An $SL(3,\mathbb{Z})$ Multiplet of 8-Dimensional Type II Supergravity Theories and the Gauged Supergravity Inside

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Abstract

The so-called "massive 11-dimensional supergravity" theory gives, for one Killing vector, Romans' massive 10-dimensional supergravity in 10 dimensions, for two Killing vectors an $Sl(2,\mathbb{Z})$ multiplet of massive 9-dimensional supergravity theories that can be obtained by standard generalized dimensional reduction type IIB supergravity and has been shown to contain a gauged supergravity.

We consider a straightforward generalization of this theory to three Killing vectors and a 3×3 symmetric mass matrix and show that it gives an $Sl(3,\mathbb{Z})$ multiplet of 8-dimensional supergravity theories that contain an $SO(3)$ gauged supergravity which is, in some way, the dual to the one found by Salam and Sezgin by standard generalized dimensional reduction.

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Introduction

Massive and gauged supergravity theories are fascinating theories in which there has been much interest recently due to their connections, as effective string theories, with Maldacena's AdS/CFT correspondence conjecture.

It is clear that there is a lack of understanding of many basic features of these theories. One of the main puzzles is their origin from compactification of higher-dimensional theories. This is the case of Romans' massive $N = 2A, d = 10$ supergravity [1] which, according to the standard lore, should have its origin in 11-dimensional supergravity but so far it has been impossible to obtain from it by standard methods.

To explain the 11-dimensional origin of Romans' massive $N = 2A, d = 10$ supergravity a massive 11-dimensional theory, containing a mass parameter and an explicit Killing vector in its action, was proposed in Ref. [2]. Dimensional reduction of the theory in the direction of the Killing vector gives Romans' theory. The construction of this theory was based on the fact that one could derive the effective action of some supersymmetric solitons of Romans' theory from 11-dimensional gauged sigma models, in which the gauging was associated to a single Killing isometry of the background [3] and the theory was interpreted as the theory resulting from 11-dimensional supergravity in a background consisting of a *KK-brane*, with 9 spatial worldvolume dimensions and one special isometric direction that had to be compact, just as the special isometric direction of the KK monopole has to be compact.[4] We will have more to say about the interpretation of this kind of theories later.

In Ref. [5] the massive 11-dimensional theory of Ref. [2] was generalized in order to maintain some sort of S duality covariance of theory compactified on T^2: the origin of the S duality ($Sl(2, \mathbb{Z})$) symmetry of the 9-dimensional theory is simply the reparametrization-invariance of the internal manifold. This symmetry was broken by the presence of the KK-brane in one of the compact directions but one could introduce a second KK-brane in the orthogonal internal direction. Then, $Sl(2, \mathbb{Z})$ transformations bring us from one background with two given KK-branes to an S dual background. The generalized 11-dimensional massive theory has 2 commuting Killing vectors $\hat{\hat{k}}_{(m)}{}^{\hat{\hat{\mu}}}$ $n = 1, 2$ appearing explicitly in the action and a symmetric mass matrix Q^{mn} whose entries could be related to brane charges. Dimensional reduction of this theory in the direction of the two Killing vectors gives the 9-dimensional massive theories of Ref. [5] that can also be obtained by GDR from type IIB supergravity.

Recently, Cowdall [6] realized that this 9-dimensional contains the long sought-after $D = 9$ $N = 2$ gauged supergravity theory. To show this explicitly, he had to perform a few field redefinitions and eliminate 2 of the 3 vector fields through massive gauge transformations (*i.e.* letting them be eaten up by the 2 2-forms that then become massive) and then it could be seen that the resulting action has a $SO(2)$ gauge symmetry. This $SO(2)$ is also the R-symmetry of the $d = 9$ $N = 2$ supertranslation algebra, so that the identification of the found theory with the gauged supergravity theory is obvious. It should be stressed that the local $SO(2)$ symmetry does not arise in any special singular limit but that it is

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[4]The effective theory of the KK monopole is also a gauged sigma model of the same kind [4].

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already present in the original massive theory albeit in a rather unconventional form due to the presence of additional massive parameters.

The question arises immediately as to whether this new connection between massive and gauged supergravities is more general and why there is such relation at all. Our goal in this work is to investigate this relationship by exploring new examples of massive and gauged supergravities. At the same time we will study their 11-dimensional origin.

Let us first describe the new massive supergravity theories we present in this paper and how we are going to obtain them. It should be clear that the massive 11-dimensional theory of Refs. [2, 5] can be further generalized by allowing the index n to go from 1 to 3, for instance, $i.e.$ having 3 commuting Killing vectors in 11 dimensions (all the fields being independent of the 3 associated coordinates) and a 3×3 symmetric mass matrix Q^{mn}. This theory would give a standard fully general-covariant theory in $d = 8$ upon reduction in the directions of the 3 Killing vectors that now appear explicitly in the action. This would be a massive theory (in fact a full $Sl(3,\mathbb{Z})$ multiplet of massive theories,[5] with the mass matrix Q^{mn} transforming under $Sl(3,\mathbb{Z})$), but we may ask ourselves whether such a theory really is a supergravity theory, given that only the bosonic sector was considered in Refs. [2, 5]. It should be clear that the same theory could be obtained by standard procedures known to preserve supersymmetry: first, we can perform the standard dimensional reduction of the massive 9-dimensional supergravity theories of Ref. [5] to 8 dimensions. The resulting theory is a massive supergravity theory with a 2×2 mass matrix. The massless theory has an $Sl(3,\mathbb{Z}) \times Sl(2,\mathbb{Z})$ duality group and we could simply perform general $Sl(3,\mathbb{Z})$ transformations in the massive theory to introduce new mass parameters.[6] This procedure should preserve supersymmetry and give the same massive 8-dimensional theory, but it is difficult to implement in practice. Our proof that the theory we obtain is indeed a good standard supergravity theory will be to establish its relation to a well-known supergravity theory as we are going to explain shortly.

Does this new massive supergravity theory contain a gauged supergravity? In $d = 8$ the R-symmetry group of the superalgebra is $SO(3)$ which is just the local invariance of the scalars in $\mathcal{M} \in Sl(3,\mathbb{R})/SO(3)$. An $SU(2)$ $d = 8$ gauged supergravity was obtained by Salam and Sezgin by means of Scherk-Schwarz generalized dimensional reduction [8] from 11-dimensional supergravity in Ref. [8]. Then, a gauge theory exists and we want to know if our massive 8-dimensional theory contains it. Since both 8-dimensional theories have very different 11-dimensional origins this is a non-trivial test of our ideas.

We are going to show that indeed the massive theory contains a gauge theory but we are going to go further: the massive theory is really nothing but an $SO(3)$ gauged theory in disguise in which a non-standard basis of the $so(3)$ Lie algebra is used and in which

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[5]The classical theory has continuous duality groups: $Sl(3,\mathbb{R}) \times Sl(2,\mathbb{R})$ that are broken by quantum-mechanical effects to the discrete subgroups $Sl(3,\mathbb{Z}) \times Sl(2,\mathbb{Z})$. To the same conclusion one arrives by considering the preservation of the periodic boundary conditions of the coordinates in the internal dimensions. All our results will be classical and we will not pay much attention to these details unless strictly necessary.

