Lagrangian Sp(3) BRST symmetry for Yang-Mills theory

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Abstract

The Sp(3) BRST symmetry for Yang-Mills theory is derived in the framework of the antibracket-antifield formalism.

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The BRST theory [1, 2, 3, 4, 5] has evolved until being regarded as an extremely powerful assembly of rules allowing for a systematical quantum treatment of gauge theories. In the meantime, it was observed that the analysis of theories subject to gauge invariances, like renormalizability, anomalies and structure of non-minimal sectors, might be clarified via the introduction of extended BRST symmetries, like, for instance, the Sp(2) symmetries [6, 7, 8, 9]. Recently, it appeared a growing interest for constructing even more extended symmetries, namely, the Sp(3) BRST symmetry, at both the Hamiltonian [10] and Lagrangian [11] levels.

In this paper we construct the Lagrangian Sp(3) BRST symmetry for Yang-Mills theories. For proofs and details we refer to the general results from [11]. Our procedure goes as follows. First, we triplicate the gauge transformations of Yang-Mills fields and derive the ghost and antifield spectra. Second, we solve the fundamental equation of the Lagrangian Sp(3) BRST formalism, called the extended classical master equation. Third, we implement a gauge-fixing procedure that ensures a direct link with the standard antibracket-antifield approach, and consequently obtain the gauge-fixed action. Finally, we determine the Sp(3) BRST symmetry of the resulting gauge-fixed action.

We begin with the action of pure Yang-Mills theory

$$S_0^{\mathrm{L}}\left[A_\alpha^a\right] = -\frac{1}{4}\int d^D x\, F_{\alpha\beta}^a F_a^{\alpha\beta}\,, \tag{1}$$

where the field strength of the Yang-Mills fields is defined in the usual manner by $F_{\alpha\beta}^a = \partial_\alpha A_\beta^a - \partial_\beta A_\alpha^a - f_{bc}^a A_\alpha^b A_\beta^c$. Action (1) is invariant under the gauge transformations

$$\delta_\epsilon A_\alpha^a = \left(D_\alpha\right)^a{}_b\,\epsilon^b\,, \tag{2}$$

with the covariant derivative expressed by $\left(D_\alpha\right)^a{}_b = \delta^a{}_b\partial_\alpha + f_{bc}^a A_\alpha^c$.

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In order to construct a Lagrangian Sp(3) BRST symmetry for this model we triplicate both the gauge generators and gauge parameters, and work, instead of (2), with the modified invariances

$$\delta_{\epsilon} A^{a}_{\alpha} = \begin{pmatrix} (D_{\alpha})^{a}{}_{b} & (D_{\alpha})^{a}{}_{b} & (D_{\alpha})^{a}{}_{b} \end{pmatrix} \begin{pmatrix} \epsilon^{b}_{1} \\ \epsilon^{b}_{2} \\ \epsilon^{b}_{3} \end{pmatrix} , \tag{3}$$

which are found off-shell second-stage reducible, with the first-, respectively, second-stage reducibility functions given by

$$Z^{B}{}_{C} = \begin{pmatrix} \mathbf{0} & \delta^{b}{}_{c} & -\delta^{b}{}_{c} \\ -\delta^{b}{}_{c} & \mathbf{0} & \delta^{b}{}_{c} \\ \delta^{b}{}_{c} & -\delta^{b}{}_{c} & \mathbf{0} \end{pmatrix} , Z^{C}{}_{d} = \begin{pmatrix} -\delta^{c}{}_{d} \\ -\delta^{c}{}_{d} \\ -\delta^{c}{}_{d} \end{pmatrix} . \tag{4}$$

According to the general results from [11], we can construct a Lagrangian Sp(3) BRST symmetry, and hence a BRST tricomplex generated by three anticommuting differentials $(s_m)_{m=1,2,3}$

$$s_m s_n + s_n s_m = 0 \, , m, n = 1, 2, 3 \, , \tag{5}$$

that start like

$$s_m = \delta_m + D_m + \cdots \, , \tag{6}$$

where $(\delta_m)_{m=1,2,3}$ are the three differentials from the Koszul-Tate tricomplex, that furnish a triresolution of smooth functions defined on the stationary surface of field equations, and $(D_m)_{m=1,2,3}$ represent the exterior derivatives along the gauge orbits associated with the new second-stage redundant description of the gauge orbits due to the triplication of the gauge transformations like in (3).

