

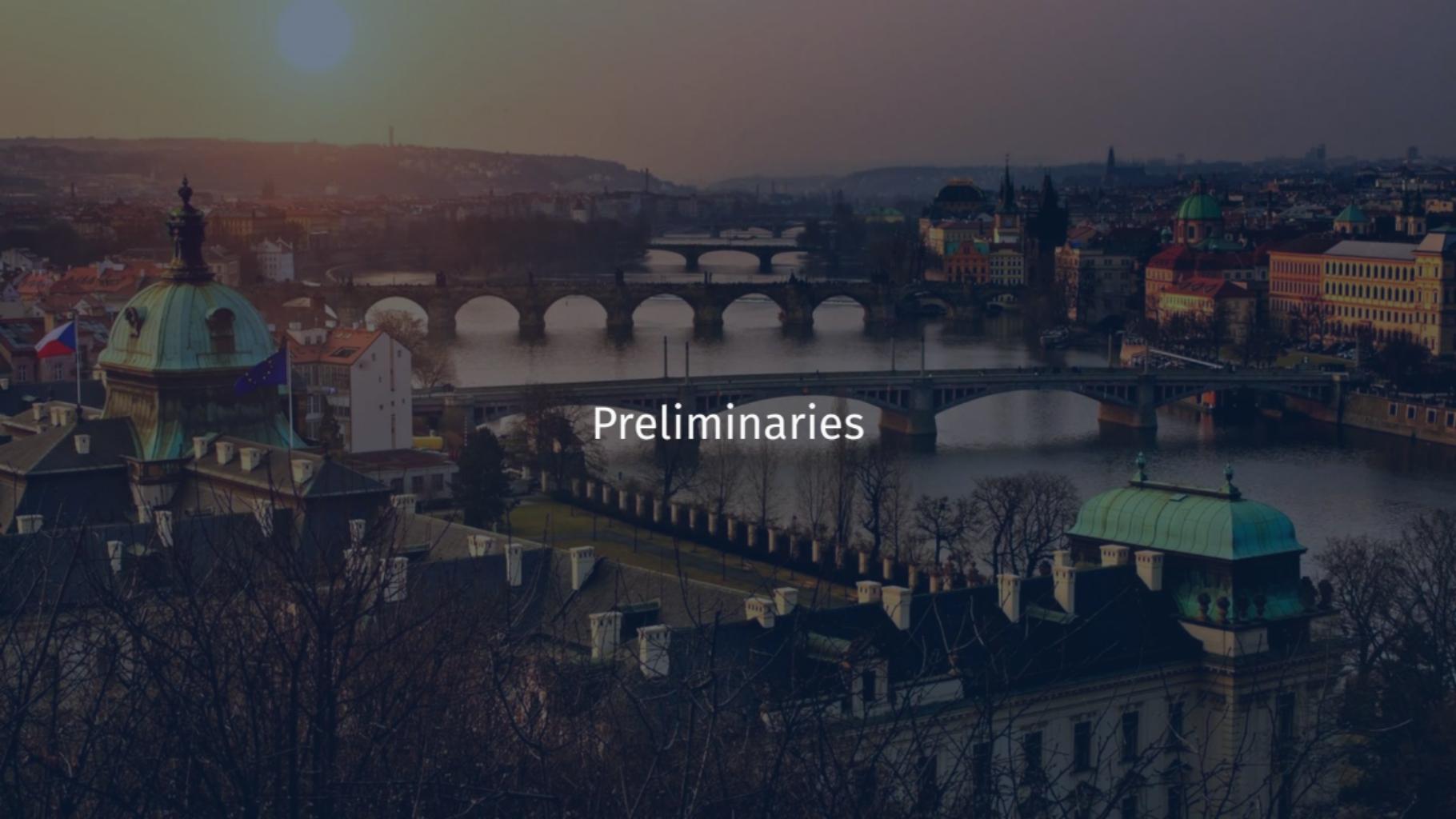
THE INFINITY MIRROR TEST FOR GRAPH GENERATORS

Satyaki Sikdar · Daniel Gonzalez · Tim Weninger

University of Notre Dame

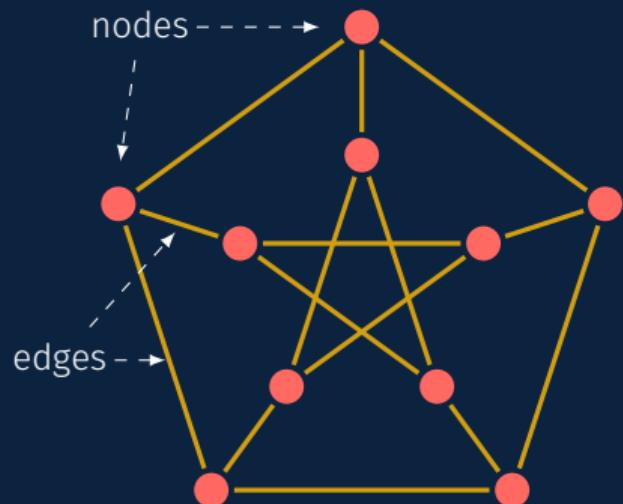
July 9, 2020



The background image shows a wide-angle view of Prague's skyline during what appears to be either sunrise or sunset. The sky is a warm orange and yellow. In the center, the Vltava River flows, with three prominent arch bridges spanning it. The city's architecture is visible, with numerous buildings featuring traditional red roofs and green domes. Bare trees in the foreground frame the lower part of the image.

Preliminaries

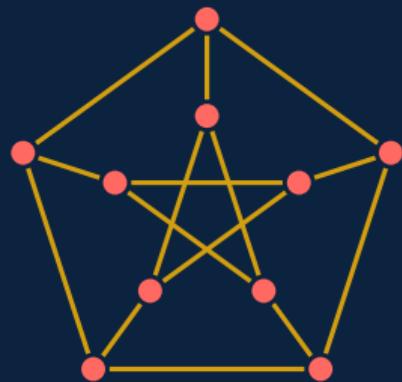
WHAT ARE GRAPHS?



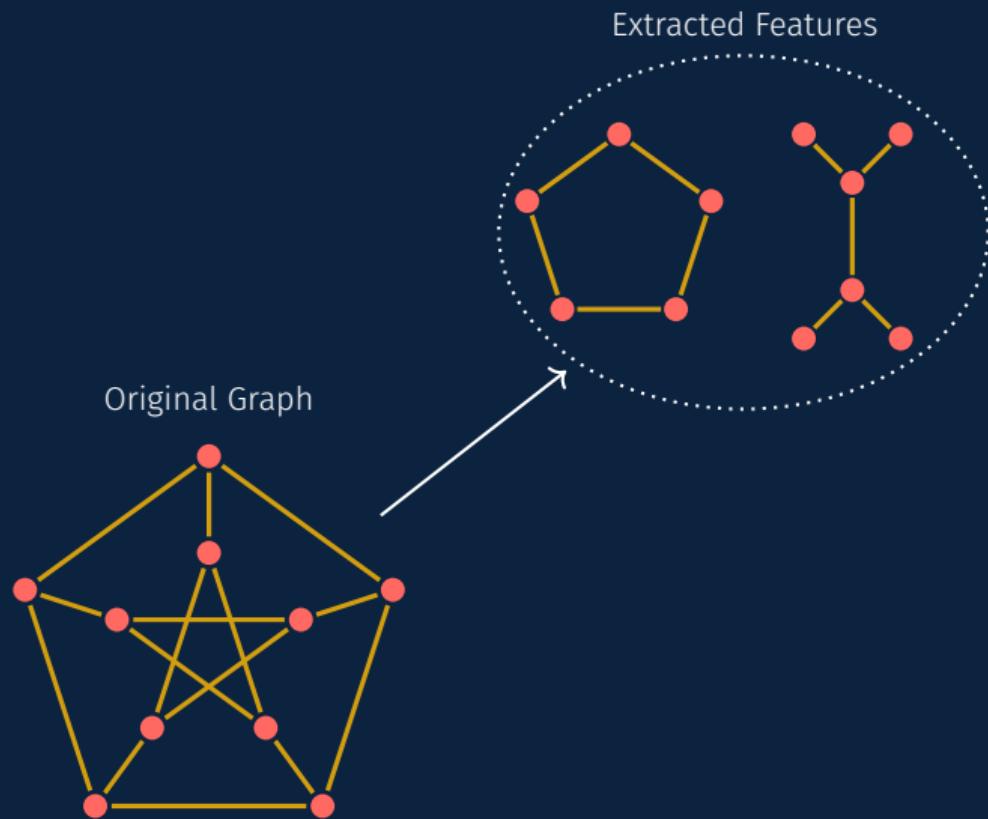
| Domain | Nodes | Edges |
|--------------------------|----------|------------------|
| <i>World Wide Web</i> | Webpages | Hyperlinks |
| <i>Scientific Papers</i> | Papers | Citations |
| <i>Flights</i> | Airports | Non-stop flights |
| <i>Facebook</i> | Users | Friendships |

WHAT ARE GRAPH GENERATIVE MODELS?

