# Regular cosmological bouncing solutions in low energy effective action from string theories.

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The possibility of obtaining singularity free cosmological solutions in four dimensional effective actions motivated by string theory is investigated. In these effective actions, besides the Einstein-Hilbert term, the dilatonic and the axionic fields are also considered as well as terms coming from the Ramond-Ramond sector. A radiation fluid is coupled to the field equations, which appears as a consequence of the Maxwellian terms in the Ramond-Ramond sector. Singularity free bouncing solutions in which the dilaton is finite and strictly positive are obtained for models with flat or negative curvature spatial sections when the dilatonic coupling constant is such that $\omega < -3/2$, and only for models with negative curvature spatial sections when $\omega > -3/2$, including the pure string case $\omega = -1$. The bounces are smoothly connected to the radiation dominated expansion phase of the standard cosmological model, and the asymptotic pasts correspond to very large flat spacetimes.

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## I. INTRODUCTION

Superstring is the most promising candidate to describe a unified theory of all interactions, gravity included. There are five consistent superstring theories in 10 dimensions, which are connected among themselves through duality transformations. To each superstring theory, there is a corresponding supergravity theory in 10 dimensions. All of them can be obtained from the 11 dimensional supergravity theory. This indicates that those superstring theories are different manifestations of a unique 11 dimensional framework, that has been named $M$-theory [1–3]. Moreover, the superstring type-IIB can be recast in a more geometrical form in a 12 dimensional model, suggesting that perhaps a yet more fundamental framework may exist in 12 dimensions, which has been called $F$-theory [4].

The physical properties of superstring theories become relevant at energy scales comparable with the Planck scale. This renders very improbable that superstring phenomenology may be tested in the near future in some laboratory experiment (see, however, Ref. [5] in which the Planck mass is lowered to TeV scale by accounting for large extra-dimensions). According to the hot big-bang scenario, however, energy scales even as high as the usual Planck scale ($M_{\rm Pl} \sim 10^{19}$GeV) may have been reached in the very early universe. Hence, for the moment, cosmology seems to be the most natural arena where the consequences of superstring theories may be tested. Perhaps, some relics of a cosmological string phase may be identified [6], opening the possibility of testing superstring models. Furthermore, superstring theories open the possibility that some typical drawbacks of the standard cosmological model, like the existence of an initial singularity, may be solved in the context of superstring cosmological models. The goal of the present paper is to show that, under certain conditions, it is possible to obtain completely regular bouncing cosmological models in the context of effective actions constructed from superstring theories (not involving, in particular, negative energies [7]), for which, moreover, the dilaton is strictly positive and never diverges.

The search of singularity free cosmology in string theories is not a new subject [8–12]. The string action at tree level does not lead in general to singularity free cosmological solutions, at least when the strict string case ($\omega = -1$, $\omega$ being the dilatonic coupling parameter) is considered. The pre-big bang model [13], which is an example of a string cosmology, requires the introduction of non-linear curvature terms in order to achieve a smooth transition from a curvature growing phase to a curvature decreasing phase. If large negative values of the dilatonic coupling parameter $\omega$ are allowed, it is possible, in some cases, to obtain completely regular models, including in the dilatonic sector [11]. This may be achieved mainly in models with spatial sections with negative curvature. Here, it will be shown that regular cosmological models

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may be obtained, even in the string case, if a radiation fluid is coupled to the string action at the tree level. Such a radiation fluid can have a fundamental motivation, for example, in the case of the superstring type IIB theory, where a 5-form appears in the Ramond-Ramond sector. Truncation and dimensional reduction of this 5-form lead to a Maxwell term in four dimensions with the desired features [14]. Hence, the model to be studied here is totally based on superstring theories. The string motivated phenomenological term included under the form of a radiation fluid makes possible to connect smoothly such string cosmological models to the radiation phase of the standard cosmological model before nucleosynthesis.

In Ref. [10], models motivated by string theory similarly including a radiation fluid have been studied, restricted to flat spatial sections and $\omega > -3/2$. In such cases, bouncing solutions have been obtained only for $\omega < -4/3$. Furthermore, for these solutions, the dilaton vanishes in the infinite past, raising doubts on the validity of the tree level action in such region. In the present paper, the curvature of the spatial sections and the value of $\omega$ are kept arbitrary. New bouncing regular solutions are then obtained, for which, as mentioned above, the dilaton remains finite and strictly positive at all times, even for the strict string case ($\omega = -1$) provided the spatial section has negative curvature.

In the following section, we derive effective string motivated actions in four dimensions, which we use in section III to derive non singular cosmological solutions which are thoroughly discussed. We end up with the conclusions in section IV.

