Field-theory results for three-dimensional transitions with complex symmetries

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Abstract

We discuss several examples of three-dimensional critical phenomena that can be described by Landau-Ginzburg-Wilson ϕ^4 theories. We present an overview of field-theoretical results obtained from the analysis of high-order perturbative series in the frameworks of the ϵ and of the fixed-dimension $d = 3$ expansions. In particular, we discuss the stability of the O(N)-symmetric fixed point in a generic N-component theory, the critical behaviors of randomly dilute Ising-like systems and frustrated spin systems with noncollinear order, the multicritical behavior arising from the competition of two distinct types of ordering with symmetry O(n_1) and O(n_2) respectively.

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1 Introduction

In the framework of the renormalization-group (RG) approach to critical phenomena, a quantitative description of many continuous phase transitions can be obtained by considering an effective Landau-Ginzburg-Wilson (LGW) theory, containing up to fourth-order powers of the field components. The simplest example is the O(N)-symmetric ϕ^4 theory,

$$\mathcal{H}_{O(N)} = \int d^d x \Big[\frac{1}{2} \sum_i (\partial_\mu \Phi_i)^2 + \frac{1}{2} r \sum_i \Phi_i^2 + \frac{1}{4!} u \sum_{ij} \Phi_i^2 \Phi_j^2 \Big], \tag{1}$$

where Φ is an N-component real field. This model describes several universality classes: the Ising one for $N = 1$ (e.g., liquid-vapor transition), the XY one for $N = 2$ (e.g., superfluid transition in ^{4}He), the Heisenberg one for $N = 3$ (isotropic magnets), and long self-avoiding walks for $N \to 0$ (dilute polymers). See, e.g., Refs. [1, 2] for recent reviews. But there are also other physically interesting transitions described by LGW theories characterized by more complex symmetries.

The general LGW Hamiltonian for an N-component field Φ_i can be written as

$$\mathcal{H} = \int d^d x \Big[\frac{1}{2} \sum_i (\partial_\mu \Phi_i)^2 + \frac{1}{2} \sum_i r_i \Phi_i^2 + \frac{1}{4!} \sum_{ijkl} u_{ijkl}\, \Phi_i \Phi_j \Phi_k \Phi_l \Big], \tag{2}$$

where the number of independent parameters r_i and u_{ijkl} depends on the symmetry group of the theory. An interesting class of models are those in which $\sum_i \Phi_i^2$ is the only quadratic invariant polynomial. In this case, all r_i are equal, $r_i = r$, and u_{ijkl} satisfies the trace condition [3] $\sum_i u_{iikl} \propto \delta_{kl}$. In these models, criticality is driven by tuning the single parameter r. Therefore, they describe critical phenomena characterized by one (parity-symmetric) relevant parameter, which often corresponds to the temperature. Of course, there is also (at least one) parity-odd relevant parameter that corresponds to a term $\sum_i h_i \Phi_i$ that can be added to the Hamiltonian (2). For symmetry reasons, criticality is observed for $h_i \to 0$. There are several physical systems whose critical behavior can be described by this type of LGW Hamiltonians with two or more quartic couplings, see, e.g., Refs. [1, 4]. More general LGW Hamiltonians, that allow for the presence of independent quadratic parameters r_i, must be considered to describe multicritical behaviors arising from the competition of distinct types of ordering. A multicritical point can be observed at the intersection of two critical lines with different order parameters. In this case the multicritical behavior is achieved by tuning two relevant scaling fields, which may correspond to the temperature and to an anisotropy parameter [5].

In the field-theory (FT) approach the RG flow is determined by a set of RG equations for the correlation functions of the order parameter. In the case of a continuous transition, the critical behavior is determined by the stable fixed point (FP) of the theory, which characterizes a universality class. The absence of a stable FP is instead an indication for a first-order transition, even in those cases in which the mean-field approximation predicts a continuous transition. But, even in the presence of a stable FP, a first-order transition may still occur for systems that are outside its attraction domain. The RG flow can be studied by perturbative methods, by performing an expansion in powers of $\epsilon \equiv 4 - d$ [6] or a fixed-dimension (FD) expansion in powers of appropriate zero-momentum quartic couplings [7]. In these perturbative approaches, the computation and the resummation of high-order series is essential to obtain reliable results for three-dimensional transitions (see Refs. [1, 2] for reviews). Beside improving the accuracy, in some cases high-order calculations turn out to be necessary to determine the correct physical picture in three dimensions.

