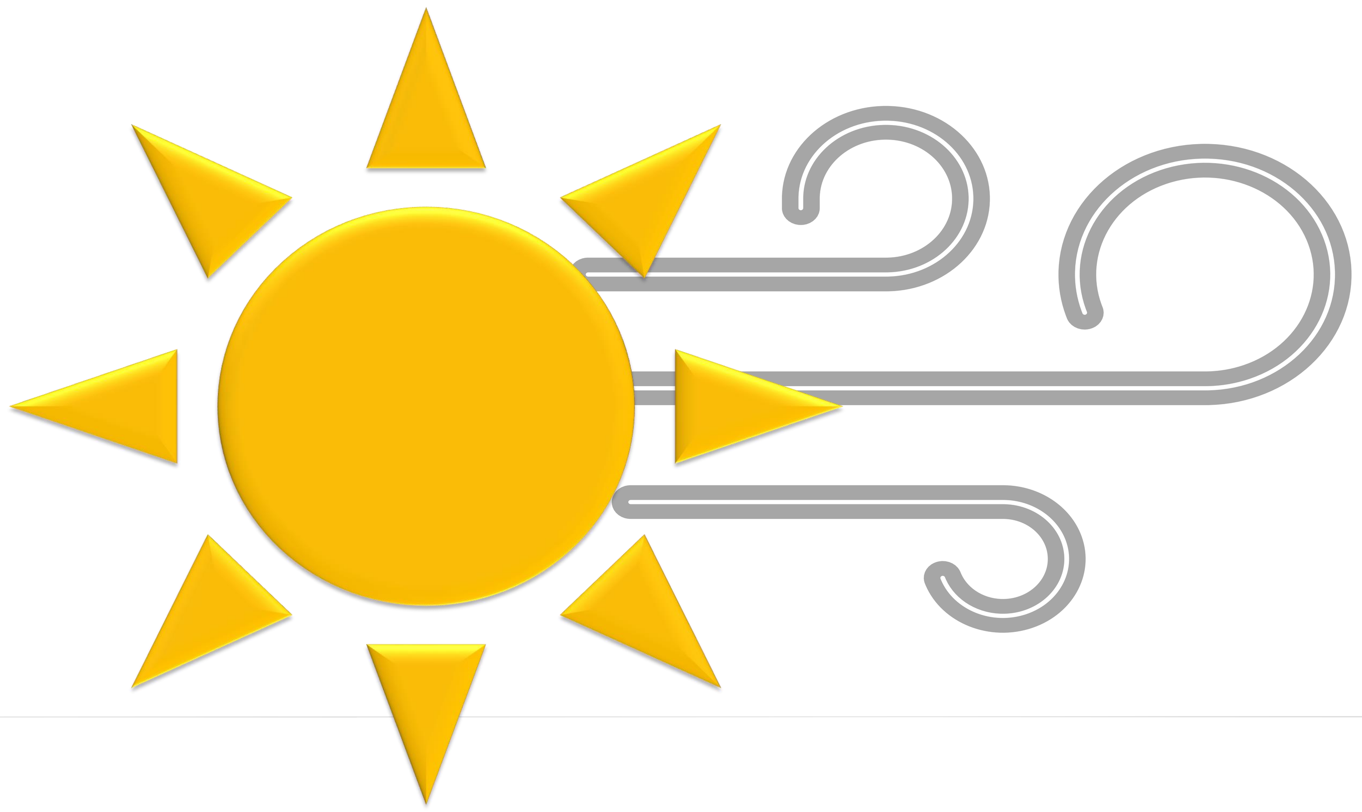


$Re_{SOLAR\ WIND} = ?$

A Taylor-made approach

Daniel Wrench *Victoria University of Wellington (VUW)*
Tulasi Nandan Parashar *VUW*
Sean Oughton *University of Waikato*
Kevin de Lange *VUW*

KEY POINTS

- The Reynolds number is an important quantity for describing a turbulent flow, but is not easily calculated for a collisionless fluid like the solar wind.
- Calculating an effective Reynolds number is subject to differing methodologies and formalisms, such as using the Taylor scale or ion inertial length.
- We compare three formulations using nearly 20 years of observations from Wind at 1 au and find considerable disagreement and variability.

Background

$$\frac{uL}{\nu} = \frac{\boldsymbol{u}\cdot\nabla\boldsymbol{u}}{\nu\nabla^2\boldsymbol{u}}$$

The **Reynolds number, Re,** is a fundamental dimensionless quantity used to describe a fluid system, and can be expressed in several different ways. In hydrodynamics, we typically see the version on the left, a function of characteristic velocity u, length scale L, and viscosity ν.

