Conformal Anomalies in Noncommutative Gauge Theories

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Abstract

We calculate conformal anomalies in noncommutative gauge theories by using the path integral method (Fujikawa's method). Along with the axial anomalies and chiral gauge anomalies, conformal anomalies take the form of the straightforward Moyal deformation in the corresponding conformal anomalies in ordinary gauge theories. However, the Moyal star product leads to the difference in the coefficient of the conformal anomalies between noncommutative gauge theories and ordinary gauge theories. The β (Callan-Symanzik) functions which are evaluated from the coefficient of the conformal anomalies coincide with the result of perturbative analysis.

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1 Introduction

Gauge theories on noncommutative space-time (noncommutative gauge theories) have recently attracted much attention (for a review, see [1]). This is partly due to the realization that such theories actually occur in string theory with a constant NS-NS two-form field [2]. The noncommutative character of the Moyal star product leads to noncommutative gauge theories. For example, noncommutative $U(1)$ gauge theories have a character similar to ordinary non-Abelian gauge theories, although the gauge group is commutative. It is shown from perturbative analysis of the β function that the noncommutative $U(1)$ Yang–Mills theory is asymptotically free [15, 13]. An intrinsic feature of noncommutative gauge theories is the so-called UV/IR mixing [13]. The planar diagrams controlled the UV properties, while nonplanar diagrams generally lead to new IR phenomena through the mixing.

Axial anomalies and chiral gauge anomalies have been actively studied in noncommutative gauge theories [3]-[12]. These anomalies can be calculated by perturbative analysis and the path integral formulation (Fujikawa's method). Chiral gauge anomalies can also be described using generalized descent equations [5]. It is known that these anomalies take the form of a straightforward Moyal deformation in the corresponding anomalies in ordinary gauge theories. However, this modification includes physical consequences to the chiral gauge anomalies. The noncommutative character of the Moyal star product actually leads to more restrictive conditions for anomaly cancellation [4, 5]. The noncommutative chiral gauge theories with fermions in the fundamental representation are anomalous. The chiral gauge anomalies only come from planar diagrams in this representation. On the other hand, the noncommutative chiral gauge theories with fermions in the adjoint representation are anomaly-free (in four dimensions) [6, 7, 11]. Although not only planar diagrams but also nonplanar diagrams contribute to the chiral gauge anomalies in this representation, they cancel in each sector. Therefore, nonplanar diagrams do not contribute to the chiral gauge anomalies (in a single gauge group) [6, 12].

The noncommutative field theories include the noncommutativity parameter θ of dimension $[\text{length}]^2$. Therefore, it is expected that the scale (or dilatation) invariance of the field theories is broken at the classical level even if the field theories are massless field theories. The breaking for scale invariance at the classical level was actually investigated in the Moyal deformed massless scalar field theory. In the classical scalar field theory, the variation of the action under the infinitesimal scale transformation is proportional to the change in the noncommutativity parameter induced by infinitesimal scale transformation [23]. Therefore, the Moyal deformed massless scalar field theory is invariant under the scale transformation including the change in the noncommutativity parameter. On the other hand, the Weyl symmetry, which is closely related to the scale invariance, is broken as a result of quantum corrections in the ordinary field theories. This phenomenon is well known as conformal (or Weyl) anomalies. It is an interesting problem to study how conformal anomalies are deformed by the Moyal star products in the noncommutative field theories.

In this paper, we have calculated conformal anomalies in four dimensional noncommutative gauge theories (on flat space) with fermions in a fundamental representation. Variants of the path integral method (Fujikawa's method) will be found to be suited for the calculations. The calculation in the path integral method is simple, although we needs some knowledge of Weyl transformations and breaking. We advance calculation of the conformal anomaly to Abelian gauge theory, the noncommutative QED, first. The generalization to non-Abelian gauge theory, the noncommutative QCD, is straightforward. The paper is organized as follows. In Sec. 2, we state the method of calculation for the conformal anomaly in the path integral method based on Ref. [22] after introducing the background field method for noncommutative QED. In Sec. 3, we calculate the conformal anomaly in noncommutative QED at the one-loop level. In ordinary gauge theories, there is a relation between the conformal anomaly and the β function. Based on the relation, we evaluate the β function in noncommutative QED and compare with the result from perturbative analysis. The calculating method shown in Sec. 3 is generalizably straightforward in the noncommutative QCD. It is stated by Sec. 4. Sec. 5 is devoted to a summary and discussion.

2 The background field method for noncommutative QED

Noncommutative gauge theories can be obtained by replacing the ordinary products of fields in the actions of their commutative counterparts by the Moyal star products,

$$\begin{aligned} f(x) * g(x) &= e^{\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial\xi_\mu}\frac{\partial}{\partial\zeta_\nu}} f(x+\xi)g(x+\zeta)\Big|_{\xi=\zeta=0} \\ &= \int \frac{d^4p}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4} e^{-\frac{i}{2}p_\mu\theta^{\mu\nu}q_\nu} e^{i(p_\mu+q_\mu)x^\mu}\hat{f}(p)\hat{g}(q) \ , \end{aligned} \tag{2.1}$$

where $\theta^{\mu\nu} = -\theta^{\nu\mu}$ is an antisymmetric real matrix. It is known that the matrix $\theta^{\mu\nu}$ is constrained by imposing unitarity on a noncommutative quantum field theories. The only allowed types of the matrix $\theta^{\mu\nu}$ are spacelike and lightlike [14].