In this paper we give an overview of the perturbative FT results obtained for a number of three-dimensional transitions described by LGW Hamiltonians. In Sec. 2 we discuss the stability of the O(N)-symmetric fixed point under generic perturbations in three-dimensional N-component systems. In Sec. 3 we discuss the critical behavior of Ising-like systems with quenched disorder effectively coupled to the energy, for instance the randomly dilute Ising model. In Sec. 4 we consider frustrated spin models with noncollinear order, whose critical behavior is effectively described by an O(M)⊗O(N)-symmetric Hamiltonian with $M = 2$. Finally, in Sec. 5 we discuss the predictions of the O(n_1)⊕O(n_2)-symmetric ϕ^4 theory for the multicritical behavior observed near the point where two critical lines with symmetry O(n_1) and O(n_2) meet.

2 Stability of the O(N)-symmetric fixed point

In order to discuss the stability of the O(N) FP in a generic N-component system, it is convenient to consider polynomial perturbations $P^{a_1,\ldots,a_l}_{m,l}$, $m \geq l$, which are of degree m in the N-component field Φ^a and transform as the l-spin representation of the O(N) group. Explicitly formulae can be found in Ref. [8]. In addition, one should also consider perturbations containing derivatives of the field. At least near four dimensions, one can use standard RG arguments to show that, beside the O(N)-symmetric terms $P_{2,0} = \Phi^2$ and $P_{4,0} = (\Phi^2)^2$, only three other perturbations should be considered, $P^{ab}_{2,2}$, $P^{ab}_{4,2}$, and $P^{abcd}_{4,4}$. The stability properties of the O(N) FP depend on the RG dimensions $y_{m,l}$ of these perturbations.[1]

In Table 1 we report FT estimates of the RG dimensions $y_{m,l}$ for $N = 2, 3, 4, 5$, obtained from the analysis of six-loop FD and five-loop ϵ series [8–10].[2] The quadratic perturbations $P^{ab}_{2,2}$ are relevant for the description of the breaking of the O(N) symmetry down to O(M)⊕O($N - M$). Since $y_{2,2} > 0$, they are always relevant. The RG dimension $y_{4,2}$ is negative for any N, so that the corresponding spin-2 perturbation $P^{ab}_{4,2}$ is always irrelevant. On the other hand, the sign of $y_{4,4}$ depends on N: it is clearly negative for $N = 2$ and positive for $N \geq 4$. For $N = 3$ it is marginally positive, suggesting the instability of the O(3) FP under generic spin-4 quartic perturbations. Actually the stability of the O(N) FP can be inferred from the RG flow of the cubic-symmetric LGW Hamiltonian for an N-component field

$$\mathcal{H}_c = \int d^d x \left\{ \frac{1}{2} \sum_{i=1}^{N} \Big[(\partial_\mu \Phi_i)^2 + r \Phi_i^2 \Big] + \frac{1}{4!} \Big[u (\sum_i^N \Phi_i^2)^2 + v \sum_i^N \Phi_i^4 \Big] \right\}. \tag{3}$$

The point is that the cubic-symmetric perturbation $\sum_i \Phi_i^4$ is a particular combination of the spin-4 operators $P^{abcd}_{4,4}$ and of the spin-0 term $P_{4,0}$. The RG flow for the cubic-symmetric theory has been much investigated using various FT and lattice techniques [1]. The O(N)

[1]Note that $P^{ab}_{2,2}$ and $P^{abcd}_{4,4}$ are RG eigenoperators, while $P^{ab}_{4,2}$ must be in general properly subtracted, i.e. the RG eigenoperator is $P^{ab}_{4,2} + z P^{ab}_{2,2}$ for a suitable value of z.

[2]Results obtained in other theoretical approaches and in experiments can be found in Refs. [1, 8] and references therein.

Table 1: Three-dimensional estimates of the RG dimensions $y_{m,l}$ from ϵ and FD expansions.

N		$y_{2,0} = \nu^{-1}$	$y_{2,2}$	$\phi_T \equiv y_{2,2}\nu$	$y_{4,0}$	$y_{4,2}$	$y_{4,4}$	$\phi_{4,4} \equiv y_{4,4}\nu$
2	ϵ	1.497(8)	1.766(6)	1.174(12)	−0.802(18)	−0.624(10)	−0.114(4)	−0.077(3)
	FD	1.493(3)		1.184(12)	−0.789(11)		−0.103(8)	−0.069(5)
3	ϵ	1.419(11)	1.790(3)	1.260(11)	−0.794(18)	−0.550(14)	0.003(4)	0.002(3)
	FD	1.414(7)		1.27(2)	−0.782(13)		0.013(6)	0.009(4)
4	ϵ	1.357(15)	1.813(6)	1.329(16)	−0.795(30)	−0.493(14)	0.105(6)	0.079(5)
	FD	1.350(11)		1.35(4)	−0.774(20)		0.111(4)	0.083(3)
5	ϵ	1.333(36)	1.832(8)	1.40(3)	−0.783(26)	−0.441(13)	0.198(11)	0.151(9)
	FD	1.312(12)		1.40(4)	−0.790(15)		0.189(10)	0.144(8)
∞		1	2	2	−1	0	1	1

