

Topological Offsets

DANIEL ZINT, New York University, USA

ZHOUYUAN CHEN, New York University, USA

YIFEI ZHU, New York University, USA

DENIS ZORIN, New York University, USA

TESEO SCHNEIDER, University of Victoria, Canada

DANIELE PANZZO, New York University, USA



Fig. 1. A collection of offsets computed from models in the Thingi10k dataset. Our algorithm provably computes manifold, watertight, and self-intersection-free offsets homeomorphic to an infinitesimally small offset (blue) and, with small changes, can also produce traditional offsets (green), and multiple non-intersecting layers (multicolor).

We introduce *Topological Offsets*, a novel approach to generate manifold and self-intersection-free offset surfaces that are topologically equivalent to an offset infinitesimally close to the surface. Our approach, by construction, creates a manifold, watertight, and self-intersection-free offset surface strictly enclosing the input, while doing a best effort to move it to a prescribed distance from the input. Differently from existing approaches, we embed the input in a background mesh and insert a topological offset around the input with purely combinatorial operations. The topological offset is then inflated/deflated to match the user-prescribed distance while enforcing that no intersections or non-manifold configurations are introduced.

We evaluate the effectiveness and robustness of our approach on the Thingi10k dataset, and show that topological offsets are beneficial in multiple graphics applications, including (1) converting non-manifold surfaces to manifold ones, (2) creating layered offsets, and (3) reliably computing finite offsets.

CCS Concepts: • Computing methodologies → Shape modeling.

Additional Key Words and Phrases: offsets, meshing

Authors' addresses: Daniel Zint, daniel.zint@nyu.edu, New York University, USA; Zhouyuan Chen, zc2952@nyu.edu, New York University, USA; Yifei Zhu, yz6994@nyu.edu, New York University, USA; Denis Zorin, dzorin@cs.nyu.edu, New York University, USA; Teseo Schneider, teseoch@uvic.ca, University of Victoria, Canada; Daniele Panuzzo, panozzo@nyu.edu, New York University, USA.

1 INTRODUCTION

Surface offsets, i.e., the regions at a fixed distance from the input surface, are a fundamental modeling tool in graphics and CAD. They are used for designing shapes, computing collision clearances for manufacturing, morphological operators, collision proxies, boundary layers, nested cages, and many more.

Despite their simple definition, their computation is still an unsolved challenge. While exact computation in 2D is possible, it is still an open problem in 3D. This led to a plethora of algorithms computing approximated versions of offsets: unfortunately, they all lose crucial properties such as lack of self-intersections, topological correctness, and geometrical precision and are often restricted to manifold and non-self-intersecting inputs (Section 2).

Robust and accurate algorithms exist for the special case of offsets with infinite radius, which have the topology of a sphere: shrink-wrapping algorithms deform that infinite offset until it tightly wraps a shape. This is useful for many applications, especially in 3D printing, as it provides a reliable way to inflate shapes with zero thickness or to topologically repair broken meshes. However, it uncontrollably loses the internal holes and handles.

We consider the reciprocal case: we introduce offsets with an infinitesimally small distance from the input and allow them to

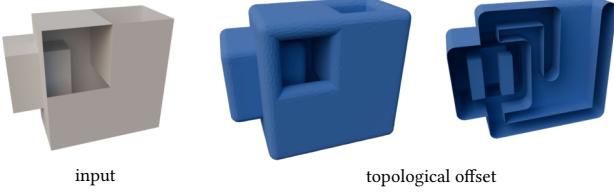


Fig. 2. Our method generates manifold meshes with topological guarantees even if the input (left) is open, non-manifold, non-orientable, and self-intersecting. The right image is a cut-view of the offset.



Fig. 3. Topological offsets (blue) always have the same topology for any given input (white), while the topology of finite offsets (green) depend on the offset distance

expand. We call the resulting surface a *topological offset* (Figure 1, blue).

We observe and prove that the topology of such an offset is unique and only depends on the input surface. This contrasts traditional offsets, which we will call *finite offsets* from now on (Figure 1, green), where the topology depends on the offset distance. Interestingly, these offsets are also uniquely defined for non-manifold, non-orientable, and self-intersecting meshes (Figure 2).

We formally define topological offsets (Section 4) and introduce a purely topological (and thus unconditionally robust) algorithm to compute it from a tetrahedral background mesh containing the input (Section 5). The geometry of the resulting offset is then optimized using a combination of local operations and interior point optimization, with a set of conservative topological and geometrical predicates (accounting for floating point rounding errors) to ensure that the resulting offset keeps the same topology and does not contain self-intersections.

Our construction is guaranteed to produce offsets with the following properties: (1) no intersections, (2) manifold, (3) same topology as an infinitesimal offset, and (4) strictly enclosing/containing the input (Figure 3).

Topological offsets enjoy wide practical applicability (Section 6): Non-manifold simplices can be removed by generating a topological offset around them and tagging all simplices in the offset region as outside (see inset). Self-intersections can be repaired by replacing a self-intersecting surface with its topological offset, which is always intersection-free. With a minor modification, topological offsets become finite ones while enjoying the same robustness, ensuring manifoldness and lack of self-intersections for any offset distance.

We compare our modified version that produces finite offsets against the state-of-the-art (finite) offset method [Zint et al. 2023]: while being robust, this approach produces intersecting offsets on about 5% of the models, hindering their usability in downstream applications. We also analyze our topological offsets on a large data set but we are not aware of any method that produces similar results that we could compare against.

Our major contributions are:

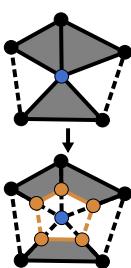
- (1) We formally define *topological offsets*.
- (2) We introduce a robust algorithm to compute them and control their geometry.
- (3) We show that they can be converted into *finite offsets*, while ensuring manifoldness and lack of intersection.
- (4) We produce layered offsets that strictly enclose each other by repeatedly adding topological or finite offsets.
- (5) We introduce (topological and finite) offsets with spatially varying distances.
- (6) We apply the topological offset to remove non-manifold edges and vertices while keeping the topological and geometrical changes minimal.
- (7) We provide an open-source reference implementation¹ to ensure the reproduction of our results and foster the adoption of this new offset type.

2 RELATED WORK

Voronoi Diagram. Generating an exact offset can be considered a sub-problem of generating a generalized Voronoi diagram. While this approach is feasible in 2D, with robust algorithms for computing 2D Voronoi diagrams [Karavelas 2022], the reliable generation of such a diagram in 3D is still an open problem [Boada et al. 2008; Yap et al. 2012]. Hemmer et al. compute the exact Voronoi diagram for arbitrary lines in 3D [Hemmer et al. 2010]. Aubry et al. use the generalized spherical Voronoi diagram around vertices (which is 2D and therefore easier to compute) to extract boundary layers [Aubry et al. 2017]. However, this method cannot handle self-intersections of the boundary layer, limiting its use to shapes where two non-adjacent elements are further away than twice the offset distance.

Discrete Offset and Morphological Operations. A volume can be discretized as a collection of voxels, and a discrete offset can be defined using morphological operations [Suriyababu et al. 2023], which are robust and efficient. However, using a grid (uniform or adaptive) introduces staircase artifacts and inherently limits feature size, as the memory and computation cost to store the data is high, even when Dexel data structures are used [Chen et al. 2019]. Other methods sidestep that issue by performing morphological operations on point clouds [Calderon and Boubekeur 2014]. However, the conversion into a point cloud induces a sampling error. [Sellán et al. 2020] proposes an approach for performing opening and closing operations on meshes without performing dilation and erosion explicitly. It does not generate offsets.

Approximate Distance Offsets. A popular compromise between *exact* and *discrete* offsets is the use of uniform grids, adaptive grids, or particles to discretize a distance field from the input surface [Liu and



¹<https://github.com/wildmeshing/topological-offsets>

Wang 2010; Meng et al. 2018; Pavić and Kobbelt 2008; Qu et al. 2004; Wang and Manocha 2013; Zint et al. 2023] and extract the offset as an isosurface [Ju et al. 2002; Lorensen and Cline 1987]. After the surface is extracted, a remeshing procedure is applied to improve mesh quality, reduce element count, and remove self-intersections [Botsch and Kobbelt 2004]. The advantage of these approaches is their efficiency in computation cost and memory, as they rely on decades of work on isosurface extraction to get the initial offset with high resolution, and they perform the expensive mesh optimization only on the resulting, possibly adaptive, surface. However, similarly to discrete methods, they cannot guarantee that the extracted offset is homeomorphic to the exact offset. Additionally, post-processing the extracted surface might introduce self-intersections [Zint et al. 2023]. Another approach is to construct linear approximations for each input triangle per cell and compute plane intersections [Wang et al. 2024]. This approximation is sufficient to produce sharp features in convex regions but the offset might intersect the input for small offset distances.

Our approach computes an offset guaranteed to be manifold, intersection-free, and with the unique topology of an infinitesimal offset. Furthermore, our method can also compute finite offsets and still enjoys the guarantee of producing a manifold and intersection-free output.

Approximate Minkowski Sums. An alternative approximation is to define the offset as the Minkowski sum of an input surface with a discretized sphere [Campen and Kobbelt 2010a,b; Martinez et al. 2015; Varadhan and Manocha 2004]. These methods create very dense meshes when using an accurate discretization of the sphere, making them a good fit only for applications where a coarse approximation of the offset is sufficient (see Figure 5 of [Wang et al. 2020] for an analysis of memory and time used by this type of algorithms).

Contrarily, the offset distance for topological offsets is adaptive while keeping a small distance error for finite offsets (Section 6).

Shrink-Wrapping. Shrink-wrapping algorithms [Huang et al. 2020; Juretić and Putz 2011; Kobbelt et al. 1999; Lee et al. 2009; Martineau et al. 2016; Suriyababu et al. 2023] shrink an infinite offset containing the full mesh until it tightly fits the mesh. This approach can be used to repair meshes reliably [Dai et al. 2024; Portaneri et al. 2022; Stuart et al. 2013] and it is a popular method to prepare models for 3D printing. The topology of the resulting surface, by construction, ignores internal holes: while this is a desirable property for mesh repair, the resulting surfaces are not offsets.

Our approach builds upon ideas in [Portaneri et al. 2022] to use a tetrahedral background mesh, but uses it to build an infinitesimal offset instead.

Enclosing Volumes and Boundary Layers. The closest works to ours are algorithms to construct enclosing volumes around an input, which are often used for animation cages, shell maps, or interface tracking in graphics [Brodersen et al. 2008; Jiang et al. 2020; Misztal and Bærentzen 2012; Porumbescu et al. 2005; Sacht et al. 2015], or boundary layers in engineering simulation [Garimella and Shephard 2000; Loseille and Löhner 2013]. These methods rely on a displacement in the normal direction (either directly, or via a geometric flow) which is not well-defined, in general (Figure 4).

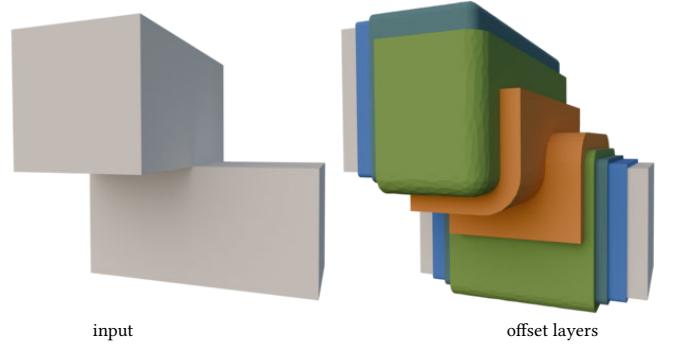


Fig. 4. The corner between the two boxes does not have a unique vertex normal, i.e., there is no direction in which the vertex can be offset without intersecting the input. Our method does not rely on vertex normals and can, therefore, handle this case properly. See [Jiang et al. 2020] for a more detailed discussion.



Fig. 5. Our method utilizes local subdivisions instead of a uniform refinement, leading to merely a fraction of triangles. Both, our method and [Guo et al. 2024], are guaranteed to construct an offset that is homeomorphic to a closed and manifold input mesh without self-intersections.

Instead of using normals, other approaches construct cages from volumetric representations of the input. [Calderon and Boubekeur 2017] rely on voxel grids, that are known to be fast and efficient, but they are not guaranteed to capture the input topology correctly, as input surfaces can be arbitrarily close. A robust method for constructing cages is presented in [Guo et al. 2024]. The method requires closed and manifold meshes without self-intersections as input, and constructs cages that are guaranteed to be homeomorphic to the input. Like our method, it computes the cage topology using a tetrahedral embedding of the input. However, the tetrahedral mesh is uniformly refined and eventually discarded and therefore no longer available for downstream applications. In contrast, our method keeps the tetrahedral mesh and ensures that it stays inversion-free. Additionally, we only perform local subdivisions instead of a uniform refinement, resulting in a much coarser initial mesh (Figure 5). Finally, our method can also handle open, non-manifold, and non-orientable input with self-intersections (Figure 2).

Multi-Material Remeshing. Our method processes a tetrahedral background mesh whose faces represent the input geometry: this is useful to avoid self-intersections in the offset without requiring explicit collision checks. Our algorithm uses a multi-material mesh

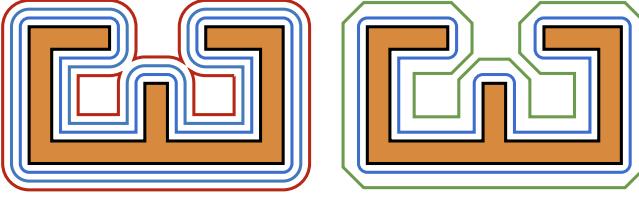


Fig. 6. An infinitesimal offset family (blue curves). If ϵ becomes too large, the offset changes topology (red curve). We are interested in a topological offset (green curve) that admits a continuous bijective map to the infinitesimal one.

optimization similar to [Faraj et al. 2016; Tournois et al. 2023], relying on the multi-material link condition proposed in [Thomas et al. 2011]. We provide more details on our remeshing algorithm in Section 5.3.

