Projection Operators and D-branes
in Purely Cubic Open String Field Theory

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Abstract

We study a matrix version of the purely cubic open string field theory as describing the expansion around the closed string vacuum. Any D-branes in the given closed string background can appear as classical solutions by using the identity projectors. Expansion around this solution gives the correct kinetic term for the open strings on the created D-branes while there are some subtleties in the unwanted degree of freedom.

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1 Projectors in String Field Theory

Recently Rastelli, Sen and Zwiebach [1, 2] pioneered the open string field theory around the nonperturbative (closed string) vacuum. They conjectured Witten's open string field theory action [3] should be modified in the following form,

$$S = \frac{1}{2} \int \Psi \star \mathcal{Q}\Psi + \frac{1}{3} \int \Psi \star \Psi \star \Psi \ , \tag{1}$$

with the "universal" BRST operator $\mathcal{Q}$ written in terms of the ghost fields only. Under this assumption one may factorize the string fields into the ghost and the matter parts, $\Psi = \Psi^{matter} \otimes \Psi^{ghost}$ and the equation for the matter part becomes simplified,

$$\Psi^{matter} \star \Psi^{matter} = \Psi^{matter} \ . \tag{2}$$

This is the defining equation for the projection operators with respect to Witten's star product $\star$. Since the ghost part is universal, the solutions to this equation are supposed to describe the various backgrounds of the open strings, namely D-branes. Motivated by this observation there is a rapid development in the construction of the projectors of the string field theory [4, 5, 6, 7, 8, 9, 10, 11, 12].

In particular there is direct argument [7, 9] in terms of the boundary conformal field theory that there are at least two types of the projection operators (the identity and the sliver states) associated with every Cardy state which describes a D-brane. A critical property of Cardy state is that it has a well-defined field content in the open string channel. In particular, there is always an identity operator in the channel connecting the same brane. From the background independent formulation of the open string field theory [13], one may always construct the wedge state as follows,

$$|n\rangle_a \equiv U_n |\mathrm{vac}; a, a\rangle \ , \tag{3}$$

where U_n describes the conformal transformations [13, 7],

$$\tilde{z} = \tan\left(\frac{2}{n}\arctan(z)\right) , \tag{4}$$

written in terms of the Virasoro generators. $|\mathrm{vac}; a, a\rangle$ is the vacuum state in the open string sector with the both ends attached to the same D-brane labeled as a. It was shown in [13] that they satisfy

$$|n, a\rangle \star |m, a\rangle = |n + m - 1, a\rangle \ . \tag{5}$$

Clearly $n = 1$ (the identity) and $n = \infty$ (the sliver) give the projection operators.

The sliver state turns out to be rank one and considered as a good candidate to describe the Chan-Paton factor of a single D-brane. On the other hand, in our previous work [12], we indicated that the identity projector also enjoys the following properties,

1. It defines the projection to the open string states with both ends attached to a particular D-brane. Namely if we denote $\mathcal{C}$ as the algebra generated by the open string fields with all possible boundary conditions, $\mathcal{I}^a = |1\rangle_a$ picks up the open string field algebra $\mathcal{A}^a$ with the particular boundary condition a. Namely

$$\mathcal{I}^a \star \mathcal{C} \star \mathcal{I}^a = \mathcal{A}^a \ . \tag{6}$$

2. While the trace of $\mathcal{I}^a$ is apparently infinite, there is a regularization which is common in BCFT that

$$\mathrm{Tr}\,(\mathcal{I}^a) = \langle \mathrm{vac}|a\rangle \ . \tag{7}$$

Here $\langle \mathrm{vac}|$ is the closed string vacuum and $|a\rangle$ is the Cardy state. The quantity in the right hand side is called as the boundary entropy and can be identified with the D-brane tension [14].

Since these two properties are quite desirable, we conjectured that the identity projector may be also a good candidate to describe the noncommutative soliton of the string field theory which describes D-branes.

