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From Big Crunch To Big Bang – Is It Possible?

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ABSTRACT

We discuss the possibility of a transition from a contracting flat space – big crunch – to an expanding flat space – big bang. Talk given at the Francqui Colloquium, 2001, "Strings and Gravity: Tying the Forces Together."

1 Introduction

The purpose of this talk is to review the work reported in [1]. Unlike other talks which report on well understood results, this talk should be viewed more as a *research proposal*. We will try to motivate a line of research and to suggest a certain conjecture.

The recent exciting advances in string theory were limited to strings in time independent backgrounds. We know very little about backgrounds which depend on time. More specifically, our understanding is limited to backgrounds which admit in asymptotic spatial infinity a global timelike Killing vector. This condition rules out interesting backgrounds like those which are important in cosmology.

Understanding cosmological solutions and especially cosmological singularities in the context of string theory is interesting both from a conceptual and from a pragmatic point of view. We can hope that through cosmology the much desired connection between string theory and experiment can materialize.

When cosmological singularities are considered two situations immediately come to mind:

- *Big bang singularity.* Here the Universe starts from a point and expands from it. A commonly held point of view is that it is meaningless to ask "What happened before the big bang?" because the big bang is the beginning of time. According to this point of view we should understand the initial conditions of the Universe or its wave function at the time of the big bang. Another point of view, which appears for example in the work on pre-big bang scenarios [2], holds that the Universe had an interesting history before the big bang. Here we will follow this point of view, but our pre-big bang history differs significantly from that in [2].

- *Big crunch singularity.* Here the Universe collapses to a point. This is simply the time reversal process of the big bang singularity, and therefore if the big bang is the beginning of time, the big crunch is the end of time. In this case special final conditions for the Universe or its final wave function at the time of the big crunch have to be specified. Having a specific final condition independent of the details of the system might lead to violations of causality – no matter what the state of the Universe is at a given time, it always ends at the same state. Therefore, it is perhaps more intuitive to expect that there is future beyond the big crunch. If this is the case, then, by time reversal, there must also be past before the big bang.

2 Singularity in our Past

In this section we will review the well known argument showing that if the Universe expands today, it always expanded and must have started at a singularity (our discussion is similar to that in [3]). In other words, it is impossible for a flat Universe like ours to undergo a period of contraction and then a period of expansion.

Using Weyl rescaling, the Einstein term in the four dimensional Lagrangian can be made canonical and all the terms in the Lagrangian with at most two derivatives are

$$\mathcal{L} = \mathcal{R} - \frac{1}{2} G_{IJ}(\phi) \partial\phi^I \partial\phi^J - V(\phi) \tag{2.1}$$

(for simplicity we neglect gauge fields). For homogeneous and isotropic configurations the energy density and the pressure are

$$\rho = T_{00} = \frac{1}{2} G_{IJ}(\phi) \dot{\phi}^I \dot{\phi}^J + V(\phi) \tag{2.2}$$

$$p = -\frac{1}{3} g^{ij} T_{ij} = \frac{1}{2} G_{IJ}(\phi) \dot{\phi}^I \dot{\phi}^J - V(\phi) \tag{2.3}$$

From these we derive that in unitary theories, where the scalar field kinetic term is non-negative

$$p + \rho = G_{IJ}(\phi) \dot{\phi}^I \dot{\phi}^J \geq 0 \tag{2.4}$$

This is known as the null energy condition.

For metrics of the form

$$ds^2 = -dt^2 + a(t)^2 \sum_i (dx^i)^2 \tag{2.5}$$

with zero spatial curvature and $H \equiv \frac{\dot{a}}{a}$ the gravity equations of motion lead to

$$\dot{H} = -\frac{1}{4}(p + \rho) \leq 0 \tag{2.6}$$

Therefore, reversal from contraction ($H < 0$) to expansion ($H > 0$) is impossible. (Here we assume that the time coordinate t covers the whole space.) We note that this statement is similar to the proof of the c theorem in the context of the AdS/CFT correspondence. There it is also shown that c is monotonic.

