

# PROBLEM SET 4

DISCRETE MATHEMATICS

Due: 22<sup>nd</sup> of February, 2023

1. (a) Show that  $\forall x(\emptyset \subseteq x)$ .  
(b) Show that  $\forall x(x \subseteq x)$ .  
(c) Show that  $\forall x(x \subseteq \emptyset \Rightarrow x = \emptyset)$ .  
(d) Show that  $(x = y) \Leftrightarrow (x \subseteq y \wedge y \subseteq x)$  for all  $x, y$ .
2. (a) Show that  $x \cap y \subseteq x$  for all  $x, y$ .  
(b) Show that  $x \subseteq x \cup y$  for all  $x, y$ .  
(c) Show that  $\forall x \forall y (x \cap y \subseteq x \cup y)$ .
3. Prove that  $\forall x \forall y \forall z ((x \subseteq y \wedge y \subseteq z) \Rightarrow x \subseteq z)$ .
4. The *difference* of two sets  $x$  and  $y$  is given by  $x \setminus y := \{a \mid a \in x \wedge a \notin y\}$ .  
(a) Show that  $x \setminus y$  exists for any two sets  $x, y$ .  
(b) Is it the case that  $(x \setminus y \subseteq x)$  for any sets  $x, y$ ? Prove your assertion in either case.
5. Recall that  $A \times B := \{(a, b) \mid a \in A \wedge b \in B\}$ .  
(a) Show that  $\forall A (A \times \emptyset = \emptyset)$ .  
(b) Is it the case that  $A \times B = B \times A$  for any sets  $A, B$ ? Prove your assertion in either case.
6. Prove that  $x \subseteq \mathcal{S}(x)$  for any  $x$ .