Deep Reinforcement Learning in Real-Time Bidding

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Abstract

Real-time bidding is getting increasingly popular for buying and selling online display advertisement. This has spurred a research interest into how to design optimal bidding algorithms, with advances during the last two to three years focusing heavily on reinforcement learning. This thesis focuses on creating a bidding agent using recent innovations in combining reinforcement learning and deep learning, drawing heavily from a recent paper by Wu et al. (2018). However, the final algorithm presented in this thesis, called *(Batch) Deep Reinforcement Learning to Bid* (`Batch-DRLB`) deviates quite a bit from their algorithm. `Batch-DRLB` shows superior results to two simple benchmark algorithms and compares very well to current state-of-the-art algorithms.

This project has been done in collaboration with Adform, which is one of the world's largest advertising technology companies, based in Copenhagen. They have provided fantastic support throughout the project. In addition to providing great resources for developing and testing the algorithm, they've provided continuous help in getting a better understanding of RTB and computational advertisement.The final algorithm is something like a thousand lines of code. Hence, I've chosen not to include it in here and have instead provided all of the code in a `GitHub` repository: https://github.com/Ostigland/dqn-rtb

Acknowledgements

First of all, I want to thank Vlad and Morten. They have been *fantastic* supervisors, from start to end. I would not have been able to complete this project without their help, patience and constant encouragement. Every day has been challenging and so much fun. I would also like to thank my academic supervisor, Alexandros, who has also been full of help and encouragement since day one. Pursuing reinforcement learning was his idea and I'm very happy and thankful that he brought it up. I also want to thank Matt Jacobs at UCLA who let me take his course on machine learning not only once, but twice, and who was always encouraging, helpful and open to discussion. I don't think I would have been able to complete this project without the freedom he gave me to challenge myself.

Finally, I would like to thank all of my friends and family, for supporting me, for everything, always. Thank you!

Abbreviations

AdX = Ad Exchange
DSP = Demand-Side Platform
RTB = Real-Time Bidding
CTR = Click-Through Rate
eCPC = effective Cost Per Click
eCPI = effective Cost Per Impression
ML = Machine Learning
RL = Reinforcement Learning
MDP = Markov Decision Process
CMDP = Constrained Markov Decision Process
NN = Neural Network
ReLU = Rectified Linear Unit
SGD = Stochastic Gradient Descent

`DQN` = Deep Q-Network
`DRLB` = Deep Reinforcement Learning to Bid
`Batch-DRLB` = Batch Deep Reinforcement Learning to Bid
`LinBid` = Linear Bidding
`RandBid` = Random Bidding

`tf` = TensorFlow
`np` = NumPy

Contents

Chapter 1

Introduction

Today, online display advertisement is increasingly sold and bought through a process known as real-time bidding (RTB). In the last decade, spending on RTB has increased dramatically. The supposed reason for this impressive growth is the overall efficiency benefits from RTB (Yuan et al., 2014). By utilizing information from cookies, advertisers can target specific users who might be more susceptible to a given advertisement campaign. More specifically, an advertiser can target *only* these desirable users. In this sense, traditional display advertisement methods, e.g. billboards and newspaper ads, are extremely inefficient since the advertiser is paying the same amount for every impression, regardless of the effect. Even more recent innovations such as buying keywords or time-slots on websites appear inefficient compared to RTB.

Whenever a user logs onto a website with available ad slots, the owner of the domain (often referred to as a *publisher*) sends out a request to a so-called *ad exchange* (AdX). The AdX then sends out bid requests to a number of so-called *demand-side platforms* (DSPs) and holds an auction in which the DSPs submit bids to win the impression. The DSP that submits the highest price wins the ad slot and pays the second-highest price. A simple summary of the process is provided in a figure below.

So, why are we talking about DSPs and where are the actual advertisers? The process of auctioning out an ad slot and subsequently showing it to a user takes less than 100 milliseconds. To put this in perspective, blinking your eye takes about 300 to 400 milliseconds. Hence, the ad-buying procedure is entirely algorithmic. Bids submitted by DSPs have to be computed instantly when a bid request is received and a typical DSP handles billions of auctions on a daily basis.

There are many areas of research in the RTB ecosystem, ranging from bidding

strategy to auction design. Yuan et al. (2014) provide a good overview of different research problems. This thesis will focus on the problem of creating a bidding strategy for an ad campaign and, more specifically, on employing an algorithmic bidding agent which incorporates reinforcement-learning techniques to bid intelligently given campaign-relevant parameters.

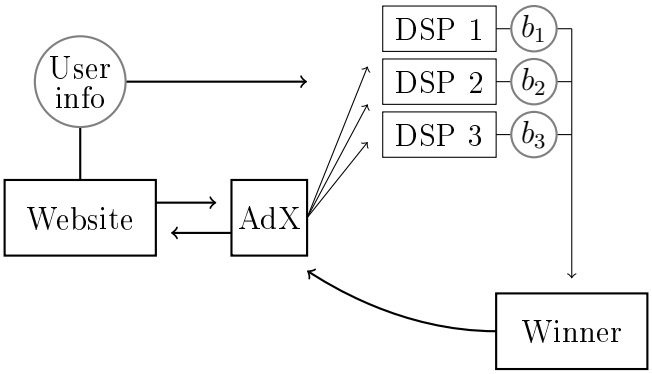


Figure 1.1: A simplified RTB ecosystem

Reinforcement learning has been introduced into RTB in the last two to three years, with two of the most prominent works being Du et al. (2017) and Cai et al. (2017). This thesis will focus on the approach used by Wu et al. (2018), where a bidding agent is built using a relatively recent innovation in combining reinforcement learning and deep learning. Chapter 2 will be devoted to an introduction to reinforcement learning, both conceptual and mathematical, while Chapter 3 will focus on how deep learning has been implemented in reinforcement learning, as well as how these implementations have been used in RTB. Chapter 4 will discuss method, including attempts to replicate the paper by Wu et al. (2018). Finally, Chapter 5 will focus on the experiments and their results, along with some discussions. The conclusion follows in Chapter 6.

It should be noted that the content of this thesis is highly specialized and that the reader is thus expected to have some knowledge of stochastic processes and standard machine-learning techniques, specifically the neural network. While it would be desirable to include a section devoted to explaining the neural network in more detail, as well as adding more content on reinforcement learning, it is simply not feasible when also having to balance readability and the time constraint. However, I have done my best to present the material as clearly and pedagogically as possible and hope that this will suffice.

Chapter 2

Reinforcement Learning

The fundamental purpose of reinforcement learning is to design *agents* with the ability to successfully navigate through *environments* from which they have no prior experience. This does not necessarily mean that they have no prior knowledge of the environment whatsoever, although this is often the case as we shall see later, but rather that they haven't taken any actions in the environment previously; they have no idea of what kind of consequences or rewards follow from different actions.

Imagine a kid trying to learn how to ride a bike. The kid might understand how a bike works, e.g. that turning the handlebars to the right makes the bike turn right and that pushing the bike pedals makes the bike accelerate and go forward, and so on. However, there are a few things that only experience can teach. For example, it's difficult, if not impossible, to understand beforehand just how much the handlebar will make the front wheel turn. Similarly, it's hard to understand how much the bike will accelerate if we push the bike pedals forward or how harshly it will brake if we push the pedals backwards. Most importantly, it's impossible to know how much it will actually hurt to hit the ground, or if it will even hurt, when you fall off the bike if you haven't already done it, or how exhilarating it is to bike fast.

The latter example is of importance for reinforcement learning, since consequence and reward are how we make sure that an agent learns to behave in an optimal way in some environment. Just like the kid experiences pain and failure when it falls off the bike, we make sure our agent receives negative or low numerical rewards when choosing "bad" actions and, conversely, that it receives positive numerical rewards when it acts in a "good" way.

2.1 Markov Decision Processes

In any reinforcement-learning problem, we have an agent and an environment. These two interact with each other through *states*, *actions* and *rewards*. The environment provides a state, to which the agents responds with an action. Then, the action receives a numerical reward while the environment provides the next state, as a response to the agent's action. The goal of the agent is to maximize the cumulative reward over a number of states and actions, often referred to as an *episode*.

The most common way to model a reinforcement-learning problem is through the *Markov Decision Process* (MDP), which models state transitions using the Markov property. In this thesis, and in reinforcement learning in general, the finite MDP is of most importance, where there is a finite number of combinations of situations (or *states*) and actions as well as a finite (discretized) interval of rewards. In defining the general framework of a finite MDP, and the notation to be used later in this thesis, I will follow Sutton and Barto (2018).

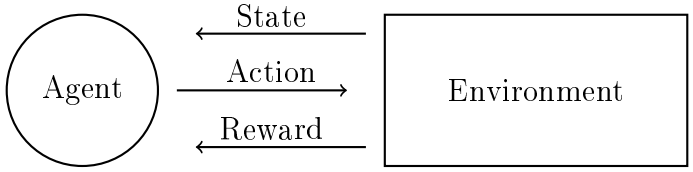


Figure 2.1: Illustration of a simple agent-environment relationship

We consider a finite series of time steps, $t = 1, 2, \ldots, T$, where T is the time of termination for the episode, and denote the action, state and reward at time t by A_t, S_t and R_t, respectively. We define a finite set for each: $A_t \in \mathcal{A}$, $S_t \in \mathcal{S}$ and $R_t \in \mathcal{R} \subset \mathbb{R}$. First, we want to consider the joint probability of some state, s', and some reward, r, following the choice of a certain action, a, in a certain state, s:

$$p(s', r, s, a) \triangleq P(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a)$$

$\forall\, s', s \in \mathcal{S}$, $\forall\, r \in \mathcal{R}$ and $\forall\, a \in \mathcal{A}$. We have that $p : \mathcal{S} \times \mathcal{S} \times \mathcal{A} \times \mathcal{R} \to [0, 1]$. That is, when considering the reward and the next state, we are only concerned with the previous state-action pair. We do not care about state-action pairs further back in the chain of transitions. Using $p(s', r, s, a)$, we want to define value of taking a certain action, a, in a certain state, s. In a probabilistic environment, this value should correspond to the sum of all possible immediate rewards, weighted by their

respective probabilities. Hence, we define the function

$$r(s,a) \triangleq \mathbb{E}\left[R_{t+1}|S_t = s, A_t = a\right] = \sum_{r\in\mathcal{R}} r \sum_{s'\in\mathcal{S}} p(s', r, s, a)$$

such that $r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$. Considering our biking kid, we might have a state in which the bike is on the crest of a hill and is just about to start rolling downwards. Our function, $r(s, a)$, then maps different actions, e.g. accelerating and breaking, to their perceived value. When finding these values, there's an important aspect to consider. For example, acceleration might yield some short-term exhilaration, but it also means some future risk as the kid will eventually have less control over the bike.

Making a choice isn't just about weighing different immediate rewards against each other, it's also about weighing the present against the future, balancing the short-term and the long-term. This is also true for a reinforcement-learning agent, which we formalize using a *discount factor*, $0 \leq \gamma \leq 1$. A low γ means that the agent is *myopic*, prioritizing short-term rewards, while a high γ means that the agent will also give consideration to future rewards. Using the discount factor, we formulate the expected return at time t as

$$G_t \triangleq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots + \gamma^{T-t-1} R_T = \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$

which gives us a recursive relationship, since

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots + \gamma^{T-t-1} R_T$$

$$= R_{t+1} + \gamma(R_{t+2} + \gamma R_{t+3} + \cdots + \gamma^{T-t-2} R_t) = R_{t+1} + \gamma G_{t+1}$$

such that $G_t = R_{t+1} + \gamma G_{t+1}$. We define $R = G_0$, i.e. such that the expected return for a whole period is denoted by R. Hence, the goal of an agent is to find the actions that maximize R. Thus, instead of just considering the immediate reward form a state-action pair, $r(s, a)$, the goal of an agent should be to consider the entire reward following a state-action pair. For this purpose, we define

$$q(s,a) = \mathbb{E}\left[G_t|S_t = s, A_t = a\right] = \mathbb{E}\left[\sum_{k=t+1}^{T} \gamma^{k-t-1} R_k \middle| S_t = s, A_t = a\right]$$

at time $t = 1, 2, \ldots, T$, such that $q : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}$. The function $q(s, a)$ is called the *action-value function* and will be one of the most important conceptual features of this thesis.

2.1.1 An Optimal Policy

In reinforcement learning, a policy can be strictly defined as a mapping from states, s, to probabilities of selecting certain actions, a, in those states. A policy is usually denoted by π and we can hence express it as

$$\pi(a, s) = P(A_t = a | S_t = s)$$

such that $\pi : \mathcal{A} \times \mathcal{S} \rightarrow [0, 1]$, $\forall\, a \in \mathcal{A}$, $\forall\, s \in \mathcal{S}$, and for $t = 1, 2, \ldots, T$. A policy can be strictly deterministic, meaning that the agent picks an action, a, with probability 1. If an agent follows a policy π, we say that $q_\pi(s, a)$ is the *action-value function for policy* π, since the subsequent state-action pairs are more or less deterministic, meaning that we can estimate the expected return if we're following a certain policy after (s, a). If a particular policy π_* has the property that $q_{\pi_*}(s, a) \geq q_\pi(s, a)$ for all $s \in \mathcal{S}, a \in \mathcal{A}$ and for all other policies π, we call it the *optimal policy*. For this policy, we denote the action-value function by $q_*(s, a)$ and define it as

$$q_*(s, a) \triangleq \max_\pi q_\pi(s, a), \quad \forall\, a \in \mathcal{A}, \forall\, s \in \mathcal{S}$$

Hence, the purpose of the optimal policy is to maximize the expected return, G_t, at any time $t = 0, 1, 2, \ldots, T$.

