*Problem Set 1*

*Discrete Structures*

*Due on the 1ˢᵗ day of February of the year of our Lord 2026 at 11:59 pm*

1. Prove each of the following statements *without truth tables.*[1]

   a. Prove $p \rightarrow p$ is a tautology for any proposition $p$.

   b. Prove $p \rightarrow q \equiv \neg q \rightarrow \neg p$ for all propositions $p, q$.

   c. Prove $p \rightarrow (q \rightarrow p)$ is a tautology for all propositions $p, q$.

   d. Prove $(p \wedge (p \rightarrow q)) \rightarrow q$ is a tautology for all propositions $p, q$.

   e. Prove $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$ for all propositions $p, q, r$.

2. Prove each of the following statements for all propositions $\varphi, \psi, \xi$.

   a. $(\varphi \rightarrow \psi), (\psi \rightarrow \xi) \vdash \varphi \rightarrow \xi$      *Hypothetical Syllogism*

   b. $\vdash \varphi \rightarrow \varphi$

   c. $\vdash \top$

   d. $\varphi, \psi \vdash \varphi \wedge \psi$      *Conjunction Introduction*

   e. $\varphi \wedge \psi \vdash \varphi$      *Conjunction Elimination*

[1]In addition to the axioms of propositional logic and the uniqueness of complements, you may assume the following:
1. $\top \equiv \neg\bot$ and $\bot \equiv \neg\top$
2. Double Negation
3. Idempotence
4. Domination
5. De Morgan's Laws