

Ramond-Ramond Field Transformation

Yungui Gong¹

Physics Department, University of Texas at Austin, Austin, TX 78712, U.S.A.

Abstract

We find that the mixture of Ramond-Ramond fields and Neveu-Schwarz two form are transformed as Majorana spinors under the T-duality group $O(d, d)$. The Ramond-Ramond field transformation under the group $O(d, d)$ is realized in a simple form by using the spinor representation. The Ramond-Ramond field transformation rule obtained by Bergshoeff et al. is shown as a specific simple example. We also give some explicit examples of the spinor representation.

¹Email: ygong@physics.utexas.edu

1 Introduction

The transformation of the fields from the Neveu-Schwarz (NS) sector under T-duality is well established. The Ramond-Ramond (RR) field transformation was first given in [1]. The authors in [1] got the RR field transformation by identifying the same RR fields and RR moduli in $d = 9$ supergravity coming from both ten dimensional type IIA and type IIB supergravity theories compactified down to nine dimensions. Unfortunately this method gives only a specific T-duality transformation, namely Buscher's T-duality transformation [2]. It is hard to get the generalized T-duality group $O(d, d)$ transformations by this method. Recently, Hassan derived the RR field transformation under $SO(d, d)$ group by working on the worldsheet theory [3]. The RR field transformation under Buscher's T-duality was also discussed by Cvetič, Lü, Pope and Stelle using the Green-Schwarz formalism [4]. If we compactify the $d = 9$ supergravity further down to lower dimensions, we know that the lower dimensional solution has $O(d, d, R)$ transformation and how the NS-NS fields which are assumed to be independent of d coordinates transform under this group [5]. Therefore, we need to find the RR field transformation under the general $O(d, d, R)$ group.

The RR fields transform as the Majorana-Weyl spinors of $SO(d, d)$ [6]. RR fields transforming as the spinors of $O(d, d)$ group is discussed in more detail from the algebraic decomposition of U duality group in [7]². The spinor representation idea was further developed by Fukuma, Oota and Tamaka [8]. It is not the RR potentials that transform as the Majorana-Weyl spinor of $SO(d, d)$; it is the mixed fields of RR potentials and NS-NS two form that transform as the Majorana-Weyl spinor of $SO(d, d)$. However, since the full T-duality group is $O(d, d)$, we expect to use the Majorana spinor representation of $O(d, d)$. Note also that $SO(d, d)$ transformations cannot interchange type IIA and type IIB theories. In this paper, we use RR fields to construct spinors of $O(d, d)$ explicitly. As a simple application we use the Majorana spinor representation to show the RR field transformations between type IIA and type IIB under T-duality. By using the Majorana spinor and the tensor representation of $O(d, d, R)$ group, we can get more general solution generating rules.

We define the RR potentials $C_{p+1} = (1/(p+1)!) C_{\mu_1 \dots \mu_{p+1}} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_{p+1}}$. Following the definition given by [8], we define the new mixed fields as

$$\begin{aligned} D_0 &\equiv C_0, & D_1 &\equiv C_1, \\ D_2 &\equiv C_2 + B_2 \wedge C_0, & D_3 &\equiv C_3 + B_2 \wedge C_1, \\ D_4 &\equiv C_4 + \frac{1}{2} B_2 \wedge C_2 + \frac{1}{2} B_2 \wedge B_2 \wedge C_0. \end{aligned} \quad (1)$$

The RR field strengths are $F = e^{-B} \wedge dD$ [8] [9], here

$$D \equiv \sum_{p=0}^4 D_p, \quad F \equiv \sum_{p=1}^5 F_p. \quad (2)$$

More explicitly, we have

$$\begin{aligned} F_1 &= dD_0, & F_2 &= dD_1, \\ F_3 &= dD_2 - B_2 \wedge dD_0, & F_4 &= dD_3 - B_2 \wedge dD_1, \\ F_5 &= dD_4 - B_2 \wedge dD_2 + \frac{1}{2} B_2 \wedge B_2 \wedge dD_0. \end{aligned} \quad (3)$$

We also use the convention

$$\int d^d x \sqrt{-g} |F_p|^2 = \int d^d x \frac{\sqrt{-g}}{p!} g^{\mu_1 \nu_1} \dots g^{\mu_p \nu_p} F_{\mu_1 \mu_p} F_{\nu_1 \nu_p}. \quad (4)$$

²The author thanks S. Ferrara for pointing out this reference.

