University of Stuttgart
Germany

INSTITUTE FOR COMPUTATIONAL PHYSICS

ist

SimTech

Robustness of quantum algorithms against coherent control errors

Julian Berberich

Daniel Fink

International Conference on Data-Integrated Simulation Science

October 6th, 2023

joint work with Prof. Christian Holm

Quantum Computing

Nature isn't classical, […] and if you want to make a simulation of nature, you'd better make it quantum mechanical […].

- Richard Feynman, 1981

Quantum Computing

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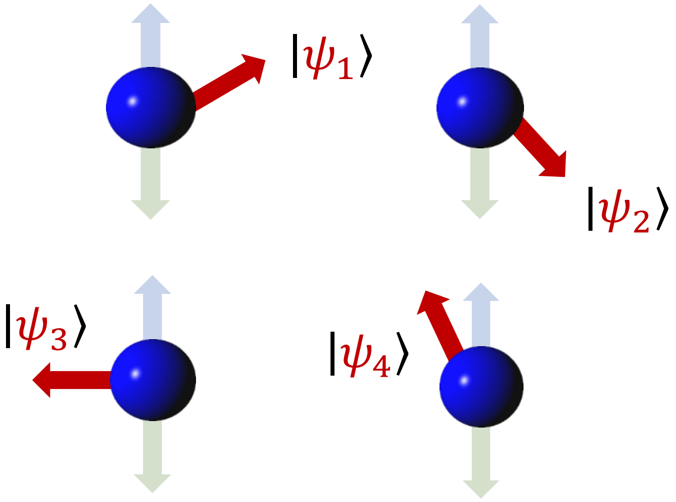
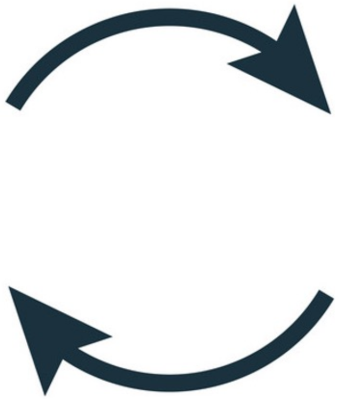
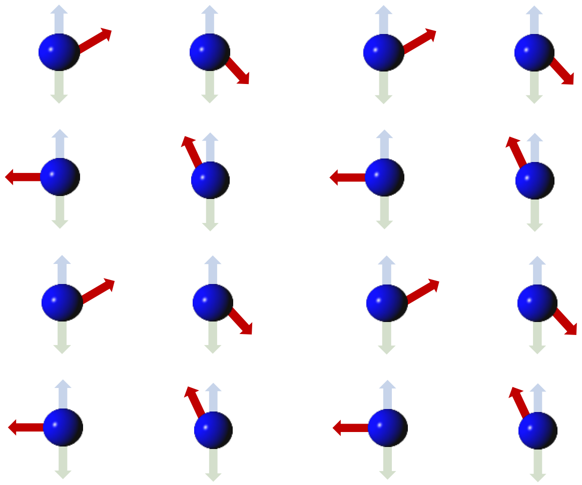
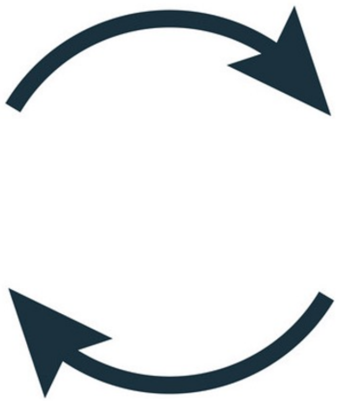


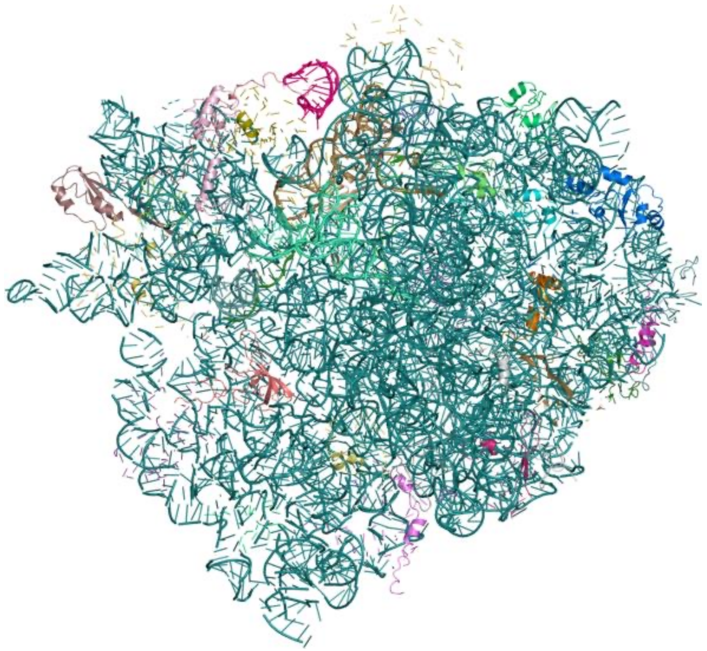


Y. W. Lin, et al., PNAS 107 8581-6 (2010).

Quantum Computing







M. Selmer, et al., Science 313 1935-42 (2006).

Main problem: current quantum computers are small-scale and noisy

Agenda

Part I

Introduction to Quantum Computing

Qubits

Quantum Algorithms

Coherent Control Errors

Part II

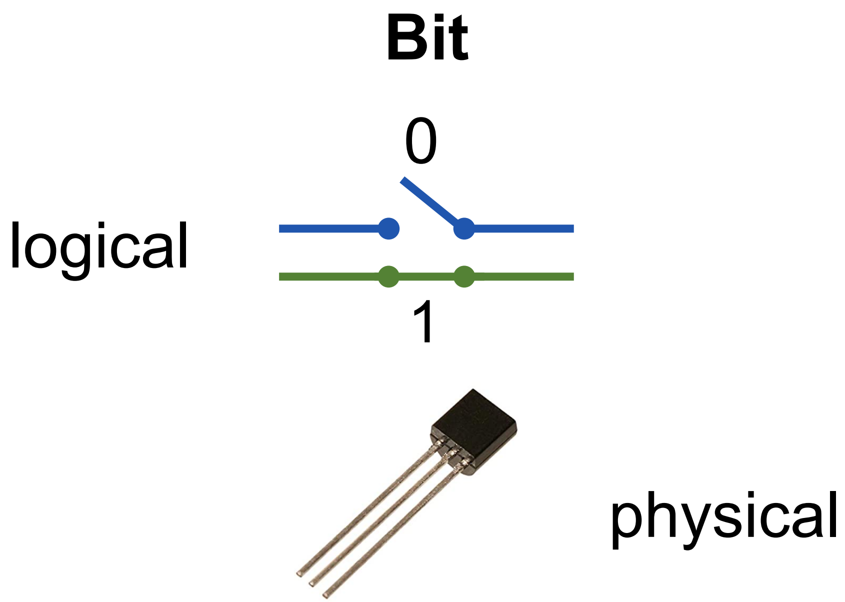
Robustness Analysis

Theoretical results

Applications

Quantum
Bits

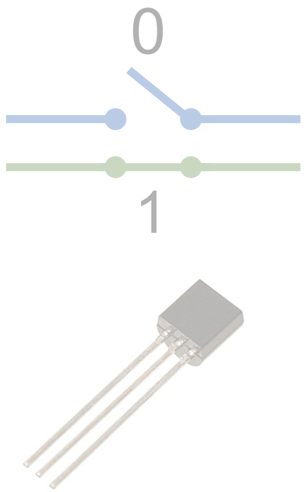
Qubits



Qubits

Bit

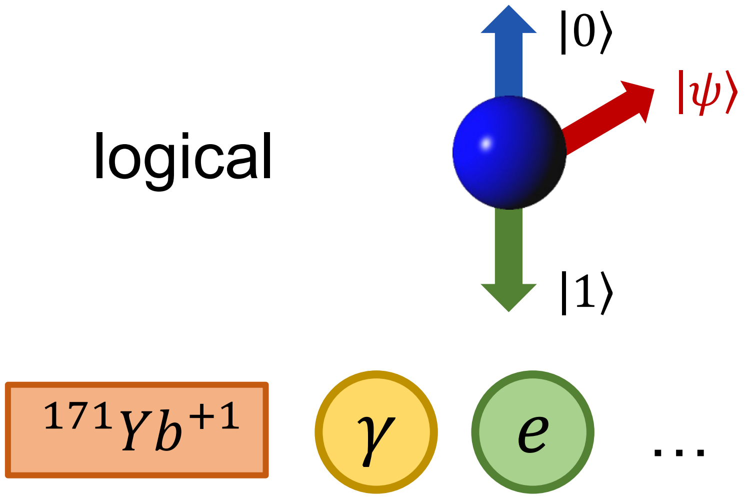
logical



physical

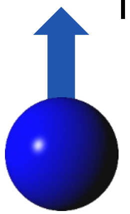
Qubit

logical



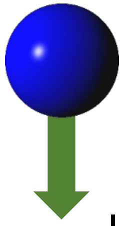
physical

Qubits



$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in \mathbb{C}^2$$

computational
basis states



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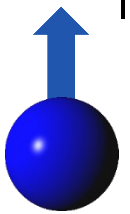

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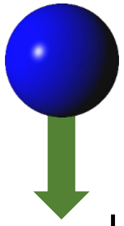
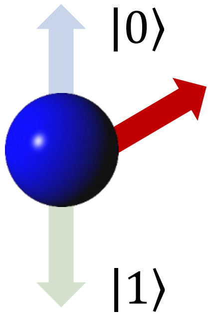




$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \in \mathbb{C}^2$, with $|\alpha|^2 + |\beta|^2 = 1$

