# MEEN40060 Fracture Mechanics – Lecture Notes and Module Related Content

*Daniel Jakob* *December 13, 2022*

Coordinator: Dr Neal Murphy (neal.murphy@ucd.ie)
Lecturers: Dr Neal Murphy
TAs: Dr Ehsan Rezvani

**Lecture Timetable:**
36 lectures, 3 per week during Semester 1

Monday 10am Room 216/Zoom

Wednesday 10am Room 216/Zoom

Friday 9am Room 216/Zoom

Weeks 1–4 §8.0 External Flow Dr. K. Nolan
Weeks 5–8 §9.0 Compressible Flow Dr. K. Nolan
Weeks 9–12 Turbomachinery Dr. W. Smith
**Laboratory:** 2 laboratory exercises each of 2 hour duration

Wednesday 3pm Room 005

Thursday 3pm Room 005

Laboratory exercises support section 8.0 & 9.0

1. External Flow: Wind Tunnel Testing
2. Compressible Flow: De Laval Nozzle Testing

**Assessment:**
Midterm In-Class Test:

| | |
|---|---|
| Friday 25th October 2020 | 15% |

Laboratory — Express format

| | |
|---|---|
| Full report (due Friday 6th November 2020) | 10% |
| Short report (due Friday 4th December 2019) | 5% |

No laboratory marks will be given if a student fails to sign in at a laboratory.

Final Open Book Examination:

| | |
|---|---|
| During weeks 14 and 15 | 70% |

**Recommended Texts:**
"Fluid Mechanics" – Frank M. White
McGraw-Hill International Editions, 5th Edition
"Fundamentals of Fluid Mechanics" – Bruce .R. Munson, Donald. F. Young and Theodore. H. Okiishi
John Wiley & Sons, 5th Edition
"Introduction to Fluid Mechanics" – Edward. J. Shaughnessy Jr, Ira. M. Katz and James. P. Schaffer
Oxford University Press, 1st Edition

# Part I Lecture Notes

## 1 History and Overview

People:

- A. A. Griffith (1920)
- George Irwin (1948)
- Egon Orowan
- Nevill Mott (1948)
- Alan Wells (1961)
- Jim Rice (1968)

# Part II (Extended) Formulae Sheet

## Mechanics

$v = u + at$
$s = \frac{v+u}{2}t$
$v^2 = u^2 + 2as$
$s = ut + \frac{1}{2}at^2$
$p = mv$
$\sum F = \frac{dp}{dt}$
$\int_{t_1}^{t_2} F dt = p_2 - p_1$ - Impulse
$P = \frac{F}{A}$
$F = ma$
$E_k = 1/2mv^2$
$\mu = Fd_\perp$ - moment
$E_p = mgh$
$W = \int F \cdot dx = F\Delta x$
$P_{avg} = \frac{\Delta W}{\Delta t}$
$P_{inst} = \frac{dW}{dT} = F \cdot v$

## Torque

$\tau = Fd_\perp$ - About midpoint
$\tau = r \times F$ - vector torque
$W = \tau(\theta_2 - \theta_1) = \tau\Delta\theta$
$P = \tau\omega$
$L = r \times p = mr \times v$ - AM particle
$L = I\omega$ - AM rigid body
$\sum \tau = \frac{dL}{dt}$

## Rotational motion

$\omega = \frac{d\theta}{dt}$
$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
Equations for constant $\alpha$:
$\theta = \theta_o + \omega_o t = \frac{1}{2}\alpha t^2$
$\omega = \omega_o + \alpha t$
$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$
$E_k = \frac{1}{2}I\omega^2$
$I = \int r^2 dm$

## SHM

$\omega = 2\pi f = \frac{2\pi}{T}$
$F = -kx$
$E_s = \frac{1}{2}kx^2$
$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const$
$T_s = 2\pi\sqrt{\frac{m}{k}}$
$T_{sp} = 2\pi\sqrt{\frac{L}{g}}$
$T_{physP} = 2\pi\sqrt{\frac{I}{mgd}}$

## Waves

$v = f\lambda$
$k = \frac{2\pi}{\lambda}$
$\omega = 2\pi f$
General Wavefunction for free wave:
$y(x,t) = Asin(\omega t - kx)$
Wave Equation:
$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$
$E = hf = \hbar\omega = \frac{hc}{\lambda}$
$eV_o = hf - \varphi$ - photoelectric effect
$f_L = \frac{v \pm v_L}{v \pm v_s} f_s$
$c = 1/\sqrt{\mu_o \varepsilon_o}$
$n = c/v$
$n_a sin(\theta_a) = n_b sin(\theta_b)$
$sin(\theta_c) = \frac{n_b}{n_a}$
$dsin(\theta) = m\lambda$ - constructive
$dsin(\theta) = (m + \frac{1}{2})\lambda$ - destructive
$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ - object,image distance

