Quantum Gravity Solution To The Cosmological Constant Problem

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Abstract

A nonlocal quantum gravity theory is presented which is finite and unitary to all orders of perturbation theory. Vertex form factors in Feynman diagrams involving gravitons suppress graviton and matter vacuum fluctuation loops by introducing a low-energy gravitational scale, $\Lambda_{\rm Gvac} < 2.4\times 10^{-3}$ eV. Gravitons coupled to non-vacuum matter loops and matter tree graphs are controlled by a vertex form factor with the energy scale, $\Lambda_{GM} < 1 - 10$ TeV.

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1 Gravitational Coupling to Vacuum Energy

We can define an effective cosmological constant [1].

$$\lambda_{\rm eff} = \lambda_0 + \lambda_{\rm vac}, \tag{1}$$

where λ_0 is the "bare" cosmological constant in Einstein's classical field equations, and $\lambda_{\rm vac}$ is the contribution that arises from the vacuum density $\lambda_{\rm vac} = 8\pi G\rho_{\rm vac}$.

Already at the standard model electroweak scale $\sim 10^2$ GeV, a calculation of the vacuum density $\rho_{\rm vac}$, based on local quantum field theory, results in a discrepancy of order 10^{55} with the observational bound

$$\rho_{\rm vac} \leq 10^{-47}\,({\rm GeV})^4. \tag{2}$$

This results in a severe fine-tuning problem of order 10^{55}, since the virtual quantum fluctuations giving rise to $\lambda_{\rm vac}$ must cancel λ_0 to an unbelievable degree of accuracy. This is the "particle physics" source of the cosmological constant problem.

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2 Nonlocal Quantum Gravity

Let us consider a model of nonlocal gravity with the action $S = S_g + S_M$, where ($\kappa^2 = 32\pi G$) [2]:

$$S_g = -\frac{2}{\kappa^2}\int d^4x\sqrt{-g}\Big\{R[g,\mathcal{G}^{-1}] + 2\lambda_0\Big\} \tag{3}$$

and S_M is the matter action, which for the simple case of a scalar field ϕ is given by

$$S_M = \frac{1}{2}\int d^4x\sqrt{-g}\mathcal{G}^{-1}\Big(g^{\mu\nu}\nabla_\mu\phi\mathcal{F}^{-1}\nabla_\nu\phi - m^2\phi\mathcal{F}^{-1}\phi\Big). \tag{4}$$

Here, $\mathcal{G}$ and $\mathcal{F}$ are nonlocal regularizing, *entire* functions and ∇_μ is the covariant derivative with respect to the metric $g_{\mu\nu}$. As an example, we can choose the covariant functions $\mathcal{G}(x) = \exp\Big[-\mathcal{D}(x)/\Lambda_G^2\Big]$, and $\mathcal{F}(x) = \exp\Big[-(\mathcal{D}(x)+m^2)/\Lambda_M^2\Big]$, where $\mathcal{D} \equiv \nabla_\mu\nabla^\mu$, and Λ_G and Λ_M are gravitational and matter energy scales, respectively [2, 3].

We expand $g_{\mu\nu}$ about flat Minkowski spacetime: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$. The propagators for the graviton and the ϕ field in a fixed gauge are given by

$$\bar{D}^\phi(p) = \frac{\mathcal{G}(p)\mathcal{F}(p)}{p^2 - m^2 + i\epsilon}, \tag{5}$$

$$\bar{D}^G_{\mu\nu\rho\sigma}(p) = \frac{(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma})\mathcal{G}(p)}{p^2 + i\epsilon}. \tag{6}$$

Unitarity is maintained for the S-matrix, because $\mathcal{G}$ and $\mathcal{F}$ are *entire* functions of p^2, preserving the Cutkosky rules. Gauge invariance can be maintained by satisfying certain constraint equations for $\mathcal{G}$ and $\mathcal{F}$ in every order of perturbation theory. This guarantees that $\nabla_\nu T^{\mu\nu} = 0$.

3 Resolution of the CCP

Due to the equivalence principle *gravity couples to all forms of energy*, including the vacuum energy density $\rho_{\rm vac}$, so we cannot ignore these virtual quantum fluctuations in the presence of a non-zero gravitational field. Quantum corrections to λ_0 come from loops formed from massive standard model (SM) states, coupled to external graviton lines at essentially zero momentum.

[2]The present version of a nonlocal quantum gravity and field theory model differs in detail from earlier published work [2, 3]. A paper is in preparation in which more complete details of the model will be provided.

