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T-duality in the string theory effective action with a string source

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Abstract

We consider the T-duality transformations of the low-energy quantum string theory effective action in the presence of classical fundamental string source and demonstrate explicitly that T-duality still holds.

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1 Introduction

Target-Space duality (or T-duality) is the one of the dualities set of superstring theory which can be viewed already at the perturbative level. As it has been pointed out in papers by Kikkawa and Yamasaki [1] and Sakai and Senda [2] it arises the formal symmetry of quantum effective potential of free toroidally compactified string theory under interchange of a torus radii with their inverse. In its turn this phenomenon occurs in the firstly quantized string theory by virtue of symmetry of the mass^2 operator under $R \leftrightarrow 1/R$ interchange supplemented by interchange of Kaluza-Klein and winding modes arising under compactification. The next important step was made in paper by Nair, Shapere, Strominger and Wilczek [3] where this symmetry was justified in the interacting case as well, and therefore received the status of exact symmetry of firstly quantized string theory. However only after the seminal papers by Buscher [4] it became clear that at the level of classical string dynamics this symmetry is realized as impossibility to differentiate (special kind of or so-called dual) backgrounds with (one or many) isometry directions in which string propagates. Connection between such kind of backgrounds coming from the target-space metric, the 2-form antisymmetric tensor field and the dilaton is given by the set of Buscher's formulae [4] [4].

Consideration of the long-wave limit of superstring theory corresponding to supergravity action also leads to the establishing the symmetry between dimensionally reduced dual theories [6]–[9] and allows to exhibit more clearly the space-time interpretation of the duality transformations as ones acting only on the matter fields and leaving the gravitational sector unchanged. Also, the target-space geometry adapted to the T-duality transformations was constructed in [10].

What concerns supersymmetric string theory, the T-duality transformations were intensively investigated in the NSR (worldsheet SUSY) formulation (see, for instance [11]–[14] and Refs. therein), although the absence of direct connection between worldsheet and target-space supersymmetries, which can be established with application of the CFT technique [15], hampered the problem of getting the T-duality rules for the space-time fermions (see, however, [16]). This drawback naturally overcomes in the manifestly target-space supersymmetric formulation of the GS superstring, however the progress in the construction of T-duality rules in the GS formulation [17]–[19] was achieved only recently.

Therefore, superstring and supergravity encourage the exact symmetry under the T-duality transformations. Moreover, as it has been pointed out in literature (see e.g. [20]) the sum of supergravity action and fundamental superstring source action should be T-duality covariant. The goal of this communication is to push forward this observation and to establish the invariance of dual theories under the T-duality transformations for the toy model of closed bosonic string being the source for gravity and 2-form antisymmetric tensor field. Because our consideration is pure bosonic we will to refer in what follows this bulk configuration as low energy quantum string theory effective action [21] or effective action for shortness. It is clear that such a configuration is the part of the bosonic sector of supergravity being the superstring theory effective action. Another point of our consideration consists in the restriction of our analysis to the pure classical frames. It means that we shall not consider the dilaton appearance in the string action and, consequently, in the set of the T-duality rules, because it is well-known that the dilaton shift

[4] The analog of these formulas for the target-space metric was derived independently by Hitchin, Karlhede, Lindström and Roček in [5] addressed to investigation of d-dimensional non-linear sigma-model.

appearing in the Buscher's formulae can be described correctly only in the frame of quantum approach [22], [23] (and Refs. therein). However, since supergravity contains the information about quantum corrections of the worldsheet, we have to take into account the dilaton shift as far as the effective action will be considered.

To reach the goal we analyse two different approaches which lead to the result. The first approach is the standard one and it demonstrates the role of worldvolume duality in establishing the T-duality invariance in the bulk. The second approach we propose is based on the gauging of the translational invariance along isometry direction simultaneously in the bulk and in the source parts of the effective action with source. This forces us, in particular, to introduce new additional target-space field instead of worldvolume one as it is made in the first case.

The paper is organized as follows. In Section 2 we give a brief review of derivation of the Buscher's formulae in the standard sigma-model-like [4] and gauged sigma-model-like [24]–[29] approaches. Section 3 devoted to the recalling the T-duality rules in Kaluza-Klein picture [7, 8], which simplify calculations (see, for instance, [30] for non-Abelian case, and Ref. [10], where such kind of transformations arise in the geometry of a target-space adapted to the presence of isometries) and in Section 4 we outline the application of these rules to the gauged-sigma-model. After these preliminaries in Section 5 we demonstrate the invariance between dual effective theories in presence of the fundamental string source. Discussion of the obtained results and concluding remarks are collected in the last section.

