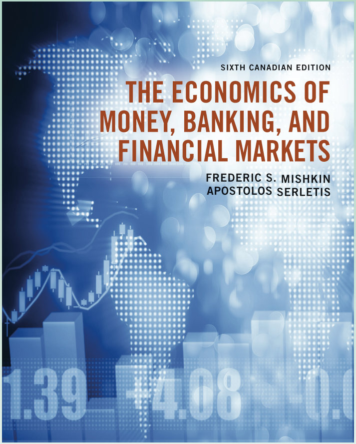
Mishkin/Serletis

The Economics of Money, Banking, and Financial Markets

Sixth Canadian Edition



Chapter 4

The Meaning of Interest Rates

Learning Objectives

1. Calculate the present value of future cash flows and the yield to maturity on the four types of credit market instruments
2. Recognize the distinctions among yield to maturity, current yield, rate of return, and rate of capital gain
3. Interpret the distinction between real and nominal interest rates

Measuring Interest Rates

How can we compare cash payments (**cash flows**) of different amounts and with different timing?

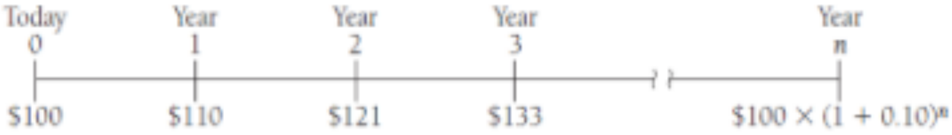
- **Present Value**:
  - *a dollar paid to you one year from now is less valuable than a dollar paid to you today*
  - *Consider a simple loan: Loan \$100 today and require \$110 repayment in one year. The **simple interest rate** is 10%.*
  - *Equivalently: At an interest of 10%, the present value of \$110 in one year's time is \$100 today.*

Discounting the Future

Repeat simple loan example for other repayment dates

- *If repay in one year: $100 x (1+0.1) = $110*
- *If repay in two years: $100 x (1+0.1)2 = $121*
- *If repay in three years: $100 x (1+0.1)3 = $133*
- *In general: $100 x (1+i)n where i is the interest rate and n is the number of years*

$\$100(1+i)^n$



Simple Present Value

PV = today's present value

CF = future cash flow or payment

i = interest rate

$$PV = \frac{CF}{(1+i)^n}$$

If the interest rate is 10%, what is PV of a security that pays \$1100 next year. \$1200 the year after. and \$1400 the year after that

$$PV = \frac{\$1100}{1.1} + \frac{\$1200}{1.1^2} + \frac{\$1400}{1.1^3}$$

$$= 3043.58$$

Lend. \$100 for 5 y at 10% interest what is the repayment amount.

$$\$100\,(1.1)^5 = 161.05$$

Four Types of Credit Market Instruments

1. **Simple Loan**
   – *One payment at the maturity date*
2. **Fixed Payment Loan** OMIT Calculation.
   – *Multiple fixed payments at pre-specified dates*
3. **Coupon Bond** OMIT Calculation.
   – *A bond that pays fixed amounts (the coupons) at fixed dates, plus a final payment (the face value) at maturity*
4. **Discount Bond**
   – *A bond that pays zero coupons, only a final payment at maturity. "Discount" since price typically less than face value.*

Yield to Maturity

- There are several common ways to calculate interest rates; the most important is the **yield to maturity**
  - *It is the interest rate that equates the present value of all cash flow payments received from a debt instrument with its value today (the current price)*

- Simple loan example: If today's value is $100 and the payment due in one year's time is $110 then the yield to maturity is 10%

For simple loans, the simple interest rate is equal to the yield to maturity.

Simple Loan

PV = amount borrowed = \$100

CF = cash flow in one year = \$110

n = number

$$\$100 = \frac{\$110}{(1+i)^1}$$

$$(1+i)^1 x\$100 = \$110$$

$$(1+i) = \frac{\$110}{\$100}$$

Fixed Payment Loan

LV = loan value

FP = fixed yearly payment

N = number of years until maturity

$$LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \ldots + \frac{FP}{(1+i)^n}$$

Cannot solve this by hand, requires a computer

Coupon Bond

P = price of coupon bond

C = yearly coupon payment

F = face value of the bond

n = years to maturity

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

Cannot solve this by hand, requires a computer

Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

TABLE 4-1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in 10 Years (Face Value = $1000)

Price of Bond ($)	Yield to Maturity (%)
1200	7.13
1100	8.48
1000	10.00
900	11.75
800	13.81

Three Facts About Coupon Bonds

1. When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate

2. The price of a coupon bond and the yield to maturity are negatively related
   - *As the yield to maturity rises, the price of the bond falls*

3. The yield to maturity is greater than the coupon rate when the bond price is below its face value

Consol or Perpetuity

Pc = price of the consol

C = yearly interest payment

ic = yield to maturity of the consol

$$i_c = \frac{C}{P_c}$$

- *A bond with no maturity date that does not repay principal but pays fixed coupon payments forever*

Discount Bond

F = face value of the discount bond

P = current price of the discount bond

$$i = \frac{F - P}{P}$$

\$1000 face-value discount bond maturing in one year that sells for \$800.

$$i = \frac{1000 - 800}{800} = 25\%$$

Note:

P is the current price of the discount bond.

