

N=2 Gauge Theories from Wrapped Five-branes

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Abstract

We present string duals of four dimensional $N = 2$ pure $SU(N)$ SYM theory. The theory is obtained as the low energy limit of D5-branes wrapped on non-trivial two-cycles. Using seven dimensional gauged supergravity and uplifting the result to ten dimensions, we obtain solutions corresponding to various points of the $N = 2$ moduli space. The more symmetric solution may correspond to a point with rotationally invariant classical vevs. By turning on seven dimensional scalar fields, we find a solution corresponding to a linear distribution of vevs. Both solutions are conveniently studied with a D5-probe, which also confirms many of the standard expectations for $N = 2$ solutions.

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1 Introduction

Supergravity duals of non-conformal $N=2$ gauge theories have been recently discussed in the literature from several complementary points of view. They can be obtained as mass deformations of $N = 4$ SYM [1, 2, 3, 4], using fractional branes at orbifold singularities [5, 6, 7, 8, 9, 10, 11] or M5-branes wrapped on Riemann surfaces [12]. In this paper, we analyze the realization of pure $SU(N)$ $N = 2$ gauge theories using wrapped type IIB NS-branes, an approach which proved successful in the study of $N = 1$ gauge theories [13, 14].

Pure $SU(N)$ $N = 2$ SYM can be realized as the low energy theory on D5-branes wrapped on a non-trivial cycle of an ALE space¹. We will also consider the S-dual configuration with NS5-branes. Differently from $AdS_5 \times S^5$ deformations and systems with fractional branes, at high energy the $N = 2$ theory is embedded in the six dimensional theory living on the NS5. Such six dimensional theory decouples from gravity when the string coupling is sent to zero, it is not conformal and not even local [16], but admit an holographic description in terms of a linear dilaton background [17]. We will find $N = 2$ solutions of the appropriate seven dimensional gauged supergravity which are asymptotic (in the ultraviolet) to the linear dilaton background and we will uplift them to ten dimensions. The resulting solutions have a complex one dimensional moduli space for the motion of a D5-probe, as expected for the dual of the $SU(N)$ $N = 2$ SYM theory. We will find symmetric solutions corresponding to classical vevs distributed in a rotationally invariant way. A particular solution in this family may correspond to the strongly coupled theory with zero (or smaller than the dynamically generated scale) classical vevs. We will also find solutions corresponding to linear distributions of vevs. The moduli space structure will be studied using a D5-probe, which, as usual, captures the quantum field theory one-loop result. The supergravity solutions we find are all singular. In $N = 2$ theories, an enhançon mechanism [18] is usually invoked for resolving the singularity. Some of the features usually associated with the enhançon mechanism are at work here.

In Section 2 we discuss our approach, which uses seven dimensional gauged supergravity. Since only the bosonic equations of motion of the relevant theory are known [19], we will perform the singular limit described in [20] on the fermionic shifts of the maximally gauged supergravity. Evidence of the $N = 2$ supersymmetry of the solution will be given by the probe analysis. In Section 3 we will consider the uplifting to ten dimensions of the most symmetric solution. By using a D5-probe, we will identify this solution as the point in moduli space of $N = 2$ SYM with zero classical vevs. In Section 4 we will turn on scalar fields corresponding to the chiral operators parameterizing the Coulomb branch. With the probe analysis, we will identify this solution as corresponding

¹Another approach to supergravity solutions for wrapped D5-branes can be found in [15].

to a linear distribution of vevs in the gauge theory.

While this work was being written, a paper [21] appeared where the solution in Section 2-3 was independently discussed.

2 Twisting the NS5 brane

In order to obtain $N = 2$ supersymmetry in four dimensions, we wrap N NS5 branes over an S^2 of an ALE space, and then twist the normal bundle as in [22]. The ten dimensional spacetime is locally of the form $\mathbf{R}^4 \times S^2 \times \mathbf{R}^2 \times \mathbf{R}^2$, where the first \mathbf{R}^2 is part of the ALE space and the second one is in the transverse flat space. These give an $U(1) \times U(1)$ normal bundle. Working with seven dimensional gauged supergravity we can perform the twist by identifying the gauge fields in the theory with the spin connection on the sphere as in [13, 14]. We thus choose the $U(1) \times U(1)$ truncation [23] of the $SO(5)$ seven dimensional gauged supergravity [24]. As in [13], the right choice to retain $N = 2$ supersymmetry is to take one of the abelian gauge fields equal to the spin connection on the sphere, setting the other one to zero.

