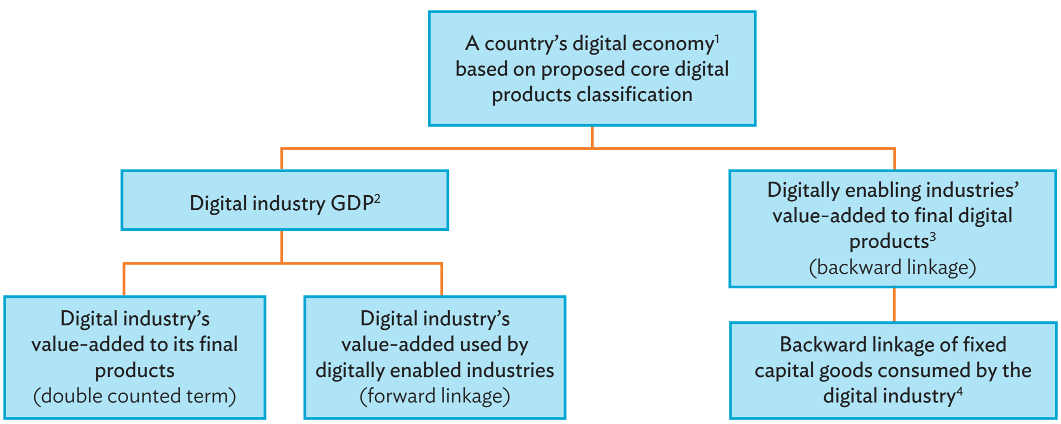
Measurement Framework

The models involved in the proposed measurement framework are rooted in input-output analysis, mainly using Leontief coefficients (Leontief 1936), as well as forward and backward linkages to directly measure the sector interdependencies in terms of value-added contributions.

In this section, a step-by-step derivation of the digital GDP equation is shown.[17] The components of the digital economy measurement framework are summarized in Figure 2. Given that each term pertains to a specific measure, users applying this framework may choose to calculate only certain terms for their purposes (e.g., only term 2 is needed to obtain the forward linkages of digital industries). Moreover, adjustments or extensions to the framework may be made to suit specific analyses, such as the measurement of specific global value chain (GVC) indicators, which is also covered in a later section.

Figure 2: Proposed Digital Economy Measurement Framework



GDP = gross domestic product.

1 Given by the $\mathrm{GDP}_{\mathrm{digital}}$ equation, $\mathrm{i}^{\mathrm{T}}\, \hat{\mathrm{V}}\mathrm{B}\hat{\mathrm{Y}}\boldsymbol{\varepsilon}_1 + \mathrm{i}^{\mathrm{T}}\, (\hat{\mathrm{V}}\mathrm{B}\hat{\mathrm{Y}})^{\mathrm{T}}\, \boldsymbol{\varepsilon}_1 - [\mathrm{diag}(\hat{\mathrm{V}}\mathrm{B}\hat{\mathrm{Y}})]^{\mathrm{T}}\, \boldsymbol{\varepsilon}_1 + (\mathrm{i} - \boldsymbol{\varepsilon}_1)^{\mathrm{T}}\, \hat{\mathrm{V}}\mathrm{B}\hat{\mathrm{Y}}\hat{\mathrm{r}}\, \boldsymbol{\varepsilon}_2$.

2 Given by the second term of the $\mathrm{GDP}_{\mathrm{digital}}$ equation.

3 Given by the first term of the $\mathrm{GDP}_{\mathrm{digital}}$ equation.

4 Given by the fourth term of the $\mathrm{GDP}_{\mathrm{digital}}$ equation.

Source: Methodology of the Digital Economy Measurement Framework study team, using Leontief (1936) coefficients.

[17] Throughout this report, digital GDP (or $\mathrm{GDP}_{\mathrm{digital}}$) refers to the gross value-added (GVA) of the digital sector. In a strict sense, digital GDP and digital GVA are similar, except that digital GDP includes net taxes on digital products. Despite the difference, digital GDP and digital GVA are expected to follow the same trends when only shares of digital GVA to total GVA are being examined, as was done in this report.

Deriving Gross Domestic Product in Terms of Leontief Inverse Coefficients

In Appendix 1, it is shown through Equations 1 to 3, that gross outputs $\mathbf{x}$ in a standard input-output table (IOT) can be concisely represented as a function of the Leontief Inverse, $(\mathbf{I} - \mathbf{A})^{-1}$, and final demand, $\mathbf{y}$. Equation 4 describes this relationship.

