

# Gauged extended supergravity without cosmological constant: no-scale structure and supersymmetry breaking.

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## Abstract

We consider the interplay of duality symmetries and gauged isometries of supergravity models giving  $N=2$ -extended, spontaneously broken supergravity with a no-scale structure. Some examples motivated by superstring and M-theory compactifications are described.

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# 1 Introduction

An important feature of a generic supergravity theory is the possibility of undergoing spontaneous supersymmetry breaking without a cosmological constant. By studying the universal coupling of a Goldstone fermion to supergravity, one can see that in a spontaneously broken supergravity theory the contributions to the vacuum energy could in principle cancel [1]. The first concrete example, based on a field theory lagrangian, was given by Polony [2, 3]. He considered  $N=1$  supergravity coupled to a single chiral multiplet with canonical kinetic term and linear superpotential and showed that it is possible to fine tune the parameters ( $\alpha$  and  $\beta$ ) of the superpotential  $W = \alpha z + \beta$  in such a way that the potential stabilizes the scalar fields with vanishing vacuum energy. The scalar field masses satisfy the sum rule  $m_A^2 + m_B^2 = 4m_{3/2}^2$  [3]. Polony type superpotentials were used in the first phenomenological studies of broken supergravity. They generate the soft breaking terms of the observable sector of standard (electroweak and strong) interactions in the supersymmetric extension of the standard model and of grand unified theories (For a review, see Ref. [4]).

The Polony classical potential is rather unnatural because it requires an ad hoc superpotential. Subsequent studies of the superHiggs sector of supergravity models lead to the introduction of a more appealing class of theories, the so called no-scale supergravities [5, 6]. In these models, the vanishing vacuum energy of the classical potential is obtained without stabilizing the scalar superpartner of the Goldstino. Instead, there is an exact cancellation, prior to minimization, of the positive Goldstino contribution against the negative gravitino contribution to the vacuum energy without the need of fine-tuning the parameters. The no-scale structure of these models poses further constraints on the soft-breaking terms which enter in the phenomenological Lagrangians [7].

The first construction of an extended supergravity exhibiting a no-scale structure was in the context of  $N=2$  supergravity coupled to abelian vector multiplets in presence of a Fayet-Iliopoulos term [8]. For a certain choice of the geometry of the scalar manifold, a spontaneous breaking of  $N=2$  to  $N=0$  with flat potential takes place. Later, models with  $N=2$  supersymmetry partially broken to  $N=1$  were found. In these models the vector multiplets gauge particular isometries of the quaternionic variety pertaining to the hypermultiplets. The breaking of supersymmetry with naturally vanishing vacuum energy was achieved by gauging two traslational isometries of the quaternionic manifold [9, 10]. One unbroken supersymmetry required a relation between the gauge coupling constants of the two translational isometries. To the  $U_1$  translational isometries of the quaternionic manifold

correspond  $n_t$  axion fields  $b^i$  transforming by a shift. We can express the shift corresponding to the gauged isometries as

$$b^i(x) \longrightarrow b^i(x) + g_1^i \xi^1(x) + g_2^i \xi^2(x),$$

so the covariant derivatives are

$$\mathcal{D}_\mu b^i = \partial_\mu b^i - g_1^i A_\mu^1(x) - g_2^i A_\mu^2(x).$$

The simplest model [9, 10] is based on the quaternionic manifold of quaternionic dimension  $n_H = 1$

$$\frac{\mathrm{SO}(1, 4)}{\mathrm{SO}(4)} \simeq \frac{\mathrm{USp}(2, 2)}{\mathrm{USp}(2) \times \mathrm{USp}(2)}.$$

It has three translational isometries,  $i = 1, \dots, 3$ . By choosing a gauging such that  $g_1^1 = g$ ,  $g_2^2 = g'$  and zero otherwise, one unbroken supersymmetry implies that  $|g| = |g'|$ .

