Stage 3, School of Mechanical & Materials Engineering, UCD
MEEN30020
Mechanics of Solids II
Lab Report
Combined Bending & Torsion

Daniel Jakob
18409686

22nd April 2021

Module Coordinator:	Dr. Neal Murphy
Date Due:	23rd April 2021
Lab T.A.:	Mr. Daniel Dreelan
Date of Lab:	5th April 2021

Abstract

In this experiment, a force is applied to copper specimen attached to a loading plate where bending, torsion, and any combination of the two can be applied. The resulting deflections and remaining deflections are measured for corresponding proportions of bending and torsion. The results are compared to two theories regarding bending and torsion in materials, max shear stress yield criterion and maximum principal stress failure criterion. Ultimately, the former reflects the material's behaviour better.

Contents

Assessment Submission Form

Student Name	Daniel Jakob
Student Number	18409686
Assessment Title	Combined Bending & Torsion
Module Code	MEEN30020
Module Title	Mechanics of Solids II
Module Coordinator	Dr. Neal Murphy
Date Submitted	22nd April 2021
Date Received	
Grade/Mark	

A SIGNED COPY OF THIS FORM MUST ACCOMPANY ALL SUBMISSIONS FOR ASSESSMENT.

STUDENTS SHOULD KEEP A COPY OF ALL WORK SUBMITTED.

Procedures for Submission and Late Submission
Ensure that you have checked the Adult Education Centre's procedures for the submission of assessments.
Note: There are penalties for the late submission of assessments. For further information please see the Adult Education *Assessment Guidelines* publication or the website at www.ucd.ie/adulted.

Plagiarism is the unacknowledged inclusion of another person's writings or ideas or works, in any formally presented work (including essays, examinations, projects, laboratory reports or presentations). The penalties associated with plagiarism are designed to impose sanctions that reflect the seriousness of the University's commitment to academic integrity. Ensure that you have read the University's ***Briefing for Students on Academic Integrity and Plagiarism*** and the UCD ***Plagiarism Statement, Plagiarism Policy and Procedures***, (http://www.ucd.ie/registrar/).

Declaration of Authorship
I declare that all material in this assessment is my own work except where there is clear acknowledgement and appropriate reference to the work of others.

Signed Daniel Jakob **Date** 22nd April 2021

Nomenclature

Physics Constants

c	Speed of light in a vacuum	$299\,792\,458\,\mathrm{m\,s^{-1}}$
G	Gravitational constant	$6.674\,30 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
h	Planck constant	$6.626\,070\,15 \times 10^{-34}\,\mathrm{J\,Hz^{-1}}$

Symbols

δ	Deflection [mm]
$\mathcal{F}$	Form factor [−]
σ	stress [MPa]
ε	deformation [mm]
F	Force [N]
I	Second moment of area $\left[\mathrm{m}^4\right]$
M	Moment [Nm]
W	Load [N]

Sub- and Superscripts

p	Plastic
y	Yield

Acronyms, Abbreviations and Initialisms

DRY	Don’t Repeat Yourself
API	Application Programming Interface

1 Introduction

Bending and torsion are important natural phenomena which must be understood thoroughly by engineers when designing structures (buildings, bridges, etc.). Bending and torsion depend on many factors including material, geometry and nature of force being applied. Lots of theory has been developed over the last couple of centuries attempting to describe these mathematically; however, theory is useless if it does not apply to real life. This lab experiment seeks out to determine the relationship between bending and torsion. A little copper specimen is being examined in a specialised test rig produced by GUNT GmbH where forces can be applied to the specimen in pure bending, pure torsion and all combinations in between. The portion of force acting in torsion and the portion acting in bending is determined by the angle at which angle the force is placed at on the loading plate (see: Subsec. 2.1). According to the *max shear stress yield criterion*, the *effective* stress of the specimen is independent of the angle at which the force is applied, however according the the *maximum principal stress failure criterion* the stress *is* dependent on the angle.

