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Nonlinear Realization of Partially Broken $N = 2$ AdS Supersymmetry in Two and Three Dimensions

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Abstract

We investigate the nonlinear realization of partially broken N=2 global supersymmetry in the $D = 2$ and 3 anti de-Sitter (AdS) space. We particularly study Nambu-Goldstone degrees of freedom for the N=2 AdS supersymmetry partially broken down to N=1 AdS supersymmetry, where we observe a NG fermion of the broken supersymmetry and a NG boson of the internal symmetry which form a NG multiplet. Based on the nonlinear realization method, we construct a superspace formalism for 2 and 3 dimensional AdS space and evaluate the covariant derivatives and supervielbeins for the AdS superspace. Finally we obtain the nonlinear transformation laws and the lowest order of effective Lagrangians.

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1. Introduction

The $\mathcal{N}$=1 supersymmetry in the minimal supersymmetric standard model is thought to be realized as the low energy limit of a more fundamental theory formulated in higher dimensions with 4-dimensional $\mathcal{N}$-extended supersymmetry, which has to be spontaneously broken to $\mathcal{N} = 1$. In such a context, partial breaking of extended supersymmetry has been studied in the literatures [1, 2, 3, 4, 5] in the last several years.

Hughes, Liu and Polchinski [1] first pointed out the possibility of the partial supersymmetry breaking and constructed an example arising from the 4-dimensional supermembrane solution of 6-dimensional supersymmetric gauge theory. While, Antoniadis, Partouche and Taylor [2] introduced the electric and magnetic Fayet-Iliopoulos terms in the $\mathcal{N}$=2 gauge theory of abelian vector multiplet and have shown the spontaneous breaking of $\mathcal{N}$=2 to $\mathcal{N}$=1. This partial breaking induced by the Fayet-Iliopoulos terms has also been obtained by taking the flat limit of the $\mathcal{N}$=2 supergravity theories [3].

On the other hand, Bagger and Galperin [4, 5] have studied the nonlinear realization of $\mathcal{N}$=2 supersymmetry partially broken down to $\mathcal{N}$=1 supersymmetry in D=4 flat space [6, 7]. They obtained the Nambu-Goldstone (NG) multiplet both for the cases; chiral multiplet [4] and vector-multiplet [5], and discussed the nonlinear transformation laws as well as the low-energy effective Lagrangians [8].

Now the partial breaking of supersymmetry in flat space can be extended to the anti-de Sitter (AdS) space, which has recently attracted much attention in the context of AdS/CFT correspondence as well as brane-world scenario. By unHiggsing the $\mathcal{N}$=1 massive spin-3/2 multiplet, Altendorfer and Bagger studied the partial breaking of the $\mathcal{N} = 2$ AdS supersymmetry $OSp(2,4)$ [9]. For AdS background, Zumino and Deser [10, 11] have investigated the nonlinear realization of $\mathcal{N} = 1$ AdS supersymmetry. They have found that there appears the "mass" term of the NG fermion which is proportional to the radius of AdS space in the effective Lagrangian and considered its relation to the super-Higgs effect.

Here in this paper, we shall investigate the nonlinear realization of D=2 and 3, $\mathcal{N}$=2 global AdS supersymmetry. We study the spontaneous breaking of this symmetry down to $\mathcal{N}$=1 AdS supersymmetry using the AdS superfield method. We find that there appears only one $\mathcal{N}$=1 NG multiplet which consists of a NG fermion of the broken second supersymmetry and a NG boson of the broken internal symmetry. We next consider the nonlinear transformation laws and then construct the effective Lagrangian up to the second order of fields. It is intriguing to examine whether a peculiar value of "mass" term would emerge as N=1 case studied in [10].

2. Algebras and Partial Breaking of AdS supersymmetry

In this section, we consider the N=2 AdS superalgebra and its spontaneous breaking. We then determine the relations among the NG fields associated with the broken generators.

2.1. D=2 and 3 AdS superalgebras

The two and three dimensional AdS superalgebras [12] are the algebras obtained by adding N spinorial generators to the bosonic AdS algebras, and are given as following (see Appendix for the notations):

$$[P_\mu, P_\nu] = -im^2 M_{\mu\nu}, \quad [P_\mu, M_{\rho\sigma}] = i\eta_{\mu\rho}P_\sigma - i\eta_{\mu\sigma}P_\rho,$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i\eta_{\mu\rho}M_{\nu\sigma} - i\eta_{\nu\sigma}M_{\mu\rho} + i\eta_{\mu\sigma}M_{\nu\rho} + i\eta_{\nu\rho}M_{\mu\sigma},$$

$$[Q^{i\alpha}, P_\mu] = \frac{m}{2}(\gamma_\mu Q^i)_\alpha, \quad [Q^i_\alpha, M_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu}Q^i)_\alpha, \quad (i = 1, 2, \cdots, N),$$

