
Math, Science, and Engineering Handbook

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1 Math

1.1 Integrals

- 1 $\int x^n dx = \frac{x^{n+1}}{n+1}$
- 2 $\int \frac{dx}{x} = \ln x$
- 3 $\int e^x dx = e^x$
- 4 $\int \cos(x) dx = \sin(x)$
- 5 $\int \sin(x) dx = -\cos(x)$
- 6 $\int \sec^2(x) dx = \tan(x)$
- 7 $\int \csc^2(x) dx = -\cot(x)$
- 8 $\int \sec(x) \cdot \tan(x) dx = \sec(x)$
- 9 $\int \csc(x) \cdot \cot(x) dx = -\csc(x)$
- 10 $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$
- 11 $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$
- 12 $\int \tan(x) dx = -\ln(\cos(x))$
- 13 $\int \cot(x) dx = \ln(\sin(x))$
- 14 $\int \sec(x) dx = \ln(\sec(x) + \tan(x))$
- 15 $\int \csc(x) dx = -\ln(\csc(x) + \cot(x))$

1.2 Formulas

Quadratic Approximation	$f(u+x) = f(u) + f'(u) \cdot x + f''(u) \cdot x^2/2$
FTC2	$d/dx \int_0^x f(t)dt = f(x)$
FTC2 Chain Rule	$d/dx \int_0^{g(x)} f(t)dt = g'(x) \cdot f(g(x))$
Weighted Average	$\int_a^b f(x)w(x)dx / \int_a^b w(x)dx$

1.3 L'Hôpital's Rule

$$\lim_{x \rightarrow a} f(x)/g(x) = \lim_{x \rightarrow a} f'(x)/g'(x)$$

$0/0$	Straight up
∞/∞	Straight up
$0 \cdot \infty$	Rewrite as quotient
0^0	Rewrite as $e^{\ln(f)}$
∞^0	Rewrite as $e^{\ln(f)}$
1^∞	Rewrite as $e^{\ln(f)}$
$\infty - \infty$	Good luck
Otherwise	Forget it.

1.4 Vector Products

Dot Product

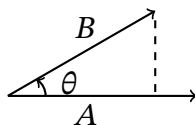


Figure 1: Dot Product

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos(\theta)$$

The scalar value of the dot product is the sum of the product of the vector components $\sum a_i \cdot b_i$

Geometrically, the scalar value is the length of the projection of \vec{B} onto \vec{A} .

Cross Product

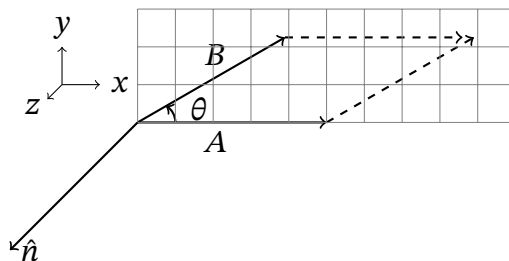


Figure 2: Cross Product

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin(\theta) \hat{n}$$

Geometrically, the vector value of the cross product is the area of the parallelogram formed by \vec{B} and \vec{A} times the unit vector \hat{n} normal to the plane of the parallelogram following the right hand rule.

Special Values

$\vec{A} \cdot \vec{B} > 0$	θ is acute.
$\vec{A} \cdot \vec{B} < 0$	θ is obtuse.
$\vec{A} \cdot \vec{B} = 0$	Vectors are orthogonal.
$\vec{A} \times \vec{B} = 0$	Vectors are parallel.

