A simple mechanical analog of the field theory of tachyon matter

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In this brief note we show that the zero dimensional version of the field theory of tachyon matter, proposed by Sen, provides an action integral formulation for the motion of a particle in the presence of Newtonian gravity and nonlinear damping (quadratic in velocity).

Recently, there has been a lot of interest around a proposal by Sen [1], of a new field theory — the field theory of tachyon matter. The derivation of the action is based on involved arguments in string and string field theory [1, 2]. This action has also been proposed earlier, independently, by several authors in [3], [4], though, it is only recently that it has been analysed in greater detail[1, 2]. Apart from its importance in the context of string theory, it is useful to investigate its consequences by considering it as a field theory in its own right. It goes without saying, that, at first sight, one is somewhat surprised by the expression for the action, given as :

$$S = -\int d^{p+1}x V(T)\sqrt{1+\eta^{ij}\partial_i T\partial_j T} \tag{1}$$

where $\eta_{00} = -1$ and $\eta_{\alpha\beta} = \delta_{\alpha\beta}$ with α, $\beta = 1, 2, ...p$, $T(x)$ is the scalar tachyon field and V(T) is the tachyon potential, which, from string field theory arguments is found to be given by :

$$V(T) \sim e^{-\alpha T/2} \tag{2}$$

where $\alpha = 1$ (bosonic, $p = 25$) and ,$\alpha = \sqrt{2}$ (superstring, $p = 9$).

The tachyon potential appears, in the action, as a multiplicative factor to a term with a square root. This is what makes this theory unfamiliar in comparison to usual field theories known to us. This field theory has been analysed extensively in recent papers by Sen [2]. There has been a flurry of recent work on the cosmological relevance/implications of tachyonic matter [5] motivated by the fact that the effective energy–momentum tensor is equivalent to that of noninteracting, nonrotating dust [1].

At a pedagogical level, it is certainly true, to some extent atleast, that lower dimensional analogs of a field theory helps our understanding in some way, though working with such correspondences may sometimes be entirely useless (a prime example is lower dimensional gravity which doesn't seem to have much connection with actual 4D gravity despite the volumes that have been written on it). It also provides an useful method of introducing the theory to undergraduates or non-experts. Prominent examples include the lower dimensional mechanical analogs of massive Klein–Gordon theory (harmonic oscillator), the Higgs model (anharmonic oscillator/double well), sine Gordon theory (particle on a circle/periodic potential) and many others. With this is mind, we are tempted to ask the question –what is the zero dimensional/mechanical analog of the field theory of tachyon matter?

To understand this let us rewrite the action in zero dimensions obtained via the correspondence :- $x^i \to t$, $T \to x$ and $V(T) \to V(x)$:

$$S_0 = -\int dt V(x)\sqrt{1-\dot{x}^2} \tag{3}$$

The equation of motion for this action (with the assumption that $1 - \dot{x}^2 \neq 0$–the equality corresponds, as evident from the discussion below, of the particle reaching a terminal velocity) gives :

$$\ddot{x} + f(x)\dot{x}^2 = f(x) \tag{4}$$

where $f(x) = -\frac{1}{V}\frac{\partial V}{\partial x}$. Using the zero dimensional version of the tachyon potential, i.e. $V(x) = e^{-\alpha x}$ (we do away with the $\frac{1}{2}$ factor in the potential for the field theory version) we obtain the equation of motion :

$$\ddot{x} + \alpha\dot{x}^2 = \alpha \tag{5}$$

Redefining $x = \gamma y$ we obtain :

$$\ddot{y} + \alpha\gamma\dot{y}^2 = \frac{\alpha}{\gamma} \tag{6}$$

The above equation is familiar to all of us. Replacing $\alpha\gamma = \frac{\beta}{m}$ and $\frac{\alpha}{\gamma} = g$ we get back :

$$m\ddot{y} + \beta\dot{y}^2 = mg \tag{7}$$

which is the equation of motion of a particle of mass m moving in a Newtonian gravitational field in the presence of quadratic damping. One can therefore get back this equation from the action :

$$S_0 = -\int dt e^{-\frac{\beta y}{m}}\sqrt{1-\frac{\beta}{mg}\dot{y}^2} \tag{8}$$

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This Lagrangian is ofcourse in a dimensionless form. One may scale the coordinate and multiply by appropriate factors to have an action with the required dimensions. This is a straightforward exercise. Thus the zero dimensional version of the field theory of tachyon matter provides an action integral formulation of the motion under Newtonian gravity in the presence of quadratic damping.

