# Robustness of quantum algorithms against coherent control errors

University of Stuttgart
Cluster of Excellence in Data-integrated Simulation Science

Julian Berberich[1], Daniel Fink[2], and Christian Holm[2]
[1]Institute for Systems Theory and Automatic Control
[2]Institute for Computational Physics

Daniel Fink

## Motivation

Consider an **ideal quantum circuit** $|\hat{\psi}\rangle = \hat{U}_1 \cdots \hat{U}_N |\psi_0\rangle$ with $\hat{U}_i = e^{-iH_i}$,

$$|\psi_0\rangle - \boxed{\hat{U}_N} - \cdots - \boxed{\hat{U}_1} - |\hat{\psi}\rangle$$

For **coherent control errors**, we replace $\hat{U}_i = e^{-iH_i}$ by $U_i(\varepsilon_i) = e^{-i(1+\varepsilon_i)H_i}$ with unknown noise $\varepsilon_i \in \mathbb{R}$, leading to the **noisy circuit**

$$|\psi_0\rangle - \boxed{U_N(\varepsilon_N)} - \cdots - \boxed{U_1(\varepsilon_1)} - |\psi(\varepsilon)\rangle$$

- **Origin:** imprecise classical control, e.g., due to miscalibration or imperfect actuation
- Coherent control errors are a **crucial source of errors** on current quantum hardware [1, 2, 3]
- There exist different approaches to deal with such errors, but a general theoretical understanding is missing

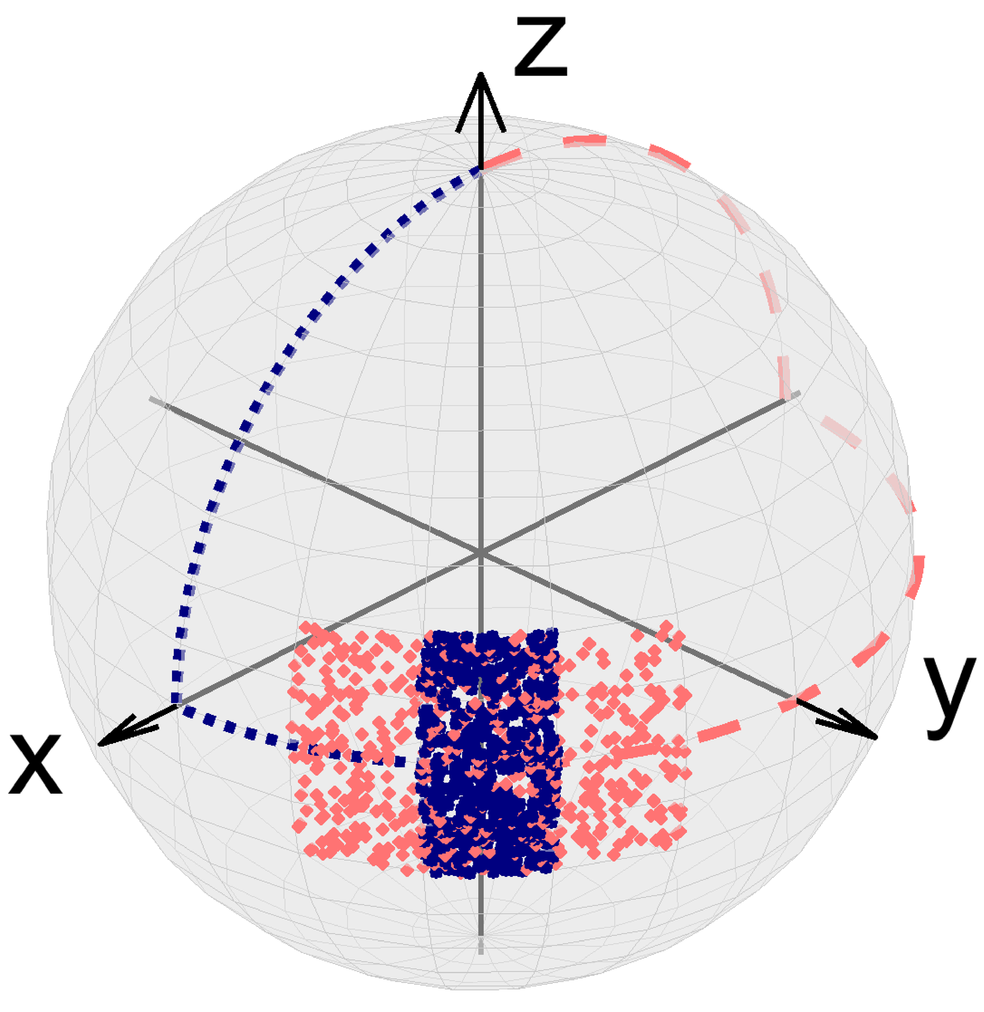
## Illustrative example

Comparison of two quantum circuits:

$$\hat{U} = R_z\left(\frac{\pi}{4}\right) R_y\left(\frac{\pi}{2}\right) \quad \text{vs.} \quad \hat{U}' = R_z\left(-\frac{3\pi}{4}\right) R_y\left(-\frac{\pi}{2}\right)$$

Both produce the same final state $|\hat{\psi}\rangle$, but $\hat{U}$ is more robust against coherent control errors than $\hat{U}'$.

