Branes from Matrix Theory
in PP-Wave Background

Seungjoon Hyun[a]* and Hyeonjoon Shin[b]†

[a]*Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea*
[b] *BK 21 Physics Research Division and Institute of Basic Science*
Sungkyunkwan University, Suwon 440-746, Korea

Abstract

Based on the recently proposed action for Matrix theory describing the DLCQ M theory in the maximally supersymmetric pp-wave background, we obtain the supersymmetry algebra of supercharge density. Using supersymmetry transformation rules for fermions, we identify BPS states with the central charges in the supersymmetry algebra, which can be activated only in the large N limit. They preserve some fraction of supersymmetries and correspond to rotating transverse membranes and longitudinal five branes.

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*hyun@phya.yonsei.ac.kr
†hshin@newton.skku.ac.kr

1 Introduction

The matrix theory has been a candidate for the microscopic description of the eleven dimensional M theory in the infinite momentum [1] or light cone frame [2, 3, 4] (or under the discrete light cone quantization (DLCQ)). Though we are still far from the complete understanding of M theory, the matrix theory has been successful in uncovering some aspects of M theory. For a review, see Ref. [5]. In view of its successes, quite a number of supersymmetries has played a crucial role in exploring the matrix theory.[1]

In its original form, the matrix theory is for the DLCQ M theory on the flat eleven dimensional Minkowskian space-time of maximal 32 supersymmetries as the background geometry. Recently, it has been proposed in Ref. [7] that the original matrix theory is a special case of a newly constructed matrix theory which is interpreted as the description of DLCQ M theory on the eleven dimensional pp-wave background. The pp-wave geometry [8, 9, 10] has 32 supersymmetries and the limiting case of the eleven dimensional AdS type geometries under the Penrose limit [11] as shown in [12]. In our notation, it is given by

$$ds^2 = -2dx^+dx^- - \left(\sum_{i=1}^{3}\frac{\mu^2}{9}(x^i)^2 + \sum_{i'=4}^{9}\frac{\mu^2}{36}(x^{i'})^2\right)(dx^+)^2 + \sum_{I=1}^{9}(dx^I)^2 ,$$
$$F_{+123} = \mu , \tag{1}$$

where μ is a parameter of the geometry and characterizes the matrix theory of Ref. [7]. The matrix theory based on this geometry has different structure compared to the original matrix theory and hence opens the possibility for us to get some more insights for the structure of the M theory. In particular, the fact that the geometry (1) is maximally supersymmetric means that the full power of supersymmetry can still be utilized as in the case of the flat space-time.

The action for the matrix theory proposed in Ref. [7] has been constructed starting from the action for the eleven dimensional superparticles on the above geometry and requiring supersymmetry. In a subsequent work, [13], the same action has been obtained through the light cone gauge formulation of the eleven dimensional super membrane in the pp-wave geometry by using the action given in [14, 15]. The resulting theory depends on the parameter μ which makes the theory to be massive and has cubic interaction term called the Myers term [16]. Interestingly, the presence of the cubic interaction allows one to have 1/2 BPS spherical membrane even for *finite* N [7, 13]. Here N measures the light cone momentum of DLCQ M theory and the dimension of the matrix theory quantities.

For finite N, the spherical membrane and its dialects may be the only extended objects present in the matrix theory. However, there may be other types of supersymmetric objects

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[1]As a typical example, see [6].

which are infinitely extended and thus activated only when N goes to infinity. In the flat case, one can read off the presence of the extended objects by looking at the supercharge density algebra [17]. The central charges are identified as the charges of the corresponding extended objects. In this paper, following the recipes given in [17], we will obtain the super-charge density algebra of the matrix theory of [7, 13] and investigate what types of extended objects appear from the theory. In fact, the supersymmetry algebra has already been given in Ref. [13], which is however valid only for finite N. We also find various rotating BPS brane solutions, which exists only in the large N limit. They include transverse membranes stretched in the $x^{i'}$-directions, longitudinal five branes spanning in the $x^{i'}$-directions and longitudinal five branes filling 1, 2, 3-directions and spanning one direction in $x^{i'}$.

