one allows both electric and magnetic charges. We would like to extend the dual symmetry of Eq. (5) down to the level of the four-vector potential, $A_\mu = (\Phi, \mathbf{A})$. However, since the expression for the electric and magnetic fields in terms of the potentials is not symmetric ($\mathbf{E} = -\nabla\Phi - \partial_t\mathbf{A}$ and $\mathbf{B} = \nabla \times \mathbf{A}$) this is difficult. In the context of electromagnetism with magnetic charge one can introduce a second four-vector potential, $C_\mu = (\Phi_m, \mathbf{C})$, [4] [5] in terms of which the electric and magnetic fields take on the more symmetric form, $\mathbf{E} = -\nabla\Phi - \partial_t\mathbf{A} - \nabla \times \mathbf{C}$ and $\mathbf{B} = -\nabla\Phi_m - \partial_t\mathbf{C} + \nabla \times \mathbf{A}$. Ref. [6] contains an elementary overview of this two-potential approach, as well as references to the extensive work that has been done on this approach to electromagnetism with magnetic charge. In this two-potential approach the dual symmetry can be extended to the level of the potentials by changing $F_{\mu\nu} \to A_\mu$ and $\tilde{F}_{\mu\nu} \to C_\mu$ in Eq. (5) (see Eq. (5.17b) in Ref. [5]). This global, dual symmetry for the potentials can be put in a form similar to the phase transformation of the scalar fields of Eq. (2) of the previous section by defining a complex four-potential, $W_\mu = A_\mu + iC_\mu$, in terms of which Eq. (5) for the potentials can be written as

$$W_\mu \to e^{-i\Lambda}W_\mu \tag{6}$$

In this form the electric-magnetic dual symmetry is similar to the phase symmetry of the scalar fields in Eq. (2). W_μ gives a complex field strength tensor, $G_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, which will be useful in the next section. Although, we have associated the transformation in Eq. (6) with the electric-magnetic duality of Maxwell's equations, one could argue that Eq. (6) is just a phase transformation for a complex, matter, vector field, W_μ. However, in the next section we will give a procedure for making this symmetry local which is distinct from the standard gauge procedure. Applying the standard gauge procedure to W_μ would simply lead to another vector field (this is done, for example, on pg. 124 of Ref. [2].) In contrast our dual gauge procedure will lead to the introduction of a scalar field. Thus, regardless of the interpretation of the symmetry in Eq. (6) the gauging procedure presented in the next section is different from the standard method of gauging a symmetry.

III. MAKING DUAL SYMMETRY LOCAL

We now want to allow the dual symmetry of Eq. (6) to become local, $\Lambda \to g\Lambda(x)$. We have introduced the constant, g, which will be seen to be the coupling constant between the vector field, W_μ, and the scalar field. In our development we build up our Lagrange density one piece at a time using an infinitesimal version (*i.e.* taking $\Lambda(x)$ infinitesimal) of Eq. (6) namely

$$\delta W_\mu = -ig\Lambda W_\mu \quad \delta W_\mu^* = ig\Lambda W_\mu^* \tag{7}$$

We will also need the infinitesimal variations of the partial derivatives of the complex vector potential

$$\delta(\partial_\mu W_\nu) = -ig\partial_\mu(\Lambda W_\nu) \quad \delta(\partial_\mu W_\nu^*) = ig\partial_\mu(\Lambda W_\nu^*) \tag{8}$$

and the variations of the complex field strengths

$$\delta G_{\mu\nu} = -ig\Lambda G_{\mu\nu} - ig(\partial_\mu\Lambda W_\nu - \partial_\nu\Lambda W_\mu)$$
$$\delta G_{\mu\nu}^* = ig\Lambda G_{\mu\nu}^* + ig(\partial_\mu\Lambda W_\nu^* - \partial_\nu\Lambda W_\mu^*) \tag{9}$$

Now we start with a "kinetic" energy Lagrangian for the vector fields

$$\mathcal{L}_1 = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu*} \tag{10}$$

in the same way that in the introduction we began with a kinetic energy term for the complex scalar field. This Lagrangian in Eq. (10) is invariant under a generalized version of the gauge transformation in Eq. (3) (*i.e.* a

