

## Problem Set 0

### Discrete Structures

Due on the 25<sup>th</sup> day of January of the year of our Lord 2026 at 11:59 pm

Justify each of your answers with an argument. An answer provided with bad or no justification is worse than a wrong answer.

1. Determine the truth value, if any, of each of the following sentences.

- a. "Every integer is either even or odd."<sup>1</sup>
- b. "Every number is either even or odd."
- c. "Every non-empty set of integers has a smallest element."<sup>2</sup>
- d. "Every non-empty set of natural numbers has a smallest element."<sup>3</sup>
- e. "Every natural number is boring."<sup>4</sup>
- f. "No natural number is boring."

2. Determine the truth value, if any, of each of the following sentences.<sup>5</sup>

- a. "This sentence is *true*."<sup>6</sup>
- b. "This sentence is *false*."
- c. "The set of all sets that contain themselves contains itself."
- d. "The set of all sets that do not contain themselves does not contain itself."
- e. "If this sentence is *false*, then 7 is a prime number."
- f. "If this sentence is *true*, then 7 is not a prime number."

3. Consider the infinite sequence of sentences indexed by natural numbers

$$S_0, S_1, S_2, \dots, S_i, \dots$$

where each sentence asserts that all of the subsequent sentences are *false*.<sup>7</sup>

$$S_i := "S_j \text{ is } \textit{false} \text{ for all } j > i"$$

For example,  $S_3 := "S_j \text{ is } \textit{false} \text{ for all } j > 3,"$  which means  $S_3$  is equivalent to the assertion that "the sentences  $S_4, S_5, S_6, \dots$  are all *false*."

What are the truth values, if any, of the sentences in this sequence?<sup>89</sup>

<sup>1</sup>Informally, an *integer* is either a positive whole number 1, 2, 3, ... or a negative whole number -1, -2, -3, ... or zero 0.

<sup>2</sup>Informally, a *set* is a "collection of objects."

<sup>3</sup>Informally, a *natural number* is a non-negative whole number 0, 1, 2, ...

<sup>4</sup>For some reasonable definition of "boring."

<sup>5</sup>You may that assume 7 is actually a prime number.

<sup>6</sup>You may find it helpful to give a sentence like this a name. For example, define  $S := "S \text{ is } \textit{true}."$  The sentence  $S$  is equivalent to "This sentence is *true*.", so reason about  $S$  instead.

<sup>7</sup>The variable  $i$  in the definition of  $S_i$  ranges over all of the natural numbers 0, 1, 2, ...

<sup>8</sup>Your argument should describe the truth values, if any, of *every* sentence in this sequence.

<sup>9</sup>HINT: you should first determine what the truth value of  $S_0$  is; once you have worked this out, then you can think about generalizing your argument to an arbitrary sentence  $S_i$  in the sequence.