The Near-Horizon Limit of the Extreme Rotating $d = 5$ Black Hole as a Homogenous Spacetime

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Abstract

We show that the spacetime of the near-horizon limit of the extreme rotating $d = 5$ black hole, which is maximally supersymmetric in $N = 2, d = 5$ supergravity for any value of the rotation parameter $j \in [-1, 1]$, is locally isomorphic to a homogeneous non-symmetric spacetime corresponding to an element of the 1-parameter family of coset spaces $\frac{SO(2,1) \times SO(3)}{SO(2)_j}$ in which the subgroup $SO(2)_j$ is a combination of the two $SO(2)$ subgroups of $SO(2,1)$ and $SO(3)$.

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Introduction

The vast majority of the known maximally supersymmetric solutions of supergravity theories seem to be symmetric spaces: Minkowski or AdS spacetimes, products of AdS spacetimes and spheres $AdS_m \times S^n$ or Hpp-wave spacetimes. Their Killing vectors and spinors and their relations that determine their supersymmetry algebras can be found by simple geometrical methods [1].

The only exception seems to be the near-horizon limit of the extreme rotating $d = 5$ black holes [2, 3, 4, 5]. This solution can be written in the form [6]

$$\begin{cases} ds^2 & = & R^2 d\Pi_{(2)}^2 - R^2 d\Omega_{(2)}^2 - R^2(d\psi + \cos\alpha\cos\theta d\varphi - \sin\alpha\sinh\chi d\phi)^2 \,, \\ \\ F & = & \sqrt{3}R\cos\alpha\cosh\chi d\chi \wedge d\phi - \sqrt{3}R\sin\alpha\sin\theta \, d\theta \wedge d\varphi \,, \end{cases} \quad (1)$$

where

$$\begin{aligned} d\Pi_{(2)}^2 & = & \cosh^2\chi \, d\phi^2 - d\chi^2 \,, \\ \\ d\Omega_{(2)}^2 & = & d\theta^2 + \sin^2\theta \, d\varphi^2 \,, \end{aligned} \quad (2)$$

are respectively the metrics of the unit radius AdS_2 spacetime and the unit radius 2-sphere S^2. The rotation parameter j is here $\cos\alpha$.

The metric of this solution looks like a sort of twisted product $AdS_3 \times S^3$ in which the sphere and the AdS spacetime share a common direction parametrized by ψ. Actually, when $\cos\alpha = 1$ (the purely electric solution), the dimension ψ belongs only to the sphere and the metric is exactly that of $AdS_2 \times S^3$ and, when $\cos\alpha = 0$, the dimension ψ belongs entirely to the AdS spacetime and the metric is exactly that $AdS_3 \times S^2$. These are singular limits, though, because the isometry group is the 7-dimensional $SO(2,1) \times SO(3) \times SO(2)$ for generic values of $\cos\alpha$ but becomes the 9-dimensional $SO(2,1) \times SO4)$ or $SO(2,2) \times SO(3)$ in the two limits

Not surprisingly, the solution can be obtained by dimensional reduction of the $AdS_3 \times S^3$ solution of $N = 2, d = 6$ supergravity along a direction which is a linear combination of the two S^1 fibers of the Hopf fibrations $AdS_3 \overset{S^1}{\to} AdS_2$ and $S^3 \overset{S^1}{\to} S^2$ [6]. It can also be obtained by dimensional oxidation of the dyonic Robinson-Bertotti solution [7, 8] of $N = 2, d = 4$ supergravity [6], (whose metric is that of $AdS_2 \times S^2$ and is also maximally supersymmetric [9, 10]) and these dimensional relations give us very important clues about the geometry of the solution and how to find a coset construction of its metric [11].