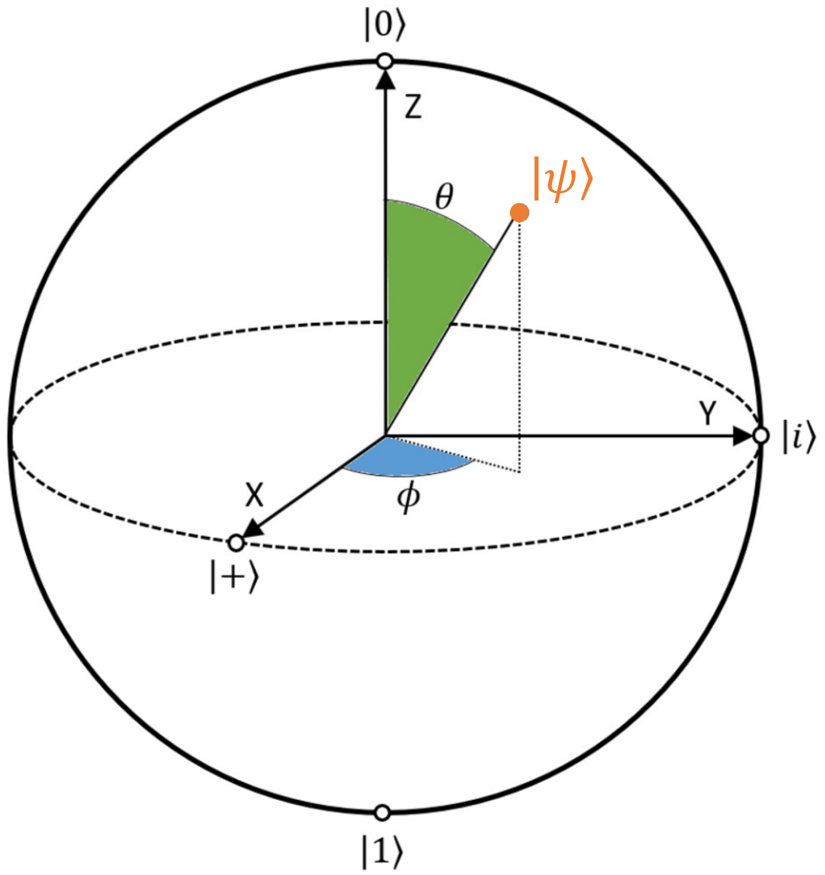
Born's rule

Qubits

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

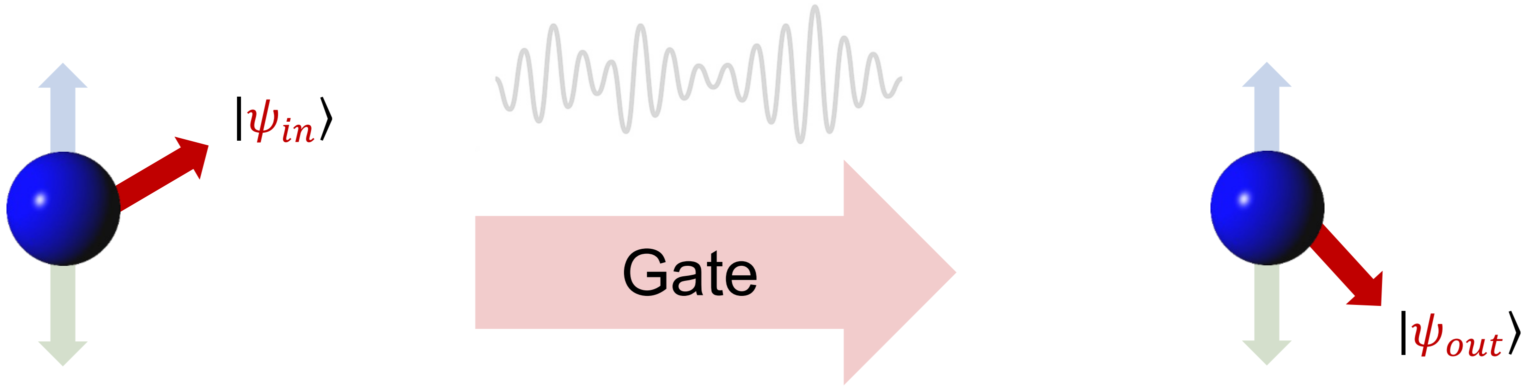
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

Bloch Sphere



Quantum Operations

Gates



Unitary operator $U: \mathbb{C}^d \to \mathbb{C}^d, \ U|\psi_{in}\rangle = |\psi_{out}\rangle$

$U = e^{-iH}$, with Hermitian generator $H = H^\dagger$

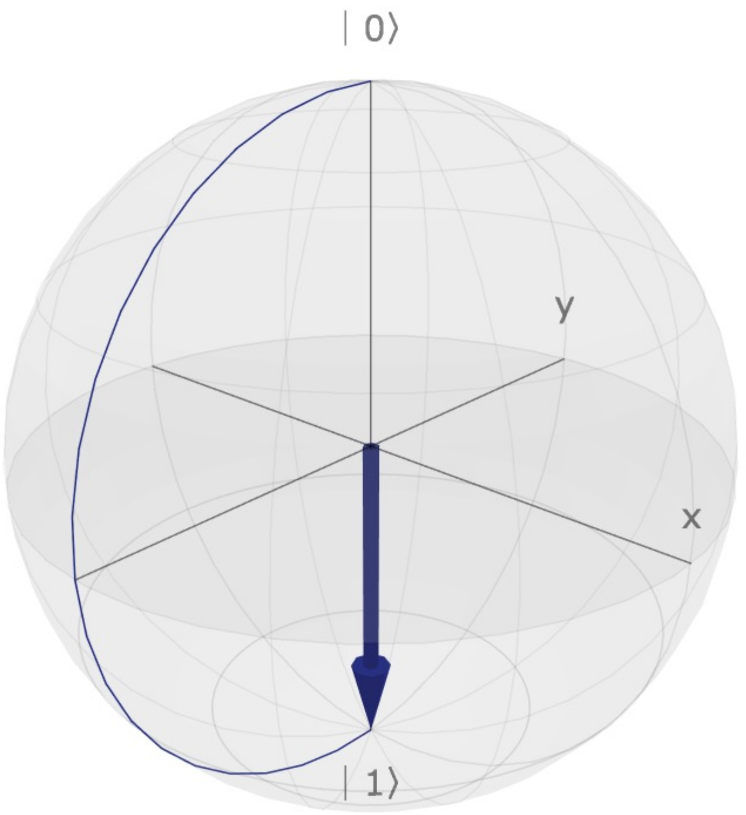
Gates

Common gates:

Pauli X (or NOT): $|0\rangle$ ⊕ $|1\rangle$

Pauli X Rotation: $|0\rangle$ RX_θ $\cos\frac{\theta}{2}|0\rangle - i\sin\frac{\theta}{2}|1\rangle$

Hadamard: $|0\rangle$ H $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$



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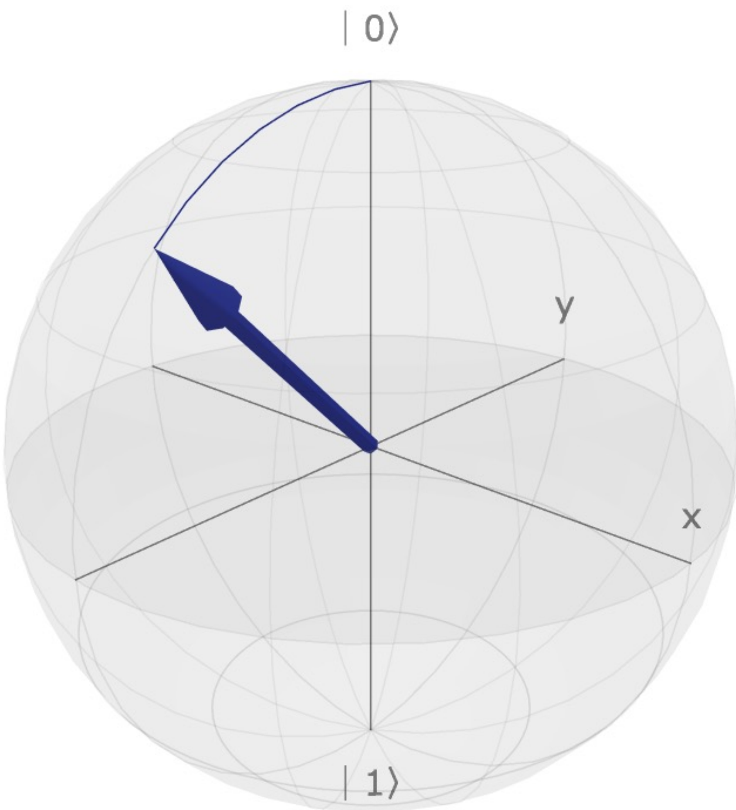