2 T-duality in bosonic string theory

Consider Polyakov's action for bosonic (closed) string [31]

$$S = \int d^2\xi \sqrt{-\gamma}\gamma^{ij}\partial_i X^{\underline{m}}\partial_j X^{\underline{n}} g_{\underline{mn}}(X). \tag{1}$$

To reach T-dual action, let's suppose that we have an isometry direction, say $X^{\underline{0}}$ [5], and evidently

$$S = \int d^2\xi \sqrt{-\gamma}\gamma^{ij}(\partial_i X^{\underline{0}}\partial_j X^{\underline{0}} g_{\underline{00}} + \partial_i X^{\underline{0}}\partial_j X^{\underline{\tilde{m}}} g_{\underline{0\tilde{m}}} + \partial_i X^{\underline{\tilde{m}}}\partial_j X^{\underline{0}} g_{\underline{\tilde{m}0}} + \partial_i X^{\underline{\tilde{m}}}\partial_j X^{\underline{\tilde{n}}} g_{\underline{\tilde{m}\tilde{n}}}). \tag{2}$$

After that one can write down (2) in the first order form [5], [32] as

$$S = \int d^2\xi [\sqrt{-\gamma}\gamma^{ij}(C_i C_j g_{\underline{00}} + C_i \partial_j X^{\underline{\tilde{m}}} g_{\underline{0\tilde{m}}} + \partial_i X^{\underline{\tilde{m}}} C_j g_{\underline{\tilde{m}0}} + \partial_i X^{\underline{\tilde{m}}}\partial_j X^{\underline{\tilde{n}}} g_{\underline{\tilde{m}\tilde{n}}})$$

$$-2\epsilon^{ij} C_i \partial_j \hat{X}^{\underline{0}}]. \tag{3}$$

Eq. of motion for $\hat{X}^{\underline{0}}$ field enforces the curl free condition for worldvolume variables C_i, and, therefore, locally

$$C_i = \partial_i \tilde{X}^{\underline{(0)}}. \tag{4}$$

Strictly speaking $\tilde{X}^{\underline{(0)}}$ is different from $X^{\underline{0}}$. However, we can always sew their by virtue of the invariance of (2) under $X^{\underline{0}} \to X^{\underline{0}} + const$, so it can be supposed that $C_i = \partial_i X^{\underline{0}}$ and the action (2) is recovered.

[5] This notation rises to the notations of Ref. [4] and we would like to keep it suggesting that there will not be a confusion between isometry and time-like coordinates.

Another story begins if one integrates out the new variables C_i. To see it, it is convenient to record the action (2) in the form of

$$S = \int_{\mathcal{M}^2} dX^{\underline{m}} \wedge *dX^{\underline{n}} g_{\underline{mn}} \tag{5}$$

and to make the same procedure as in (3). Therefore

$$S = \int_{\mathcal{M}^2} C \wedge *C g_{\underline{00}} + C \wedge *dX^{\underline{\tilde{n}}} g_{\underline{0\tilde{n}}} + dX^{\underline{\tilde{m}}} \wedge *C g_{\underline{\tilde{m}0}} + dX^{\underline{\tilde{m}}} \wedge *dX^{\underline{\tilde{n}}} g_{\underline{\tilde{m}\tilde{n}}} - 2C \wedge d\hat{X}^{\underline{0}}. \tag{6}$$

Integrating out the C field we derive

$$\frac{\overleftarrow{\delta}\,\mathcal{L}}{\delta C} = *C g_{\underline{00}} + *dX^{\underline{\tilde{n}}} g_{\underline{0\tilde{n}}} - d\hat{X}^{\underline{0}} = 0,$$

$$C = -\frac{1}{g_{\underline{00}}}(i_0 g - *d\hat{X}^{\underline{0}}), \qquad *C = -\frac{1}{g_{\underline{00}}}(*i_0 g - d\hat{X}^{\underline{0}}) \tag{7}$$

and

$$S = \int_{\mathcal{M}^2} dX^{\underline{\tilde{m}}} \wedge *dX^{\underline{\tilde{n}}} g_{\underline{\tilde{m}\tilde{n}}} - \frac{1}{g_{\underline{00}}} i_0 g \wedge *i_0 g + \frac{1}{g_{\underline{00}}} d\hat{X}^{\underline{0}} \wedge *d\hat{X}^{\underline{0}} + \frac{2}{g_{\underline{00}}} i_0 g \wedge d\hat{X}^{\underline{0}}. \tag{8}$$

Important point is that the action (8) written in the dual with respect to the C field variables $d\hat{X}^{\underline{0}}$ still allows for eq. (4), however it becomes "Bianchi identity" for the solution to the equation of motion for C following from the original action [5] (see also Refs. [33] and [34]).

