Math, Science, and Engineering Handbook

Daniel Kelley
<daniel.kelley@gmp.san-jose.ca.us>

June 3, 2022

1 Math

1.1 Integrals

1 $\int x^n \, dx = \frac{x^{n+1}}{n+1}$
2 $\int \frac{dx}{x} = lnx$
3 $\int e^x \, dx = e^x$
4 $\int cos(x) \, dx = sin(x)$
5 $\int sin(x) \, dx = -cos(x)$
6 $\int sec^2(x) \, dx = tan(x)$
7 $\int csc^2(x) \, dx = -cot(x)$
8 $\int sec(x) \cdot tan(x) \, dx = sec(x)$
9 $\int csc(x) \cdot cot(x) \, dx = -csc(x)$
10 $\int \frac{dx}{\sqrt{a^2-x^2}} = sin^{-1}(\frac{x}{a})$
11 $\int \frac{dx}{a^2+x^2} = \frac{1}{a} tan^{-1}(\frac{x}{a})$
12 $\int tan(x) \, dx = -ln(cos(x))$
13 $\int cot(x) \, dx = ln(sin(x))$
14 $\int sec(x) \, dx = ln(sec(x) + tan(x))$
15 $\int csc(x) \, dx = -ln(csc(x) + cot(x))$

1.2 Formulas

Quadratic Approximation	$f(u + x) = f(u) + f'(u) \cdot x + f''(u) \cdot x^2/2$
$FTC2$	$d/dx \int_0^x f(t)dt = f(x)$
$FTC2$ Chain Rule	$d/dx \int_0^{g(x)} f(t)dt = g'(x) \cdot f(g(x))$
Weighted Average	$\int_a^b f(x)w(x)dx / \int_a^b w(x)dx$

1.3 L'Hôpital's Rule

$$\lim_{x \to a} f(x)/g(x) = \lim_{x \to a} f'(x)/g'(x)$$

$0/0$	Straight up
∞/∞	Straight up
$0 \cdot \infty$	Rewrite as quotient
0^0	Rewrite as $e^{ln(f)}$
∞^0	Rewrite as $e^{ln(f)}$
1^∞	Rewrite as $e^{ln(f)}$
$\infty - \infty$	Good luck
Otherwise	Forget it.

1.4 Parametric Vector Calculus

Position			$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$	$\int \vec{v}(t)dt$	
Velocity	$d\vec{r}(t)/dt$		$\vec{v}(t) = x'(t)\hat{\imath} + y'(t)\hat{\jmath}$	$\int \vec{a}(t)dt$	$\frac{ds}{dt}\vec{T}$
Acceleration	$d\vec{v}(t)/dt$	$d^2\vec{r}(t)/dt^2$	$\vec{v}(t) = x''(t)\hat{\imath} + y''(t)\hat{\jmath}$		
Arc Length	$\frac{ds}{dt} = \sqrt{x'(t)\hat{\imath} + y'(t)\hat{\jmath}}$				
Unit Tangent	$\vec{T} = \vec{v}/\lvert\vec{v}\rvert$				

1.5 Partial Differentiation

Tangent Plane to $f(x_0, y_0)$	$z - z_0 = (\frac{\partial f}{\partial x})_{x_0}(x - x_0) + (\frac{\partial f}{\partial y})_{y_0}(y - y_0)$
Approximation	$f(x, y) = z_0 + \frac{\partial f}{\partial x}(x - x_0) + \frac{\partial f}{\partial y}(y - y_0)$

1.6 Least Square Line

$$\begin{pmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{pmatrix}^{-1} \begin{pmatrix} \sum x_i y_i \\ \sum y_i \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \text{ for } y = ax + b \text{ given } n \text{ points } (x_i, y_i)$$

1.7 Second Derivative Test

Given $f(x, y)$ critical points (x_c, y_c) where $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

$A = \frac{\partial^2 f}{\partial x^2} @ (x_c, y_c)$

$B = \frac{\partial^2 f}{\partial x \partial y} @ (x_c, y_c)$

$C = \frac{\partial^2 f}{\partial y^2} @ (x_c, y_c)$

$AC - B^2 > 0$, $A > 0$ or $C > 0$	Minimum point
$AC - B^2 > 0$, $A < 0$ or $C < 0$	Maximum point
$AC - B^2 < 0$	Saddle point
$AC - B^2 = 0$	Need higher order terms to conclude

1.8 Differential Chain Rule

$f(x(t), y(t), z(t))$; $\frac{df}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt} + f_z \frac{dz}{dt}$

1.9 Level Curves and Surfaces

The *level curve* for a function $f(x, y)$ is the set of points (x, y) where $f(x, y) = C$ for constant C.

1.10 Gradient

The *gradient* ∇f of function f is a vector of the partial derivatives of f for each independant variable; e.g. $\nabla f(x, y) = \langle f_x, f_y \rangle$. $\nabla f \perp f(x, y)$, i.e. *gradient* $\perp$ *level curve*.

The *directional derivitive* of f at the point P in the direction of $\vec{u}$ is $\left.\frac{df}{ds}\right|_{P,\vec{u}} = \nabla f(P) \cdot \vec{u}$.

Given an *objective* function f and a *constraint* function $g = C$ for constant C, the *extrema* of f are found when $\nabla f \parallel \nabla g$. The *Lagrange multiplier* λ is $\frac{\nabla f}{\nabla g}$.

