

Supersymmetric theories with compact extra dimensions in $N=1$ superfields

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Abstract

We present an $N=1$ superfield formulation of supersymmetric gauge theories with a compact extra dimension. The formulation incorporates the radion superfield and allows to write supersymmetric theories on warped gravitational backgrounds. We apply it to study the breaking of supersymmetry by the F -term of the radion, and show that, for flat extra dimensions, this leads to the same mass spectrum as in Scherk-Schwarz models of supersymmetry breaking. We also consider scenarios where supersymmetry is broken on a boundary of a warped extra dimension and compare them with anomaly mediated models.

1 Introduction

Supersymmetry and extra dimensions are well-motivated extensions of the Standard Model. They could play a role in the hierarchy problem, or be crucial ingredients in a quantum description of gravity, as is the case in string theory. Also extra dimensions and supersymmetry could be connected to the origin of the electroweak symmetry breaking [1].

To study supersymmetric theories it is very useful to have a superfield description where supersymmetry invariance is manifest, and nonrenormalization theorems are easily derived [2]. The $\mathcal{N} = 1$ superfield formalism has been extensively analyzed in four dimensions. In higher dimensions, however, supersymmetry is usually presented in component fields, since a superfield description is usually not known. A first attempt to write higher-dimensional supersymmetric theories in superfields was presented in Ref. [3] for theories in 10 dimensions, and has been recently extended to other dimensions in Ref. [4]. The formulation of Refs. [3, 4] is based on writing higher-dimensional supersymmetric theories using $\mathcal{N} = 1$ four-dimensional superfields. These are the ordinary superfields defined in a 4D superspace. Higher-dimensional supersymmetric theories contain the 4D supersymmetry and therefore it is always possible to write them using $\mathcal{N} = 1$ superfields. With this formulation, only the $\mathcal{N} = 1$ supersymmetry is manifest. In spite of this limitation, we think that the formulation is already very useful for supersymmetric theories with extra dimensions. The effective action is simpler to write than in component fields, and the bulk-boundary couplings are easily obtained. Nonrenormalization theorems can also be derived.

Here we will use the $\mathcal{N} = 1$ superfield formulation to write the action of a 5D supersymmetric theory with a compact extra dimension. The important new ingredient of our formulation is that it incorporates the radion superfield T . This will allow us to write the supersymmetric action for fields living in either a flat or a warped extra dimension. In particular, we will consider a gauge theory with 5D vector multiplets and charged hypermultiplets in an extra dimension (1) flat and compactified in a circle S^1 or orbifold S^1/\mathbb{Z}_2 , (2) warped as in the Randall-Sundrum (RS) scenario [5].

We will apply this superfield formulation to study supersymmetry breaking induced by the T -term of the radion superfield. This occurs when a constant superpotential is present in the bulk of the extra dimension. For a flat extra dimension we will show that this is equivalent to break supersymmetry by boundary conditions (Scherk-Schwarz (SS) mechanism [6]). In particular, we will derive the mass spectrum of the models of Refs. [7, 8]. Our formulation therefore will provide a superfield description of the SS mechanism and will show that the SS breaking of supersymmetry is spontaneous.

For a warped extra dimension, we will consider the breaking of supersymmetry induced by a boundary superpotential, and derive the mass spectrum in the gauge sector. We will show the similarities of this breaking with models of anomaly mediated supersymmetry breaking (AMSB) [9].

A final comment is in order. In our formulation we will be only considering part of the 5D supergravity multiplet, the radion superfield T . Therefore our approach does not incorporate the full 5D gravitational sector. The action derived below must be considered as that of supersymmetric theories on nontrivial gravitational backgrounds. A complete formulation with the full 5D supergravity multiplet is a subject of future research.

2 5D superfield action with a flat and compact extra dimension

Let us consider a 5D theory in $\mathcal{M}^4 \times S^1$. The metric is given by

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2, \quad (1)$$

where R is the radion of the extra dimension labeled by y , which ranges from 0 to 2π . We want to derive the action of superfields living on the 5D gravitational background of Eq. (1). For this purpose, we need to promote R to a superfield. This corresponds to a 4D chiral superfield T that, together with R , it is known to contain the fifth-component of the graviphoton B_5 , the fifth-component of the right-handed gravitino Ψ_R^5 and a complex auxiliary field F_T , and we will write it as

$$T = R + iB_5 + \theta\Psi_R^5 + \theta^2 F_T. \quad (2)$$

2.1 Vector supermultiplet

The off-shell 5D $N=1$ vector supermultiplet consists of a 5D vector A_M , two Weyl gauginos $\lambda_{1,2}$, a real scalar Σ , and a real and complex auxiliary field D and F_χ respectively. Under the $N=1$ supersymmetry, they form a vector supermultiplet V and a chiral supermultiplet χ ¹:

$$\begin{aligned} V &= -\theta\sigma^\mu\bar{\theta}A_\mu - i\bar{\theta}^2\theta\lambda_1 + i\theta^2\bar{\theta}\bar{\lambda}_1 + \frac{1}{2}\bar{\theta}^2\theta^2 D, \\ \chi &= \frac{1}{\sqrt{2}}(\Sigma + iA_5) + \sqrt{2}\theta\lambda_2 + \theta^2 F_\chi, \end{aligned} \quad (3)$$

where V is given in the Wess-Zumino gauge. Let us first consider an Abelian theory. Under a gauge transformation, the superfields transform as

$$\begin{aligned} V &\rightarrow V + \Lambda + \Lambda^\dagger, \\ \chi &\rightarrow \chi + \sqrt{2}\partial_5\Lambda, \end{aligned} \quad (4)$$

where Λ is an arbitrary chiral field. The gauge invariant action is given by

$$S_5 = \int d^5x \left[\frac{1}{4g_5^2} \int d^2\theta TW^\alpha W_\alpha + \text{h.c.} + \frac{2}{g_5^2} \int d^4\theta \frac{1}{(T + T^\dagger)} \left(\partial_5 V - \frac{1}{\sqrt{2}}(\chi + \chi^\dagger) \right)^2 \right]. \quad (5)$$

