MAP 5615 - Monte Carlo Methods

Homework 1 (Due Feb 1, 2019)

1. Given an LCG with parameters a, c, m, prove that (use mathematical induction)

$$x_{n+k} \equiv a^k x_n + \frac{(a^k - 1)}{a-1} c \mod m; \; (a \geq 2, k \geq 0)$$

which shows that the $(n+k)$th term can be computed directly from the nth term.

2. Write a code that implements RANDU. For debugging purposes, print x_{1000} when the seed is $x_0 = 1$.

   (a) Using RANDU generate $u_1, ..., u_{20,002}$, where $u_n = x_n/M$. For all triplets in your sequence, (u_i, u_{i+1}, u_{i+2}),in which $0.5 \leq u_{i+1} \leq 0.51$, plot u_i versus u_{i+2}. Comment on the pattern of your scatterplot.

   (b) Generate a sequence of length 1002. Use a program that plots points in 3 dimensions and rotates the axes to rotate the points until you can see the 15 planes.

3. Download a code for Mersenne twister written by Mutsuo Saito and Makoto Matsumoto.[1]. Generate 1002 numbers, and plot pairs and triples of successive numbers for a visual inspection of randomness. Discuss your conclusions.

4. Write a computer code for the Halton sequence. The input should be the dimension s and n, and the output should be the nth Halton vector where the bases are the first s prime numbers. Then, generate 1000 two-dimensional Halton vectors and plot them. Also plot 1000 two-dimensional vectors obtained from the Mersenne twister. Can you make any observations comparing the two plots visually?

5. Consider the quadrature problem $\int_0^1 e^x dx$. You will estimate this integral using MC and RQMC methods by computing $\theta_k = \frac{1}{N}\sum_{i=1}^{N} e^{x_i}$ where subscript k refers to the kth estimate for the integral.

   (a) Use Mersenne twister to obtain $\theta_1, ..., \theta_m$ where $m = 40$ and $N = 1,000$. Then compute the mean and standard deviation of the estimates $\theta_1, ..., \theta_m$.

   (b) Use random shifted Halton sequences to obtain $\theta_1, ..., \theta_m$ where $m = 40$ and $N = 1,000$. Compute the mean and standard deviation of the estimates $\theta_1, ..., \theta_m$.

   (c) Repeat parts (a) and (b) with $N = 10,000$.

   (d) Compute the exact value of the integral. Then find a good way to present your results in (a) through (c) (either forming a table or a plot) together with the exact value, so that you can make convincing conclusions on which method is better in approximating the integral.

[1] www.math.sci.hiroshima-u.ac.jp/~m-mat/MT/emt.html. Also see the links at Wikipedia for twister.