Seoul National University

Seyeon

Donghyeon Son, Jaechan Lee, Seonghyeon Jo

2023-10-13

Data structures (1)

1.1 GCC Extension Data Structures

OrderStatisticTree.h — 29 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> ordered_set;

void test()
{
   ordered_set X;
   X.insert(1);
   X.insert(2);
   X.insert(4);
   X.insert(8);
   X.insert(16);

   cout<<*X.find_by_order(1)<<endl; // 2
   cout<<*X.find_by_order(2)<<endl; // 4
   cout<<*X.find_by_order(4)<<endl; // 16
   cout<<(end(X)==X.find_by_order(6))<<endl; // true

   cout<<X.order_of_key(-5)<<endl;  // 0
   cout<<X.order_of_key(1)<<endl;   // 0
   cout<<X.order_of_key(3)<<endl;   // 2
   cout<<X.order_of_key(4)<<endl;   // 2
   cout<<X.order_of_key(400)<<endl; // 5
}
```

HashMap.h — 19 lines

```
#pragma once
#pragma GCC optimize ("O3")
#pragma GCC target ("avx2")
#include <bits/extc++.h> /** keep-include */
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
  const uint64_t C = ll(4e18 * acos(0)) | 71;
  ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};
__gnu_pbds::gp_hash_table<ll,int,chash> h({},{},{},{},{1<<16});

// For CodeForces, or other places where hacking might be a
     problem:

const int RANDOM = chrono::high_resolution_clock::now().
     time_since_epoch().count();
struct chash { // To use most bits rather than just the lowest
     ones:
  const uint64_t C = ll(4e18 * acos(0)) | 71; // large odd
       number
  ll operator()(ll x) const { return __builtin_bswap64((x^
       RANDOM)*C); }
};
__gnu_pbds::gp_hash_table<ll, int, chash> h({},{},{},{}, {1 <<
     16});
```

1.2 Interval Tree structures

SegmentTree.h

Description: Point modification, interval sum query on $[l, r)$.
Time: $\mathcal{O}(\log N)$

30 lines

```
struct segtree {
    using elem = int;
    int n;
    elem T[2*N];

    inline elem agg(elem a, elem b) {
        return max(a, b);
    }

    void build(vector<elem> &v) {
        n = v.size();
        for (int i = 0; i<n; i++)
            T[n+i] = v[i];
        for (int i = n-1; i>0; i--)
            T[i] = agg(T[i<<1], T[(i<<1)|1]);
    }
    void modify(int pos, elem val) {
        for (T[pos += n] = val; pos > 1; pos >>= 1)
            T[pos >> 1] = agg(T[pos], T[pos^1]);
    }
    // query is on [l, r)!!
    elem query(int l, int r){
        elem res = 0;
        for (l += n, r += n; l < r; l >>=1, r>>=1) {
            if (l & 1) res = agg(T[l++], res);
            if (r & 1) res = agg(res, T[--r]);
        }
        return res;
    }
};
```

LazySegmentTree.h

Description: Interval incremental modification, interval sum query on $[l, r)$.
Time: $\mathcal{O}(\log N)$

78 lines

```
const int N = 20000;

struct Segtree {
    int n, h;
    int T[2 * N];
    int Lazy[N];
    int32_t Len[2 * N];

    void apply(int pos, int val) {
        T[pos] += val * Len[pos];
        if (pos < n) Lazy[pos] += val;
    }

    void build(vector<int> &v) {
        n = v.size();
        h = sizeof(int) * 8 - __builtin_clz(n);
        for (int i = 0; i < n; i++) {
            T[n + i] = v[i];
            Len[n + i] = 1;
        }
        for (int i = n - 1; i > 0; i--) {
            T[i] = T[i << 1] + T[(i << 1) | 1];
            Len[i] = Len[i << 1] + Len[(i << 1) | 1];
        }
    }
```

```
    void pupd(int p) {
        while (p > 1) {
            p >>= 1;
            T[p] = (T[p << 1] + T[(p << 1) | 1] + (Lazy[p] *
                 Len[p]));
        }
    }

    void propagate(int p) {
        for (int s = h; s > 0; s--) {
            int i = p >> s;
            if (!i) continue;
            if (Lazy[i] != 0) {
                apply(i << 1, Lazy[i]);
                apply((i << 1) | 1, Lazy[i]);
                Lazy[i] = 0;
            }
        }
    }

    void modify(int pos, int val) {
        for (T[pos += n] = val; pos > 1; pos >>= 1) {
            T[pos >> 1] = T[pos] + T[pos ^ 1];
        }
    }

    void modifyRange(int l, int r, int val) {
        l += n, r += n;
        int l0 = l, r0 = r;
        for (; l < r; l >>= 1, r >>= 1) {
            if (l & 1) apply(l++, val);
            if (r & 1) apply(--r, val);
        }
        pupd(l0), pupd(r0 - 1);
    }

    // query is on [l, r)!!
    int query(int l, int r) {
        l += n, r += n;
        propagate(l);
        propagate(r - 1);
        int res = 0;
        for (; l < r; l >>= 1, r >>= 1) {
            if (l & 1) {
                res += T[l++];
            }
            if (r & 1) {
                res += T[--r];
            }
        }
        return res;
    }
} S;
```

LazySegAssignment.h

Description: Lazy segtree with assignment modifications, sum queries. Parameter k stands for the length of the interval corrsponding to node p.

51 lines

```
void calc(int p, int k) {
  if (d[p] == 0) t[p] = t[p<<1] + t[p<<1|1]; // Assumes that 0
       is never used for modification
  else t[p] = d[p] * k;
}

void apply(int p, int value, int k) {
  t[p] = value * k;
  if (p < n) d[p] = value;
}

void build(int l, int r) {
  int k = 2;
  for (l += n, r += n-1; l > 1; k <<= 1) {
    l >>= 1, r >>= 1;
    for (int i = r; i >= l; --i) calc(i, k);
  }
}

void push(int l, int r) {
  int s = h, k = 1 << (h-1);
  for (l += n, r += n-1; s > 0; --s, k >>= 1)
    for (int i = l >> s; i <= r >> s; ++i) if (d[i] != 0) {
      apply(i<<1, d[i], k);
      apply(i<<1|1, d[i], k);
      d[i] = 0;
    }
}

void modify(int l, int r, int value) {
  if (value == 0) return;
  push(l, l + 1);
  push(r - 1, r);
  int l0 = l, r0 = r, k = 1;
  for (l += n, r += n; l < r; l >>= 1, r >>= 1, k <<= 1) {
    if (l&1) apply(l++, value, k);
    if (r&1) apply(--r, value, k);
  }
  build(l0, l0 + 1);
  build(r0 - 1, r0);
}

int query(int l, int r) {
  push(l, l + 1);
  push(r - 1, r);
  int res = 0;
  for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
    if (l&1) res += t[l++];
    if (r&1) res += t[--r];
  }
  return res;
}
```

PersistentSegmentTree.h

Description: Interval incremental modification, interval sum query on $[l, r)$.
Time: $\mathcal{O}(\log N)$

<algorithm>, <vector> 53 lines

```
template<class T> T defaultValue(){ return T(); }
template<class T> T defaultOperation(T a,T b){return a+b;}
template <class S,S (*e)() = defaultValue<S> >
struct Node{
    Node *l, *r;
    long long v;
    Node(){l=r=nullptr;v=e();}
};
template <class S,S (*op)(S,S) = defaultOperation<S>,S (*e)() =
      defaultValue<S> >
class PST{ private:
    using NN = Node<S,e>;
    std::vector< NN* > d;
    int log, sz, _n;
    void build(NN *node, const std::vector<S>& x,int start,int
         end){
        if(start==end && start<x.size()){ node->v = x[start];
             return;}
        else if(start==end){node->v = e(); return;} //
        int m = (start+end)>>1;
        node->l = new NN(); node->r = new NN();
        build(node->l,x,start,m); build(node->r,x,m+1,end);
        node->v = op(node->l->v,node->r->v);
    }
    void _a(NN *prv, NN *now, int start, int end, int x, int v)
         {
        if(start==end){now->v=v; return;}
        int m=(start+end)>>1;
        if(x<=m){
            now->l = new NN(); now->r = prv->r;
            _a(prv->l, now->l, start, m, x, v);
        } else {
            now->l = prv->l; now->r = new NN();
            _a(prv->r,now->r,m+1,end,x,v);
        }
        now->v = op(now->l->v,now->r->v);
    }
    S _q(NN* node, int start, int end, int l, int r){ //query,
         [l,r)
        if(r<start || end<l) return 0;
        if(l<=start && end<=r) return node->v;
        int m = (start+end)>>1;
        return op(_q(node->l, start, m, l, r),_q(node->r,m+1,
             end,l,r));
    }
    public:
    explicit PST(int n=100000): PST(std::vector<S>(n,e())) {}
    explicit PST(const std::vector<S>& x) : _n(int(x.size())){
        log=0;
        while( (1U << log) < (unsigned int)(_n)) log++;
        sz = 1<<log;
        d.emplace_back(new NN()); build(d[0],x,0,sz-1);
    }
    void add(int loc, int v){
        d.emplace_back(new NN()); _a(d[d.size()-2],d.back(), 0,
             sz-1, loc, v);
    }
    S query(int treeidx, int l, int r){ return _q(d[treeidx],
         0, sz-1, l, r-1); }
    int size(){return d.size();}
};
```

LazySegRecursive.h

47 lines

```
struct SegTree {
  vector<ll> tree;
  vector<ll> lazy;
  SegTree(int n) {
    tree.resize(4*n, 0);
    lazy.resize(4*n, 0);
  }
  ll init(vector<ll>& v, int node, ll l, ll r) {
    if(l==r)
      return tree[node] = v[l];
    else {
      ll mid = (l+r) >> 1;
      return tree[node] = init(v, 2*node, l, mid) + init(v, 2*
           node+1, mid+1, r);
    }
  }
  void propagate(int node, ll l, ll r) {
    tree[node] += lazy[node] * (r-l+1LL);
    lazy[2*node] += lazy[node];
    lazy[2*node + 1] += lazy[node];
    lazy[node] = 0;
  }
    // add on [s, e]
  ll update(int node, ll l, ll r, ll s, ll e, ll v) {
    if(s <= l && e >= r) {
      lazy[node] += v;
      return tree[node] + lazy[node] * (r-l+1LL);
```

```
    }
    else if(e < l || s > r)
      return tree[node] + lazy[node] * (r-l+1LL);
    else {
      propagate(node, l, r);
      ll mid = (l+r) >> 1;
      return tree[node] = update(2*node, l, mid, s, e, v) +
           update(2*node+1, mid+1, r, s, e, v);
    }
  }
  ll query(int node, ll l, ll r, ll s, ll e) {
    if(s <= l && e >= r)
      return tree[node] + lazy[node] * (r-l+1LL);
    else if(e < l || s > r)
      return 0;
    else {
      propagate(node, l, r);
      ll mid = (l+r) >> 1;
      return query(2*node, l, mid, s, e) + query(2*node+1, mid
           +1, r, s, e);
    }
  }
};
```

2DSegmentTree.h

Description: Compute sum of rectangle $[a,b)\times[c,d)$ and point modification

Time: Both operations are $\mathcal{O}(\log^2 N)$.

71 lines

```
auto gif = [](int a, int b) { return a + b; };

class SEG2D {
public:
    int n;
    int m;
    vector<vector<int>> tree;

    SEG2D(int n = 0, int m = 0) {
        tree.resize(2 * n);
        for (int i = 0; i < 2 * n; i++) tree[i].resize(2 * m);
        this->n = n;
        this->m = m;
    }

    SEG2D(int n, int m, vector<vector<int>> &data) {
        tree.resize(2 * n);
        for (int i = 0; i < 2 * n; i++) tree[i].resize(2 * m);
        this->n = n;
        this->m = m;
        init(data);
    }

    void init(vector<vector<int>> &data) {
        n = data.size();
        m = data.front().size();
        tree = vector<vector<int>>(2 * n, vector<int>(2 * m, 0)
            );
        for (int i = 0; i < n; i++)
            for (int j = 0; j < m; j++)
                tree[i + n][j + m] = data[i][j];
        for (int i = n; i < 2 * n; i++)
            for (int j = m - 1; j > 0; j--)
                tree[i][j] = gif(tree[i][j * 2], tree[i][j * 2
                    + 1]);
        for (int i = n - 1; i > 0; i--)
            for (int j = 1; j < 2 * m; j++)
                tree[i][j] = gif(tree[i * 2][j], tree[i * 2 +
                    1][j]);
    }

    void update(int x, int y, int val) {
        tree[x + n][y + m] = val;
        for (int i = y + m; i > 1; i /= 2)
            tree[x + n][i / 2] = gif(tree[x + n][i], tree[x + n
                ][i ^ 1]);
        for (int i = x + n; i > 1; i /= 2)
            for (int j = y + m; j >= 1; j /= 2)
                tree[i / 2][j] = gif(tree[i][j], tree[i ^ 1][j
                    ]);
    }

    int query_1D(int x, int yl, int yr) {
        int res = 0;
        int u = yl + m, v = yr + m + 1;
        for (; u < v; u /= 2, v /= 2) {
            if (u & 1)
                res = gif(res, tree[x][u++]);
            if (v & 1)
                res = gif(res, tree[x][--v]);
        }
        return res;
    }

    int query_2D(int xl, int xr, int yl, int yr) {
        int res = 0;
        int u = xl + n, v = xr + n + 1;
        for (; u < v; u /= 2, v /= 2) {
            if (u & 1)
                res = gif(res, query_1D(u++, yl, yr));
            if (v & 1)
                res = gif(res, query_1D(--v, yl, yr));
        }
        return res;
    }
};
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.

