The Effective Average Action Beyond First Order

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A derivative expansion of the effective average action beyond first order yields renormalization group functional flow equations which are used for the computation of critical exponents of the Ising universality class. The critical exponent ν in $D = 3$ is consistent with high-precision methods.

The effective average action

The effective average action $\Gamma_k[\varphi]$ (see [1] and [2–5]) is a functional of the fields φ. It interpolates between the classical action S at some microscopic ultraviolet scale Λ and the effective action Γ: $\Gamma_\Lambda = S$, $\lim_{k\to 0}\Gamma_k = \Gamma$. Its dependence on the momentum scale k is described by an exact renormalization group functional flow equation:

$$\partial_k\Gamma_k = \frac{1}{2}\mathrm{Tr}\{(\Gamma_k^{(2)} + R_k)^{-1}\partial_k R_k\}$$

where $\Gamma_k^{(2)}$ is the second functional derivative of Γ_k with respect to the fields and R_k denotes a momentum cutoff function with the properties $\lim_{k\to 0} R_k = 0$, $\lim_{k\to\Lambda} R_k \to \infty$ and $\lim_{q^2\to 0} R_k(q^2) > 0$ ensuring the above limit properties of Γ_k. Universal properties of the renormalization group flow, however, are independent of the actual choice of R_k.

The flow of $\Gamma_k^{(2)}$ and higher n-point-functions can be derived from the flow of Γ_k by functional derivation, e.g.

$$\begin{aligned}\partial_k\Gamma_k^{(2)} &= \mathrm{Tr}\{\Gamma_k^{(3)2}(\Gamma_k^{(2)} + R_k)^{-3}\partial_k R_k\}\\ &\quad - \frac{1}{2}\mathrm{Tr}\{\Gamma_k^{(4)}(\Gamma_k^{(2)} + R_k)^{-2}\partial_k R_k\}\end{aligned}$$

where the flow of $\Gamma_k^{(3)}$ would involve $\Gamma_k^{(4)}$ and $\Gamma_k^{(5)}$ and so on. In order to end up with a closed system of functional flow equations one needs to do a finite truncation of Γ_k.

In the following we concentrate on a one-component real scalar field $\varphi : \mathbb{R}^D \to \mathbb{R}$. There are the invariants $\rho = \frac{1}{2}\varphi\varphi$ and $\rho q^2 = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi$ featuring the $O(1)$ symmetry of the Ising universality class. Here the lowest order truncation of the effective average action reads

$$\Gamma_k = \int d^Dx\{U_k(\rho) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi Z_k + O(\partial^2)\}$$

in terms of the effective average potential $U_k(\rho)$ and a field independent wave function renormalization Z_k. The effective average propagator

$$\Gamma_k^{(2)}(q^2) + R_k(q^2) = U_k'(\rho) + 2\rho U_k''(\rho) + q^2 Z_k + R_k(q^2)$$

gives the flow of the effective average potential

$$\partial_k U_k(\rho) = \frac{1}{2}\int\frac{d^Dq}{(2\pi)^D}\frac{\partial_k R_k}{U_k'(\rho) + 2\rho U_k''(\rho) + q^2 Z_k + R_k(q^2)}$$

and $\partial_k Z_k$ can be determined from $\partial_{q^2}\partial_k\Gamma_k^{(2)}(q^2)\mid_{q^2=0}$.