$$\begin{aligned} \{Q^i_\alpha, \bar{Q}^{j\beta}\} &= m\delta^{ij}(\sigma^{\mu\nu})_\alpha{}^\beta M_{\mu\nu} + 2\delta^{ij}(\gamma^\mu)_\alpha{}^\beta P_\mu + i\delta_\alpha^\beta T^{ij}(D=2), \\ &= m\delta^{ij}(\sigma^{\mu\nu})_\alpha{}^\beta M_{\mu\nu} + 2\delta^{ij}(\gamma^\mu)_\alpha{}^\beta P_\mu + 2i\delta_\alpha^\beta T^{ij}(D=3), \end{aligned} \tag{1}$$

$$[T^{ij}, Q^k_\alpha] = im\delta^{ik}Q^j_\alpha - im\delta^{jk}Q^i_\alpha,$$

$$[T^{ij}, T^{kl}] = -i\delta^{jk}T^{il} + i\delta^{ik}T^{jl} + i\delta^{jl}T^{ik} - i\delta^{il}T^{jk},$$

where T^{ij}'s are $SO(N)$ generators of the internal symmetry of the supercharges and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] = i\epsilon^{\mu\nu}\gamma_5\,(D=2), -\epsilon^{\mu\nu\rho}\gamma_\rho\,(D=3)$. We can recover $SO(d-1,2)$ algebra which is the isometry of the d dimensional AdS space, given by hyperboloid $\eta_{AB}y^Ay^B = 1/m^2$ with $\eta_{AB} = (\eta_{\mu\nu}, +1)$, $\eta^{\mu\nu} = (+1, -1, \ldots -1)$, by rewriting the generators of pseudo-translation P_μ as $P_\mu \equiv mM_{\mu d}$, where m is the reciprocal of the radius of AdS space. Note that by the Majorana properties of the supercharges Q^i_α, the internal symmetry becomes $SO(N)$. For N=2, there is only one internal symmetry generators T^{12} which we shall denote by T in the following. This algebra is contracted to the super-Poincaré algebra in the flat limit $m \to 0$.

2.2. Nambu-Goldstone multiplet

Now we identify the N=1 NG multiplets for the partially broken N=2 AdS superalgebra. In our case where N=2 AdS superalgebra is spontaneously broken down to N=1 AdS superalgebra, we have the broken generators Q^2_α and T. We assign a fermion ψ_α and a boson ϕ to the corresponding NG particles, respectively. Because of the unbroken $N=1$ supersymmetry, these particles must have its own superpartners. But in this case it turns out that ψ and ϕ are the superpartners of each other, which can be seen from the following argument [13, 14].

Let us consider the Jacobi identity for Q^1, T and ψ given by

$$\{[Q^1_\alpha, T], \psi\} - \{[T, \psi], Q^1_\alpha\} - [\{\psi, Q^1_\alpha\}, T] = 0. \tag{2}$$

Taking the vacuum expectation value of (2) we have

$$\langle 0|[\{\psi, Q^1_\alpha\}, T]|0\rangle = -im\langle 0|\{Q^2_\alpha, \psi\}|0\rangle \neq 0, \tag{3}$$

where we used the fact $Q^1|0\rangle = 0$ and $\langle 0|\{Q^2_\alpha, \psi\}|0\rangle \neq 0$. Since $\langle 0|[\phi, T]|0\rangle \neq 0$ for ϕ, the NG boson of T, this means that

$$\phi \sim \{\psi, Q^1_\alpha\}, \tag{4}$$

i.e. the supersymmetry transformation of the NG fermion of the second supersymmetry, ψ, is related to the NG boson of the internal symmetry, ϕ.

3. Coset Construction and AdS Superspace

When we apply the framework of the nonlinear realization [15] to the analysis of the partially broken AdS supersymmetry, we must represent the NG field as a superfield. In this section we formulate the AdS superfield using a group theoretical method [16].

For $D = 2$ and 3, the $N = 1$ scalar multiplet can be represented as a real scalar function $\Phi(x^\mu, \theta_\alpha)$

$$\Phi(x, \theta) = A(x) + \bar{\theta}\psi(x) + \frac{1}{2}\bar{\theta}\theta F(x), \tag{5}$$

where $A(x)$, $\psi(x)$ and $F(x)$ are a real scalar, a Majorana spinor and an auxiliary scalar, respectively. Here we note that the Φ represents an irreducible N=1 scalar multiplet.