[6]This is analogous to the procedure used to construct new duality-covariant families of black-hole solutions. see $e.g.$ [7].

Stückelberg fields are present so the mass terms of some fields look like field strengths. The theory is directly $SO(3)$ gauge invariant and, at the same time it has massive gauge invariance. There is no need to take any particular limit and removing the Stückelberg fields and going to a standard basis of $so(3)$ is just a matter of esthetics and choice and, at the same time, of maintaining $Sl(3,\mathbb{R})$ covariance in the sense of Refs. [5, 9].

What is the relation of this theory to Salam and Sezgin's? The theory turns out to be very similar but, there are two subtle, but important, differences: first of all, the $SO(3)$ vectors in Salam and Sezgin's (SS) theory are the Kaluza-Klein (KK) vectors while in the theory that we are going to derive from the 11-dimensional construction of Refs. [2, 5], the $SO(3)$ vectors are associated to membranes wrapped on 2 cycles of the T^3 we are compactifying on. Secondly, the potential in the SS theory comes with a factor $1/\Im m(\tau)$ where τ is the complex scalar field in the $Sl(2,\mathbb{R})/SO(2)$ coset while in our case the potential carries a factor $|\tau|^2/\Im m(\tau)$. This factor is precisely the S dual of the one in SS (*i.e.* they are related by the $Sl(2,\mathbb{R})$ transformation $\tau \to -1/\tau$) in the ungauged, massless theory and we will see that the KK vectors are also the S dual of the vectors coming from the 11-dimensional 3-form (also in the ungauged, massless, theory because the gauging explicitly breaks S duality).

It is clear, then, that if we wanted to obtain gauged 8-dimensional supergravity from the ungauged theory by the standard gauging procedure, we could have chosen to gauge the KK vectors and we would have obtained the SS theory or we could have chosen to gauge the vectors associated to membranes wrapped on 2 cycles and we would have obtained our theory. Both theories are related by an S duality transformation *which is not a symmetry,* neither of the theory nor of the equations of motion but related two different theories. The S duality transformation can also be understood as a field redefinition. In the first case we could have explained its 11-dimensional origin in more or less standard terms but in the second only by appealing to the ideas of Refs. [2, 5] we could get some 11-dimensional understanding. This is essentially our main result.

The rest of the paper is organized as follows: in Section 1 we perform the dimensional reduction of 11-dimensional supergravity to $d=8$. In Section 2 we perform the dimensional reduction of *massive* 11-dimensional supergravity to $d=8$ dimensions to obtain an $Sl(3,\mathbb{R})$ multiplet of massive $d=8$ supergravity theories with $SO(3)$ gauge symmetry. In Section 3 we study the vacua of this theory. In Section 4 we present our conclusions, comment on possible interpretations and future directions of work.

1 Direct dimensional reduction of $D=11$ Supergravity on T^3

Our goal in this section is to perform the standard dimensional reduction of 11-dimensional supergravity to obtain the theory describing the massless fields that arise in the compactification of that theory on T^3. 11-dimensional supergravity was compactified to 8 dimensions by Salam and Sezgin in Ref. [10], but they used Scherk and Schwarz's GDR [8] to com-

pactify on an $SU(2)$ internal manifold to obtain a $SU(2)$ gauged $d = 8$ supergravity. The gauged theory does not exhibit all the duality symmetries of the ungauged one. In particular, gauging usually break all electric-magnetic dualities. Since we are interested in the theory with all its dualities and its corresponding massive version, we start by the standard dimensional reduction of 11-dimensional supergravity.

1.1 $d = 11$ Supergravity

The bosonic fields of $N = 1, d = 11$ supergravity are the Elfbein and a 3-form potential[7]

$$\left\{ \hat{\hat{e}}_{\hat{\hat{\mu}}}{}^{\hat{\hat{a}}}, \hat{\hat{C}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}\hat{\hat{\rho}}} \right\} . \tag{1.1}$$

The field strength of the 3-form is

$$\hat{\hat{G}} = 4\, \partial \hat{\hat{C}} , \tag{1.2}$$

and is obviously invariant under the gauge transformations

$$\delta \hat{\hat{C}} = 3\, \partial \hat{\hat{\chi}} , \tag{1.3}$$

where $\hat{\hat{\chi}}$ is a 2-form.
The action for these bosonic fields is

$$\hat{\hat{S}} = \int d^{11}\hat{\hat{x}} \sqrt{|\hat{\hat{g}}|} \left[\hat{\hat{R}} - \frac{1}{2 \cdot 4!} \hat{\hat{G}}^2 - \frac{1}{6^4} \frac{1}{\sqrt{|\hat{\hat{g}}|}} \hat{\hat{\epsilon}}\, \partial \hat{\hat{C}}\, \partial \hat{\hat{C}}\, \hat{\hat{C}} \right] . \tag{1.4}$$

1.2 Dimensional Reduction

The KK Ansatz for the Elfbein is

$$\left(\hat{\hat{e}}_{\hat{\hat{\mu}}}{}^{\hat{\hat{a}}} \right) = \begin{pmatrix} e_\mu{}^a & e_m{}^i A^m{}_\mu \\ \\ 0 & e_m{}^i \end{pmatrix} , \qquad \left(\hat{\hat{e}}_{\hat{\hat{a}}}{}^{\hat{\hat{\mu}}} \right) = \begin{pmatrix} e_a{}^\mu & -A^m{}_a \\ \\ 0 & e_i{}^m \end{pmatrix} , \tag{1.5}$$

where $A^m{}_a = e_a{}^\mu A^m{}_\mu$. We define the internal metric on T^3 by

$$G_{mn} = e_m{}^i e_{nj} = -e_m{}^i e_n{}^j \delta_{ij} . \tag{1.6}$$

Under global transformations in the internal space

$$x^{m\,\prime} = \left(R^{-1\,T} \right)^m{}_n x^n + a^m , \qquad R \in GL(3, \mathbb{R}) , \tag{1.7}$$

objects with internal space indices (the internal metric $G = (G_{mn})$ and the KK vectors $\vec{A}_\mu = (A^m{}_\mu)$) transform as follows:

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[7]Index conventions: $\hat{\hat{\mu}}$ ($\hat{\hat{a}}$) are curved (flat) 11-dimensional, μ (a) are curved (flat) 8-dimensional, and m (i) are curved (flat) 3-dimensional (compact space). Our signature is $(+ - \cdots -)$.

