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# DISCRETE MATHEMATICS

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These notes are intended for CS173 at the University of Illinois Urbana-Champaign.

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# Contents

# *Notation*

Table 1. An overview of some important notation used throughout this text. Most of this notation is standard, but some things (*e.g.*, $\mathbb{P}$, $\mathfrak{suc}$, $[\![n]\!]$) are not yet standardized.

| SYNTAX | PRONUNCIATION | TYPE | SEMANTICS |
|---|---|---|---|
| $x \coloneqq y$ | *"Define $x$ to be $y$."* | *def.* | Lets the label "$x$" refer to $y$. |
| $x = y$ | *"$x$ equals $y$."* | *pred.* | Asserts $x$ and $y$ are identical. |
| $x \in y$ | *"$x$ is an element of $y$."* | *pred.* | Asserts that $x$ is inside of $y$. |
| $p \vdash q$<br>$p \Rightarrow q$ | *"$p$ proves $q$."*<br>*"$p$ implies $q$."* | *judg.* | We can syntactically derive $q$ from the assumption $p$. |
| $p \equiv q$<br>$p \Leftrightarrow q$ | *"$p$ is equivalent to $q$."*<br>*"$p$ if and only if $q$."* | *judg.* | The statements $p$ and $q$ are semantically interchangeable. |
| $\top$ | *"True."* | *prop.* | Asserts truth. |
| $\bot$ | *"False."* | *prop.* | Asserts falsity. |
| $\neg p$ | *"Not $p$."* | *prop.* | Asserts the negation of $p$. |
| $p \wedge q$ | *"$p$ and $q$."* | *prop.* | Asserts both $p$ and $q$. |
| $p \vee q$ | *"$p$ or $q$."* | *prop.* | Asserts at least one of $p$ or $q$. |
| $p \to q$ | *"If $p$, then $q$."* | *prop.* | Asserts $q$ whenever $p$ is true. |
| $\forall x(\varphi(x))$ | *"For all $x$, $x$ satisfies $\varphi$."* | *prop.* | Asserts $\varphi$ about every object. |
| $\exists x(\varphi(x))$ | *"There exists an $x$ such that $x$ satisfies $\varphi$."* | *prop.* | Asserts $\varphi$ about some object. |
| $(\forall x \in y)(\varphi(x))$ | *"For all $x$ in $y$, $x$ satisfies $\varphi$."* | *prop.* | Asserts $\varphi$ about every $x$ in $y$. |
| $(\exists x \in y)(\varphi(x))$ | *"There exists an $x$ in $y$ such that $x$ satisfies $\varphi$."* | *prop.* | Asserts $\varphi$ about some $x$ in $y$. |
| $\{x \mid \varphi(x)\}$ | *"the collection of all $x$ such that $x$ satisfies $\varphi$"* | *obj.* | The collection of all and only the objects with property $\varphi$. |
| $\{x \in y \mid \varphi(x)\}$ | *"the set of all $x$ in $y$ such that $x$ satisfies $\varphi$"* | *obj.* | The set of all and only the elements of $y$ with property $\varphi$. |
| $f : x \to y$ | *"$f$ is a function from $x$ to $y$"* | *judg.* | A function $f$ whose domain is $x$ and codomain is $y$. |
| $\mathbb{P}(x)$ | *"the power set of $x$"* | *obj.* | The set of all subsets of $x$. |
| $\mathfrak{suc}(x)$ | *"the successor of $x$"* | *obj.* | The *"next thing after"* $x$. |
| $[\![n]\!]$ | *"bracket $n$"* | *obj.* | The set $\{m \in \mathbb{N} \mid m < n\}$ of the first $n$ natural numbers. |

Short for $\forall x(x \in y \to \varphi(x))$.

Short for $\exists x(x \in y \wedge \varphi(x))$.

We only refer to a collection as a *"set"* when we can prove it exists.

Depending on context, the expression $f : x \to y$ can refer to an *object* (*i.e.*, a *function* named $f$ whose domain is the set $x$ and codomain is the set $y$), or it can be a *proposition* (*i.e.*, the *assertion* that $f$ is a function whose domain is the set $x$ and codomain is the set $y$).