2.1.2 The Bellman Optimality Equation

We consider the definition of $q(s, a)$ above. We also consider the recursive relationship for the expected return. Hence, we can actually define the action-value function as

$$q(s, a) = \mathbb{E}\left[G_t | S_t = s, A_t = a\right] = \mathbb{E}\left[R_{t+1} + \gamma G_{t+1} | S_t = s, A_t = a\right]$$

Now, let's assume that we are following the optimal policy, π_*. If we have found the optimal policy, we are no longer exploring but only exploiting. In other words, we're only taking greedy action with probability 1. This means that, if $S_{t+1} = s'$, we know that $G_{t+1} = \max_a q_*(s', a)$. Hence, we can describe the optimal action-value function using a recursive relationship:

$$\begin{aligned} q_*(s, a) &= \mathbb{E}\left[R_{t+1} + \gamma G_{t+1} | S_t = s, A_t = a\right] \\ &= \mathbb{E}\left[R_{t+1} + \gamma \max_{a'} q_*(S_{t+1}, a') \middle| S_t = s, A_t = a\right] \\ &= \sum_{s', r} p(s', r, s, a) \left[r + \gamma \max_{a'} q_*(s', a')\right] \end{aligned}$$

This equation says that the expected return from taking an action a in a state s when following the optimal policy corresponds to the immediate expected reward and the expected return from following the optimal policy in the next state. This is known as the *Bellman optimality equation* for the action-value function. While this is analytically nice and solvable if we have knowledge of all transition probabilities and action values, it's applicability is constrained by computational complexity. Hence, many reinforcement-learning techniques, such as Monte Carlo methods, aim to approximate $q_*(s, a)$. This is also true for the method which will later be introduced as the *Deep Q-Network*.

2.1.3 Constrained Markov Decision Processes

So far, we've been concerned with an agent who's making decisions to maximize a single metric: the expected reward. In this case, we want to choose a policy π such that

$$\pi = \arg\max_{\pi} \mathbb{E}[R|\pi] = \arg\max_{\pi} \left[\sum_{t=1}^{T} \gamma^{t-1} R_t \middle| \pi \right]$$

which we've previously defined as the optimal policy, π_*. However, we've only been concerned with unconstrained maximization. It is not clear that π_* is an optimal policy when we impose constraints on the agents. We refer to such a case as a *Constrained Markov Decision Process* (CMDP). We define $C = \sum_{t=1}^{T} \gamma^{t-1} C_t$, where C_t is the cost at time t, and instead consider the problem of finding π such that

$$\begin{aligned} \max_{\pi} \quad & \mathbb{E}\left[R|\pi\right] \\ \text{s.t.} \quad & \mathbb{E}\left[C|\pi\right] \leq c \end{aligned}$$

where c is our cost constraint. How do we make this fit into the reinforcement-learning framework? Geibel (2007) discusses a number of methods fitted to different CMDP problems, one of which is to expand the state space, $\mathcal{S}$, to include the cost constraint. Instead of just considering the normal state-relevant parameters when taking an action, we also consider the cost incurred and if the constraint has been reached the agent will either be incapacitated or the period will be terminated. This is the approach that will be followed in this thesis.

2.2 Exploration and exploitation

As mentioned in the introduction to this chapter, reinforcement-learning techniques aim to deploy agents into new environments. This means that they have to

explore the environment and essentially take random actions to see what happens, before being able to act intelligently. When the agent is acting intelligently and taking the decisions that it knows maximizes the expected return, we say that it is *exploiting*. When the agent is only exploiting and not exploring, we call it *greedy*. Hence, the optimal policy, as defined by $q_*(s, a)$, is greedy since we're always choosing the actions that will maximize the expected return.

The exploration-exploitation trade-off is one of the most important aspects of reinforcement learning. On the one hand we want the agent to be as well-informed as possible about the actions it's taking, but on the other hand we want it to gain as much reward as possible; more exploration means less exploitation, and vice versa. This is usually solved by a so-called ϵ-greedy policy, which means that the agents exploits with a probability $1 - \epsilon$ and explores with a probability ϵ, where $0 \leq \epsilon \leq 1$. This is illustrated in the figure below with two possible actions.

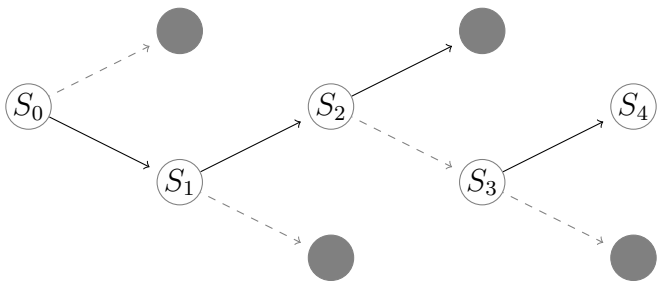


Figure 2.2: Illustration of an ϵ-greedy policy, where greedy actions are whole arrows and random actions are dotted arrows.

When the agent is learning, we want it to explore as much as possible, i.e. to have a high ϵ. Conversely, when the agent has finished learning, we want it to exploit, i.e. having a low ϵ or even $\epsilon = 0$. In practice, this can be solved by a number of ways. Often, it is the case that the agent starts out with $\epsilon \geq 0.9$ and then lets ϵ decay over time, e.g. linearly or exponentially, according to some fixed rate. Configuring an ϵ-greedy policy is by no means an exact science. Rather, it's a result of testing and applicability to the problem at hand. In this thesis, we will be working with a limited action space with seven distinct actions, i.e. $|\mathcal{A}| = 7$, and $\mathcal{S} \subset \mathbb{R}^5$. We will attempt to use an ϵ which decays linearly according to some fixed rate.

2.3 Q-Learning

One of the most famous reinforcement-learning methods is the *Q-learning algorithm*, which, as the name suggests, is concerned with directly estimating the action-value function. While Q-learning deserves a longer theoretical background and discussion of its convergence properties, this will make for a shorter introduction as we will ultimately not be concerned with the Q-learning algorithm, but rather with a variant of Q-learning for which we can't make assertions about stability and convergence.

We start by initializing some arbitrary action-value function $Q_0(s, a)$ for all $s \in \mathcal{S}, a \in \mathcal{A}$ and as the agent is exploring the environment (as well as when it's exploiting), we're continually making incremental updates for the action-value function:

$$Q_{n+1}(S_t, A_t) = Q_n(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \max_a Q_n(S_{t+1}, a) - Q_n(S_t, A_t) \right]$$

where α is the learning rate. That is, when observing S_t, A_t and R_{t+1}, we update the action-value function by the scaled difference between the old value, $Q_n(S_t, A_t)$, and the sum of the immediate reward and the discounted expected return for the next state under a greedy policy, $R_{t+1} + \gamma \max_a Q_n(S_{t+1}, a)$. If we look closely, this is actually a familiar sight. Setting $\alpha = 1$, we get

$$\begin{aligned} Q_{n+1}(S_t, A_t) &= Q_n(S_t, A_t) + \left[R_{t+1} + \gamma \max_a Q_n(S_{t+1}, a) - Q_n(S_t, A_t) \right] \\ &= R_{t+1} + \gamma \max_a Q_n(S_{t+1}, a) \end{aligned}$$

which is analogous to the Bellman optimality equation for the action-value function for a greedy policy, i.e. where we choose the best action with probability 1. However, we're not concerned with transition probabilities anymore, as we're estimating action values with purely numerical methods (e.g. taking averages from large samples). It's important to note that the Q-learning update occurs at almost every step, meaning that the algorithm keeps updating the values for all state-action pairs even as ϵ decreases and the agent exits the exploring phase.

One of the problems with Q-learning is that we might run into trouble in large-scale systems since it's hard to visit every state-action pair a sufficient amount of times to get a good estimate of their values, especially when we have to balance exploration and exploitation. This is why we are now turning to the next chapter, where we will get a grasp of how we can approximate our action-value function and give our agent the power to generalize over its experiences.

Chapter 3

Deep Reinforcement Learning

While traditional reinforcement learning (RL) methods, e.g. the Q-learning algorithm, have nice properties with respect to stability and convergence, they're not always applicable when dealing with high-dimensional sensory inputs. Imagine that we want to create an agent with the purpose of playing 64-pixel arcade games, simply by "looking at" and analyzing the screen. This means that the input, i.e. the states, have dimensionality on the order of $64 \times 64 = 4096$. Hence, the number of unique states in the state space, $\mathcal{S}$, is potentially enormous, which means that Q-learning is likely to be computationally infeasible due to the number of state-action pairs.

Since the 90s, attempts have been made to find a more slick solution to estimating the value of action-value pairs in systems with large state-spaces. To date, the most prominent of these attempts is arguably the combination of deep learning and RL through the approximation of the action-value function with a neural network. This idea culminated in the projects by Google DeepMind, presented in Mnih et al. (2013, 2015), in which an RL agent, combined with a deep convolutional neural network, exceeded human-level performance in a number of arcade games by representing the state with 210×160 RGB visual inputs, i.e. dimensionality corresponding to $210 \times 160 \times 3 = 100800$.

3.1 Q-learning Powered by Deep Learning

While the approximation mechanism in a Q-learning algorithm is strictly local, i.e separate estimates for each state-action pair, the approximation mechanism in a deep neural network is global. In other words, combining an RL agent with a deep neural network gives it the ability to generalize its estimates. To consider an example, let's go back to our favorite biking kid. Imagine that the kid leans

too much to the right, causing the whole bike to fall over and hit the ground. Now, if our young biker's brain was wired like a Q-learning algorithm, the fall to the right, and the pain that came with it, would say nothing about what would happen in a similar situation where the bike was instead tilted to the left. This is of course absurd, and we should feel lucky we don't have Q-learning algorithms running the brain department, because the human brain has a powerful ability to generalize and draw comparisons. This is essentially the ability we want to give our RL agent; instead of having to thoroughly experience everything, we want it to be able to use limited experiences to get a comprehensive, general understanding of the environment.

One of the predecessors of DeepMind's arcade-game master was presented by Riedmiller (2005) under the name of *neural-fitted Q-iteration*. Riedmiller presented the problem as finding a tool to balance the positive and negative effects of using a global approximation, rather than a local one. While a global approximation might nullify the training from an older experience when adjusting the approximation based on a recent experience, it can also accelerate training significantly by exploiting generalization. The principal method proposed to achieve this goal is to store experiences and reuse them whenever the approximation of the Q-function is updated. This is based on the idea of *experience replay*, presented by Lin (1992)

In the previous chapter the update rule for the Q-learning algorithm was used, which was based on making incremental updates to the action-value estimates using the Bellman optimality equation:

$$Q_{n+1}(S_t, A_t) = Q_n(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \max_a Q_n(S_{t+1}, a) - Q_n(S_t, A_t) \right]$$

Now, we consider similar update rule, but with a Q-function approximated by a neural network and parametrized by a set of weights, θ, such that we have $Q_n(s, a) = Q(s, a : \theta_n)$ and want to make updates by minimizing a loss:

$$\left(\left[r + \gamma \max_{a'} Q(s', a'; \theta_n) \right] - Q(s, a; \theta_n) \right)^2$$

with respect to the set of weights, θ_n, using e.g. a stochastic gradient descent (SGD) algorithm on previous experiences. In other words, we're concerned with finding a set of weights, θ, such that $Q(s, a; \theta) \approx Q_*(s, a)$. Riedmiller used this technique successfully on a number of simple control problems where the neural-fitted Q-algorithm found good policies relatively fast, compared to analytical model-based techniques. However, it was the introduction of two additional algorithmic features by the DeepMind team that eventually created the Atari-playing RL agent with superhuman game performance.

3.2 The Deep Q-Network

In 2013, a group of researchers from DeepMind, including the aforementioned Martin Riedmiller, released a paper which presented an algorithm that could successfully learn how to play a number of Atari games using a Q-learning agent powered by a deep, convolutional neural network (Mnih et al, 2013). The algorithm, called *Deep Q-Learning*, is very similar to Riedmiller's neural-fitted Q-iteration, except that it uses a convolutional neural network and a more efficient type of experience replay. It also incorporates a so-called *target network*, which is used to train the local network, i.e. the one used for decision making. The weights of the local network are then copied on to the target network at some pre-determined frequency.

Similarly to Riedmiller's algorithm, Deep Q-Learning fits an estimator to the Bellman optimality equation using the loss function

$$L_n(\theta_n) = \mathbb{E}_{s,a\sim\rho}\left[y_n - Q(s,a;\theta_n)\right]^2$$

where $\rho(s,a)$ is a probability distribution over states, s, and actions, a, and

$$y_n = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q(s',a';\theta_n^-)\middle| s,a\right]$$

where θ^- are the weights of the target network. Instead of using the entire replay memory, Mnih et al. (2013) take a mini-batch of samples from the experience replay memory and perform (SGD), minimizing $L(\theta)$ with respect to θ using these samples. This is done at every step of the algorithm, using the target network. When a deep convolutional neural network is used, the estimator of Q is called a *Deep Q-Network* (`DQN`).

In 2015, Mnih et al. released another paper where the `DQN` was applied to 49 different Atari games. The agent achieved more than 75% of human score in more than half of the games, as well as beating humans in several games. To give an idea of how the problem was set up, and how the problem in this thesis will be set up, the authors used, for example, a mini-batch size of 32, a replay memory size of 100000, a target network update frequency of 10000, and a discount factor, γ, of 0.99.

3.2.1 Experience replay

The notion of experience replay is in no way new to Mnih et al. (2013, 2015), but the authors find a more efficient use of it by combining it with random sampling and SGD. However, the use of experience replay is primarily due to stability. In

games, and control problems in general, certain states are often interconnected, meaning that they are correlated and hence not independent (Mnih et al., 2013). In other words, a bias will appear when the agent is learning. Sampling from the replay memory remedies this by continuously letting the agent re-experience old state-action pairs. More specifically, we never let the agent improve its estimations with immediate experiences; we always sample from the replay memory when updating the network, at every step. Riedmiller (2005) noted significant improvements in stability and training time from using the replay memory, making the learning process more stable and data efficient.

