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D-branes and Cosmological Singularities

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Abstract

The motion of a test Dq-brane in a Dp-brane background is studied. The induced metric on the test brane is then interpreted as the cosmology of the test brane universe. One is then able to resolve the resulting cosmological singularities. In particular, for a D3-brane in a D5-brane background, one finds a 3+1 dimensional FRW universe with equation of state $p = \varepsilon$. It has been argued that this may have been the dominant form of matter at very early times.

1 Introduction

It has been shown that singularities are generic features of cosmological and collapse solutions in general relativity. These singularities are representative of the breakdown of the classical theory at short distances. It is hoped that string theory will allow us to resolve these singularities, and indeed for many classical solutions involving time-like singularities this is the case. In particular, consider the time-like singularities associated with D-branes. The geometry outside a D-brane is given by the supergravity (SUGRA) solution for an extremal black p-brane with Q units of R-R charge (see section 2). These solutions ($p \neq 3$) are singular. However, as curvatures become large the supergravity solutions breakdown and stringy effects become important. One should then replace the space-time description with the description of the gauge theory living on the brane. Another interesting example is provided by M-theory. One can show that the singular geometry produced by a D6-brane can be obtained from the dimensional reduction of the a smooth 11 dimensional geometry, analogous to the Kaluza-Klein monopole in 5 dimensions.

There has been much less progress in understanding the space-like singularities which are present in cosmological models. A recent idea which has received a lot of attention is that of an Ekpyrotic Universe [1]. In this model our universe is created by the collision of a slowly moving bulk brane with the visible brane on which we (will) live. Prior to the collision our universe is a cold, empty place. Upon collision the branes fuse together creating a "hot, thermal bath of radiation and matter" living on the brane. The scenario takes advantage of the non-locality of branes to solve the horizon problem, supersymmetry to address flatness, and quantum fluctuations to explain large scale density fluctuations.

In this note a toy model is proposed which also uses coincident branes to describe the origins of our universe. A test Dq-brane is allowed to fall into, or be produced from, a stack of Dp-branes. The induced metric on the test brane is then interpreted as the cosmology on the test brane universe. Moreover, the cosmology has a space-like singularity at the point when the branes become coincident. This differs from the Ekpyrotic universe in many respects. Most importantly, in this model the universe exists before the collision with (or after the production of) the probe brane. In Ekpyrotic scenario the "hot" universe exists only after the bulk brane fuses with the visible brane. The authors of [1] used the properties of branes to give new solutions to many cosmological questions. In this note our interest is simply to use the current understanding of how to resolve the time-like singularities associated with D-branes to learn how string theory might resolve the space-like singularities associated with our early universe. Indeed, for a Dp-brane background the singularity induced on the Dq-brane can be resolved. Namely, for $p \neq 3$ the proper picture at short distances, where the induced metric is becoming singular, is that of the gauge theory living on the brane. For $p = 3$ the space-time is non-singular and the resulting cosmological singularity induced on the brane can be understood entirely from the SUGRA perspective.

Section 2 gives a brief review of D-brane metrics and some details about how the cosmology of the test brane will be interpreted. Section 3 will discuss a Dp-brane moving in a Dq-brane background and section 4 applies the results of section 3 to the specific case of a D3-brane moving in D5-brane background.

2 D-brane Metrics

The geometry produced by flat D-branes is homogeneous and isotropic parallel to the brane and spherically symmetric in the directions transverse to the brane. Therefore the induced metric on a test brane, which lies parallel to the D-brane, should also be homogeneous and isotropic, and is thus described by a FRW cosmology. The metric for Q Dp-branes sitting at the origin is given by [2],

$$ds^2 = Z(r)^{-1/2}\eta_{\mu\nu}dX^\mu dX^\nu + Z(r)^{1/2}dX^m dX^m \quad .$$

$$\begin{aligned} \text{Here} \quad e^{2\phi} &= Z(r)^{(3-p)/2} \\ Z(r) &= 1 + \frac{\rho^{7-p}}{r^{7-p}} \\ r^2 \equiv X^m X^m \qquad & \rho^{7-p} \propto \alpha'^{(7-p)/2}\, g\, Q \quad . \end{aligned}$$