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$$G' = RGR^T, \qquad \vec{A}'_\mu = (R^{-1})^T \vec{A}_\mu. \tag{1.8}$$

We know that $GL(3,\mathbb{R})$ can be decomposed in $Sl(3,\mathbb{R}) \times \mathbb{R}^+ \times \mathbb{Z}_2$ and any matrix R, forgetting its $\mathbb{Z}_2$ part (we will focus on $GL(3,\mathbb{R})/\mathbb{Z}_2 \sim Sl(3,\mathbb{R}) \times \mathbb{R}^+$), can therefore be decomposed into

$$R = c\Lambda, \quad \Lambda \in Sl(3,\mathbb{R}), \quad c \in \mathbb{R}^+. \tag{1.9}$$

We want to separate fields that transform under the different factors. First we define the symmetric $Sl(3,\mathbb{R})$ matrix

$$\mathcal{M} = -G/|\det G|^{1/3}, \tag{1.10}$$

and the scalar

$$\sqrt{|\det G|} = e^{-\varphi}. \tag{1.11}$$

Now, under $Sl(3,\mathbb{R})$ only $\mathcal{M}$ and $\vec{A}_\mu$ transform:

$$\mathcal{M}' = \Lambda \mathcal{M} \Lambda^T, \quad \vec{A}'_\mu = (\Lambda^{-1})^T \vec{A}_\mu, \tag{1.12}$$

that is, $\vec{A}_\mu$ transforms contravariantly, while under $\mathbb{R}^+$ rescalings only φ and $\vec{A}_\mu$ transform:

$$\varphi' = \varphi - \log c, \quad \vec{A}'_\mu = c\vec{A}_\mu. \tag{1.13}$$

For future convenience, we label the KK vector with an upper index 1, *i.e.* $A^{1\,m}{}_\mu$.

Using the standard techniques, the above Elfbein Ansatz, and rescaling the resulting 8-dimensional metric to the Einstein frame

$$g_{\mu\nu} = e^{\varphi/3} g_{E\,\mu\nu}, \tag{1.14}$$

one finds

$$\int d^{11}\hat{x} \sqrt{|\hat{\hat{g}}|} \left[\hat{\hat{R}} \right] = \int d^8 x \sqrt{|g_E|} \left[R_E + \tfrac{1}{2} (\partial\varphi)^2 \right.$$
$$\left. + \tfrac{1}{4}\mathrm{Tr}\left(\partial\mathcal{M}\mathcal{M}^{-1}\right)^2 - \tfrac{1}{4} e^{-\varphi} F^{1\,m} \mathcal{M}_{mn} F^{1\,n} \right], \tag{1.15}$$

where

$$F^{1\,m} = 2\partial A^{1\,m}. \tag{1.16}$$

The kinetic term for $\mathcal{M}$ is just an $Sl(3,\mathbb{R})/SO(3)$ sigma model.

The fields arising from $\hat{\hat{C}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}\hat{\hat{\rho}}}$ are $\{C_{\mu\nu\rho}, B_{\mu\nu\,m}, A^{2\,m}{}_\mu, a\}$. We decompose the 11-dimensional 3-form by identifying objects with flat 11- and 8-dimensional flat indices (up to factors coming from the rescaling of the metric) as

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$$\hat{\hat{C}}_{abc} = e^{-\varphi/2}C_{abc}\,,$$

$$\hat{\hat{C}}_{ab\,m} = e^{-\varphi/3}B_{m\,ab}\,,$$ (1.17)

$$\hat{\hat{C}}_{a\,mn} = \epsilon_{mnp}e^{-\varphi/6}A^{2\,p}{}_a\,,$$

which implies, for curved components

$$\hat{\hat{C}}_{\mu\nu\rho} = C_{\mu\nu\rho} + 3A^{1\,m}{}_{[\mu}B_{|m|\,\nu\rho]} + 3\epsilon_{mnp}A^{1\,m}{}_{[\mu}A^{1\,n}{}_\nu A^{2\,p}{}_{\rho]} + \epsilon_{mnp}\,a\,A^{1\,m}{}_{[\mu}A^{1\,n}{}_\nu A^{1\,p}{}_{\rho]}\,,$$

$$\hat{\hat{C}}_{\mu\nu\,m} = B_{m\,\mu\nu} + 2\epsilon_{mnp}A^{1\,n}{}_{[\mu}A^{2\,p}{}_{\nu]} + \epsilon_{mnp}\,a\,A^{1\,n}{}_{[\mu}A^{1\,p}{}_{\nu]}\,,$$

$$\hat{\hat{C}}_{\mu\,mn} = \epsilon_{mnp}A^{2\,p}{}_\mu + \epsilon_{mnp}a\,A^{1\,p}{}_\mu\,,$$

$$\hat{\hat{C}}_{mnp} = \epsilon_{mnp}\,a\,.$$

(1.18)

These fields inherit the following gauge transformations from 11-dimensional gauge and general coordinate transformations of $\hat{\hat{C}}$:

$$\delta C = 3\partial\Lambda - 6A^{1\,m}\partial\Lambda_m + 3\epsilon_{mnp}A^{1\,m}A^{1\,n}\partial\Lambda^{2\,p}\,,$$

$$\delta B_m = 2\partial\Lambda_m - 2\epsilon_{mnp}A^{1\,n}\partial\Lambda^{2\,p}\,,$$ (1.19)

$$\delta A^{2\,m} = \partial\Lambda^{2\,m}\,.$$

In particular we see that this choice implies that these potentials do not transform under reparametrizations of the internal torus $\delta A^{1\,m} = \partial\Lambda^{1\,m}$. The gauge-invariant field strengths of the above fields are

$$G = 4\partial C + 6F^{1\,m}B_m\,,$$

$$H_m = 3\partial B_m + 3\epsilon_{mnp}F^{1\,n}A^{2\,p}\,,$$ (1.20)

$$F^{2\,m} = 2\partial A^{2\,m}\,,$$

and lead to the following non-trivial Bianchi identities:

$$\partial G = 2F^{1\,m}H_m\,,$$

$$\partial H_m = \tfrac{3}{2}\epsilon_{mnp}F^{1\,n}F^{2\,p}\,,$$ (1.21)

Using now

$$e_i{}^m e_j{}^n e_k{}^p \epsilon_{mnp} = \det(e^{-1})\,\epsilon_{ijk}\,,\tag{1.22}$$

with $\det(e^{-1}) = e^{\varphi}$, we get the following decomposition of the 11-dimensional 4-form field strength into the above 8-dimensional field strengths:

$$\hat{\hat{G}}_{abcd} = e^{-2\varphi/3}G_{abcd}\,,$$

$$\hat{\hat{G}}_{abc\,i} = e^{-\varphi/2}e_i{}^m\,H_{m\,abc}\,,$$

$$\hat{\hat{G}}_{ab\,ij} = e^{2\varphi/3}\,\epsilon_{ijk}\,e_p{}^k\left[F^{2\,p}{}_{ab} + a\,F^{1\,p}{}_{ab}\right]\,,\tag{1.23}$$

$$\hat{\hat{G}}_{a\,ijk} = e^{5\varphi/6}\,\epsilon_{ijk}\,\partial_a\,a\,.$$

We can now reduce the kinetic term of the 11-dimensional 3-form. The result can be combined with the result of the reduction of the Einstein-Hilbert term, giving

$$\int d^{11}\hat{\hat{x}}\sqrt{|\hat{\hat{g}}|}\left[\hat{\hat{R}} - \tfrac{1}{2\cdot4!}\hat{\hat{G}}^2\right] = \int d^8x\sqrt{|g_E|}\left[R_E + \tfrac{1}{4}\mathrm{Tr}\left(\partial\mathcal{M}\mathcal{M}^{-1}\right)^2 + \tfrac{1}{4}\mathrm{Tr}\left(\partial\mathcal{W}\mathcal{W}^{-1}\right)^2\right.$$
$$\left. -\tfrac{1}{4}F^{i\,m}\mathcal{M}_{mn}\mathcal{W}_{ij}F^{j\,n} + \tfrac{1}{2\cdot3!}H_m\mathcal{M}^{mn}H_n - \tfrac{1}{2\cdot4!}e^{-\varphi}G^2\right]\,,\tag{1.24}$$

