Irreducible Freedman-Townsend vertex and Hamiltonian BRST cohomology

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Abstract

The irreducible Freedman-Townsend vertex is derived by means of the Hamiltonian deformation procedure based on local BRST cohomology.

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1 Introduction

The key point in the development of the BRST formalism [1]–[13] is the recursive pattern of homological perturbation theory [14]–[20]. The Hamiltonian BRST formulation was proved to be a natural background for analysing various topics, such as the construction of such a symmetry in quantum mechanics [8] (Chapter 14), or an appropriate correlation between the BRST symmetry itself and canonical quantization methods [21]. Its cohomological understanding helped at clarifying some new aspects, like the Hamiltonian analysis of anomalies [22], the precise relation between local Lagrangian and Hamiltonian BRST cohomologies [23], the construction of irreducible BRST symmetries for reducible theories [24], or the study of consistent Hamiltonian interactions that can be added among fields with gauge freedom [25]–[28].

In this paper we investigate the consistent interactions that can be introduced among a system of two-form gauge fields in four dimensions by means

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of an irreducible Hamiltonian BRST deformation procedure, in spite of the reducibility of the initial model. Our strategy goes as follows. First, we start with a system of abelian two-forms in four dimensions, described by a reducible BRST differential, s_R, to whom we associate an irreducible BRST differential s_I, with the properties

$$s_R^2 = 0 = s_I^2, \; H^0\left(s_R\right) \simeq H^0\left(s_I\right), \tag{1}$$

where $H^k\left(s\right)$ denotes the kth order cohomological space of the differential s. The relations (1) allow us to replace the reducible BRST differential with the irreducible one from the point of view of the basic equations of the BRST formalism. Second, we apply the Hamiltonian deformation technique to the irreducible version of the model under study. It has been shown in [25]–[28] that the Hamiltonian problem of constructing consistent interactions can be reformulated as a deformation problem of the BRST charge (canonical generator of the BRST symmetry) and BRST-invariant Hamiltonian of a given free theory. In turn, the Hamiltonian deformation scheme reduces to two towers of equations involving the free BRST differential. The free irreducible BRST differential splits in our case as $s_I = \delta_I + \gamma_I$, where δ_I stands for the Koszul-Tate differential, and γ_I represents the exterior derivative along the gauge orbits. In order to generate the first-order deformation of the non-integrated BRST charge density, we perform its expansion according to an auxiliary degree, called antighost number, and assume that we can take the last representative of this expansion to be annihilated by γ_I. In consequence, we have to know $H\left(\gamma_I\right)$. In the meantime, the computation of the before last term of this expansion requires the knowledge of $H\left(\delta_I | \tilde{d}\right)$, where $\tilde{d}$ is the spatial part of the exterior space-time derivative, $\tilde{d} = dx^i \partial_i$. After the computation of these cohomologies, we appropriately solve the deformation equations, finally obtaining the BRST charge and BRST-invariant Hamiltonian of the interacting theory. Third, from these quantities we identify the structure of the deformed gauge theory, hence the first-class constraints, the first-class Hamiltonian, and the accompanying gauge algebra. The resulting model is precisely the irreducible version of the non-abelian Freedman-Townsend model [29]. It is well-known that the Freedman-Townsend model plays a special role due on the one hand to its link with Witten's string theory [30], and, on the other hand, to its equivalence to the non-linear σ-model [29].

Our paper is structured in five sections. Section 2 is devoted to the construction of an irreducible Hamiltonian BRST differential for a four-dimensional system of abelian two-forms. In Section 3 we deal with the deformation of this irreducible BRST symmetry on account of cohomological techniques. Section 4 realizes the identification of the resulting deformed

model, while Section 5 ends the paper with some conclusions.

2   Irreducible BRST differential of the free model

In this section we construct an irreducible BRST differential for a set of abelian two-forms in four dimensions.

2.1   The undeformed model

We begin with the first-order Lagrangian action that describes a system of free two-forms

$$S_0^L \left[A_\mu^a, B_a^{\mu\nu} \right] = \frac{1}{2} \int d^4x \left(-B_a^{\mu\nu} F_{\mu\nu}^a + A_\mu^a A_a^\mu \right), \tag{2}$$

where $B_a^{\mu\nu}$ stands for a set of antisymmetric tensor fields, and the field strength of A_μ^a reads as $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$. Eliminating the second-class constraints with the help of the Dirac bracket built with respect to themselves (the independent 'co-ordinates' of the reduced phase-space are $\bar{z}^A = \left(A_i^a, B_a^{0i}, B_a^{ij}, \pi_{ij}^a \right)$), we obtain the first-class constraints

$$\gamma_{1i}^a \equiv \epsilon_{0ijk} \pi^{ajk} \approx 0, \ G_{2i}^a \equiv \frac{1}{2} \epsilon_{0ijk} F^{ajk} \approx 0, \tag{3}$$

and the first-class Hamiltonian

$$H_0' = \frac{1}{2} \int d^3x \left(B_a^{ij} F_{ij}^a - A_i^a A_a^i + \left(\partial^i B_{0i}^a \right) \left(\partial_j B_a^{0j} \right) \right) \equiv \int d^3x \, h_0', \tag{4}$$

where the non-vanishing Dirac brackets among the independent components are expressed by

$$\left[B_a^{0i} \left(x \right), A_j^b \left(y \right) \right]_{x^0=y^0}^* = \delta_a{}^b \delta^i{}_j \delta^3 \left(\mathbf{x} - \mathbf{y} \right), \tag{5}$$

$$\left[B_a^{ij} \left(x \right), \pi_{kl}^b \left(y \right) \right]_{x^0=y^0}^* = \frac{1}{2} \delta_a{}^b \delta_k^{[i} \delta_l^{j]} \delta^3 \left(\mathbf{x} - \mathbf{y} \right). \tag{6}$$

