# A Ramond-Neveu-Schwarz string with one end fixed

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**Abstract**

We study an RNS string with one end fixed on a $D0$-brane and the other end free as a qualitative guide to the spectrum of hadrons containig one very heavy quark. The mixed boundary conditions break half of the world-sheet supersymmetry. Boson-fermion masses can still be matched if space-time is 9 dimensional; thus $SO(8)$ triality still plays a role in the spectrum, although full space-time supersymmetry does not survive. We quantize the system in a temporal-like gauge where $X^0 \sim \tau$. Only odd $\alpha$ and even $d$ R modes remain, while the NS oscillators $b$ become odd-integer moded. Although the gauge choice eliminates negative-norm states at the outset, there are still even-moded Virasoro and even(odd) moded super-Virasoro constraints to be imposed in the NS(R) sectors. The Casimir energy is now positive for R states, but vanishes for NS; there are no tachyons. States for $\alpha' M^2 \leq 3$ are explicitly constructed and found to be organized into $SO(8)$ irreps by (super)constraints, which include a novel "$\sqrt{L_0}$" operator in the NS and $\Gamma^0 \pm I$ in the R sectors. GSO projections are not allowed. The distinctive physical signatures of the system are (i) a slope twice that of the open string, (ii) absence of all but leading trajectories, yielding an asympotic level density $\sim N^{\frac{D-1}{2}}$ instead of $\exp\sqrt{N}$. When both ends are fixed, all leading and subleading trajectories are eliminated instead, resulting in a spectrum qualitatively similar to the $J/\psi$ and $\Upsilon$ particles.

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# 1 Introduction

Bosonic String theory was originally discovered [1, 2] while trying to account for observed properties of hadron dynamics, and it was indeed qualitatively successful in reproducing features of hadron physics such as linear Regge trajectories for mesons, amplitudes with Dolen-Horn-Schmid duality [3] and desired high-$s$ and $t$ behaviors and poles [4]. The nearly-massless up and down quarks and antiquarks were incorporated into the string picture by being placed at open string endpoints, where Neumann boundary conditions ensured their moving at the speed of light. Harari-Rosner diagrams [5, 6] were useful for keeping track of internal quantum numbers and visualising duality between Regge pole exchanges and resonances in the $s$-channel.

There was an obvious need to find an extension of the string model that could encompass baryons, which had masses similar to those of the mesons (except for the very light pion), and also lay on Regge trajectories with approximately the same slope. These similarities suggested a new kind of symmetry (albeit partially broken, in view of the non-vanishing mass differences) between bosons and fermions, and string models featuring such a symmetry were constructed by Ramond [7] and Neveu and Schwarz [8]. It gradually became clear, however, that hadron dynamics was governed by a gauge theory, and superstring theory was elevated to the status of a promising candidate for describing quantum gravity and other fundamental physical phenomena at distances of $10^{-34}$ m. The partial success of strings in modelling hadrons was attributed to gluons forming string-like flux tubes between quarks.

It may be instructive to return to this oldest use of string theory as a phenomenological guide for the study of new generations of baryons and mesons containing at least one heavy (of the top, bottom or charmed variety) and one light quark. In an earlier work [9] we modeled such mesons as excitations of an open string with one end fixed on a $D0$-brane and the other end free. This gave rise to testable predictions such as a restriction to odd oscillator modes and a doubled Regge slope. Whether the description is of any merit will become apparent when higher spin excitations of heavy quark bearing mesons are found and studied. Strings with mixed boundary conditions have also been considered in [10, 11, 12, 13].

In this note, we examine the Ramond-Neveu-Schwarz (RNS) version of [9]. Our main motivation and hope is that this may provide qualitative hints about the spectrum of baryons and mesons with a single $c, b$ or a $t$ quark, although we of course do not expect it to provide a heavy-hadron model that is realistic in all its details. Apart from its possible phenomenological usefulness, the system is also of some intrinsic string-theoretic interest in a number of respects. One of these is that its most natural treatment requires a gauge halfway between the light-cone and the old covariant treatments. The old covariant method of quantization of even the standard RNS string is described only very sketchily in standard reference works such as [14, 15, 16]; the detailed implementation of the constraints peculiar to our problem and the subsequent emergence of the physical spectrum turns out to be surprisingly intricate. Another interesting feature is the partial breaking of world-sheet supersymmetry through mixed boundary conditions and the way this is manifested in the space-time picture.

The plan of the paper is as follows: In section 2 we obtain the mode expansions and the restricted SUSY transformations allowed by our novel boundary conditions. We then impose a gauge we might call "the rest-frame gauge". Section 3 is devoted to the Virasoro and super-Virasoro constraints in the NS and R sectors. Some of the new features that emerge include the limitation of the Virasoro algebra to even modes, while the modes of the R and NS constraints can only be odd and even respectively. The Virasoro and super-Virasoro algebras turn out to have non-vanishing anomalies in all dimensions for reasons we will discuss. In section 4, we work out the low-lying members of the NS and R spectra. The organization and projection of physical states into specific $SO(D-1)$ irreps via the constraints involves novelties such as an NS constraint that may be regarded as "a bosonic-sector square root of $L_0$" and an R constraint which is a truncated form of the usual Ramond-Dirac operator $F_0$. The spectrum does not exhibit full space-time supersymmetry, but it is at least possible to match the masses of bosons and fermions if $D-1=8$, which means aspects of $SO(8)$ triality are still reflected in the states. Because of changes in the modes, the NS Casimir energy now vanishes, while the R ground state becomes massive. For higher states, the principal feature of the model distinguishing it from the ordinary RNS one is the absence of daughter trajectories. This leads to a $(D-1)^N$ behavior in the multiplicity of states (at mass level $M \sim \sqrt{N}$) instead of the usual $e^{\sqrt{N}}$. The paper ends with concluding remarks in section 5.