In fact, these relations immediately suggest that the metric could be constructed as an invariant metric over the coset $\frac{SO(2,1) \times SO(3)}{SO(2)_j}$, in which the subgroup $SO(2)_j$ is a combination of he two $SO(2)$ subgroups of $SO(2,1)$ and $SO(3)$, that is: the group manifold $SO(2,1)$ equipped with the bi-invariant metric can be identified with the AdS_3 spacetime and the coset $SO(2,1)/SO(2)$ with the left-invariant metric can be identified (locally) with the AdS_2 spacetime. Analogously, the group manifold $SO(3)$ equipped with the bi-invariant

metric can be identified (locally) with the S^3 spacetime and the coset $SO(3)/SO(2)$ with the left-invariant metric can be identified with the S^2 spacetime. In the product $AdS_3 \times S^3$ there are two $SO(2)$ subgroups available for taking the quotient (which is equivalent to dimensional reduction) and one choice gives, in $d = 5$ $AdS_2 \times S^3$ and the other $AdS_3 \times S^2$. One could also take the quotient over the $SO(2)_j$ subgroup generated by a linear combination of the generators of the two above-mentioned $SO(2)$ subgroups and the left-invariant metric should be the one in Eq. (1).

There is another $SO(2)$ subgroup present, generated by the orthogonal linear combination. This $SO(2)$ commutes with the other one and belongs to its normalizer, which is $SO(2) \times SO(2)$. It is a well-known fact [11] that the isometry group of the left-invariant metric over a coset G/H is, generically $G \times N(H)/H$, where $N(H)$ is the normalizer of H and $N(H)/H$ is the right isometry group. Here $N(H)/H = SO(2)_j$ and then the full isometry group should be the 7-dimensional $SO(2,1) \times SO(3) \times SO(2)$, as we want. In the two singular limits, there is enhancement of the isometry group as explained above.

In this paper we are going to prove that our proposal is indeed correct by explicitly constructing first the metric in Eq. (1) as a left-invariant metric over the coset[4] $\frac{SO(2,1) \times SO(3)}{SO(2)_j}$. The spacetime, is, thus, homogeneous, but it is not symmetric. Secondly, we are then going to use this construction to find the Killing vectors and spinors, although we will find difficulties to relate them, due to the fact that in our construction we will not use the Killing metric, but instead we will use the Minkowski metric, which is also $SO(2)$-invariant: the Killing metric of the real form $so(2,1) \times so(3)$ has the signature $(--+---)$, i.e. the $so(2,1)$ part has the wrong signature in our conventions (mostly minus signature), but this can not be corrected by means of analytic continuation (one gets complex metrics or metrics with wrong signature). Fortunately, the Minkowski metric has the necessary properties.

Construction of the Metric and Killing Vectors

The Lie algebra of $SO(2,1)$ can be written in the form

$$[T_i, T_j] = -\epsilon_{ijk}Q^{kl}T_l, \quad i,j,\cdots = 1,2,3,,, \qquad Q = \text{diag}\,(++-),$$ (3)

and its Killing metric is $K = 2\text{diag}\,(++-)$. To construct AdS_2, one has to take the coset $SO(2,1)/SO(2)$ where the subgroup $SO(2)$ is generated indistinctly by T_1 or T_2. We will choose for the sake of definiteness T_1. The projection of the Killing metric on the orthogonal subspace generated by T_2, T_3 $\text{diag}\,(+-)$ has the right signature to give AdS_2. Actually, the signature is the opposite to our mostly minus conventions, but a global factor is immaterial and the time coordinate, compact, is associated to T_3 (the $-$ sign in the Killing metric).

It is important to observe that there is no real form of this algebra with Killing metric $K = \text{diag}\,(--+)$. Also, we are forced to associate the time coordinate with T_3.

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[4]Our identification of the near-horizon limit of the rotating extreme black hole and the coset space is only local. We will not be concerned with global issues here.

The Lie algebra of $SO(3)$ can be written in the form

$$[\tilde{T}_i, \tilde{T}_j] = -\epsilon_{ijk}\tilde{T}_k, \quad i, j, \cdots = 1, 2, 3, , \tag{4}$$

and its Killing metric is $K = 2\text{diag}\,(- - -)$. To construct S^2, one has to take the coset $SO(3)/SO(2)$ where the subgroup $SO(2)$ is generated by any of the generators $\tilde{T}_i$. We will choose T_3 for definiteness. Observe that there is no real form with Killing metric $K = 2\text{diag}\,(+ + +)$.