The indices μ, ν correspond to directions along the brane while m, n run transverse to the brane. Throughout this paper we will use the gauge, $\xi^0 = \tau$ $\xi^i = X^i$. Here ξ^a are the coordinates on the brane. Since the branes are parallel the indices a, b are a subset of the μ, ν. τ will be chosen so that the induced metric is in the standard FRW form, i.e. $\tilde{G}_{00} = -1$. The motion of the test brane is assumed to be radial (in the transverse space) and lie along X^{p+1}, $(r \equiv |X^{p+1}|)$. Furthermore, it will be assumed that $Q \gg 1$ so that the back reaction of the test brane on the geometry can be ignored. The induced metric on the Dq-brane, $\tilde{G}_{ab} = G_{MN}\frac{\partial X^M}{\partial \xi^a}\frac{\partial X^N}{\partial \xi^b}$, is therefore,

$$\tilde{G}_{ab} = Z^{-1/2}\begin{pmatrix} -\dot{t}^2 + \dot{r}^2 Z & \\ & \mathbf{1}_{(q\times q)} \end{pmatrix} \quad .$$

We will find that it is possible to make the gauge choice $\tilde{G}_{00} = -1$, which can be written as,

$$Z^{-1/4}\sqrt{\dot{t}^2 - \dot{r}^2 Z} = 1 \; . \tag{2.1}$$

This brings the induced metric to that of a flat Robertson Walker geometry,

$$ds^2 = -d\tau^2 + a^2(\tau)dX^i dX^i \; .$$

Here $a(\tau) = Z^{-1/4}[r(\tau)]$. $\tilde{G}_{ab}$, will be interpreted as the cosmology on the $q{+}1$ dimensional universe.

At this point we shall assume the naive perspective of a $q+1$ dimensional classical observer, and thus interpret the expansion of the brane universe as if it were due to a perfect fluid content in standard Einstein gravity. It is worth noting that in the toy models discussed below there is no real matter living on the brane. Thus the expansion of the universe, which is actually being caused by the curvature of the higher dimensional space, is incorrectly perceived to be the result of dark matter living on the brane.

3 Solving Equations of motion

The motion of a Dq-brane moving in a Dp-brane background is governed by the DBI action[2],

$$S_q = -\mu_q \int d^{q+1}\xi \; e^{-\phi}\sqrt{-det(\tilde{G}_{ab} + \tilde{\mathfrak{F}}_{ab})} \; + \mu_q \int_{q+1} \tilde{C}_{q+1} \; . \tag{3.2}$$

For simplicity it will be assumed that there is no gauge field living on the test brane and that there is no antisymmetric closed string background living in the bulk; thus $\tilde{\mathfrak{F}}_{ab} = 0$. For parallel branes if $p \neq q$ then $\tilde{C}_{q+1}$, the pullback of the Ramond-Ramond potential, vanishes. However, for $p = q$ [3][1],

$$\tilde{C}_{q+1} = (Z_p^{-1} - 1)\; \dot{t}\; d\tau \wedge dx^1 \wedge \ldots \wedge dx^q \; .$$

Our action is then,

$$S_q = -\mu_q \int d^{q+1}\xi \left[Z^{(p-q-4)/4}\left(\sqrt{\dot{t}^2 - Z\dot{r}^2} \; - \; \delta_{p,q}\; \dot{t}\;\right) + \delta_{p,q}\; \dot{t}\right] . \tag{3.3}$$

We can now find the classical equations of motion. Defining $s \equiv -(p-q-4)/4$ Lagrangian takes the form,

$$\mathcal{L} = Z^{-s}(\sqrt{\dot{t}^2 - Z\dot{r}^2} - \dot{t}\delta_{s,1}) + \dot{t}\delta_{s,1} \; .$$

[1] In [3] $\tilde{C}_{q+1}$ is given in static gauge. Here we have changed coordinates $\xi^0 = X^0 \to \xi^0 = \tau$.

