

A Small Note on PP-Wave Vacua in 6 and 5 Dimensions

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abstract

We discuss Kowalski-Glikman type pp-wave solutions with unbroken supersymmetry in 6 and 5 dimensional supergravity theories.

In this small note we want to discuss pp-wave solutions with unbroken supersymmetry, the so-called Kowalski-Glikman solutions, in lower dimensional supergravity theories. The known KG solutions [1, 2, 3] consist of some covariantly constant fieldstrength and a metric which has the form of a pp-wave, *i.e.*

$$ds_d^2 = 2du(dv + Adu) + dx^i dx_i, \quad (1)$$

where $A = x^i A_{ij} x^j$ ($i, j = 1 \dots d-2$). For this metric, the only non-vanishing component of the Ricci curvature is $R_{uu} = 2\eta^{ij} A_{ij}$ and by introducing light-cone coordinates in the tangent-space we find that the only non-vanishing component of the spin-connection is $\phi_\mu = 2\delta_{u\mu} \partial_i A \gamma^{i+}$.¹

The basic problem one is faced with when looking for non-trivial solutions of supergravity theories which preserve all supersymmetry are the dilatino equations, since they are algebraic in nature. Clearly, the easiest way to avoid such trouble is by not having dilatino equations in the first place. Sometimes however, one can make use of special properties of the dilatino equations in order to find non-trivial vacua. The first example is $d = 10$, $N = 1$ supergravity where one can use the chirality of the theory [4] in order to find a solution which preserves all supersymmetry. Another example is type IIB supergravity, where one can find such solutions, notably the $aDS_5 \otimes S^5$ solution and the Kowalski-Glikman type solution presented in [2], with RR 5-form flux since the dilatino equation does not contain a contribution of the 5-form fieldstrength [5]. The fact that the type IIB dilatino variation does not depend on the 5-form fieldstrength is however due to the fact that it is self-dual and that the spinors are chiral. One is therefore tempted to say that non-trivial solutions with unbroken supersymmetry exist whenever there are no dilatinos or when the theory is chiral and it is these kind of theories we are going to examine.

¹We use the mostly minus signature for the metric, the γ matrices satisfy $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ and the covariant derivative on spinors is taken to be $\nabla\epsilon = d\epsilon - 4^{-1}\phi\epsilon$. We also introduce the light-cone combinations $\gamma^+ = \gamma^u$ and $\gamma^- = \gamma^v + A\gamma^u$, which satisfy $\{\gamma^+, \gamma^-\} = 2$ and $\{\gamma^\pm, \gamma^\pm\} = \{\gamma^\pm, \gamma^i\} = 0$.

There are not many supergravity theories that are chiral or have no dilatinos, so that the investigation of the existence of KG solutions is rather limited. The highest dimensional possibilities have already been presented in the literature, namely by Kowalski-Glikman in the case of M-theory [1] and by Blau *et. al.* for type IIB [2]. The next on the list is $N = 1$ $d = 10$ supergravity. Such an investigation was carried out by Kowalski-Glikman [4] who showed that the solution is not of the pp-wave type, but rather has geometry $aDS_3 \otimes S^3 \otimes \mathbb{E}_4$. A similar analysis was performed on the $N = 2$ $d = 4$ supergravity [3] showing that the only supersymmetric solutions are the Robinson-Bertotti and the KG solution. This means that the only remaining candidates are $d = 6$ $(2, 0)$ or $(4, 0)$ supergravity and $d = 5$ $N = 2$ supergravity. Although $N = 1$ $d = 4$ supergravity matches the profile, it can be discarded since the integrability condition for the Killing spinor equation implies that the space must be Riemann flat, *i.e.* Minkowski.

The $d = 6$ $(2, 0)$ supergravity comprises of the graviton, e_μ^a , two symplectic Majorana-Weyl Rarita-Schwinger fields, combined into the $USp(2)$ vector Ψ_μ , and a 2-form B whose fieldstrength, $H = dB$, is self-dual. As such one is faced with the same problem as in type IIB supergravity. A Lorentz invariant action can however be written down by introducing a Lagrange multiplier field [6], by writing a non-self-dual action as in [7] or by adding an anti-symmetric tensor multiplet [8]. However, using the conventions of [9] the equation of motion for the metric reads

$$R_{\mu\nu} = \frac{1}{4} H_{\mu\kappa\rho} H_\nu{}^{\kappa\rho} . \quad (2)$$

Choosing the self-dual Ansatz $H = \lambda du \wedge (dx^1 dx^2 + dx^3 dx^4)$ the above equation is solved by choosing $2\eta^{ij} A_{ij} = -\lambda^2$.