We remark that the functions G_{2i}^a from (3) are not independent (first-stage reducible)

$$\partial^i G_{2i}^a = 0, \tag{7}$$

while the entire first-class constraint set (3) remains abelian in terms of the Dirac bracket.

2.2 Irreducible Hamiltonian BRST symmetry

Here, we derive an irreducible Hamiltonian BRST symmetry associated with the model under study by following the general line exposed in [24]. Thus, for a system subject to the first-class constraints

$$G_{a_0}\left(\bar{z}^A\right) \approx 0, \ a_0 = 1, \ldots, M_0, \tag{8}$$

that are first-stage reducible

$$Z^{a_0}{}_{a_1} G_{a_0} = 0, \ a_1 = 1, \ldots, M_1, \tag{9}$$

we can construct a corresponding theory, based on the irreducible first-class constraints

$$\gamma_{a_0} \equiv G_{a_0} + A_{a_0}{}^{a_1} \pi_{a_1} \approx 0, \tag{10}$$

where (y^{a_1}, π_{a_1}) denote some new canonical pairs that extend the original phase-space, and $A_{a_0}{}^{a_1}$ are some functions (that may involve at most the original variables $\bar{z}^A$), taken to satisfy the condition

$$rank\left(Z^{a_0}{}_{a_1} A_{a_0}{}^{b_1}\right) = M_1. \tag{11}$$

In order to infer an irreducible Lagrangian formulation that is manifestly covariant, it is necessary to add some supplementary canonical pairs $\left(y^{(1)a_1}, \pi_{a_1}^{(1)}\right)$ and $\left(y^{(2)a_1}, \pi_{a_1}^{(2)}\right)$, subject to the constraints

$$\gamma_{1a_1} \equiv \pi_{a_1} - \pi_{a_1}^{(1)} \approx 0, \ \gamma_{2a_1} \equiv -\pi_{a_1}^{(2)} \approx 0, \tag{12}$$

which together with (10) form an irreducible first-class set. Under these circumstances, it is shown [24] that we can construct the Hamiltonian BRST symmetry s_I of the irreducible theory based on the first-class constraints (10) and (12), such that $s_I^2 = 0 = s_R^2$ and $H^0\left(s_I\right) \simeq H^0\left(s_R\right)$, where s_R stands for the Hamiltonian BRST symmetry within the initial reducible formulation. The last formulas indicate that the BRST symmetries s_I and s_R are equivalent from the point of view of the fundamental equations of the Hamiltonian BRST formalism, namely, the nilpotency of the BRST operator and the isomorphism between the zeroth-order cohomological space of the BRST differential and the algebra of physical observables. As a consequence, it is permissible to replace the reducible BRST symmetry with the irreducible one. In this light, we further analyse the construction of the irreducible Hamiltonian BRST symmetry for abelian two-forms.

Initially, we determine the concrete form of the irreducible first-class constraints. In this respect, we observe that in the case of the model under study

we have that $Z^{a_0}_{a_1} \to \delta^a{}_b \partial^i$. If we take $A^{a_1}_{a_0} \to -\delta^b{}_c \partial_i$, then the requirement (11) is indeed fulfilled. Accordingly, the irreducible first-class constraints (10) and (12) take the concrete form

$$\gamma^a_{1i} \equiv \epsilon_{0ijk} \pi^{ajk} \approx 0, \; \gamma^a_{2i} \equiv \frac{1}{2} \epsilon_{0ijk} F^{ajk} - \partial_i \pi^a \approx 0, \tag{13}$$

$$\gamma^a_1 \equiv \pi^a - \pi^{(1)a} \approx 0, \; \gamma^a_2 \equiv -\pi^{(2)a} \approx 0, \tag{14}$$

where (φ_a, π^a), $\left(\varphi^{(1)}_a, \pi^{(1)a}\right)$ and $\left(\varphi^{(2)}_a, \pi^{(2)a}\right)$ represent the supplementary canonical pairs mentioned in the above. Thus, the irreducible constraint set preserves the abelian behaviour (in the Dirac bracket) of the former reducible one. We will work with a first-class Hamiltonian of the type

$$H_0 = \int d^3x \left(\frac{1}{2} B^{ij}_a \left(F^a_{ij} + \epsilon_{0ijk} \partial^k \pi^a \right) - \frac{1}{2} A^a_i A^i_a + \right.$$
$$\left. \varphi_a \pi^{(2)a} + \frac{1}{2} \left(\partial^i B^a_{0i} \right) \partial_j B^{0j}_a + \left(\partial_i \varphi^{(2)}_a \right) \partial^i \pi^a \right), \tag{15}$$

with the help of which we infer the gauge algebra relations

$$[H_0, \gamma^a_{1i}]^* = \gamma^a_{2i}, \; [H_0, \gamma^a_{2i}]^* = \partial_i \gamma^a_2, \tag{16}$$

$$[H_0, \gamma^a_1]^* = -\gamma^a_2, \; [H_0, \gamma^a_2]^* = -\partial^i \gamma^a_{2i}. \tag{17}$$

