Problem Set 7

Discrete Mathematics

Due: 11th of April, 2023

1. For this problem, let A, B, and C be sets and consider two functions $f : A \to B$ and $g : B \to C$.
   (a) Show that, if f and g are both injections, then $g \circ f$ is an injection.
   (b) Show that, if f and g are both surjections, then $g \circ f$ is a surjection.
   (c) Show that, if f and g are both bijections, then $g \circ f$ is a bijection.
2. Prove that every finite set is countable.
3. Let A and B be countable sets.
   (a) Show that $A \cap B$ is countable.
   (b) Show that $A \cup B$ is countable.
4. **Bonus:** Suppose $A_0, A_1, \ldots A_i, \ldots$ is a countably-infinite collection of disjoint countable sets. Show that their union is countable.
5. Let $X \coloneqq \left\{x_0, \ldots x_{k-1}\right\}$ be a set of cardinality $k \in \mathbb{N}$. How many strings of length $n \in \mathbb{N}$ over X are there? Once you have a conjecture, *define* the appropriate bijection (you do not need to prove its bijectivity).
6. Let X be a finite set with $|X| = |n| \in \mathbb{N}$. Show that $|\mathcal{P}(X)| = |2^n|$.
7. Recall that $|\mathbb{N}| = \aleph_0$. Is the cardinality of its power set $\mathcal{P}(\mathbb{N})$ lesser, greater, or equal? Prove your answer.
8. Prove the set of infinite strings over the Hawaiian alphabet $\mathscr{H} \coloneqq \{a, e, i, o, u, h, k, l, m, n, p, w\}$ is uncountable.