Listing 1: `makelink.m` function m-file

```
function [] = makelink(x1, y1, z1, x2, y2, z2, t, c)
%makelink(x1, y1, z1, x2, y2, z2, thickness)
%   MAKELINK plots a square based bar in 3D space
%   takes 8 inputs:
%   x1, y2, z1 are the 3D coordinates of the start of the link
%   x2, y2, z2 are the 3D coordinates of the end of the link
%   t is the thickness of the bar
%   c is the colour (RGB array) of the bar

% Author: Daniel Jakob (18409686), 06/03/2021, Version 1.2

t = t/2; % bar thickness halved, centering bar
if z1 == z2 % flat members
    if y1 == y2 % going in x-direction
        vertices =  [x1 y1+t z1-t;
                     x1 y1+t z1+t;
                     x1 y1-t z1+t;
                     x1 y1-t z1-t;
                     x2 y2+t z2-t;
                     x2 y2+t z2+t;
                     x2 y2-t z2+t;
                     x2 y2-t z2-t];

    else % going in y-direction (must rotate by 90 deg)
        vertices =  [x1+t y1 z1-t;
                     x1+t y1 z1+t;
                     x1-t y1 z1+t;
                     x1-t y1 z1-t;
                     x2+t y2 z2-t;
                     x2+t y2 z2+t;
                     x2-t y2 z2+t;
                     x2-t y2 z2-t];
    end
else % non-flat members
    vertices=   [x1+t y1-t z1;
                 x1+t y1+t z1;
                 x1-t y1+t z1;
                 x1-t y1-t z1;
                 x2+t y2-t z2;
                 x2+t y2+t z2;
                 x2-t y2+t z2;
                 x2-t y2-t z2];

end
```

```

faces = [1 2 3 4;
         1 2 6 5;
         2 3 7 6;
         3 4 8 7;
         4 1 5 8;
         5 6 7 8];
patch('Vertices', vertices, 'Faces', faces, 'FaceColor', c);
end
```

1.1 Objectives

The objective of this experiment is to examine 2 copper specimens in the test rig by inserting one specimen and iteratively applying more load at a certain angle — which dictates whether the specimen is placed under bending or torsion — until the material experiences non-elastic deformation (i.e., has exceeded its elastic limit) and then unloading the weights and then measuring what the change in length of the specimen is, its remaining deformation, Δw. This is repeated for the other angles desired to be examined and then the test is run in reverse angle order for a second specimen. Once all of the data has been collected, the forces and remaining deformations are used to determine whether or not the angle played a role on the deflection of the specimen. Don't Repeat Yourself (DRY)

1.2 Theory

The max shear stress yield DRY criterion says:

$$\tau_{\text{max}} = \frac{F}{2A} \tag{1}$$

where: τ = shear stress
F = force applied
A = area
$\frac{F}{A} = \sigma_{\text{axial}}$

at the yield point, σ_{axial} equals σ_{yield} and in 3D state of stress $\tau_{\text{max, 3D}} = {}^{\sigma_{\text{yield}}}/_{2}$, therefore: $2\tau_{\text{max, 3D}} = \sigma_{\text{yield}}$, this is also called the effective stress. In this test the effective stress σ_{eff} equals ${}^{4FR}/_{\pi r^3}$ which is independent from ϕ meaning the angle at which the force was applied at on the loading plate does not affect the stress on the specimen Application Programming Interface (API).

For brittle materials, the maximum principal stress failure criterion says:

$$\sigma_1 = \frac{2FR}{\pi r^3}(1 + \cos\phi) \tag{2}$$

which means the angle does affect the stress in the specimen. As discussed in Subsec. 2.1, pure API bending takes place at $\phi = 0° \Rightarrow \cos\phi = 1$, meaning there is yielding at $F_{\text{bend}} = \left({}^{\pi r^3}/_{4R}\right)\sigma_{\text{yield}}$, conversely, pure torsion takes place at $\phi = 90° \Rightarrow \cos\phi = 0$ meaning yielding at $F_{\text{tor}} = \left({}^{\pi r^3}/_{2R}\right)\sigma_{\text{yield}} = 2F_{\text{bend}}$.