We have all the elements necessary at the construction of the main ingredients of the Hamiltonian BRST formulation of the new model. The BRST charge and BRST-invariant Hamiltonian of this irreducible free theory are expressed by

$$\Omega_0 = \int d^3x \sum_{\Delta=1}^{2} \left(\eta^i_{\Delta a} \gamma^a_{\Delta i} + \eta_{\Delta a} \gamma^a_\Delta \right) \equiv \int d^3x \omega_0, \tag{18}$$

$$H_{0_B} = H_0 + \int d^3x \left(\eta^i_{1a} \mathcal{P}^a_{2i} - \eta_{1a} \mathcal{P}^a_2 - \eta_{2a} \partial^i \mathcal{P}^a_{2i} + \eta^i_{2a} \partial_i \mathcal{P}^a_2 \right) \equiv \int d^3x h_{0_B}, \tag{19}$$

where $\eta^\Gamma \equiv (\eta^i_{1a}, \eta^i_{2a}, \eta_{1a}, \eta_{2a})$ are fermionic ghost number one ghosts, and $\mathcal{P}_\Gamma \equiv (\mathcal{P}^a_{1i}, \mathcal{P}^a_{2i}, \mathcal{P}^a_1, \mathcal{P}^a_2)$ denote their corresponding ghost number minus one antighosts. The ghost number (gh) is defined like the difference between the pure ghost number (pgh) and the antighost number (antigh), with

$$\text{pgh} \left(z^A \right) = 0, \; \text{pgh} \left(\eta^\Gamma \right) = 1, \; \text{pgh} \left(\mathcal{P}_\Gamma \right) = 0, \tag{20}$$

$$\text{antigh} \left(z^A \right) = 0, \; \text{antigh} \left(\eta^\Gamma \right) = 0, \; \text{antigh} \left(\mathcal{P}_\Gamma \right) = 1, \tag{21}$$

where

$$z^A = \left(A^a_i, B^{0i}_a, B^{ij}_a, \pi^a_{ij}, \varphi_a, \pi^a, \varphi^{(1)}_a, \pi^{(1)a}, \varphi^{(2)}_a, \pi^{(2)a} \right). \tag{22}$$

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From (18) it is easy to see that the irreducible BRST symmetry $s_I \bullet = [\bullet, \Omega_0]^*$ of the free theory decomposes like $s_I = \delta_I + \gamma_I$, where δ_I is the irreducible Koszul-Tate differential, and γ_I represents the irreducible exterior derivative along the gauge orbits. These operators act on the generators of the free irreducible BRST complex through the relations

$$\delta_I z^A = 0, \ \delta_I \eta^\Gamma = 0, \ \delta_I \mathcal{P}_{1i}^a = -\epsilon_{0ijk}\pi^{jka}, \tag{23}$$

$$\delta_I \mathcal{P}_{2i}^a = -\frac{1}{2}\epsilon_{0ijk}F^{jka} + \partial_i \pi^a, \ \delta_I \mathcal{P}_1^a = -\pi^a + \pi^{(1)a}, \ \delta_I \mathcal{P}_2^a = \pi^{(2)a}, \tag{24}$$

$$\gamma_I A_i^a = 0, \ \gamma_I B_a^{0i} = \epsilon^{0ijk}\partial_j \eta_{2ak}, \ \gamma_I B_a^{ij} = \epsilon^{0ijk}\eta_{1ak}, \ \gamma_I \pi_{ij}^a = 0, \tag{25}$$

$$\gamma_I \varphi_a = \eta_{1a} + \partial_i \eta_{2a}^i, \ \gamma_I \varphi_a^{(1)} = -\eta_{1a}, \ \gamma_I \varphi_a^{(2)} = -\eta_{2a}, \tag{26}$$

$$\gamma_I \pi^a = \gamma_I \pi^{(1)a} = \gamma_I \pi^{(2)a} = 0, \ \gamma_I \eta^\Gamma = 0, \ \gamma_I \mathcal{P}_\Gamma = 0. \tag{27}$$

The last definitions will be used in the sequel during the deformation procedure.

3 Construction of irreducible deformations

In this section we determine the consistent interactions that can be added to action (2) in the framework of the irreducible Hamiltonian BRST background established before. Based on the results from [25]–[28], we can reformulate the general problem of constructing consistent Hamiltonian interactions as a deformation problem of the BRST charge and BRST-invariant Hamiltonian associated with a given free theory. If we expand the BRST charge Ω of the interacting theory in the powers of the deformation parameter g, $\Omega = \Omega_0 + g\Omega_1 + g^2\Omega_2 + \cdots$, and ask that $[\Omega, \Omega]^* = 0$, we find the equations

$$[\Omega_1, \Omega_0]^* = 0, \ \frac{1}{2}[\Omega_1, \Omega_1]^* + [\Omega_2, \Omega_0]^* = 0, \ \cdots, \tag{28}$$