2 $d = 10$ Type IIA and Type IIB Reduction to $d = 9$

The action of ten dimensional type IIA supergravity can be written as

$$S_{10}^{IIA} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} e^{-2\Phi} \left[R(G) + 4G^{MN} \partial_M \Phi \partial_N \Phi - \frac{1}{2} |H_3|^2 \right] - \frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-G} (|F_2|^2 + |F_4|^2) + \frac{1}{4\kappa_{10}^2} \int d^{10}x B_2 \wedge dC_3 \wedge dC_3, \quad (5)$$

where $H_3 = dB_2$, $F_2 = dC_1 = dD_1$, $F_4 = dC_3 + H_3 \wedge C_1 = dD_3 - B_2 \wedge dD_1$ and the subscript number of a form denotes the degree of the form. Now we dimensionally reduce the action (5) to nine dimensions by the vielbein,

$$E_M^A = \begin{pmatrix} e_\mu^a & e A_\mu^{(1)} \\ 0 & e \end{pmatrix}, \quad E_A^M = \begin{pmatrix} e_a^\mu & -e_a^\nu A_\nu^{(1)} \\ 0 & e^{-1} \end{pmatrix}. \quad (6)$$

The dimensionally reduced nine dimensional action for the NS and R sector is

$$S_9 = \frac{1}{2\kappa_9^2} \int d^9x \sqrt{-g} e^{-2\phi} \left[R(g) + 4g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - e^{-2} g^{\mu\nu} \partial_\mu e \partial_\nu e - \frac{1}{2} e^2 |F_2^{(1)}|^2 - \frac{1}{2} e^{-2} |F_2^{(2)}|^2 - \frac{1}{2} |H_3^{(1)}|^2 \right] - \frac{1}{4\kappa_9^2} \int d^9x \sqrt{-g} \left(e |F_2|^2 + e^{-1} g^{\mu\nu} \partial_\mu D_x \partial_\nu D_x + e^{-1} |H_3^{(2)}|^2 + e |F_4|^2 \right), \quad (7)$$

where

$$e^2 = G_{xx}, \quad g_{\mu\nu} = G_{\mu\nu} - G_{xx} A_\mu^{(1)} A_\nu^{(1)}, \quad (8a)$$

$$A_\mu^{(1)} = \frac{G_{\mu x}}{G_{xx}}, \quad A_\mu^{(2)} = B_{\mu x} \quad (8b)$$

$$A_\mu = D_\mu - A_\mu^{(1)} D_x, \quad F_{\mu\nu}^i = \partial_\mu A_\nu^{(i)} - \partial_\nu A_\mu^{(i)}, \quad (8c)$$

$$B_{\mu\nu}^{(1)} = B_{\mu\nu} + \frac{1}{2} A_\mu^{(1)} A_\nu^{(2)} - \frac{1}{2} A_\nu^{(1)} A_\mu^{(2)}, \quad B_{\mu\nu}^{(2)} = D_{\mu\nu x}, \quad (8d)$$

$$\phi = \Phi - \ln G_{xx}/4, \quad \mathcal{D}_{\mu\nu\rho} = D_{\mu\nu\rho}, \quad (8e)$$

$$H_3^{(1)} = dB_2^{(1)} - \frac{1}{2} (A_1^{(1)} \wedge F_2^{(2)} + A_1^{(2)} \wedge F_2^{(1)}), \quad (8f)$$

$$H_3^{(2)} = dB_2^{(2)} - B_2^{(1)} \wedge dD_x + \frac{1}{2} A_1^{(2)} \wedge A_1^{(1)} \wedge dD_x - A_1^{(2)} \wedge (F_2 + F_2^{(1)} D_x), \quad (8g)$$

$$F_4 = d\mathcal{D}_3 - B_2^{(1)} \wedge dD_1 + \frac{1}{2} A_1^{(1)} \wedge A_1^{(2)} \wedge dD_1 + H_3^{(2)} \wedge A_1^{(1)}, \quad (8h)$$

