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Non-semisimple gaugings of $D = 5 \ \mathcal{N} = 8$ Supergravity and FDA.s[†]

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Abstract

We reformulate maximal $D = 5$ supergravity in the consistent approach uniquely based on Free Differential Algebras and the solution of their Bianchi identities (= rheonomic method). In this approach the lagrangian is unnecessary since the field equations follow from closure of the supersymmetry algebra. This enables us to explicitly construct the non–compact gaugings corresponding to the non–semisimple algebras $\mathrm{CSO(p, q, r)}$, irrespectively from the existence of a lagrangian. The use of Free Differential Algebras is essential to clarify, within a cohomological set up, the dualization mechanism between one–forms and two–forms. Our theories contain $12 - r$ self–dual two–forms and $15 + r$ gauge vectors, r of which are abelian and neutral. These theories, whose existence is proved and their supersymmetry algebra constructed hereby, have potentially interesting properties in relation with domain wall solutions and the trapping of gravity.

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1 Introduction

Gauged supergravity with a maximal compact group, $\mathcal{G} = \mathrm{SO}(6)$ in $D = 5$ [1, 2], $\mathcal{G} = \mathrm{SO}(8)$ in $D = 4$ [3] and $\mathcal{G} = \mathrm{USp}(4)$ in $D = 7$ [4] has attracted much renewed attention in the last two years because of the $\mathrm{AdS_{p+2}/CFT_{p+1}}$ correspondence (for a general review see [5] and references therein; for the case $D = p + 2 = 4$ see also [6] and references therein). Indeed the maximally supersymmetric vacuum of these gauged supergravities is the AdS_D space and the compact gauge group $\mathcal{G}_{gauge}$ is the $\mathcal{R}$-symmetry of the corresponding maximally extended supersymmetry algebra.

However the compact gaugings are not the only ones for extended supergravities. There exist also versions of these theories where the gauge group $\mathcal{G}_{gauge}$ is non–compact. Unitarity is preserved because in all possible extrema of the corresponding scalar potential the non–compact gauge symmetry is broken to some residual compact subgroup. Furthermore, there are models in which the gauge group is non–semisimple. For $\mathcal{N} = 8$ in $D = 4$, they were particularly studied by Hull [7, 8] and an exhaustive classification of these gaugings was more recently obtained by some of us [9].

The non–semisimple gauged supergravities are relevant for a close relative of the $\mathrm{AdS/CFT}$ correspondence namely the

$$\text{Domain Wall}/QFT \;\; \text{correspondence} \tag{1.1}$$

between gauged supergravities and quantum field theories realized on domain wall solutions of either string theory or M–theory. This generalization of the Maldacena conjecture was introduced by Boonstra, Skenderis and Townsend [10] and has been further developed in recent times [11, 12]. Indeed after the challenging proposal by Randall and Sundrum [13] that *compactification of extra dimensions* can be traded for the *trapping of gravity* on 4–dimensional branes, much interest has gone into finding supergravity theories that can accomodate the Randall Sundrum scenario [14], [15] . These have been related to domain–walls in [11], and hence to non–semisimple gauged supergravities [10].

For all these reasons it is interesting to study the non–semisimple gaugings of $D = 5$ supergravity, both in the case of lower and maximal supersymmetry. For maximal $\mathcal{N} = 8$ supergravity in five–dimensions the analogue of the $D = 4$ exhaustive classification derived in [9] has not been obtained so far. Günaydin, Romans and Warner have constructed the $\mathrm{SO(6-q,q)}$ gaugings [1] that are the analogues of the $\mathrm{SO(8-q,q)}$ gaugings in four–dimensions but so far no gauging based on the so called $\mathrm{CSO(p,q,r)}$ contracted algebras (with $p + q + r = 6$) has been produced. These gaugings exist in 4–dimensions (with $p + q + r = 8$) and it would be natural to assume that they also exist in 5–dimensions. The difficulty met by the authors who have so far investigated this problem resides in the novel five–dimensional feature of one–form/two–form duality. As long as all vector fields are abelian we can consider them as one–form or two–form gauge potentials at our own will. Yet when we introduce a certain degree of non–abelian gauge symmetry matters become more complicated, since only 1–forms can gauge non–abelian groups while 2–forms cannot. On the other hand 1–forms that transform in a non–trivial representation of a non abelian gauge group which is not the adjoint representation are equally inconsistent. They have to be replaced by 2–forms and some other mechanism, different from gauge symmetry has to be found to half their degrees of freedom. This is self–duality between the 2–form and its field strength. Hence gauged supergravity can only exist with an appropriate mixture of 1–forms and self–dual 2–forms. While this mixture was mastered in the case of compact and non–compact but semisimple gaugings, the case of $\mathrm{CSO(p,q,r)}$ algebras that are not semisimple seemed to be unreachable in the existing literature.

In the present paper we show that the $\mathrm{CSO(p,q,r)}$ gaugings do exist and are fairly simple. The catch is the use of the geometric approach (based on Free Differential Algebras [1]) where the mechanism of one–form/two–form dualization receives a natural algebraic formulation and explanation.

The final result is that in the case of the $\mathrm{CSO(p,q,r)}$ gaugings there are $15+r$ gauge vectors and $12-r$ self–dual two–forms. $\mathbf{15}$ of the vectors gauge the contracted algebra while $\boldsymbol{r}$ of them have an abelian gauge symmetry with respect to which no field in the theory is charged. At the same time these vectors are neutral with respect to the transformations of the gauge algebra. Furthermore how many fields are true vectors and how many are replaced by self–dual two–forms is decided by a cohomological argument clearly formulated in the Free Differential Algebra set up.

The price one might be forced to pay in the case of $r>0$ extra neutral vector fields is that, although field equations can be normally derived from closure of the supersymmetry algebra, yet a lagrangian of conventional type might not exist, just as it happens for type IIB supergravity in $D=10$ (after all, this is not terribly surprising since $\mathcal{N}=8$ supergravities in five dimensions should eventually be interpreted in terms of brane mechanisms and compactifications from type IIB superstring). This whould make impossible the construction of the theory by means of lagrangian–based techniques. However in our construction, based on the closure of the supersymmetry algebra, the existence of a Lagrangian is not fundamental, the existence of the theory following from the consistent closure of Bianchi-Identities.

The scalar potential of these supergravities can be systematically derived, together with the complete field equations, from the closure of the supersymmetry algebra we have determined in the present paper. This is completely algorithmic and straightforward, but it involves lengthy calculations that are postponed to a forthcoming publication [17], where a full–fledged investigation of the properties of the potential and of its implications for the correspondence (1.1) will be given.

2 $D=5\quad\mathcal{N}=8$ supergravity

In this section we recall the main features of $D=5\quad\mathcal{N}=8$ supergravity theory [1], [2], fitting its formulation into the framework of the rheonomic constructions [16] and of the general discussion of duality symmetries [18] and central charges [19]. While adopting where possible the conventions of [1], recasting $D=5\ \mathcal{N}=8$ supergravity into the general framework of [19] is also a matter of notations since the names given to the various types of indices must reflect their interpretation within the framework. Specifically the notations are as follows. By $A,B=1,\dots,8$ we denote the indices labeling the supercharges and acted on by the *isotropy* subgroup H of the scalar coset G/H. In our case they are in the fundamental $\mathbf{8}$ of $\mathrm{USp(8)}$. The indices $\Lambda,\Sigma=1,\dots,27$ label instead the vector fields and correspond to the linear representation of the scalar *isometry* group G to which the vectors are assigned. In our case they run in the $\mathbf{27}$ of $\mathrm{E}_{6(6)}$. Next we need a notation for the electric subgroup $\mathrm{SL}(6,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})\subset\mathrm{E}_{6(6)}$ within which the generators of the gauge group can be chosen. It is as follows. $I,J=1,\dots,6$ are indices in the fundamental $\mathbf{6}$ both

[1]for comprehensive reviews of these topics see vol.2 of [16]

of $\mathrm{SL}(6,\mathbb{R}) \subset \mathrm{E}_{6(6)}$ and of $\mathrm{SO}(6)$ (or of its non–compact/non–semisimple versions); the indices $\alpha, \beta = 1, 2$ run in the fundamental $\mathbf{2}$ of $\mathrm{SL}(2,\mathbb{R}) \subset \mathrm{E}_{6(6)}$. Finally $\mu, \nu = 0, \ldots, 4$ are the usual curved spacetime indices, while we call $a, b = 0, \ldots, 4$ the flat indices of the fünfbein. The conventions for the gamma matrices, the spinors and the symplectic metric as those used in [1] and [20].