The trigrading of the Sp(3) BRST algebra is governed by the ghost tridegree ($\mathrm{trigh} = (\mathrm{gh}_1, \mathrm{gh}_2, \mathrm{gh}_3)$), and we have that $\mathrm{trigh}\,(s_1) = (1,0,0)$, $\mathrm{trigh}\,(s_2) = (0,1,0)$, and $\mathrm{trigh}\,(s_3) = (0,0,1)$. The ghost spectrum contains, due to the triplication, the fields

$$\overset{(1,0,0)}{\eta}{}^{a}_{1}, \overset{(0,1,0)}{\eta}{}^{a}_{2}, \overset{(0,0,1)}{\eta}{}^{a}_{3}, \overset{(0,1,1)}{\pi}{}^{a}_{1}, \overset{(1,0,1)}{\pi}{}^{a}_{2}, \overset{(1,1,0)}{\pi}{}^{a}_{3}, \overset{(1,1,1)}{\lambda}{}^{a} , \tag{7}$$

where we denoted an element F with the ghost tridegree equal to (i,j,k) by $\overset{(i,j,k)}{F}$, and set $\mathrm{trigh}\left(A^{a}_{\mu}\right) = (0,0,0)$. The Grassmann parities of the ghosts are: $\varepsilon\left(\eta^{a}_{m}\right) = \varepsilon\left(\lambda^{a}\right) = 1$, $\varepsilon\left(\pi^{a}_{m}\right) = 0$, $m = 1,2,3$. The ghost spectrum can be understood by using the properties of the exterior derivatives along the gauge orbits [11].

The Sp(3) formalism relies on the presence of three antibrackets, denoted by $(,)_m$, $m = 1,2,3$, which implies that we have to introduce three antifields conjugated to each field/ghost, one for each antibracket. Consequently, the antifield spectrum reads as

$$\overset{(-1,0,0)}{A}{}^{*(1)\alpha}_{a}, \overset{(0,-1,0)}{A}{}^{*(2)\alpha}_{a}, \overset{(0,0,-1)}{A}{}^{*(3)\alpha}_{a}, \overset{(-2,0,0)}{\eta}{}^{*(1)}_{1a}, \overset{(-1,-1,0)}{\eta}{}^{*(2)}_{1a}, \overset{(-1,0,-1)}{\eta}{}^{*(3)}_{1a}, \tag{8}$$

$$\overset{(-1,-1,0)}{\eta}{}^{*(1)}_{2a}, \overset{(0,-2,0)}{\eta}{}^{*(2)}_{2a}, \overset{(0,-1,-1)}{\eta}{}^{*(3)}_{2a}, \overset{(-1,0,-1)}{\eta}{}^{*(1)}_{3a}, \overset{(0,-1,-1)}{\eta}{}^{*(2)}_{3a}, \overset{(0,0,-2)}{\eta}{}^{*(3)}_{3a}, \tag{9}$$

$$\overset{(-1,-1,-1)}{\pi}{}^{*(1)}_{1a},\ \overset{(0,-2,-1)}{\pi}{}^{*(2)}_{1a},\ \overset{(0,-1,-2)}{\pi}{}^{*(3)}_{1a},\ \overset{(-2,0,-1)}{\pi}{}^{*(1)}_{2a},\ \overset{(-1,-1,-1)}{\pi}{}^{*(2)}_{2a},\ \overset{(-1,0,-2)}{\pi}{}^{*(3)}_{2a}, \tag{10}$$

$$\overset{(-2,-1,0)}{\pi}{}^{*(1)}_{3a},\ \overset{(-1,-2,0)}{\pi}{}^{*(2)}_{3a},\ \overset{(-1,-1,-1)}{\pi}{}^{*(3)}_{3a},\ \overset{(-2,-1,-1)}{\lambda}{}^{*(1)}_{a},\ \overset{(-1,-2,-1)}{\lambda}{}^{*(2)}_{a},\ \overset{(-1,-1,-2)}{\lambda}{}^{*(3)}_{a}. \tag{11}$$

