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Exam 2.p.2024.spring

Discrete Mathematics

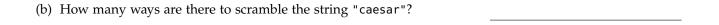
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26	of April, 2024	
A	nswer the following questions by marking either True or False.	
(a)	Every function is either injective or surjective. True	
	○ False	
(b)	If $ X = n \in \mathbb{N}$, then there are $n!$ surjections from X to X .	
	TrueFalse	
(c)	There is a set the same size as its power set.	
	TrueFalse	
(d)	Every random number generator must eventually repeat a number.	
	TrueFalse	
(e)	$\forall A \forall B \Big(A = B \Leftrightarrow (\forall f : A \to B) \big((\forall a_1, a_2 \in A) (a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)) \Leftrightarrow (\forall b \in B) (\exists a \in A) (f(a) = b) \Big) \Big)$	
	TrueFalse	
(f)	There are countably many eventually periodic decimal strings.	A string $f: \mathbb{N} \to X$ is called <i>periodic</i>
	TrueFalse	if $(\forall n \in \mathbb{N})(f(n) = f(n+p))$ for some $p \in \mathbb{N}$ called the <i>period</i> of f . A string f is called <i>eventually periodic</i> if $f = s + t$
(g)	If $P : \mathbb{N} \to 10$ is <i>aperiodic</i> , every finite decimal string appears in P .	where s is finite and t is periodic. A string that is not eventually periodic is <i>aperiodic</i> .
	TrueFalse	
(h)	Let <i>X</i> and <i>Y</i> be sets. If $f: X \to Y$, then $ f = X $.	
	TrueFalse	
(i)	If φ_e is Euler's totient function, then $\varphi_e(99) = 80$.	
	TrueFalse	
(j)	There are countably many propositions.	
	TrueFalse	

2 Answer the following questions without proof.

(a) What is
$$\left| \left\{ s : k \to \{0,1\} \mid \sum_{i=0}^{k-1} s(i) = n \right\} \right|$$
 when $k, n \in \mathbb{N}$?



(c) How many even natural numbers can be written using
$$k \in \mathbb{N}$$
 decimal digits such that $k \ge 2$ and no consecutive digits are repeated?

(d) What is
$$\left| \left\{ S \subseteq \{1, 2, \dots, 50\} \mid (\forall x, y \in S) \left(x \neq y \Rightarrow |x - y| > 25 \right) \right\} \right|$$
?

(e) Given
$$k \in \mathbb{N}$$
 such that $k \ge 3$, how many ways are there to write k as a sum of positive integers *without* using the numbers 1 and 2?

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3 You may rely on any theorems we have proven or studied.

An archaeologist on a recent expedition has discovered, for each $n \in \mathbb{N}_+$, a manuscript written in a language called nglish, seemingly used by the native inhabitants of ngland in ancient times. Each manuscript is exactly n pages long and contains precisely 2n distinct words.

Prove each manuscript contains a page with two distinct words on it.

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4 You may rely on any theorems we have proven or studied.

Prove there are uncountably many infinite strings of prime numbers.