and x is the compactified coordinate. Here we follow the general prescription of dimensional reduction given in [10]. For example, the lower dimensional field strength comes from the higher dimensional field strength as $H_{\mu\nu\rho}^{(1)} = e_\mu^a e_\nu^b e_\rho^c E_a^M E_b^N E_c^P H_{MNP}$. The action (7) can be obtained from the type IIB supergravity in ten dimensions also if we use the following vielbein for the IIB theory [1]

$$\mathcal{E}_M^A = \begin{pmatrix} e_\mu^a & e^{-1} A_\mu^{(2)} \\ 0 & e^{-1} \end{pmatrix}, \quad \mathcal{E}_A^M = \begin{pmatrix} e_a^\mu & -e_a^\nu A_\nu^{(2)} \\ 0 & e \end{pmatrix}, \quad (9)$$

together with the following definitions,

$$e^{-2} = \mathcal{G}_{xx}, \quad g_{\mu\nu} = \mathcal{G}_{\mu\nu} - \mathcal{G}_{xx} A_\mu^{(2)} A_\nu^{(2)}, \quad (10a)$$

$$A_\mu^{(1)} = \mathcal{B}_{\mu x}, \quad A_\mu^{(2)} = \frac{\mathcal{G}_{\mu x}}{\mathcal{G}_{xx}}, \quad D = D_x, \quad (10b)$$

$$A_\mu = D_{\mu x} - \mathcal{B}_{\mu x} D = D_{\mu x} - A_\mu^{(1)} D, \quad (10c)$$

$$B_{\mu\nu}^{(1)} = \mathcal{B}_{\mu\nu} - \frac{1}{2} A_\mu^{(1)} A_\nu^{(2)} + \frac{1}{2} A_\nu^{(1)} A_\mu^{(2)}, \quad B_{\mu\nu}^{(2)} = D_{\mu\nu}, \quad (10d)$$

$$\phi = \hat{\Phi} - \ln \mathcal{G}_{xx}/4, \quad \mathcal{D}_{\mu\nu\rho} = D_{\mu\nu\rho x}. \quad (10e)$$

The type IIB ten dimensional supergravity action we use is

$$S_{10}^{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-\mathcal{G}} e^{-2\hat{\Phi}} \left[R(\mathcal{G}) + 4\mathcal{G}^{MN} \partial_M \hat{\Phi} \partial_N \hat{\Phi} - \frac{1}{2} |\mathcal{H}_3|^2 \right] \\ - \frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-\mathcal{G}} \left(|F_1|^2 + |F_3|^2 + \frac{1}{2} |F_5|^2 \right) + \frac{1}{4\kappa_{10}^2} \int d^{10}x \mathcal{B}_2 \wedge dC_4 \wedge dC_2, \quad (11)$$

together with the self dual constraint on F_5 . Now we can get Buscher's T-duality transformations [2] from Eqs. (8a)-(8e) and Eqs. (10a)-(10e) as follows

$$\tilde{g}_{xx} = \frac{1}{g_{xx}}, \quad \tilde{g}_{\mu x} = \frac{B_{\mu x}}{g_{xx}}, \quad \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{g_{\mu x} g_{\nu x} - B_{\mu x} B_{\nu x}}{g_{xx}}, \quad (12a)$$

$$\tilde{B}_{\mu x} = \frac{g_{\mu x}}{g_{xx}}, \quad \tilde{B}_{\mu\nu} = B_{\mu\nu} - \frac{B_{\mu x} g_{\nu x} - B_{\nu x} g_{\mu x}}{g_{xx}}, \quad (12b)$$

$$\tilde{\phi} = \phi - \frac{1}{2} \ln g_{xx}, \quad (12c)$$

$$\tilde{D}_x = D, \quad \tilde{D}_\mu = D_{\mu x}, \quad \tilde{D}_{\mu\nu x} = D_{\mu\nu}, \quad \tilde{D}_{\mu\nu\rho} = D_{\mu\nu\rho x}. \quad (12d)$$

From the above transformation rules (12a)- (12d), we have the following transformations in terms of the original RR potentials,

$$\tilde{C}_x = C, \quad \tilde{C}_\mu = C_{\mu x} + B_{\mu x} C, \quad \tilde{C}_{\mu\nu x} = C_{\mu\nu} + \frac{g_{\mu x} C_{\nu x} - g_{\nu x} C_{\mu x}}{g_{xx}}, \quad (13a)$$

$$\tilde{C}_{\mu\nu\rho} = C_{\mu\nu\rho x} - \frac{3}{2} B_{[\mu\nu} C_{\rho]x} - \frac{3}{2} B_{x[\mu} C_{\nu\rho]} - \frac{6g_{x[\mu} B_{\nu|x]} C_{\rho]x}}{g_{xx}}. \quad (13b)$$