where the equation for power zero in g was omitted as it is automatically obeyed. Let $H_B = H_{0_B} + gH_1 + g^2H_2 + \cdots$ be the BRST-invariant Hamiltonian of the interacting theory. The BRST-invariance of H_B with respect to Ω generates the equations

$$[H_1, \Omega_0]^* + [H_{0_B}, \Omega_1]^* = 0, \ [H_2, \Omega_0]^* + [H_1, \Omega_1]^* + [H_{0_B}, \Omega_2]^* = 0, \ \cdots, \tag{29}$$

where the zeroth order equation in g is verified by assumption, and was therefore discarded. Thus, the relations (28-29) completely describe the BRST approach to the construction of consistent Hamiltonian interactions, and will

accordingly be named the main equations of the Hamiltonian deformation procedure. The equation that governs the first-order deformation of the BRST charge (the first relation in (28)), demands that Ω_1 should be an s_I-co-cycle. Trivial co-cycles lead to trivial deformations (that can be absorbed by a field redefinition), and will be removed. The second equation in (28) asks that $[\Omega_1, \Omega_1]^*$ should be BRST-exact in order to ensure the existence of the second-order deformation Ω_2. In this context, we are interested in local deformations only, i.e., in the solutions $\Omega_k = \int d^3x\omega_k$, $H_k = \int d^3xh_k$, with ω_k and h_k local functions, so the relevant cohomology space in terms of the integrands is $H\left(s_I|\tilde{d}\right)$.

The first equation in (28) holds if and only if ω_1 is a BRST-co-cycle modulo $\tilde{d}$, i.e.,

$$s_I\omega_1 = \partial_i n^i, \tag{30}$$

for some n^i. In order to solve (30), we expand ω_1 according to the antighost number, $\omega_1 = \overset{(0)}{\omega}_1 + \overset{(1)}{\omega}_1 + \cdots + \overset{(J)}{\omega}_1$, where the last term can be assumed to be annihilated by γ_I, so $\gamma_I \overset{(J)}{\omega}_1 = 0$. Then, in order to compute the first-order deformation of the BRST charge, we need to know $H(\gamma_I)$. Analysing the definitions (25-27), we remark that the fields B_a^{ij} and $\varphi \equiv \left(\varphi_a, \varphi_a^{(1)}, \varphi_a^{(2)}\right)$ are not γ_I-invariant, while the ghosts η_{1a}^i, η_{1a}, η_{2a}, together with their spatial derivatives are trivial in the cohomology of γ_I (as they are γ_I-exact). The ghosts η_{2a}^i and their derivatives are γ_I-closed, but their antisymmetrized first-order derivatives are γ_I-exact, as are also their subsequent derivatives. Thus, the cohomology of γ_I will be generated by A_i^a, $\partial_i B_a^{0i}$, $\pi \equiv \left(\pi_{ij}^a, \pi^a, \pi^{(1)a}, \pi^{(2)a}\right)$, $\mathcal{P}_\Gamma$, and their spatial derivatives, as well as by the undifferentiated ghosts η_{2a}^i and their symmetrized first-order derivatives, $\partial^{(j}\eta_{2a}^{i)}$. Consequently, the general solution of the equation $\gamma_I a = 0$, can be written as

$$a = a_M\left([A_i^a], \left[\partial_i B_a^{0i}\right], [\pi], [\mathcal{P}_\Gamma]\right) e^M\left(\eta_{2a}^i, \partial^{(j}\eta_{2a}^{i)}\right) + \gamma_I b, \tag{31}$$

where $e^M\left(\eta_{2a}^i, \partial^{(j}\eta_{2a}^{i)}\right)$ constitutes a basis in the (finite-dimensional) space of the polynomials in the ghosts η_{2a}^i and their symmetrized first-order derivatives, while the notation $a[q]$ means that a depends on q and its spatial derivatives up to a finite order. As antigh $\left(\overset{(J)}{\omega}_1\right) = J$ and gh $\left(\overset{(J)}{\omega}_1\right) = 1$, we have that pgh $\left(\overset{(J)}{\omega}_1\right) = J + 1$. From (31) it results that the solution to the equation $\gamma_I \overset{(J)}{\omega}_1 = 0$ is (up to a trivial term)

$$\overset{(J)}{\omega}_1 = a_J\left([A_i^a], \left[\partial_i B_a^{0i}\right], [\pi], [\mathcal{P}_\Gamma]\right) e^{J+1}\left(\eta_{2a}^i, \partial^{(j}\eta_{2a}^{i)}\right), \tag{32}$$

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where antigh $(a_J) = J$. The equation (30) projected on antighost number $(J-1)$ takes the form

$$\delta_I \overset{(J)}{\omega}_1 + \gamma_I \overset{(J-1)}{\omega}_1 = \partial_i \sigma^i. \tag{33}$$