1.5 Parametric Vector Calculus

Position	$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$	$\int \vec{v}(t)dt$
Velocity	$d\vec{r}(t)/dt$	$\vec{v}(t) = x'(t)\hat{i} + y'(t)\hat{j}$
Acceleration	$d\vec{v}(t)/dt$	$\vec{a}(t) = x''(t)\hat{i} + y''(t)\hat{j}$
Arc Length	$\frac{ds}{dt} = \sqrt{x'(t)^2 + y'(t)^2}$	$\frac{ds}{dt} \vec{T}$
Unit Tangent	$\vec{T} = \vec{v}/ \vec{v} $	

1.6 Partial Differentiation

Tangent Plane to $f(x_0, y_0)$	$z - z_0 = \left(\frac{\partial f}{\partial x}\right)_{x_0}(x - x_0) + \left(\frac{\partial f}{\partial y}\right)_{y_0}(y - y_0)$
Approximation	$f(x, y) \approx z_0 + \frac{\partial f}{\partial x}(x - x_0) + \frac{\partial f}{\partial y}(y - y_0)$

1.7 Least Square Line

$$\begin{pmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{pmatrix}^{-1} \begin{pmatrix} \sum x_i y_i \\ \sum y_i \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \text{ for } y = ax + b \text{ given } n \text{ points } (x_i, y_i)$$

1.8 Second Derivative Test

Given $f(x, y)$ critical points (x_c, y_c) where $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

$$A = \frac{\partial^2 f}{\partial x^2} @ (x_c, y_c)$$

$$B = \frac{\partial^2 f}{\partial x \partial y} @ (x_c, y_c)$$

$$C = \frac{\partial^2 f}{\partial y^2} @ (x_c, y_c)$$

$$AC - B^2 > 0, A > 0 \text{ or } C > 0 \quad \text{Minimum point}$$

$$AC - B^2 > 0, A < 0 \text{ or } C < 0 \quad \text{Maximum point}$$

$$AC - B^2 < 0 \quad \text{Saddle point}$$

$$AC - B^2 = 0 \quad \text{Need higher order terms to conclude}$$

1.9 Differential Chain Rule

$$f(x(t), y(t), z(t)); \frac{df}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt} + f_z \frac{dz}{dt}$$

1.10 Level Curves and Surfaces

The *level curve* for a function $f(x, y)$ is the set of points (x, y) where $f(x, y) = C$ for constant C .

1.11 Gradient

The *gradient* ∇f of (*potential*) function f is a vector of the partial derivatives of f for each independent variable; e.g. $\nabla f(x, y) = \langle f_x, f_y \rangle$. $\nabla f \perp f(x, y)$, i.e. *gradient* \perp *level curve*.

The *directional derivative* of f at the point P in the direction of \vec{u} is $\left. \frac{df}{ds} \right|_{P, \vec{u}} = \nabla f(P) \cdot \vec{u}$.

Given an *objective* function f and a *constraint* function $g = C$ for constant C , the *extrema* of f are found when $\nabla f \parallel \nabla g$. The *Lagrange multiplier* λ is $\frac{\nabla f}{\nabla g}$.

1.12 Center of Mass

M Mass
 δ Density Function
 \bar{x} x center
 \bar{y} y center

$$\begin{aligned} M &= \int \int_R \delta \, dA \\ \bar{x} &= \frac{1}{M} \int \int_R x \delta \, dA \\ \bar{y} &= \frac{1}{M} \int \int_R y \delta \, dA \end{aligned}$$

1.13 Moment of Inertia

I_x Moment about x axis
 I_y Moment about y axis

$$\begin{aligned} I_x &= \int \int_R \delta y^2 \, dy \\ I_y &= \int \int_R \delta x^2 \, dx \end{aligned}$$

1.14 Change of Variables

$$\int \int_R f(x, y) dx dy = \int \int_R g(u, v) |J| du dv$$

$$g(u, v) = f(x(u, v), y(u, v))$$

$$|J| = \left| \frac{\partial(x, y)}{\partial(u, v)} \right| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$\left| \frac{\partial(x, y)}{\partial(u, v)} \right| \cdot \left| \frac{\partial(u, v)}{\partial(x, y)} \right| = 1$$