It should be noted that in the version of experience replay used by Mnih et al. (2013, 2015), there is a pre-determined size of the experience replay memory, whereas Riedmiller (2005) and Lin (1992) use all of the previous transitions. The main reason for this seems to be memory usage and computational feasibility; the simulations run on the Atari games can have millions of transitions. In the `DQN`-based RTB algorithm introduced later, we will also be using a pre-determined size for the experience replay memory. Whenever the memory is full, we remove the earliest experience when adding new experiences.

3.2.2 Target network

Together with experience replay, the target network is what really makes the `DQN` an efficient RL agent. When updating the decision-making network at every time-step, the risk of policy divergence and instability increases. If an update increases $Q(s_t, a_t)$, it is likely that $Q(s_{t+1}, a)$ also increases for all a, even though it is not a good estimate of the optimal policy (Mnih et al., 2015). Similarly to experience replay, the target network makes the learning process more stable and efficient. The authors show the effect of using experience replay and a target network for a number of games and where the effect on the agent's performance is astounding. While experience replay accounts for the greatest part, the use of a target network improves the performance of the agent many times over in several cases.

3.2.3 Summary

As mentioned previously, we refrained from discussing convergence and stability properties of Q-learning since the method we'd be using cannot make guarantees on convergence. Boyan and Moore (1995) were early to discuss this problem and showed that the combination of function approximation and certain RL methods could lead to serious instabilities and bad policies. It's evident that this problem is still pervasive, but in using the contributions by Lin (1992), Riedmiller (2005), and Mnih et al. (2015), we arrive at the `DQN` which does a good job in maintaining

both stability and efficiency. With exception for the convolutional neural network used by Mnih et al. (2013, 2015), this is the approach which will be followed when we construct a bidding agent.

3.3 Deep Reinforcement Learning in RTB

Reinforcement Learning was recently introduced into RTB when Du et al. (2017) and Cai et al. (2017) independently proposed RL-based bidding agents. The principal reason for this is the need for dynamism; instead of setting parameters for a whole batch of bids, e.g. which average bid and CTR estimation to use, we want a bidding agent which can achieve more strategic granularity, e.g. by observing campaign-relevant parameters and adjust its behavior in real time or by being able to make more fine-grained valuations of individual impressions.

Both RL-based methods use the demographic user information and the CTR estimations to capture the state dynamics, while the agent acts by setting bid prices. For example, Du et al. (2017) use historical data to estimate the winning probability of a certain bid price and the CTR, which then gives the probability of a click given a certain bid price. The goal of the agent is then to maximize the number of clicks under the budget constraint. While the methods from both papers outperform state-of-the-art linear bidding algorithms, they will not be the focus of this thesis. They use model-based training, which has problems with scalability due to computational complexity and with non-stationarity - and RTB is a highly non-stationary environment (Wu et al., 2018).

We will focus on creating a bidding agent using the approach of Wu et al. (2018), which uses historical data to train a bidding agent with a variant of the `DQN`; although, in this case there is no convolution, but a feed-forward neural network with three hidden layers, each having 100 neurons. Going forward, we will use the name '`DQN`' to describe the approximated Q-function used by the bidding agent.

3.3.1 Real-Time Bidding with a Deep Q-Network

In *Budget Constrained Bidding by Model-free Reinforcement Learning in Display Advertising* (2018), researchers from Alibaba Group discuss how they try to solve the problem of optimal bidding by using a `DQN`. This approach is completely different from the other RL-based approaches mentioned above. Instead of formulating bids by creating a model of the click probability from a given bid and then using this model to solve the constrained optimization problem of maximizing the

number of clicks under a given budget, the bids are formulated as:

$$b_{t,k} = \frac{\phi_{t,k}}{\lambda_t}$$

Let's consider what this expression means and how it explains the model. $b_{t,k}$ is the bid at step k, for $k = 1, 2, \ldots, K$, in time-step t, for $t = 1, 2, \ldots, T$. That is, one episode has T time-steps, where T is the time of termination. In each of these time-steps, the agent participates in K different auctions. $\phi_{t,k}$ is the CTR estimation for a particular auction, while λ_t is the bid-scaling parameter for that particular time-step. That is, all the bids in one time-step are scaled with the same parameter. The action of the agent is then to regulate the scaling parameter λ_t at each time step, t, depending on the state, S_t, which is described by

- the time step, t,
- the remaining budget, B_t,
- the number of regulation opportunities left at time t, ROL_t,
- the budget consumption rate, β_t, where $\beta_t = \frac{B_{t-1} - B_t}{B_{t-1}}$,
- the cost of the impressions won between time $t-1$ and t, CPM_t,
- the auction win rate, WR_t, and
- the total value of winning impressions, e.g. the number of clicks, at time step $t-1$, r_{t-1}.

Hence, the state can be describe as

$$S_t = (t, B_t, \mathrm{ROL}_t, \beta_t, \mathrm{CPM}_t, \mathrm{WR}_t, r_{t-1})$$

The agent thus considers campaign-relevant parameters rather than how the auction environment will react to it placing a bid and eventually winning an impression. For example, if the remaining budget is low and and the number of remaining budget regulation opportunities is high, the agent should ideally increase λ in order to decrease the bid scaling and, hence, bid less aggressively.

The auction space, $\mathcal{S}$, then consists of all of the possible values of the tuple, S_t. Since we're aiming for approximating our Q-function with a deep neural network, we don't have to consider discretization of the continuous parameters in S_t. The actions, i.e. the possible adjustments to λ_t, are $\mathcal{A} = \{-8\%, -3\%, -1\%, 0, 1\%, 3\%, 8\%\}$, such that

$$\lambda_t = \lambda_{t-1} \times (1 + A_t), \quad A_t \in \mathcal{A}$$

The reward, R_{t+1}, after some action A_t in some state S_t, is then given by

$$R_{t+1} = \sum_{k=1}^{K} X_{t,k}\phi_{t,k}$$

where $X_{t,k} = 1$ if the kth impression was won and $X_{t,k} = 0$ otherwise, while $\phi_{t,k}$ is the aforementioned CTR estimation for the kth bid. Similarly, the cost after some action A_t in some state S_t, is given by

$$C_t = \sum_{k=1}^{K} X_{t,k}c_{t,k}$$

where $c_{t,k}$ is the cost for the kth impression during time t. As mentioned previously, the cost constraint will be incorporated into the MDP by expanding the state when including the cost incurred and the budget, i.e. since $B_t = B_{t-1} - C_{t-1}$. Finally, we set the discount factor, γ, to $\gamma = 1$, since we consider all impressions to be equally important over the course of one episode.

Given S_t, the agent will estimate the value of taking different actions, i.e. using the Q-function. As previously mentioned, the Q-function will be approximated using a feed-forward neural network with three hidden layers, each having 100 neurons. It is not explicitly stated by Wu et al. (2018) which type of activation function is used in the network, but it's assumed here that they are using *ReLU activation functions*, as this is common for deep reinforcement learning applications, and for machine learning in general. The goal of the network is thus to evaluate all possible adjustments to λ given $S_t = (t, B_t, \mathrm{ROL}_t, \beta_t, \mathrm{CPM}_t, \mathrm{WR}_t, r_{t-1})$. The network is illustrated in figure 3.1.

The authors follow the same procedure as Mnih et al. (2015), using experience replay and a target network. They use a replay memory size of 100000 together with a mini-batch size of 32. For all of the samples in the mini-batch, they take the tuple (s, a, s', r) and set

$$y_n = \begin{cases} r & \text{if n+1 is terminal} \\ r + \gamma \max_{a'} Q(s', a'; \theta_n^-) & \text{otherwise} \end{cases}$$

and perform a gradient descent step on $(y_n - Q(s, a; \theta))^2$ with respect to θ. They use a target network update frequency of 100. The episode length is set to $T = 96$, with one day of bidding constitutes one episode. The authors use 7 days of bidding for training and 3 days of bidding for testing, using a number of different ad campaigns. The bids are thus allocated to the different time steps, most likely based

on their time stamps in the auction data. However, it's unclear what the "normal" step length is for any given campaign. It's also unclear whether or not they let their agent train on all of the campaigns and then test it on separate campaigns, or if the agent is trained and tested on separate campaigns.

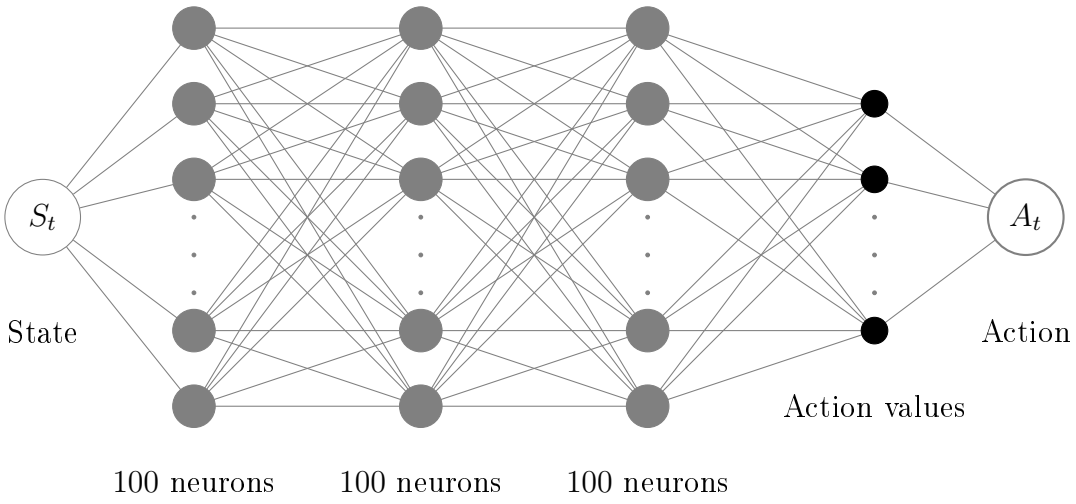


Figure 3.1: A feed-forward neural network with three hidden layers

The learning rate for the gradient descent algorithm when training the `DQN` is set to 0.001. For the ϵ-greedy policy, the starting value is set to $\epsilon_{\max} = 0.9$ which decays linearly to $\epsilon_{\min} = 0.05$. The authors also introduce two additional algorithmic features: an adaptive ϵ-greedy policy and an innovative reward function. Both of these features aim to increase the efficiency of the training and hence the performance of the network. The reward function also has the purpose of preventing the agent from getting stuck in suboptimal policies where it's just depleting the budget instantly. We will discuss both of these in more detail later in the methods and experiments sections. The authors call this method *Deep Reinforcement Learning to Bid* (`DRLB`). The interaction between the agent and the environment is illustrated below along with the pseudocode for `DRLB`.

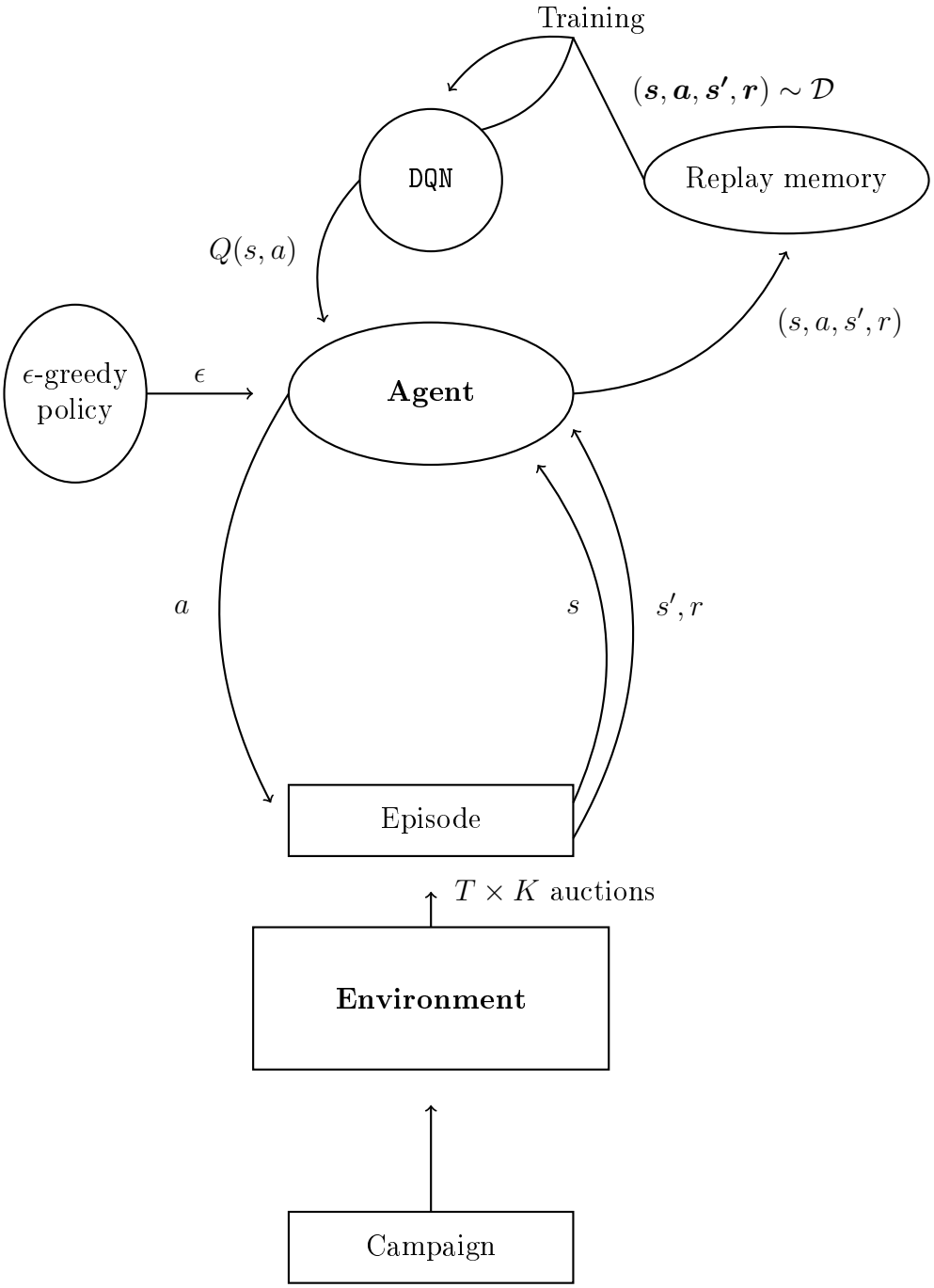


Figure 3.2: Illustration of how the bidding agent interacts with the RTB environment.