## II. THE EFFECTIVE ACTION

Our analysis is based on the following effective action at tree level:

$$L = \sqrt{-\tilde{g}}\mathrm{e}^{-\tilde{\sigma}}\left(\tilde{R} - \omega\tilde{\sigma}_{;A}\tilde{\sigma}^{;A} - \frac{1}{12}H_{ABC}H^{ABC}\right) - \sqrt{-\tilde{g}}\left(\frac{1}{2}\xi_{;A}\xi^{;A} + \frac{1}{240}F_{ABCDE}F^{ABCDE}\right) \qquad (1)$$

where $\tilde{\sigma}$ is the dilatonic field, $H_{ABC}$ is the axionic field, and $\omega$ is the dilatonic coupling constant. The two last terms come from the Ramond-Ramond sector of superstring type IIB. The tildes indicate that all quantities are considered in a $D$-dimensional space-time, $D = 10$ in the pure superstring context.

The dilatonic coupling constant is $\omega = -1$ for usual superstring theory. However, this may not necessarily be the case for some ten dimensional theories stemming from a more fundamental one in higher dimensions. In some specific situations, the value of $\omega$ can be found to be even less than $-3/2$. As an example, the superstring type IIB action may be reformulated in 12 dimensions, in the context of the so-called $F$-theory. A low energy limit of the $F$-theory has been studied by [4], where an action in 12 dimensions has been established, which was shown to lead to the low energy limit of the superstring type IIB in 10 dimensions through truncation and dimensional reduction. Let us consider this twelve dimensional action, given by [4]

$$L_{12} = \sqrt{-\tilde{g}}\biggl(\tilde{R} - \frac{1}{2}\Psi_{;A}\Psi^{;A} - \frac{1}{48}\mathrm{e}^{a\Psi}F_{ABCD}F^{ABCD} - \frac{1}{240}\mathrm{e}^{b\Psi}G_{ABCDE}G^{ABCDE} + \lambda B_4\wedge \mathrm{d}A_3\wedge \mathrm{d}A_3\biggr), \qquad (2)$$

with $a^2 = -1/5$ and $b^2 = -4/5$, $\lambda$ being a coupling parameter for the Chern-Simons type term involving the potentials of the five and four-forms. Writing the metric as

$$\mathrm{d}s_{12}^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{e}^{2\beta}\mathrm{d}x_i\mathrm{d}x^i, \qquad (3)$$

with Greek indices $\mu, \nu$ running from 0 to 9 and Latin indices $i \in [10, 11]$, and setting the five-form equal to zero, we obtain the following Lagrangian

$$L_{10} = \sqrt{-g}\mathrm{e}^{2\beta}\biggl(R + 2\beta_{;\rho}\beta^{;\rho} - \frac{1}{2}\Psi_{;\rho}\Psi^{;\rho} - \frac{1}{12}\mathrm{e}^{a\Psi-2\beta}F_{\mu\nu\lambda}F^{\mu\nu\lambda} - \frac{1}{8}\mathrm{e}^{a\Psi-4\beta}F_{\mu\nu}F^{\mu\nu}\biggr), \qquad (4)$$

where we have retained just the two and three-forms coming from the four-form in the original action. The term originating the three-form was made purely imaginary in 12 dimensions. Choosing $\Psi = 2\beta/a$ and defining $\phi = \mathrm{e}^{2\beta}$, one ends up with the following action in 10 dimensions:

$$L_{10} = \sqrt{-g}\left[\phi\left(R + 3\frac{\phi_{;\rho}\phi^{;\rho}}{\phi^2} - \frac{1}{12}F_{\mu\nu\lambda}F^{\mu\nu\lambda}\right) - \frac{1}{8}F_{\mu\nu}F^{\mu\nu}\right]. \qquad (5)$$

One can see that, in this case, we obtain an action with $\omega = -3$ together with a Maxwell term (which generates the radiation fluid). This is a remarkable example on how an effective string action with $\omega \neq -1$ (in this case, $\omega = -3 < -3/2$) can be realized. That is why we will maintain the value of $\omega$ in Eq. (1) arbitrary in what follows, unless otherwise specified.

The $D$-dimensional metric is written as

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{e}^{2\beta}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \qquad (6)$$

where $g_{\mu\nu}$ is the four dimensional metric, $\mathrm{e}^\beta$ is the scale factor of the $d = D - 4$ dimensional internal space which we suppose to be homogeneous and flat. For now on, we will consider a static internal space. This is not obligatory in some of the cases to be analyzed latter, but such a restriction considerably simplifies the unified presentation of many different cases allowed by the action given by Eq. (1).

