

$$\int \int_R f(x, y) \, dx \, dy = \int \int_R g(u, v) \, |J| \, du \, dv$$

$$g(u, v) = f(x(u, v), y(u, v))$$

$$|J| = \left| \frac{\partial(x, y)}{\partial(u, v)} \right| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$\left| \frac{\partial(x, y)}{\partial(u, v)} \right| \cdot \left| \frac{\partial(u, v)}{\partial(x, y)} \right| = 1$$