Time: Both operations are $\mathcal{O}(\log N)$.

22 lines

```
struct FT {
  vector<ll> s;
  FT(int n) : s(n) {}
  void update(int pos, ll dif) { // a[pos] += dif
    for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
  }
  ll query(int pos) { // sum of values in [0, pos)
    ll res = 0;
    for (; pos > 0; pos &= pos - 1) res += s[pos-1];
    return res;
  }
  int lower_bound(ll sum) {// min pos st sum of [0, pos] >= sum
    // Returns n if no sum is >= sum, or -1 if empty sum is.
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) {
      if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
        pos += pw, sum -= s[pos-1];
    }
    return pos;
  }
};
```

FenwickTree2d.h

Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j].

Time: $\mathcal{O}(\log^2 N)$.

"FenwickTree.h" 22 lines

```
struct FenwickTree2D {
    vector<vector<int>> bit;
    int n, m;

  FenwickTree2D(int n, int m) : n(_n), m(_m) {
    bit.assign(n, vector<int>(m, 0));
  }

    int sum(int x, int y) {
        int ret = 0;
        for (int i = x; i >= 0; i = (i & (i + 1)) - 1)
            for (int j = y; j >= 0; j = (j & (j + 1)) - 1)
                ret += bit[i][j];
        return ret;
    }

    void add(int x, int y, int delta) {
        for (int i = x; i < n; i = i | (i + 1))
            for (int j = y; j < m; j = j | (j + 1))
                bit[i][j] += delta;
    }
};
```

MergeSortTree.h

Description: greater(s,e,k,1,0,n) returns number of elements strictly greater than k in range $[s,e]$. Pay attention to INTERVAL INCLUSIVENESS!!!

Time: $\mathcal{O}(\log N)$.

38 lines

```
#define MAXN (1<<18)
#define ST (1<<17)
struct merge_sort_tree
{
  vector <int> tree[MAXN];
  int n;
  void construct (vector <int> data)
  {
    n = 1;
    while(n < data.size()) n <<= 1;
    for (int i = 0; i<data.size(); i++)
      tree[i+n] = {data[i]};
    for (int i = data.size(); i<n; i++)
      tree[i+n] = {};
    for (int i = n-1; i>0; i--)
    {
      tree[i].resize(tree[i*2].size()+tree[i*2+1].size());
      for (int p = 0, q = 0, j = 0; j < tree[i].size(); j++)
      {
        if (p == tree[i*2].size() ||
        (q<tree[i*2+1].size() && tree[i*2+1][q]<tree[i*2][p]))
          tree[i][j] = tree[i*2+1][q++];
        else tree[i][j] = tree[i*2][p++];
      }
    }
  }
  //greater(s,e,k,1,0,n)
  int greater(int s, int e, int k, int node, int ns, int ne)
  {
    if (ne <= s || ns >= e)
      return 0;
    if(s <= ns && ne <= e)
      return tree[node].end() - upper_bound(all(tree[node]), k)
          ;
    int mid = (ns+ne)>>1;
    return greater(s,e,k,node*2,ns,mid) +
     greater(s,e,k,node*2+1,mid,ne);
  }
};
```

1.3 Miscellaneous

EraseableHeap.h

Description: Heap with rmv() function.
Usage: push top pop size and rmv()

13 lines

```
struct iHeap {
    static const int inf = 1e9 + 5;
    priority_queue<int> q, qr;
    void rset() { while(!q.empty()) q.pop(); while(!qr.empty())
         qr.pop(); }
    inline int size() { return q.size()-qr.size(); }
    inline void rmv(int x) { qr.push(x); }
    inline int top() {
        while(q.size() && qr.size() && q.top() == qr.top()) { q
            .pop(); qr.pop(); }
        return q.size()?q.top():-inf;
    }
    inline void push(int x) { q.push(x); }
    inline int pop(int x) {int t = top(); rmv(x); return t;}
}
```

UnionFind.h

Description: Disjoint-set data structure.
Time: $\mathcal{O}(\alpha(N))$

21 lines

```
struct DSU
{
    int par[V], sz[V];
    DSU(){init(V);}
    void init(int n) {
        for (int i = 0; i<n; i++)
        par[i] = i, sz[i] = 1;
    }
    int find(int x) {
        return x == par[x] ? x : (par[x] = find(par[x]));
    }
    bool unite(int x, int y) {
        int u=find(x), v=find(y);
        if(u==v) return false;
        if(sz[u]>sz[v]) swap(u, v);
        sz[v]+=sz[u];
        sz[u] = 0;
        par[u] = par[v];
        return true;
    }
};
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
Usage: int t = uf.time(); ...; uf.rollback(t);
Time: $\mathcal{O}(\log(N))$

21 lines

```
struct RollbackUF {
  vi e; vector<pii> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return sz(st); }
  void rollback(int t) {
    for (int i = time(); i --> t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a]});
    st.push_back({b, e[b]});
    e[a] += e[b]; e[b] = a;
    return true;
  }
};
```

Treap.h

Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: $\mathcal{O}(\log N)$

55 lines

```
struct Node {
  Node *l = 0, *r = 0;
  int val, y, c = 1;
  Node(int val) : val(val), y(rand()) {}
  void recalc();
};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {
  if (n) { each(n->l, f); f(n->val); each(n->r, f); }
}

pair<Node*, Node*> split(Node* n, int k) {
  if (!n) return {};
  if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(k)
    auto pa = split(n->l, k);
    n->l = pa.second;
    n->recalc();
    return {pa.first, n};
  } else {
    auto pa = split(n->r, k - cnt(n->l) - 1); // and just "k"
    n->r = pa.first;
    n->recalc();
    return {n, pa.second};
  }
}

Node* merge(Node* l, Node* r) {
  if (!l) return r;
  if (!r) return l;
  if (l->y > r->y) {
    l->r = merge(l->r, r);
    l->recalc();
    return l;
  } else {
    r->l = merge(l, r->l);
    r->recalc();
    return r;
  }
}

Node* ins(Node* t, Node* n, int pos) {
  auto pa = split(t, pos);
  return merge(merge(pa.first, n), pa.second);
}

// Example application: move the range [l, r) to index k
void move(Node*& t, int l, int r, int k) {
  Node *a, *b, *c;
  tie(a,b) = split(t, l); tie(b,c) = split(b, r - l);
  if (k <= l) t = merge(ins(a, b, k), c);
  else t = merge(a, ins(c, b, k - r));
}
```

SubMatrix.h

Description: Calculate submatrix sums quickly, given upper-left and lower-right corners (half-open).
Usage: SubMatrix<int> m(matrix);
m.sum(0, 0, 2, 2); // top left 4 elements
Time: $\mathcal{O}\left(N^2 + Q\right)$

13 lines

```
template<class T>
struct SubMatrix {
  vector<vector<T>> p;
  SubMatrix(vector<vector<T>>& v) {
    int R = sz(v), C = sz(v[0]);
    p.assign(R+1, vector<T>(C+1));
    rep(r,0,R) rep(c,0,C)
      p[r+1][c+1] = v[r][c] + p[r][c+1] + p[r+1][c] - p[r][c];
  }
  T sum(int u, int l, int d, int r) {
    return p[d][r] - p[d][l] - p[u][r] + p[u][l];
  }
};
```

SparseTableRMQ.h

Description: Computes minimum of a range in $O(1)$ time.
Time: $\mathcal{O}(1)$.

18 lines

```
// update
int st[K + 1][MAXN];

std::copy(array.begin(), array.end(), st[0]);

for (int i = 1; i <= K; i++)
    for (int j = 0; j + (1 << i) <= N; j++)
        st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i -
             1))]);

// query with [L, R]
int log2_floor(unsigned long long i) {
    return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
}

int minimum(int L, int R) {
    int i = log2_floor(R - L + 1);
    return min(st[i][L], st[i][R - (1 << i) + 1]);
}
```

MoQueries.h

Description: Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change step to add/remove the edge (a, c) and remove the initial add call (but keep in).
Time: $\mathcal{O}\left(N\sqrt{Q}\right)$

49 lines

```
void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer

vi mo(vector<pii> Q) {
  int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s;
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) {
    pii q = Q[qi];
    while (L > q.first) add(--L, 0);
    while (R < q.second) add(R++, 1);
    while (L < q.first) del(L++, 0);
    while (R > q.second) del(--R, 1);
    res[qi] = calc();
  }
```

```
  return res;
}

vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root=0){
  int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
  add(0, 0), in[0] = 1;
  auto dfs = [&](int x, int p, int dep, auto& f) -> void {
    par[x] = p;
    L[x] = N;
    if (dep) I[x] = N++;
    for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
    if (!dep) I[x] = N++;
    R[x] = N;
  };
  dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) rep(end,0,2) {
    int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                  else { add(c, end); in[c] = 1; } a = c; }
    while (!(L[b] <= L[a] && R[a] <= R[b]))
      I[i++] = b, b = par[b];
    while (a != b) step(par[a]);
    while (i--) step(I[i]);
    if (end) res[qi] = calc();
  }
  return res;
}
```

Mathematics (2)

2.1 Matrices

Matrix.h

Description: Basic operations on square matrices.

Usage: `Matrix<int, 3> A;`
`A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};`
`vector<int> vec = {1,2,3};`
`vec = (A^N) * vec;`

26 lines

```
template<class T, int N> struct Matrix {
  typedef Matrix M;
  array<array<T, N>, N> d{};
  M operator*(const M& m) const {
    M a;
    rep(i,0,N) rep(j,0,N)
      rep(k,0,N) a.d[i][j] += d[i][k]*m.d[k][j];
    return a;
  }
  vector<T> operator*(const vector<T>& vec) const {
    vector<T> ret(N);
    rep(i,0,N) rep(j,0,N) ret[i] += d[i][j] * vec[j];
    return ret;
  }
  M operator^(ll p) const {
    assert(p >= 0);
    M a, b(*this);
    rep(i,0,N) a.d[i][i] = 1;
    while (p) {
      if (p&1) a = a*b;
      b = b*b;
      p >>= 1;
    }
    return a;
  }
};
```

GaussianElimination.h

Time: $\mathcal{O}(n^3)$

43 lines

```
const double EPS = 1e-10;
typedef vector<vector<double>> VVD;

// Gauss-Jordan elimination with full pivoting.
// solving systems of linear equations (AX=B)
// INPUT:    a[][] = an n*n matrix
//           b[][] = an n*m matrix
// OUTPUT:   X      = an n*m matrix (stored in b[][])
//           A^{-1} = an n*n matrix (stored in a[][])
// O(n^3)
bool gauss_jordan(VVD& a, VVD& b) {
    const int n = a.size();
    const int m = b[0].size();
    vector<int> irow(n), icol(n), ipiv(n);

    for (int i = 0; i < n; i++) {
        int pj = -1, pk = -1;
        for (int j = 0; j < n; j++) if (!ipiv[j])
            for (int k = 0; k < n; k++) if (!ipiv[k])
                if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])
                    ) { pj = j; pk = k; }
        if (fabs(a[pj][pk]) < EPS) return false; // matrix is
            singular
        ipiv[pk]++;
        swap(a[pj], a[pk]);
        swap(b[pj], b[pk]);
        irow[i] = pj;
        icol[i] = pk;

        double c = 1.0 / a[pk][pk];
        a[pk][pk] = 1.0;
        for (int p = 0; p < n; p++) a[pk][p] *= c;
        for (int p = 0; p < m; p++) b[pk][p] *= c;
        for (int p = 0; p < n; p++) if (p != pk) {
            c = a[p][pk];
            a[p][pk] = 0;
            for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c
                ;
            for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c
                ;
        }
    }
    for (int p = n - 1; p >= 0; p--) if (irow[p] != icol[p]) {
        for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][
            icol[p]]);
    }
    return true;
}
```

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.

Time: $\mathcal{O}(N^3)$

15 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
}
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.

Time: $\mathcal{O}(n^2m)$

38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.

Time: $\mathcal{O}(n^2m)$

34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
```

```
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$ — 35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

RankOfMatrix.h

Description: Search for the rank using Gaussian elimination.
Time: $\mathcal{O}(n^3)$ — 30 lines