Algorithm - `DRLB`

Initialize replay memory $\mathcal{D}$ to capacity $n_{\mathcal{D}}$
Initialize Q_{local} with random weights θ
Initialize Q_{target} with weights $\theta^- = \theta$
for Episode = 1 to N **do**
    Initialize λ_0
    for k = 1 to K **do**
        Bid with λ_0, s.t. $b_{0,k} = \frac{\phi_{0,k}}{\lambda_0}$
    end for
    for t = 1 to T **do**
        Observe state s_t
        Update ϵ
        Get action a_t from ϵ**-greedy policy**
        Set $\lambda_t = \lambda_{t-1} \times (1 + a_t)$
        for k = 1 to K **do**
            Bid with λ_t, s.t. $b_{t,k} = \frac{\phi_{t,k}}{\lambda_t}$
        end for
        Observe reward r_t and next state s_{t+1}
        Store (s_t, a_t, r_t, s_{t+1}) in replay memory $\mathcal{D}$
        Sample mini-batch from replay memory $\mathcal{D}$
        for j = 1 to 32 **do**
            if s_{j+1} is terminal **then**
                set $y_j = r_j$
            else
                set $y_j = r_j + \gamma \max_{a'} Q(s_{j+1}, a'; \theta^-)$
            end if
            Perform a gradient descent step on $(y_j - Q(s_j, a_j; \theta))^2$ w.r.t. θ
        end for
        Every 100 steps set $\theta^- = \theta$
    end for
end for

Figure 3.3: Pseudocode for the `DRLB` algorithm

Chapter 4

Method

While it might seem straightforward to set up and solve the problem given the lengthy descriptions in the previous chapter, this is far from the case. As machine learning is not an exact science, the difference between success and failure can lie in small, seemingly innocuous, details. Many of these kinds of details are not provided by Wu et al. (2018) for the `DRLB` algorithm and, in some cases, where they are provided they give rise to more confusion than clarity. For example, they do not disclose how they initialize the bid-scaling parameter λ, e.g. whether they use a fixed value for every episode or if they initialize the parameter randomly. They also omit seemingly important details, such as what kind of activation function they use in their neural network, or if they have had to deal with typical training problems, such as diverging action values. Most importantly, they haven't released any code for the article, meaning that I've had to build the project from scratch.

This chapter will be devoted to describing how I've tried to replicate the `DRLB` agent. First, I will describe how I set up the problem in terms of creating the RL agent and the RTB-auction environment. Then, I will describe the parts of the `DRLB` algorithms which are not entirely clear in the article and what kind of problems I've had with them, as well as how I've tried to solve them. Then, I will describe the methods that will be used for comparison and benchmarking in order to evaluate the bidding agent and then the data. Finally, I will discuss in more detail the process of training and testing the agent.

4.1 Setting up the problem

The first step towards creating and implementing a auction simulation environment was to create a working agent. This basically means creating an RL agent which has a deep neural network as a function approximator for the action-value

function, $Q(s, a)$, an experience replay memory, a target network and an ϵ-greedy policy with which it picks actions. The agent also needs to have the ability to train its deep neural network, as well as the ability to manage the target network. In order to create an agent with these properties, I'm using `python` programming. For all of the machine-learning aspects, I've relied on Google's `tensorflow` library, especially `tf.layers` and `tf.train`.

However, much of the challenge in creating a functioning `DQN` agent is in making the whole machine work together. Another dimension of complexity is added when the agent also has to function together with a simulated environment. Thus, I start by creating a `DQN` agent for a much simpler problem, to get a better grasp of how it works and how to incorporate it into a more complicated setting. Once I have succeeded in creating an agent for this simple environment which is learning and completing the given task, I move on to building the auction simulation environment and trying to replicate the `DRLB` agent.

4.1.1 Mountain Car test

The simple problem I chose to work on is OpenAI's environment '`MountainCar-v0`', where an agent has to learn how to drive a small car up a hill such that the car reaches a flag on top of the hill. However, in order to get up the hill, the car needs momentum. Hence, the agent has to learn to generate momentum from the left hill before attempting to ascend the right hill.

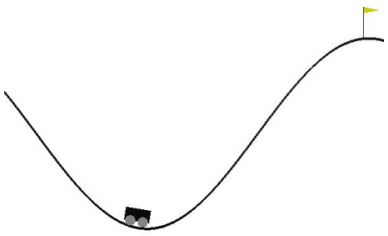


Figure 4.1: OpenAI's `MontainCar-v0` environment

There are three possible actions for the agent: to push left, to push right and to not push at all. The state is two-dimensional since it includes the horizontal position and the velocity. Additionally, we're concerned with a completely stationary and deterministic problem; for example, the position of the flag won't move and we know what happens if we push left (the car will go to the left). This is quite different from the RTB environment, where for example the market price for impressions can change dramatically over the course of a day.

Consequently, driving the mountain car doesn't require a neural network like the `DRLB` network illustrated previously. Instead, I use a much simpler architecture with two hidden layers, the first of which has 64 neurons while the second one has 32 neurons. The reason for this is to manage an accuracy-efficiency trade-off. Since we're working with a simple, low-dimensional problem, it's unlikely that we're going to reap any benefits in terms of accuracy by adding extra layers. At the same time, adding layers and neurons also means that we're adding training time. The network architecture is illustrated in the figure below.

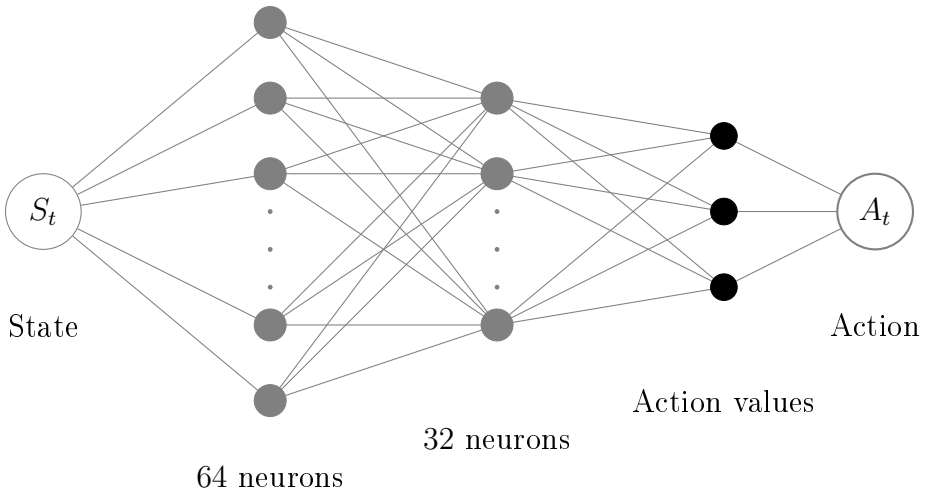


Figure 4.2: A feed-forward neural network with two hidden layers

Every episode is 200 steps and for each episode the code is similar to that presented above for the `DRLB` algorithm. I use an exponentially decaying ϵ, with $\epsilon_{\max} = 1.0$ and $\epsilon_{\min} = 0.01$. I also use a replay memory size of 50000, a mini-batch size of 50, a target network update frequency of 200 and a completely random selection of actions for 25000 simulated steps (in order to fill the replay memory). For every step that the car does not pass the flag, it gets a reward of -1, else it gets a reward of 0. The pseudocode is illustrated in figure 4.3. The actual code has also been included in the `GitHub` repository. It should be noted that the exponential decay used in the above example is relatively slow to converge, taking about 200 to 300 episodes. It is possible to create a `DQN`-based agent which learns how to master the mountain car a lot faster. However, the example was primarily for seeing that the algorithm worked in a simple environment and since it did, I move on to creating the RTB auction simulation environment.

Algorithm - `DQN` for `MountainCar-v0`

Initialize replay memory $\mathcal{D}$ to capacity $n_{\mathcal{D}}$
Initialize Q_{local} with random weights θ
Initialize Q_{target} with weights $\theta^- = \theta$
for Episode = 1 to N **do**
  Observe the initial state s_0
  for t = 1 to 200 **do**
    Choose action a_t from ϵ-**greedy policy**
    Observe reward r_t and next state s_{t+1}
    Store (s_t, a_t, r_t, s_{t+1}) in replay memory $\mathcal{D}$
    Sample a mini-batch from replay memory $\mathcal{D}$ as (s_j, a_j, s'_j, r_j)
    for j = 1 to 50 **do**
      if s'_j is terminal **then**
        set $y_j = r_j$
      else
        set $y_j = r_j + \gamma \max_{a'} Q(s_j, a'; \theta^-)$
      end if
      Perform a gradient descent step on $(y_j - Q(s_j, a_j; \theta))^2$ w.r.t. θ
    end for
    Every 200 steps set $\theta^- = 0.05 \times \theta^- + 0.95 \times \theta$
  end for
end for

Figure 4.3: Pseudocode for a `DQN`-inspired algorithm

4.1.2 Building the environment

The ultimate goal of this project is to use data from real auctions to simulate an environment in which the performance of a bidding agent can be tested. I solve this by creating a class which mimics the structure of environments in OpenAI's `Gym` library. The environment defines all of the possible actions, tracks all of the state-relevant parameters, the time step, and so on. When creating the state and the environment's behavior when responding to the agent's actions, I have to start using the paper by Wu et al. (2018).

First, the environment defines $\mathcal{A} = \{-8\%, -3\%, -1\%, 0\%, 1\%, 3\%, 8\%\}$ as the set of possible actions. Then, the environment tracks the values defining the state, i.e. the tuple $S_t = (t, B_t, \text{ROL}_t, \beta_t, \text{CPM}_t, \text{WR}_t, r_{t-1})$. The environment also manages all of the data, fetching the necessary bids and CTR estimations from the dataset when a new step is taken, as well as tracking the step and time of termination. I formalize all of this in a `python` class, which has the functions of resetting the

environment with an initial budget and λ_0 when a new episode is started, taking steps given a certain action, and returning results for a period, e.g. when testing the performance of the agent on a certain campaign. The class is itself initialized by taking predetermined episode and step lengths and a `python` dictionary containing campaign-relevant information (or information on several campaigns), e.g. the bid data, the available budget, the number of impressions, and so on.

Wu et al. (2018) explicitly describe the state using the tuple above. However, as mentioned in the previous chapter, the problem with equipping RL agents with function approximators is that a global approximation might complicate the training process, rather than to make it more efficient. Let's consider how the state dynamics are described. The time step, t, will always be between 0 and 96. The budget, B_t, can be in the hundreds of thousands, or even millions. The budget consumption rate, β_t, will of course always be between 0 and 1; as will the winning rate. It's also the case that the remaining regulation opportunities, ROL_t, is just inversely proportional to the time step. That is, $\mathrm{ROL}_t = 96 - t$. Similarly, the cost, CPM_t, is already incorporated into the budget and the budget consumption rate. Hence, we have two considerations to make:

(i) are some parameters unnecessary?

(ii) are the different parameter ranges prone to create instabilities in the approximation?

Intuitively, the answer to both questions is a clear *yes*. I initially tried to use the state dynamics described by Wu et al. (2018), but the agent didn't work very well. Most of the time, it actually performed worse than a form of random bidding. I suspected that the agent might have problems when learning from parameters with too much variation. I thus changed $S_t = (t, B_t, \mathrm{ROL}_t, \beta_t, \mathrm{CPM}_t, \mathrm{WR}_t, r_{t-1})$ to another tuple: $S_t = (B_t/B_0, \mathrm{ROL}_t, \beta_t, \mathrm{WR}_t, r_{t-1})$.

Instead of including the absolute budget, I consider the current budget in relation to the initial budget. I also discard the cost and the time step. This made a massive difference, and this is what I'll be using in chapter 5. Finally, it's not entirely clear how many bids are processed at each time step. Wu et al. (2018) most likely use the time stamps in the auction data to allocate different bids to time steps and episodes. This information is not available here, meaning that the step length has to be fixed. However, even when adapting the step length to how the bids might be allocated, it's unclear how the authors manage to have an ϵ-greedy policy that explores sufficiently. In my experiments, I will let the agent train on all the campaigns and then test the performance on separate campaigns, using and experimenting with different step lengths.

4.2 Comparisons and benchmark

While RTB spending has grown explosively, the development of bidding strategies has not been as explosive. For all of the progress being made in machine learning, DSPs are usually restricted to a relatively simple technique known as *linear bidding*, while often estimating values of different impressions with logistic regression models.

There are several reasons for this. For example, there has been a lack of large, publicly available datasets for research and benchmarking until a few years ago when a Chinese RTB company, iPinYou, released a large dataset for research purposes (Zhang et al., 2015). There are also some natural constraints in an RTB setting, e.g. the time constraint which means that any bidding algorithm has to be able to formulate a bid within 100 milliseconds of receiving a bid request from an AdX, as well as the non-stationarity of impression markets which makes it even more difficult to design efficient, general models.