We begin with the noncommutative $U(1)$ Yang–Mills action,

$$S_{gauge}[A_\mu] = -\frac{1}{4g^2}\int d^4x \, F_{\mu\nu}(x) * F^{\mu\nu}(x) \ . \tag{2.2}$$

Here the field strength $F_{\mu\nu}(x)$ is

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - i[\, A_\mu(x), A_\nu(x)\,]_M \ , \tag{2.3}$$

with the Moyal bracket $[\, A, B\,]_M = A * B - B * A$. The action (2.2) is invariant under the infinitesimal gauge transformation $\delta A_\mu(x) = \partial_\mu\lambda(x) - i[\, A_\mu(x), \lambda(x)\,]_M$ with the infinitesimal gauge transformation parameter $\lambda(x)$.

In order to compute the effective action including the quantum effect of the gauge field, we introduce the background field method for noncommutative gauge theories [20, 18]. We decompose the gauge field A_μ into a background field B_μ and a fluctuating field a_μ,

$$A_\mu(x) = B_\mu(x) + a_\mu(x) \ . \tag{2.4}$$

Then the field strength decomposes as follows:

$$F_{\mu\nu} = F_{\mu\nu}[B] + D_\mu[B]a_\nu - D_\nu[B]a_\mu - i[\, a_\mu, a_\nu \,]_M \ , \tag{2.5}$$

where $F_{\mu\nu}[B]$ is the field strength of the background field B_μ, and $D_\mu[B]$ is the covariant derivative acting on the fluctuating field a_μ,

$$F_{\mu\nu}[B](x) \ \equiv \ \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x) - i[\, B_\mu(x), B_\nu(x) \,]_M \ , \tag{2.6}$$

$$D_\mu[B]a_\nu(x) \ \equiv \ \partial_\mu a_\nu(x) - i[\, B_\mu(x), a_\nu(x) \,]_M \ . \tag{2.7}$$

Substitution in the action (2.2) of the field strength (2.5) and integration by parts yields

$$\begin{aligned} S_{gauge}[a_\mu, B_\mu] \ = \ -\frac{1}{4g^2}\int d^4x \Big\{ & \ F_{\mu\nu}[B] * F^{\mu\nu}[B] + 4ia^\mu * [\ F_{\mu\nu}[B], \ a^\nu \ ]_M \\ & -2a_\mu * D_\nu[B]D^\nu[B]a^\mu - 2D_\mu[B]a^\mu * D_\nu[B]a^\nu + \mathcal{O}(a_\mu^3) \Big\} \ . \end{aligned} \tag{2.8}$$

In deriving this action, we have used the classical equation of motion for the background field B_μ. If the background field B_μ is regarded as fixed, the action (2.8) has the following local symmetry:

$$\delta a_\mu(x) = D_\mu[B]\lambda(x) - i[\ a_\mu(x), \ \lambda(x) \ ]_M \ . \tag{2.9}$$

In order to define the functional integral, we need to perform the gauge fixing for the local gauge symmetry implemented by the transformation (2.9). We choose a gauge fixing (GF) term and Faddeev–Popov (FP) ghost term (in the 't Hooft–Feynman gauge) as follows:

$$\begin{aligned} S_{GF+FP}[a_\mu, \ c, \ \bar{c}] \ = \ -\frac{1}{2g^2}\int d^4x \Big\{ & \ D_\mu[B]a^\mu * D_\nu[B]a^\nu \\ & -i\bar{c} * D^\mu[B](\ D_\mu[B]c - i[\ a_\mu, c \,]_M \ ) \Big\} \ , \end{aligned} \tag{2.10}$$

where $c(x)$ and $\bar{c}(x)$ are the ghost fields. Here the gauge fixing condition has been taken to be covariant with respect to the background field. We can obtain the gauge fixed action for the fluctuating field a_μ by adding Eqs. (2.8) and (2.10).

We next introduce the matter fields. The action for the massless fermion interacting with a background $U(1)$ gauge field B_μ is given by

$$S_{matter}[\bar{\psi}, \psi] = \int d^4x \, \bar{\psi}(x) * (i\not{D}[B])\psi(x) \ , \tag{2.11}$$

where the covariant derivative acting on the fermion is defined by [4, 6, 7]

$$\not{D}[B]\psi(x) \equiv \gamma^\mu \partial_\mu \psi(x) - i\gamma^\mu B_\mu(x) * \psi(x) \ . \tag{2.12}$$

The gauge-fixed action in the noncommutative QED is given by $S_{gauge} + S_{GF+FP} + S_{matter}$. Note that the gauge fixed action is still invariant under a local transformation:

$$\begin{aligned}
&\delta\psi(x) = -ig\lambda(x) * \psi(x) \ , \qquad \delta\bar{\psi}(x) = +ig\bar{\psi}(x) * \lambda(x) \ , \\
&\delta B_\mu(x) = \partial_\mu \lambda(x) - i[\ B_\mu(x),\ \lambda(x)\ ]_M \ , \qquad \delta a_\mu(x) = i[\ a_\mu(x),\ \lambda(x)\ ]_M \ , \\
&\delta c(x) = i[\ c(x),\ \lambda(x)\ ]_M \ , \qquad \delta\bar{c}(x) = i[\ \bar{c}(x),\ \lambda(x)\ ]_M \ .
\end{aligned} \tag{2.13}$$

The Wilsonian effective action is obtained by functional integration over the fluctuating field. The one-loop effective action $W[B]$ for the background field B_μ can be written as

$$\exp(-W[B]) = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}a_\mu\mathcal{D}c\mathcal{D}\bar{c}\ \exp\Big(S_{quad}[a_\mu] + S_{quad}[c,\ \bar{c}] + S_{matter}[\bar{\psi},\psi]\Big)\ , \tag{2.14}$$

with

$$S_{quad}[a_\mu] \ = \ \frac{1}{2g^2}\int d^4x\ \{\ a_\mu * D_\nu[B]D^\nu[B]a^\mu - 2ia_\mu * [\ F^{\mu\nu}[B],\ a_\nu\ ]_M\ \}\ , \tag{2.15}$$

$$S_{quad}[c,\ \bar{c}] \ = \ \frac{1}{4g^2}\int d^4x \Big\{\ i\bar{c} * D_\mu[B]D^\mu[B]c\ \Big\}\ . \tag{2.16}$$

Here, we perform a Wick rotation into Euclidean space-time with the metric $g_{\mu\nu} = -\delta_{\mu\nu}$ for the actual calculations.