Our ansatz for the string frame metric in seven dimensions is:

$$ds_7^2 = dx_4^2 + N[d\rho^2 + e^{2h}(d\theta^2 + \sin^2 \theta d\varphi^2)], \quad (1)$$

which in the Einstein frame reads:

$$ds_7^2 = e^{2f}(dx_4^2 + d\rho^2) + e^{2g}(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (2)$$

with $f = -\frac{2}{5}\Phi_7$, $g = -\frac{2}{5}\Phi_7 + h$ (Φ_7 is the seven dimensional dilaton). We also chose $N=1$ for simplicity. We look for solutions of the equations of motion of $U(1) \times U(1)$ gauged supergravity that also preserve $N = 2$ supersymmetry. The theory at hand contains, apart from the metric, two scalars, two abelian gauge fields, a tree-form potential (which we take equal to zero in the following) and the corresponding fermions. The $U(1) \times U(1)$ truncation was used in [13] to find $N = 2$ M-theory solutions interpolating between AdS_7 and AdS_5 , corresponding to wrapped M5-branes. In order to study NS5 branes in Type IIB, we need to perform a singular limit in the theory, which reduces M theory to type II, as discussed in [20]. The bosonic part of the Lagrangian thus becomes:

$$2\kappa^2 e^{-1} L = R + 4m^2 e^{(2\lambda_1 + 2\lambda_2)} - 5\partial_\mu(\lambda_1 + \lambda_2)^2 - \partial_\mu(\lambda_1 - \lambda_2)^2 - e^{-4\lambda_1} F_{\mu\nu}^{(1)2} - e^{-4\lambda_2} F_{\mu\nu}^{(2)2}. \quad (3)$$

While the maximally gauged supergravity (M theory compactified on S^4) has AdS vacua, corresponding to the $(2, 0)$ CFT, the new theory (type II on S^3) has only run-away vacua. There is instead a solution corresponding to NS5 branes. The same singular limit on the

supersymmetry variations for the fermions gives ($k = 2m$ is the gauge coupling)²:

$$\begin{aligned}
\delta\psi_\mu &= [\nabla_\mu + \frac{k}{2}(A_\mu^{(1)}\Gamma^{12} + A_\mu^{(2)}\Gamma^{34}) + \frac{1}{2}\gamma_\mu\gamma^\nu\partial_\nu(\lambda_1 + \lambda_2) + \\
&\quad + \frac{1}{2}\gamma^\nu(e^{-2\lambda_1}F_{\mu\nu}^{(1)}\Gamma^{12} + e^{-2\lambda_2}F_{\mu\nu}^{(2)}\Gamma^{34})]\epsilon, \\
\delta\lambda^{(1)} &= [\frac{m}{4}e^{2\lambda_1} - \frac{1}{4}\gamma^\mu\partial_\mu(3\lambda_1 + 2\lambda_2) - \frac{1}{8}\gamma^{\mu\nu}e^{-2\lambda_1}F_{\mu\nu}^{(1)}\Gamma^{12}]\epsilon, \\
\delta\lambda^{(2)} &= [\frac{m}{4}e^{2\lambda_2} - \frac{1}{4}\gamma^\mu\partial_\mu(2\lambda_1 + 3\lambda_2) - \frac{1}{8}\gamma^{\mu\nu}e^{-2\lambda_2}F_{\mu\nu}^{(2)}\Gamma^{34}]\epsilon.
\end{aligned} \tag{4}$$