```
const double EPS = 1E-9;

int compute_rank(vector<vector<double>> A) {
    int n = A.size();
    int m = A[0].size();

    int rank = 0;
    vector<bool> row_selected(n, false);
    for (int i = 0; i < m; ++i) {
        int j;
        for (j = 0; j < n; ++j) {
            if (!row_selected[j] && abs(A[j][i]) > EPS)
                break;
        }

        if (j != n) {
            ++rank;
            row_selected[j] = true;
            for (int p = i + 1; p < m; ++p)
                A[j][p] /= A[j][i];
            for (int k = 0; k < n; ++k) {
                if (k != j && abs(A[k][i]) > EPS) {
                    for (int p = i + 1; p < m; ++p)
                        A[k][p] -= A[j][p] * A[k][i];
                }
            }
        }
    }
    return rank;
}
```

2.2 FFT, Berlekamp

FastFourierTransform.h

Description: $O(N \log N)$ Polynomial multiplication
Time: $\mathcal{O}(N \log N)$ — 46 lines

```
#define _USE_MATH_DEFINES

#define sz(v) ((int)(v).size())
#define all(v) (v).begin(),(v).end()
typedef complex<double> base;

void fft(vector <base> &a, bool invert)
{
    int n = sz(a);
    for (int i=1,j=0;i<n;i++){
        int bit = n >> 1;
        for (;j>=bit;bit>>=1) j -= bit;
        j += bit;
        if (i < j) swap(a[i],a[j]);
    }
    for (int len=2;len<=n;len<<=1){
        double ang = 2*M_PI/len*(invert?-1:1);
        base wlen(cos(ang),sin(ang));
        for (int i=0;i<n;i+=len){
            base w(1);
            for (int j=0;j<len/2;j++){
                base u = a[i+j], v = a[i+j+len/2]*w;
                a[i+j] = u+v;
                a[i+j+len/2] = u-v;
                w *= wlen;
            }
        }
    }
    if (invert){
        for (int i=0;i<n;i++) a[i] /= n;
    }
}

vector<int> multiply(vector<int>& a, vector<int>& b)
{
    vector<base> fa(all(a)), fb(all(b));
    int n = 1, m = sz(a)+sz(b)-1;
    while (n < m) n <<= 1;
    fa.resize(n); fb.resize(n);
    fft(fa, false); fft(fb, false);
    for (int i=0;i<n;i++) fa[i] *= fb[i];
    fft(fa, true);
    vector<int> ret(m);
    for (int i=0;i<m;i++) ret[i] = fa[i].real()+(fa[i].real()
        >0?0.5:-0.5); // removed casting to int here.. should
        be fine
    return ret;
}
```

NumberTheoreticTransform.h

Description: For NTT, change second loop of above FFT Code as:
Time: $\mathcal{O}(N \log N)$ — 26 lines

```
vector<base> root(n/2);
int ang = modpow(3, (mod - 1) / n);
if(invert) ang = modpow(ang, mod - 2);
root[0] = 1;
for(int i = 1; i<n/2; i++)
    root[i] = (root[i-1]*ang)%mod;
for (int len = 2; len <= n; len <<= 1)
{
    int step = n / len;
    for (int i = 0; i<n; i+= len)
    {
        for (int j = 0; j<len/2; j++)
        {
            base u = a[i+j], v = (a[i+j+len/2]*root[step*j])%
                mod;
            a[i+j] = (u+v)%mod;
            a[i+j+len/2] = (u-v)%mod;
        }
    }
}
if (invert)
{
    for (int i = 0; i<n; i++)
        a[i] = frac(a[i],n);
}
for (int i = 0; i<n; i++)
    a[i] = (a[i]+10*mod)%mod;
```

BerlekampMassey.h

Description: Find linear recurrence when 3n terms are given.
Usage: `guess_nth_term({1, 1, 2, 3, 5, 8}, 10000000);`
Time: $\mathcal{O}(N^2)$ — 116 lines

```
struct Berlekamp_Massey
{
  const int mod = 1000000007;
  using lint = long long;
  lint ipow(lint x, lint p){
    lint ret = 1, piv = x;
    while(p){
      if(p & 1) ret = ret * piv % mod;
      piv = piv * piv % mod;
      p >>= 1;
    }
    return ret;
  }
  vector<int> berlekamp_massey(vector<int> x){
    vector<int> ls, cur;
    int lf, ld;
    for(int i=0; i<x.size(); i++){
      lint t = 0;
```

```
      for(int j=0; j<cur.size(); j++){
        t = (t + 1ll * x[i-j-1] * cur[j]) % mod;
      }
      if((t - x[i]) % mod == 0) continue;
      if(cur.empty()){
        cur.resize(i+1);
        lf = i;
        ld = (t - x[i]) % mod;
        continue;
      }
      lint k = -(x[i] - t) * ipow(ld, mod - 2) % mod;
      vector<int> c(i-lf-1);
      c.push_back(k);
      for(auto &j : ls) c.push_back(-j * k % mod);
      if(c.size() < cur.size()) c.resize(cur.size());
      for(int j=0; j<cur.size(); j++){
        c[j] = (c[j] + cur[j]) % mod;
      }
      if(i-lf+(int)ls.size()>=(int)cur.size()){
        tie(ls, lf, ld) = make_tuple(cur, i, (t - x[i]) % mod);
      }
      cur = c;
    }
    for(auto &i : cur) i = (i % mod + mod) % mod;
    return cur;
  }
  int get_nth(vector<int> rec, vector<int> dp, lint n){
    int m = rec.size();
    vector<int> s(m), t(m);
    s[0] = 1;
    if(m != 1) t[1] = 1;
    else t[0] = rec[0];
    auto mul = [&rec](vector<int> v, vector<int> w){
      int m = v.size();
      vector<int> t(2 * m);
      for(int j=0; j<m; j++){
        for(int k=0; k<m; k++){
          t[j+k] += 1ll * v[j] * w[k] % mod;
          if(t[j+k] >= mod) t[j+k] -= mod;
        }
      }
      for(int j=2*m-1; j>=m; j--){
        for(int k=1; k<=m; k++){
          t[j-k] += 1ll * t[j] * rec[k-1] % mod;
          if(t[j-k] >= mod) t[j-k] -= mod;
        }
      }
      t.resize(m);
      return t;
    };
    while(n){
      if(n & 1) s = mul(s, t);
      t = mul(t, t);
      n >>= 1;
    }
    lint ret = 0;
    for(int i=0; i<m; i++) ret += 1ll * s[i] * dp[i] % mod;
    return ret % mod;
  }
  int guess_nth_term(vector<int> x, lint n){
    if(n < x.size()) return x[n];
    vector<int> v = berlekamp_massey(x);
    if(v.empty()) return 0;
    return get_nth(v, x, n);
  }
  struct elem{int x, y, v;};
  vector<int> get_min_poly(int n, vector<elem> M)
  {
    vector<int> rnd1, rnd2;
    mt19937 rng(0x14004);
    auto randint = [&rng](int lb, int ub){
      return uniform_int_distribution<int>(lb, ub)(rng);
    };
    for(int i=0; i<n; i++){
      rnd1.push_back(randint(1, mod - 1));
      rnd2.push_back(randint(1, mod - 1));
    }
    vector<int> gobs;
    for(int i=0; i<2*n+2; i++){
      int tmp = 0;
      for(int j=0; j<n; j++){
        tmp += 1ll * rnd2[j] * rnd1[j] % mod;
        if(tmp >= mod) tmp -= mod;
      }
      gobs.push_back(tmp);
      vector<int> nxt(n);
      for(auto &i : M){
        nxt[i.x] += 1ll * i.v * rnd1[i.y] % mod;
        if(nxt[i.x] >= mod) nxt[i.x] -= mod;
      }
      rnd1 = nxt;
    }
    auto sol = berlekamp_massey(gobs);
    reverse(sol.begin(), sol.end());
    return sol;
  }
  // Usage : guess_nth_term(first_values, n);
};
```

Number Theory (3)

3.1 Primes

```
< 10^k    prime    # of prime
-----------------------------
1             7             4
2            97            25
3           997           168
4          9973          1229
5         99991          9592
6        999983         78498
7       9999991        664579
8      99999989       5761455
9     999999937      50847534
```

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

3.2 Estimates

$\sum_{d|n} d = O(n \log\log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

3.3 Modular arithmetic

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h

8 lines

```
const int mod = 16769023; // faster if const

int modpow(int b, int e) {
  int ans = 1;
  for (; e; b = b * b % mod, e /= 2)
    if (e & 1) ans = ans * b % mod;
  return ans;
}
```

ModLog.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod m$, or -1 if no such x exists. modLog(a,1,m) can be used to calculate the order of a.

Time: $\mathcal{O}(\sqrt{m})$

11 lines

```
ll modLog(ll a, ll b, ll m) {
  ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<ll, ll> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j;
  if (__gcd(m, e) == __gcd(m, b))
    rep(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.

modsum(to, c, k, m) $= \sum_{i=0}^{\text{to}-1} (ki + c)\%m$. divsum is similar but for floored division.

Time: $\log(m)$, with a large constant.

16 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).

Time: $\mathcal{O}(\log^2 p)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModPow.h" 24 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
```

```
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

3.4 Primality

FastEratosthenes.h

Description: Prime sieve for generating all primes smaller than LIM.
Time: LIM=1e9 $\approx$ 1.5s

20 lines

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
  const int S = (int)round(sqrt(LIM)), R = LIM / 2;
  vi pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)*1.1));
  vector<pii> cp;
  for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
    cp.push_back({i, i * i / 2});
    for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
  }
  for (int L = 1; L <= R; L += S) {
    array<bool, S> block{};
    for (auto &[p, idx] : cp)
      for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] = 1;
    rep(i,0,min(S, R - L))
      if (!block[i]) pr.push_back((L + i) * 2 + 1);
  }
  for (int i : pr) isPrime[i] = 1;
  return pr;
}
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" 18 lines

```
ull pollard(ull n) {
  auto f = [n](ull x) { return modmul(x, x, n) + 1; };
  ull x = 0, y = 0, t = 0, prd = 2, i = 1, q;
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), all(r));
  return l;
}
```

3.5 Divisibility

euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in `__gcd` instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (b) { ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d; }
  return x = 1, y = 0, a;
}
```

CRT.h

Description: Chinese Remainder Theorem.
`crt(a, m, b, n)` computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \mathrm{lcm}(m, n)$. Assumes $mn < 2^{62}$.
Time: $\log(n)$

"euclid.h" 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
}
```

3.5.1 Bézout's identity

For $a \neq$, $b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

3.6 Fractions

ContinuedFractions.h

Description: Given N and a real number $x \ge 0$, finds the closest rational approximation p/q with $p, q \le N$. It will obey $|p/q - x| \le 1/qN$.
For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.
Time: $\mathcal{O}(\log N)$

21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
       a = (ll)floor(y), b = min(a, lim),
       NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that gives us a
      // better approximation; if b = a/2, we *may* have one.
      // Return {P, Q} here for a more canonical approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
        make_pair(NP, NQ) : make_pair(P, Q);
    }
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ};
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
}
```

FracBinarySearch.h

Description: Given f and N, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \le N$.
Usage: `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}`
Time: $\mathcal{O}(\log(N))$

25 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
  if (f(lo)) return lo;
  assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
}
```

3.7 Mobius / Dirichlet

Dirichlet Convolution For $f, g : \mathbb{N} \to \mathbb{C}$, we define

$$(f * g)(n) = \sum_{d|n} f(d)g(n/d) = \sum_{ab=n} f(a)g(b)$$

Set of arithmetic functions form a commutative ring under pointwise addition and Dirichlet convolution, equipped with the multiplicative identity $\epsilon(n) = \chi_{\{1\}}(n)$. In this ring, we have $\mathbf{1} * \mu = \epsilon$ for the Mobius function μ:

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

And also sum of divisor function $\sigma = id * \mathbf{1}$, number of divisors $d = 1 * 1$ gives $id = \sigma * \mu, \mathbf{1} = d * \mu$.

Finally, the Mobius inversion formula

$$g = f * \mathbf{1} \iff f = g * \mu$$

$$g(n) = \sum_{d|n} f(d) \iff f(n) = \sum_{d|n} \mu(d)g(n/d)$$

Xudyh's Sieve (1) For a multiplicative function f ($f(ab) = f(a)f(b)$ when $\gcd(a, b) = 1$), s_f can be computed in linear time (2) There exists a function g such that, s_g and s_{f*g} can be evaluated super fast.

