Problem Set 6

Discrete Mathematics

Due on the 5th of March, 2024

All basic arithmetic and algebraic facts about $\mathbb{N}$ and $\mathbb{Z}$ are now yours to use.

(20 pts) 1.(a) Show that $\big(c \neq 0 \wedge ac \mid bc\big) \Rightarrow \big(a \mid b\big)$ for all $a, b, c \in \mathbb{Z}$.

(b) Show that $\big(n \mid x \wedge n \mid y\big) \Rightarrow \big(n \mid ax + by\big)$ for all $n, x, y, a, b \in \mathbb{Z}$.

(20 pts) 2. For all $z \in \mathbb{Z}$, show that z is even implies z is not odd.

(20 pts) 3.(a) For all $n \in \mathbb{N}$, show that n is even implies $n + 1$ is odd.

(b) For all $n \in \mathbb{N}$, show that n is odd implies $n + 1$ is even.

(20 pts) 4. Show that $3 \mid n^3 - n$ for all $n \in \mathbb{N}$.[1]

[1] *Hint: try a proof by induction.*

(20 pts) 5. The *Fibonacci sequence* is the recursive function $\mathcal{F} : \mathbb{N} \to \mathbb{N}$ below.

$$\begin{aligned} \mathcal{F}(0) &:= 0 \\ \mathcal{F}(1) &:= 1 \\ \mathcal{F}(n+2) &:= \mathcal{F}(n+1) + \mathcal{F}(n) \end{aligned}$$

Show that $1 + \sum_{i=0}^{n} \mathcal{F}(i) = \mathcal{F}(n+2)$ for all $n \in \mathbb{N}$.