Make Paper

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1 INTRODUCTION

woot-woot

2 FORMULA'S WITH π

The first formula for π I encountered used the Taylor expansion of the arctan function evaluated at $\frac{\pi}{4}$:

$$\pi = 4 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}. \tag{1}$$

Not long after that encounter, I came across the Basel series which is even more surprising:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}. \tag{2}$$

To me, the only *unsuprising* thing about the Basel series is that Euler figured it out (in the first half of the eighteenth century at that). Although proving (1) is relatively straightforward after some exposure to calculus, it wasn't until I took a course in complex variables that I learned the technology needed to give a proof of (2). Note that there are other ways to prove (2), this is just a demonstration of the way I first learned how to prove it.

If you let $C(z) = \pi \cot \pi z$ and put $f(z) = \frac{1}{z^2}$, then you can show that

$$\int_{\gamma_k} f(z)C(z)dz \to 0 \tag{3}$$

in the limit as $k \to \infty$, where $k \in \mathbb{N}$ and the curve γ_k is a square with vertices at $\pm(k + \frac{1}{2}) \pm i(k + \frac{1}{2})$. Applying the Residue Theorem then gives you (2).

3 WITTEN ON FEYNMAN'S $i\varepsilon$ PRESCRIPTION IN STRING THEORY

I wish I could say I understoond Ed Witten's *The Feynman $i\varepsilon$ in String Theory* [2], but alas string theory is beyond me. Perhaps if I find the time soonish I'll write up an analysis of how the $i\varepsilon$ arises in scalar quantum field theory[1]. It's been quite a time since I've done any QFT calculations, but the way I remember proceeding is starting from the two-point correlation function[1]

$$\langle \Omega | \, T\phi(x_1)\phi(x_2) \, | \Omega \rangle = \lim_{T \to \infty(1-i\varepsilon)} \frac{\int \mathcal{D}\phi \; \phi(x_1)\phi(x_2) \exp\left[i \int_{-T}^{T} d^4x \mathcal{L} \right]}{\int \mathcal{D}\phi \; \exp\left[i \int_{-T}^{T} d^4x \mathcal{L} \right]}. \tag{4}$$

Note that here $\mathcal{D}\phi$ is a measure for a functional integral. We can already see some hints as to where we'll pick up an $i\varepsilon$ in our propagator by looking at the limit for large time. Fair enough, but that doesn't explain why we do that…again maybe in the future I'll revist this some more. At some point you'll need to invoke

$$\int dx \exp\left[-kx^2\right] = \sqrt{\frac{\pi}{k}}, \tag{5}$$

which is pretty much the only integral you ever need to remember. This is known as a Gaussian integral and could probably get discussed in Section 2.

[1]The result is more general than the case of scalar quantum field theories, but the way I remember seeing it is through deriving the Feynman rules for scalar quantum field theory.

REFERENCES

[1] Michael E. Peskin and Daniel V. Schroeder. 1995. *An Introduction to quantum field theory.* Addison-Wesley, Reading, USA.

[2] Edward Witten. 2013. The Feynman i in String Theory. (2013). arXiv:hep-th/1307.5124