The organization of this paper is as follows. In section 2, we will compute the supercharge density algebra. In section 3, we will obtain various rotating transverse membranes and longitudinal five branes. In section 4 we draw some conclusions.

2   Supercharge Density Algebra

The Lagrangian [7, 13] for the matrix theory in the pp-wave background Eq. (1) is given by

$$L = \text{Tr}\mathcal{L} \,. \tag{2}$$

$\mathcal{L}$ is the Lagrangian density in the sense that it is not traced, and its expression is

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{2R}D_0 X^I D_0 X^I - \frac{1}{2R}\left(\frac{\mu}{3}\right)^2 (X^i)^2 - \frac{1}{2R}\left(\frac{\mu}{6}\right)^2 (X^{i'})^2 + \frac{i}{2}\theta^\alpha D_0 \theta^\alpha - \frac{i\mu}{8}\theta^\alpha \Pi_{\alpha\beta}\theta^\beta \\
& + \frac{R}{4}[X^I, X^J]^2 + \frac{R}{2}\theta^\alpha \gamma^I_{\alpha\beta}[X^I, \theta^\beta] - \frac{i\mu}{3}\epsilon_{ijk}X^i X^j X^k \,,
\end{aligned}
\tag{3}
$$

where $D_0 = \partial_0 - i[A_0,]$ and we have defined

$$\Pi = \gamma^{123} \,. \tag{4}$$

R is the radius of the longitudinal x^- direction. X^I, θ^α ($\alpha = 1, ..., 16$), and A_0 are hermitian $N \times N$ matrices which have upper and lower $SU(N)$ indices ($a, b, ... = 1, ..., N$). The multiplication between matrices is then given by

$$(AB)^b_a = A^c_a B^b_c \,. \tag{5}$$

The matrix theory is supersymmetric and, by definition, describes the eleven dimensional M theory in the infinite momentum or light cone frame. In the light cone frame, it is natural to split the eleven dimensional 32 supersymmetries into 16 kinematical and 16 dynamical

supersymmetries. If we take $A_0 = 0$ gauge, the corresponding transformation rules ($\tilde{\delta}$) for the former one are

$$\tilde{\delta}X^I = 0 \ ,$$
$$\tilde{\delta}\theta = \tilde{\eta} \ , \tag{6}$$

where, unlike the flat case, the transformation parameter depends on time and, with constant $\tilde{\epsilon}$, is as follows:

$$\tilde{\eta} = e^{\frac{\mu}{4}\Pi t}\tilde{\epsilon} \ . \tag{7}$$

As for the dynamical supersymmetries, the transformation rules (δ) are

$$\delta X^I = i\theta\gamma^I\eta \ ,$$
$$\delta\theta = \left(P^I\gamma^I + \frac{i}{2}[X^I, X^J]\gamma^{IJ} + \frac{\mu}{3R}X^i\Pi\gamma^i - \frac{\mu}{6R}X^{i'}\gamma^{i'}\Pi\right)\eta \ , \tag{8}$$

where P^I is the canonical conjugate momentum of X^I, and η has different time dependence from that of $\tilde{\eta}$ as

$$\eta = e^{-\frac{\mu}{12}\Pi t}\epsilon \ , \tag{9}$$

with constant ϵ.

The Lagrangian (2) has fermionic sector which is first order in time derivative. Thus the conjugate momentum of fermion gives constraint which is in second class. Before we compute the supersymmetry algebra, we should first take it into account properly. First of all, we obtain the Hamiltonian corresponding to the Lagrangian, which in the $A_0 = 0$ gauge is

$$H = R\mathrm{Tr}\mathcal{H} \ . \tag{10}$$

Here the Hamiltonian density $\mathcal{H}$ is given by

$$\begin{aligned}
\mathcal{H} &= \frac{1}{2}(P^I)^2 + \frac{1}{2}\left(\frac{\mu}{3R}\right)^2(X^i)^2 + \frac{1}{2}\left(\frac{\mu}{6R}\right)^2(X^{i'})^2 - \frac{1}{4}[X^I, X^J]^2 \\
&\quad + i\frac{\mu}{3R}\epsilon_{ijk}X^iX^jX^k + i\frac{\mu}{8R}\theta^\alpha\Pi_{\alpha\beta}\theta^\beta + \frac{1}{4}\gamma^I_{\alpha\beta}[\theta^\alpha, [\theta^\beta, X^I]] \ .
\end{aligned} \tag{11}$$