FP turns out to be unstable for $N > N_c$ with $N_c \approx 3$. The most accurate results have been provided by analyses of high-order FT perturbative expansions, six-loop FD and five-loop ϵ series, see e.g. Refs. [11, 12], which find $N_c \lesssim 2.9$ in three dimensions, and the existence of a stable FP characterized by a reduced cubic symmetry for $N \geq N_c$. These results imply that the O(N)-symmetric FP is unstable under spin-4 quartic perturbations for $N \geq 3$, and can be applied to establish the stability of the O(N) FP in any physical critical phenomenon that is effectively described by a generic LGW Hamiltonian for an N-component field.[3]

3 Randomly dilute Ising model

In the last few decades many theoretical and experimental studies have investigated the critical properties of statistical models in the presence of quenched disorder. A typical example is obtained by mixing a uniaxial antiferromagnet with a nonmagnetic material, such as $Fe_xZn_{1-x}F_2$ and $Mn_xZn_{1-x}F_2$. These materials can be modeled by the randomly dilute Ising model (RDIM)

$$\mathcal{H}_{\rm RDIM} = J \sum_{<ij>} \rho_i \, \rho_j \; s_i s_j, \tag{4}$$

where the sum is extended over all nearest-neighbor sites of a lattice, $s_i = \pm 1$ are the spin variables, ρ_i are uncorrelated quenched random variables, which are equal to one with probability x (the spin concentration) and zero with probability $1-x$ (the impurity concentration). Above the percolation threshold of the spin concentration, the critical behavior of the RDIM belongs to a new universality class that is distinct from the Ising universality class of pure systems, and that is shared by all systems with quenched disorder effectively coupled to the energy. See, e.g., Refs. [1, 13, 14] for recent reviews.

Using the FT approach and the replica method, one arrives at the effective LGW Hamiltonian $\mathcal{H}_c$ [15], cf. Eq. (3), which is expected to describe the critical properties of the RDIM in the limit $N \to 0$. The most precise FT results for the critical exponents have been obtained by analyzing the FD six-loop expansions [16, 17]. The major drawback of the FT

[3]Note that the condition that $\sum \Phi_i^2$ is the only quadratic invariant forbids the presence in the Hamiltonian of any spin-2 term $P_{2,2}^{ab}$. Analogously, the trace condition $\sum_i u_{iikl} \propto \delta_{kl}$ forbids quartic polynomials transforming as the spin-2 representation of the O(N) group, i.e. the operators $P_{4,2}^{ab}$.

Table 2: Critical exponents for the RDIM universality class.

		γ	ν	α	β
six-loop FD	[16]	1.330(17)	0.678(10)	−0.034(30)	0.349(5)
Monte Carlo	[18]	1.342(10)	0.684(5)	−0.051(16)	0.3546(28)
$Fe_xZn_{1-x}F_2$	[13]	1.31(3)	0.69(1)	−0.10(2)	0.359(9)

perturbative approach is the non-Borel summability of the series due to a more complicated analytic structure of the field theory corresponding to quenched disordered models. Nevertheless, series analyses seem to provide sufficiently robust estimates, which are in good agreement with experiments and recent Monte Carlo simulations. The results of the six-loop analysis are reported in Table 2, where they are compared with estimates obtained in Monte Carlo simulations of the RDIM and in experiments on uniaxial magnets. The values of the exponents are definetely different from those of the pure Ising universality class, where, e.g., $\nu = 0.63012(16)$ [19].

Using the FT approach, one can also compute the critical exponent ϕ describing the crossover from random-dilution to random-field critical behavior in Ising systems, and in particular the crossover observed in dilute anisotropic antiferromagnets when an external magnetic field is applied [13]. The crossover exponent ϕ is related to the RG dimensions of the quadratic operator $\Phi_i\Phi_j$ $(i \neq j)$ in the limit $N \to 0$ [20]. Six-loop computations [21] provide the estimate $\phi = 1.43(1)$, which turns out to be in good agreement with the available experimental estimates, for example $\phi = 1.42(3)$ for $Fe_xZn_{1-x}F_2$ [13].

