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# BRST Invariance of the Non-Perturbative Vacuum in Bosonic Open String Field Theory

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**Abstract**

Tachyon condensation on a bosonic D-brane was recently demonstrated numerically in Witten's open string field theory with level truncation approximation. This non-perturbative vacuum, which is obtained by solving the equation of motion, has to satisfy furthermore the requirement of BRST invariance. This is indispensable in order for the theory around the non-perturbative vacuum to be consistent. We carry out the numerical analysis of the BRST invariance of the solution and find that it holds to a good accuracy. We also mention the zero-norm property of the solution. The observations in this paper are expected to give clues to the analytic expression of the vacuum solution.

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# 1 Introduction

Recently, string field theories, especially Witten's open string field theory [1], have attracted much interest from the viewpoint of tachyon condensation. In the course of studies on non-BPS states in string theory [2, 3, 4], Sen has made the following conjectures.

- The open string tachyon is a sign of instability of the system, and it condenses into the non-perturbative vacuum, where the space-time filling D25-brane completely disappears.
- Kink solutions on the non-perturbative vacuum represent various lower dimensional D-branes (descent relations).

To prove these conjectures, especially the first one, we need an off-shell formulation of string theory, and a unique candidate at present is string field theory. In fact, Sen and Zwiebach [5] showed that the first conjecture holds to a miraculously good accuracy by making use of string field theory with level truncation scheme [6]. After this breakthrough, a lot of works in this direction have come out. For the first conjecture, there have appeared works on higher order level truncations [7], superstring extension [8, 9, 10, 11], and approaches toward the analytic solution [12, 13, 14, 15, 16]. As for the second conjecture, there are approaches in the level truncation scheme [17, 18, 19, 20, 21] and the analysis in the large non-commutativity limit [22, 23, 24, 25]. In particular, the latter has given a decisive answer to the descent relations.

Because Witten's open string field theory is formulated as a gauge theory, we must fix the gauge invariance to find classical solutions. To fix the gauge we use the BRST method, and then the gauge-fixed action has an invariance under the BRST transformation. The BRST invariance of the vacuum as well as of the action is by all means necessary to deal with theories with unphysical negative-norm states [26]. The perturbative vacuum with vanishing string field is trivially BRST-invariant. However, the BRST invariance of the tachyon condensed vacuum is a non-trivial matter since the vacuum solution is obtained by solving only the equation of motion. So we must check the BRST invariance separately to guarantee that the perturbation theory around the vacuum makes sense physically.

The purpose of the present paper is to examine the BRST invariance of the non-perturbative vacuum solution found in [5, 7] in the level truncation approximation. We find that the BRST invariance for the lower level states hold to a very good accuracy. We also analyze the "fake" vacuum belonging to a different branch from the "true" one and show that the BRST invariance is spontaneously broken there. Though we analyze the BRST invariance only numerically here, the BRST invariance of the full theory is expected to give crucial information for constructing the analytic expression of the vacuum solution.

The rest of this paper is organized as follows. In section 2, we recapitulate Witten's open string field theory to fix our conventions. Section 3, the main part, is devoted to the analysis of the BRST invariance of the solution. Lastly in section 4, we conclude our results and make some comments on the numerical results of [5, 7].

# 2 String field theory action and BRST invariance

In this section we briefly review Witten's open bosonic string field theory [1]. Especially we pay attention to gauge invariance, gauge fixing and BRST invariance. Since we are interested only in the translation invariant states, we set the space-time momentum equal to zero.

**Action**

The gauge invariant action of Witten's string field theory is given by*

$$S = -\frac{V_{26}}{g_{\mathrm{o}}^2}\left(\frac{1}{2}\int db_0\,\langle\,\Phi\,|\,Q_{\mathrm{B}}\,|\,\Phi\,\rangle + \frac{1}{3}\int db_0^{(3)}\int db_0^{(2)}\int db_0^{(1)}\,{}_1\langle\,\Phi\,|\,{}_2\langle\,\Phi\,|\,{}_3\langle\,\Phi\,|\,V_3\,\rangle_{123}\right). \quad (2.1)$$