If we investigate the constraint structure with this Hamiltonian, we see that no more constraint is generated. The Dirac procedure for the constrained system then leads us to the following Dirac brackets for the elementary matrix quantities:

$$\{X_a^{Ib}, P_c^{Jd}\}_{DB} = \delta^{IJ}\delta_a^d\delta_c^b \ ,$$
$$\{\theta_a^{\alpha b}, \theta_c^{\beta d}\}_{DB} = -i\delta^{\alpha\beta}\delta_a^d\delta_c^b \ . \tag{12}$$

From now on, we will keep the subscript DB to distinguish the Dirac bracket from the commutator and the anti-commutator between matrices.

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We now turn to the calculation of the algebra between supercharge densities. We denote Q_α ($\tilde{Q}_\alpha$) the 16 dynamical (kinematical) supercharges that generate the dynamical (kinematical) supersymmetry transformation δ ($\tilde{\delta}$). Then the supercharges for the matrix theory in the pp-wave background are

$$Q_\alpha = \mathrm{Tr}q_\alpha ,$$
$$\tilde{Q}_\alpha = \mathrm{Tr}\tilde{q}_\alpha , \tag{13}$$

where the supercharge densities q_α and $\tilde{q}_\alpha$ are $N \times N$ matrices and given by

$$q_{\alpha a}^b = \sqrt{\frac{R}{2}}\Big\{P^I\gamma_{\alpha\alpha'}^I - \frac{i}{2}[X^I, X^J]\gamma_{\alpha\alpha'}^{IJ} - \frac{\mu}{3R}X^i(\Pi\gamma^i)_{\alpha\alpha'} + \frac{\mu}{6R}X^{i'}(\Pi\gamma^{i'})_{\alpha\alpha'}, \theta^{\alpha'}\Big\}_a^b ,$$
$$\tilde{q}_{\alpha a}^b = \sqrt{\frac{2}{R}}\delta_{\alpha\alpha'}\theta_a^{\alpha'b} . \tag{14}$$

The purpose of computing the Dirac brackets between supercharge densities is to see the central charges of extended objects which disappear for finite N if we trace the matrix indices. However, the actual calculation is fairly complicated and hence some simplification is required. As pointed out in Ref. [17], if we drop pieces of the resulting answer which are antisymmetric in the spinor indices, we can freely trace on one of the terms in the Dirac bracket. This simplifies and reduces the calculation and furthermore does not touch the structure of the central charge densities. What we are doing is thus the computation of Dirac brackets between supercharge density and supercharge.

In the process of calculation, we need the following $SO(9)$ Fierz identities.

$$\theta_\alpha\theta_\beta = \frac{1}{32}(\gamma^I\gamma^J)_{\alpha\beta}\theta\gamma^I\gamma^J\theta + \frac{1}{96}(\gamma^I\gamma^J\gamma^J)_{\alpha\beta}\theta\gamma^I\gamma^J\gamma^K\theta ,$$
$$\gamma_{\alpha\alpha'}^I\gamma_{\beta\beta'}^{IJ} + \gamma_{\alpha\alpha'}^{IJ}\gamma_{\beta\beta'}^I + (\alpha \leftrightarrow \beta) = 2\gamma_{\alpha'\beta'}^J\delta_{\alpha\beta} - 2\gamma_{\alpha\beta}^J\delta_{\alpha'\beta'} . \tag{15}$$