Finally, we mention that six-loop perturbative series for multicomponent systems with quenched disorder, taking also into account a possible cubic anisotropy, have been computed and analyzed in Refs. [16, 22].

4 Frustrated spin models with noncollinear order

In physical magnets noncollinear order is due to frustration that may arise either because of the special geometry of the lattice, or from the competition of different kinds of interactions [23]. Typical examples of systems of the first type are stacked triangular antiferromagnets (STA's), where magnetic ions are located at each site of a three-dimensional stacked triangular lattice. On the basis of the structure of the ground state, in an N-component STA one expects a transition associated with a breakdown of the symmetry from O(N) in the HT phase to O($N-2$) in the LT phase. The nature of the transition is still controversial. In particular, the question is whether the critical behavior belongs to a new chiral universality class, as originally conjectured by Kawamura [24]. On this issue, there is still much debate, FT methods, Monte Carlo simulations, and experiments providing contradictory results in many cases (see, e.g., Ref. [1] for a recent review of results). Overall, experiments on STA's favor a continuous transition belonging to a new chiral universality class.

The determination of an effective LGW Hamiltonian describing the critical behavior leads to the O(M)⊗O(N)-symmetric theory [24]

$$\mathcal{H}_{ch} = \int d^d x \Big\{ \frac{1}{2} \sum_a \Big[(\partial_\mu \phi_a)^2 + r\phi_a^2 \Big] + \frac{1}{4!} u \Big(\sum_a \phi_a^2 \Big)^2 + \frac{1}{4!} v \sum_{a,b} \Big[(\phi_a \cdot \phi_b)^2 - \phi_a^2 \phi_b^2 \Big] \Big\}, \quad (5)$$

where ϕ_a, $a = 1, \ldots, M$, are N-component vectors. The case $M = 2$ with $v > 0$ describes frustrated spin models with noncollinear order; [4] $N = 2$ and $N = 3$ correspond to XY and Heisenberg systems, respectively. Recently the Hamiltonian (5) has been also considered to discuss the phase diagram of Mott insulators [25]. See Refs. [1,23] for other applications.

Six-loop calculations [26] in the framework of the $d = 3$ FD expansion provide a rather robust evidence for the existence of a new stable FP in the XY and Heisenberg cases corresponding to the conjectured chiral universality class, and contradicting earlier studies based on much shorter (three-loop) series [27]. It has also been argued [28] that the stable chiral FP is actually a focus, due to the fact that the eigenvalues of its stability matrix turn out to have a nonzero imaginary part. The new chiral FP's found for $N = 2, 3$ should describe the apparently continuous transitions observed in STA's. The FT estimates of the critical exponents are in satisfactory agreement with the experimental results, including the chiral crossover exponent related to the chiral degrees of freedom [29]. We also mention that high-order FT analyses of two-dimensional systems have been reported in Ref. [30].

On the other hand, other FT studies, see, e.g., Ref. [31], based on approximate solutions of continuous RG equations, do not find a stable FP, thus favoring a weak first-order transition. Monte Carlo simulations have not been conclusive in setting the question, see, e.g., Refs. [32–34]. Since all the above approaches rely on different approximations and assumptions, their comparison and consistency is essential before considering the issue substantially understood.

5 Multicritical behavior in O(n_1)⊕O(n_2) theories

The competition of distinct types of ordering gives rise to multicritical behavior. More specifically, a multicritical point (MCP) is observed at the intersection of two critical lines characterized by different order parameters. MCP's arise in several physical contexts, for instance in anisotropic antiferromagnets, in high-T_c superconductors, in ^{4}He, etc. The multicritical behavior arising from the competition of two orderings characterized by O(n) symmetries is determined by the RG flow of the most general O(n_1)⊕O(n_2)-symmetric LGW Hamiltonian involving two fields ϕ_1 and ϕ_2 with n_1 and n_2 components respectively, i.e. [5]

$$\mathcal{H}_{mc} = \int d^d x \Big[\tfrac{1}{2} (\partial_\mu \phi_1)^2 + \tfrac{1}{2} (\partial_\mu \phi_2)^2 + \tfrac{1}{2} r_1 \phi_1^2 + \tfrac{1}{2} r_2 \phi_2^2 + u_1 (\phi_1^2)^2 + u_2 (\phi_2^2)^2 + w \phi_1^2 \phi_2^2 \Big]. \quad (6)$$

The critical behavior at the MCP is determined by the stable FP when both r_1 and r_2 are tuned to their critical value. An interesting possibility is that the stable FP has O(N) symmetry, $N \equiv n_1 + n_2$, so that the symmetry gets effectively enlarged approaching the MCP.