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} \tag{4}$$

Further mathematical manipulations would also allow derivation of a similar equation for economy-wide GDP. For brevity, let the Leontief inverse, $(\mathbf{I} - \mathbf{A})^{-1} \equiv \mathbf{B}$. A direct value-added coefficient vector is defined as

$$\mathbf{v} = (\mathrm{v}_1\ \mathrm{v}_2 \ldots \mathrm{v}_n) = \left(\frac{\mathrm{gva}_1}{\mathrm{x}_1}\ \frac{\mathrm{gva}_2}{\mathrm{x}_2} \ldots \frac{\mathrm{gva}_n}{\mathrm{x}_n}\right) \tag{5}$$

where, gva_j, $j = 1, 2, \ldots, n$, refers to the gross value-added (GVA) generated by industry j and x_j refers to the gross output of the same industry j. Thus, each entry in $\mathbf{v}$ is the ratio of industry j's GVA to its own output. It is shown below that pre-multiplying $\mathbf{v}$ from Equation 5 to $\mathbf{x}$ from Equation 4 would yield an expression that calculates economy-wide GDP via the production approach (Equation 6).[18] Knowing how to derive economy-wide GDP using the $\mathbf{vBy}$ formulation in Equation 6 is the first step in understanding how a more disaggregated digital GDP is quantified.

$$\begin{gathered}\mathbf{vx} = \mathbf{vBy}^{19} \\ \rightarrow \mathrm{gva}_1 + \mathrm{gva}_2 + \ldots + \mathrm{gva}_n = \sum_{i=1}^{n} \sum_{j=1}^{n} \mathrm{v}_i\, \mathrm{b}_{ij}\, \mathrm{y}_j \\ = \text{economy-wide GDP}\end{gathered} \tag{6}$$

[18] GDP via the production approach is calculated by summing value-added generated by all economic sectors.

[19] In expanded matrix form, $\mathbf{vx} = \mathbf{vBy}$

$$\rightarrow (\mathrm{v}_1\quad \mathrm{v}_2\quad \cdots\quad \mathrm{v}_n)\begin{pmatrix}\mathrm{x}_1\\ \mathrm{x}_2\\ \vdots\\ \mathrm{x}_n\end{pmatrix} = (\mathrm{v}_1\quad \mathrm{v}_2\quad \cdots\quad \mathrm{v}_n)\begin{bmatrix}\mathrm{b}_{11} & \mathrm{b}_{12} & \cdots & \mathrm{b}_{1n}\\ \mathrm{b}_{21} & \mathrm{b}_{22} & \cdots & \mathrm{b}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \mathrm{b}_{n1} & \mathrm{b}_{n2} & \cdots & \mathrm{b}_{nn}\end{bmatrix}\begin{pmatrix}\mathrm{y}_1\\ \mathrm{y}_2\\ \vdots\\ \mathrm{y}_n\end{pmatrix}$$

$$\rightarrow \left(\frac{\mathrm{gva}_1}{\mathrm{x}_1}\quad \frac{\mathrm{gva}_2}{\mathrm{x}_2}\quad \cdots\quad \frac{\mathrm{gva}_n}{\mathrm{x}_n}\right)\begin{pmatrix}\mathrm{x}_1\\ \mathrm{x}_2\\ \vdots\\ \mathrm{x}_n\end{pmatrix} = (\mathrm{v}_1\quad \mathrm{v}_2\quad \cdots\quad \mathrm{v}_n)\begin{bmatrix}\mathrm{b}_{11} & \mathrm{b}_{12} & \cdots & \mathrm{b}_{1n}\\ \mathrm{b}_{21} & \mathrm{b}_{22} & \cdots & \mathrm{b}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \mathrm{b}_{n1} & \mathrm{b}_{n2} & \cdots & \mathrm{b}_{nn}\end{bmatrix}\begin{pmatrix}\mathrm{y}_1\\ \mathrm{y}_2\\ \vdots\\ \mathrm{y}_n\end{pmatrix}$$

Disaggregating Gross Domestic Product across Users and Suppliers of Value-Added

The economy-wide GDP that is calculated using Equation 6 can be further disaggregated to an $n \times n$ matrix where an industry's backward and forward linkages can be derived. In particular, this matrix will show an industry's sources (backward linkages) and destination (forward linkages) of value-added. In the context of the digital economy, these sources and destinations respectively refer to industries on which digital sectors are dependent (digitally enabling industries), and industries that are enabled by digital sectors (digitally enabled industries).

