Observer-independent quantum of mass

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Abstract

It has been observed recently by Giovanni Amelino-Camelia [3, 4] that the hypothesis of existence of a minimal observer-independent (Planck) length scale is hard to reconcile with special relativity. As a remedy he postulated to modify special relativity by introducing an observer-independent length scale. In this letter we set forward a proposal how one should modify the principles of special relativity, so as to assure that the value of mass scale is the same for any inertial observer. It turns out that one can achieve this by taking dispersion relations such that the speed of light goes to infinity for finite momentum (but infinite energy), proposed in the framework of the quantum κ-Poincaré symmetry. It follows that at the Planck scale the world may be non-relativistic.

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In the recent years we face a growing mass of evidence (see e.g., [1]), coming both from loop quantum gravity, where one finds that area and volume are quantized [2], and from many aspects of string theory that space is quantized, i.e., there exists in nature a minimal length, usually identified with the Planck length $\mathcal{L}_P$. However, as pointed out recently by Giovanni Amelino-Camelia in a series of remarkable papers [3], [4] the hypothesis of existence of the fundamental minimal length is by itself puzzling. If there is something fundamental about the Planck length (i.e., if it has a status similar to that of the speed of light in special relativity), then it must have the same value for all inertial observers, which is hard to reconcile with one of the most basic results of special relativity, the FitzGerald-Lorentz contraction. So if we believe in the modern evidences, there are two choices: either to assume that the existence of the length scale reflects a property of some background field configuration which furnishes, what we call our universe[1], or, following [3], [4], to assume that the existence of the scale reflects fundamental, kinematical properties of space-time. In the latter case it follow from the relativity postulate (see below) that one should assume that that the fundamental scale has to be the same for all inertial observers. If we make such assumption, there is no choise, but to modify the principles of special relativity. Such a modification has been proposed in [3], [4]. In these papers the author proposes to promote a minimal length to the status of the speed of light in the standard special relativity, i.e., to assume that the value of the minimal length is observer-independent. To do so Amelino-Camelia proposes to consider a theory with non-standard dispersion relation for light (and thus with variable speed of light) and to illustrate this proposal he presents some simple, leading order computations. His proposal is a starting point of our analysis presented below.

Before turning to our investigations, let us make the following observation. There are three dimensionful constants in fundamental physics: speed of light c (as it will turn out this constant is only a long-wavelength limit of velocities of massless particles), Newton's gravitational constant G, and the Planck constant $\hbar$. All these constants should play a fundamental role in the quantum theory of gravity. Putting another way, in physics we have three fundamental scales, of length $\mathcal{L}_P = \sqrt{\hbar G/c^3} \sim 10^{-35}\, m$, time $\mathcal{T}_P = \sqrt{\hbar G/c^5} \sim 10^{-43}\, s$, and mass $\mathcal{M}_P = \sqrt{\hbar/Gc} \sim 10^{-8}\, kg$. Now, the problem is that if we believe in what special relativity teaches us, these scales are not observer-independent. We encounter therefore a paradox: On the one hand one would like the fundamental scales behave very much like the

[1] And thus the emergence of the scale has a dynamical origin.

speed of light behaves in special relativity (and this is the heart of Amelino-Camelia's observation), i.e., if any inertial observer attempts to measure them, he/she gets the same result, and on the other, any Lorentz-boosted observer would attribute to them different values.

In this paper, instead of considering the minimal length we will concentrate on a related problem of maximal mass, and only briefly comment on the minimal length issue.

Our starting point would be a set of the following postulates, being an slightly modified version of the postulates presented in [3]

1. (*Relativity principle*) The laws of physics take the same form in all inertial frames.

2. (*Speed of light*) The laws of physics involve the fundamental velocity scale c. This scale can be measured by each inertial observer as a speed of light with wavelength much longer than the fundamental length $\lambda\mathcal{L}_P$. The speed of light depends on the wavelength λ in such a way that it becomes infinity for finite λ (of order of Planck length $\mathcal{L}_P$.)

3. (*Mass scale*) The laws of physics involve fundamental mass scale, $\mathcal{M}_P$ which is the same for all inertial observers. This mass scale is related to the length scale as follows. For a photon of momentum $\mathcal{M}_P c$, the wavelength $\lambda = \mathcal{L}_P$.