Derivative expansion beyond first order

In a second order derivative expansion of the effective average action there are contributions from up to four nonvanishing momenta. These are parametrized by the effective average potential and one respectively three linearly independent wave function renormalizations in first respectively second order:

$$\begin{aligned}\Gamma_k = \int d^Dx\{&U_k(\rho)\\ &+ Z_{A,k}(\rho)\cdot\frac{1}{2!}\partial_\mu\varphi\partial^\mu\varphi\\ &+ Z_{B,k}(\rho)\cdot\frac{1}{4!}\partial_\mu\varphi\partial^\mu\varphi\partial_\nu\varphi\partial^\nu\varphi\\ &+ Z_{C,k}(\rho)\cdot\frac{1}{4!}\varphi\partial_\mu\partial^\mu\varphi\partial_\nu\varphi\partial^\nu\varphi\\ &+ Z_{D,k}(\rho)\cdot\frac{1}{4!}\varphi\varphi\partial_\mu\partial^\mu\varphi\partial_\nu\partial^\nu\varphi + O(\partial^6)\}\end{aligned}$$

This truncation has been the starting point for a recent investigation in [6]. There an additional field expansion of the potential and the wave function renormalizations in powers of ρ is done. Here we keep the full field dependence but instead go only one step beyond first order and take only U_k, $Z_{A,k}$ and $Z_{D,k}$ into account due to the following reasoning:

While $Z_{A,k}$ and $Z_{D,k}$ directly contribute to $\Gamma_k^{(2)}$ and thus enter all flow equations as q^2 and q^4 terms in the denominators of the integrands, $Z_{B,k}$ and $Z_{C,k}$ only give additive contributions to the four-point-vertex and the three-point-vertex respectively. Also, their flow requires $\partial_k\Gamma_k^{(4)}$ and $\partial_k\Gamma_k^{(3)}$. This is why we neglect $Z_{B,k}$ and $Z_{C,k}$ in this paper which corresponds to a truncation beyond first and below second order.

We hope that this improves the critical exponent ν compared to first order calculations. On the other hand, the anomalous dimension η is related to the momentum dependence of the wave function renormalization and will probably suffer from the missing contributions.

Substituting $U := U_k$, $Z_0 := Z_{A,k}$ and $Z_1 := \frac{2!}{4!}\varphi\varphi Z_{D,k}$ the truncation thus reads:

$$\begin{aligned}\Gamma_k = \int d^Dx\{&U(\rho)\\ &+ Z_0(\rho)\cdot\frac{1}{2!}\partial_\mu\varphi\partial^\mu\varphi\\ &+ Z_1(\rho)\cdot\frac{1}{2!}\partial_\mu\partial^\mu\varphi\partial_\nu\partial^\nu\varphi + O(\partial^4)\}\end{aligned}$$

and the next step is to derive $\partial_k U$, $\partial_k Z_0$ and $\partial_k Z_1$:

Renormalization group flow equations

Expanding the fields $\varphi(x) = \bar{\varphi} + \chi(x)$ in small fluctuations $\chi(x) = \sum_q e^{iqx}\chi(q)$ in momentum space around a constant background $\bar{\varphi}$ yields the n-point-functions $\Gamma^{(n)}(q_1, ..., q_n) = \frac{\delta}{\delta\varphi(q_1)} \cdots \frac{\delta}{\delta\varphi(q_n)}\,\Gamma = \frac{\delta}{\delta\chi(q_1)} \cdots \frac{\delta}{\delta\chi(q_n)}\,\Gamma\,|_{\chi=0}$:

$$
\begin{aligned}
&\Gamma^{(2)}(q,-q) = U' + 2\rho U'' + q^2 \cdot Z_0 + q^4 \cdot Z_1 \\
&\Gamma^{(2)}(p,-p) = U' + 2\rho U'' + p^2 \cdot Z_0 + p^4 \cdot Z_1 \\
&\Gamma^{(3)}(p,q,-p-q) = \\
&\quad \Gamma^{(3)}(p+q,-q,-p) = \sqrt{2\rho}\,\{\,3U'' + 2\rho U''' \\
&\qquad + (p^2 + (pq) + q^2) \cdot Z_0' \\
&\qquad + (p^4 + 2p^2(pq) + 3p^2q^2 + 2(pq)q^2 + q^4) \cdot Z_1'\,\} \\
&\Gamma^{(4)}(p,q,-q,-p) = 3U'' + 12\rho U''' + 4\rho^2 U'''' \\
&\qquad + (p^2 + q^2) \cdot (Z_0' + 2\rho Z_0'') \\
&\qquad + (p^4 + 4p^2q^2 + q^4) \cdot (Z_1' + 2\rho Z_1'')
\end{aligned}
$$