3 OVERVIEW

We introduce an algorithm for creating topological offsets. Unlike many other offset methods, we rely on a volumetric meshing algorithm to embed the input into a tetrahedral background mesh before processing: this increases our running time but provides strong guarantees on topology, lack of self-intersections, and termination. Our guarantees hold even when our algorithm is implemented using floating point arithmetic (Section 5.4).

Input. The input to our algorithm is a simplicial complex (e.g., a non-manifold triangular surface) embedded within a manifold tetrahedral background mesh embedded in \mathbb{R}^3 without inverted elements, plus an offset distance δ . The manifoldness condition is only required on the background mesh, not on the input. Additionally, we require that the simplicial complex is in the interior of the tetrahedral background mesh, i.e., there is a layer of tetrahedra completely enclosing the input.

Output. The output of our algorithm is a tetrahedral mesh with the offset (and input) embedded. The embedded triangle mesh of the offset is guaranteed to be manifold, intersection-free, and watertight. Our algorithm strives to improve this mesh's quality and keep a distance δ from the input (Section 6). We note that the tetrahedral mesh is useful in many downstream applications and is easy to discard if unnecessary.

Summary. We first introduce the theoretical concept of topological offsets and prove that their topology is unique for a sufficiently small ϵ (Section 4). This proof requires us to define a locality condition that strongly relies on a specific type of mesh embedding that we call simplicial embedding (Definition 2). We then propose an algorithm to compute them and optionally improve their quality (Section 5). Crucially, our construction *does not require selecting a sufficiently small ϵ* : it is purely topological and provably produces an offset homeomorphic to an offset with infinitesimally small ϵ (Theorem 4).

4 INFINITESIMAL AND TOPOLOGICAL OFFSET

Consider a smooth, simple, manifold, closed surface C with a single connected component, and bounded curvature embedded in a domain Ω . A finite offset $O(C, \epsilon)$ is the set of points at a distance $\epsilon > 0$ from the surface,

$$O(C, \epsilon) = \{x \in \Omega \mid \|c(x) - x\|_2 = \epsilon\},$$

where $c(x)$ is the function returning the point of C closest to the point x . We can always find an ϵ_C such that the offset does not intersect the medial axis of C , and thus, for such an ϵ_C , the function $c(x)$ is bijective. By repeating the same construction for a simplicial complex S (Figure 6), an ϵ for which the function c is bijective does not exist anymore since the medial axis extends to the sharp points of S . However, as we will show in the following, there is still an $\hat{\epsilon}$ such that all offsets with a distance smaller than $\hat{\epsilon}$ have the same topology, are manifold, and are free of self-intersections.

DEFINITION 1 (INFINITESIMAL OFFSET FAMILY). *We call $\tilde{O}(S, \hat{\epsilon})$ the infinitesimal offset family of a simplicial complex S the 1-parameter family of offsets parametrized by a distance parameter $\epsilon \in (0, \hat{\epsilon})$.*

To streamline the explanation, we denote with $\mathcal{S}_\epsilon(s)$ the ϵ -inflation of the simplex s , i.e., the Minkowski sum of s and the ball of radius $\epsilon > 0$. Similarly, we use $\mathcal{S}_\epsilon(S)$ to denote the ϵ -inflation of all simplices in S .

THEOREM 1 (INFINITESIMAL OFFSETS). *For a simplicial complex S , there is an $\hat{\epsilon} > 0$ such that all offset surfaces $\partial\mathcal{S}_\epsilon(S)$, $0 < \epsilon \leq \hat{\epsilon}$ are manifold surfaces, assuming that faces of S are in a general position.*

PROOF. The distance function to a convex set is C_1 [Rockafellar and Wets 2009]. The distance function to a union of a finite number of convex sets is $\min(d_0 \dots d_n)$ where d_i is the distance to i -th convex set, so clearly piecewise C_1 and Lipschitz, as \min is Lipschitz and the composition preserves the Lipschitz property. The offsets are the level sets of this function.

The specialization of Clarke's theorem [Borwein and Lewis 2006] to the scalar functions $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ provides a criterion when the resulting offset is a manifold. Let $U \in \mathbb{R}^3$ be an open neighborhood of a point $p = (x, y, z)$, and let $c = f(p)$. If there is a matrix 2×3 matrix B , such that for any $A \in \partial f(p)$ (the generalized differential of f) The matrix $\begin{bmatrix} A \\ B \end{bmatrix}$ is an invertible 3×3 matrix, then the level set $c = f(p)$ is locally a Lipschitz submanifold.

The generalized differential is the convex hull of gradients of all distance functions "active" at p . Geometrically, this corresponds to the convex hull of the normal vectors to all simplex offset surfaces intersecting at p .

We do not need to consider a larger number of vectors in the generalized differential, as such points, if they exist, can be eliminated by a small change in the offset ϵ . In a general position (i.e., for almost all values of ϵ and general position of faces of S), we can assume that the level-set surfaces of each face intersect transversally, except if the point is on a patch shared by two or more such surfaces (e.g., a part of the sphere centered at a common vertex of two triangles), which we exclude from consideration as all offsets share a normal in this case. In other words, two vectors, in the case of intersection of

two surfaces and 3 vectors, in the case of three surfaces, are linearly independent.

Note that if two vectors in the generalized differential are linearly dependent and point in opposite directions (tangential contact), then a zero vector is in the convex hull and the matrix B required by the theorem does not exist, but such contact can be removed by a small perturbation of ϵ .

The statement of Clarke's theorem requires finding two vectors, v_1 , and v_2 (the rows of B) such that for any $w \in \partial f$, $[v_1, v_2, w]$ are linearly independent. If there is only one vector in $\partial f(p)$ (C_1 point) then any two orthogonal vectors would suffice. If there are two vectors, ∂f is a segment of a line in \mathbb{R}^3 not passing through zero. We just need to take any two independent vectors in a plane passing through zero that is parallel to this line. Then $(v_1, v_2, (1-t)w_1 + tw_2)$ are linearly independent for any $0 \leq t \leq 1$, as no vector with the endpoint on the line is in the span of v_1 and v_2 . Similarly, the convex hull of 3 independent vectors w_i in $\partial f(p)$ is a subset of a plane, not containing zero, so we take the vectors v_1 and v_2 to be a basis of a plane through zero, parallel to this plane. This proves the existence of B in all cases. \square

The infinitesimal offsets are all homeomorphic to each other, which makes their topology unique for all distances smaller than $\hat{\epsilon}$. To prove this statement, we assume that the simplicial complex S is simplicially embedded (Definition 2) in a tetrahedral background mesh M . If the input is a simplicial complex, such embedding can always be constructed, possibly with some refinement of S , which does not change its geometry [Diazzi et al. 2023]. The additional benefit of our proof relying on the background mesh is that it immediately leads to a constructive algorithm for topology-preserving offset approximation.

DEFINITION 2 (SIMPLICIAL EMBEDDING). A tetrahedral mesh M is a simplicial embedding of a simplicial complex $S \subset M$ if for any tetrahedron $t \in M$ the intersection of S and t is either empty, or is a vertex, an edge, or a triangle of S and M .

Locality. We now partition the space into a collection of convex cells, enabling us to localize the definition of the offset surface. We subdivide every tetrahedron into four convex regions, using the pattern shown in Figure 7. We denote with $\mathcal{V}_t(v)$ the convex cell corresponding to a vertex $v \in t$. We define a convex cell for each edge e_{ij} between the vertices v_i and v_j by taking the intersection of the convex cells of its vertices $\mathcal{V}_t(e_{ij}) = \mathcal{V}_t(v_i) \cap \mathcal{V}_t(v_j)$, and similarly for every face $\mathcal{V}_t(f_{ijk}) = \mathcal{V}_t(v_i) \cap \mathcal{V}_t(v_j) \cap \mathcal{V}_t(v_k)$ (f_{ijk} has vertices v_i, v_j , and v_k). We denote as $\tau_S(s)$ the open star of the simplex s , which contains s and all other simplices in S containing s . For example, the star of an edge is the edge itself, plus all the triangles incident to it.

Consider a tetrahedral mesh M that is a simplicial embedding of a simplicial complex S . We show that for any choice of S , there is a sufficiently small ϵ for which only $\tau_S(v)$ of a vertex v contributes to the offset in $\mathcal{V}_t(v)$. To do this, we consider the offset we would obtain by considering the maximal surface that could be embedded in M , which we denote as \hat{M} , composed of all vertices, edges, and triangles in M . We prove that for this worst case, the locality holds – in practice, S would be a subset of \hat{M} . This property is crucial, as

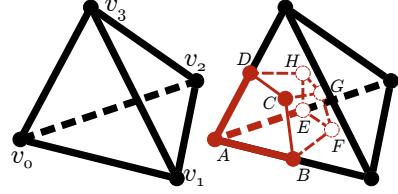


Fig. 7. Illustration of a tetrahedron with its four vertices (left) and a convex cell in red (right).

it allows us to construct the offset locally, unlike traditional offsets whose geometry and topology are global properties.

LEMMA 1 (OFFSET LOCALITY). For any tetrahedral mesh M embedded in \mathbb{R}^3 , there exists an $\hat{\epsilon} > 0$ such that, for every simplex $s \in \hat{M}$ and every tetrahedron $t \in M$ incident to s :

$$(\mathcal{S}_\epsilon(\hat{M}) \setminus \mathcal{S}_\epsilon(\tau_{\hat{M}}(s))) \cap \mathcal{V}_t(s) = \emptyset$$

PROOF. All simplices in $\tau_{\hat{M}}(s)$ contain s , and there is always a point $p \in s$ contained in $\mathcal{V}_t(s)$ since t is incident to s , and therefore the distance between all simplices in $\mathcal{S}_\epsilon(\tau_{\hat{M}}(s))$ and $\mathcal{V}_t(s)$ is zero. All other simplices in $\hat{M} \setminus \tau_{\hat{M}}(s)$ have a positive distance from $\mathcal{V}_t(s)$ as: (1) they cannot intersect the interior of t since M is embedded and (2) they cannot be in the boundary of $\mathcal{V}_t(s)$ as the intersection of $\mathcal{V}_t(s)$ and t is a subset of $\tau_{\hat{M}}(s)$. For any ϵ smaller than the minimal distance between $\mathcal{S}_\epsilon(\hat{M}) \setminus \mathcal{S}_\epsilon(\tau_{\hat{M}}(s))$ and $\mathcal{V}_t(s)$ their intersection is empty. If we pick the smallest ϵ for all $s \in \hat{M}$ and $t \in M$, then the locality condition holds for all convex cells $\mathcal{V}_t(s)$. \square

We note that an ϵ satisfying the locality property for \hat{M} also trivially satisfies the same property for any other embedded simplicial complex $S \subset \hat{M}$.

LEMMA 2. Let X and Y be connected closed bounded submanifolds of \mathbb{R}^n with boundaries. Let $f : X \rightarrow Y$ be a continuous injective map, bijectively mapping ∂X to ∂Y , and its inverse on $f(X)$ is continuous. Then f is a homeomorphism between X and Y .

PROOF. As f is continuous, its image $f(X)$ in Y is closed and bounded, as X is closed and bounded. As $f : X \rightarrow f(X)$ is a homeomorphism, f maps ∂X to $\partial f(X)$, so $\partial f(X) = \partial Y$. Suppose $f(X)$ does not coincide with the whole Y . As $f(X)$ is closed, $Y \setminus f(X)$ is open, by assumption nonempty, and is contained in the interior of Y ($\text{Int}(Y)$). $\text{Int}(Y)$ is connected because the interior of a connected manifold is connected. Let y_0 be an interior point of $f(X)$. Consider a continuous simple path $p : [0, 1] \rightarrow Y$ connecting y_0 and a point in $\text{Int}(Y) \setminus f(X)$ (as X has interior points and $f(\partial X) = \partial Y$, there are interior points in $f(X)$, so y_0 exists).

Consider the subset Z of $[0, 1]$ such that $f(Z)$ is contained in $f(X)$ and its complement in $[0, 1]$. As $f(X)$ is closed, then $Z = p^{-1}(f(X) \cap \text{Im}(p))$ is closed, and not coinciding with $\text{Im}(p)$, so it has a boundary at an interior point q of $[0, 1]$, for which $p(q)$ is contained in $f(X)$. As it is a boundary point of Z , any neighborhood of $p(q)$ in Y contains both points of $f(X)$ and $Y \setminus f(X)$, i.e., is on the boundary of $f(X)$. But by construction of the path, $p(q)$ must be an interior point of Y , which contradicts $f(\partial X) = \partial Y$. \square

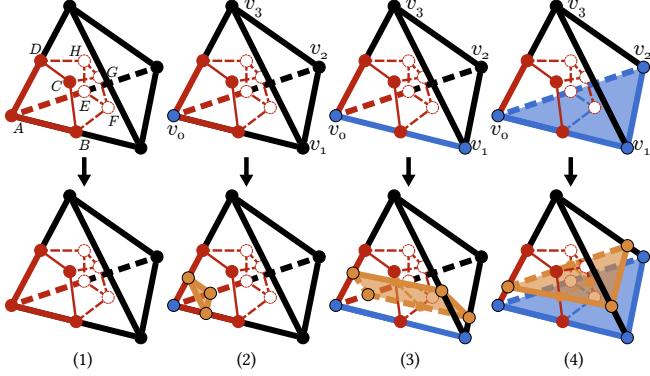


Fig. 8. Rules to generate the topological offset (orange) for an embedded simplicial complex (blue). For illustration purposes, we omit the simplicial decomposition of the polyhedra.

LEMMA 3. *There is an $0 < \epsilon \leq \hat{\epsilon}$, such that the union of the ϵ -inflations of the elements of $\tau_S(s)$ of a simplex $s \in S$,*

$$\mathcal{S}_\epsilon(\tau_S(s)) = \bigcup_{c \in \tau_S(s)} \mathcal{S}_\epsilon(c),$$

is a star domain with s in its kernel and its boundary $\partial\mathcal{S}_\epsilon(\tau_S(s))$ is manifold.