# 2 The classical RNS string with one end fixed

## 2.1 Equation of motion:

We start with standard material to set our notation and conventions. In the superconformal gauge the action reads

$$S = -\frac{1}{2\pi}\int d^2\sigma \left\{\partial_\alpha X_\mu \partial^\alpha X^\mu - i\overline{\Psi}^\mu \rho^\alpha \partial_\alpha \Psi_\mu\right\}, \tag{1}$$

where $\rho^\alpha$ represents the two dimensional Dirac matrices and a convenient basis satisfying $\{\rho^\alpha, \rho^\beta\} = -2\eta^{\alpha\beta}$ is

$$\rho^0 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \; \rho^1 = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}.$$

We will denote the upper and lower components of $\Psi$ as $\Psi_-$ and $\Psi_+$, respectively. We should note that the Dirac operator, $i\rho^\alpha\partial_\alpha$, is real in this representation; thus, the components of the Majorana world sheet spinor may be taken as real. In this paper, our focus will be on these spinor degrees of freedom since the bosonic fields have already been treated in [9].

The fermion equation of motion derived from (1) is simply the two dimensional Dirac equation and in the chosen basis it splits up into the following equations:

$$\frac{1}{2}\partial_+ \Psi^\mu_- \equiv (\frac{\partial}{\partial\sigma} + \frac{\partial}{\partial\tau})\Psi^\mu_- = 0, \tag{2}$$

$$-\frac{1}{2}\partial_- \Psi^\mu_+ \equiv (\frac{\partial}{\partial\sigma} - \frac{\partial}{\partial\tau})\Psi^\mu_+ = 0. \tag{3}$$

The equation of motion for the bosonic field in the same basis is

$$\partial_+(\partial_- X^\mu) = \partial_-(\partial_+ X^\mu) = 0. \tag{4}$$

## 2.2 Boundary conditions and mode expansions:

The boundary conditions for the the bosonic fields are as in [9]: We locate the infinitely massive "quark" at the $\sigma = 0$ end and identify this point with the origin of target space coordinates through the Dirichlet boundary condition

$$X^i(0,\tau) = 0, \tag{5}$$

while we adopt the Neumann boundary condition

$$\partial_\sigma X^\mu(\sigma_{max},\tau) = 0 \tag{6}$$

for the massless end. If we use the usual integrally moded oscillators to expand $X^\mu$, these two conditions lead to the requirement that $\alpha_n^\mu = 0$ for even n and $\sigma_{max} = \pi/2$ (another option, adopted in [10] among others, is to maintain the usual range $[0,\pi]$ for $\sigma$, in which case the $\alpha_n^\mu$ oscillators become half-integrally moded; our choice is to work with the odd-numbered subset of the original integral modes rather than to introduce an entirely new set of oscillators). As mentioned in [9], the disappearance of the even modes is quite similar to the same phenomenon in the elementary quantum mechanics problem of a half-oscillator potential $V = \frac{1}{2}kx^2$ for $x \geq 0$, $V = \infty$ for $x < 0$. As in the usual RNS string, vanishing of the surface term requires $\Psi_+^\mu = \pm\Psi_-^\mu$ at each end, and without loss of generality, we set $\Psi_+^\mu(0,\tau) = \Psi_-^\mu(0,\tau)$ for $\sigma = 0$ and then consider the two cases depending on the relative sign at $\sigma_{max} = \pi/2$. Leaving the question of how to reconcile the choice at $\sigma = 0$ with world-sheet supersymmetry to the next section, the two cases arising from the sign choices are the following:

(i) The Ramond (R) boundary condition

$$\Psi_+^i(\frac{\pi}{2},\tau) = \Psi_-^i(\frac{\pi}{2},\tau) \tag{7}$$

leads to the exclusively even-moded expansion

$$\Psi_-^i(\sigma,\tau) = \frac{1}{\sqrt{2}}\sum_{n:even} d_n^i e^{-in(\tau-\sigma)}, \tag{8}$$

$$\Psi_+^i(\sigma,\tau) = \frac{1}{\sqrt{2}}\sum_{n:even} d_n^i e^{-in(\tau+\sigma)}. \tag{9}$$

(ii) The Neveu-Schwarz (NS) boundary condition

$$\Psi_+^i(\frac{\pi}{2},\tau) = -\Psi_-^i(\frac{\pi}{2},\tau) \tag{10}$$

results in the odd-integer modes

$$\Psi_-^i(\sigma,\tau) = \frac{1}{\sqrt{2}}\sum_{r:odd} b_r^i e^{-ir(\tau-\sigma)}, \tag{11}$$

$$\Psi_+^i(\sigma,\tau) = \frac{1}{\sqrt{2}}\sum_{r:odd} b_r^i e^{-ir(\tau+\sigma)}. \tag{12}$$

We note that while the boundary conditions merely restrict the Ramond oscillators $d_n^i$ to even and the $\alpha_n^i$ to odd modes, the Neveu-Schwarz sector actually changes from half-integral to odd integral oscillators. This will lead to a massless NS and a massive R ground state, reversing the usual RNS results.

### 2.3 Broken global world-sheet SUSY:

The action is invariant under the infinitesimal world-sheet supersymmetry transformations

$$\delta X^\mu = \bar{\epsilon}\Psi^\mu, \tag{13}$$

$$\delta\Psi^\mu = -i\rho^\alpha \partial_\alpha X^\mu \epsilon. \tag{14}$$

with $\epsilon$ a constant anticommuting spinor. Let us examine the space components of (13) and (14) first. For fermionic fields (13) combined with (5) gives

$$\delta X^i(0,\tau)_{\sigma=0} = (\epsilon_+^\dagger \Psi_-^i - \epsilon_-^\dagger \Psi_+^i)_{\sigma=0} = 0. \tag{15}$$

This would normally be satisfied either by

(i)

$$\Psi_+^i(0,\tau) = \Psi_-^i(0,\tau), \tag{16}$$

and

$$\epsilon_+ = \epsilon_-, \tag{17}$$

or by

(ii)

$$\Psi_+^i(0,\tau) = -\Psi_-^i(0,\tau), \tag{18}$$

and

$$\epsilon_+ = -\epsilon_-. \tag{19}$$

However, having ruled out (ii) by our choice of boundary conditions in the previous subsection, we are restricted to (i), which eliminates half of the world-sheet supersymmetry. We could have reversed the fermion boundary conditions at the two ends and consequently chosen (ii), but the obvious conclusion is that our mixed Dirichlet-Neumann conditions are compatible with only half of the usual supersymmetry transformations. This breaking of worldsheet SUSY will have to manifest itself in the space-time spectrum, as we will see later. Since Poincaré invariance has been explicitly broken, we do not expect spacetime supersymmetry operators, which are essentially square-roots of the Poincaré generators, to be effective.