It is easy to check that this superfield action leads to the right action in component fields. We must perform the integrals over θ taking $\langle T \rangle = R$, eliminate the auxiliary fields using their equation of motion

$$F_\chi = 0, \quad D = -\frac{\partial_5 \Sigma}{R^2}, \quad (6)$$

and rescale the fields according to $\Sigma \rightarrow R\Sigma$, $\lambda_2 \rightarrow -iR\lambda_2$. We finally obtain

$$S_5 = \frac{1}{g_5^2} \int d^5x \sqrt{-g} \left[-\frac{1}{2} \partial_M \Sigma \partial^M \Sigma - \frac{1}{4} F_{MN} F^{MN} + \frac{i}{2} \bar{\lambda}_i \gamma^M \partial_M \lambda_i \right], \quad (7)$$

¹We will follow the notation of Ref. [2].

where we have defined the symplectic-Majorana spinors $[\lambda_i]^T \equiv (\lambda_i, \epsilon_{ij} \bar{\lambda}_j)$ in order to make Eq. (7) manifestly invariant under the $SU(2)$ automorphism group [10].

For the non-Abelian case, the second term of Eq. (5) must be replaced by

$$\frac{2}{g_5^2} \int d^4\theta \frac{1}{(T + T^\dagger)} \text{Tr} \left[\{e^{V/2}, \partial_5 e^{-V/2}\} + \frac{1}{\sqrt{2}} (e^{V/2} \chi^\dagger e^{-V/2} + e^{-V/2} \chi e^{V/2}) \right]^2, \quad (8)$$

that is gauge invariant under the gauge transformation

$$\begin{aligned} \chi &\rightarrow U^{-1}(\chi - \sqrt{2}\partial_5)U, & e^V &\rightarrow U^{-1}e^V U^{-1\dagger}, \\ \chi^\dagger &\rightarrow U^\dagger(\chi^\dagger + \sqrt{2}\partial_5)U^{-1\dagger}, & e^{-V} &\rightarrow U^\dagger e^{-V} U, \end{aligned} \quad (9)$$

where $U = e^{-\Lambda}$, $U^\dagger = e^{-\Lambda^\dagger}$, $\Lambda = \Lambda^a T^a$, $\chi \equiv \chi^a T^a$ and $V \equiv V^a T^a$. For $T = \text{constant}$, Eq. (8) differs from Ref. [4] only in chiral or antichiral terms which vanish under the integration over the whole superspace $\int d^4\theta$. Nevertheless, these terms are nonzero for the case in which T is a superfield with a nontrivial θ dependence, and they must be taken into account.

2.2 Hypermultiplet

The off-shell 5D hypermultiplet consists in two complex scalars, ϕ and ϕ^c , a Dirac fermion Ψ and two complex auxiliary fields F_Φ and F_{Φ^c} . It can be arranged in two $N = 1$ chiral superfields, Φ and Φ^c . Assuming that they are charged under some gauge group and transform as $\Phi \rightarrow U^{-1}\Phi$ and $\Phi^c \rightarrow \Phi^c U$, we have that the 5D action for the hypermultiplet is given by

$$S_5 = \int d^5x \left\{ \int d^4\theta \frac{1}{2}(T + T^\dagger) (\Phi^\dagger e^{-V} \Phi + \Phi^c e^V \Phi^{c\dagger}) + \int d^2\theta \Phi^c \left[\partial_5 - \frac{1}{\sqrt{2}} \chi \right] \Phi + \text{h.c.} \right\}. \quad (11)$$

For $\langle T \rangle = R$, the auxiliary fields are given by

$$\begin{aligned} F_\Phi &= \frac{1}{R} \left(\partial_5 + \frac{1}{2}(\Sigma - iA_5) \right) \phi^{c\dagger}, & F_\chi &= \frac{g_5^2 R}{\sqrt{2}} \phi^\dagger T^a \phi^{c\dagger}, \\ F_{\Phi^c}^\dagger &= -\frac{1}{R} \left(\partial_5 - \frac{1}{2}(\Sigma + iA_5) \right) \phi, & D^a &= -\frac{\partial_5 \Sigma^a}{R^2} + \frac{g_5^2}{2} (\phi^\dagger T^a \phi - \phi^c T^a \phi^{c\dagger}). \end{aligned} \quad (12)$$

By rescaling Σ and λ_2 as in Section 2.1, Eq. (11) gives

$$\begin{aligned} S_5 &= \int d^5x \sqrt{-g} \left\{ -|D_M \phi|^2 - |D_M \phi^c|^2 + i\bar{\Psi} \gamma^M D_M \Psi - \frac{1}{\sqrt{2}} (\phi^c \bar{\lambda}_1 \Psi - \phi^\dagger \bar{\lambda}_2 \Psi) + \text{h.c.} \right. \\ &\quad \left. - \frac{i}{2} \bar{\Psi} \Sigma \Psi - \frac{1}{4} (\phi_i^\dagger \Sigma^2 \phi_i) - \frac{g_5^2}{8} \sum_{m,a} (\phi_i^\dagger (\sigma^m)_{ij} T^a \phi_j)^2 \right\}, \end{aligned} \quad (13)$$

where in the last two terms we have defined $\{\phi_1, \phi_2\} \equiv \{\phi, \phi^{c\dagger}\}$ and the gauge covariant derivative as $D_M = \partial_M - \frac{i}{2} A_M^a T^a$. A supersymmetric mass for the hypermultiplet,

$$\int d^2\theta \Phi^c m \Phi + \text{h.c.}, \quad (14)$$

can be easily included by performing in Eq. (11) the shift $\chi \rightarrow \chi - \sqrt{2}m$ that in Eq. (13) corresponds to $\Sigma \rightarrow \Sigma - 2m$.

