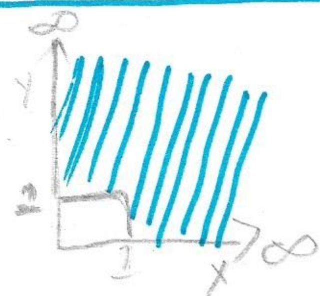
1.- a) $P(X>1,\ Y>1) = \int_1^\infty \int_1^\infty \frac{1}{8} x e^{-(x+y)/2}\, dy\, dx$



$\frac{x}{8}\int_1^\infty e^{-\frac{x}{2}-\frac{y}{2}}\, dy \qquad u=-\frac{x}{2}-\frac{y}{2},\ du=-\frac{1}{2}dy$

$-\infty,\ -\frac{x}{2}-\frac{1}{2}$

$= -\frac{x}{4}\int_{-\frac{x}{2}-\frac{1}{2}}^{-\infty} e^u\, du$

$= -\frac{x}{4} e^u \Big|_{-\frac{x}{2}-\frac{1}{2}}^{-\infty} = -\frac{x}{4} e^{-\infty} - \left(-\frac{x}{4} e^{-x/2-1/2}\right)$

$= \frac{x}{4} e^{-x/2-1/2}$

$\int_1^\infty \frac{x}{4} e^{-x/2-1/2}\, dx \qquad u=-\frac{x}{2}-\frac{1}{2},\ du=-\frac{1}{2}dy$

$\frac{1}{2}\int_{-\infty}^{-1} -2e^u\left(u+\frac{1}{2}\right) du = -\int_{-\infty}^{1} e^u u+\frac{e^u}{2}\, du = -\int_{-\infty}^{-1} e^u\, du - \frac{1}{2}\int_{-\infty}^{-1} e^u\, du$

$= \left(-e^u u\right)\Big|_{-\infty}^{1} + \frac{1}{2}\int_{-\infty}^{-1} e^u\, du$

$= \frac{1}{e} - \left(\lim_{b\to-\infty} -e^b b\right) + \frac{1}{2}\int_{-\infty}^{-1} e^u\, du$

$= \frac{1}{2} + \frac{1}{2}\int_{-\infty}^{-1} e^u\, du$

$= \frac{1}{e} + \frac{e^u}{2}\Big|_{-\infty}^{-1}$

$\frac{e^u}{2}\Big|_{-\infty}^{-1} = \frac{1}{2e} - \frac{e^{-\infty}}{2} = \frac{1}{2e}$

$= \frac{1}{e} + \frac{1}{2e} = \boxed{\frac{3}{2e} \approx 0.551}$

1.b $P(y>2) = \int_2^{\infty} f_Y(y)\,dy$

$$f_Y(y) = \int_0^{\infty} \frac{1}{8} x e^{-(x+y)/2}\,dx$$

$$= \frac{e^{-y/2}}{2}$$

$$\int_2^{\infty} \frac{e^{-y/2}}{2}\,dy = \frac{1}{2}\int_2^{\infty} e^{-y/2}\,dy = \frac{1}{2} - e^{-y/2}\Big|_2^{\infty}$$

$$= 0 - \frac{1}{2} - e^{-2/2}$$

$$= \frac{1}{e} \approx 0.367$$

2.a) $f_{XY}(0,1) = \frac{2}{9}$

Suponiendo X son filas y Y son columnas

Es la probabilidad que la empresa B tenga un contrato y la empresa A cero contratos

2b)

$f_X(x)$	$f_Y(y)$
$f_X(0) = \frac{4}{9}$	$f_Y(0) = \frac{4}{9}$
$f_X(1) = \frac{4}{9}$	$f_Y(1) = \frac{4}{9}$
$f_X(2) = \frac{1}{9}$	$f_Y(2) = \frac{1}{9}$

2 c) $f_{Y|X}(0|2) = \frac{1/9}{1/9}$

$f_{Y|X}(1|2) = \frac{0}{1/9}$

$f_{Y|X}(2|2) = \frac{0}{1/9}$

2 d) $f_{X|Y}(0|1) = \frac{2/9}{4/9}$

$f_{X|Y}(1|1) = \frac{2/9}{4/9}$

$f_{X|Y}(2|1) = \frac{0}{4/9}$

2e) $f_{Y|X}(2|1) = 0$

2 f) $f_{XY}(0,0) = \frac{1}{9}$

$f_X(0) = \frac{4}{9}$

$f_Y(0) = \frac{4}{9}$

$\left(\frac{4}{9}\right)\left(\frac{4}{9}\right) = \frac{16}{81}$

$\frac{1}{9} \neq \frac{16}{81}$

no son independientes porque para toda X y Y no se cum

$f_{XY}(x,y) = f_X(x)\, f_Y(y)$

3 c) $F(x,y) = \int_0^x \int_0^y 2xe^{-y}\,dy\,dx$

$= \int_0^x 2x \int_0^y e^{-y}\,dy\,dx = \int_0^x 2x - e^{-y}\Big|_0^y dx$

$= \int_0^x 2x - e^{-y}\,dx = \frac{2x^2}{2} - e^{-y}\Big|_0^x$

$= -x^2e^{-y}$

$$\boxed{F(x,y) = \begin{cases} -x^2e^{-y} & 0<x<1,\ 0<y<\infty \\ 0 & \text{otros} \end{cases}}$$

