

Problem Set 4

Discrete Mathematics

Due on the 20th of February, 2024

(20 pts) 1.(a) Show $\forall x(\emptyset \subseteq x)$.

(b) Show $\forall x(x \subseteq x)$.

(c) Show $\forall x(\emptyset \in \mathbb{P}(x))$.

(d) Show $\forall x(x \in \mathbb{P}(x))$.

(e) Show $\forall x \forall y \forall z ((x \subseteq y) \wedge (y \subseteq z)) \Rightarrow x \subseteq z$.

(10 pts) 2. We define the *intersection* and *difference* of any two sets x and y below.

$$x \cap y := \{z \mid z \in x \wedge z \in y\}$$

$$x \setminus y := \{z \mid z \in x \wedge z \notin y\}$$

(a) Show $\forall x \forall y \exists z (z = x \cap y)$.

(b) Show $\forall x \forall y \exists z (z = x \setminus y)$.

(20 pts) 3. We define the *union* of any two sets x and y below.

$$x \cup y := \{z \mid z \in x \vee z \in y\}$$

(a) Show $\forall x \forall y (x \cap y \subseteq x)$.

(b) Show $\forall x \forall y (x \subseteq x \cup y)$.

(c) Show $\forall x \forall y (\mathbb{P}(x) \cup \mathbb{P}(y) \subseteq \mathbb{P}(x \cup y))$.

(d) Show $\forall x \forall y (x \cap y = x \Leftrightarrow x \in \mathbb{P}(y))$.

(50 pts) 4. We define the *union over x* and *intersection over x* for any set x below.

$$\cup x := \{z \mid \exists y (y \in x \wedge z \in y)\}$$

$$\cap x := \{z \mid \forall y (y \in x \Rightarrow z \in y)\}$$

(a) Show that $\forall x (\cup \mathbb{P}(x) = x)$.

(b) What is $\cup \emptyset$? Justify your answer with a proof.

(c) What is $\cap \emptyset$? Justify your answer with a proof.

(d) Is $\emptyset = \{z \mid z \in \emptyset\}$? Justify your answer with a proof.

(e) Is $\emptyset = \{z \mid z \notin \emptyset\}$? Justify your answer with a proof.