In ordinary gauge theories, the one-loop conformal (or Weyl) anomalies can be simply evaluated by using the background field method in the path integral approach [22]. In this approach, the conformal anomalies are characterized as the Jacobian for the functional measure $\mathcal{D}\hat{\psi}\mathcal{D}\hat{\bar{\psi}}\mathcal{D}\hat{a}_\mu\mathcal{D}\hat{c}\mathcal{D}\hat{\bar{c}}$ with the field variables in flat space,

$$\begin{aligned}
&\hat{\psi}(x) \equiv \sqrt[4]{|g|}\psi(x)(=\psi(x))\ , \qquad \hat{\bar{\psi}}(x) \equiv \sqrt[4]{|g|}\bar{\psi}(x)(=\bar{\psi}(x))\ , \\
&\hat{a}_\mu(x) \equiv \sqrt[4]{|g|}e^i_\mu a_i(x)(=a_i(x))\ , \\
&\hat{c}(x) \equiv \sqrt[4]{|g|}c(x)(=c(x))\ , \qquad \hat{\bar{c}}(x) \equiv \sqrt[4]{|g|}\bar{c}(x)(=\bar{c}(x))\ ,
\end{aligned} \tag{2.17}$$

respectively. Here g is the determinant of the metric and e^i_μ is the vielbein in flat (Euclidean) space. The scale transformation can be regarded as a combination of the Weyl transformation and the coordinates transformation. The choice of the functional measure is dictated by the manifest covariance under the (general) coordinate transformation in curved space. Note that the redefinition of the field variables modifies Weyl transformations laws. We take the following transformation laws as Weyl transformation laws on noncommutative space:

$$
\begin{aligned}
&\psi(x) \to \tilde{\psi}(x) = \exp\left(-\tfrac{1}{2}\alpha(x)\right) * \psi(x) \ , \quad \bar{\psi}(x) \to \tilde{\bar{\psi}}(x) = \bar{\psi}(x) * \exp\left(-\tfrac{1}{2}\alpha(x)\right) \ , \\
&a_\mu(x) \to \tilde{a}_\mu(x) = \exp\left(-\alpha(x)\right) * a_\mu(x) \ , \\
&c(x) \to \tilde{c}(x) = c(x) * \exp\left(-2\alpha(x)\right) \ , \qquad \bar{c}(x) \to \tilde{\bar{c}}(x) = \bar{c}(x) \ ,
\end{aligned}
\tag{2.18}
$$

where $\alpha(x)$ is an infinitesimal arbitrary function. When the Moyal star products in Eqs. (2.18) are restored to the ordinary (commutative) products, the transformation laws (2.18) are also restored to the ordinary Weyl transformation laws for the field variables (2.17). In the next section, we derive the conformal anomaly in noncommutative QED (in the flat space limit) on the basis of the path integral approach. For this purpose, we will evaluate the associated Jacobian of functional measure in Eq. (2.14) under the transformation laws (2.18). For convenience, however, we suppose $\alpha(x)$ is an infinitesimal arbitrary constant hereafter. Namely, we treat the global Weyl transformations.

3 The conformal anomaly in noncommutative QED

3.1 The contribution from matter fields

We first evaluate the contribution from the matter fields to the conformal anomaly. The global Weyl transformation laws for the matter fields are given by

$$
\begin{aligned}
\psi(x) \quad &\longrightarrow \quad \tilde{\psi}(x) = \exp\left(-\frac{1}{2}\alpha\right)\psi(x) \ , \\
\bar{\psi}(x) \quad &\longrightarrow \quad \tilde{\bar{\psi}}(x) = \bar{\psi}(x)\exp\left(-\frac{1}{2}\alpha\right) \ ,
\end{aligned}
\tag{3.1}
$$

where α is a constant parameter. In order to define the integral measure of the fermionic fields more accurately, we decompose $\psi(x)$ and $\bar{\psi}(x)$ into eigenfunctions of the Dirac operator defined in Eq. (2.12),

$$
\psi(x) = \sum_n a_n \varphi_n(x) \ , \qquad \bar{\psi}(x) = \sum_n \bar{b}_n \varphi_n^\dagger(x) \ . \tag{3.2}
$$

The coefficients a_n and $\bar{b}_n$ are Grassmann numbers. The Dirac operator $\not{D}[B]$ has real eigenvalues λ_n

$$
\not{D}[B]\varphi_n(x) = \lambda_n \varphi_n(x) \ , \tag{3.3}
$$

and the set of eigenfunctions $\{\varphi_n(x)\}$ is complete. We assume that the set of eigenfunctions is orthonormal:

$$\int d^4x\, \varphi_n^\dagger(x) * \varphi_m(x) \; (= \int d^4x\, \varphi_n^\dagger(x)\varphi_m(x)) \;= \delta_{nm} \;. \tag{3.4}$$

