Logarithmic Conformal Field Theories Near a Boundary

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Abstract

We consider logarithmic conformal field theories near a boundary and derive the general form of one and two point functions. We obtain results for arbitrary and two dimensions. Application to two dimensional magnetohydrodynamics is discussed.

1 Introduction

Gurarie has pointed out [1] that it is possible to construct consistent conformal field theories which have logarithmic terms in their correlation functions. In these theories representation of the Virasoro algebra is not diagonalizable. Logarithmic conformal field theories (LCFTs) contain logarithmic operators which have logarithms as well as powers in their operator product expansion. Such operators do not appear in unitary CFTs. However this does not mean that they are not relevant. By now a large number of examples of the applications of such theories have been found. For some examples see citations in [2]. Extensive work has been done on the structure of LCFTs [2, 3, 4], although a lot remains to be worked out.

Conformal invariance in physical systems arises at a fixed point. It is also known that critical behavior near boundaries is affected by the geometry of the boundary. Indeed correlators of conformal field theories near boundaries have extra structure which gives rise to the surface critical behavior [5]. Conformal theories of turbulence near a boundary were discussed by Chung et al [6].

In this paper we investigate such theories near a boundary. For definiteness, we consider a semi-infinite d-dimensional system bounded by a $(d-1)$-dimensional plane surface. The results of this paper fall into two classes. For arbitrary dimensions d, we show that conformal invariance determines the one-point functions up to some

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constants and restricts the form of two-point functions up to some unknown functions of a single scaling variable. In two-dimensions our result is much stronger. We show that in this case the n-point functions in the semi-infinite geometry satisfy the same equations as the 2n-point functions in the bulk. In the end, we apply the results derived in sections II and III, to the 2D MHD problem and find the mean values of velocity and magnetic field.

2 Arbitrary Dimensions

In logarithmic conformal field theories, there exist blocks of fields which constitute a non-diagonalizable representation of the conformal group. Under a conformal transformation $\mathbf{r} \to \mathbf{r}' = f(\mathbf{r})$ fields of a block trasmform as [3]:

$$\Phi'(\mathbf{r}') = \left|\frac{\partial \mathbf{r}'}{\partial \mathbf{r}}\right|^{T} \Phi(\mathbf{r}) \tag{1}$$

where

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{pmatrix} \tag{2}$$

and T is Jordanian matrix

$$T = \begin{pmatrix} -\frac{\Delta_\phi}{d} & 0 & \dots & 0 \\ 1 & -\frac{\Delta_\phi}{d} & \dots & \vdots \\ 0 & 1 & \dots & 0 \\ \vdots & \dots & \ddots & 0 \\ 0 & \dots & 1 & -\frac{\Delta_\phi}{d} \end{pmatrix} . \tag{3}$$

Here d is dimension of space-time and Δ_ϕ is conformal dimension of ϕ_i's. As an example, for a two-dimensional Jordan cell $\Phi = \phi_1$ and $\Psi = \phi_2$, equation (1) reduces to:

$$\begin{aligned} \Phi'(\mathbf{r}') &= \left|\frac{\partial \mathbf{r}'}{} \partial \mathbf{r}\right|^{-\frac{\Delta_\phi}{d}} \Phi(\mathbf{r}) \\ \Psi'(\mathbf{r}') &= \left|\frac{\partial \mathbf{r}'}{\partial \mathbf{r}}\right|^{-\frac{\Delta_\phi}{d}} \left(\Psi(\mathbf{r}) + \log\left|\frac{\partial \mathbf{r}'}{\partial \mathbf{r}}\right|\Phi(\mathbf{r})\right) . \end{aligned} \tag{4}$$

In d dimensions, invariance under conformal transformations determines two- and three-point functions up to some constants and four-point functions are determined up to functions of crossing ratios. These invariances exist only if the geometry of the problem is not changed by the transformations. So if there is a boundary in the problem, correlation functions are invariant only under transformations which preserve the boundary. In this case we have less symmetries and correlation functions are not easily determined . To be more specific, we investigate correlation functions of a two-dimensional Jordanian cell with a simple boundary – a $(d-1)$-dimensional hyper-plane. The symmetry group that preserves the geometry is made up of translations along the boundary, rotations in the boundary, dilatation and special conformal transformation along the boundary. We take as the boundary, the hyper-plane $y=0$.