Using these identities and the Dirac brackets, Eq. (12), we obtain after slightly tedious manipulation

$$
\begin{aligned}
\{\tilde{q}_{\alpha a}^b, \tilde{Q}_\beta\}_{DB} &= -i\frac{2}{R}\delta_{\alpha\beta}\delta_a^b , \\
\{q_{\alpha a}^b, \tilde{Q}_\beta\}_{DB} &= -2i\Big(P^I\gamma_{\alpha\beta}^I - \gamma_{\alpha\beta}^{IJ}z^{IJ} - \frac{\mu}{3R}X^i(\Pi\gamma^i)_{\alpha\beta} + \frac{\mu}{6R}X^{i'}(\Pi\gamma^{i'})_{\alpha\beta}\Big)_a^b , \\
\{q_{a(\alpha)}^b, Q_\beta\}_{DB} &= -4iR\mathcal{H}_a^b\delta_{\alpha\beta} + i\frac{2\mu}{3}(\Pi\gamma^{ij})_{\alpha\beta}J_a^{ijb} - i\frac{\mu}{3}(\Pi\gamma^{i'j'})_{\alpha\beta}J_a^{i'j'b} \\
&\quad - 2i\gamma_{\alpha\beta}^I z_a^{Ib} - 2i\gamma_{\alpha\beta}^{IJKL}z_a^{IJKLb} \\
&\quad - \frac{\mu}{3}(\Pi\gamma^{j'})_{\alpha\beta}[2(X^i)^2 - (X^{i'})^2, X^{j'}]_a^b \\
&\quad - \frac{\mu}{6}\epsilon_{ijk}\gamma_{\alpha\beta}^{iji'j'}[X^{i'}, \{X^k, X^{j'}\}]_a^b .
\end{aligned} \tag{16}
$$

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where we have symmetrized over α and β in the last Dirac bracket. Some definitions and remarks are as follows: J^{ij} and $J^{i'j'}$ are $SO(3)$ and $SO(6)$ rotation generators respectively, which are given by

$$
\begin{aligned}
J^{ij} &= X^i P^j - P^i X^j - \frac{i}{4}\theta\gamma^{ij}\theta \,, \\
J^{i'j'} &= X^{i'} P^{j'} - P^{i'} X^{j'} - \frac{i}{4}\theta\gamma^{i'j'}\theta \,.
\end{aligned}
\tag{17}
$$

The z's represent the central charge densities for the matrix theory in the flat case [17] and are given by

$$
\begin{aligned}
z^I &= iR\{P^J, [X^J, X^I]\} - \frac{R}{2}[\theta^{\alpha'}, [\theta^{\alpha'}, X^I]] \\
z^{IJ} &= \frac{i}{2}[X^I, X^J] \\
z^{IJKL} &= RX^{[I}X^J X^K X^{L]}
\end{aligned}
\tag{18}
$$

These charges are interpreted as those of the wrapped membrane, membrane, and wrapped (or longitudinal) fivebrane, respectively. The wrapped objects extend along the light cone x^- direction and hence we have R dependence in the corresponding expressions for them. For finite N, they do not activate when we take matrix trace. The vanishing of $\text{Tr}z^{IJ}$ and $\text{Tr}z^{IJKL}$ for finite N is obvious. For the central charge z^I, as noted in Ref. [17], we should use the Gauss law constraint Φ which is obtained as

$$
\Phi_a^b = \frac{\partial L}{\partial A_{0b}^a} = i[P^I, X^I]_a^b - (\theta^\alpha\theta^\alpha)_a^b
\tag{19}
$$

In terms of this constraint, z^I can be rewritten as

$$
z^I = R\{\Phi, X^I\} + iR[X^J, \{P^J, X^I\}] + \frac{R}{2}\{\theta^\alpha, \{\theta^\alpha, X^I\}\}
\tag{20}
$$

In the $A_0 = 0$ gauge, $\Phi = 0$ and $\text{Tr}z^I$ vanishes for finite N. As a final remark, we would like to note that, for finite N, the algebra (16) reduces to that obtained in [13] or the super pp-wave algebra [18] in the context of the eleven dimensional supergravity.

