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Intersecting Membranes from Charged Macroscopic Strings

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Abstract

We present a class of orthogonal membrane configurations which preserve 1/4 of the full type IIA supersymmetry. These membrane configurations carry additional F-string charges. We further analyze the $D1-D3$ configuration after applying T- duality along the world volume directions of the above orthogonal membranes.

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Various configurations of D-branes and their bound states [1, 2, 3, 4] have emerged as important objects in string theory. They are very useful in understanding string theory and gauge theory beyond their perturbative regime. They are also important in understanding black holes from a microscopic point of view [5]. It is well known by now that a system of multiple D- branes related by $SU(N)$ rotation, preserves unbroken supersymmetry [6] and, consequently, it is interesting to study their properties and applications in order to test various conjectures in string theory. Explicit supergravity solutions for these branes and their bound states are studied at length in literature [7, 8]. A special class of such branes are orthogonally intersecting branes. The purpose of this paper is to construct a class of orthogonally intersecting supersymmetric branes in ten dimensional type IIA/IIB string theory. We generalize these configurations starting from charged macroscopic string solution.

Charged macroscopic strings have been useful in the past in establishing various duality conjectures in string theory. Like the neutral solutions, BPS nature of these configurations, allow us to construct stable multi-string configurations. Charged string solutions are in general constructed from neutral strings by means of a solution generating technique. In particular, the solutions presented in [9], are parametrized by a group $O(1,1;1,1)$. In this case, the complete solution is represented by two nontrivial parameters α and β. The supersymmetry of the solutions while embedded in type II string theories, is explicitly shown in [10] for particular values of α and β. In general, these strings carry vectorial charges and currents in addition to the string charges. The bound states constructed from these strings are interpreted as D- brane bound sates with F- string charges being dissolved in different orthogonal directions [11]. Starting with a particular class of D-branes of [11], we construct orthogonal membrane configurations in ten dimensions. They are different from the ones constructed in [12, 13], as our solutions contain NS-NS 2-form potentials, indicating the presence of F-string charges as well. Indeed in certain limits, our solution reduces to [12, 13]. It preserves 1/4 of the full IIA supersymmetry. We then apply T- duality along one of the world volume directions to generate bound state configuration of orthogonal (D1, D3) branes. The existence and stability of $(D1 \perp D3)$ is checked by examining the mass-charge relationship explicitly. We start by writing down the supergravity solution of a D2-brane constructed from the charged macroscopic string[11]. This membrane carries NS-NS two form charge along with the R-R three form potential. These NS-NS charges are parameterized by the solution generating parameter α. One can obtain the charge neutral D2-brane solution[14, 15] by taking $\alpha = 0$ limit.

$$\begin{aligned}
ds^2 &= \frac{(1+X)^{3/2}}{1+X\cosh^2\alpha}\Big[\frac{1}{1+X}\{-(dt)^2+(dx^9)^2+\frac{1+X\cosh^2\alpha}{1+X}(dx^8)^2\} \\
&+ \frac{1+X\cosh^2\alpha}{1+X}\sum_{i=1}^{7}(dx^i)^2\Big], \\
e^{\phi(10)} &= \frac{(1+X)^{3/2}}{1+X\cosh^2\alpha}, \quad A^{(3)} = -\frac{X\cosh\alpha}{1+X\cosh^2\alpha}dt\wedge dx^8\wedge dx^9, \\
A^{(1)} &= -\frac{X\sinh\alpha}{1+X}dx^8, \quad B^{(2)} = \frac{X\sinh\alpha\cosh\alpha}{1+X\cosh^2\alpha}dt\wedge dx^9,
\end{aligned} \tag{1}$$

where $X = C(\frac{l}{|\vec{x}|})^5$ is the harmonic function in the transverse space and l is the length parameter. Now, we present the supergravity solution of two such membranes orthogonal to each other. To start with, both membranes (1) are lying in $x^8 - x^9$ plane and are parallel to each other. A rotation between $x^8 - x^9$ and $x^1 - x^2$ plane is then performed following [12]. The rotation angles in our case are $(\theta_1, \theta_2) = (0, \pi/2)$, where θ_1 and θ_2 are the angles with which the two membranes are being rotated. The second rotated brane now lies in $x^1 - x^2$ plane which is orthogonal to the $x^8 - x^9$ plane. The final solution is given by:

$$\begin{aligned}
ds^2 &= \frac{(1+X)^{3/2}}{1+X\cosh^2\alpha}\Big[\frac{1}{1+X}\{-(dt)^2+(1+X_2\cosh^2\alpha)(dx^9)^2 \\
&+ \frac{1+X\cosh^2\alpha}{1+X}(1+X_2)(dx^8)^2+(1+X_1\cosh^2\alpha)(dx^1)^2 \\
&+ \frac{1+X\cosh^2\alpha}{1+X}(1+X_1)(dx^2)^2\} + \frac{1+X\cosh^2\alpha}{1+X}\sum_{i=3}^{7}(dx^i)^2\Big], \\
e^{\phi(10)} &= \frac{(1+X)^{3/2}}{1+X\cosh^2\alpha},
\end{aligned}$$