It was later shown [11, 12] that it is possible to couple this  $N = 2$  hidden sector to observable matter for a suitable choice of the vector and hypermultiplet geometry and for appropriate gauge groups.

The no-scale structure for  $N > 2$  extended supergravity is encountered in the context of eleven dimensional supergravity with Scherk-Schwarz generalized dimensional reduction. This produces spontaneously broken supergravity theories in four dimensions [13, 14]. The four dimensional interpretation of these theories [15] is an  $N = 8$  gauged supergravity whose gauge algebra (a “flat” algebra according to Ref. [13]) is a 28 dimensional Lie subalgebra of  $E_{7,7}$  obtained in the following way: Consider the decomposition

$$\mathfrak{e}_{7,7} \xrightarrow[\mathfrak{e}_{6,6} + \mathfrak{so}(1,1)]{} \mathfrak{e}_{6,6} + \mathfrak{so}(1,1) + \mathbf{27}^+ + \mathbf{27}^-,$$

Then the flat subalgebra is the semidirect sum of a factor  $\mathfrak{u}(1)$  in the Cartan subalgebra of  $\mathfrak{usp}(8)$  (maximal compact subalgebra of  $\mathfrak{e}_{6,6}$ ) with the 27 translational subalgebra  $\mathbf{27}^-$ . The commutation rules are

$$\begin{aligned} [X_\Lambda, X_0] &= f_{\Lambda 0}^\Sigma X_\Sigma, \\ [X_\Lambda, X_\Sigma] &= 0 \quad \Lambda = 1, \dots, 27, \end{aligned}$$

with  $f_{\Lambda 0}^\Sigma = M_\Lambda^\Sigma$  in the CSA of  $\mathfrak{usp}(8)$ .

The 27 axions  $a^\Lambda$  in  $E_{7,7}/\mathrm{SU}(8)$  transform under the gauge algebra as follows:

$$\delta a^\Lambda = M_\Sigma^\Lambda \xi^\Sigma + \xi^0 M_\Sigma^\Lambda a^\Sigma,$$

and their covariant derivatives, in terms of the gauge fields  $B_\mu$  and  $Z_\mu^\Sigma$ , are

$$\mathcal{D}_\mu a^\Lambda = \partial_\mu a^\Lambda - M_\Sigma^\Lambda a^\Sigma B_\mu - M_\Sigma^\Lambda Z_\mu^\Sigma.$$

The gauge fields transform as

$$\begin{aligned}\delta Z_\mu^\Lambda &= \partial_\mu \xi^\Lambda + \xi^0 M_\Sigma^\Lambda Z_\mu^\Sigma - \xi^\Sigma M_\Sigma^\Lambda B_\mu \\ \delta B_\mu &= \partial_\mu \xi^0\end{aligned}$$

With respect to  $\text{USp}(8)$  the representation **27** of  $\text{E}_{7,7}$  is the two fold anti-symmetric  $\Omega$ -traceless representation, so we can write  $\Lambda \rightarrow (a_1, a_2)$  and

$$M_\Sigma^\Lambda \rightarrow M_{[b_1]^{a_1} \delta_{b_2}^{a_2}}^{[a_1] \delta_{b_2}^{a_2}} - \Omega\text{-traces}$$

where  $M_{b_1}^{a_1}$  turns out to be the gravitino  $\text{U}(1)$ -charge matrix,

$$\begin{pmatrix} m_1 \epsilon & 0 & 0 & 0 \\ 0 & m_2 \epsilon & 0 & 0 \\ 0 & 0 & m_3 \epsilon & 0 \\ 0 & 0 & 0 & m_4 \epsilon \end{pmatrix} \quad (1)$$

with  $\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ . The gravitino mass matrix is the symmetric  $8 \times 8$  matrix

$$S_{ab} = e^{-3\phi} (M\Omega)_{ab},$$

where  $\phi$  is the radion field [15, 16]. This is the constant term in the fifth component of the  $\text{USp}(8)$  connection of  $\text{E}_{6,6}/\text{USp}(8)$  upon generalized dimensional reduction [17].

