Feedback-Based Design Method for Spatially-Informed and Structurally-Performative Column Placement in Multi-Story Construction

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Abstract. This paper presents a feedback-based computational method for the placement of columns in the early design phase of complex multi-story structures. The method integrates a circle packing algorithm, a spring system, and structural engineering simulations within a single script for the reciprocal and informed arrangement of columns in the space. While allowing the users to have an explorative approach, it empowers diverse potentials in multi-story constructions including additional cantilevering spaces around the boundary, increased spatial qualities with large span possibilities, multidirectional structural arrangements, and multi-purpose use of space. As a result, the developed algorithm allows for flexibility by leveraging the design possibilities of grid-based and irregular column arrangements and promotes the integration of structural and design-related constraints in the spatial organization of various building typologies.

Keywords: Computational design · Column placement · Complex networks · Organization of space · Multi-story buildings

1 Introduction

1.1 Integration in Multi-Story Construction

Construction is regarded as a slow-to-change sector since technological advancements often take several decades to be significantly implemented (Drewer & Gann, 1994; Grübler et al., 1999; Stoneman, 2001; Kuklina et al., 2021). The labor-productivity growth of the building industry has been one percent a year over the past two decades, even when other sectors such as manufacturing or agriculture have displayed remarkable development (Barbosa et al., 2017). In essence, construction involves sophisticated,

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project-based activities that include interdependent subgroups collaborating on tasks over time (Mahapatra & Gustavsson, 2008). As a result of its normative rules, however, the sector becomes more rigid (Geels, 2004). Stable perceptions regarding roles and responsibilities lead to predefined boundaries between disciplines. In spite of it being the world's second-largest industry, the low amount of sharing, concealment of knowledge, and lack of integration hinder innovation in construction.

At the same time, urbanization rates have been increasing in all geographic regions over the last seven decades (United Nations: Department of Economic and Social Affairs, 2019). The rise is to such an extent that the number of multi-story construction in the last two decades is more than in the previous 115 years (Oldfield et al., 2014). Column-slab systems, in particular, have gained increasing attention due to their impact on material use and longevity of buildings (Hueste et al., 2007; Georgopoulos et al., 2014; Nandy, 2016; Meibodi et al., 2018; Santhosh & Kumar, 2021; Krtschil et al., 2022). From a design perspective, those systems require several spatial and structural aspects to be considered. This involves the properties of the building materials, the loads to be carried, the arrangement of the linear and surface elements in each story, and how forces are transferred on the structure (Grünbaum, 2008). On a global level, it is expected from the design team to balance varying demands including the spatial decisions, the client's interests, the projects' cost, and the overall performance of the proposed design (RIBA, 2020). Conventional practices often follow a linear approach despite the need for integrated knowledge. The involvement of sophisticated and standalone software programs favors the gap between disciplines.

1.2 Computational Design for Integration

Advancements in computational design have formed a novel paradigm in the building industry. Geometry-based tools and their integrated scripting environments have developed new design thinking with generative rule sets, parameters, and logical relationships (Barrios Hernandez, 2006; Oxman & Gu, 2015). Finite element analysis tools have helped define stresses, deflections, and dynamic behavior even for intricate geometries using sophisticated techniques (Mueller, 2014).

Despite the advanced computer technologies, the fundamental concept of the existing processes has remained unchallenged, displaying the computerized version of traditional modes (Menges, 2016). Design tools mostly prioritize the generation of articulated geometric shapes regardless of their multifaceted constraints. Similarly, analysis tools mainly analyze predefined geometries and are therefore unsuited for simultaneously informing the design process. In the early design stages of multi-story construction, designers are limited in how to evaluate their design options beyond architectural constraints. Even in the most prestigious architectural projects, engineers end up having subservient positions (MacDonald, 2001). The late consideration of structural concerns or the needs of users results in changes that increase the time and cost of the project.

2 Research Aim and Scope

The need for integrated thinking is particularly apparent in the design of slab layouts in multi-story construction. Slab design requires the consideration of various domains, such as building codes and regulations, load-bearing capacity, the structural performance of the slab, the accessibility of the space, energy performance, serviceability, aesthetics, cost, and material usage, all in a holistic way. Within that context, the placement of columns directly influences the placement of the beams, walls, or other structural elements, the span and structural depth of the building, the spatial organization of the defined spaces and rooms, as well as the arrangement of the circulation elements and service shafts. Hence, the positioning of the columns has a prominent impact on the holistic domains of multi-story construction. This research aims at enhancing informed and creative thinking in early spatial design while ensuring the structural performance of the slab system. It focuses on the development of a feedback-based computational workflow for the placement of columns in the design of complex multi-story structures.