- Ideal state trajectories $\hat{U}|\psi_0\rangle$ (blue, dotted) and $\hat{U}'|\psi_0\rangle$ (red, dashed)
- State trajectories with coherent control errors $U(\varepsilon)|\psi_0\rangle$ (blue) and $U'(\varepsilon')|\psi_0\rangle$ (red)
- $\varepsilon, \varepsilon' \in \mathbb{R}^2$ are sampled uniformly from $[-0.2, 0.2]$ (500 samples)




**Summary:** The worst-case fidelity of $U$ is significantly larger
$\rightarrow$ $U$ is more robust against coherent control errors

## Theoretical results

For any $\varepsilon, \varepsilon' \in \mathbb{R}^N$ and any initial state $|\psi_0\rangle$, we have

- Lipschitz bound:

$$\| |\psi(\varepsilon)\rangle - |\psi(\varepsilon')\rangle \|_2 \leq \sum_{i=1}^{N} \|H_i\|_2 \|\varepsilon - \varepsilon'\|_\infty$$

- Worst-case fidelity bound:

$$|\langle\psi(\varepsilon)|\hat{\psi}\rangle| \geq 1 - \left(\sum_{i=1}^{N} \|H_i\|_2\right)^2 \frac{\|\varepsilon\|_\infty^2}{2}$$

- A fidelity guarantee of $\mathcal{F} = |\langle\psi(\varepsilon)|\hat{\psi}\rangle|$ when

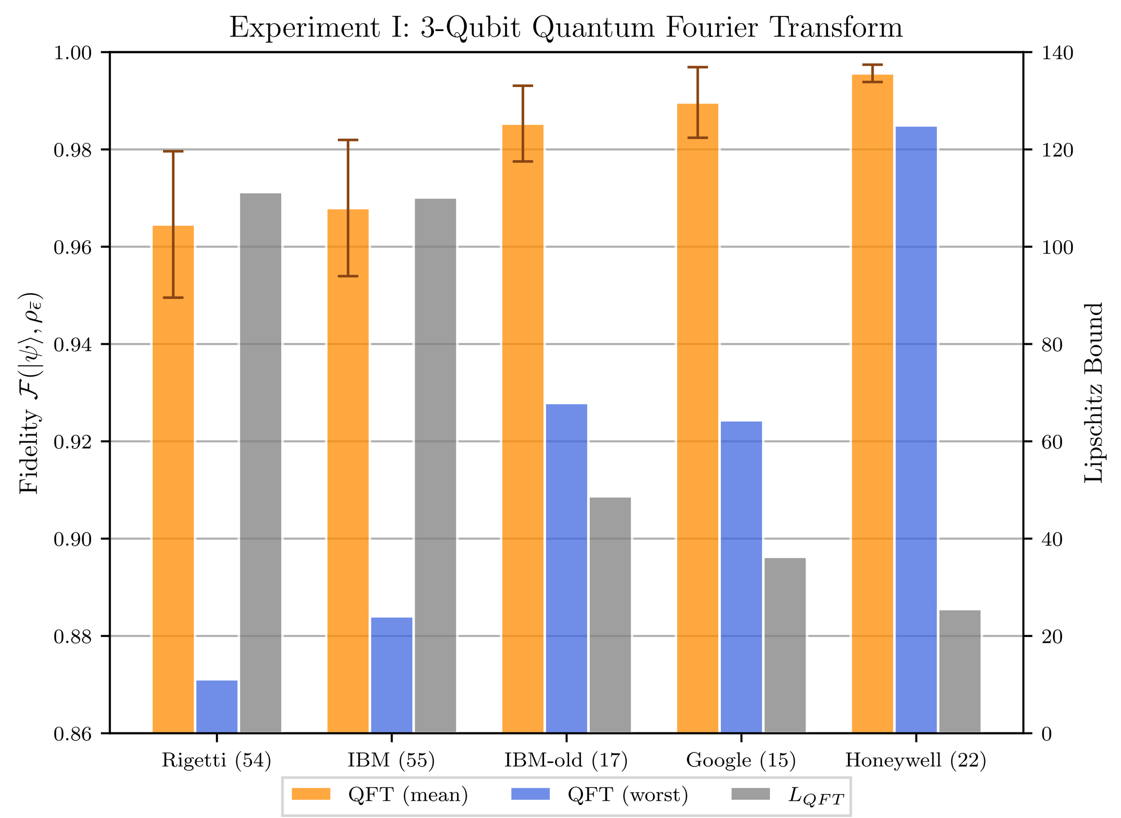
$$\|\varepsilon\|_\infty \leq \frac{\sqrt{2}}{\sum_{i=1}^{N} \|H_i\|_2} \sqrt{1 - \mathcal{F}}$$