Consider the one-point functions $\langle\Phi(\mathbf{r})\rangle = f_1(\mathbf{r})$, $\langle\Psi(\mathbf{r})\rangle = f_2(\mathbf{r})$ near the boundary. Invariance under translations along the surface implies that f_1 and f_2 depend only on y. Also under an infinitesimal dilatation transformation:

$$\mathbf{r}' = \mathbf{r} + \epsilon\mathbf{r} \tag{5}$$

we must have:

$$\begin{aligned} f_1(y) &= (1+\epsilon)^{\Delta_\phi} f_1(y') \ , \\ f_2(y) &= (1+\epsilon)^{\Delta_\phi}\Big(f_2(y') + \log(1+\epsilon)^d f(y')\Big) \ . \end{aligned} \tag{6}$$

Expanding these equations to the first order in ϵ, f_1 and f_2 satisfy the differential equations:

$$\begin{aligned} y\frac{\partial f_1}{\partial y} + \Delta_\phi f_1 &= 0 \\ y\frac{\partial f_2}{\partial y} + \Delta_\phi f_2 + d\, f &= 0 \end{aligned} \tag{7}$$

which yield:

$$\begin{aligned} \langle\Phi(\mathbf{r})\rangle &= \frac{C_1}{y^{\Delta_\phi}} \\ \langle\Psi(\mathbf{r})\rangle &= \frac{1}{y^{\Delta_\phi}}(C_2 - d\, C_1 \log y) \end{aligned} \tag{8}$$

Invariance under rotation and special conformal transformations reveals no other constraint on these one-point functions.

Now consider the two-point function $G_1(\mathbf{r}_1,\mathbf{r}_2) = \langle\Phi(\mathbf{r}_1)\Phi(\mathbf{r}_2)\rangle$. The points $\mathbf{r}_1$ and $\mathbf{r}_2$ lie in a unique plane perpendicular to the surface. Within this plane we can specify the points by coordinates (x_1,y_1), (x_2,y_2). By translational invariance parallel to the surface, G_1 depends on x_1-x_2, y_1 and y_2. Invariance under dilatation implies:

$$G_1(x_1-x_2,y_1,y_2) = (1+\epsilon)^{\Delta_\phi}(1+\epsilon)^{\Delta_\phi} G(x_1'-x_2',y_1',y_2') \ . \tag{9}$$

An infinitesimal special conformal transformation along x is

$$\begin{aligned} x' &= x + \epsilon(x^2-y^2) \\ y' &= y + 2\epsilon x y, \end{aligned} \tag{10}$$

and under such a transformation we have

$$G_1(x_1-x_2,y_1,y_2) = (1+2\epsilon x_1)^{\Delta_\phi}(1+2\epsilon x_2)^{\Delta_\phi} G(x_1'-x_2',y_1',y_2') \ . \tag{11}$$

By expanding equations (9) and (11) to the first order in ϵ, we arrive at :

$$\begin{aligned} u\frac{\partial G_1}{\partial u} + y_1\frac{\partial G_1}{\partial y_1} + y_2\frac{\partial G_1}{\partial y_2} + 2\Delta_\phi G_1 &= 0 \\ (y_1^2 - y_2^2)\frac{\partial G_1}{\partial u} + u\left(y_1\frac{\partial G_1}{\partial y_1} - y_2\frac{\partial G_1}{\partial y_2}\right) &= 0 \end{aligned} \quad (12)$$

in which $u = x_1 - x_2$. The first equation states that G_1 is a homogeneous function of dimension $2\Delta_\phi$:

$$G_1 = \frac{1}{(u)^{2\Delta_\phi}} g_1(\alpha, \beta) \quad (13)$$

where $\alpha = y_1/u$ and $\beta = y_2/u$. Substituting this in the second line of equation (12) one finds [5]

$$\left[\alpha + \frac{\alpha}{\alpha^2 - \beta^2}\right]\frac{\partial g_1}{\partial \alpha} + \left[\beta + \frac{\beta}{\beta^2 - \alpha^2}\right]\frac{\partial g_1}{\partial \beta} + 2\Delta_\phi g_1 = 0\ . \quad (14)$$

