

## Problem Set 8

### Discrete Mathematics

Due on the 4<sup>th</sup> of April, 2024

- (10 pts) 1. Including the instructor, there are 32 people in our class. Prove that two of these people were born on the same day of the month.
- (10 pts) 2. As of the 28<sup>th</sup> of March, 2024, there are over 8.1 billion people living on Earth.<sup>1</sup> A person's heart will beat no more than  $7 \times 10^9$  times over their lifespan. Show that there are two currently-living people on Earth whose hearts have beat the exact same number of times.
- (20 pts) 3. Let  $n \in \mathbb{N}_+$  and consider  $\mathcal{A} \subseteq \mathbb{N}$  such that  $|\mathcal{A}| = n + 1$ . Prove there exist  $x, y \in \mathcal{A}$  with  $x \neq y$  such that  $n \mid x - y$ .
- (20 pts) 4. Consider  $\mathcal{S} := \{3, 4, 7, 8, 9, 10, 12, 15, 18, 19, 27, 28\}$  and  $\mathcal{X} \subseteq \mathcal{S}$  with  $|\mathcal{X}| \geq 9$ . Show that there exist three *distinct* elements  $x_1, x_2, x_3 \in \mathcal{X}$  such that  $x_1 + x_2 + x_3 = 40$ .
- (20 pts) 5. Recall that  $\binom{n}{0} = \binom{n}{n} = 1$  for all  $n, k \in \mathbb{N}$  when  $k \leq n$ .
- (a) Show  $\binom{n}{k} = \binom{n}{n-k}$  for all  $n, k \in \mathbb{N}$  where  $k \leq n$ .
- (b) Show  $\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$  for all  $n, k \in \mathbb{N}$  where  $k \leq n$ .
- (20 pts) 6. Prove that  $|\mathbb{P}(X)| = 2^{|X|}$  for any finite set  $X$ .

<sup>1</sup> For simplicity, you may assume the human population will only monotonically increase over time from the point this problem set was assigned.