Problem Set 3

Discrete Mathematics

Due: 13^{th} of February, 2023

1. Implement the following sentences as functions using `Python`. Each function should take as many inputs (of type `bool`) as there are propositional variables in the sentences, and should return the correct truth value for the sentence when its variables are replaced by your function's inputs.

   For example, $\neg(p \land q)$ could be rewritten as `f` in two ways:

   ```
   def f(p, q):
       return not(p and q)
   ```

   ```
   f = lambda p, q: not(p and q)
   ```

   Your functions *must be named* `ps03pr1a`, `ps03pr1b`, `ps03pr1c`, `ps03pr1d`, and `ps03pr1e`, respectively. Turn in your code via email as *one* separate `Python` file named `ps03-<lastname>-<firstname>.py` with your work.

   (a) $(\neg p \to \bot) \to p$

   (b) $(p \to q) \leftrightarrow (\neg q \to \neg p)$

   (c) $p \to (q \to r)$

   (d) $(p \to q) \to r$

   (e) $\big((p \lor q) \land (\neg p \lor r)\big) \to (q \lor r)$

2. Show that the following arguments are valid.

   Remark. Every application of a rule of inference must be explicitly referenced by name for these subproblems.

   (a) $\dfrac{\varphi \to \psi}{\varphi \Rightarrow \psi}$ known as **Conditional Elimination**

   (b) $\dfrac{\begin{array}{l}\varphi \to \psi \\ \psi \to \chi\end{array}}{\varphi \to \chi}$ known as the **Hypothetical Syllogism**

   (c) $\dfrac{\begin{array}{l}\varphi \\ \psi\end{array}}{\varphi \land \psi}$ known as **Adjunction**, *a.k.a.* **Conjunction Introduction**

   (d) $\dfrac{\varphi \land \psi}{\varphi}$ known as **Simplification**, *a.k.a.* **Conjunction Elimination**

   (e) $\dfrac{\varphi}{\varphi \lor \psi}$ known as **Addition**, *a.k.a.* **Disjunction Introduction**

   (f) $\dfrac{\begin{array}{l}\varphi \to \chi \\ \psi \to \chi \\ \varphi \lor \psi\end{array}}{\chi}$ known as **Proof by Cases**, *a.k.a.* **Disjunction Elimination**

   (g) $\dfrac{\begin{array}{l}\varphi \lor \psi \\ \neg\varphi\end{array}}{\psi}$ known as the **Disjunctive Syllogism**

   (h) $\dfrac{\begin{array}{l}\varphi \to \chi \\ \psi \to \xi \\ \varphi \lor \psi\end{array}}{\chi \lor \xi}$ known as the **Constructive Dilemma**

   (i) $\dfrac{\begin{array}{l}\varphi \\ \neg\varphi\end{array}}{\psi}$ known as the **Ex Falso Quodlibet**, *a.k.a.* the **Principle of Explosion**

   (j) $\dfrac{\begin{array}{l}\varphi \leftrightarrow \psi \\ \varphi\end{array}}{\psi}$ known as **Biconditional Elimination**

3. Imagine a universe of discourse consisting of the collection of sentient humanoid beings (*e.g.*, people, humans, androids) in the year 2029 in Japan. Now, consider the following facts:

   I. Every ghost in a shell is an android.
   II. Some androids are ghosts in shells.
   III. All humans are people.
   IV. No androids are people.
   V. Major Kusanagi (草薙素子) is an android.
   VI. Togusa (トグサ) is a human.

   (a) Translate these six statements into the first-order logic by defining appropriate predicates.

   (b) Translate each of the following English sentences into the first-order logic using the definitions made in subproblem 2(a) and then determine whether or not they follow from the facts given. If you think they do follow, then prove the argument is valid. Otherwise, provide an informal argument of invalidity.

      i. Major Kusanagi is a person.
      ii. Someone is a ghost in a shell.
      iii. Togusa is an android if he is a ghost in a shell.
      iv. No ghost in a shell is a person.
      v. There is a human who is not an android.