As shown by a re-formulation using terms from the Navier-Stakes equation, non-linear inertial forces dominate dissipative forces at large Re, promoting turbulence and its ensuing energy cascade.

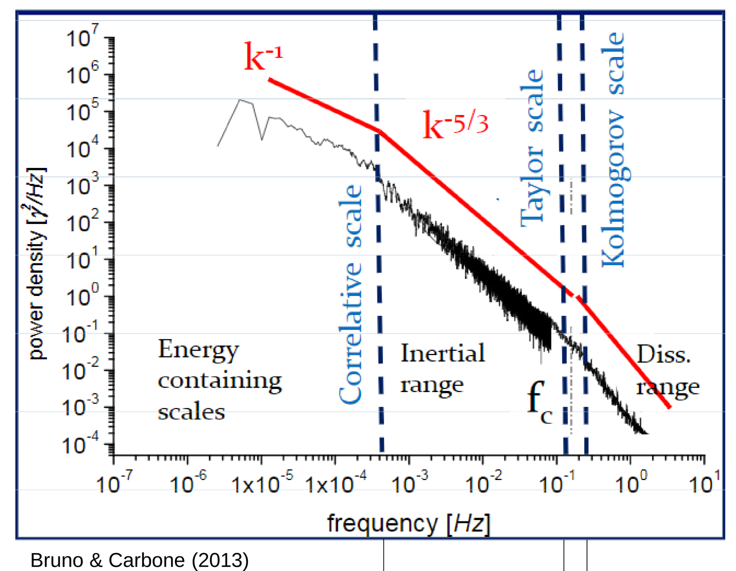
$$\left(\frac{\lambda_C}{\eta}\right)^{\frac{4}{3}}$$

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

Another useful interpretation of Re is as the bandwidth of the inertial range.

This becomes clear with the formulation on the left, as the ratio of the correlation scale λ_C (equivalent to L) and the Kolmogorov scale η.

In lieu of a viscosity, this version allows us to suggest reasonable proxies for the inner scale, in place of η.



Bruno & Carbone (2013)

λ_C λ_T η

$$\left(\frac{\lambda_C}{\lambda_T}\right)^2$$

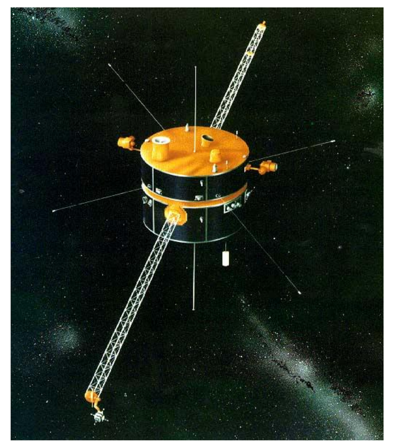
$$\lambda_T = \sqrt{\frac{\langle \boldsymbol{u}^2\rangle}{\langle(\nabla\times\boldsymbol{u})^2\rangle}}$$

Finally, Re can be expressed with reference to the Taylor scale λ_T. This quantity is associated with the mean square gradients of the velocity or magnetic field.

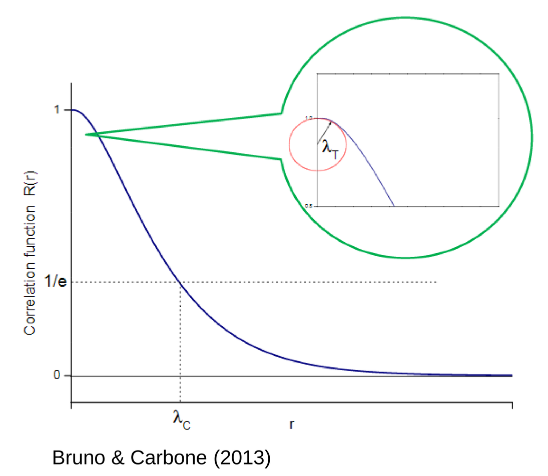
λ_T is typically used for calculating Re in the solar wind. It requires a high-resolution autocorrelation (see figure on right), as well as making certain numerical assumptions.

Data

In order to calculate 3 different versions of Re, power spectra and autocorrelation functions were computed from 12-hour intervals of magnetic field data from *Wind* at 1 au. These intervals covered the period 2004-2022.



