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Exam 2.p.2024.spring

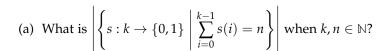
Discrete Mathematics

26th of April, 2024



2	6" of April, 2024	
1 A	nswer the following questions by marking either True or False.	
(a)	Every function is either injective or surjective.	
	O True	
	○ False	
(b)	If $ X = n \in \mathbb{N}$, then there are $n!$ surjections from X to X .	
	TrueFalse	
(c)	There is a set the same size as its power set.	
	○ True○ False	
(d)	Every random number generator must eventually repeat a number.	
	○ True○ False	
(e)	$\forall A \forall B \Big(A = B \Leftrightarrow (\forall f : A \to B) \big((\forall a_1, a_2 \in A) (a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_1) \big) \Big)$	(a_2) \Leftrightarrow $(\forall b \in B)(\exists a \in A)(f(a) = b))$
	TrueFalse	,
(f)	There are countably many eventually periodic decimal strings.	A string $f: \mathbb{N} \to X$ is called <i>periodic</i>
	○ True	if $(\forall n \in \mathbb{N})(f(n) = f(n+p))$ for some $p \in \mathbb{N}$ called the <i>period</i> of f . A string
	○ False	f is called <i>eventually periodic</i> if $f = s + t$ where s is finite and t is periodic. A string
(g)	If $P : \mathbb{N} \to 10$ is <i>aperiodic</i> , every finite decimal string appears in P .	that is not periodic is called <i>aperiodic</i> .
	○ True○ False	
(h)	Let <i>X</i> and <i>Y</i> be sets. If $f: X \to Y$, then $ f = X $.	
	TrueFalse	
(i)	If φ_e is Euler's totient function, then $\varphi_e(99) = 80$.	
	TrueFalse	
(j)	There are countably many propositions.	
	○ True○ False	

2 Answer the following questions without proof.



(b) How many ways are there to scramble the string "caesar"?

(c) How many even natural numbers can be written using $k \in \mathbb{N}$ decimal digits such that $k \ge 2$ and no consecutive digits are repeated?

(d) What is $\left| \left\{ S \subseteq \{ z \in \mathbb{N} \mid 1 \leqslant z \leqslant 50 \} \mid (\forall x, y \in S) \left(|x - y| > 25 \right) \right\} \right|$?

(e) Given $k \in \mathbb{N}$ such that $k \ge 3$, how many ways are there to write k as a sum of positive integers *without* using the numbers 1 and 2?

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3 You may rely on any theorems we have proven or studied.

An archaeologist on a recent expedition has discovered, for each $n \in \mathbb{N}_+$, a manuscript written in a language called nglish, seemingly used by the native inhabitants of ngland in ancient times. Each manuscript is exactly n pages long and contains precisely 2n distinct words.

Prove each manuscript contains a page with two distinct words on it.

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4 You may rely on any theorems we have proven or studied.

Prove there are uncountably many infinite strings of prime numbers.