In order to ensure the nilpotency, as well as the acyclicity of the Koszul-Tate differentials, we further enlarge the antifield spectrum with the bar and tilde variables [11]

$$\overset{(0,-1,-1)}{\bar{A}}{}^{(1)\alpha}_{a},\ \overset{(-1,0,-1)}{\bar{A}}{}^{(2)\alpha}_{a},\ \overset{(-1,-1,0)}{\bar{A}}{}^{(3)\alpha}_{a},\ \overset{(-1,-1,-1)}{\bar{\eta}}{}^{(1)}_{1a},\ \overset{(-2,0,-1)}{\bar{\eta}}{}^{(2)}_{1a},\ \overset{(-2,-1,0)}{\bar{\eta}}{}^{(3)}_{1a}, \tag{12}$$

$$\overset{(0,-2,-1)}{\bar{\eta}}{}^{(1)}_{2a},\ \overset{(-1,-1,-1)}{\bar{\eta}}{}^{(2)}_{2a},\ \overset{(-1,-2,0)}{\bar{\eta}}{}^{(3)}_{2a},\ \overset{(0,-1,-2)}{\bar{\eta}}{}^{(1)}_{3a},\ \overset{(-1,0,-2)}{\bar{\eta}}{}^{(2)}_{3a},\ \overset{(-1,-1,-1)}{\bar{\eta}}{}^{(3)}_{3a}, \tag{13}$$

$$\overset{(0,-2,-2)}{\bar{\pi}}{}^{(1)}_{1a},\ \overset{(-1,-1,-2)}{\bar{\pi}}{}^{(2)}_{1a},\ \overset{(-1,-2,-1)}{\bar{\pi}}{}^{(3)}_{1a},\ \overset{(-1,-1,-2)}{\bar{\pi}}{}^{(1)}_{2a},\ \overset{(-2,0,-2)}{\bar{\pi}}{}^{(2)}_{2a},\ \overset{(-2,-1,-1)}{\bar{\pi}}{}^{(3)}_{2a}, \tag{14}$$

$$\overset{(-1,-2,-1)}{\bar{\pi}}{}^{(1)}_{3a},\ \overset{(-2,-1,-1)}{\bar{\pi}}{}^{(2)}_{3a},\ \overset{(-2,-2,0)}{\bar{\pi}}{}^{(3)}_{3a},\ \overset{(-1,-2,-2)}{\bar{\lambda}}{}^{(1)}_{a},\ \overset{(-2,-1,-2)}{\bar{\lambda}}{}^{(2)}_{a},\ \overset{(-2,-2,-1)}{\bar{\lambda}}{}^{(3)}_{a}, \tag{15}$$

$$\overset{(-1,-1,-1)}{\tilde{A}}{}^{\alpha}_{a},\ \overset{(-2,-1,-1)}{\tilde{\eta}}{}_{1a},\ \overset{(-1,-2,-1)}{\tilde{\eta}}{}_{2a},\ \overset{(-1,-1,-2)}{\tilde{\eta}}{}_{3a}, \tag{16}$$

$$\overset{(-1,-2,-2)}{\tilde{\pi}}{}_{1a},\ \overset{(-2,-1,-2)}{\tilde{\pi}}{}_{2a},\ \overset{(-2,-2,-1)}{\tilde{\pi}}{}_{3a},\ \overset{(-2,-2,-2)}{\tilde{\lambda}}{}_{a}. \tag{17}$$

The Grassmann parities of the antifields (8–11) and (16–17) are opposite to those of the corresponding fields/ghosts, while those of (12–15) coincide with them. The antifields (8–11) bear a supplementary superscript between parentheses, which signifies in what bracket they are conjugated to the corresponding fields/ghosts. The antifield sector is also graded by a supplementary tridegree, named resolution tridegree, and defined by $\mathrm{trires} = (\mathrm{res}_1, \mathrm{res}_2, \mathrm{res}_3) = -\mathrm{trigh}$. The induced simple grading, called total resolution degree, $\mathrm{res} = \mathrm{res}_1 + \mathrm{res}_2 + \mathrm{res}_3$, will be useful in the sequel when solving the fundamental equation of the Sp(3) formalism, namely, the extended classical master equation.