First, we consider supercovariant derivatives for AdS superfield. The AdS superspace is given as the coset space of the AdS algebra (1) represented by the coset representive

$$L(x,\theta) = e^{iz^\mu(x)P_\mu} e^{i\bar\theta^\alpha Q_\alpha}, \tag{6}$$

where $z^\mu(x) = \frac{2}{mx}\tan^{-1}\frac{mx}{2}x^\mu, x = (x^\mu x_\mu)^{1/2}$. Here x^μ's are the coordinates of AdS space and θ^α's are the fermionic coordinates. In these coordinates, the zweibein (dreibein) of the AdS space and the spin connection take the form

$$e^a_\mu = \frac{\delta^a_\mu}{1+\frac{m^2x^2}{4}}, \quad \omega^{ab}_\mu = \frac{m^2}{2}\frac{x^a\delta^b_\mu - x^b\delta^a_\mu}{1+\frac{m^2x^2}{4}}. \tag{7}$$

We calculate the Cartan differential 1-form as [10]

$$L^{-1}dL \equiv iDx^a P_a + iD\bar\theta^\alpha Q_\alpha + \frac{i}{2}\Delta\Omega^{ab}M_{ab}, \tag{8}$$

where

$$\begin{aligned}
Dx^a &= \left(1+\frac{m}{2}\bar\theta\theta\right)\nabla x^a + i(d\bar\theta\gamma^a\theta) \quad (D=2)\\
&= \left(1+\frac{m}{2}\bar\theta\theta\right)\nabla x^a - \frac{1}{4}\epsilon^{bca}\Delta\omega^{bc}(\bar\theta\theta) + i(d\bar\theta\gamma^a\theta) \quad (D=3),\\
D\bar\theta^\alpha &= -i\frac{m}{2}\nabla x^a(\bar\theta\gamma_a)^\alpha + \frac{1}{4}\Delta\omega^{ab}\epsilon_{ab}(\bar\theta\gamma_5)^\alpha + \left(1-\frac{m}{4}(\bar\theta\theta)\right)d\bar\theta^\alpha \quad (D=2)\\
&= -i\frac{m}{2}\nabla x^a(\bar\theta\gamma_a)^\alpha + \frac{i}{4}\Delta\omega^{ab}\epsilon_{abc}(\bar\theta\gamma^c)^\alpha + \left(1-\frac{m}{2}(\bar\theta\theta)\right)d\bar\theta^\alpha \quad (D=3),\\
\frac{1}{2}\Delta\Omega^{ab} &= \left(1+\frac{m}{2}\bar\theta\theta\right)\frac{1}{2}\Delta\omega^{ab} - \frac{m}{2}\epsilon^{ab}(d\bar\theta\gamma_5\theta) \quad (D=2)\\
&= \left(1+\frac{m}{2}\bar\theta\theta\right)\frac{1}{2}\Delta\omega^{ab} - \frac{1}{4}m^2\nabla x_c\epsilon^{cab}(\bar\theta\theta) - i\frac{m}{2}\epsilon^{abc}(d\bar\theta\gamma_c\theta) \quad (D=3),
\end{aligned} \tag{9}$$

where we use the notations $\nabla x^a = e^a_\mu dx^\mu$, $\Delta\omega^{ab} = -\omega^{ab}_\mu dx^\mu$.

The superzweibein (dreibein) $W_M^{\ A}$ is defined as

$$Dz^A = dz^M W_M^{\ A}, \quad Dz^A = (Dx^a, D\bar\theta^\alpha), \quad dz^M = (dx^\mu, d\theta^\beta). \tag{10}$$

From the formula of Cartan form (9), we obtain $W_M^{\ A}$ and its inverse becomes

$$\begin{aligned}
(W^{-1})_A^{\ M} &= \begin{pmatrix} (W^{-1})_a^{\ \mu} & (W^{-1})_a^{\ \beta} \\ (W^{-1})_\alpha^{\ \mu} & (W^{-1})_\alpha^{\ \beta} \end{pmatrix}\\
&= \begin{pmatrix} e^\mu_a & -\frac{im}{2}(\bar\theta\gamma_a)^\beta + \frac{1}{2}\omega_a(\bar\theta\gamma_5)^\beta \\ -ie^\mu_a(\gamma^a\theta)_\alpha & \left(1-\frac{m}{4}(\bar\theta\theta)\right)\delta^\beta_\alpha - \frac{i}{4}(\bar\theta\theta)\omega_a\epsilon^{ad}(\gamma_d)^\beta_\alpha \end{pmatrix} \quad (D=2)
\end{aligned} \tag{11}$$

$$= \begin{pmatrix} e^{\mu}_a & \frac{im}{2}(\bar\theta\gamma_a)^\beta + \frac{i}{2}\omega_{ac}(\bar\theta\gamma^c)^\beta \\ -ie^{\mu}_a(\gamma^a\theta)_\alpha & \left(1-\frac{m}{4}(\bar\theta\theta)\right)\delta^\beta_\alpha - \frac{1}{4}(\bar\theta\theta)\omega^c_\mu(\gamma_c\gamma^\mu)^\beta_\alpha \end{pmatrix} \quad (D=3),$$