2.1 The ungauged theory

The supersymmetry algebra for the ungauged theory is the superPoincaré superalgebra, whose external automorphism symmetry (the $\mathcal{R}$–symmetry) is $\mathrm{USp}(8)$. The theory is invariant under local $\mathrm{ISO}(4,1) \times \mathrm{USp}(8)$ and global $\mathrm{E}_{6(6)}$ transformations, and under local supersymmetry transformations, generated by 32 real supersymmetry charges, organized in the eight pseudo–Majorana spinors

$$\mathrm{Q}^A = \Omega^{AB}\, \mathcal{C}\, \left(\overline{\mathrm{Q}}_A\right)^T \qquad A = 1, \ldots, 8\,. \tag{2.1}$$

Here $\Omega^{AB} = -\Omega^{BA}$ is the $\mathrm{USp}(8)$ invariant metric and $\mathcal{C}$ is the 5–dimensional charge conjugation matrix. The theory contains: the graviton field, namely the fünfbein 1–form V^a, eight gravitinos $\psi^A \equiv \psi^A_\mu\, dx^\mu$ in the $\mathbf{8}$ representation of $\mathrm{USp}(8)$, 27 vector fields $A^\Lambda \equiv A^\Lambda_\mu\, dx^\mu$ in the $\mathbf{27}$ of $\mathrm{E}_{6(6)}$, 48 dilatinos χ^{ABC} in the $\mathbf{48}$ of $\mathrm{USp}(8)$, and 42 scalars ϕ that parametrize the coset manifold $E_{(6)6}/\mathrm{USp}(8)$, and appear in the theory through the coset representative $\mathbb{L}_\Lambda{}^{AB}(\phi)$, in the $(\mathbf{27}, \overline{\mathbf{27}})$ of $\mathrm{USp}(8) \times \mathrm{E}_{6(6)}$. The local $\mathrm{USp}(8)$ symmetry is gauged by the composite connection built out of the scalar fields. The connection (in the $\mathbf{36}$ of $\mathrm{USp}(8)$) and the vielbein (in the $\mathbf{42}$ of $\mathrm{USp}(8)$) of the scalar manifold are defined through the following relation:

$$\mathbb{L}^{-1}_{AB}{}^\Lambda d\mathbb{L}_\Lambda{}^{CD} = \mathcal{Q}_{[A}{}^{[C}\delta^{D]}_{B]} + \mathcal{P}_{AB}{}^{CD}\,. \tag{2.2}$$

The isometry of the scalar manifold, $E_{(6)6}$, is a global symmetry of the theory.

2.2 The gauging

In maximal supergravities, where no matter multiplets can be added, *gauging* corresponds to the addition of suitable interaction terms that turn a subgroup $\mathcal{G}$ of the global $E_{(6)6}$ duality group into a local symmetry. This is done by means of vectors chosen among the 27 A^Λ. The $E_{(6)6}$ symmetry is broken to the normalizer of $\mathcal{G}$ in $E_{(6)6}$, and after this operation the new theory has a local symmetry $\mathrm{USp}(8) \times \mathcal{G}$ and a global symmetry $N(\mathcal{G}, E_{(6)6})$. The choice of $\mathcal{G}$ is strictly constrained by the request that the vectors which gauge this symmetry should transform in the coadjoint representation of $\mathcal{G}$, so that the following branching must be true:

$$\mathbf{27} \xrightarrow{\mathcal{G} \subset E_{(6)6}} Coadj\,(\mathcal{G}) \oplus \text{ rep. of } \mathcal{G}\,. \tag{2.3}$$

It turns out that this request is satisfied if and only if $\mathcal{G}$ is a fifteen–dimensional subgroup of $\mathrm{SL}(6,\mathbb{R}) \subset E_{(6)6}$ whose adjoint is identified with the $\mathbf{15}$ representation of $\mathrm{SL}(6,\mathbb{R})$. Indeed the $\mathbf{27}$ of $E_{(6)6}$ decomposes under

$$\mathrm{SL}(6,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R}) \subset E_{(6)6} \tag{2.4}$$

as

$$\mathbf{27} \longrightarrow (\overline{\mathbf{15}}, \mathbf{1}) \oplus (\mathbf{6}, \mathbf{2}) \tag{2.5}$$

(for example, $\mathbb{L}_\Lambda{}^{AB} \longrightarrow (\mathbb{L}^{IJAB}, \mathbb{L}_{I\alpha}{}^{AB})$) so that the property (2.3) is satisfied. The subgroups of $\mathrm{SL}(6,\mathbb{R})$ whose adjoint is the $\mathbf{15}$ of $\mathrm{SL}(6,\mathbb{R})$ are the $\mathrm{SO(p,q)}$ groups with $p+q=6$ and their contractions $\mathrm{CSO(p,q,r)}$, which will be discussed in section 4 (see [7, 8, 9] for definitions). The possible gaugings are then restricted to these groups. The normalizer in $E_{(6)6}$ of all these groups is the same as the normalizer of $\mathrm{SL}(6,\mathbb{R})$, namely $\mathrm{SL}(2,\mathbb{R})$. Therefore this latter is the residual global symmetry for all possible gaugings. The 27 vectors A^Λ are then decomposed into the vectors A_{IJ} in the $(\overline{\mathbf{15}}, \mathbf{1})$, that gauge $\mathcal{G}$, and the vectors in the $(\mathbf{6}, \mathbf{2})$, which do not gauge anything and are then forced (as we will see later) to be dualized into two–forms $B^{I\alpha}$. The fifteen generators G^{IJ} of $\mathcal{G}$ can be expressed as linear combinations of the $\mathbf{35}$ generators G_ℓ ($\ell=1,\ldots 35$) of $\mathrm{SL}(6,\mathbb{R})$: $G^{IJ}=G_\ell\, e^{\ell IJ}$ where $e^{\ell IJ}$ is the *embedding matrix* [9] which describes the embedding of $\mathcal{G}$ into $\mathrm{SL}(6,\mathbb{R})$. For all the admissible cases in the fundamental $\mathbf{6}$–dimensional representation the generators of the gauge group $\mathcal{G}$ take the form [1]

$$(G^{IJ})^K{}_L = \delta_L^{[I}\eta^{J]K} \tag{2.6}$$

where η^{JK} is a diagonal matrix with p eigenvalues equal to $\mathbf{1}$, q eigenvalues equal to (-1) and, only in the case of contracted groups, r null eigenvalues. This signature completely characterizes the gauge groups and correspondingly the gauged theory. From (2.6) one can build the generators of $\mathcal{G} \subset E_{(6)6}$ in the $\mathbf{27}$ representation of $E_{(6)6}$, namely some suitable matrices $(G^{IJ})_\Lambda{}^\Sigma$. According to the general framework of [18, 16, 9], in presence of gauging, the composite H–connection of $\mathrm{USp(8)}$ and scalar vielbein, defined in (2.2) are replaced by their gauged analogues:

$$\mathbb{L}^{-1}_{AB}{}^\Lambda d\mathbb{L}_\Lambda{}^{CD} + g(\mathbb{L}^{-1})^\Lambda_{AB}(G^{IJ})_\Lambda{}^\Sigma\mathbb{L}_\Sigma{}^{CD}A_{IJ} = \hat{\mathcal{Q}}_{[A}{}^{[C}\delta^{D]}_{B]} + \hat{\mathcal{P}}_{AB}{}^{CD}\,, \tag{2.7}$$

where g is the gauge coupling constant. The covariant $\mathrm{USp(8)}$ derivative of a field V_A is defined as

$$\nabla V_A = \mathcal{D}V_A + \hat{\mathcal{Q}}_A{}^B \wedge V_B \tag{2.8}$$

where $\mathcal{D}$ is the Lorentz–covariant exterior derivative. The covariant derivative with respect to $\mathcal{G}$ of a field V^I in the $\mathbf{6}$ of $\mathrm{SL}(6,\mathbb{R})$ is instead defined as follows:

$$DV^I \equiv \nabla V^I + g(G^{KL})^I{}_J A_{KL} \wedge V^J\,. \tag{2.9}$$

The field content of the gauged supergravity theory is the following

#	Field	$(SU(2)\times SU(2))$–spin rep.	USp(8) rep.	$\mathcal{G}$ rep.
1	V^a	$(1,1)$	$\mathbf{1}$	$\mathbf{1}$
8	ψ^A	$(1,1/2)\oplus(1/2,1)$	$\mathbf{8}$	$\mathbf{1}$
15	A_{IJ}	$(1/2,1/2)$	$\mathbf{1}$	$\mathbf{15}$
12	$B^{I\alpha}$	$(1,0)\oplus(0,1)$	$\mathbf{1}$	$\mathbf{6}\oplus\overline{\mathbf{6}}$
48	χ^{ABC}	$(1/2,0)\oplus(0,1/2)$	$\mathbf{48}$	$\mathbf{1}$
42	$\mathbb{L}_\Lambda{}^{AB}(\phi)$	$(0,0)$	$\mathbf{27}$	$\overline{\mathbf{27}}$

(2.10)

3 Gauged supergravities from Free Differential Algebras and Rheonomy

Gauged maximal supergravities in $D = 5$ were originally constructed within the framework of Noëther coupling and component formalism [1],[2]. As we pointed out in the introduction the gaugings corresponding to the contracted groups $\mathrm{CSO(p, q, r)}$ were left open in that approach. We are able to construct explicitly all these theories by reverting to our preferred approach based on Free Differential Algebras (FDA.s) and the principle of rheonomy [16]. Indeed, within this approach the non-semisimple theories are shown to exist and explicitely constructed irrespectively on the existence or not of a lagrangian formulation. Moreover, all the subtle points concerning the role of two–form dualization are naturally resolved in the Free Differential Algebra rheonomic approach. As far as five dimensions are concerned this was already noted in [14] where the hypermultiplets were coupled to $\mathcal{N} = 2$ supergravity. Similarly the essential role of FDA.s in gauging theories with p–form gauge fields was made evident in [23] where the unique six–dimensional $F(4)$–supergravity was finally constructed.

