Problem Set 4
Discrete Mathematics
Due: 20$^{\text{th}}$ of February, 2023

1.   (a) Show that $\forall x(\varnothing \subseteq x)$.

     (b) Show that $\forall x(x \subseteq x)$.

     (c) Show that $\forall x(x \subseteq \varnothing \Rightarrow x = \varnothing)$.

     (d) Show that $(x = y) \Leftrightarrow (x \subseteq y \wedge y \subseteq x)$ for all x, y.

2.   (a) Show that $x \cap y \subseteq x$ for all x, y.

     (b) Show that $x \subseteq x \cup y$ for all x, y.

     (c) Show that $\forall x \forall y(x \cap y \subseteq x \cup y)$.

3. Prove that $\forall x \forall y \forall z\big((x \subseteq y \wedge y \subseteq z) \Rightarrow x \subseteq z\big)$.

4. The *difference* of two sets x and y is given by $x \setminus y := \{a \mid a \in x \wedge a \notin y\}$.

     (a) Show that $x \setminus y$ exists for any two sets x, y.

     (b) Is it the case that $(x \setminus y \subseteq x)$ for any sets x, y? Prove your assertion in either case.

5. Recall that $A \times B := \big\{(a, b) \mid a \in A \wedge b \in B\big\}$.

     (a) Show that $\forall A(A \times \varnothing = \varnothing)$.

     (b) Is it the case that $A \times B = B \times A$ for any sets A, B? Prove your assertion in either case.

6. Prove that $x \subseteq \mathcal{S}(x)$ for any x.