Simple matrix operations involving the **v**, **B**, and **y** matrices are performed to get an industry's backward and forward linkages. Diagonalizing the direct value-added coefficient vector from Equation (5) and the final demand vector results in matrices $\hat{\mathbf{v}}$ and $\hat{\mathbf{y}}$ below.

$$\hat{\mathbf{v}} = \begin{bmatrix} v_1 & 0 & \dots & 0 \\ 0 & v_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & v_n \end{bmatrix}; \quad \hat{\mathbf{y}} = \begin{bmatrix} y_1 & 0 & \dots & 0 \\ 0 & y_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & y_n \end{bmatrix}$$

Pre-multiplying $\hat{\mathbf{v}}$ to **B** and then post-multiplying the matrix product to $\hat{\mathbf{y}}$ gives the $\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}$ matrix in Equation 7, which is an $n \times n$ matrix that disaggregates the scalar economy-wide GDP across all industries that use and supply value-added.

$$\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}} = \begin{bmatrix} v_1 & 0 & \dots & 0 \\ 0 & v_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & v_n \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix} \begin{bmatrix} y_1 & 0 & \dots & 0 \\ 0 & y_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & y_n \end{bmatrix}$$

$$\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}} = \begin{bmatrix} v_1 b_{11} y_1 & v_1 b_{12} y_2 & \dots & v_1 b_{1n} y_n \\ v_2 b_{21} y_1 & v_2 b_{22} y_2 & \dots & v_2 b_{2n} y_n \\ \vdots & \vdots & \ddots & \vdots \\ v_n b_{n1} y_1 & v_n b_{n2} y_2 & \dots & v_n b_{nn} y_n \end{bmatrix} \quad (7)$$

On the one hand, the rows of the $\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}$ matrix correspond to the distribution of the use of the value-added created from a particular industry across all industries in the economy. Therefore, adding all row entries gives an industry's GDP. Analogously, tracing the $\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}$ matrix row-wise corresponds to the forward linkages of the industry. The columns, on the other hand, correspond to the breakdown of value-added contributions of all industries in an economy to final goods and services production of a particular industry. Thus, summing all entries in a column result in the value of an industry's final products. In parallel, tracing the $\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}$ matrix column-wise shows the backward linkages of the industry.

Quantifying the Digital Economy in a Two-Industry Economy

For simplicity, it can first be assumed that there are two industries in a given economy, with Industry 1 being the digital industry. This will result in the 2 × 2 $\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}$ matrix below.

$$\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}} = \begin{bmatrix} v_1 b_{11} y_1 & v_1 b_{12} y_2 \\ v_2 b_{21} y_1 & v_2 b_{22} y_2 \end{bmatrix}$$

As mentioned, the sums of the first and second rows are equal to the GDP totals of the digital and nondigital industries, respectively.

$$\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}} = \begin{bmatrix} v_1 b_{11} y_1 & v_1 b_{12} y_2 \\ v_2 b_{21} y_1 & v_2 b_{22} y_2 \end{bmatrix} \quad \text{GDP of digital industry}$$

$$\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}} = \begin{bmatrix} v_1 b_{11} y_1 & v_1 b_{12} y_2 \\ v_2 b_{21} y_1 & v_2 b_{22} y_2 \end{bmatrix} \quad \text{GDP of nondigital industry}$$

In measuring the digital economy, the entirety of the digital industry's GDP must be obtained. The term $v_1\, b_{11}\, y_1$ accounts for the digital industry's value-added contribution to its own final products. The second term, $v_1\, b_{12}\, y_2$, is the value-added originating from the digital industry that is required by the nondigital industry. This also happens to be the contribution of the digital industry to the value of the nondigital industry's final products. Assuming that $v_1\, b_{12}\, y_2$ is not zero, even if the second industry does not produce digital goods and services, its production is enabled by the digital industry.[20] In this sense, Industry 2 is digitally enabled through forward linkage.