In general we should consider the $O(d, d, R)$ transformations. The group element Ω of $O(d, d, R)$ satisfies

$$\Omega^T J \Omega = J, \quad J = \begin{pmatrix} 0 & \mathbb{1}_d \\ \mathbb{1}_d & 0 \end{pmatrix}. \quad (14)$$

If we put the NS sector fields in a $2d$ by $2d$ matrix

$$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix} = \begin{pmatrix} \mathbb{1} & 0 \\ B & \mathbb{1} \end{pmatrix} \begin{pmatrix} G^{-1} & 0 \\ 0 & G \end{pmatrix} \begin{pmatrix} \mathbb{1} & -B \\ 0 & \mathbb{1} \end{pmatrix}, \quad (15)$$

where $G = [G_{ij}]$ and $B = [B_{ij}]$ are $d \times d$ matrices, i and j run over the compactified or independent d coordinates. Let

$$A_{\mu m}^{(1)} = G_{\mu m}, \quad A_{\mu}^{(1)m} = G^{mn} A_{\mu n}^{(1)}, \quad (16)$$

$$A_{\mu m}^{(2)} = B_{\mu m} + B_{mn} A_{\mu}^{(1)n}, \quad \mathcal{A}_{\mu}^i = \begin{pmatrix} A_{\mu}^{(1)m} \\ A_{\mu m}^{(2)} \end{pmatrix}, \quad (17)$$

$$g_{\mu\nu} = G_{\mu\nu} - G_{mn} A_{\mu}^{(1)m} A_{\nu}^{(1)n}, \quad (18)$$

$$\phi = \Phi - \frac{1}{4} \ln \det(G_{mn}), \quad (19)$$

$$B_{\mu\nu} = \hat{B}_{\mu\nu} + \frac{1}{2} A_{\mu}^{(1)m} A_{\nu m}^{(2)} - \frac{1}{2} A_{\nu}^{(1)m} A_{\mu m}^{(2)} - A_{\mu}^{(1)m} B_{mn} A_{\nu}^{(1)n}, \quad (20)$$

where Φ , $G_{\mu m}$, $G_{\mu\nu}$, G_{mn} , $\hat{B}_{\mu\nu}$, $B_{\mu m}$ and B_{mn} are the original NS fields. The $O(d, d)$ transformations for the NS fields are

$$M \rightarrow \Omega M \Omega^T, \quad \mathcal{A}_{\mu}^i \rightarrow \Omega_{ij} \mathcal{A}_{\mu}^j, \quad g_{\mu\nu} \rightarrow g_{\mu\nu}, \quad \phi \rightarrow \phi, \quad B_{\mu\nu} \rightarrow B_{\mu\nu}. \quad (21)$$

3 Spinor Representation

In this section, we will show that the RR fields transform as the Majorana spinors. We can write the general group element Ω of $O(d, d, R)$ as

$$\Omega = \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix}, \quad (22)$$

with $\mathcal{A}\mathcal{B}^T + \mathcal{B}\mathcal{A}^T = \mathcal{C}\mathcal{D}^T + \mathcal{D}\mathcal{C}^T = 0$, $\mathcal{A}\mathcal{D}^T + \mathcal{B}\mathcal{C}^T = \mathcal{C}\mathcal{B}^T + \mathcal{D}\mathcal{A}^T = 1$, \mathcal{A} , \mathcal{B} , \mathcal{C} and \mathcal{D} are $d \times d$ matrices. We can also show that $\mathcal{D} = \mathcal{C}\mathcal{A}^{-1}\mathcal{B} + (\mathcal{A}^{-1})^T$. The $O(d, d, R)$ group can be generated by the following three matrices [11]

