Instituto Superior de Ciencias y Tecnología Nucleares

Dissertation Diploma Thesis

About an Alternative Vacuum State for Perturbative QCD

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Havana 1999

Abstract

A particular initial state for the construction of a perturbative QCD expansion is investigated. It is formed as a coherent superposition of zero momentum gluon pairs and shows Lorentz as well as global $SU(3)$ symmetries. The general form of the Wick theorem is discussed, and it follows that the gluon and ghost propagators determined by the proposed vacuum state, coincides with the ones used in an alternative of the usual perturbation theory proposed in a previous work, and reviewed here. Therefore, the ability of such a procedure of producing a finite gluon condensation parameter already in the first orders of perturbation theory is naturally explained. It also follows that this state satisfies the physicality condition of the BRST procedure in its Kugo and Ojima formulation. A brief review of the canonical quantization for gauge fields, developed by Kugo and Ojima, is done and the value of the gauge parameter α is fixed to $\alpha = 1$ where the procedure is greatly simplified. Therefore, after assuming that the adiabatic connection of the interaction does not take out the state from the interacting physical space, the predictions of the perturbation expansion for the physical quantities, at the value $\alpha = 1$, should have meaning. The validity of this conclusion solves the gauge dependence indeterminacy remained in the proposed perturbation expansion.

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Chapter 1

Introduction

Quantum Chromodynamics (QCD) was discovered in the seventies and it has been considered as the fundamental theory for the strong interactions. A theory of such sort, showing a non abelian invariance group, was first suggested by Yang and Mills [1]. The main idea in it is the principle of local gauge invariance, which for example, in Quantum Electrodynamics (QED) means that the phase of the wave function can be defined in an arbitrary way at any point of the space-time. In a non abelian theory, the arbitrary phase is generalized to an arbitrary transformation in an internal symmetry group, for QCD the internal symmetry group is $SU(3)$. The discovery of this theory generated radical changes in the character of the Modern Theoretical Physics and as a consequence it has been deeply investigated in the last years. It is believed at the present time that all the interactions in Nature are gauge invariant [2].

In one limit, the smallness of the coupling constant at high momentum (asymptotic freedom) made possible the theoretical investigation of the so-called hard processes by using the familiar perturbative language. This so called perturbative QCD (PQCD) was satisfactorily developed. However, relevant phenomena associated with the strong interactions can't be described by the standard perturbative methods and the development of the so-called non-perturbative QCD is at the moment one of the great challenges of Theoretical Physics.

One of the most peculiar characteristics of the strong interactions is the color confinement. According to this philosophy colored objects, like quarks and gluons, can't be observed as free particles in contrast with hadrons that are colorless composite states and effectively detected. The physical nature of such phenomenon remains unclear. Qualitatively, it is compared with the Meissner effect in superconductors, in which the magnetic field is expelled from the bulk in which the Cooper pairs condensate exists. It is considered that the QCD vacuum expels the color fields from it. Numerous attempts to explain this property have been made, for example explicit calculations in which the theory is regularized in a spatial lattice [3], and also through the construction of phenomenological models. One of this models, the so called MIT Bag Model [4], assumes that a bag or bubble is formed around the objects having color in such a manner that they could not escape from it, because their effective mass is smaller inside the bag volume and very high outside. The dimensional quantity introduced in this model is B, the bag constant, which is the pressure that the vacuum makes on the color field. Another approach is the so called String Model [5] which is based in the assumption that the interaction forces between quarks and antiquarks grow with the distance, in such a way that the energy increases linearly with the string length $E(L) = kL$. The main parameter introduced in this theory is the string tension k which determines the strength of the confining interaction potential.

A fundamental problem in QCD is the nature of its ground state [6]. This state is imagined as a very dense state of matter, composed of gluons and quarks interacting in a complicated way. Its properties are not easy accessible in experiments, because quarks and gluons fields can't be directly observed, only the color neutral hadrons are detected. Furthermore, the interactions between quarks can't be directly determine, because their scattering amplitudes can't be measured. It is known, from the experience in solid-state physics, that a good understanding of the ground state structure implies a natural explanation of many of the phenomenological facts concerning to its excitations. The theory of superconductivity is a good example, up to the moment in which a good theory of the ground state was at hand the description of its excitations remained basically phenomenological.

It is already accepted that in QCD the zero-point oscillations of the coupled modes produce a finite energy density, such effects are called non-perturbative ones. Obviously such an energy density can be subtracted by definition, however this procedure does not solve the problem, because soft modes are rearranged in the excited states and the variation of their energy should be unavoidable considered. This energy density is determined phenomenologically and its numerical estimate is [6]

$$E_{vac} \simeq -f\langle 0 \mid g^2G^2 \mid 0\rangle \simeq 0.5GeV/fm^3,$$

where the so-called non-perturbative gluonic condensate $\langle 0 \mid g^2G^2 \mid 0\rangle$ was introduced and phenomenologically evaluated by Shiftman, Vainshtein and Zakharov [7]. The negative sign of E_{vac} means that the non-perturbative vacuum energy is lower that the one associated to the perturbative vacuum.

Some of the QCD vacuum models, developed to explain the above-mentioned properties, are mentioned below. These models can be classified considering the dimension of the manifold in which the non-perturbative field fluctuations are concentrated.

1- The "instanton" model, in which it is assumed that the field is gathered in some localized regions of the space and time as instantaneous fluctuations. These are considered as fluctuations concentrated in zero dimensional manifolds.

2- The "soliton" model, in which it is assumed that the non linear gauge fields create some kind of stable particles or solitons (i.e. glueballs [8] or monopoles [9]) in the space. The space-time manifold to be considered for these models is one-dimensional.

3- The "string" model, in which closed strings (field created between color charges shows a form resembling a flux tube or string) are present in the vacuum. In space-time the history of these strings is a 2-dimensional surface, so in this picture the fluctuations are concentrated in closed surfaces.

4- The last model to be mentioned is the simplest one. It will be discussed here in more detail because it furnished the starting roots of the present discussion. This model is the "homogeneous" vacuum model, in which it is assumed that a magnetic field exist in the vacuum [10].

In the homogeneous vacuum field model, the existence of a constant magnetic abelian field H is assumed. A simple calculation in the one loop approximation gives as result the following energy density [6]

$$E\left(H\right)=\frac{H^{2}}{2}\left(1+\frac{bg^{2}}{16\pi^{2}}\ln\left(\frac{H}{\Lambda^{2}}\right)\right).$$

This formula predicts negative energy values for small values of the field H, so the usual perturbative ground state with $H=0$ is unstable with respect to the formation of a state with a non vanishing field intensity [6]

$$H_{vac}=\Lambda^{2}\exp\left(-\frac{16\pi^{2}}{bg^{2}}-\frac{1}{2}\right),$$

at which the energy $E\left(H\right)$ has a minimum.

With the use of this model an extensive number of physical problems, related with the hadron structure, confinement, etc. have been investigated. Nevertheless, after some time its intense study was abandoned. The main reason were:

1. The perturbative relation giving E_{vac} would be only valid if the second order of the perturbative expansion is relatively small.

2. The specific spatial and color directions of the magnetic field break the now seemingly indispensable Lorentz and $SU(3)$ invariance of the ground state.

3. The magnetic moment of the vector particle (gluon) is such that its energy in the presence of the field has a negative eigenvalue, which also makes unstable the homogeneous magnetic field H.

Before presenting the objectives of the present work it should be stressed that QCD quantization [11] is realized in the same way as that in QED, and it can be shown that QCD is renormalizable. The quadratic field terms in the QCD Lagrangian (L_{QCD}), which depend on the quark and gluon fields, have the same form that the ones corresponding to the electrons and photons in QED. However, in connection with the interaction, there appears a substantial difference due to the coupling of the gluon to itself. In order to assure the unitarity of the quantum theory of gauge fields, it was necessary to introduce fictitious particles called the Faddeev-Popov ghosts, which carry color charge, behaves as fermions (their fields anticommute) in spite of their boson like propagation. These particles cancel out the contributions of the non-physical gauge field degrees of freedom, and in physical calculations only appear as internal lines of the Feynman diagrams.

As it is well known, a perturbative expansion depends on the initial conditions at $t \to \pm\infty$ or what is the same on the states in which the expansion is based. The perturbation theory at finite orders differs in attention to the ground state selected, or from a functional point of view what boundary conditions are chosen. The perturbation theory in QED (PQED) is in excellent correspondence with the experimental facts. In this theory the expansion is based on a perturbative vacuum state that is the empty of the Fock space, excluding the presence of fermion and boson particles. This is a radical simplification of the exact perturbative ground state that should be a complex combination of states on the Fock basis. Formally the expansion around the Fock vacuum contains all the effects associated to the exact vacuum, but it would require from infinite orders of the expansion in the coupling constant for describing them. The rapid convergence of the perturbative series in PQED indicates that the higher excited states of the Fock basis expansion, in the real vacuum, have a short life and a small influence on physical observable. In QCD the color confinement indicates that the ground state has a non-trivial structure, which in terms of a Fock expansion could be represented with the formation of a gluon "condensate". Therefore, it should not be surprising that the PQCD fails to describe the low energy physics where the propagator of gluons could be affected by the presence of the condensate, even under the validity of a modified perturbative expansion. Such a perturbative condensate could generate all the effects over the physical observable, which in the standard expansion could require an infinite number of terms of the series.

In a previous work [12], following the above ideas, the construction of a modified perturbation theory for QCD was implemented. This construction retained the main invariance of the theory (the Lorentz and $SU(3)$ ones), and it was also able to reproduce some of the main physical predictions of the chromomagnetic field models. The central idea in that work was to modify the perturbative expansion in such a way that the effects of a gluon condensate could be incorporated. Such a modification is needed to be searched through the connection of the interaction on an alternative state in the Fock space designed to incorporate the presence of the gluon condensate. It is not excluded that this procedure could be also a crude approximation of the reality as in the case in which the connection is done on the Fock vacuum (QED). However, this procedure could produce a reasonable if not good description of the low energy physics. If such is the case the low and high-energy descriptions of QCD could be unified in a common unified perturbation theory. In particular, in that previous work [12] the results had the interesting outcome of producing a non vanishing mean value for the relevant quantity G^2. In addition the effective potential, in terms of the condensation parameter at a first order approximation, showed a minimum at non-vanishing values of that parameter. Therefore, the procedure was able to reproduce at least some central predictions of the chromomagnetic models and general QCD analysis.

The main objective of the present work is to search the foundations of the mentioned perturbation theory. The concrete aim is to find a physical state in the Fock space of the non-interacting theory being able to generate that expansion. The canonical quantization formalism for gauge fields, developed by Kugo and Ojima is employed.

The exposition will be organized as follows: The Chapter 2 is divided in three sections. In the first one a review of the former work [12] is done, by also establishing the needs for the present one and the objectives which are planned to be analyzed and solved. In the second section the operational quantization method for gauge fields developed by Kugo and Ojima is discussed. Starting from it, in the third section it is exposed the ansatz for the Fock space state that generates the desired form of the perturbative expansion. The proof that the state satisfies the physical state condition is also given in this section. The Chapter 3 is divided in three sections. In the first one an analysis for the general form of the generating functional in an arbitrary ground state is made. In the second section it is shown that the proposed state can generate the desired modification for the gluon propagator by a proper selection of the parameters at hand. In the third section the modification of the propagator for the ghost particles is investigated, such propagator was not modified in the work [12] and here this procedure is justified as compatible within the present description. Finally, two appendices are introduced for a detailed analysis of the most elaborated parts in the calculation of transverse, longitudinal and scalar modes contribution to the gluon propagator modification.

Chapter 2

Ground State Ansatz

The previous work [12] is reviewed, as motivation for the present discussion, and the objectives for the present work stated. It is also reviewed the canonical quantization method for gauge fields developed by Kugo and Ojima (K.O.). Finally the QCD modified vacuum state is proposed and it is shown that this state satisfies the BRST physicality conditions imposed by the K.O. formalism.

2.1 Motivation

In this section a review of a previous work [12] is made. The main properties of this approach, as was mentioned in the introduction, were:

a) The ability to produce a gluon-condensation parameter value $\langle G^2 \rangle$ directly in the first approximation.

b) The prediction of a minimum of the effective action for non-vanishing values of the condensation parameter.

The discussion in [12] opened the possibility of reproducing some interesting physical implications of the early chromomagnetic field models for the QCD vacuum [10, 13] by also solving some of their main shortcoming: The breaking of Lorentz and $SU(3)$ invariance. However, the discussion in [12] had also a limitation; that is it was unknown if the state that generated the proposed modification to the gluon propagator was a physical state of the theory. This shortcoming, could be expressed in the gauge parameter dependence of the calculated gluon mass. Below it is reminded the main analysis in [12].

The exposition was referred to the Euclidean space and the followed conventions were used,

$$\begin{aligned}
\nabla_\mu^{ab} &= \delta^{ab}\partial_\mu + g f^{abc} A_\mu^c, \\
F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu{}^a + f^{abc} A_\mu^c A_\nu^b,
\end{aligned}$$

where g is the coupling constant and f^{abc} are the structure constant of $SU(3)$.