where we have introduced the symmetric $Sl(2,\mathbb{R})/SO(2)$ matrix

$$\mathcal{W} = \frac{1}{\Im\mathrm{m}(\tau)}\begin{pmatrix} |\tau|^2 & \Re\mathrm{e}(\tau) \\ \Re\mathrm{e}(\tau) & 1 \end{pmatrix}\,,\tag{1.25}$$

whet τ is the standard complex combination

$$\tau = a + ie^{-\varphi}\,.\tag{1.26}$$

Let us now reduce the Chern-Simons term also using tangent space indices and taking into account the definition, in any dimension

$$\epsilon^{\mu_1\cdots\mu_d} = \sqrt{|g|}\,\epsilon^{a_1\cdots a_d}e_{a_1}{}^{\mu_1}\ldots e_{a_d}{}^{\mu_d}\,.\tag{1.27}$$

The final result is[8]

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[8] One of our coefficients differs from the corresponding in Ref. [10]. Our Chern-Simons term has been explicitly checked to be gauge-invariant.

$$S = \int d^8 x \sqrt{|g_E|} \left\{ R_E + \tfrac{1}{4}\mathrm{Tr}\left(\partial\mathcal{M}\mathcal{M}^{-1}\right)^2 + \tfrac{1}{4}\mathrm{Tr}\left(\partial\mathcal{W}\mathcal{W}^{-1}\right)^2 \right.$$

$$-\tfrac{1}{4}F^{i\,m}\mathcal{M}_{mn}\mathcal{W}_{ij}F^{j\,n} + \tfrac{1}{2\cdot 3!}H_m\mathcal{M}^{mn}H_n - \tfrac{1}{2\cdot 4!}e^{-\varphi}G^2 \,,$$

$$-\tfrac{1}{6^3\cdot 2^4}\tfrac{1}{\sqrt{|g_E|}}\,\epsilon\left[GGa - 8GH_mA^{2\,m} + 12G(F^{2\,m}+aF^{1\,m})B_m \right.$$

$$\left.\left. -8\epsilon^{mnp}H_mH_nB_p - 8G\partial aC - 16H_m(F^{2\,m}+aF^{1\,m})C\right]\right\} \,. \tag{1.28}$$

The kinetic terms (except for that of C) are explicitly invariant under $Sl(2,\mathbb{R})$ transformations

$$\mathcal{W}' = \Lambda\mathcal{W}\Lambda^T \,, \qquad F^{i\,m\,\prime} = F^{j\,m}\left(\Lambda^{-1}\right)_j{}^i \,, \qquad \Lambda \in Sl(2,\mathbb{R}) \,, \tag{1.29}$$

and $Sl(3,\mathbb{R})$ transformations

$$\mathcal{M}' = K\mathcal{M}K^T \,, \quad F^{i\,m\,\prime} = F^{i\,n}\left(K^{-1}\right)_n{}^m \,, \quad H'_m = K_m{}^n H_n \,, \quad K \in Sl(3,\mathbb{R}) \,. \tag{1.30}$$

The kinetic term of C and the Chern-Simons term are not invariant as a matter of fact. However, let us look into the equations of motion of C. We can write them, together with the Bianchi identity, in the following form:

$$\partial G^i = 2F^{i\,m}H_m \,, \tag{1.31}$$

where

$$G^1 \equiv G \,, \qquad G^2 \equiv -e^{-\varphi}\star G - aG \,. \tag{1.32}$$

G^i transforms as a doublet under $Sl(2,\mathbb{R})$ (just like the doublet $F^{i\,m}$) and therefore, the above equation of motion is covariant under $Sl(2,\mathbb{R})$ electric-magnetic duality transformations. The remaining equations of motion are covariant under $Sl(2,\mathbb{R})$ transformations as well. The structures are very similar to those of $N = 4, d = 4$ supergravity (see *e.g.* Ref. [11]), the obvious difference being that in four dimensions we dualize 2-form field strengths and in eight dimensions we dualize 4-form field strengths. This duality was first described in Ref. [12] and is part of a series of electric-magnetic dualities present in type II theories in any dimension (the 6-dimensional version was studied in Ref. [13] and a general discussion can be found in Ref. [14]).

Let us summarize our results: the 8-dimensional supergravity theory we have just obtained has, then, the bosonic fields

$$\{g_{\mu\nu}, C, B_m, A^{1\,m}, A^{2\,m}, a, \varphi, \mathcal{M}_{mn}\} \,, \tag{1.33}$$

with field strengths given by Eqs. (1.16, 1.20) and action given by Eq. (1.28). The scalars parametrize $Sl(3,\mathbb{R})/SO(3)$ and $Sl(2,\mathbb{R})/SO(2)$ sigma models. The action has the global

invariance group $Sl(3, \mathbb{R})$ but the equations of motion are also invariant under $Sl(2, \mathbb{R})$ electric-magnetic duality transformations.

2   Dimensional Reduction of Massive 11-Dimensional Supergravity

In this section we are going to perform the dimensional reduction to eight dimensions of a further generalization of the massive 11-dimensional theory proposed in Refs. [2, 5]. We are going to use the same field definitions as in the previous section since we want to recover that theory in the massless limit. First we start by describing the massive 11-dimensional theory.

2.1   Massive 11-Dimensional Supergravity

The theory we are considering has three Killing vectors $\hat{\hat{k}}_{(n)}$, which are defined by

$$\hat{\hat{k}}_{(m)}{}^{\hat{\hat{\mu}}}\partial_{\hat{\hat{\mu}}} = \partial_m \qquad \rightarrow \qquad \hat{\hat{k}}_{(m)}{}^{\hat{\hat{\mu}}}\hat{\hat{k}}_{(n)}{}^{\hat{\hat{\nu}}}\hat{\hat{g}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}} = G_{nm} \ , \tag{2.1}$$

and a symmetric matrix Q^{mn}, which we will leave arbitrary for the moment. With these elements and the 2-form gauge parameter $\hat{\hat{\chi}}$ we construct the massive gauge parameter[9]

$$\hat{\hat{\lambda}}^{(n)} \equiv -i_{\hat{\hat{k}}_{(m)}}\hat{\hat{\chi}}Q^{nm} \ , \tag{2.2}$$

and define the massive gauge transformations of the two 11-dimensional fields:

$$\begin{cases} \delta_{\hat{\hat{\chi}}}\hat{\hat{g}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}} & = \ 2\hat{\hat{\lambda}}^{(n)}{}_{(\hat{\hat{\mu}}}\hat{\hat{k}}_{(n)}{}^{\hat{\hat{\rho}}}\hat{\hat{g}}_{\hat{\hat{\nu}})\hat{\hat{\rho}}} \ , \\[2ex] \delta_{\hat{\hat{\chi}}}\hat{\hat{C}} & = \ 3\partial\hat{\hat{\chi}} + 3\hat{\hat{\lambda}}^{(n)}\left(i_{\hat{\hat{k}}_{(n)}}\hat{\hat{C}}\right) \ . \end{cases} \tag{2.3}$$

The 4-form field strength is given by

$$\hat{\hat{G}} = 4\partial\hat{\hat{C}} + 3\left(i_{\hat{\hat{k}}_{(m)}}\hat{\hat{C}}\right)Q^{mn}\left(i_{\hat{\hat{k}}_{(n)}}\hat{\hat{C}}\right) \ . \tag{2.4}$$

The proposed 11-dimensional massive supergravity then reads[10]

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[9] $i_{\hat{\hat{k}}}\hat{\hat{T}}$ stands for the contraction of the last index of the tensor $\hat{\hat{T}}$ with the vector $\hat{\hat{k}}$.

