Collapsing sphere on the brane radiates

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We study the analogue of the Oppenheimer-Snyder model of a collapsing sphere of homogeneous dust on the Randall-Sundrum type brane. We show that the collapsing sphere has the Vaidya radiation envelope which is followed by the brane analogue of the Schwarzschild solution described by the Reissner-Nordström metric. The collapsing solution is matched to the brane generalized Vaidya solution and which in turn is matched to the Reissner-Nordström metric. The mediation by the Vaidya radiation zone is the new feature introduced by the brane. Since the radiating mediation is essential, we are led to the remarkable conclusion that a collapsing sphere on the brane does indeed, in contrast to general relativity, radiate null radiation.

PACS numbers: 04.50.+h; 04.70.Bw; 98.80.Cq; 12.10.-g; 11.25.Mj

The seat of high energy modifications to general relativity (GR) would be the neighbourhoods of singularities occurring in gravitational collapse and the cosmological big-bang explosion. The study of situations leading to such singular events would therefore be most pertinent in the models that claim to incorporate such modifications. The Randall-Sundrum (RS) brane world model [1] is one such model which has currently attracted great attention and activity. In this paper, we wish to study gravitational collapse of a homogeneous dust sphere on the RS type brane. In GR, it is described by the well-known Oppenheimer-Snyder (OS) model [2] in which the interior of collapsing sphere is given by the Friedmann metric while the exterior is the static Schwarzschild solution. The two are matched continuously on the moving boundary. This reflects the well-known classical result that a collapsing sphere does not radiate. In contrast it turns out that a collapsing dust sphere on the brane would require in general a non static exterior [3], and that could be the Vaidya radiating solution on the brane [4]. That is, a collapsing sphere must have a radiation envelope described by the brane analogue of the Vaidya radiating solution [4], which could finally be matched to the brane analogue of the Schwarzschild solution described by the Reissner-Nordström (RN) metric [5]. We shall thus establish that the analogue of the Oppenheimer-Snyder collapse on the brane is a Vaidya radiating sphere.

String theory and M-theory represent one of the routes to find a covering theory for GR, which would possibly be a theory of quantum gravity. In this approach, gravitation becomes a higher dimensional interaction with the 4-D conventional GR as its low energy limit. In the brane world scenario, the matter fields remain confined to the 3-brane, which is our 4-D Universe we live in, while gravity can propagate in higher dimensional spacetime called the bulk. In the RS brane model, gravity is localized on the brane by the curvature of the bulk spacetime which is an Einstein space with negative Λ. What essentially happens is that gravitational field gets "reflected" back onto the brane through the Weyl curvature of the bulk as a trace free matter field. It is termed the Weyl (dark) radiation on the brane. This is the non-local effect mediated by the bulk Weyl curvature while the local effect manifests through the extrinsic curvature of the brane resulting into square of the stresses. This leads to an effective Einstein equation on the brane [6] which incorporates both local as well as non-local effects representing the high energy modifications of GR. At the very basic level the brane modification to the Newtonian potential goes as r^{-3}, which is symptomatic of the anti de-Sitter (AdS_5) bulk.

A solution of the effective vacuum equation on the brane for a static black hole was obtained [5], and it turned out to be the RN metric of a charged black hole in GR. Here charge refers not to electric charge but instead to the Weyl tidal charge as a measure of the reflected gravitational field energy from the bulk. (We would refer it as the Weyl RN (WRN) solution.) Finding the corresponding solution in the bulk and then to match it onto the black hole solution on the brane with proper boundary conditions is a very difficult and formidable task. The evolution and extension of the brane solution into the bulk numerically as well as analytically is also wrought with considerable difficulties ([7] and [8]). As a first step, the consideration is therefore focussed on solving the effective equation on the brane without reference to the bulk spacetime except that it is taken to have non-zero Weyl curvature.

The question of gravitational collapse of a homoge-

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neous dust sphere on the brane has very recently been addressed [3] in which it was concluded that the exterior spacetime has to be non-static unless the collapsing sphere is of pure Weyl radiation. A model of a star in hydrostatic equilibrium on the brane has also been considered [9]. In that the interior is the brane analogue of the Schwarzschild interior solution of uniform density while the exterior is no longer unique as was the case in GR. This is because the exterior spacetime is now not vacuum. In the context of the cosmic censorship conjecture, collapse of the Vaidya null radiation onto an empty cavity on the brane has also been studied [4]. It is shown that the brane effects favour formation of a black hole against a naked singularity. This is in line with the expectation that the brane effects tend to strengthen the gravitational field. In all these studies, only the effective equation on the brane has been solved without reference to the bulk spacetime except that it is assumed to have non-zero Weyl curvature to project dark radiation on the brane. We would also resort to the same strategy and would only address the effective Einstein equation on the brane.