3 Relevant Work

In the last two decades, computational methods became increasingly popular for designing floor layouts in buildings. Several have focused on determining the structural and architectural schemes for certain layout conditions. For instance, Shaw et al. developed an evolutionary algorithm utilizing the sweep line method to derive column layouts for orthogonal buildings (Shaw et al., 2008). Nimtawat and Nanakorn suggested a coding scheme that identifies beam-slab layouts with rectangular slabs as binary chromosome strings with given column and wall positions (Nimtawat & Nanakorn, 2010). Herr and Fischer provided a strategy for the generation of structural column and beam layouts for reinforced concrete structures in China (Herr & Fischer, 2013). Muresan et al. optimized the stiffness distribution in a slab layout while preventing the oversizing of elements using a set of floor outlines and column layouts (Mureşan et al., 2018). By dividing the rooms repeatedly, Mondal proposed an automation process for placing the columns and beams in single-story convex orthogonal floor plans (Mondal, 2018, 2021). However, the boundaries or layouts generated in all the above research have been limited to regular and rectangular configurations. Furthermore, they either lacked continuous structural integration or were insufficient to address spatial complexities such as cantilevered spaces. Similarly, other computational workflows have been initiated to self-organize architectural elements of a structure (Alvarez et al., 2019; Schwinn & Menges, 2015). Considering the early design phase of multi-story construction, Orozco et al. developed methods for arranging the panel segmentation and the reinforcement of timber slab structures (Orozco et al., 2021, 2022; Krtschil et al., 2022). Nevertheless, these methods excluded the arrangement of columns.

Some research has also highlighted the importance of automated column placement for less traditional configurations in the early design phase and has been influential in the development of this research. Scheuer used agent-based modeling (ABM) to define the configuration of arbitrarily positioned columns in a large concrete structure.

However, because of the dynamic shrinking and growing behavior, the system tended to be heavier and less prone to change. Besides, the involvement of many interdependent parameters made the decision-making process more sensitive (Scheurer, 2005). Questioning the linearity of the structural elements, Vierlinger et al. included inclined columns in the system and established a symbiosis of functional and architectural variables. Nevertheless, the involvement of thousands of elements and their elimination required some other optimization steps as well as integrated post-processing (Vierlinger et al., 2013). Preisinger used multi-objective optimization to place inclined columns under a roof while avoiding some predefined volumes. In this example, however, instead of approaching the problem with a more organized and informed methodology, a level of randomness has been involved in the process (Preisinger, 2022).

4 Methodology

This paper presents a feedback-based computational method for the early-stage spatial and structural design of slab layouts, focusing on the column placement of complex multi-story structures. Unlike standalone software packages, it allows users to integrate several constraints and simultaneously observe the implications of their design considerations in regular or irregular layout conditions. The methodology utilizes those constraints as inputs and outputs while benefiting from algorithmic design thinking.

4.1 Input Variables

Column positions are often prescribed by architects with consideration of the outside boundary of the slab, the desired span, and the arrangement of used spaces and access areas. In this relatively heuristic approach, designers are limited in the involvement of structural requirements.

Following that, the developed method takes several variables and considerations as inputs (Fig. 1), such as:

- Boundary of the slab: The continuous line that limits the area of the slab. The
 overall form can vary from rectangular layouts to more complex or curvilinear
 shapes.
- Number of columns: The desired number of columns to be used in the space. The
 results can be simultaneously checked to meet the building requirements.
- Column distribution area: The area within the boundary of the slab where the columns should be distributed. This feature is convenient when certain cantile-vering or balcony spaces around the boundary are considered. In case the user wants to place the columns within the entire boundary of the slab, this feature can be disabled.
- Span range: The expected optimal span between the columns after they are placed by the developed algorithm. More specifically, it identifies the diameters of the circles around the columns. This feature helps the users think beyond the otherwise limiting spatial opening possibilities of certain material systems.

- Fixed columns and walls: Predefined locations of specific columns or walls such as those around the main circulation areas or shafts of a building. If not required, this feature can be disabled.
- No-column areas: The spaces to have no columns within the column distribution area. These can include the locations around the openings and exit routes or the areas involving elevator shafts, stairwells, or spatial rooms.