PROOF. Each $\mathcal{S}_\epsilon(c)$ is the Minkowski sum of two convex primitives (a simplex and a sphere), and it is thus convex. For two star-shaped domains r_1 and r_2 , every point p in the intersection of the two kernels ($p \in \ker(r_1) \cap \ker(r_2)$) has visibility to all points in both r_1 and r_2 and thus belongs to the kernel of the union of r_1 and r_2 ($p \in \ker(r_1 \cup r_2)$). Since s is a face of all $c_j \in \tau_S(s)$, s is in the kernel of all $\mathcal{S}_\epsilon(c_i)$, and therefore in the kernel of their union. This proves that $\mathcal{S}_\epsilon(\tau_S(s))$ is a star domain with s in its kernel. From Theorem 1, its boundary is manifold as $\tau_S(s)$ is a simplicial complex. \square

Using the result of the previous three lemmas, we will show that a convex cell's intersection with any infinitesimal offset is either empty or a disk.

THEOREM 2 (LOCAL DISK TOPOLOGY). *For any $\epsilon \in (0, \hat{\epsilon})$ and for any cell $\mathcal{V}_t(v_i)$, the intersection of the cell and the offset $\mathcal{S}_\epsilon(S) \cap \mathcal{V}_t(v_i)$ is empty iff $S \cap \mathcal{V}_t(v_i)$ is empty, otherwise it is a topological disk.*

PROOF. We consider the four rules in Figure 8 and restrict them to individual convex cells $\mathcal{V}_t(v_i)$ of their vertices. We observe that, after factoring out symmetries, there are only four distinct configurations (Figure 8): (1) the convex cell does not contain any simplex of S , (2) it contains only a vertex of S , (3) it contains a vertex and intersects an edge of S , and (4) it contains a vertex and intersects two edges and a triangle of S .

Case (1): Due to locality (Lemma 1), the convex cell does not contain $\partial\mathcal{S}_\epsilon(S)$.

Case (2): Due to locality (Lemma 1), the convex cell $\mathcal{V}_t(A)$ contains a part of $\partial\mathcal{S}_\epsilon(S)$ which, inside the cell, is identical to $\partial\mathcal{S}_\epsilon(\tau_S(A))$.

Points. We will show that $\partial\mathcal{S}_\epsilon(\tau_S(A))$ intersects the three edges AB , AE , and AD at one point per edge. Due to locality (Lemma 1), $\epsilon < \|B - A\|$, as the offset of A would otherwise contribute to

another convex cell. As $\mathcal{S}_\epsilon(\tau_S(A))$ is a star domain with A in its kernel ($\epsilon > 0$) and B outside ($\epsilon < \|B - A\|$), the edge AB must intersect $\partial\mathcal{S}_\epsilon(\tau_S(A))$ exactly once. The same holds for AE and AD . From locality, it also follows that $\partial\mathcal{S}_\epsilon(\tau_S(A))$ does not intersect any edge of $\mathcal{V}_t(A)$ that is not incident to A , as they are also part of other convex cells.

Curves. We will show that $\partial\mathcal{S}_\epsilon(\tau_S(A))$ intersects the three quadrilateral faces $ABCD$, $ABFE$, and $AEHD$ in three simple curves, one for each face, whose boundary is composed of the vertices identified before. Due to locality, the sphere $\partial\mathcal{S}_\epsilon(A)$ intersects AB and AD , but not BC or CD . Therefore, the intersection $\partial\mathcal{S}_\epsilon(A) \cap ABCD = \gamma$ is a circular segment. Let r be a ray from A to a point on γ . r intersects $\partial\mathcal{S}_\epsilon(\tau_S(A)) \cap ABCD$ in a unique point as $\mathcal{S}_\epsilon(\tau_S(A)) \cap ABCD$ is star-shaped with A in its kernel (Lemma 3), and all intersection points are contained in the interior of $ABCD$ due to Lemma 1. Therefore, r establishes an injective mapping between γ and $\partial\mathcal{S}_\epsilon(\tau_S(A)) \cap ABCD$. Due to Lemma 2, this map is bijective. As γ is a simple curve, it follows that $\partial\mathcal{S}_\epsilon(\tau_S(A)) \cap ABCD$ is also a simple curve. The same argument can be applied to $ABFE$ and $AEHD$.

Patch. The intersection $\Gamma = \partial\mathcal{S}_\epsilon(A) \cap \mathcal{V}_t(A)$ is a surface with disk topology (a spherical, simply connected patch) due to locality. Its boundary $\partial\Gamma$ maps via ray casting, with A as origin, to the points and curves identified before. Ray casting with A as origin defines an injective map from the interior of Γ to $\Gamma_{\tau S} = \partial\mathcal{S}_\epsilon(\tau_S(A)) \cap \mathcal{V}_t(A)$, as $\Gamma_{\tau S}$ is a star-shaped domain with A in its kernel, and $\partial\mathcal{S}_\epsilon(\tau_S(A))$ must not intersect $BFGC$, $FGHE$, or $CGHD$ (locality).

Both Γ and $\Gamma_{\tau S}$ are connected closed 2-manifold with metric inherited from \mathbb{R}^3 with a single boundary (the intersection of both with the boundary of \mathcal{V}_t is a 1-manifold as we show above). Both are connected: if it consisted of several connected components with boundary, then we would have several separate boundary loops. So only one connected component can have a boundary; the rest need to be without boundary. But then these connected components would also be separate components of the entire offset surface, as there cannot be a continuous path connecting them to any point of the offset outside \mathcal{V}_t , not intersecting \mathcal{V}_t boundary. From Lemma 2, it follows that the projection is a homeomorphism, and thus $\Gamma_{\tau S}$ is a disk.

Case (3): Points. $\partial\mathcal{S}_\epsilon(\tau_S(A))$ intersects the 4 edges AE , AD , BF , and BC at one point per edge, and it does not intersect any of the other edges of $\mathcal{V}_t(A)$. The proof is the same as for Case (2) for AE and AD . For BF , we observe that $\partial\mathcal{S}_\epsilon(\tau_S(A)) \cap BF = \partial\mathcal{S}_\epsilon(\tau_S(e_01)) \cap BF$ (Lemma 1) and $\partial\mathcal{S}_\epsilon(\tau_S(e_01))$ is a star domain with B in its kernel ($\epsilon > 0$) and F outside ($\epsilon < \|F - B\|$), the edge BF must intersect $\partial\mathcal{S}_\epsilon(\tau_S(A))$ exactly once. The proof for BC is similar.

Curves. $\partial\mathcal{S}_\epsilon(\tau(A))$ intersects the 4 quadrilateral faces $ABCD$, $ABFE$, $BFGC$, and $AEHD$ in four curves, one for each face, whose boundary is composed of the vertices identified before. The intersection of $\partial\mathcal{S}_\epsilon(B) \cap BFGC$ is a circular segment, thus a simple curve. We establish an injective mapping between $\partial\mathcal{S}_\epsilon(B) \cap BFGC$ and $\partial\mathcal{S}_\epsilon(\tau_S(A)) \cap BFGC$ via ray casting as in Case (2), using B as the origin and noting that B is in the kernel of $\partial\mathcal{S}_\epsilon(\tau_S(e_01))$ (Lemma 3) and $\partial\mathcal{S}_\epsilon(\tau_S(e_01)) \cap BFGC = \partial\mathcal{S}_\epsilon(\tau_S(A)) \cap BFGC$. This map is bijective due to Lemma 2. The proof for $AEHD$ is the same as for Case (2), using A as the ray origin.

For $ABCD$ and $ABFE$, the curves of interest intersect two opposite sides of the quads, so the polar argument is slightly different. In both cases, we need to consider A as the projection point: the bijection is between a part of a circle cut out by the sides of the polygon and the segment of the boundary of $\mathcal{S}_\epsilon(S) \cap V_t(A)$. Consider points C' and D' – intersection points on edges BC and AD . On a small circle around A , these project to C'' and D'' . C'' is in the interior of $ABCD$, not on an edge. Any ray through A in the interior of $ABCD$ and to the same side from AC'' as D'' , passes through exactly one point of the circle and one point of $\partial\mathcal{S}_\epsilon(S) \cap V_t(A)$. It remains to prove that no ray on the other side of AC'' intersects $\partial\mathcal{S}_\epsilon(S) \cap V_t(A)$. Suppose it does: then if we extend it further, it will intersect the segment BC on the side closer to B , i.e. in the interior of $\mathcal{S}_\epsilon(S) \cap V_t(A)$, this means that it has to intersect $\partial\mathcal{S}_\epsilon(S) \cap V_t(A)$ twice on its way from A to BC'' which contradicts the fact that it is star-shaped. We conclude that there is a bijection.

Patch. Same proof as Case (2).

Case (4): Points. $\partial\mathcal{S}_\epsilon(\tau_S(A))$ intersects the 4 edges AD , BC , FG , and EH at one point per edge, and it does not intersect any of the other edges of $\mathcal{V}_t(A)$. The proof is the same as Case (2) for AD with A in the kernel, same as Case (3) for BC and EH , with B and E in the kernel, respectively. For FG , we observe that $\partial\mathcal{S}_\epsilon(\tau_S(A)) \cap FG = \partial\mathcal{S}_\epsilon(\tau_S(f_{012})) \cap BF$ (Lemma 1) and $\partial\mathcal{S}_\epsilon(\tau_S(f_{012}))$ is a star domain with F in its kernel ($\epsilon > 0$) and G outside ($\epsilon < \|G - F\|$), the edge FG must intersect $\partial\mathcal{S}_\epsilon(\tau_S(A))$ exactly once.

Curves. $\partial\mathcal{S}_\epsilon(\tau(A))$ intersects 4 quadrilateral faces $ABCD$, $BFGC$, $FGHE$, and $AEHD$, in four simple curves, one for each face, whose boundary is composed of the vertices identified before. The proof for all 4 faces is similar to face $ABCD$ of Case (3), using A , B , E , and A as projection points.

Patch. Same proof as Case (2). \square

From Theorem 2, it follows that all infinitesimal offsets are homeomorphic to each other, as a bijective map can be defined between each disk inside each convex cell.

Topological Offset. We established that infinitesimal offsets of a simplicial complex are all manifold surfaces homeomorphic to each other: their topology is unique for a given input surface. To decouple geometry and topology, we introduce the concept of a *topological offset* (Figure 6, green curve), which is a surface that is homeomorphic to an infinitesimal offset, but whose geometric embedding is arbitrary (as long as it does not intersect the input surface). The reason for not using the infinitesimal offset directly is practical: representing it in a mesh might lead to infinitesimally small elements: on the other hand, topological offsets can be computed reliably and robustly.

DEFINITION 3 (TOPOLOGICAL OFFSET). A simple surface O is a topological offset of a non-intersecting simplicial complex $S \subset \Omega$ if O and S do not intersect and O is homeomorphic to an infinitesimal offset of S , i.e., there exists a continuous and bijective function $\hat{c}: O \rightarrow \mathcal{S}_\epsilon(S)$ for any $0 < \epsilon < \hat{\epsilon}$.

The function \hat{c} is a topological version of the closest point function c used in the offset definition.

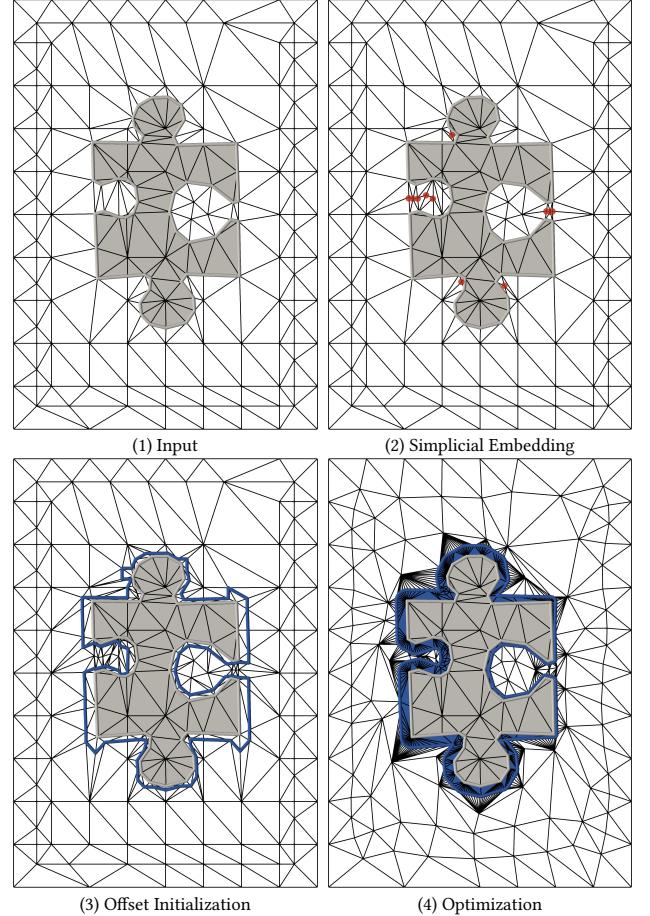


Fig. 9. The full 2D pipeline for generating the offset: (1) we start with a simplicial complex embedded in a background mesh, (2) we make the embedding simplicial (see highlighted vertices), which allows us to (3) generate an initial topological offset, and (4) we optionally optimize the offset while keeping the background mesh and offset valid.

5 TOPOLOGICAL OFFSET CONSTRUCTION

We now describe an algorithm (Figure 9) to compute a topological offset of a simplicial complex S . The algorithm has two steps (Section 5.1 and Section 5.2) that are parameter-free, followed by an optional geometrical optimization to improve the offset quality and embedding, which we will describe in Section 5.3.

Input Preprocessing. If the input simplicial complex is not embedded in a tetrahedral mesh, we embed it using TetWild [Hu et al. 2018]. Note that other algorithms could be used for this step, such as TetGen [Si 2015] or Robust CDT [Diazzi et al. 2023]. The embedded complex is a subset of the triangles, edges, or vertices in the tetrahedral mesh tagged as belonging to the complex.