One might ask – is this the unique action which gives the above equation of motion? To answer it, we look for a counterexample. Raising the Lagrangian to its nth power (recall the fact that the square root and the squared action for the motion of relativistic, massive test particles in a background gravitational field give the same equations of motion) we write an action of the form :

$$S_n = -\int dt e^{-\frac{n\beta y}{m}} \left(1 - \frac{\beta}{mg}\dot{y}^2\right)^{\frac{n}{2}} \tag{9}$$

From the equations of motion for this action we find that it is only for $n = 1$ (and no other value of n)we get the equation for a particle moving in a Newtonian gravitational field in the presence of quadratic damping. Another possibility is replacing $\sqrt{1-\dot{x}^2}$ by $\sqrt{1+\dot{x}^2}$. This yields a wrong sign in the right hand side $m\ddot{y} + \beta\dot{x}^2 = -mg$. It is true that there may be other actions which yield the same equation of motion, but, for the moment, we prefer to remain satisfied with the one above.

Though the equations of motion and solutions for the above problem are well known we are not entirely sure if the action integral reformulation exists. Moreover, the connection with the field theory of tachyon matter is new and perhaps worth mention. Additionally, this action integral formulation could be useful if one wishes to quantise the theory in the language of path integrals. The task is quite formidable in view of the exponential and the square root in the action. However, it is possible to expand the square root and exponential for small β and arrive at an action of the form (for small β) :

$$S_0 \sim -\int dt \left(1 - \frac{\beta y}{m} - \frac{\beta}{mg}\dot{y}^2\right) \tag{10}$$

For the above action it is easy to write out the path integral following the standard procedure for quadratic actions. Quadratic damping can be treated as a perturbation over classical solutions of $m\ddot{y} = mg$ and its effect on the kernel can be ascertained through its β dependence.

We also note that the Hamiltonian arising from the Lagrangian in (Eqn (7)) is given as:

$$H = \sqrt{\frac{mg}{\beta}} \left(p^2 + \frac{\beta}{mg} e^{-\frac{2\beta y}{m}}\right)^{\frac{1}{2}} \tag{11}$$

The overall negative sign in the action given in (1) or (7) is crucial in order to have a positive definite Hamiltonian. Using the above one might attempt at deriving a quantum mechanical or microscopic reason behind quadratic damping.

In addition, the generality of the approach manifest through the freedom in choosing $V(x)$ may be useful in situations with more complicated , spatially dependent, but quadratic in velocity, damping forces. Extensions to higher (two or three) dimensions in space is possible though the equations of motion there are no longer uncoupled – the presence of the nonlinear damping term being the cause behind this feature.

It is tempting, as a follow-up exercise, to extend the above ideas to $1+1$ dimensional field theories, in particular, nonlinear field theories. For static solutions of a nonlinear scalar field theory, the equations of motion can be written down by a straightforward mapping of the variables of the mechanical system in Euclidean time to the field theoretic variables. It is obvious that the ϕ'^2 term, which will be present in the equations of motion, will damp the solutions in a way similar to the mechanical system discussed above. However, even though static solutions for the field system can indeed be written down, it is not clear to us what they actually mean in the context of a field theory. These and similar issues provide avenues of further study.

Finally, it is a rather pleasant surprise that a field theory arising out of such an involved and complicated enterprise as string field theory does have a connection (albeit through an analogy) with a system which we all know about and also experience in our everyday life. This, like other theories with similar analogs, makes the field theory of tachyon matter perhaps somewhat closer to reality. As mentioned in the beginning of this article, what we have discussed here is just a zero dimensional analog– its input in the actual field theoretic context is, to us, at this moment, largely vacuous.

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