Problem 1 - Question a

We are solving Problem 1, and the question is to analyze the function $10n^4 + 7n^2 + 3n$.

Proof By Definition:

By definition, a function $t(n)$ is said to be in $\Theta(g(n))$ if $t(n)$ is bounded both above and below by some positive constant multiples of $g(n)$ for all large n, i.e.,

$$c_2 g(n) \leq t(n) \leq c_1 g(n)$$

for all $n \geq n_o$.

Proof by Limit:

When computing the limit of two functions in question, three principal cases may arise:

$$\lim_{n\to\infty} \frac{t(n)}{g(n)} = \begin{cases} 0 & : \text{implies that } t(n) \text{ has a smaller order of growth than } g(n) \\ c & : \text{implies that } t(n) \text{ has the same order of growth as } g(n) \\ \infty & : \text{implies that } t(n) \text{ has a larger order of growth than } g(n) \end{cases}$$

Proof for $t(n) = 10n^4 + 7n^2 + 3n$ and $g(n) = n^4$

Proof by Definition:

$$c_2 n^4 \leq 10n^4 + 7n^2 + 3n \leq c_1 n^4$$

Let's first prove the right inequality:

$$\frac{10n^4 + 7n^2 + 3n}{n^4} \leq \frac{c_1 n^4}{n^4}$$

$$10 + 7\left(\frac{n^2}{n^4}\right) + 3\left(\frac{n}{n^4}\right) = c_1$$

$$10 + \frac{7}{n^2} + \frac{3}{n^3} \leq c_1$$

Choose $n = n_0 = c_1$:

$$10 + \frac{7}{1^2} + \frac{3}{1^3} \leq c_1$$

$$20 \leq c_1$$

$c_1 \geq 20$, so $c_1 = 20$
Now, let's prove the left inequality.