1.15 Vector Field

\vec{F} Field

M Field component in x direction (\hat{i}) F_x

N Field component in y direction (\hat{j}) F_y

C Curve $\vec{r}(t) = \langle x(t), y(t) \rangle$

$$\vec{F}(x, y) = \langle M, N \rangle$$

$$\vec{F}(x, y) = M(x, y)\hat{i} + N(x, y)\hat{j}$$

$$\text{curl } \vec{F} = N_x - M_y$$

$$\text{div } \vec{F} = M_x + N_y$$

1.16 Rectangular/Polar Conversion

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

$$\theta = \tan^{-1}(y/x)$$

$$r = \sqrt{x^2 + y^2}$$

$$dx dy = r dr d\theta$$

1.17 Complex Arithmetic

$i = \sqrt{-1}$	Imaginary unit
$z = a + bi$	Complex number z
$\bar{z} = a - bi$	Complex conjugate
$a = \text{Re}(a + bi)$	Real part
$b = \text{Im}(a + bi)$	Imaginary part
$(a + bi) + (c + di) = (a + c) + (b + d)i$	Addition
$(a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i$	Multiplication
$\frac{a + bi}{c + di} = \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}$	Division
$ z = \sqrt{a^2 + b^2}$	Absolute value, Modulus
$\arg(z) = \tan^{-1}(b/a) = \theta$	Argument
$z\bar{z} = z ^2$	Modulus squared
$e^{i\theta} = \cos(\theta) + i \sin(\theta)$	Euler's Formula
$z = z [\cos(\theta) + i \sin(\theta)]$	Polar form I
$z = z e^{i\theta}$	Polar form II

1.18 Sinusoidal Functions

A	Amplitude
ω	Angular Frequency
ϕ	Phase lag
τ	Time delay
ν	Frequency
P	Period

$$f(t) = A \cos(\omega t - \phi)$$

$$f(t) = A \cos(\omega (t - \tau))$$

$$\tau = \phi/\omega$$

$$\nu = \omega/2\pi$$

$$P = 1/\nu$$

1.19 Sinusoidal Identity

$$a \cos(\omega t) + b \sin(\omega t) = A \cos(\omega t - \phi)$$

$a \cos(\omega t) + b \sin(\omega t)$	Rectangular (Cartesian) form
$A \cos(\omega t - \phi)$	Amplitude-phase form

$$\begin{aligned}
 A &= \sqrt{a^2 + b^2} \\
 \phi &= \tan^{-1}(b/a) \\
 a + bi &= Ae^{i\phi} \\
 a &= A \cos(\phi) \\
 b &= A \sin(\phi)
 \end{aligned}$$

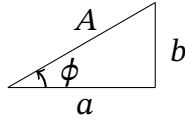


Figure 3: $a + bi = Ae^{i\phi}$

1.20 Line Integral

$$\begin{aligned}
 C &= \vec{r}(t) \\
 s &= \text{arc-length}(C) \\
 \vec{r}(t) &= \langle x(t), y(t) \rangle \\
 P(t) &= M(x, y) \\
 Q(t) &= N(x, y)
 \end{aligned}$$

Work

Force on particle along a curve.

$$\begin{aligned}
 \int_C \vec{F} \cdot d\vec{r} &= \int_C \vec{F} \cdot \hat{T} ds \\
 &= \int_C \langle M, N \rangle \cdot \langle dx, dy \rangle \\
 &= \int_C M dx + N dy \\
 &= \int_C (P + Q) dt
 \end{aligned}$$

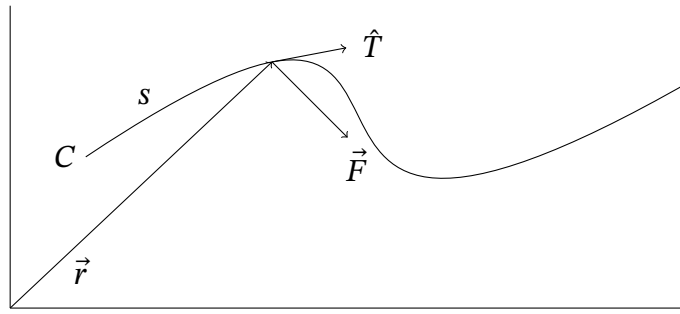


Figure 4: Work

Flow

Flow across a curve.