However, it is apparent in the first column that the value of the digital industry's final goods and services may be comprised not only of contributions from itself ($v_1\, b_{11}\, y_1$) but also from the nondigital industry ($v_2\, b_{21}\, y_1$). Assuming that $v_2\, b_{21}\, y_1$ is not zero, it is evident that the nondigital industry enables the production of the digital industry. In this sense, Industry 2 is digitally enabling through backward linkage. For this reason, $v_2\, b_{21}\, y_1$ will also be counted as part of the digital economy. The term $v_2\, b_{22}\, y_2$, on the other hand, pertains to value-added that originated from, and is used by, the nondigital industry. Since this does not involve transactions with the digital industry, it will not be counted as part of the digital economy.

Thus, the **GDP attributable to the digital economy is given by the entire GDP of the digital industry plus the portion of the nondigital industry's GDP that enables production in the digital industry**:

$$\mathrm{GDP}_{\mathrm{digital}} = \mathrm{GDP}_1 + \mathrm{GDP}_2 - v_2 b_{22} y_2$$

$$\mathrm{GDP}_{\mathrm{digital}} = v_1 b_{11} y_1 + v_1 b_{12} y_2 + v_2 b_{21} y_1$$

[20] One can say that a portion of the digital industry's value-added goes to the nondigital industry.

This can be directly calculated using the equation below:

$$\mathrm{GDP}_{\mathrm{digital}} = \mathbf{i}^{\mathrm{T}}\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\boldsymbol{\varepsilon}_1 + \mathbf{i}^{\mathrm{T}}(\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}})^{\mathrm{T}}\boldsymbol{\varepsilon}_1 - [\mathrm{diag}(\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}})]^{\mathrm{T}}\boldsymbol{\varepsilon}_1 \quad (8)$$

$$= (1 \quad 1)\begin{bmatrix} v_1b_{11}y_1 & v_1b_{12}y_2 \\ v_2b_{21}y_1 & v_2b_{22}y_2 \end{bmatrix}\begin{pmatrix}1\\0\end{pmatrix} + (1 \quad 1)\begin{bmatrix} v_1b_{11}y_1 & v_2b_{21}y_1 \\ v_1b_{12}y_2 & v_2b_{22}y_2 \end{bmatrix}\begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix} v_1b_{11}y_1 \\ v_2b_{22}y_2 \end{pmatrix}^{\mathrm{T}}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= (v_1b_{11}y_1 + v_2b_{21}y_1 \quad v_1b_{12}y_2 + v_2b_{22}y_2)\begin{pmatrix}1\\0\end{pmatrix} + (v_1b_{11}y_1 + v_1b_{12}y_2 \quad v_2b_{21}y_1 + v_2b_{22}y_2)\begin{pmatrix}1\\0\end{pmatrix} - (v_1b_{11}y_1 \quad v_2b_{22}y_2)\begin{pmatrix}1\\0\end{pmatrix}$$

$$= v_1b_{11}y_1 + v_2b_{21}y_1 + v_1b_{11}y_1 + v_1b_{12}y_2 - v_1b_{11}y_1$$

$$= v_1b_{11}y_1 + v_1b_{12}y_2 + v_2b_{21}y_1$$

The first term, $\mathbf{i}^{\mathrm{T}}\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\boldsymbol{\varepsilon}_1$, of Equation 8 directly calculates the backward linkage related to the digital industry while the second term, $\mathbf{i}^{\mathrm{T}}(\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}})^{\mathrm{T}}\boldsymbol{\varepsilon}_1$, gives the forward linkage. To account for the double-counted term, the diagonal entry in the $\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}$ matrix that corresponds to the digital industry is removed, which is why $[\mathrm{diag}(\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}})]^{\mathrm{T}}\boldsymbol{\varepsilon}_1$ is subtracted in $\mathrm{GDP}_{\mathrm{digital}}$. An "eliminator vector" $\boldsymbol{\varepsilon}_1$ is used to mathematically eliminate entries that should not be included in calculations. Such eliminator vectors will be used throughout the framework.