**Applications:**

- Computing worst-case fidelity bounds for Quantum Error Correction (QEC) **threshold theorems**
- A new **guideline** for robust quantum algorithm design and transpilation: encourage small values of $\|H_i\|_2$
- Variational quantum algorithms: **parameter regularization improves robustness**

## Robustness analysis of elementary gate sets

- We transpile the 3-Qubit Quantum Fourier Transform circuit with different native gate sets
- The transpiled circuits have different numbers of gates




**Summary:** The number of gates alone is not sufficient to explain the magnitude of the error $\rightarrow$ Lipschitz bounds can explain the gap

## Validation on a quantum computer (IBM-Nairobi)

- We implement two circuits which ideally produce the same final state
- The circuits are not simplified by the standard IBM transpiler
- The $z$-rotations are affected by coherent control errors with bound $\bar{\varepsilon}$

**Circuit $U_A$:**

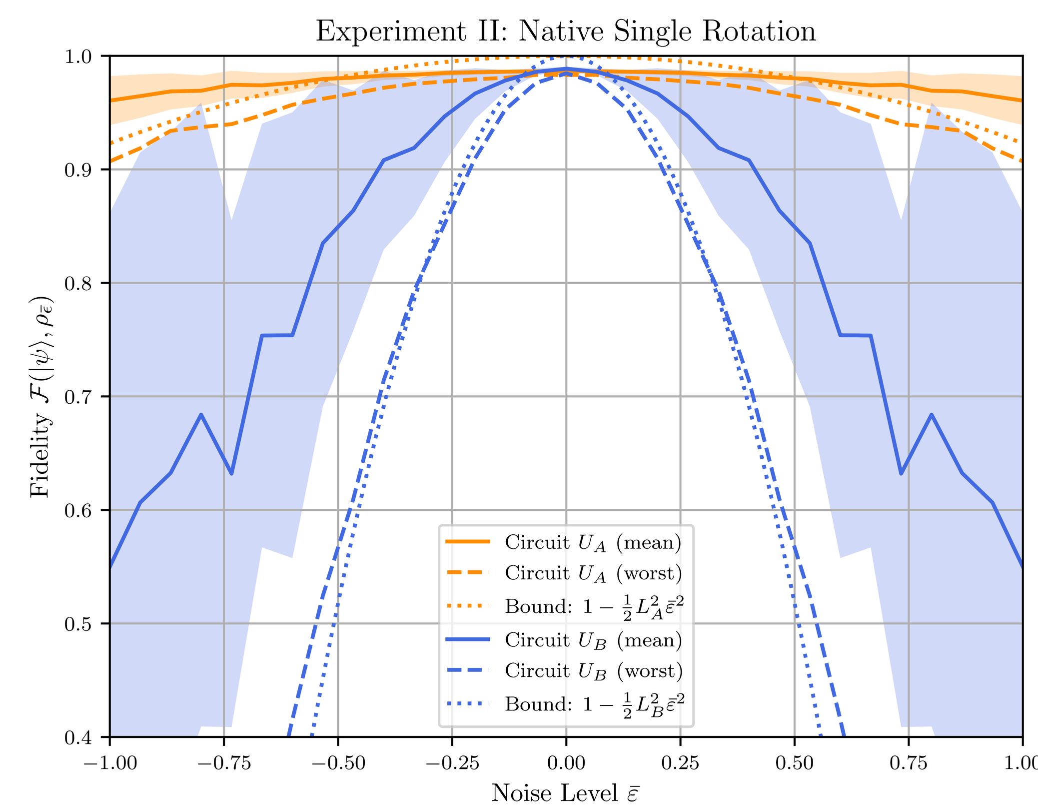
$$|0\rangle - \boxed{\sqrt{X}} - \boxed{R_z(\tfrac{\pi}{4})} - \boxed{\sqrt{X}} - |\hat{\psi}\rangle$$

Lipschitz bound $L_A = \frac{\pi}{8}$

**Circuit $U_B$:**

$$|0\rangle - \boxed{X} - \boxed{\sqrt{X}} - \boxed{R_z(\tfrac{5\pi}{4})} - \boxed{\sqrt{X}} - |\hat{\psi}\rangle$$

Lipschitz bound $L_B = \frac{5\pi}{8}$




## References

[1] F. Arute, K. Arya, R. Babbush, and et al.
Quantum supremacy using a programmable superconducting processor.
*Nature*, 574:505–510, 2019.

[2] J. P. Barnes, C. J. Trout, and D. Lucarelli.
Quantum error-correction failure distributions: comparison of coherent and stochastic error models.
*Physical Review A*, 95:062338, 2017.

[3] C. J. Trout, M. Li, M. Gutiérrez, Y. Wu, S.-T. Wang, L. Duan, and K. R. Brown.
Simulating the performance of a distance-3 surface code in a linear ion trap.
*New Journal of Physics*, 20:043038, 2018.



Institute for Computational Physics

SimTech