5D $SU(3)_W$ unification at TeV and cancellation of local gauge anomalies with split multiplets [1]

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Abstract

We consider the 5D gauge unification of $SU(2)_L \times U(1)_Y$ into $SU(3)_W$ at a TeV scale. Compactification of the extra dimension on an orbifold $S^1/(Z_2 \times Z_2')$ allows fixed points where $SU(2)_L \times U(1)_Y$ representations can be assigned. We explain the long proton lifetime and the top-bottom mass hierarchy geometrically. We also show that local gauge anomalies on the orbifold can be exactly cancelled by a 5D Chern-Simons term with a jumping coefficient.

We know that the gauge couplings of the Standard Model(SM) undergo the logarithmical runnings to high energy scales due to quantum corrections. In the Minimal Supersymmetric Standard Model(MSSM), for instance, the gauge couplings become unified at $M_{GUT} = 2 \times 10^{16}$ GeV within the experimental error bound[1]. The large hierarchy of scales is needed to get a large α_s and $\sin^2\theta_W \simeq 0.231$ at M_Z with the unification of gauge couplings at M_{GUT}[3]. In the MSSM, the so called gauge hierarchy is maintained by softly broken supersymmetry against radiative corrections to the Higgs mass.

To alleviate the gauge hierarchy problem in the GUTs with a large gauge group, we can consider the low energy unification of $SU(2)_L \times U(1)_Y$ into $SU(3)_W$[2]. Then, the lepton sector(L, e^c) and the Higgs sector are successfully embedded into $\mathbf{3}$'s of $SU(3)_W$ with the hypercharge operator $Y = \mathrm{diag.}(-1/2, -1/2, +1)$. Moreover, we have $\sin^2\theta_W = 0.25$ at the unification scale, which is so close to its experimental value at M_Z and thus does not need large logarithms to run. However, it is not possible to accommodate quark fields without an extra $U(1)$ gauge group beyond $SU(3)_W$ due to their fractional hypercharges in units of $\frac{1}{2}$.

There has been recently a lot of attention to the GUT models on orbifolds with extra dimensions[3, 4]. The main virtue of the GUT orbifolds is that the breaking of gauge symmetry and/or supersymmetry and the doublet-triplet splitting can be performed at the same time by the geometrical boundary conditions. Along this line, the 5D $SU(3)_W$ gauge theory has been considered on an orbifold $S^1/(Z_2 \times Z_2')$ with its radius R[5, 6, 7]. If we impose on the bulk gauge fields different charges under the two Z_2 reflection symmetries, then the bulk gauge symmetry can be broken down to the electroweak gauge symmetry at the TeV-sized compactification scale. The corresponding boundary conditions of the gauge fields $A_M = (A_\mu, A_5)(\mu = 0, 1, 2, 3)$ are written in terms of one relection Z_2 and one twist $T = Z_2 \times Z_2'$ as

$$A_\mu(y) = Z_2 A_\mu(-y) Z_2^{-1} = T A_\mu(y + \pi R) T^{-1}, \quad (1)$$

$$A_5(y) = -Z_2 A_5(-y) Z_2^{-1} = T A_5(y + \pi R) T^{-1} \quad (2)$$

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[3]In the case with SU(5) unification, note that $\alpha_{1,2,3} \simeq \frac{1}{25}$ and $\sin^2\theta_W = \frac{3}{8} = 0.375$.

where $Z_2 = \mathrm{diag.}(1,1,1)$ and $T = \mathrm{diag.}(1,1,-1)$. The 5D fundamental scale(M_s), which is regarded as the unification scale, is also chosen to be not far above the TeV scale such that the logarithmic Kaluza-Klein corrections to the running of gauge couplings can predict a correct $\sin^2\theta_W$ at low energies. Thus, there is no big desert between the unification scale and the weak scale but we still have to cope with the rapid proton decay in generic models with such a low fundamental scale.