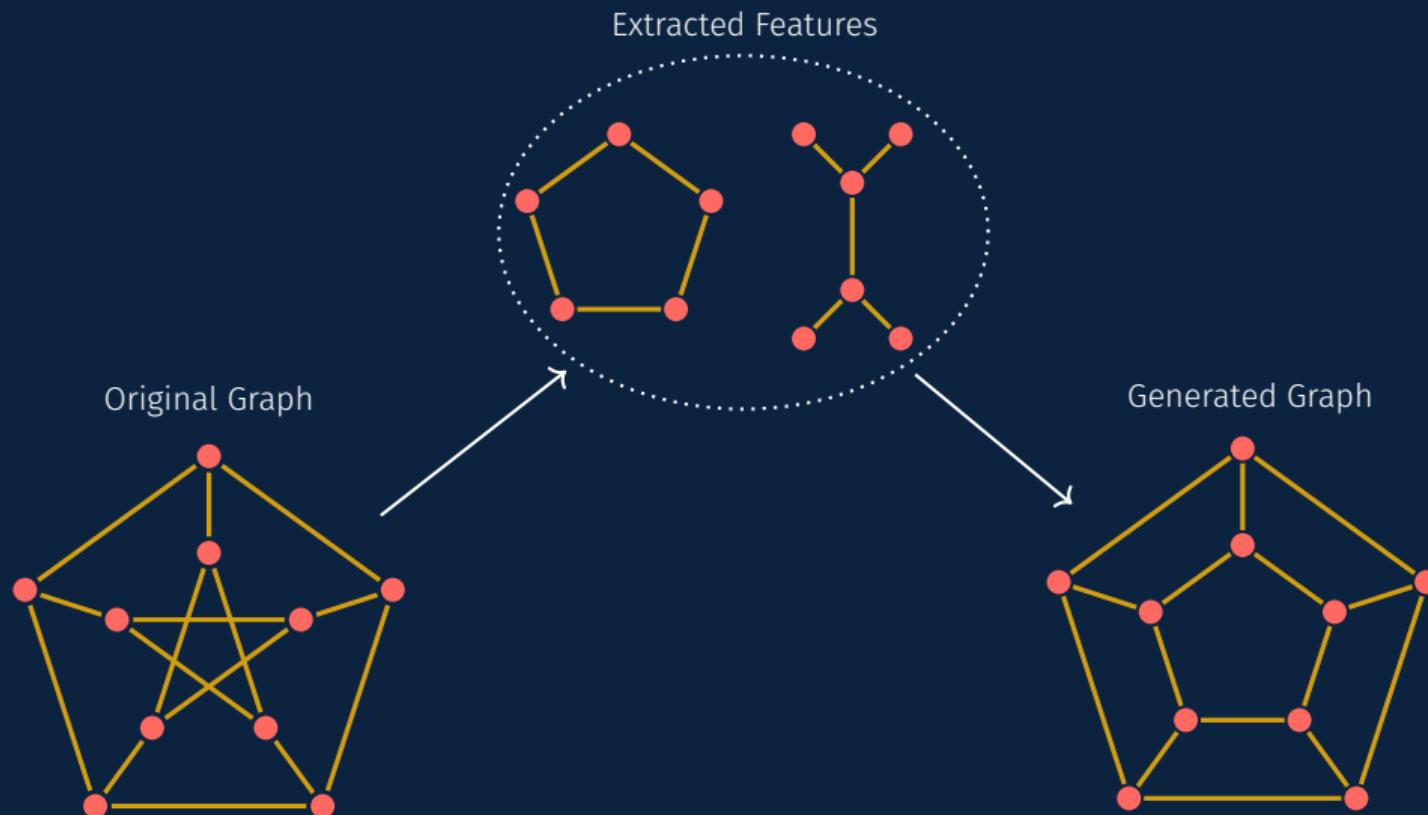
Original Graph



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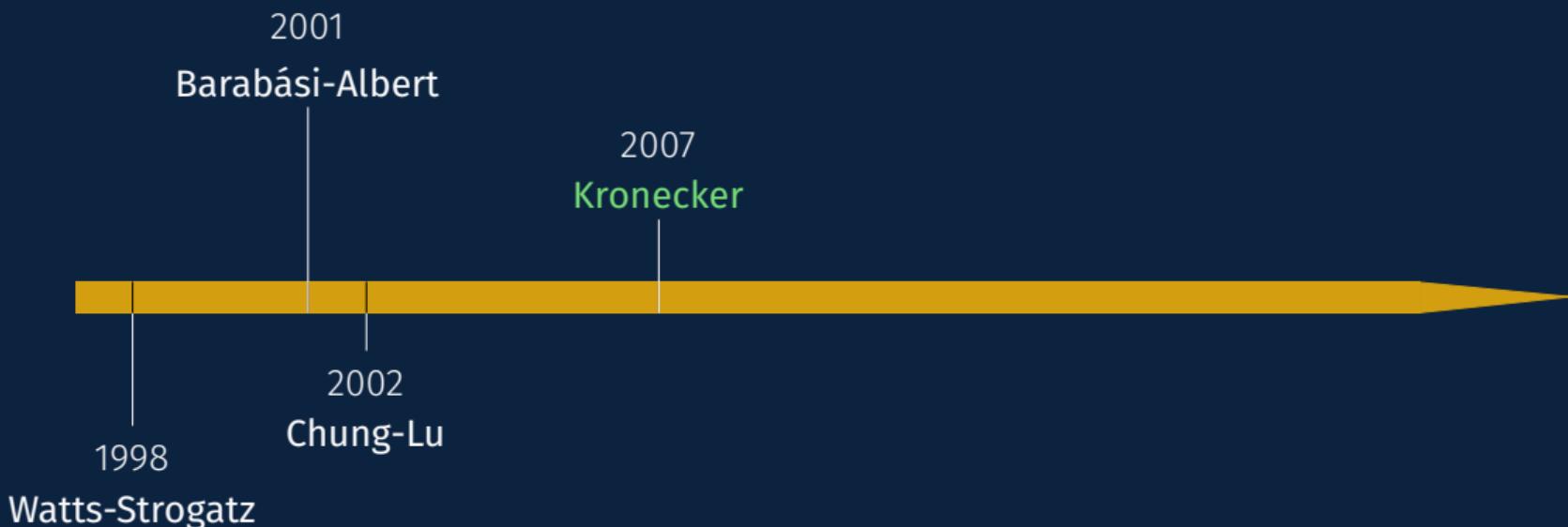
A BRIEF HISTORY OF RECENT GRAPH GENERATORS



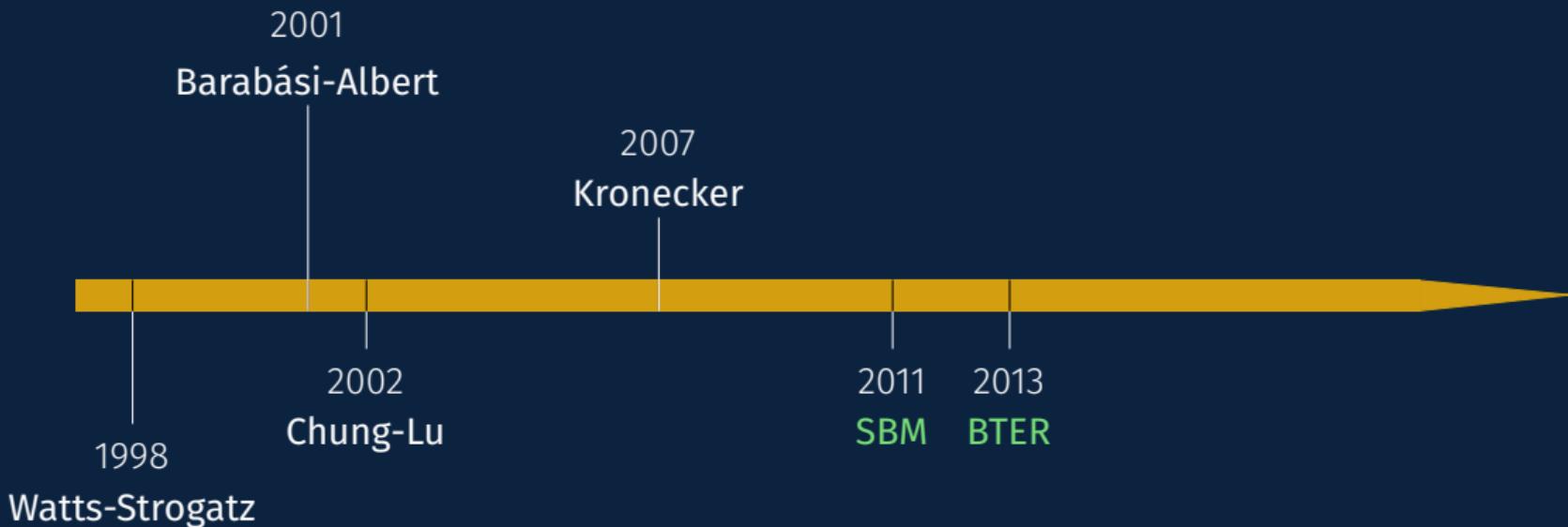
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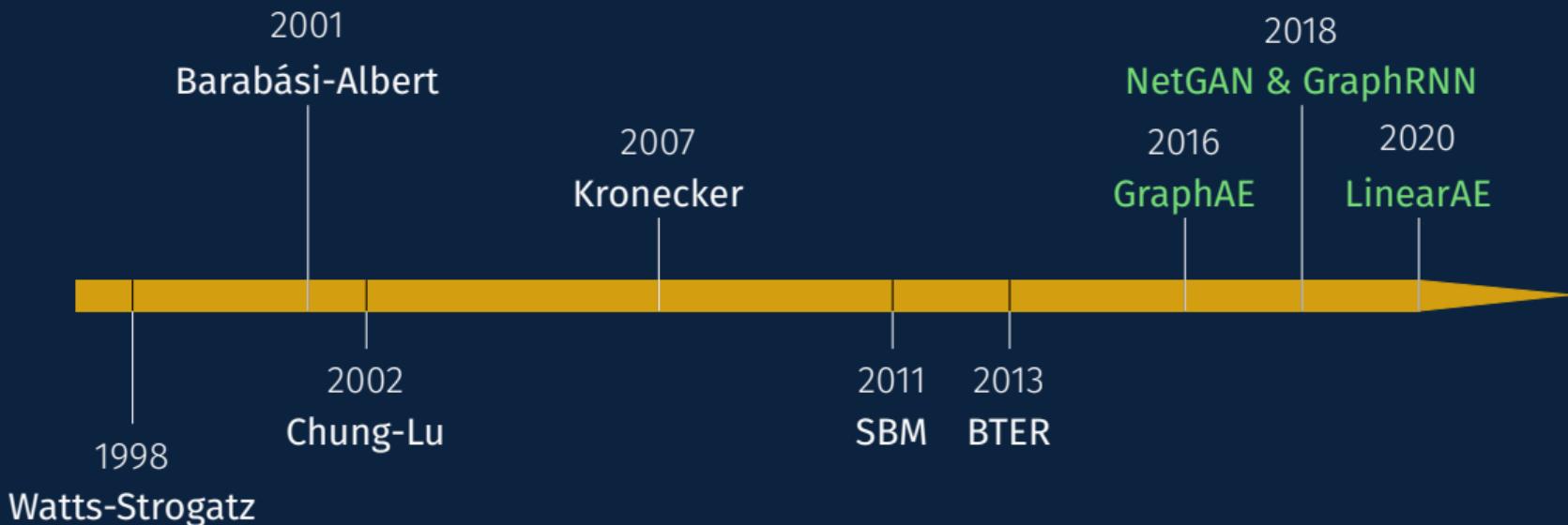
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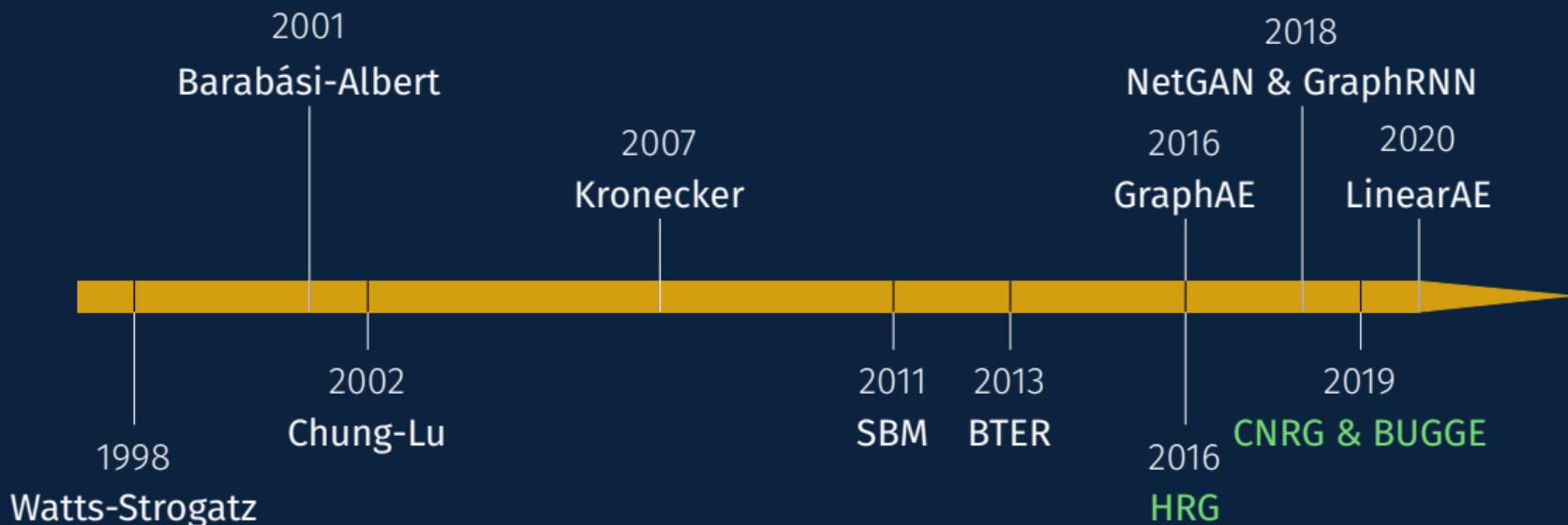
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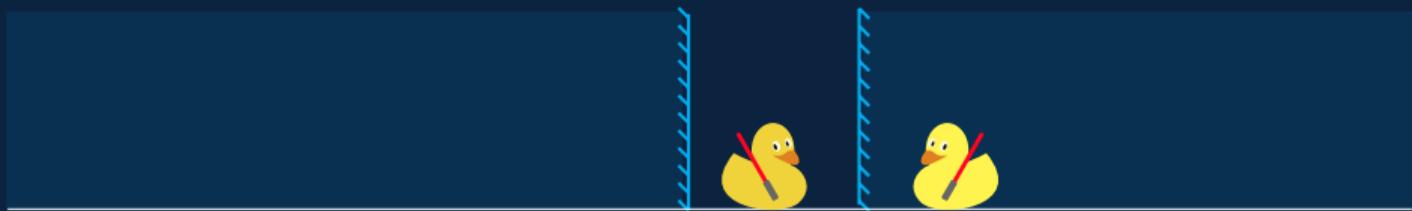
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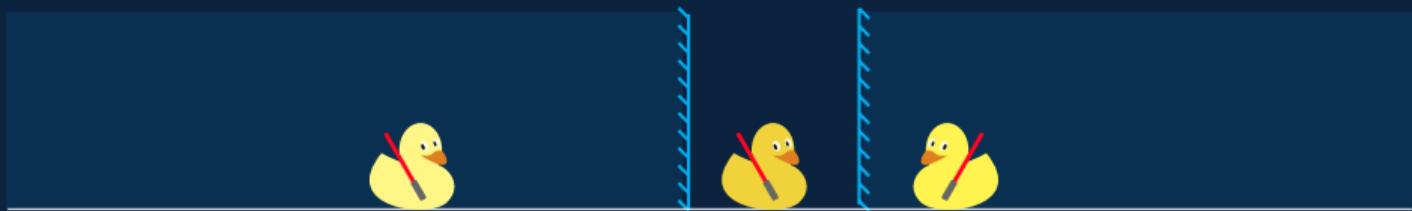