Dimensional reduction and isotropization of the Maxwellian term [which may come from the Ramond-Ramond sector, or as described in the passage from

Eq. (2) to Eq. (5)], lead to the following effective action in four dimensions

$$L=\sqrt{-g}\left[\phi\left(R-\omega\frac{\phi_{;\rho}\phi^{;\rho}}{\phi^2}-\frac{\Psi_{;\rho}\Psi^{;\rho}}{\phi^2}\right)-\frac{1}{2}\xi_{;\rho}\xi^{;\rho}\right]+L_{\rm r}, \quad (7)$$

where for now on Greek indices run from 0 to 3. In this action, $\phi = \mathrm{e}^{-\sigma}$ is the dilaton, the field $\Psi$ comes from the axionic term, and is thus called the axion, while $L_{\rm r}$ represents an ordinary radiation fluid term, which can be obtained from the five-form existing in the Ramond-Ramond sector, as was stressed above. We shall also call $\xi$ the RR-scalar as is originates from the same sector.

From Eq. (7), we obtain the field equations

$$\begin{aligned} R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R &= 8\pi\frac{T}{\phi}+\frac{\omega}{\phi^2}\left(\phi_{;\mu}\phi_{;\nu}-\frac{1}{2}g_{\mu\nu}\phi_{;\rho}\phi^{\rho}\right)\\ &+\frac{1}{\phi}\left(\phi_{\mu\nu}-g_{\mu\nu}\Box\phi\right)\\ &+\frac{1}{\phi^2}\left(\Psi_{;\mu}\Psi_{;\nu}-\frac{1}{2}g_{\mu\nu}\Psi_{;\rho}\Psi^{;\rho}\right)\\ &+\frac{1}{\phi}\left(\xi_{;\mu}\xi_{;\nu}-\frac{1}{2}g_{\mu\nu}\xi_{;\rho}\xi^{;\rho}\right), \end{aligned} \quad (8)$$

for the Einstein part,

$$\Box\phi+\frac{2}{3+2\omega}\phi^{-1}\Psi_{;\rho}\Psi^{;\rho}+\frac{1}{3+2\omega}\xi_{;\rho}\xi^{;\rho}=\frac{8\pi T}{3+2\omega}, \quad (9)$$

with $T\equiv T^{\mu}{}_{\mu}$, for the dilaton $\phi$, while we get

$$\Box\Psi-\Psi_{;\rho}\frac{\phi^{;\rho}}{\phi}=0, \quad (10)$$

to describe the dynamics of the axion $\Psi$, and finally

$$\begin{aligned} \Box\xi &= 0, & (11)\\ T^{\mu\nu}{}_{;\mu} &= 0, & (12) \end{aligned}$$

for the RR-scalar $\xi$ and the radiation fluid respectively. These equations we now implement in a cosmological context.

## III. COSMOLOGICAL SOLUTIONS

Introducing the Friedman-Robertson-Walker metric

$$\mathrm{d}s^2=\mathrm{d}t^2-a(t)^2\left[\frac{\mathrm{d}r^2}{1-kr^2}+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)\right], \quad (13)$$

$k$ being the normalized curvature of the maximally symmetric spatial sections ($k=0,\pm1$), and assuming the fields now depend only on time, the field equations derived above reduce to the following equations of motion:

$$3\left(\frac{\dot a}{a}\right)^2+3\frac{k}{a^2}=8\pi\frac{\rho}{\phi}+\frac{\omega}{2}\left(\frac{\dot\phi}{\phi}\right)^2-3\frac{\dot a}{a}\frac{\dot\phi}{\phi}+\frac{\dot\Psi^2}{2\phi^2}+\frac{\dot\xi^2}{2\phi}, \quad (14)$$

which is the generalization of the Friedman equation, and

$$\begin{aligned} \ddot\phi+3\frac{\dot a}{a}\dot\phi+\frac{2}{(3+2\omega)}\frac{\dot\Psi^2}{\phi}+\frac{\dot\xi^2}{(3+2\omega)} &= \frac{8\pi(\rho-3p)}{(3+2\omega)}, & (15)\\ \ddot\Psi+3\frac{\dot a}{a}\dot\Psi-\dot\Psi\frac{\dot\phi}{\phi} &= 0, & (16)\\ \ddot\xi+3\frac{\dot a}{a}\dot\xi &= 0, & (17)\\ \dot\rho+3\frac{\dot a}{a}(\rho+p) &= 0. & (18) \end{aligned}$$

In these expressions, $\rho$ is the energy density and $p$ is the pressure of some perfect fluid which obeys, for the sake of generality, a barotropic equation of state, $p=\lambda\rho$, with $\lambda$ an arbitrary constant. Later on, we will specialize this fluid to the case we are interested in, namely, radiation. A dot stands for a derivative with respect to the cosmic time $t$.