2.3 The orbifold S^1/\mathbb{Z}_2 and bulk-boundary couplings

The above bulk action is not modified if the extra dimension is compactified in the orbifold S^1/\mathbb{Z}_2 , which corresponds to the circle S^1 with the identification $y \leftrightarrow -y$. This identification leads to a manifold with two boundaries at $y=0$ and at $y=\pi$.

There can be fields living on these 4D boundaries. They respect an $N=1$ supersymmetry and therefore their superfield action is the ordinary one. The couplings of the boundary fields to the bulk fields are easily obtained using superfields. Assuming that \mathbf{V} and \mathbf{X} are respectively even and odd under the \mathbb{Z}_2 parity, we have that \mathbf{X} vanishes on the boundaries and therefore only \mathbf{V} couples to fields on the boundaries. For the hypermultiplet, if we assume that Φ and Φ^c are respectively even and odd under the \mathbb{Z}_2 , we have that only Φ can couple to the boundary fields. For a chiral superfield Q living on the $y=0$ boundary these couplings are simply given by

$$S_5 = \int d^5x \left[\int d^4\theta \left(Q^\dagger e^{-V} Q + e^{-V} \xi \right) + \int d^2\theta W(\Phi, Q) + \text{h.c.} \right] \delta(y), \quad (15)$$

where \mathbf{W} is a superpotential that can depend on Φ and Q , and ξ is a Fayet-Iliopoulos term that can be present for an Abelian vector supermultiplet. The boundary couplings Eq. (15) change the auxiliary field equation of motion by δ -function terms:

$$\begin{aligned} D &= -\frac{\partial_5 \Sigma}{R^2} + \frac{g_5^2}{2} \delta(y) (Q^\dagger Q + \xi), \\ F_\Phi &= \frac{1}{R} \left(\partial_5 + \frac{1}{2} (\Sigma - iA_5) \right) \phi^{c\dagger} - \delta(y) \left. \frac{\partial W}{\partial \Phi} \right|_{\Phi=\phi}. \end{aligned} \quad (16)$$

Similarly we can obtain the couplings on the boundary at $y=\pi$.

3 Superfield action in a warped 5D space

The above action can be generalized to the case where the extra dimension is warped. As an example, we will consider the RS scenario [5] where the extra dimension y is compactified on an orbifold S^1/\mathbb{Z}_2 of radius R , with $-\pi \leq y \leq \pi$. The 5D space is defined by the metric

$$ds^2 = e^{-2R\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2, \quad (17)$$

where

$$\sigma = k|y|, \quad (18)$$

and $1/k$ is the curvature radius. This space corresponds to a slice of AdS_5 . The supersymmetric version of the RS model has been recently studied in Refs. [11, 12]. The supersymmetric conditions for vector and hypermultiplets on the background of Eq. (17) were derived in Ref. [11]. Here we will present the action written in superfields.

3.1 Vector supermultiplet

The action for the Abelian vector superfield is given by

$$S_5 = \int d^5x \left[\frac{1}{4g_5^2} \int d^2\theta TW^\alpha W_\alpha + \text{h.c.} + \frac{2}{g_5^2} \int d^4\theta \frac{e^{-(T+T^\dagger)\sigma}}{(T+T^\dagger)} \left(\partial_5 V - \frac{1}{\sqrt{2}}(\chi + \chi^\dagger) \right)^2 \right]. \quad (19)$$

The auxiliary fields are given by

$$F_\chi = 0, \quad D = -\frac{e^{-2R\sigma}}{R^2} (\partial_5 - 2R\sigma') \Sigma, \quad (20)$$

where $\sigma' = \partial_5 \sigma$. After the rescaling $\Sigma \rightarrow R\Sigma$, $\lambda_1 \rightarrow e^{-3R\sigma/2} \lambda_1$, $\lambda_2 \rightarrow -iRe^{-R\sigma/2} \lambda_2$, we obtain the action

$$S_5 = -\frac{1}{g_5^2} \int d^5x \sqrt{-g} \left[\frac{1}{2} (\partial_M \Sigma)^2 + \frac{1}{2} m_\Sigma^2 \Sigma^2 + \frac{1}{4} F_{MN}^2 - \frac{i}{2} \bar{\lambda}_i \gamma^M D_M \lambda_i - m_\lambda \frac{i}{2} \bar{\lambda}_i [\sigma_3]_{ij} \lambda_j \right], \quad (21)$$

where $\sigma_3 = \text{diag}(1, -1)$, $D_M \lambda_i = \partial_M \lambda_i + \Gamma_M [\sigma_3]_{ij} \lambda_j$, being Γ_M the spin connection, $\Gamma_M = (\sigma' \gamma_5 \gamma_\mu / 2, 0)$, and where

$$m_\Sigma^2 = -4k^2 + 2\frac{\sigma''}{R}, \quad m_\lambda = \frac{1}{2}\sigma', \quad (22)$$

in agreement with Ref. [11].

