Week 2: The Rocket Equation - Solutions

The Rocket Equation Formula Summary

$\Delta v = I_{sp} g \ln(\frac{m_1}{m_2})$ $\frac{m_1}{m_2} = \exp\left(\frac{\Delta v}{I_{sp} g}\right) = e^{\left(\frac{\Delta v}{I_{sp} g}\right)}$

1. Δv is the total change in velocity that has been made over a burn. Said aloud, this is “Delta V”.
2. I_{sp} is the Specific Impulse of the engine
3. $g = \frac{9.8m}{s^2}$ is the acceleration due to gravity
4. m_1 is the mass that the rocket begins the burn with
5. m_2 is the mass that the rocket ends the burn with

Note from Mr. Wiese: I_{sp} depends on where it is being measured due to differing gravities. However, a constant value would be $I_{sp} \cdot g_{earth}$. If you wanted to find the I_{sp} of a differing planet, you could just divide $I_{sp \to earth} \cdot g_{earth}$ by the rate of gravity on the new planet.

Problem Set Level 1

1.1 - What are the units of I_{sp}? $\Delta v = I_{sp} g \ln(\frac{m_1}{m_2})$

In terms of units:

1. $\Delta v \to \frac{m}{s}$
2. $g \to \frac{m}{s^2}$
3. $\ln(\frac{m_1}{m_2}) \to (1)$

We can simplify the equation to units: $\frac{m}{s} = I_{sp} \frac{m}{s^2}$ $s = I_{sp}$

Note from Mr. Tappe: You can think of I_{sp} as the amount of time that a rocket (forgetting about all masses except for the mass of the fuel) can just hover (have neutral vertical force with gravity.

The units of I_{sp} are in seconds because it is measuring the amount of time that something can happen.

1.2 - TWR of a rocket with $90,000kg$ and a thrust of $1,500,00N$
$\frac{1{,}500{,}000N}{90{,}000kg} = 16.\bar{6}$

In order for a rocket to launch, the upwards vertical force (thrust) needs to be greater than the downwards vertical force (gravity · mass).

Therefore, the Thrust-to-Weight ratio has to be bigger than one:

$$\frac{T}{W} > 1$$

This rocket will launch.

1.3 How much do you accelerate with an $I_{sp} = 200s$, $m_1 = 10,000kg$, and $m_2 = 500kg$? $\Delta v = I_{sp} g \ln(\frac{m_1}{m_2})$ $\Delta v = 200s \frac{9.8m}{s^2} \ln(\frac{10{,}000kg}{500kg})$ $\Delta v = 200 \frac{9.8m}{s} \ln(20)$

$\ln(20) = 2.9957322736 \approx 3$

$\Delta v = \frac{600 \cdot 9.8m}{s} = 5,880 \frac{m}{s}$

1.4 How much fuel does your friend need to burn if they have a ship with $I_{sp} = 300s$, $m_1 = 10,000kg$, trying to reach $\Delta v = 5,880$? $\Delta 5,880 \frac{m}{s} = 300s \cdot \frac{9.8m}{s^2} \ln(\frac{10{,}000kg}{m_2})$

$\frac{10{,}000kg}{m_2} = \exp\left(\frac{\Delta 5{,}880 \frac{m}{s}}{300s \cdot \frac{9.8m}{s^2}}\right) = \exp\left(\frac{\Delta 5{,}880 \frac{m}{s}}{2{,}940 \frac{m}{s}}\right) = \exp(2) \approx 7.389$

$m_2 = \frac{10{,}000kg}{7.389} \approx 1,353.3631073217$

$\Delta m = 10,000kg - 1,353.363kg \approx 8,646.637kg$

1.5 Trash Disposal from Earth to the Sun $\Delta v = I_{sp} g \ln(\frac{m_1}{m_2})$ $\Delta 30 \frac{km}{s} = 542s \cdot \frac{9.8m}{s^2} \ln(\frac{m_1}{m_2})$ $\frac{m_1}{m_2} = \exp\left(\frac{\Delta 30 \frac{km}{s}}{542s \cdot \frac{9.8m}{s^2}}\right)$ $\frac{m_1}{m_2} = \exp\left(\frac{\Delta 30 \frac{km}{s}}{5.312 \frac{km}{s}}\right)$

$m_2 = 1t$

$\frac{m_1}{m_2} = \exp(5.647) \approx 283.61$ $m_1 = 283.61t$