Eqs. (16), (17) and (18) admit the first integrals

$$\dot\Psi=\frac{A\phi}{a^3},\quad \dot\xi=\frac{B}{a^3},\quad \rho=Da^{-3(1+\lambda)}, \quad (19)$$

where $A$, $B$ and $D$ are integration constants. According to the string motivated action discussed above, let us now specialize the equations for the radiation fluid case ($\lambda=1/3$). For this specific case, Eq. (15) simplifies to

$$\ddot\phi+3\frac{\dot a}{a}\dot\phi+\frac{2}{(3+2\omega)}\frac{A^2}{a^6}\phi+\frac{B^2}{(3+2\omega)a^6}=0, \quad (20)$$

which can be solved in the following way. It is convenient to define a new time coordinate $\theta$ given by the relation

$$\mathrm{d}t=a^3\mathrm{d}\theta. \quad (21)$$

In terms of this new time coordinate, Eq. (20) reads

$$\phi''+\frac{2A^2}{(3+2\omega)}\phi+\frac{B^2}{(3+2\omega)}=0, \quad (22)$$

where primes denote differentiations with respect to $\theta$. Similarly, Eq. (14), when expressed in terms of $\theta$, reads

$$\left(\frac{a'}{a^3}\right)^2+k=\frac{M}{a^2\phi}+\frac{\omega}{6}\frac{\phi'^2}{a^4\phi^2}-\frac{a'\phi'}{a^5\phi}+\frac{1}{6a^4}\left(A^2+\frac{B^2}{\phi}\right), \quad (23)$$

in which use has been made of Eq. (19), and we have set $M=8\pi D/3$, which is dimensionless in the radiation case.

Eq. (23) may be recast in a very convenient form through the redefinition $a=\phi^{-1/2}b$, which implies to change to the so-called Einstein frame. This yields

$$\left(\frac{b'}{b}\right)^2+(kb^2-M)\frac{b^2}{\phi^2}=\frac{1}{6}\left(A^2+\frac{B^2}{\phi}+\frac{3+2\omega}{2}\frac{\phi'^2}{\phi^2}\right), \quad (24)$$

whose solution we next investigate. Notice, however, that we want to keep considering the Jordan frame as the physical frame; the conformal transformation above is introduced only for technical reasons. The solutions of Eqs. (22) and (14), with the redefinition made above for the scale factor, depend on the sign of the term $3+2\omega$ and on the presence of the Ramond-Ramond scalar field. We will consider each case separately. For simplicity, we will call $3+2\omega>0$ (respectively $<0$) as the normal (resp. anomalous) case, and $\xi=$ constant (respectively $\xi\neq$ constant) as the axionic (resp. RR) case. In what follows, the quantities $\phi_0$ and $a_0$ are constants of integration subject to the constraints indicated in each case.

### A. Normal axionic case

In this first case for which $\xi$ is constant [i.e., $B=0$ in Eq. (19)] and $\omega>-3/2$, the solution of Eq. (22) is given by

$$\phi(\theta)=\phi_0\sin(\alpha\theta), \tag{25}$$

where

$$\alpha=\sqrt{\frac{2A^2}{3+2\omega}} \tag{26}$$

and we have chosen $\phi(0)=0$.

Plugging this solution into Eq. (24) yields

$$\phi_0^2\sin^2(\alpha\theta)b'^2=(C^2+Mb^2-kb^4)b^2, \tag{27}$$

where

$$C^2=\frac{1}{6}A^2\phi_0^2. \tag{28}$$

We are seeking regular bouncing solutions for which the scale factor is bounded from below but can grow arbitrarily large, while $\phi$ is non vanishing and finite. This means that the function $b$ should also grow indefinitely on both sides of the bounce. As can be seen by inspection of Eq. (27), a necessary condition for this to happen is that the curvature be non-positive. This is to be contrasted with the general relativistic case for which a positive curvature is a pre-requisite to ensure that a bounce is possible [15], and can be understood by stating that, in the case at hand, a positive curvature implies a finite scale factor at all times.

Under the assumption that both sides are positive definite, one can integrate Eq. (27), written as

$$\int_{b_0}^{b}\frac{\mathrm{d}\tilde{b}}{\tilde{b}\sqrt{C^2+M\tilde{b}^2-k\tilde{b}^4}}=\pm\int_{\theta_0}^{\theta}\frac{\mathrm{d}\tilde{\theta}}{\phi_0\sin(\alpha\tilde{\theta})}, \tag{29}$$

to provide the solution {see, e.g., Ref. [16], Eq. (2.266)}

$$\frac{g(b)}{g(b_0)}=\frac{f(\theta)}{f(\theta_0)}, \tag{30}$$

where

$$f(\theta)=\left|\tan\left(\frac{\alpha\theta}{2}\right)\right|^p, \tag{31}$$

and

$$g(b)=\frac{M}{C}+\frac{2}{b^2}\left(C+\sqrt{C^2+Mb^2-kb^4}\right), \tag{32}$$

$b_0$ and $\theta_0$ being constants of integration that we choose such that $Cg(b_0)=f(\theta_0)$ for further convenience, and

