

Punto 1- Integración:

$$\text{Se tiene } f(x) \approx P_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b).$$

$$\text{De modo que } I = \int_a^b f(x) dx \approx \int_a^b P_1(x) dx = \frac{f(a)}{b-a} \int_a^b (b-x) dx + \frac{f(b)}{b-a} \int_a^b (x-a) dx = \dots$$

$$\dots = \frac{f(a)}{b-a} \left[bx - \frac{x^2}{2} \right]_{x=a}^{x=b} + \frac{f(b)}{b-a} \left[\frac{x^2}{2} - ax \right]_{x=a}^{x=b} = \dots$$

$$\dots = \frac{f(a)}{b-a} \left(b^2 - \frac{b^2}{2} - ab + \frac{a^2}{2} \right) + \frac{f(b)}{b-a} \left(\frac{b^2}{2} - ab - \frac{a^2}{2} + a^2 \right) = \dots$$

$$\dots = \left(\frac{b^2 - 2ab + a^2}{2} \right) \left(\frac{1}{b-a} \right) (f(a) + f(b)) = \dots$$

$$\dots = \frac{(b-a)^2}{2(b-a)} (f(a) + f(b)) = \frac{b-a}{2} (f(a) + f(b))$$

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