One of the most common techniques, the aforementioned linear bidding, uses the CTR, denoted here by ϕ, which tries to capture the probability of a given user clicking on a display advertisement. I will evaluate the performance of the `DRLB`-based bidding agent using a simple form of linear bidding, `LinBid`. The kth bid, b_k, is formulated by taking the average CTR over a large number of historical cases, as well as the average bid used in these auctions, and the current CTR estimation, ϕ_k:

$$b_k = b_{\text{average}} \times \frac{\phi_k}{\phi_{\text{average}}}$$

Together with random uniform bidding, `RandBid`, this model will be used as a benchmark. For random uniform bidding, I take the historical minimum bid and the historical maximum bid and sample bids from a uniform distribution, s.t.

$$b_k \sim \text{Uniform}(b_{\min}, b_{\max})$$

Neither method uses episodes, which is why the time step is dropped from the subscript. For both `RandBid` and `LinBid`, the set of training bids for a specific campaign will be used as historical data when evaluating the performance on that campaign. I will evaluate the algorithms by considering the number of impressions won, the number of clicks, the winning rate, the total budget spent, the effective cost per click (eCPC) and the effective cost per impression (eCPI).

4.3 Data

The principal source of data in this thesis will be the aforementioned dataset released by the chinese RTB company iPinYou in 2015. The primary reason for this is the ability to compare results to other papers and methods, since all of the three previously mentioned papers incorporating RL uses the iPinYou dataset. Zhang, Yuan and Wang (2015) released a paper together with the dataset, describing the data in detail together with some summary statistics. The dataset contains 9 different campaigns, each running over the course of a couple of days. Each campaign is divided up into one dataset for training and one for testing. In total, there are 15395258 (i.e. $\sim$ 15 million) impressions in the training data and 4100716 (i.e. $\sim$ 4 million) impressions in the testing data. The dataset is described in more detail below.

Table 4.1: iPinYou Dataset

Campaign	Train impressions	Train clicks	Test impressions	Test clicks
1458	3083056	2454	614638	543
2259	835556	280	417197	131
2261	687617	207	343862	97
2821	1322561	843	661964	394
2997	312437	1386	150063	533
3358	1742104	1358	300928	339
3386	2847802	2076	545421	496
3427	2593765	1926	536795	395
3476	1970360	1027	523848	302

Since the `DRLB`-based agent formulates bids using the CTR estimations and then takes actions according to the budget, the budget consumption rate, and so on, we only need the winning bids, the CTR estimations, the total budgets and the total number of impressions. I have used the processed data by Du et al. (2017), where CTR estimations have already been made using impression-specific information and logistic regression, according to Zhang, Yuan and Wang (2014, 2015).

Wu et al. (2018) also use the iPinYou dataset when testing their `DRLB` algorithm. However, they have not released their processed data and they use a more complicated metric when evaluating the performance of their agent. We will be using the processed data from Du et al. (2017) as this allows performance comparisons with several other methods with respect to the number of clicks won and the eCPC.

4.4 Training the agent

The challenge of this project, or any reinforcement learning task, is to train the agent. We have the data and have created the environment; the next step is to make the agent navigate through it successfully. This is, of course, easier said than done. There are many details to consider and changing seemingly unimportant parameters can have a huge effect on the agent's learning process and subsequent performance. I will be working with and testing for several different hyperparameters, e.g. the step length, K, the decay rate for the ϵ-greedy policy, r_ϵ, the initial bid-scaling, λ_0, and the learning rate, α.

As mentioned previously, Wu et al. (2018) incorporate two additional algorithmic features into the `DRLB` procedure as described above: a new type of reward function, the `RewardNet` and an adaptive ϵ-greedy policy. The `RewardNet` uses a neural network with the same architecture as the Q-function approximator to consider the reward from entire episodes when taking a certain action in a certain state, rather than considering the immediate reward from a state-action pair. The point of this is to prevent the agent from immediately depleting the budget by decreasing λ. We will not be using the `RewardNet`, but instead show that inefficient budget depletion can be avoided by considering the ϵ decay rate, the step length and the learning rate, as well as by initializing the budget for respective episodes randomly during the training phase.

The adaptive ϵ-greedy policy checks if the distribution of the action values, i.e. the outputs from the `DQN`, is unimodal, e.g. that the graph connecting the action values is first strictly increasing, then nonincreasing and then strictly decreasing. If the distribution of action values is found not to be unimodal, the ϵ is set to $\epsilon = \max(\epsilon_t, 0.5)$. The authors do not make it clear exactly how they test for unimodality. I tried creating a function which checked if the action values had a unimodal distribution, but it always found that this was false, i.e. basically setting $\epsilon = \max(\epsilon_t, 0.5)$ permanently. This, of course, led to inefficient and excessive exploration. Hence, I've discarded the use of this feature as well.

Wu et al. (2018) do not make it clear how they initialize the budget during training. In the iPinYou dataset, there is a specified cost for each campaign. It is not specified anywhere in the article by Wu et al. (2018) if they use this as their budget, or if they use some other, pre-specified budget. Neither is it clarified how they partition and initialize the budget for separate episodes during the same campaign. This is far from ideal, as the goal of the `DRLB` algorithm is partly to create an agent which can manage the spending of the budget in an optimal way. Hence, budget partition and initialization seem like important considerations to make in

both training and testing. I have partly followed Du et al. (2017) in this respect. They use the specified training budget, B_{train}, in the iPinYou dataset during the training phase and then they scale the training budget using the proportion of impressions in the testing data, N_{test}, to the number of impressions in the training data, N_{train}. Then, they use a budget-scaling parameter, b_0, to see how the agent performs with different budget sizes. The testing budget is then defined by

$$B_{\text{test}} = b_0 \times \frac{N_{\text{test}}}{N_{\text{train}}} \times B_{\text{train}}$$

This is the testing budget I will use when running experiments and performance comparisons, which allows comparisons to the results presented by Du et al. (2017). However, during the training phase I will initialize the budget randomly for different episodes using a normal distribution, setting the fraction of the total budget as the mean and testing for different variances:

$$B_{\text{episode}} \sim \mathcal{N}\left(\frac{N_{\text{episode}}}{N_{\text{train}}} \times B_{\text{train}}, \sigma_B^2\right)$$

where σ_B^2 is the variance for the budget initialization. We will see how this variance affects the performance of the agent and its ability to deplete the budget in an efficient way.

The bulk of the experimentation will be devoted to finding a good set of parameters which lead to efficient training and a good performance with respect to how the budget is spent, how much of the budget is spent, how many impressions are acquired, what the cost is per impression and how many of those impressions turned into clicks. While there are many hyperparameters to consider, I will focus on

(i) the step length, K,

(ii) the learning rate, α,

(iii) the ϵ decay rate, r_ϵ, and

(iv) the variance for the episodic budget initialization, σ_B^2.

I will test the stability of the agent by initializing it with different bid-scaling parameters, λ_0. I will be running performance tests using the budget-scaling parameter $b_0 = 1/32$, since this is the same value used by Du et al. (2017). Hence, using $b_0 = 1/32$ will allow for making broader performance comparisons, in addition to benchmarking with `LinBid` and `RandBid`. Finally, the remaining hyperparameters will be set exactly as by Wu et al. (2018). That is, I'll use

- the starting value for ϵ, $\epsilon_{\text{max}} = 0.9$,
- the end value for ϵ, $\epsilon_{\text{min}} = 0.05$,
- the target network update frequency, $C = 100$,
- the experience replay memory size, $n_{\mathcal{D}} = 100000$,
- the episode length, $T = 96$, and
- the discount factor, $\gamma = 1$.

In order to be able to compare results, I've used `np.random.seed(1)` together with `tf.set_random_seed(1)` in every test. When training and testing for different hyperparameters, I've been able to use one of Adform's 32-core machines, which has been incredibly helpful and has allowed for parallel testing.

4.5 Modeling bias and constraint

In terms of wanting to know how well the agent could perform in an actual RTB setting, this methodology is far from ideal. The main problem is that offline evaluations inevitably impose a modeling bias. Li et al. (2012) discuss this for a similar problem. That is, it's unrealistic to assume that if we introduce a new bidder to an auction environment, who bids aggressively and depletes its budget, all of the other bidders will remain passive in changing their bidding strategy and simply accept winning fewer bids. Obviously, other bidders will react by increasing their bids, since they also want to deplete their budgets and win as many impressions as possible.

However, this is the approach followed by Du et al. (2017), as well as Wu et al. (2018). While this type of offline evaluation might be problematic, it can at least give an idea of how the bidding agent will manage the budget and it's ability to form reasonable bids. It would be desirable to run a more complete simulation with several bidding agents interacting with each other, but I will settle for an offline test where the agent is interacting with a passive environment. This is partly due to comparability with Du et al. (2017) and partly due to the time constraint imposed on this thesis, since creating and evaluating a sufficiently realistic environment for simulations would take a considerable effort.

4.6 Stability testing

The bulk of the experimentation will be devoted to finding good parameters for the agent's performance. Then, we also want to test the stability of the agent. As previously mentioned, I will do this by testing its performance for different initial bid-scaling parameters, λ_0. I'll consider

$$\lambda_0 \in \{5 \cdot 10^{-2}, 10^{-2}, 5 \cdot 10^{-3}, 10^{-3}, 5 \cdot 10^{-4}, 10^{-4}, 5 \cdot 10^{-5}, 10^{-5}, 5 \cdot 10^{-6}, 10^{-6}\}$$

When searching for good parameters, I'll use $\lambda_0 = 10^{-4}$. I'll also be testing for stability by running the same experiment, i.e. training with the same parameters, but using different seeds for a number of simulations. This should give us an idea of how robust the particular agent is with respect to different random outcomes and, in some sense, to nonstationarity. It should be noted here that we're not testing the stability of the *algorithm*, but the stability of the *agent*, which is the outcome of the algorithm when using a certain set of parameters.

Chapter 5

Experiments and Results

This chapter will be devoted to optimizing the agent's performance and then comparing its performance to the simple benchmark algorithms presented above, `RandBid` and `LinBid`, as well as the results presented by Du et al. (2017), which includes a more sophisticated linear bidding algorithm, the RL algorithm presented by Cai et al. (2017), `RLB`, and their own model-based RL algorithms, `CMDP` and `Batch-CMDP`. For these latter ones, we only have information on the number of clicks and the eCPC.

First, I will present the results for the benchmarks and comparisons. Then, I will display some of the results from testing different parameters, as well as discuss the results. Finally, I will check the stability of the agent by initializing it with different λ_0, as well as by testing for different seeds in the random processes, e.g. the weight initialization for the neural networks. Since we have deviated quite a bit from the `DRLB` algorithm as it is presented by Wu et al. (2018), I will instead call the algorithm we're using here `Batch-DRLB`, since it uses finer episodes by breaking campaigns up into batches.

5.1 Comparative results

When testing `LinBid`, `RandBid` and `Batch-DRLB`, I've used

$$B_{\text{campaign, test}} = b_0 \times \frac{N_{\text{campaign, test}}}{N_{\text{total, train}}} \times B_{\text{total, train}}$$

where $N_{\text{campaign, test}}$ is the total number of impressions in the test data for the specific campaign, $N_{\text{total, train}}$ is the total number of impressions in all of the training data and $B_{\text{total, train}}$ is the total budget available in all of the training data. Since the testing data is divided up into different episodes for `Batch-DRLB`, any

budget that has not been spent during an episode spills over to the next episode. The results for `LinBid` and `RandBid` are presented in table 5.1 and table 5.2, respectively. In table 5.3, I'm presenting the results from Du et al. (2017) as they present it themselves, i.e. as clicks (eCPC), for the tuned linear bidding algorithm (`tLinBid`), `RLB`, `CMDP` and `Batch-CMDP`. The eCPC is measured in thousands.

All of the four algorithms presented from Du et al. (2017) outperform `LinBid` and `RandBid` by some distance. This is no surprise, since they are all state-of-the-art algorithms. However, I will still mainly be focusing on `LinBid` and `RandBid` as our primary comparisons, since there is no information on total cost or number of impressions won for the algorithms in table 5.3.

Table 5.1: `LinBid`

Campaign	Impressions	Clicks	Cost	Win rate	eCPC	eCPI
1458	37351	105	1541903	0.06	14.7	41.28
2259	16177	4	1046594	0.04	26.2	64.70
2261	20306	9	862623	0.06	95.8	42.48
2821	44596	15	1660619	0.07	110.7	37.24
2997	20425	36	391497	0.13	10.9	19.17
3358	11383	48	754918	0.04	15.7	66.32
3386	27672	34	1368258	0.05	40.2	49.45
3427	26117	47	1346624	0.05	28.7	51.56
3476	22793	39	1314143	0.04	33.7	57.66

Table 5.2: `RandBid`

Campaign	Impressions	Clicks	Cost	Win rate	eCPC	eCPI
1458	28024	20	1541902	0.05	77.1	55.02
2259	15388	3	1046594	0.04	34.9	68.01
2261	15346	0	862624	0.04	N/A	56.21
2821	23679	14	1660627	0.04	118.6	70.13
2997	8794	24	391501	0.06	16.3	44.52
3358	9202	3	754919	0.03	251.6	82.04
3386	22191	18	1368261	0.04	76	61.66
3427	21931	13	1346623	0.04	10.4	61.40
3476	19110	10	1314143	0.04	13.1	68.77

Table 5.3: `Results from Du et al. (2017)`

Campaign	tLinBid	RLB	CMDP	Batch-CDMP
1458	464 (1.09)	424 (3.09)	464 (2.71)	462 (2.8)
2259	7 (173.52)	12 (101.2)	13 (89.47)	10 (119.16)
2261	9 (105.67)	11 (87.39)	8 (118.10)	7 (116.46)
2821	40 (40.26)	47 (39)	39 (45.32)	41 (45.05)
2997	64 (2.73)	82 (3.7)	71 (2.95)	71 (2.98)
3358	189 (3.77)	199 (4.29)	208 (3.38)	203(4.28)
3386	55 (5.52)	61 (21.21)	92 (12.99)	91 (14.26)
3427	203 (6.55)	261 (5.14)	292 (4.47)	292 (4.36)
3476	162 (5.92)	131 (9.87)	181 (7.16)	188 (6.82)

5.2 Parameter testing

When considering different parameters, we will start by looking at campaign 1458, as this is the campaign with the most training and testing data. However, we will still be training the agent using all of the training data from all of the campaigns. I considered

$$K \in \{1000, 750, 500, 250\}$$

$$\alpha \in \{0.001, 0.0005, 0.0001, 0.000075\}$$

$$r_\epsilon \in \{0.00025, 0.0001, 0.000075, 0.00005, 0.000025\}$$

$$\sigma_B^2 \in \{0, 2500, 5000, 7500, 10000, 12500, 15000\}$$

From this, I have chosen two sets of parameters. First, I've chosen the set of parameters that resulted in the most clicks and impressions. I'll call this set `ALPHA`. However, there are two important considerations to make here:

(i) Since the clicks are (in a sense) randomly distributed in the dataset, getting a high number of clicks can be a random occurrence rather than the result of an intelligent agent. This actually seems to be the case for many sets of parameters, where the agent only spent a fraction of the budget and got a low number of impressions, but still managed to get a large number of clicks.

