
Math, Science, and Engineering Handbook

Daniel Kelley

<daniel.kelley@gmp.san-jose.ca.us>

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1 Math

1.1 Integrals

- 1 $\int x^n dx = \frac{x^{n+1}}{n+1}$
- 2 $\int \frac{dx}{x} = \ln x$
- 3 $\int e^x dx = e^x$
- 4 $\int \cos(x) dx = \sin(x)$
- 5 $\int \sin(x) dx = -\cos(x)$
- 6 $\int \sec^2(x) dx = \tan(x)$
- 7 $\int \csc^2(x) dx = -\cot(x)$
- 8 $\int \sec(x) \cdot \tan(x) dx = \sec(x)$
- 9 $\int \csc(x) \cdot \cot(x) dx = -\csc(x)$
- 10 $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$
- 11 $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$
- 12 $\int \tan(x) dx = -\ln(\cos(x))$
- 13 $\int \cot(x) dx = \ln(\sin(x))$
- 14 $\int \sec(x) dx = \ln(\sec(x) + \tan(x))$
- 15 $\int \csc(x) dx = -\ln(\csc(x) + \cot(x))$

1.2 Formulas

Quadratic Approximation	$f(u+x) = f(u) + f'(u) \cdot x + f''(u) \cdot x^2/2$
FTC2	$d/dx \int_0^x f(t)dt = f(x)$
FTC2 Chain Rule	$d/dx \int_0^{g(x)} f(t)dt = g'(x) \cdot f(g(x))$
Weighted Average	$\int_a^b f(x)w(x)dx / \int_a^b w(x)dx$

1.3 L'Hôpital's Rule

$$\lim_{x \rightarrow a} f(x)/g(x) = \lim_{x \rightarrow a} f'(x)/g'(x)$$

0/0	Straight up
∞/∞	Straight up
$0 \cdot \infty$	Rewrite as quotient
0^0	Rewrite as $e^{\ln(f)}$
∞^0	Rewrite as $e^{\ln(f)}$
1^∞	Rewrite as $e^{\ln(f)}$
$\infty - \infty$	Good luck
Otherwise	Forget it.

1.4 Parametric Vector Calculus

Position	$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$	$\int \vec{v}(t)dt$
Velocity	$d\vec{r}(t)/dt$	$\vec{v}(t) = x'(t)\hat{i} + y'(t)\hat{j}$ $\int \vec{a}(t)dt$ $\frac{ds}{dt}\vec{T}$
Acceleration	$d\vec{v}(t)/dt$ $d^2\vec{r}(t)/dt^2$	$\vec{v}(t) = x''(t)\hat{i} + y''(t)\hat{j}$
Arc Length	$\frac{ds}{dt} = \sqrt{x'(t)^2 + y'(t)^2}$	
Unit Tangent	$\vec{T} = \vec{v}/ \vec{v} $	

1.5 Partial Differentiation

Tangent Plane to $f(x_0, y_0)$	$z - z_0 = \left(\frac{\partial f}{\partial x}\right)_{x_0}(x - x_0) + \left(\frac{\partial f}{\partial y}\right)_{y_0}(y - y_0)$
Approximation	$f(x, y) = z_0 + \frac{\partial f}{\partial x}(x - x_0) + \frac{\partial f}{\partial y}(y - y_0)$

1.6 Least Square Line

$$\begin{pmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{pmatrix}^{-1} \begin{pmatrix} \sum x_i y_i \\ \sum y_i \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \text{ for } y = ax + b \text{ given } n \text{ points } (x_i, y_i)$$

