

Daniel's H3 Mathematics Notes

Daniel Ginting

December 29, 2025

To poki.

Abstract

This is my H3 Mathematics notes. This paper is nothing less than a re-formatting of Dr Jonathon Teo's H3 Slides in 2025. The contents therefore are not original to the author.

1 Nomenclature and Notations

- \mathbb{Z} denotes the set of integers.
- \mathbb{Z}^+ denotes the set of positive integers.
- \mathbb{N} denotes the set of natural numbers.
- \mathbb{Q} denotes the set of rational numbers.
- \emptyset denotes the empty set.

2 Mathematical Statements

2.1 Structure

2.1.1 Conditional Statements

2.1.2 Biconditional Statements

2.1.3 Existential Statements

2.2 Content

A (mathematical) definition is a (true) mathematical statement that gives the precise meaning of a word or phrase that represents some object, property or other concepts.

A theorem is a true mathematical statement that can be proven mathematically.

An axiom is a mathematical statement that does not require proof.

3 Proofs

A (mathematical) proof is a deductive argument for a mathematical statement, showing that the stated assumptions logically guarantee the conclusion.

$$P \xRightarrow[\text{and previously established facts}]{\text{definitions, axioms, algebraic manipulations,}} Q$$

3.1 Direct Proof

A direct proof is an approach to prove a conditional statement $P \Rightarrow Q$.

- 3.2 Counter Example
- 3.3 Proof by Cases
- 3.4 Proof by Contradiction
- 3.5 Proving Existential Statements
 - 3.5.1 Constructive Proof
 - 3.5.2 Non-constructive Proof
- 4 Congruence Modulo
 - 4.1 Properties of Congruence
 - 4.2 Modular Arithmetic
- 5 Truth Table
- 6 Negation
- 7 Sequences and Series
- 8 Pigeon Hole Principle