Price of Anarchy

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Introduction

Background

During the current COVID-19 pandemic the most prevalent means of reaction around the world was to impose social distancing. This has been done by means of rules requiring the use of masks, prohibiting large cultural and sports events, various restrictions in public places as well as working places and, in quite a few places, this even got as far as a total shut down and curfew.

Motivation

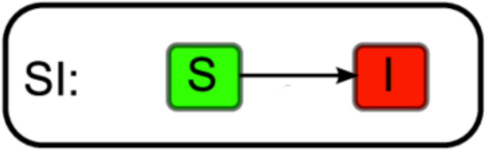
During a pandemic is it better to impose some level of protection or let people choose their own level strategically

- We compare the outcome in both cases in order to determine whether self-imposed distancing performs on par as the government imposed one.
- we will find the strategically level of protection using tool from game theory

SI Model

Main Properties

- The model takes place in periods 1, 2, ...
- Population is viewed as the continuum [0, 1]
- A contagious person infects a random number susceptible people each period with (mean) $\beta > 0$, called the passage rate
- $S + I = 1$



Model Dynamics

Basic SI model dynamics at time t:

$$S_t = S_{t-1} - \beta S_{t-1} I_{t-1}$$

$$I_t = I_{t-1} + \beta S_{t-1} I_{t-1}$$

Multi Group SI:
for each population type p from the group of all populations P

$$S_t^p = S_{t-1}^p - \beta \sum_{j \in P} a_{ij} S_{t-1}^p I_{t-1}^j$$

$$I_t^p = I_{t-1}^p + \beta \sum_{j \in P} a_{ij} S_{t-1}^p I_{t-1}^j$$

where A is the adjacency matrix, a_{ij} is the value of the matrix in ij describes the average number of people of type j meets a person of type i.

Model Dynamics

SI with Government imposing level of defence d:
for each population type p from the group of all populations P

$$S_t^p = S_{t-1}^p - \beta d^2 \sum_{j \in P} a_{ij} S_{t-1}^p I_{t-1}^j$$

$$I_t^p = I_{t-1}^p + \beta d^2 \sum_{j \in P} d_j a_{ij} S_{t-1}^p I_{t-1}^j$$

SI with level of defence d_p for each group p:

$$S_t^p = S_{t-1}^p - \beta d^p \sum_{j \in P} d^j a_{ij} S_{t-1}^p I_{t-1}^p$$

$$I_t^p = I_{t-1}^p + \beta d^p \sum_{j \in P} d^j a_{ij} S_{t-1}^p I_{t-1}^p$$

Contagion game

- every person in the population is a player
- every player is in a homogeneous group, 2 player from the same group are identical.
- each player have a risk factor for being infected $L^p > 0$
- each player can choose some defence filter on the continuum [0, 1].
- there is a cost for defence f(d): $[0, 1] \rightarrow [\infty, 0]$
- if player p choose a stronger defence filter the risk of being infected decrease an so the cost, on the other hand the cost of defence increase.
- if a player that have interactions with player p choose a stronger defence filter his probability of infection and cost decrease without the other hand increase
- each player calculate his own cost using his chance to be infected up to some time T (approximate time of vaccination)

Contagion game

All the equations are from the point of view of player p:
Cost of infection

$$C_L^p = L^p \frac{1}{{S_t^p}^2}$$

Cost of Protection

$$C_D^p = \frac{1}{{d^p}^{\frac{1}{\gamma}}} + \frac{1}{\gamma} d^p - \frac{1}{\gamma} - 1$$

where γ is the filter elasticity factor - proportionate fall in passage rate from vigilance.
$\forall d < 1 \; if \; \gamma < 0 \rightarrow \frac{\partial C_d^p}{\partial d^p} < 0$. $\gamma \approx \frac{1}{8}$ - Hall and Jones (2007)
Final Cost Function

$$C^p = C_L^p + C_D^p$$

Contagion game equilibrium

The Contagion game is a game of choosing a defence level (d) and we are looking for a nash equilibrium. the following equations describes the nash equilibrium. Each player choose his protection rate by solving this equation:

$$\frac{\partial C^p}{\partial d^p} = -2L\frac{\partial S_t^p}{\partial d^p}\frac{1}{{S_T^p}^3} - \frac{\frac{1}{\gamma}}{{d^p}^{\frac{1}{\gamma}+1}} + \frac{1}{\gamma} = 0$$