The use of the identity projector seems at least puzzling from the viewpoint of the standard conjecture [2, 4, 5, 7, 10]. Namely the rank of the projector is usually identified with the number of the D-branes. This assumption stems from the fact that the potential energy

$$V = \frac{1}{2}\int \Psi^2 - \frac{1}{3}\int \Psi^3 \tag{8}$$

expanded around a particular projector p has the $n \times n$ (n is the rank of the projector p) negative modes and is identified as the number of the tachyon states.

Naive application of this idea to the identity projector does not make any sense because the identity projector has infinite rank. However, we think that there is a possible way out. Namely in (8) one should add the kinetic term. Usually if the kinetic term is defined in terms of L_0 (Virasoro generator), there is a contribution to the mass term and the infinite degeneracy is resolved.

The problem is that in the pure ghost BRST operator there is no matter Virasoro operator in the kinetic term. It means that we can not use the above argument to save the identity projector. However, we think that this is also a problem of the current formulation of the vacuum string field theory. For example, in [7], it was observed that the expansion around D-brane background (the sliver state) has no physical modes (every variation of the sliver can be absorbed by the gauge transformation). In [10], it is also indicated that the ghost action vanishes identically. In any case, we think that there will be no hope to recover the perturbative open string spectrum without introducing the matter Virasoro algebra at some point.

Since the appearance of the projector to describe D-branes is quite natural from the viewpoint of K-theory [15, 16, 17], we would like to try the other scenario, namely the purely cubic action to describe the vacuum string theory. By combining the idea of [18] with the identity projector, we show that it gives a partially successful scenario to this problem in a sense it reproduces the correct perturbative spectrum on the survived D-branes. While it also gives at the same time some undesirable features, we think that it gives an intermediate step to understand the correct description of the vacuum string field theory.

2 SFT in the presence of several D-branes and split string method

Before we discuss the vacuum string field theory, we would like to give some definitions of the open string field theory in the presence of several D-branes. Suppose we start from N D-branes with the boundary condition specified by the Cardy state $|a\rangle$ $a = 1, \cdots, N^2$. The open string field in this setting is described by a $N \times N$ matrix Ψ whose ab-th component is written as Ψ^{ab}. Ψ^{ab} describes the open string which connects D-branes with labels a and b. We have to define the (perturbative or conventional) BRST operator Q for each sector by combining the Virasoro operator of the matter sector (which is different for each sector) and the ghost sector which is universal. We will use Q_{ab} if we want to specify the operator on a particular sector. We write the matrix $Q\Psi$ to mean the matrix whose ab-th component is $Q_{ab}(\Psi^{ab})$.

Since the integration operator and the three string vertex can be written in the universal form [13] (namely written in terms of Virasoro operators without specifying the particular representation), the matrix generalization

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[2]One may take some of them to be identical D-brane. In that case this label specifies Chan-Paton factor.

of Witten's lagrangian is well-defined,

$$S = \int \left(\frac{1}{2} \Psi \star (Q\Psi) + \frac{1}{3} \Psi \star \Psi \star \Psi \right) , \quad \int \equiv \sum_a \int_a \qquad (9)$$

Here the integration $\int_a$ is defined by the identity operator for the open string in aa channel. The summation with respect to a gives the trace over Chan-Paton factor when the D-branes are identical. The integration symbol $\int$ thus combines the trace of matrix together with usual integration of the string field. This action has the matrix generalization of the gauge symmetry,

$$\delta \Psi^{ab} = Q_{ab}\epsilon_{ab} + \sum_c (\epsilon_{ac} \star \Psi^{cb} - \Psi^{ac} \star \epsilon_{cb}) , \qquad (10)$$

where ϵ_{ab} is a string field from ab-sector. Appearance of non-abelian gauge symmetry even for the intertwining strings might sound strange. However in these cases ϵ_{ab} gives only the massive gauge symmetry and does not appear in the massless sector.