It is easy to see that this action is equivalent to the string action propagating in dual background, which is parameterized by the coordinates $\tilde{X}^{\underline{m}} = (X^{\underline{\tilde{m}}}, \hat{X}^{\underline{0}})$, with the NS 2-form potential $\tilde{B}^{(2)}$, i.e.

$$S = \int_{\mathcal{M}^2} d\tilde{X}^{\underline{m}} \wedge *d\tilde{X}^{\underline{n}} \tilde{g}_{\underline{mn}} + 2\tilde{B}^{(2)}, \tag{9}$$

where [4]

$$\tilde{g}_{\underline{\tilde{m}\tilde{n}}} = g_{\underline{\tilde{m}\tilde{n}}} - \frac{1}{g_{\underline{00}}} g_{\underline{0\tilde{m}}} g_{\underline{0\tilde{n}}}, \qquad \tilde{g}_{\underline{00}} = \frac{1}{g_{\underline{00}}},$$

$$\tilde{B}^{(2)}_{\underline{0m}} = -\tilde{B}^{(2)}_{\underline{m0}} = \frac{g_{\underline{0\tilde{m}}}}{g_{\underline{00}}}, \qquad \tilde{B}^{(2)}_{\underline{\tilde{m}\tilde{n}}} = 0. \tag{10}$$

Therefore, target-space duality switches the pure gravitational background to the dual background with gravity and the NS 2-form gauge field. However, if we start from a string in the background of the NS two-form field $B^{(2)}$

$$S = \int_{\mathcal{M}^2} dX^{\underline{m}} \wedge *dX^{\underline{n}} g_{\underline{mn}} + 2B^{(2)} \tag{11}$$

and apply the scheme noticed above again, we arrive at

$$S = \int_{\mathcal{M}^2} dX^{\underline{\tilde{m}}} \wedge *dX^{\underline{\tilde{n}}} g_{\underline{\tilde{m}\tilde{n}}} - \frac{1}{g_{\underline{00}}} i_0 g \wedge *i_0 g + \frac{1}{g_{\underline{00}}} (d\hat{X} + i_0 B) \wedge *(d\hat{X} + i_0 B)$$

$$+ \frac{2}{g_{\underline{00}}} i_0 g \wedge (d\hat{X} + i_0 B) + dX^{\underline{\tilde{m}}} \wedge dX^{\underline{\tilde{n}}} B^{(2)}_{\underline{\tilde{n}\tilde{m}}}, \tag{12}$$

which is the action for the string in dual background

$$S = \int_{\mathcal{M}^2} d\tilde{X}^{\underline{m}} \wedge *d\tilde{X}^{\underline{n}} \tilde{g}_{\underline{mn}} + 2\tilde{B}^{(2)} \tag{13}$$

with

$$\tilde{g}_{\underline{\tilde{m}\tilde{n}}} = g_{\underline{\tilde{m}\tilde{n}}} - \frac{1}{g_{\underline{00}}}(g_{\underline{0\tilde{m}}} g_{\underline{0\tilde{n}}} - i_0 B^{(2)}_{\underline{\tilde{m}}} i_0 B^{(2)}_{\underline{\tilde{n}}}), \qquad \tilde{g}_{\underline{00}} = \frac{1}{g_{\underline{00}}},$$
$$\tilde{B}^{(2)(-)} = B^{(2)(-)} + \frac{i_0 g}{g_{\underline{00}}} \wedge i_0 B^{(2)}, \quad i_0 \tilde{B}^{(2)} = \frac{i_0 g}{g_{\underline{00}}}, \quad i_0 B^{(2)} = \frac{i_0 \tilde{g}}{\tilde{g}_{\underline{00}}}, \tag{14}$$

where $i_0 g \equiv dX^{\underline{\tilde{m}}} g_{\underline{0\tilde{m}}}$, $i_0 B \equiv dX^{\underline{\tilde{m}}} B_{\underline{0\tilde{m}}}$ and $B^{(2)(-)} \equiv \frac{1}{2} dX^{\underline{\tilde{m}}} \wedge dX^{\underline{\tilde{n}}} B^{(2)}_{\underline{\tilde{n}\tilde{m}}}$.

