

Preliminary test for IST 718 Advanced Information Analytics

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This test is intended to check whether you have the necessary background to take **IST 718: Advanced Information Analytics**. If you cannot complete these questions in a reasonable amount of time (e.g., 1 hour), you need to seriously consider whether this is the right course for you or put significant extra effort to fulfill the pre-requisites. I do not have the key for this test.

1 Linear algebra

1.1 Systems of equations

Two trains on the same track move on a collision course. At time 0, one train starts at position 0 and moves east at speed v_1 . At time 0, another train starts L miles to the east and moves west at speed v_2 .

- **Question 1:** In a cartesian plane, draw the lines representing the two trajectories using the x axis as the time and the y axis as the train location.
- **Question 2:** Write the equations of both lines
- **Question 3:** What does the slope of the equations mean in terms of the problem?
- **Question 4:** Find the time of collision of both trains by solving for x .
- **Question 5:** Find the point of collision by solving for y .

1.2 Matrix algebra

Consider the following matrices:

$$X = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ -1 & -2 & 3 \\ 3 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{2} \\ -1 \\ 2 \end{bmatrix} \quad Y = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -2 \end{bmatrix}$$

- **Question 1:** What are the matrix dimensions of X , B , and Y ?
- **Question 2:** Which of the following matrix multiplications are you allowed to do: 1) XB , 2) BX , 3) XY , 4) YX , 5) BY , 6) YB
- **Question 3:** Show that $(XB)^T$ equals $B^T X^T$, where T is the matrix transposition operator
- **Question 4:** Compute the value $(XB - Y)^T(XB - Y)$

2 Programming

You should be able to answer the following questions even though the code is written in Python, which is highly legible.

- **Question 1:** What is the result of calling `f(5)` after defining the following function?:

```
def f(n):  
    if n <= 1:  
        return 1  
    return f(n-1) + f(n-2)
```

- **Question 2:** What will the execution of the following code print?:

```
x = 5  
y = 6  
print(x)  
print(y)  
def g():  
    global y  
    x = 6  
    x = x - 1  
    y = y - 1  
    print(x)  
    print(y)  
g()  
print(x)  
print(y)
```

- **Question 3:** What does the following for-loop print?

```
def m(x):  
    return x < 3  
  
for i in [1, 2, 3, 4, 5]:  
    if not m(i):  
        print(i)
```

- **Question 4:** What does calling `z(h, [1, 2, 3])` print out?:

```
def h(x):  
    return 2*x  
  
def z(f, L):  
    for i in L:  
        print(f(i))
```

3 Calculus

Given the following functions

$$f(\mu) = (\mu - x)^2$$

$$g(\mu) = \alpha(\mu - x)^2$$

$$h(\mu, \sigma) = \frac{f(\mu)}{\sigma^2}$$

and where α is a constant.

- **Question 1:** What is the derivative of f with respect to μ
- **Question 2:** Show that the value of μ that makes

$$\frac{df(\mu)}{d\mu} == 0$$

and

$$\frac{dg(\mu)}{d\mu} == 0$$

is the same.

- **Question 3:** Calculate $dh/d\mu$ and $dh/d\sigma$

4 Probability

Consider the sample space $S = \{1, 2, 3, 4\}$ and the random variable X with a discrete probability function

$$p(X = 1) = 0.5 \quad p(X = 2) = 0.1 \quad p(X = 3) = 0.3 \quad p(X = 4) = 0.2$$

- **Question 1:** What is the probability that X is not 1?
- **Question 2:** What is the probability that X is either 3 or 4?
- **Question 3 (optional):** Using the following conditional probability definition $P(X|Y) = P(X, Y)/P(Y)$, what is the probability that X is 1 given that you know that X is less than 4?

5 Statistics

Consider the following data $x = \{1, 2, 3, 4, 5\}$ and $y = \{-1, -2, 0, -4, -5\}$

- **Question 1:** What are the mean, median, and standard deviation of the data x ?
- **Question 2:** What is the covariance between x and y ?