

Lab 5

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In [75]: from itertools import combinations
from math import comb
import sympy as sp
```

Question 1

3 vertices (corners) of a regular hexagon are randomly joined. What is the probability that an equilateral triangle is formed?

```
In [76]: vertices = [1,2,3,4,5,6]

all_triangles = list(combinations(vertices,3))

equilateral_triangles = [(1,3,5), (2,4,6)]

probability = len(equilateral_triangles) / len(all_triangles)

print(f"Probability of forming an equilateral triangle: {probability}")

Probability of forming an equilateral triangle: 0.1
```

Question 2

3 persons A, B, C independently fire at a target. What is the probability that (i) Exactly one of them hits the target, (ii) At least one of them hits the target? Given: Probability of hitting the target. P(A) = 1/6, P(B) = 1/4, P(C) = 1/3

```
In [77]: P_A = 1/6
P_B = 1/4
P_C = 1/3

P_A_miss = 1-P_A #5/6
P_B_miss = 1-P_B #3/4
P_C_miss = 1-P_C #2/3

P_one_hit =(P_A*P_B_miss*P_C_miss)+( P_A_miss*P_B*P_C_miss)+(P_A_miss * P_B_miss * P_C)
P_one_hit
print(f"Probability of exactly one person hitting the target: {P_one_hit:.4f}")

Probability of exactly one person hitting the target: 0.4306
```

Question 3

The probability that a teacher gives a surprise test is 0.55. If a student remains absent for two days. What is the probability that he misses exactly one test, and at most one test?

```
In [78]: P_T=0.55
P_T_miss = 1 -P_T #0.45

P_exactly_one = (P_T * P_T_miss)+(P_T_miss*P_T)
P_at_most_one = P_no_test + P_exactly_one
P_at_most_one
print(f"Probability of missing at most one test: {P_at_most_one:.4f}")

Probability of missing at most one test: 0.6975
```

Question 4

A box contains 2 defective pens and 3 working pens. Pens are tested one by one until both defective ones are discovered. What is the probability that the testing procedure comes to an end at the end of (i) 2nd testing, (ii) 3rd testing?

```
In [79]: P_end_2nd = (2/5)*(1/4)

P_DWD = (2/5)*(3/4)*(1/3)
P_WDD = (3/5)*(2/4)*(1/3)

P_end_3rd = P_DWD+P_WDD

print(f"Probability that testing ends at 2nd test: {P_end_2nd}")
print(f"Probability that testing ends at 3rd test: {P_end_3rd}")

Probability that testing ends at 2nd test: 0.1
Probability that testing ends at 3rd test: 0.2
```

Question 5

If there are 40 people in a room, what is the probability that everyone has different birthdays? Assume 365 possible birthdays in a year.

```
In [80]: n = 40
days_in_year = 365
probability = 1

for i in range(n):
    probability *= (days_in_year-i)/days_in_year

print(f"Probability that all 40 people have different birthdays: {probability}")

Probability that all 40 people have different birthdays: 0.10876819018205097
```

Question 6

An amoeba has a 25%, 25%, and 50% chance of producing 0, 1, or 2 offspring, respectively. Each of the amoeba's descendants also has the same probabilities. What is the probability that the amoeba's lineage dies out?

```
In [81]: q = sp.Symbol('q')

eq = 0.50*q**2 - 0.75*q + 0.25

solutions = sp.solve(eq, q)
print(f"Probability of extinction: {solutions[0]: .2f}")

Probability of extinction: 0.50
```

Question 7

The entries in a 2 x 2 matrix are integers that are independently chosen for each entry. The probability that the entry is odd is p. If the probability that the value of the determinant is even is 0.5, find p.

```
In [82]: p = sp.Symbol('p')
eq = p**4 - p**2 + 0.25

solutions = sp.solve(eq, p)

print(f"Value of p: {valid_p}")

Value of p: [0.707106781186548]
```

Question 8

A drunker takes either a step forward or backward. The probability that he takes a forward step is 0.4. Find the probability that at end of 11 steps he is 1 step away from the starting point?

```
In [83]: p_f = 0.4 #step forward
p_b = 0.6 #step backwards
n = 11
k=6
probability = comb(n, k) * (p_f**k) * (p_b**(n - k))

print(f"Probability that the drunkard is 1 step away after 11 steps: {probability}")
```

Probability that the drunkard is 1 step away after 11 steps: 0.14714929152000003

Question 9

A coin is twice as likely to land head as a tail in a series of independent tosses. Find the probability that 3rd head occurs on the 5th toss.

```
In [84]: p = 2/3
q = 1 - p
n = 5
r = 3
probability = comb(n-1,r-1) * (p**r) * (q**(n-r))
print(f"Probability that the 3rd head occurs on the 5th toss: {probability}")
```

Probability that the 3rd head occurs on the 5th toss: 0.19753086419753085

Question 10

An HIV test is 99% accurate (both ways). Only 0.3% of the population is HIV +. What is the probability that a random person is HIV + given that the person tests +?

```
In [85]: P_H = 0.003
P_Hc = 1 - P_H
P_T_given_H = 0.99
P_T_given_Hc = 0.01

P_T = (P_T_given_H*P_H) + (P_T_given_Hc*P_Hc)
P_H_given_T = (P_T_given_H*P_H) / P_T

print(f"Probability that a person has HIV given they tested positive: {P_H_given_T}")
```

Probability that a person has HIV given they tested positive: 0.22952086553323028

Question 11

Cards are dealt one by one from a pack of 52 well-shuffled cards. What is the probability that exactly ‘k’ cards are dealt before the 1st ace appears?

```
In [86]: P_N = 12/13
P_A = 1/13
k = 5
prob = (P_N**k) * P_A
prob
```

Out[86]: 0.05155207094376431

Question 12

All face cards are removed from a pack of 52 well-shuffled cards. From the remaining 40 cards, 4 cards are drawn randomly. What is the probability that 4 cards are from different suits and denominations?

```
In [87]: numerator = 10*9*8*7
denominator = comb(40,4)
probability = numerator/denominator
print(f"Probability that all 4 cards are from different suits and ranks: {probability}")
```

Probability that all 4 cards are from different suits and ranks: 0.05514826567458146

```
In [ ]:
```