In this language, the description of the identity projector becomes very simple. Namely for each aa-sector, one may define the identity operator $|1\rangle_a \otimes \mathcal{I}^{gh}$ ($\mathcal{I}^{gh}$ is the universal identity operator of the ghost field). The projector $\mathcal{I}^a$ is then defined as putting it at aa-th component and putting zero in other entries. It is obvious that it satisfies

$$\mathcal{I}^a \star \mathcal{I}^b = \delta_{ab}\mathcal{I}^b , \qquad \sum_a \mathcal{I}^a = \mathcal{I}_\mathcal{C} \qquad (11)$$

($\mathcal{I}_\mathcal{C}$ is the identity for the whole system) and (6).

For the discussion in the next section, it will be useful to introduce the intuitive picture given by the split string formalism [5, 10, 6, 19]. Since the ghost sector is common and the same for any boundary conditions we concentrate our attention to the matter sector. We use the notation of [5, 10] in the following.

We start from the discussion of N D-25 branes in the flat background. Suppose that the open string field Ψ^{aa} whose both ends are connected to the a-th ($1 \leq a \leq N$) D-brane is described by the embedding function $x^\mu_{aa}(\sigma)$ ($0 \leq \sigma \leq \pi$, $\mu = 0, \ldots, 25$) with the Neumann boundary condition at $\sigma = 0, \pi$. In the split string formalism, we divide it into the left and the right halves,

$$\ell^\mu_a(\sigma) = x^\mu_{aa}(\sigma), \quad r^\mu_a(\sigma) = x^\mu_{aa}(\pi - \sigma), \quad (0 \leq \sigma < \frac{\pi}{2}). \qquad (12)$$

The boundary condition at the midpoint should be Dirichlet type in order to properly connect the left and the right halves. $\ell(\sigma)$ and $r(\sigma)$ thus have the

boundary condition of Dirichlet-Neumann type and can be expanded as,

$$\frac{\ell_a^\mu(\sigma)}{\sqrt{2}} = \sum_{n=0}^{\infty} \ell_{a,2n+1}^\mu \cos(2n+1)\sigma, \quad \frac{r_a^\mu(\sigma)}{\sqrt{2}} = \sum_{n=0}^{\infty} r_{a,2n+1}^\mu \cos(2n+1)\sigma. \quad (13)$$

At the midpoint, since we fix $\ell(\frac{\pi}{2}) = r(\frac{\pi}{2}) = 0$, we need an extra degree of freedom (the zero mode) and we write it as $\bar{x}$ (note that it does not have the index of the boundary).

To describe the open string that intertwine different D-branes (say a and b), we define the embedding function by combining ℓ_a and r_b,

$$x_{ab}^\mu(\sigma) = \begin{cases} \ell_a^\mu(\sigma) + \bar{x} & 0 \leq \sigma \leq \frac{\pi}{2} \\ r_b^\mu(\pi - \sigma) + \bar{x} & \frac{\pi}{2} \leq \sigma \leq \pi \end{cases} \quad (14)$$

Thanks to the Dirichlet boundary condition at $\sigma = \pi/2$, such a mixed combination becomes consistent. The string field for a-b sector is defined as a matrix indexed by ℓ_a and r_b

$$\Psi^{ab}(x_{ab}) \rightarrow \hat{\Psi}^{ab}(\ell_a, r_b) , \quad (15)$$

and the multiplication is defined through the contraction,

$$\hat{\Psi}^{ab} \star \hat{\Phi}^{bc}(\ell_a, r_b) = \int \prod_\sigma dy(\sigma) \hat{\Psi}^{ab}(\ell_a, y_b) \hat{\Phi}^{bc}(y_b, r_c). \quad (16)$$

The zero mode can be incorporated by multiplying $e^{ip\bar{x}}$ if the momentum of the open string is p.