The most recent example of no-scale extended supergravity is the  $\text{N} = 4$  spontaneously broken theory [16, 20] which is the low energy effective action for type IIB superstrings on type IIB orientifolds in presence of D3-branes and with three-form fluxes turned on [18]. In presence of  $n$  D3-branes this theory corresponds to a gauged supergravity with gauge group the direct product  $T_{12} \times \text{U}(n)$  which are a particular set of isometries of the sigma model  $\text{SO}(6, 6+n^2)/\text{SO}(6) \times \text{SO}(6+n^2)$ . The latter is a sigma model of an  $\text{N} = 4$  supergravity theory coupled to  $6+n^2$  vector multiplets. In the superstring interpretation, six of the vector multiplets come from the supergravity fields on the bulk and the rest comes from a non abelian D3-brane Born-Infeld action coupled to supergravity. The twelve bulk vectors gauge the  $T_{12}$  factor and the gauge vectors living on the brane gauge the  $\text{U}(n)$  Yang-Mills group. The full action has been recently constructed [21]. It is a no-scale  $\text{N} = 4$

supergravity with four arbitrary gravitino masses and its moduli space is a product of three non compact projective spaces  $U(1, 1+n)/U(1) \times U(n)$  [22]. If we formally integrate out step by step the three massive gravitino multiplets (in this process  $N = 4 \rightarrow N = 3 \rightarrow N = 2 \rightarrow N = 1$ ) we end up with an  $N = 1$  no-scale supergravity with a particular simple form which falls in the class of no-scale models studied in the literature [7].

The paper is organized as follows. In Section 2 we review the scalar potential in  $N$ -extended supergravity and outline the properties of no-scale supergravities regardless of the specific matter content and of the number of supersymmetries. In Section 3 we formulate the  $N = 8$  and  $N = 4$  spontaneously broken theories discussed so far in the context of no-scale gauged supergravities. In Section 4 we briefly review the  $N = 1$  no scale models and consider the  $N = 1$  type IIB orientifold model in this framework.

## 2 Scalar potential in $N$ -extended supergravity: vacua without cosmological constant

We consider an  $N$ -extended supergravity theory in  $D = 4$ . We will denote by  $\psi_{\mu A}$ ,  $A = 1, \dots, N$  the spin 3/2 gravitino fields and by  $\lambda^I$  the spin 1/2 fields (the spinor indices are not shown explicitly). They are all taken to be left handed, and the right handed counterparts are denoted by  $\bar{\psi}_{\mu}^A$  and  $\bar{\lambda}_I$ . The scalar fields will be denoted by  $q^u$ , and are coordinates on a Riemann manifold  $\mathcal{M}$ . Supersymmetry requires that  $\mathcal{M}$  has a restricted holonomy group  $H = H_R \times H_M$ , with  $H_R$  being  $U(N)$  or  $SU(N)$  ( $U(N)$  being the R-symmetry group) and  $H_M$  varying according to the different matter multiplet species. It also requires that on  $\mathcal{M}$  there is an  $H_R$ -bundle with a connection whose curvature is related to the geometric structure of  $\mathcal{M}$  [23].

For  $N = 1$  supergravity coupled to  $n$  chiral multiplets we have a Kähler-Hodge manifold of complex dimension  $n$ , with  $H_M = SU(n)$  and  $H_R = U(1)$ . On  $\mathcal{M}$  there is a  $U(1)$  bundle whose Chern class is equal to the Kähler class.