### 2.4 Gauge Choice:

For the bosonic string with one end fixed at the origin, it was natural to choose the gauge condition

$$X^0 = \alpha_0^0 \tau = l p^0 \tau \tag{20}$$

just as the light-cone gauge $X^+ \sim \tau$ is suited to a string with both ends moving at the speed of light. Indeed, the first excited state of such a free open string is a massless vector boson with $D-2$ polarizations, which is the number of transverse oscillators used in the light-cone gauge. Our problem, in contrast, leads to a massive vector with $D-1$ polarization states, which is in accord with (20). Combining this gauge choice with (13), we get

$$\delta X^0 = \bar{\epsilon}\Psi^0 = 0. \tag{21}$$

This is similar to setting all the $\Psi^+$ and $X^+$ oscillators to zero in the light-cone gauge treatment of the usual RNS string. The condition (21) kills all the $b^0_n$ modes of the NS-sector. On the other hand, for the R-sector, all $d^0_n$ modes are zero except $d^0_0$. To see why this mode survives, consider the explicit form of (21) with the $\Psi^i$ in (15) replaced with $\Psi^0$ and $\epsilon_+ = \epsilon_-$. This is only compatible with $d^0_0$ being non-zero. This is just as well, since we would not want to lose an element of the Clifford algebra in $(D-1,1)$.

# 3 Quantization

With the "rest-frame gauge" chosen above, the $\alpha^0_n$ modes and their associated negative norm states [9] are discarded at the outset. However, unlike the situation in the light-cone gauge, this does not mean that all the states obtained by hitting the ground state with the spacelike oscillators are automatically physical. In the light-cone gauge, the constraints are implemented at the operatorial level, allowing one to solve for the $\alpha^-_n$ in terms of transverse oscillators. Our gauge choice does not involve an off-diagonal space-time metric and does not offer a similar possibility. There are still constraints which must be imposed on all possible states, and only the states annihilated by these constraints are the final physical ones. This is similar to non-covariant treatments of gauge theories where one first sets $A_0 = 0$, but then also imposes $\nabla \cdot \mathbf{A} = 0$ to isolate the physical degrees of freedom.

## 3.1 Canonical Quantization:

We will follow the conventional canonical quantization procedure. The canonical anticommutation relations for the fermionic fields are

$$\left\{\Psi^i_\alpha(\sigma,\tau), \Psi^j_\beta(\sigma',\tau)\right\} = \pi\delta(\sigma-\sigma')\delta^{ij}\delta_{\alpha\beta}. \tag{22}$$

Substituting the known mode expansions, we obtain anticommutation relations for the modes. These are

$$\left\{b^i_r, b^j_s\right\} = \delta^{ij}\delta_{r+s}(r,s:odd) \tag{23}$$

for the NS-sector, and

$$\{d^i_m, d^j_n\} = \delta^{ij}\delta_{m+n}(m,n : even) \tag{24}$$

for the R-sector. Finally, the $d^0_0$ mode obeys

$$\{d^0_0, d^0_0\} = -I. \tag{25}$$

As in the ordinary RNS string, the $d^i_0$ and $d^0_0$ are proportional to the Dirac gamma matrices with $\Gamma^\mu = i\sqrt{2}d^\mu_0$ in $(D-1,1)$ dimensional Minkowski space.

### 3.2 Super-Virasoro constraints

The unphysical modes are eliminated by imposing $T^+_{\alpha\beta} \mid \Psi_{phys}\rangle = 0$ and $J^+_\alpha \mid \Psi_{phys}\rangle = 0$ on the states, where $T^+_{\alpha\beta}$ and $J^+_\alpha$ are the positive-frequency components of the energy-momentum tensor and the supercurrent, respectively.

The energy-momentum tensor for the fermionic part is

$$T_{\alpha\beta} = \frac{i}{4}\bar{\Psi}^\mu\rho_\alpha\partial_\beta\Psi_\mu + \frac{i}{4}\bar{\Psi}^\mu\rho_\beta\partial_\alpha\Psi_\mu. \tag{26}$$

and, in the chosen gauge, its components in conformal coordinates are

$$T_{++} = \frac{1}{2}(T_{00} + T_{01}) \quad = \quad \frac{i}{2} : \Psi^i_+\partial_+\Psi_{+i} :, \tag{27}$$

$$T_{--} = \frac{1}{2}(T_{00} - T_{01}) \quad = \quad \frac{i}{2} : \Psi^i_-\partial_-\Psi_{-i} : . \tag{28}$$

The supercurrent is

$$J_\alpha = \frac{1}{2}\rho^\beta\rho_\alpha\Psi^\mu\partial_\beta X_\mu \tag{29}$$

The nonzero components of the supercurrent, again in conformal coordinates, are

$$J_+ \quad = \quad \Psi^i_+\partial_+X_i - \Psi^0_+\partial_+X_0, \tag{30}$$

$$J_- \quad = \quad \Psi^i_-\partial_-X_i - \Psi^0_-\partial_-X_0. \tag{31}$$

The conservation law of the supercurrent yields

$$\partial_-J_+ = \partial_+J_- = 0. \tag{32}$$

We now examine the implications of these constraints in the Neveu-Schwarz and Ramond sectors.

#### 3.2.1 Neveu-Schwarz constraints

The super-Virasoro operators are the Fourier coefficients of the energy-momentum tensor and supercurrent. To obtain these coefficients we use the fact that $T_{++}(\sigma) = T_{--}(-\sigma)$ and the "doubling trick" of Polchinski [15], extending the region of integration to $[-\frac{\pi}{2}, \frac{\pi}{2}]$. We can then write

$$L^{(b)}_m = \frac{1}{2}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} e^{im\sigma}T_{++}(\sigma)d\sigma. \tag{33}$$

Substituting the mode expansions, we get

$$L_m^{(b)} = \frac{1}{2} \sum_{r:odd} (r + \frac{m}{2}) : b_{-r}^i b_{m+r}^i : \tag{34}$$

where $m$ is seen to be necessarily even. The $L_m^{(b)}$ thus complement the $L_m^{(\alpha)}$ which are also even. This is an indication of the consistency of the mode elimination scheme imposed by our mixed boundary conditions.