Source: NASA



Bruno & Carbone (2013)

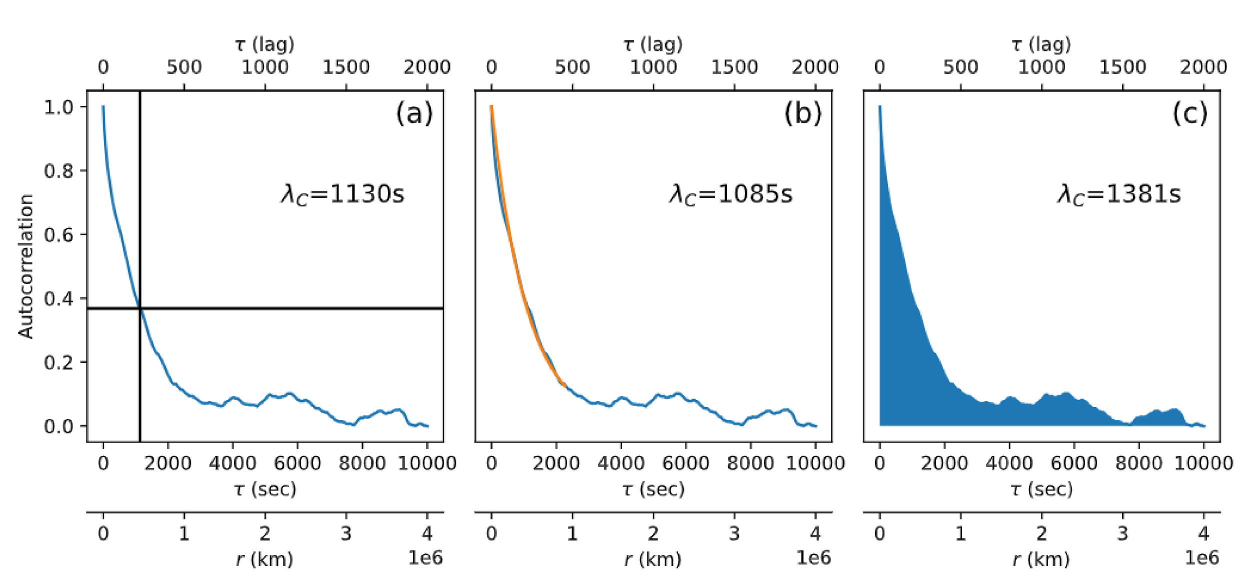
We used 11Hz data to estimate the Taylor scale and 0.2Hz data to estimate the correlation scale.

We invoked the Taylor hypothesis to convert to spatial lags.

Method

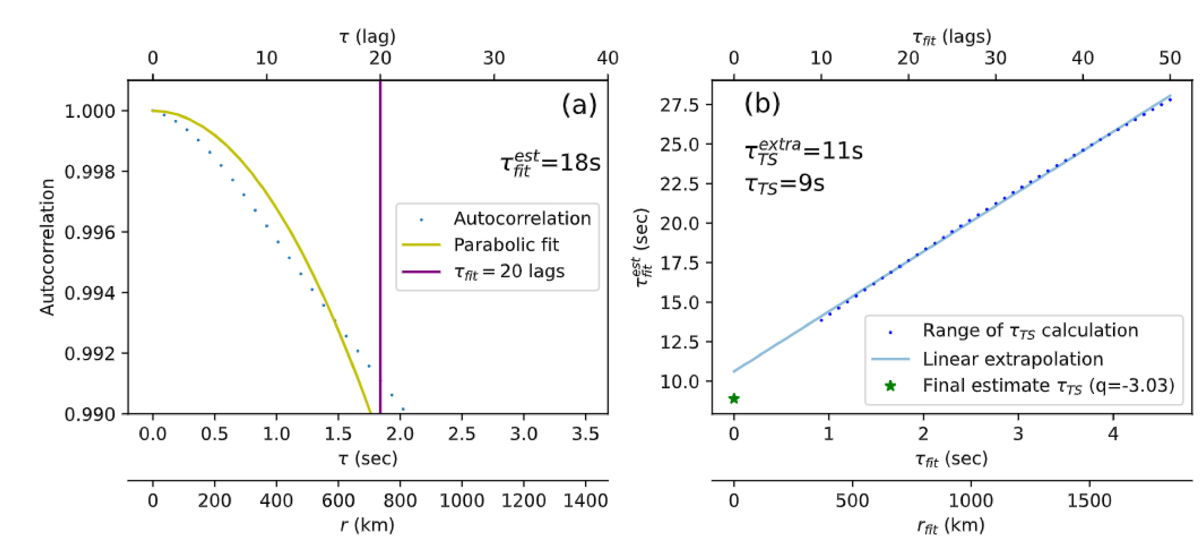
λ_C

We found good agreement between three methods of calculating the correlation scale: e-folding, exponential fit, and the integral of the correlation function. Average values are between 8.53-9.13 × 10^5 km, slightly smaller than previously-reported values of about 10^6. The exponential fit value is used for calculating Re.