In order to ensure a solution to (33) (or, in other words, the existence of $\overset{(J-1)}{\omega}_1$), it is necessary that a_J belongs to $H_J\left(\delta_I|\tilde{d}\right)$. With the help of the Lagrangian results [31] translated at the Hamiltonian level, it follows that the cohomology of δ_I modulo $\tilde{d}$ is vanishing in the case of the model under study for all antighost numbers strictly greater that one, namely,

$$H_J\left(\delta_I|\tilde{d}\right) = 0, \text{ for } J > 1. \tag{34}$$

In the meantime, the general representative of $H_1\left(\delta_I|\tilde{d}\right)$ can be written as

$$\lambda = \lambda^i{}_a \mathcal{P}^a_{2i}, \tag{35}$$

with $\lambda^i{}_a$ some constants, such that

$$\delta_I \lambda = \partial^j \left(\lambda^i{}_a \left(-\epsilon_{0ijk} A^{ka} + g_{ij}\pi^a \right) \right). \tag{36}$$

On behalf of (34), it follows simply that the expansion of ω_1 stops after the first two terms, $\omega_1 = \overset{(0)}{\omega}_1 + \overset{(1)}{\omega}_1$, where $\overset{(1)}{\omega}_1 = a_1 e^2 \left(\eta^i_{2a}, \partial^{(j}\eta^{i)}_{2a} \right)$, and a_1 pertains to $H_1\left(\delta_I|\tilde{d}\right)$. Taking into account that pgh $\left(e^2 \left(\eta^i_{2a}, \partial^{(j}\eta^{i)}_{2a} \right) \right) = 2$, there are only three possibilities, namely, $\eta^i_{2a}\eta^j_{2b}$, $\eta^i_{2a}\partial^{(j}\eta^{k)}_{2b}$ and $\partial^{(i}\eta^{j)}_{2a}\partial^{(k}\eta^{l)}_{2b}$, such that

$$\overset{(1)}{\omega}_1 = a^{ab}_{ij}\eta^i_{2a}\eta^j_{2b} + a^{ab}_{ijk}\eta^i_{2a}\partial^{(j}\eta^{k)}_{2b} + a^{ab}_{ijkl}\partial^{(i}\eta^{j)}_{2a}\partial^{(k}\eta^{l)}_{2b}, \tag{37}$$

with a^{ab}_{ij}, a^{ab}_{ijk} and a^{ab}_{ijkl} from $H_1\left(\delta_I|\tilde{d}\right)$, hence linear combinations of $\mathcal{P}^a_{2i}$ (see (35))

$$a^{ab}_{ij} = a^{abk}_{ijc}\mathcal{P}^c_{2k}, \ a^{ab}_{ijk} = a^{abl}_{ijkc}\mathcal{P}^c_{2l}, \ a^{ab}_{ijkl} = a^{abi'}_{ijklc}\mathcal{P}^c_{2i'}, \tag{38}$$

where a^{abk}_{ijc}, a^{abl}_{ijkc} and $a^{abi'}_{ijklc}$ are constants. Due to the covariance, these constants must vanish, $a^{abk}_{ijc} = 0$, $a^{abl}_{ijkc} = 0$, $a^{abi'}_{ijklc} = 0$, so $\overset{(1)}{\omega}_1 = 0$. So far, we deduced that the only nonvanishing piece of ω_1 is given by

$$\overset{(0)}{\omega}_1 = a^a{}_i \left([A^a_i], \left[\partial_i B^{0i}_a \right], [\pi] \right) \eta^i_{2a}. \tag{39}$$

Strictly speaking, $\overset{(0)}{\omega}_1$ should have contained also a term of the type $a^a{}_{ij}\partial^{(i}\eta^{j)}_{2a}$, with $a^a{}_{ij} = a^a{}_{ji}$. This term can be rewritten under the form $a^a{}_{ij}\partial^{(i}\eta^{j)}_{2a} =$

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$\partial^i \left(2a^a{}_{ij} \eta^j_{2a} \right) - 2 \left(\partial^j a^a{}_{ij} \right) \eta^i_{2a}$. As the non-integrated density $\overset{(0)}{\omega}_1$ is defined up to a total divergence, we can omit the term $\partial^i \left(2a^a{}_{ij} \eta^j_{2a} \right)$, while the piece $-2 \left(\partial^j a^a{}_{ij} \right) \eta^i_{2a}$ is of the same type like that appearing in (39) modulo the identification $a^a{}_i = -2 \partial^j a^a{}_{ij}$.

Let us investigate now the first-order deformation of the BRST-invariant Hamiltonian. On account of (19) and (39), we find that

$$
[H_{0_B}, \Omega_1]^* = \int d^3x \left(-a^a{}_i \left(\eta^i_{1a} + \partial^i \eta_{2a} \right) + \right.
$$
$$
\int d^3y \eta^i_{2b}(y) \left(\left[\varphi_a(x), a^b{}_i(y) \right]^* \pi^{(2)a}(x) - \left[\varphi_a^{(2)}(x), a^b{}_i(y) \right]^* \partial_i \partial^i \pi^a(x) - \right.
$$
$$
\left. \left[B_a^{0j}(x), a^b{}_i(y) \right]^* \partial_j \partial^k B^a_{0k}(x) \right) . \tag{40}
$$