Quantifying the Digital Economy in a Simple Three-Industry Economy without Capital Formation

Implementing the method in the example above results in double counting if there are two or more digital industries that interact with each other. To demonstrate, let there be three industries in an economy, represented by the $\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}$ matrix below.

$$\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}} = \begin{bmatrix} v_1b_{11}y_1 & v_1b_{12}y_2 & v_1b_{13}y_3 \\ v_2b_{21}y_1 & v_2b_{22}y_2 & v_2b_{23}y_3 \\ v_3b_{31}y_1 & v_3b_{32}y_2 & v_3b_{33}y_3 \end{bmatrix}$$

Assume that Industry 1 and Industry 2 are digital. Applying Equation (8), $\mathrm{GDP}_{\mathrm{digital}}$ is expanded as a linear equation below.

$$\mathrm{GDP}_{\mathrm{digital}} = \mathbf{i}^{\mathrm{T}}\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\boldsymbol{\varepsilon} + \mathbf{i}^{\mathrm{T}}(\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}})^{\mathrm{T}}\boldsymbol{\varepsilon} - [\mathrm{diag}(\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}})]^{\mathrm{T}}\boldsymbol{\varepsilon}$$

$$\mathrm{GDP}_{\mathrm{digital}} = v_1b_{11}y_1 + v_2b_{21}y_1 + v_3b_{31}y_1 + v_1b_{12}y_2 + v_2b_{22}y_2 + v_3b_{32}y_2 + v_1b_{12}y_2 + v_1b_{13}y_3 + v_2b_{21}y_1 + v_2b_{23}y_3$$

As seen above, the terms, $v_2b_{21}y_1$ and $v_2b_{12}y_2$ are double counted since all value-added use of and value-added contribution to all digital industries in the economy are recorded. For example, $v_2b_{21}y_1$ is the value-added generated by Industry 2, which is used by Industry 1 and is therefore counted from a forward perspective. However, this also happens to be a source of value-added for Industry 1's final products and is then counted from a backward perspective.

From here, further adjustments are made in the framework to account for the interdependence of digital industries. A neat and simple solution is by aggregating similarly classified industries and treating them as a single sector, i.e., "the digital sector," since the two-industry case reveals that the $\mathrm{GDP}_{\mathrm{digital}}$ equation precludes any double counting when there is only a single digital industry.

In the framework, carrying out aggregations for the **Z**, **x**, **f**, and **gva** matrices makes use of "aggregator matrices." The full demonstration of how these matrices work can be found in Appendix 2. Therefore, after aggregating digital subsectors into one digital sector, the procedure in the two-industry case is still preserved, except that aggregator matrices are integrated into the framework. Thus, only some notational changes are necessary given by the following:

$$\mathbf{x}_{\mathrm{agg}} = \mathbf{Z}_{\mathrm{agg}}\mathbf{i} + \mathbf{y}_{\mathrm{agg}}$$

$$\mathbf{x}_{\mathrm{agg}} = \left(\mathbf{I} - \mathbf{A}_{\mathrm{agg}}\right)^{-1}\mathbf{y}_{\mathrm{agg}}$$

$$\left(\mathbf{I} - \mathbf{A}_{\mathrm{agg}}\right)^{-1} \equiv \mathbf{B}_{\mathrm{agg}}$$

$$\mathbf{v}_{\mathrm{agg}} = \begin{pmatrix} \mathrm{v}_1 & \mathrm{v}_2 & \cdots & \mathrm{v}_{n-q-1} \end{pmatrix}$$

Integrating these notational changes with Equation 8 results in the revised $\mathrm{GDP}_{\mathrm{digital}}$ equation in Equation 9.

$$\mathrm{GDP}_i = \mathrm{v}_i\mathrm{b}_{i1}\mathrm{y}_1 + \mathrm{v}_i\mathrm{b}_{i2}\mathrm{y}_2 + \cdots + \mathrm{v}_i\mathrm{b}_{i,n-q-1}\mathrm{y}_{n-q-1}, \quad i = 1,2,\ldots n-q-1$$