2 Methodology

2.1 Equipment and Instrumentation

The lab equipment consisted of the test rig, the weights and specimens. There are four weights, 1 N, 2 N, 4 N and 8 N. 2 identical copper specimens will be used during the course of the experiment, see Fig. 1. The specimen is a hollow, dumbbell-shaped cylinder. It is fitted to the rig via a clamping mechanism. The test rig has several elements: the

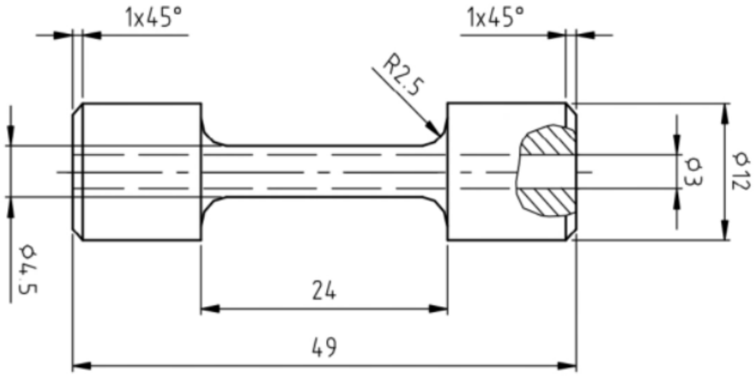

Figure 1: Specimen geometry and dimensions [1]

hook from which the load weights are hung, the counterweights, the measuring gauge, the cable pulley system and the magnetic foot which keeps in place the measuring gauge as seen in Fig. 2. The counterweight is present in the rig to mitigate the effects of a shear force, V, being applied to the specimen. A shear force would just obfuscate results and be generally a nuisance. The gauge is in units of $^{10}/_{100}$ mm which equals $1{\times}10^{-5}$ m. Fig. 3

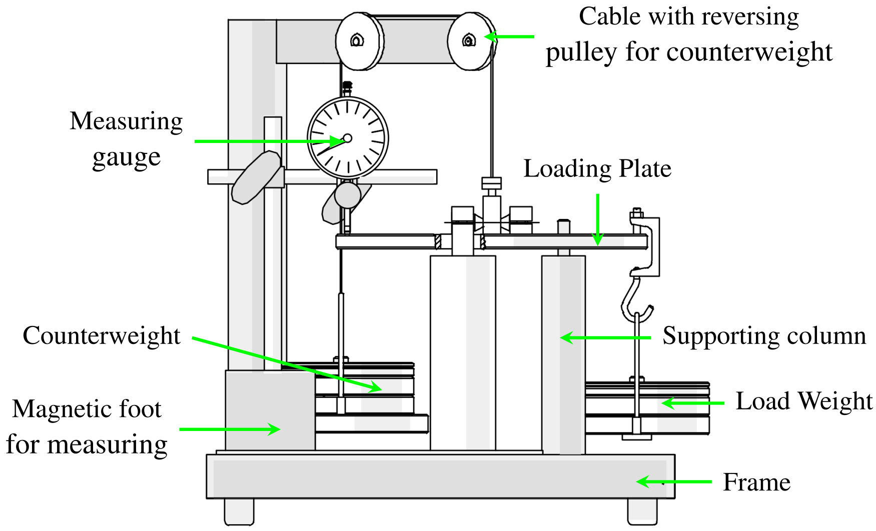

Figure 2: Rig Diagram [1]

displays a plan view of the loading plate of the test rig. The specimen is attached to the loading plate and the base of the rig by the 2 clamping bridges. The quarter circle in the bottom left of the image is where the angle of force is chosen. There are little divots for where a pin which carries the force can be placed. In the configuration shown in the figure, the angle ϕ is 0° which corresponds to File.txt pure bending in the specimen which can be intuitively seen by inspection of the how the specimen is orientated in relation to the force. If the force-pin were place at the divot which is shown at the bottom of the quarter-circle, ϕ would be 90° and pure torsion would take place in the specimen, the cylinder would be twisted.