Another way to get the same result is to consider the isometry gauging procedure [24]–[29]. To recall, remind that as we have pointed above the action (3) possesses the invariance under the constant shift in the $X^{\underline{0}}$ direction, i.e. under $X^{\underline{0}} \to X^{\underline{0}} + const$. Let us now to require the invariance of the action under the local shift with some arbitrary function $f(X^{\underline{\tilde{m}}})$. In this case the invariance is spoiled by the terms constructed out the differentials of new function $f(X^{\underline{\tilde{m}}})$. To compensate this contribution it is necessary to extend usual derivative with vector field as

$$DX^{\underline{m}} = dX^{\underline{m}} + Ck^{\underline{m}}, \tag{15}$$

where $k^{\underline{m}}$ is the Killing vector in the isometry direction. Then, in the so-called adapted frame where $k^{\underline{m}} = \delta^{\underline{m}}_{\underline{0}}$ the covariant derivative $DX^{\underline{0}}$ is inert under $\delta X^{\underline{0}} = f$ supported by $\delta C = -d\epsilon$ with local parameter $\epsilon(X^{\underline{\tilde{m}}})$. Therefore, in this picture the action (6) becomes

$$S = \int_{\mathcal{M}^2} DX^{\underline{m}} \wedge *DX^{\underline{n}} g_{\underline{mn}} - 2C \wedge d\hat{X}^{\underline{0}}. \tag{16}$$

Equation of motion for the $\hat{X}^{\underline{0}}$ gives the curl free condition for the field C and therefore, locally we can solve it as $C = d\tilde{X}^{\underline{0}}$. By virtue of the gauge symmetry for the vector field C we can fix the gauge $C = 0$ or $X^{\underline{0}} = 0$. In both cases the action (16) reduces to the action (5) (either in terms of $(X^{\underline{0}}, X^{\underline{\tilde{m}}})$ coordinates or $(\tilde{X}^{\underline{0}}, X^{\underline{\tilde{m}}})$ ones). In its turn the integration over the C field in the adapted coordinate frame leads after imposing the gauge fixing $X^{\underline{0}} = 0$ to the Buscher's rules (10).

3 T-duality in Kaluza-Klein picture

As it has been outlined in Introduction, at the level of firstly quantized string theory T-duality deals with compactification and interchange of Kaluza-Klein (KK) and winding modes. Let's analyse this picture from the point of view of T-duality rules (14). Namely, the first rule for $\tilde{g}_{\underline{\tilde{m}\tilde{n}}}$ can be represented as

$$\tilde{g}_{\underline{\tilde{m}\tilde{n}}} - \frac{1}{g_{\underline{00}}} i_0 B^{(2)}_{\underline{\tilde{m}}} i_0 B^{(2)}_{\underline{\tilde{n}}} = g_{\underline{\tilde{m}\tilde{n}}} - \frac{1}{g_{\underline{00}}} g_{\underline{0\tilde{m}}} g_{\underline{0\tilde{n}}} \qquad \Longrightarrow$$
$$\tilde{g}_{\underline{\tilde{m}\tilde{n}}} - \tilde{g}_{\underline{00}} \frac{i_0 \tilde{g}_{\underline{\tilde{m}}}}{\tilde{g}_{\underline{00}}} \frac{i_0 \tilde{g}_{\underline{\tilde{n}}}}{\tilde{g}_{\underline{00}}} = g_{\underline{\tilde{m}\tilde{n}}} - g_{\underline{00}} \frac{i_0 g_{\underline{\tilde{m}}}}{g_{\underline{00}}} \frac{i_0 g_{\underline{\tilde{n}}}}{g_{\underline{00}}}, \tag{17}$$

where we have used the rules for other fields. The expressions in both sides of (17) are nothing but the elements $\hat{g}_{\underline{\tilde{m}\tilde{n}}}$ and $\hat{\tilde{g}}_{\underline{\tilde{m}\tilde{n}}}$ of the metric tensor matrix decomposition

$$\overline{g}_{\underline{\tilde{m}\tilde{n}}} = \begin{pmatrix} \hat{g}_{\underline{\tilde{m}\tilde{n}}} := g_{\underline{\tilde{m}\tilde{n}}} - \frac{1}{g_{\underline{00}}} i_0 g_{\underline{\tilde{m}}} i_0 g_{\underline{\tilde{n}}} & g_{\underline{00}} \frac{i_0 g_{\underline{\tilde{m}}}}{g_{\underline{00}}} \\ g_{\underline{00}} \frac{i_0 g_{\underline{\tilde{n}}}}{g_{\underline{00}}} & g_{\underline{00}} \end{pmatrix},$$

