

Vortices in Theories with Flat Directions

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In theories with flat directions containing vortices, such as supersymmetric QED, there is a vacuum selection effect in the allowed asymptotic configurations. We explain the role played by gauge fields in this effect and give a simple criterion for determining what vacua will be chosen, namely those that minimise the vector mass. We then consider the effect of vacuum selection on stable (BPS) non-topological vortices in a simple Abelian model with $N=2$ supersymmetry which occurs as a low energy limit of Calabi–Yau compactifications of type II superstrings. In this case the magnetic flux spreads over an arbitrarily large area. We discuss the implications for cosmology and for superstring inspired magnetic confinement scenarios.

I. INTRODUCTION

Flat directions of the scalar potential are a feature of a large class of supersymmetric (SUSY) theories, for instance in Abelian theories where the gauge symmetry is broken with a Fayet-Iliopoulos (FI) D-term. In the last few years these have undergone intense study, in particular in the context of effective actions for SUSY non-Abelian gauge theories and confinement (see e.g. [1, 2]).

A generic consequence of SUSY theories with a D-term is the formation of cosmic strings [3, 4]. However, the string solution is richer than the usual Nielsen–Olesen (NO) vortex [5, 6] for two distinct reasons. Firstly, the existence of flat directions means that there is a moduli of degenerate vacua and, at first sight, it is unclear which of the vacuum states give rise to stable cosmic string solutions. Secondly, the presence of supersymmetry gives rise to fermion zero modes in the string core [4], rendering the string current-carrying. This typically leads to the formation of *vortons*, long-lived remnants whose observational signatures can severely constrain, and in some cases rule out, such models[7].

Topological strings arising from the bosonic sector of an $N=1$ SUSY model have been analysed in [3], showing that even though there is a one-parameter choice of vacuum for the moduli field, only one choice permits cosmic string solutions which could be stable. Their conclusion was based on the behaviour of the modulus field well outside the core of the vortex. This vacuum selection effect is generic in Abelian theories with flat directions in that, as we will show, it is driven by the tendency to minimise the vector mass at the core of the vortex. Thus, one expects it to affect non-topological as well as topological strings, and it could conceivably improve their stability.

In this paper we analyse this vacuum selection effect in the context of (BPS) non-topological strings. In particular, we investigate the nature of BPS vortices in $N=2$ supersymmetric QED with two hypermultiplets of opposite charge. This model has been analysed previously in connection with Calabi–Yau compactifications of type II superstrings [8], where the existence of vortices is linked to the confinement of magnetic charges. We will find that, due to the vacuum selection effect, the structure of the BPS vortices is identical to that of so-called semilocal strings[9].

Apart from its physical significance, this model has the advantage that all calculations can be performed explicitly, which is important because the stability of semilocal strings is somewhat counter-intuitive. We should stress that although the vacuum manifold is simply connected, the gauge boson is massive and magnetic flux is topologically conserved and quantized; the vortices analysed here are *bona-fide* BPS states carrying one unit of magnetic flux.

And yet we will find that, *in practice, there are no stable vortices in this model*. To be more precise, the vortices are only neutrally stable (which is consistent with the BPS condition) and degenerate with a whole family of other BPS “magnetic flux tubes” of arbitrarily large width. The slightest perturbation excites the zero mode and makes the vortex expand. The vacuum selection effect does not single out the narrow Nielsen–Olesen vortex over the other, more extended, BPS flux tubes. In a cosmological context, this suppresses vorton production. In the context of superstring compactifications, it casts serious doubts on the mechanism of magnetic charge confinement proposed in [8].

The paper is organized as follows. We first review the vacuum selection effect in the model of [3]. We explicitly show that the resulting cosmic string saturates the Bogomol’nyi bound, so is strictly stable, and that it is no accident that the selected vacuum is the one minimising the vector boson mass. We argue that this gives a generic criterion to determine which vacuum or vacua are selected in a given theory, provided all charges are equal in absolute value.