The action for the problem, including the auxiliary sources for all the fields was taken as,

$$\begin{aligned}
S_T\left[A,\overline{C},C\right] &= \int d^4x \left\{ -14 F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2\alpha}\partial_\mu A_\mu^a \partial_\nu A_\nu^a + \overline{C}^a \nabla_\mu{}^{ab} \partial_\mu C^b \right. \\
&\qquad \left. + J_\mu^a A_\mu^a + \overline{\xi}^a C^a + \overline{C}^a \xi^a \right\},
\end{aligned}$$

where A_μ, $\overline{C}, C$ are the gauge and ghost fields and α is the gauge fixing parameter [14] for the Lorentz gauge.

The generating functional for the Green functions was expressed in the form

$$Z_T\left[J,\xi,\overline{\xi}\right] = \frac{1}{N}\int D\left(A,\overline{C},C\right) \exp\left\{S_T\left[A,\overline{C},C\right]\right\},$$

which through the usual Legendre transformation led to the effective action,

$$\Gamma\left[\Phi\right] = \ln Z\left[J\right] - J_i \Phi_i, \qquad \text{with} \quad \Phi_i = \frac{\delta \ln Z\left[J\right]}{\delta J_i}, \tag{2.1}$$

Φ_i denoted the mean values of the fields, and the compact notation of DeWitt [15],

$$\Phi_i \equiv \left(A,\overline{C},C\right); \qquad J_i \equiv \left(J,\xi,\overline{\xi}\right),$$

was used. In which Φ_i and J_i indicate all the fields and sources at a space-time point, respectively. Repeated indices imply space-time integration as well as summation over all the field types and over their Lorentz and color components.

The one-loop effective action and the corresponding “quantum” Lagrange equations, in the compact notation, were considered as,

$$\Gamma\left[\Phi\right] = S\left[\Phi\right] + \frac{1}{2}\ln DetD\left[\Phi\right], \tag{2.2}$$

$$\Gamma_{,i}\left[\Phi\right] = S_{,i}\left[\Phi\right] + \frac{1}{2} S_{,ikj} D_{kj} = -J_i, \tag{2.3}$$

the functional derivatives were denoted by

$$L_{,i}\left[\Phi\right] = \frac{\delta L\left[\Phi\right]}{\delta\Phi_i}$$

and the action defined by $S_T = S + J_i\Phi_i$.

The Φ dependent propagator D was defined, as usual, through

$$D_{ij} = -S_{,ij}^{-1}\left[\Phi\right], \tag{2.4}$$

After considering a null mean value for the vector field Φ, as requires the $SO(4)$ invariance, the propagator relation (2.4) took the form

$$D_{ij} = -S_{,ij}^{-1}\left[0\right]. \tag{2.5}$$

In this case the only non vanishing second derivatives of the action were,

$$\frac{\delta^2 S}{\delta A_\mu^a\left(x\right)\delta A_\nu{}^b\left(x'\right)}\left[0\right] = \delta^{ab}\left(\partial_x^2\delta_{\mu\nu} - \left(1 + 1\alpha\right)\partial_\mu^x\partial_\nu^x\right)\delta\left(x - x'\right), \tag{2.6}$$

$$\frac{\delta^2 S}{\delta C^a\left(x\right)\delta\overline{C}^b\left(x'\right)}\left[0\right] = \delta^{ab}\partial_x^2\delta\left(x - x'\right). \tag{2.7}$$

The gluon and ghost propagators are the inverse kernels of (2.6) and (2.7). Here, the alternative for a perturbative description of gluon condensation appeared. As (2.6) consist of derivatives only, the inverse kernel of the gluon propagator could include coordinate independent terms reflecting a sort of gluon condensation. It should be noticed that the propagator is a $SO\left(4\right)$ tensor (not a vector) then a constant term in it does not led necessary to a breaking of the $SO\left(4\right)$ invariance [12]. Accordingly with the above remark gluon and ghost propagators were selected as

$$D_{\mu\nu}^{ab}\left(x\right) = \int\frac{dp}{\left(2\pi\right)^4}\left[C\delta^{ab}\delta_{\mu\nu}\delta\left(p\right) + \frac{\delta^{ab}}{p^2}\left(\delta_{\mu\nu} - \left(1 + \alpha\right)\frac{p_\mu p_\nu}{p^2}\right)\right]\exp\left(ipx\right), \tag{2.8}$$

$$D_G^{ab}\left(x\right) = \int\frac{dp}{\left(2\pi\right)^4}\frac{\delta^{ab}}{p^2}\exp\left(ipx\right), \tag{2.9}$$

and it was checked that the equations of motion (2.3), considering (2.8) and (2.9) and taking vanishing gluon and ghost fields, were satisfied.

After that some implications of the modified gluon propagator, in the standard perturbative calculations, were analyzed [12].

The first interesting result obtained was the standard one loop polarization tensor. It was modified by a massive term, depending on the condensate parameter, with the form

$$m^2 = \frac{3g^2}{(2\pi)^4} C (1 - \alpha) . \tag{2.10}$$

This result had a dependence on the gauge parameter α; which as was mentioned above is one of the shortcomings of the discussion [12] because it was unknown if this mass term was generated by a non-physical vacuum state. In the present work the idea is to solve this difficulty by explicitly constructing a perturbative state leading to the considered form of the propagator, but also satisfying the BRST physical state condition in the non-interacting limit.

The mean value of the squared field intensity operator was also calculated [12], within the simplest approximation (the tree approximation), with the use of the proposed propagator. That is, it was evaluated the expression

$$\langle 0 \mid S_g [\Phi] \mid 0 \rangle \equiv \frac{1}{N} \left[\int D (\Phi) \, S_g [\Phi] \exp S_T [\Phi] \right]_{J_i = 0} ,$$

with

$$S_g [\Phi] \equiv \int d^4x \left\{ -\frac{1}{4} F^a_{\mu\nu} (x) \, F^a_{\mu\nu} (x) \right\} ,$$

and the following result was obtained,

$$\langle 0 \mid S_g [\Phi] \mid 0 \rangle = -\frac{72 g^2 C^2}{(2\pi)^8} \int d^4x. \tag{2.11}$$

Then the mean value of G^2 took the form

$$G^2 \equiv \langle 0 \mid F^a_{\mu\nu} (x) \, F^a_{\mu\nu} (x) \mid 0 \rangle = \frac{288 g^2 C^2}{(2\pi)^8} . \tag{2.12}$$

The substitution of Eq. (2.12) in Eq. (2.10) gave a rough estimate of the gluon mass. It was selected a particular value of $\alpha = 0$ and assumed the more or less accepted value of g^2G^2 in the physical vacuum

$$g^2 G^2 \cong 0.5 \left(\frac{GeV}{c^2} \right)^4 , \tag{2.13}$$

then the estimated value of the gluon mass became

$$m = 0.35 \frac{GeV}{c^2} . \tag{2.14}$$

Finally, an evaluation for the contribution to the effective potential of all the one-loop graphs, having only mass term insertions in the polarization tensor, was done. The result, in terms of G^2 (2.12), turned to be of the form [12],

$$V\left(G^2\right) = \frac{G^2}{4} + \frac{3}{16\pi^2} g^2 \frac{G^2}{32} \ln \frac{g^2 G^2}{\mu^4}, \tag{2.15}$$

where μ is the dimensional parameter included by the renormalization procedure.

As it can be noticed in (2.15), the effective potential indicates the spontaneous generation of a G^2 condensate from the usual perturbative vacuum $(G^2 = 0)$. This occurred in close analogy with the chromomagnetic fields.

Then from the reviewed functional treatment, there are some interesting features that allow believing that the above procedure could describe relevant phenomena of the low energy region through a perturbative expansion. However some questions needed to be answered and taken as objectives of the present work are:

1- To determine under what conditions the new gluon propagator (2.8) corresponds to a modified vacuum satisfying the physical state condition. This could also help in the understanding of the α dependence in the gluonic mass term.

2- To investigate the form of the ghost propagator in the modified vacuum state, because in the previous work [12] it was taken the as same of the usual perturbative theory.

2.2 Operational Quantization Formalism

As it is well known the non-abelian character of Yang-Mills fields determines the asymptotic freedom property, and the quark-confinement problem of QCD. This character simultaneously makes difficult the quantization of such theories. The first approach to this quantization was made by Faddeev and Popov in the path integral formalism [11], with the resulting correct Feynman rules including the Faddeev-Popov ghost fields and the renormalizability of the theory. But this approach has the problem of the absence of notions about the state vector space and the Heisemberg operators. In this case due the non-abelian character of the theory it is not possible to use the operators formalism developed by Gupta-Bleuler [16] or the more general Nakanishi-Lautrup version [17], which can be used only for the abelian case. This situation occurs because de S-Matrix calculated with those procedures is not unitary in the non abelian case, as it was first mentioned by Feynman [18].

In the present work the operator formalism developed by T. Kugo and I. Ojima [19], for the first consistent quantization of the Yang-Mills fields, is considered. This formulation uses the Lagrangian invariance under a global symmetry operation called the BRST transformation [20]. In the following a brief review of the K.O. work is done and the following conventions are used.

Let G be a compact Lie group, and Λ any matrix in the adjoin representation of its associated Lie Algebra. The matrix Λ can be represented as a linear combination of the form

$$\Lambda = \Lambda^a T^a,$$

were T^a are the generators $(a = 1, ...,\text{Dim}G = n)$, which can be chosen as Hermitian ones and satisfying

$$\left[T^a, T^b\right] = i f^{abc} T^c.$$

The field variations under infinitesimal gauge transformations are given by

$$\begin{aligned}
\delta_\Lambda A_\mu^a(x) &= \partial_\mu \Lambda^a(x) + g f^{acb} A_\mu^c(x)\,\Lambda^b(x) = D_\mu^{ab}(x)\,\Lambda^b, \\
D_\mu{}^{ab}(x) &= \partial_\mu \delta^{ab} + g f^{acb} A_\mu{}^c(x).
\end{aligned}$$

The metric $g_{\mu\nu}$ is taken in the convention

$$g_{00} = -g_{ii} = 1 \qquad \text{for} \quad i = 1, 2, 3.$$

The complete G.D. Lagrangian to be considered is the one employed in the operator quantization approach [21]. Its explicit form is given by

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{GF} + \mathcal{L}_{FP} \tag{2.16}$$

$$\mathcal{L}_{YM} = -\frac{1}{4} F_{\mu\nu}^a(x)\, F^{\mu\nu,a}(x), \tag{2.17}$$

$$\mathcal{L}_{GF} = -\partial^\mu B^a(x)\, A_\mu{}^a(x) + \frac{\alpha}{2} B^a(x)\, B^a(x), \tag{2.18}$$

$$\mathcal{L}_{FP} = -i \partial^\mu \overline{c}^a(x)\, D_\mu^{ab}(x)\, c^b(x), \tag{2.19}$$

where field intensity is

$$F_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + g f^{abc} A_\mu^b(x)\, A_\nu^c(x).$$

Relation (2.17) defines the standard Yang-Mills Lagrangian, Eq. (2.18) defines the gauge fixing term which can be also rewritten in the form

$$\mathcal{L}_{GF} = -\frac{1}{2\alpha}\left(\partial^\mu A_\mu^a(x)\right)^2 + \frac{\alpha}{2}\left(B^a(x) + \frac{1}{\alpha}\partial^\mu A_\mu^a(x)\right)^2 - \partial^\mu\left(B^a(x)\, A_\mu^a(x)\right),$$

equivalent to the more familiar $-\frac{1}{2\alpha}\left(\partial^{\mu} A_{\mu}^{a}\left(x\right)\right)^{2}$, at the equations of motion level [14] and Feynman diagram expansion.

Finally, Eq. (2.19) describes the non-physical Faddeev-Popov ghost sector. The definition for such fields in the Kugo and Ojima (K.O.) approach is satisfying

$$\overline{c}^{\dagger}=\overline{c}, \qquad c^{\dagger}=c.$$

That is, the ghost fields are Hermitian. In the Faddeev-Popov formalism [14] they satisfy

$$C^{\dagger}=\overline{C}, \qquad \overline{C}^{\dagger}=C.$$

However, a simple change of variables is able to transform between the ghost fields satisfying both kind of conjugation conditions. The selected conjugation properties, for this sector, allowed Kugo and Ojima to solve various formal problems existing for the application of the BRST operator quantization method to QCD, for example the hermiticity of the Lagrangian, which guarantees the unitarity of the S-Matrix.

The physical state conditions in the BRST procedure [21] are given by

$$\begin{aligned} Q_{B} \mid phys\rangle &= 0, \\ Q_{C} \mid phys\rangle &= 0, \end{aligned} \tag{2.20}$$

where

$$Q_{B}=\int d^{3}x\left[B^{a}\left(x\right)\overleftrightarrow{\partial_{0}}c^{a}\left(x\right)+gB^{a}\left(x\right)f^{abc}A_{0}^{b}\left(x\right)c^{c}\left(x\right)+\frac{i}{2}g\partial_{0}\left(\overline{c}^{a}\right)f^{abc}c^{b}\left(x\right)c^{c}\left(x\right)\right],$$

with

$$f\left(x\right)\overleftrightarrow{\partial_{0}}g\left(x\right)\equiv f\left(x\right)\partial_{0}g\left(x\right)-\partial_{0}\left(f\left(x\right)\right)g\left(x\right).$$

The BRST charge is conserved as a consequence of the BRST symmetry of the Lagrangian (2.16).

