

1) Proving Mutual Exclusion

To show that the baker's algorithm is a correct implementation of the N-thread critical section problem, we will consider the three requirements: Mutual Exclusion, Progress, and Bounded Waiting.

1. Mutual Exclusion

a. One process is in critical section, another process tries to enter: show that the second process will block in entry code

If P_k is in the critical section, it means it will have compared its $number[k]$ (or pid) with every other $number[j]$, so when P_i enters the entry code, and chooses a $number[i]$, it will be greater than $number[i]$.

If P_i already chose a $number[i]$, $number[k]$ will still be greater than, because otherwise P_k would not have entered the critical section (it would still be “spinning” on the while ($(number[i] < number[k]) \dots$)).

b. Two (or more) processes are in entry code: show that at most one will enter critical section
If, two (same argument for more than 2) processes P_k and P_i are in the entry code at the same time.

The worst case scenario would be that they both choose the same number for $choose[i]$ and $choose[k]$. This would be resolved in the final while() statement where the process with the smaller unique process id would enter the critical section, and the process with the greater process id would stay in the while loop, until the first set its $number[] = 0$.

2. Progress

a. No process in critical section, P_1 arrives: show that P_1 enters

If there are no processes in the critical section and process P_1 arrives, it will choose the next $number[1]$. It will then check this $number[1]$ against all other $number[n]$'s. At some point in time it will become the lowest number (as no processes are in the critical section, it is assumed this process will probably have the lowest), and will pass the for and while loops, and enter the critical section.

b. Two (or more) processes are in the entry code: show that at least one will enter critical section.

Let processes $P_1 \dots P_n$ enter the entry code, let us consider the first of those processes, P_k , that enters the entry code before all other processes, and there is no process in the critical section, it will receive the smallest $number[k]$ assignment, and will then be able to progress through the for and while loops to enter the critical section – because it will have the smallest $number[k]$. In the special case that another process P_i enters the entry code at the same time as P_k and has the same $number[]$ assignment, at least one (and actually only one) process will still be able to enter the critical section.

3. Bounded Waiting

a. One process in critical section, another process is waiting to enter: show that if first process exits the critical section and attempts to re-enter, the waiting process will get in first.

Assuming some process P_k is in the critical section, and another process P_i enters the entry code, assuming fair scheduling of processes, the process P_i will receive a $number[i]$, the max of all assigned $number[n]$'s. At anytime after this, if process P_k exits the critical section, and

enters the entry code, it will receive a number[k] higher than number[i]. This will guarantee that P_i will have an opportunity to enter the critical section.

Even if P_i and P_k were assigned the same number[] values, and P_k had a lower pid, P_k may beat P_i again, but eventually P_i will have an opportunity to enter the critical section, assuming the scheduler gives process P_i time to progress through the while and for loops.

2) Semaphores and Condition Variables

There is a problem with mutual exclusion, because there is a race condition. Consider the following scenario:

There are 2 utes waiting (`utes_waiting = 2`). Another ute arrives and calls `UteArrives()`, he gets the mutex, and enters the first `if()` statement, sends the 2 ute signals, calls `RideTrax`, and releases the mutex, and is context switched.

Now, two different processes are swapped in one after another that call `UteArrives()` and beat the previously waiting ute processes to the mutex. These new processes increment `utes_waiting` to 4 (each add one).

Now, before the waiting utes can context switch in and decrement, a third ute arrives, calls `UteArrives()` and beat the first two again, incrementing `utes_waiting` to 5, and waits.

Now the count is all messed up. It will just keep going up and up as utes arrive, until the game is over, no one else made it to the game, and they all go home crying.

Because even if at this point the original two utes were context switched in and got the mutex, and decremented their count, the `utes_waiting` would still be 3, which will never be caught in an `if` statement. A similar condition could occur with different permutations of `utes_waiting` and `cougars_waiting`, but the same basic problem.

One solution to this problem would be to do the decrementing in the successful `if` statement before letting the mutex go. For example, after each “`signal(utes);`” there was a “`utes_waiting--;`”.

In the above scenario, if the first successful `if()` statement, (where it tested true for `utes_waiting == 2`) would have decremented `utes_waiting` by two, before calling `V(mutex)`, it would have solved the problem (you would of course then NOT decrement it in the third `else` clause as it is currently implemented). This would solve that problem.

3) Deadlock (Book problem 7.2)

Deadlock occurs in the dinning philosopher's problem, because it fulfills each of the four necessary conditions of deadlock:

1. Mutual Exclusion. The resource (chopstick) cannot be shared, one chopstick can only be used by one philosopher at a time – in other words, two or more philosophers cannot share a chopstick – it would be pretty awkward, especially at the dinner table.
2. Hold and Wait. In the case where each philosopher grabs one chopstick to begin (let's say for this exercise they first grab the chopstick to the right), then to complete their task of eating they need another chopstick. Since all other chopsticks are being held by the other philosophers,

they must wait. In other words, each is holding a resource and waiting for another resource to be freed.