# *Writing Systems*

| GREEK | NAME | IPA |
|---|---|---|
| Α α | *alpha* | [a] |
| Β β | *beta* | [v] |
| Γ γ | *gamma* | [ɣ] |
| Δ δ | *delta* | [ð] |
| Ε ε | *epsilon* | [e] |
| Ζ ζ | *zeta* | [z] |
| Η η | *eta* | [i] |
| Θ θ | *theta* | [θ] |
| Ι ι | *iota* | [i] |
| Κ κ | *kappa* | [k] |
| Λ λ | *lambda* | [l] |
| Μ μ | *mu* | [m] |
| Ν ν | *nu* | [n] |
| Ξ ξ | *xi* | [ks] |
| Ο ο | *omicron* | [o] |
| Π π | *pi* | [p] |
| Ρ ϱ | *rho* | [ɾ] |
| Σ σ | *sigma* | [s] |
| Τ τ | *tau* | [t] |
| ϒ υ | *upsilon* | [i] |
| Φ ϕ | *phi* | [pʰ] |
| Χ χ | *chi* | [kʰ] |
| Ψ ψ | *psi* | [ps] |
| Ω ω | *omega* | [ɔː] |

| HEBREW | NAME | IPA |
|---|---|---|
| א | *aleph* | [ʔ] |
| ב | *beth* | [v] |
| ג | *gimel* | [ɣ] |
| ד | *daleth* | [ð] |
| ה | *he* | [h] |
| ו | *waw* | [v] |
| ז | *zayin* | [z] |
| ח | *cheth* | [χ] |
| ט | *teth* | [t] |
| י | *yod* | [j] |
| כ | *kaf* | [χ] |
| ל | *lamed* | [l] |
| מ | *mem* | [m] |
| נ | *nun* | [n] |
| ס | *samech* | [s] |
| ע | *ayin* | [ʕ] |
| פ | *fe* | [f] |
| צ | *tsadi* | [ts] |
| ק | *qof* | [k] |
| ר | *resh* | [ʁ] |
| ש | *shin* | [ʃ] |
| ת | *thav* | [θ] |

Table 2. The Greek alphabet. Each glyph is shown in uppercase and lowercase along with its common English name and modern IPA pronunciation.

Table 3. The Hebrew abjad. Only non-final variants of each glyph are shown.

# *Logic*

# 0

# Language

> *"No language is justly studied merely as an aid to other purposes. It will in fact better serve other purposes, philological or historical, when it is studied for love, for itself."*
>
> —J. R. R. Tolkien

The purpose of these notes is to uh…

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Figure 1. A fragment taken from the *Oxyrhynchus papyri*, dated to approximately 100 AD, depicting proposition 5 from book 2 of Euclid's *Elements*.

## 0.1 *A Brief History of…*

It is worth reflecting on $\forall$ and $\exists$ and $\omega$ and $\aleph$.

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## 0.2 *Syntax and Semantics*

A language provides a way of encoding ideas as sequences of symbols.[1]

[1] For our purposes, we will restrict our attention only to *written* languages.

## 0.3 *A Recurring Theme*

# 1

# Zeroth-Order Logic

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## 1.1 Truth Values

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## 1.2 Logical Connectives

### Negations

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### Conjunctions and Disjunctions

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### *Conditional Statements*

## 1.3 *A Modest Proposal*

### *Equivalence*

### *Axioms*

### *Rules of Inference*

### *Classical Syllogisms*

# 2

# *First-Order Logic*

## 2.1 *A More Expressive Language*

### *Forming Formulæ Well*

## 2.2 *Rules of Inference*

# 3

# Rhetoric

3.1 *Quantified Expressions*

3.2 *Conditional Expressions*

3.3 *Conjunctions*

3.4 *Disjunctions*

3.5 *Nonconstructive Arguments*

# *Mathematics*

# 4

# Foundations

*"Finally I am becoming stupider no more."*

—Paul Erdős

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## 4.1 Sets

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### Existence

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### Extensionality

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### *Pairing*

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### *Separation*

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### *Union*

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### *Power*

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### *Regularity*

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### *Replacement (?)*

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### *Choice (?)*

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# 5

# *Arithmetic*

# 6

# *Ancient Number Theory*

# 7

# *Combinatorics*

# 8

# Asymptotics

# 9

# *Infinity*

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$\rightarrow \varnothing$

$$\varnothing \sim \varnothing \sim \varnothing$$

$$(0 \leqslant 1) \wedge \left(2^{2^{2^2}} \geqslant \sqrt{\sqrt{2}}\right)$$

$$p \leftrightarrow q$$

$$\varphi \Rightarrow \psi$$

$$\varphi \Leftrightarrow \psi$$

$$\Gamma \equiv (\xi \land \chi) \lor \rho$$

$$M \models \mathbb{R}$$

$$p_0, ..., p_{n-1} \vdash q_0, ..., q_{m-1}$$

$$p_0, ..., p_{n-1} \vdash q_0, ..., q_{m-1}$$

$$(\forall n \in \mathbb{N})(s(n) > n)$$

$$(\forall n)(s(n) > n)$$