3 BPS Branes in the Matrix Model on pp-wave Background

One can read off the possible BPS solutions from the supersymmetry transformation rules (6) and (8). The only possible static solution one can get is the fuzzy sphere in the x^i-directions[7]

$$
[X^i, X^j] = i\frac{\mu}{3R}\epsilon_{ijk}X^k
\tag{21}
$$

The kinematical supersymmetry generators are completely broken while all the dynamical supersymmetry generators are preserved. Thus it is half-BPS state and describe spherical membrane. This configuration is given in [7] and extensively discussed in [13].

Our focus is to find all the BPS solutions including the non-static branes.[2] There exist non-static flat membrane solutions spanning in $X^{i'}$ directions. They are given by the configurations

$$X_4 = r_1 \cos(\mu t/6) , \quad X_7 = r_1 \sin(\mu t/6) ,$$
$$X_5 = r_2 \cos(\mu t/6) , \quad X_8 = r_2 \sin(\mu t/6) , \tag{22}$$

where

$$[r_1, r_2] = i\mathcal{F}_{12} , \tag{23}$$

where I is the unit matrix in the $SU(N)$ space and $\mathcal{F}_{ab}$ is antisymmetric. The solutions exist only in the large N limit as discussed in the previous section.

This configuration has $\frac{1}{8}$ supersymmetry and corresponds to the rotating transverse membrane expanded along $X^{i'}$ directions with angular frequency $\omega = \frac{\mu}{6}$. The four dynamical supercharges which preserve the above configuration are those satisfying

$$\Pi\epsilon = \gamma_{47}\epsilon = \gamma_{58}\epsilon = \pm i\epsilon , \ . \tag{24}$$

in which the transformation rule (8) becomes

$$\delta\theta = \frac{i}{2}[r_1, r_2]e^{\pm\frac{i\mu}{4}t}\gamma_{45}\epsilon . \tag{25}$$

These are cancelled with the kinematical supersymmetry transformations generated by $\tilde{\epsilon}$ satisfying

$$\Pi\tilde{\epsilon} = \pm i\tilde{\epsilon} , \ . \tag{26}$$

and thus given by

$$\tilde{\delta}\theta = e^{\pm\frac{i\mu}{4}t}\tilde{\epsilon} . \tag{27}$$

One can not get longitudinal five brane by superposing orthogonal membranes of the type (22), which would have membrane charges in it. But still there exist the solutions of longitudinal five brane stretched along $X^{i'}$ directions without membrane charges. It is realized by the configurations of the form

$$X^4 = x^4 \cos(\mu t/6) - x^7 \sin(\mu t/6) , \quad X^7 = x^4 \sin(\mu t/6) + x^7 \cos(\mu t/6) ,$$

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[2]Some BPS solutions related to this has been given in [19].

$$X^5 = x^5 \cos(\mu t/6) - x^8 \sin(\mu t/6) , \quad X^8 = x^5 \sin(\mu t/6) + x^8 \cos(\mu t/6) , \quad (28)$$

while all other coordinates vanish, i.e. $X^i, X^{i'} = 0$. Here $x^{i'}$ are time-independent and satisfy the condition

$$[x^{i'}, x^{j'}] = \frac{1}{2} \epsilon_{i'j'k'l'} [x^{k'}, x^{l'}] , \quad (29)$$

where $\epsilon_{i'j'k'l'}$ is the Levi-Civita symbol in four indices. The condition (29) gives rise to the condition

$$[X^{i'}, X^{j'}] = \frac{1}{2} \epsilon_{i'j'k'l'} [X^{k'}, X^{l'}] , \quad (30)$$

in which the dynamical supersymmetry transformations reduce to

$$\delta\theta = \frac{i}{2} [X^{i'}, X^{j'}] \gamma_{i'j'} \epsilon . \quad (31)$$

Therefore it describes longitudinal five-brane filling four directions in $X^{i'}$ and sticking at the origin in the transverse directions. One can easily see from (31) that it preserves $\frac{1}{4}$ supersymmetry. It does not contain membrane charges but has angular momentum on the $4 - 7$ and $5 - 8$ planes. Again this configuration is possible only in the large N limit [17]

