Daniel Perry
cs6210 – fall 2005
homework 1

3.
a. approximate the derivative of sin(x) at x=1, with the function: f'(x) = (f(x+h)-f(x))/h

Results:
The optimal value of h to minimize error was h=10^(-4).

b. same, but use the function: f'(x) = (f(x+h)-f(x-h))/2*h

Results:
The optimal value of h to minimize error was h=10^(-2)

Conclusions:

As the data indicates, the first approximation required a much smaller h value to minimize the error. Both approximations resulted a very large error at approximately the same h (10^(-7)). These results lead one to think that the h required for minimal error is affected more by the algorithm (aka the function approximating the derivative) than to the machine, while the explosion in error around 10^(-7) was probably due to machine imprecision.

Some other main differences in the data to note, are that, the second approximation, after reaching its optimal error value, slowly lost accuracy; while the first approximation lost accuracy at a much more rapid rate.

Data/Graph:

k	h	err1	err2
0	1.000000e+00	4.724758e-01	8.565357e-02
1	1.000000e-01	4.293787e-02	8.999109e-04
2	1.000000e-02	4.212141e-03	1.001358e-05
3	1.000000e-03	3.437996e-04	1.382828e-05
4	1.000000e-04	3.118515e-04	1.382828e-05
5	1.000000e-05	2.099991e-03	8.802414e-04
6	1.000000e-06	3.860474e-03	3.860474e-03
7	1.000000e-07	6.517906e-01	3.537674e-01
8	1.000000e-08	5.403023e-01	5.403023e-01
9	1.000000e-09	5.403023e-01	5.403023e-01
10	1.000000e-10	5.403023e-01	5.403023e-01
11	1.000000e-11	5.403023e-01	5.403023e-01
12	1.000000e-12	5.403023e-01	5.403023e-01
13	1.000000e-13	5.403023e-01	5.403023e-01
14	1.000000e-14	5.403023e-01	5.403023e-01
15	1.000000e-15	5.403023e-01	5.403023e-01
16	1.000000e-16	5.403023e-01	5.403023e-01

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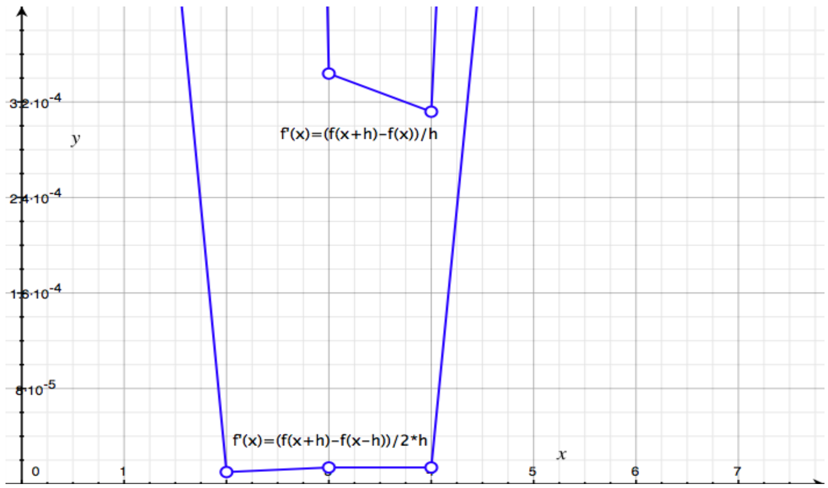

Illustration 1: graph of data from approximation of derivitive

4. Approximate the function e^(-x).
a) This approximation did well up to about x=2^3=8. But even at x=2^1 and x=2^2, it's error was much larger than the second algorithm, while at x=2^0=1, the error was about the same.

b) This approximation did well up to about x=2^6=64. The errors all stayed low. At x=2^7-128, the exp function returned 0. Comparing what the approximation returned with what Maple gives for e^(-128), the approxmation is way off – very eroneous. We can assume then, that this is a fairly good approxmiation for x<=64.

Conclusions:

The second approximation is obviously better. The difference in the errors of the first and second approximations are most likely due to the alternating sign in the first approxmation's sum. As examined in class, an alternating series has a very large relative error. This problem is a good illustration of when to use a constant sign series for an alternating sign series.

Data

x	result1	result2	actual (exp)
1.000000e+00	3.678794e-01	3.678794e-01	3.678795e-01
2.000000e+00	1.353353e-01	1.353353e-01	1.353353e-01
4.000000e+00	1.831546e-02	1.831564e-02	1.831564e-02
8.000000e+00	3.345282e-04	3.354625e-04	3.354626e-04
1.600000e+01	2.321823e-02	1.125351e-07	1.125352e-07
3.200000e+01	5.079496e+05	1.266416e-14	1.266417e-14
6.400000e+01	1.850097e+19	1.603810e-28	1.603811e-28
1.280000e+02	1.457706e+38	3.365409e-39	0.000000e+00
2.560000e+02	7.967982e+37	1.007739e-38	0.000000e+00
5.120000e+02	-1.475149e+37	6.244875e-38	0.000000e+00
1.024000e+03	2.352250e+38	4.104378e-39	0.000000e+00
2.048000e+03	-3.550415e+37	2.775615e-38	0.000000e+00
4.096000e+03	-1.462253e+37	6.795488e-38	0.000000e+00
8.192000e+03	1.904174e+38	5.236257e-39	0.000000e+00
1.638400e+04	3.838587e+35	2.601947e-36	0.000000e+00
3.276800e+04	-1.199961e+35	8.329027e-36	0.000000e+00
6.553600e+04	-6.144645e+37	1.626986e-38	0.000000e+00