3.3.1 Qualitative Predictors

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Including Qualitative Variables into a Regression Model

Qualitative or factor variables as predictors:

- Gender
- Political affiliation
- Student status

They can be included in the model by creating an indicator or dummy variable.

Let's study the following scenarios:

- Predictors with only two levels
- Predictors with more than two levels

Predictors with Only Two Levels

Consider the Credit data set

The aim here is to predict which customers will default on their credit card debt.

Based on the gender variable:

$$x_{i1} = \begin{cases} 1 & \text{if the } i \text{th person is female} \\ 0 & \text{if the } i \text{th person is male} \end{cases}$$

Regression equation:

$$y_i = \beta_0 + \beta_1 x_{i1} + \epsilon_i = \begin{cases} \beta_0 + \beta_1 + \epsilon_i & \text{if the } i \text{th person is female} \\ \beta_0 + \epsilon_i & \text{if the } i \text{th person is male} \end{cases}$$

Least squares coefficient estimates

Gender is enconded as a dummy variable in the model (Table 3.7)

Average credit card debt:

► Males: \$509.80

► Females: \$509.80 + \$19.73 = \$529.53

p-value for the dummy variable is very high

Arbitrary Coding Scheme

The main difference is the way in which the coefficients are interpreted

$$x_{i1} = \begin{cases} 1 & \text{if the } i \text{th person is female} \\ 0 & \text{if the } i \text{th person is male} \end{cases}$$

$$x_{i1} = \begin{cases} 1 & \text{if the } i \text{th person is female} \\ -1 & \text{if the } i \text{th person is male} \end{cases}$$

Regression equation:

$$y_i = \beta_0 + \beta_1 x_{i1} + \epsilon_i = \begin{cases} \beta_0 + \beta_1 + \epsilon_i & \text{if the } i \text{th person is female} \\ \beta_0 - \beta_1 + \epsilon_i & \text{if the } i \text{th person is male} \end{cases}$$

Equations

$$x_{i1} = \begin{cases} 1 & \text{if the } i \text{th person is Asian} \\ 0 & \text{if the } i \text{th person is not Asian} \end{cases}$$

$$x_{i2} = \begin{cases} 1 & \text{if the } i \text{th person is Caucasian} \\ 0 & \text{if the } i \text{th person is not Caucasian} \end{cases}$$

$$y_i = \beta_0 + \beta_2 \mathbf{Y}_{ii} + \beta_0 \mathbf{Y}_{i0} + \beta_0 \mathbf{Y}_{i0}$$

$$\begin{aligned} y_i &= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \epsilon_i \\ &= \begin{cases} \beta_0 + \beta_1 + \epsilon_i & \text{if the } i \text{th person is Asian} \\ \beta_0 + \beta_2 + \epsilon_i & \text{if the } i \text{th person is Caucasian} \\ \beta_0 + \epsilon_i & \text{if the } i \text{th person is African American} \end{cases} \end{aligned}$$

Slide with Plot