These are inserted into

$$
\begin{aligned}
&\partial_k \Gamma = \frac{1}{2}\int \frac{d^Dp}{(2\pi)^D}\,\frac{\partial_k R(p^2)}{\Gamma^{(2)}(p,-p)+R(p^2)} \\
&\partial_k \Gamma^{(2)}(q,-q) \\
&\qquad = \int \frac{d^Dp}{(2\pi)^D}\,\frac{\Gamma^{(3)}(p,q,-p-q)\Gamma^{(3)}(p+q,-q,-p)\ \partial_k R(p^2)}{(\Gamma^{(2)}(p^2)+R(p^2))^2(\Gamma^{(2)}((p+q)^2)+R((p+q)^2))} \\
&\qquad - \frac{1}{2}\int \frac{d^Dp}{(2\pi)^D}\,\frac{\Gamma^{(4)}(p,q,-q,-p)\ \partial_k R(p^2)}{(\Gamma^{(2)}(p^2)+R(p^2))^2}
\end{aligned}
$$

and yield the flow equations

$$
\begin{aligned}
&\partial_k U = \partial_k \Gamma \\
&\partial_k Z_0 = \frac{\partial}{\partial q^2}\ \partial_k \Gamma^{(2)}\ \ |_{q^2=0} \\
&\partial_k Z_1 = \frac{1}{2}\frac{\partial}{\partial q^2}\frac{\partial}{\partial q^2}\ \partial_k \Gamma^{(2)}\ \ |_{q^2=0}
\end{aligned}
$$

In order to be able to perform the q^2-derivatives one has to expand the denominator containing $(p+q)^2$. Let $N = \Gamma^{(2)}(p^2) + R(p^2)$ and $M = \Gamma^{(2)}((p+q)^2) + R((p+q)^2)$:

$$
\frac{1}{M} = \frac{1}{N} - \frac{(2pq+q^2)\dot{N}}{N^2} + \frac{(2pq+q^2)^2\dot{N}^2}{N^3} - \frac{1}{2}\frac{(2pq+q^2)^2\ddot{N}}{N^2} + \ldots
$$

Then the mixed scalar products (pq) can also be eliminated:

$$
\begin{aligned}
&\frac{1}{2}\int \frac{d^Dp}{(2\pi)^D} f(p^2,q^2) \cdot (pq)^2 = \frac{1}{2}\int \frac{d^Dp}{(2\pi)^D} f(p^2,q^2)\frac{p^2q^2}{D} \\
&\frac{1}{2}\int \frac{d^Dp}{(2\pi)^D} f(p^2,q^2) \cdot (pq)^4 = \frac{1}{2}\int \frac{d^Dp}{(2\pi)^D} f(p^2,q^2)\frac{p^4q^4}{D(D+2)} \\
&\frac{1}{2}\int \frac{d^Dp}{(2\pi)^D} f(p^2,q^2) \cdot (pq)^k = 0 \text{ for all odd } k
\end{aligned}
$$

This finally gives the explicit flow equations:

$$
\begin{aligned}
&\partial_k U = 1 \cdot I\left[\frac{1}{N}\right] \\
&\partial_k Z_0 = 4\rho C_0^2 \cdot I\left[-\frac{\dot{N}}{N^4} + \frac{4}{D}\frac{p^2\dot{N}^2}{N^5} - \frac{2}{D}\frac{p^2\ddot{N}}{N^4}\right] + \ldots \\
&\partial_k Z_1 = 2\rho C_0^2 \cdot I\left[\frac{2\dot{N}^2}{N^5} - \frac{\ddot{N}}{N^4}\right] + \ldots
\end{aligned}
$$

$$
\begin{aligned}
&I\Big[f(p^2)\Big] = \frac{1}{2}\int \frac{d^Dp}{(2\pi)^D}\ f(p^2)\ \partial_k R(p^2) \\
&C_0 = 3U'' + 2\rho U'''
\end{aligned}
$$

Scale invariant flow equations

As we are interested in critical phenomena of second order phase transitions corresponding to fix points in the renormalization group flow, we write the flow equations in explicitely scale invariant form.