$$\overline{\tilde{g}}_{\underline{\tilde{m}\tilde{n}}} = \begin{pmatrix} \hat{\tilde{g}}_{\underline{\tilde{m}\tilde{n}}} := \tilde{g}_{\underline{\tilde{m}\tilde{n}}} - \frac{1}{\tilde{g}_{\underline{00}}} i_0 \tilde{g}_{\underline{\tilde{m}}} i_0 \tilde{g}_{\underline{\tilde{n}}} & \tilde{g}_{\underline{00}} \frac{i_0 \tilde{g}_{\underline{\tilde{m}}}}{\tilde{g}_{\underline{00}}} \\ \tilde{g}_{\underline{00}} \frac{i_0 \tilde{g}_{\underline{\tilde{n}}}}{\tilde{g}_{\underline{00}}} & \tilde{g}_{\underline{00}} \end{pmatrix},$$

respecting to the following KK backgrounds [6]

$$d\overline{s}^2 = \hat{g}_{\underline{\tilde{m}\tilde{n}}} dX^{\underline{\tilde{m}}} \otimes dX^{\underline{\tilde{n}}} + g_{\underline{00}}(dX^{\underline{0}} + A) \otimes (dX^{\underline{0}} + A) \tag{18}$$

and

$$d\overline{\tilde{s}}^2 = \hat{\tilde{g}}_{\underline{\tilde{m}\tilde{n}}} dX^{\underline{\tilde{m}}} \otimes dX^{\underline{\tilde{n}}} + \tilde{g}_{\underline{00}}(dX^{\underline{0}} + \tilde{A}) \otimes (dX^{\underline{0}} + \tilde{A}) \tag{19}$$

with $A = i_0 g/g_{\underline{00}}$ and $\tilde{A} = i_0\tilde{g}/\tilde{g}_{\underline{00}}$. Therefore, in the KK picture T-duality rules look like [8], [17]

$$\begin{aligned} &\hat{g}_{\underline{\tilde{m}\tilde{n}}} = \hat{\tilde{g}}_{\underline{\tilde{m}\tilde{n}}}, \qquad g_{\underline{00}} = \frac{1}{\tilde{g}_{\underline{00}}}, \\ &\tilde{B}^{(2)-} = B^{(2)-} + A \wedge i_0 B^{(2)}, \\ &A = i_0 \tilde{B}^{(2)}, \qquad \tilde{A} = i_0 B^{(2)} \end{aligned} \tag{20}$$

and read off the interchange of the KK and winding modes. Apparently that $g_{\underline{00}}$ determines the length of the internal circle, and therefore there is also $R \leftrightarrow 1/R$.

4 Gauged sigma-model representation and T-duality

Let's turn the attention to the gauged sigma-model representation of string action (eq. (16)). One of the powerful points of such a representation consists in the possibility to arrive at the result of integration over C field (eqs. (8) and (12)) effectively without performing the integration. To this end let us regard the field C (or $\tilde{C}$ in dual picture) as *additional* worldvolume field (as it should be), and starting from the dual string action

$$S = \int_{\mathcal{M}^2} DX^{\underline{m}} \wedge *DX^{\underline{n}} \tilde{g}_{\underline{mn}} + 2\tilde{B}^{(2)}(D) - 2\tilde{C} \wedge d\hat{X}^{\underline{0}} \tag{21}$$

rewrite it as

$$\begin{aligned} S = \int_{\mathcal{M}^2} [&dX^{\underline{\tilde{m}}} \wedge *dX^{\underline{\tilde{n}}} \hat{\tilde{g}}_{\underline{\tilde{m}\tilde{n}}} + \tilde{g}_{\underline{00}}(dX^{\underline{0}} + \tilde{C} + \frac{i_0\tilde{g}}{\tilde{g}_{\underline{00}}}) \wedge *(dX^{\underline{0}} + \tilde{C} + \frac{i_0\tilde{g}}{\tilde{g}_{\underline{00}}}) \\ &+ 2\tilde{B}^{(2)-}(d) + 2i_0\tilde{B}^{(2)} \wedge (dX^{\underline{0}} + \tilde{C}) - 2\tilde{C} \wedge d\hat{X}^{\underline{0}}]. \end{aligned} \tag{22}$$

[6]Strictly speaking, these are effective "KK backgrounds" since their form is derived from D-dimensional line elements $d\overline{s}^2 = \overline{g}_{\underline{mn}} dX^{\underline{m}} \otimes dX^{\underline{n}}$ and $d\overline{\tilde{s}}^2 = \overline{\tilde{g}}_{\underline{mn}} dX^{\underline{m}} \otimes dX^{\underline{n}}$ by extracting $X^{\underline{0}}$ coordinate and rearranging the terms in T-duality invariant combinations.