$$\Lambda_C = \begin{pmatrix} \mathbb{1} & 0 \\ C & \mathbb{1} \end{pmatrix}, \quad \Lambda_R = \begin{pmatrix} (R^T)^{-1} & 0 \\ 0 & R \end{pmatrix}, \quad \Lambda_i = \begin{pmatrix} -\mathbb{1} + e_i & e_i \\ e_i & -\mathbb{1} + e_i \end{pmatrix}, \quad (e_i)_{jk} = \delta_{ij}\delta_{jk}, \quad (23)$$

where $C^T = -C$, $R \in GL(d, R)$ and i, j , and $k = 1, \dots, d$. The action of Λ_C shifts the NS two-form by the matrix C . Under the action of Λ_R , $G \rightarrow RGR^T$, $B \rightarrow RBR^T$. For the group $O(d, d, Z)$, we need to restrict the matrix elements to be integers.

The Dirac matrices satisfy $\{\Gamma_r, \Gamma_s\} = 2J_{rs}$ with r and $s = 1, \dots, 2d$. Let

$$a_i = \frac{\Gamma_{d+i}}{\sqrt{2}}, \quad a_i^{\dagger} = \frac{\Gamma_i}{\sqrt{2}}, \quad i = 1, \dots, d. \quad (24)$$

Then we have $\{a_i, a_j^{\dagger}\} = \delta_{ij}\mathbb{1}$, $\{a_i, a_j\} = \{a_i^{\dagger}, a_j^{\dagger}\} = 0$. Define the vacuum to be $a_i|0\rangle = 0$, we can get the representation (Fock) space as

$$|\alpha\rangle = (a_1^{\dagger})^{i_1} \dots (a_d^{\dagger})^{i_d} |0\rangle, \quad i_1, \dots, i_d = 0 \text{ or } 1. \quad (25)$$

The spinor representation of the $O(d, d)$ group is given by

$$S(\Omega)\Gamma_s S(\Omega)^{-1} = \sum_r \Gamma_r \Omega^r{}_s. \quad (26)$$

For convenience we can define the operator corresponding to a matrix Ω as

$$\Omega \Gamma_s = \sum_r \Gamma_r \Omega^r{}_s \Omega, \quad \Omega |\beta\rangle = \sum_\alpha |\alpha\rangle S_{\alpha\beta}(\Omega). \quad (27)$$

The operators for the three generating matrices are [6][8]

$$\Lambda_C = \exp\left(\frac{1}{2} C_{ij} a_i a_j\right), \quad \Lambda_i = \pm(a_i + a_i^\dagger), \quad (28)$$

$$\Lambda_R = (\det R)^{-1/2} \exp\left(a_i A_i{}^j a_j^\dagger\right), \quad R = R_i{}^j = \exp(A_i{}^j), \quad (29)$$

where the repeated indices are summed. We choose + sign for the Λ_i operator. The new mixed D fields form a spinor as follows: for $d = 1$,

$$\begin{aligned} \chi_\alpha &= (D, D_x), & \chi_{\mu\alpha} &= (D_\mu, D_{\mu x}), \\ \chi_{\mu\nu\alpha} &= (D_{\mu\nu}, D_{\mu\nu x}), & \chi_{\mu\nu\rho\alpha} &= (D_{\mu\nu\rho}, D_{\mu\nu\rho x}), \\ & \dots, \end{aligned}$$

with $|\alpha\rangle = (|0\rangle, a^\dagger|0\rangle)$; for $d = 2$,

$$\begin{aligned} \chi_\alpha &= (D, D_x, D_y, D_{yx}), \\ \chi_{\mu\alpha} &= (D_\mu, D_{\mu x}, D_{\mu y}, D_{\mu yx}), \\ \chi_{\mu\nu\alpha} &= (D_{\mu\nu}, D_{\mu\nu x}, D_{\mu\nu y}, D_{\mu\nu yx}), \\ & \dots, \end{aligned}$$

with $|\alpha\rangle = (|0\rangle, a_x^\dagger|0\rangle, a_y^\dagger|0\rangle, a_x^\dagger a_y^\dagger|0\rangle)$ and so on. The fields χ transform as

$$|\tilde{\chi}_{\mu_1 \dots \mu_p \alpha}\rangle = \sum_\beta S^{-1}(\Omega^T)_{\alpha\beta} |\tilde{\chi}_{\mu_1 \dots \mu_p \beta}\rangle. \quad (30)$$

For instance, the spinor representation matrix of $O(1, 1)$ for Λ_i is

$$S((\Lambda^T)^{-1}) = S(\Lambda) = \Lambda = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (31)$$

From the spinor matrix (31), it is easy to get Buscher's T-duality transformations (12a)-(12d), (13a) and (13b) by combining Eqs. (21) and (30). The spinor representation of $SO(1, 1)$ for $\Lambda_i \Lambda_j$ is

$$S(\Lambda^2) = \Lambda^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (32)$$

This is a trivial identity transformation. Furthermore it gives a Majorana-Weyl spinor representation.