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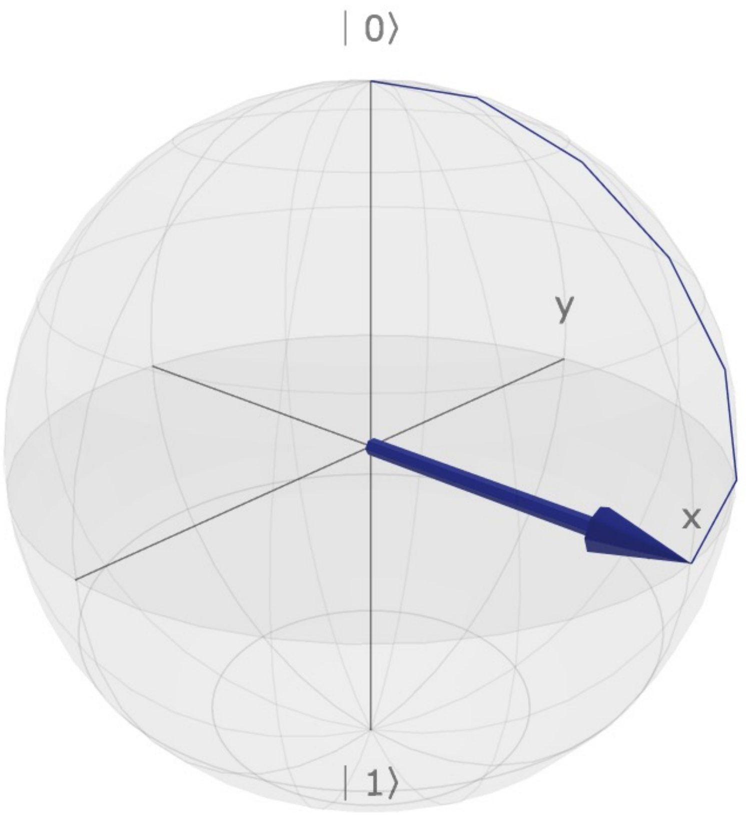
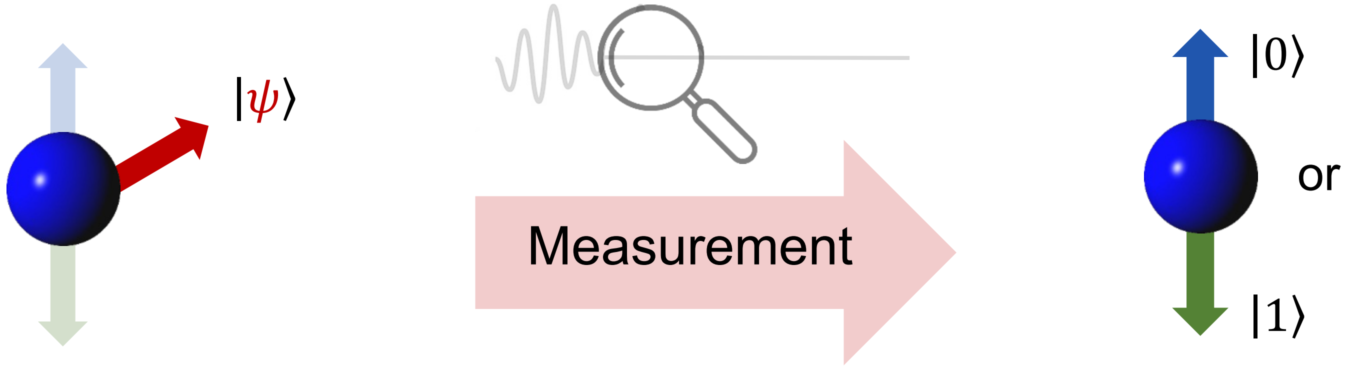


Quantum Measurements

Measurements



Observable $\mathcal{O}: \mathbb{C}^d \to \mathbb{C}^d,\ \mathcal{O} = \mathcal{O}^\dagger$