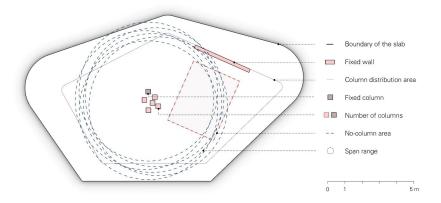


Fig. 1. Input variables displayed on a testing setup

Considering the performance-related calculations, structural inputs such as the material definitions, cross sections, height, and load should also be defined. In conclusion, a variety of inputs are proposed to enhance the flexibility of the method. As it is possible to disable some of the inputs, it is also possible to expand the number of inputs.

4.2 Algorithms, Solvers, and Outputs

According to the given constraints, the solvers of the system distribute the columns while helping reduce the displacement of the slab and allowing for simultaneous checking of the results within a single script (Fig. 2). For each feature, algorithms and solvers from different fields have been involved in the process, and integrated into the graphical algorithm editor named Grasshopper [Grasshopper 1.0.0007] in the same computer-aided design application software Rhinoceros3D [Rhino 7.0].

Algorithms

The circle-packing algorithm applies a mathematical technique for arranging circles within a given space so that they are tangent to each other and the boundaries of the space. In the construction industry, architects and engineers try to find the best column arrangement within a given boundary area. When the centers of the circles are viewed as the central points of the vertical structural elements, circle packing solves a similar column placement problem as the architects and engineers do. Therefore, the inputs and

outputs of the circle packing have a strong correlation with the ones of the column placement problem. The boundary of the circle packing area, the number of circles, and the distance between the circles' centers correspond to the column distribution area, the number of columns, and the span of the slab structure, respectively. While the outcome of the circle packing is efficiently distributed circles within a space, the outcome of the developed algorithm treats the central points of the circles as columns. For this method, the algorithm developed by Daniel Piker for Kangaroo Physics has been applied [Kangaroo 2.42]. Concerning the predominantly geometric approach of circle packing, structural considerations have been involved through a spring system and parametric structural engineering tools.

Spring system models are used in various applications, from physics simulations to robotics, for simulating mechanical systems' dynamic behavior. They can be seen as the simplest finite element method using one-dimensional elements (Kattan, 2008). When defined as networks, the model describes a position at each vertex point as well as a spring along the edges between those points with a stiffness and a length. Following this logic, the spring system between the column locations is concurrently generated and checked while the column distribution solver is running. The deformation has been the limiting factor of the slab design in the selected case study. Considering the distance between each column, the algorithm prevents the columns from exceeding the optimal span while helping reduce the displacement of the slab. The push and pull mechanism of the springs also ensures that the distributed columns are kept away from the fixed column locations and from the no-column areas.

Solvers

Structural calculations and engineering models are performed through an interactive structural design plug-in named Karamba [Karamba3D 2.2.0]. It is preferred based on its simplicity of use for non-experts and the speed with which it produces responses to different design options. Besides, to repeat a sequence of instructions multiple times, a feedback-based solver named Anemone is utilized [Anemone 0.4]. In consideration of the desired outputs, the distribution of the columns and the spring system are looped.

Outputs

Considering the nature of co-design (Knippers et al., 2021), slab layout selection is often not solely based on one domain's knowledge. Instead, the optimal solution is mostly the compromise of several constraints involved, which are mentioned in Section 2 "Research Aim and Scope". For this reason, the overall algorithm is designed to enable the users to check the outcomes continuously. These consist of spatial arrangements, the location of the columns, their corresponding structural simulations, column reaction forces, and the displacement of the slab. Furthermore, additional features are integrated with different slab systems in mind, such as generating a network of beams and eliminating the under-utilized beams, with the flexibility to expand or modify those options.

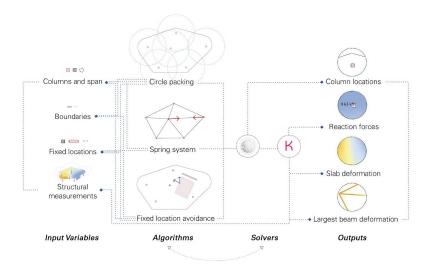


Fig. 2. Methodology chart explaining the input variables, algorithms, solvers, and outputs.