In order to ensure the compensation of the term $a^a{}_i \eta^i_{1a}$ through a similar term in $[H_1, \Omega_0]^*$, we take H_1 of the type

$$
H_1 = \int d^3x \left(-\frac{1}{2} \epsilon^{0ijk} a^a{}_i B_{ajk} + \bar{h}_1 \right), \tag{41}
$$

where $\bar{h}_1$ does not involve B_{ajk}. By means of (41), it results that

$$
[H_1, \Omega_0]^* = \int d^3x \left(a^a{}_i \eta^i_{1a} - \right.
$$
$$
\int d^3y \left(\frac{1}{4} \epsilon^{0ijk} \epsilon_{0lmn} B_{ajk}(x) \left[a^a{}_i(x), F^{bmn}(y) \right]^* \eta^l_{2b}(y) - \right.
$$
$$
\left. \left[\bar{h}_1(x), \omega_0(y) \right]^* \right) , \tag{42}
$$

such that the first equation in (29) becomes

$$
-\int d^3x a^a{}_i \partial^i \eta_{2a} + \int d^3x d^3y \left(\eta^l_{2b}(y) \left(\left[\varphi_a(x), a^b{}_l(y) \right]^* \pi^{(2)a}(x) - \right. \right.
$$
$$
\left[\varphi_a^{(2)}(x), a^b{}_l(y) \right]^* \partial_i \partial^i \pi^a(x) - \left[B_a^{0j}(x), a^b{}_l(y) \right]^* \partial_j \partial^k B^a_{0k}(x) -
$$
$$
\left. \frac{1}{4} \epsilon^{0ijk} \epsilon_{0lmn} B_{ajk}(x) \left[a^a{}_i(x), F^{bmn}(y) \right]^* \right) + \left[\bar{h}_1(x), \omega_0(y) \right]^* \right) = 0. \tag{43}
$$

The last equation is verified with the choices

$$
a^a{}_i = -f^a{}_{bc} \left(\frac{1}{2} \epsilon_{0ijk} A^{bj} A^{ck} + A^c_i \pi^b \right), \tag{44}
$$

$$
\bar{h}_1 = f^a{}_{bc} \left(-\left(\partial^i B^b_{0i} \right) \left(B_a^{0j} A^c_j + \varphi_a \pi^c + \varphi_a^{(1)} \pi^c + \varphi_a^{(2)} \pi^{(2)c} + \right. \right.
$$
$$
\left. \left. \eta^j_{2a} \mathcal{P}^c_{2j} + \eta_{2a} \mathcal{P}^c_2 \right) + A^c_i \left(\pi^b \partial^i \varphi_a^{(2)} - \eta_{2a} \mathcal{P}^{bi}_2 + \eta^i_{2a} \mathcal{P}^b_2 \right) \right), \tag{45}
$$

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where f^a_{bc} are some constants, antisymmetric in the lower indices, $f^a_{bc} = -f^a_{cb}$. Inserting (44-45) in (39) and (41), we determine the complete expressions of the first-order deformations of both BRST charge and BRST-invariant Hamiltonian.

Next, we analyse the second-order deformations. With Ω_1 at hand, we find that $[\Omega_1, \Omega_1]^* = 0$, so the second equation in (28) is obeyed for $\Omega_2 = 0$. The higher-order equations that describe the deformation of the BRST charge will be satisfied if we set $\Omega_3 = \Omega_4 = \cdots = 0$. On the one hand, as $\Omega_2 = 0$, the second equation in (29) reduces to

$$[H_2, \Omega_0]^* + [H_1, \Omega_1]^* = 0. \tag{46}$$

On the other hand, $[H_1, \Omega_1]^*$ is given by

$$[H_1, \Omega_1]^* = s_I \left(\int d^3x \left(f^a_{bc} f^d_{ea} A^c_i \pi^b A^{ei} \varphi^{(2)}_d + \right.\right.$$

$$f^a_{bc} f^d_{eg} g^{eb} \left(\left(\varphi_a + \varphi^{(1)}_a \right) \pi^c \left(\frac{1}{2} \left(\varphi_d + \varphi^{(1)}_d \right) \pi^g + B^{0j}_d A^g_j + \right.\right.$$

$$\varphi^{(2)}_d \pi^{(2)g} + \eta^j_{2d} \mathcal{P}^g_{2j} + \eta_{2d} \mathcal{P}^g_2 \right) + B^{0i}_a A^c_i \left(\frac{1}{2} B^{0j}_d A^g_j + \varphi^{(2)}_d \pi^{(2)g} + \right.$$

$$\left.\left.\eta_{2d} \mathcal{P}^g_2 \right) + \eta^i_{2a} \mathcal{P}^c_{2i} \left(B^{0j}_d A^g_j + \varphi^{(2)}_d \pi^{(2)g} + \frac{1}{2} \eta^j_{2d} \mathcal{P}^g_{2j} + \eta_{2d} \mathcal{P}^g_2 \right) \right)\right) -$$

$$f^c_{[de} f^a_{b]c} \int d^3x \left(\eta^i_{2a} \partial^l B^d_{0l} \left(A^e_i \pi^b + \frac{1}{2} \epsilon_{0ijk} A^{bj} A^{ek} \right) + \right.$$

$$\left.\frac{1}{3} \epsilon^{0ijk} \eta_{2a} A^d_i A^e_j A^b_k \right), \tag{47}$$

where $[deb]$ means antisymmetry with respect to the indices between brackets. From (47), we remark that $[H_1, \Omega_1]^*$ is indeed s_I-exact (as required by (46)) if and only if the antisymmetric constants f^a_{bc} fulfill the Jacobi identity