transformation for both A_μ and C_μ). The final Lagrange density that we find, will no longer respect this standard gauge symmetry, but it will be invariant under the local version of the dual symmetry in Eq. (6). Thus we gain one local symmetry, Eq. (6), at the cost of losing another, Eq. (3). For the scalar field case we also included a mass and quartic self interaction term since these were allowed by the phase symmetry of Eq. (2). In the same way we could include a mass term, $m^2 W_\mu W^{\mu*}$, and self interaction term, $\lambda(W_\mu W^{\mu*})^2$ to our Lagrangian. Such terms are usually forbidden by the standard gauge transformation in Eq. (3), but are allowed by Eq. (6). We could also add a term like $\epsilon_{\mu\nu\alpha\beta} G^{\mu\nu} G^{\alpha\beta*} = G^{\mu\nu} \widetilde{G}_{\mu\nu}^*$ to $\mathcal{L}_1$. However, by the anti-symmetry properties of $\epsilon_{\mu\nu\alpha\beta}$, such a term would not change the field equations derived from the Lagrangian. In the end when we arrive at the dual version of the covariant derivative it will be straightforward to show that both terms like $G_{\mu\nu} G^{\mu\nu*}$ and $G^{\mu\nu} \widetilde{G}_{\mu\nu}^*$ can be made consistent with the local dual symmetry of Eq. (5). Taking the variation of $\mathcal{L}_1$ using Eq. (9)

$$
\begin{aligned}
\delta\mathcal{L}_1 &= -\frac{1}{4}(\delta G_{\mu\nu} G^{\mu\nu*} + G_{\mu\nu} \delta G^{\mu\nu*}) \\
&= \frac{ig}{2}\partial^\mu\Lambda(W^\nu G_{\mu\nu}^* - W^{\nu*} G_{\mu\nu})
\end{aligned}
\tag{11}
$$

Since $\delta\mathcal{L}_1 \neq 0$ we continue to add terms to the Lagrangian in the hopes of building a total Lagrangian for which $\delta\mathcal{L}_{total} = 0$. We next consider

$$
\mathcal{L}_2 = \frac{g}{2}(\partial_\mu\phi W_\nu G^{\mu\nu*} + \partial_\mu\phi^* W_\nu^* G^{\mu\nu})
\tag{12}
$$

where we have introduced a complex, scalar field ϕ which we require to transform as

$$
\phi \to \phi - i\Lambda(x) \quad \phi^* \to \phi^* + i\Lambda(x)
\tag{13}
$$

The arbitrary function, $\Lambda(x)$, is the same as in Eq. (6). Just as the dual transformation of Eq. (6) was similar to the phase transformation of Eq. (2), so the transformation of Eq. (13) is similar to the gauge transformation of Eq. (3). We will call ϕ the "gauge" field for the dual symmetry or the dual gauge field. Since the transformation of the scalar field only involves an imaginary shift of the field via $i\Lambda(x)$ one could use this freedom to transform away the imaginary part of the scalar field by choosing $\Lambda(x)$ to equal the imaginary part of ϕ. This freedom will manifest itself later in that the scalar field kinetic energy term, allowed by Eq. (13), will only contain the real part of the scalar field.

The infinitesimal forms of the transformation for ϕ and its partial derivatives are given by

$$
\begin{aligned}
\delta\phi &= -i\Lambda \quad \delta(\partial_\mu\phi) = -i\partial_\mu\Lambda \\
\delta\phi^* &= i\Lambda \quad \delta(\partial_\mu\phi^*) = i\partial_\mu\Lambda
\end{aligned}
\tag{14}
$$

Using these and the transformations of Eqs. (7) - (9) we find

$$
\begin{aligned}
\delta\mathcal{L}_2 &= -\frac{ig}{2}\partial_\mu\Lambda(W_\nu G^{\mu\nu*} - W_\nu^* G^{\mu\nu}) + \frac{ig^2}{2}\partial_\mu\Lambda(\partial^\mu\phi - \partial^\mu\phi^*)W_\nu^* W^\nu \\
&\quad + \frac{ig^2}{2}\partial_\nu\Lambda(\partial_\mu\phi^* W^{\nu*} W^\mu - \partial_\mu\phi W^\nu W^{\mu*})
\end{aligned}
\tag{15}
$$

The first term in $\delta\mathcal{L}_2$ now cancels the unwanted term from $\delta\mathcal{L}_1$, but only at the cost of two new terms. Next we add

$$
\mathcal{L}_3 = -\frac{g^2}{2}\partial_\mu\phi\partial^\mu\phi^* W_\nu W^{\nu*}
\tag{16}
$$

which has the following variation from Eqs. (7) and (14)

$$
\delta\mathcal{L}_3 = -\frac{ig^2}{2}\partial_\mu\Lambda(\partial^\mu\phi - \partial^\mu\phi^*)W_\nu^* W^\nu
\tag{17}
$$

which cancels the second term from $\delta\mathcal{L}_2$ in Eq. (15). In arriving at Eq. (17) we used the result that $W_\nu^* W^\mu$ is invariant under the local dual transformation so that $\delta(W_\nu^* W^\mu) = 0$. Finally, adding