where we define $\omega_\mu = \frac{1}{2}\omega^{ab}_\mu\epsilon_{ab}\,(D=2)$, $\omega_{\mu a} = \frac{1}{2}\omega^{bc}_\mu\epsilon_{abc}\,(D=3)$. Then we introduce the supercovariant derivative of general superfield Φ which transform linearly under the AdS supersymmetry as follows:

$$D_A\Phi = \frac{D\Phi}{Dz^A} = (W^{-1})_A^{\ M}\frac{D\Phi}{dz^M}, \quad D\Phi \equiv d\Phi + \frac{i}{2}\Delta\omega^{bc}\Sigma_{bc}\Phi. \tag{12}$$

where Σ_{bc} is the matrix representation of the Lorentz group. Hence,

$$\begin{aligned} \frac{D\Phi}{Dx^a} &= \nabla_a\Phi + \frac{im}{2}\left(\bar\theta\gamma_a\frac{\partial\Phi}{\partial\bar\theta}\right) + \frac{1}{2}\omega_a\left(\bar\theta\gamma_5\frac{\partial\Phi}{\partial\bar\theta}\right) \quad (D=2) \\ &= \nabla_a\Phi + \frac{im}{2}\left(\bar\theta\gamma_a\frac{\partial\Phi}{\partial\bar\theta}\right) + \frac{i}{2}\omega_{ac}\left(\bar\theta\gamma^c\frac{\partial\Phi}{\partial\bar\theta}\right) - \frac{im}{4}(\bar\theta\theta)\omega^{bc}_a\Sigma_{bc}\Phi \qquad (13) \\ &- \frac{im^2}{4}\epsilon^{abc}(\bar\theta\theta)\Sigma_{bc}\Phi + \frac{im^2}{4}\epsilon^{dbc}(\bar\theta\gamma_a\gamma_c\theta)\Sigma_{db}\Phi + \frac{im}{4}\omega_{ae}(\bar\theta\gamma^e\gamma_c\theta)\epsilon^{dbc}\Sigma_{db}\Phi \quad (D=3), \\ \frac{D\Phi}{D\bar\theta^\alpha} &= -i(\gamma^a\theta)_\alpha\frac{D\Phi}{Dx^a} + \left(1+\frac{m}{4}(\bar\theta\theta)\right)\frac{\partial\Phi}{\partial\bar\theta^\alpha} - \frac{im}{2}\epsilon^{bc}(\gamma_5\theta)_\alpha\Sigma_{bc}\Phi \quad (D=2) \\ &= -i(\gamma^a\theta)_\alpha\frac{D\Phi}{Dx^a} + \left(1+\frac{m}{2}(\bar\theta\theta)\right)\frac{\partial\Phi}{\partial\bar\theta^\alpha} + \frac{m}{2}\epsilon^{bca}(\gamma_a\theta)_\alpha\Sigma_{bc}\Phi \quad (D=3), \qquad (14) \end{aligned}$$

where ∇_a is the AdS covariant derivative. As $m \to 0$, these operators are reduced to the usual supercovariant derivatives in the flat superspace.

Next we consider the transformation laws. Under the multiplication of a group element g of AdS supersymmetry group, the left-coset $L(x,\theta)$ transforms as [10]

$$L \to gL = L'h \quad , \tag{15}$$

h is a group element of Lorentz group. In particular, for an infinitesimal transformation, we take $g = 1 + i\bar\epsilon^\alpha Q^1_\alpha$, $L' = L + \delta L$, $h = 1 + i\delta u^{\mu\nu}M_{\mu\nu}$ and (15) becomes

$$e^{-i\bar\theta Q}e^{-iz(x)\cdot P}(i\bar\epsilon Q)e^{iz(x)\cdot P}e^{i\bar\theta Q} - e^{-i\bar\theta Q}e^{-iz(x)\cdot P}\delta(e^{iz(x)\cdot P}e^{i\bar\theta Q}) = i\delta u^{\mu\nu}M_{\mu\nu}. \tag{16}$$

To get the transformation laws of x^μ and θ^α we must evaluate the left hand side of (16) and set the coefficients of P_a and Q_α to zero. Then we obtain the equations of