## Elasticity

$Stress = F/A$
$Strain = \delta l/l_o$
$Youngs\ Mod = Stress/Strain$

## Thermodynamics

$pV = nRT$
$moles = m/A$
Kinetic energy per molecule
$E_k = n/2k_bT$- n=Degrees of freedom
$v_{rms} = \sqrt{3k_bT/m} = \sqrt{3RT/A}$
$\gamma = C_p/C_v$
$\gamma = 5/3$ - Monatomic
$\gamma = 7/5$ - Diatomic

$v = \sqrt{\frac{\gamma P}{\rho}}$

## Entropy-Heat

$dS = \frac{dQ}{T}$
$Q = mc\Delta T$
$Q = mL$
$dQ = Ldm$ - if $m$ changes
Consider what changes, i.e sign on $Q$.
Also for a large bath, $\Delta S = \frac{\Delta Q}{T}$
$\frac{dQ}{dt} = k\frac{\Delta T}{x}$

## Relativity

$\beta = v/c$
$\gamma = 1/\sqrt{1-\beta^2}$
$p = \gamma mv$
$E = \gamma mc^2$
$E^2 = (pc)^2 + (mc^2)^2$
$\Delta t = \gamma \Delta t_o$
$\Delta L = \frac{\Delta L_o}{\gamma}$
$E_k = (1-\gamma)mc^2$

## Electromagnetism

$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}$
$E = \frac{Q}{4\pi\varepsilon_o r^2}\hat{r}$
$p = q \cdot l$ - Dipole moment
$\tau = p \times E$ - Torque
$u = -p \cdot E$ - Potential Energy
$E = -\nabla V$
$V = -\int E \cdot dl$
$\int E \cdot dA = \frac{Q_{encl}}{\varepsilon_o} = \frac{\sum_i q_i}{\varepsilon_o}$
$C = Q/V$
$C = \varepsilon_o \frac{A}{d}$ - parallel plate
$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + .. + \frac{1}{C_n}$ - Series
$C = C_1 + C_2 + .. + C_n$ - parallel
$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$ - stored E
$u_e = \frac{1}{2}\varepsilon_o\varepsilon_r E^2$ - $E_E$ density
$u_b = \frac{1}{2}\frac{B^2}{\mu_o\mu_r}$ - $E_B$ density
$J = \sum_i n_i q_i v_{d_i}$
$\rho = \frac{E}{J}$ - resistivity
$\sigma = \frac{1}{\rho}$ - conductivity
$R = \frac{\rho L}{A}$
$V = \varepsilon - Ir$ - across a Battery
$P = VI = I^2R = \frac{V^2}{R}$
$I = \frac{dQ}{dt} = nqAv_d$
$R = R_1 + R_2 + .. + R_n$ - series
$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + .. + \frac{1}{R_n}$ - parallel
$F = qE$
$F = q\, v \times B$
$F = Il \times B$
$B = \frac{\mu_o I}{2\pi r}$ - current carrying wire
$B = \frac{\mu_o I}{2r}$ - center of a loop
Multiply by N for N loops. $B = \frac{\mu_o I}{2(x^2+r^2)^{3/2}}$ - $x$ from center of loop
$\mu_e = \frac{e\hbar}{2m_e}$
$\mu_N = \frac{e\hbar}{2m_N}$
$\varepsilon = -\frac{d\Phi_b}{dt} = -\frac{B \cdot dA}{dt}$
$E_b = -\mu_{N/e} \cdot B$
$\int B \cdot dl = \mu_o I_{encl}$

## Capacitors

Charging:
$Q = Q_o(1 - e^{-t/RC})$
$I = I_o e^{-t/RC}$
Discharging:
$Q = Q_o e^{-t/RC}$
$I = Ie^{-t/RC}$
Where $RC = \tau$ is the time constant

## Gravity

$F = G\frac{Mm}{r^2}$
$g = G\frac{M}{r^2}$

## Quantum mechanics

$\left(-\frac{\hbar}{2m}\nabla + V\right)\psi = E\psi$
$\Delta p \Delta x \geq \frac{\hbar}{2}$
$f(E) = \frac{1}{e^{(E-E_f)/k_bT}+1}$
$\lambda_d b = \frac{h}{p}$