The Infinity Mirror Test

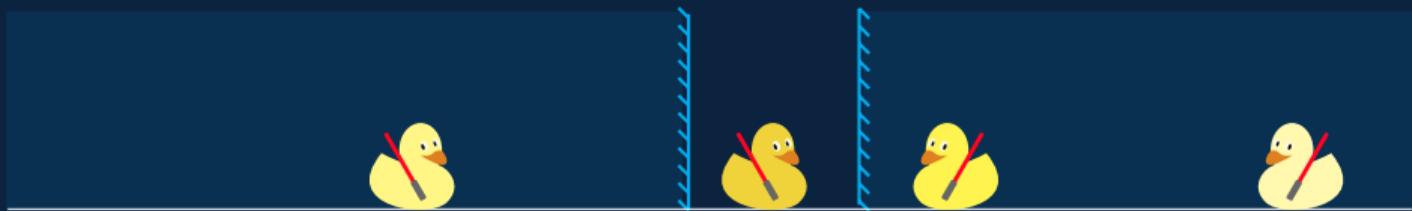
THE INFINITY MIRROR TEST FOR DUCK VADER



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THE INFINITY MIRROR TEST: A STRESS TEST FOR GRAPH GENERATORS

Key Idea

Iteratively fitting and generating from a model will amplify the model's implicit biases.



Figure 1: The Infinity Mirror Test

THE INFINITY MIRROR TEST: A STRESS TEST FOR GRAPH GENERATORS

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Methodology

Given an initial graph G_0 and a model \mathcal{M} , compute 50 independent chains of graphs $\langle G_1, G_2, \dots, G_n \rangle$.



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Evaluation

Compare graph G_i with G_0 in a given chain to expose different biases.

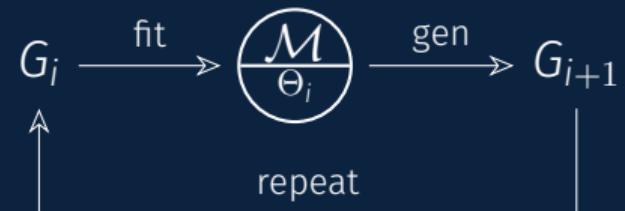
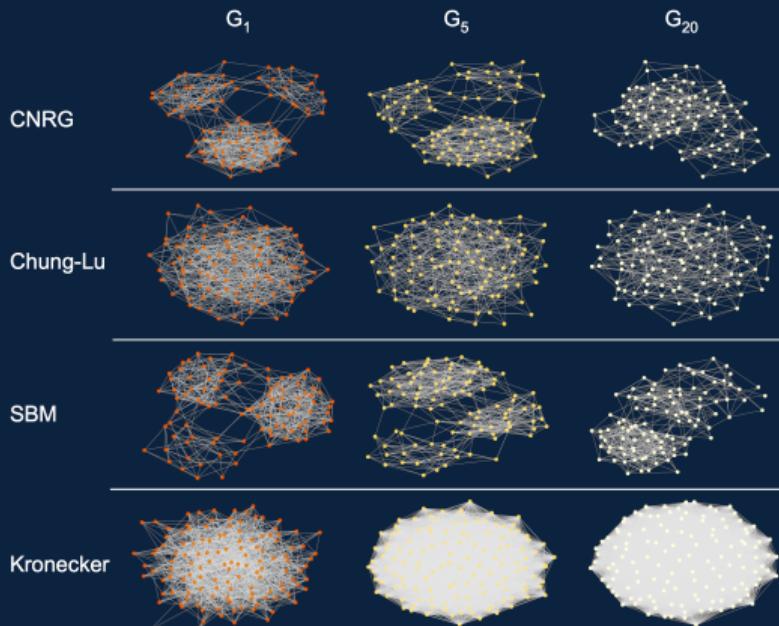
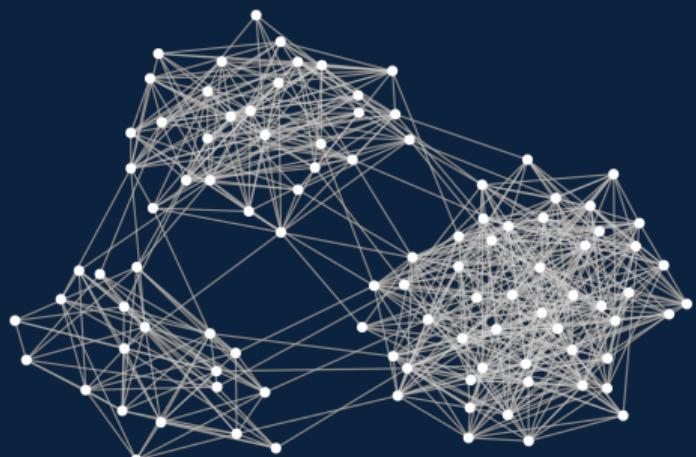


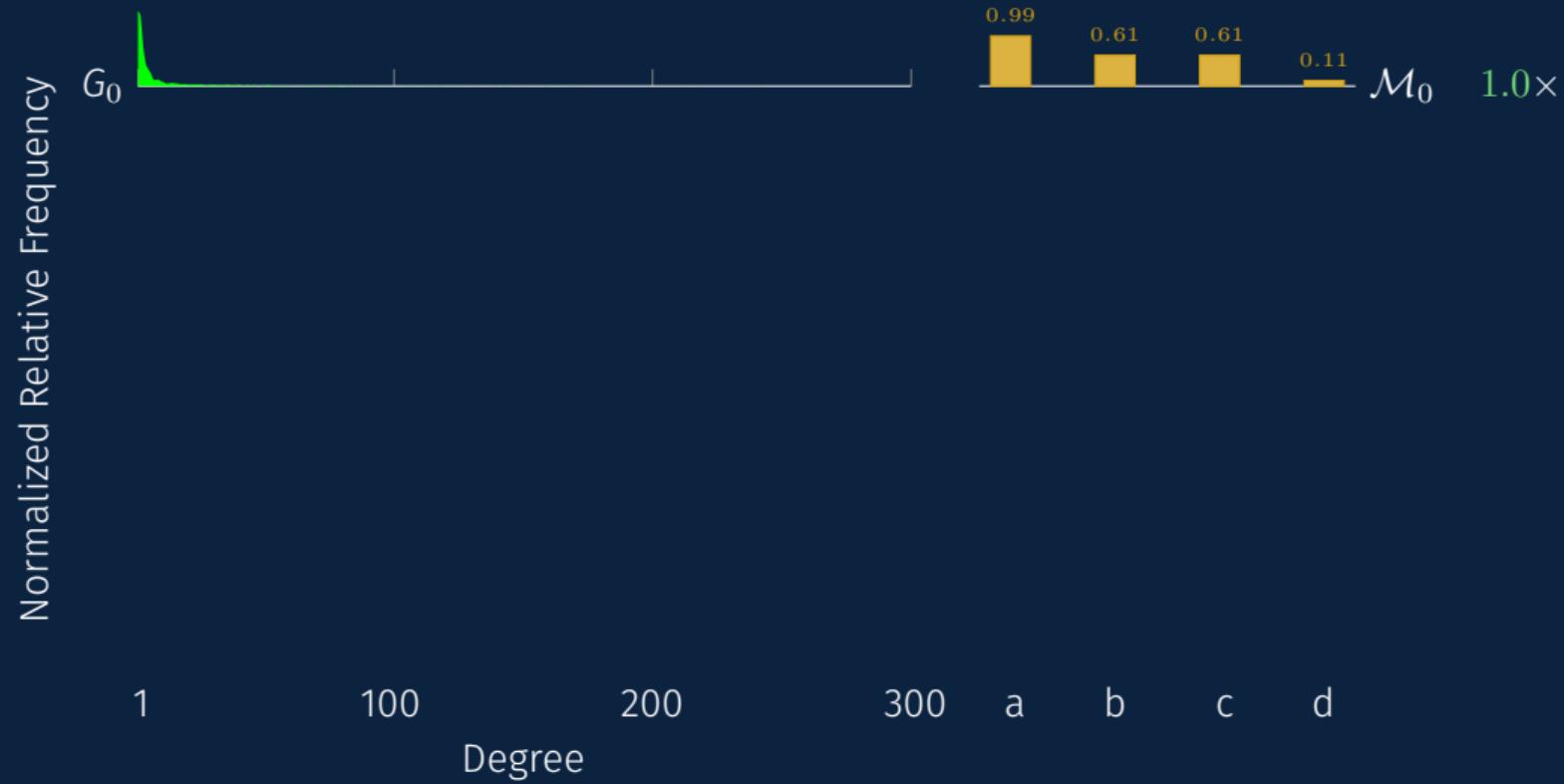
Figure 1: The Infinity Mirror Test

EXAMPLE INFINITY MIRROR



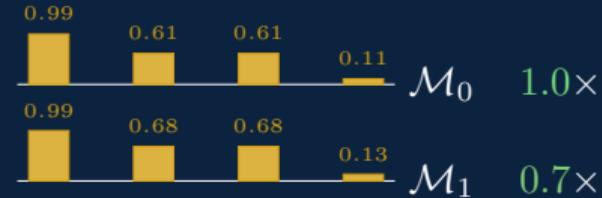
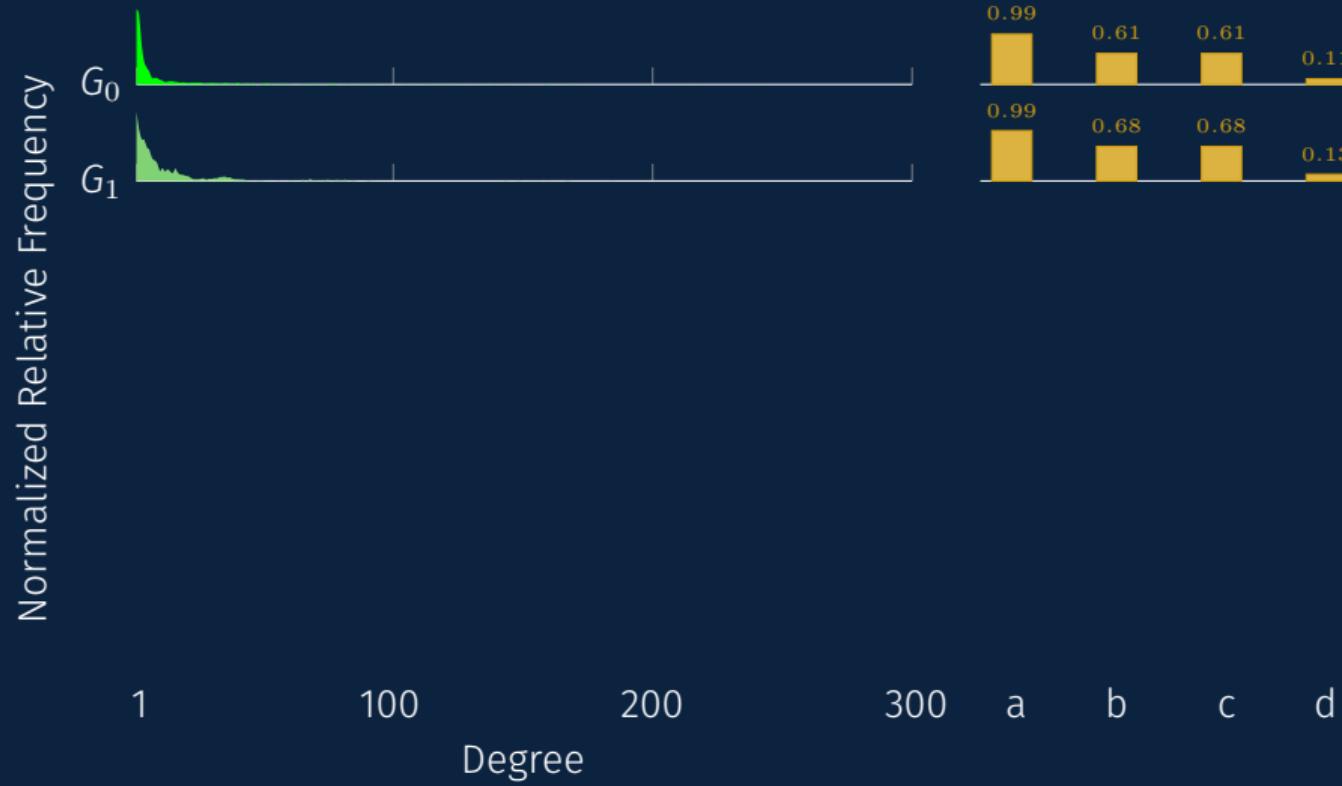
KRONECKER DEGREE DISTRIBUTION RIDGE PLOT