The subgroup $SO(2)$ that we will use will be the one generated by the combination

$$M \equiv \cos\alpha T_1 + \sin\alpha\tilde{T}_3. \tag{5}$$

We now make the following redefinitions

$$P_0 = \tfrac{1}{R}T_3, \quad P_1 = \tfrac{1}{R}T_2, \quad P_2 = \tfrac{1}{R}\tilde{T}_1, \quad P_3 = \tfrac{1}{R}\tilde{T}_2, \quad P_4 = -\tfrac{\sin\alpha}{R}T_1 + \tfrac{\cos\alpha}{R}\tilde{T}_3. \tag{6}$$

The subalgebra $\mathfrak{h}$ is generated by M and the orthogonal subspace $\mathfrak{k}$ by the P_as. The non-vanishing commutators

$$[M, P_0] = \cos\alpha P_1, \quad [M, P_1] = \cos\alpha P_0, \quad [M, P_2] = -\sin\alpha P_3, \quad [M, P_3] = \sin\alpha P_2,$$

$$[P_4, P_0] = -\tfrac{\sin\alpha}{R}P_1, \quad [P_4, P_1] = -\tfrac{\sin\alpha}{R}P_0, \quad [P_4, P_2] = -\tfrac{\cos\alpha}{R}P_3, \quad [P_4, P_3] = -\tfrac{\cos\alpha}{R}P_2,$$

$$[P_0, P_1] = \tfrac{\cos\alpha}{R^2}M - \tfrac{\sin\alpha}{R}P_4, \quad [P_2, P_3] = -\tfrac{\sin\alpha}{R^2}M - \tfrac{\cos\alpha}{R}P_4,$$
$$\tag{7}$$

indicate that $[\mathfrak{k}, \mathfrak{h}] \subset \mathfrak{k}$ (reductivity) but $[\mathfrak{k}, \mathfrak{k}] \not\subset \mathfrak{h}$, so we do not have a symmetric pair and we will not have a symmetric space.

The Killing metric of the product group manifold $SO(2,1) \times SO(3)$ in the new basis $\{P_a, M\}$ $a = 0, \cdots, 4$ is

$$(K_{IJ}) = \frac{2}{R^2} \begin{pmatrix} -1 & & & & & \\ & +1 & & & & \\ & & -1 & & & \\ & & & -1 & & \\ & & & & \sin^2\alpha - \cos^2\alpha & -2R\sin\alpha\cos\alpha \\ & & & & -2R\sin\alpha\cos\alpha & R^2(\cos^2\alpha - \sin^2\alpha) \end{pmatrix}, \tag{8}$$

but we are not going to use it to construct the left-invariant metric. Instead, we will use the 5-dimensional Minkowski metric η_{ab}, which gives a metric invariant under the left action of G since

$$f_{M(a}{}^c\eta_{b)c} = 0. \tag{9}$$

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The coset representative is chosen to be

$$u(x) = e^{x^0 P_0} \cdots e^{x^4 P_4} \,,$$ (10)

and the left-invariant Maurer-Cartan 1-form $V = -u^{-1}du$ is

$$
\begin{aligned}
-V &= T_I \Gamma_{\mathrm{Adj}}(e^{-x^4 P_4})^I{}_J \Gamma_{\mathrm{Adj}}(e^{-x^1 P_1})^I{}_{P_0} dx^0 + T_I \Gamma_{\mathrm{Adj}}(e^{-x^4 P_4})^I{}_{P_1} dx^1 \\
&\quad + T_I \Gamma_{\mathrm{Adj}}(e^{-x^4 P_4})^I{}_J \Gamma_{\mathrm{Adj}}(e^{-x^3 P_3})^I{}_{P_2} dx^2 + T_I \Gamma_{\mathrm{Adj}}(e^{-x^4 P_4})^I{}_{P_3} dx^3 + P_4 dx^4 \,,
\end{aligned}
$$ (11)

and, with the definitions

$$V = e^a P_a + \vartheta M \,,$$ (12)