3.2 Hypermultiplet

The action is given by

$$S_5 = \int d^5x \left\{ \int d^4\theta \frac{1}{2} (T + T^\dagger) e^{-(T+T^\dagger)\sigma} (\Phi^\dagger e^{-V} \Phi + \Phi^c e^V \Phi^{c\dagger}) + \int d^2\theta e^{-3T\sigma} \Phi^c \left[\partial_5 - \frac{1}{\sqrt{2}} \chi - \left(\frac{3}{2} - c \right) T\sigma' \right] \Phi + \text{h.c.} \right\}, \quad (23)$$

where following Ref. [11] we have parametrized the hypermultiplet mass as $c\sigma'$. Neglecting for simplicity the gauge sector, we have that the auxiliary fields are given by

$$F_\Phi = \frac{e^{-R\sigma}}{R} \left[\partial_5 + \left(\frac{3}{2} - c \right) R\sigma' \right] \phi^{c\dagger}, \quad F_{\Phi^c}^\dagger = -\frac{e^{-R\sigma}}{R} \left[\partial_5 - \left(\frac{3}{2} - c \right) R\sigma' \right] \phi, \quad (24)$$

that after the rescaling $\Psi \rightarrow e^{-R\sigma/2} \Psi$ yields,

$$S_5 = \int d^5x \sqrt{-g} \left\{ (-|\partial_M \phi|^2 - |\partial_M \phi^c|^2 - m_\phi^2 |\phi|^2 - m_{\phi^c}^2 |\phi^c|^2 + i\bar{\Psi} \gamma^M (\partial_M + \Gamma_M) \Psi - im_\Psi \bar{\Psi} \Psi) \right\}, \quad (25)$$

where

$$m_{\phi, \phi^c}^2 = \left(c^2 \pm c - \frac{15}{4} \right) k^2 + \left(\frac{3}{2} \mp c \right) \frac{\sigma''}{R}, \quad m_\Psi = c\sigma'. \quad (26)$$

3.3 Low-energy 4D effective theory

At energies below the KK masses (that in the warped case are of order $ke^{-Rk\pi}$), the KK can be integrated out and the effective theory can be written with only the massless sector. For the 5D vector superfield this corresponds to the zero mode of the superfield \mathbf{V} . Its wave-function $f_0(y)$ is determined by $\partial_5 f_0 = 0$, so it is y -independent [13]. The 4D effective lagrangian for the massless mode of \mathbf{V} is therefore the same as the one with a flat extra dimension:

$$\mathcal{L}_{4D} = \frac{\pi}{2g_5^2} \int d^2\theta TW^\alpha W_\alpha + \text{h.c.} . \quad (27)$$

For the hypermultiplet only the chiral superfield Φ (even under \mathbb{Z}_2) has a massless mode. Its wave-function satisfies $[\partial_5 - (\frac{3}{2} - c)T\sigma']f_0 = 0$ that yields $f_0 = e^{(\frac{3}{2}-c)T\sigma}$. After integrating over y , we get the effective lagrangian

$$\mathcal{L}_{4D} = \int d^4\theta \frac{1}{(\frac{1}{2} - c)k} \left(e^{(\frac{1}{2}-c)(T+T^\dagger)k\pi} - 1 \right) \Phi^\dagger e^{-V} \Phi . \quad (28)$$

3.4 Bulk-boundary couplings

For a chiral superfield Q living on the $y=0$ boundary the bulk-boundary couplings are the same as those in Eq. (15). On the other boundary, at $y=\pi$, we have

$$S_5 = \int d^5x \left[\int d^4\theta e^{-(T+T^\dagger)k\pi} \left(Q^\dagger e^{-V} Q + e^{-V} \xi \right) + \int d^2\theta e^{-3T k\pi} W(\Phi, Q) + \text{h.c.} \right] \delta(y - \pi) . \quad (29)$$

4 Supersymmetry breaking by the radion \mathbf{F} -term

As an application of the action derived above, we will calculate the spectrum of soft masses that is obtained when supersymmetry is broken by a nonzero \mathbf{F} -term of the radion superfield. We will first consider the case of a flat extra dimension with a constant (y -independent) \mathbf{F}_T . We will show that the resulting mass spectrum is the same as that in models with SS supersymmetry breaking. For a warped extra dimension we will consider the case when \mathbf{F}_T is induced on the boundary. We will see that \mathbf{F}_T is exponentially suppressed, generating small soft masses.

4.1 A flat extra dimension

The simplest model to break supersymmetry by the radion \mathbf{F} -term is the no-scale model [14]. In superfields this is given by

$$S_5 = \int d^5x \left[-3M_5^3 \int d^4\theta \varphi^\dagger \varphi \frac{(T + T^\dagger)}{2} + \int d^2\theta \varphi^3 W + \text{h.c.} \right] , \quad (30)$$

where \mathbf{W} is a “superpotential” that can arise in gauged supergravity theories [15]. We are considering that \mathbf{W} is constant (y -independent) and real. Eq. (30) corresponds to the effective action of the radion of a flat extra dimension. We have introduced the conformal compensator

φ [16]. This is a non-propagating superfield useful to incorporate the supergravity effects to the effective radion potential. The superfield φ makes the 4D supergravity action manifestly invariant under a conformal transformation. The breaking of the superconformal group down to the super-Poincare group is parametrized by the scalar component of φ and the R -term of φ corresponds to the auxiliary component of the supergravity multiplet:

$$\varphi = 1 + \theta^2 F_\varphi. \quad (31)$$

From Eq. (30) we obtain

$$F_T = \frac{2W}{M_5^3}, \quad F_\varphi = 0, \quad (32)$$

that leads to a vanishing potential for the radion (at tree level). Therefore if the vacuum expectation value of W is nonzero, supersymmetry is broken by a nonzero F_T with zero cosmological constant. This is a very simple example of breaking supersymmetry by F_T . This model, however, does not stabilize the radius R . For this purpose, one could need some extra massive matter fields, as for example in Ref. [17]. Other scenarios have been proposed in Ref. [18]. In what follows, we will assume that the radius is stabilized without affecting Eq. (32).

