SET 10   1

Problem Set 10

Discrete Mathematics

Due on the 22nd of April, 2024

(40 pts)   1. Construct—*with proof*—the *explicit* functions requested below.

   (a) A bijection from $\{x \in \mathbb{R} \mid -1 < x < 1\}$ to $\{x \in \mathbb{R} \mid -\pi < x < \pi\}$.

   (b) A surjection from $\mathbb{N}$ to $\{p \mid p \text{ is prime}\}$.

   (c) An injection from X to $\mathbb{P}(X)$ for *every* set X.

   (d) A surjection from $\mathbb{P}(X)$ to X for *every* set X.

(30 pts)   2. Let A be an arbitrary finite set of cardinality $|A| = n$, where $n \in \mathbb{N}$. How many finite strings over A are there?

(30 pts)   3. Imagine that, one day, you encounter a library. At the entrance of this library is an enormous tome $\mathscr{B}$ listing *all* of the *possible* sentences in the English language, indexed by natural numbers.

   Walking past the entrance, you see that the library has rows of bookshelves numbered $0, 1, 2, \ldots$, so that there is exactly one row of books for each $n \in \mathbb{N}$. A great owl—perched on the pedestal that supports $\mathscr{B}$—informs you that, for each $n \in \mathbb{N}$, the nth row of bookshelves contains *all* of the books that could possibly ever be that begin with the nth sentence in $\mathscr{B}$. With the sound of granite scraping against marble, the doors to the library close behind you. The owl makes you the following proposal: you will be free to go *if and only if* you can read every book in the library *in a countable amount of time.*

   Will you be set free? The owl demands a proof to justify your answer.

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