XudyhSieve.h

Description: Sums of mod'ed arithmetic progressions.
Prefix sum of multiplicative functions : p_f : the prefix sum of $f(x)$ (1 <= x <= th). p_g : the prefix sum of $g(x)$ (0 <= x <= N). p_c : the prefix sum of $f * g(x)$ (0 <= x <= N). th : the thereshold, generally should be $n^{(2/3)}$

31 lines

```
struct prefix_mul {

  typedef long long (*func) (long long);

  func p_f, p_g, p_c;
  long long n, th;
  std::unordered_map <long long, long long> mem;

  prefix_mul (func p_f, func p_g, func p_c) : p_f (p_f), p_g (
       p_g), p_c (p_c) {}

  long long calc (long long x) {
    if (x <= th) return p_f (x);
    auto d = mem.find (x);
    if (d != mem.end ()) return d -> second;
    long long ans = 0;
    for (long long i = 2, la; i <= x; i = la + 1) {
      la = x / (x / i);
      ans = ans + (p_g (la) - p_g (i - 1) + mod) * calc (x / i)
           ;
    }
    ans = p_c (x) - ans; ans = ans / inv;
    return mem[x] = ans;
  }

  long long solve (long long n, long long th) {
    if (n <= 0) return 0;
    prefix_mul::n = n; prefix_mul::th = th;
    inv = p_g (1);
    return calc (n);
  }

};
```

Numerical (4)

4.1 Polynomials and recurrences

Polynomial.h

17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$.
Time: $\mathcal{O}\left(n^2\right)$

13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  rep(k,0,n-1) rep(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  rep(k,0,n) rep(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
  return res;
}
```

4.2 Optimization

IntegrateAdaptive.h

Description: Fast integration using an adaptive Simpson's rule.
Usage:
```
double sphereVolume = quad(-1, 1, [](double x) {
return quad(-1, 1, [&](double y) {
return quad(-1, 1, [&](double z) {
return x*x + y*y + z*z < 1; });});});
```

15 lines

```
typedef double d;
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
  d c = (a + b) / 2;
  d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
  if (abs(T - S) <= 15 * eps || b - a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
}
template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
  return rec(f, a, b, eps, S(a, b));
}
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \le b$, $x \ge 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
Usage:
```
vvd A = {{1,-1}, {-1,1}, {-1,-2}};
vd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);
```
Time: $\mathcal{O}\left(NM * \#pivots\right)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}\left(2^n\right)$ in the general case.

68 lines

```
typedef double T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
      rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];}
      rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
    }

  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      rep(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    }
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      rep(i,0,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                     < MP(D[r][n+1] / D[r][s], B[r])) r = i;
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  }

  T solve(vd &x) {
```

```
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      rep(i,0,m) if (B[i] == -1) {
        int s = 0;
        rep(j,1,n+1) ltj(D[i]);
        pivot(i, s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

Combinatorics (5)

5.1 Permutations

Cycles Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

Derangement Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

Burnside's lemma Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

5.2 Partitions and subsets

5.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

5.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k}\binom{n_i}{m_i} \pmod p$.

5.2.3 Binomials

multinomial.h

Description: Computes $\binom{k_1+\cdots+k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \ldots k_n!}$. 6 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
}
```

5.3 Miscellaneous Sequences

5.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).

$B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

5.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

5.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

5.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

5.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

5.3.6 Labeled unrooted trees

\# on n vertices: n^{n-2}
\# on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
\# with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

5.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (6)

6.1 Theorems

Konig's Theorem In any bipartite graph,
|max flow| = |max matching| = |min vertex cover|. Also,
V − |max indep set|.

In any graph without isolated vertices,
|min edge cover| + |max matching| = V

⇒ For bipartite graph of no isolated edge,
|min edge cover| = |max indep set|

Flow complexity When all edges are unit capacities (or more generally for all intermediate vertices, there are one in/out edge of unit capacity), Dinic's algorithm runs in $O(E\min(V^{2/3}, E^{1/2}))$

6.2 Trees

LCA.h

Description: LCA in $O(N\log N + Q\log N)$ 65 lines

```
int n, k;
bool visited[101010];
int par[101010][21], maxedge[101010][21], minedge[101010][21];
int d[101010];
vector <pii> graph[101010]; // {destination, weight}
void dfs(int here, int depth) { // run dfs(root,0)
    visited[here] = true;
    d[here] = depth;
    for (auto there: graph[here]) {
        if (visited[there.first])
```

```
            continue;
        dfs(there.first, depth + 1);
        par[there.first][0] = here;
        maxedge[there.first][0] = there.second;
        minedge[there.first][0] = there.second;
    }
}

void precomputation() {
    for (int i = 1; i < 21; i++) {
        for (int j = 1; j <= n; j++) {
            par[j][i] = par[par[j][i - 1]][i - 1];
            maxedge[j][i] = max(maxedge[j][i - 1],
                                maxedge[par[j][i - 1]][i - 1]);
            minedge[j][i] = min(minedge[j][i - 1],
                                minedge[par[j][i - 1]][i - 1]);
        }
    }
}

pii lca(int x, int y) {
    int maxlen = INT_MIN;
    int minlen = INT_MAX;
    if (d[x] > d[y])
        swap(x, y);
    for (int i = 20; i >= 0; i--) {
        if (d[y] - d[x] >= (1 << i)) {
            minlen = min(minlen, minedge[y][i]);
            maxlen = max(maxlen, maxedge[y][i]);
            y = par[y][i];
        }
    }
    if (x == y)
        return {minlen, maxlen};
    // x is lca point
    for (int i = 20; i >= 0; i--) {
        if (par[x][i] != par[y][i]) {
            minlen = min(minlen, min(minedge[x][i], minedge[y][
                i]));
            maxlen = max(maxlen, max(maxedge[x][i], maxedge[y][
                i]));
            x = par[x][i];
            y = par[y][i];
        }
    }
    minlen = min(minlen, min(minedge[x][0], minedge[y][0]));
    maxlen = max(maxlen, max(maxedge[x][0], maxedge[y][0]));

    int lca_point = par[x][0];
    return {minlen, maxlen};
    // lca_point is lca point
}

void lca_construction() {
    dfs(1, 0);
    precomputation();
}
```

HLD.h

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges.
Time: $\mathcal{O}\left((\log N)^2\right)$

72 lines

```
const int N = 2e5+5;
const int D = 19;
const int S = (1<<D);

int n, q, v[N];
vector<int> adj[N];

int sz[N], p[N], dep[N];
int st[S], id[N], tp[N];

void update(int idx, int val) {
  st[idx += n] = val;
  for (idx /= 2; idx; idx /= 2)
    st[idx] = max(st[2 * idx], st[2 * idx + 1]);
}

int query(int lo, int hi) {
  int ra = 0, rb = 0;
  for (lo += n, hi += n + 1; lo < hi; lo /= 2, hi /= 2) {
    if (lo & 1)
      ra = max(ra, st[lo++]);
    if (hi & 1)
      rb = max(rb, st[--hi]);
  }
  return max(ra, rb);
}

int dfs_sz(int cur, int par) {
  sz[cur] = 1;
  p[cur] = par;
  for(int chi : adj[cur]) {
    if(chi == par) continue;
    dep[chi] = dep[cur] + 1;
    p[chi] = cur;
    sz[cur] += dfs_sz(chi, cur);
  }
  return sz[cur];
}

int ct = 1;

void dfs_hld(int cur, int par, int top) {
  id[cur] = ct++;
  tp[cur] = top;
  update(id[cur], v[cur]);
  int h_chi = -1, h_sz = -1;
  for(int chi : adj[cur]) {
    if(chi == par) continue;
    if(sz[chi] > h_sz) {
      h_sz = sz[chi];
      h_chi = chi;
    }
  }
  if(h_chi == -1) return;
  dfs_hld(h_chi, cur, top);
  for(int chi : adj[cur]) {
    if(chi == par || chi == h_chi) continue;
    dfs_hld(chi, cur, chi);
  }
}

int path(int x, int y){
  int ret = 0;
  while(tp[x] != tp[y]){
    if(dep[tp[x]] < dep[tp[y]])swap(x,y);
    ret = max(ret, query(id[tp[x]],id[x]));
    x = p[tp[x]];
  }
  if(dep[x] > dep[y])swap(x,y);
  ret = max(ret, query(id[x],id[y]));
  return ret;
}
```

LinkCutTree.h

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.
Time: All operations take amortized $\mathcal{O}(\log N)$.

90 lines

```
struct Node { // Splay tree. Root's pp contains tree's parent.
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted)
  }
  void pushFlip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
  }
  int up() { return p ? p->c[1] == this : -1; }
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y : x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      z->c[h ^ 1] = b ? x : this;
    }
    y->c[i ^ 1] = b ? this : x;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
  }
  void splay() {
    for (pushFlip(); p; ) {
      if (p->p) p->p->pushFlip();
      p->pushFlip(); pushFlip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);
    }
  }
  Node* first() {
    pushFlip();
    return c[0] ? c[0]->first() : (splay(), this);
  }
};

struct LinkCut {
  vector<Node> node;
  LinkCut(int N) : node(N) {}

  void link(int u, int v) { // add an edge (u, v)
    assert(!connected(u, v));
    makeRoot(&node[u]);
    node[u].pp = &node[v];
  }
  void cut(int u, int v) { // remove an edge (u, v)
    Node *x = &node[u], *top = &node[v];
    makeRoot(top); x->splay();
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;
      x->fix();
    }
```

```
  }
  bool connected(int u, int v) { // are u, v in the same tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();
  }
  void makeRoot(Node* u) {
    access(u);
    u->splay();
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;
      u->fix();
    }
  }
  Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; }
      pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
  }
};
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 60 lines

```
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
  RollbackUF uf(n);
  vector<Node*> heap(n);
  for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1,-1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  rep(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) {
        Node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      }
    }
    rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
  }

  for (auto& [u,t,comp] : cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  }
  rep(i,0,n) par[i] = in[i].a;
  return {res, par};
}
```

CentroidDecomp.h

Description: JusticeHui implementation of Centroid Decomposition 11 lines

```
int sz[101010]; // subtree size
vector<int> g[101010]; // adj list
int getSize(int v, int b = -1){ // get sz
    sz[v] = 1;
    for(auto i : g[v]) if(i != b) sz[v] += getSize(i, v);
    return sz[v];
}
int getCent(int v, int b = -1, int cap = n){ // find centroid
    for(auto i : g[v]) if(&& i != b && sz[i]*2 > cap) return
        getCent(i, v, cap);
    return v;
}
```

6.3 Strongly Connected Components

SCC.h

Description: Finds sccs in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.
Usage: `build graph only with add_edge. use C.find_scc();`
Time: $\mathcal{O}(E + V)$ 37 lines

```
vector<int> adj[10020];
vector<int> revadj[10020];
vector<vector<int>> scc;
stack<int> s;
int visited[10020];
void DFS(int node) {
  visited[node] = 1;
  for(auto N: adj[node]) {
    if(!visited[N]) {
      DFS(N);
    }
  }
  s.push(node);
}
void revDFS(int node, vector<int>& v) {
  visited[node] = 1;
  for(auto N: revadj[node]) {
    if(!visited[N]) {
      revDFS(N, v);
    }
  }
  v.push_back(node);
}
void findSCC() {
  fill(visited, visited+10002, 0);
  while(!s.empty()) {
    int tmp = s.top();
    s.pop();
    if(visited[tmp]) {
      continue;
    }
    vector<int> v;
    revDFS(tmp, v);
    sort(v.begin(), v.end());
    scc.push_back(v);
  }
}
```

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.
Usage: `int eid = 0; ed.resize(N);`
`for each edge (a,b) {`
`ed[a].emplace_back(b, eid);`
`ed[b].emplace_back(a, eid++); }`
`bicomps([&](const vi& edgelist) {...});`
Time: $\mathcal{O}(E + V)$ 33 lines

```
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F& f) {
  int me = num[at] = ++Time, e, y, top = me;
  for (auto pa : ed[at]) if (pa.second != par) {
    tie(y, e) = pa;
    if (num[y]) {
      top = min(top, num[y]);
      if (num[y] < me)
        st.push_back(e);
    } else {
      int si = sz(st);
      int up = dfs(y, e, f);
      top = min(top, up);
      if (up == me) {
        st.push_back(e);
        f(vi(st.begin() + si, st.end()));
        st.resize(si);
      }
      else if (up < me) st.push_back(e);
      else { /* e is a bridge */ }
    }
  }
  return top;
}

template<class F>
void bicomps(F f) {
  num.assign(sz(ed), 0);
  rep(i,0,sz(ed)) if (!num[i]) dfs(i, -1, f);
}
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a\|\|b)\&\&(!a\|\|c)\&\&(d\|\|!b)\&\&\ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

Usage: `TwoSat ts(number of boolean variables);`
`ts.either(0, ~3); // Var 0 is true or var 3 is false`
`ts.setValue(2); // Var 2 is true`
`ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true`
`ts.solve(); // Returns true iff it is solvable`
`ts.values[0..N-1] holds the assigned values to the vars`
Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.