With the help of this equation

$$\frac{\partial S_t^p}{\partial d^p} = \frac{\partial S_{t-1}^p}{\partial d^p} - \beta d^j a_{ij}\sum_{j\in P} S_{t-1}^p I_{t-1}^j - \alpha_{ij} d^p (\frac{\partial S_{t-1}^p}{\partial d^p} I_{t-1}^j + S_{t-1}^p \frac{\partial I_{t-1}^j}{\partial d^i})$$

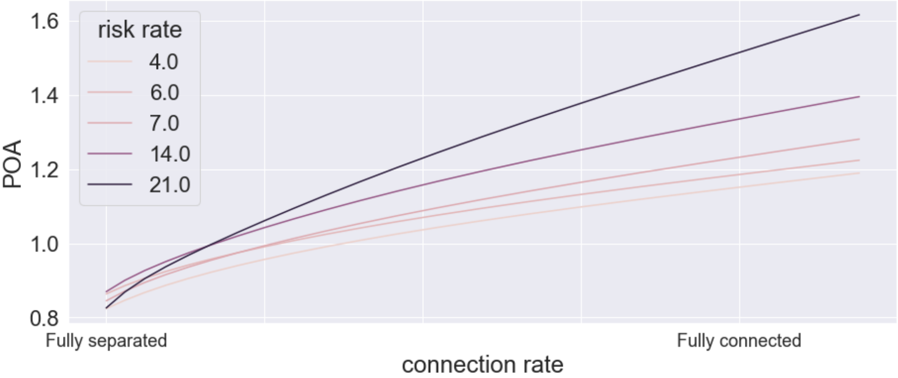
where $i \neq j \rightarrow \alpha_{ij} = 1$. $i = j \rightarrow \alpha_{i,j} = 2$
This equation can be solved with simulation with similar equations for d_j for each $j \in P$ and similarly for I. It is possible because $I_0^i = I_0$, $S_0^i = 1 - I_0^i$ for all $i \in P$, I_0 is exogenic.

Government or equilibrium

- In the situation that government impose some defence level they can only impose one level for all the population.
- For a single homogeneous population there is no difference between Government or equilibrium (both minimize the same equations).
- Previous point implies that if 2 homogeneous populations are fully separated (meaning $a_{11} = a_{22} = 1$ and $a_{12} = a_{21} = 0$ where $a_{ij} \in A$ describe the interactions between i and j) then policy of giving all the information to the public and let them choose the level of defence strategically is better then government decision.
- On the other hand if the population are fully connected (meaning $a_{11} = a_{12} = a_{12} = a_{21} = 0.5$) it is likely that government decision is better than letting players decide on their own
- What is happening between the two?

results

The Price of Anarchy (PoA) is the ratio between the worst case of individual behavior and the case of government containment policy.



Discussion

- As we can see from the graph there is a clear trend and correlation between the advantage of government decision and the connectivity of the network, implying that if the network is less connected between types and more within types then government interference may have negative effect.
- Another observation is regards the ratio between groups in the cost of getting sick (L). as the ratio get higher the effect described above gets stronger. we can interpret this as the increase in 'free riding' of the less vulnerable population (young) on the expanse of others (old) as the ration increase

SIR Model

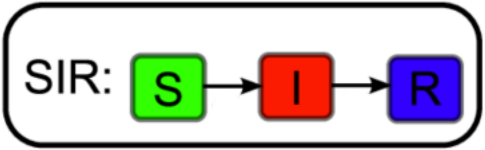
Basic Model Dynamics - SIR

Basic SI model dynamics at time t:

$$S_t = S_{t-1} - \beta S_{t-1} I_{t-1}$$
$$I_t = I_{t-1} + \beta S_{t-1} I_{t-1} - r I_{t-1}$$
$$R_t = R_{t-1} + r I_{t-1}$$

where r is the recovery rate. 'recovers' (or dies / is removed from the infected pool)