3 Look for a Loophole

In this section we will look for a possible loophole in the previous discussion. We will show that if the system goes through a singularity perhaps it is possible to reverse from contraction to expansion; i.e. from a big crunch to a big bang.

We simplify the Lagrangian (2.1) to a single scalar field ϕ. Then we can define ϕ such that the kinetic term is canonical. For simplicity we ignore the potential and then the Lagrangian is

$$\mathcal{L} = \mathcal{R} - \frac{1}{2}\partial\phi\partial\phi \tag{3.1}$$

We will consider the theory based on this Lagrangian in d spacetime dimensions.

In the minisuperspace description there are three degrees of freedom $a(\eta)$, $N(\eta)$ and $\phi(\eta)$ defined through

$$ds^2 = a^2(\eta)[-N^2(\eta)d\eta^2 + \sum_i (dx^i)^2] \qquad \phi = \phi(\eta) \tag{3.2}$$

It is convenient to define the combinations

$$a_\pm = a^{\frac{d-2}{2}} e^{\mp\gamma\phi} = \frac{1}{2}(a_0 \pm a_1) \qquad \gamma = \sqrt{\frac{d-2}{8(d-1)}} \tag{3.3}$$

which are constrained to satisfy $a_0 > |a_1|$. In terms of these variables the Lagrangian (3.1) becomes

$$\mathcal{L} \sim \frac{1}{N(\eta)} \left[-\left(\frac{da_0}{d\eta} \right)^2 + \left(\frac{da_1}{d\eta} \right)^2 \right] \tag{3.4}$$

This theory is invariant under reparametrization of the time η with an appropriate transformation of $N(\eta)$. We can choose a gauge $N(\eta) = 1$ and impose the equation of motion of N; i.e. $\left(\frac{da_0}{d\eta}\right)^2 = \left(\frac{da_1}{d\eta}\right)^2$ as a constraint. This constraint eliminates the negative mode associated with a_0. (This negative mode is common in gravity and is familiar in the worldsheet description of string theory.) The coordinates a_0 and a_1 are free except that they are bounded: $a_0 > |a_1|$.

The solutions of the equations of motion and the constraint up to shifts of η are [2]

$$a(\eta) = a(1)|\eta|^{\frac{1}{d-2}} \qquad\qquad \phi(\eta) = \phi(1) \pm \frac{1}{2\gamma} \log|\eta| \tag{3.5}$$

$a(1)$ and $\phi(1)$ are two integration constants. We will focus on the solutions with the plus sign. One solution exists only for negative η and the other only for positive η. The solution with negative η represents contraction to a singularity with $a = 0$ and $\phi = -\infty$ which happens at $\eta = 0$. The other solution which exists for positive η represents expansion from a singularity with $a = 0$ and $\phi = -\infty$ which takes place at $\eta = 0$. In terms of the coordinates a_0 and a_1 the motion is free; i.e. linear in η. These variables are finite at $\eta = 0$, where they reach their bound $a_0 = a_1$.

It is natural to conjecture that the two solutions can be connected at the singularity at $\eta = 0$. Then the free classical motion of a_0 and a_1 simply bounces off the bound $a_0 = a_1$, such that both for η negative and for η positive $a_0 > a_1$. We thus conjecture that our system contracts for negative η and then expands for positive η.

It is important to stress that this assumption does not violate the no-go theorem about the impossibility of such reversal because the system goes through a singularity. At that point the simple description of the dynamics in terms of the Lagrangian (3.1) breaks down and the no-go theorem does not apply.

In order to determine whether such reversal is possible we must go beyond the Lagrangian (3.1) and understand better its microscopic origin.

4 Higher Dimensional Perspective

In this section we will present a higher dimensional field theoretic extension of the theory (3.1) which leads to a geometric picture of the reversal from contraction to expansion.