(ii) Even if the agent gets a large number of both clicks and impressions, *and* depletes the budget, it might be a case of *overfitting* to a specific budget, a specific campaign, and so on.

Thus, I've also chosen another set of parameters, `BETA`, which displays a more stable behavior with less clicks and impressions. The results for both agents,

Batch-DRLB-α and Batch-DRLB-β, is given below, as well as the actual parameters.

Table 5.4: Parameters for Batch-DRLB, version ALPHA and BETA

Version	K	α	r_ϵ	σ_B^2
ALPHA	500	0.0001	0.0001	2500
BETA	500	0.0001	0.00005	10000

Table 5.5: Batch-DRLB for campaign 1458

Version	Impressions	Clicks	Cost (% of budget)	Win rate	eCPC	eCPI
ALPHA	54085	465	1499421 (97.2%)	0.09	3.22	27.72
BETA	50302	429	1528742 (99.1%)	0.08	3.56	30.39

For both ALPHA and BETA, ϵ converged to $\epsilon_{\min} = 0.05$. Intuitively, BETA should be more stable as it's been exploring for a longer time and since it has a much greater variance in the budget initialization. While both versions outperformed the benchmarks by some distance, ALPHA gets even better result than the state-of-the-art algorithms tested by Du et al. (2017), albeit marginally. When testing the performance, I remodeled the state dynamics as described in section 4.4, i.e. discarding the cost and the absolute budget, as well as the time step, and instead using relative measures to decrease the dimensionality and to make the parameter ranges more uniform. To get a better understanding of how the two agents actually behaved, I have illustrated how they spend their budget and regulate the bid-scaling parameter over the testing episodes in figure 5.1 and figure 5.2.

Both agents display interesting behavior. First of all, both agents seem obsessed with decreasing the bid-scaling parameter in order to bid more aggressively. Secondly, it's interesting to note how both agents, to varying extent, seem to regulate the bid-scaling parameter as the budget is being depleted, effectively managing the budget-consumption rate as the budget is running out. It's especially interesting to note how BETA seems to react to budget spill-overs (e.g. the blue, green and red lines), by bidding more aggressively to eventually deplete the budget. This behavior is a contrast to many other agents, where the budget is being depleted more or less immediately, without any obvious concern for long-term gains.

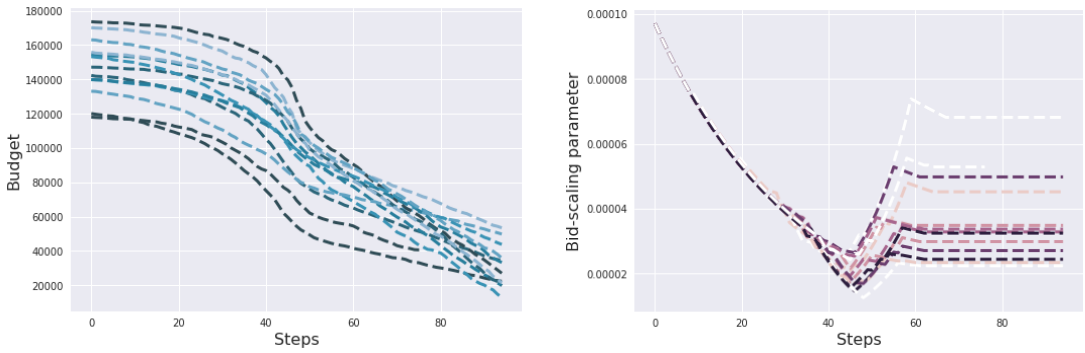


Figure 5.1: Budget spending and bid scaling for `ALPHA`

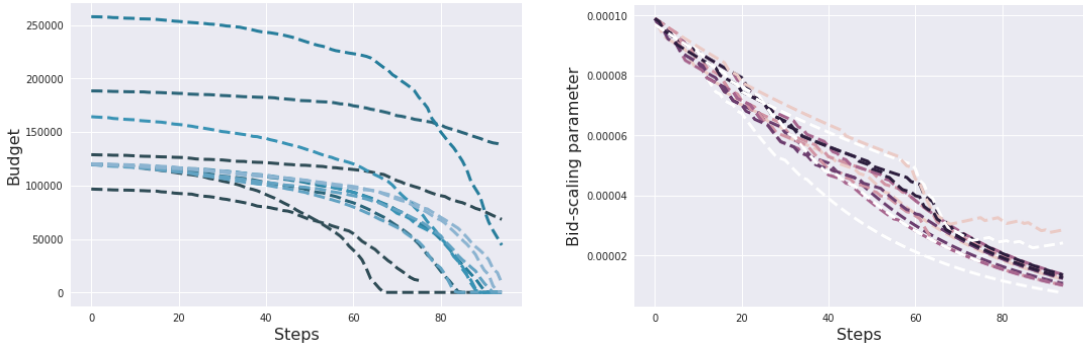


Figure 5.2: Budget spending and bid scaling for `BETA`

We have two agents who both produce good results with respect to budget consumption, number of impressions and number of clicks, and who also seem to display at least some intelligent behavior. Now, we want to see how they perform when varying the initial bid-scaling parameter. Then, if we find a better initial bid-scaling parameter, I'll use this to run several simulations with different random seeds and see how the performance turns out. Finally, I'll test the "best" agent against the benchmarks on all campaigns.

5.3 Stability tests with initial bid scaling, λ_0

The problem with initialization in reinforcement learning is that a "bad" initialization of some parameter can lead to inefficient training where the agent immediately learns a suboptimal behavior, or doesn't really learn anything at all. For example, let's consider what a "too small" λ_0 would mean for our bidding agents. There are cases where the initial bidding is so aggressive that the agent is winning every

auction and depleting the budget in just a couple of steps. In such a situation, exploring has little effect as the agent will never be able to regulate the bid-scaling fast enough not to deplete the entire budget immediately.

Similarly, if λ_0 is "too big", the agent will encounter few situations where it actually spends a sufficient amount of the budget to get a noticeable reward. This is, of course, strongly linked to the learning rate. If the agent wins a couple of bids in a step, the experience will be stored in the memory. Then, the agent will use this in a mini-batch where each experience is trained on with only one gradient descent step, and where the other experiences are likely with a higher bid-scaling and hence a small reward. In other words, even when the agent actually does bid more aggressively, these experiences will likely have a marginal impact on the training; especially, if we have a low learning rate. We consider the results for both `ALPHA` and `BETA` in table 5.6 below.

Table 5.6: λ_0-test for campaign 1458

λ_0	`ALPHA` cost (% of budget)	`ALPHA imp.`	`BETA` cost (% of budget)	`BETA` imp.
0.05	88 (0.0%)	8	273275 (17.7%)	10157
0.01	43 (0.0%)	4	1352751 (87.7%)	34239
0.005	4279 (0.3%)	121	175909 (11.4%)	7451
0.001	1530223 (99.2%)	38176	85806 (5.6%)	3074
0.0005	1541478 (100%)	40687	1541478 (100%)	42095
0.0001	1499421 (97.2%)	54085	1528742 (99.1%)	50302
0.00005	1534536 (99.5%)	42620	1402779 (91.0%)	37125
0.00001	1416183 (91.8%)	27921	1416183 (91.8%)	29771
0.000005	1313476 (85.2%)	24586	1313475 (85.2%)	24216
0.000001	1123690 (72.9%)	16695	1123685 (72.9%)	16471

There are some interesting observations to be made from the tests with λ_0. First of all, both agents seem to be stable and perform well for $\lambda_0 \in [0.00005, 0.0005]$. Outside this interval, there are some serious instabilities. For example, for the last three rows, the agents perform almost identically. This is because they are both depleting their entire budgets in a matter of steps. Even though `BETA` has a higher budget initialization variance and a smaller learning rate, it seems to conform to the same behavior as `ALPHA`. `BETA` also exhibit more instability for $\lambda_0 \in [0.001, 0.05]$.

While there might be a more optimal initial bid-scaling parameter to be found in the interval $[0.00005, 0.005]$, it seems that the arbitrary λ_0 we chose in the beginning actually does pretty well. Hence, I'll use it going forward. However, it should be noted that this choice of λ_0 is probably specific to `ALPHA` and `BETA`. In fact, there were several agents in the parameter search that displayed the type of behavior that `ALPHA` and `BETA` were displaying with too small or too large initial bid-scaling parameters. This suggests that λ_0 is also a crucial parameter to consider when training and optimizing the agent. In a more rigorous parameter search, each parameter set should have been varied using different λ_0 as well. This result also adds mystery to why Wu et al. (2018) do not discuss this seemingly important feature of the `DRLB` algorithm in their paper.

Moving on to the stability testing with different random seeds, I'll be using $\lambda_0 = 0.0001$. However, in a project with even broader scope, finding an optimal λ_0 should be included in a more rigorous and intelligent parameter search.

5.4 Stability tests with random seeds

There are four important instances of randomness when the agent interacts with the environment: the random initialization of the network weights, the randomness in the ϵ-greedy policy, the random sampling from the experience replay memory and the budget initialization during the training phase. So far, I have been using the same random seeds, `np.random.seed(1)` and `tf.set_random_seed(1)`, in all simulations in order to have comparable tests. Changing these seeds will change the "randomness" in how e.g. the experience samples are chosen and the greedy actions are taken. An agent with a robust training algorithm should be able to achieve comparable results with different seeds. In the table 5.7, I've listed the results for a number of different random seeds, for both `ALPHA` and `BETA`.

It seems that none of the agents are entirely stable with respect to varying random seeds. This is of course not ideal, as we would like the agent to conform to a similar behavior, despite changes in random outcomes. However, `ALPHA` actually performs pretty well. It depletes more than 80% of the budget in 9 out of 12 cases. This might seem a bit unexpected, since `BETA` was initially assumed to be more stable and less fitted to specific environments. However, if we think about it, it should be quite the opposite. Since `BETA` has a larger budget initialization variance and a much longer exploration time, it should be more prone to instabilities with respect to different random outcomes. That is, in addition to the budget variance being higher, the "inherent variance" in the results should also be greater, as we're essentially giving the agent a broader array of random inputs to train on.

Table 5.7: Repeated tests for campaign 1458, $\lambda_0 = 0.0001$

seed	ALPHA cost (% of budget)	ALPHA imp.	BETA cost (% of budget)	BETA imp.
1	1499421 (97.2%)	54085	1528742 (99.1%)	50302
2	1316505 (85.4%)	48996	203361 (13.2%)	6479
3	597440 (38.7%)	18436	34388 (2.2%)	1079
4	1539999 (99.9%)	39623	1400626 (90.8%)	52508
5	72832 (4.7%)	2361	92942 (6.0%)	3344
6	1322969 (85.8%)	37262	1539999 (99.9%)	39609
7	1486883 (96.4%)	37474	114412 (7.4%)	4347
8	1498941 (97.2%)	45046	1147601 (74.4%)	32765
9	1539999 (99.9%)	43527	38166 (2.5%)	1108
10	1540000 (99.9%)	44590	281970 (18.3%)	11404
11	1540001 (99.9%)	55022	1540001 (99.9%)	49748
12	272452 (17.7%)	9960	385760 (25.0%)	13271

One observation of interest is the result from `seed(5)`. Suddenly, `ALPHA` only spends a small fraction of its budget. While the weight initialization might make a difference, it's likely that the random sampling from the experience replay memory, the budget initialization and the ϵ-greedy policy are the dominant forces affecting the outcomes. This is also what is suggested by the overall results. `ALPHA` is *clearly* more stable than `BETA` for the different random seeds. So, what are the main differences between `ALPHA` and `BETA`? The latter has a much larger variance for budget initialization, as well as a smaller ϵ decay rate, meaning that it explores for a longer time. Any or both of these factors could have a significant impact on the difference between `ALPHA` and `BETA`, or on what happens for `seed(5)`. However, discussing this further is only part of a much larger problem: to understand the relationships between the different parameters and to find a set of parameters which are robust with respect to randomness. Such a discussion could easily be a thesis project in its own right, and I will thus not pursue it further here.