The phase diagram of the model with Hamiltonian (6) has been investigated within the mean-field approximation in Ref. [35]. If the transition at the MCP is continuous, one may

[4] Negative values of v correspond to magnets with sinusoidal spin structures.

observe either a bicritical or a tetracritical behavior. But it is also possible that the transition at the MCP is of first order. $O(\epsilon)$ calculations in the framework of the ϵ expansion [5] show that the isotropic O(N)-symmetric FP ($N \equiv n_1 + n_2$) is stable for $N < N_c = 4 + O(\epsilon)$. With increasing N, a new FP named biconal FP (BFP), which has only O(n_1)$\oplus$O(n_2) symmetry, becomes stable. Finally, for large N, the decoupled FP (DFP) is the stable one. In this case, the two order parameters are effectively uncoupled at the MCP, giving rise to a tetracritical behavior.

The $O(\epsilon)$ computations provide useful indications on the RG flow in three dimensions, but a controlled extrapolation to $\epsilon = 1$ requires much longer series and an accurate resummation exploiting their Borel summability. For this purpose we have extended the ϵ expansion to $O(\epsilon^5)$ [8]. A robust picture of the RG flow predicted by the O(n_1)$\oplus$O(n_2)-symmetric LGW theory can be achieved by supplementing the analysis of the ϵ series with the results for the stability of the O(N) FP (cf. Sec. 2), which were also obtained by analyzing six-loop FD series, and with nonperturbative arguments allowing to establish the stability of the DFP [36]. Since the Hamiltonian (6) contains spin-4 quartic perturbations with respect to the O(N) FP, the results for the spin-4 RG dimension $y_{4,4}$ (cf. Table 1) imply that the O(N) FP is stable only for $N = 2$, i.e. when two Ising-like critical lines meet. It is unstable in all cases with $N \geq 3$. This implies that for $N \geq 3$ the enlargement of the symmetry O(n_1)$\oplus$O(n_2) to O(N) does not occur, unless an additional parameter is tuned beside those associated with the quadratic perturbations. For $N = 3$, i.e. for $n_1 = 1$ and $n_2 = 2$, the critical behavior at the MCP is described by the BFP, whose critical exponents turn out to be very close to those of the Heisenberg universality class. For $N \geq 4$ and for any n_1, n_2 the DFP is stable, implying a tetracritical behavior. This can also be inferred by using nonperturbative arguments [36] that allow to determine the relevant stability eigenvalue from the critical exponents of the O(n_i) universality classes.

Anisotropic antiferromagnets in a uniform magnetic field $H_\parallel$ parallel to the anisotropy axis present a MCP in the $T - H_\parallel$ phase diagram, where two critical lines belonging to the XY and Ising universality classes meet [5]. The above results predict a multicritical behavior described by the BFP, contradicting the $O(\epsilon)$ calculations that suggested the stability of the O(3) FP. Notice that it is hard to distinguish the biconal from the O(3) critical behavior. For instance, the correlation-length exponent ν differs by less than 0.001 in the two cases.

The case $N = 5$, $n_1 = 2$, $n_2 = 3$ is relevant for the SO(5) theory [37,38] of high-T_c superconductors, which proposes a description of these materials in terms of a three-component antiferromagnetic order parameter and a d-wave superconducting order parameter with U(1) symmetry, with an approximate O(5) symmetry. Within the SO(5) theory, it has been speculated that the antiferromagnetic and superconducting transition lines meet at a MCP in the temperature-doping phase diagram, which is bicritical and shows an effectively enlarged O(5) symmetry. There are also recent claims in favor of the stability of the O(5) FP based on Monte Carlo simulations of three-dimensional five-component systems [39]. Our results on the RG flow of the O(2)$\oplus$O(3) theory show that the O(5) FP cannot describe the asymptotic critical behavior at the MCP, unless a further tuning of the parameters is performed. Therefore, the O(5) symmetry is not effectively realized at the point where the antiferromagnetic and superconducting transition lines meet. The multicritical behavior is either governed by the tetracritical decoupled fixed point or is of first-order type if the system is outside its attraction domain. The predicted tetracritical behavior may explain a number of recent experiments that provided evidence of a coexistence region of the antiferromagnetic and superconducting phases, see, e.g., Ref. [40]. The O(5) FP is unstable with a crossover exponent $\phi_{4,4} \approx 0.15$, which, although rather small, is nonetheless sufficiently large not to exclude the possibility of observing the RG flow towards the eventual asymptotic behavior for reasonable values of the reduced temperature, even in systems with a moderately small breaking of the O(5) symmetry, such as those described by the projected SO(5) model discussed in Refs. [38, 41].

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