With the ghost and antifield spectra at hand, we can give the boundary conditions on the solution to the extended classical master equation. They take the form

$$\overset{[0]}{S} = S_0^{\mathrm{L}}\left[A^a_\alpha\right],\ \overset{[1]}{S} = \int d^D x\, A^{*(m)\alpha}_a \left(D_\alpha\right)^a_{\ b} \eta^b_m, \tag{18}$$

$$\overset{[2]}{S} = \int d^D x \left(\left(\varepsilon_{mnp}\eta^{*(m)}_{na} + \bar{A}^{(p)\alpha}_b \left(D_\alpha\right)^b_{\ a}\right)\pi^a_p + \cdots\right), \tag{19}$$

$$\overset{[3]}{S} = \int d^D x \left(-\left(\pi^{*(m)}_{ma} + \bar{\eta}^{(m)}_{ma} - \tilde{A}^\alpha_b \left(D_\alpha\right)^b_{\ a}\right)\lambda^a + \cdots\right), \tag{20}$$

where the supplementary superscript between brackets in $\overset{[0]}{S}$, $\overset{[1]}{S}$, etc., refers to a decomposition of the solution to the master equation via the total resolution degree, and ε_{mnp} is completely antisymmetric, with $\varepsilon_{123} = +1$. The boundary conditions (18–20) can be understood by means of homological arguments [11].

The construction of the Sp(3) algebra (5) is completely equivalent to the construction of the anticanonical generator S of this symmetry, which is solution to the extended classical master equation

$$\frac{1}{2}(S,S)+VS=0\,, \tag{21}$$

and is required to have the ghost tridegree $\mathrm{trigh}\,(S)=(0,0,0)$. The symbol $(,)$ denotes the total antibracket, written as the sum among the three antibrackets, and V represents the noncanonical part of the total Koszul-Tate differential, $\delta=\delta_1+\delta_2+\delta_3$. The master equation (21) projected on independent components reads as $\frac{1}{2}(S,S)_m+V_mS=0$, with $(,)_m$ and V_m obviously meaning the antibracket, respectively, the noncanonical part of the total Koszul-Tate differential corresponding to the component 'm' of the Sp(3) BRST symmetry, $\delta_m=\delta_{\mathrm{can}m}+V_m$. We mention that the individual antibrackets $(,)_m$, as well as the total one, satisfy the usual properties of the antibracket-antifield formalism, while the operators V_m and V behave like derivations with respect to the antibrackets. Their features are $\mathrm{trigh}\,((,)_m)=\mathrm{trigh}\,(s_m)$, $\mathrm{trigh}\,(V_m)=\mathrm{trigh}\,(s_m)=-\mathrm{trires}\,(V_m)$. The operators V_m act only on the bar and tilde variables through

$$V_m\bar{\Phi}_A^{(n)}=(-)^{\varepsilon_A}\varepsilon_{mnp}\Phi_A^{*(p)}\,,V_m\tilde{\Phi}_A=(-)^{\varepsilon_A+1}\bar{\Phi}_A^{(m)}\,, \tag{22}$$

where ε_A stands for the Grassmann parity of a generic field/ghost Φ^A.

In order to solve the equation (21), we develop S according to the total resolution degree, $S=\sum_{k\geq 0}\overset{[k]}{S}$, $\mathrm{res}\left(\overset{[k]}{S}\right)=k$, $\mathrm{trigh}\left(\overset{[k]}{S}\right)=(0,0,0)$, where the boundary terms are given in (18–20). Following this line, we find that the solution to (21) reads as