Since the exterior has to be non-static as well as free of collapsing matter, it could only have radial energy flux which could sustain the non-static character. The two possiblities are the Vaidya null radiation and the heat flux. The latter is ruled out because there cannot exist a spherically symmetric solution with heat flux alone as the source. This leaves only the null radiation for the exterior, which is described by the Vaidya solution on the brane [4]. In this paper we wish to show that the brane analogue of the Oppenheimer-Snyder (OS) collapse of homogeneous dust sphere is: Friedmann collapsing metric for the interior which is matched to the Vaidya radiating envelope, which in turn matches finally to the asymptotically flat WRN metric. Here and henceforth all solutions would refer to their brane analogues, while the analogue of the Schwarzschild would be referred explicitly as WRN. This is the canonical paradigmical picture of gravitational collapse on the brane.

The effective Einstein field equation on the brane is given by [6]

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu} + \frac{48\pi G}{\lambda} S_{\mu\nu} - \mathcal{E}_{\mu\nu} \qquad (1)$$

where $\lambda > 10^8 GeV^4$ is the brane tension, Λ is the brane cosmological constant, $S_{\mu\nu}$ represents the quadratic stresses and $\mathcal{E}_{\mu\nu}$ is the projection of the Weyl curvature on the brane giving the Weyl radiation. The Bianchi identities imply the following conservation laws:

$$\nabla^\nu T_{\mu\nu} = 0, \qquad \nabla^\nu \mathcal{E}_{\mu\nu} = \frac{48\pi G}{\lambda} \nabla^\nu S_{\mu\nu}, \qquad {\mathcal{E}_\mu}^\mu = 0 \quad (2)$$

The above equations do not however close because of the presence of 5-D degrees of freedom in $\mathcal{E}_{\mu\nu}$. For this, the Lie derivatives of the brane should be considered [6], which would then close the system. We shall however not address the full 5-D equations and would rather confine ourselves to the brane degrees of freedom.

Now we consider a collapsing dust cloud described by the Friedmann metric in isotropic coordinates

$$ds^2 = -d\tau^2 + \frac{a(\tau)^2}{(1+\frac{1}{4}kr^2)^2}\left[dr^2 + r^2 d\Omega^2\right] \qquad (3)$$

where τ is the proper time. The proper radius is given by

$$R(\tau) = \frac{ra(\tau)}{1+\frac{1}{4}kr^2}$$

. The modified Friedman equation can be written as

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho\left[1+\frac{\rho}{2\lambda}\right] + \frac{C}{\lambda a^4} - \frac{k}{a^2} + \frac{\Lambda}{3} \qquad (4)$$

where C is the Weyl radiation constant fixed by the bulk Weyl curvature, and the ρ^2 term is the local high energy correction. The ususal Friedmann collapse is recovered when $\lambda^{-1} \to 0$. From the conservation equation (2), it follows that $\rho = \rho_0(a_0/a)^3$ where a_0 is the intial radius from which the collapse began.

Since the collapsing boundary surface Σ is free falling we can write

$$R_\Sigma(\tau) = \frac{r_0 a(\tau)}{(1+\frac{1}{4}kr_0^2)^2}$$

where $r = r_0 = constant$. This allows us to recast (4) into

$$\dot{R}^2 = \frac{2GM}{R} + \frac{3GM^2}{4\pi\lambda R^4} + \frac{Q}{\lambda R^2} + E + \frac{\Lambda}{3}R^2. \qquad (5)$$

In the above equation the total energy per proper stellar volume, M and the total "tidal charge" Q are given by [3]

$$M = \frac{4\pi a_0^3 r_0^3 \rho_0}{3(1+\frac{1}{4}kr_0^2)^3}, \qquad Q = C\frac{r_0^4}{(1+\frac{1}{4}kr_0^2)^4} \qquad (6)$$

and the "energy" per unit mass assumes the following form

$$E = -\frac{kr_0^2}{(1+\frac{1}{4}kr_0^2)^2}, \qquad E > -1. \qquad (7)$$

The brane contribution to the collapse comes through ρ^2 and the Weyl radiation. It would turn out that it is the latter which is responsible for the non-static exterior. As argued above, the only possible choice for the exterior is the Vaidya null radiating spacetime. On the brane it would be obtained by solving the equation,

$$G_{\mu\nu} = \Lambda g_{\mu\nu} - 8\pi\sigma k_\mu k_\nu + \mathcal{E}_{\mu\nu} \qquad (8)$$

where $k_\mu k^\mu = 0$ which implies $S_{\mu\nu} = 0$. For the exterior, we seek the general solution (for the Weyl stresses, as for WRN, the null energy condition, $\mathcal{E}_{\mu\nu}k^\mu k^\nu = 0$ is assumed) of this equation which is given by [4],

