Propositional Logic

Discrete Mathematics

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Definition 1 (Proposition)**.**

A *proposition* is a sentence (in our language) that has one (and only one) definite, consistent truth value.

Definition 2 (Negation)**.**

Given a proposition p, the *negation* of p, denoted $\neg p$, is defined by

p	$\neg p$
$\top$	$\bot$
$\bot$	$\top$

Some possible readings of $\neg p$:

- Not p.
- p does not hold.
- It is not the case that p.
- We do not have that p.

Definition 3 (Conjunction)**.**

Given two propositions p and q, the *conjunction* of p with q, denoted $p \wedge q$, is defined by

p	q	$p \wedge q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$

Some possible readings of $p \wedge q$:

- p, and q.
- p, but q.
- p; also, q.
- p; further, q.
- In addition to p, we also have q.

Definition 4 (Disjunction)**.**

Given two propositions p and q, the *disjunction* of p with q, denoted $p \vee q$, is defined by

p	q	$p \vee q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$

Some possible readings of $p \vee q$:

- p, or q.
- Either p, or q.

Definition 5 (Material Implication)**.**

Given two propositions p and q, the *conditional* formed by assuming p and concluding q, denoted $p \to q$, is defined by

p	q	$p \to q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\top$

Some possible readings of $p \to q$:

- If p, then q.
- p implies q.
- q is conditioned on p.
- q only if p.
- p is sufficient for q.
- q is necessary for p.