The fermionic generators of this sector are obtained similarly using $J_+(\sigma) = J_-(-\sigma)$, which results in

$$G_m = \frac{\sqrt{2}}{\pi} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} e^{im\sigma} J_+(\sigma) d\sigma. \tag{35}$$

In terms of the mode expansions one finds

$$G_m = \sum_{r:odd} \alpha_{-r}^i b_{m+r}^i \tag{36}$$

where $m$ is again forced to be even .

The super-Virasoro algebra for this sector is

$$\begin{aligned} [L_m, L_n] &= (m-n) L_{m+n} + A(m) \delta_{m+n}, && (37) \\ [L_m, G_n] &= (\frac{m}{2} - n) G_{m+n}, && (38) \\ \{G_m, G_n\} &= 2L_{m+n} + (\alpha_0^0)^2 \delta_{m+n} + B(m) \delta_{m+n} && (39) \end{aligned}$$

where $L_m = L_m^{(\alpha)} + L_m^{(b)}$ and

$$\begin{aligned} L_0^{(\alpha)} &= \frac{-1}{2} (\alpha_0^0)^2 + \frac{1}{2} \sum_{r:odd} : \alpha_r^i \alpha_{-r}^i :, && (40) \\ L_m^{(\alpha)} &= \frac{1}{2} \sum_{r:odd} : \alpha_r^i \alpha_{m-r}^i :, && (41) \end{aligned}$$

where, in the second line, m is even and nonzero.

We see that all super-Virasoro operators have their usual forms, say as in [14], except for the fact that they are written in terms of odd modes only. We must now see how this changes the central extension terms. The $\alpha$-mode anomaly $A(m)$ was calculated in [9] with the result

$$A^\alpha(m) = \frac{(D-1)}{24} (m^3 + 2m). \tag{42}$$

Thus we will only calculate the anomaly due to the $b$-modes, and this can be easily done via

$$\begin{aligned} A^{(b)}(m) &= \langle 0 \mid [L_m^{(b)}, L_{-m}^{(b)}] \mid 0\rangle & (43)\\ &= \langle 0 \mid L_m^{(b)} L_{-m}^{(b)} \mid 0\rangle & (44)\\ &= \frac{1}{4}\langle 0 \mid \sum_{r,s:odd} (r+\frac{m}{2})(s-\frac{m}{2}) b^i_{-r} b^i_{m+r} b^j_{-s} b^j_{-m+s} \mid 0\rangle & (45)\\ &= \frac{1}{4}\langle 0 \mid \sum_{r,s:odd} (r+\frac{m}{2})(s-\frac{m}{2}) b^i_{m+r} b^i_{-r} b^j_{-m+s} b^j_{-s} \mid 0\rangle, & (46) \end{aligned}$$

where m is, of course, even and positive. The expectation value in the third line vanishes for $r > 0$ and $s > m$, whereas the expectation value in the fourth line vanishes for $s < 0$ and $m < -r$. This yields

$$\begin{aligned} A^{(b)}(m) &= \frac{1}{4}\langle 0 \mid \sum_{r=-m+1}^{-1} \sum_{s=1}^{m-1} (r+\frac{m}{2})(s-\frac{m}{2}) b^i_{-r} b^i_{m+r} b^j_{-s} b^j_{-m+s} \mid 0\rangle, & (47)\\ &= 2(D-1)\sum_{s=1}^{m-1} (s-\frac{m}{2})^2 & (48) \end{aligned}$$

where all the summations above are over odd integers and $\eta_{ij}\eta^{ij} = D-1$ is the number of space coordinates. The summation in the second line, when taken over all integers, produces the well-known NS-sector central extension term. On the other hand, our odd integer summation gives

$$A^{(b)}(m) = \frac{(D-1)}{48}(m^3 - 4m). \quad (49)$$

Hence the total central extension term of the first commutator is

$$A(m) = \frac{(D-1)}{16} m^3. \quad (50)$$

Can this anomaly be cancelled by ghost contributions? There are general reasons to expect the answer to be negative: Our Dirichlet boundary condition reduces the usual full open string disc to half a disc with a diameter that is not allowed to change under conformal mappings (the appearance of only even Virasoro modes is related to this constraint; in classical terms, we only have the operators $L_{2n} \sim z^{2n+1}\partial_z$, which can only effect conformal transformations invariant under $z \to -z$). Furthermore, world-sheet and space-time properties are always intimately linked: the symmetry of the world-sheet action under diffeomorphisms and Weyl symmetry ensures Poincaré invariance in space-time; similarly, world-sheet supersymmetry is transformed into space-time supersymmetry after the GSO projection. Reasoning in the other direction, our breaking Poincaré invariance down to $SO(D-1)$ by choosing a special fixed point suggests a breakdown in conformal symmetry on the world-sheet. As a simple explicit example, we will show below how ghosts indeed fail to remove the anomaly in the bosonic string with one end fixed.

We will use the notation of [14]. The main new feature is that the open string boundary conditions $c^+ = c^-$ and $b_{++} = b_{--}$ at $\sigma = 0$ and $\sigma = \frac{\pi}{2}$ lead to the exclusively even-moded expansions

$$c^+ = \sum_{n:even} c_n e^{-in(\tau+\sigma)}, \tag{51}$$

$$c^- = \sum_{n:even} c_n e^{-in(\tau-\sigma)}, \tag{52}$$

$$b_{++} = \sum_{n:even} b_n e^{-in(\tau+\sigma)}, \tag{53}$$

$$b_{--} = \sum_{n:even} b_n e^{-in(\tau-\sigma)}. \tag{54}$$

The ghost Virasoro operators are

$$L_m^{(gh)} = \sum_{k:even} (m-k) b_{m+k} c_{-k} \tag{55}$$

where m is again even. The $L_m^{(gh)}$ satisfy the usual Virasoro algebra

$$[L_m^{(gh)}, L_n^{(gh)}] = (m-n) L_{m+n}^{(gh)} + A_m^{(gh)} \delta_{m+n} \tag{56}$$

with an anomaly term

$$A_m^{(gh)} = -\frac{27}{12} m^3 + \frac{9}{2} m^2. \tag{57}$$

The complete Virasoro generators corresponding to $S_0 + S_{gh}$ are $L_m^{(\alpha)} + L_m^{(gh)} - \epsilon_c \delta_m$, where we have shifted $L_0$ so that it will annihilate physical states. The total anomaly is now

$$A_m = \frac{D-1}{24}(m^3 + 2m) - \frac{27}{12} m^3 + \frac{9}{2} m^2 + 2\epsilon_c m. \tag{58}$$

The presence of the term quadratic in $m$ shows that no choice of $D$ and $\epsilon_c$ can make the total anomaly disappear. In this respect, our system resembles an ordinary string in a lower than critical dimension.