$$\mathcal{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



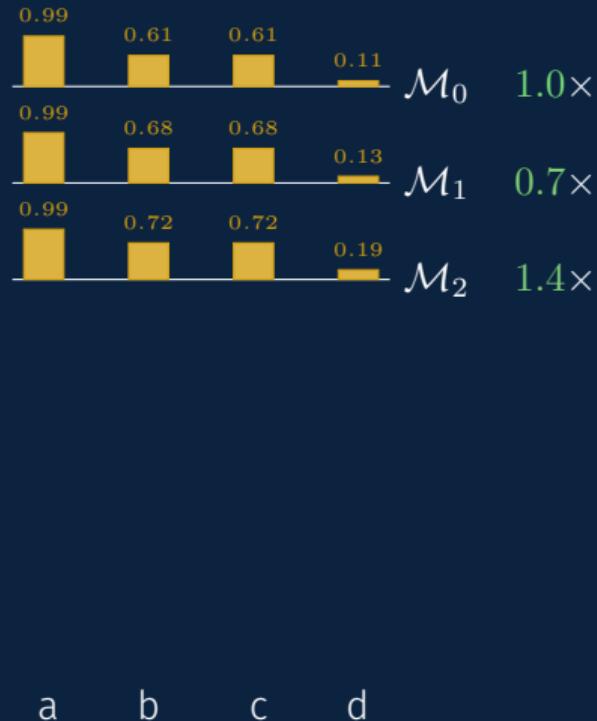
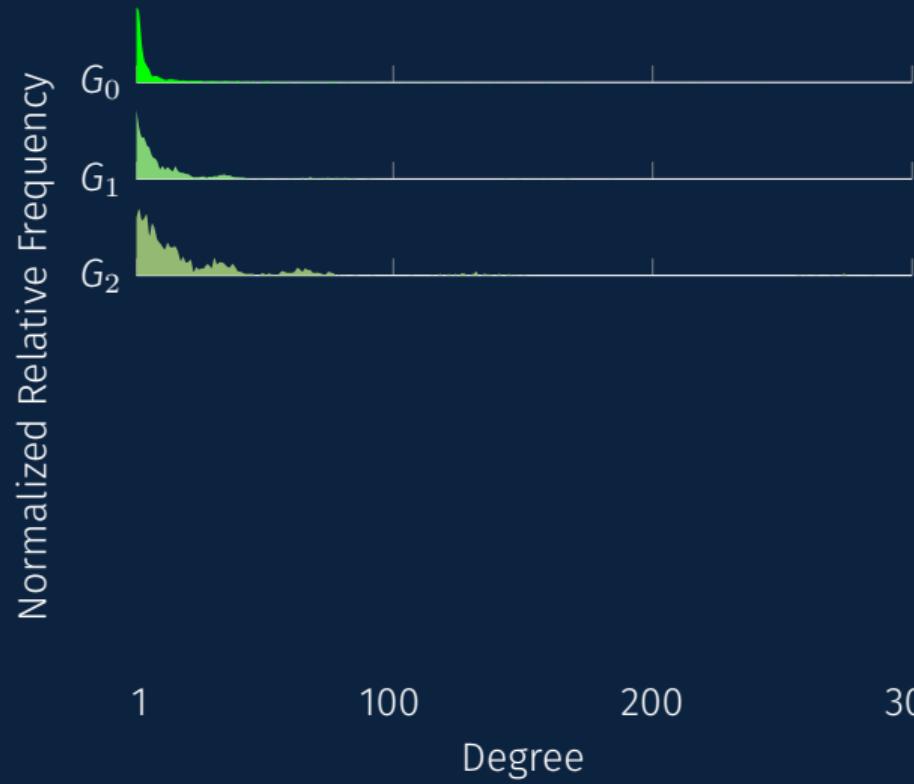
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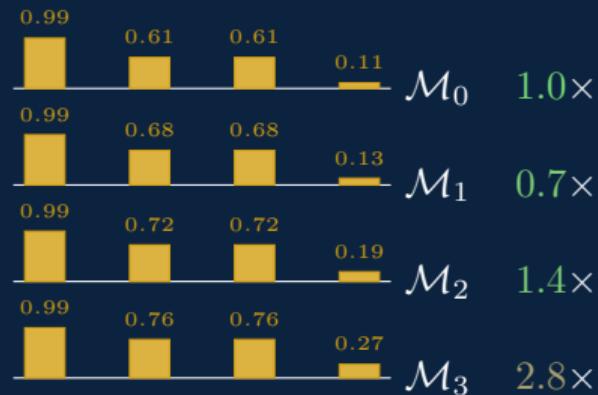
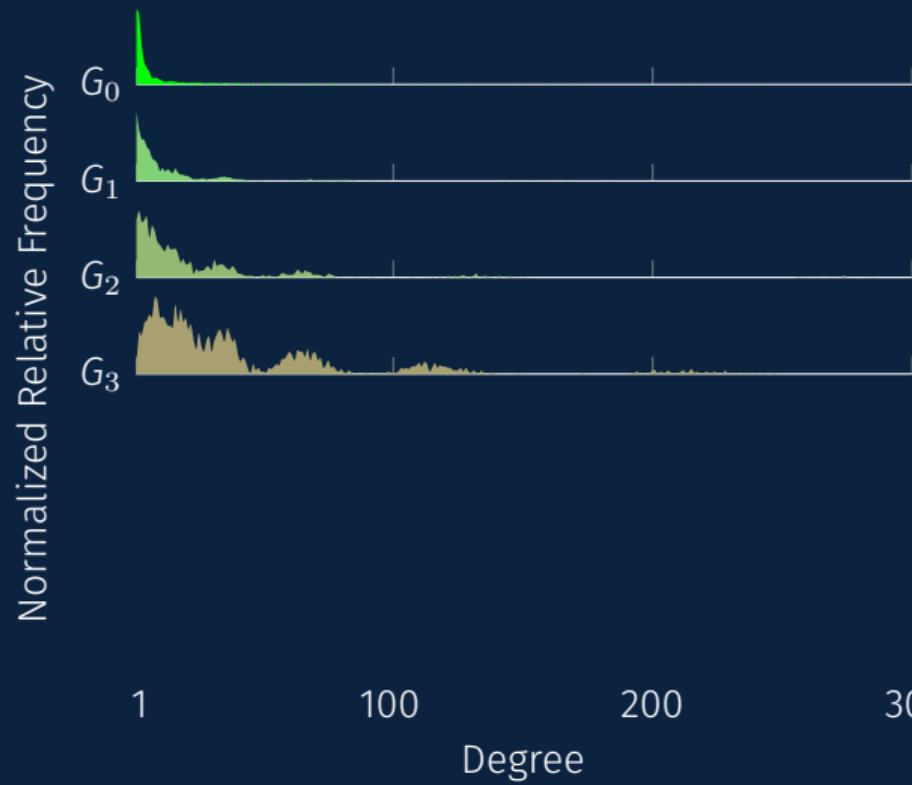
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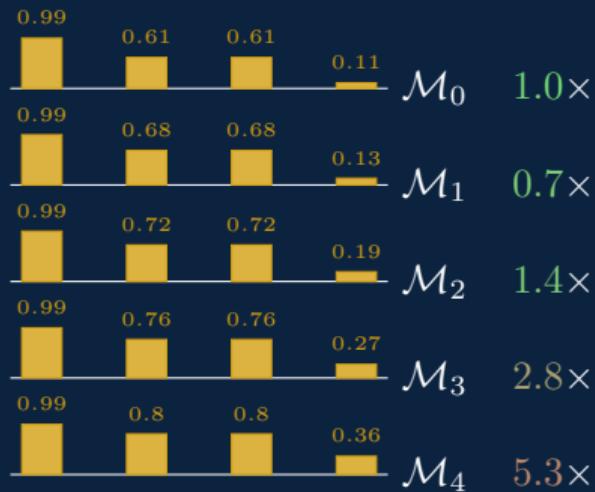