In this paper we write string field $|\,\Phi\,\rangle$ as a vector in the Fock space of first quantized string except ghost zero modes, $b_0$ and $c_0 = \frac{\partial}{\partial b_0}$. String field $|\,\Phi\,\rangle$, a ghost number $-1$ state, is expanded in terms of $b_0$ as

$$|\,\Phi\,\rangle = b_0|\,\phi\,\rangle + |\,\psi\,\rangle, \quad (2.2)$$

where $|\,\phi\,\rangle$ and $|\,\psi\,\rangle$ carry ghost number 0 and $-1$ respectively. The BRST operator $Q_{\mathrm{B}}$ is represented as

$$Q_{\mathrm{B}} = c_0 L + b_0 M + \widetilde{Q}_{\mathrm{B}}, \quad (2.3)$$

with

$$L = \sum_{n=1}^{\infty}\left(\alpha_{-n}\alpha_n + nc_{-n}b_n + nb_{-n}c_n\right) - 1, \quad (2.4)$$

$$M = -2\sum_{n=1}^{\infty} nc_{-n}c_n, \quad (2.5)$$

*We have factored out the space-time volume $V_{26}$ since $\Phi$ is translation invariant. $g_{\mathrm{o}}$ is the open string coupling constant.

$$\widetilde{Q}_{\rm B}=\sum_{n\neq 0}c_{-n}\sum_{m=-\infty}^{\infty}\frac{1}{2}\alpha_{n-m}\alpha_m+\sum_{n\neq 0,m\neq 0}\frac{m-n}{2}c_m c_n b_{-m-n}. \tag{2.6}$$

The vertex $|\,V_3\,\rangle_{123}$ representing the three-string midpoint interaction is defined by [27, 28]

$$|\,V_3\,\rangle_{123}=\exp\left[-\sum_{r,s=1}^{3}\sum_{n=1}^{\infty}X^{rs}_{n0}c^{(r)}_{-n}b^{(s)}_0\right]|\,v_3\,\rangle_{123}\,, \tag{2.7}$$

$$|\,v_3\,\rangle_{123}=\mu\exp\left[\sum_{r,s=1}^{3}\left(\frac{1}{2}\sum_{n,m=1}^{\infty}N^{rs}_{nm}\alpha^{(r)}_{-n}\cdot\alpha^{(s)}_{-m}-\sum_{n,m=1}^{\infty}X^{rs}_{nm}c^{(r)}_{-n}b^{(s)}_{-m}\right)\right]|\,0\,\rangle_{123}\,. \tag{2.8}$$

Here we adopt the convention of [7] and take $\mu=3^{9/2}/2^7$. The Neumann coefficients, $N^{rs}_{nm}, X^{rs}_{nm}$ and $X^{rs}_{n0}$ are defined using the six-string Neumann coefficients $\bar{N}^{RS}_{nm}$ $(R,S=1,\ldots,6)$ as

$$N^{rs}_{nm}=\bar{N}^{rs}_{nm}+\bar{N}^{r\,(s+3)}_{nm}, \tag{2.9}$$

$$X^{rs}_{nm}=(-)^{r+s+1}\,n\left(\bar{N}^{rs}_{nm}-\bar{N}^{r\,(s+3)}_{nm}\right), \tag{2.10}$$

where $\bar{N}^{RS}_{nm}$ are given by the contour integrals as

$$\begin{aligned}
\bar{N}^{RS}_{nm}&=\frac{1}{nm}\oint_{Z_R}\frac{dz}{2\pi i}\oint_{Z_S}\frac{dw}{2\pi i}\frac{1}{(z-w)^2}(-)^{n(R-1)+m(S-1)}[f(z)]^{n(-)^R}[f(w)]^{m(-)^S},\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad n>0,\ m>0,\\
\bar{N}^{RS}_{n0}&=\frac{1}{n}\oint_{Z_R}\frac{dz}{2\pi i}\frac{1}{z-Z_S}(-)^{n(R-1)}[f(z)]^{n(-)^R},\qquad n>0,\\
f(z)&=\frac{z(z^2-3)}{3z^2-1},\qquad Z_{R=1,\ldots,6}=\left\{\sqrt{3},\ \frac{1}{\sqrt{3}},\ 0,\ -\frac{1}{\sqrt{3}},\ -\sqrt{3},\ \infty\right\}.
\end{aligned} \tag{2.11}$$