$$\hat{\mathbf{v}}_{\mathrm{agg}}\mathbf{B}_{\mathrm{agg}}\hat{\mathbf{y}}_{\mathrm{agg}} = \begin{bmatrix} \mathrm{v}_1\mathrm{b}_{11}\mathrm{y}_1 & \mathrm{v}_1\mathrm{b}_{12}\mathrm{y}_2 & \cdots & \mathrm{v}_1\mathrm{b}_{1,\mathrm{n-q-1}}\mathrm{y}_{n-q-1} \\ \mathrm{v}_2\mathrm{b}_{21}\mathrm{y}_1 & \mathrm{v}_2\mathrm{b}_{22}\mathrm{y}_2 & \cdots & \mathrm{v}_2\mathrm{b}_{2,n-q-1}\mathrm{y}_{n-q-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{v}_{n-q-1}\mathrm{b}_{n-q-1,1}\mathrm{y}_1 & \mathrm{v}_{n-q-1}\mathrm{b}_{n-q-1,2}\mathrm{y}_2 & \cdots & \mathrm{v}_{n-q-1}\mathrm{b}_{n-q-1,n-q-1}\mathrm{y}_{n-q-1} \end{bmatrix}$$

$$\mathrm{GDP}_{\mathrm{digital}} = \mathbf{i}^{\mathrm{T}}\hat{\mathbf{v}}_{\mathrm{agg}}\mathbf{B}_{\mathrm{agg}}\hat{\mathbf{y}}_{\mathrm{agg}}\boldsymbol{\varepsilon}_1 + \mathbf{i}^{\mathrm{T}}\left(\hat{\mathbf{v}}_{\mathrm{agg}}\mathbf{B}_{\mathrm{agg}}\hat{\mathbf{y}}_{\mathrm{agg}}\right)^{\mathrm{T}}\boldsymbol{\varepsilon}_1 - \left[\mathrm{diag}\left(\hat{\mathbf{v}}_{\mathrm{agg}}\mathbf{B}_{\mathrm{agg}}\hat{\mathbf{y}}_{\mathrm{agg}}\right)\right]^{\mathrm{T}}\boldsymbol{\varepsilon}_1 \tag{9}$$

Integrating Gross Fixed Capital Formation of the Digital Economy in a Three-Industry Economy

Equation 9 captures all contemporaneous input-output transactions with respect to exogenous final demand. However, if in the current year an industry purchases capital goods from a nondigital industry to use as inputs for future production, the **Z** matrix will not be able to capture this, as formation of fixed capital is reflected in the final demand vector, **y**.[21]

[21] Capital goods refer to fixed assets, or assets intended for use in the production of other goods and services for a period of more than 1 year, as defined by the System of National Accounts (SNA) 2008.

While the contribution of fixed capital formation to current year's production is reflected in the **gva** matrix as consumption of fixed capital, it fails to account for the various sector contributions required to produce said fixed capital as an output in the market. To illustrate, suppose there is a three-industry economy with Industry 1 as a digital industry and Industry 2 and Industry 3 as nondigital. Suppose further that Industry 1 purchases capital goods from Industry 3. In a standard input-output framework, this purchase by Industry 1 will be reflected in **y**. To show this, if **y** is disaggregated across three final demand components, for simplicity: household final consumption expenditure (hfce), general government consumption expenditure (ggce), and gross fixed capital formation (gfcf), then:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \mathbf{h} + \mathbf{g} + \mathbf{k} = \begin{bmatrix} \text{hfce}_1 \\ \text{hfce}_2 \\ \text{hfce}_3 \end{bmatrix} + \begin{bmatrix} \text{ggce}_1 \\ \text{ggce}_2 \\ \text{ggce}_3 \end{bmatrix} + \begin{bmatrix} \text{gfcf}_1 \\ \text{gfcf}_2 \\ \text{gfcf}_3 \end{bmatrix}$$

Further disaggregating vector **k** into a matrix with columns as the purchaser of capital and the rows as the seller of capital results in matrix **K**, where Industry 1's purchase of fixed capital from Industry 3 is equal to gfcf_{31}. Suppose gfcf_{31} is the only capital investment in the economy for the period.

$$\mathbf{K} = \begin{bmatrix} \text{gfcf}_{11} & \text{gfcf}_{12} & \text{gfcf}_{13} \\ \text{gfcf}_{21} & \text{gfcf}_{22} & \text{gfcf}_{23} \\ \text{gfcf}_{31} & \text{gfcf}_{32} & \text{gfcf}_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \text{gfcf}_{31} & 0 & 0 \end{bmatrix}$$

While matrix **K** shows which industry sold the capital, it does not show how said capital was produced. Therefore, without explicitly integrating the production of gross fixed capital purchased by digital Industry 1, the computation of $\text{GDP}_{\text{digital}}$ will be understated. This is because the capital goods produced by Industry 3 and purchased by Industry 1 also derived value from other industries in the economy. Thus, other industries' value-added shares to Industry 3's final products indirectly enable the digital economy and should therefore be counted as part of $\text{GDP}_{\text{digital}}$. The $\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}$ matrix already contains this information, but it still needs to be explicitly augmented to Equation 9.