# Part III Past Exams

## 2019/2020 Semester 1 Exam

### *Q1 (Short Questions)*

(a) Using a simple atomistic model, show that the theoretical cohesive strength of an ideal elastic solid is of the order of its Young's modulus. Is this upper limit ever approached in real materials? Using the same model, also show that the cohesive strength may be written as

$$\sigma_c = \sqrt{\frac{E\gamma_s}{x_o}}$$

where $E$ is Young's modulus, $\gamma_s$ is the surface energy per unit area and $x_o$ is the equilibrium lattice spacing. Comment on the accuracy of your result. (10 marks)

(b) Describe the energy balance approach to fracture as originally proposed by Griffith. Explain how Griffith used this approach to successfully predict the fracture strength of glass. Why does the direct application of Griffith's result grossly underestimate the fracture strength of metals? (10 marks)

(c) What is meant by the 'plane strain fracture toughness of a material', $K_{Ic}$? When carrying out a standard laboratory test to determine $K_{Ic}$ for a given material, discuss the restrictions that apply to the dimensions of the test specimens in order to obtain a valid result. How are these restrictions overcome for tough materials in more recent standards such as ASTM E1820:2001? (10 marks)

(d) What is meant by an '$R$-Curve' in the context of the fracture resistance characteristics of a material. Discuss the factors influencing the shape of the $R$-Curve, giving examples of materials exhibiting flat, rising and falling $R$-Curves and the underlying fracture mechanisms which cause this behaviour. (10 marks)

(e) Briefly describe how linear elastic fracture mechanics may be used to characterize fatigue crack growth under constant amplitude cyclic loading conditions. Explain how the occurrence of variable amplitude loading influences the propagation of a fatigue crack. (10 marks)

(f) Describe the typical variation of triaxiality in the vicinity of a mode I crack front and show how different levels of constraint in plane stress and plane strain lead to relatively large differences in the size and shape of the plastic zone under these conditions. (10 marks)

(g) Briefly discuss the role of the $J$-integral in elastic-plastic fracture mechanics. In particular, describe how the $J$-integral may be viewed as both an energy parameter and a stress intensity parameter, and discuss the relationship between the $J$-integral and the other widely used elastic-plastic parameter, the crack tip opening displacement (CTOD). (10 marks)

(Total: 70 marks)

## *Q1 Solution*

(a) The potential energy-atomic separation relationship for a pair of atoms typically takes the following form: The force-separation relationship is

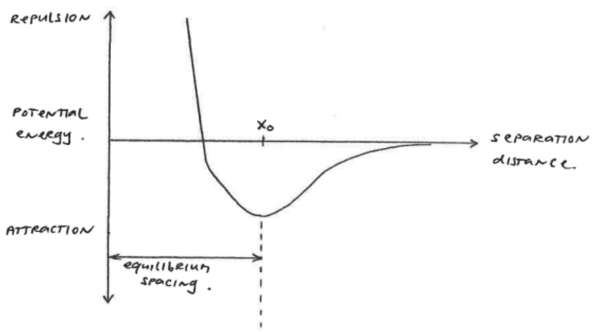


**Figure 1.1.** caption

as follows: Approximate the force-separation curve as half the period of

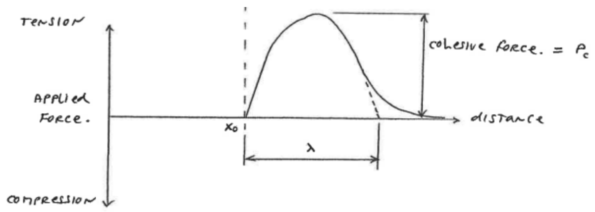


**Figure 1.2.** caption

a sine wave. Take origin at $x = x_0$, then we have:

$$P = P_c \sin^2 \left( \frac{\pi x}{\lambda} \right) \tag{1.1}$$

For small displacements, $\sin x \approx x$

$$\Rightarrow P \simeq P_c \left( \frac{\pi x}{\lambda} \right) \tag{1.2}$$

The bond stiffness is the slope of the force-displacement diagram

$$k = \frac{P}{x} = \frac{P_c \pi}{\lambda} \tag{1.3}$$

Relating this to the bulk properties of the material:

$$E = \frac{\sigma}{\varepsilon}, \quad \sigma = \frac{F}{A}, \quad \varepsilon = \frac{\text{displacement}}{\text{gauge length}} = \frac{x}{x_0} \tag{1.4}$$

(b) fdsfa