$$\begin{aligned}
S=\int d^Dx\Bigg(&-\frac{1}{4}F^a_{\alpha\beta}F_a^{\alpha\beta}+A_a^{*(m)\alpha}\left(D_\alpha\right)^a{}_b\,\eta^b_m+\left(\varepsilon_{mnp}\eta^{*(m)}_{na}+\bar{A}_b^{(p)\alpha}\left(D_\alpha\right)^b{}_a\right)\pi^a_p+\\
&\left(-\pi^{*(m)}_{ma}-\bar{\eta}^{(m)}_{ma}+\tilde{A}^\alpha_b\left(D_\alpha\right)^b{}_a\right)\lambda^a+\frac{1}{2}\eta^{*(m)}_{na}f^a_{bc}\eta^c_m\eta^b_n-\frac{1}{2}\pi^{*(m)}_{na}f^a_{bc}\eta^c_m\pi^b_n+\\
&\frac{1}{12}\varepsilon_{npr}\pi^{*(m)}_{na}f^a_{bc}f^c_{de}\eta^e_m\eta^d_p\eta^b_r-\frac{1}{2}\varepsilon_{mnp}\bar{A}_a^{(m)\alpha}f^a_{bc}\left(\left(D_\alpha\right)^c{}_d\,\eta^d_n\right)\eta^b_p+\\
&\lambda^{*(m)}_a\left(\frac{1}{2}f^a_{bc}\eta^c_m\lambda^b-\frac{1}{12}\left(f^a_{bc}f^c_{de}+f^a_{dc}f^c_{be}\right)\eta^e_m\eta^d_n\pi^b_n\right)+\frac{1}{6}\varepsilon_{mnp}\tilde{A}^\alpha_a f^a_{bc}f^c_{de}\left(\left(D_\alpha\right)^e{}_f\eta^f_m\right)\eta^d_n\eta^b_p-\\
&\frac{1}{2}\tilde{A}^\alpha_a f^a_{bc}\left(\left(\left(D_\alpha\right)^c{}_d\,\eta^d_m\right)\pi^b_m-\eta^c_m\left(D_\alpha\right)^b{}_d\,\pi^d_m\right)-\frac{1}{2}\bar{\eta}^{(m)}_{ma}f^a_{bc}\eta^c_n\pi^b_n+\bar{\eta}^{(m)}_{na}f^a_{bc}\eta^c_n\pi^b_m-\\
&\frac{1}{6}\varepsilon_{mnp}\bar{\eta}^{(m)}_{ra}f^a_{bc}f^c_{de}\eta^e_r\eta^d_n\eta^b_p+\frac{1}{2}\tilde{\eta}_{ma}f^a_{bc}\left(\frac{1}{12}\varepsilon_{npr}f^c_{de}f^e_{fg}\eta^g_m\eta^f_n\eta^d_p\eta^b_r-\varepsilon_{mnp}\pi^c_n\pi^b_p\right)+\\
&\tilde{\eta}_{ma}f^a_{bc}\left(\eta^c_m\lambda^b-\frac{1}{2}f^c_{de}\eta^e_m\eta^b_n\pi^d_n\right)+\frac{1}{2}\varepsilon_{mnp}\bar{\pi}^{(m)}_{na}f^a_{bc}\left(\eta^c_p\lambda^b-\frac{1}{2}f^c_{de}\eta^e_p\eta^b_r\pi^d_r\right)+\\
&\frac{1}{2}\bar{\pi}^{(m)}_{na}\left(-f^a_{bc}\pi^c_m\pi^b_n+\frac{1}{12}\left(f^a_{bc}\left(f^c_{de}f^e_{fg}+f^c_{fe}f^e_{dg}\right)+f^a_{gc}\left(f^c_{de}f^e_{fb}+f^c_{fe}f^e_{db}\right)\right)\eta^g_n\eta^f_m\eta^d_p\eta^b_p\right)+\\
&\frac{1}{2}\bar{\lambda}^{(m)}_a\left(f^a_{bc}\left(\lambda^c\pi^b_m-\frac{1}{6}\varepsilon_{mnp}f^c_{de}\lambda^e\eta^d_n\eta^b_p\right)+\frac{1}{6}\left(f^a_{bc}f^c_{de}+f^a_{dc}f^c_{be}\right)\pi^e_m\eta^d_n\pi^b_n\right)+\\
&\frac{1}{24}\bar{\lambda}^{(m)}_a\left(\varepsilon_{mnp}f^c_{de}\left(f^a_{bc}f^e_{fg}+f^a_{fc}f^e_{bg}\right)\eta^g_n\eta^f_r\eta^d_p\pi^b_r+\frac{1}{12}M^a_{bdfhi}\eta^i_n\eta^h_n\eta^f_m\eta^d_p\eta^b_p\right)+
\end{aligned}$$

$$\frac{1}{2}\tilde{\lambda}_a \left(f^a_{bc}\lambda^c\lambda^b - \frac{1}{3}\left(f^a_{bc}f^c_{de} + f^a_{dc}f^c_{be}\right)\lambda^e\eta^d_m\pi^b_m + \frac{1}{6}f^c_{de}\left(f^a_{bc}f^e_{fg} + f^a_{gc}f^e_{fb}\right)\eta^g_m\eta^f_n\pi^d_n\pi^b_m \right) +$$
$$\frac{1}{144}\varepsilon_{mnp}\tilde{\lambda}_a \left(\bar{M}^a_{bdfhi}\eta^i_m\eta^h_m\pi^f_m\eta^d_n\eta^b_p - \frac{1}{12}\tilde{M}^a_{bdfhjk}\eta^k_m\eta^j_m\eta^h_n\eta^f_n\eta^d_p\eta^b_p \right)\Bigg) , \tag{23}$$

