

PROBLEM SET 4

DISCRETE MATHEMATICS

Due: 24th of February, 2023

1. (a) Show that $\forall x(\emptyset \subseteq x)$.
(b) Show that $\forall x(x \subseteq x)$.
(c) Show that $\forall x(x \subseteq \emptyset \Rightarrow x = \emptyset)$.
(d) Show that $(x = y) \Leftrightarrow (x \subseteq y \wedge y \subseteq x)$ for all x, y .
2. (a) Show that $x \cap y \subseteq x$ for all x, y .
(b) Show that $x \subseteq x \cup y$ for all x, y .
(c) Show that $\forall x \forall y (x \cap y \subseteq x \cup y)$.
3. Prove that $\forall x \forall y \forall z ((x \subseteq y \wedge y \subseteq z) \Rightarrow x \subseteq z)$.
4. The *difference* of two sets x and y is given by $x \setminus y := \{a \mid a \in x \wedge a \notin y\}$.
(a) Show that $x \setminus y$ exists for any two sets x, y .
(b) Is it the case that $(x \setminus y \subseteq x)$ for any sets x, y ? Prove your assertion in either case.
5. Recall that $A \times B := \{(a, b) \mid a \in A \wedge b \in B\}$.
(a) Show that $\forall A (A \times \emptyset = \emptyset)$.
(b) Is it the case that $A \times B = B \times A$ for any sets A, B ? Prove your assertion in either case.
6. Prove that $x \subseteq \mathcal{S}(x)$ for any x .