PROBLEM SET 2

DISCRETE MATHEMATICS

Due: 3rd of February, 2023

- 1. Show that $p \to q$ is logically equivalent to $\neg q \to \neg p$.
- 2. Show that $(\neg p \to \bot) \to p$ is a tautology.
- 3. Show that $(p \to r) \lor (q \to r)$ is equivalent to $(p \land q) \to r$.
- 4. Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.
- 5. Using the predicate definitions below, let our universe of discourse be the collection of all characters in the play Macbeth and translate the following propositions of the first-order logic into English sentences.

$$\kappa(x) \coloneqq \text{``}x \text{ is a king.''}$$

$$\vartheta(x) \coloneqq \text{``}x \text{ is a thane.''}$$

$$\psi(x) \coloneqq \text{``}x \text{ is a witch.''}$$

$$\omega(x) \coloneqq \text{``}x \text{ is man woman-borne.''}$$

$$\mu(x,y) \coloneqq \text{``}x \text{ murders }y.\text{''}$$

- (a) Macbeth is a king, and Banquo is a thane.
- (b) Every king is murdered by someone.
- (c) Someone murders every king.
- (d) No witches murder anyone.
- (e) No kings murder any thanes.
- (f) Macbeth is not murdered by any man woman-borne.
- (g) Every king is a thane.
- (h) Banquo is not a king, yet someone murders him.
- (i) Any witch could not possibly be a king.
- (i) The only kings are those woman-borne.
- 6. Using the predicate definitions below, let our universe of discourse be the collection of all living beings on the Obra-Dinn and translate the following propositions of the first-order logic into English sentences.

$$\kappa(x) \coloneqq$$
 " x is the Captain." $\delta(x,y) \coloneqq$ " x is drowned by y ." $\rho(x,y) \coloneqq$ " x was terribly ravaged by y ." $\sigma(x) \coloneqq$ " x is a sea monster." $\gamma(x,y) \coloneqq$ " x was shot by y ."

- (a) κ ("Richard Witterel") $\wedge \forall x (\mu(x) \to \gamma(x, \text{"Richard Witterel"}))$.
- (b) $\neg \forall x \exists y (\sigma(y) \land \rho(x,y)).$
- (c) $\exists x \forall y (\sigma(x) \land (\mu(y) \rightarrow \delta(y, x))).$

(d)
$$\left(\neg \exists x \big(\sigma(x)\big) \land \left(\neg \exists x \big(\sigma(x)\big) \rightarrow \exists x \big(\gamma(\text{``Richard Witterel''}, x)\big)\right)\right) \rightarrow \exists x \big(\gamma(\text{``Richard Witterel''}, x)\big).$$

- 7. Let φ and ψ be predicates. Rewrite the following first-order sentences so that no quantifier has a \neg to its left.
 - (a) $\forall x \exists y (x \lor y)$
 - (b) $\forall x \forall y (\neg x \lor (x \leftrightarrow y))$
 - (c) $\exists x \forall y \forall z ((x \rightarrow y) \rightarrow z)$

(d)
$$x \Big(\varphi(x) \lor \forall y \exists z \big(\psi(y, z) \to \neg x \big) \Big)$$