Leaving ghosts aside for the rest of paper, we now return to the central extension term appearing in the anti-commutator of the $G_r$. This can be similarly computed with the result

$$B(m) = \langle 0 \mid G_m G_{-m} \mid 0 \rangle, \tag{59}$$

$$= \frac{(D-1)}{4} m^2. \tag{60}$$

At this point, we note that the anomaly-free $OSp(1 \mid 2)$ closed subalgebra of the NS-sector of the usual RNS string is now reduced to the Abelian supersubalgebra of $L_0$ and $G_0$.

The mass-shell condition comes from the zero-frequency part of the Virasoro constraint $T_{\alpha\beta} = 0$. At the quantum level, the masses of the states follow from

$$(L_0 + \epsilon_c) \mid \Psi_{phys}\rangle = 0 \tag{61}$$

where $\epsilon_c$ is the Casimir energy resulting from normal ordering. In the NS sector, the mass shell condition reads

$$(L_0^{(\alpha)} + L_0^{(b)} + \epsilon_c) \mid \Psi_{phys}\rangle = 0 \tag{62}$$

Substituting the mode expansions, this becomes

$$\{\frac{-l^2M^2}{2} + N^{(\alpha)} + N^{(b)} + \frac{1}{2}\sum_{i=1}^{D-1}\sum_{m=1}^{\infty} m\delta^{ii} - \frac{1}{2}\sum_{i=1}^{D-1}\sum_{k=1}^{\infty} k\delta^{ii}\} \mid \Psi_{phys}\rangle = 0 \tag{63}$$

where

$$N^{(\alpha)} = \sum_{i=1}^{D-1}\sum_{m=1,odd}^{\infty} \alpha_{-m}^i \alpha_m^i, \tag{64}$$

$$N^{(b)} = \sum_{i=1}^{D-1}\sum_{k=1,odd}^{\infty} k b_{-k}^i b_k^i. \tag{65}$$

The Casimir energy is now

$$\frac{1}{2}\sum_{i=1}^{D-1}\sum_{m=1,odd}^{\infty} m\delta^{ii} - \frac{1}{2}\sum_{i=1}^{D-1}\sum_{k=1,odd}^{\infty} k\delta^{ii} = 0, \tag{66}$$

showing that the normal ordering constant for our NS-sector vanishes. Hence the usual tachyonic ground state in the NS spectrum has now been replaced by a massless scalar.

Although the Casimir energy is zero for this sector, we calculate the sum $\sum_{i=1}^{D-1}\sum_{m=1,odd}^{\infty} m\delta^{ii}$ for later use. Using $\zeta(-1) = \frac{-1}{12}$, the sum over odd integers is simply $\zeta(-1) - 2\zeta(-1) = \frac{1}{12}$, and hence the result is $\frac{D-1}{12}$. This shortcut calculation agrees with the result obtained from regularization schemes.

### 3.2.2 Ramond constraints

The super-Virasoro operators of this sector are

$$L_m^{(d)} = \frac{1}{2}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} e^{im\sigma} T_{++}(\sigma) d\sigma \tag{67}$$

and

$$F_m = \frac{\sqrt{2}}{\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} e^{im\sigma} J_+(\sigma) d\sigma. \tag{68}$$

Substituting the known mode expansions of $T_{++}$ and $J_+$, we find

$$L_m^{(d)} = \frac{1}{2} \sum_{n:even} (n + \frac{m}{2}) : d_{-n}^i d_{m+n}^i : \tag{69}$$

where m must be even, and

$$F_s = \sum_{r:odd} \alpha_{-r}^i d_{r+s}^i \tag{70}$$

where s is necessarily odd. An exception is the operator

$$f_0 = \alpha_0^0 d_0^0 \tag{71}$$

which is all that survives here from the generalized Dirac operator $F_0$ of the standard RNS string. The super-Virasoro algebra of this sector is as follows:

$$\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + A(m)\delta_{m+n}, & (72)\\
[L_m, F_r] &= (\frac{m}{2} - r)F_{m+r}, & (73)\\
\{F_r, F_s\} &= 2L_{r+s} + (\alpha_0^0)^2\delta_{r+s} + B(r)\delta_{r+s}, & (74)\\
[L_m, f_0] &= 0, & (75)\\
\{f_0, f_0\} &= -(\alpha_0^0)^2 & (76)
\end{aligned}$$

where $L_m = L_m^{(\alpha)} + L_m^{(d)}$ and the commutators (or anticommutators, as appropriate) of $(\alpha_0^0)^2$ or $(f_0)$ with other operators vanish.

The central extension terms of this algebra can be calculated as in the previous section, giving

$$A^{(d)}(m) = \frac{(D-1)}{24}\{\frac{m^3}{2} - 3m^2 + 4m\}. \tag{77}$$

Then the total central extension term of the first commutator, including the contribution of the $\alpha$ modes becomes

$$A(m) = \frac{(D-1)}{8}(\frac{m^3}{2} - m^2 + 2m). \tag{78}$$

A similar treatment for the central extension term of anti-commutators of fermionic generators yields

$$B(r) = \frac{(D-1)}{4}r^2. \tag{79}$$

To calculate the normal ordering constant of this sector, we proceed exactly as in the previous section. The mass-shell condition for this sector is

$$(L_0^{(\alpha)} + L_0^{(d)} + \epsilon_c) \mid \Psi_{phys}\rangle = 0 \tag{80}$$

In terms of modes, this becomes

$$\{\frac{-l^2M^2}{2}+N^{(\alpha)}+N^{(d)}+\frac{1}{2}\sum_{i=1}^{D-1}\sum_{m=1,odd}^{\infty}m\delta^{ii}-\frac{1}{2}\sum_{i=1}^{D-1}\sum_{k=1,even}^{\infty}k\delta^{ii}\}\mid\Psi_{phys}\rangle=0 \tag{81}$$

where $N^{(\alpha)}$ has already been given in (64) and

$$N^{(d)}=\sum_{i=1}^{D-1}\sum_{k=1,even}^{\infty}kd_{-k}^{i}d_{k}^{i}. \tag{82}$$

The Casimir energy is now

$$\epsilon_c = \frac{1}{2}\sum_{i=1}^{D-1}\sum_{m=1,odd}^{\infty}m\delta^{ii}-\frac{1}{2}\sum_{i=1}^{D-1}\sum_{k=1,even}^{\infty}k\delta^{ii}, \tag{83}$$

$$= (D-1)\{\frac{1}{2}[\zeta(-1)-2\zeta(-1)]-\zeta(-1)\}=\frac{D-1}{8}. \tag{84}$$

We note that the normal ordering constant here is nonzero, unlike that of the R-sector of the usual RNS string.

