Fractional spin through quantum affine algebras

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Abstract

In this paper, we study the fractional decomposition of the quantum enveloping affine algebras $U_Q(\hat{A}(n))$ and $U_Q(\widehat{C}(n))$ in the $Q \to q = e^{\frac{2i\pi}{k}}$ limit. This decomposition is based on the bosonic representation and can be related to the fractional supersymmetry and k-fermionic spin. The equivalence between the quantum affine algebras and the classical ones in the fermionic realization is also established.

1 Introduction

The concept of quantum group and algebras [1, 2], have enriched the arena of mathematics and theoretical physics. Quantum groups were appeared in studying Yang-Baxter equations [3] as well as scattering method [4]. In [5, 6] the quantum analogous of Lie superalgebras was constructed. The quantized enveloping algebras associated to affine algebras and superalgebras are given in [1, 7]. It is well known that the boson realization is a very powerful and elegant method for studying quantum algebras representations. Based on this method, the representation theory of quantum affine algebras has been an object of intensive studies, namely, the results for the oscillator representations of affine algebras. There are obtained [8 − 10] through consistent realizations involving deformed Bose and Fermi operators [11, 12].

To make a connection with the quantum group theory, a new geometric interpretation of fractional supersymmetry has been introduced in [13 − 17]. In these latters, the authors show that the one-dimensional superspace is isomorphic to the braided line when the deformation parameter goes to a root of unity. The similar technics are used, in [18], to show how internal spin arises naturally in certain limit of the Q-deformed momentum algebras $U_Q(sl(2))$.

Indeed, using Q-Schwinger realization, it is proved that the decomposition of the $U_Q(sl(2))$ into a direct product $U(sl(2))$ and the deformed $U_q(sl(2))$ (note that $U_Q(sl(2)) = U_q(sl(2)$ at $Q = q)$. The property of splitting quantum algebras A_n, B_n, C_n and D_n and quantum superalgebras $C(n)$, $B(n,m)$, $C(n+1)$ and $D(n,m)$ in the limit $Q \to q$ limit is investigated in [19].

We also notice that the case of deformed Virasoro algebras and some other particular quantum (Super) algebras is given in [20].

The aim of this paper is to investigate the decomposition property of the quantum affine algebras $U_Q(\hat{A}(n))$ and $U_Q(\hat{C}(n))$ in the $Q \to q$ limit. We start in section 2 by defining k-fermionic algebra. In section 3, we discuss the decomposition property of Q-boson oscillator in the $Q \to q$ limit. We introduce the way in which one obtains two independent objects, an ordinary boson and a k-fermion, from one $Q-$deformed boson when $Q \to q$. We establish also the equivalence between a Q-deformed fermion and conventional (ordinary) one. Using these results, we analyze the $Q \to q$ limit of the quantum affine algebra $U_Q(\hat{A}(n))$ (section 4) and the quantum affine algebra $U_Q(\widehat{C}(n))$ (section 5). Some concluding remarks are given in section 6.

2 Preliminaries about k-fermionic algebra.

The q-deformed bosonic algebra Σ_q generated by A^+, A^- and number operator N is given by:

$$A^-A^+ - qA^+A^- = q^{-N} \tag{1}$$

$$A^-A^+ - q^{-1}A^+A^- = q^N \tag{2}$$

$$q^N A^{\pm} q^{-N} = q^{\pm 1} A^{\pm} \tag{3}$$

$$q^N q^{-N} = q^{-N} q^N = 1, \tag{4}$$

where the deformation parameter:

$$q = e^{\frac{2i\pi}{l}},\ l \in N - \{0,1\}, \tag{5}$$

is a root of unity.