56 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false, 1 = true

  TwoSat(int n = 0) : N(n), gr(2*n) {}

  int addVar() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }

  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f].push_back(j^1);
    gr[j].push_back(f^1);
  }
  void setValue(int x) { either(x, x); }

  void atMostOne(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i,2,sz(li)) {
      int next = addVar();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }

  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    for(int e : gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = low;
      if (values[x>>1] == -1)
        values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;
  }

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i,0,2*N) if (!comp[i]) dfs(i);
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
};
```

6.4 Network flow

Dinic.h

Description: Maxflow

74 lines

```
struct Edge {
    int u, v;
    int cap, flow;

    Edge() {}

    Edge(int u, int v, int cap) : u(u), v(v), cap(cap), flow(0)
        {}
};

struct Dinic {
    int N;
    vector <Edge> E;
    vector <vector<int>> g;
    vector<int> d, pt;

    Dinic(int N) : N(N), E(0), g(N), d(N), pt(N) {}

    void AddEdge(int u, int v, int cap) {
        if (u != v) {
            E.push_back(Edge(u, v, cap));
            g[u].push_back(E.size() - 1);
            E.push_back(Edge(v, u, 0));
            g[v].push_back(E.size() - 1);
        }
    }

    bool BFS(int S, int T) {
        queue<int> q({S});
        fill(d.begin(), d.end(), N + 1);
        d[S] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            if (u == T) break;
            for (int k: g[u]) {
                Edge &e = E[k];
                if (e.flow < e.cap && d[e.v] > d[e.u] + 1) {
                    d[e.v] = d[e.u] + 1;
                    q.push(e.v);
                }
            }
        }
        return d[T] != N + 1;
    }

    int DFS(int u, int T, int flow = -1) {
        if (u == T || flow == 0) return flow;
        for (int &i = pt[u]; i < g[u].size(); i++) {
            Edge &e = E[g[u][i]];
            Edge &oe = E[g[u][i] ^ 1];
            if (d[e.v] == d[e.u] + 1) {
                int amt = e.cap - e.flow;
                if (flow != -1 && amt > flow) amt = flow;
                if (int pushed = DFS(e.v, T, amt)) {
                    e.flow += pushed;
                    oe.flow -= pushed;
                    return pushed;
                }
            }
        }
        return 0;
    }

    int MaxFlow(int S, int T) {
        int total = 0;
        while (BFS(S, T)) {
            fill(pt.begin(), pt.end(), 0);
            while (int flow = DFS(S, T)) {
                total += flow;
            }
        }
        return total;
    }
};
```

MinCostMaxFlow.h

Description: Min cost max flow
Time: Fast

69 lines

```
struct Edge{
    int u, v, w, cap, flow;
};
const int INF = 0x3f3f3f3f;
struct MCMF {
    int n;
    vector<Edge> edges;
    vector<vector<int>> adj;
    vector<int> par, dist, inque;
    MCMF(int _n) {
        n = _n;
        adj.resize(n+1);
        par.resize(n+1);
        dist.resize(n+1);
        inque.resize(n+1);
    }
    void addedge(int u, int v, int cap, int w) {
        adj[u].push_back(edges.size());
        edges.push_back({u, v, w, cap, 0});
        adj[v].push_back(edges.size());
        edges.push_back({v, u, -w, cap, cap});
    }
    void spfa(int src) {
        fill(par.begin(), par.end(), -1);
        fill(dist.begin(), dist.end(), INF);
        fill(inque.begin(), inque.end(), 0);
        dist[src] = 0;
        queue<int> que({src});
        while(que.size()) {
            int q = que.front();
            que.pop();
            inque[q] = 0;
            for(int i: adj[q]) {
                Edge& e = edges[i];
                if(e.flow < e.cap && dist[e.v] > dist[e.u]+e.w)
                    {
                    dist[e.v] = dist[e.u]+e.w;
                    par[e.v] = i;
                    if(!inque[e.v]) {
                        inque[e.v] = 1;
                        que.push(e.v);
                    }
                }
            }
        }
    }
    pair<int, int> solve(int src, int sink) {
        int mc = 0, mf = 0;
        while(1) {
            spfa(src);
            if(par[sink] == -1)
                return {mc, mf};
            int flow = INF, c = sink;
            while(c != src) {
```

```
            Edge &e = edges[par[c]];
            flow = min(flow, e.cap-e.flow);
            c = e.u;
        }
        c = sink;
        while(c != src) {
            Edge &e = edges[par[c]], &ie = edges[par[c]^1];
            e.flow += flow;
            ie.flow -= flow;
            c = e.u;
        }
        mf += flow;
        mc += dist[sink]*flow;
    }
  }
};
```

L-R MaxFlow (1) MCMF solution. For edge (a→b) with bound $[l, r]$, make two edges as $(l, -1), (r - l, 0)$. MCMF forces flow to remaining -1 edges. COST : is there possible lr-flow. FLOW : actual lr-flow. (2) Dinic solution. Make new SRC and SNK. Make (SNK, SRC, INF) edge. For edge (a→b) with bound $[l, r]$, make $(a \to \text{NEWSNK}, l)$, $(\text{NEWSRC} \to b, l)$, $(a \to b, r - l)$. Is sum of maxflow NEWSRC → NEWSNK equal to sum of l? To find actual flow, run dinic again with original SRC-SNK (without resetting).

EdmondsKarp.h

Description: Flow algorithm with guaranteed complexity $O(VE^2)$. To get edge flow values, compare capacities before and after, and take the positive values only.

35 lines

```
template<class T> T edmondsKarp(vector<unordered_map<int, T>>&
    graph, int source, int sink) {
  assert(source != sink);
  T flow = 0;
  vi par(sz(graph)), q = par;

  for (;;) {
    fill(all(par), -1);
    par[source] = 0;
    int ptr = 1;
    q[0] = source;

    rep(i,0,ptr) {
      int x = q[i];
      for (auto e : graph[x]) {
        if (par[e.first] == -1 && e.second > 0) {
          par[e.first] = x;
          q[ptr++] = e.first;
          if (e.first == sink) goto out;
        }
      }
    }
    return flow;
out:
    T inc = numeric_limits<T>::max();
    for (int y = sink; y != source; y = par[y])
      inc = min(inc, graph[par[y]][y]);

    flow += inc;
    for (int y = sink; y != source; y = par[y]) {
      int p = par[y];
      if ((graph[p][y] -= inc) <= 0) graph[p].erase(y);
      graph[y][p] += inc;
    }
  }
}
```

MinCut.h

Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s, only traversing edges with positive residual capacity.

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph. (Stoer-Wagner) MinCut mc; mc.init(n); for (each edge) mc.addEdge(a,b,weight); mincut = mc.solve(); mc.cut = $\{0, 1\}^n$ describing which side the vertex belongs to.
Time: $\mathcal{O}\left(V^3\right)$

63 lines

```
struct MinCutMatrix {
    int n;
    vector<vector<int>> graph;

    void init(int _n) {
        n = _n;
        graph = vector<vector<int>>(n, vector<int>(n, 0));
    }
    void addEdge(int a, int b, int w) {
        if (a == b) return;
        graph[a][b] += w;
        graph[b][a] += w;
    }

    pair<int, pair<int, int>> stMinCut(vector<int> &active) {
        vector<int> key(n);
        vector<int> v(n);
        int s = -1, t = -1;
        for (int i = 0; i < active.size(); i++) {
            int maxv = -1;
            int cur = -1;
            for (auto j : active) {
                if (v[j] == 0 && maxv < key[j]) {
                    maxv = key[j];
                    cur = j;
                }
            }
            t = s; s = cur;
            v[cur] = 1;
            for (auto j : active) key[j] += graph[cur][j];
        }
        return make_pair(key[s], make_pair(s, t));
    }

    vector<int> cut;

    int solve() {
        int res = numeric_limits<int>::max();
        vector<vector<int>> grps;
        vector<int> active;
        cut.resize(n);
        for (int i = 0; i < n; i++) grps.emplace_back(1, i);
        for (int i = 0; i < n; i++) active.push_back(i);
        while (active.size() >= 2) {
            auto stcut = stMinCut(active);
            if (stcut.first < res) {
                res = stcut.first;
                fill(cut.begin(), cut.end(), 0);
                for (auto v : grps[stcut.second.first]) cut[v]
                     = 1;
            }

            int s = stcut.second.first, t = stcut.second.second
                ;
            if (grps[s].size() < grps[t].size()) swap(s, t);

            active.erase(find(active.begin(), active.end(), t))
                ;
            grps[s].insert(grps[s].end(), grps[t].begin(), grps
                [t].end());
            for (int i = 0; i < n; i++) { graph[i][s] += graph[
                i][t]; graph[i][t] = 0; }
            for (int i = 0; i < n; i++) { graph[s][i] += graph[
                t][i]; graph[t][i] = 0; }
            graph[s][s] = 0;
        }
        return res;
    }
};
```

6.5 Matching

hopcroftKarp.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: vi btoa(m, -1); hopcroftKarp(g, btoa);
Time: $\mathcal{O}\left(\sqrt{V}E\right)$

42 lines

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi& B) {
  if (A[a] != L) return 0;
  A[a] = -1;
  for (int b : g[a]) if (B[b] == L + 1) {
    B[b] = 0;
    if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B))
      return btoa[b] = a, 1;
  }
  return 0;
}

int hopcroftKarp(vector<vi>& g, vi& btoa) {
  int res = 0;
  vi A(g.size()), B(btoa.size()), cur, next;
  for (;;) {
    fill(all(A), 0);
    fill(all(B), 0);
    cur.clear();
    for (int a : btoa) if(a != -1) A[a] = -1;
    rep(a,0,sz(g)) if(A[a] == 0) cur.push_back(a);
    for (int lay = 1;; lay++) {
      bool islast = 0;
      next.clear();
      for (int a : cur) for (int b : g[a]) {
        if (btoa[b] == -1) {
          B[b] = lay;
          islast = 1;
        }
        else if (btoa[b] != a && !B[b]) {
          B[b] = lay;
          next.push_back(btoa[b]);
        }
      }
      if (islast) break;
      if (next.empty()) return res;
      for (int a : next) A[a] = lay;
      cur.swap(next);
    }
    rep(a,0,sz(g))
      res += dfs(a, 0, g, btoa, A, B);
  }
}
```

Hungarian.h

Description: Min-cost matching on bipartite graphs, in $O(n^3)$ time

108 lines

```
int w[N][N];
int match_x[N];
int match_y[N];

int l_x[N], l_y[N];
bool s[N], t[N];
int slack[N];
int slack_x[N];

int tree_x[N];
int tree_y[N];

int hungarian(int n) {
  memset(match_x, -1, sizeof(match_x));
  memset(match_y, -1, sizeof(match_y));
  int ret = 0;

  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
      l_x[i] = max(l_x[i], w[i][j]);
    }
  }
  memset(l_y, 0, sizeof(l_y));

  int m = 0;
  while (m != n) { // repeat at most V times
    memset(tree_x, -1, sizeof(tree_x));
    memset(tree_y, -1, sizeof(tree_y));
    memset(s, 0, sizeof(s));
    memset(t, 0, sizeof(t));

    int s_start;
    for (int i = 0; i < n; ++i) { // O(V)
      if (match_x[i] == -1) {
        s[i] = 1;
        s_start = i;
        break;
      }
    }

    for (int i = 0; i < n; ++i) { // init slack
      slack[i] = l_x[s_start] + l_y[i] - w[s_start][i];
      slack_x[i] = s_start;
    }

    here:
    int y = -1;
    for (int i = 0; i < n; ++i) { // compare: O(V)
      if (slack[i] == 0 && !t[i]) y = i;
    }

    if (y == -1) { // n_l = t
      // update label
      int alpha = INF;
      for (int i = 0; i < n; ++i) { // O(V)
        if (!t[i]) {
          alpha = min(alpha, slack[i]);
        }
      }
      for (int i = 0; i < n; ++i) { // O(V)
        if (s[i]) l_x[i] -= alpha;
        if (t[i]) l_y[i] += alpha;
      }
      for (int i = 0; i < n; ++i) { // O(V)
        if (!t[i]) {
          slack[i] -= alpha;
          if (slack[i] == 0) {
            y = i;
          }
        }
      }
    }

    // n_l != t is guaranteed
    if (match_y[y] == -1) { // free
      tree_y[y] = slack_x[y];
      while (y != -1) {
        int x = tree_y[y];
        match_y[y] = x;
        int next_y = match_x[x];
        match_x[x] = y;
        y = next_y;
      }
      m++;
    }
    else { // matched
      int z = match_y[y];
      tree_x[z] = y;
      tree_y[y] = slack_x[y];
      s[z] = 1;
      t[y] = 1;
          // z is added - update slack and n_l
      for (int i = 0; i < n; ++i) { // O(V)
        if (l_x[z] + l_y[i] - w[z][i] < slack[i]) {
          slack[i] = l_x[z] + l_y[i] - w[z][i];
          slack_x[i] = z;
        }
      }
      goto here;
    }
  }

  for (int i = 0; i < n; ++i) {
    ret += l_x[i];
    ret += l_y[i];
  }
  return ret;
}
```

DFSMatching.h

Description: Simple bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and *btoa* should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. *btoa*[*i*] will be the match for vertex i on the right side, or -1 if it's not matched.