$$\begin{aligned} ds^2 &= -\left(1 - \frac{2Gm(v)}{\mathsf{r}} - \frac{Q(v)}{\mathsf{r}^2} - \frac{\Lambda^2}{3}\right)dv^2 \\ &+2dvd\mathsf{r} + \mathsf{r}^2 d\Omega^2 \end{aligned} \tag{9}$$

where v is the retarded Eddington null coordinate. We would employ the standard 4-D Israel matching conditions on the brane which would require continuity of the metric and the extrinsic curvature of the boundary surface Σ. That would mean continuity of the metric components and of $\dot{R}$. Apart from these conditions, we also have one more condition coming from vanishing of fluid pressure, $p = 0$.

In matching of metrics (3) and (9) we employ the procedure due to Santos [10]. For the metric (9) the equation of the surface Σ is given by

$$f(\mathsf{r}, v) = \mathsf{r} - \mathsf{r}_\Sigma(v) = 0$$

. The first junction condition requires that the metric functions match smoothly across the boundary surface Σ. For the metrics (3) and (9) this yields

$$\begin{aligned} d\tau &= \left(1 - \frac{2Gm(v)}{\mathsf{r}_\Sigma} - \frac{\Lambda \mathsf{r}_\Sigma^2}{3} - \frac{Q(v)}{\mathsf{r}_\Sigma^2} + 2\frac{d\mathsf{r}}{dv}\right)^{\frac{1}{2}} dv \\ R_\Sigma &= \mathsf{r}_\Sigma(v). \end{aligned}$$

The second matching condition requires that the extrinsic curvature components K_{ij} are continuous across Σ which would be equivalent to continuity of $\dot{R}$. For the metric (9), we write

$$\dot{R}^2 = \frac{2Gm(v)}{R} + E + \frac{\Lambda}{3}R^2 + \frac{Q(v)}{\lambda R^2}$$

on Σ, which when compared to (5) yields the condition

$$m(v) = M + \frac{3M^2}{8\pi\lambda R^3} + \frac{1}{2RG\lambda}\left(Q - Q(v)\right) \tag{10}$$

where we have also used the continuity of the metric. Eqn. (5) is the same as eqn. (7) of [3] while eqn. (10) is the analogue of eqn. (15) of [3].

Apart from this we also have fluid pressure $p = 0$, which would further yield the condition

$$Q(v) = \left[\frac{3GM^2}{4\pi R^2} + Q\right]_\Sigma \tag{11}$$

Now eqn. (10) simply gives $m(v) = M$, which is the analogue of the OS condition $m(R) = M$ in GR. The new condition which marks the brane effects is the above equation (11). It is this condition which implies that a collapsing sphere on the brane must radiate because $Q(v)$ represents propagation of nonlocal stresses. On the other hand $Q(v) = 0$ would imply both $Q = 0$ (if projection of the bulk Weyl vanishes in the exterior so should for the interior as well) and $M = 0$, as predicted in [3]. There it was obtained by requiring $R = 4\Lambda$ in the exterior. This is the main result which simply follows from the matching conditions and the pressure free condition. The result would be true even when pressure is non-zero because on the boundary it will always vanish. A collapsing fluid sphere on the brane would therefore radiate and its exterior will have the radiation envelope described by the Vaidya solution.

Finally we shall match the Vaidya solution to the WRN static metric which could be done in a very straight forward manner. We now match the metric (9) to the WRN metric given by

$$ds^2 = -Adt^2 + \frac{dr^2}{A} + r^2 d\Omega^2 \tag{12}$$

where

$$A = 1 - 2\frac{G\mathcal{M}}{r} - \frac{q}{r^2} - \frac{\Lambda}{3}r^2$$

. The junction conditions required for the smooth matching of metrics (9) and (12) and their associated extrinsic curvature components yield

$$m(v) = \mathcal{M} + \frac{q}{2Gr} - \frac{Q(v)}{2Gr} \tag{13}$$

$$m(v) = \mathcal{M} + \frac{q}{Gr} - \frac{Q(v)}{Gr}. \tag{14}$$

Comparing (13) and (14) we obtain the expected result

$$m(v) = \mathcal{M}, \ Q(v) = q \tag{15}$$

This completes the matching exercise. It shows that a collapsing Friedmann sphere (OS collapse) on the brane has Vaidya radiation envelope followed by the WRN metric.