1.11 Center of Mass

M	Mass
δ	Density Function
$\bar{x}$	x center
$\bar{y}$	y center

$$M = \int\int_R \delta \, dA$$
$$\bar{x} = \frac{1}{M}\int\int_R x\delta \, dA$$
$$\bar{y} = \frac{1}{M}\int\int_R y\delta \, dA$$

1.12 Moment of Inertia

I_x	Moment about x axis
I_y	Moment about y axis

$$I_x = \int\int_R \delta y^2 \, dy$$
$$I_y = \int\int_R \delta x^2 \, dx$$

1.13 Change of Variables

$$\int\int_R f(x,y)\,dx\,dy = \int\int_R g(u,v)\,|J|\,du\,dv$$
$$g(u,v) = f(x(u,v),y(u,v))$$
$$|J| = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$
$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| \cdot \left|\frac{\partial(u,v)}{\partial(x,y)}\right| = 1$$

1.14 Vector Field

$\vec{F}$	Field
M	Field component in x direction ($\hat{\imath}$)
N	Field component in y direction ($\hat{\jmath}$)
C	Curve $\vec{r}(t) = \langle x(t), y(t)\rangle$

$$\vec{F}(x,y) = \langle M, N\rangle$$
$$\vec{F}(x,y) = M(x,y)\hat{\imath} + N(x,y)\hat{\jmath}$$

1.15 Line Integral

$$\begin{aligned} C &= \vec{r}(t) \\ \vec{r}(t) &= \langle x(t), y(t)\rangle \\ P(t) &= M(x,y) \\ Q(t) &= N(x,y) \\ \int_C \vec{F}\cdot d\vec{r} &= \int_C \langle M, N\rangle \cdot \langle dx, dy\rangle \\ &= \int_C M\,dx + N\,dy \\ &= \int_C (P+Q)\,dt \end{aligned}$$

1.16 Gradient Field

If $\vec{F}(x, y) == \nabla f$

$\int_a^b \vec{F} \cdot d\vec{r} = f(b) - f(a)$	Fundamental Theorem for Line Integrals
$\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$	Path independence
$\oint \vec{F} \cdot d\vec{r} = 0$	If $\vec{r}$ is a closed path

2 Science

2.1 Units

Quantity	MKS	Name	Abbrev.
Angle		radian	rad
Solid Angle		steradian	*sr*
Area	m^2		
Volume	m^3		
Frequency	s^{-1}	Hertz	Hz
Velocity	$m \cdot s^{-1}$		
Acceleration	$m \cdot s^{-2}$		
Angular Velocity	$rad \cdot s^{-1}$		
Angular Acceleration	$rad \cdot s^{-2}$		
Density	$kg \cdot m^{-3}$		
Momentum	$kg \cdot m \cdot s^{-1}$		
Angular Momentum	$kg \cdot m^2 \cdot s^{-1}$		
Force	$kg \cdot m \cdot s^{-2}$	Newton	N
Work, Energy	$kg \cdot m^2 \cdot s^{-2}$	Joule	J
Power	$kg \cdot m^2 \cdot s^{-3}$	Watt	W
Torque	$kg \cdot m^2 \cdot s^{-2}$		
Pressure	$kg \cdot m^{-1} \cdot s^{-2}$	Pascal	Pa

2.2 Lab Reports

Abstract
Objective
Method
Data
Analysis
Conclusion
Bibliography

2.3 Laws

Newton's 1st Law	$\sum \mathbf{F} = 0 \Leftrightarrow \dot{\mathbf{v}}$
Newton's 2nd Law	$\mathbf{F} = m\mathbf{a} = \dot{\mathbf{p}}$
Newton's 3rd Law	$\mathbf{F}_a = -\mathbf{F}_a$
Gravity	$\mathbf{F} = m\mathbf{g}$; $\mathbf{g} = 9.81$ m/s
Hooke's Law	$F_x = -k\Delta x$; $k =$ spring constant
Force	$N = kg \cdot m \cdot s^{-2}$
Energy	$J = N \cdot m$
Power	$W = J \cdot s^{-1}$
Momentum	$\mathbf{p} = m \cdot \mathbf{v}$
Kinetic Energy	$K = \frac{1}{2}m\mathbf{v}^2 = \frac{\mathbf{p}^2}{2m}$
Momentum is conserved	
Energy is conserved	

2.4 Mechanics Problem Workflow

Draw a good picture.
Decorate with forces with a *free body diagram for each body.*
Choose a suitable coordinate system.
Decompose forces on each body.
Determine acceleration for each body.
Determine 1d equations of motion for each body, including necessary constraints.
Reconstruct multidimensional motion vectors.
Algebraically determine kinematics as needed.

3 Engineering

3.1 DC Ohm's Law

$$
\begin{aligned}
I &= \frac{E}{R} &&= \frac{P}{E} &&= \sqrt{\frac{P}{R}} \\
R &= \frac{E}{I} &&= \frac{E^2}{P} &&= \frac{P}{I^2} \\
E &= IR &&= \frac{P}{I} &&= \sqrt{PR} \\
P &= EI &&= I^2R &&= \frac{E^2}{R}
\end{aligned}
$$

3.2 AC Ohm's Law

$$
\begin{aligned}
I &= \frac{E}{Z} &&= \frac{P}{Ecos(\theta)} &&= \sqrt{\frac{P}{Zcos(\theta)}} \\
Z &= \frac{E}{I} &&= \frac{E^2cos(\theta)}{P} &&= \frac{P}{I^2cos(\theta)} \\
E &= IZ &&= \frac{P}{Icos(\theta)} &&= \sqrt{\frac{PZ}{cos(\theta)}} \\
P &= EIcos(\theta) &&= I^2Zcos(\theta) &&= \frac{E^2cos(\theta)}{Z}
\end{aligned}
$$

Bibliography

[1] Robert G. Brown, *Introductory Physics I*. http://webhome.phy.duke.edu/ rgb/Class/intro-physics-1/intro-physics-1.pdf

[2] Allied Radio Corporation, *Allied's Electronics Data Handbook*. https://archive.org/details/AlliedsElectronicsDataHandbook