$$
\mathcal{L}_4 = \frac{g^2}{4}(\partial^\nu\phi\partial_\mu\phi^* W_\nu^* W^\mu + \partial^\nu\phi^*\partial_\mu\phi W_\nu W^{\mu*})
\tag{18}
$$

gives a variation of

$$\delta\mathcal{L}_4 = \frac{ig^2}{2}\partial_\nu\Lambda(\partial_\mu\phi W^\nu W^{\mu*} - \partial_\mu\phi^* W^{\nu*} W^\mu) \tag{19}$$

where we have renamed indices to get this form. Again, we have used $\delta(W_\nu^* W^\mu) = 0$ and $\delta(W_\nu W^{\mu*}) = 0$. This variation of $\mathcal{L}_4$ cancels the third term from $\delta\mathcal{L}_2$ in Eq. (15), and by adding all four terms together ($\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4$) we arrive at a Lagrange density which is invariant under the local dual transformation (*i.e.* $\delta\mathcal{L} = 0$). In the standard application of the gauge principle sketched in the introduction, one finishes by adding a term to the Lagrange density which contains only the vector fields (*i.e.* the last term in Eq. (4)). The same thing is possible for the local dual symmetry with the Lagrange density of the form

$$\mathcal{L}_5 = \frac{1}{2}(\partial_\mu\phi + \partial_\mu\phi^*)(\partial^\mu\phi + \partial^\mu\phi^*) \tag{20}$$

It is easy to see that under the infinitesimal transformation, Eq. (14), $\delta\mathcal{L}_5 = 0$. The Lagrange density of Eq. (20) looks similar to the standard kinetic energy terms of a scalar field namely $\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$. Collecting these five terms together the total Lagrange density can be written in the simplified form

$$\begin{aligned}\mathcal{L}_{total} &= -\frac{1}{4}(G_{\mu\nu} - g\partial_\mu\phi W_\nu + g\partial_\nu\phi W_\mu)(G^{\mu\nu*} - g\partial^\mu\phi^* W^{\nu*} + g\partial^\nu\phi^* W^{\mu*}) \\ &+ \frac{1}{2}(\partial_\mu\phi + \partial_\mu\phi^*)(\partial^\mu\phi + \partial^\mu\phi^*)\end{aligned} \tag{21}$$

This is invariant ($\delta\mathcal{L}_{total} = 0$) under the dual "gauge" transformations of Eqs. (7) (8) (14). The scalar-vector theory associated with the Lagrange density of Eq. (21) is distinct from scalar electrodynamics. The scalar-vector couplings given in Eqs. (12) (16) (18) are all derivative couplings, whereas scalar electrodynamics also has polynomial couplings between the scalar and vector field. The coupling g in the Lagrangian of Eq. (21) also has a mass dimension -1, whereas scalar electrodynamics has a dimensionless coupling. If the Lagrange density in Eq. (21) is to have a mass dimension 4, and if ϕ, W_μ and ∂_μ have the conventional mass dimension of 1, then g must have mass dimension -1. This last comment raises the question as to the renormalizability of the scalar-vector theory of Eq. (21). The fact that g has a negative mass dimension indicates that the theory associated with Eq. (21) is non-renormalizable. However, theories which are non-renormalizable can still be useful when treated as effective theories [7]. In any case for the present paper we are focused on the task of constructing a classical Lagrangian which respects the dual symmetry locally. We leave the technical and complex question of the renormalization of the theory in Eq. (21) for a possible future investigation.

The derivative coupling which arises between the vector and scalar fields from the dual gauge principle (Eqs. (12) (16) (18)) can be compared to the derivative couplings which occur in an effective Lagrangian for pions [8]

$$\mathcal{L}_{eff} = \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} + \frac{1}{6F^2}\left[(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 - \vec{\pi}^2(\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi})\right] + ... \tag{22}$$

where F is a coupling constant with mass dimension 1, and $\vec{\pi}$ is the pion triplet. Here the derivative couplings are between scalar fields, while in Eq. (21) the couplings are between scalar and vector fields. Also just as with the Lagrangian in Eq. (21) the effective Lagrangian in Eq. (22) is non-renormalizable.