Now solving for the equation for $\hat{X}^{\underline{0}}$

$$\tilde{C} = dy, \quad (can \;\; be \;\; gauged \;\; out)$$

and applying the T-duality rules (20) we are left with expression (12) (in the coinciding original and dual coordinates basis), which was obtained from the original string action (1)–(3) by use of equation of motion for the C field, i.e.

$$S = \int_{\mathcal{M}^2} \mathcal{L}^{dual\;\;gaug.s.mod.}_{|KK\;\;T-duality\;\;rules} = \int_{\mathcal{M}^2} \mathcal{L}^{orig.}_{|\frac{\delta\mathcal{L}^{orig.}}{\delta C}=0}. \tag{23}$$

Actually, this statement is trivial, because it says about equivalence of the original theory written in terms of dual variables (after resolving for the equation of motion for C field) to its dual theory written in terms of the same variables (after application of T-duality rules). However, this notion becomes crucial in the consideration of T-duality in the effective action constructed out the part of the bosonic sector of supergravity with the fundamental string as a source.

5 Effective action with a string source and T-duality

After preliminaries made in the sections before we are in position to consider the model describing the effective action with string as a source for gravity and 2-form antisymmetric tensor field. This action has the following form [35]

$$S = \int_{\mathcal{M}^D} e^{-2\phi}\left[\frac{1}{(D-2)!}R^{\underline{a}_1\underline{a}_2}E^{\underline{a}_3}\dots E^{\underline{a}_D}\epsilon_{\underline{a}_1\dots\underline{a}_D} + \frac{1}{2}d\phi * d\phi + \frac{1}{2}H^{(3)} * H^{(3)}\right]$$

$$-\frac{1}{2}\int_{\mathcal{M}^D} d^Dx\sqrt{-g}\frac{\delta^D(x-X(\xi))}{\sqrt{-g}}\int_{\mathcal{M}^2} d^2\xi[\sqrt{-\gamma}\gamma^{ij}\partial_i X^{\underline{m}}\partial_j X^{\underline{n}} g_{\underline{mn}}(x) + \epsilon^{ij}\partial_i X^{\underline{m}}\partial_j X^{\underline{n}} B^{(2)}_{\underline{nm}}], \tag{24}$$

where $H^{(3)} = dB^{(2)}$ and the wedge product is assumed. The variation of this action over $g_{\underline{mn}}$ and $B^{(2)}_{\underline{mn}}$ leads to the Einstein equation for gravity with the dilaton and the NS 2-form gauge field in the presence of matter source and to the equation for the NS 2-form field strength with the string source

$$R_{\underline{mn}} - \frac{1}{2}Rg_{\underline{mn}} - \frac{1}{2}(\partial_{\underline{m}}\phi\partial_{\underline{n}}\phi - \frac{1}{2}g_{\underline{mn}}(\partial\phi)^2) - \frac{1}{4}(H^{(3)}_{\underline{mpq}}H^{(3)\underline{pq}}_{\underline{n}} - \frac{1}{6}g_{\underline{mn}}H^{(3)2}) = e^{2\phi}T_{\underline{mn}}, \tag{25}$$

$$(-)^{(D-4)}\partial_{\underline{m}}(e^{-2\phi}\sqrt{-g}H^{(3)\underline{mnp}}) = \int_{\mathcal{M}^D} d^Dx\sqrt{-g}\frac{\delta^D(x-X(\xi))}{\sqrt{-g}}\int_{\mathcal{M}^2} d^2\xi\epsilon^{ij}\partial_i X^{\underline{n}}\partial_j X^{\underline{p}} \tag{26}$$

with $T_{\underline{mn}}$ being the energy-momentum tensor of the string

$$T^{\underline{mn}} = \int_{\mathcal{M}^D}\frac{\delta^D(x-X(\xi))}{\sqrt{-g}}\int d^2\xi\sqrt{-\gamma}\gamma^{ij}\partial_i X^{\underline{m}}\partial_j X^{\underline{n}}.$$

Now try to implement the consideration above to the case of the effective action (24). There are at least two different ways to establish the T-duality covariance.