For  $N = 2$  coupled to  $n$  vector multiplets we have a special Kähler-Hodge manifold. If we have  $n_h$  hypermultiplets, then  $\mathcal{M}$  is a quaternionic manifold of quaternionic dimension  $n_h$ . The holonomy is  $H = H_R \times H_M$  with  $H_R = SU(2)$  and  $H_M = USp(2n_h)$ . On  $\mathcal{M}$  there is an  $SU(2)$ -bundle with curvature equal to the triplet of hyperKähler forms on  $\mathcal{M}$ .

For  $N > 2$  the manifolds of the scalars are maximally symmetric spaces  $\mathcal{M} = G/H$  with  $H = H_R + H_M$ . Then there is also an  $H_R$ -bundle on  $\mathcal{M}$  whose connection is the  $H_R$  part of the spin connection.

For  $N=3$  with  $n_v$  vector multiplets  $H_M = \text{SU}(n_v)$  and  $H_R = \text{U}(3)$ . For  $N \geq 4$  the supergravity multiplet itself contains scalars. For  $N=4$  with  $n_v$  vector multiplets  $H_M = \text{SO}(n_v)$  and  $H_R = \text{SU}(4) \times \text{U}(1)$ . For  $N > 4$  there are no matter multiplets and  $H = H_R = \text{U}(N)$  except for  $N=8$  where  $H = \text{SU}(8)$  [24].

The above considerations imply that the covariant derivative of the supersymmetry parameter,  $\mathcal{D}_\mu \epsilon_A$ , contains, in presence of scalar fields, an  $H_R$  connection in addition to the spacetime spin connection.

The supersymmetry variations of the fermions in a generic supergravity theory can be expressed as [25]

$$\delta \psi_{A\mu} = \mathcal{D}_\mu \epsilon_A + \frac{1}{2} S_{AB} \gamma_\mu \epsilon^B + \dots \quad (2)$$

$$\delta \lambda^I = i P_{\mu A}^I \gamma^\mu \epsilon^A + N^{IA} \epsilon_A + \dots, \quad (3)$$

where  $S_{AB} = S_{BA}$ , and  $N^{IA}$  are sections of  $H_R$  bundles on  $\mathcal{M}$  which depend on the specific model under consideration. The dots stand for terms which contain vector fields.  $P_{\mu A}^I dx^\mu$  is pullback into spacetime of  $P_{uA}^I dq^u$ , the vielbein one-form on  $\mathcal{M}$ , so

$$P_{\mu A}^I = P_{uA}^I \partial_\mu q^u.$$

The variation of the scalars is then given by

$$\delta q^u P_{uA}^I = \bar{\lambda}^I \epsilon_A.$$

The supergravity lagrangian contains the following terms

$$\frac{1}{\sqrt{-g}} \mathcal{L} = \dots + S_{AB} \bar{\psi}_\mu^A \sigma^{\mu\nu} \psi_\nu^B + i N^{IA} \bar{\lambda}_I \gamma^\mu \psi_{\mu A} + M^{IJ} \bar{\lambda}_I \lambda_J + \text{c.c.} - V(q)$$

where  $M^{IJ}$  is the mass matrix of the spin  $1/2$  fields and  $V(q)$  is the potential of the scalar fields. The potential must be such that the supersymmetry variation of all these terms cancel. This implies [26, 27]

$$\begin{aligned} \delta_B^A V(q) &= -3 S^{AC} S_{BC} + N^{IA} N_{IB} \\ \frac{\partial V}{\partial q^u} P_{uA}^I &= 2i N^{IB} S_{BA} + 2 M^{IJ} N_{JA}, \end{aligned} \quad (4)$$

where  $N_{IA} = (N^{IA})^*$  and  $S^{AC} = S_{AC}^*$ .

Flat space requires that on the extremes  $\partial V / \partial q^u = 0$  the potential vanishes, so

$$3 \sum_C S^{AC} S_{CA} = \sum_I N^{IA} N_{IA}, \quad \forall A.$$

The first term in the potential (4) is the square of the gravitino mass matrix. It is hermitian, so it can be diagonalized by a unitary transformation. Assume that it is already diagonal, then the eigenvalue in the entry  $(A_0, A_0)$  is non zero if and only if  $N^{IA_0} \neq 0$  for some  $I$ . On the other hand, if the gravitino mass matrix vanishes then  $N^{IA}$  must be zero.