The annihilation operator A^- is hermetic conjugated to creation operator A^+ and N is hermetic also. From $[1-4]$, it is easy to have the following relations:

$$A^-(A^+)^n = [[n]]q^{-N}(A^+)^{n-1} + q^n(A^+)^n A^- \tag{6}$$

$$(A^-)^n A^+ = [[n]](A^-)^{n-1}q^{-N} + q^n A^+(A^-)^n, \tag{7}$$

where the notation $[[\ ]]$ is defined by:

$$[[n]] = \frac{1-q^{2n}}{1-q^2} \tag{8}$$

We introduce a new variable k defined by:

$$k = l \ \text{ for odd values of } \ l, \tag{9}$$

$$k\ =\ \frac{l}{2} \ \text{ for even values of } \ l, \tag{10}$$

such that for odd l (resp. even l), we have $q^k = 1$ (resp. $q^k = -1$). In the particular case $n = k$, $eqs(6,\ 7)$ permit us to have:

$$A^-(A^+)^k = \pm(A^+)^k A^- \tag{11}$$

$$(A^-)^k A^+ = \pm A^+ (A^-)^k, \tag{12}$$

and the eqs $(1-5)$ yield to:

$$q^N (A^+)^k = (A^+)^k q^N \tag{13}$$

$$q^N (A^-)^k = (A^-)^k q^N \tag{14}$$

One can show that the elements $(A^-)^k$ and $(A^+)^k$ are the elements of the centre of $\sum_q$ algebra (odd values for l); and the irreducible representations are k-dimensional. These two properties allows to:

$$(A^+)^k = \alpha I \tag{15}$$

$$(A^-)^k = \beta I \tag{16}$$

The extra possibilities parameterized by:

$$(1) \;\; \alpha = 0, \;\; \beta \neq 0$$

$$(2) \;\; \alpha \neq 0, \;\; \beta = 0$$

$$(3) \;\; \alpha \neq 0, \;\; \beta \neq 0,$$

are not relevant for the considerations of this paper. In the two cases (1) and (2) we have the so-called semi-periodic (semi-cyclic) representation and the case (3) correspond to the periodic one. In what follows, we are interested to a representation of the algebra $\sum_q$ such that the following:

$$(A^{\mp})^k = 0,$$

is satisfied. We note that the algebra $\sum_{-1}$ obtained for $k=2$, correspond to ordinary fermion operators with $(A^+)^2 = 0$ and $(A^-)^2 = 0$ which reflects the exclusion's Pauli principle. In the limit case where $k \to \infty$, the algebra $\sum_1$ correspond to the ordinary bosons. For other values of k, the k-fermions operators interpolate between fermions and bosons, these are also called anyons with fractional spin in the sense of Majid [21, 22].

3 Fractional spin through Q-boson.

In the previous section, we have worked with q at root of unity. In this case, quantum oscillator $(k$-fermionic$)$ algebra exhibit a rich representation with very special properties different from the case where q is generic. So, in the first case the Hilbert space is finite dimensional. In contrast, where q is generic, the Fock space is infinite dimensional. In order to investigate the decomposition of Q-deformed boson in the limit $Q \to e^{\frac{2i\pi}{k}}$ we start by recalling the Q-deformed algebra Δ_Q.

The algebra Δ_Q generated by an annihilation operator B^-, a creation operator B^+ and a number operator N_B:

$$B^- B^+ - Q B^+ B^- = Q^{-N_B} \tag{17}$$

$$B^- B^+ - Q^{-1} B^+ B^- = Q^{N_B} \tag{18}$$

$$Q^{N_B} B^+ Q^{-N_B} = Q B^+ \tag{19}$$

$$Q^{N_B} B^- Q^{-N_B} = Q^{-1} B^- \tag{20}$$

$$Q^{N_B} Q^{-N_B} = Q^{-N_B} Q^{+N_B} = 1. \tag{21}$$

From the above equations, we obtain:

$$[Q^{-N_B} B^-, [Q^{-N_B} B^-, [....[Q^{-N_B} B^-, (B^+)^k]_{Q^{2k}}...]_{Q^4}]_{Q^2}] = Q^{\frac{k(k-1)}{2}} [k]! \tag{22}$$

where the Q-deformed factorial is given by:

$$[k]! = [k][k-1][k-2]..............[1], \tag{23}$$

and:

$$[0]! = 1$$

$$[k] = \frac{Q^k - Q^{-k}}{Q - Q^{-1}}$$

The Q-commutator, in $eq(22)$, of two operators A and B is defined by:

$$[A, B]_Q = AB - QBA$$

The aim of this section is to determine the limit of Δ_Q algebra when Q goes to the root of unity q. The starting point is the limit $Q \to q$ of the $eq(22)$,

$$lim_{\,Q\to q}\frac{1}{k}Q^{-N_B}[Q^{-N_B}B^-,[Q^{-N_B}B^-,[....[Q^{-N_B}B^-,(B^+)^k]_{Q^{2k}}...]_{Q^4}]_{Q^2}]$$

$$= lim_{\,Q\to q}\frac{Q^{\frac{k(k-1)}{2}}}{[k]!}[Q^{-N_B}(B^-)^k,(B^+)^k] = q^{\frac{k(k-1)}{2}} \qquad (24)$$

This equation can be reduced to:

$$lim_{\,Q\to q}[\frac{Q^{\frac{kN_B}{2}}(B^-)^k}{([k]!)^{\frac{1}{2}}},\frac{(B^+)^kQ^{\frac{kN_B}{2}}}{([k]!)^{\frac{1}{2}}}] = 1. \qquad (25)$$

Since q is a root of unity, it is possible to change the sign on the exponent of $q^{\frac{kN_B}{2}}$ terms in the above equation.

We define the operators as in [18]:

$$b^- = lim_{\,Q\to q}\frac{Q^{\pm\frac{kN_B}{2}}}{([k]!)^{\frac{1}{2}}}(B^-)^k,\; b^+ = lim_{Q\to q}\frac{(B^+)^kQ^{\pm\frac{kN_B}{2}}}{([k]!)^{\frac{1}{2}}}, \qquad (26)$$

which lead to an ordinary boson algebra noted Δ_0, generated by:

$$[b^-,b^+] = 1. \qquad (27)$$

The number operator of this new bosonic algebra defined as the usual case, $N_b = b^+b^-$. At this stage we are in position to discuss the splitting of Q-deformed boson in the $Q \to q$ limit. Let us introduce the new set of generators given by:

$$A^- = B^-q^{-\frac{kN_b}{2}} \qquad (28)$$

$$A^+ = B^+q^{-\frac{kN_b}{2}} \qquad (29)$$

$$N_A = N_B - kN_b, \qquad (30)$$

which define a k-fermionic algebra:

$$[A^+,A^-]_{q^{-1}} = q^{N_A} \qquad (31)$$

$$[A^-,A^+]_q = q^{-N_A} \qquad (32)$$

$$[N_A, A^{\pm}] = \pm A^{\pm}. \tag{33}$$

It is easy to verify that the two algebras generated by the set of operators $\{b^+, b^-, N_b\}$ and $\{A^+, A^-, N_A\}$ are mutually commutative. We conclude that in the $Q \to q$ limit, the Q-deformed bosonic algebra oscillator decomposes into two independent oscillators, an ordinary boson and k-fermion; formally one can write:

$$\lim_{Q \to q} \Delta_Q \equiv \Delta_0 \otimes \Sigma_q,$$

where Δ_0 is the classical bosonic algebra generated by the operators $\{b^+, b^-, N_b\}$.

Similarly, we want to study the Q-fermion algebra at root of unity. To do this, we start by considering the Q- deformed fermionic algebra, noted Ξ_Q:

$$F^- F^+ + Q F^+ F^- = Q^{N_F} \tag{34}$$

$$F^- F^+ + Q^{-1} F^+ F^- = Q^{-N_F} \tag{35}$$

$$Q^{N_F} F^+ Q^{-N_F} = Q F^+ \tag{36}$$

$$Q^{N_F} F^- Q^{-N_F} = Q^{-1} F^- \tag{37}$$

$$Q^{N_F} Q^{-N_F} = Q^{-N_F} Q^{N_F} = 1 \tag{38}$$

$$(F^+)^2 = 0, \quad (F^-)^2 = 0 \tag{39}$$

In the case $n = 2$ $(q = -1)$, we define the new fermionic operators as follow:

$$f^+ = \lim\ _{Q \to 1} F^+ Q^{-\frac{N_F}{2}} \tag{40}$$

$$f^- = \lim\ _{Q \to 1} Q^{-\frac{N_F}{2}} F^-. \tag{41}$$

By a direct calculus, we obtain the following anti-commutation relation:

$$\{f^-, f^+\} = 1. \tag{42}$$

Moreover, we have the nilpotency condition:

$$(f^-)^2 = 0, \quad (f^+)^2 = 0. \tag{43}$$

Thus, we see that the Q-deformed fermion reproduce the conventional (ordinary) fermion. The same convention notation permits us to write:

$$lim_{Q\to q} \Xi_Q \equiv \Sigma_{-1}$$

4 Quantum affine algebra $U_Q(\hat{A}(n))$ at Q a root of unity

We apply the above results to derive the property of decomposition of quantum affine algebra $U_Q(\hat{A}(n))$ in the $Q \to q$ limit. Recalling that the $U_Q(\hat{A}(n))$ algebra is generated by the set of generators $\{e_i$, f_i, $k_i^{\pm} = Q^{\pm d_i h_i}$, $0 \leq i$ $\langle$ $n\}$ satisfying the following relations:

$$[e_i, f_j] = \delta_{ij} \frac{k_i - k_i^{-1}}{Q_i - Q_i^{-1}} \tag{44}$$

$$k_i e_j k_i^{-1} = Q_i^{a_{ij}} e_j, \; k_i f_j k_i^{-1} = Q_i^{a_{ij}} f_j \tag{45}$$

$$k_i k_i^{-1} = k_i^{-1} k_i = 1, \; k_i k_j = k_j k_i. \tag{46}$$

The quantum affine algebra $U_Q(\hat{A}_n)$ admits two Q-oscillators representations: bosonic and fermionic ones; in the bosonic realization, the generators of $U_Q(\hat{A}_n)$ can be constructed by introducing $(n+1)$ Q-deformed bosons as follows:

$$e_i = B_i^- B_{i+1}^+, 1 \leq i \leq n$$

$$f_i = B_i^+ B_{i+1}^-, 1 \leq i \leq n$$

$$k_i = Q^{-N_i + N_{i+1}}, 1 \leq i \leq n$$

$$e_0 = B_{n+1}^- B_1^+$$

$$f_0 = B_1^- B_{n+1}^+$$

$$k_0 = Q^{N_1 - N_{n+1}}.$$

The fermionic realization of $U_Q(\hat{A}(n))$ is given by:

$$e_i = F_i^+ F_{i+1}^-, 1 \leq i \leq n$$

$$f_i = F_i^- F_{i+1}^+, 1 \leq i \leq n$$

$$k_i = Q^{N_i - N_{i+1}}, 1 \leq i \leq n$$

$$e_0 = F_{n+1}^+ F_1^-$$

$$f_0 = F_1^+ F_{n+1}^-$$

$$k_0 = Q^{-N_1 + N_{n+1}}.$$

At this stage, our aim is to investigate the limit $Q \rightarrow q$ of the affine algebra $U_Q(\hat{A}_n)$. As it is already mentioned in the introduction, our analysis is based on the Q-oscillator representation based on Q-Schwinger realization. In the $Q \rightarrow q$, the splitting of Q-deformed bosons leads to classical bosons $\{b_i^+, b_i^-, N_{b_i}, 1 \leq i \leq n\}$ given by the $eqs(26, 27)$ and k-fermionic algebra $\{A_i^+, A_i^-, N_{A_i}, 1 \leq i \leq n\}$given by $eqs(31-33)$. From the classical bosons, we define for $i = 1, ..., n$ the operators:

$$e_i = b_i^- b_{i+1}^+ \tag{47}$$

$$f_i = b_i^+ b_{i+1}^- \tag{48}$$

$$h_i = -N_{b_i} + N_{b_{i+1}} \tag{49}$$

$$e_0 = b_1^- b_{n+1}^+ \tag{50}$$

$$f_0 = b_1^+ b_{n+1}^- \tag{51}$$

$$h_0 = -N_{b_1} + N_{b_{n+1}}, \tag{52}$$

the set $\{e_i, f_i, h_i, 0 \leq i \leq n\}$ generate the classical algebra $U(\hat{A}(n))$. From the remaining generators $\{A_i^+, A_i^-, N_{A_i}, 1 \leq i \leq n+1\}$, we can realize $U_q(\hat{A}(n))$, generated by E_i, F_i, K_i, E_0, F_0 and K_0 where:

$$E_i = A_i^- A_{i+1}^+, 1 \leq i \leq n \tag{53}$$

$$F_i = A_i^+ A_{i+1}^-, 1 \leq i \leq n \tag{54}$$

$$K_i = q^{-N_{A_i}+N_{A_{i+1}}}, 1 \leq i \leq n \tag{55}$$

$$E_0 = A_1^+ A_{n+1}^- \tag{56}$$

$$F_0 = A_1^- A_{n+1}^+ \tag{57}$$

$$K_0 = q^{N_{A_1}-N_{A_{n+1}}}. \tag{58}$$

The algebra $U_q(\hat{A}(n))$ is the same version of $U_Q(\hat{A}_n)$ obtained by simply taking $Q = q$ and $B_i \sim A_i$. Due to the commutativity of elements of $U_q(\hat{A}(n))$ and $U(\hat{A}_n)$, we obtain the following decomposition of the quantum affine algebra $U_Q(\hat{A}_n)$ in the bosonic realization

$$lim_{\,Q\to q} U_Q(\hat{A}_n) \equiv U_q(\hat{A}(n)) \otimes U(\hat{A}(n)).$$

We discuss now the equivalence between $U_Q(\hat{A}_n)$ and $U(\hat{A}(n))$ algebras in the fermionic realization. Indeed, we have discussed in section 2, how one can identify the conventional fermions with Q-deformed fermions. Consequently, due to this equivalence, it is possible to construct Q-deformed affine algebras $U_Q(\hat{A}_n)$ using ordinary fermions. It is also possible to construct the affine algebra $U(\hat{A}_n)$ by considering Q-deformed fermions. So, in the fermionic realization we have equivalence between $U(\hat{A}_n)$ and $U_Q(\hat{A}_n)$. To be more clear, we consider the $U_Q(\hat{A}_n)$ in the Q-fermionic representation. Where the generators are given by:

$$e_i = F_i^- F_{i+1}^+, 1 \leq i \leq n \tag{59}$$

$$f_i = F_i^+ F_{i+1}^-, 1 \leq i \leq n \tag{60}$$

$$k_i = Q^{N_{F_i} - N_{F_{i+1}}}, 1 \leq i \leq n \tag{61}$$

$$e_0 = F^+_{n+1} F^-_1 \tag{62}$$

$$f_0 = F^+_1 F^-_{n+1} \tag{63}$$

$$k_0 = Q^{-N_{F_1} + N_{F_{n+1}}}. \tag{64}$$

Due to the equivalence fermion Q-fermion, the operators f^-_i, f^+_i are defined as a constant multiple of conventional fermion operators:

$$f^+_i = F^+_i Q^{\frac{-N_{F_i}}{2}} \tag{65}$$

$$f^-_i = Q^{\frac{-N_{F_i}}{2}} F^-_i, \tag{66}$$

from which we can realize the generators:

$$E_i = f^-_i f^+_{i+1},\ 1 \leq i \leq n \tag{67}$$

$$F_i = f^+_i f^-_{i+1},\ 1 \leq i \leq n \tag{68}$$

$$H_i = N_{f_i} - N_{f_{i+1}},\ 1 \leq i \leq n \tag{69}$$

$$E_0 = f^+_{n+1} f^-_1 \tag{70}$$

$$F_0 = f^+_1 f^-_{n+1} \tag{71}$$

$$H_0 = -N_{f_1} + N_{f_{n+1}}. \tag{72}$$

The set $\{E_i, F_i, H_i\ ;\ 0 \leq i \leq n\}$ generate the classical affine algebra $U(\hat{A}_n)$ in the fermionic representation and we have