δx^μ and $\delta\bar\theta^\alpha$, which leads to the transformation laws

$$\begin{aligned}
\delta x^\mu &= i(\bar\eta\gamma^\mu\theta) \quad (D=2,3), \\
\delta\bar\theta^\alpha &= \bar\eta^\alpha - \frac{i}{4}(\bar\theta\theta)\omega_a(\bar\eta\gamma^a\gamma_5)^\alpha \quad (D=2) \\
&= \left(1-\frac{m}{4}(\bar\theta\theta)\right)\bar\eta^\alpha - \frac{i}{4}(\bar\theta\theta)\omega_{ab}(\bar\eta\sigma^{ab})^\alpha \quad (D=3).
\end{aligned} \tag{17}$$

Here we difine η_α by

$$e^{-iz(x)\cdot P}(i\bar\epsilon^\alpha Q_\alpha)e^{iz(x)\cdot P} \equiv i\bar\eta^\alpha Q_\alpha. \tag{18}$$

For the transformation laws, the parameter ϵ always appears in this form, so we conclude that $\epsilon = \eta$ is the global supersymmetric parameter in the AdS space background. This spinor is known as Killing spinor (see for example [17]) satisfying the equation:

$$\nabla_\mu\eta = -\frac{im}{2}\gamma_\mu\eta \quad (D=2,3), \tag{19}$$

which implies that the supergravity transformation of a gravitino, $\delta\psi_\mu = 0$.

Here we make some comments on $D=2$ and 3, $N=1$ supergravity multiplet $({e_\mu}^a, \psi_\mu, S)$, where ${e_\mu}^a$ is the zweibein (dreibein) or "graviton", ψ_μ is the Rarita-Schwinger field or "gravitino", and S is the auxiliary scalar field, transforms under an infinitesimal parameter ϵ as [18]

$$\delta {e_\mu}^a = \bar\epsilon\gamma^a\psi_\mu,\ \delta\psi_\mu = 2(\partial_\mu + \frac{1}{2}\omega_\mu\gamma_5 + \frac{i}{2}\gamma_\mu S)\epsilon,\ \delta S = -\frac{1}{2}S\bar\epsilon\gamma^\mu\psi_\mu + \frac{i}{2}e^{-1}\epsilon^{\mu\nu}\bar\epsilon\gamma_5\psi_{\mu\nu} \quad (D=2), \tag{20}$$

$$\delta {e_\mu}^a = \bar\epsilon\gamma^a\psi_\mu,\ \delta\psi_\mu = 2(\partial_\mu + \frac{1}{2}{\omega_\mu}^c\gamma_c + \frac{i}{4}\gamma_\mu S)\epsilon,\ \delta S = -\frac{1}{2}S\bar\epsilon\gamma^\mu\psi_\mu + \frac{i}{2}e^{-1}\epsilon^{\mu\nu\lambda}\bar\epsilon\gamma_\lambda\psi_{\mu\nu} \quad (D=3), \tag{21}$$

where $\psi_{\mu\nu} \equiv \nabla_\mu\psi_\nu - \nabla_\nu\psi_\mu = (\partial_\mu + \frac{1}{2}\omega_\mu\gamma_5)\psi_\nu - (\mu\leftrightarrow\nu)$ for $D=2$, with $\omega_\mu\gamma_5$ repalced by ${\omega_\mu}^c\gamma_c$ for $D=3$. To realize the global AdS supersymmetry, we take the supergravity multiplet to the AdS fixed background: [19, 20]

$${e_\mu}^a = {e_\mu}^a(\mathrm{AdS}), \quad \psi_\mu = 0, \quad S = m\ (D=2),\ 2m\ (D=3) \tag{22}$$

Now let us go back to the transformation laws for the component fields given as

$$\delta A = \bar\eta\psi,$$

$$\begin{aligned}
\delta\psi &= (F - i\gamma^\mu \partial_\mu A)\eta, \\
\delta F &= -i(\bar{\eta}\gamma^\mu \nabla_\mu \psi) \quad (D=2) \\
&= -i(\bar{\eta}\gamma^\mu \nabla_\mu \psi) - \frac{m}{2}(\bar{\eta}\psi) \quad (D=3).
\end{aligned} \tag{23}$$

These are in agreement with the fixed background result for the supergravity transformation [18].

To construct an invariant action, we also need the invariant volume element, which is given by

$$\begin{aligned}
\mathrm{sdet} W_M^{\ A} &= e\left(1 + \frac{m}{2}(\bar{\theta}\theta)\right) \quad (D=2) \\
&= e\left(1 + m(\bar{\theta}\theta)\right) \quad (D=3), \qquad e = \mathrm{det} e_\mu^a.
\end{aligned} \tag{24}$$

The invariant action is obtaind as a generalization of the one in a flat case. For a scalar superfield Φ, the Kinetic term is obtained as

$$\mathcal{L}(x) = \int d^2\theta (\mathrm{sdet} W) \frac{1}{4} \bar{D}^\alpha \Phi D_\alpha \Phi. \tag{25}$$