Usage: `vi btoa(m, -1); dfsMatching(g, btoa);`

Time: $\mathcal{O}(VE)$

22 lines

```
bool find(int j, vector<vi>& g, vi& btoa, vi& vis) {
  if (btoa[j] == -1) return 1;
  vis[j] = 1; int di = btoa[j];
  for (int e : g[di])
    if (!vis[e] && find(e, g, btoa, vis)) {
      btoa[e] = di;
      return 1;
    }
  return 0;
}
int dfsMatching(vector<vi>& g, vi& btoa) {
  vi vis;
  rep(i,0,sz(g)) {
    vis.assign(sz(btoa), 0);
    for (int j : g[i])
      if (find(j, g, btoa, vis)) {
        btoa[j] = i;
        break;
      }
  }
  return sz(btoa) - (int)count(all(btoa), -1);
}
```

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"DFSMatching.h" 20 lines

```
vi cover(vector<vi>& g, int n, int m) {
  vi match(m, -1);
  int res = dfsMatching(g, match);
  vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] = false;
  vi q, cover;
  rep(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1) {
      seen[e] = true;
      q.push_back(match[e]);
    }
  }
  rep(i,0,n) if (!lfound[i]) cover.push_back(i);
  rep(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(sz(cover) == res);
  return cover;
}
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost.

Time: $\mathcal{O}(N^2M)$

31 lines

```
pair<int, vi> hungarian(const vector<vi> &a) {
  if (a.empty()) return {0, {}};
  int n = sz(a) + 1, m = sz(a[0]) + 1;
  vi u(n), v(m), p(m), ans(n - 1);
  rep(i,1,n) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vi dist(m, INT_MAX), pre(m, -1);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1, delta = INT_MAX;
      rep(j,1,m) if (!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      }
      rep(j,0,m) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    }
  }
  rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
```

```
}
```

GeneralMatching.h

Description: Matching for general graphs. Fails with probability N/mod.
Time: $\mathcal{O}(N^3)$

"../numerical/MatrixInverse-mod.h" 40 lines

```
vector<pii> generalMatching(int N, vector<pii>& ed) {
  vector<vector<ll>> mat(N, vector<ll>(N)), A;
  for (pii pa : ed) {
    int a = pa.first, b = pa.second, r = rand() % mod;
    mat[a][b] = r, mat[b][a] = (mod - r) % mod;
  }

  int r = matInv(A = mat), M = 2*N - r, fi, fj;
  assert(r % 2 == 0);

  if (M != N) do {
    mat.resize(M, vector<ll>(M));
    rep(i,0,N) {
      mat[i].resize(M);
      rep(j,N,M) {
        int r = rand() % mod;
        mat[i][j] = r, mat[j][i] = (mod - r) % mod;
      }
    }
  } while (matInv(A = mat) != M);

  vi has(M, 1); vector<pii> ret;
  rep(it,0,M/2) {
    rep(i,0,M) if (has[i])
      rep(j,i+1,M) if (A[i][j] && mat[i][j]) {
        fi = i; fj = j; goto done;
    } assert(0); done:
    if (fj < N) ret.emplace_back(fi, fj);
    has[fi] = has[fj] = 0;
    rep(sw,0,2) {
      ll a = modpow(A[fi][fj], mod-2);
      rep(i,0,M) if (has[i] && A[i][fj]) {
        ll b = A[i][fj] * a % mod;
        rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod;
      }
      swap(fi,fj);
    }
  }
  return ret;
}
```

6.6 Heuristics

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.
Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

MaximumClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.
Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

49 lines

```
typedef vector<bitset<200>> vb;
struct Maxclique {
  double limit=0.025, pk=0;
  struct Vertex { int i, d=0; };
  typedef vector<Vertex> vv;
  vb e;
  vv V;
  vector<vi> C;
  vi qmax, q, S, old;
  void init(vv& r) {
    for (auto& v : r) v.d = 0;
    for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
    sort(all(r), [](auto a, auto b) { return a.d > b.d; });
    int mxD = r[0].d;
    rep(i,0,sz(r)) r[i].d = min(i, mxD) + 1;
  }
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (sz(R)) {
      if (sz(q) + R.back().d <= sz(qmax)) return;
      q.push_back(R.back().i);
      vv T;
      for(auto v:R) if (e[R.back().i][v.i]) T.push_back({v.i});
      if (sz(T)) {
        if (S[lev]++ / ++pk < limit) init(T);
        int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
        C[1].clear(), C[2].clear();
        for (auto v : T) {
          int k = 1;
          auto f = [&](int i) { return e[v.i][i]; };
          while (any_of(all(C[k]), f)) k++;
          if (k > mxk) mxk = k, C[mxk + 1].clear();
          if (k < mnk) T[j++].i = v.i;
          C[k].push_back(v.i);
        }
        if (j > 0) T[j - 1].d = 0;
        rep(k,mnk,mxk + 1) for (int i : C[k])
          T[j].i = i, T[j++].d = k;
        expand(T, lev + 1);
      } else if (sz(q) > sz(qmax)) qmax = q;
      q.pop_back(), R.pop_back();
    }
  }
  vi maxClique() { init(V), expand(V); return qmax; }
  Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) {
    rep(i,0,sz(e)) V.push_back({i});
  }
};
```

MaximumIndependentSet.h

Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.

6.7 Other Stuff

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.
Time: $\mathcal{O}(V+E)$

15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
  int n = sz(gr);
  vi D(n), its(n), eu(nedges), ret, s = {src};
  D[src]++; // to allow Euler paths, not just cycles
  while (!s.empty()) {
    int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
    if (it == end){ ret.push_back(x); s.pop_back(); continue; }
    tie(y, e) = gr[x][it++];
    if (!eu[e]) {
      D[x]--, D[y]++;
      eu[e] = 1; s.push_back(y);
    }}
  for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
  return {ret.rbegin(), ret.rend()};
}
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where $m[i][j]$ = inf if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, inf if no path, or -inf if the path goes through a negative-weight cycle.
Time: $\mathcal{O}(N^3)$

12 lines

```
const ll inf = 1LL << 62;
void floydWarshall(vector<vector<ll>>& m) {
  int n = sz(m);
  rep(i,0,n) m[i][i] = min(m[i][i], 0LL);
  rep(k,0,n) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) {
      auto newDist = max(m[i][k] + m[k][j], -inf);
      m[i][j] = min(m[i][j], newDist);
    }
  rep(k,0,n) if (m[k][k] < 0) rep(i,0,n) rep(j,0,n)
    if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
}
```

TopoSort.h

Description: Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than n – nodes reachable from cycles will not be returned.
Time: $\mathcal{O}(|V|+|E|)$

14 lines

```
vi topoSort(const vector<vi>& gr) {
  vi indeg(sz(gr)), ret;
  for (auto& li : gr) for (int x : li) indeg[x]++;
  queue<int> q; // use priority queue for lexic. smallest ans.
  rep(i,0,sz(gr)) if (indeg[i] == 0) q.push(-i);
  while (!q.empty()) {
    int i = -q.front(); // top() for priority queue
    ret.push_back(i);
    q.pop();
    for (int x : gr[i])
      if (--indeg[x] == 0) q.push(-x);
  }
  return ret;
}
```

ArticulationBridge.h

Time: $\mathcal{O}(V+E)$

23 lines

```
const int MX = 100000;
vector<int> adj[MX];
int num[MX], low[MX], parent[MX];
int ind = 1;
vector<pair<int, int>> ans;
int dfs(int u, bool isroot) {
```

```
  num[u] = low[u] = ind++;
  int childcnt = 0;
  for(int a: adj[u]) {
    if(a == parent[u]) continue;
    if(num[a]) {
      low[u] = min(low[u], num[a]);
    } else {
      childcnt++;
      parent[a] = u;
      low[u] = min(low[u], dfs(a, false));
      if(low[a] > num[u])
        ans.emplace_back(min(a,u), max(a,u));
    }

  }
  return low[u];
}
```

ArticulationPoint.h

Time: $\mathcal{O}(V+E)$ 23 lines

```
const int MX = 100000;
vector<int> adj[MX];
int num[MX], low[MX], parent[MX];
bool isarti[MX];
int ind = 1;
int dfs(int u, bool isroot) {
  num[u] = low[u] = ind++;
  int childcnt = 0;
  for(int a: adj[u]) {
    if(a == parent[u]) continue;
    if(num[a]) {
      low[u] = min(low[u], num[a]);
    } else {
      parent[a] = u;
      childcnt++;
      low[u] = min(low[u], dfs(a, false));
      if(low[a] >= num[u])
        isarti[u] = true;
    }
  }
  if(isroot) isarti[u] = (childcnt>=2);
  return low[u];
}
```

dijkstra.h

Description: Computes shortest path from a single source vertex to all of the other vertices in a weighted digraph.
Time: $\mathcal{O}(E \log V)$ 28 lines

```
const int MX = 2e5 + 5;
struct edge {
  int v, w;
  bool operator<(const edge &p) const{
    return w > p.w;
  }
};
vector<edge> adj[MX];
int dist[MX];

void dijkstra(int s) {
    memset(dist, 0x3f, sizeof dist);
  dist[s] = 0;
  priority_queue<edge> pq;
  pq.push({s, 0});
  while(!pq.empty()) {
    auto t = pq.top();
    int v = t.v, w = t.w;
    pq.pop();
    if(w > dist[v]) continue;
    for(auto p: adj[v]) {
      if(dist[p.v] > dist[v]+p.w) {
        dist[p.v] = dist[v]+p.w;
        pq.push({p.v, dist[p.v]});
      }
    }
  }
}
```

Geometry (7)

7.1 Formulae

Triangles

Semiperimeter: $p = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles):
$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc\left[1 - \left(\dfrac{a}{b+c}\right)^2\right]}$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\dfrac{\alpha+\beta}{2}}{\tan\dfrac{\alpha-\beta}{2}}$

Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

Spherical coordinates

$$x = r\sin\theta\cos\phi \qquad r = \sqrt{x^2 + y^2 + z^2}$$

$$y = r\sin\theta\sin\phi \qquad \theta = \text{acos}(z/\sqrt{x^2 + y^2 + z^2})$$

$$z = r\cos\theta \qquad \phi = \text{atan2}(y, x)$$

7.2 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.) 28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
};
```

lineDistance.h

Description: Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

"Point.h" 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description: Returns the shortest distance between point p and the line segment from point s to e.
Usage: `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`

"Point.h" 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h

Description: If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.
Usage: `vector<P> inter = segInter(s1,e1,s2,e2);`
`if (sz(inter)==1)`
`cout << "segments intersect at " << inter[0] << endl;`

"Point.h", "OnSegment.h" 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
```

```
  return {all(s)};
}
```

lineIntersection.h

Description: If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.

Usage: `auto res = lineInter(s1,e1,s2,e2);`
`if (res.first == 1)`
`cout << "intersection point at " << res.second << endl;`

"Point.h" 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 9 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

"Point.h" 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h

Description: Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

"Point.h" 6 lines

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab insted. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h" 5 lines

```
template<class P>
P lineProj(P a, P b, P p, bool refl=false) {
  P v = b - a;
  return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
`int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }`
`// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`

35 lines

```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

7.3 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 11 lines

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" 9 lines

```
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
  P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
  double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
  if (h2 < 0) return {};
  if (h2 == 0) return {p};
  P h = ab.unit() * sqrt(h2);
  return {p - h, p + h};
}
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.