The also conserved charge Q_C is given by

$$Q_{C}=i\int d^{3}x\left[\overline{c}^{a}\left(x\right)\overleftrightarrow{\partial_{0}}c^{a}\left(x\right)+g\overline{c}^{a}\left(x\right)f^{abc}A_{0}^{b}\left(x\right)c^{c}\left(x\right)\right],$$

its conservation comes from the Noether theorem, due to the Lagrangian invariance (2.16) under the phase transformation $c\rightarrow e^{\theta}c,\ \overline{c}\rightarrow e^{-\theta}\overline{c}$. This charge defines the so called "ghost number" as the difference between the number of ghost c and $\overline{c}$.

The analysis here is restricted to the Yang-Mills Theory without spontaneous breaking of the gauge symmetry. The quantization for the theory defined by the Lagrangian

(2.16), considering the interacting free limit $g \rightarrow 0$, leads to the following commutation relations between the free fields,

$$\begin{aligned}
\left[A_{\mu}^{a}(x), A_{\nu}^{b}(y)\right] &= \delta^{ab}\left(-i g_{\mu\nu} D(x-y)+i(1-\alpha)\partial_{\mu}\partial_{\nu} E(x-y)\right), \\
\left[A_{\mu}^{a}(x), B^{b}(y)\right] &= \delta^{ab}\left(-i\partial_{\mu} D(x-y)\right), \\
\left[B^{a}(x), B^{b}(y)\right] &= \left\{\overline{c}^{a}(x), \overline{c}^{b}(y)\right\}=\left\{c^{a}(x), c^{b}(y)\right\}=0, \\
\left\{c^{a}(x), \overline{c}^{b}(y)\right\} &= -D(x-y),
\end{aligned} \tag{2.21}$$

The E functions are defined by [21]

$$E_{(.)}(x)=\frac{1}{2}\left(\nabla^{2}\right)^{-1}\left(x_{0}\partial^{0}-1\right) D_{(.)}(x).$$

The equations of motion for the non-interacting fields take the simple form

$$\square A_{\mu}^{a}(x)-(1-\alpha)\partial_{\mu} B^{a}(x) = 0, \tag{2.22}$$

$$\partial^{\mu} A_{\mu}^{a}(x)+\alpha B^{a}(x) = 0, \tag{2.23}$$

$$\square B^{a}(x)=\square c^{a}(x)=\square \overline{c}^{a}(x) = 0. \tag{2.24}$$

This equations can be solved for an arbitrary values of the α parameter. However, the discussion will be restricted to the case $\alpha=1$ which corresponds to the situation in which all the gluon components satisfy the D'Alambert equation. This selection, as considered in the framework of the usual perturbative expansion, implies that you are not able to check the α independence of the physical quantities. This simplification is a necessary requirement. In the present discussion, the aim is to construct a perturbative state that satisfies the BRST physical state condition, in order to connect adiabatically the interaction. Then, the physical character of all the prediction will follow whenever the former assumption that adiabatic connection do not take the state out of the physical subspace at any intermediate state. The consideration of different values of α, would be also a convenient recourse for checking the α independent perturbative expansion. However, at this stage it is preferred to delay this more technical issue for future work.

In that way the field equations for the $\alpha=1$ are

$$\square A_{\mu}^{a}(x)=\square B^{a}(x)=\square c^{a}(x)=\square \overline{c}^{a}(x) = 0, \tag{2.25}$$

$$\partial^{\mu} A_{\mu}^{a}(x)+B^{a}(x) = 0. \tag{2.26}$$

The solutions of the set (2.25), (2.26) can be written as

$$\begin{aligned}
A_{\mu}^{a}\left(x\right) &= \sum_{\vec{k},\sigma}\left(A_{\vec{k},\sigma}^{a}f_{k,\mu}^{\sigma}\left(x\right)\,+A_{\vec{k},\sigma}^{a+}f_{k,\mu}^{\sigma *}\left(x\right)\right),\\
B^{a}\left(x\right) &= \sum_{\vec{k}}\left(B_{\vec{k}}^{a}g_{k}\left(x\right)\,+B_{\vec{k}}^{a+}g_{k}^{*}\left(x\right)\right),\\
c^{a}\left(x\right) &= \sum_{\vec{k}}\left(c_{\vec{k}}^{a}g_{k}\left(x\right)+c_{\vec{k}}^{a+}g_{k}^{*}\left(x\right)\right),\\
\overline{c}^{a}\left(x\right) &= \sum_{\vec{k}}\left(\overline{c}_{\vec{k}}^{a}g_{k}\left(x\right)+\overline{c}_{\vec{k}}^{a+}g_{k}^{*}\left(x\right)\right).
\end{aligned}\tag{2.27}$$

The wave packets system, for non-massive scalar and vector fields, are taken in the form

$$\begin{aligned}
g_{k}\left(x\right) &= \frac{1}{\sqrt{2Vk_{0}}}\exp\left(-ikx\right),\\
f_{k,\mu}^{\sigma}\left(x\right) &= 1\sqrt{2Vk_{0}}\epsilon_{\mu}^{\sigma}\left(k\right)\exp\left(\,-\,ikx\right).
\end{aligned}\tag{2.28}$$

The polarization vectors, in Eq. (2.28) are defined by

$$\vec{k}\cdot\vec{\epsilon}_{\sigma}\left(k\right)=0,\;\;\epsilon_{\sigma}^{0}\left(k\right)=0,$$

and satisfy

$$\vec{\epsilon}_{\sigma}\left(k\right)\cdot\vec{\epsilon}_{\tau}\left(k\right)=\delta_{\sigma\tau},$$

where $\sigma,\tau=1,2$ are the transverse modes. For the longitudinal L and scalar S modes the definitions are

$$\begin{aligned}
\epsilon_{L}^{\mu}\left(k\right) &= -ik^{\mu}=-i\left(\left|\vec{k}\right|,\vec{k}\right),\;\;\epsilon_{L}^{\mu *}\left(k\right)=-\epsilon_{L}^{\mu}\left(k\right),\\
\epsilon_{S}^{\mu}\left(k\right) &= -i\frac{\overline{k}^{\mu}}{2\left|\vec{k}\right|^{2}}=\frac{-\,i\left(\left|\vec{k}\right|,-\vec{k}\right)}{2\left|\vec{k}\right|^{2}},\;\;\epsilon_{S}^{\mu *}\left(k\right)\,=-\epsilon_{S}^{\mu}\left(k\right),
\end{aligned}$$

and satisfy

$$\begin{aligned}
\epsilon_{L}^{\mu *}\left(k\right)\cdot\epsilon_{L,\mu}\left(k\right) &= \epsilon_{S}^{\mu *}\left(k\right)\cdot\epsilon_{S,\mu}\left(k\right)=0,\\
\epsilon_{L}^{\mu *}\left(k\right)\cdot\epsilon_{S,\mu}\left(k\right) &= 1.
\end{aligned}$$

The scalar product of the defined polarizations define the metric matrix

$$\widetilde{\eta}_{\sigma\tau} = \epsilon_{\sigma}^{\mu *}(k) \cdot \epsilon_{\tau,\mu}(k) \equiv \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Now it is possible to introduce the contravariant (in the polarization index) polarizations

$$\epsilon^{\sigma,\mu}(k) = \sum_{1,2,L,S} \widetilde{\eta}^{\sigma\tau} \cdot \epsilon_{\tau}^{\mu}(k),$$

satisfying

$$\sum_{\sigma} \epsilon^{\sigma,\mu}(k) \cdot \epsilon_{\sigma}^{\nu *}(k) = \sum_{\sigma,\tau} \widetilde{\eta}^{\sigma\tau} \cdot \epsilon_{\tau}^{\mu}(k) \cdot \epsilon_{\sigma}^{\nu *}(k) = g^{\mu\nu}$$

and

$$\begin{aligned} \epsilon^{\sigma,\mu}(k) \cdot \epsilon_{\mu}^{\tau *}(k) &= \widetilde{\eta}^{\sigma\tau}, \\ \widetilde{\eta}^{\sigma\tau'} \cdot \widetilde{\eta}_{\tau'\tau} &= \delta_{\tau}^{\sigma}. \end{aligned}$$

After that, it follows for the vector functions

$$\sum_{\vec{k},\sigma} f_{k,\sigma}^{\mu}(x) \cdot f_{k}^{\sigma,\nu *}(y) = g^{\mu\nu} D_{+}(x-y).$$

As it can be seen from (2.26) the $A_{\vec{k},\sigma}^{a}$ and $B_{\vec{k}}^{a}$ modes are not all independent. Indeed, it follows from (2.26) that

$$B_{\vec{k}}^{a} = A_{\vec{k}}^{S,a} = A_{\vec{k},L}^{a}.$$

Then excluding the scalar mode, the free Heisemberg fields expansion takes the form

$$A_{\mu}^{a}(x) = \sum_{\vec{k}} \left(\sum_{\sigma=1,2} A_{\vec{k},\sigma}^{a} f_{k,\mu}^{\sigma}(x) + A_{\vec{k}}^{L,a} f_{k,L,\mu}(x) + B_{\vec{k}}^{a} f_{k,S,\mu}(x) \right) + h.c., \tag{2.29}$$

where $h.c.$ represents the Hermitian conjugate of the first term.

In order to satisfy the commutations relations (2.21) the creation and annihilation operator, associated to the Fourier components of the field, should obey

$$\begin{aligned}\left[A^{a}_{\vec{k},\sigma}, A^{a'+}_{\vec{k}',\sigma'}\right] &= -\delta^{aa'}\delta_{\vec{k}\vec{k}'}\eta_{\sigma\sigma'},\\ \left\{c^{a}_{\vec{k}}, \overline{c}^{a'+}_{\vec{k}'}\right\} &= i\delta^{aa'}\delta_{\vec{k}\vec{k}'},\\ \left\{\overline{c}^{a}_{\vec{k}}, c^{a'+}_{\vec{k}'}\right\} &= -i\delta^{aa'}\delta_{\vec{k}\vec{k}'}\end{aligned} \tag{2.30}$$

and all the other vanish.

In a symbolic matrix form these relations can be arranged as follows

$$\begin{array}{cccccc} & A^{a'+}_{\vec{k}',\sigma'} & A^{L,a'+}_{\vec{k}'} & B^{a'+}_{\vec{k}'} & c^{a+}_{\vec{k}'} & \overline{c}^{a+}_{\vec{k}'} \\ A^{a}_{\vec{k},\sigma} & \delta^{aa'}\delta_{\vec{k}\vec{k}'}\delta_{\sigma\sigma'} & 0 & 0 & 0 & 0 \\ A^{L,a}_{\vec{k}} & 0 & 0 & -\delta^{aa'}\delta_{\vec{k}\vec{k}'} & 0 & 0 \\ B^{a}_{\vec{k}} & 0 & -\delta^{aa'}\delta_{\vec{k}\vec{k}'} & 0 & 0 & 0 \\ c^{a}_{\vec{k}} & 0 & 0 & 0 & 0 & i\delta^{aa'}\delta_{\vec{k}\vec{k}'} \\ \overline{c}^{a}_{\vec{k}} & 0 & 0 & 0 & -i\delta^{aa'}\delta_{\vec{k}\vec{k}'} & 0 \end{array} \tag{2.31}$$

The above commutation rules and equation of motions define the quantized non-interacting limit of G.D. Then, it is possible now to define the alternative interacting free ground state to be considered for the adiabatic connection of the interaction. As discussed before, the expectation is that the physics of the perturbation theory to be developed will be able to furnish good description of some low energy physical effects.

It is interesting to comment now that one of the first tasks proposed for the present work was to construct a state, in quantum electrodynamics, able to generate a modification for the photon propagator similar to the one proposed in [12] for gluons. It was used the quantification operator method developed by Gupta and Bleuler (GB), however was impossible to find any state generating this covariant propagator modification and satisfying the physical state condition imposed by this formalism.

In the GB formalism the physical state condition is given by

$$\partial^{\mu}A^{+}_{\mu}(x) \mid \Phi\rangle = 0,$$

or in terms of the annihilation operators [22], by

$$k_0\left(A_{\vec{k},3} - A_{\vec{k},0}\right) \mid \Phi\rangle = 0.$$

The more general state satisfying this condition is [23]

$$| \Phi\rangle = \sum_{m,n_1,n_2} B_{n_1,n_2,m} \mid \Phi\left(n_1,n_2,m\right)\rangle,$$

with

$$| \Phi\left(n_1,n_2,m\right)\rangle = (m!)^{-\frac{1}{2}} \left(A^{+}_{\vec{k},3} - A^{+}_{\vec{k},0}\right)^{m} (n_1!n_2!)^{-\frac{1}{2}} \left(A^{+}_{\vec{k},1}\right)^{n_1} \left(A^{+}_{\vec{k},2}\right)^{n_2} \mid 0\rangle,$$

where $B_{n_1,n_2,m}$ are arbitrary constants. This general form of the state is the one that disabled to find a covariant modification to the propagator.

2.3 The alternative vacuum state

In the present section the construction of a relativistic invariant ground state in the non-interacting limit of QCD is considered. It will be required that the proposed state satisfies the BRST physical state conditions. Then this state will have an opportunity to furnish the gluodynamics ground state under the adiabatic connection of the interaction.