In component fields, this leads to

$$\mathcal{L}(x) = e\left(\frac{1}{2}\nabla^\mu A \nabla_\mu A + \frac{i}{2}(\bar{\psi}\gamma^\mu \nabla_\mu \psi) + \frac{1}{2}F^2\right). \tag{26}$$

4. Nonlinear Transformation Laws and Effective Lagrangians

In this section we consider the nonlinear transformation laws and effective lagrangians for NG fields for partial breaking of N=2 AdS supersymmetry using the nonlinear realization method [15]. The relevant coset representive is given by

$$L(x) = e^{iz(x)\cdot P} e^{i\bar{\theta}Q^1} e^{i\bar{\chi}(x^\mu,\theta)Q^2} e^{iv(x^\mu,\theta)T}. \tag{27}$$

Here we note that the coset space is parametrized by the N=1 AdS superspace coordinates (x^μ, θ) as well as by the NG superfields $\chi_\alpha(x^\mu, \theta)$ and $v(x^\mu, \theta)$, the relevant component of which correspond to NG fields discussed in the previous section. Cartan 1-form can be evaluated as

$$\begin{aligned}
L^{-1}\, dL &= e^{-ivT} e^{-i\bar{\chi}Q^2} e^{-i\bar{\theta}Q^1} e^{-iz(x)\cdot P} d\left(e^{iz(x)\cdot P} e^{i\bar{\theta}Q^1} e^{i\bar{\chi}Q^2} e^{ivT}\right) \\
&\equiv i\mathcal{D}x^a P_a + i\mathcal{D}\bar{\theta}^\alpha Q^1_\alpha + i\mathcal{D}\bar{\chi}^\alpha Q^2_\alpha + i\mathcal{D}vT + \frac{i}{2}\Delta\tilde{\Omega}^{ab} M_{ab},
\end{aligned} \tag{28}$$

where

$$
\begin{aligned}
\mathcal{D}x^a &= Dx^a\left(1+\frac{m}{2}(\bar\chi\chi)\right)+i(d\bar\chi\gamma^a\chi) \quad (D=2)\\
&= Dx^a\left(1+\frac{m}{2}(\bar\chi\chi)\right)+i(d\bar\chi\gamma^a\chi)-\frac{1}{4}\Delta\Omega^{bc}\epsilon_{bca}(\bar\chi\chi) \quad (D=3),\\
\mathcal{D}\bar\theta^\alpha &= i\frac{m}{2}Dx^a(\bar\chi\gamma_a)^\alpha\sin(mv)+\left(1+\frac{m}{4}(\bar\chi\chi)\right)D\bar\theta^\alpha\cos(mv)\\
&\quad -\frac{1}{4}\Delta\Omega^{ab}\epsilon_{ab}(\bar\chi\gamma_5)^\alpha\sin(mv)-\left(1-\frac{m}{4}(\bar\chi\chi)\right)d\bar\chi^\alpha\sin(mv) \quad (D=2)\\
&= i\frac{m}{2}Dx^a(\bar\chi\gamma_a)^\alpha\sin(mv)+\left(1+\frac{m}{2}(\bar\chi\chi)\right)D\bar\theta^\alpha\cos(mv)\\
&\quad +\frac{i}{4}\Delta\Omega^{ab}(\bar\chi\sigma_{ab})^\alpha\sin(mv)-\left(1-\frac{m}{2}(\bar\chi\chi)\right)d\bar\chi^\alpha\sin(mv) \quad (D=3),\\
\mathcal{D}v &= -(D\bar\theta\chi)+dv \quad (D=2)\\
&= -2(D\bar\theta\chi)+dv \quad (D=3).
\end{aligned} \tag{29}
$$

From these we obtain the superzweibein (dreibein) modified by NG fields in the similar manner of N=1 case,

$$\mathcal{D}z^A = dz^M\mathcal{W}_M^{\ A}. \tag{30}$$

The supercovariant derivatives of NG fields are also modified by the existence of NG fields and they are defined as

$$\mathcal{D}_A\Phi=\frac{\mathcal{D}\Phi}{\mathcal{D}z^A}=(\mathcal{W}^{-1})_A^{\ M}\frac{\mathcal{D}\Phi}{dz^M}, \qquad D\Phi\equiv d\Phi+\frac{i}{2}\Delta\omega^{bc}\Sigma_{bc}\Phi. \tag{31}$$

Now we determine the constraints which realize the relationship among the NG fields discussed in the previous section. In general the constraints should be invariant under the nonlinear transformations. The modified supercovariant derivatives of NG fields satisfy this condition. So we take the constraint as follows,

$$\frac{\mathcal{D}v}{\mathcal{D}\bar\theta^\alpha}=(\mathcal{W}^{-1})_\alpha^{\ \mu}\frac{\mathcal{D}v}{dx^\mu}+(\mathcal{W}^{-1})_\alpha^{\ \beta}\frac{\mathcal{D}v}{d\bar\theta^\beta}=0. \tag{32}$$