3.1 The rheonomy principle

For completeness, let us briefly recall the main steps in the *"rheonomy approach"* to supergravity. The starting point is to consider as fundamental fields the set of 1–forms $\mu^A \equiv \{\omega^{ab}, \psi^\alpha, V^a\}$, that constitute a cotangent frame dual to the Poincaré super–Lie algebra generators $\{J_{ab}, Q_\alpha, P_a\}$. The ordinary space–time parametrized by $\{x^\mu\}$ coordinates can be extended to a superspace parametrized also by the fermionic $\{\theta^\alpha\}$ spinor coordinates. We can give to the space–time fields $\mu^A(x)$ a θ-dependence through an appropriate extension mapping:

$$\begin{cases} \omega^{ab}(x) & \rightarrow & \omega^{ab}(x,\theta) \\ \psi^\alpha(x) & \rightarrow & \psi^\alpha(x,\theta) \\ V^a(x) & \rightarrow & V^{ab}(x,\theta)\,. \end{cases} \tag{3.1}$$

In such a way the bosonic space-time fields $\mu^A(x)$ are the boundary values at $\theta^\alpha = 0$ of these superspace fields

$$\mu^A(x) \equiv \mu^A(x,\theta)|_{\theta=d\theta=0}\,. \tag{3.2}$$

The same extension holds also for the set of curvature 2–forms defined through the structural equations:

$$R^A(x,\theta) \equiv d\mu^A(x,\theta) + \tfrac{1}{2}C^A{}_{BC}\mu^B(x,\theta)\wedge\mu^C(x,\theta) \equiv R^A_{LM}dZ^L\wedge dZ^M \tag{3.3}$$

that generalize Maurer–Cartan equations obtained by setting $R^A = 0$. In eq. (3.3) R^A denotes the multiplet $\{R^{ab}, \rho, R^a\}$ of super–Poincaré curvatures and $dZ^L \equiv \{dx^\mu, d\theta^\alpha\}$ is the set of coordinates which span the cotangent space to superspace.

In order to be completely determined as functions of x^μ, θ^α, the fields μ^A must be equipped with a complete set of Cauchy boundary conditions, namely we have to specify both the space–time configurations $\mu^A_\mu(x,0)$ on the boundary $\theta = 0$ and the first–order derivatives along the theta directions $\partial_{\theta^\alpha}\mu^A_\mu(x,\theta)|_{\theta=0}$ on the same boundary. These derivatives can be expressed in terms of θ projections $R^A_{\alpha,L}$ of the R^A curvatures. The *extension* *map* (3.1) can thus be determined by specifying the following two sets of boundary values:

$$\mu^A(x,0) \quad ; \quad R^A_{\alpha,L}(x,0)\,. \tag{3.4}$$

The former are the space–time configurations for the fields $\{\omega^{ab}(x)$, $\psi^\alpha(x)$, $V^a(x)\}$. In order to determine the latter, namely the so called *outer* components of the curvatures, we make use of the *rheonomy principle*, which states that the outer components of the curvatures are linear combinations of the *inner* ones, i.e. of the space–time configurations $\{R^{ab}(x), \rho(x), R^a(x)\}$:

$$R^A_{\alpha,L} = C^{A|\mu\nu}_{\alpha L|B} R^B_{\mu\nu} \tag{3.5}$$

where the C's are suitable constant tensors. This expansion is called the *rheonomic parametrization* of the curvatures. The values of the constants C can be determined by imposing the closure of Bianchi identities

$$dR^A + C^A_{BC}\mu^B \wedge R^C = 0\,. \tag{3.6}$$

Since by definition we have:

$$R^A_{\mu\nu} = \partial_{[\mu}\mu^A_{\nu]} + \tfrac{1}{2}C^A{}_{BC}\mu^B_{[\mu} \wedge \mu^C_{\nu]} \tag{3.7}$$

it turns out that the knowledge of the pure space-time configurations $\{\mu^A_\nu(x,0),\partial_\mu\mu^A_\nu(x,0)\}$ completely determines the superspace extensions defined in (3.1). It is worth noting that in this context Bianchi identities are not identically satisfied. This is not surprising since supersymmetry is an on–shell symmetry, and therefore it closes only modulo the equations of motion. Bianchi identities are actually equations of the theory, determining its dynamics. Not only they give the rheonomic parametrizations, but they also fix the geometry of the scalar manifold and give the equations of motion satisfied by the spacetime fields. In this framework, a supersymmetry transformation of the fields μ^A is given by a diffeomorphism along a fermionic direction in superspace $\varepsilon = \varepsilon^\alpha\partial_\alpha$ and is expressed by means of a Lie derivative along ε:

$$\delta\mu^A(x,\theta) = l_\varepsilon\mu^A(x,\theta) = (\nabla\varepsilon)^A + 2\varepsilon^\alpha C^{A|\mu\nu}_{\alpha L|B} R^B_{\mu\nu} dZ^L \tag{3.8}$$

from which one can retrieve the supersymmetry transformation rules of the fields $\delta\psi_\mu, \ldots$ as given in the usual component formalism. Note that the Lie derivatives l_ε close a super–Lie algebra, namely:

$$[l_{\varepsilon_1}, l_{\varepsilon_2}] = l_{[\varepsilon_1,\varepsilon_2]} \tag{3.9}$$

if the integrability condition $d^2 = 0$ is used. Of course this requirement is equivalent to enforcing the closure of Bianchi identities for the curvatures R^A.

Summarizing, to construct a supergravity theory we use the rheonomic conditions (3.5) for the curvatures and then we solve Bianchi Identities in two steps. In the *first step* we analyse the sectors that determine the unknown coefficients C's. Once a set of C's satisfying Bianchi Identities has been found, the corresponding supergravity theory has been *proven to exist* and its on–shell closed supersymmetry algebra has been constructed. Indeed, we have an explicit and consistent form for all the susy transformation rules.

Moreover, the classical dynamics of the theory is completely determined, since the classical equations of motion uniquely follow from closure of the susy algebra. To work them out explicitly corresponds to the *second step*. It suffices to complete the analysis of B-I in the remaining sectors by means of calculations that are completely straightforward and guaranteed, although somewhat lengthy. From this viewpoint, the explicit construction of the Lagrangian $\mathcal{L}$ is not really needed. Simply, when $\mathcal{L}$ exists, the determination of the field equations is more easily obtained by $\delta\mathcal{L}$ variations than through the analysis of the remaining sectors of Bianchi Identities [2]. When the Lagrangian exists, it can be obtained by means of a straightforward procedure starting from the rheonomic parametrizations [16].

What we sketched above describes minimal supergravity, containing only the graviton, the gravitino and the spin connection, but it is easily generalized to all supergravity theories [16], where also other fields are present, such as dilatinos, scalars, vectors and higher order forms. In these cases, the whole construction can be repeated with the μ^A defined to include all the 1–forms of the theory. Scalars and spin–one half fields are introduced by including their covariant derivatives as additional curvatures of the theory. When higher order forms are present, the super-Lie algebra has to be enlarged to a Free Differential Algebra expressing the occurrence of a higher order cohomology.