After beginning to work in the K.O. formalism some indications were found, that the appropriate state obeying the physical state conditions in this procedure, and with possibilities for generating the modification to the gluon propagator proposed in the previous work, could have the general structure

$$| \phi\rangle = \exp \sum_{a} \left(\sum_{\sigma=1,2} \frac{1}{2} C_{\sigma}\left(|\vec{p}|\right) A^{a+}_{\vec{p},\sigma} A^{a+}_{\vec{p},\sigma} + C_3\left(|\vec{p}|\right) \left(B^{a+}_{\vec{p}} A^{L,a+}_{\vec{p}} + i\bar{c}^{a+}_{\vec{p}} c^{a+}_{\vec{p}} \right) \right) \mid 0\rangle, \quad (2.32)$$

where $\vec{p}$ is an auxiliary momentum chosen as one of the few smallest momenta of the quantized theory in a finite volume V. This value will be taken later in the limit $V \to \infty$ for recovering Lorentz invariance. From here the sum on the color index a will be explicit. The parameters $C_i\left(|\vec{p}|\right)$, $i = 1, 2, 3,$ will be fixed below from the condition that the free propagator associated to a state satisfying the BRST physical state condition, coincides with the one proposed in the previous work [12]. The solution of this problem, then would give a more solid foundation to the physical implications of the discussion in that work.

It should also be noticed that the state defined by Eq. (2.32) has some similarity with the coherent states [24]. However, in the present case, the creation operators appear in squares. Thus, the argument of the exponential creates pairs of physical and non-physical particles. An important property of this function is that its construction in terms of creation operator pairs determines that the mean value of an odd number of field operators vanishes. This at variance with the standard coherent state in which the mean values of the fields are nonzero. The vanishing of the mean fields is a property in common with the standard perturbative vacuum, in which Lorentz invariance could be broken by a non-vanishing expectation value of a 4-vector the gauge field. It should be also stressed that this state as formed by the superposition of gluons state pairs suggests a connection with some recent works in the literature that consider the formation gluons pairs due to the color interactions.

Now it is checked that the state (2.32) satisfies the BRST physical state conditions

$$\begin{aligned} Q_B \mid \Phi\rangle &= 0, \\ Q_C \mid \Phi\rangle &= 0. \end{aligned} \tag{2.33}$$

The expressions for these charges in the interaction free limit [21] are

$$\begin{aligned} Q_B &= i \sum_{\vec{k},a} \left(c^{a+}_{\vec{k}} B^a_{\vec{k}} - B^{a+}_{\vec{k}} \ c^a_{\vec{k}} \right), \\ Q_C &= i \sum_{\vec{k},a} \left(\overline{c}^{a+}_{\vec{k}} c^a_{\vec{k}} + c^{a+}_{\vec{k}} \overline{c}^a_{\vec{k}} \right). \end{aligned} \tag{2.34}$$

Considering first the action of Q_B on the proposed state,

$$\begin{aligned} Q_B \mid \Phi\rangle &= i \exp\left\{ \sum_{\sigma,a} \ 12 C_\sigma \left(|\vec{p}|\right) A^{a+}_{\vec{p},\sigma} A^{a+}_{\vec{p},\sigma} \right\} \times \\ &\times \left(\exp\left\{ \sum_a C_3 \left(|\vec{p}|\right) i \overline{c}^{a+}_{\vec{p}} c^{a+}_{\vec{p}} \right\} \sum_{\vec{k},b} c^{b+}_{\vec{k}} B^b_{\vec{k}} \exp\left\{ \sum_a C_3 \left(|\vec{p}|\right) B^{a+}_{\vec{p}} A^{L,a+}_{\vec{p}} \right\} \right. \\ &- \left. \exp\left\{ \sum_a C_3 \left(|\vec{p}|\right) B^{a+}_{\vec{p}} A^{L,a+}_{\vec{p}} \right\} \sum_{\vec{k},b} B^{b+}_{\vec{k}} c^b_{\vec{k}} \exp\left\{ \sum_a C_3 \left(|\vec{p}|\right) i \overline{c}^{a+}_{\vec{p}} c^{a+}_{\vec{p}} \right\} \right) \mid 0\rangle = 0, \end{aligned} \tag{2.35}$$

where the identity

$$\left[B^b_{\vec{k}}, \exp \sum_a C_3 \left(|\vec{p}|\right) B^{a+}_{\vec{p}} A^{L,a+}_{\vec{p}} \right] = -C_3 \left(|\vec{p}|\right) B^{b+}_{\vec{p}} \delta_{\vec{k},\vec{p}} \exp \sum_a C_3 \left(|\vec{p}|\right) B^{a+}_{\vec{p}} A^{L,a+}_{\vec{p}}, \tag{2.36}$$

was used.

For the action of Q_C on the considered state it follows

$$Q_C \mid \Phi\rangle = i\exp\left\{\sum_{\sigma,a} 12C_\sigma\left(|\vec{p}|\right) A^{a+}_{\vec{p},\sigma}A^{a+}_{\vec{p},\sigma} + \sum_a C_3\left(|\vec{p}|\right) B^{a+}_{\vec{p}} A^{L,a+}_{\vec{p}}\right\} \tag{2.37}$$
$$\times\left[\sum_{\vec{k},b} \overline{c}^{b+}_{\vec{k}} c^{b}_{\vec{k}}\left(1+\sum_a iC_3\left(|\vec{p}|\right)\overline{c}^{a+}_{\vec{p}} c^{a+}_{\vec{p}}\right) + \sum_{\vec{k},b} c^{b+}_{\vec{k}}\overline{c}^{b}_{\vec{k}}\left(1+\sum_a iC_3\left(|\vec{p}|\right)\overline{c}^{a+}_{\vec{p}} c^{a+}_{\vec{p}}\right)\right] \mid 0\rangle = 0$$

which vanishes due to the commutation rules of the ghost operators (2.31).

Next, the evaluation of norm of the proposed state is considered, which due to the commutation properties of the operator can be written as

$$\begin{aligned}\langle\Phi \mid \Phi\rangle = \prod_{a=1,..,8}\ \prod_{\sigma=1,2} & \langle 0 \mid \exp\left\{ 12C^*_\sigma\left(|\vec{p}|\right) A^{a}_{\vec{p},\sigma}A^{a}_{\vec{p},\sigma}\right\}\exp\left\{\frac{1}{2}C_\sigma\left(|\vec{p}|\right) A^{a+}_{\vec{p},\sigma}A^{a+}_{\vec{p},\sigma}\right\} \mid 0\rangle \\ \times\quad & \langle 0 \mid \exp\left\{C^*_3\left(|\vec{p}|\right) A^{L,a}_{\vec{p}} B^{a}_{\vec{p}}\right\}\exp\left\{C_3\left(|\vec{p}|\right) B^{a+}_{\vec{p}} A^{L,a+}_{\vec{p}}\right\} \mid 0\rangle \\ \times\quad & \langle 0 \mid \left(1 - iC^*_3\left(|\vec{p}|\right) c^{a}_{\vec{p}}\overline{c}^{a}_{\vec{p}}\right)\left(1 + iC_3\left(|\vec{p}|\right)\overline{c}^{a+}_{\vec{p}} c^{a+}_{\vec{p}}\right) \mid 0\rangle. \end{aligned} \tag{2.38}$$

For the product of the factors associated with transverse modes and the eight values of the color index, after expanding the exponential in series, it follows that

$$\begin{aligned} & \left[\langle 0 \mid \exp\left\{\frac{1}{2}C^*_\sigma\left(|\vec{p}|\right) A^{a}_{\vec{p},\sigma}A^{a}_{\vec{p},\sigma}\right\}\exp\left\{\frac{1}{2}C_\sigma\left(|\vec{p}|\right) A^{a+}_{\vec{p},\sigma}A^{a+}_{\vec{p},\sigma}\right\} \mid 0\rangle\right]^8 \\ &= \left[\langle 0 \mid \sum_{m=0}^{\infty}\left|\frac{1}{2}C_\sigma\left(|\vec{p}|\right)\right|^{2m} \frac{\left(A^{a}_{\vec{p},\sigma}\right)^{2m}\left(A^{a+}_{\vec{p},\sigma}\right)^{2m}}{(m!)^2} \mid 0\rangle\right]^8 \\ &= \left[\sum_{m=0}^{\infty}\left|\frac{1}{2}C_\sigma\left(|\vec{p}|\right)\right|^{2m}\frac{(2m)\,!}{(m!)^2}\right]^8, \end{aligned} \tag{2.39}$$

where the identity

$$\langle 0 \mid \left(A^{a}_{\vec{p},\sigma}\right)^{2m}\left(A^{a+}_{\vec{p},\sigma}\right)^{2m} \mid 0\rangle = (2m)!,$$

was used.

The factors linked with the scalar and longitudinal modes can be transformed as follows

$$
\begin{aligned}
&\left[\langle 0 \mid \exp\left\{C_3^*\left(|\vec{p}|\right) A_{\vec{p}}^{L,a} B_{\vec{p}}^{a}\right\} \exp\left\{C_3\left(|\vec{p}|\right) B_{\vec{p}}^{a+} A_{\vec{p}}^{L,a+}\right\} \mid 0\rangle\right]^{8} \\
&= \left[\langle 0 \mid \sum_{m=0}^{\infty} \left|C_3\left(|\vec{p}|\right)\right|^{2m} \frac{\left(A_{\vec{p}}^{L,a} B_{\vec{p}}^{a}\right)^{m} \left(B_{\vec{p}}^{a+} A_{\vec{p}}^{L,a+}\right)^{m}}{(m!)^{2}} \mid 0\rangle\right]^{8} \\
&= \left[\sum_{m=0}^{\infty} \left|C_3\left(|\vec{p}|\right)\right|^{2m}\right]^{8} = \left[\frac{1}{\left(1-\left|C_3\left(|\vec{p}|\right)\right|^{2}\right)}\right]^{8} \text{ for } \quad \left|C_3\left(|\vec{p}|\right)\right| < 1,
\end{aligned}
\tag{2.40}
$$

in which the identity

$$
\langle 0 \mid \left(A_{\vec{p}}^{L,a} B_{\vec{p}}^{a}\right)^{m} \left(B_{\vec{p}}^{a+} A_{\vec{p}}^{L,a+}\right)^{m} \mid 0\rangle = (m!)^{2},
$$

was employed.

Finally the factor connected with the ghost fields can be calculated as follows

$$
\begin{aligned}
&\left[\langle 0 \mid \left(1 - i C_3^*\left(|\vec{p}|\right) c_{\vec{p}}^{a} \overline{c}_{\vec{p}}^{a}\right) \left(1 + i C_3\left(|\vec{p}|\right) \overline{c}_{\vec{p}}^{a+} c_{\vec{p}}^{a+}\right) \mid 0\rangle\right]^{8} \\
&= \left[1 + \left|C_3\left(|\vec{p}|\right)\right|^{2} \langle 0 \mid c_{\vec{p}}^{a} \overline{c}_{\vec{p}}^{a} \overline{c}_{\vec{p}}^{a+} c_{\vec{p}}^{a+} \mid 0\rangle\right] = \left[1 - \left|C_3\left(|\vec{p}|\right)\right|^{2}\right]^{8}.
\end{aligned}
\tag{2.41}
$$

After substituting all the calculated factors, the norm of the state can be written as

$$
N = \langle \Phi \mid \Phi \rangle = \prod_{\sigma=1,2} \left[\sum_{m=0}^{\infty} \left|C_\sigma\left(|\vec{p}|\right)\right|^{2m} \frac{(2m)!}{(m!)^{2}}\right]^{8}.
\tag{2.42}
$$

Therefore, it is possible to define the normalized state

$$
\mid \widetilde{\Phi}\rangle = \frac{1}{\sqrt{N}} \mid \Phi\rangle.
\tag{2.43}
$$

Note that, as it should be expected, the norm is not dependent on the $C_3\left(|\vec{p}|\right)$ parameter which defines the non-physical particle operators entering in the definition of the proposed vacuum state.

Chapter 3

Propagator Modifications

The general form for generating functionals and propagators, for boson and fermion particles in an arbitrary vacuum state, are analyzed. The modification for the gluon and ghost propagators, introduced by the vacuum state defined in the previous chapter, are calculated.

3.1 General Form of the Propagator

As it is well known in the Quantum Field Theory to calculate any element of the S-Matrix, after applying the reduction formulas, it is necessary to obtain the vacuum expectation value of the temporal ordering of Heisemberg operators [25]. That is it is needed to calculate

$$\langle \Psi \mid T\left(\hat{A}_H(x_1)\,\hat{A}_H(x_2)\,\hat{A}_H(x_3)\,...\right) \mid \Psi \rangle, \tag{3.1}$$

where $\mid \Psi \rangle$ is the real vacuum of the interacting theory. For simplifying the exposition it is considered a scalar field, the generalization for vector fields is straightforward.