Step 1: Simplicial Embedding. We create a simplicial embedding (Section 5.1) using a sequence of local operations. The pseudocode for this step is described in Algorithm 1. This condition is essential

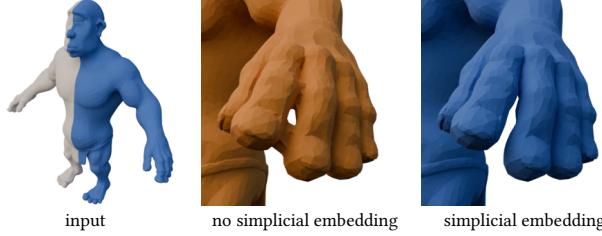


Fig. 10. By making the background mesh a simplicial embedding of the input (white), we guarantee that the topological offset (blue) is homeomorphic to the infinitesimal offset family. Without simplicial embedding, the topology might be different (orange).

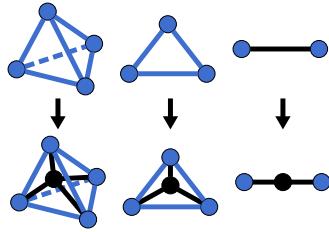


Fig. 11. Split operation for a tetrahedron, triangle, and edge.

to ensure the existence of the discrete counterpart of a topological offset (Figure 10).

Step 2: Offset Insertion. We insert a topological offset in the background mesh using a topological variant of marching tetrahedra (Section 5.2).

Step 3: Offset Optimization. Optionally, we optimize the tetrahedral mesh (and consequently the offset) by increasing its quality and adapting for various applications, as described in Section 5.3.

Output Postprocessing. The output of our algorithm is a tetrahedral mesh with an embedded offset surface. The background mesh can be discarded if the downstream application only needs the surface mesh.

5.1 Step 1. Simplicial Embedding

We propose an algorithm to convert a simplicial complex embedded in a tetrahedral background mesh M (such as the one generated by [Hu et al. 2018]) into a simplicial embedding by performing a sequence of edge splits (Figure 11) to update M . Algorithm 1 iterates over every tetrahedron $t \in M$, and checks if the boundary of t is in S (line 2). In that case, it splits the tetrahedron (line 4). It then iterates the same procedure on all triangles (line 5) and edges (line 8) that are not in S (Figure 12 shows some examples). Note that a split to an edge or a face is propagated to the tetrahedra intersecting it.

THEOREM 3 (SIMPLICIAL EMBEDDING OF A SIMPLICIAL COMPLEX). *Algorithm 1 produces a mesh M such that M is a simplicial embedding of S .*

PROOF. After the first loop of Algorithm 1 completes, no tetrahedron has four faces in S . If a tetrahedron has three or two faces

Algorithm 1 SIMPLICIAL_EMBEDDING(M, S):

Input: A mesh M with an embedded simplicial complex S

Output: M' is a simplicial embedding of S

```

1:  $M' \leftarrow M$ 
2: for each tetrahedron  $t \in M'$  do
3:   if  $t \cap S$  contains four triangles then
4:      $M' \leftarrow \text{SPLIT\_TETRAHEDRON}(M', t, S)$ 
5:   for each triangle  $t \in M'$  do
6:     if  $t \notin S$  and  $t \cap S$  contains three edges then
7:        $M' \leftarrow \text{SPLIT\_TRIANGLE}(M', t, S)$ 
8:   for each edge  $e \in M'$  do
9:     if  $e \notin S$  and  $e \cap S$  contains two vertices then
10:       $M' \leftarrow \text{SPLIT\_EDGE}(M', e, S)$ 
11: return  $M'$ 

```

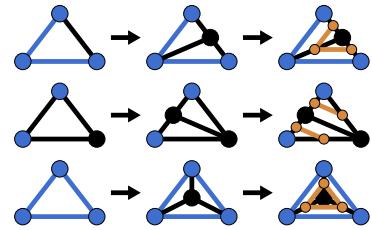


Fig. 12. 2D exemplary results for applying Algorithm 1 to make M a simplicial embedding of S (blue), and the resulting topological offsets (orange). For illustration purposes, we omit the simplicial decomposition of the polygons.

in S , then there is a face or faces that have three edges in S , and these faces will be split in the second loop. Therefore, after the first two loops, all tetrahedra have no more than one face in S . Suppose a tetrahedron has a face and two or more edges not contained in this face in S . Then there are triangles not in S with three edges in S that will be split in the second loop, i.e., there is no more than one edge in S after the second loop. Thus, after the first two loops, a tetrahedron may contain at most one face of S and at most one edge not contained in this face, and some number of vertices of S . Similarly, in the last loop, if there is a face and an edge, or a face and a vertex, or an edge and a vertex, or two vertices, there will be always an edge not in S with two vertices in S that will be split in the third loop. We conclude that the output is a simplicial embedding of S . \square

5.2 Step 2. Offset Insertion

We insert the discrete topological offset into the background mesh by using a binary version of marching tetrahedra [Guéziec and Hummel 1995]: every tetrahedron that contains a vertex, an edge, or a triangle of the input simplicial complex is partitioned using the rules in Figure 8. We do not perform an interpolation as suggested in [Guéziec and Hummel 1995], but we just place the inserted vertices at the midpoint of the split edges.

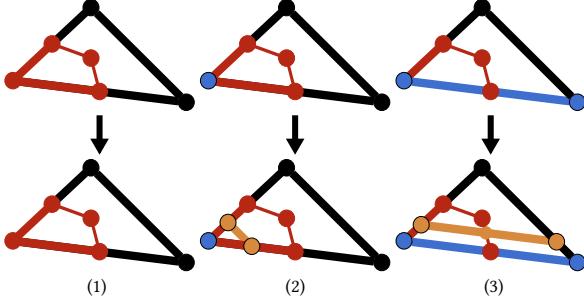


Fig. 13. Rules to generate the topological offset in 2D. For illustration purposes, we omit the simplicial decomposition of the polygons. A convex cell as it is used in Theorem 2 is depicted in red.

THEOREM 4. *The application of the rules in Figure 8 to a simplicial complex S simplicially embedded in a background mesh M is a discrete topological offset homeomorphic to a continuous one.*

PROOF. Case (1): No elements are added in this case.

Case (2): $\mathcal{S}_\epsilon(S) \cap \mathcal{V}_t(v_0)$ is homeomorphic to the orange triangle in Figure 8 case (2) because it is also a disk intersecting the same simplices. All other cells in the tetrahedron t do not intersect $\mathcal{S}_\epsilon(S)$. Thus, the rule constructs a disk homeomorphic to $\mathcal{S}_\epsilon(S)$ within t .

Case (3): The cells $\mathcal{V}_t(v_0)$ and $\mathcal{V}_t(v_1)$ have a non-empty intersection with $\mathcal{S}_\epsilon(S)$, and the offset $\partial\mathcal{S}_\epsilon(S)$ within every cell of the tetrahedron t is a disk (Theorem 2). The cells share the face $BFGC = \mathcal{V}_t(e_{01})$ and the disks' boundaries intersect that face in the curve $\partial\mathcal{S}_\epsilon(S) \cap \mathcal{V}_t(e_{01})$. It follows that the union of the two disks is also a disk. The orange polygon (that can be decomposed into two triangles) in Figure 8 case (3) is also a disk and thus homeomorphic to $\mathcal{S}_\epsilon(S)$ within t .

Case (4): The cells $\mathcal{V}_t(v_0)$, $\mathcal{V}_t(v_1)$, and $\mathcal{V}_t(v_2)$ have a non-empty intersection with $\mathcal{S}_\epsilon(S)$, and the offset $\partial\mathcal{S}_\epsilon(S)$ within every cell of the tetrahedron t is a disk (Theorem 2). The cells share faces in the same manner as in case (2), e.g., $\mathcal{V}_t(v_0)$ and $\mathcal{V}_t(v_2)$ share the face $EFGH = \mathcal{V}_t(e_{02})$. It follows that the union of the three disks is also a disk. The orange triangle in Figure 8 case (4) is also a disk and thus homeomorphic to $\mathcal{S}_\epsilon(S)$ within t . \square

The mesh, composed of all polygons constructed from the rules in Figure 8, is manifold, as it is homeomorphic to an infinitesimal offset (Theorem 4) which is manifold (Theorem 1).

While we prove the correctness of our method only for a simplicial complex embedded in a tetrahedral mesh, it is possible to prove a similar result in 2D, which leads to the patterns in Figure 13. We show an example of the 2D algorithm in Figure 14, where the input is a sequence of marked edges in a triangle mesh.

We note that Algorithm 1 is purely topological. The coordinates of the vertices of M are never used, which makes our algorithm unconditionally robust. We note, however, that the computation of the vertex positions after a split might lead to inverted elements if the rounding error is larger than the length of the edge. We show an example of such a problematic case in Figure 25, and note that

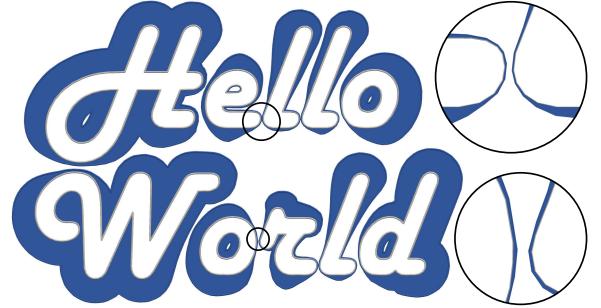


Fig. 14. Our method can also construct topological offsets in 2D.



Fig. 15. A varying offset distance applied to the armadillo.

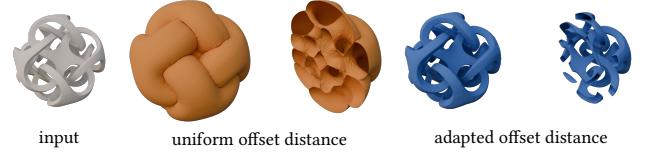


Fig. 16. We prevent offset from getting too close (orange) by locally adapting the offset distance (blue).

this only happened once in our large-scale evaluations (Section 6). See Section 5.4 for additional details on this problem.

5.3 Step 3. Offset Optimization

The inserted topological offset depends on the resolution of the background mesh: a finer mesh will create an offset closer to the input surface while a coarser mesh will have an offset further away. To remove this dependence from the background mesh and to allow customization of the embedded offset, we introduce an optimization algorithm that modifies the connectivity and geometry of the offset while preserving its topology and avoiding self-intersections and intersections with the input.

User Parameters. While the generation of a topological offset does not require any parameters, we introduce a primary parameter to control desired distance δ of the offset from the input simplicial complex (which could be spatially adaptive, Figure 15). We note that this is a desideratum: it is, in general, impossible to guarantee an arbitrary distance while preserving the infinitesimal offset topology (Figure 16). The other parameters control the tradeoff between geometrical accuracy and running time: (1) the maximum normal deviation $\sigma_{\max} = 15^\circ$ which controls how well the mesh should

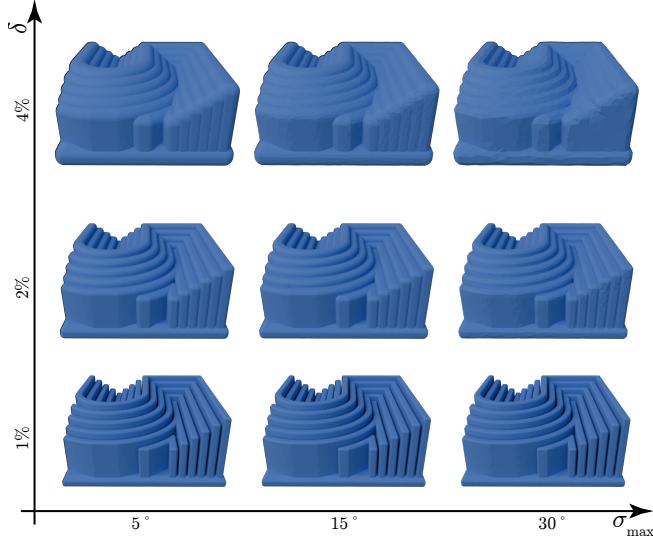
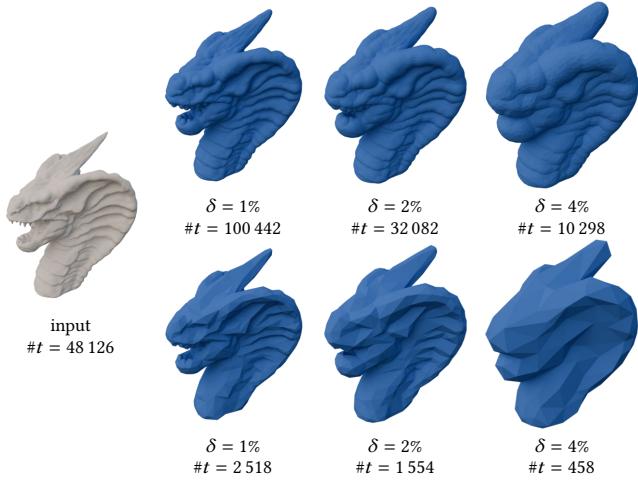


Fig. 17. Effect of the offset distance and normal deviation on the output.

Fig. 18. Setting $\ell_{\min} = 100\%$ (relative to the bounding box diagonal) and $\sigma_{\max} = 90^\circ$ leads to coarse topological offsets. The top row shows the offset meshes with default parameters.

adapt to the offset curvature (Appendix A), (2) $\sigma_{\min} = 2^\circ$ which controls when the offset curvature is considered planar, (3) $\ell_{\max} = \infty$ and (4) $\ell_{\min} = 2\delta \sin(\sigma_{\max})$ the maximum/minimum edge length.

Increasing the maximum normal deviation leads to a less smooth offset surface (Figure 17), while increasing ℓ_{\min} leads to coarse results (Figure 18). Finally, our method can generate *outside* and *inside* offsets if the input is closed and oriented (Figure 19).

Termination. The optimization terminates if the average σ_{\max} does not decrease by more than 0.5° across iterations, if both the maximum and mean distance error do not decrease by more than $0.5\%\delta$, or after 10 iterations.



Fig. 19. For closed surfaces, our method can generate inside (left) or outside (right) offsets.

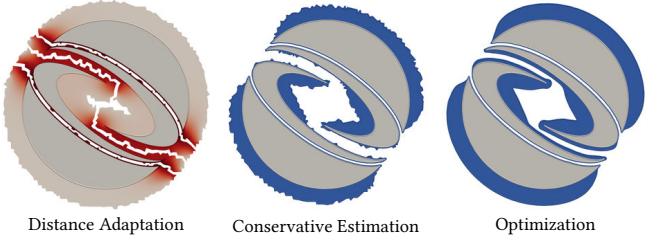


Fig. 20. Overview of our topological offset optimization.

