New Developments in String Gravity and String Cosmology. A Summary Report

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CONTENTS

1 Minimal String Driven Cosmology and its Predictions

In refs. [1], [2] we constructed a minimal model for the Universe evolution fully extracted from effective String Theory.

By linking this model to a minimal but well established observational information, we proved that it gives realistic predictions on early and current energy density and its results are compatible with General Relativity.

Interestingly enough, this model predicts the current energy density Omega=1 and a lower limit Omega larger or equal 4/9. On the other hand, the energy density at the exit of inflationary stage is also predicted $\text{Omega}_{\text{infl}} = 1$.
This result shows agreement with General Relativity (spatially flat metric gives critical energy density) within an unequivalent Non-Eistenian context (string low energy effective equations).

The order of magnitude of the energy density-dilaton coupled term at the beginning of radiation dominated stage agrees with GUT scale.

Whithout solving the known problems about higher order corrections and graceful exit of inflation, we find this model closer to the observational Universe properties than the current available string cosmology scenarii.

At a more fundamental level, this model is by its construction close to the standard cosmological evolution, and it is driven selfconsistently by the evolution of the string equation of state itself.

The inflationary String Driven stage is able to reach an enough amount of inflation, describing a Big Bang like evolution for the metric.

2 The Primordial Gravitational Wave Background in String Cosmology

In ref.[1] we found the spectrum P(w)dw of the gravitational wave background produced in the early universe in string theory.

We work in the framework of String Driven Cosmology, whose scale factors are computed with the low-energy effective string equations as well as selfconsistent solutions of General Relativity with a gas of strings as source.

The scale factor evolution is described by an early string driven inflationary stage with an instantaneous transition to a radiation dominated stage and successive matter dominated stage. This is an expanding string cosmology always running on positive proper cosmic time.

A careful treatment of the scale factor evolution and involved transitions is made. A full prediction of the power spectrum of gravitational waves without any free-parameters is given.

We study and show explicitly the effect of the dilaton field, characteristic to this kind of cosmologies.

We compute the spectrum for the same evolution description with three differents approachs.

Some features of gravitational wave spectra, as peaks and asymptotic behaviours, are found direct consequences of the dilaton involved and not only of the scale factor evolution.

3 Non-Singular String-Cosmologies From Exact Conformal Field Theories

In ref. [3] we constructed non-singular two and three dimensional string cosmologies using the exact conformal field theories corresponding to SO(2,1)/SO(1,1) and SO(2,2)/SO(2,1) coset models.

All semi-classical curvature singularities are canceled in the exact theories for both of these cosets, but some new curvature singularities emerge in the quantum models.
However, considering different patches of the global manifolds, allows the construction of non-singular spacetimes with cosmological interpretation.

In both, two and three dimensions, we constructed non-singular oscillating cosmologies, non-singular expanding and inflationary cosmologies including a de Sitter (exponential) stage with positive scalar curvature. Non-singular contracting and deflationary cosmologies were also constructed.

We analyse these cosmologies in detail with respect to the behaviour of the scale factors, the scalar curvature and the string-coupling.

The sign of the scalar curvature turns out to be changed by the quantum corrections in oscillating cosmologies and evolves with time in the non-oscillating cases.

Similarities between the two and three dimensional cases suggest a general picture for higher dimensional coset cosmologies :
(i) Anisotropy seems to be a generic unavoidable feature,
(ii) cosmological singularities are generically avoided and
(iii) it is possible to construct non-singular cosmologies where some spatial dimensions are experiencing inflation while the others experience deflation.
De Sitter stage can be achieved asymptotically at early times or late times, but there is not a conformal coset model of this type describing a de Sitter background globally.

4 Quantum Field Theory, String Temperature and the String Phase of De Sitter Spacetime

The density of mass levels ρ(m) and the critical temperature for strings in de Sitter space-time were found in ref. [5].

Quantum Field Theory (QFT) and string theory in de Sitter space have been compared in refs. [5] and [7].

A 'Dual'-transform is introduced which relates classical to quantum string lengths, and more generally, QFT and string domains.

Interestingly, the string temperature in De Sitter space turns out to be the Dual transform of the QFT-Hawking-Gibbons temperature.