It is instructive to change variables to

$$\bar{g}_{\mu\nu} = e^{-\frac{4}{d-2}\gamma\phi} g_{\mu\nu} \qquad\qquad \psi = e^{\gamma\phi} \tag{4.1}$$

In terms of these variables the action of the Lagrangian (3.1) is

$$S = \int d^d x \sqrt{-\bar{g}} \psi^2 \mathcal{R}(\bar{g}) \tag{4.2}$$

Up to rescaling of the coordinates the classical solution (3.5) is

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu} \qquad\qquad \psi = \psi(1)\sqrt{|\eta|} \tag{4.3}$$

We see that the metric $\bar{g}_{\mu\nu}$ is independent of η. In particular, it is smooth at the transition point $\eta = 0$, it is nonsingular and does not exhibit reversal from contraction to expansion. The solution of ψ is singular at $\eta = 0$ and since $\psi(0) = 0$, the Planck scale goes to zero there. In these variables the singularity does not appear as a short distance singularity (because $\bar{g}_{\mu\nu}$ is finite) but as a strong coupling singularity.

We again see the need for better knowledge of the microscopic theory at the singularity.

This theory can originate from the compactification of a $d+1$ dimensional theory on a circle or an interval. Then ϕ (or ψ) represents the radion – the size of the compact direction. More explicitly, let the metric of the $d+1$ dimensional theory be

$$ds^2 = g^{(d+1)}_{\alpha\beta} dx^\alpha dx^\beta = \psi^4 dw^2 + \bar{g}_{\mu\nu} dx^\mu dx^\nu \qquad\qquad w \sim w + 1 \tag{4.4}$$

We neglect the gauge field arising from the off diagonal components of the compactification and the higher Kaluza-Klein modes (w dependence in ψ and $\bar{g}_{\mu\nu}$). Then, the action (4.2) is simply the standard Hilbert-Einstein action in $d+1$ dimensions of the metric $g^{(d+1)}_{\alpha\beta}$ of (4.4)

$$S = \int d^d x dw \sqrt{-g^{(d+1)}} \mathcal{R}(g^{(d+1)}) \tag{4.5}$$

In terms of the metric $g^{(d+1)}$ up to rescaling the coordinates our classical solution is

$$ds^2 = A^2 t^2 dw^2 + \eta_{\mu\nu} dx^\mu dx^\nu \qquad\qquad t = x^0 \tag{4.6}$$

with a constant A. We see here a flat $d-1$ dimensional space times a two dimensional space $\mathcal{M}^2$.

The metric of $\mathcal{M}^2$ is

$$ds^2 = -dt^2 + A^2 t^2 dw^2 = dx^+ dx^- \qquad\qquad x^\pm = \pm t e^{\pm Aw} \tag{4.7}$$

The expression in terms of $x^\pm$ shows that the space is locally flat. But the identification $w \sim w+1$ identified $x^\pm \sim e^{\pm A} x^\pm$; i.e. the identification is by a boost. Without the identification this is Milne Universe which gives a description of a quadrant of two dimensional Minkowski space (a quadrant of the whole $x^\pm$ space). With the identification by a boost the space $\mathcal{M}^2$ is more subtle. It has already been discussed by Horowitz and Steif [4].

One can take various points of view about the singularity of this space:

- Only the past cone with an end of time or only the future cone with a beginning of time should be kept. In terms of the classical solutions (3.5) this corresponds to taking one of the solutions without assuming our conjecture.

- We can view the space as the quotient of two dimensional Minkowski space by the boost. This space is not Hausdorff. Furthermore, it has closed timelike loops. It is not clear whether string theory on such an orbifold is consistent.

- Our conjecture amounts to keeping only the past cone, the future cone and the singularity that connects them. In other words, our space is characterized by the metric (4.7) with $-\infty < t < \infty$; i.e. time flows from minus infinity to plus infinity, and the singularity is a bridge from a big crunch to a big bang.