**Gauge fixing and BRST invariance**

Since the action (2.1) has an invariance under the stringy gauge transformation

$$\delta_\Lambda|\,\Phi\,\rangle_3=Q_{\rm B}|\,\Lambda\,\rangle_3+{}_1\langle\,\Phi\,|\,{}_2\langle\,\Lambda\,|\,V_3\,\rangle_{123}-{}_1\langle\,\Lambda\,|\,{}_2\langle\,\Phi\,|\,V_3\,\rangle_{123}, \tag{2.12}$$

we have to fix the gauge and introduce the ghost fields to quantize the system. This procedure is accomplished by imposing the gauge condition (here we take the Siegel gauge),

$$b_0|\,\Phi\,\rangle=0\quad\Leftrightarrow\quad|\,\psi\,\rangle=0, \tag{2.13}$$

and allowing the component fields with an arbitrary ghost number in $|\,\phi\,\rangle$. Then the action (2.1) with the gauge condition (2.13) gains, instead of the gauge invariance, the invariance under the BRST transformation $\delta_{\rm B}\phi=(\delta S/\delta\psi)_{\psi=0}$.

Let us summarize the concrete expressions of the equation of motion and the BRST transformation in the gauge-fixed string field theory. First, the equation of motion of the gauge invariant action (2.1) reads

$$Q_{\mathrm{B}}^{(3)} | \Phi \rangle_3 + \int db_0^{(2)} \int db_0^{(1)} \; {}_1\langle \Phi |\, {}_2\langle \Phi | V_3 \rangle_{123} = 0. \tag{2.14}$$

Imposing the gauge condition (2.13), the $b_0^{(3)}$-independent part of (2.14) gives the equation of motion of the gauge-fixed system:

$$L^{(3)} | \phi \rangle_3 + {}_1\langle \phi |\, {}_2\langle \phi | v_3 \rangle_{123} = 0. \tag{2.15}$$

On the other hand, the $b_0^{(3)}$ part of the left-hand-side of (2.14) is the BRST transformation:

$$\delta_{\mathrm{B}} | \phi \rangle_3 = \widetilde{Q}_{\mathrm{B}}^{(3)} | \phi \rangle_3 - \sum_{r=1}^{3} \sum_{n=1}^{\infty} X_{n0}^{r3} \; {}_1\langle \phi |\, {}_2\langle \phi | c_{-n}^{(r)} | v_3 \rangle_{123}. \tag{2.16}$$

The equation of motion (2.15) was studied numerically in [5, 7] using the level truncation scheme, and they found a surprisingly good agreement with the conjectured result [4] for the tachyon potential. So the tachyon-condensed vacuum $\phi_{\mathrm{c}}$ calculated there is believed to be the true vacuum in which D25-brane completely disappears and there are no degrees of freedom of open strings.

However, besides being a solution to the equation of motion (2.15), $\phi_{\mathrm{c}}$ should be BRST invariant, $\delta_{\mathrm{B}} | \phi_{\mathrm{c}} \rangle = 0$ (namely, the $b_0^{(3)}$ part of the original equation of motion (2.14) should also hold). This requirement of the unbroken BRST invariance is independent of the equation of motion (2.15). Moreover, it is necessary in order for the perturbation theory around $\phi = \phi_{\mathrm{c}}$ to be a consistent one where unphysical negative-norm states are controlled by the BRST symmetry. The BRST invariance should, in particular, play an important role in showing that there are no open string excitations on the non-perturbative vacuum. Poles in the propagators are allowed if they correspond to unphysical states confined by the BRST symmetry [26].

Therefore, it is an important and non-trivial matter to check the BRST invariance of the numerical solution obtained in [5, 7]. We shall carry out this task in the next section.

# 3 BRST invariance of the non-perturbative vacuum

In this section we examine the BRST invariance of the non-perturbative vacuum of [7] for the first few levels in $\delta_{\mathrm{B}} | \phi_{\mathrm{c}} \rangle$. We shall find more and more precise cancellations among the terms in $\delta_{\mathrm{B}} | \phi_{\mathrm{c}} \rangle$ as we incorporate higher level fields in the quadratic term in (2.16).