# 4 The Spectrum

## 4.1 Mass formulas in the NS- and R-Sectors:

With the calculated normal ordering constants, we can write down the mass-shell conditions. For the NS-sector, we have

$$(-\alpha'_{ND}M^2+N^{(\alpha)}+N^{(b)})\mid\Psi_{phys}\rangle=0, \tag{85}$$

while for the R-sector

$$(-\alpha'_{ND}M^2+N^{(\alpha)}+N^{(d)}+\frac{(D-1)}{8})\mid\Psi_{phys}\rangle=0, \tag{86}$$

where $\alpha'_{ND}=\frac{l^2}{2}$, as calculated in [9]. This doubling of the slope relative to $\alpha'_{NN}$ of the usual open string is one of the principal distinguishing features of our system. A quick way to understand this result is to consider a classical string with one end fixed in its highest angular momentum (leading Regge trajectory) mode, where it rotates rigidly. This can also be viewed as an ordinary open string in the same mode with its center of mass at rest and then throwing away one half. In order to preserve the relation $J\sim\alpha'M^2$ with $J$ and $M$ both being halved, the slope has to be doubled.

Since half of the world-sheet supersymmetry has been broken at the beginning by the restriction $(\epsilon_+=\epsilon_-)$, we do not expect space-time supersymmetry to appear the way it does in the RNS string subjected to the GSO projection [17]. Actually, the halving of supersymmetry via $D$-branes is a familiar phenomenon [16], and the fixed end here is nothing but a $D0$-brane. While the spectrum is thus not expected to be fully supersymmetric, we have two further options in the degree of superymmetry breaking:

(i) $D-1=8n$: The masses of the states in the NS-sector have the the same values as those in the R-sector for $\alpha'_{ND}M^2 \geq n$, partially preserving SUSY in the mass spectrum.

(ii) $D-1 \neq 8n$: Mass values in the two sectors are completely different and SUSY is completely broken.

In the following, we wil work with the minimal supersymmetry breaking option $n=1$, $D=9$. All states except the NS scalar being massive, we expect to see irreps of $SO(8)$, which is clearly a remnant of the space-time SUSY enjoyed by the ten-dimensional superstring. We now examine the spectra of the two sectors sectors separately.

### 4.2 Neveu-Schwarz spectrum:

A physical state in the NS-sector must satisfy

$$
\begin{aligned}
G_{2m} \mid \phi\rangle &= 0, m>0 \qquad (87)\\
L_{2n} \mid \phi\rangle &= 0, n>0 \qquad (88)\\
L_0 \mid \phi\rangle &= 0, \qquad (89)
\end{aligned}
$$

and the last condition leads to $\alpha'_{ND}M^2 = N^{(\alpha)} + N^{(b)}$.

There is also a $G_0$ constraint that has to be handled separately. An examination of the NS constraint algebra (37-39) shows that all the physical state conditions can be obtained from $G_2$, $G_0$ and $L_0$. The latter does not in fact annihilate physical states; it instead requires them to be eigenstates with the mass as the eigenvalue. From (39) and (85) (recall $\epsilon_c(NS)=0$), we see that

$$
G_0 \mid \phi\rangle = \sqrt{\alpha'_{ND}}M \mid \phi\rangle \qquad (90)
$$

We note that this amounts to taking the square root of the Klein-Gordon-like operator $L_0$. The novelty is that this happens not in the fermionic but in the bosonic sector!

We now examine how low-lying physical states are obtained. Since negative-metric states have been barred from the beginning, it is not immediately obvious what role is left for the above constraints to play. If we use the $L_n$ directly, the answer in the bosonic NS sector turns out to be that all $\alpha^i_{-n}$ and $b^i_{-n}$ oscillators for $n>1$ are ruled out, and the surviving states are automaticaly organized into $SO(8)$ irreps. Hence the daughter trajectories are eliminated from the spectrum. The $G_n$ constraints prune the remaining states even further; for example, a potential $N=3$ state of the form $\alpha^i_{-1}\alpha^j_{-1}b^k_{-1} \mid 0\rangle$ is prohibited by the $G_2$ constraint. Finally, the $G_0$ constraint allows only specific linear combinations of the states that have survived that far. This is obviously different from what happens in the light-cone gauge in the usual RNS string, where all combinations of $\alpha^i_{-n}, b^i_{-n}, (i = 1, ..., 8)$ oscillators on the vacuum are guaranteed to produce physical states, which then combine with the others at the same mass to give $SO(9)$ irreps. There being no obvious pattern to the allowed states beyond what we have just mentioned, we limit ourselves to displaying below the contents of the first four levels.

- $N = 0, M^2 = 0$

$$| 0\rangle$$

ia a massless scalar, providing a stable vacuum for this sector.

- $N = 1$, $\alpha'_{ND} M^2 = 1$: We start with the two massive vector states

$$| \alpha\rangle \equiv \alpha^i_{-1} \, | 0\rangle, \mathbf{8_v}$$

$$| \beta\rangle \equiv b^i_{-1} \, | 0\rangle, \mathbf{8_v}$$

allowed by the $G_2$ constraint. However, the only combination permitted by $G_0$ is $| \alpha\rangle + | \beta\rangle$.