[10] This action is the same as in Ref. [5] but here we have taken the contorsion part out of the connection used in the curvature.

$$\hat{\hat{S}} = \int d^{11}\hat{\hat{x}}\sqrt{|\hat{\hat{g}}|} \left\{ \hat{\hat{R}}\left(\hat{\hat{g}}\right) + \tfrac{1}{2}\left(d\hat{\hat{k}}_{(n)}\right)_{\hat{\mu}\hat{\nu}} Q^{nm}\left(i_{\hat{\hat{k}}_{(m)}}\hat{\hat{C}}\right)^{\hat{\mu}\hat{\nu}} - \tfrac{1}{2\cdot 4!}\hat{\hat{G}}^{\,2} \right.$$

$$- \hat{\hat{K}}_{\hat{\mu}\hat{\nu}\hat{\rho}}\hat{\hat{K}}^{\hat{\nu}\hat{\rho}\hat{\mu}} + \tfrac{1}{2}\left(\hat{\hat{k}}_{(n)\ \hat{\mu}} Q^{nm}\hat{\hat{k}}_{(m)}{}^{\hat{\mu}}\right)^2 - \left(\hat{\hat{k}}_{(n)\ \hat{\mu}} Q^{nm}\hat{\hat{k}}_{(m)\ \hat{\nu}}\right)^2$$

$$- \tfrac{1}{6^4}\frac{\hat{\hat{\epsilon}}}{\sqrt{|\hat{\hat{g}}|}}\left\{ \partial\hat{\hat{C}}\,\partial\hat{\hat{C}}\,\hat{\hat{C}} + \tfrac{9}{8}\partial\hat{\hat{C}}\,\hat{\hat{C}}\left(i_{\hat{\hat{k}}_{(n)}}\hat{\hat{C}}\right)Q^{nm}\left(i_{\hat{\hat{k}}_{(m)}}\hat{\hat{C}}\right) \right.$$

$$\left.\left. + \tfrac{27}{80}\hat{\hat{C}}\left[\left(i_{\hat{\hat{k}}_{(n)}}\hat{\hat{C}}\right)Q^{nm}\left(i_{\hat{\hat{k}}_{(m)}}\hat{\hat{C}}\right)\right]^2 \right\}\right\}, \qquad (2.5)$$

where we have defined a *contorsion* tensor

$$\hat{\hat{K}}_{\hat{a}\hat{b}\hat{c}} = \tfrac{1}{2}\left(\hat{\hat{T}}_{\hat{a}\hat{c}\hat{b}} + \hat{\hat{T}}_{\hat{b}\hat{c}\hat{a}} - \hat{\hat{T}}_{\hat{a}\hat{b}\hat{c}}\right), \qquad (2.6)$$

and where the *torsion* is defined by

$$\hat{\hat{T}}_{\hat{\mu}\hat{\nu}}{}^{\hat{\rho}} = -\left(i_{\hat{\hat{k}}_{(n)}}\hat{\hat{C}}\right)_{\hat{\mu}\hat{\nu}} Q^{nm}\hat{\hat{k}}_{(m)}{}^{\hat{\rho}}. \qquad (2.7)$$

Note that when considering the above theory defined with n Killing vectors, we should take Q to be invertible, since otherwise we could diagonalize Q and end up with a theory defined with less Killing vectors. *E.g.* in the case at hand, taking Q to have 2 zero eigenvalues, would lead to Romans' theory compactified over a 2-torus.

2.2 Dimensional Reduction

We start by reducing the massive gauge parameters, the massive gauge transformations of the fields (whose definitions are the same as in the massless case) and the field strengths. We first make the following definitions inspired by $SO(3)$ gauge theory (see Appendix A) of which $A^{2\,m}$ is going to play the role of gauge field:

$$f_{mn}{}^{p} \equiv \epsilon_{mnq}Q^{qp}, \qquad \sigma^{m}{}_{n} = \Lambda^{2\,p}f_{pn}{}^{m}. \qquad (2.8)$$

We are also going to use the definitions of $SO(3)$ gauge covariant derivative that can be found in that appendix. We get

$$\begin{cases} \hat{\hat{\lambda}}^{(m)}{}_{\mu} = -\Lambda_{n\,\mu}Q^{nm}, \\[2mm] \hat{\hat{\lambda}}^{(m)}{}_{n} = \sigma^{m}{}_{n}. \end{cases} \qquad (2.9)$$

The massive gauge transformations are

$$\delta \mathcal{M}_{mn} = -\mathcal{M}_{pn}\sigma^p{}_m - \mathcal{M}_{mp}\sigma^p{}_n \,,$$

$$\delta A^{1\,m} = \sigma^m{}_n A^{1\,n} - Q^{mn}\Lambda_n \,,$$

$$\delta A^{2\,m} = \mathcal{D}\Lambda^{2\,m} \,, \tag{2.10}$$

$$\delta B_m = 2\partial\Lambda_m - B_n\sigma^n{}_m + 2\partial\sigma^q{}_m (Q^{-1})_{qn} A^{1\,n} \,,$$

$$\delta C = 3\partial\Lambda_{(2)} - 3\partial\sigma^m{}_n (Q^{-1})_{mp} A^{1\,n} A^{1\,p} - 6\Lambda_m \partial A^{1\,m} \,.$$

We see that there are two kinds of massive gauge transformations: those generated by the 1-form parameters Λ_m which are standard massive gauge transformations that shift the vectors $A^{1\,m}$ so they can be completely gauged away, and those generated by the scalar parameters $\Lambda^{2\,m}$ that take the form of $SO(3)$ gauge transformations in a non-standard basis (see Appendix A).

The gauge-covariant field strengths are

$$G = 4\partial C + 6 F^{1\,m} B_m - 3 B_m Q^{mn} B_n \,,$$

$$H_m = 3\partial B_m + 3\epsilon_{mnp} F^{1\,n} A^{2\,p} \,,$$

$$F^{2\,m} = 2\partial A^{2\,m} - f_{pq}{}^m A^{2\,p} A^{2\,q} \,, \tag{2.11}$$

$$F^{1\,m} = 2\partial A^{1\,m} + Q^{mn} B_n \,,$$

$$\mathcal{D}\mathcal{M}_{mn} = \partial\mathcal{M}_{mn} + f_{pm}{}^q A^{2\,p} \mathcal{M}_{qn} + f_{pn}{}^q A^{2\,p} \mathcal{M}_{mq} \,,$$

where the $F^{1\,m}$ appearing in G and H_m is the massive one.