### 3.1 BRST invariance at level two

To begin with, we expand the string field following [7] up to level four, assuming that only zero-ghost-number, Lorentz-scalar and even-level fields acquire non-zero vacuum expectation values†:

$$
\begin{aligned}
|\,\phi\,\rangle = \quad & \psi_1\,|\,0\,\rangle + \psi_2\,(\alpha_{-1}\cdot\alpha_{-1})|\,0\,\rangle + \psi_3\,b_{-1}c_{-1}|\,0\,\rangle \\
&+ \psi_4\,(\alpha_{-1}\cdot\alpha_{-3})|\,0\,\rangle + \psi_5\,(\alpha_{-2}\cdot\alpha_{-2})|\,0\,\rangle + \psi_6\,(\alpha_{-1}\cdot\alpha_{-1})(\alpha_{-1}\cdot\alpha_{-1})|\,0\,\rangle \\
&+ \psi_7\,(\alpha_{-1}\cdot\alpha_{-1})b_{-1}c_{-1}|\,0\,\rangle + \psi_8\,b_{-1}c_{-3}|\,0\,\rangle + \psi_9\,b_{-2}c_{-2}|\,0\,\rangle + \psi_{10}\,b_{-3}c_{-1}|\,0\,\rangle.
\end{aligned} \tag{3.1}
$$

Because the BRST transformation $\delta_{\rm B}$ raises the ghost number by one, we should examine the BRST transformation of the component fields with ghost number $-1$. Explicitly, we shall consider the following five component fields in $|\,\phi\,\rangle$ with levels 2 and 4:

$$
\begin{aligned}
&\bar{C}_2\,c_{-2}|\,0\,\rangle + \bar{C}_{4a}\,c_{-4}|\,0\,\rangle + \bar{C}_{4b}\,b_{-1}c_{-1}c_{-2}|\,0\,\rangle \\
&\quad + \bar{C}_{4c}\,(\alpha_{-1}\cdot\alpha_{-1})c_{-2}|\,0\,\rangle + \bar{C}_{4d}\,(\alpha_{-1}\cdot\alpha_{-2})c_{-1}|\,0\,\rangle.
\end{aligned} \tag{3.2}
$$

First let us consider $\delta_{\rm B}\bar{C}_2$. As an example of the calculation of the quadratic term in $\delta_{\rm B}|\,\phi\,\rangle$ of (2.16) contributing to $\delta_{\rm B}\bar{C}_2$, we present the level-0×level-0 and level-0×level-2 terms using the Neumann coefficients:

$$
\Big[-X^{33}_{20}\,(\psi_1)^2 + 2\cdot 26 N^{11}_{11}X^{33}_{20}\psi_1\psi_2 - 2\left(X^{11}_{11}X^{33}_{20} - X^{31}_{21}X^{13}_{10}\right)\psi_1\psi_3\Big]\,c_{-2}|\,0\,\rangle. \tag{3.3}
$$

Collecting the linear term, $\widetilde{Q}_{\rm B}|\,\phi\,\rangle$ of (2.16), and all the quadratic terms with the level sum equal to or less than four, we get the following expression for $\delta_{\rm B}\bar{C}_2$:

$$
\begin{aligned}
\delta_{\rm B}\bar{C}_2 = {} & 26\,\psi_2 + 3\,\psi_3 \\
& - \mu\Bigg[\frac{16\,{\psi_1}^2}{27} - \frac{4160\,\psi_1\,\psi_2}{729} + \frac{32\,\psi_1\,\psi_3}{81} + \frac{483392\,{\psi_2}^2}{19683} - \frac{4160\,\psi_2\,\psi_3}{2187} - \frac{11248\,{\psi_3}^2}{19683} \\
& \qquad + \frac{26624\,\psi_1\,\psi_4}{6561} + \frac{21632\,\psi_1\,\psi_5}{6561} + \frac{582400\,\psi_1\,\psi_6}{19683} - \frac{4160\,\psi_1\,\psi_7}{2187} \\
& \qquad + \frac{9856\,\psi_1\,\psi_8}{19683} + \frac{800\,\psi_1\,\psi_9}{2187} - \frac{3200\,\psi_1\,\psi_{10}}{19683}\Bigg].
\end{aligned} \tag{3.4}
$$

Now let us substitute into (3.4) the numerical values of $\psi_k$ for the non-perturbative vacuum obtained in [7]. The values of $\psi_k$ in the (10,20) approximation of [7] is listed in Table 1. Table 2 summarizes the result after substituting these values into the BRST transformation