Final configuration we will describe is the longitudinal five brane filling X^1, X^2, X^3-directions and one direction in $X^{i'}$. This five brane has infinite number of membranes embedded ellipsoidally. One can take the coordinates of the form

$$X^4 = r \cos(\mu t/6) , \quad X^5 = r \sin(\mu t/6) , \quad (32)$$

and

$$X_\pm = X^1 \pm iX^2 = e^{\pm i\mu t/6} x_\pm , \quad X^3 = x^3 , \quad (33)$$

where $r, x_\pm$, and x^3 are time independent. All other coordinates should be taken to be zero. The configuration is characterized by

$$[x^3, x_\pm] = \pm \frac{\mu}{2R} , \quad [x_+, x_-] = \frac{2\mu}{3R} x^3 , \quad (34)$$

and

$$[x_\pm, r] = i\mathcal{F}_\pm , \quad [x^3, r] = 0 , \quad (35)$$

with $\mathcal{F}_- = \mathcal{F}_+^\dagger$. Note that the commutation relations (34) are not those of fuzzy sphere, rather they are of fuzzy ellipsoid with

$$\frac{(X^1)^2}{6} + \frac{(X^2)^2}{6} + \frac{(X^3)^2}{4} = (\frac{\mu}{R})^2 j(j+1) \quad (36)$$

for $(2j + 1)$-dimensional representation of $SU(2)$. Therefore one may regard this configuration as the one of longitudinal five branes in which ellipsoidal membranes are embedded. For the dynamical supercharges of the type

$$\Pi\epsilon = \gamma_{12}\epsilon = \gamma_{45}\epsilon = \pm i\epsilon , \quad (37)$$

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the dynamical supersymmetry transformation rule reduces to

$$\delta\theta \quad = \quad \frac{i}{2}[X_\pm, r]e^{\pm\frac{i\mu}{4}t}\gamma_{14}\epsilon .$$

(38)

Therefore the configuration has $\frac{1}{8}$ supersymmetry which are generated by the combination of dynamical supersymmetry transformation (38) and kinematical supersymmetry transformation (27).

4   Discussions

One immediate question in exploring M/string theory on some background is what kind of nonperturbative BPS objects survive in that particular background. The same question has been explored in the IIB superstring theory on the pp-wave background [20, 21, 22] in the works [23, 24] by studying the boundary conditions of open strings on D-branes.[3] On the other hand, in the IIA superstring theory side, we rather have to consider M theory on eleven dimensional pp-wave background. Matrix theory, so far, is the only tractable candidate to describe M theory. Hence, it seems natural to probe M theory on pp-wave background using the corresponding massive matrix theory.

In this paper we analyzed supersymmetry algebra of the matrix model and found some central charges which can be activated only in the large N limit. Then using the supersymmetry transformation rules of fermions we identified some transverse membrane solutions and longitudinal five brane solutions. They exists only in the large N limit and can not be static, but rather they are rotating solutions with specific angular frequency. The angular frequency is proportional to the mass μ and is thus due to the non-trivial pp-wave background. One may wonder how it would be possible for the infinitely stretched branes to have angular momentum. However one should note that, as pointed out in [13], the eleventh coordinate of the pp-wave background geometry loose its meaning as a spatial coordinate in the asymptotic region. This makes questionable using the matrix theory as a proper description in that region. One may hope quadratic potential terms confine X in the finite region giving effective cut-off. In any case our results give the existence of such solution in general terms which would be realized in the case when the transverse spaces are compactified.

It would be interesting to probe BPS branes in IIB superstring theory [23, 24] in the context of IIB matrix string theory [27, 28, 29].

After the completion of this paper, there appeared a paper on the web [30] which is closely related to section 2 in our paper.

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[3]See also [25, 26] for the pp-wave background originating from $Ads_3 \times S^3$ geometry.

Acknowledgments

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