F is the face value of the discount bond.

There is an inverse relationship between current price and interest rate.

If the interest rate rises, the price falls and vice versa.

$$PV = \frac{CF}{1+i}$$ CF: Cash Flow in one Y.

$$PV = \frac{1000}{1.1} = 909.09 \qquad PV = \frac{\$1000}{1.2} = 833.33$$

The Distinction Between Interest Rates and Returns

How well a person does financially by holding a bond for some period of time is the **rate of return**

– *The return (R) depends on coupons received (C) and the price for which the bond is eventually sold:*

$$R = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

$$\frac{C}{P_t} = \text{current yield} = i_c$$

$$\frac{P_{t+1} - P_t}{P_t} = \text{rate of capital gain} = g$$

Rate of Return - For any security, the amount of each payment to the owner + the change in the security's value, as a fraction of the purchase price.

$$R = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

Current yield. : $i = \frac{C}{P_t}$ Yearly payment / price.

The Distinction Between Interest Rates and Returns (cont'd)

- The return equals the yield to maturity only if the holding period equals the time to maturity
- A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if time to maturity is longer than the holding period
- The more distant a bond's maturity, the greater the size of the percentage price change associated with an interest-rate change

The Distinction Between Interest Rates and Returns (cont'd)

- The more distant a bond's maturity, the lower the rate of return the occurs as a result of an increase in the interest rate
- Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise

One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

TABLE 4-2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity when Bond is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return [col (2) + col (5)] (%)
30	10	1000	503	−49.7	−39.7
20	10	1000	516	−48.4	−38.4
10	10	1000	597	−40.3	−30.3
5	10	1000	741	−25.9	−15.9
2	10	1000	917	−8.3	+1.7
1	10	1000	1000	0.0	+10.0

*Calculated with a financial calculator, using Equation 3.

P_{t+1} given in COL 4. calculated. using Equation 3.

Example: first row 10% → 20%

$$R = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

$$= 10 + \frac{503 - 1000}{1000}$$

$$= 10 + (-49.7)$$

$$= -39.7$$

Interest-Rate Risk

The risk level associated with an asset's return that results from interest-rate changes is its **interest-rate risk**

- Prices and returns for long-term bonds are more volatile than those for shorter-term bonds
- There is no interest-rate risk for any bond whose time to maturity matches the holding period

Real and Nominal Interest Rates

- **Nominal interest rates** make no allowance for inflation
- **Real interest rates** adjust for changes in price level
  - *more accurately reflects the cost of borrowing*

- Ex ante real interest rate is adjusted for expected changes in the price level

$r = i - \pi^e$

- Ex post real interest rate is adjusted for actual changes in the price level

$r = i - \pi$

Fisher Equation

i = nominal interest rate

r = real interest rate

πe = expected inflation rate

$$i = r + \pi^e$$

The real interest rate will be low or negative if the expected inflation rate is high, for a given nominal rate

Example:

The lender wants a real return of 8% and expects the inflation to be 2%. Therefore, the nominal rate is 10%.

If inflation goes up to 4%, the real return is 6%.

Example 2:

For nominal rate is 10% inflation is 12%. the real return is -2%? The lender receive 10% more dollars, but goods are 12% more expensive. the lender is actually worse off.

Fisher Equation (cont'd)

- When the real interest rate is low, there are greater incentives to borrow $i = r + \pi^e$ $\downarrow$
- Low interest rates reduces the incentives to lend
- The real interest rate is a better indicator of the incentives to borrow or lend

Real and Nominal Interest Rates

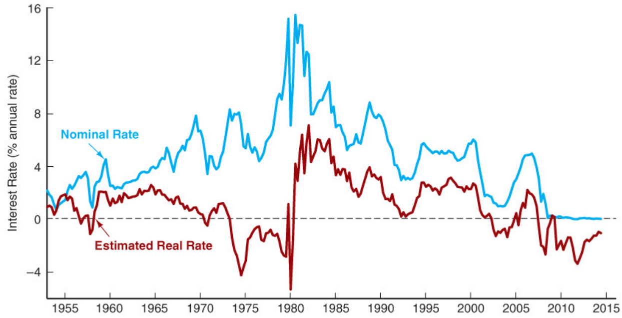


FIGURE 4-1 Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2014

Nominal and real interest rates often do not move together. When nominal rates were high in the 1970s, real rates were actually extremely low—often negative.

Sources: Nominal rates from **www.federalreserve.gov/releases/H15**. The real rate is constructed using the procedure outlined in Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," *Carnegie-Rochester Conference Series on Public Policy* 15 (1981): 151–200. This procedure involves estimating expected inflation as a function of past interest rates, inflation, and time trends, and then subtracting the expected inflation measure from the nominal interest rate.