leads to the Fünfbeins e^a and to the H-connection ϑ

$$
\begin{aligned}
-e^0 &= \cosh\left(\tfrac{x^1}{R}\right)\cosh\left(\tfrac{\sin\alpha}{R}x^4\right) dx^0 + \sinh\left(\tfrac{\sin\alpha}{R}x^4\right) dx^1 \,, \\
-e^1 &= \cosh\left(\tfrac{x^1}{R}\right)\sinh\left(\tfrac{\sin\alpha}{R}x^4\right) dx^0 + \cosh\left(\tfrac{\sin\alpha}{R}x^4\right) dx^1 \,, \\
-e^2 &= \cos\tfrac{x^3}{R}\cos\left(\tfrac{\cos\alpha}{R}x^4\right) dx^2 - \sin\left(\tfrac{\cos\alpha}{R}x^4\right) dx^3 \,, \\
-e^3 &= \cos\tfrac{x^3}{R}\sin\left(\tfrac{\cos\alpha}{R}x^4\right) dx^2 + \cos\left(\tfrac{\cos\alpha}{R}x^4\right) dx^3 \,, \\
-e^4 &= -\sin\alpha\sinh(\tfrac{x^1}{R}) dx^0 - \cos\alpha\sin\tfrac{x^3}{R}dx^2 + dx^4 \,, \\
-\vartheta &= \tfrac{\cos\alpha}{R}\sinh(\tfrac{x^1}{R}) dx^0 - \sin\alpha\sin\tfrac{x^3}{R}dx^2 \,.
\end{aligned}
$$ (13)

Redefining the coordinates

$$x^0/R = \phi \,,\quad x^1/R = \chi \,,\quad x^2/R = \varphi \,,\quad x^3/R = \theta + \pi/2 \,,\quad x^4/R = \psi \,,$$ (14)

it is easy to see that the metric

$$ds^2 = \eta_{ab} e^a \otimes e^b \,,$$ (15)

is precisely that of Eq. (1).

According to the general results on homogeneous spaces the Killing vectors $k_{(I)}$ associated to the left isometry group $G = SO(2,1) \times SO(3)$ are given by

$$k_{(I)} = \Gamma_{\mathrm{Adj}}(u^{-1})^a{}_I e_a \,.$$ (16)

Their explicit expressions are

$$
\begin{aligned}
k_{(P_0)} &= -\partial_{x^0}\,, \\[2ex]
k_{(P_1)} &= \operatorname{tgh}(x^1/R)\sin\left(x^0/R\right)\partial_{x^0} - \cos\left(x^0/R\right)\partial_{x^1} - \sin\alpha\,\frac{\sin\left(x^0/R\right)}{\cosh(x^1/R)}\partial_{x^4}\,, \\[2ex]
k_{(P_2)} &= -\partial_{x^2}\,, \\[2ex]
k_{(P_3)} &= -\tan(x^3/R)\sin\left(x^2/R\right)\partial_{x^2} - \cos\left(x^2/R\right)\partial_{x^3} - \cos\alpha\,\frac{\sin\left(x^2/R\right)}{\cos\left(x^3/R\right)}\partial_{x^4}\,,
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
k_{(P_4)} &= \sin\alpha\left[\operatorname{tgh}(x^1/R)\cos\left(x^0/R\right)\partial_{x^0} + \sin\left(x^0/R\right)\partial_{x^1}\right] \\[1.5ex]
&\quad - \cos\alpha\left[\tan\left(x^3/R\right)\cos\left(x^2/R\right)\partial_{x^2} - \sin\left(x^2/R\right)\partial_{x^3}\right] \\[1.5ex]
&\quad - \left\{\frac{\cos\left(x^0/R\right)}{\cosh(x^1/R)} - \cos^2\alpha\left[\frac{\cos\left(x^2/R\right)}{\cos\left(x^3/R\right)} - \frac{\cos\left(x^0/R\right)}{\cosh(x^1/R)}\right]\right\}\partial_{x^4}\,, \\[1.5ex]
k_{(M)} &= -R\cos\alpha\left[\operatorname{tgh}(x^1/R)\cos\left(x^0/R\right)\partial_{x^0} + \sin\left(x^0/R\right)\partial_{x^1}\right] \\[1.5ex]
&\quad - R\sin\alpha\left[\tan\left(x^3/R\right)\cos\left(x^2/R\right)\partial_{x^2} - \sin\left(x^2/R\right)\partial_{x^3}\right] \\[1.5ex]
&\quad - R\sin\alpha\cos\alpha\left[\frac{\cos\left(x^2/R\right)}{\cos\left(x^3/R\right)} - \frac{\cos\left(x^0/R\right)}{\cosh(x^1/R)}\right]\partial_{x^4}\,.
\end{aligned}
\tag{18}
$$