Time: $\mathcal{O}(n)$

"../../content/geometry/Point.h" 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  };
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
}
```

circumcircle.h

Description: The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

"Point.h" 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h" 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
}
```

7.4 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`
Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 11 lines

```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  }
  return cnt;
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" 8 lines

```
// ! Results can be negative. use abs() if needed.
template<class T>
T polygonArea2(vector<Point<T>>& v) {
    T a = v.back().cross(v[0]);
    for (int i = 0; i < v.size()-1; i++)
        a += v[i].cross(v[i+1]);
    return a;
}
```

PolygonCenter.h

Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

"Point.h" 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

PolygonCut.h

Description: Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h", "lineIntersection.h" 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

PolygonUnion.h

Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
Time: $\mathcal{O}(N^2)$, where N is the total number of points

"Point.h", "sideOf.h" 33 lines

```
typedef Point<double> P;
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y; }
double polyUnion(vector<vector<P>>& poly) {
  double ret = 0;
  rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
    P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
    vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
    rep(j,0,sz(poly)) if (i != j) {
      rep(u,0,sz(poly[j])) {
        P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])];
        int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
        if (sc != sd) {
          double sa = C.cross(D, A), sb = C.cross(D, B);
          if (min(sc, sd) < 0)
            segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
        } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))>0){
          segs.emplace_back(rat(C - A, B - A), 1);
          segs.emplace_back(rat(D - A, B - A), -1);
        }
      }
    }
    sort(all(segs));
    for (auto& s : segs) s.first = min(max(s.first, 0.0), 1.0);
    double sum = 0;
    int cnt = segs[0].second;
    rep(j,1,sz(segs)) {
      if (!cnt) sum += segs[j].first - segs[j - 1].first;
      cnt += segs[j].second;
    }
    ret += A.cross(B) * sum;
  }
  return ret / 2;
}
```

ConvexHull.h

Description: Returns a vector of the points of the convex hull in counterclockwise order. Points on the edge of the hull between two other points are not considered part of the hull.
Time: $\mathcal{O}(n \log n)$

"Point.h" 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(all(pts)))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/colinear points).

"Point.h" 12 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    }
  return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no colinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 14 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no colinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1, -1)$ if no collision, • $(i, -1)$ if touching the corner i, • (i, i) if along side $(i, i+1)$, • (i, j) if crossing sides $(i, i+1)$ and $(j, j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i, i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(N + Q \log n)$

"Point.h" 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  }
  return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P> poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

7.5 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
}
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" 63 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;

  T distance(const P& p) { // min squared distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  }

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if width >= height (not ideal...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" 88 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  bool mark; Q o, rot; P p;
  P F() { return r()->p; }
  Q r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
};

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}
Q makeEdge(P orig, P dest) {
  Q q[] = {new Quad{0,0,0,orig}, new Quad{0,0,0,arb},
           new Quad{0,0,0,dest}, new Quad{0,0,0,arb}};
  rep(i,0,4)
    q[i]->o = q[-i & 3], q[i]->rot = q[(i+1) & 3];
  return *q;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
```

```
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
  return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
  q.push_back(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  return pts;
}
```

Strings (8)

KMP.h

32 lines

```
vector<int> getPi(string p) {
  int j = 0, plen = p.length();
  vector<int> pi;
  pi.resize(plen);
  for(int i = 1; i< plen; i++) {
    while((j > 0) && (p[i] != p[j]))
      j = pi[j-1];
    if(p[i] == p[j]) {
      j++;
      pi[i] = j;
    }
  }
  return pi;
}
vector <int> kmp(string s, string p) {
  vector<int> ans;
  auto pi = getPi(p);
  int slen = s.length(), plen = p.length(), j = 0;
  for(int i = 0 ; i < slen ; i++) {
    while(j>0 && s[i] != p[j])
      j = pi[j-1];
    if(s[i] == p[j]) {
      if(j==plen-1) {
        ans.push_back(i-plen+1);
        j = pi[j];
      }
      else
        j++;
    }
  }
  return ans;
}
```

Zfunc.h

Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

Time: $\mathcal{O}(n)$

12 lines

```
vi Z(string S) {
  vi z(sz(S));
  int l = -1, r = -1;
  rep(i,1,sz(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  }
  return z;
}
```

Manacher.h

17 lines

```
int N, A[MAXN];
char S[MAXN];

void Manachers() {
    int r = 0, p = 0;
    for (int i = 1; i <= N; i++) {
        if (i <= r)
            A[i] = min(A[2*p-i],r-i);
        else
            A[i] = 0;
        while (i-A[i]-1 > 0 && i+A[i]+1 <= N
        && S[i-A[i]-1] == S[i+A[i]+1])
            A[i]++;
        if (r < i+A[i])
            r = i+A[i], p = i;
    }
}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.

Usage: `rotate(v.begin(), v.begin()+minRotation(v), v.end());`

Time: $\mathcal{O}(N)$

8 lines

```
int minRotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
    if (s[a+k] > s[b+k]) { a = b; break; }
  }
  return a;
}
```

SuffixArray.h

Description: Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i] = lcp(sa[i], sa[i-1])`, `lcp[0] = 0`. The input string must not contain any zero bytes.

Time: $\mathcal{O}(n \log n)$

38 lines

```
struct SuffixArray {
    vector <int> buildSA(string &s) {
        int n = s.length(), m = max(256, n) + 1;
        vector <int> sa(n), rank(n*2), new_rank(n*2), cnt(m),
            idx(n);
        for (int i = 0; i < n; i++) {
            rank[i] = s[i]; sa[i] = i;
        }
        for (int d = 1; d < n; d = d * 2) {
            auto cmp = [&](int i,int j){
                return rank[i] < rank[j] || (rank[i]==rank[j]
                    && rank[i+d]<rank[j+d]);};
            fill(cnt.begin(), cnt.end(), 0);
            for (int i = 0; i < n; i++) cnt[rank[i+d]]++;
            for (int i = 1; i < m; i++) cnt[i] += cnt[i-1];
            for (int i = n-1; i >= 0; i--) idx[--cnt[rank[i+d
                ]]] = i;
            fill(cnt.begin(), cnt.end(), 0);
            for (int i = 0; i < n; i++) cnt[rank[i]]++;
            for (int i = 1; i < m; i++) cnt[i] += cnt[i-1];
            for (int i = n-1; i >= 0; i--) sa[--cnt[rank[idx[i
                ]]]] = idx[i];
            new_rank[sa[0]] = 1;
            for (int i = 1; i < n; i++) new_rank[sa[i]] =
                new_rank[sa[i-1]] + cmp(sa[i-1], sa[i]);
            rank = new_rank;
            if (rank[sa[n-1]] == n) break;
        }
        return sa;
    }

    vector <int> buildLCP(string &s, vector<int> &sa) {
        int n = s.length();
        vector <int> lcp(n), isa(n);
        for (int i = 0; i < n; i++) isa[sa[i]] = i;
        for (int k = 0, i = 0; i < n; i++) {
            if (!isa[i]) continue;
            for(int j = sa[isa[i]-1]; s[i+k]==s[j+k]; k++);
            lcp[isa[i]] = (k ? k-- : 0);
        }
        return lcp;
    }
};
```

SuffixTree.h

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).

Time: $\mathcal{O}(26N)$

50 lines

```
struct SuffixTree {
  enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
  int toi(char c) { return c - 'a'; }
  string a; // v = cur node, q = cur position
  int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;

  void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
      if (t[v][c]==-1) { t[v][c]=m;  l[m]=i;
        p[m++]=v; v=s[v]; q=r[v];  goto suff; }
      v=t[v][c]; q=l[v];
    }
    if (q==-1 || c==toi(a[q])) q++; else {
      l[m+1]=i;  p[m+1]=m;  l[m]=l[v];  r[m]=q;
      p[m]=p[v];  t[m][c]=m+1;  t[m][toi(a[q])]=v;
      l[v]=q;  p[v]=m;  t[p[m]][toi(a[l[m]])]=m;
      v=s[p[m]];  q=l[m];
      while (q<r[m]) { v=t[v][toi(a[q])];  q+=r[v]-l[v]; }
      if (q==r[m])  s[m]=v;  else s[m]=m+2;
      q=r[v]-(q-r[m]);  m+=2;  goto suff;
    }
  }

  SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
```

```
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
  }

  // example: find longest common substring (uses ALPHA = 28)
  pii best;
  int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
      mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
      best = max(best, {len, r[node] - len});
    return mask;
  }
  static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
  }
};
```

Hashing.h

Description: Rabin-Karp rolling hash, always use multiple modulos and bases when unsure.

18 lines

```
vector <ll> RabinKarp(string &s, ll window, ll mod, ll p) {
    vector <ll> hash_values;
    ll cur = 0;
    ll hash_offset = 1;
    for (int i = 0; i < window; i++) {
        cur *= p;
        cur += s[i];
        cur %= mod;
        if (i > 0) hash_offset = (hash_offset * p)%mod;
    }
    hash_values.push_back(cur);
    for (int i = window; i < s.length(); i++) {
        cur = (mod + cur - (s[i-window] * hash_offset)%mod)%mod
            ;
        cur = (cur * p + s[i]) % mod;
        hash_values.push_back(cur);
    }
    return hash_values;
}
```

AhoCorasick.h

75 lines

```
struct Trie {
    int score = 0, id = 0;
    string prefix;
    Trie *next[26];
    Trie *fail;
    Trie(string s) {
        score = 0;
        id = trie_node_list.size();
        prefix = s;
        memset(next, 0, sizeof(next));
        trie_node_list.push_back(this);
    }
    void insert(string &s, int p) {
        if (p >= s.length())
            return;
        else {
            int cur = s[p] - 'A';
            if (next[cur] == NULL) {
                string nxt_str = prefix;
                nxt_str.push_back(s[p]);
                next[cur] = new Trie(nxt_str);
            }
            next[cur]->insert(s, p+1);
        }
    }
};
Trie* root = new Trie("");

Trie* follow(Trie* node, int ind) {
    Trie* u = node;
    while (u != root && !u->next[ind])
        u = u -> fail;
    if (u -> next[ind])
        u = u -> next[ind];
    return u;
}

void build_failure_links() {
    queue<Trie*> q;
    root->fail = root;
    q.push(root);
    while (!q.empty()) {
        Trie *cur = q.front();
        q.pop();
        for (int i = 0; i < 26; i++) {
            Trie *nxt = cur->next[i];
            if (!nxt) continue;
            if (cur == root) nxt->fail = root;
            else {
                Trie * prev = cur -> fail;
                prev = follow(prev, i);
                nxt->fail = prev;
            }
            q.push(nxt);
        }
    }
}

bool match(string text) {
    Trie* current = root;
    bool result = false;
    for(char _c : text){
        int c = _c - 'a';
        while(current != root && !current->next[c])
            current = current->fail;
        // move if next node is present.
        if(current->next[c])
            current = current->next[c];
        // found output!
        if(current->output){
            result = true;
            break;
        }
    }
}
```

DP Optimization (9)

9.1 Convex Hull Trick

점화식이 다음과 같은 형태를 만족할 때,

$$D(i) = \min_{j<i}(D(j) + B(j) \times A(i))$$

이를 직선 $l_j : y = D(j) + B(j)x$ 들을 삽입한 상황에서, $x = A(i)$를 쿼리하여 최솟값을 찾는 문제로 본다. $O(n^2) \to O(n \log n)$ (Linecontainer, Li-Chao Tree), 추가로 $B(j)$ 가 단조성을 갖는 경우 $O(n)$.

CHT.h

Description: insertion queries come in decreasing v.y/v.x order, queries come in increasing x order.

22 lines

```
const int MEM=50003;
struct cht{
    int s=0,e=0,id[MEM];
    pll f[MEM];
    double cross(int a,int b){
        return 1.0*(f[a].y-f[b].y)/(f[b].x-f[a].x);
    }
    void insert(pll v,int l){
        f[e]=v;
        id[e]=l;
        while(s+1<e && cross(e-2,e-1)>cross(e-1,e)){
            f[e-1]=f[e];
            id[e-1]=id[e];
            --e;
        }
        ++e;
    }
    ll query(ll x){
        while(s+1<e && f[s+1].y-f[s].y <= x*(f[s].x-f[s+1].x))
            ++s;
        return f[s].x * x + f[s].y;
    }
} CHT;
```

LiChaoTree.h

Description: Convex hull trick. Current implementation is for max query. Be especially aware of overflow. Let M be maximum x coordinate, $aM + b$ should be less than LLMAX.
Time: $\mathcal{O}(\log N)$

87 lines

```
struct LiChao
{
  struct Line // Linear function ax + b
  {
    int a, b;
    int eval(int x)
    {
      return a*x + b;
    }
  };
  struct Node // [start, end] has line f
  {
    int left, right;
    int start, end;
    Line f;
  };

  Node new_node(int a, int b)
  {
    return {-1,-1,a,b,{0,-INF}};
    // for min, change -INF to INF
  }

  vector <Node> nodes;

  void init(int min_x, int max_x)
  {
    nodes.push_back(new_node(min_x, max_x));
  }
  void insert(int n, Line new_line)
  {
    int xl = nodes[n].start, xr = nodes[n].end;
    int xm = (xl + xr)/2;
    Line llo, lhi;
    llo = nodes[n].f, lhi = new_line;
    if (llo.eval(xl) >= lhi.eval(xl))
```

```
    swap(llo, lhi);
  if (llo.eval(xr) <= lhi.eval(xr))
  {
    nodes[n].f = lhi;
    // for min, lhi -> llo
    return;
  }
  else if (llo.eval(xm) > lhi.eval(xm))
  {
    nodes[n].f = llo;
    // for min, llo -> lhi
    if (nodes[n].left == -1)
    {
      nodes[n].left = nodes.size();
      nodes.push_back(new_node(xl,xm));
    }
    insert(nodes[n].left, lhi);
    // for min, lhi -> llo
  }
  else
  {
    nodes[n].f = lhi;
    // for min, lhi -> llo
    if (nodes[n].right == -1)
    {
      nodes[n].right = nodes.size();
      nodes.push_back(new_node(xm+1,xr));
    }
    insert(nodes[n].right,llo);
    // for min, llo -> lhi
  }
}
void insert(Line f)
{
  insert(0, f);
}
int get(int n, int q)
{
  // for min, max -> min, -INF -> INF
  if (n == -1) return -INF;
  int xl = nodes[n].start, xr = nodes[n].end;
  int xm = (xl + xr)/2;
  if (q > xm)
    return max(nodes[n].f.eval(q), get(nodes[n].right, q));
  else return max(nodes[n].f.eval(q), get(nodes[n].left, q));
}
int get(int pt)
{
  return get(0, pt);
}
};
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