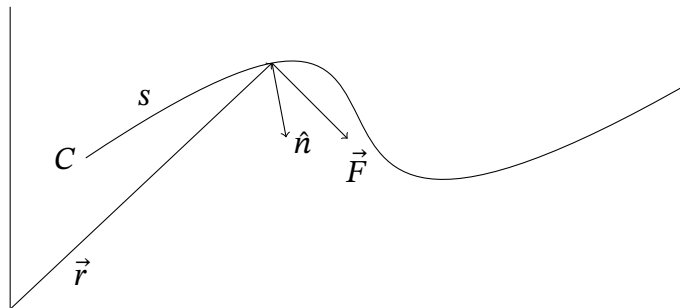


Figure 5: Flow

$$\begin{aligned}
 \int_C \vec{F} \cdot \hat{n} \, ds &= \int_C \langle M, N \rangle \cdot \langle dy, -dx \rangle \\
 &= \int_C -N \, dx + M \, dy \\
 &= \int_C (P - Q) \, dt
 \end{aligned}$$

Area

Area of a simply connected closed curve.

$$A = \frac{1}{2} \oint_C -y \, dx + x \, dy$$

1.21 Gradient Field

If $\vec{F}(x, y) = \nabla f$ then the field \vec{F} is *conservative*.

$\int_a^b \vec{F} \cdot d\vec{r} = f(b) - f(a)$	Fundamental Theorem for Line Integrals
$\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$	Path independence
$\oint \vec{F} \cdot d\vec{r} = 0$	If \vec{r} is a closed path

1.22 Green's Theorem

$\oint_C \vec{F} \cdot d\vec{r} = \int \int_R \text{curl}(\vec{F}) \, dA$	tangential
$\oint_C \vec{F} \cdot \hat{n} \, ds = \int \int_R \text{div}(\vec{F}) \, dA$	normal

1.23 Differential Equations

Separation of Variables

$$\begin{aligned}\frac{dy}{dx} &= f(x)g(y) \\ \frac{dy}{g(y)} &= f(x) \, dx \\ \int \frac{dy}{g(y)} &= \int f(x) \, dx + c\end{aligned}$$

Integrating Factors

$$\begin{aligned}\frac{dy}{dx} + P(x)y &= Q(x) \\ \frac{d}{dx}(e^{\int P(x) dx} y) &= Q(x) e^{\int P(x) dx} \\ \int \frac{d}{dx}(e^{\int P(x) dx} y) &= \int Q(x) e^{\int P(x) dx} \\ e^{\int P(x) dx} y &= \int Q(x) e^{\int P(x) dx} \\ y &= e^{-\int P(x) dx} \int Q(x) e^{\int P(x) dx}\end{aligned}$$

Step and Delta Functions

$u(t) = (t < 0) ? 0 : 1$	Unit step function
$u(t - a) = (t < a) ? 0 : 1$	Unit step function at a
$u(0) = \frac{1}{2}$	Heaviside step function
$u_{ab}(t) = u(t - a) - u(t - b)$	Box function $a < t < b$
$\delta(t) = (t = 0) ? \infty : 0$	Dirac delta function
$f(t)\delta(t) = f(0)\delta(t)$	
$f(t)\delta(t - a) = f(a)\delta(t)$	
$\frac{d}{dt}u(t) = \delta(t)$	
$\int_c^d \delta(t) dt = (c < 0 < d) ? 1 : 0$	
$\int_c^d f(t)\delta(t) dt = (c < 0 < d) ? f(0) : 0$	
$\int_c^d f(t)\delta(t - a) dt = (c < a < d) ? f(a) : 0$	

1.24 Fourier Series

Fourier Coefficients

L = half period

t = dependent variable, generally time

$f(t)$ = given function

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nt) + b_n \sin(nt)$$

$$a_0 = \frac{1}{L} \int_{-L}^L f(t) dt$$

$$a_n = \frac{1}{L} \int_{-L}^L f(t) \cos\left(n\frac{\pi}{L}t\right) dt$$

$$b_n = \frac{1}{L} \int_{-L}^L f(t) \sin\left(n\frac{\pi}{L}t\right) dt$$

$$a_n = \frac{2}{L} \int_0^L f(t) \cos\left(n\frac{\pi}{L}t\right) dt$$

$$b_n = \frac{2}{L} \int_0^L f(t) \sin\left(n\frac{\pi}{L}t\right) dt$$

$b_0 = 0, f(t) = f(-t)$, even function.

