# *Problem Set 1*

*Discrete Structures*

*Due on the 1st day of February of the year of our Lord 2026 at 11:59 pm*

1. Prove each of the following statements *without truth tables.*[1]
   a. Prove $p \to p$ is a tautology for any proposition $p$.
   b. Prove $p \to q \equiv \neg q \to \neg p$ for all propositions $p$, $q$.
   c. Prove $\neg(p \to q) \equiv p \land \neg q$ for all propositions $p$, $q$.
   d. Prove $(p \land (p \to q)) \to q$ is a tautology for all propositions $p$, $q$.
   e. Prove $(p \lor q) \to r \equiv (p \to r) \land (q \to r)$ for all propositions $p$, $q$, $r$.
2. Prove each of the following statements for all propositions $\varphi$, $\psi$, $\xi$.
   a. $(\varphi \to \psi), (\psi \to \xi) \vdash \varphi \to \xi$ — *Hypothetical Syllogism*
   b. $\vdash \varphi \to \varphi$
   c. $\vdash \top$
   d. $\varphi, \psi \vdash \varphi \land \psi$ — *Conjunction Introduction*
   e. $\varphi \land \psi \vdash \varphi$ — *Conjunction Elimination*

[1] In addition to the axioms of propositional logic and the uniqueness of complements, you may assume the following:
1. $\top \equiv \neg\bot$ and $\bot \equiv \neg\top$
2. Double Negation
3. Idempotence
4. Domination
5. De Morgan's Laws