where

$$M^a_{bdfhi} = \left(f^a_{ec}f^c_{bd} + f^a_{dc}f^c_{be}\right)\left(f^e_{fg}f^g_{hi} + f^e_{hg}f^g_{fi}\right) + \left(f^a_{ec}f^c_{bf} + f^a_{fc}f^c_{be}\right)\left(f^e_{dg}f^g_{hi} + f^e_{hg}f^g_{di}\right) +$$
$$\left(f^a_{ec}f^c_{bh} + f^a_{hc}f^c_{be}\right)\left(f^e_{fg}f^g_{di} + f^e_{dg}f^g_{fi}\right) , \tag{24}$$

$$\bar{M}^a_{bdfhi} = f^a_{ce}f^c_{dg}\left(f^e_{hf}f^g_{bi} - f^e_{if}f^g_{bh} + 3f^e_{bf}f^g_{hi}\right) + f^e_{cf}\left(f^c_{bg}\left(f^a_{he}f^g_{di} - f^a_{ie}f^g_{dh}\right) + 3f^a_{be}f^c_{dg}f^g_{hi}\right) , \tag{25}$$

$$\tilde{M}^a_{bdfhjk} = f^c_{be}\left(f^a_{gc}f^e_{df} + f^a_{dc}f^e_{gf}\right)\left(f^g_{hi}f^i_{kj} + f^g_{ki}f^i_{hj}\right) +$$
$$f^c_{de}\left(f^a_{gc}f^e_{kh} + f^a_{kc}f^e_{gh}\right)\left(f^g_{bi}f^i_{fj} + f^g_{fi}f^i_{bj}\right) +$$
$$f^c_{de}\left(f^a_{gc}f^e_{fh} + f^a_{fc}f^e_{gh}\right)\left(f^g_{ki}f^i_{bj} + f^g_{bi}f^i_{kj}\right) . \tag{26}$$

Now, we develop a gauge-fixing procedure that ensures a direct equivalence with the standard antibracket-antifield BRST formalism. To this end, we begin by restoring an anticanonical structure for all the variables (including the bar and tilde ones) in order to bring the classical master equation of the Sp(3) BRST formalism to a more familiar form. We focus, for example, on the first antibracket, and discard the other two. As we cannot declare the existing variables excepting Φ^A (original fields and ghosts) and $\Phi^{*(1)}_A$ conjugated in the first antibracket, we need to extend the algebra of the Sp(3) BRST tricomplex [11] by adding the variables $\left(\rho^A_2, \rho^A_3, \kappa^A_1, \mu^A_2, \mu^A_3, \nu^A_1\right)$, which are respectively conjugated in the first antibracket to $\left(\Phi^{*(3)}_A, \Phi^{*(2)}_A, \bar{\Phi}^{(1)}_A, \bar{\Phi}^{(3)}_A, \bar{\Phi}^{(2)}_A, \tilde{\Phi}_A\right)$. As explained in [11], it is useful to still add some more variables in order to realize a proper connection with the gauge-fixing procedure from the standard antibracket-antifield formalism. In view of this, we introduce the fermionic fields $\overset{(0,0,0)}{\varphi}{}^a$ that do not enter the original action, and hence are purely gauge, endowed with the gauge invariances $\delta_\xi\varphi^a = \xi^a$. Consequently, we further enlarge the ghost sector with the fields

$$\left(\overset{(1,0,0)}{C}{}^a_1, \overset{(0,1,0)}{C}{}^a_2, \overset{(0,0,1)}{C}{}^a_3, \overset{(0,1,1)}{p}{}^a_1, \overset{(1,0,1)}{p}{}^a_2, \overset{(1,1,0)}{p}{}^a_3, \overset{(1,1,1)}{l}{}^a\right) , \tag{27}$$

displaying the Grassmann parities $\varepsilon\left(C^a_m\right) = \varepsilon\left(l^a\right) = 0$, $\varepsilon\left(p^a_m\right) = 1$. For notational simplicity, we make the collective notation $\varphi^I = \left(\varphi^a, C^a_m, p^a_m, l^a\right)$. Thus, the additional antifield spectrum will contain the variables $\left(\varphi^{*(m)}_I, \bar{\varphi}^{(m)}_I, \tilde{\varphi}_I\right)$, $m = 1, 2, 3$. As the sector corresponding to the new fields does not interfere in any point with the original one, the solution to the master equation of the Sp(3) BRST formalism associated with the overall gauge theory will be

$$\bar{S} = S + \int d^Dx \left(\varphi^{*(m)}_a C^a_m + \left(\varepsilon_{mnp}C^{*(m)}_{na} + \bar{\varphi}^{(p)}_a\right)p^a_p - \left(p^{*(m)}_{ma} + \bar{C}^{(m)}_{ma} - \tilde{\varphi}_a\right)l^a \right) . \tag{28}$$