Using the relations between the operators in the Interaction and Heisemberg representations

$$\begin{aligned}
\hat{A}_H(x) &= \hat{U}(0,t)\,\hat{A}_I(x)\,\hat{U}(t,0)\,, \\
\hat{U}(t_1,t_2)\,\hat{U}(t_2,t_3) &= \hat{U}(t_1,t_3)
\end{aligned}$$

and assuming that the real vacuum interacting state can be obtained from the non-interacting one under the adiabatic connection of the interaction. The expression (3.1) takes the form [25]

$$\frac{\langle\Phi \mid T\left\{\hat{A}_I(x_1)\,\hat{A}_I(x_2)\,\hat{A}_I(x_3)\ldots\exp\left(-\int\limits_{-\infty}^{\infty}H_i(t)\,dt\right)\right\}\mid\Phi\rangle}{\langle\Phi \mid T\left\{\exp\left(-\int\limits_{-\infty}^{\infty}H_i(t)\,dt\right)\right\}\mid\Phi\rangle}, \tag{3.2}$$

where Φ is the non interacting vacuum of the theory.

To evaluate these quantities it is needed to develop the exponential in series of perturbation theory and calculate the vacuum expectation values of the temporal ordering of fields in the interaction representation ($\hat{A}_I(x)$), but in this representation the field operators are like free fields ($\hat{A}^0(x)$) about which much is known.

$$\hat{A}_I(x) = \hat{A}^0(x).$$

And it is necessary to evaluate terms of the form

$$\langle\Phi \mid T\left(\hat{A}^0(x_1)\,\hat{A}^0(x_2)\,\hat{A}^0(x_3)\ldots\right)\mid\Phi\rangle. \tag{3.3}$$

Introducing the auxiliary generating functional

$$Z[J] \equiv \langle\Phi \mid T\left(\exp\left\{i\int d^4x J(x)\,A^0(x)\right\}\right)\mid\Phi\rangle, \tag{3.4}$$

it is possible to write for the relevant expectation values the expression

$$\langle\Phi \mid T\left(\hat{A}^0(x_1)\,\hat{A}^0(x_2)\,\hat{A}^0(x_3)\ldots\right)\mid\Phi\rangle = \left(\frac{1}{i}\frac{\delta}{\delta J(x_1)}\frac{1}{i}\frac{\delta}{\delta J(x_1)}\frac{1}{i}\frac{\delta}{\delta J(x_1)}\ldots Z[J]\right)_{J=0}. \tag{3.5}$$

Considering now the auxiliary functional

$$Z[J;t] \equiv \langle\Phi \mid T\left(\exp\left\{i\int\limits_{-\infty}^{t}dt\int d^3x J(x)\,A^0(x)\right\}\right)\mid\Phi\rangle \tag{3.6}$$

and defining $W(t)$ through the relation

$$
\begin{aligned}
&T\left(\exp\left\{i\int\limits_{-\infty}^{t}dt\int d^3xJ\left(x\right)A^0\left(x\right)\right\}\right)\\
&=T\left(\exp\left\{i\int\limits_{-\infty}^{t}dt\int d^3xJ\left(x\right)A^{0-}\left(x\right)\right\}\right)W\left(t\right),
\end{aligned}
\tag{3.7}
$$

where $A^{0-}\left(x\right)$ and $A^{0+}\left(x\right)$ are the negative (creation) and positive (annihilation) frequency parts, respectively.

The t differentiation on the expression (3.7), takes the form

$$
\begin{aligned}
&i\int\limits_{x_0=t}d^3xJ\left(x\right)A^0\left(x\right)T\left(\exp\left\{i\int\limits_{-\infty}^{t}dt\int d^3xJ\left(x\right)A^{0-}\left(x\right)\right\}\right)W\left(t\right)\\
&=\ T\left(\exp\left\{i\int\limits_{-\infty}^{t}dt\int d^3xJ\left(x\right)A^{0-}\left(x\right)\right\}\right)\frac{dW\left(t\right)}{dt}+\\
&\quad+i\int\limits_{x_0=t}d^3xJ\left(x\right)A^{0-}\left(x\right)T\left(\exp\left\{i\int\limits_{-\infty}^{t}dt\int d^3xJ\left(x\right)A^{0-}\left(x\right)\right\}\right)W\left(t\right).
\end{aligned}
\tag{3.8}
$$

Keeping in mind that the free field creation operators commute, for all times, the following relation holds

$$
\left[A^{0-}\left(x\right),A^{0-}\left(y\right)\right]=0,
\tag{3.9}
$$

then the T instruction can be eliminated and after some algebra is obtained

$$
\begin{aligned}
\frac{dW\left(t\right)}{dt}&=i\exp\left\{-i\int\limits_{-\infty}^{t}dt\int d^3xJ\left(x\right)A^{0-}\left(x\right)\right\}\int\limits_{x_0=t}d^3xJ\left(x\right)A^{0+}\left(x\right)\times\\
&\quad\times\exp\left\{i\int\limits_{-\infty}^{t}dt\int d^3xJ\left(x\right)A^{0-}\left(x\right)\right\}W\left(t\right)\\
&=i\int\limits_{y_0=t}d^3yJ\left(y\right)\left\{A^{0+}\left(y\right)-i\int\limits_{-\infty}^{t}d^4xJ\left(x\right)\left[A^{0-}\left(x\right),A^{0+}\left(y\right)\right]\right\}W\left(t\right).
\end{aligned}
\tag{3.10}
$$

The initial condition on $W(t)$ is

$$W(-\infty) = 1. \tag{3.11}$$

Then the solution of (3.10) is

$$\begin{aligned} W(t) &= \exp\left\{ i \int_{-\infty}^{t} d^4y J(y) A^{0+}(y) \right\} \\ &\times \exp\left\{ \int_{-\infty}^{t} d^4y \int_{-\infty}^{y_0} d^4x J(y) J(x) \left[A^{0-}(x), A^{0+}(y) \right] \right\}, \end{aligned}$$

when $t \to \infty$ this expression takes the form

$$\begin{aligned} W(\infty) &= \exp\left\{ i \int d^4y J(y) A^{0+}(y) \right\} \times \\ &\times \exp\left\{ \int d^4x d^4y \theta(y_0 - x_0) J(y) J(x) \left[A^{0-}(x), A^{0+}(y) \right] \right\}. \end{aligned}$$

Therefore, the generating functional (3.4) can be written in the following way [25]

$$\begin{aligned} Z[J] &\equiv \langle \Phi \mid \exp\left\{ i \int d^4x J(x) A^{0-}(x) \right\} \exp\left\{ i \int d^4y J(y) A^{0+}(y) \right\} \mid \Phi \rangle \\ &\times \exp\left\{ \frac{i}{2} \int d^4x d^4y J(x) D(x-y) J(y) \right\}, \end{aligned} \tag{3.12}$$

where $D(x-y)$ is the usual propagator for an scalar particle.

In case that is needed to calculate a similar matrix element for fermions the following functional is defined

$$Z[\eta, \bar{\eta}] \equiv \langle \Phi \mid T\left(\exp\left\{ i \int d^4x \left[\bar{\eta}(x) \psi^0(x) + \bar{\psi}^0(x) \eta(x) \right] \right\} \right) \mid \Phi \rangle. \tag{3.13}$$

Because of the anticommuting properties of $\bar{\psi},\ \psi$ fields the introduced sources $\bar{\eta},\ \eta$ satisfy anticommuting relations between then and with the field operators.

Here is assumed the left differentiation convention, then the S-Matrix element can be calculate by the following expression

$$\begin{aligned}&\langle \Phi \mid T\left(\psi^{0}\left(y_{1}\right)\bar{\psi}^{0}\left(z_{1}\right)\psi^{0}\left(y_{2}\right)...\bar{\psi}^{0}\left(z_{k}\right)\right)\mid \Phi\rangle\\&=\left(\frac{1}{i}\frac{\delta}{\delta\eta\left(z_{k}\right)}...\frac{1}{i}\frac{\delta}{\delta\bar{\eta}\left(y_{2}\right)}\frac{1}{i}\frac{\delta}{\delta\eta\left(z_{1}\right)}\frac{1}{i}\frac{\delta}{\delta\bar{\eta}\left(y_{1}\right)}Z\left[\eta,\bar{\eta}\right]\right)_{\eta,\bar{\eta}=0}.\end{aligned}\tag{3.14}$$

Now, in the same way that for the bosons, the following auxiliary functional is defined by

$$Z\left[\eta,\bar{\eta};t\right]\equiv\langle\Phi\mid T\left(\exp\left\{i\int\limits_{-\infty}^{t}dt\int d^{3}x\left[\bar{\eta}\left(x\right)\psi^{0}\left(x\right)+\bar{\psi}^{0}\left(x\right)\eta\left(x\right)\right]\right\}\right)\mid\Phi\rangle\tag{3.15}$$

and the corresponding $G\left(t\right)$ functional by

$$\begin{aligned}&T\left(\exp\left\{i\int\limits_{-\infty}^{t}dt\int d^{3}x\left[\bar{\eta}\left(x\right)\psi^{0}\left(x\right)+\bar{\psi}^{0}\left(x\right)\eta\left(x\right)\right]\right\}\right)\\&=T\left(\exp\left\{i\int\limits_{-\infty}^{t}dt\int d^{3}x\left[\bar{\eta}\left(x\right)\psi^{0-}\left(x\right)+\bar{\psi}^{0-}\left(x\right)\eta\left(x\right)\right]\right\}\right)G\left(t\right).\end{aligned}\tag{3.16}$$

Manipulations completely parallel to those leading to (3.10) give

$$\begin{aligned}\frac{dG\left(t\right)}{dt}=i\Bigg[&\int\limits_{y_{0}=t}d^{3}y\left(\bar{\eta}\left(y\right)\psi^{0+}\left(y\right)+\bar{\psi}^{0+}\left(y\right)\eta\left(y\right)\right)\\&+i\int\limits_{-\infty}^{t}d^{4}x\int\limits_{y_{0}=t}d^{3}y\bar{\eta}\left(y\right)\left\{\psi^{0+}\left(y\right),\bar{\psi}^{0-}\left(x\right)\right\}\eta\left(x\right)\\&-i\int\limits_{-\infty}^{t}d^{4}x\int\limits_{y_{0}=t}d^{3}y\bar{\eta}\left(x\right)\left\{\psi^{0-}\left(x\right),\bar{\psi}^{0+}\left(y\right)\right\}\eta\left(y\right)\Bigg]G\left(t\right),\end{aligned}\tag{3.17}$$

This equation is easily integrated to obtain the solution

$$
\begin{aligned}
G(t) &= \exp\left\{ i \int\limits_{-\infty}^{t} d^4y \left(\bar{\eta}(y)\,\psi^{0+}(y) + \bar{\psi}^{0+}(y)\,\eta(y)\right) \right\} \\
&\times \exp\left\{ - \int\limits_{-\infty}^{t} d^4y \int\limits_{-\infty}^{y_0} d^4x \bar{\eta}(y) \left\{ \psi^{0+}(y), \bar{\psi}^{0-}(x) \right\} \eta(x) \right\} \\
&\times \exp\left\{ \int\limits_{-\infty}^{t} d^4x \int\limits_{-\infty}^{x_0} d^4y \bar{\eta}(y) \left\{ \psi^{0-}(y), \bar{\psi}^{0+}(x) \right\} \eta(x) \right\},
\end{aligned}
\tag{3.18}
$$

where in the last term the dummy variables x and y were interchanged. Consequently the following expression for the generating functional arise [25]

$$
\begin{aligned}
Z[\eta,\bar{\eta}] \equiv \langle \Phi \mid & \exp\left\{ i \int d^4x \left[\bar{\eta}(x)\,\psi^{0-}(x) + \bar{\psi}^{0-}(x)\,\eta(x)\right] \right\} \\
& \times \exp\left\{ i \int d^4x \left[\bar{\eta}(x)\,\psi^{0+}(x) + \bar{\psi}^{0+}(x)\,\eta(x)\right] \right\} \mid \Phi \rangle \\
& \times \exp\left\{ i \int d^4x d^4y \bar{\eta}(x)\, S(x-y)\, \eta(y) \right\}
\end{aligned}
\tag{3.19}
$$

where $S(x-y)$ is the standard fermion propagator.

As much for the case of bosons as for fermions the term related with the vacuum expectation value for the usual vacuum is one. This is so because the annihilation operators are located to the right and to the left those of creation. However in the present work the vacuum expectation values generate the propagator modifications, because the vacuum state considered is not the trivial one. The other term in the generating functional expression, that is expressed by a simple exponential of c numbers, gives the usual propagator and it has the same form when is calculated by this operational method or alternatively by the functional method.

Then, starting from the analysis in the present section it can be concluded that from an operation formalism point of view any modification to the usual propagators is only generated by a change in the vacuum state of the theory. And these modifications can be determined through the vacuum expectation values in (3.12) and (3.19). From a functional formalism point of view, the propagator modifications are generated by changes in the boundary conditions.

3.2 Modified Gluon Propagator

As it follows from the general form of the Wick Theorem, analyzed in the previous section, the modification of the gluon propagator introduced by the modified vacuum state (2.32) is defined by the expression

$$\langle\widetilde{\Phi} \mid \exp\left\{i\int d^4x J^{\mu,a}\left(x\right)A_\mu^{a-}\left(x\right)\right\}\exp\left\{i\int d^4x J^{\mu,a}\left(x\right)A_\mu^{a+}\left(x\right)\right\} \mid \widetilde{\Phi}\rangle, \tag{3.20}$$

for each value of the color index a. All the different colors can be worked out independently because of the commutation relations between the annihilation and creation operators for the free theory. At the necessary point of the analysis all the color contributions will be included.

