Punto 2 - Integración

Se tiene $\epsilon(x) = \frac{f''(\xi)}{2}(x-a)(x-b)$. De modo que el error asociado será:

$$E = \int_a^b \epsilon(x)\,dx = \frac{f''(\xi)}{2}\int_a^b (x^2 - xa - xb + ab)\,dx$$

$$= \frac{f''(\xi)}{2}\left[\frac{x^3}{3} - \frac{x^2}{2}a - \frac{x^2}{2}b + abx\right]_{x=a}^{x=b}$$

$$= \frac{f''(\xi)}{2}\left(\frac{b^3}{3} - \frac{b^2a}{2} - \frac{b^3}{2} + b^2a - \frac{a^3}{3} + \frac{a^2}{2} + \frac{a^2b}{2} - a^2b\right)$$

$$= \frac{f''(\xi)}{2}\left(-\frac{b^3}{6} + \frac{b^2a}{2} - \frac{a^2b}{2} + \frac{a^3}{6}\right)$$

$$= -\frac{f''(\xi)}{2}\left(\frac{-b^3 + 3b^2a + 3ba^2 + a^3}{6}\right)$$

$$= -\frac{f''(\xi)}{12}(b-a)^3 = -\frac{h^3}{12}f''(\xi)$$

—P