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# Math, Science, and Engineering Handbook

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## 1 Math

### 1.1 Integrals

- 1  $\int x^n dx = \frac{x^{n+1}}{n+1}$
- 2  $\int \frac{dx}{x} = \ln x$
- 3  $\int e^x dx = e^x$
- 4  $\int \cos(x) dx = \sin(x)$
- 5  $\int \sin(x) dx = -\cos(x)$
- 6  $\int \sec^2(x) dx = \tan(x)$
- 7  $\int \csc^2(x) dx = -\cot(x)$
- 8  $\int \sec(x) \cdot \tan(x) dx = \sec(x)$
- 9  $\int \csc(x) \cdot \cot(x) dx = -\csc(x)$
- 10  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$
- 11  $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$
- 12  $\int \tan(x) dx = -\ln(\cos(x))$
- 13  $\int \cot(x) dx = \ln(\sin(x))$
- 14  $\int \sec(x) dx = \ln(\sec(x) + \tan(x))$
- 15  $\int \csc(x) dx = -\ln(\csc(x) + \cot(x))$

### 1.2 Formulas

Quadratic Approximation	$f(u+x) = f(u) + f'(u) \cdot x + f''(u) \cdot x^2/2$
FTC2	$d/dx \int_0^x f(t)dt = f(x)$
FTC2 Chain Rule	$d/dx \int_0^{g(x)} f(t)dt = g'(x) \cdot f(g(x))$
Weighted Average	$\int_a^b f(x)w(x)dx / \int_a^b w(x)dx$

### 1.3 L'Hôpital's Rule

$$\lim_{x \rightarrow a} f(x)/g(x) = \lim_{x \rightarrow a} f'(x)/g'(x)$$

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0/0	Straight up
$\infty/\infty$	Straight up
$0 \cdot \infty$	Rewrite as quotient
$0^0$	Rewrite as $e^{\ln(f)}$
$\infty^0$	Rewrite as $e^{\ln(f)}$
$1^\infty$	Rewrite as $e^{\ln(f)}$
$\infty - \infty$	Good luck
Otherwise	Forget it.

## 1.4 Parametric Vector Calculus

Position	$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$	$\int \vec{v}(t)dt$
Velocity	$d\vec{r}(t)/dt$	$\vec{v}(t) = x'(t)\hat{i} + y'(t)\hat{j}$ $\int \vec{a}(t)dt$ $\frac{ds}{dt}\vec{T}$
Acceleration	$d\vec{v}(t)/dt$	$d^2\vec{r}(t)/dt^2$ $\vec{v}(t) = x''(t)\hat{i} + y''(t)\hat{j}$
Arc Length	$\frac{ds}{dt} = \sqrt{x'(t)^2 + y'(t)^2}$	
Unit Tangent	$\vec{T} = \vec{v}/ \vec{v} $	

## 1.5 Partial Differentiation

Tangent Plane to $f(x_0, y_0)$	$z - z_0 = \left(\frac{\partial f}{\partial x}\right)_{x_0}(x - x_0) + \left(\frac{\partial f}{\partial y}\right)_{y_0}(y - y_0)$
Approximation	$f(x, y) = z_0 + \frac{\partial f}{\partial x}(x - x_0) + \frac{\partial f}{\partial y}(y - y_0)$

## 1.6 Least Square Line

$$\begin{pmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{pmatrix}^{-1} \begin{pmatrix} \sum x_i y_i \\ \sum y_i \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \text{ for } y = ax + b \text{ given } n \text{ points } (x_i, y_i)$$

## 1.7 Second Derivative Test

Given  $f(x, y)$  critical points  $(x_c, y_c)$  where  $\frac{\partial f}{\partial x} = 0$  and  $\frac{\partial f}{\partial y} = 0$

$$A = \frac{\partial^2 f}{\partial x^2} @ (x_c, y_c)$$

$$B = \frac{\partial^2 f}{\partial x \partial y} @ (x_c, y_c)$$

$$C = \frac{\partial^2 f}{\partial y^2} @ (x_c, y_c)$$

$$AC - B^2 > 0, A > 0 \text{ or } C > 0 \quad \text{Minimum point}$$

$$AC - B^2 > 0, A < 0 \text{ or } C < 0 \quad \text{Maximum point}$$

$$AC - B^2 < 0 \quad \text{Saddle point}$$

$$AC - B^2 = 0 \quad \text{Need higher order terms to conclude}$$

## 1.8 Differential Chain Rule

$$f(x(t), y(t), z(t)); \frac{df}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt} + f_z \frac{dz}{dt}$$

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## 1.9 Level Curves and Surfaces

The *level curve* for a function  $f(x, y)$  is the set of points  $(x, y)$  where  $f(x, y) = C$  for constant  $C$ .

### 1.10 Gradient

The *gradient*  $\nabla f$  of function  $f$  is a vector of the partial derivatives of  $f$  for each independent variable; e.g.  $\nabla f(x, y) = \langle f_x, f_y \rangle$ .  $\nabla f \perp f(x, y)$ , i.e. *gradient*  $\perp$  *level curve*.