The annihilation and creation fields in (3.20) are given by

$$\begin{aligned} A_\mu^{a+}\left(x\right) &= \sum_{\vec{k}}\left(\sum_{\sigma=1,2}A_{\vec{k},\sigma}^{a}f_{k,\mu}^{\sigma}\left(x\right)+A_{\vec{k}}^{L,a}f_{k,L,\mu}\left(x\right)+B_{\vec{k}}^{a}f_{k,S,\mu}\left(x\right)\right), \\ A_\mu{}^{a-}\left(x\right) &= \sum_{\vec{k}}\left(\sum_{\sigma=1,2}A_{\vec{k},\sigma}^{a+}f_{k,\mu}^{\sigma *}\left(x\right)+A_{\vec{k}}^{L,a+}f_{k,L,\mu}^{*}\left(x\right)+B_{\vec{k}}^{a+}f_{k,S,\mu}^{*}\left(x\right)\right). \end{aligned}$$

In what follows it is calculated explicitly, for each color, the action of the exponential operators

$$\begin{aligned} &\exp\left\{i\int d^4x J^{\mu,a}\left(x\right)A_\mu^{a+}\left(x\right)\right\} \mid \Phi\rangle \\ &= \exp\left\{i\int d^4x J^{\mu,a}\left(x\right)\sum_{\vec{k}}\left(\sum_{\sigma=1,2}A_{\vec{k},\sigma}^{a}f_{k,\mu}^{\sigma}\left(x\right)+A_{\vec{k}}^{L,a}f_{k,L,\mu}\left(x\right)+B_{\vec{k}}^{a}f_{k,S,\mu}\left(x\right)\right)\right\} \\ &\quad\times\exp\left\{\sum_{\sigma=1,2}\frac{1}{2}C_\sigma\left(|\vec{p}|\right)A_{\vec{p},\sigma}^{a+}A_{\vec{p},\sigma}^{a+}+C_3\left(|\vec{p}|\right)\left(B_{\vec{p}}^{a+}A_{\vec{p}}^{L,a+}+i\overline{c}_{\vec{p}}^{a+}c_{\vec{p}}^{a+}\right)\right\} \mid 0\rangle. \end{aligned} \tag{3.21}$$

After a systematic use of the commutation relations among the annihilation and creation operators, the exponential operators can be decomposed in products of exponential for each space-time mode. This fact allows to perform the calculation for each mode independently. Then the expression (3.21) takes the form

$$\prod_{\sigma=1,2} \exp\left\{ i\int d^4x J^{\mu,a}(x) \sum_{\vec{k}} A^a_{\vec{k},\sigma} f^{\sigma}_{k,\mu}(x) \right\} \exp\left\{ 12 C_\sigma(|\vec{p}|) A^{a+}_{\vec{p},\sigma} A^{a+}_{\vec{p},\sigma} \right\} \mid 0\rangle$$
$$\times \ \exp\left\{ i\int d^4x J^{\mu,a}(x) \sum_{\vec{k}} \left(B^a_{\vec{k}} f_{k,S,\mu}(x) + A^{L,a}_{\vec{k}} f_{k,L,\mu}(x) \right) \right\}$$
$$\times \ \exp\left\{ C_3(|\vec{p}|) B^{a+}_{\vec{p}} A^{L,a+}_{\vec{p}} \right\} \mid 0\rangle \exp\left\{ C_3(|\vec{p}|) i\bar{c}^{a+}_{\vec{p}} c^{a+}_{\vec{p}} \right\} \mid 0\rangle. \tag{3.22}$$

For a transverse component, it is necessary to calculate

$$\exp\left\{ i\int d^4x J^{\mu,a}(x) \sum_{\vec{k}} A^a_{\vec{k},\sigma} f^{\sigma}_{k,\mu}(x) \right\} \exp\left\{ \frac{1}{2} C_\sigma(|\vec{p}|) A^{a+}_{\vec{p},\sigma} A^{a+}_{\vec{p},\sigma} \right\} \mid 0\rangle \qquad \text{for } \sigma = 1,2 \tag{3.23}$$

The following recourse is used to calculate this expression; calling U the first exponential in (3.23) this expression can be written as

$$\exp\left\{ \frac{1}{2} C_\sigma(p) \left(U A^{a+}_{\vec{p},\sigma} U^{-1} \right) \left(U A^{a+}_{\vec{p},\sigma} U^{-1} \right) \right\} \mid 0\rangle, \tag{3.24}$$

since

$$U^{-1} \mid 0\rangle = \mid 0\rangle.$$

The inverse U^{-1} is the same U when in the exponential argument the sign is changed. Using the Baker-Hausdorf formula

$$\exp[\hat{F}]\hat{G}\exp[-\hat{F}] = \exp\left\{ [\hat{F},\ ] \right\} \hat{G} = \sum \frac{1}{n!} \left[\hat{F}, \left[\hat{F}, ..., \left[\hat{F}, \hat{G} \right] ..... \right] \right]$$

and noticing that only the first and the second term in the expansion are non-vanishing when $\hat{F}$ and $\hat{G}$ are linear functions of annihilation and creation operators, it follows

$$\exp[\hat{F}]\hat{G}\exp[-\hat{F}] = \hat{G} + \left[\hat{F}, \hat{G} \right].$$

Therefore, for the relevant commutators appearing in (3.24) it follows

$$\left[i\int d^4x J^{\mu,a}(x) \sum_{\vec{k}} A^a_{\vec{k},\sigma} f^{\sigma}_{k,\mu}(x), A^{a+}_{\vec{p},\sigma} \right] = i\int d^4x J^{\mu,a}(x) f^{\sigma}_{p,\mu}(x).$$

Then for the expression (3.23) the following result is obtained

$$\exp\left\{\frac{1}{2}C_\sigma\left(|\vec{p}|\right)\left(A^{a+}_{\vec{p},\sigma}+i\int d^4xJ^{\mu,a}\left(x\right)f^{\sigma}_{p,\mu}\left(x\right)\right)^2\right\}\mid 0\rangle \tag{3.25}$$

For the longitudinal and scalar modes, following the above procedure, the result obtained is

$$\begin{aligned}&\exp\left\{i\int d^4xJ^{\mu,a}\left(x\right)\sum_{\vec{k}}\left(B^a_{\vec{k}}f_{k,S,\mu}\left(x\right)+A^{L,a}_{\vec{k}}f_{k,L,\mu}\left(x\right)\right)\right\}\\ &\times\exp\left\{C_3\left(|\vec{p}|\right)B^{a+}_{\vec{p}}A^{L,a+}_{\vec{p}}\right\}\mid 0\rangle\\ &=\exp\left\{C_3\left(|\vec{p}|\right)\left(B^{a+}_{\vec{p}}-i\int d^4xJ^{\mu,a}\left(x\right)f_{p,L,\mu}\left(x\right)\right)\right.\\ &\qquad\qquad\left.\times\left(A^{L,a+}_{\vec{p}}-i\int d^4xJ^{\mu,a}\left(x\right)f_{p,S,\mu}\left(x\right)\right)\right\}\mid 0\rangle.\end{aligned} \tag{3.26}$$

where the expressions below were used

$$\left[\left(i\int d^4xJ^{\mu,a}\left(x\right)\sum_{\vec{k}}A^{L,a}_{\vec{k}}f_{k,L,\mu}\left(x\right)\right),B^{a+}_{\vec{p}}\right]=-i\int d^4xJ^{\mu,a}\left(x\right)f_{p,L,\mu}\left(x\right)$$

$$\left[\left(i\int d^4xJ^{\mu,a}\left(x\right)\sum_{\vec{k}}B^{a}_{\vec{k}}f_{k,S,\mu}\left(x\right)\right),A^{L,a+}_{\vec{p}}\right]=-i\int d^4xJ^{\mu,a}\left(x\right)f_{p,S,\mu}\left(x\right).$$

For the full modification calculation (3.20), it is necessary to evaluate

$$\langle\Phi\mid\exp\left\{i\int d^4xJ^{\mu,a}\left(x\right)A^{a-}_{\mu}\left(x\right)\right\}=\left(\exp\left\{-i\int d^4xJ^{\mu,a}\left(x\right)A^{a+}_{\mu}\left(x\right)\right\}\mid\Phi\rangle\right)^{\dagger}, \tag{3.27}$$

which can be easily obtained by conjugating the result for the right hand side, through (3.25) and (3.26).

Then, substituting (3.25), (3.26) and (3.27) in (3.20), the following expression should be calculated

$$
\begin{aligned}
&\frac{1}{N}\langle 0 \mid \exp\left\{\sum_{\sigma=1,2}\frac{1}{2}C_{\sigma}^{*}\left(|\vec{p}|\right)\left(A_{\vec{p},\sigma}^{a}+i\int d^{4}xJ^{\mu,a}\left(x\right)f_{p,\mu}^{\sigma *}\left(x\right)\right)^{2}\right\}\\
&\qquad\times\exp\left\{\sum_{\sigma=1,2}12C_{\sigma}\left(|\vec{p}|\right)\left(A_{\vec{p},\sigma}^{a+}+i\int d^{4}xJ^{\mu,a}\left(x\right)f_{p,\mu}^{\sigma}\left(x\right)\right)^{2}\right\}\mid 0\rangle\\
&\times\langle 0\mid \exp\left\{C_{3}^{*}\left(|\vec{p}|\right)\left(B_{\vec{p}}^{a}-i\int d^{4}xJ^{\mu,a}\left(x\right)f_{p,L,\mu}^{*}\left(x\right)\right)\right.\\
&\qquad\qquad\left.\times\left(A_{\vec{p}}^{L,a}-i\int d^{4}xJ^{\mu,a}\left(x\right)f_{p,S,\mu}^{*}\left(x\right)\right)\right\}\\
&\qquad\times\exp\left\{C_{3}\left(|\vec{p}|\right)\left(B_{\vec{p}}^{a+}-i\int d^{4}xJ^{\mu,a}\left(x\right)f_{p,L,\mu}\left(x\right)\right)\right.\\
&\qquad\qquad\left.\times\left(A_{\vec{p}}^{L,a+}-i\int d^{4}xJ^{\mu,a}\left(x\right)f_{p,S,\mu}\left(x\right)\right)\right\}\mid 0\rangle\\
&\times\langle 0\mid \exp\left(-iC_{3}^{*}\left(|\vec{p}|\right)c_{\vec{p}}^{a}\bar{c}_{\vec{p}}^{a}\right)\exp\left(iC_{3}\left(|\vec{p}|\right)\bar{c}_{\vec{p}}^{a+}c_{\vec{p}}^{a+}\right)\mid 0\rangle
\end{aligned}
\tag{3.28}
$$

In the expression (3.28) the calculated contribution, for each transverse mode, is

$$
\exp\left\{-\int\frac{d^{4}xd^{4}y}{2Vp_{0}}J^{\mu,a}\left(x\right)J^{\nu,a}\left(y\right)\epsilon_{\mu}^{\sigma}\left(p\right)\epsilon_{\nu}^{\sigma}\left(p\right)\frac{\left(C_{\sigma}\left(|\vec{p}|\right)+C_{\sigma}^{*}\left(|\vec{p}|\right)+2\left|C_{\sigma}\left(|\vec{p}|\right)\right|^{2}\right)}{2\left(1-\left|C_{\sigma}\left(|\vec{p}|\right)\right|^{2}\right)}\right\},
\tag{3.29}
$$

and the longitudinal and scalar mode contribution is

$$
\exp\left\{-\int\frac{d^{4}xd^{4}y}{2Vp_{0}}J^{\mu,a}\left(x\right)J^{\nu,a}\left(y\right)\epsilon_{S,\mu}\left(p\right)\epsilon_{L,\nu}\left(p\right)\frac{\left(C_{3}\left(|\vec{p}|\right)+C_{3}^{*}\left(|\vec{p}|\right)+2\left|C_{3}\left(|\vec{p}|\right)\right|^{2}\right)}{\left(1-\left|C_{3}\left(|\vec{p}|\right)\right|^{2}\right)}\right\},
\tag{3.30}
$$

The detailed analysis of these calculations can be found in the Appendixes 1 and 2.

Therefore, after collecting the contributions of all the modes, assuming $C_1\left(|\vec{p}|\right)=C_2\left(|\vec{p}|\right)=C_3\left(|\vec{p}|\right)$ (which follows necessarily in order to obtain Lorentz invariance) and using the properties of the defined vectors basis, the modification to the propagator becomes

$$
\exp\left\{\frac{1}{2}\int\frac{d^{4}xd^{4}y}{2p_{0}V}J^{\mu,a}\left(x\right)J^{\nu,a}\left(y\right)g_{\mu\nu}\left[\frac{\left(C_{1}\left(|\vec{p}|\right)+C_{1}^{*}\left(|\vec{p}|\right)+2\left|C_{1}\left(|\vec{p}|\right)\right|^{2}\right)}{\left(1-\left|C_{1}\left(|\vec{p}|\right)\right|^{2}\right)}\right]\right\}.
\tag{3.31}
$$

In the expression (3.31), the combination of the $C_1(|\vec{p}|)$ constant is always real and nonnegative, for all $|C_1(|\vec{p}|)| < 1$.