The right isometry group is given by the vectors dual to the Maurer-Cartan 1-forms e_a associated to the generators of $N(H)/H$, and commute with the left Killing vectors. In this case, the generator of $N(H)/H$ is P_4 and the associated Killing vector denoted $k_{(N)}$ turns out to be

$$
k_{(N)} = e_4 = -\partial_{x^4}\,.
\tag{19}
$$

Construction of the Killing Spinors and the Superalgebra

The Killing spinor equation of $N=2, d=5$ Supergravity is (choosing $s(\alpha) = +1$) [12, 6]

$$
\left\{\nabla_a - \tfrac{1}{8\sqrt{3}}(\gamma^{bc}\gamma_a + 2\gamma^b g^c{}_a)\mathcal{F}_{bc}\right\}\kappa = 0\,.
\tag{20}
$$

κ is an unconstrained Dirac spinor (one component of a pair of symplectic-Majorana spinors). We contract this equation with the Maurer-Cartan 1-forms e^a to write is in the form:

$$\left\{ d - \tfrac{1}{4}\omega^a{}_b\gamma_a{}^b - \tfrac{1}{8\sqrt{3}}(\gamma^{bc}\mathcal{F}_{bc}\gamma_a + 2\gamma^b\mathcal{F}_{ba})e^a \right\}\kappa = 0 \,. \tag{21}$$

In homogeneous spaces, the spin connection is given by

$$\omega^a{}_b = \vartheta^i f_{ib}{}^a + \tfrac{1}{2}e^c f_{cb}{}^a \,, \tag{22}$$

and we obtain a spinorial representation of the vertical generators M_i

$$\Gamma_s(M_i) = \tfrac{1}{4}f_{ib}{}^a\gamma_a{}^b \,. \tag{23}$$

In symmetric spaces the structure constants $f_{cb}{}^a = 0$ and the contribution of the spin connection to the Killing spinor equation is just $-\vartheta^i\Gamma_s(M_i)$ [1], but in this case we have extra terms

$$-\tfrac{1}{4}\omega^a{}_b\gamma_a{}^b = -\vartheta\Gamma_s(M) - \tfrac{1}{8}e^c f_{cb}{}^a\gamma_a{}^b \,, \qquad \Gamma_s(M) \equiv \tfrac{1}{2}(\cos\alpha\gamma^{01} - \sin\alpha\gamma^{23}) \,. \tag{24}$$

The extra terms do not give by itself $-e^a\Gamma_s(P_a)$, but it can be checked that, combined with the terms that depend on the vector field strength, they do, and the Killing spinor equation take the form

$$\left\{ d - \vartheta\Gamma_s(M) - e^a\Gamma_s(P_a) \right\}\kappa = 0 \,, \qquad \Gamma_s(P_a) = -\tfrac{1}{2R}(\cos\alpha\gamma^{01} - \sin\alpha\gamma^{23})\gamma_a \,, \tag{25}$$

which leads to

$$\kappa = \Gamma_s(u^{-1})\kappa_0 \,, \tag{26}$$

where κ_0 is a constant spinor. The matrix $\Gamma_s(u^{-1})^\beta{}_\alpha$ can be used as a basis of Killing spinors $\kappa_{(\alpha)}{}^\beta$ to which we associate supercharges $Q_{(\alpha)}$.

The commutators of the bosonic generators P_a, M, N of the superalgebra (associated to the Killing vectors) with the supercharges is given immediately by the spinorial Lie-Lorentz derivative of the Killing spinor with respect to the associated Killing spinors [13, 14]. For the generators associated to the left isometry group $\{T_I\} = \{P_a, M\}$ we can use Eq. (2.23) of Ref. [1]

$$\mathbb{L}_{k_{(I)}}\Gamma_s(u^{-1}) = -\Gamma_s(u^{-1})[\mathbb{L}_{k_{(I)}}\Gamma_s(u)]\Gamma_s(u^{-1}) = -\Gamma_s(u^{-1})\Gamma_s(T_I) \,, \tag{27}$$

which implies the commutators

$$[Q_{(\alpha)}, T_I] = -Q_{(\beta)}\Gamma_s(T_I)^\beta{}_\alpha \,. \tag{28}$$

The other commutators with N are trivial.