Up to this point, it is (at least intuitively) clear that our argument does not depend on the fact that we start from the N identical D-25 branes. Indeed the definition of (14,16) is always consistent even if we have the mixed boundary conditions. In this case, however, there is a subtlety in the definition dynamical degree of freedom at the midpoint. The dynamical degree of freedom of the open string x^{ab} belongs to the twisted sector. We have to split it into two pieces and the question is whether one may limit the boundary condition at $\sigma = \pi/2$ as Dirichlet. In the generic BCFT, one may further need some sort of discrete summation over sectors even at $\sigma = \pi/2$. Our argument in the next section will depend on the splitting the dynamical degree of freedom to some extent. To be strict, it will be safer to assume that the discussion in the next section is made only for the N identical D-branes while we will use the universal notation.

In the split string formalism, the definition of the identity projector becomes quite intuitive. We introduce the $N \times N$ matrix valued string field p. It is defined that we put

$$\mathcal{I}_{gh} \otimes \pi_{ab} \prod_\sigma \delta(\ell_a(\sigma) - r_b(\sigma)) \quad (17)$$

at a-b th entry where π_{ab} is a constant complex number and $\mathcal{I}_{gh}$ is the identity operator of the ghost sector [19, 10]. By the matrix multiplication, it satisfies $p^2 = p$ if π_{ab} satisfies $\sum_b \pi_{ab} \pi_{bc} = \pi_{ac}$. We note that in the definition of the identity operator, we have to equate the boundary conditions in order that the delta function functional $\delta(\ell_a - r_b)$ is consistent.

We comment that in the tachyon condensation process from D-$(p+2)$ brane to D-p branes [20, 21] in the Seiberg-Witten limit [22], we have already met the matrix π_{ab}. Indeed in this example one may take $p+1$ directions to be Neumann for ℓ_a^μ and $r_a\mu$ and the zero mode algebra gives the matrix π_{ab}. It is the situation where the string field algebra becomes the direct product of $\mathcal{B}(\mathcal{H}) \otimes \mathcal{A}$ where $\mathcal{A}$ is the open string algebra between D-p branes. The matrix π_{ab} can be precisely identified with GMS soliton [21] after Weyl correspondence. The appearance of GMS soliton was also discussed in the direct string amplitude calculation of $p - p'$ system in [23].

3   Identity projector in Purely Cubic Theory

In the following, we consider the purely cubic system for the matrix extended open string fields,

$$S = \frac{1}{3} \int \Psi^3 \ . \tag{18}$$

Compared to (1) it corresponds to the choice $\mathcal{Q} = 0$, as the BRST operator which is in a sense universal (purely ghost). This action was the first example of the background independent formulation of the open string field theory [24, 18] (see also [25, 26, 27, 28]).

It was suggested [1] that it will not likely to be the closed string vacuum which is discussed in the literature [32, 33]. Indeed we will meet a certain difficulty which seems to be originated from this fact in our later discussion.

While these facts are discouraging, it has certainly a merit that it was known already [18, 27] that one can reproduce the conventional perturbative string field theory theory as the fluctuation around a solution to $\Psi^2 = 0$,

$$\Psi_0 = Q_L \mathcal{I} \ , \tag{19}$$

for the single component theory. Here $\mathcal{I}$ is the identity operator of the whole system (matter+ghost) and Q_L is the perturbative BRST current integrated over the left half of the open string. By putting $\Psi = \Psi_0 + \psi$ into (18) gives,

$$S = \frac{1}{2} \int \psi \star Q\psi + \frac{1}{3} \int \psi^3 \ , \tag{20}$$

which is the correct open string field theory on a D-25 brane.

In the following, we would like to construct a similar solution in the multi-component theory. Matrix generalization is essential to discuss the tachyon condensation since in such theory we assume that there are infinite number of D-branes which is annihilated (or melted) in the closed string vacuum. In a sense, the situation is similar to consider "Dirac sea" for D-branes. In order to discuss the creation/annihilation of D-branes, we need to start from the (in general infinite dimensional) matrix generalization.

Here we have a comment on the background independence of the string field theory. In our formalism, we need to start from a particular background for the closed string. From that data, one can in principle construct the possible boundary states and determine the matrix algebra of the open strings. Since we introduce the all possible boundary conditions to construct Ψ, it is background independent in the open string sense. On the other hand, since we need the information of the closed string background, it is not universal in the closed string sense at least at the tree level. We expect there are some consistency conditions for the closed string background at the quantum level.