Under the infinitesimal transformations (3.1), the integration measure of the fermionic fields transforms as

$$\mathcal{D}\widetilde{\psi}\mathcal{D}\widetilde{\bar{\psi}} = J_\psi[\alpha]\mathcal{D}\psi\mathcal{D}\bar{\psi} \;, \tag{3.5}$$

with the Jacobian

$$\begin{aligned} J_\psi[\alpha] \;&=\; \det\Big[\; \delta_{nm} - \frac{1}{2}\alpha \int d^4x\, \varphi_n^\dagger(x) * \varphi_m(x) \;\Big]^{-2} \\ &=\; \exp\Big[\; \alpha \sum_n \int d^4x\, \varphi_n^\dagger(x) * \varphi_n(x) \;\Big] \;. \end{aligned} \tag{3.6}$$

In deriving the second line, we have used the identity $\ln\det = \mathrm{Tr}\ln$. In the same way as the evaluation of chiral anomalies, we regularize the Jacobian (3.6) with a Gaussian damping factor at hand,

$$\begin{aligned} J_\psi[\alpha] \;&\equiv\; \lim_{\epsilon\longrightarrow 0} \exp\Big[\; \alpha \sum_n \int d^4x\, \exp\Big(-\,\epsilon\, \lambda_n^2\;\Big)\, \varphi_n^\dagger(x) * \varphi_n(x) \;\Big] \\ &=\; \lim_{\epsilon\longrightarrow 0} \exp\Big[\; \alpha \sum_n \int d^4x\, \Big(\, \exp_*(-\,\epsilon\, \not{D} * \not{D}) * \varphi_n^\dagger(x) \,\Big) \;* \varphi_n(x) \;\Big] \;. \end{aligned} \tag{3.7}$$

Here the damping factor $\exp_*$ is defined by $\exp_* x \equiv 1 + x + \frac{1}{2!}x * x + \cdots$. By expanding $\varphi_n(x)$ in plane waves, we can rewrite the Jacobian $J_\psi[\alpha]$ into the form

$$J_\psi[\alpha] = \exp\Big[\; \alpha \int d^4x\, \mathcal{A}_\psi(x) \;\Big] \;,$$

with

$$\int d^4x\, \mathcal{A}_\psi(x) \equiv \lim_{\epsilon\longrightarrow 0} \int d^4x \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\Big[\, \Big(\, \exp_*(-\,\epsilon\, \not{D} * \not{D}) * e^{ik\cdot x} \,\Big) \;* e^{-ik\cdot x} \;\Big] \;, \tag{3.8}$$

where tr[ ] denotes a trace over the Dirac matrices γ^μ. By using the identity $\gamma^\mu\gamma^\nu = g^{\mu\nu} + \sigma^{\mu\nu}(\equiv \frac{1}{2}[\gamma^\mu,\ \gamma^\nu])$, we obtain

$$\begin{aligned} &\int d^4x\, \mathcal{A}_\psi \\ &= \lim_{\epsilon\longrightarrow 0} \int d^4x \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\Big[\, \Big(\, \exp_*\Big\{ -\epsilon \,\Big(\; D_\mu * D^\mu - \frac{i}{2}\sigma^{\mu\nu}F_{\mu\nu}(x) \,\Big) \Big\} * e^{ik\cdot x} \,\Big) * e^{-ik\cdot x} \,\Big] \\ &= \lim_{\epsilon\longrightarrow 0} \int d^4x \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\Big[\, \exp_*\Big\{ -\epsilon \,\Big(\, (ik_\mu + D_\mu) * (ik^\mu + D^\mu) - \frac{i}{2}\sigma^{\mu\nu}F_{\mu\nu}(x) \,\Big) \Big\} \,\Big] \;. \end{aligned} \tag{3.9}$$

In deriving the second line of Eq. (3.9), we have utilized the fact that $(\partial_\mu e^{ik\cdot x})*e^{-ik\cdot x} = ik_\mu + \partial_\mu$ and $e^{ip\cdot x} * e^{ik\cdot x} * e^{-ik\cdot x} = e^{ip\cdot x}$. Note that the background gauge field in the covariant derivative D_μ and its field strength do not depend on the momentum k_μ.

After rescaling the momentum $k_\mu \to k_\mu/\sqrt{\epsilon}$, we have

$$\int d^4x\, \mathcal{A}_\psi = \lim_{\epsilon\longrightarrow 0} \frac{1}{\epsilon^2} \int d^4x \int \frac{d^4k}{(2\pi)^4}\, e^{k_\mu k^\mu} \\ \mathrm{tr}\left[\, \exp_* \left\{ -2i\sqrt{\epsilon} k^\mu D_\mu - \epsilon D_\mu * D^\mu - \frac{i}{2}\epsilon \sigma^{\mu\nu} F_{\mu\nu}(x) \right\} \right] . \qquad (3.10)$$

We can easily perform the momentum integration for Eq. (3.10) with the aid of the Gaussian integral. By using the formulas

$$\int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} e^{k_\lambda k^\lambda} = \frac{1}{(4\pi)^2} , \quad \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} e^{k_\lambda k^\lambda} k^\mu k^\nu = -\frac{1}{2}\frac{1}{(4\pi)^2}\delta^{\mu\nu} , \\ \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} e^{k_\lambda k^\lambda} k^\mu k^\nu k^\rho k^\sigma = \frac{1}{4}\frac{1}{(4\pi)^2}(\delta^{\mu\rho}\delta^{\nu\sigma} + \delta^{\mu\sigma}\delta^{\nu\rho} + \delta^{\mu\sigma}\delta^{\nu\rho}) , \qquad (3.11)$$

we obtain

$$\int d^4x\, \mathcal{A}_\psi(x) = \lim_{\epsilon\longrightarrow 0} \frac{1}{(4\pi)^2} \int d^4x \left(\frac{4}{\epsilon^2} + \frac{2}{3} F_{\mu\nu}(x) * F^{\mu\nu}(x) \right) . \qquad (3.12)$$