Let us derive the soft masses in the gauge sector for this scenario. Taking $T = R + \theta^2 F_T$ in Eq. (5), we obtain an extra mass term for the 5D gauginos:

$$\mathcal{L}_{soft} = \frac{1}{2} \frac{F_T}{R} \lambda_i^T C \lambda_i, \quad (33)$$

where C is the charge-conjugation matrix in 5D. Eq. (33) is a Majorana mass term. In the case of the orbifold S^1/\mathbb{Z}_2 where we decompose the 5D fields in KK states, $\lambda_1 = \sum \cos(ny) \lambda_1^{(n)}(x)$ and $\lambda_2 = \sum \sin(ny) \lambda_2^{(n)}(x)$, the gaugino masses are given by

$$\mathcal{L}_{mass} = \frac{F_T}{2R} \lambda_1^{(0)} \lambda_1^{(0)} + \frac{1}{R} \sum_{n=1}^{\infty} \begin{pmatrix} \lambda_1^{(n)} & \lambda_2^{(n)} \end{pmatrix} \begin{pmatrix} F_T/2 & n \\ n & F_T/2 \end{pmatrix} \begin{pmatrix} \lambda_1^{(n)} \\ \lambda_2^{(n)} \end{pmatrix} + \text{h.c.}, \quad (34)$$

which give rise to Majorana gaugino masses $F_T/2 \pm n/R$. This is the same spectrum as the one obtained by SS supersymmetry breaking with an R -symmetry [7]. The correspondence is $F_T/2 = q_R$ that implies $W/M_5^3 = q_R$, where q_R is the R -charge defined in Ref. [7]. For $F_T = 1$ it also corresponds to the gaugino mass spectrum of Ref. [8]. Notice that the scalar Σ does not get a soft mass as it is also predicted by SS supersymmetry breaking since the R -charge of Σ is zero.

For the hypermultiplet a nonzero F_T gives in Eq. (11) an extra contribution to the R -terms of the scalars. These are now given by

$$\begin{aligned} F_\Phi &= \frac{1}{R} \left([\partial_5 + i\alpha] \phi^{c\dagger} - \frac{1}{2} F_T \phi \right), \\ F_{\Phi^c}^\dagger &= -\frac{1}{R} \left([\partial_5 + i\alpha] \phi + \frac{1}{2} F_T \phi^{c\dagger} \right), \end{aligned} \quad (35)$$

where, for later use, we have also included a supersymmetric mass, Eq. (14), with $m = i\alpha$. After KK reduction in a circle S^1 (imposing periodic boundary conditions), the mass term for

the scalars is given by

$$\mathcal{L}_{mass} = -\frac{1}{R^2} \sum_{n=-\infty}^{\infty} (\phi^{(n)\dagger} \quad \phi^{c(n)\dagger}) \begin{pmatrix} (n+\alpha)^2 + F_T^2/4 & i(n+\alpha)F_T \\ -i(n+\alpha)F_T & (n+\alpha)^2 + F_T^2/4 \end{pmatrix} \begin{pmatrix} \phi^{(n)} \\ \phi^{c(n)} \end{pmatrix}, \quad (36)$$

that corresponds to scalars with masses $|n + \alpha \pm F_T/2|/R$. Fermions do not get soft masses and the mass spectrum is given by $|n + \alpha|/R$. Let us compare the mass spectrum of Eq. (36) with that in the models of Refs. [7] and [8].

In Ref. [8] quarks and leptons were associated to hypermultiplets that would correspond to the above hypermultiplet with $\alpha = 0$, $F_T = 1$ and, after orbifolding, $n = 0, 1, 2, \dots$. To derive the mass spectrum of the Higgs of Ref. [8], we must proceed as follows. Let us absorb the supersymmetric mass $m = i\alpha$ of the hypermultiplet by the redefinition ²

$$\Phi' = e^{i\alpha y} \Phi, \quad \Phi^{c'} = e^{-i\alpha y} \Phi^c, \quad (37)$$

and assign the following \mathbb{Z}_2 parities: $\Phi'(y) \rightarrow \Phi'(-y)$ and $\Phi^{c'}(y) \rightarrow -\Phi^{c'}(-y)$. For $\alpha = 1/2$, we have that the KK decomposition in the S^1/\mathbb{Z}_2 orbifold is given by $\Phi' = \sum_n \cos[(n + 1/2)y] \Phi^{(n)}(x)$ and $\Phi^{c'} = \sum_n \sin[(n + 1/2)y] \Phi^{c(n)}(x)$ where $n = 0, 1, 2, \dots$. The mass spectrum is supersymmetric and is given by $|n + 1/2|/R$. Let us now turn on the F_T and break supersymmetry. The scalar mass matrix will be given by Eq. (36) with $\alpha = 1/2$ and $n = 0, 1, 2, \dots$, and therefore the scalar masses will be $|n + 1/2 \pm F_T/2|/R$. For $F_T = 1$ we obtain the same mass spectrum as that of the Higgs in Ref. [8]. There is a single massless scalar that is associated with the SM Higgs.