3 d)

$f_X(x) = \int_0^\infty 2xe^{-y}\,dy = 2x\int_0^\infty e^{-y}\,dy = 2x(-e^{-y})\Big|_0^\infty$

$= -(-2xe^0)$

$= 2x$

$f_Y(y) = \int_0^1 2xe^{-y}\,dx = e^{-y}\int_0^1 2x\,dx$

$= e^{-y}\frac{2x^2}{2}\Big|_0^1 = e^{-y}$

3 e) $f_{Y|X}(x) = \frac{2xe^{-y}}{2x}$ $\qquad f_{X|Y}(y) = \frac{2xe^{-y}}{e^{-y}}$

3 f) $f_X(x) f_Y(y) = 2x(e^{-y}) = 2xe^{-y}$

$$F_{XY}(x,y) = f_X(x) f_Y(y)$$

$2xe^{-y} = 2xe^{-y}$ son independientes

4 $$f(x;\theta) = \begin{cases} \frac{1}{\theta} r x^{r-1} e^{\frac{-x^r}{\theta}} & \theta > 0,\ x > 0 \\ 0 & \text{e.o.c} \end{cases}$$

r = constante conocida positiva
X = muestra aleatoria mayor a 0
θ = parametro mayor a 0

a) $$L(x;\theta) = \prod_{i=1}^{n} \frac{1}{\theta} r x_i^{r-1} e^{\frac{-x_i^r}{\theta}}$$

$$= \frac{1}{\theta} r x_1^{r-1} e^{\frac{-x_1^r}{\theta}} \cdot \frac{1}{\theta} r x_2^{r-1} e^{\frac{-x_2^r}{\theta}} \cdots \frac{1}{\theta} r x_n e^{\frac{-x_n^r}{\theta}}$$

$$= \frac{r^n}{\theta^n} \prod_{i=1}^{n} x_i^{r-1} e^{\frac{-1}{\theta}(x_1^r + x_2^r + \dots x_n^r)}$$

$$= \frac{r^n}{\theta^n} \prod_{i=1}^{n} x_i^{r-1} e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i^r}$$

$$\ln L(x;\theta) = \ln\left(\frac{r^n}{\theta^n} \prod_{i=1}^{n} x_i^{r-1} e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i^r}\right)$$

$$= \ln\left(\frac{r^n}{\theta^n}\right) + \ln\left(\prod_{i}^{n} x_i^{r-1}\right) + \ln\left(e^{-\frac{1}{\theta}\sum_{i}^{n} x_i^r}\right)$$

$$= \ln(r^n) - \ln(\theta^n) + \left(-\frac{1}{\theta}\sum_{i=1}^{n} X_i^r\right) + \ln\left(\prod_{i=1}^{n} X_i^{r-1}\right)$$

$$(n\ln(r) - n\ln(\theta)) - \frac{1}{\theta}\sum X_i^r + \ln\left(\prod_{i=1}^{n} X_i^{r-1}\right)$$

$$\frac{d\ln(L(X;\theta))}{d\theta} = -\frac{n}{\theta} + \frac{1}{\theta^2}\sum_{i=1}^{n} X_i^r$$

$$0 = -\frac{n}{\theta} + \frac{1}{\theta^2}\sum_{i=1}^{n} X_i^r$$

$$\frac{n}{\theta} = \frac{1}{\theta^2}\sum$$

$$\frac{\theta^2}{\theta} = \frac{1}{n}\sum_{i=1}^{n} X_i^r$$

$$\theta = \frac{1}{n}\sum_{i=1}^{n} X_i^r$$

b) $$\frac{\partial^2 \ln(L(x;\theta))}{\partial^2\theta} = \frac{\theta n - 2\sum_{i=1}^{n} X_i^r}{\theta^3}$$

$$= \frac{\frac{1}{n}\sum_{i=1}^{n} X_i^r n - 2\sum_{i=1}^{n} X_i^r}{\frac{1}{n}\sum X_i^r}$$

la segunda derivada es menor que cero

Por lo tanto es un máximo