Consider the dominant contributions to the vacuum density arising from the graviton-standard model loop corrections. We shall adopt a model consisting of a photon loop coupled to gravitons, which will contribute to the vacuum polarization loop cooection to the bare cosmological constant λ_0. The covariant photon action is [4]:

$$S_A = -\frac{1}{4}\sqrt{-g}g^{\mu\nu}g^{\alpha\beta}\mathcal{G}^{-1}F_{\mu\alpha}\mathcal{F}^{-1}F_{\nu\beta}, \tag{7}$$

with $F_{\mu\alpha} = \partial_\mu A_\alpha - \partial_\alpha A_\mu$.

The lowest order correction to the graviton-photon vacuum loop will have the form (in Euclidean momentum space):

$$\Pi^{\rm Gvac}_{\mu\nu\rho\sigma}(p) = \kappa^2 \int \frac{d^4q}{(2\pi)^4} V_{\mu\nu\lambda\alpha}(p, -q, -q-p)$$

$$\times \mathcal{F}^\gamma(q^2) D^\gamma_{\lambda\beta}(q^2) V_{\rho\sigma\beta\gamma}(-p, q, p-q) \mathcal{F}^\gamma((p-q)^2) D^\gamma_{\alpha\gamma}((p-q)^2) \mathcal{G}^{\rm Gvac}(q^2), \tag{8}$$

where $V_{\mu\nu\rho\sigma}$ is the photon-photon-graviton vertex and in a fixed gauge: $D^\gamma_{\mu\nu} = -\delta_{\mu\nu}/q^2$ is the free photon propagator. Additional contributions to $\Pi^{\rm Gvac}_{\mu\nu\rho\sigma}$ come from tadpole graphs [4].

This leads to the vacuum polarization tensor

$$\Pi^{\rm Gvac}_{\mu\nu\rho\sigma}(p) = \kappa^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2[(q-p)^2]}$$

$$\times K_{\mu\nu\rho\sigma}(p, q) \exp\biggl\{-q^2/\Lambda_M^2 - [(q-p)^2]/\Lambda_M^2 - q^2/\Lambda_{\rm Gvac}^2\biggr\}. \tag{9}$$

For $\Lambda_{\rm Gvac} \ll \Lambda_M$, we observe that from power counting of the momenta in the loop integral, we get

$$\Pi^{\rm Gvac}_{\mu\nu\rho\sigma}(p) \sim \kappa^2 \Lambda^4_{\rm Gvac} N_{\mu\nu\rho\sigma}(p^2) \sim \frac{\Lambda^4_{\rm Gvac}}{M^2_{\rm PL}} N_{\mu\nu\rho\sigma}(p^2), \tag{10}$$

where $N(p^2)$ is a finite remaining part of $\Pi^{\rm Gvac}(p)$ and $M_{\rm PL} \sim 10^{19}$ GeV is the Planck mass.

We now have

$$\rho_{\rm vac} \sim M^2_{\rm PL} \Pi^{\rm Gvac}(p) \sim \Lambda^4_{\rm Gvac}. \tag{11}$$

If we choose $\Lambda_{\rm Gvac} \leq 10^{-3}$ eV, then the quantum correction to the bare cosmological constant λ_0 is suppressed sufficiently to satisfy the observational bound on λ, *and it is protected from large unstable radiative corrections.*

This provides a solution to the cosmological constant problem at the energy level of the standard model and possible higher energy extensions of the standard model. The universal fixed gravitational scale $\Lambda_{\rm Gvac}$ corresponds to the fundamental length $\ell_{\rm Gvac} \leq 1$ mm at which virtual gravitational radiative corrections to the vacuum energy are cut off.

The gravitational form factor $\mathcal{G}$, *when coupled to non-vacuum SM gauge boson or matter loops*, will have the form in Euclidean momentum space $\mathcal{G}^{\rm GM}(q^2) = \exp\Bigl[-q^2/\Lambda^2_{GM}\Bigr]$. If we choose $\Lambda_{GM} = \Lambda_M > 1 - 10$ TeV, then we will reproduce the standard model experimental results, including the running of the standard model coupling constants, and $\mathcal{G}^{GM}(q^2) = \mathcal{F}^M(q^2)$ becomes $\mathcal{G}^{GM}(0) = \mathcal{F}^M(q^2 = m^2) = 1$ on the mass shell. *This solution to the CCP leads to a violation of the WEP for coupling of gravitons to vacuum energy and matter.* This could be checked experimentally in a satellite Eötvös experiment on the Casimir vacuum energy.