From (2) it follows that the non trivial components of the spin connection are:

$$\omega_{\hat{\alpha}}^{\alpha\rho} = f', \quad \omega_{\hat{\theta}}^{\theta\rho} = g'e^{g-f}, \quad \omega_{\hat{\phi}}^{\varphi\rho} = g'e^{g-f}\sin\theta, \quad \omega_{\hat{\phi}}^{\varphi\theta} = \cos\theta, \tag{5}$$

where $\alpha = 0, 1, 2, 3$ labels the four dimensional coordinates in (2), and the hats distinguish the curved coordinates from the flat ones. We impose the ansatz:

$$\gamma_\rho\epsilon = -\epsilon, \quad \gamma_{\varphi\theta}\epsilon = i\epsilon, \quad \Gamma^{12}\epsilon = i\epsilon, \quad \Gamma^{34}\epsilon = i\epsilon, \quad \partial_{\alpha,\theta,\varphi}\epsilon = 0. \tag{6}$$

As explained above, we take $A^{(1)} = -\frac{1}{k}\cos\theta d\varphi$, $A^{(2)} = 0$, so that inserting (2) in (4) we obtain (equating the variations (4) to zero):

$$\begin{aligned}
f' &= -(\lambda_1' + \lambda_2'), \\
g' &= -(\lambda_1' + \lambda_2') + \frac{1}{k}e^{f-2g-2\lambda_1}, \\
3\lambda_2' + 2\lambda_1' &= -me^{f+2\lambda_2}, \\
3\lambda_1' + 2\lambda_2' &= -me^{f+2\lambda_1} + \frac{1}{k}e^{f-2g-2\lambda_1},
\end{aligned} \tag{7}$$

whose solutions are:

$$\begin{aligned}
f &= -(\lambda_2 + \lambda_1), \\
e^{2g-2f} &= u, \\
e^{\lambda_2-\lambda_1} &= \sqrt{1 - \frac{1}{2u} + \frac{2Ke^{-2u}}{u}}, \\
e^{\lambda_2+\lambda_1} &= e^{-\frac{2}{5}u} \left[1 - \frac{1}{2u} + \frac{2Ke^{-2u}}{u} \right]^{-\frac{1}{10}}
\end{aligned} \tag{8}$$

with:

$$\frac{du}{d\rho} \equiv e^{\lambda_2-\lambda_1}. \tag{9}$$

²It is possible to verify that the same ansatz gives, in the $N = 1$ case, the results of [14].

We have chosen the integration constants for e^{-2g-2f} and f in order that u ranges in the interval $[0, \infty)$ and the seven dimensional dilaton has the canonical NS5 asymptotic behaviour (for $\rho \rightarrow \infty$) $\phi \approx -\rho$. We also fixed to one the integration constant for $e^{\lambda_2+\lambda_1}$ which has no physical relevance, appearing as an overall factor. We will mainly consider the case $K \geq \frac{1}{4}$ where $u \in [0, \infty)$. For solutions with $K < \frac{1}{4}$, u can never reach zero.

We explicitly checked that the second order equations in [19] are satisfied. We will have further evidence about the supersymmetry of the solution in the next section, when we will find a moduli space for D5 probes.

3 Ten dimensional solution

We refer to [20] to lift the previous solution to ten dimensions. The string frame metric is (ds_7^2 is given by (1) with $N=1$):

$$ds^2 = ds_7^2 + \frac{1}{m^2} e^{2\lambda_2+2\lambda_1} \Delta^{-1} \left[e^{-2\lambda_1} [d\tilde{\mu}_1^2 + \tilde{\mu}_1^2 (d\phi_1 + \cos \theta d\varphi)^2] + e^{-2\lambda_2} [d\tilde{\mu}_2^2 + \tilde{\mu}_2^2 d\phi_2^2] \right] \quad (10)$$

and the dilaton reads:

$$e^{2\Phi} = e^{6\lambda_2+6\lambda_1} \Delta^{-1}, \quad (11)$$

with

$$\Delta = e^{2\lambda_1} \tilde{\mu}_1^2 + e^{2\lambda_2} \tilde{\mu}_2^2 \quad (12)$$

and $\phi_{1,2}, \tilde{\mu}_{1,2} = (\sin \theta', \cos \theta')$ being angular coordinates of the transverse three-sphere.