Optimization Overview. The optimization algorithm proceeds in 3 steps (Figure 20): we estimate a spatially adaptive distance $\hat{\delta}$ that is as close as possible to δ (Section 5.3.1), we then use a topological marching front algorithm to modify the offset to be as close as possible to $\hat{\delta}$ (Section 5.3.2), and finally optimize it with a set of topological and geometrical local operations (Section 5.3.3).

5.3.1 Distance Adaptation. As we enforce the infinitesimal offset topology, it might happen that two different parts of the offset collide during optimization (Figure 16). We do the best effort to avoid such collisions by locally reducing the offset distance. After initializing the topological offset, we greedily expand the offset volume without changing its topology. We then compute the distance of the expanded volume's boundary to the input, propagate it back to the input, and eventually use this newly computed distance as our spatially varying distance $\hat{\delta}$.

Greedy Expansion. To estimate the maximum possible offset distance, we grow the offset volume using a marching front algorithm. We add tetrahedra to the offset volume (the volume enclosed by the offset surface and the input) if they do not modify the topology of the offset surface. That is, a tetrahedron may only be added to the offset volume if it intersects the surface in one, two, or three faces and their incident edges and vertices. We initialize a queue

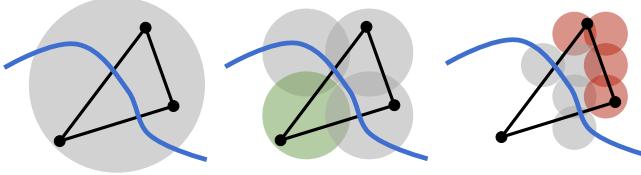


Fig. 21. Circle approximation of a triangle to check if it is inside (green) or outside (red) the offset volume. A circle (grey) is subdivided if it intersects the offset surface (blue).

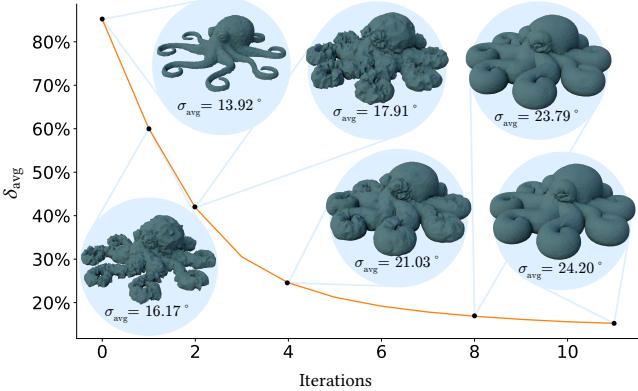


Fig. 22. Iterations of the optimization of a topological offset without proper initialization. While the algorithm manages to lower δ_{avg} , it plateaus around 15%. With proper initialization, it converges to 1.6% in 5 iterations (Figure 23).

with all tetrahedra that are face adjacent to the offset volume. If a tetrahedron is added to the volume, its face-adjacent neighbors are added to the queue.

A tetrahedron t can only be added to the offset volume if it is within the offset distance δ . We conservatively approximate δ by enclosing t into a sphere (Figure 21). If the sphere is farther than δ , t is outside. In the other case, we decompose t into 8 sub-spheres and repeat the procedure until either all spheres touching t are outside, one sphere is inside, or the radius is smaller than $10\%\delta$. In the latter cases, we conservatively consider t to be inside. The output of this procedure is a region of space around the input that is homeomorphic to the offset volume.

Distance Propagation. We start by assigning the target distance δ to every vertex on the boundary of the expanded offset volume. Then, we select every tetrahedron vertex adjacent to the expanded offset volume, compute the distance between its barycenter and the input, and update the assigned distance value by selecting the smallest of the values. To propagate this distance from the boundary to the input surface, we use harmonic interpolation on the background mesh. We store the solution on the vertices of the input simplicial complex as adapted offset distance $\hat{\delta}$.

We discard the expanded offset volume after the distance propagation and re-compute it using the new $\hat{\delta}$ to improve the geometry of the topological offset before optimization.

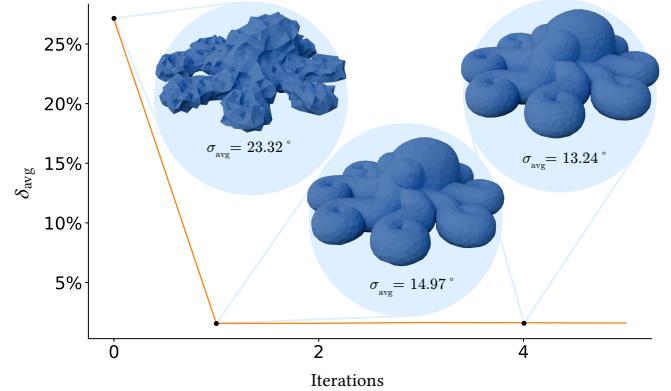


Fig. 23. Iterations of the optimization of a topological offset. The average relative distance error δ_{avg} and normal deviation σ_{avg} converge very fast when the offset was properly initialized.

5.3.2 Conservative Estimation. Our goal is to expand the offset volume such that its boundary (the offset surface) is as close as possible to the adapted offset distance $\hat{\delta}$. We expand the offset volume using the marching front method described in Section 5.3.1, but we modify the distance approximation to make it conservative. A tetrahedron t is only added if it is completely within distance $\hat{\delta}$. Similar to Section 5.3.1, we measure the distance by enclosing t into a sphere. If the sphere is within $\hat{\delta}$, t is inside. In the other case, we decompose t into 8 sub-spheres and repeat the procedure until either all spheres touching t are inside, one sphere is outside, or the radius is smaller than $10\%\delta$. In the latter cases, we conservatively consider t to be outside. Note that here, the expansion is conservative and not greedy as in Section 5.3.1.

The conservative estimation does not just improve performance, but also avoids the optimization to get stuck in local minima (Figure 22).

5.3.3 Optimization. Our optimization algorithm is composed of three steps, which are repeated until convergence: (1) we update a sizing field, (2) we modify the offset mesh, improving its quality and moving its vertices to the desired distance from the input, and (3) we modify the background mesh, increasing its quality. In all iterations, the meshes are modified using a set of local operations, following [Botsch and Kobbelt 2004]: our algorithm iterates passes of splits, collapse, swaps, and vertex relocation. These operations are always performed on the background mesh: if the algorithm tries to split an edge of the offset mesh, this operation is applied to the corresponding edge of the background mesh or vice versa. Note that average normal deviation and relative distance error converge very fast due to the conservative estimation (Figure 23).

Invariants. The operations are executed only if their effect on the mesh satisfies the following invariants: (I1) the input surface and the boundary are not modified, (I2) the orientation of all tetrahedra in the background mesh is preserved (tested using the exact predicate in [Shewchuk 1996]) and (I3) the topology of the offset and input surface is preserved [Vivodtzev et al. 2010].

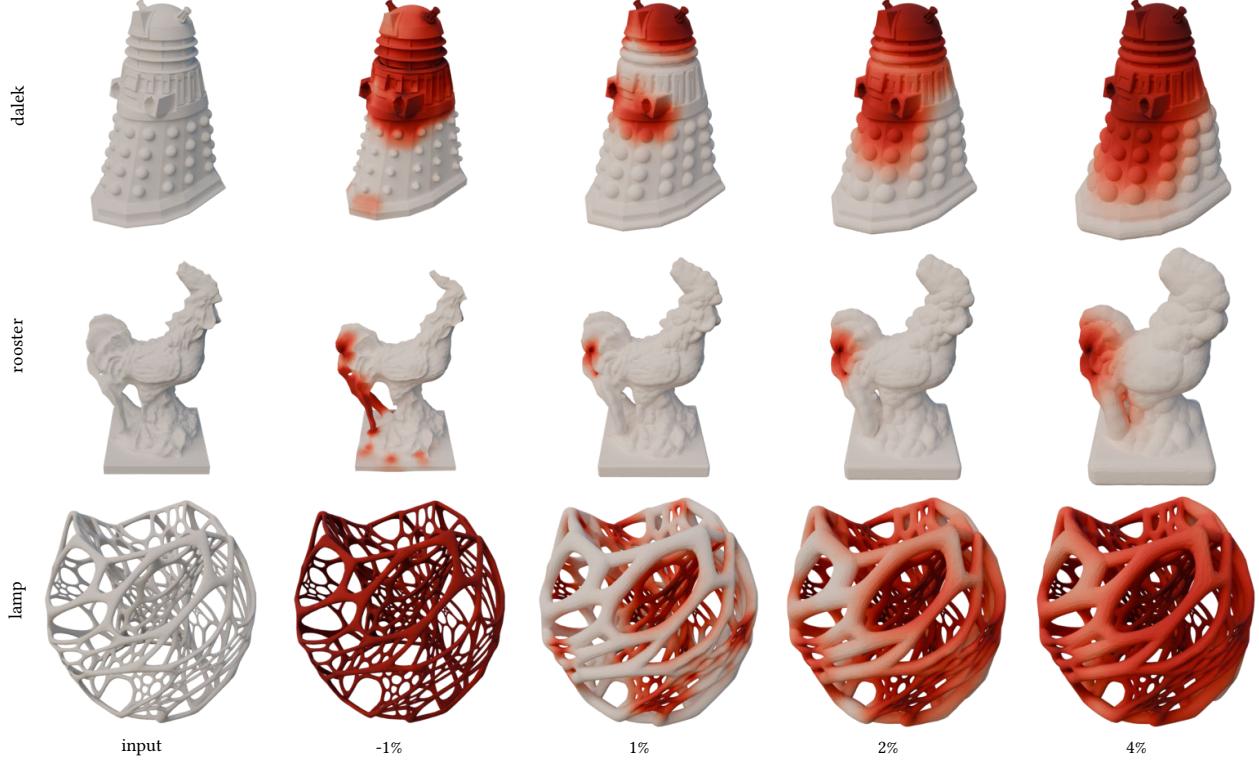


Fig. 24. Our topological offsets can handle a large variety of offset distances. Regions where the offset distance was adapted are colored in red.

THEOREM 5. Let M be a background mesh containing a simplicial embedding S and an offset surface O . The mesh M' computed after performing any sequence of local operations satisfying the invariants I1, I2, and I3 contains a new surface O' homeomorphic to O that does not intersect S .

PROOF. O' is homeomorphic to O due to the explicit avoidance of operations changing its topology (I3), we refer to [Vivodtzev et al. 2010] for details. O' cannot intersect S because I1 and I2 imply that there is always a continuous bijection between the points in M and M' [Lipman 2014]. \square

It follows from Theorem 5 that topological offsets are preserved by our mesh optimization.

Step 1: Sizing Field Update. Edge split and collapse operations are driven by a sizing field that is defined on each edge of the offset mesh and is initialized with the current length of each edge.

In each update pass, if one of the incident triangles has a shape regularity below 0.5 or a normal deviation above the user-defined maximum σ_{\max} , we divide the target length by two. If shape regularity is below 0.5 and the normal deviation is above σ_{\min} , we increase the target length by 1.5. After this update, to keep the sizing field smooth, we cap the target length to 1.5 times the length of any adjacent edge.

Step 2: Local operations. Every edge that has a length greater than $4/3$ of its target length is split. The new vertex is positioned at

the center of the edge. All edges that are shorter than $3/4$ of their target length are collapsed. We perform half-edge collapses. While there are no extra conditions for splits, a collapse is only performed if the user-defined maximum normal deviation is not exceeded. Both operations are scheduled according to the current edge length but for splits, long edges are prioritized, while for collapse, the short ones are considered first. Edges are swapped if the operation increases the minimal triangle shape regularity (Appendix A) of the two adjacent triangles. The swap is not performed if the normal deviation before the operation is below the user-defined maximum and would be above afterward. Long edges are prioritized in the swap operation. We adapt the vertex relocation method proposed in [Zint et al. 2023] to work with a spatially varying offset distance. We compute the offset distance for a vertex as the area-weighted average of offset distances at the sample points of the adjacent triangles. If the computed position would cause a tetrahedron from the background mesh to be flipped, we perform a binary search along the way to the computed position to find a valid position. If no valid position can be found, we do not move the vertex.

Figure 24 shows different results of our topological offsets for different distances on 3 models from Thingi10k [Zhou and Jacobson 2016] where the distance was locally adapted (Table 1 shows the statistics). All the depicted examples have an average relative distance error below 3%, an average normal deviation below 20° , and an average triangle shape regularity of at least 0.78. The quality metrics are explained in Appendix A.

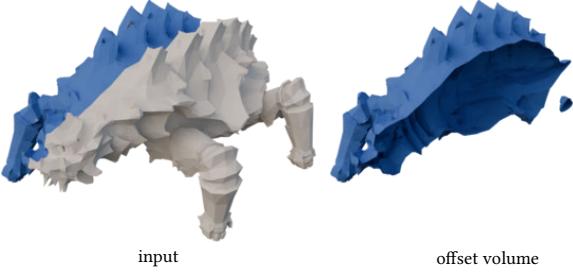


Fig. 25. Our method fails if an edge split during the offset initialization causes tetrahedra to invert due to numerical inaccuracy. In this example, we try to compute two offsets, one at 10^{-11} and a second one at 10^{-12} .

Step 3: Embedding Optimization. We use the optimization scheme that was presented in [Hu et al. 2018] with two minor modifications (in addition to the aforementioned invariants) to make it more efficient, as we are not interested in obtaining a background mesh of very high quality: we only want the background mesh to not hinder the movement of the offset. First, we trigger the update of the sizing field if the tetrahedron AMIPS energy is above 100 (instead of 8) and limit the target edge length to three times the length of any adjacent edge. Second, we only optimize the two-ring neighborhood of a tetrahedron with AMIPS above 100.

5.4 Robustness and Failure Cases

We now analyze our algorithm from a robustness perspective, discussing precisely which guarantees it provides and what are the potential failures.