The field strength of $A^{2\,m}$ is just an $SO(3)$ gauge field strength while the field strength of the scalar matrix $\mathcal{M}_{mn}$ is the $SO(3)$ gauge covariant derivative of an object with two covariant vector indices. The field strength of $A^{1\,m}$ has a different form, typical of massive theories and it is ready to give mass terms (a mass matrix) for the 2-forms B_m when the $A^{1\,m}$s which are nothing but Stückelberg fields for the B_ms, are gauged away. The field strength of B_m could also be written in this way

$$H_m = 3\mathcal{D}B_m + 6\epsilon_{mnp}\partial A^{1\,n} A^{2\,p} \,. \tag{2.12}$$

The first term is the $SO(3)$ gauge covariant derivative of an $SO(3)$-covariant object (anti-symmetrized, as usual in all three lower indices). The second term restores the invariance under Λ_m transformations which is broken in the covariant derivative term.

Thus, all field strengths are invariant under Λ_m transformations and covariant under $\Lambda^{2\,m}$ (*i.e.* $SO(3)$) transformations

$$\delta H_m = -H_n \sigma^n{}_m \,,$$

$$\delta F^{1,2\,m} = \sigma^m{}_n F^{1,2\,n} \,,$$

$$(2.13)$$

except for G, which is invariant.

Let us have a look at the part of the 11-dimensional massive theory that, naively, will lead to the potential for the $Sl(3,\mathbb{R})/SO(3)$ scalar fields. This term is easily found to be

$$\sqrt{|\hat{\hat{g}}|}\left[\tfrac{1}{2}\left(\hat{\hat{k}}_{(n)\,\hat{\mu}}Q^{nm}\hat{\hat{k}}_{(m)}{}^{\hat{\mu}}\right)^2 - \left(\hat{\hat{k}}_{(n)\,\hat{\mu}}Q^{nm}\hat{\hat{k}}_{(m)\,\hat{\nu}}\right)^2\right] =$$

$$= \sqrt{|g_E|}\left\{\tfrac{1}{2}e^{-\varphi}\left[\mathrm{Tr}\left(Q\mathcal{M}\right)\right]^2 - 2\mathrm{Tr}\left(\mathcal{M}Q\mathcal{M}Q\right)\right\} \,.$$

$$(2.14)$$

The two terms in the eleven dimensional Lagrangian that are modifying the field strengths of the objects coming from the metric are,

$$\tfrac{1}{2}\sqrt{|\hat{\hat{g}}|}\left(d\hat{\hat{k}}_{(n)}\right)_{\hat{\mu}\hat{\nu}}Q^{nm}\left(i_{\hat{\hat{k}}_{(m)}}\hat{\hat{C}}\right)^{\hat{\mu}\hat{\nu}} =$$

$$= \sqrt{|g_E|}\left\{-\tfrac{1}{2}e^{-\varphi}\left(dA^{1\,n}\right)_{\mu\nu}\mathcal{M}_{np}Q^{pm}B_m^{\mu\nu} + \epsilon_{qmr}Q^{mn}\left(\partial_\mu\mathcal{M}\cdot\mathcal{M}^{-1}\right)_n{}^q A^{2\,r\,\mu}\right\} \,,$$

$$(2.15)$$

and the contorsion

$$-\sqrt{|\hat{\hat{g}}|}\,\hat{\hat{K}}_{\hat{\mu}\hat{\nu}\hat{\rho}}\hat{\hat{K}}^{\hat{\mu}\hat{\nu}\hat{\rho}} = \sqrt{|g_E|}\left\{-\tfrac{1}{4}e^{-\varphi}B_{m\,\mu\nu}(Q\mathcal{M}Q)^{mn}B_n{}^{\mu\nu}\right.$$

$$+\tfrac{1}{2}\left[(Q\mathcal{M}Q)^{mv}\mathcal{M}^{px}\epsilon_{pms}\epsilon_{xuw} + Q^{pv}Q^{mx}\epsilon_{pms}\epsilon_{xvw}\right]A^{2\,s}{}_\mu A^{2\,w\,\mu}$$

$$\left.-\tfrac{1}{4}e^{\varphi}\,a^2\left[(Q\mathcal{M}Q)^{ms}\mathcal{M}^{ru}\mathcal{M}^{pv}\epsilon_{rpm}\epsilon_{uvs} - 2Q^{mn}Q^{sp}\mathcal{M}^{ru}\epsilon_{mpr}\epsilon_{nsu}\right]\right\} \,.$$

$$(2.16)$$

It can be seen that the terms in Eq. (2.15) and the first two terms in Eq. (2.16) combine in just the right way with the $F^{1\,n}$ and the $\partial_\mu\mathcal{M}\cdot\mathcal{M}^{-1}$ terms in Eq. (1.15), as to promote them to their gauge-covariant equivalents in Eq. (2.11).

Note that one also finds a potential for a, which however can be rewritten as

$$+ \sqrt{|g_E|}\tfrac{1}{2}e^{\varphi}a^2\left\{\left[\mathrm{Tr}\left(Q\mathcal{M}\right)\right]^2 - 2\mathrm{Tr}\left(Q\mathcal{M}Q\mathcal{M}\right)\right\} \,,$$

$$(2.17)$$

so that it can be combined with the result in Eq. (2.14) to complete the potential for the scalars in the $d=8$ theory as

$$\mathcal{V} = -\tfrac{1}{2}\frac{|\tau|^2}{\Im\mathrm{m}(\tau)}\left\{\left[\mathrm{Tr}\left(Q\mathcal{M}\right)\right]^2 - 2\mathrm{Tr}\left(Q\mathcal{M}Q\mathcal{M}\right)\right\} \,.$$

$$(2.18)$$

The complete $d=8$ massive action can be written as

$$
\begin{aligned}
S \;=\; \int d^8x \sqrt{|g_E|} \Big\{ & R_E + \tfrac14 \mathrm{Tr}\,(\mathcal{DMM}^{-1})^2 + \tfrac14 \mathrm{Tr}\,(\partial\mathcal{WW}^{-1})^2 \\[4pt]
& -\tfrac14 F^{i\,m}\mathcal{M}_{mn}\mathcal{W}_{ij}F^{j\,n} + \tfrac{1}{2\cdot 3!}H_m\mathcal{M}^{mn}H_n - \tfrac{1}{2\cdot 4!}e^{-\varphi}G^2 - \mathcal{V} \\[4pt]
& -\tfrac{1}{6^3\cdot 2^4}\tfrac{1}{\sqrt{|g_E|}}\,\epsilon\Big[GGa - 8GH_m A^{2\,m} + 12GG^m_{(2)}B_m - 16H_m G^m_{(2)}C \\[4pt]
& -8G\partial aC - 8\epsilon^{mnp}H_m H_n B_p + 2H_m Q^{mn}B_n\left(Ca + 6B_p A^{2\,p}\right) \\[4pt]
& -3B_m Q^{mn}B_n\left(Ga + 2H_m A^{2\,m} + 3B_m G^m_{(2)} + 2C\partial a\right) \\[4pt]
& +4Cf_{pq}{}^m A^{2\,p}A^{2\,q}H_m - 12Cf_{pq}{}^m A^{2\,p}G^q_{(2)}B_m \\[4pt]
& +\tfrac{3^2\cdot 11}{2}\left(B_n Q^{nm}B_m\right)^2 a - \tfrac{2^2\cdot 3^3\cdot 11}{5}B_n Q^{nm}B_m f_{pq}{}^r A^{2\,p}A^{2\,q}B_r \Big]\Big\}\,,
\end{aligned}
\tag{2.19}
$$