4 More Examples

In order to discuss the solution generating transformations, we focus on $O(d) \otimes O(d)$ group in this section. We embed the $O(d)$ matrices R and S into $O(d, d)$ matrix Ω . Because the metric J of $O(d, d)$ is rotated from the diagonal metric η by

$$J = \mathcal{R} \eta \mathcal{R}, \quad \eta = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}, \quad \mathcal{R} = \frac{\sqrt{2}}{2} \begin{pmatrix} -\mathbb{1} & \mathbb{1} \\ \mathbb{1} & \mathbb{1} \end{pmatrix},$$

so

$$\Omega = \mathcal{R}^{-1} \begin{pmatrix} S & 0 \\ 0 & R \end{pmatrix} \mathcal{R} = \frac{1}{2} \begin{pmatrix} R+S & R-S \\ R-S & R+S \end{pmatrix}.$$

Note that Ω is also an element of $O(2d)$, so $(\Omega^T)^{-1} = \Omega$.

For example, if we take $R = -\mathbb{1} + 2e_i$, $S = -\mathbb{1}$, then we recover the T-duality Λ_i discussed before. If we choose [3]

$$S = \mathbb{1}, \quad R = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix},$$

then we have the spinor representation

$$S(\Omega) = \begin{pmatrix} \cos \frac{\theta}{2} & 0 & 0 & -\sin \frac{\theta}{2} \\ 0 & \cos \frac{\theta}{2} & \sin \frac{\theta}{2} & 0 \\ 0 & -\sin \frac{\theta}{2} & \cos \frac{\theta}{2} & 0 \\ \sin \frac{\theta}{2} & 0 & 0 & \cos \frac{\theta}{2} \end{pmatrix}. \quad (33)$$

For flat background with zero B field, RR fields transform the same way as the D fields. This result is consistent with that obtained in [3].

If one of the coordinate is timelike, we have

$$\Omega = \frac{1}{2} \begin{pmatrix} \eta(S+R)\eta & \eta(R-S) \\ (R-S)\eta & S+R \end{pmatrix}, \quad \mathcal{R} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\eta & \mathbb{1} \\ \eta & \mathbb{1} \end{pmatrix}, \quad (34)$$

here η is the Minkowski metric, S and R are $O(d-1, 1)$ matrices satisfying $S\eta S^T = \eta$ and $R\eta R^T = \eta$. For example,

$$R = S = \begin{pmatrix} \cosh \alpha & \sinh \alpha \\ \sinh \alpha & \cosh \alpha \end{pmatrix}$$

generate the boost transformation along t - x coordinates,

$$S^{-1}(\Omega_b^T) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cosh \alpha & \sinh \alpha & 0 \\ 0 & \sinh \alpha & \cosh \alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (35)$$

More explicitly, the boost transformation for the RR field and B field is

$$\tilde{B}_{\mu t} = B_{\mu t} \cosh \alpha + B_{\mu x} \sinh \alpha, \quad \tilde{B}_{\mu x} = B_{\mu t} \sinh \alpha + B_{\mu x} \cosh \alpha, \quad (36a)$$

$$\tilde{C}_{\mu \dots \nu t} = C_{\mu \dots \nu t} \cosh \alpha + C_{\mu \dots \nu x} \sinh \alpha, \quad \tilde{C}_{\mu \dots \nu x} = C_{\mu \dots \nu t} \sinh \alpha + C_{\mu \dots \nu x} \cosh \alpha, \quad (36b)$$

$$\tilde{B}_{tx} = B_{tx}, \quad \tilde{C}_{\mu \dots \nu tx} = C_{\mu \dots \nu tx}, \quad \tilde{B}_{\mu \nu} = B_{\mu \nu}, \quad \tilde{C}_{\mu \dots \nu} = C_{\mu \dots \nu}. \quad (36c)$$

