

PROBLEM SET 7

DISCRETE MATHEMATICS

Due: 7th of April, 2023

1. For this problem, let A , B , and C be sets and consider two functions $f : A \rightarrow B$ and $g : B \rightarrow C$.
 - (a) Show that, if f and g are both injections, then $g \circ f$ is an injection.
 - (b) Show that, if f and g are both surjections, then $g \circ f$ is a surjection.
 - (c) Show that, if f and g are both bijections, then $g \circ f$ is a bijection.
2. Prove that every finite set is countable.
3. Let A and B be countable sets.
 - (a) Show that $A \cap B$ is countable.
 - (b) Show that $A \cup B$ is countable.
4. Suppose $A_0, A_1, \dots, A_i, \dots$ is a countably-infinite collection of countable sets. Show that $\bigcup_{i=0}^{\infty} A_i$ is countable.
5. Let $X := \{x_0, \dots, x_{k-1}\}$ be a set of size $k \in \mathbb{N}$. How many strings of length $n \in \mathbb{N}$ over X are there? Prove it.
6. Recall that $|\mathbb{N}| = \aleph_0$. Is the cardinality of its power set $\mathcal{P}(\mathbb{N})$ lesser, greater, or equal? Prove your answer.
7. Prove the set of infinite strings over the Hawaiian alphabet $\mathcal{H} := \{a, e, i, o, u, h, k, l, m, n, p, w\}$ is uncountable.