$$U_q(\hat{A}(n)) \equiv U(\hat{A}(n)).$$

5 Quantum affine algebra $U_Q(\widehat{C}(n))$ at a root of unity.

Let $Q \in C - \{0\}$ be the deformation parameter. We shall use also $Q_i = Q^{d_i}$ with d_i are numbers that symmetries the Cartan matrix (a_{ij}). The quantum affine algebra $U_Q(\widehat{C}(n))$ is described in the Serre-Chevalley basis in terms of the simple root e_i, f_i and Cartan generators h_i, where $i = 0, ...n$, satisfy the following commutation relations:

$$[e_i, f_j] = \delta_{ij} \frac{Q^{d_i h_i} - Q^{-d_i h_i}}{Q_i - Q_i^{-1}} \tag{73}$$

$$[h_i, h_j] = 0 \tag{74}$$

$$[h_i, e_j] = a_{ij} e_j, \; [h_i, f_j] = -a_{ij} f_j. \tag{75}$$

Introducing the quantities $k_i = Q^{d_i h_i}$ which permit to rewrite the $eqs(73-75)$ as follows:

$$[e_i, f_j] = \delta_{ij} \frac{k_i - k_i^{-1}}{Q_i - Q_i^{-1}} \tag{76}$$

$$\begin{aligned} k_i e_j k_i^{-1} &= Q_i^{a_{ij}} e_j && (77) \\ k_i f_j k_i^{-1} &= Q_i^{a_{ij}} f_j && (78) \\ k_i k_i^{-1} &= k_i^{-1} k_i = 1 && (79) \\ k_i k_j &= k_j k_i. && (80) \end{aligned}$$

Explicitly the generators of the quantum algebra $U_Q(\widehat{C}(n))$ are given in the bosonic case by:

$$e_i = B_i^+ B_{i+1}^- + B_{2n-i}^+ B_{2n-i+1}^-, 1 \leq i \leq n-1 \tag{81}$$

$$f_i = B_i^- B_{i+1}^+ + B_{2n-i}^- B_{2n-i+1}^+, 1 \leq i \leq n-1 \tag{82}$$

$$h_i = N_{B_i} - N_{B_{i+1}} + N_{B_{2n-i}} - N_{B_{2n-i+1}}, 1 \leq i \leq n-1 \tag{83}$$

$$e_n = B_{n+1}^- B_n^+ \tag{84}$$

$$f_n = B_{n+1}^+ B_n^- \tag{85}$$

$$h_n = N_{B_n} - N_{B_{n+1}} \tag{86}$$

$$e_0 = B_{2n}^+ B_1^- \tag{87}$$

$$f_0 = B_1^+ B_{2n}^- \tag{88}$$

$$h_0 = N_{B_{2n}} - N_{B_1}. \tag{89}$$

Due to the property of Q-boson decomposition in the $Q \to q$ limit, each Q-boson $\{B_i^-, B_i^+, N_{B_i}\}$ reproduce an ordinary bosonic algebra $\{b_i^-, b_i^+, N_{b_i}\}$ and k-fermion operators $\left\{A_i^-, A_i^+, N_{A_i}\right\}$.