† We can further truncate matter CFT states to the descendant states of unit operator [12]. But we do not take this property into account in this paper.

| | | | | | |
|---|---|---|---|---|---|
| $\psi_1$ | 1.09259 | $\psi_4$ | $-0.01148$ | $\psi_7$ | 0.00786 |
| $\psi_2$ | 0.05723 | $\psi_5$ | $-0.00509$ | $\psi_8$ | 0.11654 |
| $\psi_3$ | $-0.43373$ | $\psi_6$ | $-0.00032$ | $\psi_9$ | 0.06990 |
| | | | | $\psi_{10}$ | 0.03885 |

Table 1: The classical solution $\psi_k$ in the (10,20) approximation

| lin. or quad. | term | raw value | partial sum 1 | partial sum 2 | total sum |
|---|---|---|---|---|---|
| linear | $\psi_2$ | 1.48798 | 0.18682 | 0.00774 | $-0.00216$ |
| | $\psi_3$ | $-1.30116$ | | | |
| quadratic | level-0×level-0 | $-0.77537$ | $-0.17908$ | | |
| | level-0×level-2 | 0.59629 | | | |
| | level-2×level-2 | $-0.02209$ | $-0.00990$ | $-0.00990$ | |
| | level-0×level-4 | 0.01219 | | | |

Table 2: The values of the terms in $\delta_{\mathrm{B}}\bar{C}_2$ and their cancellations

(3.4). For example, the row [linear : $\psi_3$] is the value of the second term of (3.4), and the row [quadratic : level-0×level-2] is the value of the sum of the $\psi_1\psi_2$ and the $\psi_1\psi_3$ terms including the factor $-\mu$.

From Table 2 we find that the cancellations occur every time we sum up higher level contributions. First, each term in the linear part of (3.4) is of order $10^0$, and they cancel each other to become 0.18682, of order $10^{-1}$. The field $\psi_2$ is constructed by the matter oscillators $\alpha$, and $\psi_3$ by the ghost ones, $b$ and $c$. So it can be considered as a kind of matter-ghost cancellation. Next adding to the linear part the level-0×level-0 and level-0×level-2 contributions in the quadratic part gives a value of order $10^{-2}$. Finally adding the contributions from level-2×level-2 and level-0×level-4, $\delta_{\mathrm{B}}\bar{C}_2$ becomes of order $10^{-3}$.[‡] Thus we have observed a marvelous cancelation among various terms in $\delta_{\mathrm{B}}\bar{C}_2$, in particular, between the linear and the quadratic terms. This analysis gives a strong evidence for $\delta_{\mathrm{B}}\bar{C}_2 = 0$ at the non-perturbative vacuum.

## 3.2 Fake vacuum

In the level truncation scheme there generally appear more than one non-trivial solutions for the equations of motion, for example, there are two solutions in the (2,6) approximation. The

[‡]This should be compared with the deviation of the potential height from the expected value, which is of order $10^{-3}$ in the (10,20) approximation [7].

non-perturbative vacuum obtained in [5, 7] was chosen by the condition that it lies on the same branch as the trivial solution $\phi = 0$.

In this subsection, let us take the (2,6) approximation and study the BRST invariance of the "fake" vacuum on a different branch. The potential in the (2,6) approximation is

$$\begin{aligned} V = &-\frac{1}{2}{\psi_1}^2 + 26\,{\psi_2}^2 - \frac{1}{2}{\psi_3}^2 \\ &+ \frac{\mu}{3}\Bigg[{\psi_1}^3 - \frac{2178904}{6561}{\psi_2}^3 - {\psi_1}^2\left(\frac{130}{9}\psi_2 + \frac{11}{9}\psi_3\right) - \frac{332332}{6561}{\psi_2}^2\,\psi_3 \\ &\qquad - \frac{2470}{2187}\psi_2\,{\psi_3}^2 - \frac{1}{81}{\psi_3}^3 + \psi_1\left(\frac{30212}{243}{\psi_2}^2 + \frac{2860\,\psi_2\,\psi_3}{243} + \frac{19}{81}{\psi_3}^2\right)\Bigg]. \end{aligned} \tag{3.5}$$

