

Problem Set 8

Discrete Mathematics

Due on the 4th of April, 2024

- (10 pts) 1. Including the instructor, there are 32 people in our class. Prove that two of these people were born on the same day of the month.
- (10 pts) 2. As of the 28th of March, 2024, there are over 8.1 billion people living on Earth.¹ A person's heart will beat no more than 7×10^9 times over their lifespan. Show that there are two currently-living people on Earth whose hearts have beat the exact same number of times.
- (20 pts) 3. Let $n \in \mathbb{N}$ and consider $\mathcal{A} \subseteq \mathbb{N}$ such that $|\mathcal{A}| = n + 1$. Prove there exist $x, y \in \mathcal{A}$ with $x \neq y$ such that $n \mid x - y$.
- (20 pts) 4. Consider $\mathcal{S} := \{3, 4, 7, 8, 9, 10, 12, 15, 18, 19, 27, 28\}$ and $\mathcal{X} \subseteq \mathcal{S}$ with $|\mathcal{X}| \geq 9$. Show that there exist three *distinct* elements $x_1, x_2, x_3 \in \mathcal{X}$ such that $x_1 + x_2 + x_3 = 35$.
- (20 pts) 5. Recall that $\binom{n}{0} = \binom{n}{n} = 1$ for all $n, k \in \mathbb{N}$ when $k \leq n$.
- (a) Show $\binom{n}{k} = \binom{n}{n-k}$ for all $n, k \in \mathbb{N}$ where $k \leq n$.
- (b) Show $\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$ for all $n, k \in \mathbb{N}$ where $k \leq n$.
- (20 pts) 6. Prove that $|\mathbb{P}(X)| = 2^{|X|}$ for any finite set X .

¹ For simplicity, you may assume the human population will only monotonically increase over time from the point this problem set was assigned.