Here we have used the trace properties of the Dirac matrices,

$$\mathrm{tr}\mathbf{1} = 4, \qquad \mathrm{tr}(\gamma^\mu\gamma^\nu) = 4\delta^{\mu\nu}, \qquad \mathrm{tr}(\sigma^{\mu\nu}\sigma^{\rho\sigma}) = -4(\delta^{\mu\nu}\delta^{\rho\sigma} + \delta^{\mu\rho}\delta^{\nu\sigma}) ,$$

with $\mathrm{tr}\gamma^\mu = \mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho) = 0$. Ignoring the term which becomes infinite in the limit $\epsilon \longrightarrow 0$ [22], we obtain the conformal anomaly coming from the quantum effect of the fermions:

$$\mathcal{A}_\psi(x) = \frac{1}{(4\pi)^2}\frac{2}{3} F_{\mu\nu}(x) * F^{\mu\nu}(x) . \qquad (3.13)$$

3.2 The contribution from the gauge field and ghost fields

We next evaluate the contribution from the gauge field and the ghost fields to the conformal anomaly. The (global) Weyl transformation laws for the fluctuating field and the ghost fields are given by

$$a_\mu(x) \longrightarrow \tilde{a}_\mu(x) = \exp(-\alpha)\, a_\mu(x) , \qquad (3.14)$$

$$c(x) \longrightarrow \tilde{c}(x) = \exp(-2\alpha)\, c(x) , \qquad \bar{c}(x) \longrightarrow \tilde{\bar{c}}(x) = \bar{c}(x) , \qquad (3.15)$$

where α is a constant parameter. We decompose $a_\mu(x)$, $c(x)$, and $\bar{c}(x)$ as

$$a_\mu(x) = \sum_n c_n V_{\mu,n}(x) \ , \qquad (3.16)$$

$$c(x) = \sum_n \alpha_n S_n(x) \ , \qquad \bar{c}(x) = \sum_n \beta_n S_n(x) \ , \qquad (3.17)$$

respectively. Here the coefficients c_n are the ordinary numbers and α_n and β_n are the Grassmann numbers. For the explicit evaluation of the Jacobian, the basis vectors $V_{\mu,n}$ and the scalar basis S_n are chosen to be the eigenfunctions,

$$D_\nu[B]D^\nu[B]V^\mu{}_n(x) - 2i[\ F^{\mu\nu}[B],\ V_{\nu,n}(x)\ ]_M = \lambda_n V^\mu{}_n(x) \ , \qquad (3.18)$$

$$D_\nu[B]D^\nu[B]S_n(x) = \lambda_n S_n(x) \ , \qquad (3.19)$$

with the covariant derivative defined in Eq. (2.7). We also assume that the sets of eigenfunctions $\{V^\mu{}_n(x)\}$ and $\{S_n(x)\}$ are orthonormal and complete, respectively.

Under the infinitesimal transformations (3.14) and (3.15), the integration measure of the fluctuating field and ghost fields transforms as

$$\mathcal{D}\tilde{a}_\mu \mathcal{D}\tilde{c}\mathcal{D}\tilde{\bar{c}} = J_a[\alpha]J_c[\alpha]\mathcal{D}a_\mu \mathcal{D}c\mathcal{D}\bar{c} \ , \qquad (3.20)$$

with the Jacobians

$$J_a[\alpha] \ = \ \exp\left[-\alpha \sum_n \int d^4x\, V_{n\mu}(x) * V_n^\mu(x) \right] \ , \qquad (3.21)$$

$$J_c[\alpha] \ = \ \exp\left[+2\alpha \sum_n \int d^4x\, S_n(x) * S_n(x) \right] \ . \qquad (3.22)$$

We shall evaluate from the Jacobian (3.22) first. For convenience, let us introduce the notation with respect to the covariant derivative $D_\mu S_n(x) \equiv \int \frac{d^4k}{(2\pi)^4} D_\mu[;k]\hat{S}_n(k)e^{ik\cdot x}$, where

$$D_\mu[;k] \equiv \partial_\mu(= ik_\mu) - 2\int \frac{d^4p}{(2\pi)^4}\hat{B}_\mu(p)e^{ip\cdot x}\sin\left(\frac{1}{2}p\wedge k\right) \ , \qquad (3.23)$$

with $p\wedge k \equiv p_\rho\theta^{\rho\sigma}k_\sigma$. The background gauge field in the covariant derivative D_μ depends on the momentum k_μ via the sine functions $\sin\left(\frac{1}{2}p\wedge k\right)$, since the covariant derivative contains the Moyal bracket. As we shall see, the sine function $\sin\left(\frac{1}{2}p\wedge k\right)$ corresponds to the structure constants in ordinary gauge theories. The covariant derivatives $D_\mu[;k]$ satisfy the following commutation relation:

$$[\, D_\mu[;k],\ D_\nu[;k]\,] = -iF_{\mu\nu}(x;k) \equiv \int \frac{d^4p}{(2\pi)^4}\hat{F}_{\mu\nu}(p)e^{ip\cdot x}(-2)\sin\left(\frac{1}{2}p\wedge k\right) \ , \qquad (3.24)$$

where $\hat{F}_{\mu\nu}(p)$ is the Fourier transformation of $F_{\mu\nu}$: $F_{\mu\nu}(x) \equiv \int \frac{d^4p}{(2\pi)^4} \hat{F}_{\mu\nu}(p) e^{ip\cdot x}$. In deriving this expression, we have used the relations
$\sin\left(p \wedge (q+k)\right) \sin\left(q \wedge k\right) - \sin\left(q \wedge (p+k)\right) \sin\left(p \wedge k\right) = \sin\left(p \wedge q\right) \sin\left((p+q) \wedge k\right)$.