This potential has two non-trivial extrema[§], which we call "true" vacuum and "fake" one respectively (see Table 3). Table 4 shows the values of $\delta_{\mathrm{B}}\bar{C}_2$ for these two vacua. In the case

| | $\psi_1$ | $\psi_2$ | $\psi_3$ |
|---|---|---|---|
| "true" | 1.0884 | 0.05596 | $-0.3804$ |
| "fake" | $-17.8041$ | $-1.68453$ | $-28.6253$ |

Table 3: Field values at the two extrema

| | lin. or quad. | term | raw value | partial sum | total sum |
|---|---|---|---|---|---|
| T | linear | $\psi_2$ | 1.45506 | 0.31391 | |
| R | | $\psi_3$ | $-1.14114$ | | 0.10472 |
| U | quadratic | level-0×level-0 | $-0.76944$ | $-0.20919$ | |
| E | | level-0×level-2 | 0.56025 | | |
| F | linear | $\psi_2$ | $-43.798$ | $-129.674$ | |
| A | | $\psi_3$ | $-85.876$ | | $-368.661$ |
| K | quadratic | level-0×level-0 | $-205.889$ | $-238.987$ | |
| E | | level-0×level-2 | $-33.098$ | | |

Table 4: $\delta_{\mathrm{B}}\bar{C}_2$ for the "true" and "fake" vacua

of the "true" vacuum, there occur neat cancellations as before and total sum is close to zero. In contrast, for the "fake" vacuum, all values are of order $10^2$ and there are no significant cancellations at all. From these facts, we see that we can distinguish the true vacuum out of other extrema of the potential by requiring the BRST invariance.

[§]Because all the coefficient fields are real, we discard complex solutions.

### 3.3 BRST invariance at level four

The BRST transformation of the level-4 fields in (3.2), $\bar{C}_{4a}$, $\bar{C}_{4b}$, $\bar{C}_{4c}$ and $\bar{C}_{4d}$, are given as follows:

$$\begin{aligned}
\delta_{\rm B}\bar{C}_{4a} &= 78\,\psi_4 + 104\,\psi_5 + 5\,\psi_8 + 6\,\psi_9 + 7\,\psi_{10} \\
&\quad - \mu\bigg[-\frac{352\,{\psi_1}^2}{729} + \frac{91520\,\psi_1\,\psi_2}{19683} - \frac{2240\,\psi_1\,\psi_3}{19683} - \frac{10634624\,{\psi_2}^2}{531441} + \frac{291200\,\psi_2\,\psi_3}{531441} \\
&\qquad + \frac{18848\,{\psi_3}^2}{59049} - \frac{585728\,\psi_1\,\psi_4}{177147} - \frac{475904\,\psi_1\,\psi_5}{177147} - \frac{12812800\,\psi_1\,\psi_6}{531441} \\
&\qquad + \frac{291200\,\psi_1\,\psi_7}{531441} + \frac{262400\,\psi_1\,\psi_8}{531441} - \frac{27328\,\psi_1\,\psi_9}{531441} + \frac{256\,\psi_1\,\psi_{10}}{177147}\bigg],
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
\delta_{\rm B}\bar{C}_{4b} &= 26\,\psi_7 - \psi_8 - 3\,\psi_9 + 5\,\psi_{10} \\
&\quad - \mu\bigg[\frac{176\,{\psi_1}^2}{729} - \frac{45760\,\psi_1\,\psi_2}{19683} + \frac{5216\,\psi_1\,\psi_3}{19683} + \frac{5317312\,{\psi_2}^2}{531441} - \frac{678080\,\psi_2\,\psi_3}{531441} \\
&\qquad - \frac{592\,{\psi_3}^2}{6561} + \frac{292864\,\psi_1\,\psi_4}{177147} + \frac{237952\,\psi_1\,\psi_5}{177147} + \frac{6406400\,\psi_1\,\psi_6}{531441} \\
&\qquad - \frac{678080\,\psi_1\,\psi_7}{531441} + \frac{13184\,\psi_1\,\psi_8}{59049} + \frac{181600\,\psi_1\,\psi_9}{531441} - \frac{4480\,\psi_1\,\psi_{10}}{531441}\bigg],
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
\delta_{\rm B}\bar{C}_{4c} &= 3\,\psi_4 + 56\,\psi_6 + 3\,\psi_7 + \frac{1}{2}\,\psi_9 \\
&\quad - \mu\bigg[-\frac{40\,{\psi_1}^2}{729} + \frac{18592\,\psi_1\,\psi_2}{19683} - \frac{80\,\psi_1\,\psi_3}{2187} - \frac{670432\,{\psi_2}^2}{177147} + \frac{18592\,\psi_2\,\psi_3}{59049} \\
&\qquad + \frac{28120\,{\psi_3}^2}{531441} - \frac{74752\,\psi_1\,\psi_4}{177147} - \frac{31168\,\psi_1\,\psi_5}{531441} - \frac{138880\,\psi_1\,\psi_6}{19683} \\
&\qquad + \frac{18592\,\psi_1\,\psi_7}{59049} - \frac{24640\,\psi_1\,\psi_8}{531441} - \frac{2000\,\psi_1\,\psi_9}{59049} + \frac{8000\,\psi_1\,\psi_{10}}{531441}\bigg],
\end{aligned} \tag{3.8}$$