For no-scale models, there is a subset of fields  $\lambda^I$  for which

$$3 \sum_C S^{AC} S_{CA} = \sum_{I'} N^{I'A} N_{I'A}, \quad \forall A \quad (5)$$

in all  $\mathcal{M}$ . This implies that the potential is given by

$$V(q) = \sum_{I \neq I'} N^{IA} N_{IA},$$

and it is manifestly positive definite. Zero vacuum energy on a point of  $\mathcal{M}$  implies that  $N^{IA} = 0$ ,  $I \neq I'$  at that point. This happens independently of the number of unbroken supersymmetries, which is controlled by  $N^{I'A}$  (gravitino mass matrix).

In  $N$  extended supergravities, the axion couplings to the gauge fields

$$\mathcal{D}_\mu a^i = \partial_\mu a^i - g_\Lambda^i A_\mu^\Lambda$$

are related to the gravitino mass matrix  $S_{AB}$  through the existence, for each pair of indices  $i, \Lambda$  of a section  $X_{i,AB}^\Lambda$  of an  $H_R$  bundle over  $\mathcal{M}$  such that

$$S_{AB} = g_\Lambda^i X_{i,AB}^\Lambda, \quad X_{i,AB}^\Lambda = X_{i,BA}^\Lambda.$$

In the next section we will give the particular form of  $S_{AB}$  in  $N=8$  and  $N=4$ .

### 3 No-scale $N=8$ and $N=4$ theories

#### 3.1 $N=8$ Scherk-Schwarz spontaneously broken supergravity

In  $N=8$  spontaneously broken supergravity à la Scherk-Schwarz, the  $R$ -symmetry that is manifest is  $\text{USp}(8) \subset \text{SU}(8)$ . The spin 3/2 gravitinos are in the fundamental representation of  $\text{USp}(8)$  (8), while the spin 1/2 fermions are in the 8 and 48 (threefold  $\Omega$ -traceless antisymmetric representation). We will denote them as  $\psi_{\mu a}, \lambda_a$  and  $\lambda_{abc}$ .

From a dimensional reduction point of view, the scalar potential is originated by the five dimensional  $\mathfrak{e}_6$ -model kinetic energy term

$$\sqrt{-g}g^{\mu\nu}P_\mu^{abcd}P_{\nu abcd}, \quad \text{for } \mu = \nu = 5,$$

where  $P_\mu^{abcd}$  is the pullback on spacetime of the vielbein one form of the coset  $E_{6,6}/\text{USp}(8)$ .

From the generalized dimensional reduction, the four dimensional scalar potential is

$$V = \frac{1}{8}e^{-6\phi}P_5^{abcd}P_{5abcd}, \quad (6)$$

where  $\phi$  is the radion field. This term would not appear in a standard dimensional reduction. The five dimensional supersymmetry variations are

$$\begin{aligned} \delta\psi_{\mu a} &= \mathcal{D}_\mu \epsilon_a + \cdots \\ \delta\lambda_{abc} &= P_{\mu abcd}\gamma^\mu \epsilon^d + \cdots \end{aligned}$$

We denote by  $Q_{\mu ab}$  the  $\text{USp}(8)$  connection in five dimensions. The functions  $S_{AB}$  and  $N^{IA}$  of the previous section (3) are then

$$S_{ab} = \frac{1}{\sqrt{3}}e^{-3\phi}Q_{5ab}, \quad N^{ab} = e^{-3\phi}Q_5^{ab}, \quad N^{abcd} = e^{-3\phi}P_5^{abcd}$$