After the orbifold compactification, there appear two fixed points where the local gauge symmetry is different. Only the $SU(2)_L \times U(1)_Y$ gauge symmetry remains at one of fixed points($y = \pi R/2$), which is denoted as A, where quark fields can be located. On the other hand, the full bulk gauge symmetry is respected at the other fixed point($y = 0$), which is denoted as O. Therefore, the location of the lepton sector or the Higgs sector in this model, is not determined from a field theoretic point of view. In this paper, to avoid the rapid proton decay, we take the minimal embedding by splitting leptons and quarks maximally in the extra dimension: leptons at O and quarks at A[5, 6, 7]. Furthermore, to give the top-bottom mass hierarchy, we take the asymmetric embedding for the Higgs sector: a up-type Higgs(H_u) at A and a down-type Higgs(H_d) in the bulk[4, 7]. Here we note that H_d should come from a bulk $\bar{\mathbf{6}}_{\mathbf{H}}$ to give reasonable lepton masses at O.

With the embedding of the SM particles on the orbifold, we consider the running of gauge couplings above the compactification scale($M_c = R^{-1}$) due to KK modes. The electroweak gauge symmetry allows arbitrary brane gauge kinetic terms at A, which could spoil the low energy prediction of this orbifold model. But, with the assumption of strong coupling constants at the compactification scale, i.e. $M_s R = \mathcal{O}(100)$, we can make a prediction on the Weinberg angle at M_Z with KK corrections above M_c[6, 7]:

$$\sin^2\theta_W(M_Z) = 0.25 - \frac{3}{8\pi}\alpha_{em}\left[\tilde{B}\ln\frac{M_s}{M_c} + B\ln\frac{M_c}{M_Z}\right] \tag{3}$$

where $B = b_g - b_{g'}/3$ and $\tilde{B} = \tilde{b}_g - \tilde{b}_{g'}/3$. Here b's denote the beta function coefficients for the SM zero modes and $\tilde{b}$'s denote those for the KK modes. As a result, in the non-SUSY case, the fundamental scale becomes $70-80$ TeV, depending on one or two Higgs fields. In the MSSM case, we obtain $M_s = 1.9 - 3.4\times 10^4$ TeV, depending on asymmetric or symmetric embedding of the Higgs sector.

Now let us look into the local gauge anomalies in our model for consistency. Since we have leptons and quarks located at different fixed points, it seems that there could exist corresponding local gauge anomalies at each fixed point. Moreover, the gauge anomalies coming from a single bulk Higgsino field are equally splitted at both fixed points[8], so that there could be the remaining local gauge anomalies even after a brane Higgsino field with opposite charge is taken into account. In any case, it turns out that the total remaining local gauge anomalies are cancelled exactly by the variation of a bulk non-abelian Chern-Simons term with a proper normalization $c = 3c_l + c_H = 7$:

$$\mathcal{L}_{CS} = \frac{c}{128\pi^2}\epsilon(y)\mathrm{tr}\left(AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5\right) \tag{4}$$

where $\epsilon(y)$ is the sign function with periodicity πR. This Chern-Simons term is parity odd and thus explicitly breaks the Z_2 parity symmetries, which implies that the gauge symmetry on the orbifold is maintained at the quantum level only at the price of the parity violation. It is also shown that gravitational mixed anomalies of $U(1)_Y$, which could appear only at A, cancel between quarks and Higgsino fields separately without the aid of a bulk Chern-Simons term[7].

To upshot, it is shown that in the 5D $SU(3)_W$ unification model compactified on $S^1/(Z_2 \times Z_2')$, the quark sector is also accommodated at the fixed point retaining only the electroweak gauge symmetry. We also considered SM multiplets split in other locations of the extra dimensions to explain the proton stability and the $t-b$ mass hierarchy. This splitting still gives rise to a consistent gauge theory with the introduction of a 5D Chern-Simons term. Our model also helps to predict $\sin^2\theta_W$ at low energies through KK modes without the need of a large hierarchy.

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