3.2 Rheonomic parametrizations for gauged $\mathcal{N}=8$ supergravity in five dimensions

In the theory we are considering, the relevant curvatures are defined below:

$$
\begin{array}{ll}
\begin{array}{l}\text{Poincaré}\\ \text{2-form}\\ \text{curvatures}\end{array} & \left\{ \begin{array}{rcl} R^a & \equiv & \mathcal{D}V^a + \frac{\mathrm{i}}{2}\bar{\psi}^A \wedge \gamma^a \psi_A \\ R^{ab} & \equiv & d\omega^{ab} - \omega^a{}_c \wedge \omega^{cb} \\ \rho_A & \equiv & \nabla\psi_A \end{array}\right. \\
\begin{array}{l}\text{vector}\\ \text{2-form}\\ \text{curvature}\end{array} & \left\{ \begin{array}{rcl} F_{IJ} & \equiv & dA_{IJ} + \frac{1}{2} g f_{IJ}{}^{KL,MN} A_{KL}\wedge A_{MN} \\ & & -\mathrm{i}\mathbb{L}^{-1}_{CDIJ}\bar{\psi}^C \wedge \psi^D \end{array}\right. \\
\begin{array}{l}\text{dilatino}\\ \text{1-form}\\ \text{curvature}\end{array} & \left\{ \begin{array}{rcl} X_{ABC} & \equiv & \nabla\chi_{ABC} \end{array}\right. \\
\begin{array}{l}\text{composite}\\ \text{1-form}\\ \text{USp(8)}\\ \text{curvature}\end{array} & \left\{ \begin{array}{rcl} \hat{R}_A{}^B & \equiv & d\hat{\mathcal{Q}}_A{}^B + \hat{\mathcal{Q}}_A{}^C \wedge \hat{\mathcal{Q}}_C{}^B \end{array}\right. \\
\begin{array}{l}\text{3–form}\\ \text{curvatures}\\ \text{of the}\\ \text{2–forms}\end{array} & \left\{ \begin{array}{rcl} H^{I\alpha} & \equiv & D\left[B^{I\alpha} + \mathrm{i}\mathbb{L}^{-1}_{AB}{}^{I\alpha}\bar{\psi}^A \wedge \psi^B\right] \end{array}\right.
\end{array}
\tag{3.10}
$$

where ∇, D denote the complete covariant differentials according to eq.s (2.8), (2.9) and the Lorentz–covariant derivatives of the vielbein and the gravitino 1–forms are defined

[2] Conceptually, the lagrangian is essential only for quantization, which however, in a modern perspective, is not the issue for theories, like supergravity, that are regarded as effective low energy theories of more fundamental microscopic quantum theories, like string theory.

below:

$$
\begin{aligned}
\mathcal{D}V^a &\equiv dV^a - \omega^a{}_b \wedge V^b \\
\mathcal{D}\psi_A &\equiv d\psi_A - \frac{1}{4}\omega^{ab}\gamma_{ab} \wedge \psi_A
\end{aligned} \tag{3.11}
$$

ω^{ab} being the spin connection. With $f_{IJ}^{KL,MN}$ we have denoted the structure constants of the gauge group $\mathcal{G}$. The curvatures (3.10) satisfy the following set of Bianchi identities:

$$
\begin{aligned}
&\mathcal{D}R^a + R^a{}_b \wedge V^b - \mathrm{i}\bar{\rho}^A \wedge \gamma^a \psi_A = 0 \\
&\mathcal{D}R^{ab} = 0 \\
&\nabla\rho_A + \tfrac{1}{4}R^{ab} \wedge \gamma_{ab}\psi_A - \hat{R}_A{}^B \wedge \psi_B = 0 \\
&\nabla X_{ABC} + \tfrac{1}{4}R^{ab}\gamma_{ab}\chi_{ABC} - 3R_{[A}{}^D \wedge \chi_{BC]D} = 0 \\
&DF_{IJ} - \mathrm{i}\mathbb{L}^{-1}_{CDIJ}\left(\hat{P}^{CDEF} \wedge \bar{\psi}_E \wedge \psi_F - 2\bar{\rho}^C \wedge \psi^D\right) = 0 \\
&DH^{I\alpha} - g(G^{KL})^I{}_J\left[F_{KL} + \mathrm{i}\mathbb{L}^{-1}_{ABKL}\bar{\psi}^A \wedge \psi^B\right] \wedge \left[B^{J\alpha} + \mathrm{i}\mathbb{L}^{-1}_{CD}{}^{J\alpha}\bar{\psi}^C \wedge \psi^D\right] = 0 \\
&\hat{R}^A_{\ B} = -\tfrac{1}{3}\hat{P}_{ACDE} \wedge \hat{P}^{BCDE} + \tfrac{1}{3}gT^B{}_{AEF}(\phi)(\mathbb{L}^{IJEF}F_{IJ} + \mathrm{i}\bar{\psi}^E \wedge \psi^F) \\
&\nabla\hat{P}_{ABCD} = gY^+_{ABCDEF}(\phi)(\mathbb{L}^{IJEF}F_{IJ} + \mathrm{i}\bar{\psi}^E \wedge \psi^F)
\end{aligned} \tag{3.12}
$$

where the USp(8)–tensors $T^B{}_{AEF}(\phi)$, $Y^+_{ABCDEF}(\phi)$ are defined as follows:

$$
Y^{AB}{}_{CDEF} \equiv \mathbb{L}^{-1}_{CD}{}^{\Lambda}(G^{IJ})_\Lambda{}^\Sigma \mathbb{L}_\Sigma{}^{AB}\mathbb{L}^{-1}_{EFIJ} \tag{3.13}
$$

$$
Y^{\pm}_{ABCDEF} \equiv \frac{1}{2}(Y_{ABCDEF} \pm Y_{CDABEF}) \tag{3.14}
$$

$$
T^A{}_{BCD} \equiv Y^{AF}{}_{BFCD}\,. \tag{3.15}
$$

The solution to the Bianchi identities (that is, the rheonomic parametrization) is given, modulo bilinears in the dilatinos, by the following expressions in terms of the inner components $R^{ab}{}_{cd}, \rho^A_{ab}, \ldots$:

$$
R^a = 0 \tag{3.16}
$$

$$
\begin{aligned}
R^{ab} &= \frac{1}{2}R^{ab}{}_{cd}V^c \wedge V^d + \\
&\quad \frac{2}{3}\mathrm{i}\mathcal{H}^{ab}_{AB}\bar{\psi}^A \wedge \psi^B + \frac{1}{6}\mathrm{i}\mathcal{H}_{AB|cd}\bar{\psi}^A \wedge \gamma_e\psi^B\epsilon^{abcde} + g\frac{2}{45}\mathrm{i}T_{AB}\bar{\psi}^A \wedge \gamma^{ab}\psi^B \\
&\quad - 2\mathrm{i}\bar{\rho}^{A\ c[a}\gamma^{b]}\psi_A \wedge V_c + \mathrm{i}\bar{\rho}^{A\ ab}\gamma^c\psi_A \wedge V_c + \mathcal{O}(\chi^2)
\end{aligned} \tag{3.17}
$$

$$
\begin{aligned}
\rho_A &= \rho_{A|ab}V^a \wedge V^b - g\frac{2}{45}T_{AB}\gamma_a\psi^B \wedge V^a \\
&\quad - \frac{2}{3}\mathcal{H}_{AB|ab}\gamma^a\psi^B \wedge V^b + \frac{1}{12}\mathcal{H}_{AB|ab}\gamma_{cd}\psi^B \wedge V_e\epsilon^{abcde} \\
&\quad + \frac{3\mathrm{i}}{4\sqrt{2}}\chi_{ABC}\bar{\psi}^B \wedge \psi^C - \frac{\mathrm{i}}{4\sqrt{2}}\gamma_a\chi_{ABC}\bar{\psi}^B\gamma^a \wedge \psi^C + \mathcal{O}(\chi^2)
\end{aligned} \tag{3.18}
$$

$$
\nabla\chi_{ABC} = (\nabla_a\chi_{ABC})V^a + \frac{1}{\sqrt{2}}gA^D_{ABC}\psi_D + \sqrt{2}\hat{P}_{ABCD|a}\gamma^a\psi^D
$$

$$
-\frac{3}{2\sqrt{2}}\mathcal{H}_{[AB|ab}\gamma^{ab}\psi_{C]}-\frac{1}{2\sqrt{2}}\Omega_{[AB}\mathcal{H}_{C]D|ab}\gamma^{ab}\psi^{D}+\mathcal{O}(\chi^2) \tag{3.19}
$$

$$
F_{IJ} = \frac{1}{2}F_{IJ|ab}V^a\wedge V^b+\frac{\mathrm{i}}{\sqrt{2}}\mathbb{L}^{-1}_{ABIJ}\bar{\chi}^{ABC}\gamma_a\psi_C\wedge V^a \tag{3.20}
$$

$$
B^{I\alpha} = \frac{1}{2}B^{I\alpha}_{ab}V^a\wedge V^b+\frac{\mathrm{i}}{\sqrt{2}}\mathbb{L}^{-1}_{AB}{}^{I\alpha}\bar{\chi}^{ABC}\gamma_a\psi_C\wedge V^a \tag{3.21}
$$

$$
\begin{aligned}
H^{I\alpha} &= H^{I\alpha}_{abc}V^a\wedge V^b\wedge V^c+g\frac{\mathrm{i}}{2}\mathbb{L}^{I\alpha}{}_{AB}\bar{\psi}^A\wedge\gamma_a\psi^B\wedge V^a\\
&\quad -g\frac{\mathrm{i}}{4\sqrt{2}}\mathbb{L}^{I\alpha}{}_{AB}\bar{\chi}^{ABC}\wedge\gamma_{ab}\psi_C\wedge V^a\wedge V^b
\end{aligned} \tag{3.22}
$$

$$
\hat{P}^{ABCD} = \hat{P}^{ABCD}_a V^a+2\mathrm{i}\sqrt{2}\,\bar{\chi}^{[ABC}\psi^{D]}+\frac{3}{2}\mathrm{i}\sqrt{2}\,\Omega^{[CD}\chi^{AB]E}\psi_E \tag{3.23}
$$