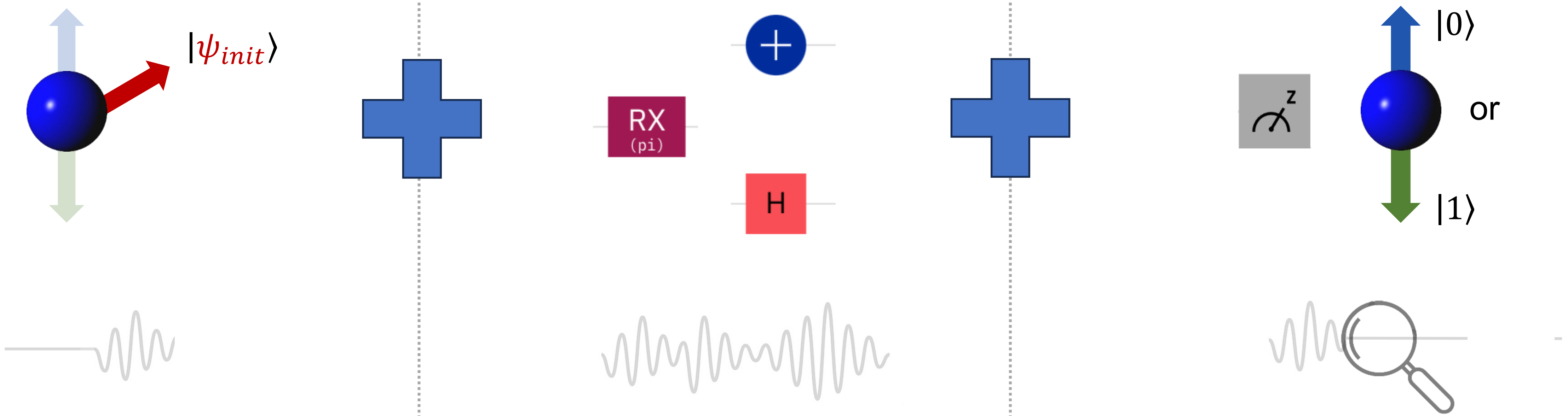
Quantum Algorithms

Quantum Algorithms

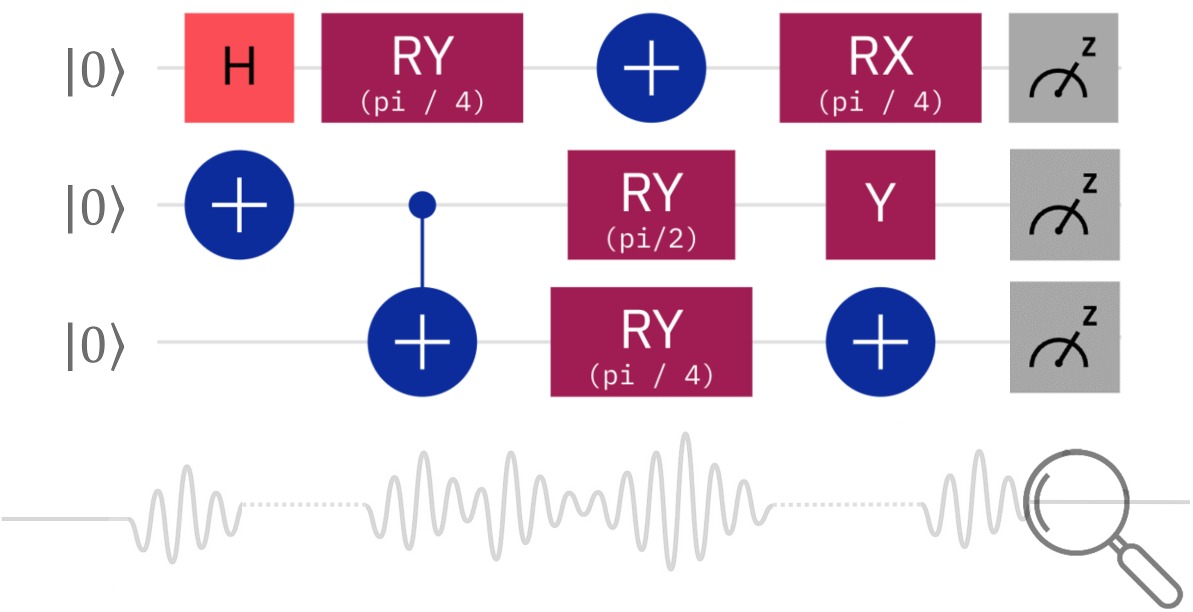
Initial state

Gates

Measurements

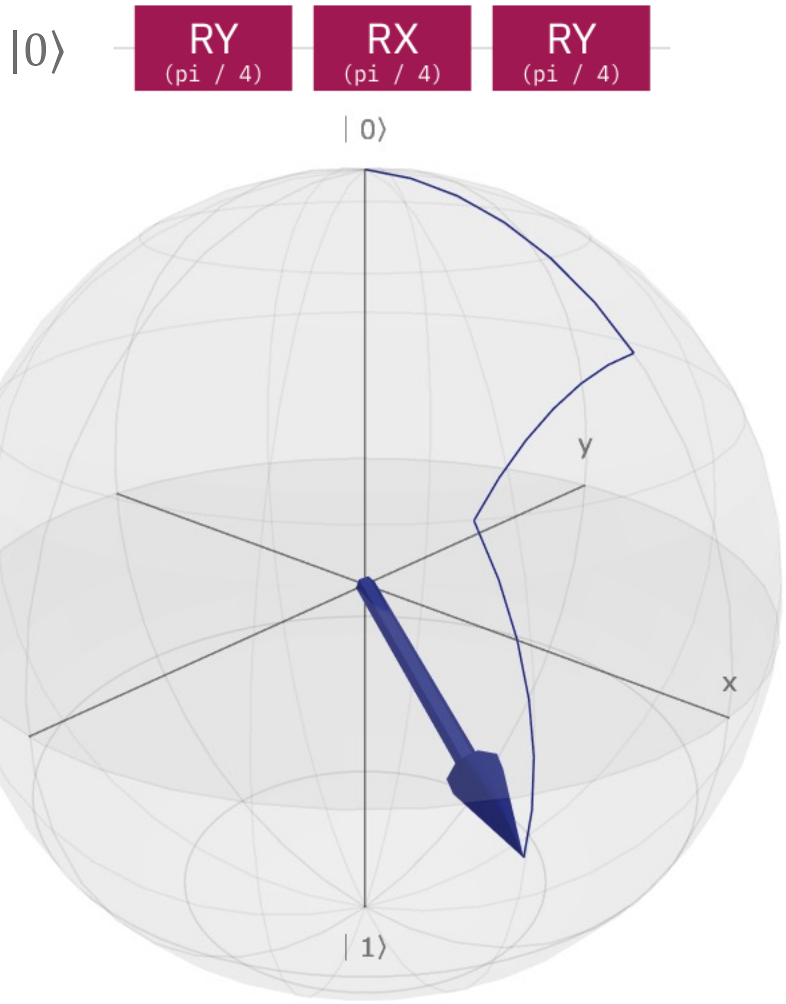


Quantum Circuits



Running Quantum Algorithms

Running Quantum Algorithms



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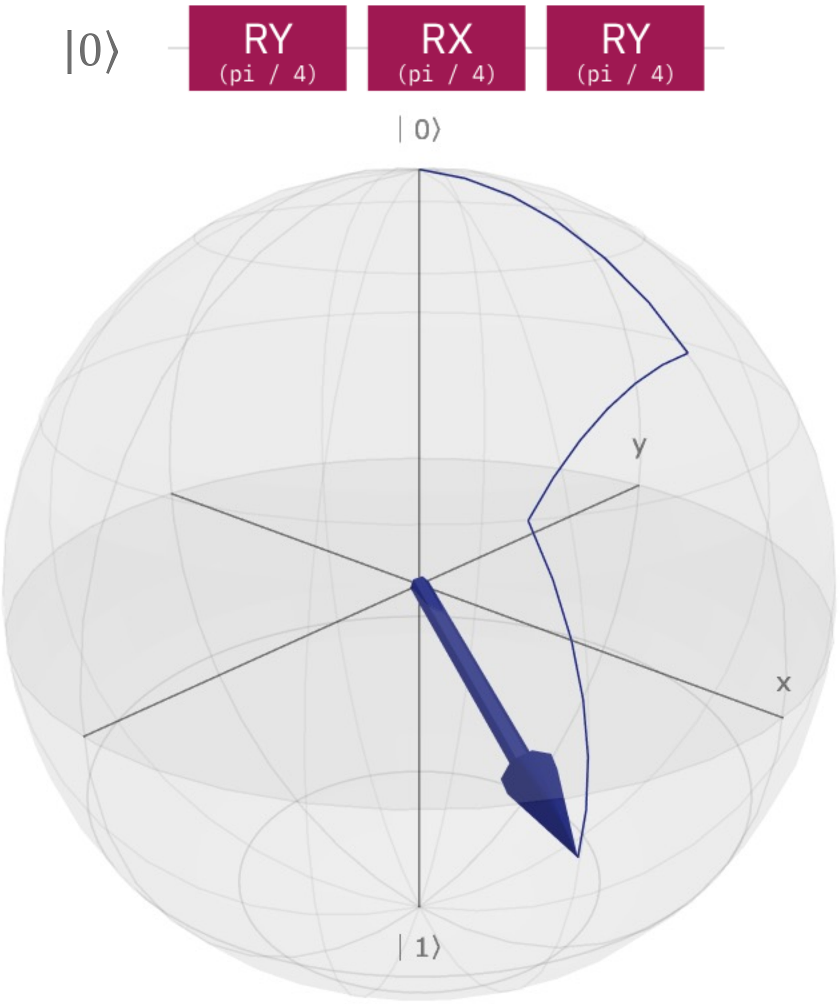

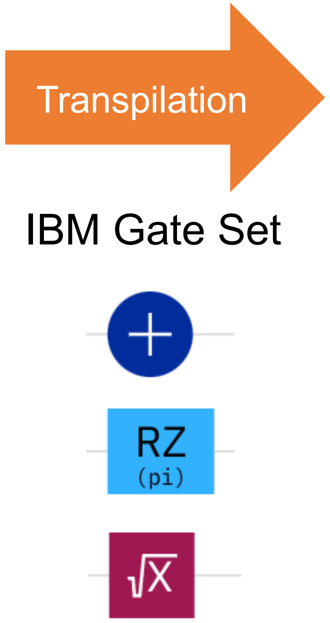

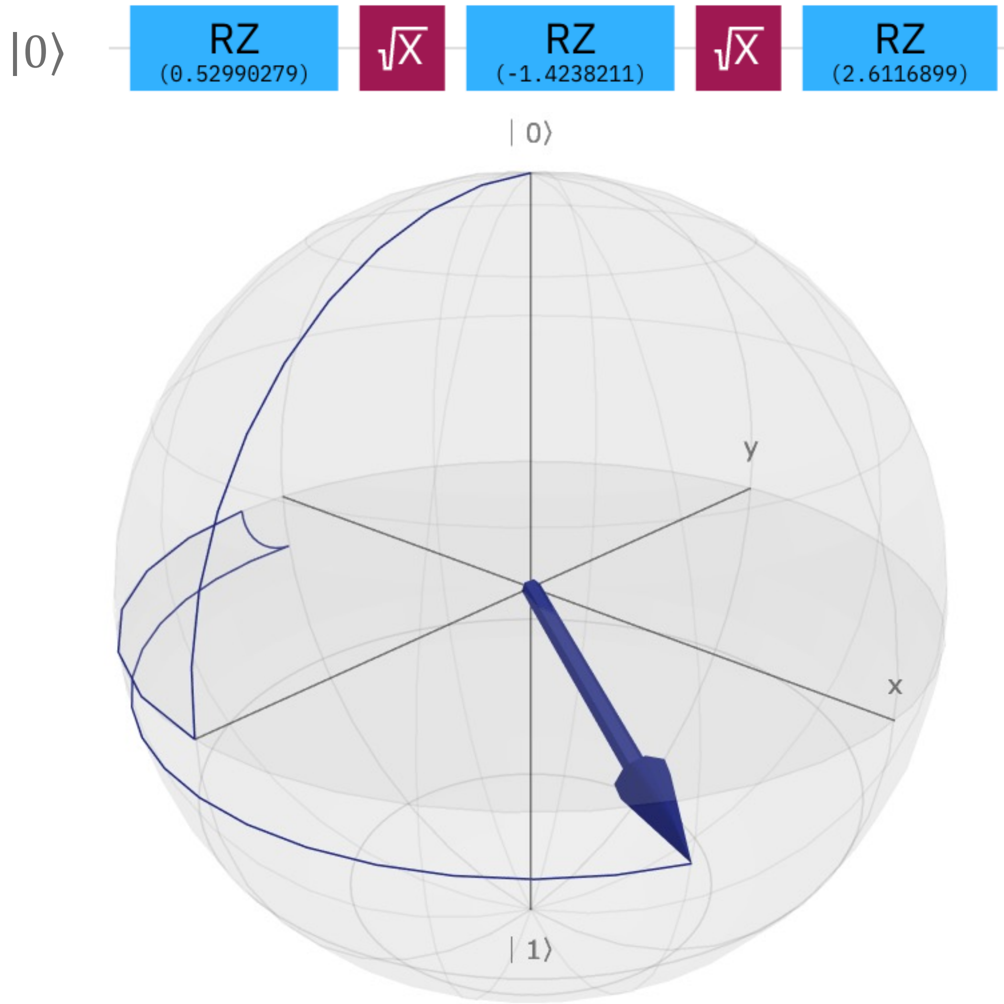

Coherent Control Errors

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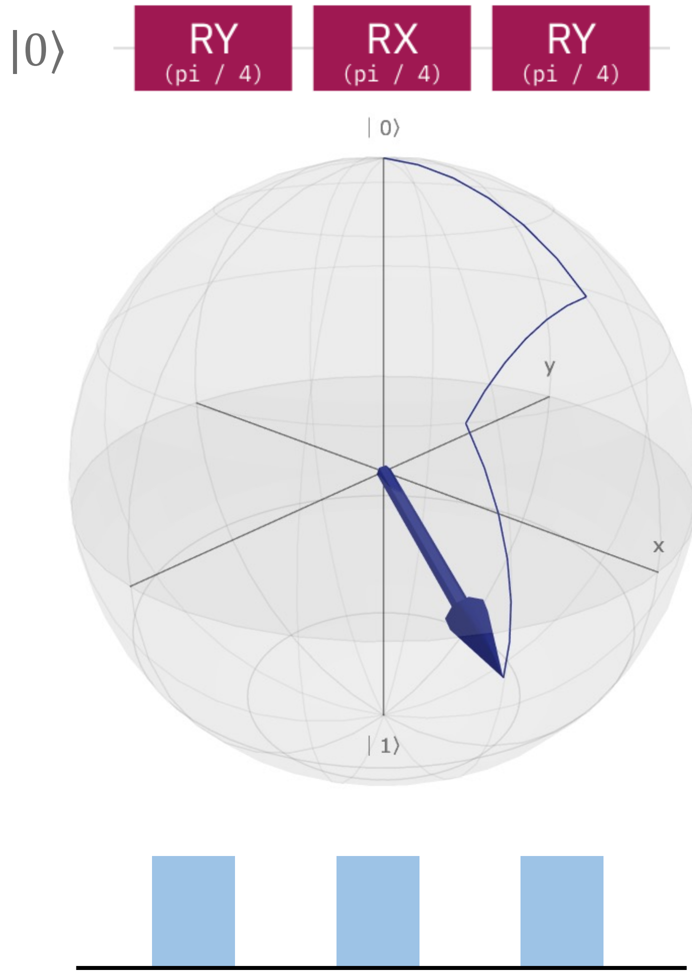

