MAP 5615 - Monte Carlo Methods in Financial Mathematics
Homework 4 (Due April 8)

1. Consider the Bernoulli probability mass function

$$f(x) = p^x(1-p)^{1-x}; x = 0, 1$$

Find the corresponding tilted mass function $f_t(x)$. Is the tilted mass function a Bernoulli mass function?

2. If $0 \leq X \leq a$, show that

   (a) $E[X^2] \leq aE[X]$

   (b) $Var(X) \leq E[X](a - E[X])$

   (c) $Var(X) \leq a^2/4$
   This result shows that to minimize variance, we can try minimizing the upper bound on X. This result can be useful in finding optimal parameters for tilted densities.

3. In Chapter 4, we discussed a control variate method to price Asian arithmetic options, using Asian geometric options as a control. Write a computer program that implements the control variate method using low-discrepancy sequences. Construct a table similar to the one given at the end of Chapter 4 lecture notes, using the same parameters, but using an RQMC method instead of Monte Carlo. (A Julia code for the control variate method with Monte Carlo is uploaded on Canvas.)

4. In the following, we will use Monte Carlo simulation to estimate $\theta = E[e^U] = \int_0^1 e^x dx$.

   (a) What is the sample mean estimator and antithetic variates estimator for θ? Find the variance of these two estimators, and find how much variance reduction is achieved by the antithetic variates estimator over the sample mean estimator.

   (b) What is the control variates estimator for θ, if we take $f(U) = U$ as the control? Find the variance of this estimator, and compare it to the ones found in (a).

   (c) Write a *computer* program for the three estimators mentioned in parts (1) and (2). Compare the estimators numerically, by comparing the absolute value of the error produced by each estimator for $N = 10000, 20000, ..., 50000$. You can use your favorite random number generator.