where the graviphoton field strength $\mathcal{H}_{AB|ab}$ is defined as

$$
\mathcal{H}^{AB}_{ab}\equiv\mathbb{L}^{IJAB}F_{IJ|ab}+\mathbb{L}_{I\alpha}{}^{AB}B^{I\alpha}_{ab} \tag{3.24}
$$

and the tensors $T_{AB}(\phi)$, $A^D_{ABC}(\phi)$ are defined as [3]

$$
T_{AB}=T^C{}_{ACB},\quad A^D_{ABC}=T^D_{[ABC]|}\,. \tag{3.25}
$$

From the definitions (3.10) and the parametrizations (3.12), applying the general procedure described above (3.8) one immediately derives the supersymmetry transformation laws of the physical fields (modulo bilinears in the dilatinos):

$$
\delta V^a_\mu = -\mathrm{i}\bar{\varepsilon}^A\gamma^a\psi_{\mu A} \tag{3.26}
$$

$$
\begin{aligned}
\delta\psi_{A\mu} &= \mathcal{D}_\mu\varepsilon_A-g\frac{2}{45}T_{AB}\gamma_\mu\varepsilon^B+\frac{2}{3}\mathcal{H}_{AB|\nu\mu}\gamma^\nu\varepsilon^B-\frac{1}{12}\mathcal{H}^{\nu\rho}_{AB}\gamma^{\lambda\sigma}\varepsilon^B\epsilon_{\mu\nu\rho\lambda\sigma}\\
&\quad+\frac{3\mathrm{i}}{2\sqrt{2}}\chi_{ABC}\bar{\varepsilon}^B\psi^C_\mu-\frac{\mathrm{i}}{2\sqrt{2}}\gamma_\nu\chi_{ABC}\bar{\varepsilon}^B\gamma^\nu\psi^C_\mu+\mathcal{O}(\chi^2)
\end{aligned} \tag{3.27}
$$

$$
\begin{aligned}
\delta\chi_{ABC} &= \frac{1}{\sqrt{2}}gA^D_{ABC}\varepsilon_D+\sqrt{2}\hat{P}_{ABCD|i}\partial_\nu\phi^i\gamma^\nu\varepsilon^D-\frac{3}{2\sqrt{2}}\mathcal{H}_{[AB|\mu\nu}\gamma^{\mu\nu}\varepsilon_{C]}\\
&\quad-\frac{1}{2\sqrt{2}}\Omega_{[AB}\mathcal{H}_{C]D|\mu\nu}\gamma^{\mu\nu}\varepsilon^D+\mathcal{O}(\chi^2)
\end{aligned} \tag{3.28}
$$

$$
\delta A_{IJ|\mu} = \mathbb{L}^{-1}_{ABIJ}\left[\frac{\mathrm{i}}{\sqrt{2}}\bar{\chi}^{ABC}\gamma_\mu\varepsilon_C+2\mathrm{i}\bar{\varepsilon}^A\psi^B_\mu\right] \tag{3.29}
$$

$$
\begin{aligned}
\delta B^{I\alpha}_{\mu\nu} &= \mathbb{L}^{I\alpha}{}_{AB}\left[-2\mathrm{i}g\bar{\varepsilon}^A\gamma_{[\mu}\psi^B_{\nu]}-\frac{\mathrm{i}}{2\sqrt{2}}g\bar{\chi}^{ABC}\gamma_{\mu\nu}\varepsilon_C\right]\\
&\quad+2\mathcal{D}_{[\mu}\left[\mathbb{L}^{-1}_{AB}{}^{I\alpha}\left(2\mathrm{i}\bar{\varepsilon}^A\psi^B_{\nu]}+\frac{\mathrm{i}}{\sqrt{2}}\bar{\chi}^{ABC}\gamma_{\nu]}\varepsilon_C\right)\right]
\end{aligned} \tag{3.30}
$$

$$
\hat{P}^{ABCD}_{,i}\delta\phi^i = 2\mathrm{i}\sqrt{2}\,\bar{\chi}^{[ABC}\varepsilon^{D]}+\frac{3\mathrm{i}}{\sqrt{2}}\,\Omega^{[CD}\chi^{AB]E}\varepsilon_E \tag{3.31}
$$

[3] $[\dots]|$ denotes the symplectic traceless antisymmetrization.

which do indeed coincide with the corresponding formulas in [1].

The B–I also give the equations of motion, which are the same as in [1]. At this point, we have in principle all the dynamical information about the theory without constructing the Lagrangian. However, as mentioned in the previous section, in order to get the equations of motion and the scalar potential, the easiest way is to derive them from the superspace Lagrangian, which, in the case of semisimple gaugings exist and we have determined to be (up to four–fermions terms):

$$
\begin{aligned}
\mathcal{L} \;=\;& \frac{1}{24}R^{ab}\wedge V^c\cdots\wedge V^e\epsilon_{abcde} - \frac{\mathrm{i}}{4}\bar\psi_A\gamma_{ab}\wedge\rho^A\wedge V^a\wedge V^b +\\
+\;& \frac{\mathrm{i}}{288}\bar\chi^{ABC}\gamma^a D\chi_{ABC}\wedge V^b\cdots\wedge V^e\epsilon_{abcde} +\\
-\;& \frac{1}{24}\mathcal{H}^{ab}_{AB}\left[\mathcal{H}^{AB} - \frac{\mathrm{i}}{\sqrt2}\bar\chi^{ABC}\gamma_\ell\psi_C\wedge V^\ell\right]\wedge V^c\cdots\wedge V^e\epsilon_{abcde} +\\
+\;& \frac{1}{288}P^{ABCD|a}\left[\,P_{ABCD} - 2\sqrt2\mathrm{i}\bar\chi_{ABC}\psi_D\right]\wedge V^b\cdots\wedge V^e\epsilon_{abcde} +\\
+\;& \left[\frac{1}{960}\mathcal{H}^{AB}_{\ell m}\mathcal{H}^{\ell m}_{AB} - \frac{1}{2880}P^{ABCD}_\ell P^\ell_{ABCD}\right]V^a\wedge\cdots\wedge V^e\epsilon_{abcde} +\\
+\;& \mathcal{H}_{AB}\wedge\left[\frac{\mathrm{i}}{2}\bar\psi^A\wedge\gamma_a\psi^B V^a + \frac{\mathrm{i}}{4\sqrt2}\bar\psi_C\wedge\gamma_{ab}\chi^{ABC}V^a\wedge V^b +\right.\\
+\;& \left.\frac{\mathrm{i}}{48}\bar\chi^{ALM}\gamma^{ab}\chi^B{}_{LM}V^c\cdots\wedge V^e\epsilon_{abcde}\right]\\
+\;& \frac{\mathrm{i}\sqrt2}{72}P_{ABCD}\bar\chi^{ABC}\gamma^{ab}\psi^D\wedge V^c\cdots\wedge V^e\epsilon_{abcde} +\\
-\;& \frac{\mathrm{i}}{180}g\left[T_{AB}\bar\psi^A\gamma^{ab}\wedge\psi^B + \frac58 A^A{}_{BCD}\bar\psi_A\gamma^a\chi^{BCD}\wedge V^b\right]\wedge V^c\cdots\wedge V^e\epsilon_{abcde} +\\
+\;& g^2\frac{1}{120}\left[\frac{6}{(45)^2}T_{AB}T^{AB} - \frac{1}{96}A_{ABCD}A^{ABCD}\right]V^a\wedge\cdots\wedge V^e\epsilon_{abcde} +\\
-\;& \frac{1}{24}\epsilon^{IJKLMN}\left[\mathcal{F}_{IJ}\wedge\mathcal{F}_{KL}\wedge A_{MN} + g\eta^{PQ}\mathcal{F}_{IJ}\wedge A_{KL}\wedge A_{MP}\wedge A_{QN} +\right.\\
+\;& \left.\frac25 g^2\eta^{PQ}\eta^{RS}A_{IJ}\wedge A_{KP}\wedge A_{QL}\wedge A_{MR}\wedge A_{SN}\right] + \frac{1}{2g}\eta_{IJ}\epsilon_{\alpha\beta}\mathcal{B}^{I\alpha}\wedge H^{J\beta} + \mathcal{L}_{4-f}
\end{aligned}
\tag{3.32}
$$

where the two–forms $\mathcal{H}_{AB}$ are defined by

$$
\mathcal{H}^{AB} \equiv \mathbb{L}^{IJAB}F_{IJ} + \mathbb{L}_{I\alpha}{}^{AB}B^{I\alpha} \tag{3.33}
$$

and we have introduced, in the Chern–Simons contributions,

$$
\begin{aligned}
\mathcal{F}_{IJ} &\equiv F_{IJ} + \mathrm{i}\mathbb{L}^{-1}_{ABIJ}\bar\psi^A\wedge\psi^B & (3.34)\\
\mathcal{B}^{I\alpha} &\equiv B^{I\alpha} + \mathrm{i}\mathbb{L}^{-1}_{AB}{}^{I\alpha}\bar\psi^A\wedge\psi^B\,. & (3.35)
\end{aligned}
$$

The projection of (3.32) onto space–time coincides with eq. (4.15) in [1].