5 Embedding in M Theory/String Theory

Before embedding our geometry in string theory, we would like to make some general comments about singularities in string theory.

For most of the singularities which have been studied in string theory the analysis starts by taking the Planck scale to infinity. By doing that, gravity decouples from the dynamics at the singularity and the remaining interactions are described by a local quantum field theory (or noncommutative field theory or little string theory). All such singularities are characterized by being at a finite distance in moduli space.

The singularity we are interested in here is quite different. First, we cannot take the Planck scale to infinity, and therefore gravity cannot be ignored. This singularity takes place at an infinite distance in moduli space ($\phi \to -\infty$), but the motion to the singularity involves a change in the scale factor a, and hence it takes only a finite time.

It is common in string theory that singularities in its General Relativity approximation become less singular when the stringy corrections are taken into account. Also, it is often the case that distinct classical spaces are connected in string theory at a point which appears singular in the General Relativity approximation. The flop transition [5] and the conifold transition [6] are well known examples of these phenomena. It is therefore possible that the past cone and the future cone are also connected in a smooth way in string theory.

The conjectured two cone geometry $\mathcal{M}^2$ can be embedded in string theory in many ways. Let us mention a few of them:

- M theory on $R^9 \times \mathcal{M}^2$. This background is flat except at $t = 0$. Therefore it is an interesting candidate to be a solution of the equations of motion. Since spacetime supersymmetry is broken, it is not obvious that $R^9 \times \mathcal{M}^2$ is an exact solution of the full equations of motion of M theory. However, for large $|t|$ the background is approximately flat $R^{10,1}$ compactified on a large circle. Since the circle is large, the equations of motion of eleven dimensional supergravity give a good approximation of the full equations of motion, and they are obviously satisfied. Therefore, at least for large $|t|$ this background is an approximate solution of the equations.

We can view this background as a time dependent background for type IIA string theory on $R^9 \times R$ where R is the time coordinate in $\mathcal{M}^2$. This description is weakly coupled and has small curvature for a certain range of t near $t \approx 0$ (but not too close to zero) and for an appropriate range of the parameter A. There it is straightforward to see that the leading order equations of motion of type IIA string theory are satisfied.

- M theory on $R^9 \times \mathcal{M}^2/Z_2$. This is a Z_2 orbifold of the previous example. It shares all the features we mentioned there. The two branes at the end of the interval move toward each other at negative t, collide at $t = 0$ and bounce off each other at positive t. The entire process looks like two banging cymbals.

- M theory on $R^8 \times S^1 \times \mathcal{M}^2$. This is an S^1 compactification of the first example and can be interpreted as type IIA string theory on $R^8 \times \mathcal{M}^2$. Here the type IIA background is flat except at $t = 0$, where the curvature is infinite. This suggests that it could be a solution (though not a supersymmetric one) of the classical equations of motion of string theory. Since the string coupling can be made arbitrarily small, the string loop corrections are small, and perhaps can be under control.

6 Conclusions

In conclusion, we conjecture a transition through a spacelike singularity from a big crunch to a big bang. Only a detailed stringy analysis can prove or disprove the conjecture.

If our conjecture is correct, in these models time has no beginning or end. In a sense this is the most conservative assumption about the evolution of such systems; i.e. they are subject to standard quantum evolution without special initial or final conditions.

This idea (perhaps not quite this model) opens the door to the possibility that the big bang singularity is preceded by a period of contraction ending in a big crunch. As in the pre-big bang models [2], if the Universe existed for a long time before the big bang, many questions in cosmology can have new solutions and have to be revisited.

The original ekpyrotic model [7] has a problem of reversal from contraction to expansion [8]. Perhaps our banging cymbals can be incorporated into the ideas of this model and make a viable model of early cosmology.

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