where we have introduced the abbreviation

$$
G^m_{(2)} \;=\; F^{2\,m} + aF^{1\,m}\,.
\tag{2.20}
$$

This is evidently a set of $SO(3)$-gauged theories (a different gauged theory for each choice of non-singular mass matrix Q^{mn}; the $SO(3)$ gauge symmetry is lost for singular choices of mass matrix), the vectors $A^{2\,m}$ associated to 11-dimensional membranes wrapped in 2 cycles of the internal T^3 playing the role of $SO(3)$ gauge fields. At the same time these are theories with massive fields: we find explicit mass terms for the 3 B_ms of the form $B_m Q^{mn}B_n$. Some of these terms are implicit in squares of the $A^{1\,m}$ vector field strengths. These vectors can be completely gauged away ("eaten" by the 2-forms) and play the role of Stückelberg fields. In other words: in the physical spectrum of the theory there are no quantum excitations associated to those vector fields and the quantum excitations associated to the 2-forms are massive.

We can wonder how this can happen in supergravity theories since all the fields in the supergravity multiplet should have the same mass (zero). The reason why is that supersymmetry is partially and spontaneously broken: massive supergravity theories as formally invariant under a certain modification of the full supersymmetry transformations of the massless theory. However, the vacuum of these theories breaks part of the symmetry and in that vacuum the supergravity multiplet becomes reducible into a massless supergravity multiplet and massive matter multiplets. In this case, the theory is written in a form in which all the gauge symmetries of the massless theory are formally present, but in the vacuum those symmetries responsible for the masslessness of the 2-forms are spontaneously broken.

This set of theories transforms into itself under global $Sl(3,\mathbb{Z})$ transformations and therefore they are an $Sl(3,\mathbb{Z})$ multiplet of theories.

At this point it is important to compare this set of theories with the $d = 8$ $SU(2)$ gauged supergravity of Salam and Sezgin [10]. To make the comparison easier we can first use a mass matrix proportional to the identity, the proportionality constant being the coupling constant g. We can immediately see that the kinetic terms in their action are identical to those in our action Eq. (2.19) up to the redefinition $\varphi_{\mathrm{SS}} = -\varphi_{\mathrm{ours}}$ but with the roles of the vector fields $A^{1\,m}$ and $A^{2\,m}$ reversed. The second difference we see is in the potential, which in their case (but in our notation) we can write in the form

$$
\begin{aligned}
\mathcal{V}_{\mathrm{SS}} &= \frac{1}{\Im\mathrm{m}(\tau)} \mathcal{V}(\mathcal{M})\,, \\[2mm]
\mathcal{V}_{\mathrm{ours}} &= \frac{|\tau|^2}{\Im\mathrm{m}(\tau)} \mathcal{V}(\mathcal{M})\,, \\[2mm]
\mathcal{V}(\mathcal{M}) &= -\tfrac{1}{2} g^2 \left\{ (\mathrm{Tr}\,\mathcal{M})^2 - 2\mathrm{Tr}\,(\mathcal{M}^2) \right\}\,.
\end{aligned}
\tag{2.21}
$$

These two potentials and the vectors $A^{1\,m}$ and $A^{2\,m}$ are related by the $Sl(2,\mathbb{Z})$ transformation

$$
\Lambda = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\,,
\tag{2.22}
$$

according to the rules Eqs. (1.29,1.32).

3 Vacua

In this Section we want to study the simplest vacuum of the theory, the one which presumably preserves more supersymmetry. A more detailed study of the vacua of this theory will be presented elsewhere.

The scalar potential of this theory is $Sl(3,\mathbb{R})$ symmetric but completely breaks the $Sl(2,\mathbb{R})$ global invariance of he massless theory. The factor $\mathcal{V}(\mathcal{M})$ has the typical form of a potential for the $Sl(3,\mathbb{R})/SO(2)$ fields and acts as a sort of Romans-like mass term for a. It is reasonable to expect that the simplest vacua will be those that minimize the factor $\mathcal{V}(\mathcal{M})$ and thus we will assume that $\mathcal{V}(\mathcal{M})$ has a minimum for some constant values $\mathcal{M} = \mathcal{M}_0$, the value of $\mathcal{V}(\mathcal{M})$ at this minimum being denoted by $\mathcal{V}_0$. Further we will take $a = 0$ (the invariance under constant shifts of a is broken and we cannot simply take any constant value as in the massless case). We are left with non-trivial metric and φ field with potential $\mathcal{V}_0 e^{-\varphi}$. The only equations of motion that remain to be solved are

$$R_{\mu\nu} + \tfrac{1}{2}\partial_\mu\varphi\partial_\nu\varphi + \tfrac{1}{12}\mathcal{V}_0 e^{-\varphi}g_{\mu\nu} = 0 \,,$$

$$\nabla^2\varphi + \tfrac{1}{2}\mathcal{V}_0 e^{-\varphi} = 0 \,.$$

(3.1)

It is clear that, unless $\mathcal{V}_0 = 0$ Minkowski spacetime is not going to be a solution. In fact these equations are almost identical to those appearing in Romans' 10-dimensional theory. This is not surprising since our theory contains the dimensional reduction of Romans'. Thus, we can look for a solution similar to the 10-dimensional D8-brane, i.e. a domain-wall-type solution. Choosing the coordinate system in which the solution takes a simpler form, we find

$$ds^2 = (z/\ell)^{2/3} g_{ij} dy^i dy^j - dz^2 \,,$$

$$e^\varphi = \mathcal{V}_0 z^2 \,,$$

(3.2)

where $i,j = 0,\ldots,6$ and ℓ is an integration constant with dimensions of length and g_{ij} is any Ricci-flat metric depending on the domain-wall worldvolume coordinates y^i.

It should be clear that this solution is not the dimensional reduction of the D8-brane since the internal metric corresponding to this solution is isotropic whereas the internal metric of the D8-brane reduced to 8 dimensions is not.

It would be interesting to find the unbroken supersymmetries of this solution but the supersymmetry transformation rules of this theory are not available yet.

More complicated vacua (in particular the reduced D8-brane) can be found using a given parametrization of the $Sl(3,\mathbb{R})/SO(3)$ coset representative and will be presented elsewhere.