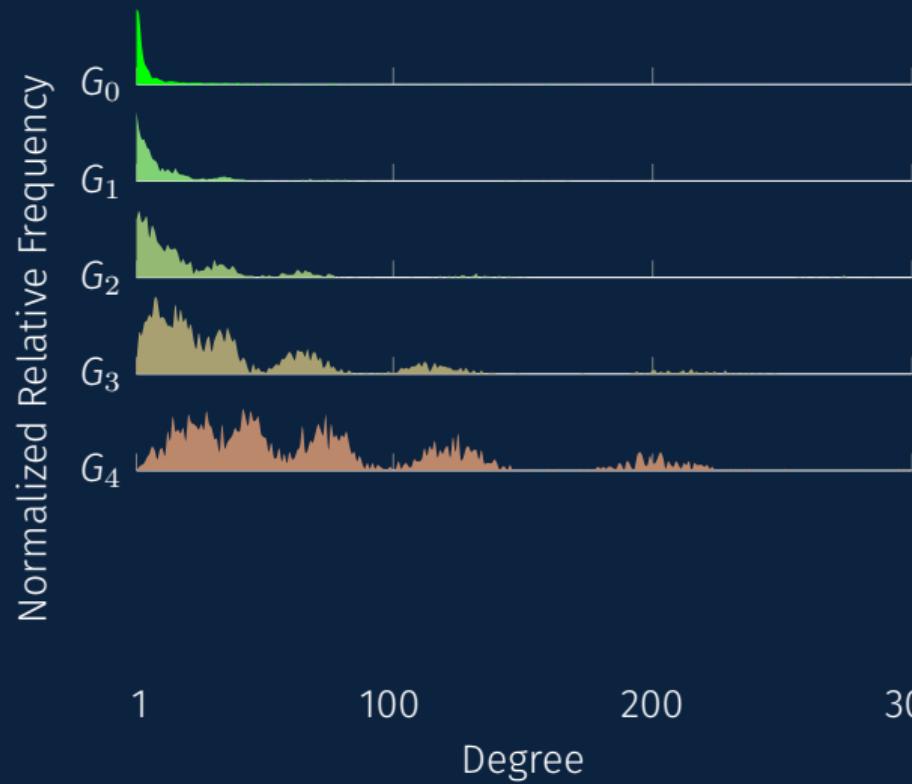
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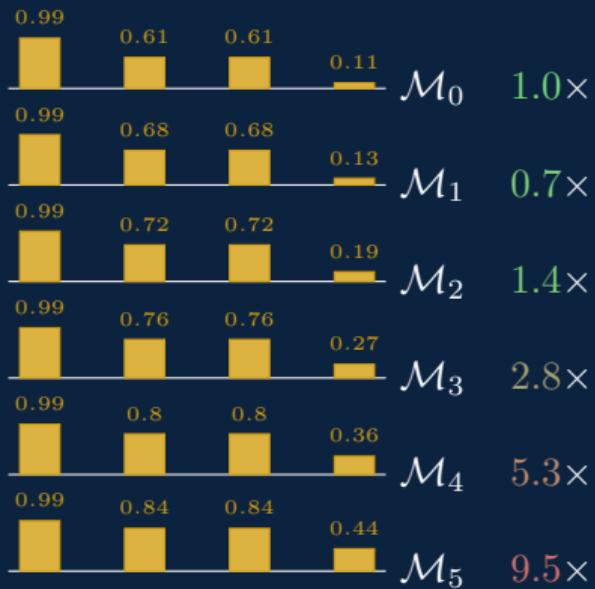
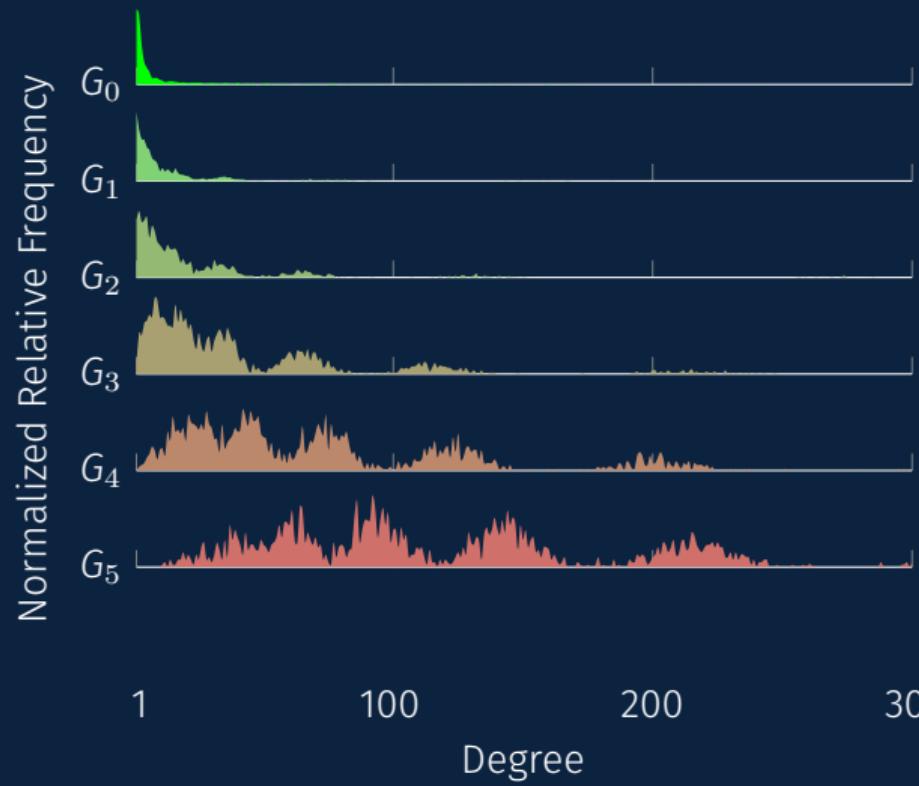
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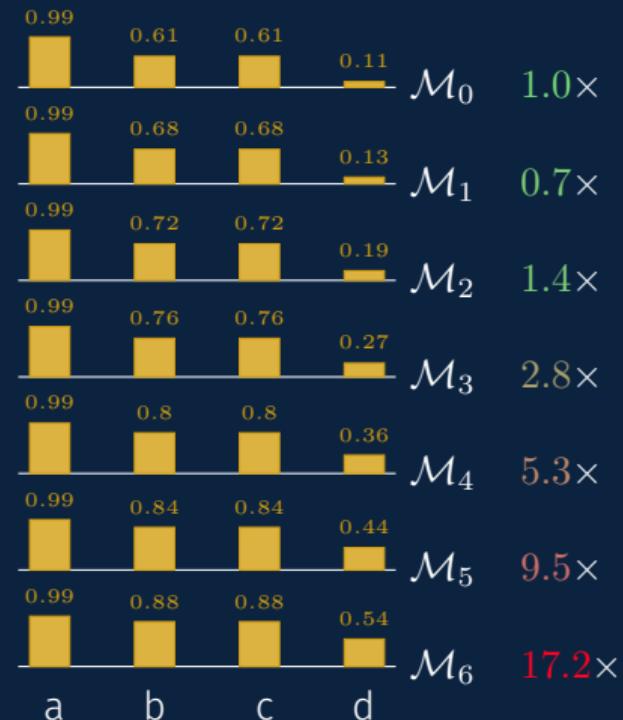
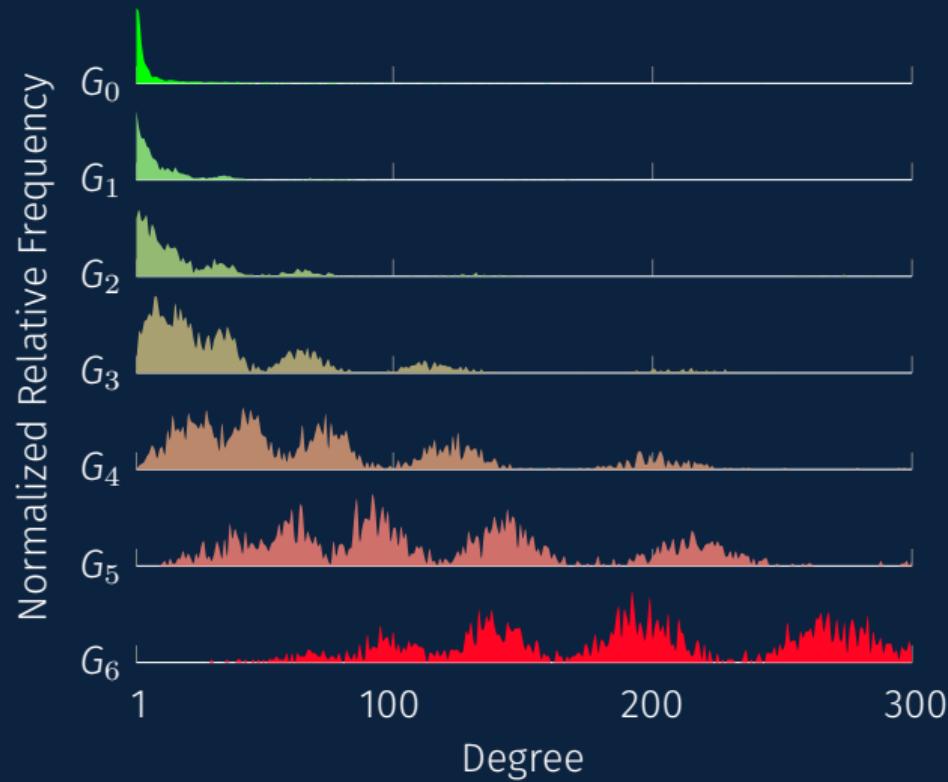
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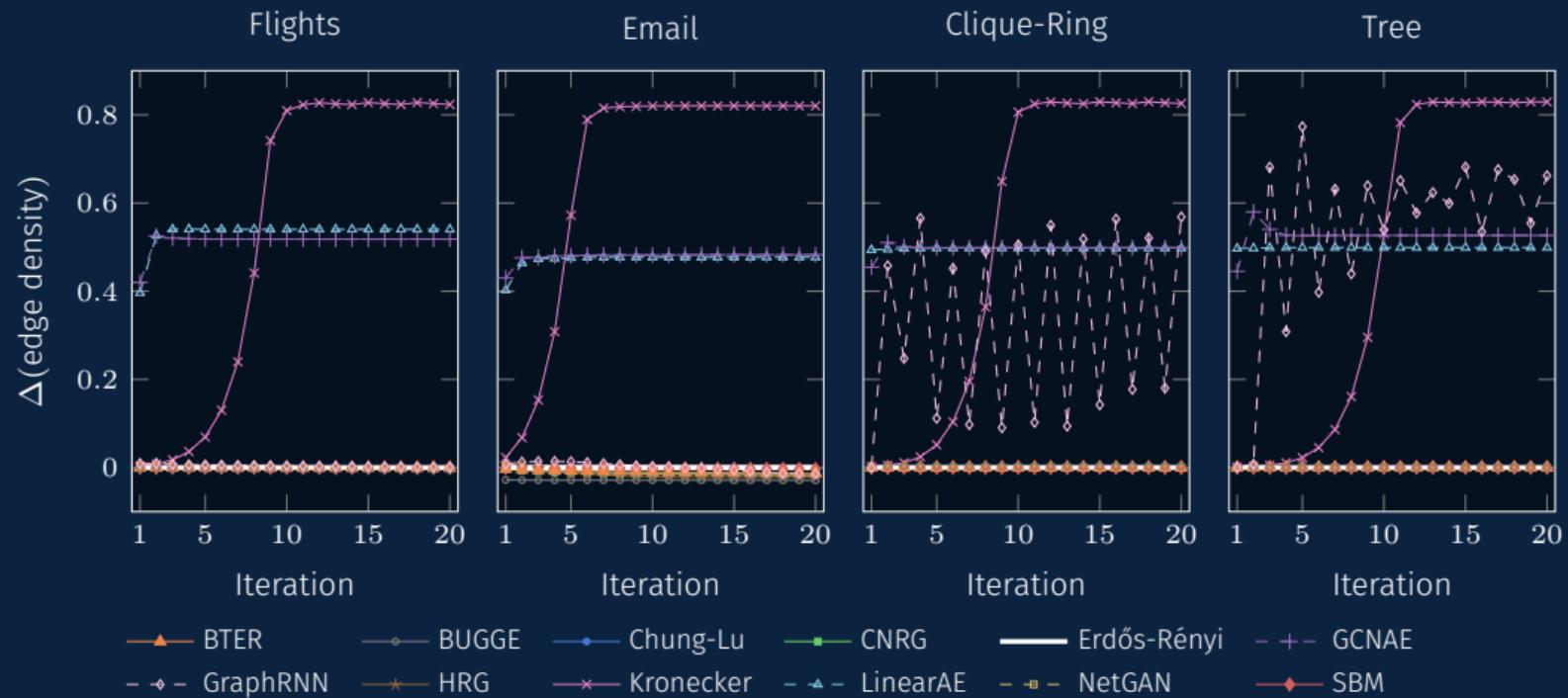