Since $t(\tau)$ is a cyclic variable we know that $\partial\mathcal{L}/\partial\dot{t} = c + \delta_{s,1}$, where c is a constant of the motion[2]. Taking the derivative and solving for $\dot{t}^2$ gives,

$$\dot{t}^2 = \frac{(Z^s c + \delta_{s,1})^2 Z \dot{r}^2}{l_{p-q}^2} \ , \tag{3.4}$$

where we have defined $l_{p-q}^2 \equiv (Z^s c+\delta_{s,1})^2 - 1$. The gauge choice (2.1) relates $r(\tau)$ and $t(\tau)$. Therefore the equation of motion coming from the variation of $\mathcal{L}$ with respect to $r(\tau)$ doesn't provide any new information. From (2.1) and (3.4) it is easy to see that,

$$d\tau = \pm \frac{Z^{1/4}}{l_{p-q}} dr \tag{3.5}$$

One can now apply these results to various cases. The specific example of a D3-brane moving in a D5-brane background yields an interesting cosmology and is discussed below.

4 An Example: D5-brane Background

Let's now consider the motion of a D3-brane in a D5-brane background. For this case (3.5) can be integrated exactly. The integral over r, however, gives rise to quartic roots and hypergeometric functions which cannot be algebraically inverted to give $r(\tau)$. Fortunately the interesting behavior, namely the big bang/big crunch takes place when the branes are nearly coincident. In this limit, $r \ll \rho$, we can see that,

$$d\tau \simeq \pm \frac{Z^{-1/4}}{c} dr \simeq \frac{1}{c}\sqrt{\frac{r}{\rho}} dr$$
$$\Longrightarrow \frac{r(\tau)}{\rho} = \mathcal{C}^2 (\frac{\tau}{\rho})^{2/3}, \quad \text{where} \quad \mathcal{C}^2 \equiv \frac{2}{3c} \ .$$

Here we have assumed $c \geq 0$, which is required for the induced metric to be Lorentzian. The induced metric on the brane is then,

$$ds^2 = -d\tau^2 + \mathcal{C}^2 (\frac{\tau}{\rho})^{2/3} dX^i dX^i \ .$$

A scale factor $a(\tau) \propto \tau^{1/3}$ corresponds to a perfect fluid with the equation of state $p = \varepsilon$. It has been argued by Banks and Fischler[4] that this may have been the dominant form of matter in the early stages of our universe. The argument is the following: $p = \varepsilon$ is the stiffest equation of state such that the velocity of sound is less than or equal to the

[2] Here we have added $\delta_{s,1}$ to c in order to simplify (3.4).

speed of light. Furthermore, Fischler and Susskind argued that for a FRW cosmology this is the stiffest equation of state satisfying the holographic bound[5]. These are taken as indications that this is the stiffest equation of state allowed by nature. For a FRW universe, with perfect fluid and $p = \gamma\varepsilon$, Einstein's equations give $\varepsilon = a(\tau)^{-3(1+\gamma)}$. So if there exists some $p = \varepsilon$ matter, and this is the stiffest equation of state allowed by nature, then this form of matter will dominate at early times.

There might be concern that the limit $r \ll \rho$ is outside the region of validity of the supergravity solutions we are using. A careful treatment of this was done in [6]. It was found that the SUGRA solutions could be trusted in the region

$$\frac{1}{\sqrt{gQ}} \ll \frac{r(\tau)}{\sqrt{\alpha'}} \ll \sqrt{\frac{Q}{g}}\,. \tag{4.6}$$

Note that for large (gQ), which is required since the back-reaction from the test brane on the geometry has been ignored, the SUGRA solutions hold very close to the brane. Similarly, as the induced geometry on the brane becomes singular there are corrections to the DBI action which become important. Thus the above solution can only be trusted as long as the induced scalar curvature on the brane, R, is much less than the string scale, $R \ll 1/\alpha'$. For our test brane universe $R \propto \tau^{-2}$. This can be seen by direct computation or by dimensional arguments. From this we can see $\alpha' \ll \tau^2$, which provides a lower bound on r. It follows that,

$$\mathcal{C}^2 (gQ)^{1/6} \ll \frac{r(\tau)}{\sqrt{\alpha'}} \ll \sqrt{gQ}\,. \tag{4.7}$$

The upper bound comes from working in the region, $r \ll \rho$. Note that the lower bound can be taken to zero by choosing $\mathcal{C} \sim 0$. This limit corresponds to having the test brane initially moving along an almost null path. Our solution is valid in the region which both (4.6) and (4.7) are satisfied.