$$
\begin{aligned}
A^{(3)} &= -\frac{\cosh\alpha}{1+X\cosh^2\alpha}dt\wedge\Big[(X_1+X_1X_2)dx^8\wedge dx^9-(X_2+X_1X_2)dx^1\wedge dx^2\Big],\\
A^{(1)} &= -\frac{\sinh\alpha}{1+X}\Big(X_1dx^8+X_2dx^2\Big),\\
B^{(2)} &= \frac{\sinh\alpha\cosh\alpha}{1+X\cosh^2\alpha}dt\wedge(X_1dx^9-X_2dx^1),
\end{aligned}
\tag{2}
$$

where $X_a = C(\frac{l_a}{|\vec{x}-\vec{x_a}|})^3$, with $a = 1, 2$, are the harmonic functions in the tranverse space, given by x^i's, and $X = (X_1 + X_2 + X_1X_2)$[12, 13] with l_a's being the arbitrary positive parameters having the dimensions of length. To clarify further the form of X appearing above, one notices that its general structure for a system of two D2-branes, when their relative orientation is restricted by SU(2) rotation is given by[12, 13, 16]:

$$
X = X_1 + X_2 + X_1X_2\sin^2(\theta_1-\theta_2), \tag{3}
$$

with θ_1 and θ_2 are the angles with which the branes being rotated. In the present context, however, $(\theta_1, \theta_2) = (0, \pi/2)$. We would like to point out here, that for a configuration of parallel membranes X is the sum of individual harmonic functions describing each of them. But, when the branes are oriented at some angle, the form of X gets modified as given above. As stated earlier, this class of solutions including NS-NS charges, are parametrized by α. We have also explicitly verified that in asymptotic limit, the above solution indeed satisfy the type IIA string equations of motion. Moreover, in $\alpha = 0$ limit of the above solution reduces to the ones given in [17, 13]. To confirm further the correctness of our solution, we present the mass-charge relation (of BPS type) in the following discussion. We have therefore obtained, an orthogonal configuration of membranes following the procedure as described in [7]. One notices that the above solution carries 3- form R-R potentials which are in $(x^8 - x^9)$ and $(x^1 - x^2)$-plane. In addition it also carries F-string charges along x^1 and x^9 directions. We now go on to check its supersymmetric property by analyzing the mass-charge relationship explicitly.

The charges associated with the membranes can be read off from the leading behaviour of the potentials, that are discussed earlier. To simplify the discussion further, a dimensional reduction along the isometry directions is performed to check the BPS properties of our solution, as described in [18, 11]. After dimensional reduction, we get charges from the following fields : $A_t^1 \sim \mathcal{B}_{t9}$, $A_t^2 \sim \mathcal{B}_{t1}$ $A_t^3 \sim \mathcal{A}_{t89}$ and $A_t^4 \sim \mathcal{A}_{t12}$, which are as follows:

$$
\begin{aligned}
Q_1 &= -3C \cosh\alpha \sinh\alpha \; {l_1}^3, \\
Q_2 &= 3C \cosh\alpha \sinh\alpha \; {l_2}^3, \\
Q_3 &= 3C \cosh\alpha \; {l_1}^3, \\
Q_4 &= -3C \cosh\alpha \; {l_2}^3,
\end{aligned} \tag{4}
$$

where, Q_i's are the charges associated with $A_t^{(i)}$ with $i = 1....4$.

The ADM mass density can be calculated by using the formula as given in [19, 7]:

$$
m = \int \sum_{i=1}^{9-p} n^i \left[\sum_{j=1}^{9-p} (\partial_j h_{ij} - \partial_i h_{jj}) - \sum_{a=1}^{p} \partial_i h_{aa} \right] r^{8-p} d\Omega, \tag{5}
$$

where n^i is a radial unit vector in the transverse space and $h_{\mu\nu}$ is the deformation of the Einstein-frame metric from flat space in the asymptotic region. The ADM mass-density for this case is found to be:

$$
m_{(2\perp 2)} = 3C \cosh^2\alpha ({l_1}^3 + {l_2}^3). \tag{6}
$$

Comparing (4) and (6), we get

$$
m^2_{(2\perp 2)} = (Q_1 - Q_2)^2 + (Q_3 - Q_4)^2. \tag{7}
$$

Now, to show this formula indeed reduces to the BPS bound that preserves certain amount of supersymmetry, one considers the most general form of the Bogomol'nyi mass matrix written explicitly in [20]. This matrix [20], corresponds to the most general charges of the brane configuration. In particular, when the charges are in 2-form tensorial representation, the vanishing eigen-value equation of the mass matrix as obtained in [12] indicates the preservation of one-quarter supersymmetry. The precise form of the mass formula [12] is given by :

$$m_{\pm}^2 = (q_{ij}q_{ij} \pm \frac{1}{2}\epsilon_{ijkl}q_{ij}q_{kl}), \tag{8}$$

where q_{ij}'s are the nonvanishing charge densities correspond to D- membranes. By suitably identifying the charges in our case, with q_{ij}'s of [12], eqn.(7) does imply the mass-formula of one-quarter BPS objects. We therefore conclude that our configuration preserves 1/4 supersymmetry.