Step 1 and 2. The decision of where to split is purely topological. However, the background mesh might flip after the split if an edge is shorter than the rounding error (Figure 25). This is not a practical concern, but a fully robust solution could be obtained by using a hybrid floating point/rational representation, following the same idea in [Hu et al. 2018].

Step 3. The invariants in the optimization are either purely topological (I1 and I3), or checked using exact predicates [Shewchuk 1996] (I2). While there are no guarantees that the prescribed distance will be obtained, this step cannot fail and will always terminate after 10 iterations.

Properties. The generated offsets are thus guaranteed to be homeomorphic to an infinitesimal offset, do not intersect the input simplicial complex, and their embedding is free of self-intersections. We show a large-scale validation of our implementation in Section 6. We note that using a tetrahedral background mesh makes these strong guarantees possible, enabling us to reduce challenging checks (self-intersections, topological correctness) to an exact Orient3D predicate.

6 RESULTS

We created topological offsets for the entire Thingi10k dataset [Zhou and Jacobson 2016], which was embedded in a background mesh using TetWild [Hu et al. 2018] with default settings. We run our experiments on cluster nodes with a Xeon E5-2690 v2 @ 3.00GHz.

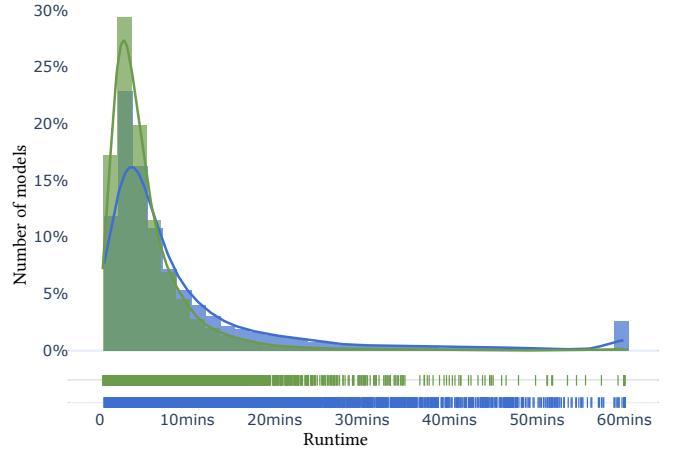


Fig. 26. Runtime of topological (blue) and finite (green) offsets.

We use the winding number to identify the outer part of the surface and create a one-sided offset. We skipped the 233 meshes where the winding number failed to identify a closed internal volume. This left 9 767 meshes in our experiments. We cap the runtime at 24 hours or stop when the convergence criteria from Section 5.3.3 (*Termination*) are met. We compute offsets on the dataset with a target distance $\delta = 4\%$ relative to the bounding box size. We are not aware of any existing method that produces topological offsets, and therefore, we cannot directly compare with any previous work. To enable direct comparisons, we introduce a minor variant of our algorithm to create finite offsets: this variant is described and compared with state-of-the-art offset methods in Section (Section 7.1).

Large-Scale Testing. Our algorithm produces a valid topological offset, embedded in a valid background mesh, for all models of the dataset except for one, where no edge split could be performed without generating inverted elements. For more details on that matter, see Section 5.4. 55.24% of the models finish within 6 minutes, and only 275 models (less than 3%) take more than one hour (Figure 26); these are highly complex models (Figure 27). The overall memory consumption is low, considering that our method processes tetrahedral meshes. 9 764 models use less than 16 GB of memory, and the remaining 3 use less than 64 GB.

Density and Quality. Our topological offsets can have many triangles (Figure 28) as they always have the topology of an infinitesimal offset and, therefore, maintain the details present in the input (Figure 27). There are few results (Figure 29) with an average normal deviation larger than 20° as the normal deviation error depends both on the edge length and the offset distance. Our experiments show that these are models where we adapt the offset distance everywhere. By reducing the edge length, we can achieve the target normal deviation even for the adapted offset distance (Figure 30).

Distance Adaptation. The distance adaptation relies on a heuristic that might fail under certain circumstances, since the greedy expansion (Section 5.3) might overestimate the offset. The geometry of the estimation and the actual topological offset might be

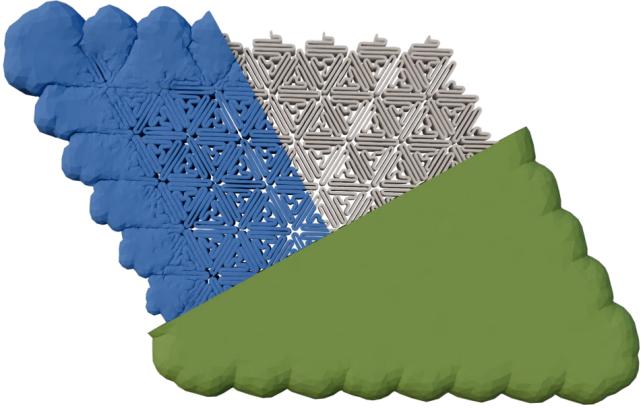


Fig. 27. For models with high geometric fidelity, the topological offset (blue) contains way more triangles, whereas finite offsets (green) remove geometric details.

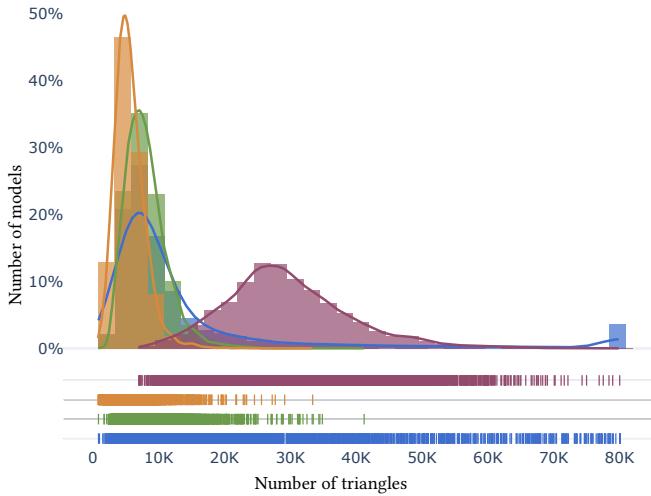


Fig. 28. Number of offset triangles of Alpha Wrapping (red), FPO (yellow), our finite offsets (green), and our topological offsets (blue).

different, and in such a case, the distance might not be adapted properly. In Figure 31, we adapted the distance for the 2% offset, while no adaptation was necessary for 7%. The greedy expansion of 4% overestimates the geometry and is more similar to the one of the 7% distance, where no adaptation is necessary; this leads to very close offsets but never intersecting, as we prohibit that with exact predicates (Section 5.3). Note that, for this example, the input and the offsets are of genus 0, so for large distances, the offset converges to a sphere.

7 APPLICATIONS

We present several applications of our topological offset: an algorithmic variant for producing finite offsets (Section 7.1), construction of multiple offset layers which guarantees that the offsets are

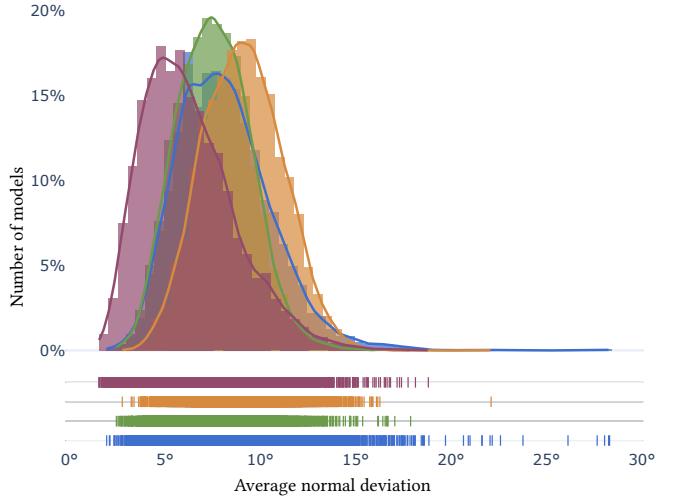


Fig. 29. Average normal deviation of Alpha Wrapping (red), FPO (yellow), our finite offsets (green), and our topological offsets (blue).

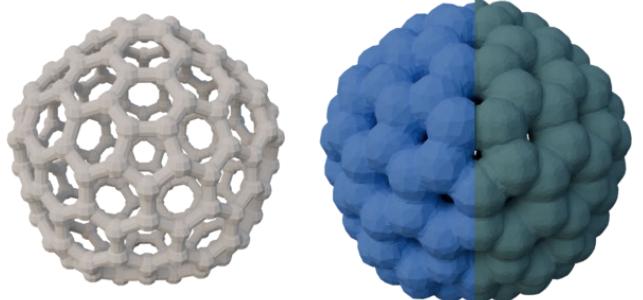


Fig. 30. The minimal edge length can be adapted (teal) if the normal deviation is large (blue) for a given offset distance and model (white).

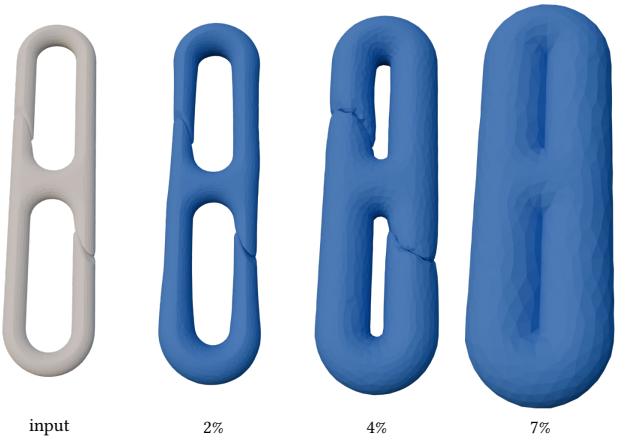


Fig. 31. Topological offsets might be very close (but never intersecting) as the local offset distance adaptation might be inaccurate.

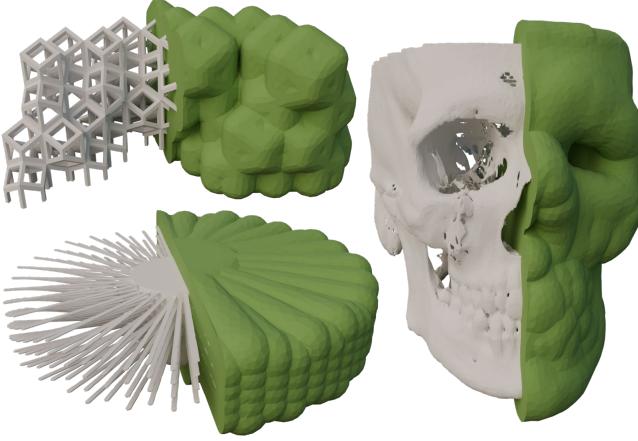


Fig. 32. With a small modification, our offsets become finite (like those produced by other methods). Our finite offsets (like our topological ones) are guaranteed to enclose the input and are manifold and self-intersection-free.

intersection-free (Section 7.2), and the use of topological offsets to remove non-manifold simplices (Section 7.3).

7.1 Finite Offsets

We can adapt our algorithm to produce finite offsets with minor modifications (Figure 32). Note that in this case our algorithm does not produce an offset homeomorphic to an infinitesimal one and, similarly to all other finite offset methods, is not guaranteed to compute an offset with the same topology as the exact finite offset with the prescribed distance, it is only an approximation. However, our finite offsets are still manifold, self-intersection-free, and enclose the input.

For finite offsets, we expand the topological offset (Section 5.3.2) without any topological condition. After we grow the region to the desired distance, we generate another topological offset around it so that we can ensure that the offset is manifold, even if the expanded offset volume is not. While the topology of the topological offsets is independent of the resolution of the embedding, this is not true for finite offsets. We compute finite offsets for the entire Thingi10k dataset and observe that the running time is reduced compared to topological offsets due to the reduced geometric complexity (Figure 26); 63.7% of the models finish within 5 minutes. Just 21 models (0.2%) take more than one hour. None of our finite offsets has a normal deviation larger than 20° (Figure 29).

7.1.1 Comparison. We compare our finite offsets (Section 7.1) with Feature-Preserving Offsets (FPO) [Zint et al. 2023], and 3D Alpha Wrapping in CGAL [Alliez et al. 2024]. We run FPO with default parameters except for the normal deviation which we set to 15° . For Alpha Wrapping, we set $\alpha = \delta/5$, which he chose to produce similar results. Note that Alpha Wrapping is not an offsetting method and, therefore, does not claim to be feature-preserving or topologically correct. We chose this method to compare against because it comes with similar guarantees to ours (watertight, orientable, and strictly contains the input).

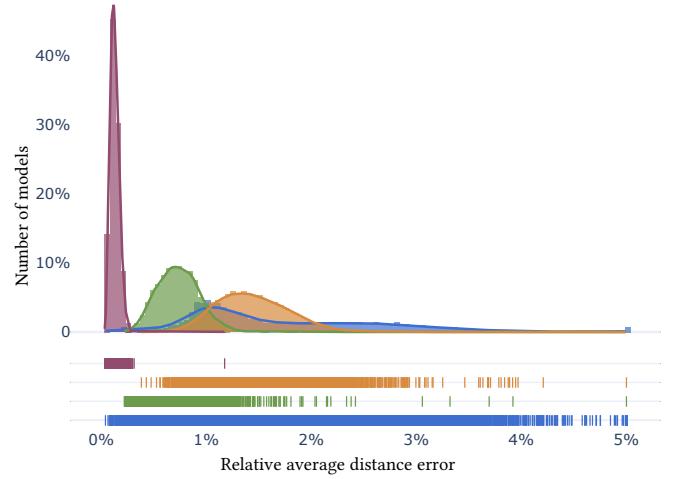


Fig. 33. Relative average distance error of Alpha Wrapping (red), FPO (yellow), our finite offsets (green), and our topological offsets (blue).