From the set $\left\{b_i^+, b_i^-, N_{b_i}, i = 0....n\right\}$ we can construct the classical affine algebra $U(\hat{C}(n))$ as follow:

$$E_i = b_i^+ b_{i+1}^- + b_{2n-i}^+ b_{2n-i+1}^-, \; 1 \le i \le n-1 \tag{90}$$

$$F_i = b_i^- b_{i+1}^+ + b_{2n-i}^- b_{2n-i+1}^+, \; 1 \le i \le n-1 \tag{91}$$

$$H_i = N_{b_i} - N_{b_{i+1}} + N_{b_{2n-i}} - N_{b_{2n-i+1}}, \; 1 \le i \le n-1 \tag{92}$$

$$E_n = b_{n+1}^- b_n^+ \tag{93}$$

$$F_n = b_{n+1}^+ b_n^- \tag{94}$$

$$H_n = N_{b_n} - N_{b_{n+1}} \tag{95}$$

$$E_0 = b_{2n}^+ b_1^- \tag{96}$$

$$F_0 = b_1^+ b_{2n}^- \tag{97}$$

$$H_0 = N_{b_{2n}} - N_{b_1}. \tag{98}$$

From the k-fermionic operators $\left\{A_i^-, A_i^+, N_{A_i}, 1 \le i \le n+1\right\}$, one can construct as in $eqs(81-89)$ the q-deformed affine algebra $U_q(\hat{C}(n))$. It is easy to verify that $U_q(\widehat{C}(n))$ and $U(\widehat{C}(n))$ are mutually commutative. As a result, we have the following decomposition of quantum algebra $U_Q(\widehat{C}(n))$ in the $Q \to q$ limit:

$$lim_{\,Q\to q} U_Q(\widehat{C}(n)) \equiv U(\widehat{C}(n)) \otimes U_q(\widehat{C}(n)).$$

The equivalence between $U_Q(\widehat{C}(n))$ and $U(\widehat{C}(n))$ algebras in the fermionic representation can be easily deduced; in fact we can construct the affine deformed algebra $U_Q(\widehat{C}(n))$ using the ordinary fermions and conversely, the classical affine algebra $U(\widehat{C}(n))$ can be realized in terms of deformed fermions. Indeed, we consider the $U_Q(\widehat{C}(n))$ in the Q-fermionic representation, where the generators are given by:

$$E_i = F_i^+ F_{i+1}^- + F_{2n-i}^+ F_{2n-i+1}^-,\ 1 \leq i \leq n-1 \tag{99}$$

$$F_i = F_i^- F_{i+1}^+ + F_{2n-i}^- F_{2n-i+1}^+,\ 1 \leq i \leq n-1 \tag{100}$$

$$K_i = Q_i^{d_i(N_{B_{i+1}} - N_{B_i} + N_{B_{2n-i+1}} - N_{B_{2n-i}})},\ 1 \leq i \leq n-1 \tag{101}$$

$$E_n = F_{n+1}^- F_n^+ \tag{102}$$

$$F_n = F_{n+1}^+ F_n^- \tag{103}$$

$$K_n = Q_n^{d_n(N_{B_{n+1}} - N_{B_n})} \tag{104}$$

$$E_0 = F_{2n}^+ F_1^- \tag{105}$$

$$F_0 = F_1^+ F_{2n}^- \tag{106}$$

$$K_0 = Q^{N_1 - N_{B_{2n}}}. \tag{107}$$

The elements d_i are the non zero integers such that $d_i a_{ij} = a_{ij} d_i$ and a_{ij} is the ij-elements of $n \times n$ generalized Cartan matrix.

As in the case of $U_Q(\hat{A}_n)$, the $Q-$deformed fermions can be identified to classical ones.

So, we can deduced that in the fermionic representation the $Q-$deformed algebra $U_Q(\widehat{C}(n))$ is equivalent to the classical affine algebra $U(\widehat{C}(n))$, one can write:

$$lim_{Q\to q} U_Q(\widehat{C}(n)) \equiv U(\widehat{C}(n)).$$

6 Conclusion

We have presented the general method leading to the investigation of the $Q \to q = e^{\frac{2i\pi}{k}}$ limit of the quantum affine algebras $U_Q(\hat{A}_n)$ and $U_Q(\widehat{C}(n))$. We note that the Q-oscillator representation is crucial in this manner of splitting in this paper. The technics and formulae used in this paper, will be useful to extend this study to the infinite deformed algebras [26], and quantum affine superalgebras [27].

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