The solution incorporates also a potential six form whose field strength is given by:

$$\begin{aligned} e^{-2\Phi} * F_3 &= 2m e^{-5\lambda_2-5\lambda_1} \epsilon_{(\tau)} + \frac{e^{-5\lambda_2-5\lambda_1}}{2m} \sum_{i=1}^2 e^{-2\lambda_i} *_{(\tau)} d e^{2\lambda_i} \wedge d(\tilde{\mu}_i^2) \\ &\quad - \frac{e^{-3\lambda_2-3\lambda_1}}{2m^3} e^{-4\lambda_1} d(\tilde{\mu}_1^2) \wedge (d\phi_1 + \cos \theta d\varphi) \wedge *_{(\tau)} (\sin \theta d\theta \wedge d\varphi). \end{aligned} \quad (13)$$

The D5 solution may be obtained from the one above by performing the S-duality transformations:

$$\begin{aligned} \Phi_D &= -\Phi, \\ ds_D^2 &= e^{\Phi_D} ds_{NS}^2, \\ dC_6 &= *F_3 = e^{-2\Phi} *_{NS} F_3. \end{aligned} \quad (14)$$

We can now examine the asymptotic behaviours of the metric and the dilaton making use of the explicit solutions (8). In NS variables, we expect a solution UV asymptotic

to the linear dilaton background [17] with a size of S^2 that grows reflecting the coupling constant running. Alternatively, in D5 variables, when $u \rightarrow \infty$ ($\rho \rightarrow \infty$) we get:

$$\begin{aligned} ds_D^2 &\approx e^u \left[dx_4^2 + du^2 + u(d\theta^2 + \sin^2 \theta d\varphi^2) + \right. \\ &\quad \left. + \frac{1}{m^2} [d\tilde{\mu}_1^2 + \tilde{\mu}_1^2 (d\phi_1 + \cos \theta d\varphi)^2 d\tilde{\mu}_2^2 + \tilde{\mu}_2^2 d\phi_2^2] \right], \\ e^{\Phi_D} &\approx e^u. \end{aligned} \quad (15)$$

As expected the dilaton diverges and the S^2 blows up.

The details of the $u \rightarrow 0$ limit³ drastically depends on the value of the integration constant K in (8). When $K > \frac{1}{4}$ we find:

$$\begin{aligned} ds_D^2 &\approx u^{-\frac{1}{2}} |\tilde{\mu}_2| \left[dx_4^2 + u du^2 + u(d\theta^2 + \sin^2 \theta d\varphi^2) + \right. \\ &\quad \left. + \frac{\tilde{\mu}_2^2}{m^2} [d\tilde{\mu}_1^2 + \tilde{\mu}_1^2 (d\phi_1 + \cos \theta d\varphi)^2 + u(d\tilde{\mu}_2^2 + \tilde{\mu}_2^2 d\phi_2^2)] \right], \\ e^{\Phi_D} &\approx u^{-\frac{1}{2}} |\tilde{\mu}_2|. \end{aligned} \quad (16)$$

The metric has a bad type singularity according to the criteria of [13] and the dilaton is diverging, so we discard this possibility. When $K = \frac{1}{4}$ we find instead:

$$\begin{aligned} ds_D^2 &\approx |\tilde{\mu}_1| \left[dx_4^2 + \frac{1}{u} du^2 + u(d\theta^2 + \sin^2 \theta d\varphi^2) + \right. \\ &\quad \left. + \frac{1}{|\tilde{\mu}_1| m^2} [u d\tilde{\mu}_1^2 + u \tilde{\mu}_1^2 (d\phi_1 + \cos \theta d\varphi)^2 + d\tilde{\mu}_2^2 + \tilde{\mu}_2^2 d\phi_2^2] \right], \\ e^{\Phi_D} &\approx |\tilde{\mu}_1|. \end{aligned} \quad (17)$$

The singularity (which is located at $u = 0$, or $\tilde{\mu}_1 = 0$) of this metric is milder, so that we can retain this solution as a dual of $N = 2$ SYM.