KRONECKER DEGREE DISTRIBUTION RIDGE PLOT

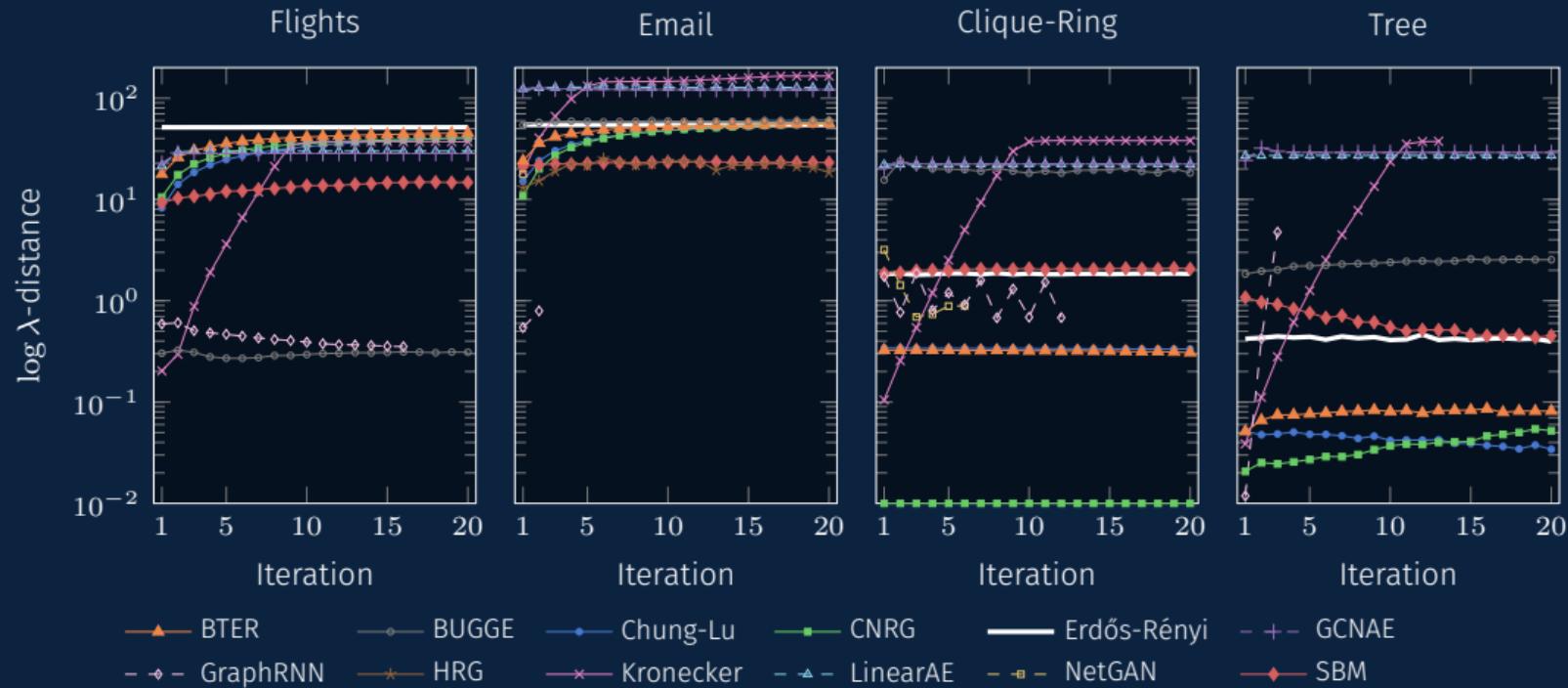
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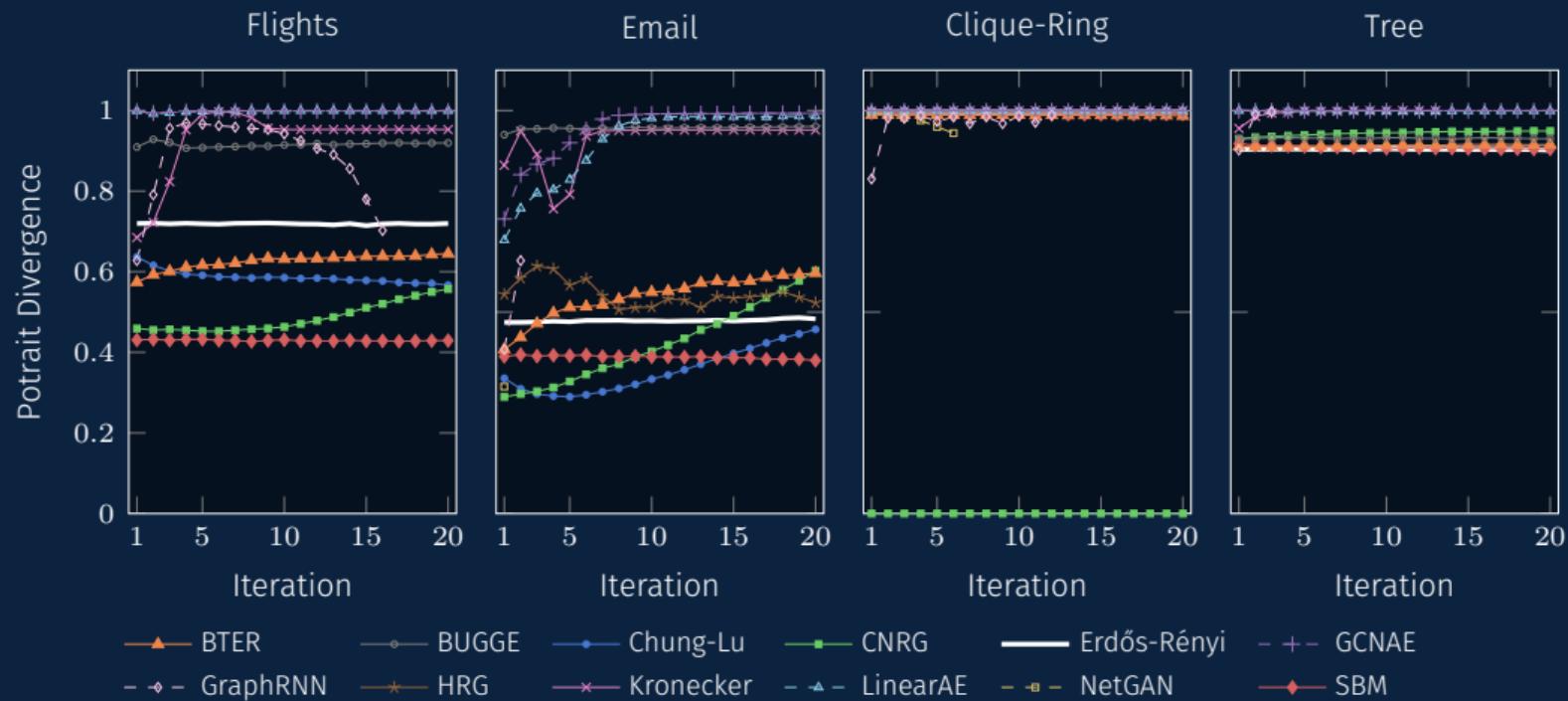
EVOLUTION OF RELATIVE EDGE DENSITY ACROSS ITERATIONS



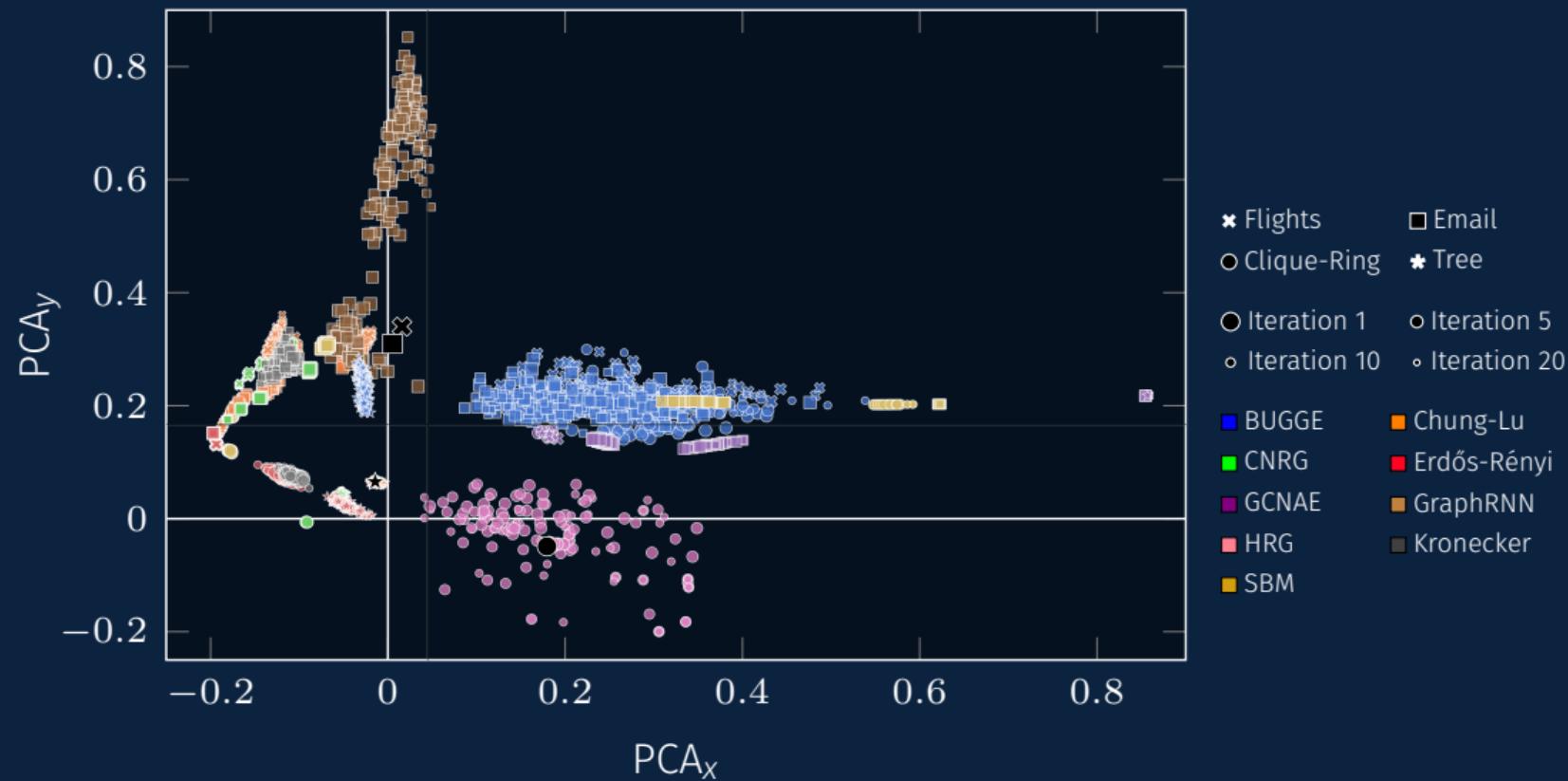
EVOLUTION OF λ -DISTANCE ACROSS ITERATIONS