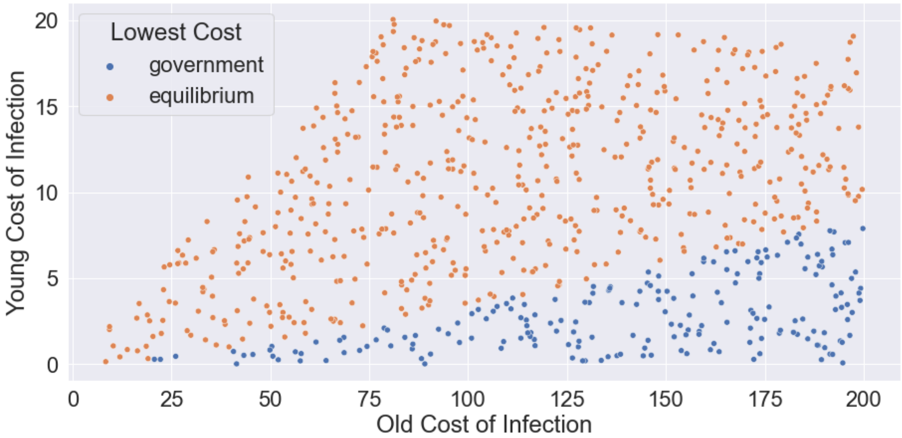
- In SIR $S + I + R = 1$, knowing that we can safely ignore R since it does not impact dynamics
- All subsequent versions of the model is similar to SI with the addition of the recovery rate



Government or equilibrium

- In the situation that government impose some defence level they can only impose one level for all the population.
- For a single homogeneous population there is no difference between Government or equilibrium (both minimize the same equations).
- We generated different defence levels (L values) for each population
- For each random generated defence levels we compare the cost in government imposed defence and strategically chosen defence
- Interaction data is taken from K Prem, AR Cook, M Jit 2017

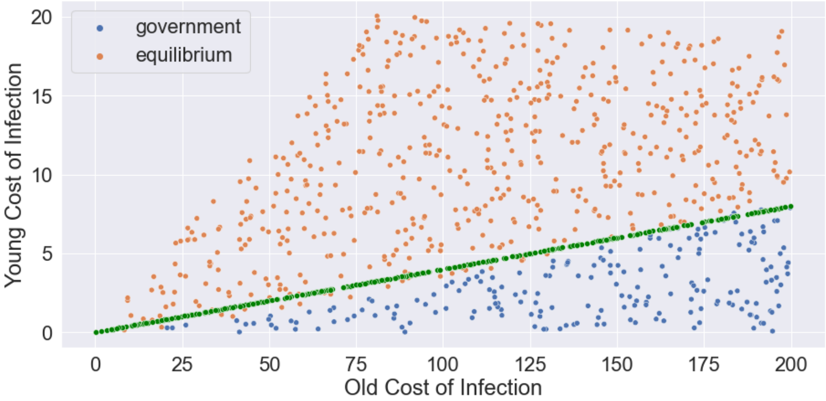
results


Lowest Cost
government
equilibrium
Young Cost of Infection
Old Cost of Infection
0
5
10
15
20
0
25
50
75
100
125
150
175
200

Discussion

As we can see from the graph the results are similar to the SI simulation such that when the rate of protection between the two population is not too big giving players to choose their own protection level leads to better policy.

we can see that the SS risk ratio is ≈ 25



Summary

Summary

- The higher the difference in cost of infection between the groups in the population the higher is the incentive to 'free ride'
- Player in our game take into account only their own risk, furthermore individuals act selfishly to maximize their own welfare
- As we saw in our simplified game there are cases when it is better to give as match information to the population and let then decide on there own.

Although more research is needed we have shown in this work that the option of letting the population decide on there own should not be ignored.

Thank you!