To see what this constraint means, let us rewrite it by N=1 notations. Here we consider the D=2 case. The D=3 case can be treated in the same way. From the result of the Cartan form,

$$\mathcal{D}v=-\left(dx^\mu W_\mu^{\ \alpha}+d\bar\theta^\beta W_\beta^{\ \alpha}\right)\chi_\alpha+dv, \tag{33}$$

so the constraint is

$$
\begin{aligned}
\frac{\mathcal{D}v}{\mathcal{D}\bar{\theta}^\alpha} &= -\left((\mathcal{W}^{-1})_\alpha{}^\mu W_\mu{}^\gamma + (\mathcal{W}^{-1})_\alpha{}^\beta W_\gamma{}^\gamma\right)\chi_\gamma + (\mathcal{W}^{-1})_\alpha{}^\mu \frac{\partial v}{\partial x^\mu} + (\mathcal{W}^{-1})_\alpha{}^\beta \frac{\partial v}{\partial \bar{\theta}^\beta} \\
&= 0.
\end{aligned} \tag{34}
$$

From (33) we obtain $(\mathcal{W}^{-1}W)_A{}^B = \delta_A{}^B + \mathcal{O}((v,\chi)^2)$ and the constraint becomes

$$
\begin{aligned}
\chi_\alpha &= \frac{Dv}{D\bar{\theta}^\alpha} + \mathcal{O}((v,\chi)^3) \quad (D=2) \\
&= \frac{1}{2}\frac{Dv}{D\bar{\theta}^\alpha} + \mathcal{O}((v,\chi)^3) \quad (D=3).
\end{aligned} \tag{35}
$$

This means that the NG ferimon corresponds to the broken Q^2, which is the superpartner of NG boson corresponding to the broken T and we can express the NG fields in terms of a scalar superfield v.

Now we evaluate nonlinear transformation laws for x^μ, θ, v, χ under broken symmetries, Q^2, T. The procedure is similar to the way we obtain N=1 supersymmetric transformation. The T transformation law turns out to be as follows

$$
\begin{aligned}
\delta x^\mu &= 0, \\
\delta\bar{\theta}^\alpha &= m\alpha(1 - \frac{m}{4}\bar{\theta}\theta)\bar{\chi}^\alpha \quad (D=2) \\
&= m\alpha(1 - \frac{m}{2}\bar{\theta}\theta)\bar{\chi}^\alpha \quad (D=3), \\
\delta\bar{\chi}^\alpha &= -m\alpha(1 + \frac{m}{4}\bar{\chi}\chi)\bar{\theta}^\alpha \quad (D=2) \\
&= -m\alpha(1 + \frac{m}{2}\bar{\chi}\chi)\bar{\theta}^\alpha \quad (D=3), \\
\delta v &= \alpha\left(1 - \frac{m}{2}\bar{\theta}\theta\right)\left(1 - \frac{m}{2}\bar{\chi}\chi\right) \quad (D=2) \\
&= \alpha\left(1 - m\bar{\theta}\theta\right)(1 - m\bar{\chi}\chi) \quad (D=3).
\end{aligned} \tag{36}
$$

Similarly, the lowest-order transformation laws under the Q^2 transformation are

$$
\begin{aligned}
&\delta x^\mu = i(\bar{\eta}\gamma^\mu\chi) + \cdots, \quad \delta\bar{\theta}^\alpha = 0 + \cdots, \quad \delta\bar{\chi}^\alpha = \bar{\eta}^\alpha + \cdots \quad , \\
&\delta v = (\bar{\eta}\theta) + \cdots \quad (D=2), \quad = 2(\bar{\eta}\theta) + \cdots \quad (D=3).
\end{aligned} \tag{37}
$$

We now consider the effective Lagrangian for the NG multiplet. In the spirit of a nonlinear realization method, the invariant phase volume $\mathrm{sdet}\mathcal{W}$ is the simplest and is possible to give the full Lagrangian as used in [10]. But in this case, unfortunately, this does not give rise to the correct kinetic terms: the ghost appears. This was expected because the ghost appears also in the case of the partially broken super-Poincaré symmetry [7]. So one has to construct the Lagrangian using the nonlinear transformation laws according to [4]. That is to say, we write down the general N=1 invariant Lagrangian and examine the condition for this Lagrangian to be invariant under Q^2 and T transformations. If the Lagrangian is invariant under T and Q^1, the Jacobi identity:

$$im\,[Q^2,\mathcal{L}] = [T,[Q^1,\mathcal{L}]] - [Q^1,[T,\mathcal{L}]] \tag{38}$$

leads to the invariance under Q^2. Thus we only have to consider the T transformation.