(the indices can be raised or lowered with the antisymmetric metric  $\Omega_{ab}$ ).  $P_5^{abcd}$  satisfies the identity [17]

$$P_5^{abcd}P_{5ebcd} = \frac{1}{8}\delta_e^a P_5^{fbcd}P_{5fbcd},$$

which is crucial to have (4) In the computation of the scalar potential using (4) there is an exact cancellation between the gravitino and the spin 1/2 fermions in the  $\mathbf{8}$  as in (5),

$$3|S_{ab}|^2 = |N^{ab}|^2,$$

so that

$$V = \frac{1}{8}|N^{abcd}|^2.$$

This explains formula (6) from a four dimensional point of view. Note that at a linearized level (near the origin of the coset, where the exponential coordinates  $\phi^{abcd}$  are small),

$$\begin{aligned} P_5^{abcd} &= M_{a'}^{[a}\phi^{a'bcd]} - \Omega\text{-traces} + \mathcal{O}(\phi^{abcd})^2, \\ Q_{5ab}\Omega^{ba'} &= M_a^{a'} + \mathcal{O}(\phi^{abcd}), \end{aligned}$$



where  $M_a^a$  was given in (1). The vacua with zero potential correspond to  $P_5^{abcd} = 0$ , while the supersymmetry breaking depends on the vanishing eigenvalues of the matrix  $Q_{5ab}$ . When all the eigenvalues  $m_i$  of (1) are different from zero, the requirement  $P_5^{abcd} = 0$  determines all but two coordinates which are the two scalars which are neutral with respect to the CSA of  $\text{USp}(8)$ . Together with the radion, they are the flat directions of the potential. There are three additional massless scalars, the three axions in the 27 of  $\text{USp}(8)$  which are neutral under the CSA. All together, they form the moduli space of the Scherk-Schwarz compactification and they parameterize the coset

$$\left(\frac{\text{SU}(1,1)}{\text{U}(1)}\right)^3.$$

If some eigenvalues  $m_i$  of  $M$  vanish, we have some unbroken supersymmetries, and the moduli space of the solution is bigger. If three masses are set to zero, then the equation  $P_5^{abcd} = 0$  leaves 14 coordinates undetermined, which parameterize

$$\frac{\text{SU}^*(6)}{\text{USp}(6)},$$

By adding the radion and 15 axions this space enlarges to

$$\frac{\text{SO}^*(12)}{\text{U}(6)},$$

which is the moduli space of the  $N = 6$  unbroken supergravity. Similar reasoning can be used for the cases with two eigenvalues equal to zero ( $N = 4$ ) and only one eigenvalue equal to zero ( $N = 2$ ). We observe that in all models the spin 1/2 fermions which cancel the negative spin 3/2 contribution to the potential are precisely the Goldstino fermions. They are in the **8** of  $\text{USp}(8)$  for the  $N = 8$  model of section 3.1 and in the **4** of  $\text{SU}(4)$  for the  $N = 4$  model of this section. When all supersymmetries are broken these fermions disappear from the spectrum to give mass to the gravitino. If some supersymmetry remains unbroken then these fermions are strictly massless.

### 3.2 $N = 4$ supergravity and type IIB orientifolds

We now consider no-scale  $N = 4$  spontaneously broken supergravity. This theory is the low energy limit of type IIB 10 dimensional supergravity compactified on orientifolds in presence of three form fluxes and  $n$  D3 branes with non commutative coordinates [18, 19]. The six  $N = 4$  vector multiplets coming from the bulk lagrangian contain 36 scalars, 21 of which are the metric deformation of the 6-torus  $T^6$   $g_{IJ}$ ,  $I, J = 1, \dots, 6$ , and 15 scalars coming

from the four form gauge field  $C_{\mu\nu\rho\lambda}$ , whose components along the 6-torus are dual to a two form

$$B^{IJ} = {}^*C^{IJ}, \quad I, J = 1, \dots, 6.$$

Turning on the three form fluxes corresponds in the effective theory to gauge particular isometries of the coset  $\text{SO}(6,6)/\text{SO}(6) \times \text{SO}(6)$  [16]. More explicitly, the gauge isometries are twelve of the fifteen translational isometries in the graded decomposition [16]

$$\mathfrak{so}(6,6) = \mathfrak{sl}(6) + \mathfrak{so}(1,1) + \mathbf{15}^+ + \mathbf{15}^-.$$

