

# Reply to Comment on Dirac spectral sum rules for QCD in three dimensions

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I reply to the comment by Dr S. Nishigaki (hep-th/0007042) to my papers Phys. Rev. D61 (2000) 056005 and Phys. Rev. D62 (2000) 016005.

Recently, Nishigaki [1] pointed out that the first sum rule for the eigenvalues of the Dirac operator given in my papers [2, 3] was incorrect. This appears to be true, and I thank Dr Nishigaki for drawing attention to this fact. That the sum rules in [2, 3] do not agree with the results by Hilmoine and Niclasen [4] had been communicated to me by these authors already before the appearance of their paper, but I was at the time unable to detect where my calculation failed.

However, I do not agree with the explanation by Dr Nishigaki of why the sum rules were not reproduced correctly. According to him, the authors of [5] “were aware that they needed to extend the saddle point manifold... in order to obtain a correct result”. Such an extension of the integral over the coset obviously can not have any effect on the result, since this procedure only involves integrating over dummy variables on which the integrand does not depend (namely the degrees of freedom of the stability subgroup), and amounts only to multiplying the integral by a numerical factor. This extension was done in [5] only for convenience. Also, to my understanding, Dr Nishigaki’s references to the papers by Kamenev and Mèzard are not relevant in this context.

Instead, the reason is quite simple. In [2], formula (6.47), and in [3], formula (8.60), in order to obtain the right sum rule, the sum should go from 1 to  $M_{as}$  and from 1 to  $M_s$  respectively, where  $M_{as}$  is the number of antisymmetric traceless generators that generate antisymmetric unimodular matrices, and  $M_s$  is the number of symmetric traceless generators that generate symmetric unimodular matrices. Taking into account the sizes of these matrices in the two cases we get

$$M_{as} = \frac{4N_f(4N_f - 1)}{2} - 1, \quad M_s = \frac{2N_f(2N_f + 1)}{2} - 1 \quad (1)$$

We then obtain for  $\beta = 1$

$$\left\langle \sum_{\lambda_k > 0} \frac{1}{(N \Sigma \lambda_k)^2} \right\rangle = \frac{2N_f}{2(2N_f - 1)(4N_f + 1)} \quad (2)$$

and for  $\beta = 4$

$$\left\langle \sum_{\lambda_k > 0} \frac{1}{(N \Sigma \lambda_k)^2} \right\rangle = \frac{2N_f}{2(2N_f - 1)(N_f + 1)} \quad (3)$$

This is in total agreement with the sum rules obtained by Hilmoine and Niclasen and by Nishigaki. I remind the reader that my definition of  $N_f$  differs by a factor of two from the other authors’ definition, and that I sum only over the positive eigenvalues. In the sum rule for  $\beta = 1$ , in addition to the above

mentioned correction, the sum rule was off by a factor of 4 due to various mistakes of factors of 2 for which I apologize. I will update the electronic versions of [2, 3] shortly.

The  $\beta$  sum rules can be summarized as follows:

$$\left\langle \sum_{\lambda_k > 0} \frac{1}{(N \Sigma \lambda_k)^2} \right\rangle = \frac{2N_f}{2(2N_f - 1)(\frac{4N_f}{\beta} + 1)} \quad (4)$$

where the number of flavors is always even and denoted by  $2N_f$ , and  $\beta$  is the Dyson index of the corresponding matrix model. The sum rule for  $\beta = 2$  was given by Verbaarschot and Zahed in [5].

## Acknowledgment

I thank Jac Verbaarschot for correspondence. I am indebted to him for assisting me in finding the mistake in my sum rules [6].

## References

- [1] S. Nishigaki, “Comment on Dirac spectral sum rules for QCD $_3$ ”, hep-th/0007042
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- [4] C. Hilmoine and R. Niclasen, “The microscopic spectral density of the Dirac operator derived from Gaussian orthogonal and symplectic ensembles”, hep-th/0004081
- [5] J.J.M. Verbaarschot and I. Zahed, Phys. Rev. Lett. 73 (1994) 2288
- [6] J. Verbaarschot, private communication.