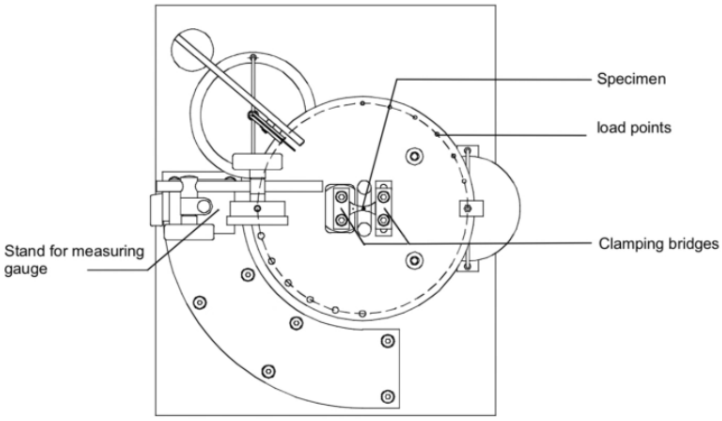

Figure 3: Plan view of loading plate [1]

2.2 Experimental Procedure

First, one side of the first specimen is inserted into the clamping mechanism connected to the loading plate and secured with an Allen key. The loading plate is lowered and the other half of the specimen is clamped ot the base machine. A close check is made to make sure the two planes of the clamp facing each other are indeed parallel. The test is started at position $\phi = 0°$, meaning pure bending, by rotating the loading plate to align the force-pin in the according divot. A load is applied to the specimen by adding weights to the counterweight, starting at 8 N. The deflection, w and remaining deflection, Δw are measured for the load applied. Then weight is added in 1 N increments, and again the deflections are measured. This is repeated until Δw reads 10 units of $^{1}/_{100}$ mm. Then, the weights are taken off, the plate is rotated so the pin is placed in the divot corresponding to 45°, this produces a combination of bending and torsion in the specimen and the measuring gauge is zeroed. The 8 N weight is applied again and the process above is repeated, and is finished by rotating the plate to 90° and repeating once more. Then, the specimen is removed and a new specimen is inserted, and the method is repeated, however, this time process is started at 90° rather than 0° as before and is done in reverse, ending at 0°. This results in six sets of data, the load, F, deflection and remaining deflection for the three angles for the 2 runs.

Once all the data has been collected, some preliminary work must be done wherein a form of normalisation must be carried out, specifically, a load value for a remaining deflection value of $^{10}/_{100}$ mm must be obtained for all loading positions and runs. This can be achieved in many ways, but in this experiment, a linear interpolation was conducted in the form of plotting a line chart for the load and the remaining deflection. This needed only to be done in one of the sets of data, by inspection of Tbl. 1, under run 2, $\phi = 0°$, it can be seen that remaining deflection skips from $9{\times}10^{-5}$ m to $12{\times}10^{-5}$ m, meaning a value of force for $10{\times}10^{-5}$ m is not obtained. Fig. 4 graphically displays the interpolation carried out: a value of ≈15.4 N is obtained.