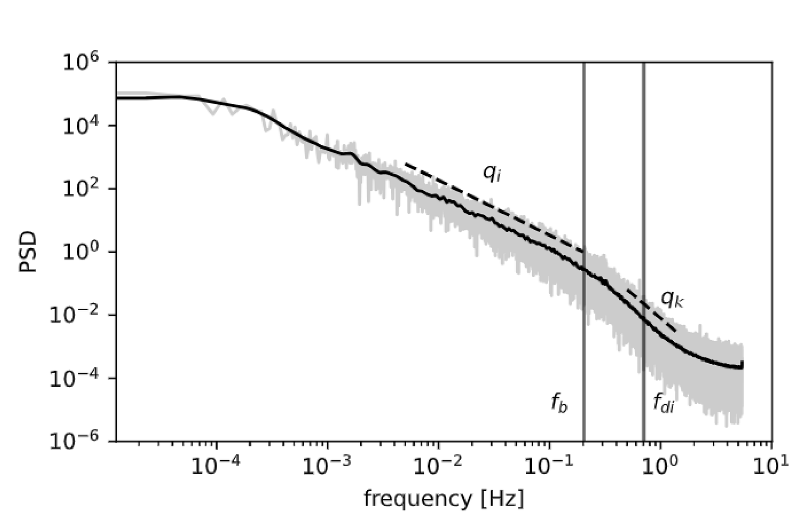
λ_T

The Richard extrapolation technique is the classic way of calculating the Taylor scale and is shown in the first panel on the right. We take the additional step of applying the correction factor developed by Chuychai et al. (2014), which accounts for the slope of the kinetic range. This resulted in a final mean of about 3000km, roughly in line with previous values in the literature.



η

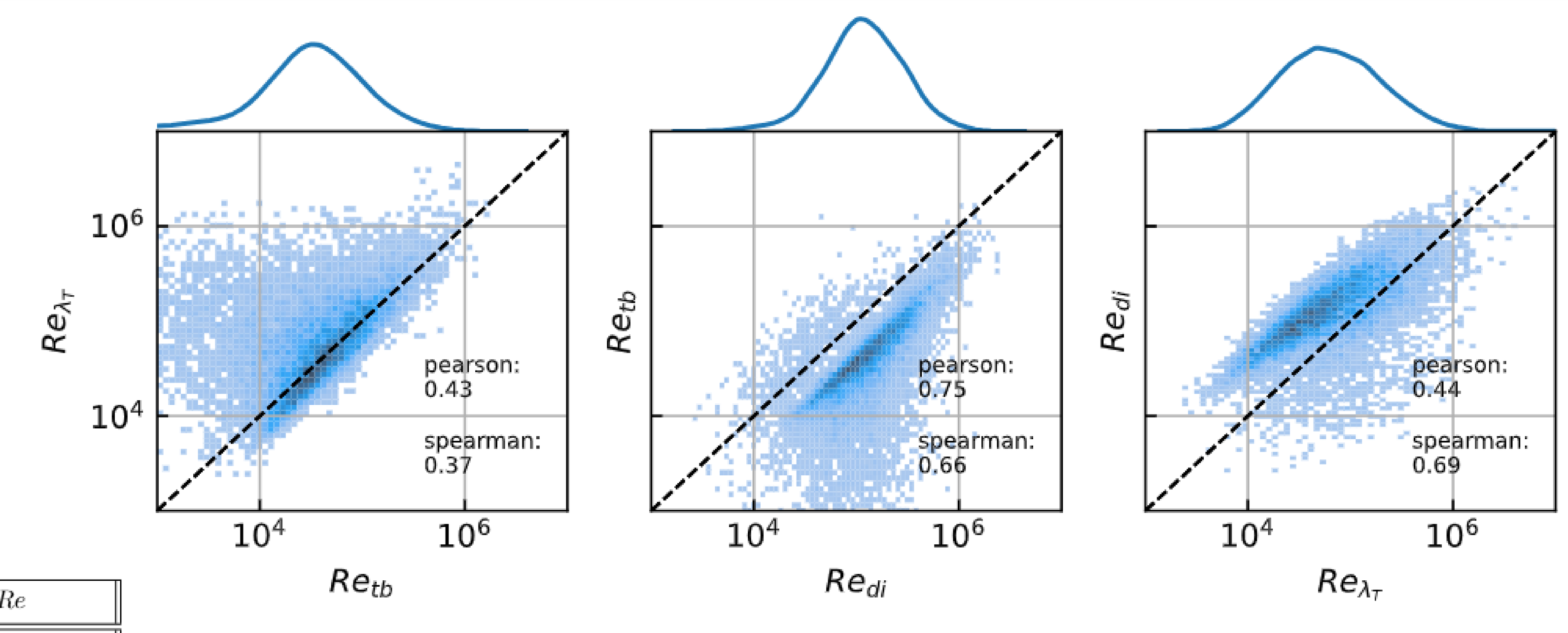
We used the ion inertial length d_i and the spectral break-scale t_b as estimates for the inner scale. d_i has been used in this way by Parashar et al. (2019) and Cuesta et al. (2022). t_b is also thought to be a reasonable estimate for the termination of the inertial range. This value was calculated as the intersection of power-laws fit to approximations of the inertial and kinetic ranges; see figure on the right.