4.3 Testing Setup

The methodology has been chiefly explored and developed on a main case study as a testing setup. The case study selection has been based on the critical overview of current multi-story buildings in regard to their environmental and design-related consequences. The timber building sector has built increasingly more multi-story structures over the last two decades (Svatoš-Ražnjević et al., 2022). The carbon sinkage potential (Churkina et al., 2020), low climatic impact (Agustí-Juan & Habert, 2017), high strength-to-weight ratio (Ramage et al., 2017), and ease of machinability of timber (Wagner et al., 2020) are among the reasons of the resurgence of the sector. In addition, several studies have discoursed the interrelationship between the use of timber in the construction industry and increased productivity of labor (Mahapatra & Gustavsson, 2008; Barbosa et al., 2017; Salvadori, 2021). However, multi-story timber construction has still been limited in its architectural vocabulary and spatial design on account of its restricted span range and unidirectional floor plans.

In order to challenge these architectural limitations, the method has been demonstrated on an irregular multi-story timber structure consisting of a curvilinear slab boundary with the potential for varying span and cantilever conditions (Orozco et al., 2021, 2022). The boundary has been designed with the largest width of 16 meters, the largest corner radius of 2 meters, and the longest cantilevering balcony condition of around 2.5 meters. The outline of the testing setup is visible in Figure 1. Even though a pavilion-scale timber building has been chosen as a case study, the proposed method can be applied to other available systems with columns such as concrete, steel, or hybrid material systems. Consequently, its implementations to other timber building layouts and material systems are also showcased.

5 Results

The initial developments of the algorithm were performed on the testing setup. Six columns were distributed on a total area of 112.3 m^2 . The column distribution boundary was intentionally designed to be challenging for the structural performance of the slab system. The algorithm was also provided with a fixed column, a fixed wall, and a nocolumn area. The cross sections of the slab, beam, column, and wall elements were defined as 30 cm, $15 \times 20 \text{ cm}$, 28 cm, and 25 cm, respectively, and the material was set as timber. Constant gravity and slab loads were applied to the structure and the simulations were displayed accordingly (Fig. 3).

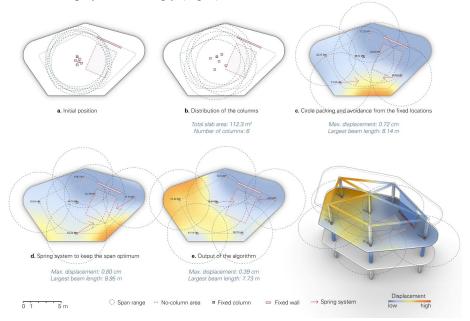


Fig. 3. Results of the column placement algorithm on a testing setup. a) Inputs of boundaries, the number of columns, fixed wall, and no-column area are given to the system. b) Given number of columns is distributed from the center of the column distribution area. c) Circle packing algorithm continuously placed the columns while avoiding no-column areas. d) Spring system kept the span in an optimal position. e) From all the generated solutions, the output of the algorithms is compared, and the desired one is selected considering the maximum displacement, largest beam length, and the reaction forces on the columns.

The columns were successfully placed on the testing setup by the algorithm. Live outcomes of each step were monitored on the same platform while the solvers were distributing the columns and trying to achieve the optimum span range. This included the maximum displacement of the slab, the length of the largest beam, the reaction forces of each column, and the columns' avoidance of the no-column area, fixed wall, and fixed column. The final slab displacement was 0.39 cm with an achieved span of 7.73 m.

To represent its flexibility, the developed algorithm was applied to several other slab boundaries with fewer inputs and different numbers of columns (Fig. 4).

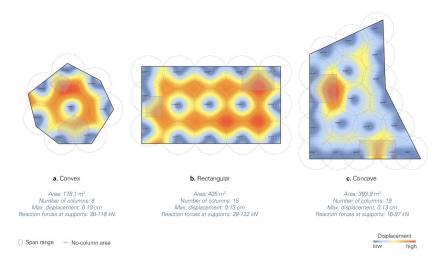


Fig. 4. Versatility of the algorithm on various boundary conditions. a) Column placement is run on a convex boundary as a heptagon. b) Rectangular boundary displayed the performance of the developed tool on regular layouts. c) Concave boundary is tested for irregular arrangements.

In addition to the multi-directional layout arrangements, it was also possible to get more regular column placements (Fig. 4b). In conclusion, regardless of the shape of the boundary, the desired output conditions were achieved.