The general solution of this equation is:

$$g_1(\alpha, \beta) = \frac{1}{(\alpha\beta)^{\Delta_\phi}} h_1\left(\frac{1 + (\alpha - \beta)^2}{\alpha\beta}\right)\ . \quad (15)$$

So the two-point correlation function is:

$$\langle \Phi(\mathbf{r}_1)\Phi(\mathbf{r}_2)\rangle = \frac{1}{(y_1 y_2)^{\Delta_\phi}} h_1\left(\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{y_1 y_2}\right)\ . \quad (16)$$

We have two other two-point functions, $G_2(\mathbf{r}_1, \mathbf{r}_2) = \langle \Phi(\mathbf{r}_1)\Psi(\mathbf{r}_2)\rangle$ and $G_3(\mathbf{r}_1, \mathbf{r}_2) = \langle \Psi(\mathbf{r}_1)\Psi(\mathbf{r}_2)\rangle$. We can follow similar steps for these two and the result is

$$\begin{aligned} G_2(\mathbf{r}_1, \mathbf{r}_2) &= \frac{1}{(y_1 y_2)^{\Delta_\phi}}\left[h_2(\eta) - d\ \log y_2\ \ h_1(\eta)\right] \\ G_3(\mathbf{r}_1, \mathbf{r}_2) &= \frac{1}{(y_1 y_2)^{\Delta_\phi}}\left[h_3(\eta) - d\ \log y_1 y_2\ \ h_2(\eta) + d^2\ \log y_1 \log y_2\ \ h_1(\eta)\right] \end{aligned} \quad (17)$$

where $\eta = [(x_1 - x_2)^2 + (y_1 - y_2)^2]/y_1 y_2$.

Far from the boundary, the effect of boundary becomes negligible and we must recover the bulk two-point functions:

$$\begin{aligned} \langle \Phi(\mathbf{r}_1)\Phi(\mathbf{r}_2)\rangle &= 0 \\ \langle \Phi(\mathbf{r}_1)\Psi(\mathbf{r}_2)\rangle &= \frac{a}{r^{2\Delta_\phi}} \\ \langle \Psi(\mathbf{r}_1)\Psi(\mathbf{r}_2)\rangle &= \frac{1}{r^{2\Delta_\phi}}(b - d\, a \log r) \end{aligned} \quad (18)$$

where $r = |\mathbf{r}_1 - \mathbf{r}_2|$ and a, b are arbitrary constants. These equations were first derived by [7] in two dimensions and were generalized to d-dimensions by [3].

To go far from the boundary one must let y_1 and y_2 tend to infinity keeping $y_1 - y_2$ and $x_1 - x_2$ finite. This means letting η tend to zero . So we can find h_1 , h_2 , h_3 in the limit $\eta \to 0$:

$$\begin{aligned} h_1(\eta) &= \frac{1}{\eta^{\Delta_\phi}} \left(\frac{4\frac{a}{d}}{\log \eta} + \frac{C_1}{(\log \eta)^2} + \ldots \right) \\ h_2(\eta) &= \frac{1}{\eta^{\Delta_\phi}} \left(-a + \frac{C_2}{(\log \eta)} + \ldots \right) \\ h_3(\eta) &= \frac{1}{\eta^{\Delta_\phi}} \left(b - d\, C_2 - \frac{d^2}{4} C_1 + \ldots \right) \end{aligned} \tag{19}$$

Here C_1 and C_2 are arbitrary constants. On the other hand the behaviour of these functions when η tends to infinity, determines the surface behaviour of correlation functions (To investigate the surface behaviour one must let $x_1 - x_2$ tend to infinity while keeping y_1 and y_2 finite).

To find the surface exponents one should know the behaviour of these functions in this limit . This requires a knowledge of the differential equations governing $h(\eta)$ which requires details of the structure of conformal field theory.