In this setup we argue that one may replace the identity in (19) by the identity projectors defined in the previous section which is BRST invariant

$$Q_{aa}(\mathcal{I}^a) = 0. \tag{21}$$

Intuitively it describes a single D-brane created from the vacuum.

In the following, we pick up one of the projector $\mathcal{I}^a$ and write it as p and the sum of the all the rest as q. It is then trivial to show that

$$p \star p = p, \quad q \star q = q, \quad p \star q = 0, \quad p + q = \mathcal{I}_\mathcal{C} . \tag{22}$$

We start to prove that

$$\Psi_0 = Q_{aL}\, p , \tag{23}$$

gives also a solution to $\Psi_0^2 = 0$. Here we assume that we can split the BRST charge for ab-sector as,

$$Q_{ab} = Q_{aL} + Q_{bR} . \tag{24}$$

Roughly speaking, Q_{aL} (resp. Q_{bR}) is the BRST charge for the half strings $\ell^a(\sigma)$ (resp. $r^b(\sigma)$). We note however that they are not nilpotent. As we have commented in the last section, this splitting may be subtle for the open strings with twisted boundary conditions because of the degree of freedom at the midpoint. In any case, this splitting of BRST charge is essential in our discussion in the following one needs modification if there is a contribution from the midpoint. For the identical D-branes, we think that there will be no modification.

Under this caution, the calculation becomes parallel to the original [18] and we need (at least formally) the following identities which was explicitly proved in [27] for the single component theory.

$$Q_{a,R}\, p = -Q_{a,L}\, p \tag{25}$$
$$(Q_{b,R} A_{ab}) \star B_{bc} = -(-1)^{|A|} A_{ab} \star Q_{b,L} B_{bc} \tag{26}$$
$$\{Q_{aa}, Q_{a,L}\} = 0 \;. \tag{27}$$

The proof of the nilpotency is given as follows. First by using (25,26)

$$Q_L\, p \star Q_L\, p = -Q_R\, p \star Q_L\, p = p \star Q_L^2\, p. \tag{28}$$

Second by using (26)

$$Q_L\, p \star Q_L\, p = (Q_R Q_L\, p) \star p. \tag{29}$$

If we note that $Q_{L,R}$ does not change the boundary conditions, $p \star Q_L^2\, p = Q_L^2\, p$ and $(Q_R Q_L\, p) \star p = Q_R Q_L\, p$. Summing up these two formulae gives

$$Q_L\, p \star Q_L\, p = \frac{1}{2} Q Q_L\, p = -\frac{1}{2} Q_L Q\, p = 0. \tag{30}$$

In this sense each identity projector indeed defines a solution to the purely cubic action.

We have to note that, unlike the situation in [18], the derivative defined by $Q_L p$ *does not* give BRST operator for the string fields in the generic sectors. Indeed,

$$D_{(Q_L\, p)} B = (Q_L\, p) \star B - (-1)^{|B|} B \star (Q_L\, p) = p \star (Q_L\, B) + (Q_R\, B) \star p \;. \tag{31}$$

If we restrict the string field to *aa*-sector, it gives of course the BRST operator. The existence of other sectors breaks such a property. Accordingly the kinetic term of the action has some undesirable terms. If we put $\Psi = Q_L\, p + \psi$ into the action, the contributions to the kinetic term takes the following form,

$$\int \psi^2 \star Q_L p = \frac{1}{2} \left(\int \psi_0 \star Q \psi_0 + \int t \star Q_L \bar{t} + \int \bar{t} \star Q_R t \right) \;. \tag{32}$$

Here $\psi_0, t, \bar{t}$ are components of ψ,

$$\psi_0 = p \star \psi \star p, \quad t = q \star \psi \star p, \quad \bar{t} = p \star \psi \star q \;. \tag{33}$$