$$p=\pm\sqrt{1+\frac{2}{3}\omega}. \tag{33}$$

Setting $a_0^2=4C^2/\phi_0$, we finally get

$$a(\theta)=\frac{a_0}{\sqrt{\sin\alpha\theta}}\left\{\frac{f(\theta)}{[M-f(\theta)]^2+4C^2k}\right\}^{1/2}, \tag{34}$$

which is the desired result for the scale factor. Note that because of the trigonometric identity

$$\tan\left[-\left(\frac{\alpha\theta+\pi/2}{2}\right)\right]=\left[\tan\left(\frac{\pi/2-\alpha\theta}{2}\right)\right]^{-1}, \tag{35}$$

the solution (34) with $p\to -p$ can be straightforwardly deduced from the original one by a mirror symmetry with respect to the point $\alpha\theta=\pi/2$. It is thus sufficient to consider $p>0$ and we shall in what follows restrict our attention to this case.

These solutions have some interesting features. As we have already discussed, for $k=1$, there are no bouncing solutions. On the other hand, for $k=0$ or $k=-1$, it is possible to choose the parameters in such a way that the extremes of the range of validity of the variable $\theta$ occur for $t\to\pm\infty$, where spacetime becomes flat.

The case $k=0$ was presented in Ref. [10]; let us recall it briefly for the sake of completeness. The denominator in Eq. (34) has only two roots if $k=0$, and the parameter $\theta$ varies from $\theta_{\mathrm{i}}=0$ to $\theta_{\mathrm{f}}=2\alpha^{-1}\arctan(M^{1/p})$. Bouncing non singular solutions are possible only when $-3/2<\omega<-4/3$. This can be seen by considering the limit for which $\theta\to\theta_{\mathrm{i}}=0$. There, the scale factor is $a\propto\theta^{(p-1)/2}$, and, from Eq. (21), $t\propto\theta^{(3p-1)/3}$, yielding $a\propto|t|^{(p-1)/(3p-1)}$. As $a(t)$ is a power law (disregarding the exceptional cases $p=1\Leftrightarrow\omega=0$, and $p=1/3\Leftrightarrow\omega=-4/3$, also discussed in Ref.[10]), the scalar curvature for $k=0$, is proportional to $t^{-2}$, which converges (in fact, goes to zero) only if $t\to\infty$ as $\theta\to 0$. This happens only for $p<1/3$, which yields $-3/2<\omega<-4/3$. Note, however, that for $\theta=0$ the dilaton $\phi$ vanishes, independently of the value of $\omega$, rendering dubious the validity of the tree level action (1) in this region.

When $k=-1$, the denominator in Eq. (34) has now three roots. One can take the parameter $\theta$ varying from $\theta_{\mathrm{f}}=2\alpha^{-1}\arctan[(M+$

$2C)^{1/p}]$. Provided[1] $2C < M$, we have $0 < \alpha\theta < \pi$, and the dilatonic field given by Eq. (25) is strictly positive and finite, taking constant values in the asymptotic regions.

Let us now consider the limit $\theta \to \theta_{\rm i}$ or $\theta \to \theta_{\rm f}$. Setting $\alpha\theta = \alpha\theta_{\rm i} + \varepsilon$ or $\alpha\theta = \alpha\theta_{\rm f} - \varepsilon$, and expanding the denominator around $\varepsilon = 0$, we get $a \propto \varepsilon^{-1/2}$, from Eq. (21) $|t| \propto \varepsilon^{-1/2}$, and finally $a \propto |t|$, independently on the value of $p$ or $\omega$. As we are considering $k = -1$, this limit corresponds to Milne flat spacetime. The scale factor (34) thus represents, with this choice of range for $\theta$, a universe contracting from a Milne spacetime to a minimum size, bouncing to an expansion phase, and ending asymptotically also in a Milne spacetime, passing smoothly through a standard cosmological model radiation dominated phase before that. It is very important to notice that these solutions are valid for $\omega > -3/2$, which includes the string case ($\omega = -1$). As the dilaton is finite and strictly positive, there are also no singularities in the string expansion parameter given by $g_s^2 = \phi^{-1}$ (which is not the case for $k = 0$), and the tree level approximation can be trusted all along. Consequently, we have obtained a perfectly regular bouncing solution in the string framework, without any singularity, even in the dilatonic field, when the curvature of the spatial section is negative.

