

PROBLEM SET 2

DISCRETE MATHEMATICS

Due: 3rd of February, 2023

1. Show that $p \rightarrow q$ is logically equivalent to $\neg q \rightarrow \neg p$.
2. Show that $(\neg p \rightarrow \perp) \rightarrow p$ is a tautology.
3. Show that $(p \rightarrow r) \vee (q \rightarrow r)$ is equivalent to $(p \wedge q) \rightarrow r$.
4. Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.
5. Using the predicate definitions below, let our universe of discourse be the collection of all characters in the play Macbeth and translate the following propositions of the first-order logic into English sentences.

$\kappa(x) :=$ “ x is a king.”

$\vartheta(x) :=$ “ x is a thane.”

$v(x) :=$ “ x is a witch.”

$\omega(x) :=$ “ x is man woman-borne.”

$\mu(x, y) :=$ “ x murders y .”

- (a) Macbeth is a king, and Banquo is a thane.
 - (b) Every king is murdered by someone.
 - (c) Someone murders every king.
 - (d) No witches murder anyone.
 - (e) No kings murder any thanes.
 - (f) Macbeth is not murdered by any man woman-borne.
 - (g) Every king is a thane.
 - (h) Banquo is not a king, yet someone murders him.
 - (i) Any witch could not possibly be a king.
 - (j) The only kings are those woman-borne.
6. Using the predicate definitions below, let our universe of discourse be the collection of all living beings on the Obra-Dinn and translate the following propositions of the first-order logic into English sentences.

$\kappa(x) :=$ “ x is the Captain.”

$\mu(x) :=$ “ x is a mate.”

$\sigma(x) :=$ “ x is a sea monster.”

$\delta(x, y) :=$ “ x is drowned by y .”

$\rho(x, y) :=$ “ x was terribly ravaged by y .”

$\gamma(x, y) :=$ “ x was shot by y .”

- (a) $\kappa(\text{“Richard Witterel”}) \wedge \forall x(\mu(x) \rightarrow \gamma(x, \text{“Richard Witterel”}))$.
 - (b) $\neg \forall x \exists y(\sigma(y) \wedge \rho(x, y))$.
 - (c) $\exists x \forall y(\sigma(x) \wedge (\mu(y) \rightarrow \delta(y, x)))$.
 - (d) $\left(\neg \exists x(\sigma(x)) \wedge \left(\neg \exists x(\sigma(x)) \rightarrow \exists x(\gamma(\text{“Richard Witterel”}, x)) \right) \right) \rightarrow \exists x(\gamma(\text{“Richard Witterel”}, x))$.
7. Let φ and ψ be predicates. Rewrite the following first-order sentences so that no quantifier has a \neg to its left.
 - (a) $\forall x \exists y(x \vee y)$
 - (b) $\forall x \forall y(\neg x \vee (x \leftrightarrow y))$
 - (c) $\exists x \forall y \forall z((x \rightarrow y) \rightarrow z)$
 - (d) $\exists x(\varphi(x) \vee \forall y \exists z(\psi(y, z) \rightarrow \neg x))$