Lastly, the algorithm was applied to real building layouts with several constraints and different materials. In reference to a timber building, the Tamedia Office Building in Zurich was selected as a base (Shigeru Ban Architects, 2013). Certain details about the building were obtained from the Multi-story Timber Buildings Database (Svatoš-Ražnjević & Menges, 2022). As a hybrid material system, 23 Dwellings' slab layout by Muoto Architects was tested (Muoto Architects, 2015) with a steel structure and a concrete elevator shaft. For both cases, the number of columns was kept the same as in the real building case (Fig. 5).

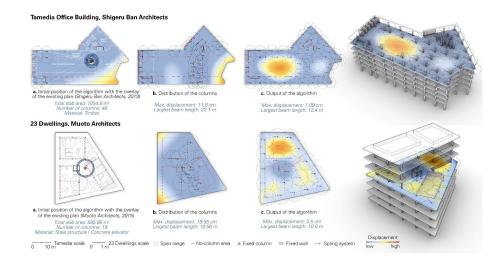


Fig. 5. Applications on the real building cases with different materials and layouts. The method is applied on a multi-story timber precedent named Tamedia Office Building by Shigeru Ban Architects which was designed with an irregular layout with 48 columns. The performance of the method is also tested on a hybrid building precedent with steel and concrete named 23 Dwellings by Muoto Architects. a) Initial position. b) Reciprocal distribution of the columns considering the given inputs. c) Output of the algorithm.

The generated results had similar layouts to the existing column arrangements. Through the process, the users could also investigate other cross-section and column arrangement possibilities with their corresponding structural simulations and calculations. At the same time, certain defects were also identified by applying the developed algorithm to real building layouts. For instance, in some cases, sharp edges on the column distribution boundary prevented the movement of the columns. Therefore, small fillets on the corners were integrated for smoother circle packing and spring systems. Besides, having a large number of fixed locations and predefined spaces on the same layout sometimes caused blockages in the distribution path of the columns. Although those were temporarily fixed, future work can identify possible software bugs and ensure the robustness of the system by testing it on several more cases. Taking everything into consideration, the algorithm successfully integrated several constraints, helped improve the decision-making process with explorative parameters, and allowed its users to approach early design cases more comprehensively.

6 Discussions and Outlook

This paper presented a feedback-based computational method for the placement of columns in the early design phase of complex multi-story structures. It integrated several design-related variables such as the boundary condition, the desired number of columns, the column distribution area, the optimal span range, the locations of fixed columns and walls, and the no-column areas as constants. Constrained by those preferences, the developed algorithm allowed its users to check the outcomes of the structural analysis live, while letting the algorithm produce their optimal layout. As a result, it empowered diverse potentials which are infrequently seen in multi-story timber building design, including additional cantilevering spaces around the boundary, increased spatial qualities with the possibility for large spans, multidirectional structural arrangements, and multi-purpose use of space. To highlight its versatility, the developed approach is then elaborated on existing slab layouts with different boundary conditions, changing numbers of columns, and different material systems.

Following the proposed method, promising fields for further research have been identified. Methodologically, the algorithm selection can go beyond circle packing and spring system, perhaps to agent-based modeling or machine learning methods. These could enable the integration of other column behaviors and user interaction while approximating optimal solutions. In addition, the column arrangements of several slabs in different levels can be simulated to better understand the seismic behavior of corresponding layouts. This can leverage the design possibilities even further, such as to systems with atriums. From a technical point of view, the single-script approach might be expanded with the full integration of a feedback-based structural solver, and with multi-objective parameters. Conceptually, the flexibility of the developed method can allow for the implementation of several other design variables of different fields. As an example, life-cycle assessment-related parameters or other performance criteria such as acoustics and vibration can be incorporated.

Overall, elaborating on the placement of columns, this method presents a reciprocal co-design approach to integrate the constraints of different disciplines involved in the early design phase of multi-story structures. It bases itself on the existing research and provides a user-friendly platform in a widely-used computer-aided design software environment. Unlike other examples, the method enables the generation and evaluation of a multitude of design options in a relatively quick, easy, and simultaneous way regardless of the regularity or irregularity of the given boundary conditions. Besides, it provides the corresponding simulations and calculations of the structural system in the same platform. The developed method has the potential to surpass the architectural and structural constraints of slab design, allowing for higher productivity, sharing, and integration in a variety of stages of multi-story construction.

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