In Ref. [7] quarks and leptons were localized on the boundaries of the orbifold while the Higgs sector was living in the bulk. To give masses to the fermions, the Higgs sector had to consist in two hypermultiplets (Φ_i, Φ_i^c) $i = 1, 2$. Assigning the \mathbb{Z}_2 parity as $\Phi(y) \rightarrow \eta \Phi(-y)$, with $\eta = +1(-1)$ for Φ_1 and Φ_2 (Φ_1^c and Φ_2^c), the Higgs sector can be written as

$$S_5 = \int d^5x \left\{ \int d^4\theta \frac{1}{2} (T + T^\dagger) (\Phi_i^\dagger \Phi_i + \Phi_i^c \Phi_i^{c\dagger}) + \int d^2\theta \Phi_i^c (\partial_5 \delta_{ij} + m \epsilon_{ij}) \Phi_j + \text{h.c.} \right\}, \quad (38)$$

where a supersymmetric mass m compatible with the \mathbb{Z}_2 parity has been introduced. For $F_T \neq 0$, the scalar masses are given by $|m - F_T/2 \pm n|/R$ and $|m + F_T/2 \pm n|/R$, while the fermion sector has masses $|m \pm n|/R$. This spectrum is identical to that of Ref. [7] with the identification $m = q_H$ and $F_T = 2q_R$.

In conclusion, we have showed that a constant (y -independent) W yields supersymmetry breaking parametrized by the F -term of the radion and gives the same spectrum as supersymmetry broken by boundary conditions. In particular, we have recovered the mass spectrum of the models of Refs. [7] and [8]. Since supersymmetry is broken by the F -term of T , the Goldstino field corresponds to the fermionic component of T that is the fifth-component of the right-handed gravitino, Ψ_R^5 (in a different way, this has also been shown in Ref. [19]).

²This y -dependence of the hypermultiplet can be the result of imposing the (supersymmetric) boundary condition $\Phi(x, y + 2\pi) = e^{i2\pi\alpha} \Phi(x, y)$ and similarly for Φ^c .

What happens when W is localized on the boundary instead of in the bulk? ³ In this case the gaugino mass term Eq. (33) is localized on the boundary and therefore will only affect the even modes ⁴:

$$\mathcal{L}_{soft} = \frac{\delta(y)}{R} \frac{W}{M_5^3} \lambda_1 \lambda_1 + \text{h.c.}, \quad (39)$$

for a canonically normalized λ_1 . This term can be thought to arise from Eq. (5) with $F_T = 2\delta(y)W/M_5^3$. The term (39) produces a mixing between the KK. To obtain the mass spectrum one has to redefine the KK states. It is much simpler, however, to obtain the mass spectrum by solving the 5D equation of motion of the gauginos with the term Eq. (39) included. This has already been done in Ref. [21]. Instead of repeating this here, we will consider this scenario of supersymmetry breaking in a warped extra dimension.

4.2 A warped extra dimension

The interesting situation for a warped extra dimension is when the breaking of supersymmetry is located on the boundary at $y = \pi$. In this case the soft masses are exponentially suppressed, explaining the hierarchy between the weak and the Planck scale [11, 22, 23, 24]. Here we will consider the effects of triggering a constant superpotential term W on the $y = \pi$ boundary. This breaks supersymmetry inducing a gaugino mass given by

$$\mathcal{L}_{soft} = \frac{\delta(y)}{R} \frac{e^{-Rk\pi} W}{M_5^3} \lambda_1 \lambda_1 + \text{h.c.}. \quad (40)$$

Eq. (40) shifts the gaugino mass spectrum from the gauge-boson one. The exact mass eigenvalues m_n of the gauginos are derived in the Appendix. For large supersymmetry breaking, $W \gg M_5^3$, the Majorana gaugino masses become independent of W :

$$\pm \sqrt{\frac{2}{Rk\pi}} k e^{-Rk\pi}, \pm \frac{5}{4} \pi k e^{-Rk\pi}, \pm \frac{9}{4} \pi k e^{-Rk\pi}, \dots \quad (41)$$

Notice that in this limit, $W/M_5^3 \rightarrow \infty$, the gauginos can be combined in Dirac fermions and the theory becomes $U(1)_R$ invariant. This is exactly the same spectrum as the one obtained when supersymmetry is broken by boundary conditions [22].

For small supersymmetry breaking, $W \ll M_5^3$, only the zero modes are substantially affected (the KK masses get small corrections as it is shown in the Appendix). We can analyze these effects by just considering the effective theory at energies below the KK masses with only the zero modes. For the vector and hypermultiplet sector the effective theory is given by Eqs. (27) and (28). For the radion, the effective lagrangian is given by [25]

$$\mathcal{L}_{4D} = -\frac{6M_5^3}{k} \int d^4\theta \varphi^\dagger \varphi (1 - e^{-(T+T^\dagger)k\pi}) + \int d^2\theta \varphi^3 [W_0 + e^{-3Tk\pi} W + \text{h.c.}], \quad (42)$$

³As far as the low-energy effective theory is concerned (the one describing physics below the KK masses and consisting of only of the zero modes), it is clear that there is no difference from the case where W is coming from the bulk as long as $W/M_5^3 \ll 1$ and the zero modes can be considered lighter than the KK.