3 Two Dimensions

In two dimensions, however, conformal group is an infinite dimensional group and any analytic function from the plane to itself is a conformal transformation . In LCFT's, under a conformal transformation $z \to w(z)$, $\bar{z} \to w(\bar{z})$ primary fields of a Jordanian cell, Φ_i's, transform as [4]:

$$\Phi_i(z,\bar{z}) \to \Phi_i(z,\bar{z}) + \left[\alpha'(z)\Delta_i^j + \delta_i^j \alpha(z)\frac{\partial}{\partial z} + \overline{\alpha'(z)}\,\bar{\Delta}_i^j + \delta_i^j \overline{\alpha(z)}\frac{\partial}{\partial \bar{z}} \right] \Phi_j(z,\bar{z}) \tag{20}$$

The effect of a small transformation may be expressed in terms of correlation functions of the fields with the energy-momentum tensor. In complex coordinates, there are two non-zero components, namely $T(z) = T_{zz}(z)$ and $\bar{T}(\bar{z}) = T_{\bar{z}\bar{z}}(\bar{z})$. In terms of these two we have:

$$\begin{aligned} &\frac{1}{2\pi i}\oint_c dz\, \alpha(z)\, \langle T(z)\Phi_{i_1}(z_1,\bar{z}_1)\ldots\rangle - \frac{1}{2\pi i}\oint_c d\bar{z}\, \overline{\alpha(z)}\, \langle \bar{T}(\bar{z})\Phi_{i_1}(z_1,\bar{z}_1)\ldots\rangle \\ &= \sum_k \sum_{j_k} \left[\alpha'(z_k)\Delta_{i_k}^{j_k} + \delta_{i_k}^{j_k}\alpha(z_k)\frac{\partial}{\partial z_k} + \overline{\alpha'(z_k)}\bar{\Delta}_{i_k}^{j_k} + \delta_{i_k}^{j_k}\overline{\alpha(z_k)}\frac{\partial}{\partial \bar{z}_k} \right] \langle \Phi_{j_1}(z_1,\bar{z}_1)\ldots\rangle \end{aligned} \tag{21}$$

where c is an arbitrary contour containing all the points z_k and k is summed over the fields in the correlators and j_k is summed over the Jordanian cell containing Φ_{i_k}. In the absence of a boundary, $\alpha(z)$ is an arbitrary analytic function, thus α and $\bar{\alpha}$ can be assumed independent. As a consequence the z and $\bar{z}$ dependence in equation (21) separates:

$$\frac{1}{2\pi i}\oint_c dz \alpha(z)\, \langle T(z)\Phi_{i_1}(z_1,\bar{z}_1)\ldots\rangle = \sum_k \sum_{j_k} \left[\alpha'(z_k)\Delta_{i_k}^{j_k} + \delta_{i_k}^{j_k}\alpha(z_k)\frac{\partial}{\partial z_k} \right] \langle \Phi_{j_1}(z_1,\bar{z}_1)\ldots\rangle \tag{22}$$

with a similar equation for $\langle \bar{T}\Phi \ldots \rangle$. Using Cauchy's theorem the correlation function $\langle T\Phi \ldots \rangle$ can be expressed in terms of linear differential operators on $\langle \Phi \ldots \rangle$:

$$\langle T(z)\Phi_{i_1}(z_1,\bar{z}_1)\ldots\rangle = \sum_k \sum_{j_k} \left[\frac{\Delta_{i_k}^{j_k}}{(z-z_k)^2} + \frac{\delta_{i_k}^{j_k}}{(z-z_k)} \frac{\partial}{\partial z_k} \right] \langle \Phi_{j_1}(z_1,\bar{z}_1)\ldots\rangle . \tag{23}$$

In the presence of a boundary, transformations are restricted to those which leave the boundary invariant, therefore the function α and $\bar{\alpha}$ are no longer independent. Let us take as boundary the real axis, thus the operators Φ_{i_k} are defined in the upper half-plane $Im(z) > 0$. The transformations which leave this boundary invariant, are the real analytic functions, i.e $\overline{\alpha(z)} = \alpha(\bar{z})$. As result of this the separation of z and $\bar{z}$ can not take place.

However, as Cardy has shown [5], we may extend the definition of $T(z)$ into the lower half-plane by:

$$T(z) := \bar{T}(z) \qquad Im(z) < 0 \tag{24}$$

and relable $\bar{z}_k = z'_k$. Changing variable $\bar{z} \to z$ in the second integral of equation (21) and using $\overline{\alpha(z)} = \alpha(\bar{z})$, it becomes:

$$\begin{aligned} &\frac{1}{2\pi i}\oint_c dz\, \alpha(z) \langle T(z)\Phi_{i_1}(z_1,z'_1)\ldots\rangle + \frac{1}{2\pi i}\oint_{\bar{c}} dz\, \alpha(z) \langle T(z)\Phi_{i_1}(z_1,z'_1)\ldots\rangle \\ &= \sum_k \sum_{j_k} \left[\alpha'(z_k)\Delta_{i_k}^{j_k} + \delta_{i_k}^{j_k}\alpha(z_k)\frac{\partial}{\partial z_k} + \alpha'(z'_k)\bar{\Delta}_{i_k}^{j_k} + \alpha(z'_k)\frac{\partial}{\partial z'_k} \right] \langle \Phi_{j_1}(z_1,z'_1)\ldots\rangle , \end{aligned} \tag{25}$$

where $\bar{c}$ is a contour in the lower half-plane[5].

The left-hand side of equation (25) can be written as one integral around a large contour containing all the points z_k and z'_k if we have $T = \bar{T}$ on the boundary. This condition is equivalent to the condition $T_{xy} = 0$ in Cartesian coordinates which means that there is no flux of energy across the boundary. Now that the left-hand side of equation (25) is an integral, one can use Cauchy's theorem to get:

$$\begin{aligned} &\langle T(z)\Phi_{i_1}(z_1,z'_1)\ldots\rangle = \\ &\sum_k \sum_{j_k} \left[\frac{\Delta_{j_k}^{i_k}}{(z-z_k)^2} + \frac{\delta_{j_k}^{i_k}}{(z-z_k)}\frac{\partial}{\partial z_k} + \frac{\bar{\Delta}_{i_k}^{j_k}}{(z-z'_k)^2} + \frac{\delta_{i_k}^{j_k}}{(z-z'_k)}\frac{\partial}{\partial z'_k} \right] \langle \Phi_{j_1}(z_1,z'_1)\ldots\rangle . \end{aligned} \tag{26}$$

Comparing equations (26) and (23), we observe that in presence of a boundary, the correlation function $\langle \Phi_{i_1}(z_1,\bar{z}_1)\ldots\Phi_{i_{2n}}(z_{2n},\bar{z}_{2n})\rangle$ regarded as a function of $(z_1,\ldots,z_n, \bar{z}_1,\ldots,\bar{z}_n)$ satisfies the same differential equations as that of the bulk correlation function $\langle \Phi_{i_1}(z_1,\bar{z}_1)\ldots\Phi_{i_{2n}}(z_{2n},\bar{z}_{2n})\rangle$ as a function of $z_1,\ldots,z_{2n}$. Note that the conformal dimension of fields $\Phi_{i_{n+1}}$ to $\Phi_{i_{2n}}$ is $\bar{\Delta}$.

As an example we calculate the one point function of such fields. Consider a 2×2 scalar Jordanian cell, composed of the fields Φ and Ψ (Ψ is the logarithmic partner). Conformal invariance of correlation functions means that acting with the set $L_0, L_{\pm 1}$ on the correlation functions yields zero. Solving the equations obtained, one arrives at:

$$\begin{aligned}
\langle\Phi(z,\bar{z})\rangle &= \frac{c}{(z-\bar{z})^{2\Delta}}\,, \\
\langle\Psi(z,\bar{z})\rangle &= \frac{1}{(z-\bar{z})^{2\Delta}}\left[c' - 2c\log\left(z-\bar{z}\right)\right].
\end{aligned} \tag{27}$$

which are the same as two-point functions without the boundary. Also note that these results are consistent with the results obtained for general dimensions in the previous section (equation (8)).

Now consider the two-point correlation functions near the boundary. Again the differential equations which are satisfied by these two-point correlation functions are the same as the equations satisfied by four-point correlation function in the bulk and so the result is:

$$\langle\Phi(z_1)\Phi(z_2)\rangle = u^{2\Delta} f_1(v)$$

$$\langle\Phi(z_1)\Psi(z_2)\rangle = u^{2\Delta}\left(f_2(v) - 2\log(z_2-\bar{z}_2) f_1(v)\right)$$

$$\begin{aligned}
\langle\Psi(z_1)\Psi(z_2)\rangle &= u^{2\Delta}(f_3(v) - 2\log\left[(z_1-\bar{z}_1)(z_2-\bar{z}_2)\right] f_2(v) \\
&\quad + 4\log(z_1-\bar{z}_1)\log(z_2-\bar{z}_2) f_1(v))
\end{aligned} \tag{28}$$

where $u = (z_1-\bar{z}_1)(z_2-\bar{z}_2)/(z_1-z_2)(\bar{z}_1-\bar{z}_2)(z_1-\bar{z}_2)(\bar{z}_1-z_2)$ and $v = (z_1-z_2)(\bar{z}_1-\bar{z}_2)/(z_1-\bar{z}_1)(z_2-\bar{z}_2)$ and f_1, f_2, f_3 are arbitrary functions (Again compare these results with the equations (16) and (17) derived for general dimensions).