3 Experimental Results

Table 1: Table of Results

	Run 1					
	0°		45°		90°	
Load [N]	Deflection $[1{\times}10^{-5}\,\mathrm{m}]$	Remaining Deflection $[1{\times}10^{-5}\,\mathrm{m}]$	Deflection $[1{\times}10^{-5}\,\mathrm{m}]$	Remaining Deflection $[1{\times}10^{-5}\,\mathrm{m}]$	Deflection $[1{\times}10^{-5}\,\mathrm{m}]$	Remaining Deflection $[1{\times}10^{-5}\,\mathrm{m}]$
8	97	0	111	0	126	1
9	111	0	126	2	142	2
10	122	0	141	3	158	3
11	135	2	155	4	175	4
12	147	2	169	5	194	6
13	160	5	185	6	211	8
14	176	8	202	7	230	10
15	195	10	219	10	–	–
	Run 2					
	90°		45°		0°	
Load [N]	Deflection $[1{\times}10^{-5}\,\mathrm{m}]$	Remaining Deflection $[1{\times}10^{-5}\,\mathrm{m}]$	Deflection $[1{\times}10^{-5}\,\mathrm{m}]$	Remaining Deflection $[1{\times}10^{-5}\,\mathrm{m}]$	Deflection $[1{\times}10^{-5}\,\mathrm{m}]$	Remaining Deflection $[1{\times}10^{-5}\,\mathrm{m}]$
8	121	2	108	1	93	0
9	138	3	121	1	106	1
10	154	5	136	2	118	1
11	171	6	151	3	131	2
12	188	8	161	4	143	3
13	206	10	180	6	157	4
14	–	–	198	8	172	6
15	–	–	214	10	187	9
16	–	–	–	–	202	12

Now the numerical analysis can take place. The yield limit of for each angle and for each run is tabulated in Tbl. 2 and the mean force values, $\overline{F}$, from run 1 and run 2 are found separately for the three angles using Eq. 3. This is done to average out possible effects caused by strain hardening, this is why run 2 is carried out starting at 90°. Lastly, the mean yield limit for each angle is normalised in terms of the mean yield force for pure bending, $\overline{F}_{\text{bend}}$, i.e., the mean force for $\phi = 0°$, as per Eq. 4.

$$\overline{F} = \frac{F_{\text{run 1}} + F_{\text{run 2}}}{2} \tag{3}$$

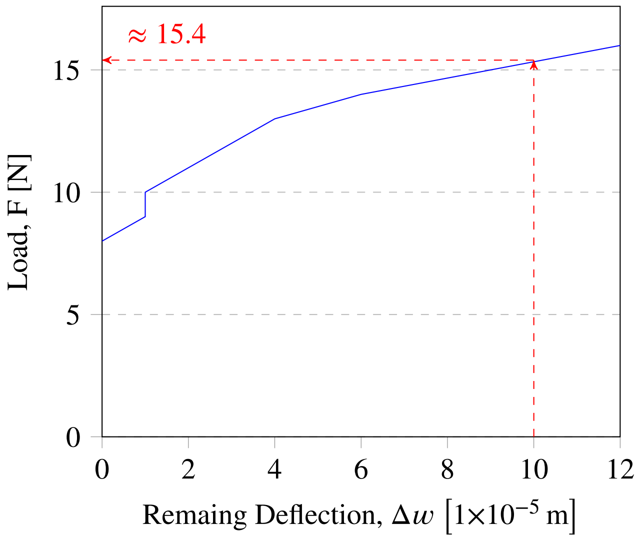

Figure 4: Run 2, Loading Angle: 0° Interpolation

$$\hat{F} = \frac{\overline{F}}{\overline{F}_{\text{bend}}} \tag{4}$$

Finally, the mean yield limits with reference to the yield limit for pure bending can be plotted against angle position, as per Fig. 5. The plot shows that the mean yield limit