The quantum back reaction problem for strings in de Sitter space is addressed selfconsistently in the framework of the 'string analogue' model (or thermodynamical approach), which is well suited to combine QFT and string studies [5], [6], [7].

We find de Sitter space-time is a self-consistent solution of the semi-classical Einstein equations in this framework. Two branches for the scalar curvature $R_{(\pm)}$ show up : a classical, low curvature solution (-), and a quantum high curvature solution (+), entirely sustained by the strings.

There is a maximal value for the curvature R_{max} due to the string back reaction.

Interestingly, our Dual relation manifests itself in the back reaction solutions : the (-) branch is a classical phase for the geometry with intrinsic temperature given by the QFT-Hawking-Gibbons temperature.

The (+) is a stringy phase for the geometry with temperature given by the intrinsic string de Sitter temperature.

2 + 1 dimensions are considered, but conclusions hold generically in D dimensions.

5 Hawking Radiation in String Theory and the String Phase of Black Holes

The quantum string emission by Black Holes is computed in the framework of the 'string analogue model' (or thermodynamical approach), which is well suited to combine QFT and string theory in curved backgrounds (particulary here, as black holes and strings posses intrinsic thermal features and temperatures).

The QFT-Hawking temperature T_H is upper bounded by the string temperature T_S in the black hole background.

The black hole emission spectrum is an incomplete gamma function of (T_H - T_S).

For $T_H \ll T_S$, the spectrum yields the QFT-Hawking emission.

For T_H near to T_S, it shows highly massive string states dominate the emission and undergo a typical string phase transition to a microscopic 'minimal' black hole of mass M_{min} or radius r_{min} (inversely proportional to T_S) and string temperature T_S.

The semiclassical QFT black hole (of mass M and temperature T_H) and the string black hole (of mass M_{min} and temperature T_S) are mapped one into another by a 'Dual' transform which links semi classical-QFT and quantum string regimes.

The string back reaction effect (selfconsistent black hole solution of the semiclassical Einstein equations with mass M_+ (radius r_+) and temperature T_+) is computed.

Both, the QFT and string black hole regimes are well defined and bounded: $r_{min} \leq r_+ \leq r_S$, $M_{min} \leq M_+ \leq M_S$, $T_{min} \leq T_+ \leq T_S$.

The string 'minimal' black hole has a life time $\tau_{min} = (K/Gh)\ T_S^{-3}$.

6 New Dual Relations between Quantum Field Theory and String Regimes in Curved Backgrounds

We introduce a R "Dual" transform which relates Quantum Field Theory and Quantum String regimes, both in a curved background.

This operation maps the characteristic length of one regime into the other and, as a consequence, maps mass domains as well.

The Hawking-Gibbons temperature and the string maximal or critical temperature are dual of each other.

If back reaction of quantum matter is included, Quantum Field and Quantum String phases appear, and duality relations between them manifest as well.

This Duality is shown in two relevant examples : Black Hole and de

Sitter space times, and appears to be a generic feature, analogous to the "wave-particle" duality.

7 New Coherent String States and Minimal Uncertainty Principle in String Theory.

We study the properties of **exact** (all level k) quantum coherent states in the context of string theory on a group manifold (WZWN models).

Coherent states of WZWN models may help to solve the unitary problem : Having positive norm, they consistently describe the very massive string states (otherwise excluded by the spin-level condition).

These states can be constructed by (at least) two alternative procedures : (i) as the exponential of the creation operator on the ground state, and (ii) as eigenstates of the annhilation operator. In the $k \to \infty$ limit, all the known properties of ordinary coherent states of Quantum Mechanics are recovered.

States (i) and (ii) (which are equivalent in the context of ordinary quantum mechanics and string theory in flat spacetime) are not equivalent in the context of WZWN models.

The set (i) was constructed by Larsen and Sanchez in ref. [4]. The construction of states (ii) was provided in ref. [8] by the same authors. We compare the two sets and discuss their properties.

We analyze the uncertainty relation, and show that states (ii) satisfy automatically the *minimal uncertainty* condition for any k; they are thus *quasiclassical*, in some sense more classical than states (i) which only satisfy it in the $k \to \infty$ limit.
The modification to the Heisenberg relation is given by $2\mathcal{H}/k$, where $\mathcal{H}$ is connected to the string energy.

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