Although, simple mass terms, like $m^2\phi\phi^*$, are forbidden by the dual gauge transformation (13), one can use the invariance of $\phi + \phi^*$ to add a term like $m^2(\phi + \phi^*)^2$, which is a mass term for the real part of the scalar field. Writing out the complex scalar dual gauge field in terms of real components ($\phi = \varphi_1 + i\varphi_2$) one notices the combination, $(\phi + \phi^*)$, contains only one real degree of freedom, φ_1. This comment also applies to $\partial_\mu\phi + \partial_\mu\phi^*$ which appears in the "kinetic" energy term for the scalar field, Eq. (20). Therefore, even though it appears that there are two scalar degrees of freedom associated with the complex scalar field, ϕ, only one degree of freedom, φ_1, has a proper kinetic energy or mass term. This result that only one degree of freedom (*i.e.* the real component) from the complex scalar field appears to be dynamical is related to the form of transformation of the scalar field given in Eq. (13). Making use of this "gauge" freedom one can chose $\Lambda(x) = \varphi_2(x)$ thus transforming away the complex degree of freedom of ϕ. There are other terms which could be added to $\mathcal{L}_{total}$ that preserve the dual gauge invariance: $(\phi + \phi^*)^n$, $(W_\mu W^{\mu*})^m$, or $(\phi + \phi^*)^n(W_\mu W^{\mu*})^m$, where n and m are arbitrary integers.

We have laboriously arrived at this Lagrange density using the infinitesimal form of the dual gauge transformation. In the next section we will show how the first term in $\mathcal{L}_{total}$ can be interpreted as a dual covariant derivative, leading much more quickly and elegantly to the invariance of the Lagrange density under the local dual transformations.

IV. COMPARISON WITH ORDINARY GAUGE SYMMETRY

In the previous section we have shown that there is a close connection between the standard gauge principle and the dual gauge principle that arises by making the electric-magnetic dual symmetry local. The difference is that the scalar and vector fields have switched roles, and are thus in some sense duals of one another. In an ordinary gauge theory one has the phase transformation $\phi \rightarrow e^{-ie\Lambda(x)}\phi$, and the gauge transformation $A_\mu \rightarrow A_\mu - \partial_\mu \Lambda$. For the dual gauge theory one has the local, dual symmetry $W_\mu \rightarrow e^{-ig\Lambda(x)}W_\mu$ and the dual gauge transformation $\phi \rightarrow \phi - i\Lambda(x)$. In an ordinary gauge theory one starts with scalar or spinor fields and introduces vector fields in order to have the phase symmetry of the scalar or spinor fields become local. For the dual gauge symmetry one starts with vector fields and introduces scalar fields so that the dual symmetry of the vector fields can become local. In an ordinary gauge theory one introduces a kinetic term for the vector fields, $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, which is invariant by itself under the gauge transformation of A_μ. In the dual gauge theory one introduces a kinetic term for the dual gauge fields, $(\partial_\mu\phi + \partial_\mu\phi^*)(\partial^\mu\phi + \partial^\mu\phi^*)$, which is invariant by itself under the dual gauge transformation of ϕ. In an ordinary gauge theory one starts with a kinetic energy Lagrangian for the matter fields $\partial_\mu\phi\partial^\mu\phi^*$, and the interaction terms between the matter and gauge fields arise as a result of the replacement of the ordinary derivative by the covariant derivative, $\partial_\mu\phi \rightarrow \partial_\mu\phi - ieA_\mu\phi$. In the dual gauge theory one can introduce a similar concept. Changing the derivatives of the vector fields in the following way

$$
\begin{aligned}
\partial_\mu W_\nu &\rightarrow \partial_\mu W_\nu - g\partial_\mu\phi W_\nu \\
\partial_\mu W_\nu^* &\rightarrow \partial_\mu W_\nu^* - g\partial_\mu\phi^* W_\nu^*
\end{aligned}
\tag{23}
$$

one has

$$
\begin{aligned}
G_{\mu\nu} &\rightarrow G_{\mu\nu} - g\partial_\mu\phi W_\nu + g\partial_\nu\phi W_\mu \\
G_{\mu\nu}^* &\rightarrow G_{\mu\nu}^* - g\partial_\mu\phi^* W_\nu^* + g\partial_\nu\phi^* W_\mu^*
\end{aligned}
\tag{24}
$$

which can be seen to give the first term in the Lagrangian of Eq. (21) starting from $-\frac{1}{4}G_{\mu\nu}G^{\mu\nu*}$. In addition one can show that the dual "covariant" derivative in Eq. (23) transforms as

$$
\partial_\mu W_\nu - g\partial_\mu\phi W_\nu \rightarrow e^{-ig\Lambda(x)}(\partial_\mu W_\nu - g\partial_\mu\phi W_\nu)
\tag{25}
$$

$$
\partial_\mu W_\nu^* - g\partial_\mu\phi^* W_\nu^* \rightarrow e^{ig\Lambda(x)}(\partial_\mu W_\nu^* - g\partial_\mu\phi^* W_\nu^*)
\tag{26}
$$

The dual covariant derivative also provides an easier and more elegant way of seeing that terms like $G_{\mu\nu}G^{\mu\nu*}$ (and also terms like $G_{\mu\nu}\widetilde{G}^{\mu\nu*}$) can be made consistent with the local dual symmetry.

