

Problem Set o

Discrete Structures

Due on the 25th day of January of the year of our Lord 2026 at 11:59 pm

Justify each of your answers with an argument. An answer provided with bad or no justification is worse than a wrong answer.

1. Determine the truth value, if any, of each of the following sentences.

- "Every integer is either even or odd."¹
- "Every number is either even or odd."
- "Every non-empty set of integers has a smallest element."²
- "Every non-empty set of natural numbers has a smallest element."³
- "Every natural number is boring."⁴
- "No natural number is boring."

2. Determine the truth value, if any, of each of the following sentences.⁵

- "This sentence is *true*."⁶
- "This sentence is *false*."
- "The set of all sets that contain themselves contains itself."
- "The set of all sets that do not contain themselves does not contain itself."
- "If this sentence is *false*, then 7 is a prime number."
- "If this sentence is *true*, then 7 is not a prime number."

3. Consider the infinite sequence of sentences indexed by natural numbers

$$S_0, S_1, S_2, \dots, S_i, \dots$$

where each sentence asserts that all of the subsequent sentences are *false*.⁷

$$S_i := "S_j \text{ is false for all } j > i"$$

For example, $S_3 := "S_j \text{ is false for all } j > 3,"$ which means S_3 is equivalent to the assertion that "the sentences S_4, S_5, S_6, \dots are all *false*."

What are the truth values, if any, of the sentences in this sequence?⁸⁹

¹Informally, an *integer* is either a positive whole number 1, 2, 3, ... or a negative whole number -1, -2, -3, ... or zero 0.

²Informally, a *set* is a "collection of objects."

³Informally, a *natural number* is a non-negative whole number 0, 1, 2, ...

⁴For some reasonable definition of "boring."

⁵You may that assume 7 is actually a prime number.

⁶You may find it helpful to give a sentence like this a name. For example, define $S := "S \text{ is true}."$ The sentence S is equivalent to "This sentence is *true*.", so reason about S instead.

⁷The variable i in the definition of S_i ranges over all of the natural numbers 0, 1, 2, ...

⁸Your argument should describe the truth values, if any, of *every* sentence in this sequence.

⁹HINT: you should first determine what the truth value of S_0 is; once you have worked this out, then you can think about generalizing your argument to an arbitrary sentence S_i in the sequence.