We then extend this idea to $N=2$ QED with two hypermultiplets of opposite charge, using a Bogomol’nyi argument on the corresponding solution [10]. We again show that it saturates a Bogomol’nyi bound, so is stable, but the presence of a cylindrically symmetric zero mode changes the dynamics completely and in particular the string expands to an

arbitrarily large area in finite time [11–13]. This occurs even though the vector mass is non-zero and the magnetic flux at infinity remains quantised, setting a constraint on viable models of confinement of magnetic charge in these $N=2$ Abelian theories. Finally we discuss the effect of SUSY breaking on the solutions and speculate on the resulting cosmology.

II. VACUUM SELECTION EFFECT AND MINIMUM GAUGE MASS

We begin by reviewing the vacuum selection effect described in [3]. Penin, Rubakov, Tinyakov and Troitsky considered a four-dimensional model with two $N=1$ chiral superfields of opposite charges coupled to a $U(1)$ vector multiplet, and a FI D-term which gives the symmetry breaking potential. The action of the bosonic sector is

$$S = \int d^4x \left[|D_\mu \phi_+|^2 + |D_\mu \phi_-|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi_+, \phi_-) \right] \\ V(\phi_+, \phi_-) = \frac{\lambda}{2} (|\phi_+|^2 - |\phi_-|^2 - \eta^2)^2 \quad (1)$$

where A_μ is a $U(1)$ gauge field and ϕ_\pm are complex scalars of opposite charges ($q_\pm = \pm q$),

$$D_\mu \phi_\pm = (\partial_\mu \pm i q A_\mu) \phi_\pm, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

The solutions for the vacuum manifold ($V = 0$) are

$$|\phi_+| = \eta \cosh u \equiv v_+, \quad |\phi_-| = \eta \sinh u \equiv v_- , \quad (2)$$

where u parametrises the moduli space. After symmetry breaking the physical spectrum consists of two massless scalars, the Goldstone boson and the modulus field; a massive scalar particle $m_s^2 = 2\lambda\eta^2 \cosh 2u$; and a massive vector particle $m_v^2 = 2q^2\eta^2 \cosh 2u$. Their masses depend on the choice of vacuum. The minimum values \bar{m}_s and \bar{m}_v correspond to $u = 0$ ($\phi_- = 0$), where we recover the Abelian Higgs model. This has static vortex solutions, the NO strings [6].

We want straight, static vortices along the z -direction so we drop the t - and z -dependence and set $A_t = A_z = 0$, defining $B = \partial_1 A_2 - \partial_2 A_1$. We can set $\eta = q = 1$ by rescaling, and we consider the Bogomol'nyi limit $\lambda = 1$ this effectively makes the model $N=2$ supersymmetric with one hypermultiplet, see also [10, 14, 15]. The conditions for finite energy include $D_\mu \phi_+ \rightarrow 0$, $D_\mu \phi_- \rightarrow 0$, $B \rightarrow 0$ faster than $1/r$ and this introduces a correlation in the phases of the fields at infinity: fields with charge q_a wind as $e^{i n q_a \theta}$ and the gauge field tends to a constant, $A_\theta \rightarrow -n$, leading to the quantisation of magnetic flux, $\int d^2 B = -2\pi n$.

In principle, one can try to construct vortices tending to any of the vacua (2) as $r \rightarrow \infty$. However, in [3] it was shown that the only static solutions are those with $u = 0$. Any other choice of boundary conditions leads to what is effectively an unstable vortex that tends to this one. The instability is very mild, and the surviving string is a NO string. We will now show that this string is stable.

In what follows we consider cylindrically symmetric configurations with $n = 1$:

$$\phi_+ = f_+(r) e^{i\theta} \quad \phi_- = f_-(r) e^{-i\theta} e^{i\Delta} \quad A_\theta d\theta = a(r) d\theta$$

$f_\pm(r)$ are real functions, with $f_+(0) = f_-(0) = a(0) = 0$ and, $f_\pm(\infty) = v_\pm$, $a(\infty) = -1$. r -dependent phases $e^{i\psi_\pm(r)}$ for ϕ_\pm are possible but minimum energy requires $\partial_r \psi_\pm = 0$, so we will ignore them. Δ is a real constant.