To this end one substitutes $z_0(\sigma) = Z_0(\rho)/Z_0(\rho_k)$ (where ρ_k is the running minimum of $U(\rho)$), $\sigma = Z_0(\rho_k)k^{2-D}\rho$, $u = k^{-d}U$, $t = -\mathrm{Ln}k$, $\eta = -\partial_t \mathrm{Ln} Z_0(\rho_k)$ etc. such that

$$
\begin{aligned}
&\partial_t u' = (-2+\eta)u' + (D-2+\eta)\sigma u'' \\
&\quad - c_0 \cdot I_2^0 \\
&\quad - c_1 \cdot I_2^1
\end{aligned}
$$

$$
\begin{aligned}
&\partial_t z_0 = \eta z + (D-2+\eta)\sigma z_0' \\
&\quad + 4\sigma c_0 c_0 \cdot (\qquad\qquad - \tfrac{0+\ D}{D}J_4^0 + \tfrac{2}{D}K_4^1) \\
&\quad + 8\sigma c_0 c_1 \cdot (\tfrac{0+1D}{D}I_3^0 - \tfrac{2+\ D}{D}J_4^1 + \tfrac{2}{D}K_4^2) \\
&\quad + 4\sigma c_1 c_1 \cdot (\tfrac{1+2D}{D}I_3^1 - \tfrac{4+\ D}{D}J_4^2 + \tfrac{2}{D}K_4^3) \\
&\quad + 8\sigma c_0 c_2 \cdot (\tfrac{0+3D}{D}I_3^1 - \tfrac{4+\ D}{D}J_4^2 + \tfrac{2}{D}K_4^3) \\
&\quad + 4\sigma c_1 c_2 \cdot (\tfrac{4+8D}{D}I_3^2 - \tfrac{12+2D}{D}J_4^3 + \tfrac{4}{D}K_4^4) \\
&\quad + 4\sigma c_2 c_2 \cdot (\tfrac{4+6D}{D}I_3^3 - \tfrac{8+\ D}{D}J_4^4 + \tfrac{2}{D}K_4^5) \\
&\quad - c_4 \cdot I_2^0 \\
&\quad - c_5 \cdot 4I_2^1
\end{aligned}
$$

$$
\begin{aligned}
&\partial_t z_1 = (2+\eta)z_1 + (D-2+\eta)\sigma z_1' \\
&\quad + 2\sigma c_0 c_0 \cdot (\qquad\qquad\qquad\qquad K_4^0) \\
&\quad + 4\sigma c_0 c_1 \cdot (\qquad\qquad - \tfrac{0+\ 2D}{D}J_4^0 + \tfrac{8+D}{D}K_4^1) \\
&\quad + 2\sigma c_1 c_1 \cdot (\tfrac{0+\ 2D}{D}I_3^0 - \tfrac{10+\ 4D}{D}J_4^1 + \tfrac{20+D}{D}K_4^2) \\
&\quad + 4\sigma c_0 c_2 \cdot (\tfrac{0+\ 2D}{D}I_3^0 - \tfrac{8+\ 6D}{D}J_4^1 + \tfrac{36+18D+D^2}{D(D+2)}K_4^2) \\
&\quad + 4\sigma c_1 c_2 \cdot (\tfrac{4+\ 8D}{D}I_3^1 - \tfrac{32+\ 8D}{D}J_4^2 + \tfrac{64+30D+D^2}{D(D+2)}K_4^3) \\
&\quad + 2\sigma c_2 c_2 \cdot (\tfrac{16+22D}{D}I_3^2 - \tfrac{72+12D}{D}J_4^3 + \tfrac{96+42D+D^2}{D(D+2)}K_4^4) \\
&\quad - c_5 \cdot I_2^0
\end{aligned}
$$