⁴See Ref. [20] for the case of the gravitino.

where W_0 and W are the superpotentials on the boundary at $y=0$ and $y=\pi$ respectively. W_0 has been introduced to cancel the cosmological constant as we will see below. From Eq. (42), we obtain the auxiliary fields

$$F_T = e^{-Rk\pi} \frac{W}{2\pi M_5^3} + \frac{W_0}{2\pi M_5^3}, \quad F_\varphi = \frac{kW_0}{2M_5^3}, \quad (43)$$

and the effective radion potential

$$V = \frac{3k}{2M_5^3} (e^{-4Rk\pi} |W|^2 - |W_0|^2). \quad (44)$$

Unlike the flat case, the vanishing of the potential is not guaranteed for a constant W . In fact, one must tune $|W_0|^2 = e^{-4Rk\pi} |W|^2$ to have a zero cosmological constant. In this case, we get

$$F_T = \frac{W}{2\pi M_5^3} e^{-Rk\pi}, \quad F_\varphi = \pi F_T e^{-Rk\pi}. \quad (45)$$

We see that, as expected, the F_φ -term of the radion is exponentially suppressed and F_φ , although nonzero, is exponentially smaller than F_T . Although this is not a realistic model since R is not stabilized, it can be considered as a simple example of a supersymmetry breaking scenario with $F_T \sim e^{-Rk\pi}$. Turning on a nonzero F_T in Eqs. (27) and (28), we obtain the supersymmetry breaking masses

$$m_{\lambda_1} = \frac{F_T}{2R}, \quad m_\phi = \left| \frac{(1/2 - c)k\pi F_T}{2 \sinh[(1/2 - c)Rk\pi]} \right|. \quad (46)$$

Notice that m_ϕ has its maximum for $c = 1/2$ and tends exponentially to zero when c deviates from this value.

The result of Eq. (46) has an interesting 4D interpretation using the AdS/CFT correspondence, that is based on the conjecture that theories on AdS_5 are dual to 4D strongly coupled conformal field theories (CFT) in the large N limit [26]. This correspondence between AdS and CFT theories has been also extended to the RS set-up, giving a useful tool to understand the physics of this 5D scenario from a 4D point of view. For example, it has been argued that placing the boundary at $y=0$ in the AdS_5 space corresponds, in the 4D dual picture, to break explicitly the conformal group by introducing an ultraviolet cutoff at R and by adding new (ultraviolet) degrees of freedom [27]. For the case of a 5D theory with gravity and a gauge sector in the bulk, these new degrees of freedom correspond to a graviton and a gauge boson coupled to the CFT (it corresponds to a gauging of the Poincare group and a global symmetry of the CFT). The boundary at $y=\pi$ has a different correspondence in the 4D dual. It corresponds to a spontaneous breaking of the conformal group at the scale $k e^{-Rk\pi}$. The radion is associated to the Goldstone boson of the broken conformal symmetry, that we will call the dilaton. This has been recently checked in different ways in Ref. [28]. Under the conformal transformation, $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$, the superfields of the effective theory of Eqs. (27)–(29) must transform according to

$$\begin{aligned} T &\rightarrow T + \frac{1}{k\pi} \ln \Omega, \\ V &\rightarrow V, \\ \Phi &\rightarrow \Omega^{(c-3/2)} \Phi, \\ Q &\rightarrow Q. \end{aligned} \quad (47)$$

The lagrangian of Eqs. (27) and (28) is, however, not fully invariant under the transformation Eq. (47) due, as we said, to the $y=0$ boundary. In Eq. (27) the appearance of \mathbf{V} coupled to the vector multiplet breaks the conformal symmetry. In the 4D dual picture this corresponds to the coupling of the dilaton to the gauge vector supermultiplet at the one-loop level due to the conformal anomaly. In 5D it appears at the tree-level according to the duality relation [28] $g_5^2 b_{CFT}/(8\pi^2) = 1/k$. For the hypermultiplet zero mode, Eq. (28), the effect of the $y=0$ boundary becomes exponentially suppressed for $c \ll 1/2$. In this limit the zero mode is localized on the boundary at $y=\pi$ and corresponds in the 4D dual picture to a bound state arising from the spontaneously broken CFT.

Our scenario where supersymmetry is broken by the \mathbf{F} -term of the radion corresponds in the 4D dual to break supersymmetry with the dilaton \mathbf{F} -term. This is similar to AMSB where the breaking of supersymmetry is parametrized by the \mathbf{F} -term of the conformal compensator φ . In AMSB models gaugino and scalar masses are generated at the loop level due to the conformal anomaly. In our case, the role of φ is played by \mathbf{V} . It is also the anomaly (in the 4D dual) responsible for the gaugino masses. The scalar mass of ϕ tends to zero at tree level in the conformal limit $c \ll 1/2$, but it is generated at the one-loop level, similarly to AMSB, due to a wave function renormalization. In the opposite limit, $c \gg 1/2$, the zero mode of the hypermultiplet is localized on the $y=0$ boundary and corresponds in the 4D picture to a new degree of freedom. In this case its tree-level mass is also small since its direct coupling to the dilaton (arising from the CFT) is exponentially suppressed.

5 Conclusions

We have presented the 5D action of a supersymmetric gauge theory with a compact extra dimension using $\mathbf{N}=\mathbf{1}$ superfields. For a flat extra dimension the action is given in Eqs. (5) and (11), while for a warped extra dimension as in the RS scenario this is given in Eqs. (19) and (23).

We have applied the above results to study the breaking of supersymmetry by the \mathbf{F} -term of the radion. We have showed that, for a flat extra dimension, this type of breaking leads to the same mass spectrum as in Scherk-Schwarz models of supersymmetry breaking. One can therefore consider our formulation as a superfield description of the SS mechanism.

We have also considered scenarios where supersymmetry is broken on a boundary of a warped extra dimension. The spectrum in this case presents certain similarities with that in AMSB models.

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Appendix

In this Appendix we will derive the mass spectrum for the gaugino sector in a theory with a warped extra dimension where the breaking of supersymmetry induces a Majorana gaugino mass on the boundary as in Eq. (40).

