EXAMEN 2

PRINCIPIOS DE OPTIMIZACIÓN

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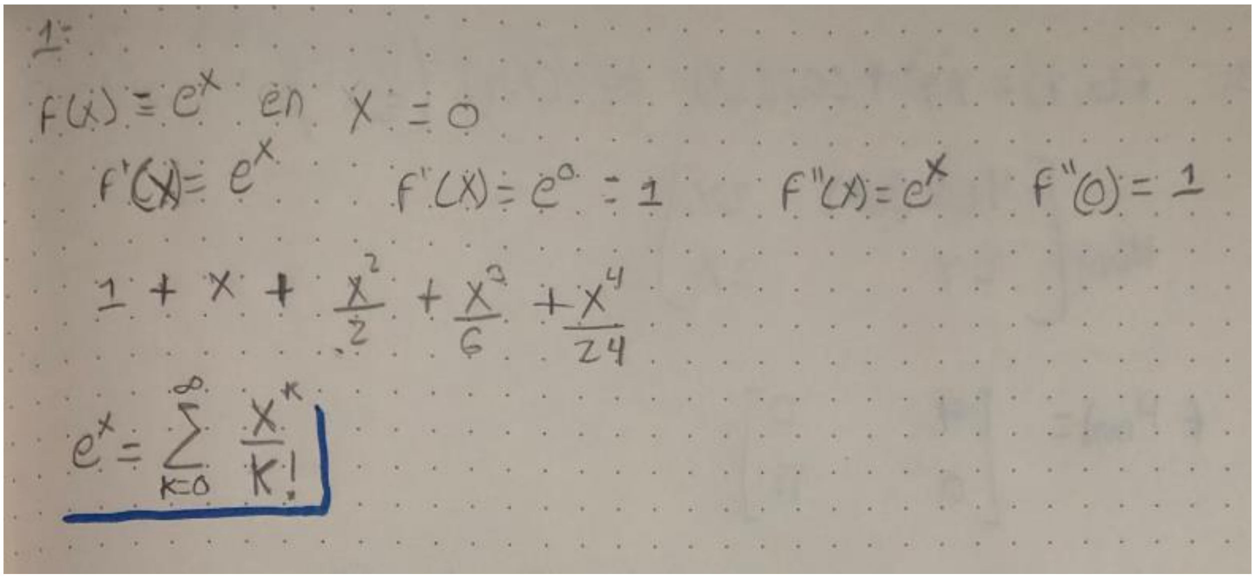
EJERCICIO 1:

Calcular el polinomio de Taylor de grado dos para la función $f(x) = e^x$ en $x = 0$.



1:

$f(x) = e^x$ en $x = 0$

$f'(x) = e^x \quad f'(x) = e^0 = 1 \quad f''(x) = e^x \quad f''(0) = 1$

$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

$$\boxed{e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}}$$

EJERCICIO 2:

Hallar los valores extremos de $f(x,y) = 2x + 5y$ sobre la elipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$.

2:

$f(x,y) = 2x + 5y \qquad g(x,y) = \frac{x^2}{4} + \frac{y^2}{3} = 1$

$\frac{\partial f}{\partial x} = 2 \qquad \frac{\partial g}{\partial x} = \frac{2}{4}x = \frac{1}{2}x$

$\frac{\partial f}{\partial y} = 5 \qquad \frac{\partial g}{\partial y} = \frac{2}{3}y = \frac{2}{3}y$

$2 = \frac{1}{2}x\lambda \quad \Big| \quad 5 = \frac{2}{3}y\lambda \quad \Big| \quad \frac{x^2}{4} + \frac{y^2}{3} - 1 = 0$

$\frac{4}{x} = \lambda, \ \frac{4}{\lambda} = x \quad \Big| \quad \frac{15}{2y} = \lambda, \ \frac{15}{2\lambda} = y$

$\frac{\left(\frac{4}{\lambda}\right)^2}{4} + \frac{\left(\frac{15}{2\lambda}\right)^2}{3} - 1 = 0$

$\frac{16}{\lambda^2 4} + \frac{225}{\lambda^2 12} - 1 = 0$

$\frac{1}{\lambda^2}\left(\frac{16}{4} + \frac{225}{12}\right) = 1$

$\lambda^2 = \frac{91}{4}$

$\lambda = \sqrt{\frac{91}{4}}$

$\frac{4}{\sqrt{\frac{91}{4}}} = x \qquad \frac{15}{2\sqrt{\frac{91}{4}}} = y$

$\pm 0.838 \approx x \qquad \pm 1.57 \approx y$

$F\left(+\frac{4}{\sqrt{\frac{91}{4}}}, +\frac{15}{2\sqrt{\frac{91}{4}}}\right) \approx 11.53 \to$ Máximo

$F(-x, +y) \approx 4.19$

$F(+x, -y) \approx -4.19$

$F(-x, -y) \approx -11.53 \to$ Mimimo

EJERCICIO 3:

Calcular los valores propios de la matriz Hessiana correspondiente a $f(x,y) = xy^2 + \cos(2x)$ en $(x,y) = (\frac{\pi}{2}, 0)$.

3.- $f(x,y) = xy^2 + \cos(2x)$ en $(x,y) = \left(\frac{\pi}{2}, 0\right)$

$$H(x,y) \begin{bmatrix} -4\cos(2x) & 2y \\ 2y & 2x \end{bmatrix}$$

$$f\ H(x,y) = \begin{bmatrix} 4 & 0 \\ 0 & \pi \end{bmatrix} \Rightarrow \begin{pmatrix} 4 & 0 \\ 0 & \pi \end{pmatrix} v = \lambda v \Rightarrow \begin{pmatrix} 4 & 0 \\ 0 & \pi \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

los valores propios son $\lambda_1 = 4$, $\lambda_2 = \pi$