Problem Set 1

Discrete Mathematics

Due: 27th of January, 2023

1. Which of the following are propositions and which are not? Justify your answer with an argument.
   (a) Santa Claus is real and he lives in the South Pole.
   (b) This sentence is true.
   (c) If this sentence is false, then 7 is a prime number.
   (d) This sentence is false.
   (e) Every proposition is either true or false.
   (f) If this sentence is true, then 12 is a prime number.
   (g) The capitial of Spain is Madrid.
   (h) There is a set x that contains itself as an element.
   (i) The set of all sets that do not contain themselves contains itself.
   (j) $\sqrt{2}$ is a morally complicated number.
2. For problem 2 and problem 3, consider the following definitions:

$$
\begin{aligned}
N &\coloneqq \text{“There is an alien aboard the Nostromo.”} \\
E &\coloneqq \text{“The crewmembers go on an expedition.”} \\
A &\coloneqq \text{“A crewmember was infected by an alien.”} \\
R &\coloneqq \text{“The crewmembers return from their expedition.”} \\
S &\coloneqq \text{“The crewmembers survive.”} \\
K &\coloneqq \text{“Ripley kills an alien.”}
\end{aligned}
$$

   Translate the following English-language sentences into the propositional logic:
   (a) If there is no alien aboard the Nostromo, then Ripley will not kill an alien.
   (b) The crewmembers went on an expedition but did not return.
   (c) Unless Ripley kills the alien, the crewmembers won't survive.
   (d) A crewmember was infected by an alien and then the crewmembers returned from their expedition.
   (e) Although there is an alien aboard the Nostromo, the crewmembers will survive.
   (f) It is sufficient that Ripley kills an alien for the crewmembers to survive.
   (g) It is necessary that Ripley kills an alien for the crewmembers to survive.
   (h) Whether or not Ripley kills an alien, the crewmembers will not survive.
   (i) It is not the case that there is an alien aboard the Nostromo or Ripley kills an alien.
   (j) The crewmembers went on an expedition – wherein a crewmember was infected by an alien – and then returned, so an alien is aboard the Nostromo; yet, the crewmembers will survive.
3. Translate the following sentences of the propositional logic into English:
   (a) $\neg K \to \neg S$
   (b) $\neg(S \to K)$
   (c) $(N \wedge K) \to S$
   (d) $(R \vee \neg R) \wedge \neg(R \wedge \neg R)$
   (e) $R \leftrightarrow N$