$a_0 = 0, f(-t) = -f(t)$, odd function.

1.25 Laplace Transform

Definitions

$F(s) = \int_{0^-}^{\infty} f(t)e^{-st} dt$	Definition
$a f(t) + b g(t) = a F(s) + b G(s)$	Linearity
$a^{zt} f(t) = F(s - z)$	s-shift
$u(t - a)f(t - a) = e^{-as}F(s)$	t-translation I
$u(t - a)f(t) = e^{-as}\mathcal{L}(f(t + a))$	t-translation II
$f'(t) = sF(s) - f(0^-)$	
$f''(t) = s^2F(s) - sf(0^-) - f'(0^-)$	
$f^{(n)}(t) = s^nF(s) - s^{n-1}f(0^-) - \dots - f^{(n-1)}(0^-)$	
$tf(t) = -F'(s)$	
$t^n f(t) = (-1)^n F^n(s)$	
$(f * g)(t) = F(s)G(s)$	
$\int_{0^-}^{t^+} f(\tau) d\tau = \frac{F(s)}{s}$	

Transforms

$1 = \frac{1}{s}$	$Re(s) > 0$
$e^{at} = \frac{1}{s-a}$	$Re(s) > a$
$t = \frac{1}{s^2}$	$Re(s) > 0$
$t^n = \frac{n!}{s^{n+1}}$	$Re(s) > 0$
$\cos(\omega t) = \frac{s}{s^2 + \omega^2}$	$Re(s) > 0$
$\sin(\omega t) = \frac{\omega}{s^2 + \omega^2}$	$Re(s) > 0$
$e^{zt} \cos(\omega t) = \frac{(s-z)}{(s-z)^2 + \omega^2}$	$Re(s) > Re(z)$
$e^{zt} \sin(\omega t) = \frac{\omega}{(s-z)^2 + \omega^2}$	$Re(s) > Re(z)$
$\delta(t) = 1$	$\forall s$
$\delta(t-a) = e^{-as}$	$\forall s$
$u(t-a) = e^{-as}/s$	$Re(s) > 0$
$\cosh(kt) = \frac{s}{s^2 - k^2}$	$Re(s) > k$
$\sinh(kt) = \frac{k}{s^2 - k^2}$	$Re(s) > k$
$\frac{\sin(\omega t) - \omega t \cos(\omega t)}{2\omega^3} = \frac{1}{(s^2 + \omega^2)^2}$	$Re(s) > 0$
$\frac{t \sin(\omega t)}{2\omega} = \frac{s}{(s^2 + \omega^2)^2}$	$Re(s) > 0$
$\frac{\sin(\omega t) + \omega t \cos(\omega t)}{2\omega} = \frac{s^2}{(s^2 + \omega^2)^2}$	$Re(s) > 0$
$t^n e^{at} = \frac{n!}{(s-a)^{n+1}}$	$Re(s) > a$
$\frac{1}{\sqrt{\pi t}} = \frac{1}{\sqrt{s}}$	$Re(s) > 0$
$t^a = \frac{\Gamma(a+1)}{s^{a+1}}$	$Re(s) > 0$

Heaviside Coverup

Decomposition of Laplace transforms into partial fractions. Denominator must be distinct linear factors.

$$\begin{aligned}G(s) &= \prod_{n=1}^k H_n(s) \\F(s)/G(s) &= \sum \frac{A_n}{H_n(s)} \\D_n(s) &= \frac{G(s)}{H_n(s)} \\A_n &= \left. \frac{F(s)}{D_n(s)} \right|_{\text{solve}(s, H_n(s)=0)}\end{aligned}$$