We observe that the required suppression of the vacuum diagram loop contribution to the cosmological constant, associated with the vacuum energy momentum tensor at lowest order, demands a low gravitational energy scale $\Lambda_{\rm Gvac} \leq 10^{-3}$ eV, which controls the coupling of gravitons to pure vacuum graviton and matter fluctuation loops.

In our finite, perturbative quantum gravity theory nonlocal gravity produces a long-distance infrared cut-off of the vacuum energy density through the low energy scale $\Lambda_{\rm Gvac} < 10^{-3}$ eV [3]. Gravitons coupled to *non-vacuum* matter tree graphs and matter loops are controlled by the energy scale: $\Lambda_{GM} = \Lambda_M > 1 - 20$ TeV

The rule is: When external graviton lines are removed from a matter loop, leaving behind *pure* matter fluctuation vacuum loops, then those initial graviton-vacuum loops are suppressed by the form factor $\mathcal{G}^{\rm Gvac}(q^2)$ where q is the internal matter loop momentum and $\mathcal{G}^{\rm Gvac}(q^2)$ is controlled by $\Lambda_{\rm Gvac} \leq 10^{-3}$ eV. On the other hand, e.g. the proton first-order self-energy graph, coupled to a graviton is controlled by $\Lambda_{GM} = \Lambda_M > 1 - 20$ TeV *and does not lead to a measurable violation of the equivalence principle.*

The scales Λ_M and $\Lambda_{\rm Gvac}$ are determined in loop diagrams by the quantum non-localizable nature of the gravitons and standard model particles. The gravitons coupled to matter and matter loops have a nonlocal scale at $\Lambda_{GM} = \Lambda_M > 1 - 20$ TeV or a length scale $\ell_M < 10^{-16}$ cm, whereas the gravitons coupled to pure vacuum energy are localizable up to an energy scale $\Lambda_{\rm Gvac} \sim 10^{-3}$ eV or down to a length scale $\ell_{\rm Gvac} > 1$ mm.

The fundamental energy scales $\Lambda_{\rm Gvac}$ and $\Lambda_{GM} = \Lambda_M$ are determined by the underlying physical nature of the particles and fields and do not correspond to arbitrary cut-offs, which destroy the gauge invariance, Lorentz invariance and unitarity of the quantum gravity theory for energies $> \Lambda_{\rm Gvac} \sim 10^{-3}$ eV. The underlying explanation of these physical scales must be sought in a more fundamental theory[3]

Let us consider the cosmological problem in the context of our results. For a spatially flat universe, the Friedmann equation is $H^2 \sim 8\pi G\bar{\rho}_{\rm vac}/3$, where $H = \dot{R}/R$, R denoting the cosmic scale and $\bar{\rho}_{\rm vac} = \lambda_0/8\pi G + \rho_{\rm vac}$ is

[3]It is interesting to note that if we choose $\Lambda_{\rm GM} = \Lambda_M = 5$ TeV, then we obtain $\Lambda_{\rm Gvac} = \Lambda_M^2/M_{\rm PL} = 2.1 \times 10^{-3}$ eV.

the effective vacuum energy density, including the bare vacuum energy density contribution, and we have assumed that the vacuum energy dominates the Friedmann equation. If we assume that the vacuum energy density has its "natural" value with a cutoff $\Lambda_c \sim M_{PL}$, then the universe never expands beyond a ridiculously small size. However, since we have suppressed the vacuum energy density contribution to the Friedmann equation by our nonlocal quantum gravity calculation, we will find that $H < H_0 \sim 10^{-33}$ eV in agreement with observations.

Our calculations are based on perturbation theory, so we cannot address possible non-perturbative contributions to the vacuum density, such as those arising from gluon condensates or spontaneous symmetry breaking phase transitions. These contributions will have to be suppressed in a non-perturbative nonlocal quantum gravity calculation.

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References

[1] Weinberg, S., Rev. Mod. Phys. **61**, 1 (1989); Straumann, N., arXive:astro-ph/0203330; Peebles, P. J. E., and Ratra, B., arXive:astro-ph/0207347; Ellwanger, U., arXive:hep-ph/0203252.

[2] Moffat, J. W., Phys. Rev. **D41**, 1177 (1990); Evens, D., Moffat, J. W., Kleppe, G., and Woodard, R. P., Phys. Rev. **D43**, 499 (1991).

[3] Moffat, J. W., arXive:hep-ph/0102088 v2.

[4] Capper, D. M., Leibbrandt, G., Medrano, M. R., Phys. Rev. **D8**, 4320 (1973); Capper, D. M., Duff, M. J., and Halpern, L., Phys. Rev. **10**, 461 (1974).