

Problem Set 1

Discrete Mathematics

Due on the 28th of January, 2024

Justify each of your answers with an argument.¹

(50 pts) 1. Determine the truth value of each sentence below.

- (a) "Madrid is the capital of Spain."
- (b) "Santa Claus lives on the north pole."
- (c) "This sentence is false."
- (d) "The set of all sets that don't contain themselves contains itself."¹
- (e) "Red is a beautiful color."
- (f) "Every declarative sentence is either *true* or *false* but not both."
- (g) "If this sentence is false, then 7 is a prime number."²
- (h) "The set of all sets contains itself."
- (i) "This sentence is *true*."
- (j) "If this sentence is *true*, then 2 is an odd number."³

¹ An answer provided with bad or no justification is as good as a wrong answer. Think carefully, and think deeply.

¹ A *set* is a collection of objects. When we talk about "*the set of all x with a property*," we mean the collection of all those x that have that property and *only* those x .

² Note that 7 really is a prime number.

³ Note that 2 is not really an odd number.

(25 pts) 2. Suppose we have an infinite sequence of sentences

$$S_0, S_1, S_2, \dots, S_i, \dots$$

where each sentence asserts that every sentence following it is *false*.

$$S_i := "S_k \text{ is false for all } k > i."$$

In this definition, i ranges over all of the natural numbers $0, 1, 2, \dots$

What are the truth values of the sentences in this sequence?

(25 pts) 3. In the sentence below, "*you*" refers to *you*, the student reading these sentences and solving this problem set. Determine the truth value of the following sentence.

"You have finitely many beliefs."