$\boxed{1)}$ $f'(x) = \frac{1}{x+1}$ $\quad f'(0) = \frac{1}{1} = 1$

$f''(x) = -\frac{1}{(x+1)^2}$ $\quad f''(0) = -\frac{1}{(0+1)^2} = 1$

$f^3(x) = \frac{2}{(x+1)^3}$ $\quad f^3(0) = \frac{2}{(x+1)^3} = 2$

$0 + 1(x-0) - \frac{1(x-0)^2}{2!} + \frac{2(x-0)^3}{6}$

$x - \frac{x^2}{2} + \frac{x^3}{3}$

$\sum_{k=1}^{\infty} \frac{(-1)^k x^k}{k}$

$\lim_{x \to -1} \ln(x+1) = -\infty$

ln(x+1) tiene una asintota vertical en -1, por lo tanto el radio de convergencia es 1 cuando la serie esta centrada en 0

$\boxed{2)}$ $f(x) = e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$g(x) = e^x + 2e^{-x} = \sum_{n=0}^{\infty} \frac{x^n + 2(-x)^n}{n!}$$

$\boxed{3)}$ $\frac{1}{x}$ $a=3$ $\qquad f(3) = \frac{1}{3}$

$f'(x) = \frac{-1}{x^2} \qquad f'(3) = \frac{-1}{3^2} = \frac{-1}{9}$

$f''(x) = \frac{2}{x^3} \qquad f''(3) = \frac{2}{3^3} = \frac{2}{27}$

$$\frac{1}{3} - \frac{1}{9}(x-3) + \frac{2}{27}(x-3)^2$$

$$\sum_{n=0}^{\infty} \frac{-1^n (x-3)^n}{3^{n+1}}$$

$$\sum_{n=0}^{9} \frac{-1^n (x-3)^n}{3^{n+1}} = \frac{1}{3} - \frac{x-3}{9} + \frac{(x-3)^2}{27} - \frac{(x-3)^3}{81} + \frac{(x-3)^4}{243} - \frac{(x-3)^5}{729} + \frac{(x-3)^6}{2187} - \frac{(x-3)^7}{6561} + \frac{(x-3)^8}{19683} - \frac{(x-3)^9}{59049}$$

$= -0.197$

$|1 + 0.197| = 1.197$

$\boxed{4)}$ $f(x) = \cos 3x \qquad a = \pi$

$\cos(3\pi) = -1$

$f'(x) = -3\text{sen}(3x) \qquad f'(\pi) = 0$

$f''(x) = -9\cos(3x) \qquad f''(\pi) = 9$

$-1 + \frac{9}{2}(x-\pi)^2 - \frac{27}{8}(x-\pi)^4$

$$\sum_{n=1}^{\infty} \frac{3^n(-\pi+x)^n \cos(\frac{n\pi}{2})}{n!}$$

$\cos(3x)$ es continua y no tiene asintotas.
por lo tanto no tiene intervalo de convergencia

$(2x - 2\pi)$

$\boxed{5)}$ $f(x) = \sin \frac{1}{x} \qquad a = 1$

$f(1) = \sin(1)$

$f'(x) = -\frac{\cos(\frac{1}{x})}{x^2} \qquad f'(1) = -\cos(1)$

$f''(x) = \frac{2x\cos(\frac{1}{x}) - \sin(\frac{1}{x})}{x^4} \qquad f''(1) =$

$f'''(x) = \frac{(1-6x^2)\cos\frac{1}{x} + 6x\sin\frac{1}{x}}{x^6}$

$\boxed{6}$ $f(x) = \frac{1}{x^2-1}$

$f(0) = \frac{1}{-1}$

$f'(x) = \frac{-2x}{(x^2-1)^2}$ $\qquad f'(0) = \frac{0}{-1^2}$

$f''(x) = \frac{8x^2}{(x^2-1)^3} - \frac{2}{(x^2-1)^2}$ $\qquad f''(0) = -\frac{2}{-1}$

$f^3(x) = \frac{24x}{(x^2-1)^3} - \frac{48x^3}{(x^2-1)^4}$ $\qquad f^3(0) = 0$

$f^4(x) = -\frac{288x^2}{(x^2-1)^4} + \frac{24}{(x^2-1)^3} + \frac{384x^4}{(x^2-1)^5}$ $\qquad f^4(0) = \frac{24}{-1}$

$-1 - \frac{2}{2}x^2 - \frac{24}{24}x^4 - \frac{720}{720}x^6$

$\sum_{n=0}^{\infty} \frac{1}{2} x^n \left(-1 + (-1)^{1+n}\right)$

$r = 1.1$

$IC = (-1.1, 1.1)$

7.

$f(x) = \sin(x)$ $\quad a=0$

$f(0) = 0$

$f'(x) = \cos(x)$ $\quad f'(0) = 1$

$f^2(x) = -sen(x)$ $\quad f^2(0) = 0$

$f^3(x) = -\cos(x)$ $\quad f^3(0) = -1$

$f^4(x) = \sin(x)$ $\quad f^4(0) = 0$

$$f(x) = 0 + 1(x-0) + \frac{0(x-0)^2}{2} + \frac{-1(x-0)^3}{6}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{1+2n}}{(1+2n)!}$$

$\boxed{8}$ $f(x) = x\cos x^3$ $\quad a=0$

$f(0)=0$

$f'(x) = \cos(x^3) - 3x^3\sin(x^3) \quad f'(0) = 1$

$f^2(x) = -3(3x^5\cos(x^3) + 4x^2\sin(x^3)) \quad f^2(0)=0$

$f^3(x) = 3x(9x^6-8)\sin(x^3) - 81x^4\cos(x^3) \quad f^3(0)=0$

$f^4(x) = 24(18x^6-1)\sin(x^3) + 9(9x^6-44)x^3\cos(x^3) \quad f^4(0)=0$

$f^5(x) = \quad f^5(0)=0$

$f^6(x) = \quad f^6(0)=0$

$f^7(x) = 27x^2(81x^{12}-6300x^6-3640)\sin(x^3) \quad f^7(0) = -2520$

$- 63(567x^{12} - 4140x^6 + 40)\cos(x^3)$

$$f(x) = \frac{1(x-0)}{} - \frac{2520(x)^7}{7!} = x - \frac{2520x^7}{5040} = x - \frac{x^7}{2}$$

$$= x\sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{2k}}{(2n)!}$$

$$\int_0^1 x\cos x^3\,dx \approx 0.440408$$

$$\int_0^1 x - \frac{x^7}{2}\,dx = 0.4375 \qquad E = 0.002908$$

$$\int_0^1 x - \frac{x^7}{2} + \frac{x^{13}}{24}\,dx = \frac{37}{84} \approx 0.44048 \qquad E = 7.2e-5$$

Dos terminos furon necesarios.