The *directional derivative* of  $f$  at the point  $P$  in the direction of  $\vec{u}$  is  $\left. \frac{df}{ds} \right|_{P, \vec{u}} = \nabla f(P) \cdot \vec{u}$ .

Given an *objective* function  $f$  and a *constraint* function  $g = C$  for constant  $C$ , the *extrema* of  $f$  are found when  $\nabla f \parallel \nabla g$ . The *Lagrange multiplier*  $\lambda$  is  $\frac{\nabla f}{\nabla g}$ .

### 1.11 Center of Mass

$$\begin{aligned} M &= \int \int_R \delta \, dA \\ \bar{x} &= \frac{1}{M} \int \int_R x \delta \, dA \\ \bar{y} &= \frac{1}{M} \int \int_R y \delta \, dA \end{aligned}$$

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## 2 Science

### 2.1 Units

Quantity	MKS	Name	Abbrev.
Angle		radian	rad
Solid Angle		steradian	sr
Area	$m^2$		
Volume	$m^3$		
Frequency	$s^{-1}$	Hertz	Hz
Velocity	$m \cdot s^{-1}$		
Acceleration	$m \cdot s^{-2}$		
Angular Velocity	$rad \cdot s^{-1}$		
Angular Acceleration	$rad \cdot s^{-2}$		
Density	$kg \cdot m^{-3}$		
Momentum	$kg \cdot m \cdot s^{-1}$		
Angular Momentum	$kg \cdot m^2 \cdot s^{-1}$		
Force	$kg \cdot m \cdot s^{-2}$	Newton	N
Work, Energy	$kg \cdot m^2 \cdot s^{-2}$	Joule	J
Power	$kg \cdot m^2 \cdot s^{-3}$	Watt	W
Torque	$kg \cdot m^2 \cdot s^{-2}$		
Pressure	$kg \cdot m^{-1} \cdot s^{-2}$	Pascal	Pa

### 2.2 Lab Reports

Abstract  
Objective  
Method  
Data  
Analysis  
Conclusion  
Bibliography

### 2.3 Laws

Newton's 1st Law	$\sum \mathbf{F} = 0 \Leftrightarrow \dot{\mathbf{v}}$
Newton's 2nd Law	$\mathbf{F} = m\mathbf{a} = \dot{\mathbf{p}}$
Newton's 3rd Law	$\mathbf{F}_a = -\mathbf{F}_b$
Gravity	$\mathbf{F} = m\mathbf{g}; \mathbf{g} = 9.81 \text{ m/s}^2$
Hooke's Law	$F_x = -k\Delta x; k = \text{spring constant}$
Force	$N = kg \cdot m \cdot s^{-2}$
Energy	$J = N \cdot m$
Power	$W = J \cdot s^{-1}$
Momentum	$\mathbf{p} = m \cdot \mathbf{v}$
Kinetic Energy	$K = \frac{1}{2}m\mathbf{v}^2 = \frac{\mathbf{p}^2}{2m}$
Momentum is conserved	
Energy is conserved	

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## 2.4 Mechanics Problem Workflow

Draw a good picture.

Decorate with forces with a *free body diagram* for each body.

Choose a suitable coordinate system.

Decompose forces on each body.

Determine acceleration for each body.

Determine 1d equations of motion for each body, including necessary constraints.

Reconstruct multidimensional motion vectors.

Algebraically determine kinematics as needed.

## 3 Engineering

### 3.1 DC Ohm's Law

$$\begin{aligned} I &= \frac{E}{R} = \frac{P}{E} = \sqrt{\frac{P}{R}} \\ R &= \frac{E}{I} = \frac{E^2}{P} = \frac{P}{I^2} \\ E &= IR = \frac{P}{I} = \sqrt{PR} \\ P &= EI = I^2R = \frac{E^2}{R} \end{aligned}$$

### 3.2 AC Ohm's Law

$$\begin{aligned} I &= \frac{E}{Z} = \frac{P}{E \cos(\theta)} = \sqrt{\frac{P}{Z \cos(\theta)}} \\ Z &= \frac{E}{I} = \frac{E^2 \cos(\theta)}{P} = \frac{P}{I^2 \cos(\theta)} \\ E &= IZ = \frac{P}{I \cos(\theta)} = \sqrt{\frac{PZ}{\cos(\theta)}} \\ P &= EI \cos(\theta) = I^2 Z \cos(\theta) = \frac{E^2 \cos(\theta)}{Z} \end{aligned}$$

## Bibliography

- [1] Robert G. Brown, *Introductory Physics I*. <http://webhome.phy.duke.edu/rgb/Class/intro-physics-1/intro-physics-1.pdf>
- [2] Allied Radio Corporation, *Allied's Electronics Data Handbook*. <https://archive.org/details/AlliedsElectronicsDataHandbook>