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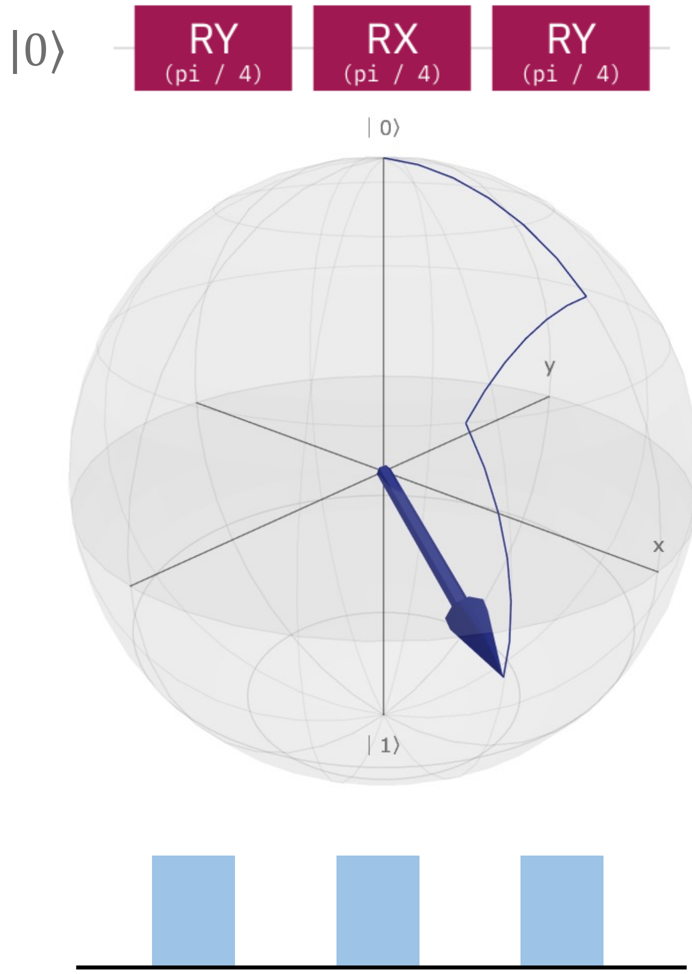
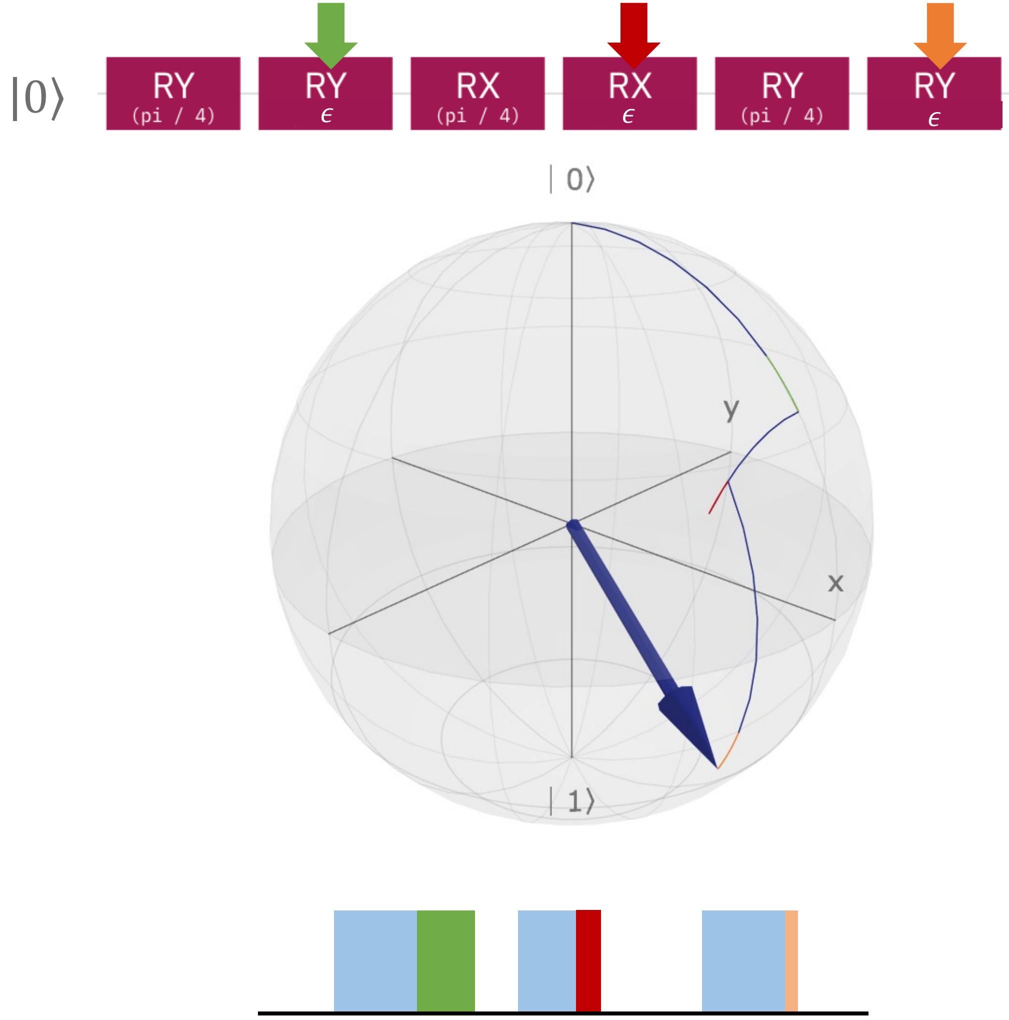


Errors

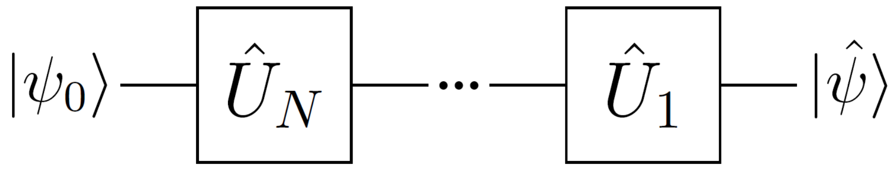


Robustness analysis

Problem setup

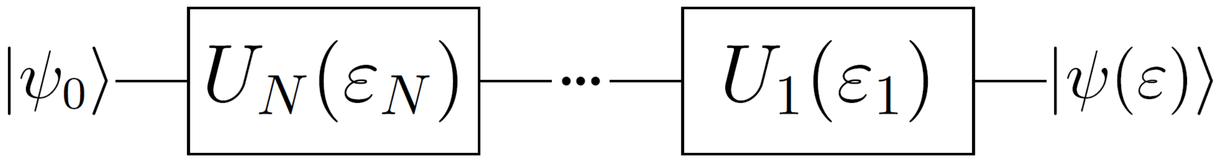
Ideal quantum algorithm

$|\hat{\psi}\rangle = \hat{U}_1 \cdots \hat{U}_N |\psi_0\rangle$ with $\hat{U}_j = e^{-iH_j}$



Noisy quantum algorithm

$|\psi(\varepsilon)\rangle = U_1(\varepsilon_1) \cdots U_N(\varepsilon_N) |\psi_0\rangle$ with
$U_j(\varepsilon_j) = e^{-i(1+\varepsilon_j)H_j}$ and $\varepsilon_j \in \mathbb{R}$



- They are related via $|\hat{\psi}\rangle = |\psi(0)\rangle$
- **Assumption:** ε is bounded, i.e., $\|\varepsilon\| \leq \bar{\varepsilon}$

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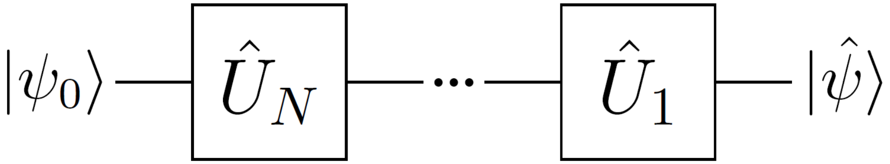

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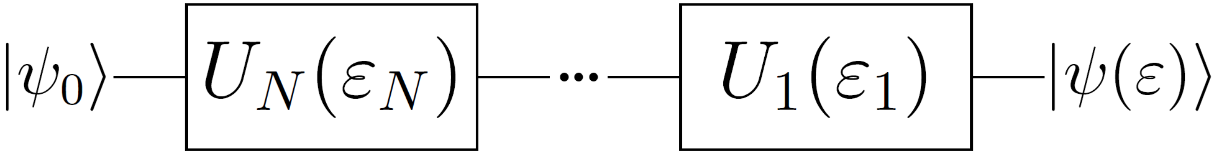

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- **Assumption:** ε is bounded, i.e., $\|\varepsilon\| \leq \bar{\varepsilon}$

Problem: Robustness analysis

Find **fidelity lower bound**: $\left|\langle\psi(\varepsilon)|\hat{\psi}\rangle\right| \geq 1 - c\bar{\varepsilon}^2$ for some $c > 0$.

Robustness analysis

Definition: $L > 0$ is a **Lipschitz bound** of $|\psi\rangle$ if

$$\||\psi(\varepsilon)\rangle - |\psi(\varepsilon')\rangle\| \leq L\|\varepsilon - \varepsilon'\| \quad \text{for all } \varepsilon, \varepsilon' \in \mathbb{R}^N.$$

Theorem

$L = \sum_{j=1}^{N} \|H_j\|$ is a Lipschitz bound of $|\psi\rangle$.

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Theorem

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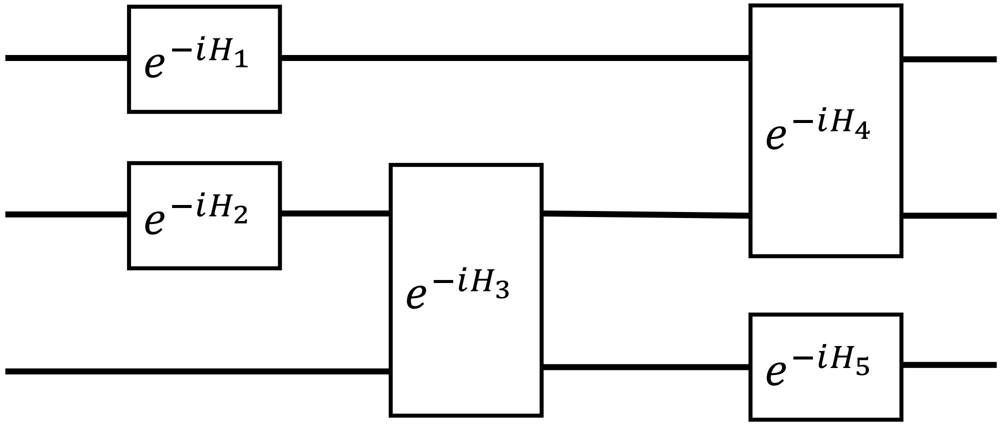
Example: $L = \|H_1\| + \cdots + \|H_5\|$



Robustness analysis

Corollary

For any ε with $\|\varepsilon\| \leq \bar{\varepsilon}$ and any initial state $|\psi_0\rangle$, it holds that

$$\left|\langle\psi(\varepsilon)|\hat{\psi}\rangle\right| \geq 1 - \left(\sum_{j=1}^{N} \|H_j\|\right)^2 \frac{\bar{\varepsilon}^2}{2}.$$

- **Fidelity loss bounded** by $\|H_j\|$ and noise bound $\bar{\varepsilon}$
- Smaller $\|H_j\|$ → better robustness

Robustness analysis

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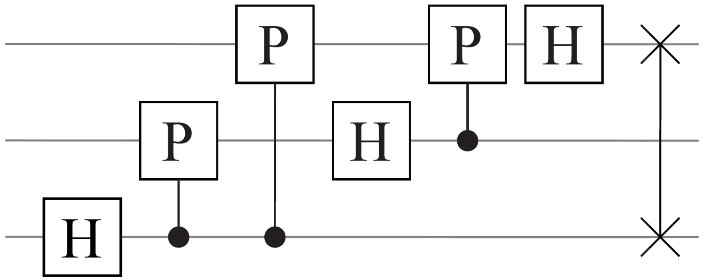
Design of the algorithm influences its robustness!

Application

Application: Quantum Fourier Transform

Problem

We study the robustness of the **transpiled circuit** with different elementary gate sets.