3.3 The problem of the two–forms

It is a known fact [2], [1] that in order to consistently gauge the $\mathcal{N}=8$ theory, one has to dualize the vectors transforming in the $(\mathbf{6},\mathbf{2})$ of $\mathrm{SO(p,q)}\times\mathrm{SL(2)}$ to massive two-forms obeying the self-duality constraint:

$$B^{I\alpha|\mu\nu} = m\epsilon^{\mu\nu\rho\sigma\lambda}\mathcal{D}_\rho B^{I\alpha}_{\rho\sigma\lambda}\,. \tag{3.36}$$

In the geometric formulation of the theory, this need for dualization emerges in a completely natural way. Indeed, let us start by considering the 12 vectors $A^{I\alpha}$. There is no way known to reconcile their abelian gauge invariance with their non-trivial transformation under the gauge group $\mathcal{G}$. Indeed, given the superspace curvatures

$$DA^{I\alpha} \equiv dA^{I\alpha} + g(G^{KL})^I{}_J A_{KL} \wedge A^{J\alpha} \tag{3.37}$$

it follows that the corresponding Bianchi identity contains a term:

$$DDA^{I\alpha} = g(G^{KL})^I{}_J \mathcal{F}_{KL} \wedge A^{J\alpha} + \ldots \tag{3.38}$$

where the vectors $A^{J\alpha}$ appear naked. Under such conditions we cannot write a rheonomic parametrization of the curvatures solving the Bianchi identities and containing as only possible terms monomials in vielbein and gravitino with coefficients expressed in terms of gauge invariant space–time curvature components. Hence we have a clash between supersymmetry and the 12 abelian gauge invariances needed to keep the vectors $A^{J\alpha}$ massless. On the other hand, making them massive would destroy the equality of the Bose and Fermi degrees of freedom. Hence, in the gauged case where the 12 vectors $A^{J\alpha}$ acquire a non–trivial transformation under the non–abelian gauge symmetry there is no way of fitting these fields into a consistent supersymmetric theory. The way out, as it was discussed in [1], is to interpret them as the duals of massive two-forms $B^{I\alpha}$ [4], obeying a self–duality constraint which halves their degrees of freedom. This construction emerges naturally in the rheonomic formulation based on Free Differential Algebras. In this context, one has to introduce superspace curvatures for the two–forms (see eq.s (3.10)) generalizing the Maurer–Cartan equations to a Free Differential Algebra [16], [22]. At first sight it seems that we cannot escape from the problem described above, that affects the vectors $A^{I\alpha}$: indeed Bianchi identities do contain the naked fields $B^{I\alpha}$. Yet we can successfully handle this fact by considering the $B^{I\alpha}$ not as gauge potentials (that is, 2-forms defined modulo 1–form gauge transformations), but as physical fields, with their own explicit parametrization (see equation (3.21)) [5]. In this way, the two–forms loose their gauge freedom and become massive, as it can be found by solving the Bianchi identities. In fact, the Bianchi identities imply also the field equations of the two–forms. In the $\bar\psi^A \wedge \gamma^a \psi_A$ sector of the $H^{I\alpha}$ sector we get the following constraint (modulo bilinears in the dilatinos):

$$D_{[a}B^{I\alpha}_{bc]} = -\frac{1}{12} g \mathbb{L}^{I\alpha}{}_{AB}\mathcal{H}^{AB\ |de}\varepsilon_{abcde} - \mathrm{i}\sqrt{2}\mathbb{L}^{-1}_{AB}{}^{I\alpha}\bar\chi^{ABC}\gamma_{[a}\rho_{bc]C}\,. \tag{3.39}$$

This is the self-duality constraint on the two-forms, that halves the number of their degrees of freedom, while gives them a mass g. Note that the algebra underlying this theory is a free differential algebra [21, 16, 22]. However, since the $B^{I\alpha}$ transform in a non trivial representation of the gauge group, it can be shown (as it follows from a theorem by

[4]In five dimensions the Hodge dual of a two–form field strength is a three–form field strength: $\epsilon^{\mu\nu\rho\lambda\sigma}D_\lambda A_\sigma = D^\mu B^{\nu\rho}$.

[5]The same happens to matter two–form fields coupled with $\mathcal{N}=2$ supergravity [14].

Chevalley–Eilenberg [21]) that the left hand side of their rheonomic parametrization at $B^{I\alpha}_{ab} = 0$ is a trivial cohomology class of the algebra spanned by the other one–forms. Said in simpler terms, in the vacuum (where the superspace curvatures are zero) the two–forms are not independent fields, rather they are names given to certain bilinears in the gravitino fields:

$$B^{I\alpha} = -\mathrm{i}\delta^{I\alpha}_{AB}\bar{\psi}^A \wedge \psi^B \,. \tag{3.40}$$

There is a drastic algebraic difference between these $\mathbf{2}$–forms and the $\boldsymbol{p}$–forms that appear in most higher dimensional supergravities. In the mathematical language of Sullivan [21] this has to do with the distinction between *non trivial minimal free differential algebras* and *trivial contractible algebras*. In the first case the exterior derivative of a $\boldsymbol{p}$–form is equated to a non–trivial cohomology class of the superalgebra spanned by the other forms, namely to a polynomial in the remaining $\mathbf{1}$–forms that cannot be written as the derivative of any other such polynomial. In the second case the derivative of the $\boldsymbol{p}$–form is equated to a trivial class. True $\boldsymbol{p}$–form gauge fields occur only when the vacuum free differential algebra (that at zero curvature) is minimal. On the other hand, if the vacuum free differential algebra is contractible then there are no true $\boldsymbol{p}$–form gauge fields since they can be traded for an expression in terms of the other $\mathbf{1}$–forms. As it was shown in [22] the contractible generators of a free differential algebra are anyhow associated with the concept of curvatures. Indeed when a minimal algebra is deformed by the introduction of curvatures it becomes contractible. So the self–duality between the field–strengths (=curvatures) and the 2–form potentials acquires in this language a natural explanation. It is just the signal that the FDA is contractible. This, in line with Chevalley-Eilenberg theorem is due to the semisimple character of the super–Lie algebra of which the FDA is the extension.

We stress that in supergravity theory one usually deals with massless $\boldsymbol{p}$–forms. This reduces their degrees of freedom by means of gauge invariance. When we gauge the theory, it often occurs that the $\boldsymbol{p}$–forms become massive and some other mechanism has to intervene in order to reduce their number of degrees of freedom and keep the balance between fermions and bosons. This mechanism can be either self–duality, as it happens in our case and in seven dimensional supergravity, or the so–called anti–Higgs mechanism, as it happens in gauged $F(4)$ gauged supergravity in six dimensions [23] [6]. Let us note that the self–duality mechanism is a relation between a form and its field strength, stating in this way the triviality of the cohomology related to that form. As it follows from the theorem in [21], this necessarily happens when the forms are in some non trivial representation of the gauge group. On the contrary, we expect the anti–Higgs mechanism, which implies a non trivial cohomology for the form, to be present only when the form is a gauge singlet.

4 Gauging the non–semisimple $\mathrm{CSO(p,q,r)}$ groups

In the original papers [1, 2] gauged versions of five–dimensional maximal supergravity were constructed where the gauge group is either $\mathrm{SO(6)}$ or one of its non compact forms $\mathrm{SO(p,q)}$ (with $p+q=6$). This is similar to what happens in four–dimensions where the semisimple gaugings of $\mathcal{N}=8$ supergravity are based on all the groups $\mathrm{SO(p,q)}$ with

[6] The former can occur only in odd dimensional space–times [25].

$p+q=8$. In fact in that theory there is an additional series of interesting non–semisimple gaugings based on contracted algebras $CSO(p,q,r)$ (with $p+q+r=8$) whose notion was introduced by Hull [7, 8] and whose classification was shown to be exaustive in [9]. It is quite natural to expect that such non–semisimple gaugings exist also in five–dimensions with $p+q+r=6$. However they were not constructed in [1, 2] because of the subtle features related with the problem of two–form gauge fields. This problem being naturally solved in the Free Differential Algebra rheonomic approach we are tempted to argue that the $\mathrm{CSO(p,q,r)}$ can be constructed in this framework. This is indeed true as we explicitly show below. Indeed the catch point is that the number of vectors dualized to two–forms is not fixed to 12 as in the semisimple gaugings rather it is variable. In the non semisimple $\mathrm{CSO(p,q,r)}$ case we have $12-r$ two–forms and $15+r$ 1–forms. However r of these latter do not gauge any transformation with non trivial action on the other fields, in other words they are associated with *central charges*. The price to be paid for that seems to be that a lagrangian formulation is not available for these theories, all the dynamical information being however encoded in the solution of the Bianchi Identities.

4.1 The $\mathrm{CSO(p,q,r)}$ algebras

We begin with a short description of the contracted algebras and in the next subsection we explain how they are gauged.