Finally let us choose[5]

$$S = \begin{pmatrix} \cosh \alpha & -\sinh \alpha \\ -\sinh \alpha & \cosh \alpha \end{pmatrix}, \quad R = \begin{pmatrix} \cosh \alpha & \sinh \alpha \\ \sinh \alpha & \cosh \alpha \end{pmatrix}. \quad (37)$$

In this case, we have

$$S^{-1}(\Omega_s^T) = \begin{pmatrix} \cosh \alpha & 0 & 0 & \sinh \alpha \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh \alpha & 0 & 0 & \cosh \alpha \end{pmatrix}. \quad (38)$$

For the background $B_{\mu\nu} = 0$, $g_{11} = 1$ and $g_{01} = 0$, the transformations of NS-NS and RR fields are

$$\tilde{g}_{00} = \frac{g_{00}}{1 + (1 + g_{00}) \sinh^2 \alpha}, \quad (39a)$$

$$\tilde{g}_{11} = \frac{1}{1 + (1 + g_{00}) \sinh^2 \alpha}, \quad (39b)$$

$$\tilde{B}_{01} = \frac{(1 + g_{00}) \sinh 2\alpha}{2[1 + (1 + g_{00}) \sinh^2 \alpha]}, \quad (39c)$$

$$\tilde{g}_{\mu 0} = \frac{g_{\mu 0} \cosh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha}, \quad (39d)$$

$$\tilde{g}_{\mu 1} = \frac{g_{\mu 1} \cosh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha}, \quad (39e)$$

$$\tilde{B}_{\mu 0} = \frac{-g_{00} g_{\mu 1} \sinh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha}, \quad (39f)$$

$$\tilde{B}_{\mu 1} = \frac{g_{\mu 0} \sinh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha}, \quad (39g)$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{(g_{\mu 0} g_{\nu 0} + g_{00} g_{\mu 1} g_{\nu 1}) \sinh^2 \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha}, \quad (39h)$$

$$\tilde{B}_{\mu\nu} = \frac{(g_{\mu 0} g_{\nu 1} - g_{\mu 1} g_{\nu 0}) \sinh \alpha \cosh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha}, \quad (39i)$$

$$\tilde{C} = C \cosh \alpha - C_{01} \sinh \alpha, \quad (39j)$$

$$\tilde{C}_0 = C_0, \quad \tilde{C}_1 = C_1, \quad \tilde{C}_\mu = C_\mu \cosh \alpha - C_{\mu 01} \sinh \alpha, \quad (39k)$$

$$\begin{aligned} \tilde{C}_{01} = & \frac{C_{01}[1 + 2(1 + g_{00}) \sinh^2 \alpha] \cosh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha} \\ & - \frac{C[1 + (1 + g_{00})(\sinh^2 \alpha + \cosh^2 \alpha)] \sinh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha}, \end{aligned} \quad (39l)$$

$$\tilde{C}_{\mu 0} = C_{\mu 0} + \frac{g_{00} g_{\mu 1} \sinh \alpha (C \cosh \alpha - C_{01} \sinh \alpha)}{1 + (1 + g_{00}) \sinh^2 \alpha}, \quad (39m)$$

$$\tilde{C}_{\mu 1} = C_{\mu 1} - \frac{C g_{\mu 0} \sinh \alpha \cosh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha} + \frac{C_{01} g_{\mu 0} \sinh^2 \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha}, \quad (39n)$$

$$\begin{aligned} \tilde{C}_{\mu\nu} = & C_{\mu\nu} \cosh \alpha - C_{\mu\nu 01} \sinh \alpha \\ & + \frac{(C_{01} \sinh \alpha - C \cosh \alpha)(g_{\mu 0} g_{\nu 1} - g_{\mu 1} g_{\nu 0}) \sinh 2\alpha}{2[1 + (1 + g_{00}) \sinh^2 \alpha]}, \end{aligned} \quad (39o)$$

$$e^{-2\tilde{\phi}} = e^{-2\phi} [1 + (1 + g_{00}) \sinh^2 \alpha]. \quad (39p)$$

5 Discussion

The RR field transformations are very simple in terms of the new mixed fields D . It is very easy to see the RR field transformations from the spinor representations. For any group element $\Omega \in O(d, d)$, we can get the spinor representation $S(\Omega)$ from Eq (26) or Eq. (27). We can introduce higher degree potentials and field strengths with some constraints as shown in [8]. With the extra potentials, the action for the RR and Chern-Simons terms can be written in a simple way. This may suggest that the D fields are the natural RR potentials. We can apply the transformation Eqs. (21) for the NS-NS fields and the transformation Eqs. (30) for the RR fields to get more general solution generating rules.