To prove stability we obtain the Bogomol'nyi equations

$$f'_\pm \mp \frac{a+1}{r} f_\pm = 0, \quad \frac{a'}{r} - (f_+^2 - f_-^2 - 1) = 0. \quad (3)$$

Note that, if $f_-(r) = 0$, eqns. (3) are the standard Bogomol'nyi equations for the Abelian Higgs model.

It can be seen that the only solution for f_- satisfying the boundary conditions at the core is $f_-(r) = 0$; thus the NO solution saturates the Bogomol'nyi bound automatically. This shows that the string is stable, since it is a global minimum of the energy, and excludes the possibility of cylindrically symmetric zero modes. In fact it turns out that the restriction to cylindrical symmetry is unnecessary: ϕ_- always vanishes on the solutions to the Bogomol'nyi equations (see, e.g. [1]). The only remaining solutions are the standard NO vortices.

As explained in [3], the key point in understanding the vacuum selection effect is the effective separation between the dynamics of the magnetic core and of the modulus field outside the core (this separation is only true for vortices, as it is a consequence of the properties of massless fields in two dimensions, the transverse dimensions of the vortex). Far from the core the magnetic field vanishes, there is no potential energy, and the modulus field (properly normalised) becomes massless, with solutions logarithmic in r – unless $u = 0$. Thus $u = 0$ is selected.

We can see this in another way: well outside the core the scalar fields can move along the moduli space from $u = 0$ to their asymptotic values, *with no appreciable cost in energy*. Indeed, outside the core,

$$E \sim \int d^2x (D_\mu \phi_+)^2 + (D_\mu \phi_-)^2 \sim \int d^2x (\partial_r u)^2 \cosh 2u$$

and the minimum energy configuration $u(r)$ that interpolates between any two distinct vacua, $u(R_1) = u_1$ and $u(R_2) = u_2$, if $R_1, R_2 \gg r_{\text{core}}$, is of the form

$$E \sim \frac{I(u_1, u_2)}{\ln R_2 - \ln R_1}, \quad \frac{I(u_1, u_2)}{2\pi} = \left[\int_{u_1}^{u_2} du \sqrt{\cosh 2u} \right]^2$$

which can be made arbitrarily small as $R_2 \rightarrow \infty$. Thus, the dynamics at the core are decoupled from the behaviour of the moduli fields; the core of the vortex is effectively free to choose its boundary conditions in order to minimise its own energy. At the core (where the magnetic field is non-zero) the field ϕ_- is suppressed and the core becomes identical to that of a NO string. But the previous argument also shows that the minimum energy is unattainable unless $I = 0$ ($u_1 = u_2$), and this is a condition for static solutions. Thus, the vacuum selection actually occurs *at the core*.

The authors of [3] offered no insight into why a particular vacuum is selected, but it is clear that $\phi_- = 0$ is chosen because it minimises the mass of the gauge field, m_v . This is an important consideration at the core of the vortex, where the magnetic field is concentrated in a region of order m_v^{-1} . Magnetic field lines repel, so a lower m_v means a larger magnetic core. Since the total magnetic flux is quantised, this lowers the energy at the core and, therefore, the total energy.

III. BPS VORTICES IN $N=2$ SQED AND VACUUM SELECTION

We now consider two $N=2$ hypermultiplets h_a , $a = 1, 2$ with charges $q_a = +q, -q$ coupled to an $N=2$ Abelian vector multiplet [16]. This model has been analysed before as a toy model for the low energy effective action of Calabi–Yau compactifications of type II superstrings by Greene, Morrison and Vafa[8], who identified magnetic configurations due to the wrapping of D-branes on cycles. In the low energy theory these would appear as monopole–antimonopole pairs joined by non-topological magnetic vortices providing magnetic confinement.

In [16] it was shown that, in the absence of FI terms, such vortices were unstable. In an attempt to stabilise the vortices, here we consider the addition of FI D-terms, the only modification still compatible with $N=2$ supersymmetry of the effective action. We will be partially successful in that the instability turns into a zero mode, but as we will show this is still not enough to provide confinement.