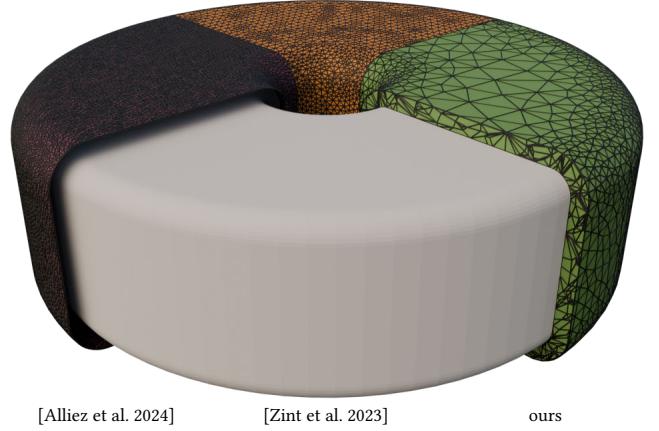


Fig. 34. All three methods perform equally well on simple models. [Alliez et al. 2024] in red, [Zint et al. 2023] in orange, and ours in green.

Normal Deviation. All four methods produce meshes with similar average normal deviation (Figure 29). Alpha Wrapping performs slightly better, which can be explained by the larger number of triangles (Figure 28).

Offset Distance. We report the average distance error relative to the user-defined target distance in Figure 33. Again, Alpha Wrapping (red) performs best, but it also has the largest amount of triangles, which influences the metric. Our method is adaptive to curvature and therefore places more triangles where normal deviation is high, while Alpha Wrapping has a uniform triangle distribution and therefore more triangles in flat regions with zero normal deviation.

Qualitative Comparison. On simple models, like the one in Figure 34, Alpha Wrapping, FPO, and our finite offsets perform equally well. However, our method requires significantly more time to produce similar results. Alpha Wrapping and FPO finish in 2.6 and



Fig. 35. Our method is the only one with a varying sizing field.

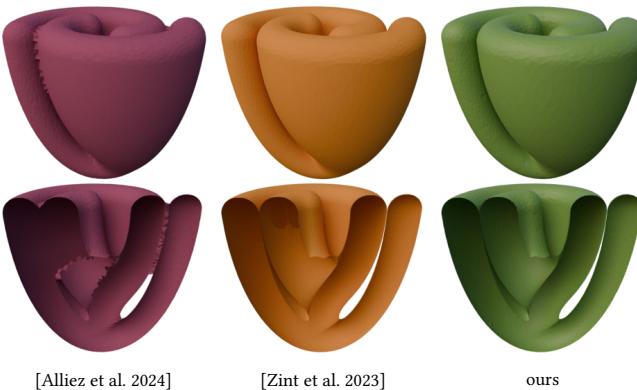
Fig. 36. Our method (green) is well suited for small offset distances. We use $\delta = 0.01\%$ and set ℓ_{\min} to the average input edge length (white) for ours and FPO (orange). For Alpha Wrapping (red), we set $\alpha = 50\delta$ to achieve comparable edge lengths.

Fig. 37. Offsets generated with different methods.

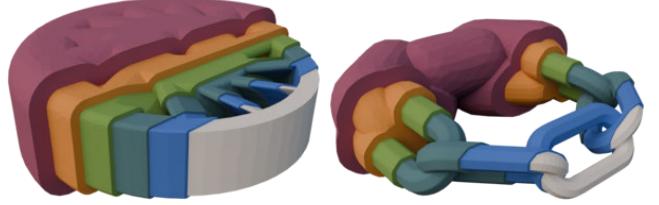


Fig. 38. Layered finite offsets at 8%, 4%, 2%, 1%, and 0.5% of the bounding box size.

33.7 seconds, respectively, while our method needs 242 seconds. This overhead is caused by the background mesh that needs to be updated. Our method returns a tetrahedral mesh in which the input and offset are embedded, while the others only generate surfaces.

Our method uses a sizing field to be adaptive to the offset curvature. Flat regions are not unnecessarily refined (Figure 35) while the mesh is denser in regions with high curvature compared to the meshes from the other methods.

Our method is well suited for small offset distances (Figure 36). In this example, we set ℓ_{\min} to the input average edge length to avoid unnecessary refinement. Alpha Wrapping introduces artifacts in convex regions (we picked α so that it creates a similar number of triangles as the input). For such a small distance, FPO produces visible intersections (white speckles in Figure 36) and overrefines the offset even with the minimal edge length restricted.

An extreme challenge for offset methods is when two offsets are almost colliding (Figure 37). Alpha Wrapping cannot enter the thin area in between the two offsets. FPO produces a good-looking result from the outside but generates self-intersections on the interior. Our method produces the desired outcome and is free of self-intersections.

7.2 Layered Offsets

We can construct multiple (finite or topological) non-intersecting offset layers by adding a simple condition to the offset initialization (Figure 38). The outermost layer must be generated first. For the next layer, the offset volume expansion (Section 5.3.2) must not touch any previously generated layer. This guarantees that all layers are intersection-free.

7.3 Manifold Extraction

A common way to remove non-manifold vertices is to duplicate them and displace them in the opposite normal direction. While this method is simple, it comes with several drawbacks. First, it is not guaranteed that a valid normal direction always exists, and therefore it can fail for certain scenarios. Second, if the mesh was embedded in a background mesh, the region around the duplicated vertex needs to be remeshed. For an in-depth discussion of related work, we refer the reader to [Attene et al. 2009].

We can remove non-manifold simplices from a mesh by constructing a topological offset around them. First, we detect all non-manifold simplices and consider them as the input for our topological offset. Second, we remove all simplices from the input mesh within the offset region, obtaining a manifold mesh.

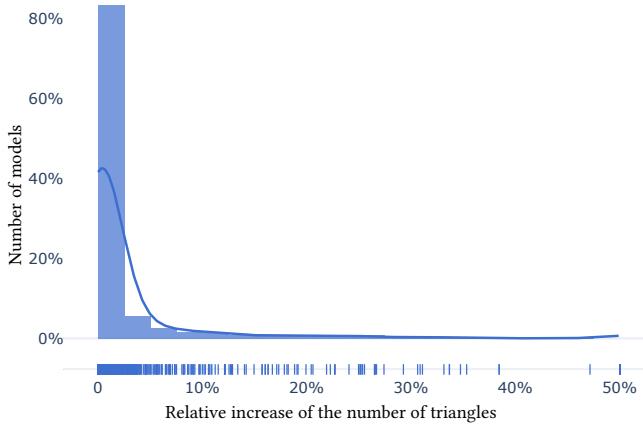


Fig. 39. Increase in the number of triangles required to make the meshes manifold.

Our algorithm is guaranteed to generate manifold meshes and to keep the embedding valid; however, it might add unnecessary vertices to the surface. To mitigate this effect, we perform a “clean-up” after inserting the offset: we collapse edges within the region of the same non-manifold vertex. We only perform collapses that maintain a valid embedding. Finally, we push all remaining vertices to a user-defined offset distance. If the desired distance would cause tetrahedral inversions, we use a binary search to find a valid position. If we cannot find a valid position, we keep the vertex where it is.

This method has similarities to one of the methods presented in [Attene et al. 2009]: this method requires specifying a radius for the removal of the non-manifold points, and different choices of this radius will create different topologies. In contrast, our method is parameter-free and can be implemented robustly using floating-point computation only.

We run the non-manifold removal algorithm on the 1053 models from Thingi10k [Zhou and Jacobson 2016], processed by TetWild [Hu et al. 2018], whose surfaces are non-manifold. The algorithm succeeds for all models, i.e., there are no issues with floating point accuracy, no models contain inverted elements, and all surfaces do not intersect and are manifold. In Figure 39, we summarize the results: for 970 out of the 1053 models (92%), our method introduces less than 10% more triangles on the surface. There are only 4 models where the element count doubles since almost all surface vertices and edges are non-manifold. Figure 40 shows how non-manifold vertices and edges are successfully removed.

8 CONCLUDING REMARKS

We introduced topological offsets, an algorithm for computing them robustly, and demonstrated their relevance in a wide range of graphics applications.

While computationally more expensive than competing finite offset methods, our algorithm generates the unique topology of an infinitesimal offset, and guarantees to produce self-intersection-free offsets that strictly enclose the input. We use these guarantees to extend our construction to multiple offsets (topological or finite)

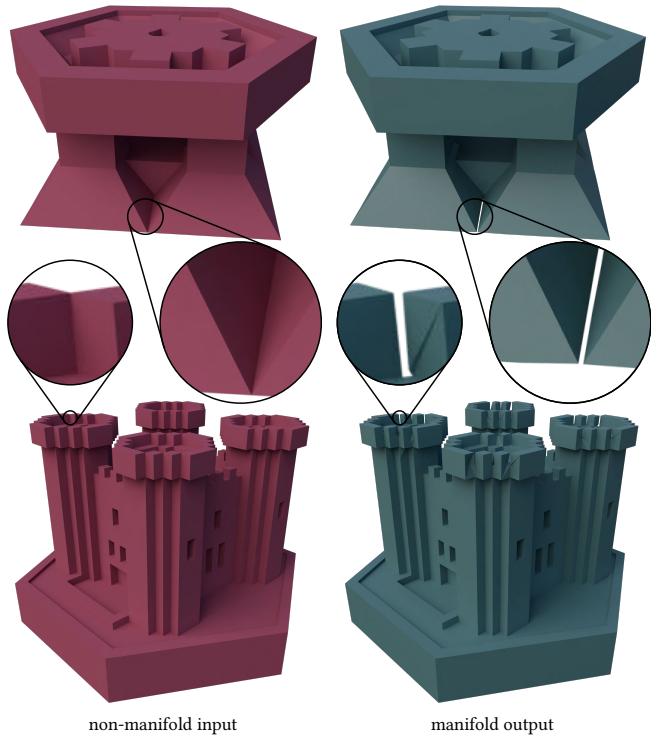


Fig. 40. Example of removal of non-manifold regions using our algorithm.

that inherit the same guarantees and therefore are strictly enclosing each other.

Exploring the use of this approach to create layered offsets (boundary layers) with exponentially increasing thickness and their use for fluid simulation is an exciting avenue for future work. Additionally, we plan to explore parallel or distributed mesh optimization methods to reduce the running time difference compared to other offset methods.

ACKNOWLEDGMENTS

This work was supported in part through the NYU IT High Performance Computing resources, services, and staff expertise. This work was also partially supported by the NSF grants OAC-2411349 and IIS-2313156, the NSERC grants DGECR-2021-00461 and RG-PIN 2021-03707, as well as a gift from Adobe Research.

REFERENCES

- Pierre Alliez, David Cohen-Steiner, Michael Hemmer, Cédric Portaneri, and Mael Rouxel-Labbé. 2024. 3D Alpha Wrapping. In *CGAL User and Reference Manual* (5.6.1 ed.). CGAL Editorial Board. <https://doc.cgal.org/5.6.1/Manual/packages.html#PkgAlphaWrap3>
- Marco Attene, Daniela Giorgi, Massimo Ferri, and Bianca Falcidieno. 2009. On converting sets of tetrahedra to combinatorial and PL manifolds. *Computer Aided Geometric Design* 26, 8 (2009), 850–864. <https://doi.org/10.1016/j.cagd.2009.06.002>
- R Aubry, S Dey, EL Mestreau, and BK Karamete. 2017. Boundary layer mesh generation on arbitrary geometries. *Internat. J. Numer. Methods Engrg.* 112, 2 (2017), 157–173.
- Randolph E Bank and R Kent Smith. 1997. Mesh smoothing using a posteriori error estimates. *SIAM J. Numer. Anal.* 34, 3 (1997), 979–997.
- Imma Boada, Narcís Coll, Narcís Mader, and J Antoni Sellares. 2008. Approximations of 2d and 3d generalized voronoi diagrams. *International Journal of Computer Mathematics* 85, 7 (2008), 1003–1022.

Table 1. Statistics for the examples presented in Figure 24. The columns display from left to right: mesh name, target offset distance, runtime in seconds, number of embedding tetrahedra, number of offset triangles, average triangle shape regularity, average normal deviation, average relative distance error.

dalek (49874)

δ	time (s)	#T	#t	qsr,avg	σ_{avg}	ϵ_{avg}
-1%	1 656	921 789	102 086	0.84	12	1.8%
1%	1 524	960 100	109 280	0.87	11	1.2%
2%	1 140	756 723	58 300	0.82	15	1.2%
4%	875	720 743	43 448	0.78	18	1.0%

rooster (57680)

δ	time (s)	#T	#t	qsr,avg	σ_{avg}	ϵ_{avg}
-1%	1 179	807 647	48 844	0.85	15	2.9%
1%	1 138	860 380	58 616	0.87	14	1.7%
2%	1 014	841 894	32 936	0.87	15	1.5%
4%	956	836 024	14 096	0.85	17	1.7%

lamp (61258)

δ	time (s)	#T	#t	qsr,avg	σ_{avg}	ϵ_{avg}
-1%	3 640	1 923 308	317 462	0.84	20	1.8%
1%	5 104	1 941 050	273 976	0.87	14	1.6%
2%	3 971	1 688 744	210 854	0.83	15	1.1%
4%	3 773	1 541 226	215 350	0.82	15	0.6%