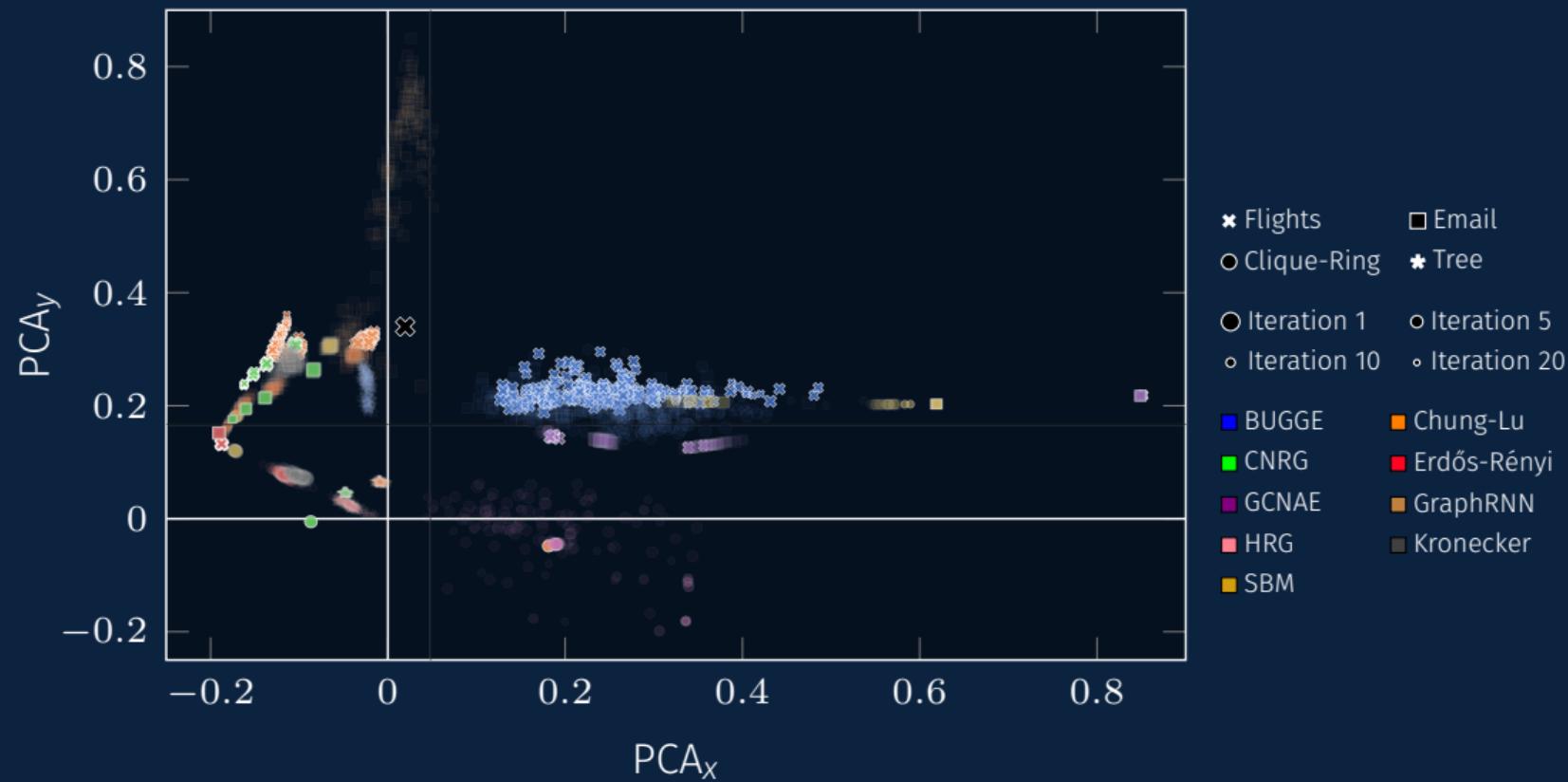
PORTRAIT DIVERGENCE OVER TIME



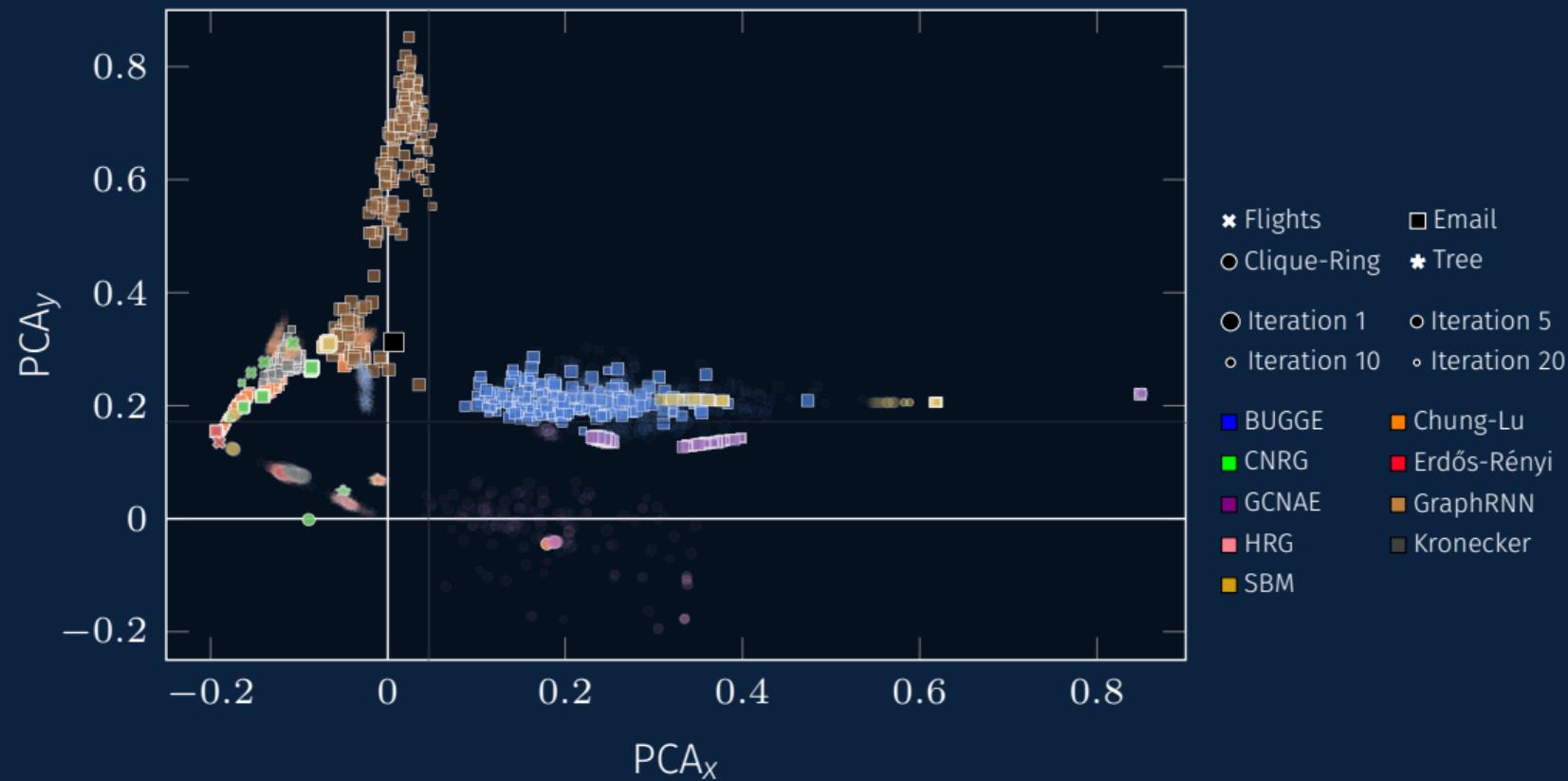
2D PCA ON GRAPHLET COUNT VECTORS



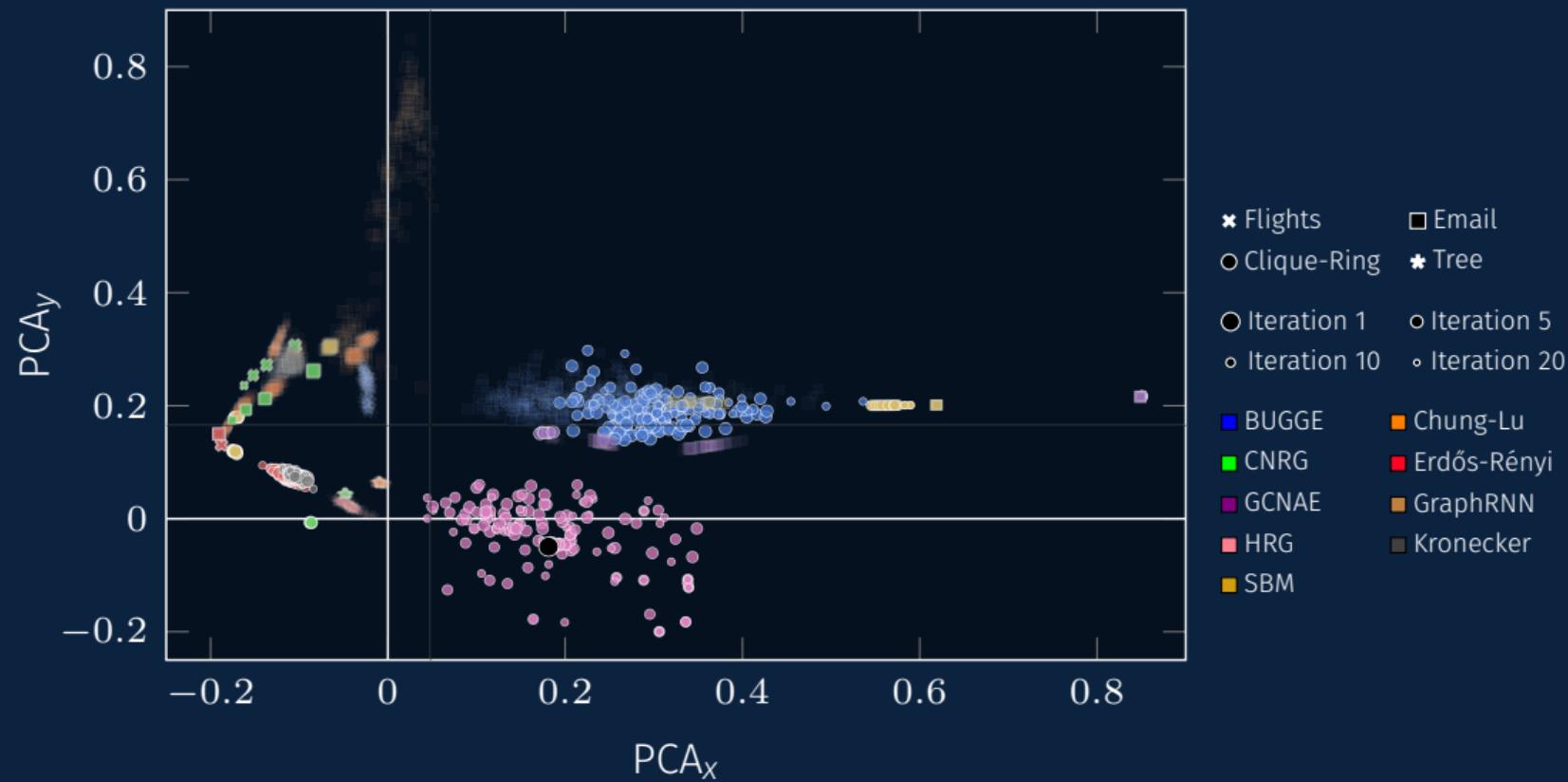
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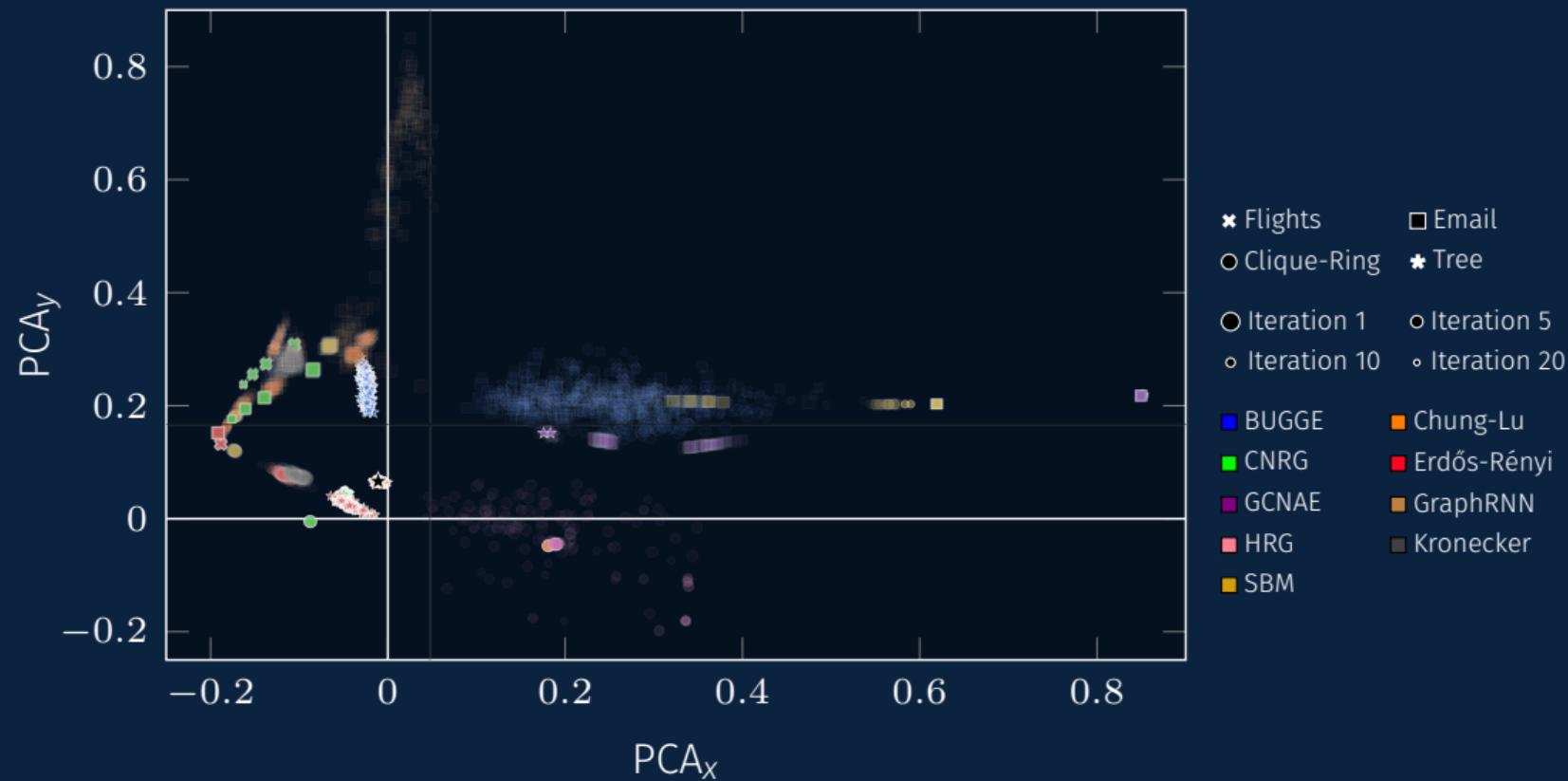
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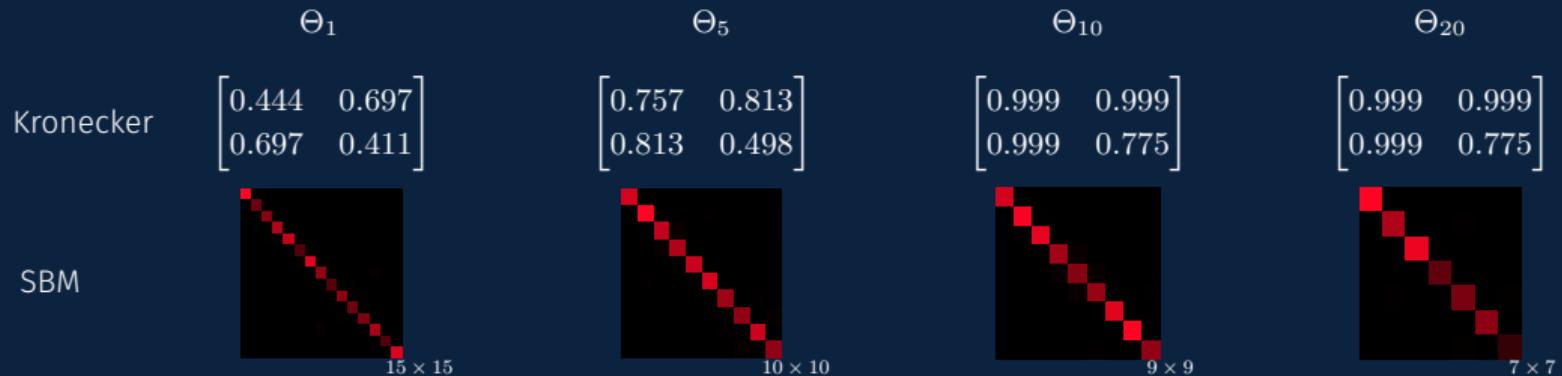
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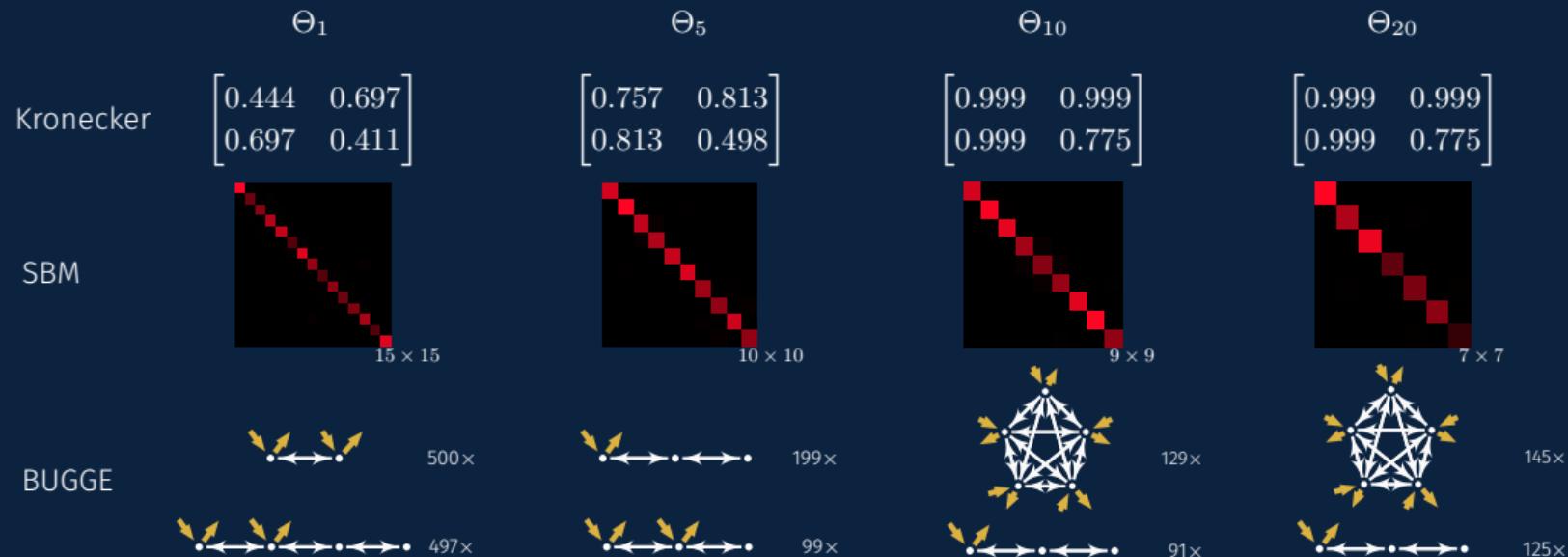
INVESTIGATING INDIVIDUAL MODEL PARAMETERS ON THE CLIQUE RING DATASET