Under the Fourier transformations for $S_n(x)$, the Jacobian (3.22) takes the following form:

$$J_c[\alpha] = \exp\left[\, +2\alpha \sum_n \int d^4x \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \hat{S}_n(k) \hat{S}_n(l) \,\right] . \tag{3.25}$$

In the same way as Eq. (3.7), we regularize the Jacobian (3.25) with a Gaussian damping factor as

$$J_c[\alpha] = \exp\left[\, \alpha \int d^4x \, \mathcal{A}_c(x) \,\right] ,$$

with

$$\begin{aligned} \int d^4x \, \mathcal{A}_c(x) &\equiv 2 \lim_{\epsilon \longrightarrow 0} \sum_n \int d^4x \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \Big(\exp\left(-\, \epsilon \, D_\mu[;k] D^\mu[;k] \,\right) \hat{S}_n(k) \Big) \hat{S}_n(l) \\ &= 2 \lim_{\epsilon \longrightarrow 0} \int d^4x \int \frac{d^4k}{(2\pi)^4} \exp\left\{-\, \epsilon \, (ik_\mu + D_\mu[;k])(ik^\mu + D^\mu[;k])\right\} . \end{aligned} \tag{3.26}$$

In deriving Eq. (3.26), we have used the identity $\partial_\mu \hat{S}_n(k) = ik_\mu \hat{S}_n(k)$. Note that the background gauge field depends on the momentum k_μ. This means that the Jacobian factor (3.26) includes the *nonplanar* contribution [20, 13]. In terms higher than the second power of the covariant derivative, however, we can decompose them into terms depending on the momentum k_μ, and terms independent of the momentum k_μ. In the fourth power of the covariant derivative, for example, we obtain

$$\begin{aligned} &\int d^4x \Big[\, D_{[\mu}[;k] \; D_{\nu]}[;k] \; D_{[\rho}[;k] \; D_{\sigma]}[;k] \,\Big] \\ &= 4 \int d^4x \left(\int \frac{d^4p}{(2\pi)^4} \hat{F}_{\mu\nu}(p) e^{ip\cdot x} \sin\left(\frac{1}{2} p \wedge (q+k)\right) \int \frac{d^4q}{(2\pi)^4} \hat{F}_{\rho\sigma}(q) e^{iq\cdot x} \sin\left(\frac{1}{2} q \wedge k\right) \right) \\ &= -2 \int d^4x \left(\int \frac{d^4p}{(2\pi)^4} \hat{F}_{\mu\nu}(p) e^{ip\cdot x} \int \frac{d^4q}{(2\pi)^4} \hat{F}_{\rho\sigma}(q) e^{iq\cdot x} \right) + (\text{the term that depends on } k_\mu) \, , \end{aligned} \tag{3.27}$$

under the integration with respect to the space-time coordinates x^μ. Here $D_{[\mu} D_{\nu]} \equiv D_\mu D_\nu - D_\nu D_\mu$. In deriving the third line of Eq. (3.27), we have made use of the Fourier inverse transform for the delta function $\int d^4x \, e^{i(p+q)\cdot x} = (2\pi)^4 \delta(p+q)$ and an identity

$$\sin^2\left(\frac{1}{2} p \wedge k\right) = \frac{1}{2}\left(1 - \cos(p \wedge k)\right) . \tag{3.28}$$

We find that the integral over space-time coordinates plays a role corresponding to a trace about the generators of gauge groups in ordinary non-Abelian gauge theories. In order for the integration to play the role of the trace, however, note that the infinitesimal parameter α in the

Weyl transformation must be a constant. The first term in the third line of Eq. (3.27) can be regarded as the *planar* contribution [20, 17, 16]. Such a *planar* contribution can be expressed as

$$\int d^4x \left(D_{[\mu}[;k]\ D_{\nu]}[;k]\ D_{[\rho}[;k]\ D_{\sigma]}[;k] \right)\Big|_{planar} = -2\int d^4x\, F_{\mu\nu}(x) * F_{\rho\sigma}(x)\ . \tag{3.29}$$

Here we have used the fact that $\int d^4x\, f(x) * g(x) = \int d^4x \int \frac{d^4p}{(2\pi)^4}\hat{f}(p)e^{ip\cdot x}\int \frac{d^4q}{(2\pi)^4}\hat{g}(q)e^{iq\cdot x}$. Contributions from the second and the third power of the covariant derivative in the planar sector cancel after the momentum integration. It is the same in the fourth power of the covariant derivative with the symmetric property for the Minkowski indices. Therefore, the momentum integration in the planar sector leads to the following result:

$$\begin{aligned}
&\int d^4x\ \mathcal{A}_c(x)\big|_{planar} \\
&= 2\lim_{\epsilon\longrightarrow 0}\frac{1}{\epsilon^2}\int d^4x\int\frac{d^4k}{(2\pi)^4}\, e^{k_\mu k^\mu}\ \exp\Big\{ -2i\sqrt{\epsilon}k^\mu D_\mu[;k] - \epsilon D_\mu[;k]D^\mu[;k]\ \Big\}\Big|_{planar} \\
&= \lim_{\epsilon\longrightarrow 0}\frac{1}{(4\pi)^2}\int d^4x\ \left(\ \frac{1}{\epsilon^2} - \frac{2}{6}F_{\mu\nu}(x) * F^{\mu\nu}(x)\ \right)\ .
\end{aligned} \tag{3.30}$$

Here we have made use of the formulas (3.11). Ignoring the infinite term, we obtain the conformal anomaly from the quantum effect of the ghost fields,

$$\mathcal{A}_c(x)\big|_{planar} = \frac{1}{(4\pi)^2}\left(-\frac{1}{3}\right)F_{\mu\nu}(x) * F^{\mu\nu}(x)\ . \tag{3.31}$$