### B. Anomalous axionic case

For $3 + 2\omega < 0$, the previous solution for Eq. (22) must be replaced by

$$\phi(\theta) = \phi_0 \sinh(\alpha\theta), \tag{36}$$

where now

$$\alpha = \sqrt{\frac{-2A^2}{3 + 2\omega}}, \tag{37}$$

and, as before, we have imposed $\phi(0) = 0$.

Again, inserting this solution into Eq. (24) yields

$$\phi_0^2 \sinh^2(\alpha\theta) b'^2 = (Mb^2 - C^2 - kb^4) b^2, \tag{38}$$

where $C$ is as before [Eq. (28)]. The same argument concerning the existence of a bouncing solution apply, namely, that such solutions cannot exist for $k = 1$. Manipulations similar to those of the previous case then lead to

$$f(\theta) \;=\; \ln \left[ \left| \tanh\left(\frac{\alpha\theta}{2}\right) \right|^p \right], \tag{39}$$

$$g(b) \;=\; \arcsin\left( \frac{Mb^2 - 2C^2}{b^2\sqrt{M^2 - 4kC^2}} \right), \tag{40}$$

where we have assumed $M^2 - 4kC^2 > 0$ (recall we are only interested in the cases $k = 0$ and $k = -1$). We now choose $f(\theta_0) = g(b_0)$ and set

$$p = \pm\sqrt{-\left(1 + \frac{2}{3}\omega\right)}, \quad \text{and} \quad a_0^2 = \frac{A^2\phi_0}{3M}, \tag{41}$$

to obtain the scale factor as

$$a(\theta) = \frac{a_0}{\sqrt{\sinh\alpha\theta}} \left[ 1 \pm \sqrt{1 - 4\frac{kC^2}{M^2}} \sin f(\theta) \right]^{-1/2}. \tag{42}$$

Regular bouncing solutions may be obtained for $k = 0$ or $k = -1$. Furthermore, differently from the previous situation, the flat case also does not exhibit any singularity in the string expansion parameter. It is interesting to note that these models can provide a quite effective way of enhancing the gravitational coupling[2]. Investigation of the asymptotic behavior reveals that, for $k = 0$, the universe displays a radiation dominated behavior in both extremities of the range ($a \propto |t|^{1/2}$ for $t \to \pm\infty$), while for $k = -1$, the curvature dominates in the asymptotic regions, leading to a Milne universe.

### C. Normal RR case

Integrating the equations of motion (22) after inclusion of $\xi$, i.e., with a non vanishing $B$ and still for $\omega > -3/2$, simply turns the solution given by Eq. (25) into

$$\phi(\theta) = \phi_0 \left( \sin\alpha\theta - s \right), \tag{43}$$

where

$$s = \frac{B^2}{2A^2\phi_0}, \tag{44}$$

[1] Considering $G_{\rm eff} = G_{\rm N}/\phi$, where $G_{\rm N}$ is the value of the gravitational coupling today, recovering the units in the string case ($\omega = -1$) by making the replacements $\phi \to \phi/G_{\rm N}$, $\Psi \to \Psi/G_{\rm N}$, $t \to a_0 t$, $a_0 \approx 1/H_0$ ($H_0$ being the present Hubble parameter, which we choose to be our inverse unit of time), and assuming the present amount of radiation ($\rho_{0\rm r} = \Omega_{0\rm r}\rho_{\rm c} \sim 10^{-4}\rho_{\rm c} \approx 10^{-33}\,{\rm g/cm}^3$, with $\rho_{\rm c}$ the critical density), we obtain from Eq. (14), assuming the radiation term to dominate at the time under consideration, that $M \sim 8\pi G\rho_{0\rm r}H_0^{-2}/3 = \Omega_{0\rm r} \approx 10^{-4}$. This implies, provided $C \approx 10^{-4} \lesssim M/2$, that $\sin(\alpha\theta_{\rm f}) \approx 10^{-6}$, so that we must set $\phi_0 \approx 10^6$ in order to have $\phi_{\rm f} = 1$ now. The value of $C$, in turn, fixes $A \approx 10^{-10}$ through Eq. (28). As a consequence, we find that the effective gravitational constant $G_{\rm N}/\phi$ was one order of magnitude larger in the past than it is today. More significant enhancements are possible but they require some fine tuning between $M$ and $2C$.

[2] To illustrate this point, let us choose $p = 1$ ($\omega = -3$), $f(\theta_{\rm i}) = -7\pi/2$ and $f(\theta_{\rm f}) = -3\pi/2$. One then obtains $\phi_{\rm i} \approx 10^{-3}$ and $\phi_{\rm f} \approx 1$, where the constant $\phi_0$ is chosen $\phi_0 \approx 10^{-2}$ in order to obtain the effective gravitational "constant" today equal to Newton constant $G_{\rm N}$. With this choice of parameters, the enhancement of the effective gravitational "constant" in the past was therefore of three orders of magnitude. Note that the dilaton is strictly positive and finite in this range.

provides the particular solution of the inhomogeneous equation, and we have assumed the same initial condition for the homogeneous part. The constant $\alpha$ is defined as in the normal axionic case.