Here we consider the second order of fields of the Lagrangian and find the existence of the characteristic "mass" term for the NG fields.

The general N=1 Lagrangian can be represented to the second order of the fields, both for D=2 and D=3 cases,

$$\mathcal{L}(x) = \int d^2\theta\, \mathrm{sdet}W \left(\frac{1}{4}\overline{D^\alpha}vD_\alpha v + \frac{\mu}{2}v^2\right), \tag{39}$$

where μ is a mass parameter to be determined. Under the T transformation, the variation is

$$\delta\mathcal{L} = \int d^2\theta\, \mathrm{sdet}W \left(\frac{1}{2}\overline{D^\alpha}vD_\alpha \delta v + \mu v\delta v\right). \tag{40}$$

Using (36) this leads to

$$\begin{aligned} \delta\mathcal{L} &= 2\left(-\frac{\alpha m}{2} + \frac{\mu}{2}\alpha\right)F \quad (D=2) \\ &= 2\left(-\alpha m + \frac{\mu}{2}\alpha\right)F \quad (D=3). \end{aligned} \tag{41}$$

Therefore if

$$\mu = m \quad (D=2), \quad \mu = 2m \quad (D=3), \tag{42}$$

we are lead to the Lagrangain invariant under T (up to second order of the fields). In fact we confirm that this is also invariant under Q^2 by using the property of the Killing spinor (19).

In terms of the component fields, the effective Lagrangian (39) for the NG fields is

$$\begin{aligned} e^{-1}\mathcal{L}(x) &= \frac{1}{2}\nabla_\mu A\nabla^\mu A + \frac{i}{2}\bar\psi\gamma^\mu\nabla_\mu\psi + \frac{1}{2}F^2 + mFA - \frac{m}{2}\bar\psi\psi + \frac{m^2}{2}A^2 \quad (D=2) \\ &= \frac{1}{2}\nabla_\mu A\nabla^\mu A + \frac{i}{2}\bar\psi\gamma^\mu\nabla_\mu\psi + \frac{1}{2}F^2 + 2mFA - \frac{3m}{4}\bar\psi\psi + 2m^2A^2 \quad (D=3). \end{aligned} \tag{43}$$

We can eliminate the auxiliary field F using an equation of motion and we get

$$
\begin{aligned}
e^{-1}\mathcal{L}(x) &= \frac{1}{2}\nabla_\mu A \nabla^\mu A + \frac{i}{2}\bar{\psi}\gamma^\mu \nabla_\mu \psi - \frac{m}{2}\bar{\psi}\psi \quad (D=2) \\
&= \frac{1}{2}\nabla_\mu A \nabla^\mu A + \frac{i}{2}\bar{\psi}\gamma^\mu \nabla_\mu \psi - \frac{3m}{4}\bar{\psi}\psi \quad (D=3).
\end{aligned} \tag{44}
$$

Note that in the Lagrangian (44) the NG fermion ψ possesses a "mass" term both for D=2 and 3 with the characteristic values depending on the space-time dimension, while the scalar A, which is the NG boson of the broken T does not have the "mass" term. Here we also note that these mass terms can be derived from the invariant Lagrangian of the supergravity coupled to matter fields by taking the the supergravity multiplet to the AdS background (22).

5. Conclusion and Discussion

In this paper we have investigated the nonlinear realization of 2 and 3-dimensional $N=2$ AdS supersymmetry which is partially broken to $N=1$ AdS supersymmetry. We have particularly studied the NG degrees of freedom, the nonlinear transformation laws and the lowest order Lagrangian with a particular attention on the "mass" term.

Finally, a comment on the 4 dimensional case is in order. The similar analysis in the present paper can be performed for $D=4$ case. The superfield formalism has already been discussed in [16]. Starting from $N=1$ supergravity, by setting the scalar auxiliary field $S=3m$, and remaining pseudo-scalar and vector auxiliary fileds to zero, we arrive at the AdS background: $\delta\psi_\mu=0$ satisfying the Killing-spinor condition: (19).

For the partially broken $N=2$ AdS denoted by $OSp(2,4)$, the on-shell degree of freedom corresponding to Q^2 is two real and the one associated with T is only one because $T^{12}\equiv T$ is hermitian $SO(2)$ generator. So we need extra one real pseudo-scalar to form a chiral multiplet or take a vector multiplet and regard the NG boson for T as a auxially field of the vector multiplet.

Note added: Recently we have noticed that a nonlinear realization of $N=2$, $D=3$ Poincaré supersymmetry down to $N=1$, $D=3$ has been studied in ref.[21] to construct the action of $N=1$, $D=4$ supermembrane, by contracting the AdS radius.

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