The model has a global $SU(2)$ symmetry that rotates the two scalar fields in each multiplet, and we introduce a triplet of FI D-terms $\vec{k} \cdot \vec{D}$. In this case the answer can be more involved: since the vacuum manifold is simply connected there is no unique vacuum that minimises the vector mass and it is not clear that the selected vacua can lead to vortices. We take $\vec{k} = (0, 0, \frac{1}{2}|q|\omega^2)$, without loss of generality; note that this breaks the $SU(2)$ symmetry.

Eliminating the auxiliary fields and setting to zero those fields that have zero vacuum expectation values, simplifies the model considerably. With a suitable rescaling $h_{ij} \rightarrow \omega h_{ij}$, $x_\mu \rightarrow x_\mu/\omega q$, $A_\mu \rightarrow \omega A_\mu$, the energy of static, straight vortex configurations can be written as

$$\begin{aligned} \frac{2E}{\omega^2} = & \int d^2x \left[|D_\mu h_{11}|^2 + |D_\mu h_{12}|^2 + |D_\mu h_{21}|^2 + |D_\mu h_{22}|^2 + \frac{1}{2}B^2 \right. \\ & \left. + (H_1^1 - H_2^2 + 1/2)^2 + (H_2^1 + H_1^2)^2 + (iH_2^1 - iH_1^2)^2 \right] \end{aligned} \quad (4)$$

where $D_\mu = \partial_\mu + i(q_a/q)A_\mu$ and $H_j^i = -(q_a/2q)h_{ai}^*h_{aj}$.

There is another $SU(2)$ symmetry between $(h_{11}$ and $h_{22}^*)$ and $(h_{12}$ and $h_{21}^*)$ which is preserved after adding the D-term. For finite energy we require $D_\mu h_{ai} = 0$ as $r \rightarrow \infty$, which correlates the phases of the multiplets at spatial infinity, giving $h_a \sim e^{inq_a\theta}$ and the quantisation of magnetic flux from $A_\theta d\theta \sim -nd\theta$; the scalars must lie in the vacuum manifold $(H_2^1 = H_1^2 = 0, H_2^2 - H_1^1 = \frac{1}{2})$.

Parametrising the scalars as $h_{ai} = r_{ai} e^{i\chi_{ai}}$, gives

$$\chi_{11} - \chi_{12} = \chi_{21} - \chi_{22} + 2m\pi, \quad (5)$$

$$r_{11}r_{12} - r_{21}r_{22} = 0, \quad (6)$$

$$(r_{11})^2 + (r_{22})^2 - (r_{12})^2 - (r_{21})^2 = 1. \quad (7)$$

The vacuum manifold is simply connected, but the set of finite energy configurations has non-contractible loops labelled by n , the (quantised) magnetic flux. This situation is familiar in the context of semilocal models (see [13] for a recent review), where the gauge field couples to two (or more) scalar fields of *equal* charges, and there is a global $SU(2)$ (or larger) symmetry between the scalars. The stability of semilocal vortices depends on the masses of the scalar and the vector particles [11]. If $m_s < m_v$ the strings are stable. In the Bogomol'nyi limit $m_s = m_v$, the resulting $n = 1$ “vortices” have a richer structure including a scalar condensate at the core which causes the magnetic flux to spread over an arbitrarily large area. The thinnest vortex is a NO string but it is not protected against the zero mode that generates the scalar condensate; as a result its energy density becomes arbitrarily close to the vacuum in a finite time and the string dissolves [12]. If $m_s > m_v$ the zero mode becomes an instability and there are no strings.

Let us consider the vacuum selection effect in model (4). The mass of the gauge field is $r_{11}^2 + r_{12}^2 + r_{21}^2 + r_{22}^2$, and the minimization of this mass, subject to eqns (6,7), predicts $h_{12} = h_{21} = 0$. To prove that this is the case, we rewrite the energy as

$$\begin{aligned} 2E = & \int d^2x [|(D_1 + iD_2)h_{11}|^2 + |(D_1 - iD_2)h_{12}|^2 \\ & + |(D_1 + iD_2)h_{21}|^2 + |(D_1 - iD_2)h_{22}|^2 \\ & + [B + (H_1^1 - H_2^2 + \frac{1}{2})]^2 + (H_2^1 + H_1^2)^2 \\ & + (iH_2^1 - iH_1^2)^2] - \int d^2x B, \end{aligned} \quad (8)$$