Application: Quantum Fourier Transform

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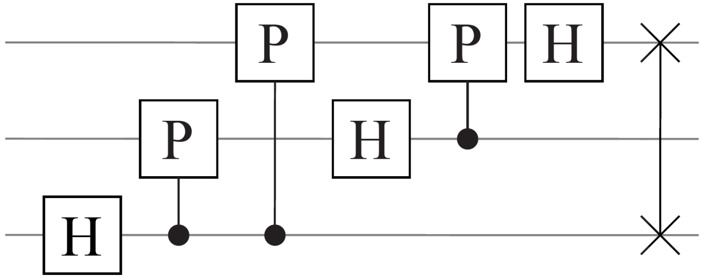
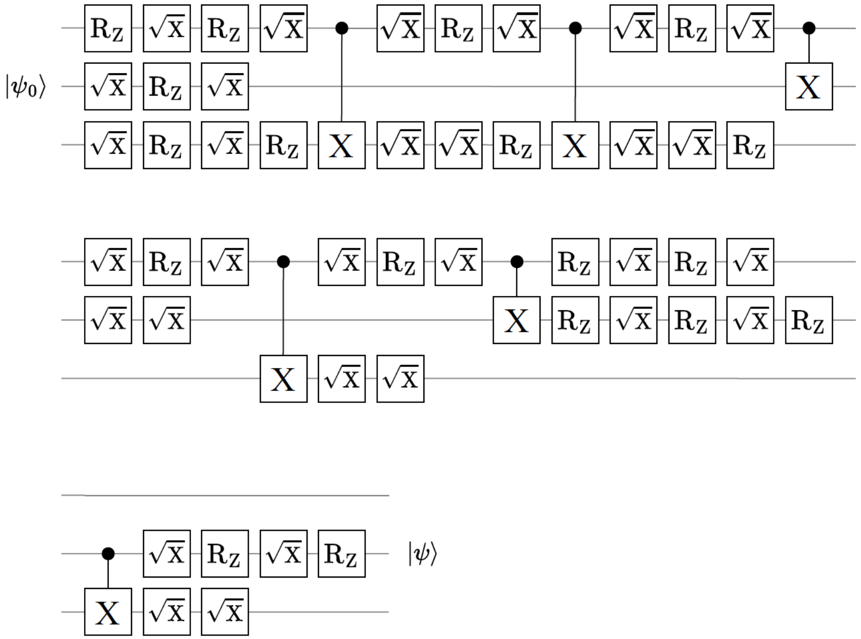
- **Gate set A**: $\sqrt{X}, X, R_z, CX$ (IBM)





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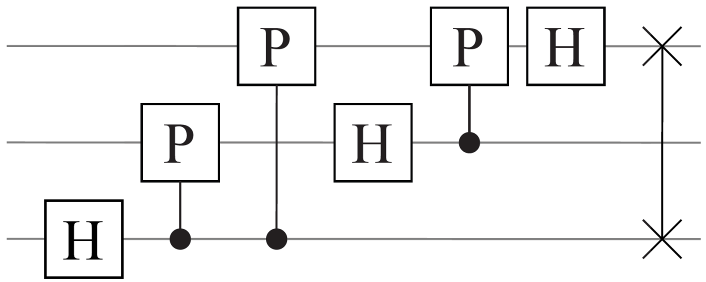

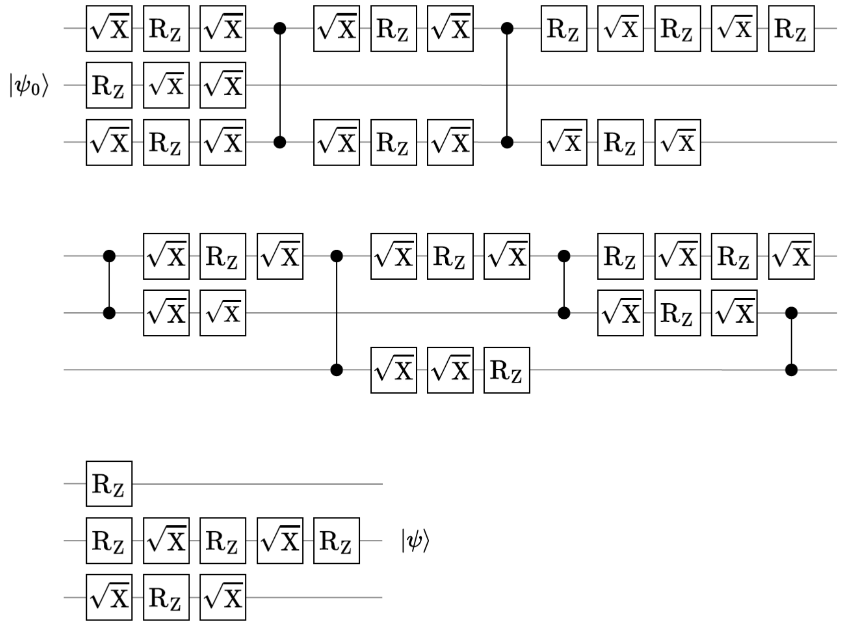

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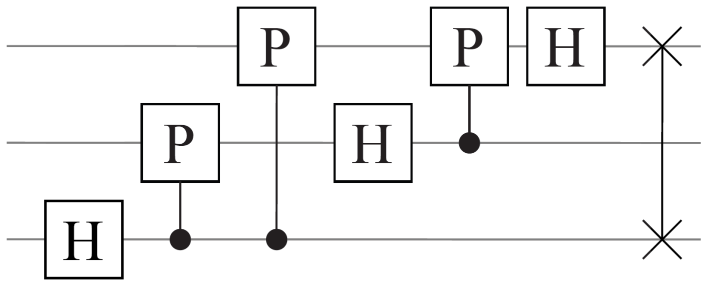

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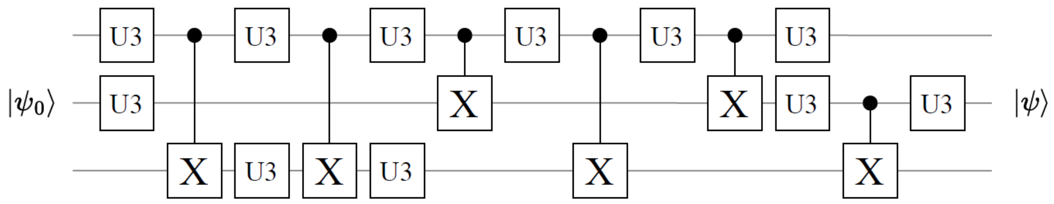

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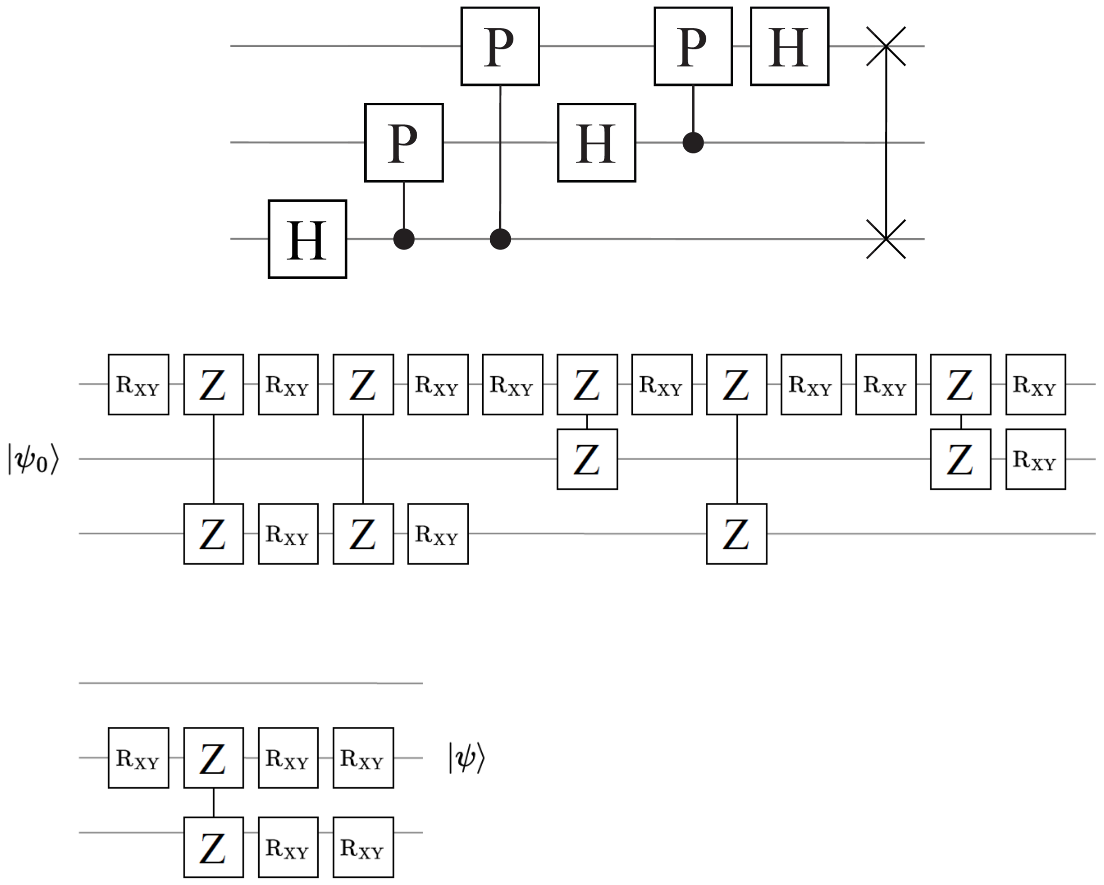
- **Gate set C:** U_1, U_2, U_3, CX (IBM old)
- **Gate set D:** $\sqrt{i\mathrm{SWAP}}, \mathrm{FSIM}, \mathrm{PhasedXZ}, X, Y, Z$ (Google)



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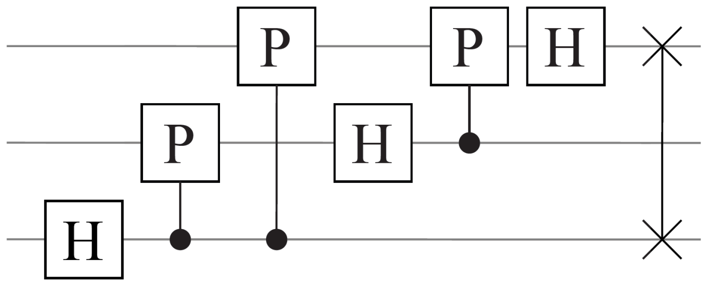