Let us observe that the first part of the second postulate is much weaker than the analogous postulate in Einstein special relativity, where it is assumed that the speed of light does not depend on the wavelength and thus defines a universal tool for measuring space-time distances between events. On the other hand we know from the quantum theory that the wavy nature of light (and all matter) has fundamental character, and one cannot avoid taking it into account while considering a theory which is supposed to describe the quantum nature of space and time.

It is clear from the second postulate that our starting point to modify special relativity would be to allow for deviation from the standard dispersion relation for photons so as to allow for variable speed of light:

$$E^2 - c^2 p^2 = 0 \tag{1}$$

is to be raplaced by

$$\mathcal{F}\left(E, p; c, \mathcal{L}_P, \mathcal{M}_P\right) = 0. \tag{2}$$

Solving this equation for $E = f(p; c, \mathcal{L}_P, \mathcal{M}_P)$ we can define the variable speed of light $\mathcal{C}$ to be

$$\mathcal{C} = \frac{\partial E}{\partial p}. \tag{3}$$

Now it is easy to see what we need in order to satisfy our postulates. In the standard special relativity, one finds that masses and distances are observer-independent in non-relativistic limit, i.e., when $V/c \to 0$. We would encounter the same effect if we assume that the variable speed of light (3) goes to infinity for some finite value of momentum carried by the light wave, that is, for some finite value of its wavelength. This means that the modified relativity has two Galilean limits: one in the non-relativistic limit $V/c \ll 1$ and $\lambda/\mathcal{L}_P \gg 1$ and the second in the Planck regime $\lambda/\mathcal{L}_P \sim 1$[2].

In what follows we will be interested in a particular form of $\mathcal{F}$ which arises in the so-called quantum κ-Poincaré theory [5, 6, 7, 8]. This theory results from applying the ideas of quantum deformations to four-dimensional Poincare algebra, and leads to modifications of relativistic symmetries at the energy scales comparable to the κ parameter of the theory, which we will identify with $\mathcal{M}_P c$.

Among different realizations of quantum κ-Poincaré, we will be particularly interested in the so called $+$-bicrossproduct basis [6, 8][3], in which (restricted to two dimension) infinitesimal action of boosts N, with parameter ω takes the form

$$\delta p = \omega \left[\frac{\mathcal{M}_P c}{2} \left(1 - e^{-2E/\mathcal{M}_P c^2} \right) - \frac{1}{2\mathcal{M}_P c} p^2 \right] \tag{4}$$

$$\delta E = \omega\, c p \tag{5}$$

One can easily check that the following dispersion relation is invariant under these transformation rules (in the case of massive particles, one should replace 0 on the right hand side with $m^2 c^4$), i.e., the expression below is a Casimir of the κ-Poincaré algebra:

$$\left(2\, \mathcal{M}_P c^2 \, \sinh \frac{E}{2\mathcal{M}_P c^2} \right)^2 - c^2 p^2 e^{E/\mathcal{M}_P c^2} = 0. \tag{6}$$

[2]It i clear that if a theory being an extension of special relativity predicts $\mathcal{C} \to \infty$ in the ultra high energy regime, this regime should be Galilean. The reason is that this theory, to be consistent, must have special relativity as its limit, which in turn reduces to Galilean physics in the limit $V/c \ll 1$. But then both limits are to be equivalent.

[3]This realization has a virtue that the Lorentz sector, as well as the action of rotations on momenta are undeformed.

Of course, in the limit of large $\mathcal{M}_P$, i.e., $E/\mathcal{M}_P c^2 \ll 1$, $p/\mathcal{M}_P c \ll 1$, from (4), (5), and (6) one obtains the standard boost action and the dispersion relation (1), respectively.

