CAN THE HIERARCHY PROBLEM BE SOLVED BY FINITE-TEMPERATURE MASSIVE FERMIONS IN THE RANDALL-SUNDRUM MODEL?

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Abstract

Quantum effects of bulk matter, in the form of massive fermions, are considered in the Randall-Sundrum AdS_5 brane world at finite temperatures. The thermodynamic energy (modulus potential) is calculated in the limiting case when the temperature is low, and is shown to possess a minimum, thus suggesting a new dynamical mechanism for stabilizing the brane world. Moreover, these quantum effects may solve the hierarchy scale problem, at quite low temperatures. The present note reviews essentially the fermion-related part of the recent article by I. Brevik, K. A. Milton, S. Nojiri, and S. D. Odintsov, Nucl. Phys. **B 599**, 305 (2001).

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1 Introduction

Consider the Randall-Sundrum (RS) scenario [1], i.e., a non-factorizable geometry with one extra fifth dimension. This dimension, called y, is compactified on an orbifold S^1/Z_2 of radius R such that $-\pi R \le y \le \pi R$. The orbifold fixed points at $y = 0$ and $y = \pi R$ are the locations of the two three-branes. We will below, instead of y, use the nondimensional coordinate ϕ, defined by $\phi = y/R$, thus lying in the interval $[-\pi, \pi]$. The RS 5D metric is

$$ds^2 = e^{-2kR|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - R^2 d\phi^2, \tag{1}$$

where $\eta_{\mu\nu}$=diag(-1,1,1,1) with $\mu = 0, 1, 2, 3$. The 5D metric is g_{MN} with capital subscripts, $M = (\mu, 5)$. The parameter $k \sim 10^{19}$ GeV is of Planck scale, related to the AdS radius of curvature, which is $1/k$. The points (x^μ, ϕ) and $(x^\phi, -\phi)$ are identified. The metric (1) is valid if the 5D cosmological constant Λ and the 5D Planck mass M_5 are related through

$$\Lambda = -6M_5^3 k^2. \tag{2}$$

The cosmological constants at the boundaries have to fulfil the constraints $\Lambda_0 = -\Lambda_\pi = -\Lambda/k$. This is the fine-tuning problem in the RS model. The 4D Planck mass M_P is related to M_5 through

$$M_P^2 = \frac{M_5^3}{k}\left(1 - e^{-2\pi kR}\right). \tag{3}$$

The $\phi = 0$ brane (Planck brane) is associated with the mass scale M_P, whereas the $\phi = \pi$ brane (TeV-brane) is associated with the scale $M_P e^{-\pi kR}$ lying in the TeV region provided that $kR \simeq 12$.

Assume now that there is a scalar field Φ in the bulk, with action

$$S_\Phi = \frac{1}{2}\int d^4x \int_{-\pi}^{\pi} d\phi\sqrt{-G}\left\{G^{AB}\partial_A\Phi\partial_B\Phi - \left(m^2 + \frac{2\alpha k}{R}(\delta(\phi) - \delta(\phi-\pi))\right)\Phi^2\right\}. \tag{4}$$

Here α is a nondimensional constant which parametrizes the mass on the boundaries. All fields in the 5D bulk can be regarded as Kaluza-Klein modes, which in turn can be considered as 4D fields on the brane with an infinite tower of masses. The mass spectrum m_n of the Kaluza-Klein modes in Φ is given [2, 3] by roots of

$$j_\nu(x_n)y_\nu(ax_n) - j_\nu(ax_n)y(x_n) = 0. \tag{5}$$

Here $x_n = m_n/ak$, $a = e^{-\pi kR}$, $\nu = \sqrt{4 + m^2/k^2}$, and the altered Bessel functions are

$$j_\nu(z) = (2-\alpha)J_\nu(z) + zJ'_\nu(z), \quad y_\nu(z) = (2-\alpha)Y_\nu(z) + zY'_\nu(z). \tag{6}$$

2 Effective potential for fermions in 5D AdS space at finite temperature

Following Ref. [4], we consider the energy (effective potential) for a bulk quantum field on a 5D AdS background at finite temperature. For a fermion of momentum $\mathbf{p}$ and mass m the partition function Z_p^f is $2\cosh(\beta E_p/2)$, where $\beta = 1/T$ is the inverse temperature and $E_p = \sqrt{\mathbf{p}^2 + m^2}$. The total fermionic partition function Z^f in a three-dimensional volume V then becomes

$$\beta F^f = -\ln Z^f = -V \int \frac{d^3p}{(2\pi)^3} \ln\left[2\cosh\left(\frac{\beta E_p}{2}\right)\right], \tag{7}$$

F^f being the free energy. The corresponding thermodynamic energy E^f is

$$E^f = \frac{\partial}{\partial\beta}(\beta F^f) = -V \int \frac{d^3p}{(2\pi)^3} \frac{E_p}{2} \tanh\left(\frac{\beta E_p}{2}\right). \tag{8}$$