In the case when Yang-Mills  $N=4$  multiplets are added (describing the D3 brane degrees of freedom), the gauge group is  $T_{12} \times \text{U}(n)$ . This theory gives rise to a no-scale supergravity with four arbitrary parameters for the gravitino masses [20]. The  $\text{SU}(4)$  (R-symmetry) representations of the bulk fermions are as follows: spin 3/2 (gravitinos) in the  $\mathbf{4}$ , spin 1/2 (dilatinos) in the  $\mathbf{4}$ , spin 1/2 (gauginos, from the 6 vector multiplets) in the  $\mathbf{20} + \mathbf{4}$ . The fermions in the brane (gauginos) form  $n^2 = \dim \text{U}(n)$  copies of the representation  $\mathbf{4}$  of  $\text{SU}(4)$ . Computing the potential (4), the subset of fields  $\lambda^I$  (5) are the bulk gauginos in the  $\mathbf{4}$ . The condition for vanishing potential [21] is then  $N^{IA} = 0$  for  $I \neq I'$ . For the bulk fermions it fixes the complex dilaton, 18 radial moduli and 12 axions. For the brane gauginos it fixes all the scalars but the ones in the CSA of  $\text{U}(n)$ .

## 4 $N=1$ no-scale supergravities

Supergravity theories with a positive definite potential have a particular convenient set up in the context of  $N=1$  supergravity.  $N=1$  theories can be obtained from an  $N$ -extended supergravity which is spontaneously broken to  $N=1$  and then integrating out the massive modes. This will be still true if  $N=1$  is itself spontaneously broken, provided the mass of the  $N=1$  gravitino is much smaller than the other gravitino masses. To compute the contribution to the scalar potential of the chiral multiplet sector of an  $N=1$  theory it is convenient to introduce some auxiliary fields [28]. We denote by  $u$  the auxiliary field associated to the gravity multiplet and by  $h^i$  the ones associated to the chiral multiplets. Let  $K$  be the Kähler potential and  $W$  be the superpotential. We introduce the function

$$\frac{\Phi}{3} = e^{-\frac{K}{3}}.$$

Then, the scalar potential is given by [29]

$$-\left(\frac{\Phi}{3}\right)^2 V = -\frac{1}{9}\Phi|u|^2 - \Phi_{i\bar{j}}h^i h^{\bar{j}} + W_i h^i + \bar{W}_{\bar{i}} h^{\bar{i}} + \frac{1}{3}u^*(3\bar{W} - \Phi_{\bar{i}}h^i) + \frac{1}{3}u(3W - \Phi_i h^{\bar{i}}). \quad (7)$$

respect to  $z^i$  and  $\bar{z}^{\bar{i}}$  are denoted by subindices  $i$  and  $\bar{i}$ ). The standard potential is easily obtained by making the field redefinition

$$\tilde{u} = u - K_i h^i$$

so that

$$-\left(\frac{\Phi}{3}\right)^2 V = -\frac{1}{9}\Phi|\tilde{u}|^2 + \frac{1}{3}\Phi K_{i\bar{j}}h^i h^{\bar{j}} + W\tilde{u} + \bar{W}\tilde{u}^* + (K_i W + W_i)h^i + (K_{\bar{i}}\bar{W} + \bar{W}_{\bar{i}})h^{\bar{i}}.$$