The generators of $\mathrm{SO(p,q)}$ (with $p+q=n$) in the vector representation are

$$(G^{IJ})^K{}_L = \delta_J^{[K}\eta^{L]I} \qquad I,J,K,L = 1,\dots,n\,, \tag{4.1}$$

where

$$\eta^{IJ} \equiv \mathrm{diag}(\overbrace{1,\dots,1}^{p},\overbrace{-1,\dots,-1}^{q})\,. \tag{4.2}$$

They satisfy

$$[G^{IJ},G^{KL}] = f^{IJ,KL}_{MN}G^{MN} \tag{4.3}$$

where

$$f^{IJ,KL}_{MN} = -2\delta^{[I}_{[M}\eta^{J][K}\delta^{L]}_{N]}\,. \tag{4.4}$$

Their generalization, studied by Hull in the context of supergravity [7],[8] are the algebras $\mathrm{CSO}(p,q,r)$ with $p+q+r=n$, defined by the structure constants (4.4) with

$$\eta^{IJ} \equiv \mathrm{diag}(\overbrace{1,\dots,1}^{p},\overbrace{-1,\dots,-1}^{q},\overbrace{0,\dots,0}^{r})\,. \tag{4.5}$$

Decomposing the indices

$$I = (\bar I,\hat I) \qquad \bar I = 1,\dots,p+q,\ \ \hat I = p+q+1,\dots,n\,, \tag{4.6}$$

we have that $G^{\bar I\bar J}$ are the generators of $\mathrm{SO(p,q)}\subset\mathrm{CSO(p,q,r)}$, while the $r(r-1)/2$ generators $G^{\hat I\hat J}$ are central charges

$$[G^{\bar I\hat J},G^{\bar K\hat L}] = \frac{1}{2}\eta^{\bar I\bar K}G^{\hat J\hat L}\,. \tag{4.7}$$

They form an abelian subalgebra, and

$$\mathrm{SO(p,q)}\times\mathrm{U(1)}^{\frac{r(r-1)}{2}}\subset\mathrm{CSO(p,q,r)}\,. \tag{4.8}$$

Notice that $\mathrm{CSO(p,q,1)} = \mathrm{ISO(p,q)}$. In the vector representation, the generators of the central charges are identically null

$$(G^{\hat{I}\hat{J}})^K{}_L = 0\,, \tag{4.9}$$

while

$$(G^{\hat{I}\bar{J}})^K{}_L = \frac{1}{2}\delta^{\hat{I}}_L \eta^{\bar{J}K} \neq 0\,. \tag{4.10}$$

It is worth noting that the Killing metric of $\mathrm{SO(p,q,r)}$ is

$$K^{IJ,KL} = f^{IJ,MN}_{PQ} f^{KL,PQ}_{MN} = -6\eta^{K[I}\eta^{J]L}\,. \tag{4.11}$$

This notation is redundant, because the adjoint representation is $n(n-1)/2$ dimensional. In the proper basis,

$$K^{IJ,KL}_{I<J,\,K<L} = -3\eta^{IK}\eta^{JL}\,. \tag{4.12}$$

This is a diagonal matrix of dimension $n(n-1)/2$, with components $\eta^{II}\eta^{JJ}$. In general, the real sections of a given group (in this case, D_3) are characterized by the signature of the Killing metric[7]. We see that, for the $\mathrm{CSO(p,q,r)}$ algebras, the signature of the Killing metric is equivalent to the signature of the matrix η^{IJ}. This explains why this tensor can give an intrinsic characterization of such groups. Notice that a similar result was found, with a different procedure, while studying the gaugings of $\mathcal{N}=8$ supergravity in four space–time dimensions [9].

4.2 The contracted gaugings

As announced above the gauged versions of $\mathcal{N}=8$, $D=5$ supergravity constructed in [1], [2] and based on a semisimple choice of the gauge group $\mathcal{G} = \mathrm{SO(p,q)}\ (p{+}q{=}6)$ can be further generalized to the non–semisimple gauge groups $\mathcal{G} = \mathrm{CSO(p,q,r)}\ (p{+}q{+}r{=}6)$.

The new gaugings can be obtained by taking for the matrix η^{IJ} the definition (4.5), with some null entries on the diagonal. Let us discuss the consequences of this in the theory, in order to see if any pathology occurs. One has

$$(G^{KL})^{\hat{I}}{}_J = \delta^{[K}_J \eta^{L]\hat{I}} = 0\,, \tag{4.13}$$

so the covariant derivative of a *contravariant field* (2.9), along the contracted directions, reduces to the ordinary $\mathrm{USp(8)}$–covariant derivative:

$$DV^{\hat{I}} = \nabla V^{\hat{I}} + g(G^{KL})^{\hat{I}}{}_J A_{KL} \wedge V^J = \nabla V^{\hat{I}}\,. \tag{4.14}$$

This, however, does not happen for the covariant derivative of a *covariant field*:

$$DV_{\hat{I}} \equiv \nabla V_{\hat{I}} - g(G^{KL})^J{}_{\hat{I}} A_{KL} \wedge V^J = \nabla V_{\hat{I}} - g\eta^{\bar{L}\bar{J}} A_{\hat{I}\bar{L}} \wedge V_{\bar{J}}\,. \tag{4.15}$$

The abelian vectors $A_{\hat{I}\hat{J}}$ do not appear in the covariant derivatives. Because of (4.7), in the field strengths

$$\mathcal{F}_{IJ} = dA_{IJ} + \frac{1}{2} f^{KL,MN}_{IJ} A_{KL} \wedge A_{MN} \tag{4.16}$$

[7] for non semisimple groups, by signature we mean the number of positive, negative and null components of the matrix η in its diagonal form

the last term is present even for the vectors of the abelian subgroup. Let us consider now the most subtle part of the theory: the two–forms. Along the contracted directions, one has

$$\mathbb{L}^{\hat{I}\alpha AB} \equiv \eta^{\hat{I}J}\varepsilon^{\alpha\beta}\mathbb{L}_{J\beta}{}^{AB} = 0\,, \tag{4.17}$$

so that the rheonomic parametrization (3.22) of $H^{\hat{I}\alpha}$ becomes

$$H^{\hat{I}\alpha} = H^{\hat{I}\alpha}_{abc} V^a \wedge V^b \wedge V^c\,. \tag{4.18}$$

The corresponding Bianchi identity reads:

$$DH^{\hat{I}\alpha} = 0\,, \tag{4.19}$$

and, substituting back the parametrization (4.18), one finds $H^{\hat{I}\alpha}_{abc} = 0$. Hence we have:

$$\begin{aligned} 0 &= H^{I\alpha} = d\left[B^{\hat{I}\alpha} + \mathrm{i}\mathbb{L}^{-1}_{AB}{}^{\hat{I}\alpha}\bar{\psi}^A \wedge \psi^B\right] + g(G^{KL})^{\hat{I}}{}_J A_{KL} \wedge \left[B^{\hat{I}\alpha} + \mathrm{i}\mathbb{L}^{-1}_{AB}{}^{\hat{I}\alpha}\bar{\psi}^A \wedge \psi^B\right] = \\ &= d\left[B^{\hat{I}\alpha} + \mathrm{i}\mathbb{L}^{-1}_{AB}{}^{\hat{I}\alpha}\bar{\psi}^A \wedge \psi^B\right]\,. \end{aligned} \tag{4.20}$$

The solution of this equation is

$$B^{\hat{I}\alpha} = \mathcal{B}^{\hat{I}\alpha} - \mathrm{i}\mathbb{L}^{-1}_{AB}{}^{\hat{I}\alpha}\bar{\psi}^A \wedge \psi^B \tag{4.21}$$

with

$$\mathcal{B}^{\hat{I}\alpha} \equiv dA^{\hat{I}\alpha}\,. \tag{4.22}$$

In other words, the Bianchi identities of the two–forms corresponding to the contracted direction (the $B^{\hat{I}\alpha}$) are cohomologically trivial, so that these fields are actually field strengths of one–form fields (4.22), having a U(1) gauge invariance, as argued in [1]. Let us stress that this explicit calculation performed in the rheonomy formalism shows that there are no consistency conflicts between the two types of gauge invariances, and therefore no need arises to introduce massive vectors as proposed in [1]. Indeed, in the FDA rheonomic approach we see in a transparent way where the consistency conflicts arise and how they are solved. Summarizing it goes as follows. When a vector field is charged with respect to the gauge group, but does not gauge any generator of the gauge algebra it appears naked in its own Bianchi Identity. This requires dualization to a two–form and the replacement of gauge invariance with self duality as a mean to reduce the number of degrees of freedom. On the other hand when a contraction is performed on some direction $\hat{I}$, in the Bianchi identity of the fields $A^{\hat{I}\alpha}$ (4.20) the naked gauge fields disappear. Therefore, the two gauge invariances are not inconsistent, and the corresponding vectors can stay massless. Note that in this case the Bianchi identities look very different from those along the non-contracted directions. Now the self–duality constraint disappears and the halving of degrees of freedom is due to the recovered U(1) gauge symmetry.