We now construct other interesting configurations starting from (2). These will be constructed by using T-duality property of string theory. In particular, we will consider the effect of T- duality along one of the world-volume direction, namely along x^9. Following the rules, that are given in [21, 7], we get the following configurations of the metric, dilaton, NS-NS 2-form, R-R 2-forms and 4-forms :

$$\begin{aligned}
ds^2 &= \frac{(1+X)^{1/2}}{(1+X\cosh^2\alpha)}\Big[-1+\frac{X_1^2\sinh^2\alpha\cosh^2\alpha}{(1+X)(1+X_2\cosh^2\alpha)}\Big](dt)^2 \\
&+ \frac{(1+X\cosh^2\alpha)}{(1+X)^{1/2}(1+X_2\cosh^2\alpha)}(dx^9)^2 + \frac{(1+X_2)}{(1+X)^{1/2}}(dx^8)^2 \\
&+ \frac{(1+X)^{1/2}(1+X_1\cosh^2\alpha)}{(1+X\cosh^2\alpha)}(dx^1)^2 + \frac{(1+X_1)}{(1+X)^{1/2}}(dx^2)^2 \\
&+ \frac{X_1\sinh\alpha\cosh\alpha}{(1+X)^{1/2}(1+X_2\cosh^2\alpha)}(dt)(dx^9) + (1+X)^{1/2}\sum_{i=3}^{7}(dx^i)^2, \\
e^{\phi^{(10)}} &= \frac{1+X}{1+X_2\cosh^2\alpha}, \\
A^{(4)}_{t189} &= \frac{X_1X_2\sinh^2\alpha\cosh\alpha}{2(1+X)(1+X\cosh^2\alpha)}, \\
A^{(4)}_{t129} &= -\Big[\frac{X_2^2\sinh^2\alpha\cosh\alpha}{2(1+X)(1+X\cosh^2\alpha)} - \frac{X_2(1+X_1)\cosh\alpha}{(1+X\cosh^2\alpha)}\Big], \\
A^{(2)}_{t8} &= -\frac{X_1(1+X_2)\cosh\alpha}{(1+X\cosh\alpha)}, \quad B^{(2)}_{t1} = -\frac{X_2\sinh\alpha\cosh\alpha}{(1+X\cosh^2\alpha)},
\end{aligned}$$

$$A^{(2)}_{98} = -\frac{X_1 \sinh\alpha}{(1+X)}, \qquad A^{(2)}_{92} = -\frac{X_2 \sinh\alpha}{(1+X)}. \tag{9}$$

Looking at the solutions, one notices that the resulting configurations is a (D1, D3)-bound state where D1 and D3 are orthogonal to each other. Apart form the R-R charges, they also carry F-string charges along a plane orthogonal to the D-string. Moreover, setting $X_1 = 0$, one can verify that the solution presented above reduces to the D3-brane solution given in [11].

Now we calculate the mass-charge relation for this configuration as well. Once again, when dimensionally reduce along all the isometry directions, charges arise from the following fields, $A^1_t \sim \mathcal{A}_{t8}$, $A^2_t \sim \mathcal{B}_{t1}$, $A^3_t \sim \mathcal{A}_{t129}$, $A^4_t \sim \mathcal{A}_{t189}$ and $A^5_t \sim g_{t9}/g_{99}$. The non-zero charges corresponding to the various field strengths and the metric components are:

$$\begin{aligned} Q_1 &= 3C\ l_1^3 \cosh\alpha, \\ Q_2 &= 3C\ l_2^3 \sinh\alpha\cosh\alpha, \\ Q_3 &= -3C\ l_2^3 \cosh\alpha, \\ P &= -3C\ l_1^3 \sinh\alpha\cosh\alpha, \end{aligned} \tag{10}$$

where Q_i's and P are the charges corresponding to the gauge fields A^i_t,(i= 1, 2, 3) and A^5_i respectively. P is the momentum in the x^9 direction of the 10- dimensional theory. The mass density of $(D1 \perp D3)$ is given by:

$$m_{(3\perp 1)} = 3C({l_1}^3 + {l_2}^3)\cosh^2\alpha. \tag{11}$$

The mass formula following from (10) and (11) can be written as:

$$m_{(3\perp 1)}{}^2 = (Q_1 - Q_3)^2 + (Q_2 - P)^2. \tag{12}$$

On the basis of our earlier observations, the above formula again indicates that a bound of $1/4$ BPS objects is saturated.

We have therefore constructed, in this paper, a general class of intersecting D- brane configuration starting from charged macroscopic strings. Preservation of one-quarter supersymmetry of these orthogonal brane configurations is shown by analyzing the BPS mass-bound explicitly. It will certainly be interesting to show the existence and supersymmetry properties of these bound states, when one of them being rotated by an arbitrary SU(2) angle. We hope to report on it in the future.

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