29 lines

```
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

9.2 Divide and Conquer Optimization

C가 Monge Array, 즉 $C(a,c)+C(b,d) \le C(a,d)+C(b,c)$ 이고,
점화식이

$$D(i,j) = \min_{k<j}(D(i-1,k)+C(k,j))$$

형태를 만족할 때, j값에 따라 최적의 k인 opt_j가 단조증가함을 관찰하여 확인할 후보를 줄인다. $D(k, s \dots e)$ 를 구하기 위해, (1) 중간값 m에 대해 $D(k,m)$ 을 구한다. (2) 각각 재귀적으로 좌우를 호출하되, 봐야 할 j의 범위를 호출과정에서 관리한다.

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i) \le k < hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.
Time: $\mathcal{O}((N + (hi - lo)) \log N)$

18 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

9.3 Monotone Queue Optimization

Deque Trick 수열에서 sliding window 형태의 최솟값을 빠르게 확인해야 할 때, deque에 pair형으로 (값, 인덱스) 를 저장한다. 이때 front가 우리가 확인하고자 하는 구간을 벗어났다면 계속 pop front 하고, 뒤에 삽입할 때 삽입하려는 값보다 deque의 back이 더 크다면 이를 제거하는 형태로 deque를 monotonic하게 유지한다.

DequeTrick.h

Description: Monotone Deque-DP for sliding-window like query.

12 lines

```
dq.push_back({-1, 0});
for (int i = 0; i < n; i++) {
    while (!dq.empty() and dq.front().first < i - d) {
        dq.pop_front();
    }
    dp[i] = max(arr[i], dq.front().second + arr[i]);
    while (!dq.empty() and dq.back().second <= dp[i]) {
        dq.pop_back();
    }
    dq.push_back({i, dp[i]});
    ans = max(ans, dp[i]);
}
```

점화식 D가 $D(i) = \min_{j<i}(D(j)+C(j,i))$ 형태이고, C가 Monge array일 때, $O(n^2)$ DP를 $O(n \log n)$ 으로 줄이는 기법.

가정 : 다음을 만족하는 cross(i,j) 를 찾을 수 있다.

$$\text{cross}(i,j) > k \iff D(i)+C(i,k) < D(j)+C(j,k)$$

Queue Q에 앞으로 계산할 점화식에서 답이 될 수 있는 후보들을 차례로 저장한다. 즉, $D(i...n)$ 이 답이 되는 j들. 따라서, Q의 모든 원소들이
cross$(Q_i, Q_{i+1}) <$ cross(Q_{i+1}, Q_{i+2}) 를 만족하고, cross$(Q_0, Q_1) \ge i$ 를 만족하도록 한다. 이때, $D(i)$ 를 $D(Q_0)+C(Q_0,i)$를 구함으로써 해결.

각 i에 대해, cross$(Q_0, Q_1) \ge i$ 을 만족시키는 것은 Q에서 필요한 만큼 pop 하여 구할 수 있고, 원소를 Q에 삽입할 때 Cross 부등식을 유지하기 위해 CHT에서와 같이 조건이 성립할때까지 맨 뒤 원소를 제거한다. Deque DP랑 비슷한 concept.

큐의 맨 뒤 원소 두개를 x, y라 할 때, cross$(x,y) \ge$ cross(y,i) 이면 y가 최적인 위치가 없으므로 이를 계속 pop한다. Cross 한번은 이분탐색으로 $O(\log n)$ 에 찾을 수 있고, 이를 n번 호출하므로 $O(n \log n)$.

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$

23 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  }
  return is.insert(before, {L,R});
}

void removeInterval(set<pii>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
  if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
  vi S(sz(I)), R;
  iota(all(S), 0);
```

```
  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
  T cur = G.first;
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at]));
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.push_back(mx.second);
  }
  return R;
}
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
Usage: `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`
Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$ 19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q;
  } else {
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  }
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);
  rec(from, to-1, f, g, i, p, q);
  g(i, to, q);
}
```

10.2 Misc. algorithms

BinarySearch.h

Description: Be careful and double check for off by 1 error
Time: $\mathcal{O}(\log n)$ 7 lines

```
while(lo + 1 < hi)
{
    int mid = (lo + hi)/2;
    if (isOK(mid))
        lo = mid;
    else hi = mid;
}
```

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \dots < f(i) \geq \dots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the < marked with (A) to <=, and reverse the loop at (B). To minimize f, change it to >, also at (B).
Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`
Time: $\mathcal{O}(\log(b-a))$ 11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)
    else b = mid+1;
  }
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

LIS.h

Description: Compute indices for the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$ 17 lines

```
template<class I> vi lis(const vector<I>& S) {
  if (S.empty()) return {};
  vi prev(sz(S));
  typedef pair<I, int> p;
  vector<p> res;
  rep(i,0,sz(S)) {
    // change 0 -> i for longest non-decreasing subsequence
    auto it = lower_bound(all(res), p{S[i], 0});
    if (it == res.end()) res.emplace_back(), it = res.end()-1;
    *it = {S[i], i};
    prev[i] = it == res.begin() ? 0 : (it-1)->second;
  }
  int L = sz(res), cur = res.back().second;
  vi ans(L);
  while (L--) ans[L] = cur, cur = prev[cur];
  return ans;
}
```

Bucket.h

Description: Be careful and double check for off by 1 error 45 lines

```
struct bset {
    // bitset is a size-64 bucket
    bitset<64> arr[1577];
    bset() {}
    // single-query
    inline int bitquery(int idx) {
        return arr[idx >> 6][idx&63]?1:0;
    }
    // single-operation
    inline void bitflip(int idx) {
        arr[idx >> 6][idx & 63].flip();
    }
    void flip(int l, int r) {
        if (r - l < 200) {
            for (int i = l; i <= r; i++)
                bitflip(i);
            return;
        }
        int u = l >> 6;
        int v = r >> 6;
        for (int i = u+1; i < v; i++) {
            // bucket-operation
            arr[i].flip();
        }
        for (int i = l; i < ((u+1) << 6); i++) bitflip(i);
        for (int i = (v << 6); i <= r; i++) bitflip(i);
    }
    int query(int l, int r) {
        int ans = 0;
        if (r - l < 200) {
            for (int i = l; i <= r; i++)
                ans += bitquery(i);
            return ans;
        }
        int u = l >> 6;
        int v = r >> 6;
        for (int i = u+1; i < v; i++) {
            // bucket-query
            ans += arr[i].count();
        }
        for (int i = l; i < ((u+1) << 6); i++) ans += bitquery(
            i);
        for (int i = (v << 6); i <= r; i++) ans += bitquery(i);
        return ans;
    }
};
```

Checkpoints (11)

11.1 Debugging

- $10^5 * 10^5 \Rightarrow$ OVERFLOW. 특히 for 문 안에서 `i * i < n` 할때 조심하기.
- If unsure with overflow, use
  `#define int long long` and stop caring.
- 행렬과 기하의 i, j 인덱스 조심. 헷갈리면 쓰면서 가기.
- 행렬에서는 (r, c), 기하에서는 (x, y) 로 문제를 표현하는 것이 많은 도움이 된다.
- Segment Tree, Trie, Fenwick 등 Struct 구현체 사용할 때는 항상 내부의 n 이 제대로 초기화되었는지 확인하기.
- Testcase가 여러 개인 문제는 항상 초기화 문제를 확인하기. 입력을 다 받지 않았는데 break나 return으로 끊어버리면 안됨.
- iterator 주의 : .end() 는 항상 맨 끝 원소보다 하나 더 뒤의 iterator. erase쓸 때는 iterator++ 관련된 문제들에 주의.
- std::sort must compare with Strict weak ordering (Codejam 2020 1A-A)
- Memory Limit : Local variable은 int 10만개 정도까지만 사용. Global Variable의 경우 128MB면 대략 int 2000만 개까지는 잘 들어간다. long long은 절반. stack, queue, map, set 같은 특이한 컨테이너는 100만개를 잡으면 메모리가 버겁지만 vector 100만개는 잡아도 된다.
- Array out of Bound : 배열의 길이는 충분한가? Vector resize를 했다면 그것도 충분할까? 배열의 -1번에 접근한 적은 없는게 확실할까?
- Binary Search : 제대로 짠 게 맞을까? 1 차이 날 때 / lo == hi 일 때 등등. Infinite loop 주의하기.
- Graph : 반례 유의하기. Connected라는 말이 없으면 Disconnected. Acyclic 하다는 말이 없으면 Cycle 넣기, 특히 $A \leftrightarrow B$ 그래프로 2개짜리 사이클 생각하기.

11.2 Thinking

- 모든 경우를 다 할 수 없나? 왜 안 되지? 시간 복잡도 잘 생각해 보기. 정해의 Target Complexity를 먼저 생각하고 주요 알고리즘들의 Complexity로 짜맞추기.
  예를들어, 쿼리가 30만개 들어온다면 한 쿼리를 적어도 $\log n$ 에 처리할 방법이 아무튼 있다는 뜻.
- 보다 쉬운 문제를 풀기. "N이 얼마 정도로 작다면..."
- 보다 특수한 문제를 풀기. "만약 쿼리가 정렬되어 있다면...", "만약 주어진 그래프가 트리 형태라면..."
- STRANGE THINGS ARE IMPORTANT
- 그 방법이 뭐지? xxxx한 일을 어떤 시간복잡도에 실행하는 적절한 자료구조가 있다면?
  - 필요한 게 정렬성이라면 힙이나 map을 쓸 수 있고
  - multiset / multimap도 사용할 수 있고.. 느리지만.
- 단조함수이며, 충분히 빠르게 검증가능한가 : Binary Search.
- 차원이 높은 문제 : 차원 내려서 생각하기. 3 → 2. 2 → 1. 2019 Codejam R1B-1 Manhattaen Crepe Cart
- 이 문제가 사실 그래프 관련 문제는 아닐까?
  - 만약 그렇다면, '간선' 과 '정점' 은 각각..?
  - 간선과 정점이 몇 개 정도 있는가?
- 이 문제에 Overlapping Subproblem이 보이나?
  → Dynamic Programming 을 적용.
- Directed Graph, 특히 Cycle에 관한 문제 : Topological Sorting? (ex : SNUPC 2019 kdh9949)
- 일반적인 directed graph를 다루기는 상당히 까다롭다. 항상 SCC + DAG를 생각하기.
- 답의 상한이 Reasonable 하게 작은가?
- output이 특정 수열/OX 형태 : 작은 예제를 Exhasutive Search. 모르는 무언가를 알기 위해서는 데이터가 필요하다.
- 그래프 문제에서, 어떤 "조건" 이 들어갔을 때 → 이 문제를 "정점을 늘림으로써" 단순한 그래프 문제로 바꿀 수 있나? (ex : SNUPC 2018 달빛 여우) 이를테면, 홀짝성에 따라 점을 2배로 늘림으로써?
- DP도 마찬가지. 어떤 조건을 단순화하기 위해 상태의 수를 사이사이에 집어넣을 수 있나? (ex : SNUPC 2018 실버런)
- DP State를 어떻게 나타낼 것인가? 첫 i개만을 이용한 답을 알면 $i+1$개째가 들어왔을 때 빠르게 처리할 수 있을까?
- 더 큰 table에서 시작해서 줄여가기. 특히 Memory가 모자라다면 Toggling으로 차원 하나 내릴 수 있는 경우도 상당히 많이 있다. 각 칸의 갱신 시간과 칸의 개수 찾기.
- Square root Decomposition : $O(n \log n)$ 이 생각나면 좋을 것 같지만 잘 생각나지 않고, 제한을 보니 $O(n\sqrt{n})$ 이면 될것도 같이 생겼을 때 생각해 보기. $O(\sqrt{n})$ 버킷 테크닉. Red Army 2020 : Queue
- 복잡도가 맞는데 왜인지 안 뚫리면 : 필요없는 long long을 사용하지 않았나? map이나 set iterator 들을 보면서 상수 커팅. 간단한 함수들을 inlining. 재귀를 반복문으로. Set과 Map.
- 마지막 생각 : 조금 추하지만 해싱이나 Random 또는 `bitset` 을 이용한 $n^2/64$ 같은걸로 뚫을 수 있나? 컴파일러를 믿고 10^8의 몇 배 정도까지는 내 봐도 될 수도. 의외로 Naive한 문제가 많다. Atcoder 158 Divisible Substring