MAP 5615 - Monte Carlo Methods

Homework 3 (Due March 27, 2019)

1. Describe in detail how to simulate the value of a random variable X such that

$$P\{X = 1\} = .3, P\{X = 2\} = .2, P\{X = 3\} = .35, P\{X = 4\} = .15$$

   using the acceptance-rejection algorithm. What is the average complexity of the algorithm? In other words, find $E[T]$ where T is the number of arithmetical operations ($\pm, \times, \div$) and comparisons (checking if a number is less than another) that need to be computed to simulate a value from X. (Ignore any operations in initialization and assignments $a := b$, and assume that generating a uniform random number is worth 3 arithmetical operations.

2. Give a method for generating the Weibull distribution function

$$F(x) = 1 - e^{-\alpha x^{\beta}}, 0 < x < \infty$$

3. Let $0 = t_0 < t_1 < ... < t_n = T$. Discuss how you would simulate $W(t)$ backwards in time, i.e., given $W(0) = 0$, simulate the Brownian motion in this order: $W(t_n), W(t_{n-1}), ..., W(t_1)$. You need to provide equations similar to ones derived in lecture notes.

4. Write a computer program that implements the Box-Muller method, and the Beasley-Springer-Moro algorithm (see Canvas for an algorithm taken from Glasserman's book) which gives an approximation to the inverse normal cdf.

   (a) What is the constructive dimension of the Box-Muller method and the Beasley-Springer-Moro algorithm?

   (b) Generate a total of 2000 i.i.d. standard normals $N(0, 1)$ using each method. Test the normality of the standard normals obtained from each method, using the Anderson-Darling test. Which data set is closer to the normal distribution? (Consult the paper by Stephens - filename **2008 Stephens.pdf** on Canvas - to find the appropriate critical points for the Anderson-Darling statistic. Clearly identify those percentiles in your soultion.)

5. Consider a European call option with parameters: T = expiry = 1, K = exercise price = 50; $r = 0.1; \sigma = 0.3; S_0 = 50$.

   (a) Compute the Black-Scholes-Merton price of the option, i.e., the exact option price.

   (b) Use random-shift Halton sequences to obtain 40 "independent" estimates for the price of the option. For each estimate, use $N = 10,000$ price paths. To simulate a path, you will simulate the geometric Brownian motion model with $\mu = r$, and using 10 time steps $t_0 = 0, t_1 = \Delta t, t_2 = 2\Delta t, ..., t_{10} = 10\Delta t = T$. Use the Box-Muller method to generate the standard normal numbers. (Note that the European call option can actually be estimated by directly generating the price at expiry, so there is no need to generate the complete price path. Nevertheless, this problem prepares you for the pricing of path dependent options.)

   (c) Repeat part (b) using Beasley-Springer-Moro algorithm.

   (d) Compare the accuracy of the estimates you obtained in parts (b) and (c) as follows: Each set of 40 estimates should be distributed according to the normal distribution whose mean is the true option price you found in part (a), and an unknown variance. Apply the Anderson-Darling statistic to test the data you obtained in parts (b) and (c), for this hypothesis. Consult Stephens' paper on Anderson-Darling statistic for critical points. Which one of the cases discussed in the paper applies to this problem? What are your conclusions for each data set? Which method is better; Box-Muller or Moro?