

## Problem Set 4

### Discrete Mathematics

Due on the 20<sup>th</sup> of February, 2024

(20 pts) 1.(a) Show  $\forall x(\emptyset \subseteq x)$ .

(b) Show  $\forall x(x \subseteq x)$ .

(c) Show  $\forall x(\emptyset \in \mathbb{P}(x))$ .

(d) Show  $\forall x(x \in \mathbb{P}(x))$ .

(e) Show  $\forall x \forall y \forall z ((x \subseteq y) \wedge (y \subseteq z) \Rightarrow x \subseteq z)$ .

(10 pts) 2. We define the *intersection* and *difference* of any two sets  $x$  and  $y$  below.

$$x \cap y := \{z \mid z \in x \wedge z \in y\}$$

$$x \setminus y := \{z \mid z \in x \wedge z \notin y\}$$

(a) Show  $\forall x \forall y \exists z (z = x \cap y)$ .

(b) Show  $\forall x \forall y \exists z (z = x \setminus y)$ .

(20 pts) 3. We define the *union* of any two sets  $x$  and  $y$  below.

$$x \cup y := \{z \mid z \in x \vee z \in y\}$$

(a) Show  $\forall x \forall y (x \cap y \subseteq x)$ .

(b) Show  $\forall x \forall y (x \subseteq x \cup y)$ .

(c) Show  $\forall x \forall y (\mathbb{P}(x) \cup \mathbb{P}(y) \subseteq \mathbb{P}(x \cup y))$ .

(d) Show  $\forall x \forall y (x \cap y = x \Leftrightarrow x \in \mathbb{P}(y))$ .

(50 pts) 4. We define the *union over  $x$*  and *intersection over  $x$*  for any set  $x$  below.

$$\cup x := \{z \mid \exists y (y \in x \wedge z \in y)\}$$

$$\cap x := \{z \mid \forall y (y \in x \Rightarrow z \in y)\}$$

(a) Show that  $\forall x (\cup \mathbb{P}(x) = x)$ .

(b) What is  $\cup \emptyset$ ? Justify your answer with a proof.

(c) What is  $\cap \emptyset$ ? Justify your answer with a proof.

(d) Is  $\emptyset = \{z \mid z \in \emptyset\}$ ? Justify your answer with a proof.

(e) Is  $\emptyset = \{z \mid z \notin \emptyset\}$ ? Justify your answer with a proof.