and obtain the Bogomol'nyi equations:

$$(D_1 + iD_2)h_{11} = 0 \quad (D_1 - iD_2)h_{22} = 0 \quad (9)$$

$$(D_1 - iD_2)h_{12} = 0 \quad (D_1 + iD_2)h_{21} = 0 \quad (10)$$

$$H_1^2 = 0 \quad (11)$$

$$B + [H_1^1 - H_2^2 + 1/2] = 0 \quad (12)$$

(Note that setting $H_1^2 = 0$ implies $D_+ H_1^2 = 0$ which holds if $D_+ h_{11} = D_+ h_{12}^* = D_+ h_{21} = D_+ h_{22}^* = 0$ so the choice of \pm signs in the gradient terms is not arbitrary).

The most general configuration for the scalar hypermultiplets compatible with cylindrical symmetry is [16]

$$\begin{aligned} h_1 \equiv \begin{pmatrix} h_{11}(r) \\ h_{12}(r) \end{pmatrix} e^{i\theta}, \quad h_2 \equiv \begin{pmatrix} h_{21}(r) \\ h_{22}(r) \end{pmatrix} e^{i\Delta} e^{-i\theta}, \\ A_\theta = a(r) \quad A_r = 0 \end{aligned} \quad (13)$$

To analyse the Bogomol'nyi equations we simply note that the pairs of fields (h_{11}, h_{21}) and (h_{22}^*, h_{12}^*) behave exactly like (ϕ_+, ϕ_-) in the $N=1$ model of the previous section: they have charges ± 1 and they appear in the square bracket of eq. (12) with signs $+1, -1$ respectively. The only way to satisfy (10) is to have $h_{12} = h_{21} = 0$, which satisfies (11) automatically. These configurations minimise the vector mass, so we conclude that the “vacuum selection” effect also works here (note that by setting the second hypermultiplet to zero the vacuum selection effect gives a topological vortex in h_{11} [15]).

This leaves eqns. (9), (12), which are precisely the Bogomol'nyi equations of semilocal strings [9] in (h_{11}, h_{22}^*) . These have been studied in [11, 17], so we simply state the results. For $n = 1$, the non-zero components of the most general cylindrically symmetric solution are

$$(h_{11}, h_{22}^*) = (f(r)e^{i\theta}, p(r)e^{i\Delta}) U$$

where U is an arbitrary global $SU(2)$ rotation, Δ is a constant and $f(r)$ and $p(r)$ are real functions with $f(0) = 0$, $f(\infty) = 1$ and $p(0) \neq 0$, $p(\infty) = 0$. They are related by $p(r) = \alpha f(r)/r$, where α is a real positive parameter which labels the solutions. For each α the gauge field is obtained from the condition $(D_1 + iD_2)h_{11} = 0$.

The lack of winding in p results in the scalar condensate at the core, $p(0) \neq 0$. Note also that the $SU(2)$ transformations between h_{11} and h_{22}^* are not related to the $SU(2)$ transformations coming from the $N=2$ supersymmetry, since they mix elements of the two hypermultiplets. The zero mode is not related to either of these global symmetries, as can be seen by the fact that it does not alter the boundary conditions at infinity. To understand its effect, we consider the asymptotic behaviour of the fields far from the core [12, 18]:

$$\begin{aligned} f(r) & \sim 1 - \frac{1}{2}(\alpha^2/r^2) + \alpha^2(3/8\alpha^2 - 2)/r^4 + \dots \\ a(r) & \sim -1 + (\alpha^2/r^2) - \alpha^2(\alpha^2 - 8)/r^4 + \dots \end{aligned} \quad (14)$$

There is a one-parameter set of magnetic “vortices” with varying widths. They all saturate the Bogomol’nyi bound, so they are all degenerate in energy, but they have different structures: most notably, they have a scalar condensate at the core and the magnetic field fall-off is a power of r (in NO vortices the fall-off is exponential). These “vortices” can be thought of as hybrids between the NO vortex and the CP^1 lumps, and α can be seen as the width above that of a NO vortex: $\alpha = 0$ corresponds to the NO string modulo $SU(2)$ rotations; but as $\alpha \rightarrow \infty$ $p(0) \rightarrow 1$ and the core expands.

