

# PROBLEM SET 1

## DISCRETE MATHEMATICS

Due: 27<sup>th</sup> of January, 2023

1. Which of the following are propositions and which are not? Justify your answer with an argument.

- (a) Santa Claus is real and he lives in the South Pole.
- (b) This sentence is true.
- (c) If this sentence is false, then 7 is a prime number.
- (d) This sentence is false.
- (e) Every proposition is either true or false.
- (f) If this sentence is true, then 12 is a prime number.
- (g) The capital of Spain is Madrid.
- (h) There is a set  $x$  that contains itself as an element.
- (i) The set of all sets that do not contain themselves contains itself.
- (j)  $\sqrt{2}$  is a morally complicated number.

2. For **problem 2** and **problem 3**, consider the following definitions:

$N :=$  “There is an alien aboard the Nostromo.”

$E :=$  “The crewmembers go on an expedition.”

$A :=$  “A crewmember was infected by an alien.”

$R :=$  “The crewmembers return from their expedition.”

$S :=$  “The crewmembers survive.”

$K :=$  “Ripley kills an alien.”

Translate the following English-language sentences into the propositional logic:

- (a) If there is no alien aboard the Nostromo, then Ripley will not kill an alien.
- (b) The crewmembers went on an expedition but did not return.
- (c) Unless Ripley kills the alien, the crewmembers won’t survive.
- (d) A crewmember was infected by an alien and then the crewmembers returned from their expedition.
- (e) Although there is an alien aboard the Nostromo, the crewmembers will survive.
- (f) It is sufficient that Ripley kills an alien for the crewmembers to survive.
- (g) It is necessary that Ripley kills an alien for the crewmembers to survive.
- (h) Whether or not Ripley kills an alien, the crewmembers will not survive.
- (i) It is not the case that there is an alien aboard the Nostromo or Ripley kills an alien.
- (j) The crewmembers went on an expedition – wherein a crewmember was infected by an alien – and then returned, so an alien is aboard the Nostromo; yet, the crewmembers will survive.

3. Translate the following sentences of the propositional logic into English:

- (a)  $\neg K \rightarrow \neg S$
- (b)  $\neg(S \rightarrow K)$
- (c)  $(N \wedge K) \rightarrow S$
- (d)  $(R \vee \neg R) \wedge \neg(R \wedge \neg R)$
- (e)  $R \leftrightarrow N$