

Problem Set 6

Discrete Mathematics

Due on the 5th of March, 2024

All basic arithmetic and algebraic facts about \mathbb{N} and \mathbb{Z} are now yours to use.

- (20 pts) 1.(a) Show that $(c \neq 0 \wedge ac \mid bc) \Rightarrow (a \mid b)$ for all $a, b, c \in \mathbb{Z}$.
 (b) Show that $(n \mid x \wedge n \mid y) \Rightarrow (n \mid ax + by)$ for all $n, x, y, a, b \in \mathbb{Z}$.
- (20 pts) 2. For all $z \in \mathbb{Z}$, show that z is even implies z is not odd.
- (20 pts) 3.(a) For all $n \in \mathbb{N}$, show that n is even implies $n + 1$ is odd.
 (b) For all $n \in \mathbb{N}$, show that n is odd implies $n + 1$ is even.
- (20 pts) 4. Show that $3 \mid n^3 - n$ for all $n \in \mathbb{N}$.¹
- (20 pts) 5. The *Fibonacci sequence* is the recursive function $\mathcal{F} : \mathbb{N} \rightarrow \mathbb{N}$ below.

¹ Hint: try a proof by induction.

$$\mathcal{F}(0) := 0$$

$$\mathcal{F}(1) := 1$$

$$\mathcal{F}(n+2) := \mathcal{F}(n+1) + \mathcal{F}(n)$$

Show that $1 + \sum_{i=0}^n \mathcal{F}(i) = \mathcal{F}(n+2)$ for all $n \in \mathbb{N}$.