1.7 Second Derivative Test

Given $f(x, y)$ critical points (x_c, y_c) where $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

$$A = \frac{\partial^2 f}{\partial x^2} @ (x_c, y_c)$$

$$B = \frac{\partial^2 f}{\partial x \partial y} @ (x_c, y_c)$$

$$C = \frac{\partial^2 f}{\partial y^2} @ (x_c, y_c)$$

$$AC - B^2 > 0, A > 0 \text{ or } C > 0 \quad \text{Minimum point}$$

$$AC - B^2 > 0, A < 0 \text{ or } C < 0 \quad \text{Maximum point}$$

$$AC - B^2 < 0 \quad \text{Saddle point}$$

$$AC - B^2 = 0 \quad \text{Need higher order terms to conclude}$$

1.8 Differential Chain Rule

$$f(x(t), y(t), z(t)); \frac{df}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt} + f_z \frac{dz}{dt}$$

1.9 Level Curves and Surfaces

The *level curve* for a function $f(x, y)$ is the set of points (x, y) where $f(x, y) = C$ for constant C .

1.10 Gradient

The *gradient* ∇f of function f is a vector of the partial derivatives of f for each independent variable; e.g. $\nabla f(x, y) = \langle f_x, f_y \rangle$. $\nabla f \perp f(x, y)$, i.e. *gradient* \perp *level curve*.

The *directional derivative* of f at the point P in the direction of \vec{u} is $\left. \frac{df}{ds} \right|_{P, \vec{u}} = \nabla f(P) \cdot \vec{u}$.

Given an *objective* function f and a *constraint* function $g = C$ for constant C , the *extrema* of f are found when $\nabla f \parallel \nabla g$. The *Lagrange multiplier* λ is $\frac{\nabla f}{\nabla g}$.

1.11 Center of Mass

M Mass
 δ Density Function
 \bar{x} x center
 \bar{y} y center

$$\begin{aligned} M &= \int \int_R \delta \, dA \\ \bar{x} &= \frac{1}{M} \int \int_R x \delta \, dA \\ \bar{y} &= \frac{1}{M} \int \int_R y \delta \, dA \end{aligned}$$

1.12 Moment of Inertia

I_x Moment about x axis
 I_y Moment about y axis

$$\begin{aligned} I_x &= \int \int_R \delta y^2 \, dy \\ I_y &= \int \int_R \delta x^2 \, dx \end{aligned}$$

1.13 Change of Variables

$$\int \int_R f(x, y) dx dy = \int \int_R g(u, v) |J| du dv$$

$$g(u, v) = f(x(u, v), y(u, v))$$

$$|J| = \left| \frac{\partial(x, y)}{\partial(u, v)} \right| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$\left| \frac{\partial(x, y)}{\partial(u, v)} \right| \cdot \left| \frac{\partial(u, v)}{\partial(x, y)} \right| = 1$$

1.14 Vector Field

\vec{F} Field

M Field component in x direction (\hat{i}) F_x

N Field component in y direction (\hat{j}) F_y

C Curve $\vec{r}(t) = \langle x(t), y(t) \rangle$

$$\vec{F}(x, y) = \langle M, N \rangle$$

$$\vec{F}(x, y) = M(x, y)\hat{i} + N(x, y)\hat{j}$$

$$\text{curl } \vec{F} = N_x - M_y$$

1.15 Line Integral

$$C = \vec{r}(t)$$

$$\vec{r}(t) = \langle x(t), y(t) \rangle$$

$$P(t) = M(x, y)$$

$$Q(t) = N(x, y)$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \langle M, N \rangle \cdot \langle dx, dy \rangle$$

$$= \int_C M dx + N dy$$

$$= \int_C (P + Q) dt$$

1.16 Gradient Field

If $\vec{F}(x,y) == \nabla f$

$$\int_a^b \vec{F} \cdot d\vec{r} = f(b) - f(a)$$
$$\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$$
$$\oint \vec{F} \cdot d\vec{r} = 0$$