We can explore the nature of the moduli space of the gauge theory using a probe D5 brane wrapped on S^2 , whose low energy effective action is (in units $2\pi\alpha' = \tau_5 = 1$):

$$S = - \int d^6 \xi e^{-\Phi_D} \sqrt{-\det(G + F)} + \int C_6 + \frac{1}{2} \int C_2 \wedge F \wedge F, \quad (18)$$

where

$$G_{\alpha\beta} = \partial_\alpha x^M \partial_\beta x^N g_{MN} \quad (19)$$

is the induced metric on the worldvolume ($\alpha, \beta = 0, 1, \dots, 5$ label the worldvolume coordinates, while $M, N = 0, 1, \dots, 9$), and F is the gauge field strength on the brane.

³This corresponds to the IR $\rho \rightarrow 0$ region if we take the integration constant in (9) equal to zero.

We now perform our calculations in static gauge choosing $\xi^0 = x^0 \equiv t, \xi^i = x^i, i = 1, \dots, 5, x^m = x^m(t), m = 6, \dots, 9$ and taking the low velocity limit. From (13) and (14) it follows that, for $\theta' = 0$:

$$\int C_6 = 4\pi V_3 \int dt (ue^{2u} - \frac{1}{2}e^{2u} + \text{const}), \quad (20)$$

which does not depend on the angular coordinates. This term contributes to the effective potential for the probe. The other contribution comes from the Dirac-Born-Infeld part of the action. The BPS configurations correspond to having zero potential. If we search for solutions with unfixed radial coordinate u , the potential vanishes when $\tilde{\mu}_1 \equiv \sin \theta' = 0$. In this case the low velocity limit of the DBI part of the action gives (taking $F = 0$ on S^2):

$$S_{DBI} = -4\pi V_3 \int dtu \left[g_{00}^2 + \frac{1}{2}g_{00}g_{mn}\dot{x}^m\dot{x}^n - \frac{1}{4}F^2 \right]. \quad (21)$$

The first contribution in the integral cancels exactly with $\int C_6$, after choosing the constant in (20) equal to $2K$. This cancellation is independent of the particular choice of K , and can be proved using only the first order equations of motion. The kinetic term in (21) reads:

$$L_{kin} = -2\pi V_3 u (e^{2u}\dot{u}^2 + e^{2u}\dot{\phi}_2^2 - \frac{1}{2}F^2) \quad (22)$$

which, introducing $r = e^u$, may be written as:

$$L_{kin} = -2\pi V_3 \log r (\dot{r}^2 + r^2\dot{\phi}_2^2 - \frac{1}{2}F^2). \quad (23)$$

This gives a complex one-dimensional moduli space as expected for the $N = 2$ gauge theory, which can be parameterized by the complex coordinate $z = re^{i\phi_2}$. We can explicitly write the holomorphic coupling after the calculation of the coefficient of $F \wedge F$ from the third term in (18), which equals $-2\pi\phi_2$.

After making explicit the dependence on the number N of D5 branes we find (apart from numerical factors) for the gauge kinetic term:

$$\mathcal{Im}(\tau(z))F^2 + \mathcal{Re}(\tau(z))F\tilde{F}; \quad \tau(z) = \frac{Ni}{\pi} \log \frac{z}{\Lambda}; \quad \Lambda \sim \sqrt{N}, \quad (24)$$

while the scalar kinetic term reduces to:

$$\mathcal{Im}(\tau(z))\partial z \partial \bar{z}. \quad (25)$$

We have thus found the structure for the moduli space expected for the $N = 2$ supersymmetric four dimensional YM theory. The supergravity description captures correctly all the perturbative contributions to the coupling. Formula (24) is compatible with points in moduli space where all the classical vevs are zero or distributed in $U(1)_R$ invariant