Our current situation is far from ideal. While we have discussed and inferred that it is reasonable to have stability only within a small range for λ_0, we've arrived at a much more problematic situation here. RTB is a highly nonstationary environment, meaning that if the agent cannot perform well with different seeds, it's not likely to be entirely robust in a real auction environment either. This is another aspect of the `DRLB` approach which is not discussed by Wu et al. (2018);

they do, however, apply their DRLB algorithm successfully to other real-world data, in addition to the iPinYou dataset. I will assume that there is some set of parameters that makes the agent more robust to different random outcomes and that our ALPHA agent is somewhat representative of what the outcome would be in such a case. Hence, I will leave the rest of the discussion of how to train a robust agent to a more rigorous analysis and instead move on to seeing how ALPHA and BETA performs on all of the campaigns.

5.5 Final results

This section will be devoted to a final comparison between ALPHA, BETA, LinBid and RandBid.. We will be using the initial random seeds, i.e. `np.random.seed(1)` and `tf.set_random_seed(1)`, the initial bid-scaling parameter $\lambda_0 = 0.0001$ and the budget-scaling $b_0 = 1/32$. Using this budget-scaling parameter allows us to compare results to Du et al. (2017). The results are given in table 5.8 and 5.9.

Table 5.8: Batch-DRLB-α

Campaign	Impressions	Clicks	Cost (% of budget)	Win rate	eCPC	eCPI
1458	54085	465	1499421 (97.2%)	0.09	3.22	27.72
2259	44983	9	944044 (90.2%)	0.11	10.5	20.99
2261	45023	8	713935 (82.8%)	0.13	89.2	15.86
2821	78763	36	1506526 (90.7%)	0.12	41.8	19.13
2997	19583	52	378317 (96.6%)	0.13	7.28	19.32
3358	12582	205	689405 (91.3%)	0.04	3.36	54.79
3386	36381	88	1340759 (98.0%)	0.07	15.2	36.85
3427	29564	296	1301828 (96.7%)	0.06	4.4	44.03
3476	26691	171	1285795 (97.8%)	0.05	7.52	48.17

Table 5.9: Batch-DRLB-β

Campaign	Impressions	Clicks	Cost (% of budget)	Win rate	eCPC	eCPI
1458	50302	429	1528742 (99.1%)	0.08	3.56	30.39
2259	21224	5	255487 (24.4%)	0.05	51.1	12.04
2261	20752	5	223573 (25.9%)	0.06	44.7	10.77
2821	63131	25	1227267 (73.9%)	0.1	49.1	19.44
2997	19588	51	378311 (96.6%)	0.13	7.42	19.31
3358	12495	197	728526 (96.5%)	0.04	3.7	58.31
3386	22393	74	696922 (50.9%)	0.04	9.42	31.12
3427	29100	277	1321650 (98.1%)	0.05	4.77	45.42
3476	25971	170	1259630 (95.9%)	0.05	7.41	48.5

ALPHA is clearly superior to BETA, with respect to impressions won, clicks won, budget spent and winning rate, for every campaign - with exception for campaign 2997 where BETA won 5 more impressions (although one click less). Thus, I will only compare ALPHA with the other algorithms. In table 5.10, I have compared number of clicks and eCPC to the algorithms tested by Du et al. (2017), and in table 5.11, I have compared the number of impressions and winning rate to LinBid and RandBid. In figures 5.3-5.12, I've plotted the budget spending and bid-scale regulations for all of the campaigns. In table 5.10, the metric I've used is 'impressions (% of budget spent)', and in table 5.11, I've used 'clicks (eCPC)'.

Table 5.10: Comparison with impressions (% of budget spent)

Campaign	LinBid	RandBid	Batch-DRLB-α
1458	37351 (100%)	28024 (100%)	**54085** (97.2%)
2259	16177 (100%)	15388 (100%)	**44983** (90.2%)
2261	20306 (100%)	15346 (100%)	**45023** (82.8%)
2821	44596 (100%)	23679 (100%)	**78763** (90.7%)
2997	**20425** (100%)	8794 (100%)	19583 (96.6%)
3358	11383 (100%)	9202 (100%)	**12582** (91.3%)
3386	27672 (100%)	22191 (100%)	**36381** (98.0%)
3427	26117 (100%)	21931 (100%)	**29564** (96.7%)
3476	22793 (100%)	19110 (100%)	**26691** (97.8%)

Table 5.11: Comparison with clicks (eCPC)

Campaign	tLinBid	RLB	CMDP	Batch-CDMP	Batch-DRLB-α
1458	464 (**1.09**)	424 (3.09)	464 (2.71)	462 (2.8)	**465** (3.22)
2259	7 (173.52)	12 (101.2)	**13** (**89.47**)	10 (119.16)	9 (104.9)
2261	9 (105.67)	**11** (**87.39**)	8 (118.10)	7 (116.46)	8 (89.2)
2821	40 (40.26)	**47** (**39**)	39 (45.32)	41 (45.05)	36 (41.8)
2997	64 (**2.73**)	**82** (3.7)	71 (2.95)	71 (2.98)	52 (7.28)
3358	189 (3.77)	199 (4.29)	**208** (3.38)	203(4.28)	205 (**3.36**)
3386	55 (**5.52**)	61 (21.21)	**92** (12.99)	91 (14.26)	88 (15.2)
3427	203 (6.55)	261 (5.14)	292 (4.47)	292 (**4.36**)	**296** (4.4)
3476	162 (**5.92**)	131 (9.87)	181 (7.16)	**188** (6.82)	171 (7.52)