$$\delta_{\rm B}\bar{C}_{4d} = 3\,\psi_4 + 4\,\psi_5 + 2\,\psi_7 + \psi_{10} - \mu\left[-\frac{65536\,\psi_2\,\psi_3}{531441} + \frac{65536\,\psi_1\,\psi_7}{531441}\right]. \tag{3.9}$$

In Tables 5 – 8, we list the numerical values of eqs. (3.6) – (3.9) for the "true" vacuum of Table 1. In every case, there occur significant cancellations.

| lin. or quad. | term | raw value | partial sum | total sum |
|---|---|---|---|---|
| linear | $\psi_4, \psi_5$ | $-1.42480$ | $-0.15075$ | |
| | $\psi_8, \psi_9, \psi_{10}$ | $1.27405$ | | |
| quadratic | level-0×level-0 | $0.63178$ | | $-0.01672$ |
| | level-0×level-2 | $-0.37778$ | $0.13403$ | |
| | level-2×level-2 | $0.02093$ | | |
| | level-0×level-4 | $-0.14090$ | | |

Table 5: $\delta_{\rm B}\bar{C}_{4a}$

| lin. or quad. | term | raw value | partial sum | total sum |
|---|---|---|---|---|
| linear | $\psi_7, \psi_{10}$ | $0.39861$ | $0.07237$ | |
| | $\psi_8, \psi_9$ | $-0.32624$ | | |
| quadratic | level-0×level-0 | $-0.31589$ | | $-0.01040$ |
| | level-0×level-2 | $0.29698$ | $-0.08277$ | |
| | level-2×level-2 | $-0.05203$ | | |
| | level-0×level-4 | $-0.01183$ | | |

Table 6: $\delta_{\rm B}\bar{C}_{4b}$

| lin. or quad. | term | raw value | partial sum | total sum |
|---|---|---|---|---|
| linear | $\psi_4, \psi_6$ | $-0.05236$ | $0.00617$ | |
| | $\psi_7, \psi_9$ | $0.05853$ | | |
| quadratic | level-0×level-0 | $0.07179$ | | $0.00225$ |
| | level-0×level-2 | $-0.08374$ | $-0.00392$ | |
| | level-2×level-2 | $0.01124$ | | |
| | level-0×level-4 | $-0.00322$ | | |

Table 7: $\delta_{\rm B}\bar{C}_{4c}$

| lin. or quad. | term | raw value | partial sum | total sum |
|---|---|---|---|---|
| linear | $\psi_4, \psi_5$ | $-0.05480$ | $-0.00023$ | |
| | $\psi_7, \psi_{10}$ | $0.05457$ | | |
| quadratic | level-0×level-0 | $0$ | | $-0.00475$ |
| | level-0×level-2 | $0$ | $-0.00452$ | |
| | level-2×level-2 | $-0.00336$ | | |
| | level-0×level-4 | $-0.00116$ | | |

Table 8: $\delta_{\rm B}\bar{C}_{4d}$

# 4 Summary and discussions

In this paper, we have shown that the non-perturbative vacuum solution obtained in [5, 7] is BRST-invariant. This property is necessary in order that there exists a consistent quantum theory of fluctuations around this vacuum. In particular, the BRST invariance would play an important role in showing the absence of physical open string excitations on the non-perturbative vacuum. We have also examined the BRST invariance of the "fake" vacuum and found that the BRST symmetry is spontaneously broken there. The fact that "fake" vacua are not BRST-invariant may indicate that they are artifacts of the level truncation and do not exist in the full theory.

