

INTEGRILRS Shanghai Spring 2025

Relay Round 1 10 minutes



Question 1 (5 points)

A Sophie Germain prime is a prime p satisfying the condition that $2p + 1$ is also prime. What is the next Sophie Germain prime after 23?

苏菲·热尔曼素数是一个素数 p ，满足条件 $2p + 1$ 也为素数。在 23 之后的下一个苏菲·热尔曼素数是什么？

Question 2 (7 points)

Let $T = TNYWR$. Rosa performs the following operation to an integer n written on the blackboard: if n is odd, she replaces n with $\frac{n-1}{2}$; if n is even, she replaces it with $n + 1$. After how many steps could she bring T down to one? For example, if she starts with $n = 17$, she would bring it down to one in 7 steps: $17 \rightarrow 8 \rightarrow 9 \rightarrow 4 \rightarrow 5 \rightarrow 2 \rightarrow 3 \rightarrow 1$.

设 $T = TNYWR$ 。罗莎对写在黑板上的整数 n 执行以下操作：如果 n 是奇数，她将其替换为 $\frac{n-1}{2}$ ；如果 n 是偶数，她将其替换为 $n + 1$ 。经过多少步操作，她可以将 T 变为 1？例如，如果她从 $n = 17$ 开始，她将在 7 步内将其变为 1： $17 \rightarrow 8 \rightarrow 9 \rightarrow 4 \rightarrow 5 \rightarrow 2 \rightarrow 3 \rightarrow 1$ 。

Question 3 (7 points)

Let $T = TNYWR$. Find the positive difference between the maximum and the minimum values of $2x + y - 3$, where x and y satisfy the equation below.

$$2x^2 + y^2 = T - 2$$

设 $T = TNYWR$ 。求 $2x + y - 3$ 的最大值与最小值之间的正差，其中 x 和 y 满足以下方程：

$$2x^2 + y^2 = T - 2$$

Question 4 (7 points)

Let $T = TNYWR$. How many rectangles are in a $T \times T$ grid? (The smallest possible rectangle could be considered as a single 1×1 cell.)

设 $T = TNYWR$ 。在一个 $T \times T$ 的网格中有多少个矩形？（最小的矩形可以视为单个 1×1 的单元格。）

END OF TEST.