where for abbreviation we have used the constants

$$
\begin{aligned}
&w = u' + 2\sigma u'' \\
&\quad c_0 = w' \\
&\quad c_1 = z_0' \\
&\quad c_2 = z_1' \\
&\quad c_3 = w' + 2\sigma w'' \\
&\quad c_4 = z_0' + 2\sigma z_0'' \\
&\quad c_5 = z_1' + 2\sigma z_1''
\end{aligned}
$$

and the momentum integrals

$$
\begin{aligned}
&I_n^m(w,z_0,z_1,\eta)[r] := \frac{1}{2}\int \frac{d^Dp}{(2\pi)^D}\,\frac{(p^2)^m\ \partial_t r}{(w+p^2z_0+p^4z_1+r)^n} \\
&J_n^m(w,z_0,z_1,\eta)[r] := \frac{1}{2}\int \frac{d^Dp}{(2\pi)^D}\,\frac{(p^2)^m\ (z_0+2p^2z_1+\dot{r})\ \partial_t r}{(w+p^2z_0+p^4z_1+r)^n} \\
&K_n^m(w,z_0,z_1,\eta)[r] := \frac{1}{2}\int \frac{d^Dp}{(2\pi)^D}\,\frac{(p^2)^m\ 2(z_0+2p^2z_1+\dot{r})^2\ \partial_t r}{(w+p^2z_0+p^4z_1+r)^{n+1}} \\
&\qquad\qquad - \frac{1}{2}\int \frac{d^Dp}{(2\pi)^D}\,\frac{(p^2)^m\ (2z_1+\ddot{r})\ \partial_t r}{(w+p^2z_0+p^4z_1+r)^n}
\end{aligned}
$$

which will be discussed next:

Momentum integrals

The momentum integrals I, J and K can be computed efficiently in case of a linear cutoff. For the choice [7] $R_k = Z_k(k^2 - q^2)\Theta(k^2 - q^2)$ one has

$$r = (1 - p^2) \cdot \Theta(1 - p^2)$$
$$\dot{r} = -\Theta(1 - p^2)$$
$$\ddot{r} = \delta(1 - p^2)$$
$$\partial_t r = (2 - \eta + p^2\eta) \cdot \Theta(1 - p^2)$$

and I, J and K are reduced to a single integral H:

$$I_n^m = (2 - \eta)H_n^m + \eta H_n^{m+1}$$
$$J_n^m = (z_0 - 1)I_n^m + 2z_1 I_n^{m+1}$$
$$K_n^m = 2(z_0 - 1)^2 I_{n+1}^m + 8(z_0 - 1)z_1 I_{n+1}^{m+1} + 8z_1^2 I_{n+1}^{m+2} - 2z_1 I_n^m - v_D(w + z_0 + z_1)^{-n}$$