The Killing spinor equations in this case read

$$0 = \delta\Psi_\mu = \nabla_\mu \epsilon - \frac{1}{8 \cdot 3!} \not{H} \gamma_\mu \epsilon . \quad (3)$$

By observing that due to the self-duality of H we have $\not{H} \gamma_\mu \epsilon = 2 \cdot 3! \lambda \gamma^+ \gamma^{12} \gamma_\mu \epsilon$, we can see that the Eq. (3) is automatically satisfied in the v direction. The equations in the i direction reads

$$0 = \partial_i \epsilon - \Omega_i \epsilon \quad : \quad \Omega_i = \frac{1}{4} \lambda \gamma^+ \gamma^{12} \gamma_i . \quad (4)$$

Following [2, 10] these equations, since $\Omega_i \Omega_j = 0$, are solved by $\epsilon = (1 + x^i \Omega_i) \xi(u)$. In the u -direction Eq. (3) reduces to

$$\partial_u \epsilon - x^i A_{ij} \gamma^j \gamma^+ \epsilon - \Omega^- \epsilon = 0 , \quad (5)$$

where the combination $\Omega^- = \frac{1}{4} \lambda \gamma^+ \gamma^{12} \gamma^-$ was used. By making the Ansatz $\xi = \exp(\Omega^- u) \epsilon_0$, with ϵ_0 an unconstrained constant symplectic-MW spinor, all x -independent terms are canceled leaving

$$x^i A_{ij} \gamma^j \gamma^+ \epsilon_0 = -\frac{1}{8} \lambda^2 x^i \gamma_i \gamma^+ \epsilon_0 . \quad (6)$$

This equation is readily solved by $A_{ij} = -\frac{1}{8} \lambda^2 \eta_{ij}$, which is compatible with the equations of motion.

The $d = 6$ $(4, 0)$ supergravity is invariant under global $USp(4) \sim SO(5)$ (See Ref. [11] and refs. therein) and its field content is a Sechsbein, e_μ^a , 4 symplectic MW spinor, which

are combined into an $SO(5)$ vector Ψ_μ , and 5 2-forms B^I which have self-dual fieldstrengths and transform as a vector under $SO(5)$. The equations of motion and the supersymmetry transformation are

$$\begin{aligned} 0 &= R_{\mu\nu} - \frac{1}{4} H_{\mu\kappa\rho}^I H_\nu^{\kappa\rho I} , \\ 0 &= \delta\Psi_\mu = \nabla_\mu\epsilon - \frac{1}{8\cdot 3!} \not{H} \gamma_\mu \Gamma^I \epsilon , \end{aligned} \quad (7)$$

where the Γ 's belong to the 5-dimensional Euclidean Clifford algebra. The $(2, 0)$ solution can be embedded into the $(4, 0)$ theory by taking only the $I = 1$ component to be different from zero. The calculations are just the same as in the $(2, 0)$ case, the only difference being that every Ω has to be multiplied by Γ^1 . The result is however the same: $(4, 0)$ supergravity admits a KG-type wave solution that breaks no supersymmetry whatsoever.

The last on the short-list is $d = 5$ $N = 2$ supergravity [12]. Its field content consists of a Fünfbein, e_μ^a , two symplectic Majorana Rarita-Schwinger fields Ψ_μ and a vector V_μ whose fieldstrength will be taken to be

$$F = du \wedge \lambda_i dx^i . \quad (8)$$

Since the chosen form for the field strength is, given the metric (1), covariantly constant and has an overall dependence on the differential du the equation of motion for the vector field is automatically satisfied. The equation of motion for the metric reads

$$R_{\mu\nu} = \frac{1}{2} [F_{\mu\kappa} F_\nu^\kappa - \frac{1}{6} g_{\mu\nu} F^2] , \quad (9)$$

and leads to the condition $4\eta^{ij} A_{ij} = \lambda_i \lambda^i$. The supersymmetry variation of the gravitino reads [12].

$$0 = \delta\Psi_\mu = \nabla_\mu\epsilon + \frac{1}{8\sqrt{3}} \not{F} \gamma_\mu \epsilon - \frac{1}{4\sqrt{3}} F_{\mu\nu} \gamma^\nu \epsilon . \quad (10)$$

The analysis is completely analogous to the one for the $(2, 0)$ theory, but for the definitions

$$\Omega_i = -\frac{1}{4\sqrt{3}} \gamma^+ [\lambda_j \gamma^j \gamma_i + \lambda_i] , \quad \Omega^- = \frac{1}{4\sqrt{3}} \lambda_i \gamma^i (\gamma^+ \gamma^- + 1) . \quad (11)$$

One finds that the analogue condition to Eq. (6) reads

$$x^i A_{ij} \gamma^j \gamma^+ \epsilon_0 = x^i [\Omega_i, \Omega^-] \epsilon_0 , \quad (12)$$

where, as before, ϵ_0 is an unconstrained symplectic Majorana spinor. After some γ -manipulations, one finds the matrix A to be

$$A_{ij} = \frac{1}{24} \{ 3\lambda_i \lambda_j + \lambda_l \lambda^l \eta_{ij} \} , \quad (13)$$

which is compatible with the equations of motion. Of course we could make use of the $SO(3)$ invariance to put $\lambda_{2,3} = 0$ and we find A to be proportional to $diag(4, 1, 1)$.

In this small note we have presented Kowalski-Glikman type solutions, solutions that do not break any supersymmetry and that look like pp-waves, in in chiral 6-dimensional supergravities and in $N = 2$ $d = 5$ supergravity. Since we used rather restrictive criteria, absence of dilatinos or chirality, it would be nice to consider other theories and see whether they admit to KG solutions. Work in this direction is in progress.

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