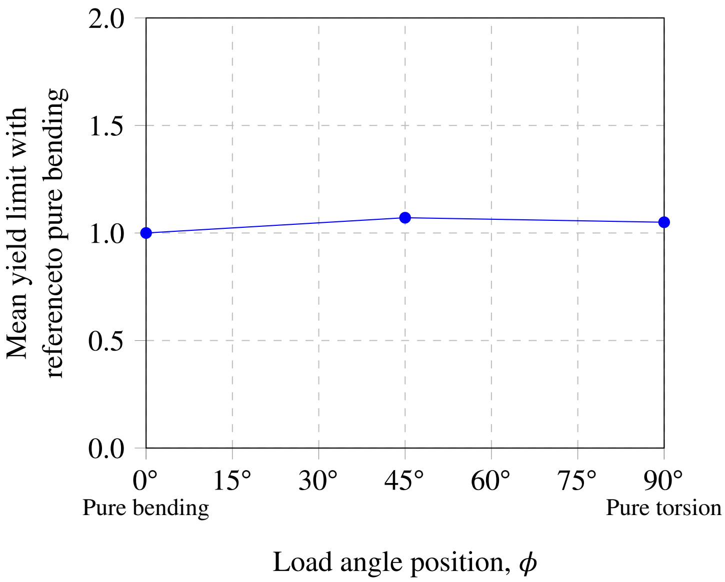

Figure 5: Mean yield limit with reference to pure bending against load angle position

with reference to pure bending, $\hat{F}$, and by extension the curve, do not deviate too much from 1.0.

3.1 Discussion

In Tbl. 1, it can be seen that the deflection at 8 N for 0° is smaller ($\approx$ 25×10^{-5} m) compared to 90° for both runs, even with strain hardening factors. This indicates that the specimen resists pure bending better than pure torsion, this may be due to any number of factors. In Tbl. 2, the values for mean yield limit with reference to pure bending, $\hat{F}$, for 45° and 90° are very close to 1.0 (at $\phi = 0°$, $\overline{F}_{\text{bend}}/\overline{F}_{\text{bend}} = 1$), this is reflected in Fig. 5.

From Fig. 5, it can be seen that the curve remains very close to 1.0, i.e., the mean yield limit does not change with angle. From this, it can be inferred that the maximum principal stress failure criterion does not apply to this material, or at least, this test. This makes sense, as this criterion specifically only applies to *brittle* materials, which copper is not generally a part of.

To improve the experimental procedure, perhaps more data points could have been collected. Data points for 22.5° and 67.5° could have been obtained, or maybe the test could have been conducted two more times, giving run 3 and run 4. Another way to improve the procedure would be to replace the specimen after every angle has been tested. This would mitigate the effects of strain hardening, and would eliminate the need for calculating the mean force for the angles over run 1 and run 2. However, this would result in, what could be seen as, a gross overuse of specimens and the experiment would take a lot longer to carry out.

Inaccuracies could stem from any part of the experiment. Reading the gauge (human error) is perhaps the greatest source of error in any experiment, as parallax is always an issue when discerning the exact position of a gauge needle. Another source of inaccuracy could be the machine itself, the most recent calibration may have been years (or decades) ago.

4 Conclusions

1. The max shear stress yield criterion matches the results better: the yield limit does NOT seem change with load angle position Appxs. A to B, Appxs. A and B.
2. The copper specimen does not behave according to the maximum principal stress failure criterion
3. The copper specimen therefore is not a brittle material Appx. A, Appx. A.1.

Table 2: Calculations

	Loading angle		
	0°	45°	90°
Yield limit Run 1 [N]	15	15	14
Yield limit Run 2 [N]	13	15	15.4
Mean force value [N]	14	15	14.7
Mean yield limit with reference to pure bending	1	1.071	1.05

References

[1] GUNT GmbH. *WP130 Verification of Stress Hypotheses Instruction Manual*. https://www.gunt.de/images/datasheet/1531/WP-130-Verification-of-stress-hypotheses-gunt-1531-pdf_1_en-GB.pdf. Hamburg, DE, 2009.

A Big Calculations

$$\frac{n!}{k!(n-k)!} = \binom{n}{k} \tag{5}$$

A.1 Hmm?

$$\frac{\begin{array}{r}\left(x_1 x_2\right)\\ \times\left(x_1' x_2'\right)\end{array}}{\left(y_1 y_2 y_3 y_4\right)} \tag{6}$$

B Smaller Calculations

$$(a), [b], \{c\}, |d|, \|e\|, \langle f\rangle, \lfloor g\rfloor, \lceil h\rceil, \ulcorner i\urcorner, /j\backslash \tag{7}$$

Eq. 7

$$A_{m,n} = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{pmatrix} \tag{8}$$