Now, we restore the anticanonical structure also with respect to the newly added variables. We give up the second and third antibracket, and introduce the variables $(r_2^I, r_3^I, k_1^I, m_2^I, m_3^I, n_1^I)$ respectively conjugated to $\left(\varphi_I^{*(3)}, \varphi_I^{*(2)}, \bar{\varphi}_I^{(1)}, \bar{\varphi}_I^{(3)}, \bar{\varphi}_I^{(2)}, \tilde{\varphi}_I\right)$. Consequently, if $\bar{S}$ is solution to the equation (21), then

$$\bar{S}_1 = \bar{S} + \int d^D x \left(\Phi_A^{*(2)} \mu_2^A + \Phi_A^{*(3)} \mu_3^A + \bar{\Phi}_A^{(1)} \nu_1^A + \varphi_I^{*(2)} m_2^I + \varphi_I^{*(3)} m_3^I + \bar{\varphi}_I^{(1)} n_1^I\right), \tag{29}$$

satisfies the equation $\left(\bar{S}_1, \bar{S}_1\right)_1 = 0$, which is precisely the standard classical master equation in the first antibracket.

With the solution (29) at hand, we can employ now the gauge-fixing procedure from the standard BRST formalism. This means that we have to choose a fermionic functional ψ_1, with the help of which we eliminate half of the variables in favour of the other half. For definiteness, we eliminate the variables $\left(\Phi_A^{*(1)}, \rho_2^A, \rho_3^A, \kappa_1^A, \bar{\Phi}_A^{(2)}, \bar{\Phi}_A^{(3)}, \tilde{\Phi}_A\right)$, together with $\left(\varphi_I^{*(1)}, r_2^I, r_3^I, k_1^I, \bar{\varphi}_I^{(2)}, \bar{\varphi}_I^{(3)}, \tilde{\varphi}_I\right)$, and, in the meantime, enforce the gauge-fixing conditions

$$\rho_2^A = \rho_3^A = \kappa_1^A = 0\,, r_2^I = r_3^I = k_1^I = 0\,, \tag{30}$$

that can be implemented by taking $\psi_1 = \psi_1\left[\Phi^\Delta, \mu_2^\Delta, \mu_3^\Delta, \nu_1^\Delta\right]$, where we employed the notations $\Phi^\Delta = \left(\Phi^A, \varphi^I\right)$, $\mu_{2,3}^\Delta = \left(\mu_{2,3}^A, m_{2,3}^I\right)$ and $\nu_1^\Delta = \left(\nu_1^A, n_1^I\right)$. It is understood that a variable is eliminated by one of the formulas $\text{antifield} = \frac{\delta^L \psi_1}{\delta(\text{field})}$, $\text{field} = -\frac{\delta^L \psi_1}{\delta(\text{antifield})}$, depending if it is a ‘field’ or an ‘antifield’. In this context, we emphasise that $\left(\Phi^\Delta, \Phi_\Delta^{*(3)}, \rho_3^\Delta, \bar{\Phi}_\Delta^{(1)}, \mu_3^\Delta, \bar{\Phi}_\Delta^{(3)}, \nu_1^\Delta\right)$ are regarded as ‘fields’, while $\left(\Phi_\Delta^{*(1)}, \rho_2^\Delta, \Phi_\Delta^{*(2)}, \kappa_1^\Delta, \bar{\Phi}_\Delta^{(2)}, \mu_2^\Delta, \tilde{\Phi}_\Delta\right)$ are viewed like their corresponding ‘antifields’, where we performed the obvious notations $\Phi_\Delta^{*(m)} = \left(\Phi_A^{*(m)}, \varphi_I^{*(m)}\right)$, $\bar{\Phi}_\Delta^{(m)} = \left(\bar{\Phi}_A^{(m)}, \bar{\varphi}_I^{(m)}\right)$, $\kappa_1^\Delta = \left(\kappa_1^A, k_1^I\right)$, $\rho_{2,3}^\Delta = \left(\rho_{2,3}^A, r_{2,3}^I\right)$, and $\tilde{\Phi}_\Delta = \left(\tilde{\Phi}_A, \tilde{\varphi}_I\right)$. For a proper link with the standard approach, we take