If a singular vector is found in such a theory, one can find some differential equations which are satisfied by f_1, f_2, f_3 (usually they are hypergeometric ones) and hence the correlation functions will be determined completely. Finding the correlation functions one can investigate the surface behaviour of the theory. In a successing paper we will consider such a theory.

4 Application to MHD

The incompressible two-dimensional magnetohydrodynamic system has two independent dynamical variables, the velocity stream function ϕ and the magnetic-flux function ψ. They obey the equations

$$\begin{aligned}
\frac{\partial w}{\partial t} &= -\epsilon_{\alpha\beta}\partial_\alpha\phi\partial_\beta w + \epsilon_{\alpha\beta}\partial_\alpha\psi\partial_\beta J + \mu\nabla^2 w \\
\frac{\partial\psi}{\partial t} &= -\epsilon_{\alpha\beta}\partial_\alpha\phi\partial_\beta\psi + \eta J\ ,
\end{aligned} \tag{29}$$

where the vorticity $w = \nabla^2\phi$ and the current $J = \nabla^2\psi$ and μ and η are viscosity and molecular resistivity. The velocity and magnetic field are given in terms of ϕ and ψ:

$$\begin{aligned}
V_\alpha &= \epsilon_{\alpha\beta}\partial_\beta\phi \\
B_\alpha &= \epsilon_{\alpha\beta}\partial_\beta\psi\ ,
\end{aligned} \tag{30}$$

where $\epsilon_{\alpha\beta}$ is the totally antisymmetric tensor with $\epsilon_{12}=1$. It has been argued that the Alf'ven effect implies that ϕ and ψ should have equal scaling dimension which naturally leads to LCFT's [8, 9].

We consider this system near a boundary and calculate the mean values of velocity and magnetic field. As we have derived in the last section the one-point function of the fields ϕ and ψ are given by equation (27), so for velocity we have:

$$\begin{aligned}\langle V_x(x,y)\rangle &= \partial_y\langle\phi(x,y)\rangle = -\frac{2\Delta C}{y^{2\Delta+1}}\\ \langle V_y(x,y)\rangle &= -\partial_x\langle\phi(x,y)\rangle = 0\ ,\end{aligned}\tag{31}$$

and for magnetic field:

$$\begin{aligned}\langle B_x(x,y)\rangle &= \partial_y\langle\psi(x,y)\rangle = -\frac{2\Delta}{y^{2\Delta}}\left[(C'+2C)-2C\log y\right]\\ \langle B_y(x,y)\rangle &= -\partial_x\langle\psi(x,y)\rangle = 0\ .\end{aligned}\tag{32}$$

A specific model is proposed by Rahimi-Tabar and Rouhani [8] with $\Delta=\frac{-5}{7}$. This theory seems to be unphysical at first sight, because V and B grow large far from boundary. However, they acquire physical meaning when this model is regularized, for example by attaching a value to the velocity field at the boundary.

Other boundaries such as strip and circle can be readily investigated by proper transformations. For example for the strip geometry with size L one obtains:

$$\begin{aligned}\langle\phi(x,y)\rangle &= \left(\frac{\pi}{L}\right)^{2\Delta}\frac{C}{(\sin\frac{\pi}{L}y)^{2\Delta}}\\ \langle\psi(x,y)\rangle &= \left(\frac{\pi}{L}\right)^{2\Delta}\frac{1}{(\sin\frac{\pi}{L}y)^{2\Delta}}\left(C'+2C\log\frac{\pi}{L}-2C\log\sin(\frac{\pi}{L}y)\right).\end{aligned}\tag{33}$$

Further development of LCFT, such as complete calculation of the four point functions, is necessary before some interesting questions such as the possible set of surface critical indices can be determined. Work in this direction is in progress.

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