Following the first way, it is necessary to write down the source part of the action (24) in the first order form by use of *worldvolume* field C_i. Then, integrating out the C field, one arrives at the T-duality transformations (14). In its turn, the bulk part of the action propagates in the background defined by the line element

$$d\overline{s}^2 = \overline{g}_{\underline{mn}} dX^{\underline{m}} \otimes dX^{\underline{n}} = \hat{g}_{\underline{\tilde{m}\tilde{n}}} dX^{\underline{\tilde{m}}} \otimes dX^{\underline{\tilde{n}}} + g_{\underline{00}}(dX^{\underline{0}} + A) \otimes (dX^{\underline{0}} + A).$$

Therefore, we should put it down in the KK manner, apply the rules (20) which is equivalent to (14) together with the dilaton shift (cf. eq. (30)) and lift it up. This sequence leads to desired result (cf. eq. (31)).

The second way allows one to realize the gauged sigma-model-like technique at the level of whole action (24) in uniform way having in mind that string propagates in special background allowing for the isometry directions. To this end, as in the sections before, extend the derivatives to the covariant ones, i.e.

$$dX^{\underline{m}} \longrightarrow DX^{\underline{m}} = dX^{\underline{m}} + Ck^{\underline{m}}$$

with *target-space* additional field $C(X^{\underline{\tilde{m}}})$, and insert it into the action [7] :

$$S = \int_{\mathcal{M}^D} e^{-2\phi} \left[\frac{1}{(D-2)!} R^{\underline{a}_1\underline{a}_2}(D) E^{\underline{a}_3}(D) \dots E^{\underline{a}_D}(D) \epsilon_{\underline{a}_1\dots\underline{a}_D} + \frac{1}{2} D\phi * D\phi + \frac{1}{2} H^{(3)}(D) * H^{(3)}(D) \right]$$

$$-\frac{1}{2} \int_{\mathcal{M}^D} d^D x \sqrt{-g} \frac{\delta^D(x - X(\xi))}{\sqrt{-g}} \times$$

$$\int_{\mathcal{M}^2} d^2\xi [\sqrt{-\gamma}\gamma^{ij} D_i X^{\underline{m}} D_j X^{\underline{n}} g_{\underline{mn}}(x) + \epsilon^{ij} D_i X^{\underline{m}} D_j X^{\underline{n}} B^{(2)}_{\underline{nm}} - 2\epsilon^{ij} C_i \partial_j \hat{X}^{\underline{0}}]. \tag{27}$$

The curl free field C does not spoil the field content of the original theory. This is due to the symmetry under the local shifts in the isometry direction which gives a possibility to fix either $C = 0$ or $X^{\underline{0}} = 0$ gauge. As in the case of the gauged sigma-model representation of string action (16), eq. (27) reduces to the eq. (24).

Consider now the first line of eq. (27) and drop out for the moment the curl free condition for C. Then, the background line element is defined by $d\overline{s}^2 = DX^{\underline{m}} \otimes DX^{\underline{n}} \overline{g}_{\underline{mn}}$ that is equivalent to the following KK-type representation

$$d\overline{s}^2 = \hat{g}_{\underline{\tilde{m}\tilde{n}}} dX^{\underline{\tilde{m}}} \otimes dX^{\underline{\tilde{n}}} + g_{\underline{00}}(dX^{\underline{0}} + C + A) \otimes (dX^{\underline{0}} + C + A). \tag{28}$$

Therefore, the symbol $\mathcal{L}^{(D)}_{grav.+dil.+NS2.}(D)$ notifies that the first line of (27) is the compressed expression for theory propagating in the KK-background (28) and having therefore the form [8]

$$S = \int d^{(D-1)}x \ e^{-2\phi} \sqrt{g_{\underline{00}}} \sqrt{-g} R + \int_{\mathcal{M}^{(D-1)}} e^{-2\phi} \sqrt{g_{\underline{00}}} [\frac{1}{2} d\phi * d\phi + \frac{1}{2} d(\log|g_{\underline{00}}|) * d(\log|g_{\underline{00}}|)$$

$$+ \frac{1}{2} g_{\underline{00}} d(C+A) * d(C+A) + \frac{1}{2} H'^{(3)} * H'^{(3)} + \frac{1}{2} g^{-1}_{\underline{00}} d(i_0 B^{(2)}) * d(i_0 B^{(2)})]. \tag{29}$$

Following our strategy it is necessary to find a solution to the equation of motion for the C field and to insert it into the action. But as it has been pointed out in Section 2 the equation $C = dy$ still holds. Therefore, effectively, (29) does not depend on C and we return to the

[7] In (27) C_i is a pullback of the bulk one-form C, i.e. $C_i = D_i X^{\underline{m}} C_{\underline{m}} \equiv \partial_i X^{\underline{\tilde{m}}} C_{\underline{\tilde{m}}}$ since in the adapted coordinate frame $i_k C = 0$.