In this way we have found new gauged $D = 5$ $\mathcal{N} = 8$ supergravities, with $(12 - r)$ two–forms, $(15 + r)$ one–forms, and gauge group $\mathrm{CSO(p,q,r)}$. It is worth noting that the r vectors $A^{\hat{I}\alpha}$ are coupled with the other fields, even if they don't gauge anything, and so are the abelian vectors $A_{\hat{I}\hat{J}}$. Indeed,

$$\mathcal{H}^{AB}_{ab} = \mathbb{L}^{IJAB}F_{ab|IJ} + \mathbb{L}_{\bar{I}\alpha}{}^{AB}B^{\bar{I}\alpha}_{ab} + \mathbb{L}_{\hat{I}\alpha}{}^{AB}B^{\hat{I}\alpha}_{ab} \tag{4.23}$$

and $\mathcal{H}^{AB}_{ab}$ does appear in the equation of motion of the two–form (3.39) along the non–contracted direction, which we have derived from the B-I and doesn't change in the contracted gaugings.

The susy transformatons rules for the new theories are obtained by substituting, for the contracted directions $\hat{I}$, eq. (3.30) with:

$$\delta A^{\hat{I}\alpha|\mu} = \mathbb{L}^{-1}_{AB}{}^{\hat{I}\alpha} \left[\frac{\mathrm{i}}{\sqrt{2}} \bar{\chi}^{ABC} \gamma_\mu \varepsilon_C + 2\mathrm{i}\bar{\varepsilon}^A \psi^B_\mu \right] \tag{4.24}$$

all other transformation laws remaining unchanged.

The new theories are completely sensible and well defined, however it seems that there is not a lagrangian formulation of them. Indeed, by looking at (3.32) describing the semisimple gaugings, one can see that it involves the inverse matrix η_{IJ} [8] in the Chern-Simons term for the two-forms. This matrix is not well defined for the contracted directions $\hat{I}$. The corresponding terms cannot therefore be present in the lagrangian. As a consequence, it seems to us that there is no way to write down new terms in the lagrangian substituting the badly defined ones in a covariant and gauge-invariant way. Technically, the obstruction is related to the fact that that Chern-Simons contribution was necessary for the vanishing of the variation of the lagrangian with respect to the vectors A_{IJ}, by use of the cubic invariant of $E_{6(6)}$. One could try to add by hand in the lagrangian the missing terms which complete the cubic invariant (even if their meaning would be quite obscure) but they would lead to not-gauge invariant field equations for the new vectors $A^{\hat{I}\alpha}$. Otherwise, one could try to interpret the badly defined terms in the lagragian as the dominant ones (since they are now infinite) through some appropriate scaling limit. However, since the connection between the covariant and contravariant representions for the $\hat{I}$ directions is lost, we were not able to implement in a covariant way the field equations through appropriate terms to be added to the lagrangian. We postpone to the concluding section some comments about this fact.

4.3 Conclusions

In this paper, we have proven the existence of $D = 5$ $\mathcal{N} = 8$ supergravities where the non–semisimple $CSO(p, q, r)$ algebras are gauged and this is a novelty since it was so far unclear in the literature whether this could be done or not. Of these theories, at the present stage, we possess the supersymmetry algebra whose closure implies the field equations. According to what we explained in section 4.2, the lagrangian formulation of these theories probably does not exist. A possible physical argument to motivate this situation that was technically illustrated above is the following. The existence of r extra neutral vectors besides the 15 charged ones implies a sort of Hodge dualization for the corresponding two–forms. Specifically, what happens here is that the field strength $H^{[3]}$ for r of the $B^{[2]}$ fields is identically zero, so that we have to interpret the $B^{[2]}$ themselves as field strengths of new gauge vectors $A^{[1]}$. In other words, we have traded r "electric" two–form fields $B^{[2]}$ for just as many "magnetic" one–forms $A^{[1]}$. In view of this, it is not too surprising if the $15 + r$ vectors are not mutually local, which would be necessary to admit a common lagrangian description.

[8] We remind that the definition of the $CSO(p, q, r)$ generators in the vector representation involves the matrix η^{IJ}.

However, all the information on the dynamics of these theories are encapsulated in the rheonomic solution of the B-I that we have presented. As explained in section 3.1, to extract the explicit form of the field equations and of the potential it suffices to analyze the remaining sectors of the B-I, through somewhat lengthy calculations, whose result, however, is a priori guaranteed.

Although these calculations and a full–fledged analysis of the field equations and their solutions are postponed to a forthcoming publication [17], we can briefly anticipate some considerations on the scalar potential that, most presumably, has the same form, in terms of the fermionic shift and gravitino mass matrix, as in the compact case [1]:

$$P = -g^2 \left[\frac{2}{675} T_{AB} T^{AB} - \frac{1}{96} A_{ABCD} A^{ABCD} \right] . \tag{4.25}$$

Indeed, expression (4.25) follows from the general structure of supersymmetry Ward identities one finds in gauged supergravities.

As it has been shown in [1], the number of supersymmetries preserved by a constant scalar configuration ϕ_0 is given by the number of eigenvalues μ of

$$W_{AB}(\phi_0) \equiv \frac{4}{15} T_{AB}(\phi_0) \tag{4.26}$$

such that

$$|\mu| = \sqrt{-\frac{3}{g^2} P(\phi_0)} . \tag{4.27}$$

The scalars are in the $\mathcal{G}$ representations

$$\mathbf{20} \oplus \mathbf{10} \oplus \bar{\mathbf{10}} \oplus \mathbf{1} \oplus \bar{\mathbf{1}} . \tag{4.28}$$

The potential is invariant under all the local and global symmetries of the theory, which in this case are $\mathcal{G} \times \mathrm{SL}(2,\mathbb{R})$. To look for minima of the potential (4.25) one can use Warner's observation [27] that, given a subgroup $\mathcal{G}' \subset \mathcal{G} \times \mathrm{SL}(2,\mathbb{R})$, and given the submanifold of the scalar manifold $\Sigma' \subset \Sigma$ invariant under the action of $\mathcal{G}'$, then the minima of Σ' are also minima of Σ. We can therefore restrict the potential to subsets of the scalars, and search their minima. In particular, the scalars invariant under $\mathcal{G}' = \mathrm{U}(1) \subset \mathrm{SL}(2,\mathbb{R})$ are those in the $\mathbf{20}$, that parametrize the coset $\mathrm{SL}(6,\mathbb{R})/\mathrm{SO}(6) \subset \mathrm{E}_{6(6)}/\mathrm{USp}(8)$. One can study the minima restricted to these scalars. In this case we can use a simple formula to find the potential, found in [1] for the $\mathrm{SO(p,q)}$ case

$$P = -\frac{g^2}{32} \left[\left(\eta^{IJ} M_{IJ}\right)^2 - 2 \left(\eta^{IJ} M_{IJ} \eta^{KL} M_{KL}\right) \right] , \tag{4.29}$$

where

$$M_{IJ} = S_I{}^K S_J{}^K \tag{4.30}$$

and $S_I{}^K$ are the $\mathrm{SL}(6,\mathbb{R})$ generator associated with the scalars we are considering. Furthermore,

$$W_{AB} = -\frac{1}{4} \Omega_{AB} \eta^{IJ} M_{IJ} . \tag{4.31}$$

The formulas (4.29), (4.31) remain valid for the contracted theories, where the η^{IJ} have zero eigenvalues. However this procedure is not very useful to find non–maximally supersymmetric minima of the potential. Indeed, since we have $W_{AB} \propto \Omega_{AB}$ all W_{AB}

eigenvalues μ are equal in modulus and differ only for the phase. This implies that restricting our attention to the scalars in the **20** representation of $SL(6,\mathbb{R})$ either there is $\mathcal{N}=8$ supersymmetry, or there is no supersymmetry. Yet a minimum with $\mathcal{N}=8$ supersymmetry should be invariant under $SU(4) = SO(6)$ and this occurs only when all the scalars are set to zero and only for the $SO(6)$ theory. Hence, in the quest for other supersymmetric minima, one must necessarily consider the scalars in the $\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{1}\oplus\overline{\mathbf{1}}$, as it was done for the only other known supersymmetric minimum, found [26] in the $SO(6)$ theory.

Such an analysis is quite involved and it is beyond the scope of the present paper. However it is a very interesting and challenging problem that we postpone to the already mentioned future investigations. We just note that the contracted gaugings yield a non supersymmetric vacuum with zero cosmological constant [9]. In the $CSO(2,0,4)$ gauging, the potential, restricted to the scalars invariant under $SO(2)\times U(1)^6$, vanishes identically. Because of Warner's argument [27] this implies that these scalars correspond to a minimum of the whole potential, which has zero cosmological constant.

Summarizing, in this paper we have shown that there exist new non–semisimple gauged supergravities in five dimensions that are potentially very interesting in the quest for brane–worlds and the DW/QFT correspondence. In perspective, it is very interesting to find an interpretation of these theories as the supergravities describing the near–brane geometry for suitable stringy branes.

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