The dispersion relation (6) has a remarkable property, that it furnishes a theory obeying the postulates presented above. Indeed if we write it in the form

$$\mathcal{M}_P^2 c^4 \left(1 - e^{-E/\mathcal{M}_P c^2}\right)^2 - c^2 p^2 = 0, \tag{7}$$

it is easy to see that when $E \to \infty$, $p \to \mathcal{M}_P c$, i.e., the energy of the wave with finite length is infinite. Putting differently, the wavelength dependent speed of light

$$\mathcal{C}(p) = \frac{dE}{dp} = c\left(1 - \frac{p}{\mathcal{M}_P c}\right)^{-1} = c\exp(E/\mathcal{M}_P c^2) \tag{8}$$

tends to infinity when $p \to \mathcal{M}_P c$. It should be stressed that in order to make the speed of light infinite one should use an infinite amount of energy, similarly to the standard special relativistic case, when one wants to make a massive particle to move with the speed of light.

Now it is easy to see that it follows from the infinitesimal transformations (4, 5) that all inertial observers would measure the same value of $\mathcal{M}_P$. Indeed

$$\delta p|_{p=\mathcal{M}_P c} = \lim_{E\to\infty} \omega \left[\frac{\mathcal{M}_P c}{2}\left(1 - e^{-2E/\mathcal{M}_P c^2}\right) - \frac{1}{2\mathcal{M}_P c} p^2\right]\Big|_{p=\mathcal{M}_P c} = 0.$$

In this way we satisfy third postulate. This is not very surprising after all, because in the limit $p \to \mathcal{M}_P c$ the speed of light becomes infinite, and the theory becomes effectively Galilean.

Let us observe now that in order to measure distances of the length ℓ we need to have in our disposal a photon of the length $\lambda \sim \ell$. Let us assume moreover that, like at low energies, momentum of the wave is inverse proportional to the wavelength $p \sim 1/\lambda$. Then simple dimensional analysis leads us to the expression $p \sim \mathcal{M}_P c\,\mathcal{L}_P/\lambda$, where $\mathcal{L}_P$ is the Planck length. But this means that if the relation (6) holds there must exist a minimal observable length equal exactly $\mathcal{L}_P$ (up to a numerical factor.) The same conclusion is true of course if one replaces (6) with any other dispersion relation with the property that energy goes to infinity for finite value of momentum. Thus the theory leads to predicting the existence of the minimal length. What we need to check is if this length would be the same, when measured by any inertial observer. Since in the relevant limit the theory is Galilean, it is almost obvious that this must be a case. However in order to prove this result one must extend the theory from the momentum sector to the whole phase space, which is non-commuting in the position sector (reflecting in this way a quantum character of the κ-Poincare group.) This will be done in a separate paper.

Let us complete this letter with a number of comments.

1. In this paper we worked in the two-dimensional framework. In $D = 4$ transformation (4) takes the form

$$\delta p_i = \omega_i \frac{\mathcal{M}_P c}{2} \left(1 - e^{-2E/\mathcal{M}_P c^2}\right) + \frac{1}{2\mathcal{M}_P c} \left(\delta_{ij} p^2 - \frac{1}{2} p_i p_j\right) \omega^j$$

   It should be noted that the second term is a conformal boost, so that the transform is a sum of a (deformed) standard boost and the conformal one. The $4D$ transformations and their physical implications will be investigated in a separate paper [10].

2. One should observe that the theory presented here is fully falsifable even at this very premature stage. First of all it is fully consistent with all present experimental data [11]. Second there are proposal of experiments to be performed in the coming years, aimed at checking the cnsequences of the moodified dispersion relations of the form (6) [12].

3. The result of this paper, namely the prediction concerning the energy dependence of the speed of light might find its application in the cosmological models in which the variable speed of light makes it possible to solve the well known problems of the standard cosmological model [13]. It should be noted that in the model presented here the speed of light grows with energy, so it should be much higher than c in the very early universe, the behavior which would in principle allow to resolve cosmological puzzles.

4. Last, but not least, it should be observed that if the main result of this letter is correct, namely that the existence of the observer-independent fundamental mass scale results from the fact that the velocity of light goes to infinity for finite wavelength, it follows that physics on Planck scale is not governed by any relativistic theory. Rather, the theory of space, time and processes at this scale, i.e., the theory of quantum gravity should be a Galilean theory possibly (given the non-commutative space-time structure resulted from the quantum

algebra structure of the κ-Poincaré algebra [6], [8]) with a discrete, non-commutative space and time. This idea in the author's opinion certainly deserves further investigations.

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