Now it is possible to perform the limit process $\vec{p} \to 0$. In doing this limit, it is considered that each component of the linear momentum $\vec{p}$ is related with the quantization volume by

$$p_x \sim \frac{1}{a},\ p_y \sim \frac{1}{b},\ p_z \sim \frac{1}{c},\ V = abc \sim \frac{1}{|\vec{p}|^3},$$

And it is necessary to calculate

$$\lim_{\vec{p}\to 0} \frac{\left(C_1(|\vec{p}|) + C_1^*(|\vec{p}|) + 2\,|C_1(|\vec{p}|)|^2\right)}{4p_0 V\left(1 - |C_1(|\vec{p}|)|^2\right)}, \tag{3.32}$$

Then, after fixing a dependence of the arbitrary constant C_1 of the form $|C_1(|\vec{p}|)| \sim \left(1 - \kappa\,|\vec{p}|^2\right), \kappa > 0$, and $C_1(0) \neq -1$ the limit (3.32) becomes

$$\lim_{\vec{p}\to 0} \frac{\left(C_1(|\vec{p}|) + C_1^*(|\vec{p}|) + 2\,|C_1(|\vec{p}|)|^2\right)|\vec{p}|^3 \frac{1}{\left(1-\left(1-\kappa|\vec{p}|^2\right)^2\right)}}{4p_0} = \frac{C}{2\,(2\pi)^4} \tag{3.33}$$

where C is an arbitrary real and nonnegative constant, determined by the also real and nonnegative constant κ.

Therefore, the total modification to the propagator including all color values turns to be

$$\begin{aligned}
&\prod_{a=1,..,8} \langle\widetilde{\Phi}\mid \exp\left\{i\int d^4x J^{\mu,a}(x)\, A_\mu^{a-}(x)\right\} \exp\left\{i\int d^4x J^{\mu,a}(x)\, A_\mu^{a+}(x)\right\} \mid \widetilde{\Phi}\rangle \\
&= \exp\left\{\sum_{a=1,..8}\int d^4x d^4y J^{\mu,a}(x)\, J^{\nu,a}(y)\, g_{\mu\nu}\frac{C}{2\,(2\pi)^4}\right\}.
\end{aligned} \tag{3.34}$$

The generating functional associated to the proposed initial state, including the usual perturbative piece for $\alpha = 1$, can be written in the form

$$Z[J] = \exp\left\{\frac{i}{2}\sum_{a,b=1,..8}\int d^4x d^4y J^{\mu,a}(x)\, \widetilde{D}^{ab}_{\mu\nu}(x-y) J^{\nu,b}(y)\right\}, \tag{3.35}$$

where

$$\widetilde{D}^{ab}_{\mu\nu}(x-y)=\int\frac{d^4k}{(2\pi)^4}\delta^{ab}g_{\mu\nu}\left[\frac{1}{k^2}-iC\delta\left(k\right)\right]\exp\left\{-ik\left(x-y\right)\right\} \tag{3.36}$$

which shows that the gluon propagator has the same form proposed in [12], for the selected gauge parameter value $\alpha=1$ (which corresponds to $\alpha=-1$ in that reference).

3.3 Modified Ghost Propagator

In the present section the possible modification to the ghost propagator will be analyzed. As was shown in Sec. 3.1 for fermionic particles the expression for the modification, introduced by a nontrivial vacuum state, is

$$\begin{aligned}&\langle\widetilde{\Phi}\mid\exp\left\{i\int d^4x\left(\overline{\xi}^a\left(x\right)c^{a-}\left(x\right)+\overline{c}^{a-}\left(x\right)\xi^a\left(x\right)\right)\right\}\\&\qquad\times\exp\left\{i\int d^4x\left(\overline{\xi}^a\left(x\right)c^{a+}\left(x\right)+\overline{c}^{a+}\left(x\right)\xi^a\left(x\right)\right)\right\}\mid\widetilde{\Phi}\rangle,\end{aligned} \tag{3.37}$$

where

$$\begin{aligned}c^{a+}\left(x\right)&=\sum_{\vec{k}}c^a_{\vec{k}}g_k\left(x\right), &\quad c^{a-}\left(x\right)&=\sum_{\vec{k}}c^{a+}_{\vec{k}}g^*_k\left(x\right),\\ \overline{c}^{a+}\left(x\right)&=\sum_{\vec{k}}\overline{c}^a_{\vec{k}}g_k\left(x\right), &\quad \overline{c}^{a-}\left(x\right)&=\sum_{\vec{k}}\overline{c}^{a+}_{\vec{k}}g^*_k\left(x\right).\end{aligned} \tag{3.38}$$

Now it is calculated explicitly the action of the exponential operator

$$\begin{aligned}&\exp\left\{i\int d^4x\left(\overline{\xi}^a\left(x\right)c^{a+}\left(x\right)+\overline{c}^{a+}\left(x\right)\xi^a\left(x\right)\right)\right\}\exp\left\{C_3\left(|\vec{p}|\right)i\overline{c}^{a+}_{\vec{p}}c^{a+}_{\vec{p}}\right\}\mid 0\rangle\\&=\left(1+i\int d^4y\overline{\xi}^a\left(y\right)\sum_{\vec{k}'}c^a_{\vec{k}'}g_{k'}\left(y\right)\right)\left(1+i\int d^4x\sum_{\vec{k}}\overline{c}^a_{\vec{k}}g_k\left(x\right)\xi^a\left(x\right)\right)\times\\&\qquad\times\left(1+C_3\left(|\vec{p}|\right)i\overline{c}^{a+}_{\vec{p}}c^{a+}_{\vec{p}}\right)\mid 0\rangle.\end{aligned} \tag{3.39}$$

The grassman character of the field and sources allowed expanding the exponential retaining only the first two terms in the expansion.

With the use of the following relations

$$\begin{aligned}
\overline{\xi}^{a}(y)\, c^{a}_{\vec{k}'}\overline{c}^{a+}_{\vec{p}}c^{a+}_{\vec{p}} \mid 0\rangle &= i\delta_{\vec{k}',\vec{p}}\overline{\xi}^{a}(y)\, c^{a+}_{\vec{p}} \mid 0\rangle, \\
\overline{c}^{a}_{\vec{k}}\xi^{a}(x)\, \overline{c}^{a+}_{\vec{p}}c^{a+}_{\vec{p}} \mid 0\rangle &= i\delta_{\vec{k},\vec{p}}\overline{c}^{a+}_{\vec{p}}\xi^{a}(x) \mid 0\rangle, \\
\overline{\xi}^{a}(y)\, c^{a}_{\vec{k}'}i\delta_{\vec{k},\vec{p}}\overline{c}^{a+}_{\vec{p}}\xi^{a}(x) \mid 0\rangle &= -\delta_{\vec{k},\vec{p}}\delta_{\vec{k}',\vec{p}}\overline{\xi}^{a}(y)\,\xi^{a}(x) \mid 0\rangle,
\end{aligned} \tag{3.40}$$

the expression (3.39) can be written as

$$\begin{aligned}
&\Big[1 + C_3(|\vec{p}|)\Big(i\overline{c}^{a+}_{\vec{p}}c^{a+}_{\vec{p}} - i\int d^4x g_p(x)\left(\overline{\xi}^{a}(x)\, c^{a+}_{\vec{p}} + \overline{c}^{a+}_{\vec{p}}\xi^{a}(x)\right) + \\
&+ i\int d^4y\int d^4x g_p(y)\, g_p(x)\,\overline{\xi}^{a}(y)\,\xi^{a}(x)\Big)\Big] \mid 0\rangle.
\end{aligned} \tag{3.41}$$

In addition the formula

$$\begin{aligned}
&\langle\widetilde{\Phi} \mid \exp\left\{i\int d^4x\left(\overline{\xi}^{a}(x)\, c^{a-}(x) + \overline{c}^{a-}(x)\,\xi^{a}(x)\right)\right\} \\
&= \left[\exp\left\{i\int d^4x\left(\overline{\xi}^{a\dagger}(x)\, c^{a+}(x) + \overline{c}^{a+}(x)\,\xi^{a\dagger}(x)\right)\right\} \mid \widetilde{\Phi}\rangle\right]^{\dagger},
\end{aligned} \tag{3.42}$$

allows to calculate the left hand side of (3.37) using (3.41).

Then the expression (3.37), substituting (3.41) and (3.42), takes the form

$$\begin{aligned}
&\langle 0 \mid \Big[1 - C^{*}_3(|\vec{p}|)\Big(ic^{a}_{\vec{p}}\overline{c}^{a}_{\vec{p}} - i\int d^4x g^{*}_p(x)\left(c^{a}_{\vec{p}}\overline{\xi}^{a}(x) + \xi^{a}(x)\,\overline{c}^{a}_{\vec{p}}\right) \\
&\quad + i\int d^4y\int d^4x g^{*}_p(y)\, g^{*}_p(x)\,\xi^{a}(y)\,\bar{\xi}^{a}(x)\Big)\Big] \\
&\times \Big[1 + C_3(|\vec{p}|)\Big(i\overline{c}^{a+}_{\vec{p}}c^{a+}_{\vec{p}} - i\int d^4x g_p(x)\left(\overline{\xi}^{a}(x)\, c^{a+}_{\vec{p}} + \overline{c}^{a+}_{\vec{p}}\xi^{a}(x)\right) \\
&\quad + i\int d^4y\int d^4x g_p(y)\, g_p(x)\,\overline{\xi}^{a}(y)\,\xi^{a}(x)\Big)\Big] \mid 0\rangle.
\end{aligned} \tag{3.43}$$

In this case, the expression (3.43) calculus is easier than the one realized for gluons. And the result of its contribution, canceling out the normalization factor, is

$$\exp\left[\frac{i\int d^4x d^4y\overline{\xi}^{a}(x)\,\xi^{a}(y)\left(C_3(|\vec{p}|) + C^{*}_3(|\vec{p}|) - 2\,|C_3(|\vec{p}|)|^2\right)}{2Vp_0\left(1 - |C_3(|\vec{p}|)|^2\right)}\right], \tag{3.44}$$

which in the limit $\vec{p} \to 0$, under the same condition considered for the gluon modification limit, takes the form

$$\exp\left\{-\sum_{a=1,..8} i \int d^4x d^4y \overline{\xi}^a(x)\, \xi^a(y) \frac{C_G}{(2\pi)^4}\right\}. \tag{3.45}$$

In this expression C_G is a real and nonnegative constant. It is interesting to note that choosing $C_3(0) = 1$, then $C_G = 0$ and there is no modification to the ghost propagator as was chosen in the previous work [12].

The ghost generating functional associated to the proposed initial state, including the usual perturbative piece for $\alpha = 1$, can be written in the form

$$Z_G[\overline{\xi}, \xi] = \exp\left\{i \sum_{a,b=1,..8} \int d^4x d^4y \overline{\xi}^a(x)\, \widetilde{D}_G^{ab}(x-y)\xi^b(y)\right\}, \tag{3.46}$$

where

$$\widetilde{D}_G^{ab}(x-y) = \int \frac{d^4k}{(2\pi)^4} \delta^{ab} \left[\frac{(-i)}{k^2} - C_G \delta(k)\right] \exp\{-ik(x-y)\}. \tag{3.47}$$

Chapter 4

Summary

By using the operational formulation for Quantum Gauge Fields Theory developed by Kugo and Ojima, a particular state vector for QCD in the non-interacting limit, that obeys the BRST physical state condition, was constructed. The general motivation for looking this wave function is to search for a reasonably good description of low energy QCD properties, through giving foundation to the perturbative expansion proposed in [12]. The high energy QCD description should not be affected by the modified perturbative initial state. In addition it can be expected that the adiabatic connection of the color interaction starting with it as an initial condition, generate at the end the true QCD interacting ground state. In case of having the above properties, the analysis would allow to understand the real vacuum as a superposition of infinite number of soft gluon pairs.

It has been checked that properly fixing the free parameters in the constructed state, the perturbation expansion proposed in the former work [12] is reproduced for the special value $\alpha = 1$ of the gauge constant. Therefore, the appropriate gauge is determined for which the expansion introduced in that work is produced by an initial state, satisfying the physical state condition for the BRST quantization procedure. The fact that the non-interacting initial state is a physical one, lead to expect that the final wave-function after the adiabatic connection of the color interaction will also satisfy the physical state condition for the interacting theory. If this assumption is correct, the results for calculations of transition amplitudes and the values of physical quantities should be also physically meaningful. In future, a quantization procedure for arbitrary values of α will be also considered. It is expected that with its help the gauge parameter independence of the physical quantities could be implemented. It seems possible that the results of this generalization will lead to α dependent polarizations for gluons and ghosts and their respective propagators, which however could produce α independent results for the physical quantities. However, this discussion will be delayed for future consideration.

It is important to mention now a result obtained during the calculation of the gluon propagator modification, in the chosen construction. It is that the arbitrary constant C is determined here to be real and nonnegative. This outcome restricts an existing arbitrariness in the discussion given in the previous work. As this quantity C is also determining the square of the generated gluon mass as positive or negative, real or imaginary, therefore it seems very congruent to arrive to a definite prediction of C as real and positive.

The modification to the standard free ghost propagator introduced by the proposed initial state, was also calculated. Moreover, after considering the free parameter in the proposed trial state as real, which it seems the most natural assumption, the ghost propagator is not be modified, as it was assumed in [12].