- $N = 2$, $\alpha'_{ND} M^2 = 2$: Under $G_2$, the massive tensor states

$$\begin{aligned}
| 1\rangle &\equiv \left\{\alpha^i_{-1}\alpha^j_{-1} - \frac{\delta^{ij}}{D-1}\alpha^k_{-1}\alpha^k_{-1}\right\} | 0\rangle, \mathbf{35_v} \\
| 2\rangle &\equiv b^i_{-1} b^j_{-1} \, | 0\rangle, \mathbf{28} \\
| 3\rangle &\equiv \left\{\alpha^i_{-1} b^j_{-1} + \alpha^j_{-1} b^i_{-1} - \frac{2\delta^{ij}}{D-1}\alpha^k_{-1} b^k_{-1}\right\} | 0\rangle, \mathbf{35_v} \\
| 4\rangle &\equiv \left\{\alpha^i_{-1} b^j_{-1} - \alpha^j_{-1} b^i_{-1}\right\} | 0\rangle, \mathbf{28}
\end{aligned}$$

are the allowed combinations. We recall that since the even $\alpha$ and b modes have been eliminated at the beginning, we cannot have excitations such as $\alpha^i_{-2} \, | 0\rangle$ and $b^i_{-2} \, | 0\rangle$. Out of these four, the condition $G_0 \, | \phi\rangle = \sqrt{2} \, | \phi\rangle$ allows only

$$\sqrt{2} \, | 1\rangle + | 3\rangle, \mathbf{35_v}$$

and

$$\sqrt{2} \, | 2\rangle + | 4\rangle, \mathbf{28_v}.$$

- $N = 3$, $\alpha'_{ND} M^2 = 3$

$$\begin{aligned}
| 1\rangle \equiv \; & \{\alpha^i_{-1}\alpha^j_{-1}\alpha^k_{-1} - \frac{\delta^{ij}}{D+1}\alpha^n_{-1}\alpha^n_{-1}\alpha^k_{-1} - \frac{\delta^{ik}}{D+1}\alpha^n_{-1}\alpha^n_{-1}\alpha^j_{-1} \\
& - \frac{\delta^{jk}}{D+1}\alpha^n_{-1}\alpha^n_{-1}\alpha^i_{-1}\} \, | 0\rangle, \mathbf{112_v} \\
| 2\rangle \equiv \; & \{b^i_{-1} b^j_{-1}\alpha^k_{-1} + b^k_{-1} b^j_{-1}\alpha^i_{-1} + b^i_{-1} b^k_{-1}\alpha^j_{-1} + \frac{2\delta^{jk}}{2-D} b^i_{-1} b^n_{-1}\alpha^n_{-1}\} \\
& - \frac{2\delta^{ik}}{2-D} b^j_{-1} b^n_{-1}\alpha^n_{-1}\} \, | 0\rangle, \mathbf{160_v}
\end{aligned}$$

$$\begin{aligned} | 3\rangle &\equiv \{b^i_{-1}b^j_{-1}b^k_{-1}\} \mid 0\rangle, \mathbf{56_v} \\ | 4\rangle &\equiv \{b^i_{-1}b^j_{-1}\alpha^k_{-1} - b^i_{-1}b^k_{-1}\alpha^j_{-1} - b^k_{-1}b^j_{-1}\alpha^i_{-1}\} \mid 0\rangle, \mathbf{56_v} \end{aligned}$$

The tensorial form of the first state is dictated by $L_2$, which is obtained from the anticommutator of $G_2$ and $G_0$. As mentioned earlier, the same $L_2$ prohibits states with higher oscillators of the form $\alpha^i_{-3} \mid 0\rangle$, The forms of the second and fourth states are determined by the action of $G_2$. Finally, only the combination

$$\sqrt{3} \mid 3\rangle + \mid 4\rangle, \mathbf{56_v}$$

obeys $G_0 \mid \phi\rangle = \sqrt{3} \mid \phi\rangle$.

### 4.3 Ramond spectrum:

The physical states in the Ramond sector must satisfy

$$\begin{aligned} F_{2m+1} \mid \psi\rangle &= 0, m > 0 & (91) \\ L_{2n} \mid \psi\rangle &= 0, n > 0 & (92) \\ (L_0 + 1) \mid \psi\rangle &= 0. & (93) \end{aligned}$$

However, using the superalgebra (72-76), one can see that these infinite set of conditions can be reduced to just the $F_1$, $F_3$ and $L_0$ constraints. In addition, taking the square root of (76), we have an $f_0$ constraint which simplifies to

$$(\Gamma^0 + I) \mid \psi\rangle = 0. \tag{94}$$

This is what remains of the Dirac equation. We must now discuss the properties of the Ramond ground state $\mid \psi\rangle$ and the meaning of (94).

Majorana spinors with 16 components (and depending on 16 real parameters) are allowed in our $(8,1)$ Minkowski space-time. These 16 components consist of linear combinations of the two independent $SO(8)$ spinors, which we will denote by $\mathbf{8_s}$ and $\mathbf{8_c}$. These are projected out of a 16-component spinor by the operators $(\Gamma^0 + I)$ and $(\Gamma^0 - I)$, respectively. Thus (94) indeed serves as a Dirac equation in halving the number of independent components.

We now apply combinations of creation operators with $N = 0, 1, 2$ on ground states $\mid \psi_0\rangle$, and then obtain the physically allowed combinations by imposing the $F_1, F_3, L_0$ and $f_0$ conditions:

- $N = 0, \alpha'_{ND}M^2 = 1$

  $$\mid 0\rangle \equiv \mid \psi_\beta\rangle, \psi_\beta \sim \mathbf{8_s}$$

  Note the $f_0$ constraint (94) has eliminated $\mathbf{8_c}$ and kept $\mathbf{8_s}$.

- $N = 1, \alpha'_{ND}M^2 = 2$

  $$\mid 1\rangle \equiv \mid \psi^{ij}_{\beta'}\rangle \equiv (\alpha^i_{-1}d^j_0 + \alpha^j_{-1}d^i_0 - \frac{\delta^{ij}}{D-1}\alpha^k_{-1}d^k_0) \mid \psi_{\beta'}\rangle \sim \mathbf{35_v} \otimes \mathbf{8_c}$$

  is the only permissible combination. Imposing $(\Gamma^0 + I) \mid 1\rangle = 0$ forces $\mid \psi_{\beta'}\rangle$ to be $\mathbf{8_c}$ since $\Gamma^0$ anticommutes with $d^i_0$.

