Problem Set 7
Discrete Mathematics
Due: 7th of April, 2023

1. For this problem, let A, B, and C be sets and consider two functions $f : A \to B$ and $g : B \to C$.

   (a) Show that, if f and g are both injections, then $g \circ f$ is an injection.

   (b) Show that, if f and g are both surjections, then $g \circ f$ is a surjection.

   (c) Show that, if f and g are both bijections, then $g \circ f$ is a bijection.

2. Prove that every finite set is countable.

3. Let A and B be countable sets.

   (a) Show that $A \cap B$ is countable.

   (b) Show that $A \cup B$ is countable.

4. Suppose $A_0, A_1, \ldots A_i, \ldots$ is a countably-infinite collection of countable sets such that $i \neq j \implies A_i \cap A_j = \varnothing$. Show that $\bigcup_{i=0}^{\infty} A_i$ is countable.

5. Let $X := \{x_0, \ldots x_{k-1}\}$ be a set of size $k \in \mathbb{N}$. How many strings of length $n \in \mathbb{N}$ over X are there? Prove it.

6. Let X be a finite set with $|X| = n \in \mathbb{N}$. Show that $|\mathcal{P}(X)| = 2^n$.

7. Recall that $|\mathbb{N}| = \aleph_0$. Is the cardinality of its power set $\mathcal{P}(\mathbb{N})$ lesser, greater, or equal? Prove your answer.

8. Prove the set of infinite strings over the Hawaiian alphabet $\mathscr{H} := \{a, e, i, o, u, h, k, l, m, n, p, w\}$ is uncountable.