

# PROBLEM SET 3

DISCRETE MATHEMATICS

Due: 10<sup>th</sup> of February, 2023

1. Show that the following arguments are valid.

**Remark.** Every application of a rule of inference must be explicitly referenced by name for these subproblems.

(a) 
$$\frac{\varphi \rightarrow \psi}{\varphi \Rightarrow q}$$
 known as **Conditional Elimination**

(b) 
$$\frac{\varphi \rightarrow \psi \quad \psi \rightarrow \chi}{\varphi \rightarrow \chi}$$
 known as the **Hypothetical Syllogism**

(c) 
$$\frac{\varphi \quad \psi}{\varphi \wedge \psi}$$
 known as **Adjunction**, *a.k.a.* **Conjunction Introduction**

(d) 
$$\frac{\varphi \wedge \psi}{\varphi}$$
 known as **Simplification**, *a.k.a.* **Conjunction Elimination**

(e) 
$$\frac{\varphi}{\varphi \vee \psi}$$
 known as **Addition**, *a.k.a.* **Disjunction Introduction**

(f) 
$$\frac{\varphi \rightarrow \chi \quad \psi \rightarrow \chi \quad \varphi \vee \psi}{\chi}$$
 known as **Proof by Cases**, *a.k.a.* **Disjunction Elimination**

(g) 
$$\frac{\varphi \vee \psi \quad \neg \varphi}{\psi}$$
 known as the **Disjunctive Syllogism**

(h) 
$$\frac{\varphi \rightarrow \chi \quad \psi \rightarrow \xi \quad \varphi \vee \psi}{\chi \vee \xi}$$
 known as the **Constructive Dilemma**

(i) 
$$\frac{\varphi \quad \neg \varphi}{\psi}$$
 known as the **Ex Falso Quodlibet**, *a.k.a.* the **Principle of Explosion**

(j) 
$$\frac{\varphi \leftrightarrow \psi \quad \varphi}{\psi}$$
 known as the **Ex Falso Quodlibet**, *a.k.a.* the **Principle of Explosion**

2. Imagine a universe of discourse consisting of the collection of sentient humanoid beings (*e.g.*, people, humans, androids) in the year 2029 in Japan. Now, consider the following facts:

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|--|---|
| I. Every ghost in a shell is an android. | IV. No androids are people.             |
| II. Some androids are ghosts in shells.  | V. Major Kusanagi (草薙素子) is an android. |
| III. All humans are people.              | VI. Togusa (トグサ) is a human.            |

- (a) Translate these six statements into the first-order logic by defining appropriate predicates.
- (b) Translate each of the following English sentences into the first-order logic using the definitions made in **subproblem 2(a)** and then determine whether or not they follow from the facts given. If you think they do follow, then prove the argument is valid. Otherwise, provide an informal argument of invalidity.
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|--|---|
| i. Major Kusanagi is a person.                         | i. No ghost in a shell is a person.         |
| ii. Someone is a ghost in a shell.                     | ii. There is a human who is not an android. |
| iii. Togusa is an android if he is a ghost in a shell. |   |