As already mentioned, the 5D Kaluza-Klein modes can be considered as 4D fields on the brane with an infinite tower of masses, so by summing up the KK modes given by Eq. (5) we get the following expression for the total KK energy:

$$E^{fKK} = -\mathcal{F}\left[\frac{\sqrt{\mathbf{p}^2 + a^2k^2x^2}}{2} \tanh\left(\frac{\beta}{2}\sqrt{\mathbf{p}^2 + a^2k^2x^2}\right)\right]. \tag{9}$$

Here the functional $\mathcal{F}$ is defined by

$$\mathcal{F}[f(p,x)] = V \int \frac{d^3p}{(2\pi)^3} \frac{i}{2\pi} \int_C dx \frac{d}{dx} f(p,x) \ln[j_\nu(x)y_\nu(ax) - j_\nu(ax)y_\nu(x)], \tag{10}$$

the contour C encircling the positive real axis.

Since the zero temperature contribution to the energy, $E^f(\infty)$, has been calculated in Ref. [5], we subtract this contribution from $E^f(\beta)$ in Eq. (8) and consider henceforth

$$\tilde{E}^f(\beta) = E^f(\beta) - E^f(\infty). \tag{11}$$

This quantity is finite. In terms of the variable q defined via $|\mathbf{p}| = q/\beta$ we can write Eq. (11) as

$$\tilde{E}^f(\beta) = \frac{V}{2\pi^2\beta^4} \int_0^\infty dq\, q^2 \frac{\sqrt{q^2 + \beta^2 m^2}}{e^{\sqrt{q^2+\beta^2m^2}} + 1}. \tag{12}$$

This expression holds for arbitrary temperatures. Assume now that the temperature is low, $\beta \gg 1$. Then it is convenient to change the variable from q to s via $q = \sqrt{s^2 + 2\beta m s}$, whereby

$$\begin{aligned} \tilde{E}^f(\beta) &= \frac{V}{2\pi^2\beta^4}\int_0^\infty ds\, \frac{(s+\beta m)^2\sqrt{s^2+2\beta m s}}{e^{s+\beta m}+1} \\ &\to \frac{V m^{5/2} e^{-\beta m}}{(2\pi\beta)^{3/2}} \end{aligned} \tag{13}$$

to leading order. We assume that $\beta m \gg 1$; then, because of the factor $e^{-\beta m}$ we need to include only the lowest root $x = x_1$ in Eq. (5), corresponding to $n = 1$. When a is small, this yields $j_\nu(x_1) = 0$ (we assume $\alpha + \nu \neq 2$), so that x_1 is of order unity. Adding the contribution from the zero-point energy we find the following effective potential:

$$V^f(a) = k^4 B_2^f \left(a^{2\mu} + \frac{B_3^f}{B_2^f}(\beta k)^{-3/2} a^{5/2} e^{-\beta k a x_1} \right), \tag{14}$$

where $\mu = \nu + 2$. We have here defined

$$B_2^f = -\frac{V}{16\pi^2}\frac{2^{1-2\nu}}{\nu\Gamma(\nu)^2}\int_0^\infty dt\, t^{2\nu+3}\frac{K_\nu'(t)}{I_\nu'(t)} > 0, \tag{15}$$

$$B_3^f = \frac{V}{(2\pi)^{3/2}}\, x_1^{5/2}. \tag{16}$$

We have now come to the main point: when βk is large, the effective potential (14) has a nontrivial *minimum*. The order of magnitude of a at the minimum, $a = a_m$, is given roughly by

$$a_m \sim \frac{1}{\beta k}\ln \beta k. \tag{17}$$

As mentioned above, taking $kR \simeq 12$ means that the $\phi = \pi$ brane is associated with the TeV region. That means, $e^{-\pi k R} \sim 10^{-17}$. With $k \sim 10^{19}$ GeV we thus see that with $1/\beta \sim 10$ GeV we get $\beta k \sim 10^{18}$, thus $a_m \sim 10^{-17}$ according to Eq. (17). The weak scale, $a_m k \sim 10^2$ GeV, can in this way be generated.

As a numerical example, assume that $\alpha = 2$. If $\nu = 5/2$ or $\mu = \nu + 2 = 9/2$ we get $x_1 = 3.6328$, $B_3^f/B_2^f = 11.9408$, and the minimum occurs at $a_m = 5.15 \times 10^{-17}$.

In conclusion, a bulk quantum fermion may generate a thermal (flat) 5D AdS brane-world with the necessary hierarchy scale. This example shows that quantum bulk effects in a brane-world AdS_5 at nonzero temperature may not only stabilize the brane-world but also provide the dynamical mechanism for the resolution of the hierarchy problem, with no fine-tuning.

Our example involving a single fermion is somewhat unsatisfactory because the minimum at $a \neq 0$ is only local. A somewhat more extended discussion, involving both bosons and fermions, is given in Ref. [4].

We mention that a very recent approach to stabilize the radius of the brane-world AdS_5 is the paper of Flachi *et al.* [6].

Finally, as a general remark, we mention that the RS scenario is of interest also for quantum gravity (for a general treatise on QG, see Ref. [7]).

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