configurations. K may distinguish in between these cases. Results in [21] suggest that the radius of the distribution is bigger than Λ for $K < 1/4$. This is compatible with a probe able to move in the region inside the distribution of branes, as found in [21]. It is tempting to associate the fine tuned value $K = 1/4$ with zero (or smaller than Λ) classical vevs. For such a strongly coupled vacuum, an enhançon mechanism [18] could be expected: even if classically at the origin, quantum mechanically the branes dispose in a spherical shell of radius Λ . It is difficult to make this more precise because the quantum field theory region of moduli space below the radius Λ is hardly seen in the solution. However, many features usually associated with the enhançon mechanism are manifest in the solution with $K = 1/4$: at $u = 0$ ($z = \Lambda$) the probe become tensionless and extra bulk fields become massless (D3-branes wrapped on the two-sphere, for example). The precise form of the singularity (and its resolution) deserves further investigation.

4 A more interesting example

We can also turn on other scalar fields in the seven dimensional gauged supergravity. The scalar fields parameterizing the sphere reduction are expressed in terms of a symmetric $SO(4)$ tensor $T_{i,j}$, $i = 1, \dots, 4$. In the previous Sections we retained the $U(1) \times U(1)$ singlets $T_{11} = T_{22} = e^{2\lambda_1}$ and $T_{33} = T_{44} = e^{2\lambda_2}$. The scalar fields T_{ij} , $i, j = 3, 4$ parameterize the motion of the NS-branes in the untwisted \mathbf{R}^2 plane, which preserves $N = 2$ supersymmetry. They are dual to the bilinear scalar operators of the $N = 2$ gauge theory. We expect that, as in similar AdS_5 examples [25, 1], we can find solutions corresponding to non-trivial points in the Coulomb branch of the $N = 2$ theory. Up to a gauge rotation, we can take $T_{11} = T_{22} = e^{2\lambda_1}$, $T_{33} = e^{2\lambda_2}$, $T_{44} = e^{2\tilde{\lambda}_2}$. This choice explicitly breaks $U(1)_{(2)}$. The equations of motion can be consistently truncated to these three scalar fields. We look for $N = 2$ solutions. We now need to consider the full $SO(5)$ gauged supergravity and perform the singular limit described in [20], in order to descend from M theory to Type II. That this is a sensible procedure will be guaranteed by the fact that the BPS first order equations satisfy the second order equations [19] for type II compactifications. The BPS equations are now

$$\begin{aligned}
f' &= -(\lambda_1' + \frac{\lambda_2' + \tilde{\lambda}_2'}{2}), \\
g' &= -(\lambda_1' + \frac{\lambda_2' + \tilde{\lambda}_2'}{2}) + \frac{1}{k}e^{f-2g-2\lambda_1}, \\
\lambda_2' + 2\tilde{\lambda}_2' + 2\lambda_1' &= -me^{f+2\tilde{\lambda}_2}, \\
2\lambda_2' + \tilde{\lambda}_2' + 2\lambda_1' &= -me^{f+2\lambda_2}, \\
3\lambda_1' + \lambda_2' + \tilde{\lambda}_2' &= -me^{f+2\lambda_1} + \frac{1}{k}e^{f-2g-2\lambda_1},
\end{aligned} \tag{26}$$

which can be solved as in previous case. The relations $e^{2h} = u, du/d\rho = e^{(\lambda_2 + \tilde{\lambda}_2)/2 - \lambda_1}$ still hold and moreover we have the important relation

$$e^{\lambda_2 - \tilde{\lambda}_2} = \frac{e^{2u} - b^2}{e^{2u} + b^2}. \quad (27)$$

For $b = 0$ we recover the previously discussed solution with $\lambda_2 = \tilde{\lambda}_2$.

