

Problem Set 10

Discrete Mathematics

Due on the 22nd of April, 2024

- (40 pts) 1. Construct—with proof—the explicit functions requested below.
- (a) A bijection from $\{x \in \mathbb{R} \mid -1 < x < 1\}$ to $\{x \in \mathbb{R} \mid -\pi < x < \pi\}$.
 - (b) A surjection from \mathbb{N} to $\{p \mid p \text{ is prime}\}$.
 - (c) An injection from X to $\mathbb{P}(X)$ for *every* set X .
 - (d) A surjection from $\mathbb{P}(X)$ to X for *every* set $X \neq \emptyset$.
- (30 pts) 2. Let A be an arbitrary finite set of cardinality $|A| = n$, where $n \in \mathbb{N}$. How many finite strings over A are there?
- (30 pts) 3. Imagine that, one day, you encounter a library.¹ At the entrance of this library is an enormous tome \mathcal{B} listing *all* of the *possible* sentences in the English language, indexed by natural numbers.

Walking past the entrance, you see that the library has rows of bookshelves numbered $0, 1, 2, \dots$, so that there is exactly one row of books for each $n \in \mathbb{N}$. A great owl—perched on the pedestal that supports \mathcal{B} —informs you that, for each $n \in \mathbb{N}$, the n^{th} row of bookshelves contains *all* of the books that could possibly ever be that begin with the n^{th} sentence in \mathcal{B} . With the sound of granite scraping against marble, the doors to the library close behind you. The owl makes you the following proposal: you will be free to go *if and only if* you can read every book in the library *in a countable amount of time*.

Will you be set free? The owl demands a proof to justify your answer.

¹ This library is so massive and so complete a collection of literary works that it even contains books of *infinite length*.