where H is a special case of a hypergeometric function

$$\begin{aligned} H_n^m(w, z_0, z_1) &= \frac{1}{2}\int \frac{d^Dp}{(2\pi)^D} \frac{(p^2)^m \ \Theta(1-p^2)}{(w+p^2z_0+p^4z_1+r)^n} \\ &= v_D \int_0^1 dx \frac{x^a}{((w+1)+(z_0-1)x+z_1x^2)^n} \\ &= \frac{v_D}{(w+1)^n} \int_0^1 dx \cdot x^a \cdot (1 - bx - cx^2)^{-n} \\ &= \frac{v_D}{(w+1)^n} \int_0^1 dx \cdot x^a \cdot \sum_{k=0}^{\infty} \frac{x^k}{k!} \sum_{l=0}^{k/2} \frac{(n+k-l-1)!k!}{(n-1)!} \frac{c^l}{l!} \frac{b^{k-2l}}{(k-2l)!} \\ &= \frac{v_D}{(w+1)^n} \sum_{k=0}^{\infty} \sum_{l=0}^{k/2} \frac{(n+k-l-1)!}{(n-1)!} \frac{c^l}{l!} \frac{b^{k-2l}}{(k-2l)!} \int_0^1 dx\, x^{a+k} \\ &= \frac{v_D}{(w+1)^n} \sum_{k=0}^{\infty} \sum_{l=0}^{k/2} \frac{(n+k-l-1)!}{(n-1)!} \frac{c^l}{l!} \frac{b^{k-2l}}{(k-2l)!} \frac{1}{a+k+1} \end{aligned}$$

with $v_D = (2^{D+1}\pi^{D/2}\Gamma(D/2))^{-1}$, $a = m + D/2 - 1$, $b = (1 - z_0)/(1 + w)$ and $c = -z_1/(1 + w)$. The series expansion and reordering is valid and convergent as long as $1 - bx - cx^2 \neq 0$ for $x \in [0; 1]$. Which is the case, as the effective average propagator stays finite at all times.

Critical exponents

Starting with a quartic potential $u = \frac{1}{2}(\rho - \kappa_\Lambda)^2$ the flow equations are discretized on a grid and numerically solved for different values of κ_Λ. During the evolution towards $k \to 0$, the running minimum of the potential ($u'(\kappa) = 0$) may either end up in the massless spontaneously broken phase ($m = 0$, $\kappa \to \infty$) or in the massive symmetric phase ($m^2 \sim k^2 u'(0)$, $\kappa = 0$). The critical κ_c leads to a second order phase transition characterized by fix points of all couplings and vanishing masses. Fine tuning κ_Λ around κ_c then yields the critical exponent ν through the scaling law $m \sim |T - T_c|^\nu$ and the established proportionality $|\kappa_\Lambda - \kappa_c| \sim |T - T_c|$. The critical exponent η may be identified with the fix point η^* of the anomalous dimension.

The critical exponents for the universality class of the three dimensional Ising model obtained in this way are compared in Table 1 to values obtained in first order ($z_1 = 0$) and lowest order ($z_1 = 0$, $z_0 = 1$). For comparison we also give results from literature. An extensive review of different techniques and recent results can be found in [8]. We adopt their overall estimates of exponents and errors. In addition we quote some of the newest and precisest results from high temperature expansions, perturbation series at fixed dimension, ϵ-expansions and Monte Carlo simulations plus some additional references.

Conclusion and outlook

The critical exponent ν has significantly improved compared to first order. This is the first calculation using a linear cutoff that is consistent with high precision methods. On the other hand, the anomalous dimension is sensitive to the neglected contributions as can be seen in comparison to [6]. Its error has even increased in comparison to first order.
Hence the inclusion of the neglected wave function renormalizations should be the next step.

	ν	η
effective average action to lowest order (this paper, $z_1 = 0$, $z_0 = 1$)	0.6271	0.1120
effective average action to first order (this paper, $z_1 = 0$)	0.6255	0.0503
effective average action beyond first order (this paper)	0.6303	0.0793
effective average action to first order (exponential cutoff [9])	0.6307	0.0470
effective average action to second order (exp. cutoff, field exp. [6])	0.632	0.033
literature values from the review [8]	0.6301(4)	0.0364(5)
25th-order high-temperature expansion [10], see also [11]	0.63012(16)	0.03639(15)
seven-loop perturbation series at fixed D=3 [12], see also [13]	0.6303(8)	0.0335(6)
five-loop order ϵ expansion with boundary conditions [13]	0.6305(25)	0.0365(50)
Monte Carlo simulation [14], see also [15, 16]	0.6297(5)	0.0362(8)

TABLE 1: Critical exponents of the three dimensional Ising universality class

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