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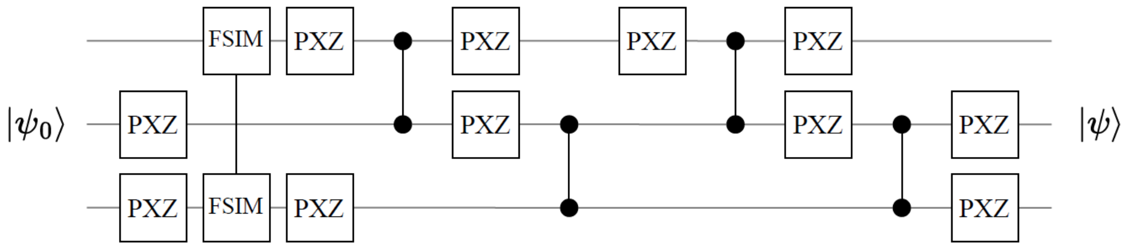
- **Gate set E:** $R_{xy}\left(\frac{\pi}{2}\right), R_{xy}(\pi), R_z, U_{zz}$ (Honeywell)



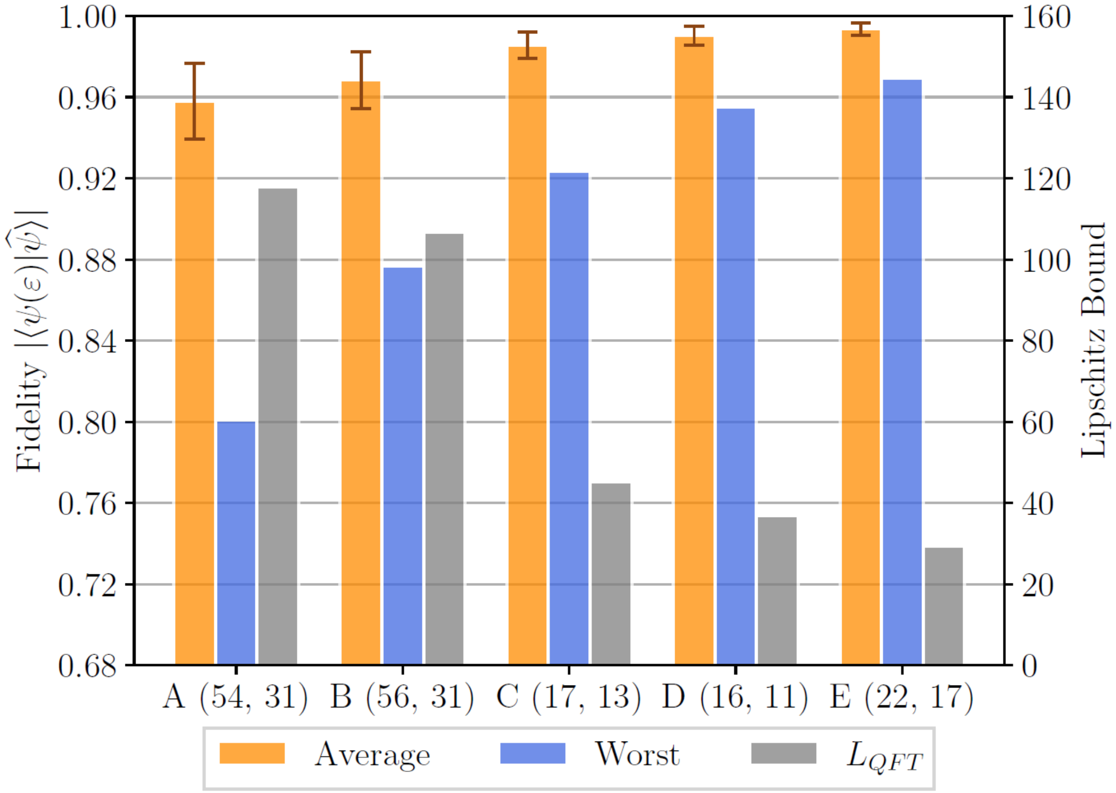
Application: Quantum Fourier Transform

For each gate set A-E, we

- **compute** the Lipschitz bound $\sum_{j=1}^{N} \|H_j\|$
- **simulate** the circuit with coherent control errors $|\varepsilon_j| \leq 0.05$ affecting each gate

Discussion

- Perfect (inverse) **correlation** between Lipschitz bound & fidelity
- Existing metrics such as gate count (left) or depth (right) **do not explain the outcome**



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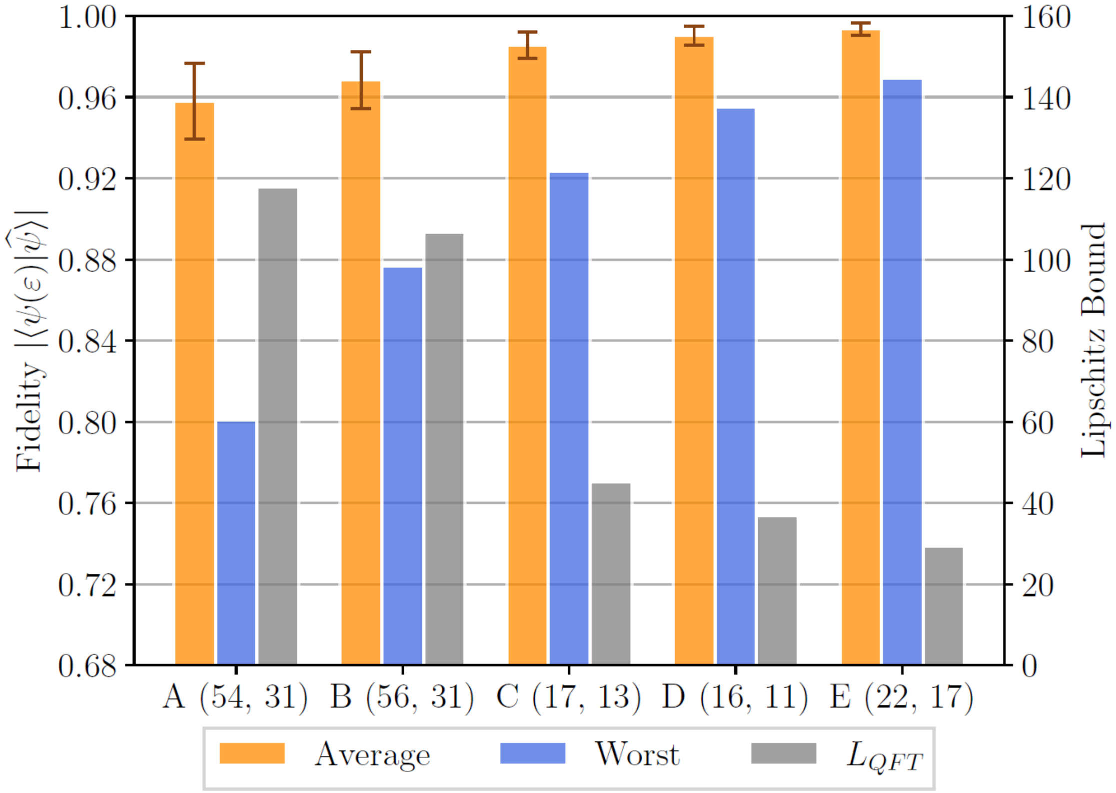
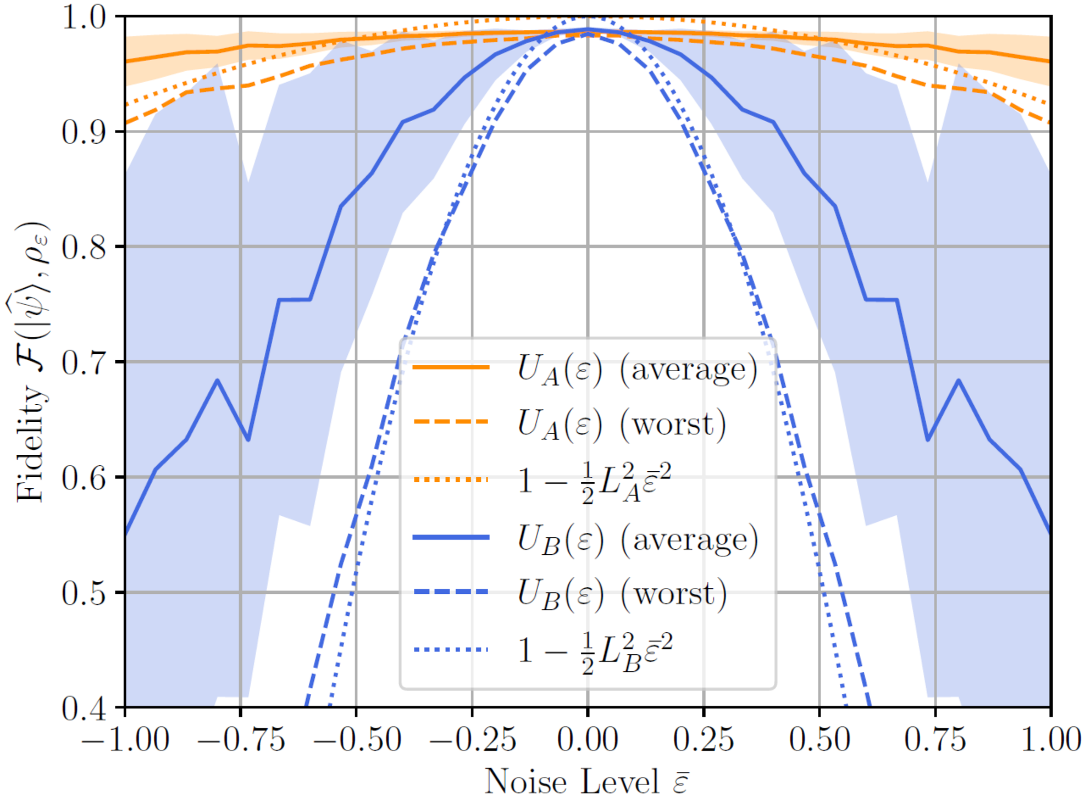
Our framework provides a priori **robustness guarantees!**



Validation on a quantum computer

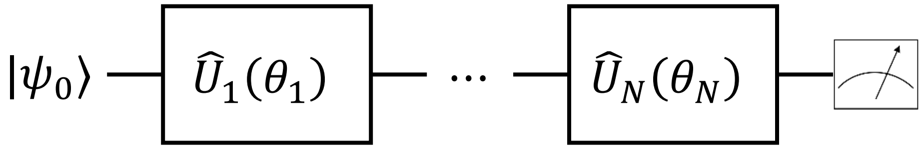
Analogous results for smaller algorithm on a **real quantum computer**!