Figure 5.3: Campaign 1458

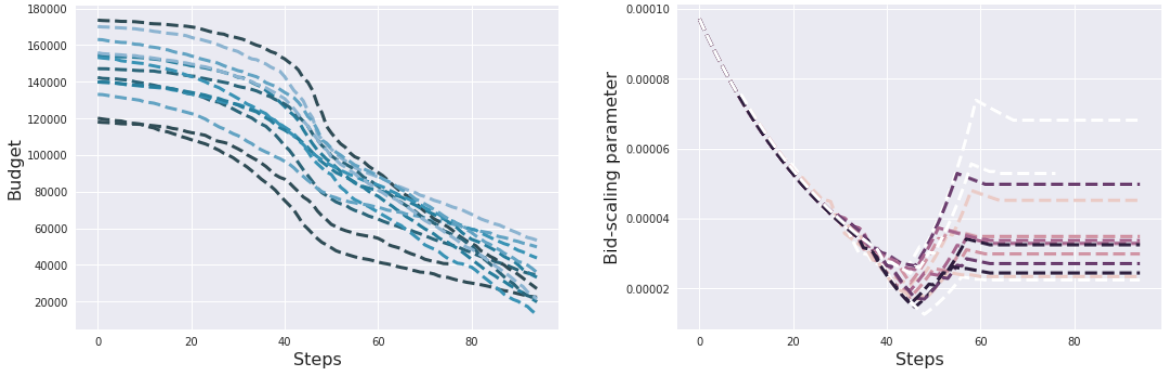


Figure 5.4: Campaign 2259

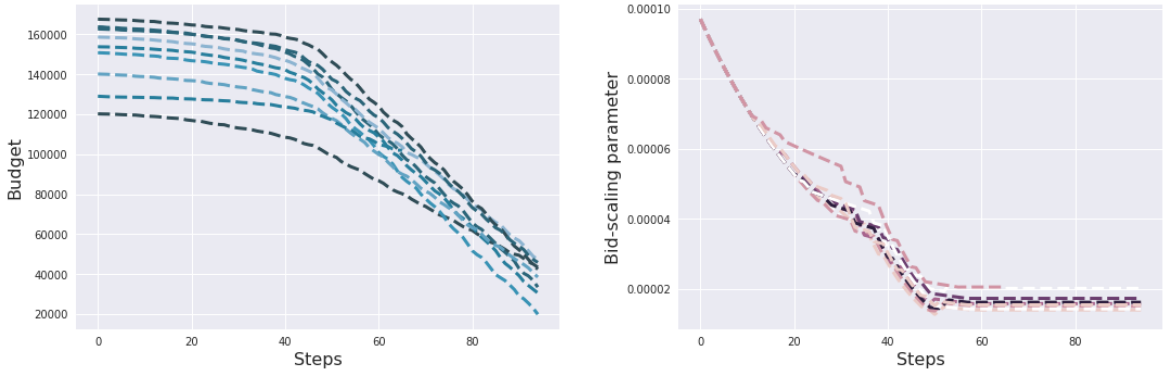


Figure 5.5: Campaign 2261

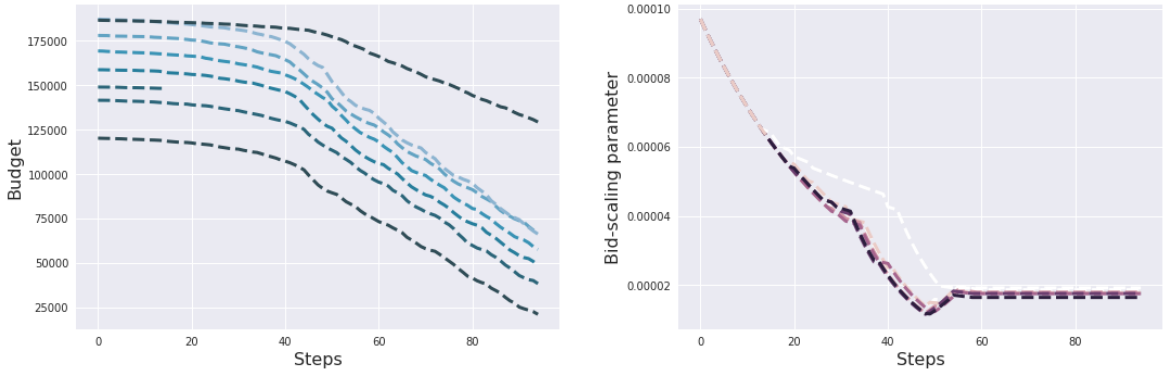


Figure 5.6: Campaign 2821

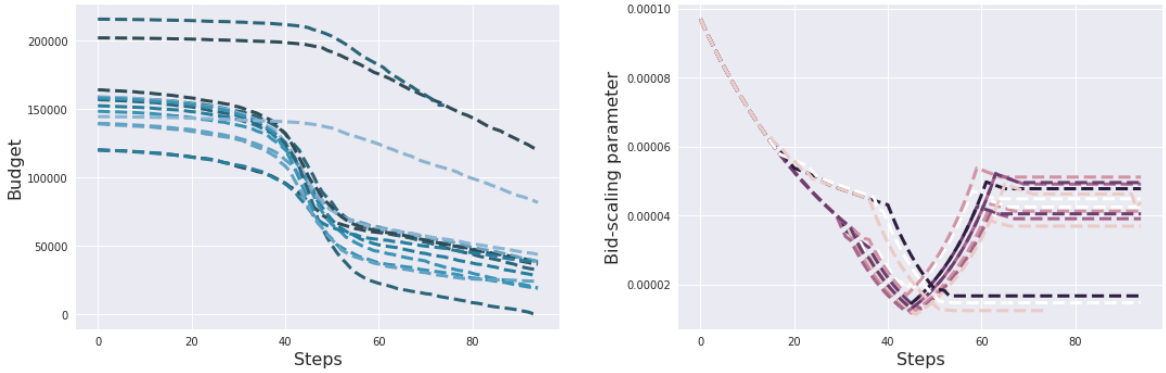


Figure 5.7: Campaign 2997

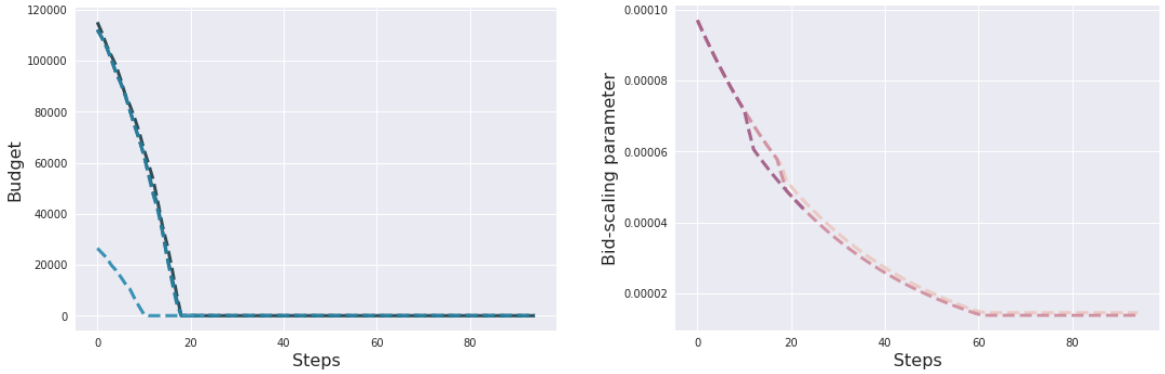


Figure 5.8: Campaign 3358

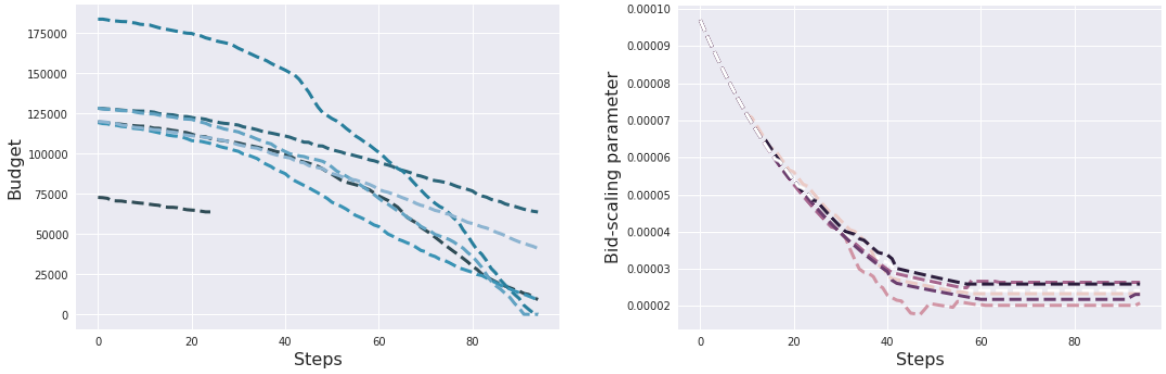


Figure 5.9: Campaign 3386

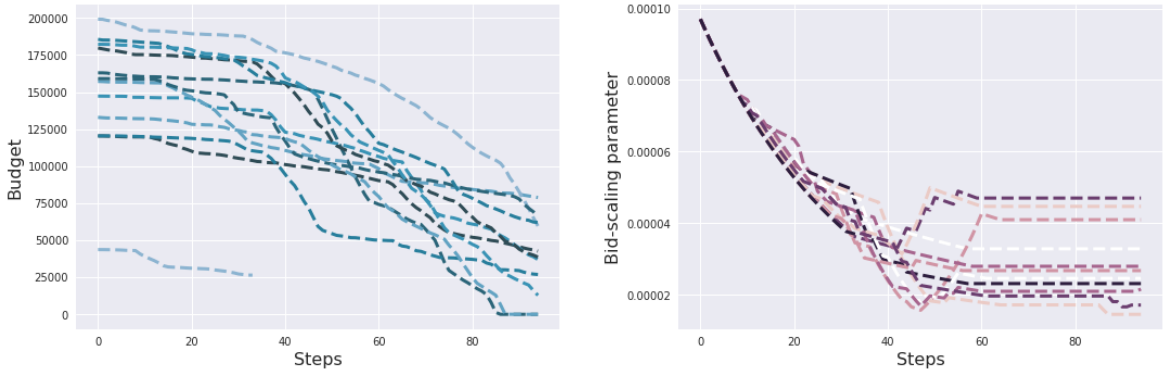


Figure 5.10: Campaign 3427

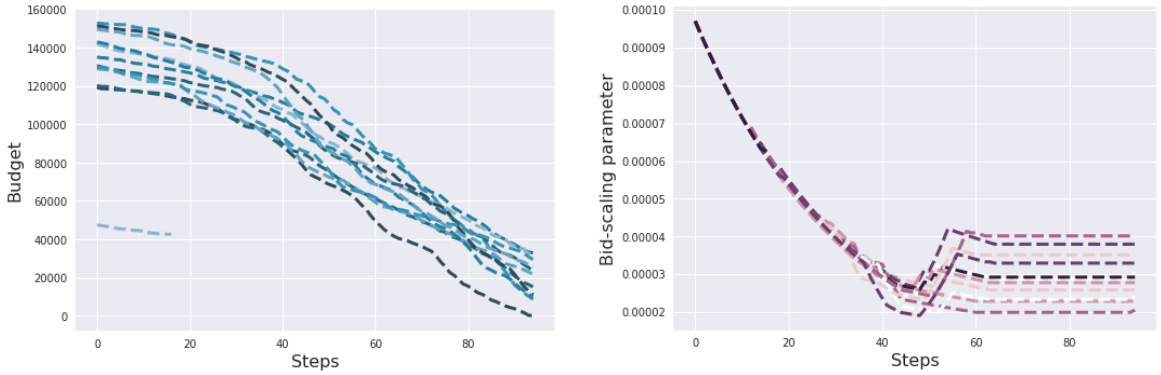
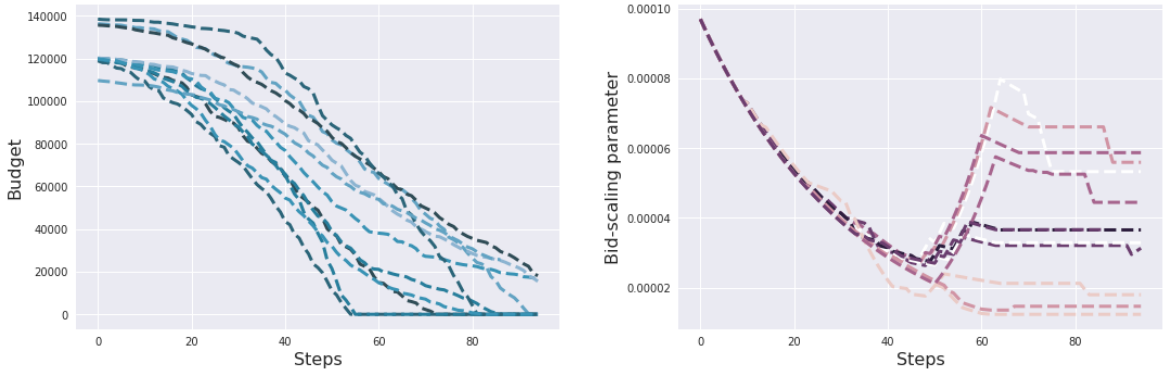


Figure 5.11: Campaign 3476



First of all, our `Batch-DRLB` is clearly superior to both `LinBid` and `RandBid`. While it's not ideal that it does not always consume the entire budget, with the notable exception of 82.8%, it still gets a lot more impressions than the other two algorithms. For example, in campaign 2821, `Batch-DRLB` gets 78763 impressions, spending 90.7% of the budget, while `LinBid` and `RandBid` get 44596 and 23679 impressions, respectively, while spending 100% of the budget. One important observation here is campaign 2997. In this case, `LinBid` actually gets the most impressions. If we look at figure 5.7, we see that the agent has learned a suboptimal behavior and is depleting the budget immediately. This is the type of behavior that both benchmark algorithms display in every campaign; they deplete the budget relatively fast and are then left incapacitated until the next campaign. This is exactly the type of "stupid" behavior that we want our agent to avoid. Looking at figures 5.3-5.11, it seems to do a pretty good job, especially as it seems to be increasing the bid-scaling parameter to bid less aggressively as the budget is being spent and as the episode is ending.

There is an interesting anomaly in the results. If we look at the budget spending for campaigns 3386, 3358 and 2261, we can see a clear cut-off point in the budget spending. This represents the final episode, which doesn't always have 96 time steps due to the amount of bids in the dataset. Considering the graph, we can see that this means that the agent is, in several cases, left with a large chunk of budget which it hasn't spent. I tried tweaking the algorithm to account for these cases. While it succeeded a couple of times, the results were a lot more volatile than for the "normal" `Batch-DRLB`. Ideally, an agent should be able to handle these cases as well. However, it seems that such adjustments only add to the complexity of training the agent and they will thus not be considered any further in this thesis. While tweaked agent did perform well at times, spending the entire budget and getting a fair amount of impressions, it was unstable and unreliable more often than not. I've included two examples of somewhat odd bid-regulation behavior in figure 5.12, which came as a result of a seemingly stable and intelligent agent being exposed to different random seeds for campaign 1458.

Looking at table 5.11, the `Batch-DRLB` algorithm actually compares very well to the state-of-the-art algorithms tested by Du et al. (2017). In campaign 1458 and campaign 3427, it actually manages to get the most clicks. These are great results. While we have to consider how the agent performed on the stability tests, especially the results in table 5.7, this does show the potential of an agent which tries to control the bid-scaling parameter in an intelligent way. Ultimately, the number of clicks won in the iPinYou dataset is a complicated indicator to use with respect to performance. As mentioned previously, there were several cases of agents getting

a very low number of impressions and a very high number of clicks at the same time. Such an agent would probably not get the same amount of clicks in some other RTB setting. Hence, I think the important comparison is how `Batch-DRLB` fares against `LinBid` and `RandBid` in terms of impressions won, especially since these are more representative of what is actually being employed in the industry today. In this comparison, with some reservation for the stability results for these particular parameters, I think it's safe to say that these results are very promising.

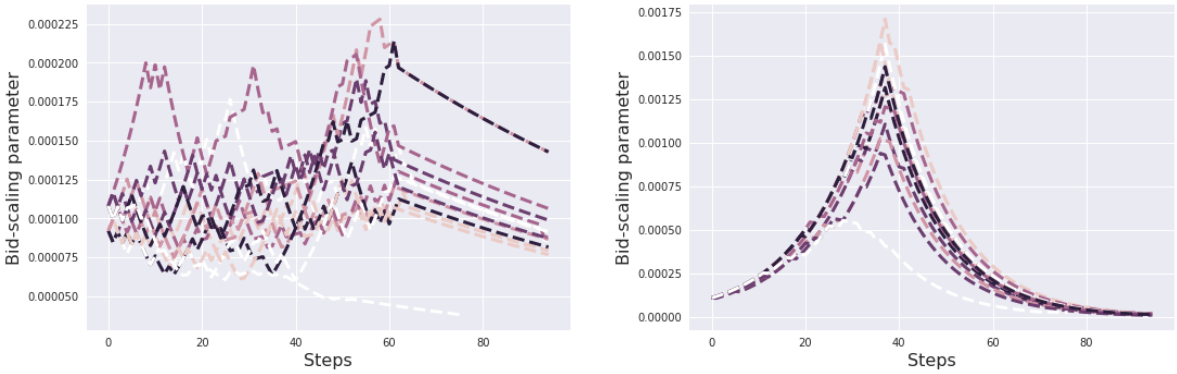


Figure 5.12: Strange bidding behavior for a "tweaked" agent bidding in campaign 1458

Chapter 6

Conclusion

This project leaves us with more questions than answers. While we initially thought that we had found a robust and superior agent in `Batch-DRLB-`α, this wasn't entirely true. The agent did perform extremely well, but it was not entirely robust with respect to different random outcomes. This is problematic in an RTB setting, as we're often dealing with nonstationary environments and hence need an agent that's able to adapt to changes in the environment. The λ_0 test also showed that the algorithm is more sensitive to different parameters than initially thought. There seems to be a strong interplay between all kinds of different input parameters. This is, of course, endemic to the field of machine learning. Successful training is often the result of experimentation rather than a complete analytical understanding of the entire system. In a more rigorous project, such experimentation would be very interesting since the algorithm shows great potential in being able to train agents with the ability to act intelligently in RTB markets.

There are also several possibilities that we haven't considered here. For example, it would be interesting to perform a more comprehensive test with more granular λ_0 values, possibly even initializing λ_0 randomly during training. We also haven't had a broader discussion about the agent's function approximator. It would definitely be interesting to experiment with different architectures and to see if there is potential for improvement in adding or removing layers and neurons. In conclusion, this is a complex problem with several intricate dependencies. Exploring it further is way beyond the scope of a Bachelor Thesis. However, the `Batch-DRLB` algorithm did extremely well against state-of-the-art algorithms, and it shows great promise in designing an even more efficient and intelligent agent.

References

Du, M, Sassioui, R., Varisteas, G., State, R., Brorsson, M., Cherkaoui, O. 2017. *Improving Real-Time Bidding Using a Constrained Markov Decision Process.* In: Cong G., Peng WC., Zhang W., Li C., Sun A. (eds) Advanced Data Mining and Applications. ADMA 2017. Lecture Notes in Computer Science, vol 10604. Springer, Cham. https://doi.org/10.1007/978-3-319-69179-4_50

Geibel P. 2006 *Reinforcement Learning for MDPs with Constraints.* In: Fürnkranz, J., Scheffer, T., Spiliopoulou, M. (eds) Machine Learning: ECML 2006. ECML 2006. Lecture Notes in Computer Science, vol 4212. Springer, Berlin, Heidelberg. https://doi.org/10.1007/11871842_63

Mnih, V., Kavukcuoglu, K., Silver, D., Graves, A., Antonoglou, I., Werstra, D,. Riedmiller, M. 2013. *Playing Atari with Deep Reinforcement Learning.* Google DeepMind: London, UK.

Mnih. V, Kavukcuoglu, K., Silver, D, Rusu, A., Veness, J, Bellemare, M., Graves, A., Riedmiller, M., Fidjeland, A., Ostrovski, G., Petersen, S., Beattie, C., Sadik, A., Antonoglou, I, King, H., Kumaran, D., Wierstra, D., Legg, S., Hassabis, D. 2015. *Human Level Control Through Deep Reinforcement Learning.* Google DeepMind: London, UK.

Sutton, R., Barto, A. 2018. *Reinforcement Learning - An Introduction.* The MIT Press: Cambridge, Massachusetts, USA.

Riedmiller, M. 2005. *Neural Fitted Q Iteration – First Experiences with a Data Efficient Neural Reinforcement Learning Method.* In: Gama J., Camacho R., Brazdil P.B., Jorge A.M., Torgo L. (eds) Machine Learning: ECML 2005. ECML 2005. Lecture Notes in Computer Science, vol 3720. Springer, Berlin, Heidelberg. https://doi.org/10.1007/11564096_32

Wu, D., Chen, X., Yang, X., Wang, H., Tan, Q., Zhang, X., Xu, J., Gai, K.

2018. *Budget Constrained Bidding by Model-free Reinforcement Learning in Display Advertising*. In: The 27th ACM International Conference on Information and Knowledge Management (CIKM 2018), October 22–26, 2018, Torino, Italy. ACM, New York, NY, USA, 9 pages. https://doi.org/10.1145/3269206.3271748

Yuan, Y., Wang, F., Li, J., Qin, R. 2014. *A Survey On Real-Time Bidding Advertising* In: 2014 IEEE International Conference on Service Operations and Logistics, and Informatics, October 8-10, 2014, Qingdao, China. IEEE, Piscataway, New Jersey, USA. https://doi.org/10.1109/ARES.2016.89

Zhang, W., Yuan, S., Wang, J. 2014. *Optimal Real-Time Bidding for Display Advertisement*. In: the 20th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, August 24-27, 2014, New York, USA. ACM, New York, NY, USA, 9 pages. https://doi.org/10.1145/2623330.2623633

Zhang, W., Yuan, S., Wang, J. 2015. *Real-Time Bidding Benchmarking with iPinYou Dataset*. UCL Technical Report 23 July, 2014. University College London, London, UK.