$$f^c_{[de} f^a_{b]c} = 0, \tag{48}$$

therefore if and only if they stand for the structure constants of a Lie algebra. Under these circumstances, we arrive at

$$h_2 = f^a_{bc} f^d_{ea} A^c_i \pi^b A^{ei} \varphi^{(2)}_d + f^a_{bc} f^d_{eg} g^{eb} \left(\left(\varphi_a + \varphi^{(1)}_a \right) \pi^c \left(\frac{1}{2} \left(\varphi_d + \varphi^{(1)}_d \right) \pi^g + \right.\right.$$

$$B^{0j}_d A^g_j + \varphi^{(2)}_d \pi^{(2)g} + \eta^j_{2d} \mathcal{P}^g_{2j} + \eta_{2d} \mathcal{P}^g_2 \right) + B^{0i}_a A^c_i \left(\frac{1}{2} B^{0j}_d A^g_j + \varphi^{(2)}_d \pi^{(2)g} + \right.$$

$$\left.\eta_{2d} \mathcal{P}^g_2 \right) + \eta^i_{2a} \mathcal{P}^c_{2i} \left(B^{0j}_d A^g_j + \varphi^{(2)}_d \pi^{(2)g} + \frac{1}{2} \eta^j_{2d} \mathcal{P}^g_{2j} + \eta_{2d} \mathcal{P}^g_2 \right)\right). \tag{49}$$

The equation that governs the third-order deformation of the BRST-invariant Hamiltonian

$$[H_3, \Omega_0]^* + [H_2, \Omega_1]^* + [H_1, \Omega_2]^* + [H_0, \Omega_3]^* = 0, \qquad (50)$$

simply becomes $[H_3, \Omega_0]^* + [H_2, \Omega_1]^* = 0$, because $\Omega_2 = \Omega_3 = 0$. Using (49), we find that $[H_2, \Omega_1]^* = 0$ due to the Jacobi identity. Accordingly, we can take $H_3 = 0$, and similarly for the higher-order deformation equations, which are satisfied for $H_4 = H_5 = \cdots = 0$. In this way, the Hamiltonian deformation procedure in the context of the irreducible BRST formulation of the model under study is completed.

4   Identification of the deformed model

In the sequel we analyse the deformed theory constructed in the above. Collecting all the results deduced so far, we can write down the complete BRST charge and BRST-invariant Hamiltonian of the deformed model, that are consistent to all orders in the deformation parameter, under the form

$$\Omega = \int d^3x \left(\epsilon_{0ijk} \pi^{jka} \eta^i_{1a} + \left(\frac{1}{2} \epsilon_{0ijk} H^{ajk} - (D_i)^a_{\ b} \pi^b \right) \eta^i_{2a} + \right.$$
$$\left. \left(\pi^a - \pi^{(1)a} \right) \eta_{1a} - \pi^{(2)a} \eta_{2a} \right), \qquad (51)$$

respectively,

$$H_B = \int d^3x \left(\frac{1}{2} B^{ij}_a \left(H^a_{ij} + \varepsilon_{0ijk} \left(D^k \right)^a_{\ b} \pi^b \right) - \frac{1}{2} A^a_i A^i_a + \varphi_a \pi^{(2)a} + \right.$$
$$\frac{1}{2} \left((D_i)^{\ b}_a B^{0i}_b - g f^c_{\ ab} \left(\varphi_c \pi^b + \varphi^{(1)}_c \pi^b + \varphi^{(2)}_c \pi^{(2)b} \right) \right)^2 -$$
$$\varphi^{(2)}_a \left((D_i)^a_{\ b} \left(D^i \right)^b_{\ c} \right) \pi^c + \eta^i_{1a} \mathcal{P}^a_{2i} - \eta_{1a} \mathcal{P}^a_2 - \eta_{2a} \left(D^i \right)^a_{\ b} \mathcal{P}^b_{2i} +$$
$$\eta^i_{2a} (D_i)^a_{\ b} \mathcal{P}^b_2 - g f^c_{\ ab} \left(\eta^i_{2c} \mathcal{P}^b_{2i} + \eta_{2c} \mathcal{P}^b_2 \right) \times$$
$$\left(\left(D^j \right)^a_{\ d} B^d_{0j} - g f^a_{\ de} \left(\pi^d \varphi^e + \pi^d \varphi^{(1)e} + \pi^{(2)d} \varphi^{(2)e} \right) \right) -$$
$$\left. \frac{1}{2} g^2 f^c_{\ ab} f^a_{\ de} \left(\eta^i_{2c} \mathcal{P}^b_{2i} + \eta_{2c} \mathcal{P}^b_2 \right) \left(\eta^d_{2j} \mathcal{P}^{je}_2 + \eta^d_2 \mathcal{P}^e_2 \right) \right), \qquad (52)$$

where we employed the notations

$$H^a_{ij} = F^a_{ij} - g f^a_{\ bc} A^b_i A^c_j, \qquad (53)$$

$$\left((D_i)^{\ b}_a B^{0i}_b - g f^c_{\ ab} \left(\varphi_c \pi^b + \varphi^{(1)}_c \pi^b + \varphi^{(2)}_c \pi^{(2)b} \right) \right)^2 \equiv$$
$$\left((D_i)^{\ b}_a B^{0i}_b - g f^c_{\ ab} \left(\varphi_c \pi^b + \varphi^{(1)}_c \pi^b + \varphi^{(2)}_c \pi^{(2)b} \right) \right) \times$$
$$\left(\left(D^j \right)^a_{\ d} B^d_{0j} - g f^a_{\ de} \left(\pi^d \varphi^e + \pi^d \varphi^{(1)e} + \pi^{(2)d} \varphi^{(2)e} \right) \right), \qquad (54)$$