2 Science

2.1 Units

Quantity	MKS	Name	Abbrev.
Angle		radian	rad
Solid Angle		steradian	sr
Area	m^2		
Volume	m^3		
Frequency	s^{-1}	Hertz	Hz
Velocity	$m \cdot s^{-1}$		
Acceleration	$m \cdot s^{-2}$		
Angular Velocity	$rad \cdot s^{-1}$		
Angular Acceleration	$rad \cdot s^{-2}$		
Density	$kg \cdot m^{-3}$		
Momentum	$kg \cdot m \cdot s^{-1}$		
Angular Momentum	$kg \cdot m^2 \cdot s^{-1}$		
Force	$kg \cdot m \cdot s^{-2}$	Newton	N
Work, Energy	$kg \cdot m^2 \cdot s^{-2}$	Joule	J
Power	$kg \cdot m^2 \cdot s^{-3}$	Watt	W
Torque	$kg \cdot m^2 \cdot s^{-2}$		
Pressure	$kg \cdot m^{-1} \cdot s^{-2}$	Pascal	Pa

2.2 Lab Reports

Abstract
Objective
Method
Data
Analysis
Conclusion
Bibliography

2.3 Laws

Newton's 1st Law	$\sum \mathbf{F} = 0 \Leftrightarrow \dot{\mathbf{v}}$
Newton's 2nd Law	$\mathbf{F} = m\mathbf{a} = \dot{\mathbf{p}}$
Newton's 3rd Law	$\mathbf{F}_a = -\mathbf{F}_b$
Gravity	$\mathbf{F} = m\mathbf{g}; \mathbf{g} = 9.81 \text{ m/s}^2$
Hooke's Law	$F_x = -k\Delta x; k = \text{spring constant}$
Force	$N = \text{kg} \cdot \text{m} \cdot \text{s}^{-2}$
Energy	$J = N \cdot m$
Power	$W = J \cdot \text{s}^{-1}$
Momentum	$\mathbf{p} = m \cdot \mathbf{v}$
Kinetic Energy	$K = \frac{1}{2}mv^2 = \frac{\mathbf{p}^2}{2m}$
Momentum is conserved	
Energy is conserved	

2.4 Mechanics Problem Workflow

Draw a good picture.

Decorate with forces with a *free body diagram for each body*.

Choose a suitable coordinate system.

Decompose forces on each body.

Determine acceleration for each body.

Determine 1d equations of motion for each body, including necessary constraints.

Reconstruct multidimensional motion vectors.

Algebraically determine kinematics as needed.

3 Engineering

3.1 DC Ohm's Law

$$\begin{aligned}
 I &= \frac{E}{R} = \frac{P}{E} = \sqrt{\frac{P}{R}} \\
 R &= \frac{E}{I} = \frac{E^2}{P} = \frac{P}{I^2} \\
 E &= IR = \frac{P}{I} = \sqrt{PR} \\
 P &= EI = I^2R = \frac{E^2}{R}
 \end{aligned}$$

3.2 AC Ohm's Law

$$\begin{aligned}
 I &= \frac{E}{Z} = \frac{P}{E \cos(\theta)} = \sqrt{\frac{P}{Z \cos(\theta)}} \\
 Z &= \frac{E}{I} = \frac{E^2 \cos(\theta)}{P} = \frac{P}{I^2 \cos(\theta)} \\
 E &= IZ = \frac{P}{I \cos(\theta)} = \sqrt{\frac{PZ}{\cos(\theta)}} \\
 P &= EI \cos(\theta) = I^2 Z \cos(\theta) = \frac{E^2 \cos(\theta)}{Z}
 \end{aligned}$$

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- [1] Robert G. Brown, *Introductory Physics I*. <http://webhome.phy.duke.edu/rgb/Class/intro-physics-1/intro-physics-1.pdf>
- [2] Allied Radio Corporation, *Allied's Electronics Data Handbook*. <https://archive.org/details/AlliedsElectronicsDataHandbook>