4   Conclusions

Following a recent observation made by Cowdall [6] that the 9-dimensional massive supergravity given in [5] contained the $d = 9$ $N = 2$ gauged supergravity, we have investigated the relation between massive and gauged supergravities in eight dimensions. Starting from a proposed eleven dimensional massive supergravity theory [2, 5], we derived, by normal dimensional reduction, an eight dimensional massive theory. The resulting massive theory can be interpreted as a gauged symmetry, since it is invariant under local $SO(3)$, which is the R-symmetry of the corresponding supertranslation algebra, transformations. Comparing this theory with the known $N = 2$ $d = 8$ gauged supergravity theory found by Sezgin and Salam [10], we find striking resemblance 8-dimensional S duality relates the axidilaton π and the gauge fields of the two theories. The theory found in this paper is then interpreted as the bosonic sector of the gauged supergravity theory one would have found if one had gauged the 3 vector fields associated to wrapped membranes instead of the 3 KK vector fields (Salam and Sezgin's choice). Even though, in order to prove completely that we have obtained the bosonic sector of a gauged supergravity theory, we need to include

the fermions and find the supersymmetry rules. It is very reasonable to expect that both things can be done.

What are the implications of this result for the massive 11-dimensional supergravity theory of Refs. [2, 5]? We think we have shown that this theory is, at the very least, an effective tool from which to obtain lower-dimensional massive supergravities that cannot be obtained either by standard or by Scherk-Schwarz generalized dimensional reduction. These theories seem to have in common gauge vector fields associated to the 11-dimensional 3-form but their more important feature is the they are related by some sort of lower-dimensional S duality transformations to those theories that can be obtained by more standard methods.

A possible interpretation of these results can be proposed based on experience in the construction of new solutions using the mechanism *reduction-S dualization-oxidation* in the cases in which the reduced theory has an S duality symmetry that the original higher dimensional theory lacks. The best-known example of the use of this mechanism is the construction of the KK-monopole [15, 16] starting from the plane-wave solution in 5-dimensional gravity: the plane-wave solution is reduced in the direction in which it propagates and then S dualized using the electric-magnetic duality symmetry present in $d = 4$ (but completely absent in the 5-dimensional theory!). Finally, it is oxidized back to $d = 5$ only to find that the new solution generated in this way (the KK monopole) does not want to be decompactified and can only live in a 5-dimensional space with a compact dimension. It is possible to construct more solutions using this mechanism and in general one finds that they have a number of dimensions that cannot be decompactified [14].

In Refs. [2, 5] it was argued that the massive 11-dimensional supergravity proposed in them could be interpreted as the 11-dimensional supergravity one gets if one places KK9-branes (referred to as M9-branes in Ref. [17]) in the vacuum. The proposed KK9-branes are objects which necessarily live in 11-dimensional spacetime with 1 compact dimension. On the other hand, being $(d - 2)$-branes, they are associated to mass parameters in the action that can be interpreted as field strengths. As soon as these mass parameters are non-vanishing in the action, one can say that the vacuum contains the corresponding $(d - 2)$-branes. This is different from lower-dimensional branes: the presence of the field strengths of the potentials to which they couple does not imply that they have to have non-trivial values and there are vacuum solutions (Minkowski spacetime, say) in which these field strengths vanish. Thus, if the 11-dimensional vacuum contains KK9-branes, then we expect mass parameters in the action as well as explicit Killing vectors, since these objects break 11-dimensional Poincaré invariance forcing one dimension to be compact.

It is clear from all this that the KK9-branes (or, perhaps, objects with more isometric dimensions) are produced when the *reduction-S dualization-oxidation* mechanism is used on 11-dimensional supergravity. The resulting theory, which we have called massive 11-dimensional supergravity cannot be decompactified.

Clearly more work is needed to confirm the interpretation of the massive 11-dimensional supergravity put forward here. In particular one would like to prove that the theory (or the family of theories one gets with N Killing vectors and arbitrary mass matrices) can be supersymmetrized directly in eleven dimensions even if under the condition of the existence

of N isometries. It would also be very interesting to see if these further straightforward generalizations to N Killing vectors lead to new gauged supergravities in lower than 8 dimensions. Especially the case with 6 Killing vectors should be interesting since then we could compare it to type IIB compactified on S^5. Work in these directions is in progress.

Acknowledgments

A $SO(3)$ in a Non-Standard Basis

The standard basis for the Lie algebra of $SO(3)$ with anti-Hermitean generators $\{T_m\}$, $m = 1, 2, 3$ is such that

$$[T_m, T_n] = \epsilon_{mnp} T_p \,, \tag{A.1}$$

so the structure constants are given by $f_{mn}{}^p = \epsilon_{mnp}$. The Killing metric for the adjoint representation $\mathrm{Adj}\,(T_m)^p{}_n = \epsilon_{mnp}$ is just

$$K_{mn} = \mathrm{Tr}\,[\mathrm{Adj}\,(T_m)\,\mathrm{Adj}\,(T_n)] = -2\delta_{mn} \,. \tag{A.2}$$

Let us now perform a change of basis and compute the new structure constants:

$$T'_m = R_m{}^n T_n \,, \qquad [T'_m, T'_n] = R_m{}^r R_n^s \epsilon_{rsp} T_p = \det R\, \epsilon_{mnq} \left(R^{-1}\right)_p{}^q \left(R^{-1}\right)_p{}^r T'_r \,, \tag{A.3}$$

where we have used that R is an arbitrary non-singular matrix. Thus

$$f_{mn}{}^p = \epsilon_{mnq} Q^{qp} \,, \qquad Q^{qp} = Q^{pq} = \det R \left(R^{-1}\right)_s{}^p \left(R^{-1}\right)_s{}^q \,. \tag{A.4}$$

The Killing metric in the new basis is simply related to the symmetric matrix Q^{mn} by

$$K_{mn} = \det Q\, (Q^{-1})_{mn} \,. \tag{A.5}$$

Fields in the fundamental (vector or adjoint) representation of $SO(3)$ behave under infinitesimal transformations with parameter

$$\sigma^m{}_n \equiv \sigma^p\,(T_p)^m{}_n = \sigma^p f_{pn}{}^m = \sigma^p \epsilon_{pnq} Q^{qm} \,, \tag{A.6}$$

either contravariantly or covariantly

$$\begin{cases} \delta\psi^m & = & \sigma^m{}_n\psi^n\,, \\ \\ \delta\xi_m & = & -\xi_n\sigma^n{}_m\,, \end{cases} \quad (A.7)$$

and their covariant derivatives are defined by[11]

$$\begin{cases} \mathcal{D}_\mu\psi^m & = & \partial_\mu\psi^m - f_{np}{}^m A^n{}_\mu\psi^p\,, \\ \\ \mathcal{D}_\mu\xi_m & = & \partial_\mu\xi_m + f_{nm}{}^p A^n{}_\mu\xi_p\,, \end{cases} \quad (A.8)$$

where the gauge field transforms according to

$$\delta A^m{}_\mu = \mathcal{D}_\mu\sigma^m = \partial_\mu\sigma^m - f_{np}{}^m A^n{}_\mu\sigma^p\,. \quad (A.9)$$

The gauge field strength is given by

$$F^m{}_{\mu\nu} = 2\partial_{[\mu}A^m{}_{\nu]} - f_{np}{}^p A^n{}_\mu A^p{}_\nu = 2\partial_{[\mu}A^m{}_{\nu]} - Q^{mq}\epsilon_{npq}A^n{}_\mu A^p{}_\nu\,. \quad (A.10)$$

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