3. No Preemption. Once a philosopher gets his chopstick he will not give it up – they are far too stubborn. In other words, once a resource (chopstick) is acquired, it will not be freed until the philosopher is done eating.
4. Circular wait. Because they are seated around a table, and each grabs one chopstick (to the right), they must each wait for the chopstick to their left. This creates a circular wait, because, say they are numbers P_1, P_2, \dots, P_N , and the chopsticks, C_1, C_2, \dots, C_N . When each grabs a chopstick, say P_1 grabs C_1 , P_2 C_2 , ... , and P_N C_N , then each must wait for the chopstick to the left – P_1 waits for C_N , P_2 for C_1 , ..., P_N for C_{N-1} . This is a circular wait – one must give up a fork for the wait to be broken.

Deadlocking in this situation could be avoided by eliminating any one of the conditions stated above, they are considered here:

1. Mutual Exclusion. If the resources (chopsticks) were not mutually exclusive – or if they chopsticks could be shared among the philosophers, the deadlock would not occur. In our example, each P_i would grab their corresponding C_i , and then instead of waiting for the chopstick to the left, simply shared that chopstick with the philosopher to the left, there would be no deadlock – theoretically they could all eat – although realistically it would be pretty funny to watch N philosophers try sharing chopsticks to eat.
2. Hold and Wait. Somewhat related to circular wait, if not all of the philosophers held a resource and was waiting for another to free up, the deadlock would not occur. As a solution, if atomically each philosopher could request both required chopsticks at once, they would not be holding a resource and waiting for another – they would have both needed chopsticks. So, in our example, if you tied two chopsticks together, and you had to grab a pack of two chopsticks to eat, it would solve the problem.
3. No Preemption. Although it may take a while to convince a philosopher to give up his chopstick before he was done eating, if it could be done, it would solve the problem. In other words, if the philosophers could be preempted to give up their resource (chopstick) so that the philosopher next to them could eat, it would end/prevent the deadlock. But good luck convincing them.
4. Circular wait. If somehow you could eliminate the circular wait, by allowing one philosopher to eat, the deadlock would end. You could do this by using the same method mentioned under “Hold and Wait” - making the allocation of two chopsticks atomic, so that when requesting resources they got all the resources required to eat. Allowing preemption or eliminating mutual exclusion would also kill the circular wait.

4) Deadlock-free synchronization (Book problem 7.14).

In this situation, the critical section of the farmers trip is the bridge. The bridge is the resource that should be mutually exclusive, so that only one farmer is on the bridge at any one time. One method to ensure this would be to establish a variable to indicate which direction the bridge was functioning at that time, let's call it `direction_var`, a counter for how many cars are on the bridge, let's call it

counter_var, and a mutex for the both those variables, called bridge_mutex.

Whenever a farmer wanted to use the bridge he encounters three situations:

1. direction_var is his direction. He would simply increase the counter, cross the bridge, and decrease the counter when he was done.
2. direction_var is not in his direction and counter_var = 0. He would change the direction_var, increase the counter_var, cross the bridge, and decrease the counter_var.
3. direction_var is not in his direction and counter_var > 0. He would wait until he arrives at situation 2 or 1.

The pseudo-code would like this:

```
bool EnterBridge(){
    P( bridge_mutex );
    if( direction_var == my_direction ){
        counter_var++;
        V( bridge_mutex );
        return false; // go cross bridge
    } else {
        if( counter == 0 ){
            direction_var = my_direction;
            counter_var++;
            V( bridge_mutex );
            return false; // go cross bridge
        } else {
            V( bridge_mutex );
            return true; // don't cross bridge yet.
        }
    }
}

void ExitBridge(){
    P( bridge_mutex );
    counter_var--;
    V( bridge_mutex );
}

main(){
    while( EnterBridge );
    CrossBridge();
    ExitBridge();
}
```

This solution does have the characteristic of starving one side or the other, because one a side gets the direction_var, it will be difficult to get it back.

5) Starvation-free synchronization (Book Problem 7.15)

To solve this problem we use the above solution with one modification, a `preferred_direction_var` on which we will test when we want to enter the bridge, and which will be switched when another counter, `counter_var_total`, which counts the total number of crossings, reaches a limit

Here is the pseudo-code (only `EnterBridge` changes):

```
bool EnterBridge(){
    P( bridge_mutex );
    if( direction_var == my_direction && preferred_direction_var == my_direction ){
        counter_var++;
        counter_var_total++;
        if( counter_var_total > limit ){
            preferred_direction_var = other_direction;
            counter_var_total = 0;
        }
        V( bridge_mutex );
        return false; // go cross bridge
    } else {
        if( counter == 0 ){
            direction_var = my_direction;
            counter_var++;
            V( bridge_mutex );
            return false; // go cross bridge
        } else {
            V( bridge_mutex );
            return true; // don't cross bridge yet.
        }
    }
}
```

This modification has the effect of stopping one side from entering once the total crossings has reached a specified limit. Then the other side will gain control and cross. Of course if either lets their `counter_var` go to zero (no more trucks crossing from that side) it can switch as well.