After some straightforward calculations, we get that Eq. (29) is modified into

$$\int_{b_0}^{b} \frac{\mathrm{d}\tilde{b}}{\tilde{b}\sqrt{\pm C^2 + M\tilde{b}^2 - k\tilde{b}^4}} = \pm \int_{\theta_0}^{\theta} \frac{\mathrm{d}\tilde{\theta}}{\phi_0 \left[\sin(\alpha\tilde{\theta}) - s\right]}, \tag{45}$$

where now

$$C^2 = \frac{1}{6} A^2 \phi_0^2 |1 - s^2| \tag{46}$$

takes into account the inhomogeneous part. In Eq. (45), the sign in front of the factor $C^2$ in the denominator of the right-hand side integrand is positive or negative depending on whether $s^2 < 1$ or $s^2 < 1$ respectively. We shall treat both cases separately.

### 1. Small RR-scalar

We assume for now on that even though we allow variations for $\xi$, those are limited in such a way that $s^2 < 1$. Eq. (45), being in a form similar to Eq. (29), yields the same result that bounces cannot be realized unless $k < 0$.

Integrating both sides of Eq. (45), we obtain the function $b$, thanks to which we can write the scale factor as

$$a(\theta) = \frac{a_0}{\sqrt{\sin\alpha\theta - s}} \left\{ \frac{f(\theta)}{\left[M - f(\theta)\right]^2 + 4C^2 k} \right\}^{1/2}, \tag{47}$$

with {see again Ref. [16], Eq. (2.251/3)}

$$f(\theta) = \left| \frac{2}{s} \left[ \frac{s\tan(\alpha\theta/2) - 1 + \sqrt{1 - s^2}}{1 + \sqrt{1 - s^2} - s\tan(\alpha\theta/2)} \right] \right|^p, \tag{48}$$

where $a_0$, $p$ and the choice for the relationship between $f(\theta_0)$ and $g(b_0)$ are the same as in the normal axionic case, except for the new definition (46) of the constant $C$. The normalization in Eq. (48) has been chosen in such a way that the limit $s \to 0$ gets indeed back to the normal axionic case (31).

The properties of these solutions are the same as in the normal axionic case, with singularity free models and regular dilatonic behavior for any $\omega > -3/2$ only when[3] $k = -1$.

[3] In string theory ($\omega = -1$), with $M \sim 10^{-4}$ and $M - 2C > 0$, one finds that $\phi > 0$ for any value of $s < 1$.

### 2. Large RR-scalar

In the opposite situation for which $s^2 > 1$, one can normalize the solution in such a way that {see Ref. [16], Eq. (2.551/3)}

$$f(\theta) = 2p \arctan\left[\frac{1 - s\tan(\alpha\theta/2)}{\sqrt{s^2 - 1}}\right], \tag{49}$$

and, provided $a_0^2 = 2C/(M\phi_0)$, the solution can be written as

$$a(\theta) = \frac{a_0}{\sqrt{s - \sin(\alpha\theta)}} \left[1 \pm \sqrt{1 - 4\frac{kC^2}{M^2}\sin f(\theta)}\right]^{-1/2}. \tag{50}$$

Bouncing solutions may be obtained but they are unstable. In fact, to obtain regular solutions, the dilatonic field must be negative, at least in a certain range of validity of the solutions. This implies a repulsive gravity effect. In order to connect such model with the real Universe, where gravity is attractive and the dilaton must be positive, the dilatonic field must pass by a zero value, and instabilities will appear.

## D. Anomalous RR case

Finally, the last situation, for which Eq. (22) is solved by

$$\phi = \phi_0 \left[\sinh(\alpha\theta) - s\right], \tag{51}$$

is very similar to the anomalous axionic case except that the hyperbolic sine in Eq. (38) is replaced by $[\sinh^2(\alpha\theta) - s]$, with the same definition for the constant $s$ and $C$ as in the anomalous axionic situation. This case is essentially similar to the normal RR one, and we obtain a different function {see Ref. [16], Eq. (2.441/3)}

$$f(\theta) = p \ln \left| \frac{2}{s} \left[ \frac{s\tanh(\alpha\theta/2) + 1 - \sqrt{1 + s^2}}{1 + \sqrt{1 + s^2} + s\tanh(\alpha\theta/2)} \right] \right|, \tag{52}$$

where the normalization again ensures that the limit $s \to 0$ is equivalent to the anomalous axionic case. With the new scale factor normalization

$$a_0^2 = \frac{A^2 \phi_0}{3M}(1 + s^2), \tag{53}$$

the new solution is expressed as

$$a(\theta) = \frac{a_0}{\sqrt{\sinh\alpha\theta - s}} \left[1 \pm \sqrt{1 - 4\frac{kC^2}{M^2}\sin f(\theta)}\right]^{-1/2}. \tag{54}$$

Again, as in the anomalous axionic case, completely non singular solutions, also with respect to the dilatonic field, are obtained for $k = 0$ or $k = -1$. The properties of both anomalous (axionic and RR) cases are very similar, even in the asymptotic regions. The significant feature of this case is that, for $k = 0$, it is not difficult to choose the free parameters in order to allow huge increases of the dilaton along the evolution of such universes.