The breakdown of the SUGRA solution marks the end of our classical knowledge. String theory, as previously mentioned, now tells us that the correct physical picture is that of the gauge theory living on the brane. Any system for which the number of ND coordinates[3] is equal to 2 has an open string tachyon which is not projected out in the GSO projection. As the D3-brane approaches the D5-branes these tachyons will form and thus prevent the D3-brane from passing through the D5-branes and escaping to infinity.

[3]the number of coordinates such that a string stretched between the branes has Neumann boundary conditions on one end and Dirichlet boundary conditions on the other

Rather, the tachyon indicates an instability whose end result is the D3-brane dissolving into the D5-branes and then spreading out as magnetic flux on the branes. For a detailed discussion see [7]. In addition to the magnetic flux there will be ripples produced from the momentum of the D3-brane. Observers living on the D3-brane falling into the D5-branes, will thus find their universe collapsing, and then suddenly, the 3+1 dimensional universe to which they are accustomed becomes a 5+1 dimensional universe. Sadly, this shocking development does not save our poor observers. Rather they find themselves being spread out as magnetic flux and ripples in their new universe. Alternatively, and perhaps more importantly, a D5-brane with incoming flux and ripples could, under very fine tuned initial conditions, cause the production of a D3-brane, giving birth to our universe.

5 Discussion

Understanding the origins of our universe should be a central part of the "theory of everything". String theory, if it is such a theory, should certainly be able to resolve the cosmological singularities associated with birth of our universe. In the above note we have discussed a toy model in which cosmological singularities can be resolved in string theory. The case of a D3-brane moving in a D5-brane background was discussed in detail. For such a scenario one is able to give a complete description of the apparent singularity. As the geometry on the probe-brane becomes singular the space-time description of the brane universe must be reinterpreted as the gauge theory living on the D5-branes; our 3+1 dimensional universe dissolves into the D5-brane and spreads out as magnetic flux and ripples on the brane. The equation of state of this universe was found to be $p = \varepsilon$, which, as discussed in section 4, may have been relevant at very early stages of our universe.

These are interesting results, but is this a realistic model? Certainly not. For starters we have not included any matter on the brane universe, which one might argue is unrealistic. Including matter would complicate the analysis but in principle it could be done. More importantly we have not confined gravity to the brane. Though the model discussed does not give a realistic description of our universe it does give us some insight into how string theory might resolve the space-like singularities associated with our early universe.

There are other resolutions of singularities which may be obtained by looking at different background geometries. In particular, consider that of a D3-brane moving in a D3-brane background. One finds that this background does not yield a physical equation of state. However, the resolution of the singularity is very interesting; the cosmological singularity can be understood as arising from the motion of the probe brane through a perfectly smooth space-time! Moreover, the singularity on a D4-brane[4] moving in the presence of a D6-brane could be understood by lifting up to 11 dimensions, where the geometry is non-singular, thus providing an M-theoretic resolution to cosmological singularities. Recall, however, that in order to avoid complications caused by back-reaction on the metric we have taken the Q, the number of D6-branes, to be very large. For multiple D6-branes the 11 dimensional geometry contains a conical singularity. One could consider a similar set up with a single D6-brane. In such a scenario, however, complications would arise from the back reaction of the metric by the D4-brane, and perhaps more importantly, the breakdown of SUGRA approximations.

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Note Added

After completing this work it was pointed out that much of what has been discussed here is contained within a paper, “Mirage Cosmology”, by A. Kehagias and E. Kiritsis [8].

[4]By dimensional reduction one is still able to obtain a model for a 3+1 dimensional universe.

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