| | Θ_1 | Θ_5 | Θ_{10} | Θ_{20} |
|-----------|--|--|--|--|
| Kronecker | $\begin{bmatrix} 0.444 & 0.697 \\ 0.697 & 0.411 \end{bmatrix}$ | $\begin{bmatrix} 0.757 & 0.813 \\ 0.813 & 0.498 \end{bmatrix}$ | $\begin{bmatrix} 0.999 & 0.999 \\ 0.999 & 0.775 \end{bmatrix}$ | $\begin{bmatrix} 0.999 & 0.999 \\ 0.999 & 0.775 \end{bmatrix}$ |

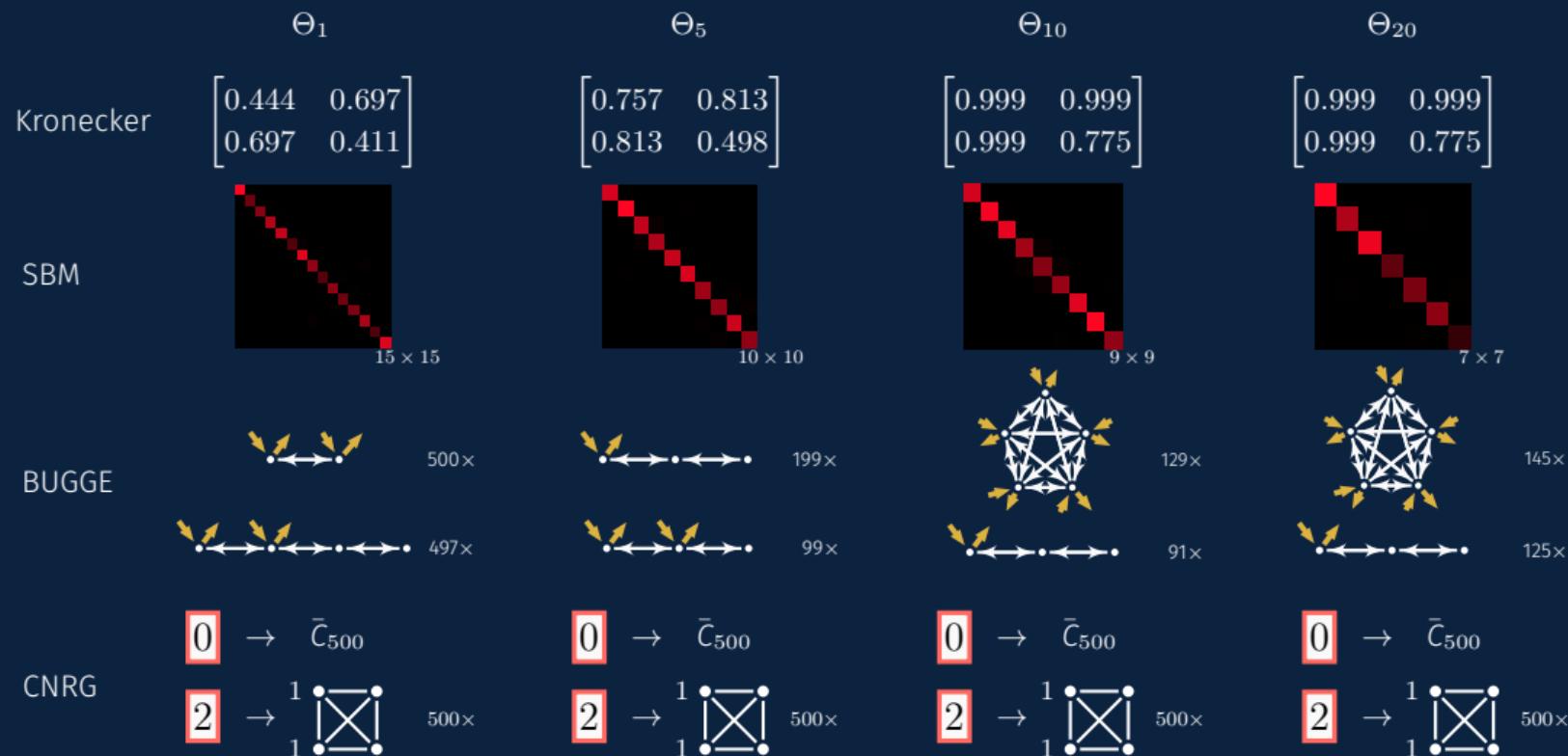
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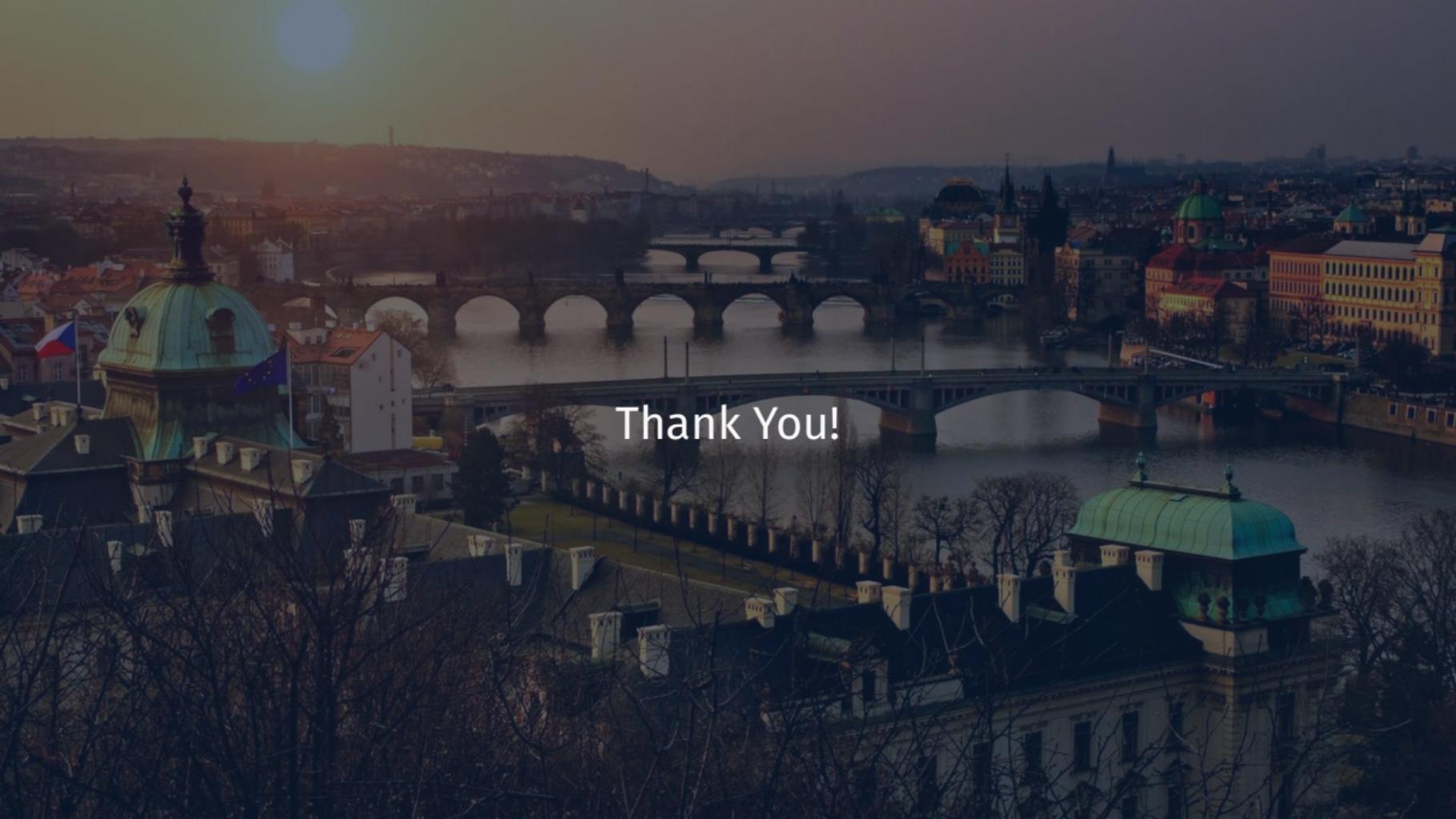
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CONCLUSIONS

Key Findings and Contributions

- Confirms previously known biases in Kronecker models.
- Uncovers distortion patterns in popularly used graph models
- Could be used as a tool to design better, more parsimonious graph models



Thank You!