$$\psi_1 = \int d^D x \left(-\mu_{3a}^{(\varphi)} \partial^\alpha \mu_{2\alpha}^a + \left(\partial^\alpha \mu_{3\alpha}^a\right) \mu_{2a}^{(\varphi)} + \left(\partial^\alpha A_\alpha^a\right) \nu_{1a}^{(\varphi)} - \left(\partial^\alpha \varphi_a\right) \nu_{1\alpha}^a\right), \tag{31}$$

where we put an extra superscript between parentheses where necessary in order to distinguish the variables of the same type that carry the same indices. After performing the gauge-fixing process, and further eliminating some auxiliary variables from the resulting action, we finally deduce the gauge-fixed action

$$\begin{aligned}
\bar{S}'_{\psi_1} = \int d^D x \Bigg(& -\frac{1}{4} F_{\alpha\beta}^a F_a^{\alpha\beta} + \left(\partial^\alpha p_{ma}\right) \left(D_\alpha\right)^a{}_b \eta_m^b + \left(\partial^\alpha C_{ma}\right) \left(D_\alpha\right)^a{}_b \pi_m^b + \\
& \left(\partial^\alpha \varphi_a\right) \left(-\left(D_\alpha\right)^a{}_b \lambda^b + \frac{1}{2} f_{bc}^a \left(\left(\left(D_\alpha\right)^c{}_d \eta_m^d\right) \pi_m^b - \eta_m^c \left(D_\alpha\right)^b{}_d \pi_m^d\right)\right) - \\
& \frac{1}{6} \varepsilon_{mnp} \left(\partial^\alpha \varphi_a\right) f_{bc}^a f_{de}^c \left(\left(D_\alpha\right)^e{}_f \eta_m^f\right) \eta_n^d \eta_p^b - \\
& \frac{1}{2} \varepsilon_{mnp} \left(\partial^\alpha C_{ma}\right) f_{bc}^a \left(\left(D_\alpha\right)^c{}_d \eta_n^d\right) \eta_p^b + l_a \left(\partial^\alpha A_\alpha^a\right) \Bigg).
\end{aligned} \tag{32}$$

The gauge-fixed action (32) can be checked to be invariant under the gauge-fixed

Sp(3) BRST transformations

$$s_m A_\alpha^a = (D_\alpha)^a{}_b \, \eta_m^b \, , s_m \varphi^a = C_m^a \, , \tag{33}$$

$$s_m \eta_n^a = \varepsilon_{mnr} \pi_r^a + \frac{1}{2} f_{bc}^a \eta_m^c \eta_n^b \, , s_m C_n^a = \varepsilon_{mnr} p_r^a \, , \tag{34}$$

$$s_m \pi_n^a = -\delta_{mn} \lambda^a - \frac{1}{2} f_{bc}^a \eta_m^c \pi_n^b + \frac{1}{12} \varepsilon_{npr} f_{bc}^a f_{de}^c \eta_m^e \eta_p^d \eta_r^b \, , s_m p_n^a = -\delta_{mn} l^a \, , \tag{35}$$

$$s_m \lambda^a = \frac{1}{2} f_{bc}^a \eta_m^c \lambda^b - \frac{1}{12} \left(f_{bc}^a f_{de}^c + f_{dc}^a f_{be}^c \right) \eta_m^e \eta_n^d \pi_n^b \, , s_m l^a = 0 \, . \tag{36}$$

This completes the Lagrangian Sp(3) BRST approach to pure Yang-Mills theory.

To conclude with, in this paper we have constructed the Lagrangian Sp(3) BRST symmetry for the Yang-Mills theory. Our procedure is based on the triplication of the gauge transformations, and subsequently, on the resolution of the extended classical master equation. With the solution of this equation at hand, we develop a gauge-fixing procedure that leads to a gauge-fixed action which is invariant under some gauge-fixed Sp(3) BRST transformations.

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