Redefining $\lambda_i \rightarrow e^{-2R\sigma} \lambda_i$ $i = 1, 2$ to absorb the spin connection term, the equations of motion for the gauginos are given by

$$\begin{aligned} ie^{R\sigma} \bar{\sigma}^\mu \partial_\mu \lambda_2 + \frac{1}{R} (\partial_5 + \frac{1}{2} R\sigma') \bar{\lambda}_1 &= 0, \\ ie^{R\sigma} \bar{\sigma}^\mu \partial_\mu \lambda_1 - \frac{1}{R} (\partial_5 - \frac{1}{2} R\sigma') \bar{\lambda}_2 - \frac{1}{2} \frac{W}{M_5^3 R} \delta(y - \pi) \bar{\lambda}_1 &= 0. \end{aligned} \quad (48)$$

We will solve these equations in the bulk, ignoring boundary effects which will only play a role when imposing boundary conditions. Looking for solutions of the form $\lambda_i(x, y) = \sum \lambda^{(n)}(x) f_i^{(n)}(y)$ and using the orthogonality condition of the modes, Eq. (48) leads to the second order differential equations

$$\left[\frac{1}{R^2} e^{R\sigma} \partial_5 (e^{-R\sigma} \partial_5) - \left(\frac{1}{4} \pm \frac{1}{2} \right) k^2 \right] f_{1,2}^{(n)} = e^{2R\sigma} m_n^2 f_{1,2}^{(n)}, \quad (49)$$

with solutions

$$f_1^{(n)}(y) = \frac{e^{R\sigma/2}}{N_n} \left[J_1 \left(\frac{m_n}{k} e^{R\sigma} \right) + b_1(m_n) Y_1 \left(\frac{m_n}{k} e^{R\sigma} \right) \right], \quad (50)$$

$$f_2^{(n)}(y) = \frac{\sigma' e^{R\sigma/2}}{k N_n} \left[J_0 \left(\frac{m_n}{k} e^{R\sigma} \right) + b_2(m_n) Y_0 \left(\frac{m_n}{k} e^{R\sigma} \right) \right], \quad (51)$$

where b_i and m_n will be determined by the boundary conditions, and N_n are normalization constants.

Taking into account the \mathbb{Z}_2 assignment, $f_i^{(n)}$ must fulfill the following conditions on the $y = 0$ boundary:

$$\begin{aligned} f_2^{(n)} \Big|_{y=0} &= 0, \\ \left(\frac{d}{dy} + \frac{R}{2} \sigma' \right) f_1^{(n)} \Big|_{y=0} &= 0, \end{aligned} \quad (52)$$

which imply

$$b_1(m_n) = b_2(m_n) = -\frac{J_0 \left(\frac{m_n}{k} \right)}{Y_0 \left(\frac{m_n}{k} \right)}. \quad (53)$$

On the other hand, the presence of the Majorana gaugino mass on the $y = \pi$ boundary in Eq. (48) requires the following condition:

$$f_2^{(n)}(\pi) = \frac{1}{2} \frac{W}{2M_5^3 R} f_1^{(n)}(\pi). \quad (54)$$

Eqs. (53) and (54) yield

$$2 \left[J_0 \left(\frac{m_n}{k} e^{Rk\pi} \right) - \frac{J_0 \left(\frac{m_n}{k} \right)}{Y_0 \left(\frac{m_n}{k} \right)} Y_0 \left(\frac{m_n}{k} e^{Rk\pi} \right) \right] = \frac{W}{2M_5^2 R} \left[J_1 \left(\frac{m_n}{k} e^{Rk\pi} \right) - \frac{J_0 \left(\frac{m_n}{k} \right)}{Y_0 \left(\frac{m_n}{k} \right)} Y_1 \left(\frac{m_n}{k} e^{Rk\pi} \right) \right], \quad (55)$$

that determines the mass spectrum. Let us look for solutions of Eq. (55) in the limit $kR \gg 1$. For the lightest modes we can take the limit $m_n e^{Rk\pi}/k \equiv \epsilon \ll 1$ in Eq. (55) that becomes

$$2 = \frac{W}{2M_5^3} \left[\frac{\epsilon}{2} - \frac{1}{\pi k R} \frac{1}{\epsilon} \right]. \quad (56)$$

If the breaking of supersymmetry is small, $W/M_5^3 \ll 1$, the only solution to Eq. (56) fulfilling $\epsilon \ll 1$ is given by

$$m \simeq \frac{W}{4M_5^3 \pi R} e^{-Rk\pi}. \quad (57)$$

This corresponds to the zero mode mass. In the strong supersymmetry breaking case, $W/M_5^3 \gg 1$, there are two solutions to Eq. (56) given by

$$m \simeq \pm \sqrt{\frac{2}{\pi k R}} k e^{-Rk\pi}. \quad (58)$$

On the other hand, the masses of the heavier KK modes are easily obtained from Eq. (55) in the limit $\epsilon \gg 1$ that becomes

$$\frac{J_0 \left(\frac{m_n}{k} e^{Rk\pi} \right)}{J_1 \left(\frac{m_n}{k} e^{Rk\pi} \right)} = \frac{W}{4M_5^3}. \quad (59)$$

If the breaking of supersymmetry is weak, $W/M_5^3 \ll 1$, the Majorana mass spectrum is approximately given by

$$m_n \simeq \left(n + \frac{3}{4} \pm \frac{W}{4\pi M_5^3} \right) \pi k e^{-Rk\pi}, \quad n = 1, 2, \dots, \quad (60)$$

whereas in the strong supersymmetry breaking limit, $W/M_5^3 \gg 1$, we have

$$m_n \simeq \left(n + \frac{1}{4} \pm \frac{4M_5^3}{\pi W} \right) \pi k e^{-Rk\pi}. \quad (61)$$

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