Finally, let us consider the anticommutators of two supercharges. These are associated to the decomposition in Killing vectors the bilinears $-i\bar{\kappa}_\alpha\gamma^a\kappa_{(\beta)}e_a$. To find this decomposition is crucial to relate the contravariant gamma matrices γ^a with the bosonic generators

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in the spinorial representation $\Gamma_s(P_a)$. In this case, it is convenient to proceed as follows. First, we find the relation

$$\gamma^a = \eta^{ab}\gamma_b = -2R\mathcal{S}\Gamma_s(P_a)\,, \qquad \mathcal{S} = \left(\cos\alpha\,\gamma^{01} + \sin\alpha\,\gamma^{23}\right), \tag{29}$$

and substitute into the bilinear

$$-i\bar{\kappa}_{(\alpha)}\gamma^a\kappa_{(\beta)}e_a = -i\Gamma_s(u^{-1})^\dagger\mathcal{D}\mathcal{S}\Gamma_s(P_b)\Gamma_s(u^{-1})\eta^{ba}e_a\,, \tag{30}$$

where $\mathcal{D} = i\gamma^0$ is th Dirac conjugation matrix. It can be checked that

$$\Gamma_s(u^{-1})^\dagger\mathcal{D}\mathcal{S} = \mathcal{D}\mathcal{S}\Gamma_s(u)\,, \tag{31}$$

and, recognizing the adjoint action of u on the $\Gamma_s(P_b)$ we have

$$-i\bar{\kappa}_\alpha\gamma^a\kappa_{(\beta)}e_a = -i\mathcal{D}\mathcal{S}\Gamma_s(T_I)\Gamma_{\text{Adj}}(u)^I{}_b\eta^{ba}e_a\,. \tag{32}$$

Now we use the following general property: for any $g \in G$, (if the Killing metric is nonsingular, as here)

$$\Gamma_{\text{Adj}}(g)^I{}_J = K_{JK}\Gamma_{\text{Adj}}(g^{-1})^K{}_L K^{LI}\,, \tag{33}$$

and the definition of the dual generators $T^I = K^{IJ}T_J$

$$-i\bar{\kappa}_\alpha\gamma^a\kappa_{(\beta)}e_a = -i\mathcal{D}\mathcal{S}\Gamma_s(T^I)\Gamma_{\text{Adj}}(u^{-1})^J{}_I K_{Jb}\eta^{ba}e_a\,. \tag{34}$$

Since the Killing metric and the Minkowski metric are different, the r.h.s. of this expression does not give the Killing vectors of the left isometry group. We have to use a non-trivial property of $\Gamma_s(u^{-1})$. Let us define the matrix η^{IJ}

$$(\eta^{IJ}) = \begin{pmatrix} 1 & & & & & \\ & -1 & & & & \\ & & -1 & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & -R^{-2} \end{pmatrix}, \tag{35}$$

and, with it and the Killing metric, the matrix

$$R^I{}_J = \eta^{IK}K_{KJ}\,. \tag{36}$$

It can be checked that

$$R^I{}_J\Gamma_{\text{Adj}}(u^{-1})^J{}_K = \Gamma_{\text{Adj}}(u^{-1})^I{}_L R^L{}_K\,, \Rightarrow \Gamma_{\text{Adj}}(u^{-1})^J{}_I K_{Jb}\eta^{ba} = \Gamma_{\text{Adj}}(u^{-1})^a{}_L R^L{}_I\,, \tag{37}$$

and

$$- i\bar{\kappa}_\alpha \gamma^a \kappa_{(\beta)} e_a = -i \left[\mathcal{DS}\Gamma_s(T^I) \right]_{\alpha\beta} R^L{}_I k_{(L)}, \tag{38}$$

that gives the anticommutators

$$\{Q_{(\alpha)}, Q_{(\beta)}\} = -i \left[\mathcal{DS}\Gamma_s(T^I) \right]_{\alpha\beta} \left(R^a{}_I P_a + R^M{}_I M \right). \tag{39}$$

Acknowledgments

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