Finally, the zero mode corresponds to a flat direction in the potential for which the vector mass is always at a minimum, so the vacuum selection effect does not single out the NO solution over the more extended vortices.

We conclude that magnetic flux in this model is still not confined to tubes of a definite size. Semilocal strings which saturate the Bogomol’nyi bound are at the limiting case between stability (for $m_s < m_v$) and instability (for $m_s > m_v$) [11, 13]. However, [12] has shown that, in fact, the flux is unconfined and eventually spreads out to flux tubes of greater and greater radius. Of crucial importance to the resulting cosmology is the time-scale for this relaxation, which is outside the scope of this investigation, but it is clear that vorton production will be suppressed.

Our conclusions are easily extended to any number of hypermultiplets, as long as their charges are equal in absolute value. They also apply in the case of several gauge fields, such as in the low energy limit of type II superstrings compactified on Calabi–Yau manifolds, which effectively contains 15 copies of the model analysed here [8].

The situation changes when considering hypermultiplets with different charges. In that case vacuum selection may be frustrated and the selected vacuum or vacua may not be the ones that minimise the vector mass. The structure of the resulting vortices can be totally different from what we described here, with the possibility of binary or multiple cores and other exotic effects and it remains a very interesting open question.

IV. DISCUSSION

We have investigated cosmic string solutions in SUSY QED with flat directions where the $U(1)$ symmetry is broken by a FI D-term. We have argued that, if all matter fields have the same absolute value of the charge, the vacuum selection effect acts to minimise the vector boson mass. This includes the original model of [3], the bosonic sector of $N=1$ SUSY QED with two chiral superfields of opposite charges, and its simplest extension to $N=2$ with one hypermultiplet, considered in [15]. In both cases the vacuum selection effect leads to topological vortices; we have shown that both solutions saturate Bogomol’nyi bounds and are consequently stable.

We then considered extensions to $N=2$ SUSY QED with several hypermultiplets of equal $|q_a|$. The case of two hypermultiplets of opposite charge was solved explicitly, and there is still a vacuum selection effect, but this case gives rise to a semilocal string instead. Again the string solution saturates the Bogomol’nyi bound, but is only critically stable as there is a zero mode (which is *not* an $SU(2)$ rotation). The solutions that saturate the Bogomol’nyi bound are parametrised by one or more internal degrees of freedom and are distinct from the string solution, in particular their core widths can be arbitrarily large. Note that our $N = 2$ results generalise to Abelian theories with M hypermultiplets provided they have equal $|q_a|$; one would still find semilocal strings, but the internal symmetry would be $SU(M)$ and not $SU(2)$ [18]. Magnetic flux would still be unconfined.

The situation changes when considering hypermultiplets with different charges. In that case vacuum selection may be frustrated and the selected vacuum or vacua may not be the ones that minimise the vector mass. The structure of the resulting vortices can be totally different from what we describe here, with the possibility of binary or multiple cores and other exotic effects and it remains a very interesting open question.

As a consequence of supersymmetry, there will be fermion zero modes in the string core for both models. The zero modes can be found using the results of [4]. For the model of [3] there will be a single zero mode, left or right moving depending on the sign of n , which is a combination of the Higgsino and gaugino. Thus the string will be chiral [19], since it has a chiral current. We can then apply the results of [7] to this model. If there are no further phase transitions to destabilise the current, the model will lead to vorton formation, restricting the scale of symmetry breaking [7]. As shown here, this is in contrast with the situation in the $N=2$ model, where vorton production is not expected to occur.

However, if we completely break SUSY, for instance via mass terms for the h_{ij} fields, we still expect a topological string with fermion zero modes in the core. The angular momentum of the resulting current can stabilise loops, giving rise to vorton production [20]. The model will then be subject to the constraints of [7].

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