Finally in ordinary gauge theories mass terms for the vector gauge bosons, $m^2 A_\mu A^\mu$, are forbidden by the gauge transformation. Under the dual gauge transformation mass terms and self interaction terms are allowed for the vector fields. One can have terms like $W_\mu W^{\mu*}$ or $(W_\mu W^{\mu*})^2$ which are invariant under the dual gauge transformation. Similar quartic terms for the vector fields can arise in ordinary gauge theories if the gauge symmetry is non-Abelian. In our example of the preceding section we started with an Abelian theory in Eq. (10), which satisfied an ordinary gauge symmetry – Eq. (3). Our final Lagrangian – Eq. (21) – no longer satisfied this original gauge symmetry, but instead satisfied a local, dual symmetry. Thus one local symmetry has been lost or exchanged in favor of another.

V. CONCLUSIONS

By gauging the electric-magnetic dual symmetry of Maxwell's field equations the roles of the vector fields and the scalar fields are to some extent exchanged. In the dual gauge theory the scalar fields arise from the dual gauge principle, in the same way that in ordinary gauge theories the vector fields arise from the ordinary gauge principle. There are other interesting formulations of the ordinary gauge principle [9] where scalar gauge fields arise in *conjunction* with the usual vector gauge fields. For the dual gauge theory, however, the roles of the scalar and vector fields are exchanged. There have been other recent attempts [10] to make the dual symmetry of the Schwarz-Sen electromagnetic action [11] local. In Ref. [10] this is done without the introduction of a scalar field. There are two obvious extensions of the dual gauge idea:

1. Non-Abelian theories have been shown to have dualities similar to the electric-magnetic duality of the Abelian Maxwell equations [12]. Thus, one could consider gauging the dual symmetry for a non-Abelian gauge theory. However, there are significant differences between the Abelian electric-magnetic duality discussed here, and the non-Abelian version given in Ref. [12].

2. In our present example of the dual gauge idea we have been able to "derive" scalar fields from the gauging of the dual symmetry. One could ask if it is possible to "derive" fermionic fields from the dual gauge idea. In our example the scalar fields were first introduced in Eq. (12) via $\mathcal{L}_2$. The dual gauge transformations of these scalar fields were then chosen as that the first term in $\delta\mathcal{L}_2$, which arose from the variation of the scalar fields, would cancel $\delta\mathcal{L}_1$. To accomplish the same thing with fermionic field we would need to replace $\partial_\mu\phi$, $\partial_\mu\phi^*$ in Eq. (12) with fermionic terms which also have one Lorentz index (*e.g.* $\partial_\mu\phi$, $\partial_\mu\phi^* \to \partial_\mu[\bar\psi\psi]$ or $\bar\psi\gamma_\mu\psi$). The fermionic fields would then need to satisfy some transformation akin to Eq. (13) so that the variation of the fermionic fields would cancel $\delta\mathcal{L}_1$.

The dual gauge principle given in this article replaces the vector, gauge field with the derivative of a scalar field in the definition of the covariant derivative (*i.e.* $\partial_\mu - ieA_\mu \to \partial_\mu - g\partial_\mu\phi$ or $ieA_\mu \to g\partial_\mu\phi$). There are other cases when a vector, gauge field can be identified with the derivative of scalar fields. For pure SU(2) gauge theory it is possible to make the following ansatz [13] [14]

$$A_\mu^a = (\epsilon_{0a\mu\nu} \mp ig_{a\mu}g_{\nu 0} \pm ig_{a\nu}g_{\mu 0})\frac{\partial^\nu\phi}{\phi} = \eta_{a\mu\nu}\frac{\partial^\nu\phi}{\phi} \tag{27}$$

where $\epsilon_{\alpha\beta\mu\nu}$ is the 4D Levi-Civita symbol, and $g_{\mu\nu}$ is the metric tensor. This ansatz turns the SU(2) Yang-Mills theory into a massless ϕ^4 theory. The relationship between the vector gauge field and scalar field given in Eq. (27) is more complicated than the relationship implied by the comparison of the ordinary and dual gauge covariant derivative (*i.e.* $ieA_\mu \to g\partial_\mu\phi$). Nevertheless both relationships involve the association/replacement of a vector field by the partial derivative of a scalar field.

VI.   ACKNOWLEDGMENT

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