Fundamental Theorem for Line Integrals

Path independence

If \vec{r} is a closed path

2 Science

2.1 Units

Quantity	MKS	Name	Abbrev.
Angle		radian	rad
Solid Angle		steradian	sr
Area	m^2		
Volume	m^3		
Frequency	s^{-1}	Hertz	Hz
Velocity	$m \cdot s^{-1}$		
Acceleration	$m \cdot s^{-2}$		
Angular Velocity	$rad \cdot s^{-1}$		
Angular Acceleration	$rad \cdot s^{-2}$		
Density	$kg \cdot m^{-3}$		
Momentum	$kg \cdot m \cdot s^{-1}$		
Angular Momentum	$kg \cdot m^2 \cdot s^{-1}$		
Force	$kg \cdot m \cdot s^{-2}$	Newton	N
Work, Energy	$kg \cdot m^2 \cdot s^{-2}$	Joule	J
Power	$kg \cdot m^2 \cdot s^{-3}$	Watt	W
Torque	$kg \cdot m^2 \cdot s^{-2}$		
Pressure	$kg \cdot m^{-1} \cdot s^{-2}$	Pascal	Pa

2.2 Lab Reports

- Abstract
- Objective
- Method
- Data
- Analysis
- Conclusion
- Bibliography

2.3 Laws

Newton's 1st Law	$\sum \mathbf{F} = 0 \Leftrightarrow \dot{\mathbf{v}}$
Newton's 2nd Law	$\mathbf{F} = m\mathbf{a} = \dot{\mathbf{p}}$
Newton's 3rd Law	$\mathbf{F}_a = -\mathbf{F}_a$
Gravity	$\mathbf{F} = m\mathbf{g}; \mathbf{g} = 9.81 \text{ m/s}$
Hooke's Law	$F_x = -k\Delta x; k = \text{spring constant}$
Force	$N = \text{kg} \cdot \text{m} \cdot \text{s}^{-2}$
Energy	$J = N \cdot m$
Power	$W = J \cdot \text{s}^{-1}$
Momentum	$\mathbf{p} = m \cdot \mathbf{v}$
Kinetic Energy	$K = \frac{1}{2}mv^2 = \frac{\mathbf{p}^2}{2m}$
Momentum is conserved	
Energy is conserved	

2.4 Mechanics Problem Workflow

- Draw a good picture.
- Decorate with forces with a *free body diagram for each body*.
- Choose a suitable coordinate system.
- Decompose forces on each body.
- Determine acceleration for each body.
- Determine 1d equations of motion for each body, including necessary constraints.
- Reconstruct multidimensional motion vectors.
- Algebraically determine kinematics as needed.

3 Engineering

3.1 DC Ohm's Law

$$\begin{aligned}
 I &= \frac{E}{R} = \frac{P}{\frac{E}{I}} = \sqrt{\frac{P}{R}} \\
 R &= \frac{E}{I} = \frac{\frac{P}{I}}{I} = \frac{P}{I^2} \\
 E &= IR = \frac{P}{I} = \sqrt{PR} \\
 P &= EI = I^2R = \frac{E^2}{R}
 \end{aligned}$$

3.2 AC Ohm's Law

$$\begin{aligned}
 I &= \frac{E}{Z} = \frac{P}{E \cos(\theta)} = \sqrt{\frac{P}{Z \cos(\theta)}} \\
 Z &= \frac{E}{I} = \frac{\frac{P}{I}}{I \cos(\theta)} = \frac{P}{I^2 \cos(\theta)} \\
 E &= IZ = \frac{P}{I \cos(\theta)} = \sqrt{\frac{PZ}{\cos(\theta)}} \\
 P &= EI \cos(\theta) = I^2 Z \cos(\theta) = \frac{E^2 \cos(\theta)}{Z}
 \end{aligned}$$

Bibliography

- [1] Robert G. Brown, *Introductory Physics I*. <http://webhome.phy.duke.edu/rgb/Class/intro-physics-1/intro-physics-1.pdf>
- [2] Allied Radio Corporation, *Allied's Electronics Data Handbook*. <https://archive.org/details/AlliedsElectronicsDataHandbook>