## IV. CONCLUSIONS

We have constructed fully regular cosmological solutions in the framework of effective actions derived from string theory principles. These solutions present bouncing behaviors for a wide range of parameters, and are singularity free; furthermore, the spacetimes they lead to are geodesically complete, thereby improving the so-called horizon problem of standard cosmology. Stemming from string theory, they have a reasonably sound basis as long as the dilaton is strictly positive and finite in all such solutions. As a consequence, it is not necessary to go beyond the tree level approximation in any part of their histories: the analytic solutions exhibited above can describe the whole history of the cosmological models they represent. Their consequences may, in turn, be used as cosmological tests.

In the normal axionic and Ramond-Ramond cases there are non singular bouncing solutions even for the string case if $k = -1$. Remembering that the radiation fluid included here has also a motivation in the superstring type IIB action, this reveals to be, to our knowledge, the first case where a complete regular bouncing cosmological solution is obtained in the strict string framework ($\omega = -1$), which is smoothly connected with the standard model radiation dominated phase. Moreover, the anomalous axionic and RR cases also exhibit complete non-singular solutions for flat and negative curvature spatial sections.

As all these models have the interesting feature to approach flat spacetime in the infinity past (either in Milne coordinates for $k = -1$, or infinitely large radiation dominated standard model with $k = 0$), there is the possibility to implement a quantum spectrum of perturbations in the initial asymptotic without any trans-Planckian problem, and, at the same time, to accomplish a smooth transition to the standard cosmological model when, after the bounce, a standard radiation dominated phase is recovered (asymptotically in the $k = 0$ case), preserving some of its main achievements like primordial nucleosynthesis and not involving some exotic matter [7].

Notice that, in all these cases, the initial value of the dilatonic field can be made smaller than its final value. Hence, the gravitational coupling has initially a greater value than it would have today. This opens the possibility to solve the hierarchical problem of the gravitational coupling, in a spirit similar to the so-called brane cosmology [17].

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[1] J. Polchinski, String theory, Cambridge University Press (Cambridge, 1998), Vols. I & II.

[2] M. B. Green, J. H. Schwarz, and E. Witten, Superstring theory, Cambridge University Press (Cambridge, 1987), Vols. I & II.

[3] E. Kiritsis, *Introduction to superstring theory*, `hep-th/9709062`.

[4] N. Khviengia, Z. Khviengia, H. Lu, and C. N. Pope, Class. Quantum Grav. **15**, 759 (1998).

[5] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998); L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).

[6] P. Binétruy, C. Deffayet and P. Peter, Phys. Lett. B **441**, 52 (1998).

[7] P. Peter and N. Pinto-Neto,Phys. Rev. D **66**, 063509 (2002).

[8] J. E. Lidsey, D. Wands, and E. J. Copeland, Phys. Rep. **337**, 343 (2000).

[9] A. Feinstein and M. A. Vazquez-Mozo, Nucl. Phys. **B568**, 405 (2000).

[10] C. P. Constantinidis, J. C. Fabris, R. G. Furtado, and M. Picco,Phys. Rev. D **61**, 043503 (2000).

[11] K. A. Bronnikov and J. C. Fabris, JHEP **209**, 62 (2002).

[12] R. Brandenberger, R. Easther, and J. Maia, JHEP **9808**, 007 (1998); D. A. Easson, R. H. Brandenberger, JHEP **9909**, 003 (1999).

[13] M. Gasperini and G. Veneziano, *The pre-big bang scenario in string cosmology*, `hep-th/0207130`.

[14] J. C. Fabris, Phys. Lett. B **267**, 30 (1991).

[15] P. Peter and N. Pinto-Neto, Phys. Rev. D **65**, 023513 (2001) and references therein.

[16] I. S. Gradshteyn, I. M. Ryshik, *Table of Integrals, Series and Products*, (Academic Press, New-York, 1980).

[17] D. Langlois, *Brane cosmology: an introduction*, `hep-th/0209261`, in *Proceedings of the YITP workshop on "Braneworld - Dynamics of spacetime boundary"*.