- $N=2, \alpha'_{ND}M^2=3$

$$\begin{aligned} |\,\psi^{ij}_{\beta}(\alpha\alpha)\rangle + |\,\psi^{ij}_{\beta}(dd)\rangle &\equiv (\alpha^i_{-1}\alpha^j_{-1} - \frac{\delta^{ij}}{D-1}\alpha^k_{-1}\alpha^k_{-1} \\ &- d^i_{-2}d^j_0 - d^j_{-2}d^i_0 + \frac{2\delta^{ij}}{D-1}d^k_{-2}d^k_0)\,|\,\psi_\beta\rangle, \sim \mathbf{35_v}\otimes\mathbf{8_s} \end{aligned}$$

The $\alpha\alpha$ and $dd$ parts of this separately satisfy the Virasoro constraints (92) and (93), but the superconstraints (91) force them into this particular combination. Because of the even number of $d$ modes, the basic spinor is $\mathbf{8_s}$. The $SO(8)$ transformation properties of these states have been indicated above, but one must be careful in distinguishing between these boldface numbers and the actual physical degrees of freedom (the NS spectrum, where the boldface numbers are identical with the number of physical states, is free of this complication). The R-sector $SO(8)$ generators are built out of $\alpha^i_n$'s and $d^i_n$'s, and thus transform the R-states exactly as indicated as in boldface. However, a look at the $N=1$ state shows that we cannot be dealing with 35x8 physical states: the $d^i_0$ merely shuffle the 8 components of the ground state spinor. Thus we have at most 8x8 = 64 states, but since the tensor is traceless, 8 spinor components corresponding to $\psi^{ii}_{\beta'}$ are absent, and the true physical content is $\mathbf{56_s}$. This is not as unfamiliar a situation as it might first appear: consider a scalar field $\theta$ and its gradient $\partial_\mu\theta$. The latter transforms as a $D$-vector, but the physical degree of freedom is still just $\theta$, which is a $D$-scalar.

Examining the physical content of the $N=2$ states, we see that $|\,\psi^{ij}_\beta(\alpha\alpha)\rangle$ contains 35x8 = 280 states, but these do not belong to a single irrep. Among the 280 states there is a $\mathbf{56_c}$ of the form $d^i_0\,|\,\psi^{ij}_\beta(\alpha\alpha)\rangle$ and the rest is $\mathbf{224_c}$. The part $|\,\psi^{ij}_\beta(dd)\rangle$ represents another $\mathbf{56_c}$, because the $d^i_0$ does not increase the number of states (bringing the number down from 280 to 64) and tracelessness in $ij$ takes off another 8. Thus the $N=2$ content is $\mathbf{224_c}+\mathbf{56_c}+\mathbf{56_c}$. Counting the $\alpha$ and $d$ modes separately and adding the numbers may seem surprising, but it is again not new: in the Higgs phenomenon, the massless photon field $A_\mu$ and $\partial_\mu\theta$ are added to form the massive vector field $B_\mu$. Although all three formally transform as 4-vectors, the final $B_\mu$ has 2+1 = 3 degrees of freedom.

# 5 Discussion

Our concern in this note has been twofold: The first was to see whether an RNS string with one fixed end would provide qualitative phenomenological hints and distinctive signatures about the spectrum of hadrons with one very heavy quark. The second concern was the more formal one of working out the novel consequences of the unusual mixed Dirichlet-Neumann boundary conditions and seeing whether the resulting system can be quantized in a consistent way. Just as in the original attempts to apply string theory to hadrons, these two aims are partially in conflict, most notably in the dimension of space-time.

Ignoring this conflict as was customarily done in the old string-based hadrodynamics, we can summarize the predictions of our model as follows: (i) There is no space-time supersymmetry except for the fact that meson and baryon Regge trajectories have the same slope, leading to equal meson and baryon masses for the higher states. (ii) The Regge slope is twice that of the one observed for lighter hadrons. (iii) There are no daughter trajectories, which means the number of states at mass $\sqrt{N}$ behave asymptotically as const. $N^{D-1}$ (in keeping with the policy of disregarding unrealistic demands of the model whenever inconvenient, $D-1$ presumably has to be taken as 3 rather than 8), rather than the $e^{\sqrt{N}}$ behavior seen for ordinary hadrons. An interesting point here is that if we consider an interaction where the free ends of two of our strings join while the heavy quark ends are kept fixed, we are led to a model in which $X^\mu \sim \Sigma\alpha_n^\mu e^{-in\tau} sin(n\sigma)$ with $\sigma$ in the range $[0,\pi]$. For such strings with both ends fixed on $D0$-branes, all $\alpha_n$ modes and, consequently, all $L_n$ are allowed. In this sector, there will be "daughter excitations", with the same spin but equally spaced masses. but not in the sense there are any leading linear Regge trajectories above them (the latter cannot be present because the balancing of centrifugal force against tension in a rigid rotation mode that defines leading trajectories is imposible when both ends are fixed). This is in qualitative, and one hopes, not entirely fortuitous accord with the multiplicity of $b\bar{b}$ and $c\bar{c}$ states of spin one. In any case, the Coulomb part of the QCD potential is known to play an important role in the dynamics of heavy quark-antiquark systems, ensuring deviations from a mass spectrum based on the purely string-based picture.

Turning to string-theoretic issues, we start with the question of whether using the more conventional $\sigma$ range $[0,\pi]$ would have made a physical difference. One may anticipate that the simultaneous doubling of the $\sigma$ range and the halving of the mode index $n$ will result in an equivalent physical system, and this is indeed what happens. The mixed boundary conditions now produce half-integral $\alpha$ and $b$ modes and integral Ramond $d$ modes, but the allowed physical spectrum is exactly as the one above except for an overall scaling. In particular, the vanishing of the NS vacuum energy and the preference for a $(8,1)$ space-time and $SO(8)$ symmetry remain. We prefer working with odd $\alpha$ and $b$ and even $d$ modes because this leads to the disappearence of odd $L_n$'s. We have broken Poincaré invariance by fixing one end of the string at a special point in space; this is consistent with discarding the operators $L_1$ and $L_{-1}$ which involve the generator of space translations.

The GSO projection turns out to be inapplicable to the mixed boundary condition superstring. This is already suggested by the fact that low-lying states do not exhibit space-time supersymmetry beyond matching masses and Regge slopes in the two sectors, but there are more basic manifestations of the incompatibility. For example, our final physical states are not homogeneous in the number of space-time fermions. Furthermore, in the R-sector we already have the remnant Dirac equation operators $\Gamma^0 \pm I$, but not in the role they play in the GSO projection. Since the mode structure here is different from the standard RNS string (for example, our $b$ operators add an integer rather than half an integer to the squared mass), one could not in any case have expected the GSO procedure to work in the usual way.

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