- Jonathan Borwein and Adrian Lewis. 2006. *Convex Analysis and Nonlinear Optimization*. Springer New York, New York, NY. <https://doi.org/10.1007/978-0-387-31256-9>
- Mario Botsch and Leif Kobbelt. 2004. A remeshing approach to multiresolution modeling. In *Proceedings of the 2004 Eurographics/ACM SIGGRAPH symposium on Geometry processing*. Association for Computing Machinery, New York, NY, USA, 185–192.
- Anders Brodersen, Ken Museth, Serban Porumbescu, and Brian Budge. 2008. Geometric Texturing Using Level Sets. *IEEE Transactions on Visualization and Computer Graphics* 14, 2 (2008), 277–288. <https://doi.org/10.1109/TVCG.2007.70408>
- Stéphane Calderon and Tamy Boubekeur. 2014. Point morphology. *ACM Trans. Graph.* 33, 4, Article 45 (jul 2014), 13 pages. <https://doi.org/10.1145/2601097.2601130>
- Stéphane Calderon and Tamy Boubekeur. 2017. Bounding proxies for shape approximation. *ACM Trans. Graph.* 36, 4, Article 57 (jul 2017), 13 pages. <https://doi.org/10.1145/3072959.3073714>
- Marcel Campen and Leif Kobbelt. 2010a. Exact and robust (self-) intersections for polygonal meshes. In *Computer Graphics Forum*, Vol. 29. Wiley Online Library, 397–406.
- Marcel Campen and Leif Kobbelt. 2010b. Polygonal boundary evaluation of Minkowski sums and swept volumes. In *Computer Graphics Forum*, Vol. 29. Wiley Online Library, 1613–1622.
- Zhen Chen, Daniele Panozzo, and Jeremie Dumas. 2019. Half-space power diagrams and discrete surface offsets. *IEEE Transactions on Visualization and Computer Graphics* 26, 10 (2019), 2970–2981.
- Jiayi Dai, Yiqun Wang, and Dong-Ming Yan. 2024. Feature-preserving shrink wrapping with adaptive alpha. *Computer Aided Geometric Design* 111 (2024), 102321. <https://doi.org/10.1016/j.cagd.2024.102321>
- Lorenzo Dazzi, Daniele Panozzo, Amir Vaxman, and Marco Attene. 2023. Constrained Delaunay Tetrahedralization: A Robust and Practical Approach. *ACM Transactions on Graphics* 42, 6 (Dec. 2023), 1–15. <https://doi.org/10.1145/3618352>
- Noura Faraj, Jean-Marc Thiery, and Tamy Boubekeur. 2016. Multi-Material Adaptive Volume Remesher. *Computer and Graphics Journal (proc. Shape Modeling International 2016)* 58 (2016), 150–160.
- Rao V. Garimella and Mark S. Shephard. 2000. Boundary layer mesh generation for viscous flow simulations. *Internat. J. Numer. Methods Engrg.* 49, 1–2 (2000), 193–218. [https://doi.org/10.1002/1097-0207\(20000910/20\)49:1<193::aid-nme929>3.0.co;2-r](https://doi.org/10.1002/1097-0207(20000910/20)49:1<193::aid-nme929>3.0.co;2-r)
- André Guéziec and Robert Hummel. 1995. Exploiting triangulated surface extraction using tetrahedral decomposition. *IEEE Transactions on visualization and computer graphics* 1, 4 (1995), 328–342.
- Jia-Peng Guo, Wen-Xiang Zhang, Chunyang Ye, and Xiao-Ming Fu. 2024. Robust Coarse Cage Construction With Small Approximation Errors. *IEEE Transactions on Visualization and Computer Graphics* 30, 7 (2024), 4234–4245. <https://doi.org/10.1109/TVCG.2023.3255207>

- Michael Hemmer, Ophir Setter, and Dan Halperin. 2010. Constructing the Exact Voronoi Diagram of Arbitrary Lines in Three-Dimensional Space. In *Algorithms – ESA 2010*, Mark de Berg and Ulrich Meyer (Eds.). Springer Berlin Heidelberg, Berlin, Heidelberg, 398–409.
- Yixin Hu, Qingshan Zhou, Xifeng Gao, Alec Jacobson, Denis Zorin, and Daniele Panozzo. 2018. Tetrahedral meshing in the wild. *ACM Trans. Graph.* 37, 4 (2018), 60–1.
- Jingwei Huang, Yichao Zhou, and Leonidas Guibas. 2020. ManifoldPlus: A Robust and Scalable Watertight Manifold Surface Generation Method for Triangle Soups.
- Zhongshi Jiang, Teseo Schneider, Denis Zorin, and Daniele Panozzo. 2020. Bijective projection in a shell. *ACM Transactions on Graphics* 39, 6 (Nov. 2020), 1–18. <https://doi.org/10.1145/3414685.3417769>
- Tao Ju, Frank Losasso, Scott Schaefer, and Joe Warren. 2002. Dual contouring of hermite data. In *Proceedings of the 29th annual conference on Computer graphics and interactive techniques*. Association for Computing Machinery, New York, NY, USA, 339–346.
- Franjo Juretić and Norbert Putz. 2011. *A Surface-Wrapping Algorithm with Hole Detection Based on the Heat Diffusion Equation*. Springer Berlin Heidelberg, Berlin, Heidelberg, 405–418. https://doi.org/10.1007/978-3-642-24734-7_22
- Menelaos Karavelas. 2022. 2D Segment Delaunay Graphs. In *CGAL User and Reference Manual* (5.5.1 ed.). CGAL Editorial Board. <https://doc.cgal.org/5.5.1/Manual/packages.html#PkgSegmentDelaunayGraph2>
- Leif P. Kobbelt, Jens Vörsatz, Ulf Labsik, and Hans-Peter Seidel. 1999. A Shrink Wrapping Approach to Remeshing Polygonal Surfaces. *Computer Graphics Forum* 18, 3 (Sept. 1999), 119–130. <https://doi.org/10.1111/1467-8659.00333>
- Y. K. Lee, Chin K. Lim, Hamid Ghazialam, Harsh Vardhan, and Erling Eklund. 2009. Surface mesh generation for dirty geometries by the Cartesian shrink-wrapping technique. *Engineering with Computers* 26, 4 (Dec. 2009), 377–390. <https://doi.org/10.1007/s00366-009-0171-0>
- Yaron Lipman. 2014. Bijective Mappings of Meshes with Boundary and the Degree in Mesh Processing. *SIAM Journal on Imaging Sciences* 7, 2 (Jan. 2014), 1263–1283. <https://doi.org/10.1137/130939754>
- Shengjun Liu and Charlie CL Wang. 2010. Fast intersection-free offset surface generation from freeform models with triangular meshes. *IEEE Transactions on Automation Science and Engineering* 8, 2 (2010), 347–360.
- William E Lorensen and Harvey E Cline. 1987. Marching cubes: A high resolution 3D surface construction algorithm. *ACM siggraph computer graphics* 21, 4 (1987), 163–169.
- Adrien Loseille and Rainald Löhner. 2013. Robust boundary layer mesh generation. In *Proceedings of the 21st International Meshing Roundtable*. Springer, Berlin, Heidelberg, 493–511.
- David Martineau, Jeremy Gould, and J. Papper. 2016. AN INTEGRATED FRAMEWORK FOR WRAPPING AND MESH GENERATION OF COMPLEX GEOMETRIES. <https://api.semanticscholar.org/CorpusID:56019523>
- Jonas Martinez, Samuel Hornus, Frédéric Claux, and Sylvain Lefebvre. 2015. Chained segment offsetting for ray-based solid representations. *Computers & Graphics* 46 (2015), 36–47.
- Wenlong Meng, Shuangmin Chen, Zhenyu Shu, Shi-Qing Xin, Hongbo Fu, and Changhe Tu. 2018. Efficiently computing feature-aligned and high-quality polygonal offset surfaces. *Computers & Graphics* 70 (2018), 62–70.
- Marek Krzysztof Misztal and Jakob Andreas Bærentzen. 2012. Topology-adaptive interface tracking using the deformable simplicial complex. *ACM Trans. Graph.* 31, 3, Article 24 (jun 2012), 12 pages. <https://doi.org/10.1145/2167076.2167082>
- Darko Pavlović and Leif Kobbelt. 2008. High-resolution volumetric computation of offset surfaces with feature preservation. In *Computer Graphics Forum*, Vol. 27. Wiley Online Library, 165–174.
- Cédric Portaneri, Mael Rouxel-Labbé, Michael Hemmer, David Cohen-Steiner, and Pierre Alliez. 2022. Alpha wrapping with an offset. *ACM Transactions on Graphics* 41, 4 (July 2022), 1–22. <https://doi.org/10.1145/3528223.3530152>
- Serban D. Porumbescu, Brian Budge, Louis Feng, and Kenneth I. Joy. 2005. Shell maps. *ACM Trans. Graph.* 24, 3 (Jul 2005), 626–633. <https://doi.org/10.1145/1073204.1073239>
- Huamin Qu, Nan Zhang, Ran Shao, Arie Kaufman, and Klaus Mueller. 2004. Feature preserving distance fields. In *2004 IEEE Symposium on Volume Visualization and Graphics*. IEEE, 39–46.
- R Tyrrell Rockafellar and Roger J-B Wets. 2009. *Variational analysis*. Vol. 317. Springer Science & Business Media.
- Leonardo Sachet, Etienne Vouga, and Alec Jacobson. 2015. Nested cages. *ACM Transactions on Graphics* 34, 6 (Nov. 2015), 1–14. <https://doi.org/10.1145/2816795.2818093>
- Silvia Sellan, Jacob Kesten, Ang Yan Sheng, and Alec Jacobson. 2020. Opening and closing surfaces. *ACM Trans. Graph.* 39, 6, Article 198 (nov 2020), 13 pages. <https://doi.org/10.1145/3414685.3417778>
- Johnathan Richard Shewchuk. 1996. Robust adaptive floating-point geometric predicates. In *Proceedings of the Twelfth Annual Symposium on Computational Geometry (Philadelphia, Pennsylvania, USA) (SCG '96)*. Association for Computing Machinery, New York, NY, USA, 141–150. <https://doi.org/10.1145/237218.237337>
- Hang Si. 2015. TetGen, a Delaunay-Based Quality Tetrahedral Mesh Generator. *ACM Trans. Math. Softw.* 41, 2, Article 11 (Feb. 2015), 36 pages. <https://doi.org/10.1145/2651359>

- 2629697
- David A. Stuart, Joshua A. Levine, Ben Jones, and Adam W. Bargteil. 2013. Automatic Construction of Coarse, High-Quality Tetrahedralizations that Enclose and Approximate Surfaces for Animation. In *Proceedings of Motion on Games (MIG '13)*. ACM. <https://doi.org/10.1145/2522628.2522648>
- Vijai Kumar Suriyababu, Cornelis Vuik, and Matthias Möller. 2023. Towards a High Quality Shrink Wrap Mesh Generation Algorithm Using Mathematical Morphology. *Computer-Aided Design* 164 (Nov. 2023), 103608. <https://doi.org/10.1016/j.cad.2023.103608>
- Dilip Mathew Thomas, Vijay Natarajan, and Georges-Pierre Bonneau. 2011. Link Conditions for Simplifying Meshes with Embedded Structures. *IEEE Transactions on Visualization and Computer Graphics* 17, 7 (July 2011), 1007–1019. <https://doi.org/10.1109/tvcg.2010.90>
- Jane Tournois, Noura Faraj, Jean-Marc Thiery, and Tammy Boubekeur. 2023. Tetrahedral Remeshing. In *CGAL User and Reference Manual* (5.6 ed.). CGAL Editorial Board. <https://doc.cgal.org/5.6/Manual/packages.html#PkgTetrahedralRemeshing>
- Gokul Varadhan and Dinesh Manocha. 2004. Accurate Minkowski sum approximation of polyhedral models. In *12th Pacific Conference on Computer Graphics and Applications, 2004. PG 2004. Proceedings*. IEEE, 392–401.
- Fabien Vivodtzev, Georges-Pierre Bonneau, Stefanie Hahmann, and Hans Hagen. 2010. *Substructure Topology Preserving Simplification of Tetrahedral Meshes*. Springer Berlin Heidelberg, 55–66. https://doi.org/10.1007/978-3-642-15014-2_5
- Bolun Wang, Teseo Schneider, Yixin Hu, Marco Attene, and Daniela Panozzo. 2020. Exact and efficient polyhedral envelope containment check. *ACM Transactions on Graphics* 39, 4 (Aug. 2020). <https://doi.org/10.1145/3386569.3392426>
- Charlie CL Wang and Dinesh Manocha. 2013. GPU-based offset surface computation using point samples. *Computer-Aided Design* 45, 2 (2013), 321–330.
- Lei Wang, Xudong Wang, Pengfei Wang, Shuangmin Chen, Shiqing Xin, Jiong Guo, Wenping Wang, and Changhe Tu. 2024. PCO: Precision-Controllable Offset Surfaces with Sharp Features. *ACM Trans. Graph.* 43, 6, Article 169 (Nov. 2024), 16 pages. <https://doi.org/10.1145/3687920>
- Chee K Yap, Vikram Sharma, and Jyh-Ming Lien. 2012. Towards exact numerical Voronoi diagrams. In *2012 Ninth International Symposium on Voronoi Diagrams in Science and Engineering*. IEEE, 2–16.
- Qingnan Zhou and Alec Jacobson. 2016. Thingi10K: A Dataset of 10,000 3D-Printing Models. *arXiv preprint arXiv:1605.04797* (2016).
- Daniel Zint, Nissim Maruani, Mael Rouxel-Labbé, and Pierre Alliez. 2023. Feature-Preserving Offset Mesh Generation from Topology-Adapted Octrees. *Computer Graphics Forum* 42, 5 (2023), 12.

A QUALITY METRICS

Throughout the offset optimization, we use 3 quality metrics to test for convergence and to determine which operations to perform.

DEFINITION 4 (TRIANGLE SHAPE REGULARITY). *The shape regularity of a triangle t as defined in [Bank and Smith 1997] is its area $A(t)$ multiplied by a normalization pre-factor of $4\sqrt{3}$ and divided by the sum of squared edge lengths,*

$$q_{sr}(t) = 4\sqrt{3}A(t)/(l_1^2 + l_2^2 + l_3^2).$$

The shape regularity is zero for a degenerate triangle and one for an equilateral one.

DEFINITION 5 (TRIANGLE NORMAL DEVIATION). *The normal deviation $\sigma(t)$ of a triangle t is the maximum angle between the offset normal $n(p)$ at the triangle center p_c and the offset normal at any other point p_i within the triangle, excluding its boundaries,*

$$\sigma(t) = \max_{p_i \in t} \angle(n(p_c), n(p_i)).$$

The offset normal can be computed for any point in space by finding the projection point on the offset and normalizing the vector from the point in space to its projection. We compute the normal deviation of a triangle by comparing $n(p_c)$ with offset normals close to the vertices p_v of the triangle, more precisely at the positions $p_i = 0.1p_c + 0.9p_v$. This choice is motivated experimentally, as using more sampling points increases the running time with negligible improvements in quality.

DEFINITION 6 (OFFSET DISTANCE ERROR). *The offset distance error of a point on the offset mesh O is the absolute value of the distance of that point to the surface S minus the targeted offset distance δ .*