We next evaluate the Jacobian (3.21). Under the Fourier transformations for $V_n^\mu(x)$, the Jacobian (3.21) takes the following form:

$$J_a[\alpha] = \exp\left[\ +2\alpha\sum_n\int d^4x\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4l}{(2\pi)^4}\hat{V}_{n\mu}(k)\hat{V}_n^\mu(l)\ \right]\ , \tag{3.32}$$

where we have used the relations $\sum_n \hat{V}_{n\mu}(k)\hat{V}_n^\mu(l) = (2\pi)^4\delta^4(k+l)$. Taking account of Eq. (3.18), we regularize the Jacobian (3.32) as follows:

$$J_a[\alpha] = \exp\left[\ \alpha\int d^4x\,\mathcal{A}_a(x)\ \right]\ ,$$

with

$$\begin{aligned}
&\int d^4x\,\mathcal{A}_a(x) \\
&\equiv -\lim_{\epsilon\longrightarrow 0}\sum_n\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4l}{(2\pi)^4}\Big(\ \exp\Big\{-\ \epsilon\ \Big(\ \delta_\mu{}^\nu D^2[;k] - 2iF_\mu{}^\nu(x;k)\ \Big)\ \Big\}\,\hat{V}_{n\nu}(k)\ \Big)\,\hat{V}_n^\mu(l) \\
&= -\lim_{\epsilon\longrightarrow 0}\int d^4x\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}\Big[\ \exp\Big\{-\ \epsilon\ \Big(\ \delta_\mu{}^\nu(ik + D[;k])^2 - 2iF_\mu{}^\nu(x;k)\ \Big)\ \Big\}\ \Big]\ ,
\end{aligned} \tag{3.33}$$

where $D^2 \equiv D_\mu D^\mu$ and tr[ ] denotes a trace with respect to the Minkowski indices. The concrete form of the field strength $F_{\mu\nu}(x;k)$ is shown in Eq. (3.24). Since the background gauge field and its field strength depend on the momentum k_μ, the Jacobian factor (3.33) also includes the *nonplanar* contribution. Selecting the *planar* contribution, we arrive at

$$\begin{aligned}
&\int d^4x \ \mathcal{A}_a(x)|_{planar} \\
&= -\lim_{\epsilon \longrightarrow 0} \frac{1}{\epsilon^2} \int d^4x \int \frac{d^4k}{(2\pi)^4} e^{k_\mu k^\mu} \ \mathrm{tr} \Big[\exp\Big\{ -2i\sqrt{\epsilon}\delta_\mu{}^\nu k^\lambda D_\lambda[;k] - \epsilon \delta_\mu{}^\nu D^2[;k] + 2iF_\mu{}^\nu(x;k) \Big\} \Big]\Big|_{planar} \\
&= \lim_{\epsilon \longrightarrow 0} \frac{1}{(4\pi)^2} \int d^4x \left(-\frac{1}{\epsilon^2} - \frac{5 \times 2}{3} F_{\mu\nu}(x) * F^{\mu\nu}(x) \right) \qquad (3.34)
\end{aligned}$$

Here we have made use of the formulas (3.11). Ignoring the infinite term, we obtain the conformal anomaly from the quantum effect of the fluctuating field,

$$\mathcal{A}_a(x)|_{planar} = \frac{1}{(4\pi)^2} \left(-\frac{10}{3} \right) F_{\mu\nu}(x) * F^{\mu\nu}(x) \ . \qquad (3.35)$$

3.3 The conformal anomaly and the β function

Adding together the contributions from matter fields, the gauge field, and ghost fields, we obtain the conformal anomaly in noncommutative QED. The explicit form of the conformal anomaly is given by

$$\begin{aligned}
\int d^4x \ \mathcal{A}(x)|_{planar} &= \int d^4x \ \Big(n_f \ \cdot \ \mathcal{A}_\psi(x) + \mathcal{A}_a(x)|_{planar} + \mathcal{A}_c(x)|_{planar} \Big) \\
&= \frac{1}{(4\pi)^2} \left(\frac{2}{3} n_f - \frac{11}{3} \right) \int d^4x \, F_{\mu\nu}(x) * F^{\mu\nu}(x) \ , \qquad (3.36)
\end{aligned}$$

where n_f is the number of flavors. This takes the same form as the conformal anomaly in ordinary QED except for the ordinary product replacing the Moyal star product. The gauge invariance of the result is guaranteed by the integration over space-time coordinates. Note that the coefficient of the conformal anomaly differs from the coefficient of the conformal anomaly in ordinary QED by a factor of 2 [22]. The difference comes from the identity (3.28) and normalization of the $U(1)$ generator.

A close relation exists between the conformal anomaly and the β function in the ordinary field theories. When quantum corrections are included, a scale transformation shifts the renormalized coupling constant. Since the variation is proportional to the β function, the corresponding change in the action is also proportional to the β function in a classically scale invariant theory. Therefore, the conformal anomaly is proportional to the β function. In ordinary QED with massless fermions, the conformal anomaly up to the one-loop correction is given by

$$\int d^4x \, \mathcal{A}(x) = \frac{\beta(e)}{2e^3} \int d^4x \, F_{\mu\nu}(x) F^{\mu\nu}(x) \ , \qquad (3.37)$$

where e denotes the coupling constant. This expression shows the relation between the conformal anomaly and β functions in ordinary QED. In analogy with the ordinary QED, we express the conformal anomaly in noncommutative QED as follows:

$$\int d^4x\, \mathcal{A}(x) = \frac{\beta(g)|_{NC-QED}}{2g^3} \int d^4x\, F_{\mu\nu}(x) * F^{\mu\nu}(x) \; , \tag{3.38}$$

where g is the coupling constant in noncommutative QED. From Eqs. (3.36) and (3.38), we can evaluate the β function in noncommutative QED up to the one-loop contribution:

$$\beta(g)|_{NC-QED} = -\frac{g^3}{(4\pi)^2} \left(\frac{22}{3} - \frac{4}{3} n_f \right) \; . \tag{3.39}$$

This is coincident with the β function obtained from the perturbative analysis [2] [21].