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$$(D_i)^a_{\ b} = \delta^a_{\ b}\partial_i + gf^a_{\ bc}A^c_i, \ (D_i)_b^{\ a} = \delta_b^{\ a}\partial_i - gf^a_{\ bc}A^c_i. \tag{55}$$

From the terms of antighost number zero in (51) we read that only the first-class constraints $\gamma^a_{2i} \equiv \frac{1}{2}\epsilon_{0ijk}F^{ajk} - \partial_i\pi^a \approx 0$ are deformed like

$$\bar{\gamma}^a_{2i} \equiv \frac{1}{2}\epsilon_{0ijk}H^{jka} - (D_i)^a_{\ b}\pi^b \approx 0, \tag{56}$$

while the others are kept unchanged. In the meantime, the resulting BRST charge contains no pieces quadratic in the ghost number one ghosts and linear in the antighosts, hence the gauge algebra (in the Dirac bracket) of the deformed first-class constraints remains abelian, being not affected by the deformation method. Analysing the structure of the pieces in (52) that involve neither ghosts, nor antighosts, we discover that the first-class Hamiltonian of the deformed theory reads as

$$H = \int d^3x \left(\frac{1}{2}B^{ij}_a \left(H^a_{ij} + \varepsilon_{0ijk}\left(D^k\right)^a_{\ b}\pi^b \right) - \frac{1}{2}A^a_i A^i_a + \varphi_a\pi^{(2)a} + $$
$$\frac{1}{2}\left((D_i)_a^{\ b}B^{0i}_b - gf^c_{\ ab}\left(\varphi_c\pi^b + \varphi^{(1)}_c\pi^b + \varphi^{(2)}_c\pi^{(2)b} \right) \right)^2 - $$
$$\varphi^{(2)}_a \left((D_i)^a_{\ b}\left(D^i\right)^b_{\ c} \right)\pi^c \bigg), \tag{57}$$

while from the components linear in the antighost number one antighosts we find that the Dirac brackets among the new first-class Hamiltonian and the new first-class constraints are modified as

$$[H, \gamma^a_{1i}]^* = \bar{\gamma}^a_{2i}, \ [H, \gamma^a_1]^* = -\gamma^a_2, \tag{58}$$

$$[H, \bar{\gamma}^a_{2i}]^* = (D_i)^a_{\ b}\gamma^b_2 - $$
$$gf^a_{\ bc}\left(\left(D^j\right)^b_{\ d}B^d_{0j} - gf^b_{\ de}\left(\pi^d\varphi^e + \pi^d\varphi^{(1)e} + \pi^{(2)d}\varphi^{(2)e} \right) \right)\bar{\gamma}^c_{2i}, \tag{59}$$

$$[H, \gamma^a_2]^* = -\left(D^i \right)^a_{\ b}\bar{\gamma}^b_{2i} - $$
$$gf^a_{\ bc}\left(\left(D^j\right)^b_{\ d}B^d_{0j} - gf^b_{\ de}\left(\pi^d\varphi^e + \pi^d\varphi^{(1)e} + \pi^{(2)d}\varphi^{(2)e} \right) \right)\gamma^c_2. \tag{60}$$

The first-class constraints and first-class Hamiltonian generated until now along the deformation scheme reveal precisely the consistent irreducible Hamiltonian interactions that can be introduced among a set of two-form gauge fields, which actually produce the irreducible version of non-abelian Freedman-Townsend model in four-dimensions. As the first-class constraints generate

gauge transformations, we can state that the added interactions deform the gauge transformations, but not the algebra of gauge transformations (due to the abelianity of the deformed first-class constraints). In order to make our result more transparent, we observe that if in (57–60) we set all the supplementary canonical pairs (φ_a, π^a), $\left(\varphi_a^{(1)}, \pi^{(1)a}\right)$, and $\left(\varphi_a^{(2)}, \pi^{(2)a}\right)$ equal to zero, we reach that the Lagrangian form of the interaction term generated by our procedure is precisely $\frac{1}{2} f_{bc}^a B_a^{\mu\nu} A_\mu^b A_\nu^c$, which is nothing but the standard Freedman-Townsend vertex, while the remaining gauge algebra relations reduce to those of the usual Freedman-Townsend model. This completes our analysis.

5 Conclusion

To conclude with, in this paper we have investigated in an irreducible manner the consistent Hamiltonian interactions among a set of two-forms in four dimensions. Our method is based on the deformation of the BRST charge and BRST-invariant Hamiltonian associated with an irreducible formulation of the free model. The main equations that control the deformed quantities are solved by using some cohomological techniques. As a result, we obtain the irreducible version of the non-abelian Freedman-Townsend model. The added interactions deform the gauge transformations (but not their algebra), as well as the Dirac brackets between the first-class Hamiltonian and the first-class constraint set. The interaction term revealed by our method is nothing but the standard Freedman-Townsend vertex.

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