Note Added: In the second version of paper [3], Hassan gave a general transformation of D field by spinor representation and discussed the equivalence of the RR field transformations between his supersymmetry method and the spinor representation.

References

- [1] E. Bergshoeff, C. Hull and T. Ortín, *Duality in The Type II Superstring Effective Action*, hep-th/9504081, Nucl. Phys. B **451**, 547 (1995); E. Bergshoeff, M. De Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, *Duality of Type II 7-branes and 8-branes*, hep-th/9601150, Nucl. Phys. B **470**, 113 (1996); P. Meessen and T. Ortín, *An $Sl(2, Z)$ Multiplet of nine-dimensional Type II Supergravity Theories*, hep-th/9806120, Nucl. Phys. B **541**, 195 (1999).
- [2] T. Buscher, *A Symmetry of the String Background Field Equations*, Phys. Lett. **194B**, 59 (1987).
- [3] S. Hassan, *T-duality, Space-time Spinors and R-R Fields in Curved Backgrounds*, hep-th/9907152; S. Hassan, *$SO(d, d)$ Transformations of Ramond-Ramond Fields and Space-time Spinors*, hep-th/9912236.
- [4] M. Cvetič, H. Lü, C.N. Pope and K.S. Stelle, *T-duality in the Green-Schwarz Formalism and the Massless/Massive IIA Duality Map*, hep-th/9907202.
- [5] G. Gibbons and D. Wiltshire, *Black Holes in Kaluza-Klein Theory*, Ann. Phys. **167**, 201 (1986); **176**, 393(E) (1987); A. Sen, *$O(d) \otimes O(d)$ Symmetry of the Space of Cosmological Solutions in String Theory, Scale Factor Duality, and Two Dimensional Black Holes*, Phys. Lett. **271B**, 295 (1991); *Twisted Black p -brane Solutions in String Theory*, **274B**, 34 (1992); S. Hassan and A. Sen, *Twisted Classical Solutions in Heterotic String Theory*, Nucl. Phys. B **375**, 103 (1992); K. Meissner and G. Veneziano, *Symmetries of Cosmological Superstring Vacua*, Phys. Lett. **267B**, 33 (1991); *Manifestly $O(d, d)$ Invariant Approach to Space-time Dependent String Vacua*, hep-th/9110004, Mod. Phys. Lett. A **6**, 3397 (1991).
- [6] D. Brace, B. Morariu and B. Zumino, *Dualities of the Matrix Model from T-duality of the Type II String*, hep-th/9810099, Nucl. Phys. B **545**, 192 (1999); *T-duality and Ramond-Ramond Backgrounds in the Matrix Model*, hep-th/9811213, Nucl. Phys. B **549**, 181 (1999); E. Witten, *String Theory Dynamics in Various Dimensions*, hep-th/9503124, Nucl. Phys. B **443**, 85 (1995).

- [7] L. Andrianopoli, R. D'Auria, S. Ferrara, P. Fré and M. Trigiante, *R-R Scalars, U-Duality and Solvable Lie Algebras*, hep-th/9611014, Nucl. Phys. B **496**, 617 (1997).
- [8] M. Fukuma, T. Oota and H. Tanaka, *Comments on T-dualities of Ramond-Ramond Potentials*, hep-th/9907132.
- [9] M.B. Green, C.M. Hull and P.K. Townsend, *D-Brane Wess-Zumino Actions, T-duality and The Cosmological Constant*, hep-th/9604119, Phys. Lett. B **382**, 65 (1996).
- [10] J. Maharana and J. Schwarz, *Noncompact Symmetries in String Theory*, hep-th/9207016, Nucl. Phys. B **390**, 3 (1993).
- [11] A. Giveon, M. Porrati and E. Rabinovici, *Target Space Duality in String Theory*, hep-th/9401139, Phys. Rep. **244**, 77 (1994).