In the recent study of OS-like collapse on the RS type brane [3], it was concluded that exterior to the collapsing sphere cannot be static unless the collapse is that of pure Weyl radiation. We have argued that the non static character of exterior spacetime could only be sustained by the Vaidya null radiation flowing out radially from the collapsing sphere. We have therefore taken for exterior the radiating generalization of the static black hole solution on the brane as given in [4]. This is at one end matched to the Friedmann solution for interior of the collapsing sphere and at the other to the WRN static metric for the black hole on the brane [5]. The matching conditions relate the mass functions at the two boundary surfaces, and the new brane effects are negotiated by the Weyl radiation and the density square contributions. It is interesting to note that for a collapsing sphere to be non-vacuous, what is required is the Vaidya null radiation along with the Weyl radiation contribution. That is, the bulk must have non-zero Weyl curvature. The distinguishing feature of spherical collapse on the brane is that *it radiates out null radiation and requires the bulk spacetime to be conformally non-flat.*

From physical considerations as alluded earlier, it is clear that the Vaidya radiating solution is perhaps the correct and natural choice for the exterior of the collapsing sphere on the brane. Since it is simply the radiating generalization of WRN, it would also not have the right weak field limit of the brane modification to the Newtonian potential going as r^{-3}. The cause of the problem lies in the non-zero Weyl curvature of the bulk, which gives rise to the Weyl tidal charge producing r^{-2} term in the potential. In the Vaidya solution, null radiation part which is brane bound and hence is correctly incorporated. The problem is with the Weyl radiation stresses which are projected from the bulk and hence cannot truly be specified without reference to the bulk spacetime. This is the crux of the problem which runs through all the solutions on the brane. Note that in the Friedmann solution too, the equation of state for radiation is imposed on the Weyl stresses. They are, without reference to bulk spacetime, only supposed to be trace-free. The question would not therefore be resolved without the knowledge of bulk spacetime. For gaining some insight into the bulk, there have been attempts to analytically and numerically Taylor expand the brane solutions into the bulk [8, 13, 14]. In the absence of full knowledge of the bulk spacetime, this is the only way open to get some useful knowledge of the bulk in the close vicinity of the brane corresponding to the brane solutions.

Further it may also be noted that the RS brane world model is very critically tuned to conformally flat ADS_5 bulk [12, 14]. For instance, it has been shown that for a conformally non-flat bulk, there exists no bound state for zero mass gravitons on the brane [12]. This is the key feature of the RS brane model which seem to be so tightly tied to asymptotically AdS bulk [15]. Also recall that the weak field limit is obtained for a Minkowski brane with conformally flat ADS_5 bulk. It would therefore be fair to say that the question in its entirity is perhaps open. This is of course an issue of viewpoint which would have various shades of belief and perception.

The Weyl tidal charge in WRN is essentially the measure of gravitational field energy being "reflected" from the bulk on the brane as a trace-free matter field. It is expected to be negative [5, 6, 11]. It turns out that the brane modifications to GR would strengthen gravity (i.e. contribute in line with GR) only when tidal charge is negative [11]. This is the most interesting aspect of the brane world formulation in which bulk contributes actively to the dynamics of the brane. For this bulk must have non-zero Weyl curvature. Also note that in the Vaidya generalization of WRN, not only the mass M but the tidal charge Q also becomes function of retarded null coordinate v. This happens because Q is essentially caused by M (through the reflected field energy from the bulk) and hence it would be related to it [5, 6, 11].

The canonical picture that emerges for the spherical collapse on the brane is that the collapsing sphere has the radially propagating null radiation envelope followed by the static spacetime of a black hole on the brane. This is in contrast to the classical picture where a spherically collapsing or oscillating system is not supposed to radiate out energy. This happens because of the bulk gravitons projected stresses on the brane and also of effective pressure not being zero on the boundary. That means dynamically the system is, in contrast to the classical case, not closed and consequently physical information could travel from interior to exterior. This is why the exterior cannot be static. Though we have only considered homogeneous dust collapse, this feature would be true in general for collapse on the brane. Since it is radiative for spherical case, it would certainly be so for more general collapse. This is the general feature of collapse on the brane.

We have however not addressed the 5-D equations for determination of the bulk spacetime. When the problem is fully solved by incorporating the bulk spacetime, the canonical picture as alluded above would hold good qualitatively. What would change would be the exact form of the solutions like WRN and its Vaidya generalization. The picture, the collapsing sphere having a null radiation envelope followed by static spacetime of a black hole on the brane, would stay. It is remarkable that without reference to bulk spacetime, we are able to deduce qualtatively the distinguishing characteristic features of gravitational collapse on the brane.

Acknowledgments

The major part of this work was done when ND was visiting the University of Natal, Durban. It was sparked off by ND's discussions with Roy Maartens at GR-16. We thank Roy for setting us on the problem through exciting discussions as well as insightful critical comments on the manuscript and also Kesh Govinder for useful discussions. ND would like to thank Sunil Maharaj and the Relativity group at Durban for warm hospitality which has facilitated this work.

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