The up-lifting to ten dimensions can be performed using formulae in [19]. The solution for wrapped D5 branes is:

$$ds_D^2 = \Delta^{\frac{1}{2}} e^{-3\lambda_1 - 3(\lambda_2 + \tilde{\lambda}_2)/2} \left\{ ds_7^2 + \frac{e^{2\lambda_1 + \lambda_2 + \tilde{\lambda}_2}}{m^2 \Delta} \left[e^{-2\lambda_1} [d\mu_1^2 + d\mu_2^2 + \cos^2 \theta (\mu_1^2 + \mu_2^2) d\varphi^2 - 2 \cos \theta (\mu_1 d\mu_2 + \mu_2 d\mu_1) d\varphi] + e^{-2\lambda_2} d\mu_3^2 + e^{-2\tilde{\lambda}_2} d\mu_4^2 \right] \right\} \quad (28)$$

where $\sum_1^4 \mu_i^2 = 1$ parameterize S^3 . The dilaton reads:

$$e^{-2\Phi_D} = e^{3\lambda_2 + 3\tilde{\lambda}_2 + 6\lambda_1} \Delta^{-1}, \quad (29)$$

with

$$\Delta = e^{2\lambda_1} (\mu_1^2 + \mu_2^2) + e^{2\lambda_2} \mu_3^2 + e^{2\tilde{\lambda}_2} \mu_4^2. \quad (30)$$

The complete formula for $F_{(3)}$ can be found in [19]. We can choose $\mu_{1,2} = \sin \theta' (\cos \phi_1, \sin \phi_1)$ and $\mu_{3,4} = \cos \theta' (\cos \phi_2, \sin \phi_2)$.

The probe computation goes over as before. We find that, for $\theta' = 0$,

$$V_{DBI} = C_{(6)} \sim \cos^2 \phi_2 e^{6f+2h+2\lambda_2} + \sin^2 \phi_2 e^{6f+2h+2\tilde{\lambda}_2} \quad (31)$$

so that there is a complex one-dimensional moduli space, which can be parameterized as before by $z = r e^{i\phi_2}, u = \log r$. In this Section we put, for simplicity, $\Lambda = 1$. The gauge field kinetic term is unchanged:

$$\mathcal{I}m(\tau(z)) F^2 + \mathcal{R}e(\tau(z)) F \tilde{F}; \quad \tau(z) = \frac{Ni}{\pi} \log z, \quad (32)$$

while the scalar kinetic term, using formulae (26), (27), reads:

$$\frac{\log |z|}{|z|^2} \left[\cos^2 \phi_2 e^{-2(2\lambda_1 + \lambda_2 + 2\tilde{\lambda}_2)} + \sin^2 \phi_2 e^{-2(2\lambda_1 + 2\lambda_2 + \tilde{\lambda}_2)} \right] dz d\bar{z} = \log |z| \left[\cos^2 \phi_2 \left(1 - \frac{b^2}{r^2} \right)^2 + \sin^2 \phi_2 \left(1 + \frac{b^2}{r^2} \right)^2 \right] dz d\bar{z}. \quad (33)$$

In the standard coordinates for the $N = 2$ effective Lagrangian, the scalar and gauge kinetic term coincide. We can obtain this by an holomorphic change of coordinates $w = z + b^2/z$. The probe coupling constant now reads:

$$\tau(w) = \frac{Ni}{\pi} \left(\text{arcosh}\left(\frac{w}{2b}\right) + \text{const} \right). \quad (34)$$

For large $|w|$, $\tau(w)$ has the standard logarithmic behaviour, while for small $|w|$ it reveals a non-trivial distribution of vevs in the gauge theory. We can explicitly compute such distribution, by approximating it with a continuum:

$$\tau(w) = \frac{i}{\pi} \sum_i \log(w - a_i) \sim i \int_{-2b}^{2b} da \mu(a) \log(w - a), \quad (35)$$

with $\mu(a) = N/(\pi\sqrt{4b^2 - a^2})$. We see that our solution represents a point in the Coulomb branch where the vevs are linearly distributed. This is somehow reminiscent of [3] with the obvious difference that the theory in [3] is a mass deformation of $N = 4$, while our starting point is (a little string UV completion of) pure $N = 2$ without matter. Curiously, this linear density of vevs is of the same type which appears for the $N = 2$ point in moduli space where all types of monopoles become massless [26]. The relation of our solution with such a particular point and with the possible soft breaking to $N = 1$ deserves further investigation.

We gave several indications that our solutions are actually $N = 2$. It would be interesting to check the supersymmetry directly in ten dimensions. We understand that this was done in [21] for the solution in Section 2 and 3.

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