Assignment 1

(1) by question, $h(x_1, x_2) = \sigma(b + w_1 x_1 + w_2 x_2)$

σ is the sigmoid $\Rightarrow$ $\sigma(x) = \frac{1}{1+e^{-x}}$

$\theta_{n+1} = \theta_n - \alpha \nabla_\theta Loss \Rightarrow \theta_1 = \theta_0 - \alpha \nabla L(\theta_0)$ (α: 每次更新的步伐大小, learning rate)

$L(\theta) = \frac{1}{2}(h(x_1, x_2) - y)^2$

↑ single data, general form is $L(\theta) = \frac{1}{N}\sum_{i=1}^{N} \frac{1}{2}\left\| y_i - h(x_1^i, x_2^i) \right\|^2$

$\sigma'(x) = \frac{d}{dx}(1+e^{-x})^{-1} = -(1+e^{-x})^{-2} \cdot -e^{-x} = e^{-x}(1+e^{-x})^{-2}$

$\Rightarrow \sigma'(x) = \left(\frac{e^{-x}}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\sigma(x)$

Let $z = b + w_1 x_1 + w_2 x_2 \Rightarrow L(\theta) = \frac{1}{2}(\sigma(z) - y)^2$

$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial z} \cdot \frac{\partial z}{\partial b} = (\sigma(z) - y) \cdot (1 - \sigma(z))\sigma(z) \cdot 1$

same way to get

$\frac{\partial L}{\partial w_1} = (\sigma(z) - y)(1 - \sigma(z))\sigma(z) \cdot x_1$

$\frac{\partial L}{\partial w_2} = (\sigma(z) - y)(1 - \sigma(z))\sigma(z) \cdot x_2$

$z = b + w_1 x_1 + w_2 x_2 = 4 + 5 \cdot 1 + 6 \times 2 = 21$

$b' = b_0 - \frac{\partial L}{\partial b} = 4 - \alpha(\sigma(21) - 3)(1 - \sigma(21))\sigma(21)$

$w_1' = 5 - \alpha(\sigma(21) - 3)(1 - \sigma(21))\sigma(21)$

$w_2' = 6 - 2\alpha(\sigma(21) - 3)(1 - \sigma(21))\sigma(21)$

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(2)

(a) by question 1 blue part

we know when $k=1 \Rightarrow \sigma'(x) = (1-\sigma(x))\sigma(x)$

$k=2 \Rightarrow \frac{d}{dx}\left[\underbrace{(1-\sigma(x))}_{u}\underbrace{\sigma(x)}_{v}\right]$

$\frac{du}{dx} = -\underbrace{\sigma(x)(1-\sigma(x))}_{\sigma'(x)}$, $\frac{dv}{dx} = \underbrace{\sigma(x)(1-\sigma(x))}_{\sigma'(x)}$

$\frac{du}{dx}v + \frac{dv}{dx}u = \sigma(x)(1-\sigma(x))\left[-\sigma(x) + (1-\sigma(x))\right]$

$= \underbrace{\sigma(x)(1-\sigma(x))}_{\sigma'(x)}(1-2\sigma(x)) = \sigma''(x)$

$\frac{d}{dx}\sigma'(x) = \frac{d}{dx}\left[\underbrace{\sigma'(x)}_{u}\underbrace{(1-2\sigma(x))}_{v}\right]$

$\frac{du}{dx} = \sigma'(x)(1-2\sigma(x))$, $\frac{dv}{dx} = -2\sigma'(x)$

$\frac{du}{dx}v + \frac{dv}{dx}u = \sigma'(x)\left[(1-2\sigma(x))^2 + (-2\sigma'(x))\right]$

$= \sigma(x)(1-\sigma(x))\left[(1-2\sigma(x))^2 - 2\sigma(x)(1-\sigma(x))\right] = \sigma'''(x)$

(b)

$\sinh(x) = \frac{e^x - e^{-x}}{2}$, $\cosh(x) = \frac{e^x + e^{-x}}{2}$

$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\sigma(x) = \frac{1}{1+e^{-x}}$, try to find same denominator

$\tanh\left(\frac{x}{2}\right) = \frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2}} = \frac{1-e^{-x}}{1+e^{-x}} \Rightarrow 1 + \tanh\left(\frac{x}{2}\right) = \frac{2}{1+e^{-x}} \Rightarrow \sigma(x) = \frac{1+\tanh\left(\frac{x}{2}\right)}{2}$

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(3)

(1) 上課有提到房價模型有可能會有負的房價

老師有提到可能可以用 $h(x_1, x_2) = \sigma(b + x_1 w_1 + x_2 w_2)$

$\sigma(x) = \frac{1}{1+e^{-x}}$，假設 $x_1 = 0$（屋齡），$x_2 = 0$（地坪）

可能房價還是 > 0，不知道 $b + w_1 x_1 + w_2 x_2$

重新設計能不能解決這問題

(2)

data 需要分
- (1) training set
- (2) validation set
- (3) test set

不知道三者佔比的比例不同，會不會導致 training 的模型結果有很大差距？