D M A

7.C

Dado que $\vec{\theta} \in \mathbb{R}^3$, se puede tomar $\frac{\partial \chi^2}{\partial \theta_j}(\vec{\theta})$ utilizando la regla de la cadena como se muestra a continuación:

i) $$\frac{\partial \chi^2}{\partial \theta_j} = \sum_{i=1}^{N} \frac{\partial \chi^2}{\partial \theta_j}\left[(y_i - M(x_i, \vec{\theta}))^2\right]$$

ii) Derivada Externa $\Rightarrow$ $$2\sum_{i=1}^{N}(y_i - M(x_i, \vec{\theta})) = A$$

Derivada Interna $\Rightarrow$ $$-\frac{\partial M(x_i, \vec{\theta})}{\partial \theta_j} = B$$

$$\frac{\partial \chi^2}{\partial \theta_j}(\vec{\theta}) = AB = -2\sum_{i=1}^{N}(y_i - M(x_i, \vec{\theta}))\frac{\partial M(x_i, \vec{\theta})}{\partial \theta_j} \quad \forall j \in [0, 2]$$

7.D

$$\nabla \chi^2 = \begin{bmatrix} \frac{\partial \chi^2}{\partial \theta_0} \\ \frac{\partial \chi^2}{\partial \theta_1} \\ \frac{\partial \chi^2}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} -2\sum_{i=1}^{N}(y_i - M(x_i, \vec{\theta})) & \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_0} \\ A & \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_1} \\ A & \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_2} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_0} \\ \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_1} \\ \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_2} \end{bmatrix}}_{= \nabla_\theta M(x_i, \vec{\theta})}$$

7.D

Luego $\vec{\theta}_{j+1} = \vec{\theta}_j - \gamma \nabla \chi^2(\vec{\theta}_j)$

$$= \vec{\theta}_j - \gamma \left(-2 \sum_{i=1}^{N} (y_i - M(x_i, \vec{\theta}_j)) \nabla_\theta M(x_i, \vec{\theta}_j) \right)$$