The appearance of three terms in the kinetic terms can be interpreted in the context of the tachyon condensation [34, 20, 21] as follows. We start from

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the D-brane system described by the various boundary conditions where the intertwining open strings describes the algebra $\mathcal{C}$. In this system, we consider the tachyon condensation where only the D-branes picked up by the projector p survives after tachyon condensation. The string field ψ_0 is then interpreted as the open strings on the survived branes. On the other hand, two string fields $t, \bar{t}$ describe the intertwiner between the survived and the disappeared ones. Whereas we have a correct kinetic term (32) for a, it is not reasonable that we have a sort of ("incomplete") kinetic terms for $t, \bar{t}$.

We would like to indicate that this is actually a similar situation to the field theory limit [20] for the noncommutative tachyon. Indeed, there also appeared the corresponding components between the created and the melted D-branes. These fields, however, are the analogue of W-bosons in the system with spontaneously broken symmetry. It was observed that they have the string scale mass and infinitely degenerate. This puzzle was solved [29, 30, 31] by noting the correct choice of variables and vanishing of the overall factor of the kinetic action.

In our case, one may also regard the intertwining modes $t, \bar{t}$ as a kind of W-bosons for the symmetry breaking. In the original action (18), the gauge symmetry (gauge group on the configuration space of the open strings) is generically $U(\infty)$ if we consider all possible D-branes which acts on the string fields as,

$$\delta_\epsilon \Psi = \epsilon \star \Psi - \Psi \star \epsilon \ . \tag{34}$$

In the expansion around the classical solution (23), the gauge symmetry is transformed to,

$$\delta \psi = (p \star (Q_{L,a}\epsilon) + (Q_{R,a}\epsilon) \star p) + \epsilon \star \psi - \psi \star \epsilon \ . \tag{35}$$

As we have commented, the first term can be written in the form $Q\epsilon$ only when ϵ belongs to aa-sector. If it belongs to the oo'-sector with $o \neq a, o' \neq a$ then this term simply vanishes. On the other hand, if it belongs to ao or oa with $o \neq a$ sectors, it can not be written as the conventional form of the gauge transformation. Therefore it may be adequate to consider that the gauge symmetry $U(\infty)$ is broken to $U(1) \oplus U(\infty - 1)$.

We may guess the off diagonal gauge transformation may remove the unwanted degree of freedom. Indeed, when $t, \bar{t}$ is infinitely small, one may approximate the gauge transformation with the explicit sector index as,

$$\delta t_{oa} = Q_{R,a}\epsilon_{oa} \ , \quad \delta \bar{t}_{ao} = Q_{L,a}\epsilon_{ao} \ . \tag{36}$$

While it is difficult to prove, the use of this gauge transformaion may remove the unwanted degree of freedoms. We note that $Q_{L,R}^2$ is written in terms of ghost fields only [27].

4　Summary

We summarize the results and the conjectures of this letter.

1. From every identity projector associated with any D-brane, one may construct the classical solution of the matrix version of the purely cubic open string field theory. Since it contains all possible open string degree of freedom, one may call it as "universal action".

2. The expansion around this classical solution gives a correct action of the open strings on the created D-brane from the vacuum. There exist some unwanted degree of freedom but it seems to be possible to gauge them out.

3. As already noticed in [1] and may be by others, the purely cubic theory has an obvious drawback that the value of the action $S(\Psi_0)$ for any classical solution Ψ_0 is always zero. In our case, we are supposed to obtain the D-brane tension from this factor but we can not. A possible resolution of this puzzle is that in the possible operation of gauging away $t, \bar{t}$, we will have a singular transformation that will reproduce a nontrivial contribution to the action. This is, of course, rather hard to confirm. There will be probably a framework where this effect is automatically produced as Rastelli, Sen and Zwiebach endeavored.

4. In any case, our work may suggest that one needs a singular classical solution to have a nontrivial BRST cohomology. We think the magic, if any, can only come from the serious examination of the notorious midpoint degree of freedom.

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