NAME:	NETID:

## Exam 2.p.2024.spring

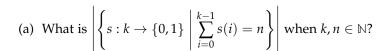
## Discrete Mathematics

1



26	of April, 2024	
A	nswer the following questions by marking either True or False.	
(a)	Every function is either injective or surjective.     True	
	○ False	
(b)	If $ X  = n \in \mathbb{N}$ , then there are $n!$ surjections from $X$ to $X$ .	
	<ul><li>True</li><li>False</li></ul>	
(c)	There is a set the same size as its power set.	
	<ul><li>True</li><li>False</li></ul>	
(d)	Every random number generator must eventually repeat a number.	
	<ul><li>True</li><li>False</li></ul>	
(e)	$\forall A \forall B \Big(  A  =  B  \Leftrightarrow (\forall f : A \to B) \big( (\forall a_1, a_2 \in A) (a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)) \Leftrightarrow (\forall b \in B) (\exists a \in A) (f(a) = b) \Big) \Big)$	
	<ul><li>True</li><li>False</li></ul>	
(f)	There are countably many eventually periodic decimal strings.	A string $f: \mathbb{N} \to X$ is called <i>periodic</i>
	<ul><li>True</li><li>False</li></ul>	if $(\forall n \in \mathbb{N})(f(n) = f(n+p))$ for some $p \in \mathbb{N}$ called the <i>period</i> of $f$ . A string $f$ is called <i>eventually periodic</i> if $f = s + t$
(g)	If $P : \mathbb{N} \to 10$ is <i>aperiodic</i> , every finite decimal string appears in $P$ .	where $s$ is finite and $t$ is periodic. A string that is not eventually periodic is <i>aperiodic</i> .
	<ul><li>True</li><li>False</li></ul>	
(h)	Let <i>X</i> and <i>Y</i> be sets. If $f: X \to Y$ , then $ f  =  X $ .	
	<ul><li>True</li><li>False</li></ul>	
(i)	If $\varphi_e$ is Euler's totient function, then $\varphi_e(99) = 80$ .	
	<ul><li>True</li><li>False</li></ul>	
(j)	There are countably many propositions.	
	<ul><li>True</li><li>False</li></ul>	

2 Answer the following questions without proof.



(b) How many ways are there to scramble the string "caesar"?

(c) How many even natural numbers can be written using  $k \in \mathbb{N}$  decimal digits such that  $k \ge 2$  and no consecutive digits are repeated?

(d) What is  $\left| \left\{ S \subseteq \{ z \in \mathbb{N} \mid 1 \leqslant z \leqslant 50 \} \mid (\forall x, y \in S) \left( |x - y| > 25 \right) \right\} \right|$ ?

(e) Given  $k \in \mathbb{N}$  such that  $k \ge 3$ , how many ways are there to write k as a sum of positive integers *without* using the numbers 1 and 2?

NAME:	NETID:

3 You may rely on any theorems we have proven or studied.

An archaeologist on a recent expedition has discovered, for each  $n \in \mathbb{N}_+$ , a manuscript written in a language called nglish, seemingly used by the native inhabitants of ngland in ancient times. Each manuscript is exactly n pages long and contains precisely 2n distinct words.

Prove each manuscript contains a page with two distinct words on it.

NAME:	NETID:

4 You may rely on any theorems we have proven or studied.

Prove there are uncountably many infinite strings of prime numbers.