4 The generalization to the $U(N)$ gauge group

It is straightforward to modify the method of the calculation in the $U(1)$ gauge group performed for the previous section to the $U(N)$ gauge group. In the $U(N)$ gauge group, the notation tr[ ] in Eq. (3.8) replaces the notation denoting the trace over the Dirac matrices γ^μ and the gauge group generators T_a with a certain irreducible representation. Similarly, the trace over the gauge group generators with the adjoint representation appears in Eq. (3.26) and the notation tr[ ] in Eq. (3.33) replaces the notation denoting the trace with respect to the Minkowski indices and the trace over the gauge group generators with the adjoint representation. Hence, the factor $C(r)$ caused from the normalization $\mathrm{tr}[T_a T_b] = C(r)\delta_{ab}$ appears in the coefficient of the corresponding expression for Eq. (3.13), and the factor $C_2(G)(= N)$ from the quadratic Casimir operator for the $U(N)$ gauge group appears in the coefficient of the corresponding expression for Eqs. (3.31) and (3.35). The corresponding conformal anomaly in noncommutative QCD with the $U(N)$ gauge group is given as follows:

$$\int d^4x\; \mathcal{A}(x)|_{planar} = \frac{1}{(4\pi)^2} \left(\frac{2}{3} n_f C(r) - \frac{11}{3} C_2(G) \right) \int d^4x\, F_{\mu\nu}{}^a(x) * F^{\mu\nu}{}_a(x) \; , \tag{4.1}$$

where $F_{\mu\nu}{}^a$ are the components of the field strength: $F_{\mu\nu} = F_{\mu\nu}{}^a T_a$. Note also that the coefficient of the conformal anomaly differs from the coefficient of the conformal anomaly in ordinary QCD with the $U(N)$ gauge group. When evaluating the β function based on the relation corresponding to Eq. (3.38) in noncommutative QCD, we obtain

$$\beta(g)|_{NC-QCD} = -\frac{g^3}{(4\pi)^2}\, 2 \left(\frac{11}{3} C_2(G) - \frac{2}{3} n_f C(r) \right) \; . \tag{4.2}$$

This is also coincident with the β function obtained from the perturbative analysis [3] [20].

[2] By the definition of a β function, the expression (3.39) is different from that in Ref. [21] only in the factor $\frac{1}{g}$.

[3] If we take a different normalization of a $U(1)$ generator, we might also obtain the one-loop β function in ordinary $U(N)$ gauge theory. For example, see Ref. [19].

5 Conclusion and Discussion

In this paper, we have evaluated the conformal anomalies in noncommutative gauge theories. As well as the axial anomalies and chiral gauge anomalies, the conformal anomalies are also calculable on the basis of the path integral formulation. Except for the difference in coefficients, the conformal anomalies (3.36) and (4.1) take the form of the straightforward Moyal deformation in the corresponding anomalies in ordinary gauge theories. The difference in coefficients is a consequence of the noncommutativity of the Moyal star product. In evaluating the conformal anomalies by the path integral formulation, the background field method has been adopted. The fluctuating fields and the ghost fields in the background field method transform as a field in the adjoint representation under the gauge transformation (2.13). The interactions with the background field and such adjoint fields include the Moyal bracket. Then the sine function $\sin(\frac{1}{2}p \wedge k)$ arising from the Moyal bracket plays the role of the structure constants in the gauge algebra. The corresponding quadratic Casimir operator takes the value of 2. It causes the difference in coefficients of the conformal anomalies between the ordinary gauge theory and noncommutative gauge theory.

There is a relation between the conformal anomaly and the β function in ordinary gauge theories. The β function in noncommutative gauge theory can be evaluated by applying the relation between the conformal anomaly and the β function. The evaluation of the β function based on this relation is in agreement with the result of the perturbative analysis.

In evaluating the conformal anomalies, we have focused attention on the planar contribution. We can confirm that the nonplanar sector in the first and second power in Taylor series of the Gaussian damping factor does not contribute in the case of $p \circ p \equiv p_\mu \theta^{\mu\rho} p^\nu \theta_{\nu\rho} < 0$. We shall report elsewhere whether the nonplanar sector does not contribute to the conformal anomalies in all order in Taylor series of the Gaussian damping factor.

We have confined our discussion to the conformal anomaly under the global Weyl transformation. If we extend the global Weyl transformation to the local Weyl transformation, then the integration over space-time coordinates does not play the role corresponding to the trace over the generators of gauge groups in ordinary non-Abelian gauge theories. Further consideration is needed in this respect. In the noncommutative field theory at the classical level, the variation of the action under the global scale (or dilatation) transformation for the fields can be expressed as the variation of the action under the global scale transformation of the noncommutativity parameter [23]. In order to argue about the conformal anomaly under the local Weyl transformation, it may be necessary to consider the noncommutative field theory with nonconstant noncommutative parameters [24]. We hope to discuss this subject in the future.

Acknowledgments

I am grateful to S. Deguchi for valuable discussions and helpful suggestions. I would like to thank A. Sugamoto for a careful reading of the manuscript and useful comments. I also would like to thank A. Armoni, J. Gomis, and R. Schiappa for useful comments.

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