Variational quantum algorithms

Variational quantum algorithms (VQAs)

Parametrized unitaries $\hat{U}(\theta) = \hat{U}_1(\theta_1) \cdots \hat{U}_N(\theta_N)$ with $\hat{U}_j(\theta_j) = e^{-i\theta_j H_j}$.



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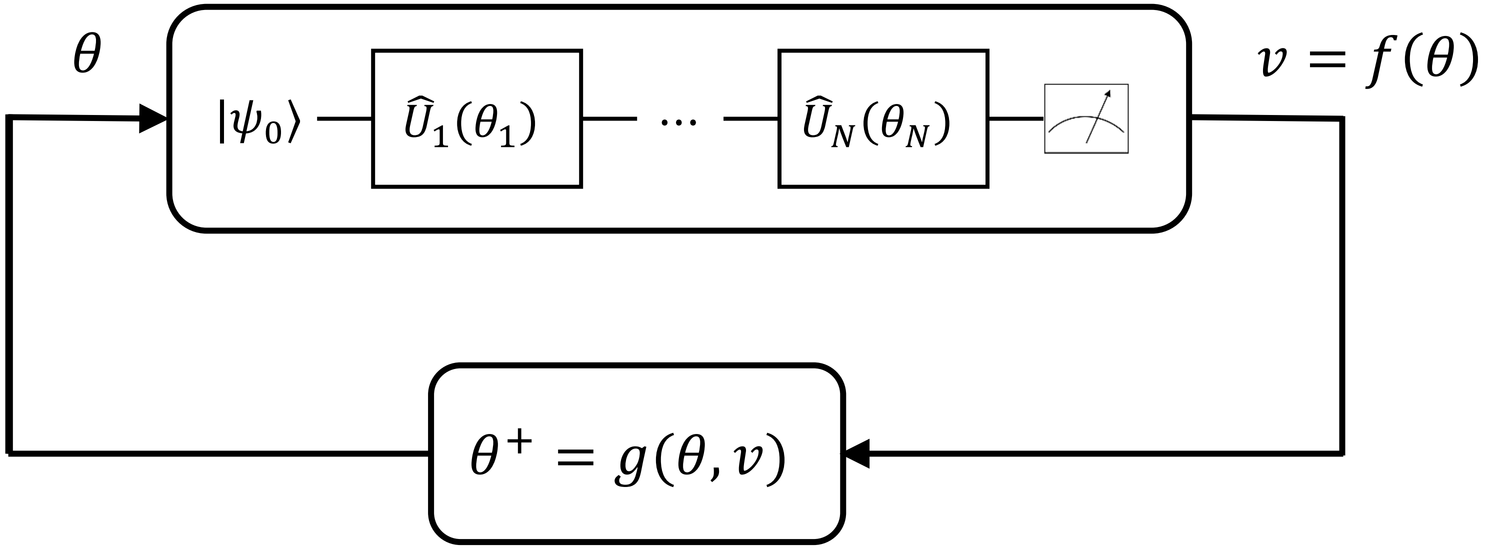
Key idea

Iteratively adapt parameter vector θ to minimize $f(\theta)$

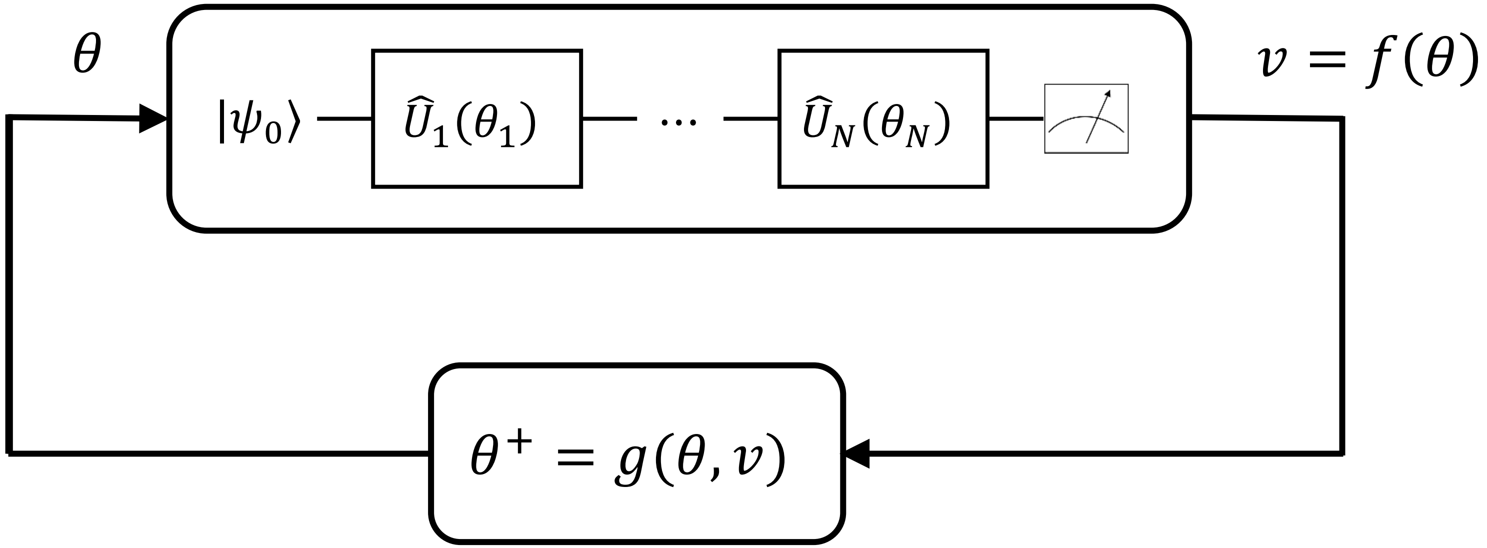


Variational quantum algorithms (VQAs)

Parametrized unitaries $\hat{U}(\theta) = \hat{U}_1(\theta_1) \cdots \hat{U}_N(\theta_N)$ with $\hat{U}_j(\theta_j) = e^{-i\theta_j H_j}$.

Key idea

Iteratively adapt parameter vector θ to minimize $f(\theta)$



- Promising approach for near-term quantum computing
- **Examples:** variational quantum eigensolver (VQE), quantum approximate optimization algorithm (QAOA), …

Regularization in VQAs

Coherent control errors

Suppose the ideal unitaries $\hat{U}_i(\theta_i)$ are affected by **coherent control errors** ε_i
→ noisy algorithm $U(\theta, \varepsilon) = e^{-i\theta_1(1+\varepsilon_1)H_1} \dots e^{-i\theta_N(1+\varepsilon_N)H_N}$.

Our analysis implies: $\sum_{j=1}^{N} |\theta_j| \|H_j\|$ is a Lipschitz bound of $\varepsilon \mapsto U(\theta, \varepsilon)|\psi_0\rangle$
→ Smaller $|\theta_j|$ implies better robustness

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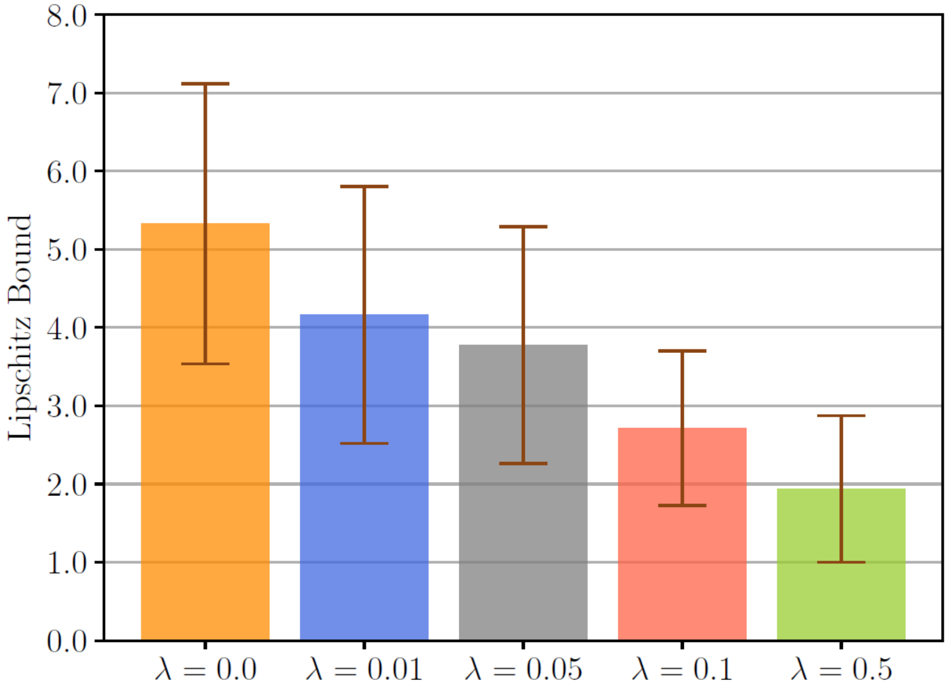
Regularization in VQAs

Solving the optimization problem $\min_\theta f(\theta) + \lambda\|\theta\|^2$ robustifies the VQA against coherent control errors!

$\lambda > 0$ ... tuning parameter

Implementation

Regularized VQA: Implementation for a simple example



Conclusion
&
Outlook

Conclusion

Summary

- **Coherent control errors** are a **major obstacle** for reliable quantum computing
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- **Applications:**
  - Robust algorithm design
  - Variational quantum algorithms

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- **Application** in circuit optimization
- **Extension** to different error classes
- Robust **quantum machine learning**

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Further details: arXiv:2303.00618

Robustness of quantum algorithms against coherent control errors

Julian Berberich[1], Daniel Fink[2], and Christian Holm[2]