Results

The 2D histograms on the right show significant disagreement between the methods. The largest linear correlation between any two methods is 0.75 between Re_{tb} and Re_{d_i}. Re_{λ_T} shows a much weaker linear association with the other two methods of only 0.44 and 0.43.

We also find a notable spread in Reynolds numbers within each estimation method, and clear log-normal distributions. This speaks to the strong variation in solar wind conditions from interval to interval.



Authors (Year)	Data	λ_C (km)	λ_T (km)	Re
Matthaeus et al. (2005)	Ace-Wind-Cluster	1,200,000	2,478 ± 702	230,000
Weygand et al. (2007)	Cluster	1,200,000 (from above)	2,400 ± 100	260,000 ± 20,000
Weygand et al. (2009)	Ace-Wind-Cluster + 6 others	2,000,000-5,600,000	700-1,400	12,600,000
Weygand et al. (2011)	Ace-Wind-Cluster + 8 others	1,000,000-2,800,000	1,200-3,500	4,000,000
Zhou et al. (2020)	Ace-Wind-Cluster	1,140,000	2,459	300,000*
Bandyopadhyay et al. (2020)	MMS	320,000	7,000	2,000**
Wrench et al. (2022)	**Wind**	**871,000**	**3,110**	**127,000**

Table 3. Previous estimates of λ_C, λ_T, and Re of the solar wind at 1 AU using the Taylor scale approximation, with the current results in bold. Here, $\lambda_C = \lambda_C^{\text{fit}}$, λ_T= Chuychai-corrected version, and $Re = Re_{\lambda_T}$. Note that some of these works grouped each scale and Re calculation by magnetic field orientation, hence why the scales are given as ranges, rather than a single number. Zhou & He (2021) and Weygand et al. (2009) calculated the ratio of Re in the field-parallel direction to the field-perpendicular direction. *This mean value reported in follow-up article Zhou & He (2021). **Re was not calculated explicitly in this article.

The table to the left shows previously-computed solar wind Re values, which span four orders of magnitude. All of these used the Taylor scale method, but Matthaeus et al. (2005) and Bandyopadhyay et al. (2020) did not use Richardson extrapolation. We are the first to use the Chuychai-corrected Taylor scale to compute the Reynolds number.

The mean value of Re_{λ_T} from the present study (127,000) is on the same order of magnitude as the results from three of these works. The ion inertial length method average is somewhat larger (174,000), whereas the spectral break method average is much lower (58,000).

Conclusion

The Reynolds number of the solar wind has been analyzed in the literature, but there is not yet a single agreed-upon method to calculate this important parameter. Our statistical comparison of three approaches revealed significant discrepancies. Typical values are on the order of 10^5 but are highly variable.

As these variations could affect our understanding of the solar wind system size and space weather predictions, we recommend further theoretical exploration to determine the most accurate method for collisionless fluids like the solar wind.

Until then, which estimator you choose depends on the resolution of your data and assumptions about hydrodynamic ordering and the role of ion-scale physics in dissipation and the termination of the inertial range.

Acknowledgements

We are grateful for financial support from the NSF and SHINE organizers, and the Department of Physics and Astronomy at the University of Delaware.

Data product

We will be sharing with the community a data product of 25 years of solar wind parameters from *Wind*, including the various scales, Re estimates, and sunspot number.

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