Now we shall comment on the zero-norm property of the non-perturbative vacuum solution in the level truncation scheme. As mentioned in [5], if we parameterize the level-2 part of the solution as

$$v \cdot \frac{1}{\sqrt{52}}(\alpha_{-1}\cdot\alpha_{-1})|\,0\,\rangle - u\cdot b_{-1}c_{-1}|\,0\,\rangle, \tag{4.1}$$

the coefficients $u$ and $v$ are almost equal[¶]. Once we assume that this equality is exact, the level-2 part of the solution has zero-norm: the norm of the first term of (4.1) is cancelled by that of the second. Next let us proceed to level 4 and observe that the $\psi_4, \psi_8$ and $\psi_{10}$ part of the solution is rewritten using the values of the (10,20) approximation in Table 1 as[‖]

$$-0.1014\times\frac{1}{\sqrt{3\cdot 26}}(\alpha_{-1}\cdot\alpha_{-3}) \quad + \quad 0.0955\times\frac{1}{\sqrt{6}}\big(3\,b_{-1}c_{-3}+b_{-3}c_{-1}\big), \tag{4.2}$$

where each state is normalized to a positive or negative unit norm. Similarly to the previous case (4.1), the two coefficients are almost equal in a few percent error:

$$\frac{(-0.1014)^2-(0.0955)^2}{(-0.1014)^2+(0.0955)^2} = 0.0599 \sim 6.0\%. \tag{4.3}$$

Though, in eqs. (4.2) and (4.3), we considered a part of the level-4 states orthogonal to the other parts, the zero-norm property holds better for the whole of the level-4 state. Evaluating the errors in the same manner as (4.3), the whole level-4 state deviates from the zero-norm combination only by 2.6%, and the whole level-6 state by 1.5%. Therefore, it is very likely

¶In terms of $\psi_2$ and $\psi_3$, this relation reads $\psi_2 = -(1/\sqrt{52})\,\psi_3$. The BRST invariance, $\delta_{\rm B}\bar{C}_2 = 0$, at the linearized level requires $\psi_2 = -(3/26)\,\psi_3$ (see eq. (3.4)). These two are close numerically, $1/\sqrt{52} = 0.139 \simeq 0.115 = 3/26$. The zero-norm property may possibly be related to the BRST invariance.

‖The classical solution $|\,\phi_{\rm c}\,\rangle$ has an exact symmetry under the exchange $(nc_{-n}, b_{-m}) \to (b_{-n}, -mc_{-m})$, which is owing to the invariance of $L$ and $|\,v_3\,\rangle$ under the exchange. We have used this fact for the second term of (4.2).

that in each level the whole state has zero norm in the full solution (or the zero norm property might hold in a more severe sense: each level is a sum of zero norm states, such as (4.2), which are orthogonal to one another). If this is the case, we can write the potential at the extremum normalized by the D25-brane tension $2/\pi^2$ as

$$\begin{aligned}\frac{\pi^2}{2}V &= \frac{\pi^2}{2}\left[\frac{1}{2}\langle\,\phi\,|\,L\,|\,\phi\,\rangle + \frac{1}{3}{}_1\langle\,\phi\,|\,{}_2\langle\,\phi\,|\,{}_3\langle\,\phi\,|\,v_3\,\rangle_{123}\right] \\ &= \frac{\pi^2}{12}\langle\,\phi\,|\,L\,|\,\phi\,\rangle = -\frac{\pi^2}{12}(\psi_1)^2, \end{aligned} \tag{4.4}$$

where we have used the equation of motion (2.15) at the second equality, and used the zero-norm assumption at the last equality. Note that the last expression is given only in terms of $\psi_1$. The numerical value of the last expression of (4.4) is $-0.9818$, very close to the desired value $-1$. This (almost) zero-norm property as well as the BRST invariance may serve as important clues to the construction of the analytic solution for the non-perturbative vacuum.

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