Problem Set 1

Discrete Mathematics

Due on the 28th of January, 2024

Justify each of your answers with an argument.[1]

[1] An answer provided with bad or no justification is as good as a wrong answer. Think carefully, and think deeply.

(50 pts) 1. Determine the truth value of each sentence below.

(a) "Madrid is the capital of Spain."

(b) "Santa Claus lives on the north pole."

(c) "This sentence is false."

(d) "The set of all sets that don't contain themselves contains itself."[1]

(e) "Red is a beautiful color."

(f) "Every proposition is either *true* or *false* but not both."

(g) "If this sentence is false, then 7 is a prime number."[2]

(h) "The set of all sets contains itself."

(i) "This sentence is *true.*"

(j) "If this sentence is *true,* then 2 is an odd number."[3]

[1] A *set* is a collection of objects. When we talk about *"the set of all x with a property,"* we mean the collection of all those x that have that property and *only* those x.

[2] Note that 7 really is a prime number.

[3] Note that 2 is not really an odd number.

(25 pts) 2. Suppose we have an infinite sequence of sentences

$$S_0, S_1, S_2, \ldots S_i, \ldots$$

where each sentence asserts that every sentence following it is *false.*

$$S_i := \text{"}S_k \text{ is } \textit{false} \text{ for all } k > i\text{."}$$

In this definition, i ranges over all of the natural numbers $0, 1, 2, \ldots$

Are any of the sentences in this sequence propositions? If so, which?

(25 pts) 3. In the sentence below, *"you"* refers to *you,* the student reading these sentences and solving this problem set. Determine the truth value of the following sentence.

"You have finitely many beliefs."