[8] In (29) $H'^{(3)} = H^{(3)} + d(i_0 B^{(2)}) A$ and the Hodge star is the $(D-1)$-dimensional operator.

situation discussed previously in the free case. After insertion of the solution for the C equation of motion (7) and application of the KK T-duality rules, which, as previously mentioned, have to be completed by the dilaton shift

$$\tilde{\phi} = \phi - \frac{1}{2} \log |g_{\underline{00}}|, \tag{30}$$

the action (27) converts into

$$S = \int_{\mathcal{M}^D} e^{-2\tilde{\phi}} \left[\frac{1}{(D-2)!} \tilde{R}^{\underline{a}_1 \underline{a}_2} E^{\underline{a}_3} \dots E^{\underline{a}_D} \epsilon_{\underline{a}_1 \cdots \underline{a}_D} + \frac{1}{2} d\tilde{\phi} * d\tilde{\phi} + \frac{1}{2} \tilde{H}^{(3)} * \tilde{H}^{(3)} \right]$$

$$-\frac{1}{2} \int_{\mathcal{M}^D} d^D x \sqrt{-g} \frac{\delta^D(x - X(\xi))}{\sqrt{-g}} \int_{\mathcal{M}^2} d^2\xi [\sqrt{-\gamma} \gamma^{ij} \partial_i X^{\underline{m}} \partial_j X^{\underline{n}} \tilde{g}_{\underline{mn}}(x) + \epsilon^{ij} \partial_i X^{\underline{m}} \partial_j X^{\underline{n}} \tilde{B}^{(2)}_{\underline{nm}}]. \tag{31}$$

Hence, T-duality indeed is the symmetry of the string effective action with a string source.

6 Discussion and conclusions

Thus, we have demonstrated that T-duality still holds for the system comprising the part of the bosonic sector of supergravity and the fundamental string source following two ways. The first one is the standard way and it shows the influence of worldvolume duality to the target-space fields. The second way emphasizes the role of isometry in T-duality consideration and gives the possibility to realize in uniform manner the statement that both bulk and string parts of the effective action with source propagate in background with isometry [9]. However, following this way, we have to consider additional target-space field rather than the worldvolume one. Due to this fact one has also to take into consideration the bulk contribution to the equation of motion for this new field. It looks surprisingly that establishing the invariance under the T-duality transformations sends in common to the uncoupling string case. At first glance one can expect to arrive to much more complicated problem especially in the part of evaluation of the solution to the equation of motion for the C field deriving from the action (27). However, let's analyse the problem from another point of view. As in the case of uncoupling string, reviewed in Section 4, one can start from the consideration of the action (31). Then, inserting the extended derivatives and gauging out the $\tilde{C}$ field, after application of T-duality rules we arrive at the intermediate result, which is schematically the same as it was sketched in eq. (23) [10]. Important point is that the KK gravity part in the l.h.s. of this picture does not contain any dependence on $d\hat{X}^{\underline{0}}$ being dual to the $\tilde{C}$ field. On the other hand, in view of (23), the KK gravity part in the r.h.s. is also free from the dependence on $d\hat{X}^{\underline{0}}$, but already with taking into account the solution to the equation of motion for the C field. Therefore, we conclude that no contribution comes from the KK gravity part to the equation of motion for the C field and therefore we can consider that $C = dy$ holds as the "Bianchi identity" for the equation of motion for its dual.

Although our consideration is restricted to the case of one isometry direction, the proposed construction generalizes straightforwardly to the case of arbitrary number of commuting isometries. In view of gauging isometries technique we may expect also that this scheme can be extended to the non-Abelian case.

[9]Other aspects of space-time symmetry gauging in application e.g. to the KK dimensional reduction can be found in [36].

[10]This result remains the same for regarding C as the bulk field as well.

Another interesting problem is to apply this scheme to the case of open string and D-branes and to evaluate the T-duality power for the effective action in presence of the D-branes sources. Also it should be very attractive to realize completely supersymmetric picture of supergravity coupling with superstring as a source and to demonstrate the invariance under the (super) T-duality transformations. We hope to return to these problems in the forthcoming publications.

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