Eliminating the auxiliary fields we get [30]

$$V = e^K [K^{i\bar{j}}\mathcal{D}_i W \mathcal{D}_{\bar{j}} \bar{W} - 3|W|^2],$$

where  $\mathcal{D}_i W = \partial_i W + K_i W$ . The simplest example of no-scale supergravity is given by a  $\mathbb{CP}^{n+1}$   $\sigma$ -model [7] for which

$$\Phi = t + \bar{t} - \sum_A c_A c_{\bar{A}}, \quad A = 1, \dots, n$$

and an arbitrary superpotential  $W(c^A)$ . From (7), since  $\Phi_{t\bar{t}} = 0$ , then the variation with respect to  $h^t$  implies  $u = 0$ , and then the potential reduces to

$$-\left(\frac{\Phi}{3}\right)^2 V = \sum_A (h^A h^{\bar{A}} + h^A W_A + h^{\bar{A}} \bar{W}_{\bar{A}}).$$

Then

$$V = e^{\frac{2}{3}K} \left| \frac{\partial W}{\partial c_A} \right|^2$$

so the extremes with vanishing vacuum energy occur for  $\partial W / \partial c_A = 0$ . In this example the gravitino mass contribution to the potential  $3e^K |W|^2$  is canceled by the  $\chi^t$  fermion contribution. The crucial point here is that the matrix  $\Phi_{i\bar{j}}$  has determinant zero and rank  $n$  [31]. Such a situation generalizes to a class of models of the following type

$$\Phi = \prod_{r=1}^m (t_r + \bar{t}_r - \sum_A c_{rA} c_{r\bar{A}})^{\frac{1}{m}}, \quad A = 1, \dots, n$$

and  $W$  a function only of  $c_{rA}$ . This expression corresponds to the Kähler potential of the product of  $m$  spaces  $\mathbb{CP}^{n+1}$  each with curvature  $3/m$ . The function  $\Phi$  is homogeneous of degree 1 in the variables  $x_r = t_r + \bar{t}_r - \sum_A c_{rA} c_{r\bar{A}}$ . This implies that the matrix of second derivatives has a null vector

$$\sum_s \frac{\partial^2 \Phi}{\partial x^r \partial x^s} x^s = 0,$$

and from the form of the potential this implies that  $u=0$ . Then the potential becomes

$$-\left(\frac{\Phi}{3}\right)^2 V = -\sum_{rs} \Phi_{rs} \tilde{h}^{t_r} \tilde{h}^{\bar{t}_s} + h^{c_{rA}} h^{\bar{c}_{r\bar{A}}} \Phi_r + h^{c_{rA}} W_{c_{rA}} + \bar{h}^{\bar{c}_{r\bar{A}}} \bar{W}_{c_{rA}},$$

where

$$\tilde{h}^{t_r} = h^{t_r} - h^{c_{rA}} \bar{c}_{r\bar{A}}.$$

Eliminating the auxiliary fields one finds

$$V = -e^K \sum_r \frac{1}{K_r} \left| \frac{\partial W}{\partial c_{rA}} \right|^2 = e^K \sum_r K^{c_{rA} \bar{c}_{s\bar{B}}} \frac{\partial W}{\partial c_{rA}} \frac{\partial \bar{W}}{\partial \bar{c}_{s\bar{B}}},$$

where we have used the inverse of the Kähler metric

$$K^{c_{rA} \bar{c}_{s\bar{B}}} = -\delta_{rA s\bar{B}} \frac{1}{K_r}.$$

An example of the above situation is realized in type IIB orientifold with fluxes if one breaks  $N=4$  to  $N=1$  [22]. In this case  $m=3$ ,  $c^{rA}$  are the brane coordinates in the adjoint of  $U(n)$ , and the superpotential is

$$W(c_{rA}) = f + g^{ABC} c_{1A} c_{2B} c_{3C},$$

where  $f$  is the constant flux that breaks  $N=1$  to  $N=0$  and  $g^{ABC}$  are the structure constants of  $SU(n)$ .

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