Some tasks which can be addressed in future works are: The study of the applicability of the Gell-Mann and Low theorem with respect to the adiabatic connection of the interaction, starting from the here proposed initial state. The investigation of zero modes quantization, that is gluon states with exact vanishing four momentum. The ability to consider them with success would allow a formally cleaner definition of the proposed state, by excluding the auxiliary momentum $\vec{p}$ recursively used in the construction carry out. Finally, the application of the proposed perturbation theory in the study of some problems related with confinement and the hadron structure.

Appendix A

Transverse Mode Contribution

The transverse mode contribution is determined by the expression

$$\begin{aligned}&\langle 0 \mid \exp\left\{\frac{1}{2}C^{*}_{\sigma}\left(|\vec{p}|\right)\left(A^{a}_{\vec{p},\sigma}+i\int d^{4}xJ^{\mu,a}\left(x\right)f^{\sigma*}_{p,\mu}\left(x\right)\right)^{2}\right\}\\&\quad\times\exp\left\{\frac{1}{2}C_{\sigma}\left(|\vec{p}|\right)\left(A^{a+}_{\vec{p},\sigma}+i\int d^{4}xJ^{\mu,a}\left(x\right)f^{\sigma}_{p,\mu}\left(x\right)\right)^{2}\right\}\mid 0\rangle\end{aligned}\tag{A.1}$$

For simplifying the exposition, the following notation is introduced

$$\begin{aligned}C^{*}&\equiv C^{*}_{\sigma}\left(|\vec{p}|\right),\ C\equiv C_{\sigma}\left(|\vec{p}|\right), &&\hat{A}^{+}\equiv A^{a+}_{\vec{p},\sigma},\ \hat{A}\equiv A^{a}_{\vec{p},\sigma}\ ,\\a_{1}&\equiv i\int d^{4}xJ^{\mu,a}\left(x\right)f^{\sigma*}_{p,\mu}\left(x\right), &&a_{2}\equiv i\int d^{4}xJ^{\mu,a}\left(x\right)f^{\sigma}_{p,\mu}\left(x\right).\end{aligned}\tag{A.2}$$

Then the expression (A.1) takes the form

$$\langle 0 \mid \exp\left\{\frac{1}{2}C^{*}\left(\hat{A}+a_{1}\right)^{2}\right\}\exp\left\{\frac{1}{2}C\left(\hat{A}^{+}+a_{2}\right)^{2}\right\}\mid 0\rangle\tag{A.3}$$

$$=\exp\left\{\frac{C^{*}}{2}a_{1}^{2}+\frac{C}{2}a_{2}^{2}\right\}\langle 0 \mid \exp\left\{\frac{C^{*}}{2}\hat{A}^{2}+C^{*}a_{1}\hat{A}\right\}\exp\left\{\frac{C}{2}\hat{A}^{+2}+Ca_{2}\hat{A}^{+}\right\}\mid 0\rangle.$$

For the action of the exponential, linear in the annihilation operator, at the left on the right the result obtained is

$$\begin{aligned}
&\exp\left\{C^* a_1 \hat{A}\right\}\exp\left\{\frac{C}{2}\hat{A}^{+2}+Ca_2\hat{A}^+\right\}\mid 0\rangle \\
&=\exp\left\{\frac{C}{2}\left(\hat{A}^+ + C^* a_1\right)^2 + Ca_2\left(\hat{A}^+ + C^* a_1\right)\right\}\mid 0\rangle,
\end{aligned} \tag{A.4}$$

where the same procedure used for calculating (3.23) is considered.

The expression (A.3), considering (A.4), can be written in the form

$$\exp\left\{\frac{C^*}{2}a_1^2+\frac{C}{2}\left(C^* a_1 + a_2\right)^2\right\}\langle 0\mid\exp\left\{\frac{C^*}{2}\hat{A}^2\right\}\exp\left\{\frac{C}{2}\hat{A}^{+2}+C\hat{A}^+\left(C^* a_1 + a_2\right)\right\}\mid 0\rangle \tag{A.5}$$

It is possible in (A.5) to act with the exponential linear in the creation operator at the right on the left and the result is

$$\begin{aligned}
&\exp\left\{\frac{C^*}{2}a_1^2+\frac{C}{2}\left(C^* a_1 + a_2\right)^2\left(1+|C|^2\right)\right\} \\
&\times\langle 0\mid\exp\left\{\frac{C^*}{2}\hat{A}^2+C^*C\left(C^* a_1 + a_2\right)\hat{A}\right\}\exp\left\{\frac{C}{2}\hat{A}^{+2}\right\}\mid 0\rangle
\end{aligned} \tag{A.6}$$

In such a way after n-steps it is possible to arrive to a recurrence relation, which can be proven by mathematical induction. This recurrence relation has the form

$$\begin{aligned}
&\exp\left\{\frac{C^*}{2}a_1^2+\frac{C}{2}\left(C^* a_1 + a_2\right)^2\sum_{m=0}^{n}\left[|C|^{2(2m)}+|C|^{2(2m+1)}\right]\right\} \\
&\times\langle 0\mid\exp\left\{\frac{C^*}{2}\hat{A}^2+C^{*n+1}C^{n+1}\left(C^* a_1 + a_2\right)\hat{A}\right\}\exp\left\{\frac{C}{2}\hat{A}^{+2}\right\}\mid 0\rangle
\end{aligned} \tag{A.7}$$

Lets probe it, acting with the exponential linear in the annihilation operator at the left on the right the result is

$$\begin{aligned}
&\exp\left\{\frac{C^*}{2}a_1^2+\frac{C}{2}\left(C^* a_1 + a_2\right)^2\left(\sum_{m=0}^{n}\left[|C|^{2(2m)}+|C|^{2(2m+1)}\right]+|C|^{4(n+1)}\right)\right\} \\
&\times\langle 0\mid\exp\left\{\frac{C^*}{2}\hat{A}^2\right\}\exp\left\{\frac{C}{2}\hat{A}^{+2}+C^{*n+1}C^{n+2}\left(C^* a_1 + a_2\right)\hat{A}^+\right\}\mid 0\rangle,
\end{aligned} \tag{A.8}$$

now acting on the left with the exponential linear in the creation operator is obtained the relation

$$\begin{aligned} &\exp\left\{\frac{C^*}{2}a_1^2+\frac{C}{2}\left(C^*a_1+a_2\right)^2\sum_{m=0}^{n+1}\left[|C|^{2(2m)}+|C|^{2(2m+1)}\right]\right\} \\ &\times\langle 0\mid \exp\left\{\frac{C^*}{2}\hat{A}^2+C^{*n+2}C^{n+2}\left(C^*a_1+a_2\right)\hat{A}\right\}\exp\left\{\frac{C}{2}\hat{A}^{+2}\right\}\mid 0\rangle \end{aligned} \quad (A.9)$$

which probe the recurrence relation (A.7).

At this point the limit $n\to\infty$ is taken, considering $|C|<1$ which implies that

$$\begin{aligned} &\lim_{n\to\infty}|C|^{2n}=0, \\ &\lim_{n\to\infty}\sum_{m=0}^{n}\left[|C|^{2(2m)}+|C|^{2(2m+1)}\right]=\frac{1}{\left(1-|C|^2\right)}, \end{aligned} \quad (A.10)$$

and the expression (A.9) in this limit has the form,

$$\exp\left\{\frac{\left(C^*a_1^2+Ca_2^2+2C^*Ca_1a_2\right)}{2\left(1-|C|^2\right)}\right\}\langle 0\mid \exp\left\{\frac{C^*}{2}\hat{A}^2\right\}\exp\left\{\frac{C}{2}\hat{A}^{+2}\right\}\mid 0\rangle \quad (A.11)$$

Finally, the notation (A.2) is substituted in (A.11). After that, the functions of $\vec{p}$ are expanded in the vicinity of $\vec{p}=0$, keeping in mind that the sources are located in a space finite region it is necessary to consider only the first terms in the expansion. Then for the expression (A.11) it is obtained the result (3.29), the renormalization factors cancel out.

Appendix B

Longitudinal and Scalar Modes Contribution

The longitudinal and scalar modes contribution is determined by the expression

$$
\begin{aligned}
&\langle 0 \mid \exp\left\{ C_3^* \left(|\vec{p}|\right) \left(B_{\vec{p}}^{a} - i \int d^4x J^{\mu,a}(x)\, f^*_{p,L,\mu}(x) \right) \right. \\
&\qquad\qquad \left. \times \left(A_{\vec{p}}^{L,a} - i \int d^4x J^{\mu,a}(x)\, f^*_{p,S,\mu}(x) \right) \right\} \\
&\quad \times \exp\left\{ C_3 \left(|\vec{p}|\right) \left(B_{\vec{p}}^{a+} - i \int d^4x J^{\mu,a}(x)\, f_{p,L,\mu}(x) \right) \right. \\
&\qquad\qquad \left. \times \left(A_{\vec{p}}^{L,a+} - i \int d^4x J^{\mu,a}(x)\, f_{p,S,\mu}(x) \right) \right\} \mid 0\rangle
\end{aligned} \tag{B.1}
$$

introducing the following notation,

$$
\begin{aligned}
C^* &\equiv C_3^* \left(|\vec{p}|\right), \; C \equiv C_3 \left(|\vec{p}|\right), \quad \hat{A}^+ \equiv A_{\vec{p}}^{L,a+}, \; \hat{A} \equiv A_{\vec{p}}^{L,a}, \quad \hat{B}^+ \equiv B_{\vec{p}}^{a+}, \; \hat{B} \equiv B_{\vec{p}}^{a}, \\
a_1 &\equiv -i \int d^4x J^{\mu,a}(x)\, f^*_{p,S,\mu}(x), \qquad a_2 \equiv -i \int d^4x J^{\mu,a}(x)\, f_{p,S,\mu}(x), \\
b_1 &\equiv -i \int d^4x J^{\mu,a}(x)\, f^*_{p,L,\mu}(x), \qquad b_2 \equiv -i \int d^4x J^{\mu,a}(x)\, f_{p,L,\mu}(x),
\end{aligned} \tag{B.2}
$$

the expression (B.1) takes the form

$$
\begin{aligned}
&\langle 0 \mid \exp\left\{C^*\left(\hat{A}+a_1\right)\left(\hat{B}+b_1\right)\right\}\exp\left\{C\left(\hat{B}^{+}+b_2\right)\left(\hat{A}^{+}+a_2\right)\right\}\mid 0\rangle \\
&=\exp\left\{C^* a_1 b_1 + C a_2 b_2\right\}\langle 0 \mid \exp\left\{C^*\left(\hat{A}\hat{B}+b_1\hat{A}+a_1\hat{B}\right)\right\} \\
&\qquad\qquad\qquad\qquad \times\exp\left\{C\left(\hat{B}^{+}\hat{A}^{+}+b_2\hat{A}^{+}+a_2\hat{B}^{+}\right)\right\}\mid 0\rangle
\end{aligned}
\tag{B.3}
$$

For the action of the exponential linear in the annihilation operator at the left on the right, is obtained

$$
\begin{aligned}
&\exp\left\{C^*\left(b_1\hat{A}+a_1\hat{B}\right)\right\}\exp\left\{C\left(\hat{B}^{+}\hat{A}^{+}+b_2\hat{A}^{+}+a_2\hat{B}^{+}\right)\right\}\mid 0\rangle \qquad \text{(B.4)}\\
&=\exp\left\{C\left[\left(\hat{B}^{+}-C^*b_1\right)\left(\hat{A}^{+}-C^*a_1\right)+b_2\left(\hat{A}^{+}-C^*a_1\right)+a_2\left(\hat{B}^{+}-C^*b_1\right)\right]\right\}\mid 0\rangle,
\end{aligned}
$$

the same procedure used for calculating (3.26) is considered.

Following the same steps described in the previous appended for transverse modes, in this case the recurrence relation obtained for longitudinal and scalar modes is

$$
\begin{aligned}
&\exp\left\{C^* a_1 b_1 + C\left(C^* a_1 - a_2\right)\left(C^* b_1 - b_2\right)\sum_{m=0}^{n}\left[|C|^{2(2m)}+|C|^{2(2m+1)}\right]\right\} \qquad \text{(B.5)}\\
&\langle 0 \mid \exp\left\{C^*\hat{A}\hat{B}+C^{*n+1}C^{n+1}\left(\left(C^*b_1-b_2\right)\hat{A}+\left(C^*a_1-a_2\right)\hat{B}\right)\right\}\exp\left\{C\hat{B}^{+}\hat{A}^{+}\right\}\mid 0\rangle.
\end{aligned}
$$

For the expression (B.5), in the limit $n \to \infty$ considering $|C| < 1$, the following relation is obtained

$$
\begin{aligned}
&\exp\left\{C^* a_1 b_1 + C\left(C^* a_1 - a_2\right)\left(C^* b_1 - b_2\right)\frac{1}{\left(1-|C|^2\right)}\right\} \\
&\langle 0 \mid \exp\left\{C^*\hat{A}\hat{B}\right\}\exp\left\{C\hat{B}^{+}\hat{A}^{+}\right\}\mid 0\rangle.
\end{aligned}
\tag{B.6}
$$

Finally, the notation (B.2) is substituted in (B.6), the functions of $\vec{p}$ are expanded in the vicinity of $\vec{p} = 0$, and the result (3.30) is obtained.

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