To derive an equation that accounts for the backward linkage of fixed capital goods consumed by the digital industry (i.e., the GDP contribution of digitally enabling industries through capital formation), a single ratio for each of the columns corresponding to industries from which the digital sector purchased capital goods can be applied.[22]

[22] A single ratio would suffice, given technical coefficients are assumed to be fixed, following the Leontief insight (Leontief 1936).

In the previous illustration, multiplying the final product of Industry 3, $v_1 b_{13} y_3 + v_2 b_{23} y_3 + v_3 b_{33} y_3$, with a ratio, say $\mathbf{r}_3$, will give the value of fixed capital investment by Industry 1, $gfcf_{31}$. Let $\mathbf{r}$ be the vector of ratios of gfcf used by the digital industry to corresponding final demand and $\hat{\mathbf{r}}$ be the diagonalized $\mathbf{r}$.

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}; \quad \hat{\mathbf{r}} = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix}$$

Post-multiplying $\hat{\mathbf{r}}$ to $\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}$ gives:

$$\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}} = \begin{bmatrix} v_1 b_{11} y_1 & v_1 b_{12} y_2 & v_1 b_{13} y_3 \\ v_2 b_{21} y_1 & v_2 b_{22} y_2 & v_2 b_{23} y_3 \\ v_3 b_{31} y_1 & v_3 b_{32} y_2 & v_3 b_{33} y_3 \end{bmatrix} \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix} = \begin{bmatrix} r_1 v_1 b_{11} y_1 & r_2 v_1 b_{12} y_2 & r_3 v_1 b_{13} y_3 \\ r_1 v_2 b_{21} y_1 & r_2 v_2 b_{22} y_2 & r_3 v_2 b_{23} y_3 \\ r_1 v_3 b_{31} y_1 & r_2 v_3 b_{32} y_2 & r_3 v_3 b_{33} y_3 \end{bmatrix}$$

All elements in the first row and column of the $\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}}$ matrix will already be accounted for by Equation 9 within forward and backward linkages, respectively, of the digital Industry 1. To prevent double counting of a portion of the forward linkage of the digital industry in $GDP_{digital}$, $(\mathbf{i} - \boldsymbol{\varepsilon}_1)^T$ is pre-multiplied to $\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}}$:

$$(\mathbf{i} - \boldsymbol{\varepsilon}_1)^T \hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}} = (0 \quad 1 \quad 1) \begin{bmatrix} v_1 b_{11} y_1 & v_1 b_{12} y_2 & v_1 b_{13} y_3 \\ v_2 b_{21} y_1 & v_2 b_{22} y_2 & v_2 b_{23} y_3 \\ v_3 b_{31} y_1 & v_3 b_{32} y_2 & v_3 b_{33} y_3 \end{bmatrix} \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix}$$

$$\rightarrow (\mathbf{i} - \boldsymbol{\varepsilon}_1)^T \hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}} = \left(\sum_{i \neq 1}^{3} vby_{i1} \quad \sum_{i \neq 1}^{3} vby_{i2} \quad \sum_{i \neq 1}^{3} vby_{i3} \right) \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix}$$

$$\rightarrow (\mathbf{i} - \boldsymbol{\varepsilon}_1)^T \hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}} = \left(r_1 \sum_{i \neq 1}^{3} vby_{i1} \quad r_2 \sum_{i \neq 1}^{3} vby_{i2} \quad r_3 \sum_{i \neq 1}^{3} vby_{i3} \right)$$

Since Industry 1 only invests in final products of Industry 3, r_2 will be equal to zero, which leaves the following:

$$(\mathbf{i} - \boldsymbol{\varepsilon}_1)^T \hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}} = \left(r_1 \sum_{i \neq 1}^{3} vby_{i1} \quad 0 \quad r_3 \sum_{i \neq 1}^{3} vby_{i3} \right)$$

Another eliminator vector, $\boldsymbol{\varepsilon}_2 = (0 \quad 0 \quad 1)^T$ is then post-multiplied to $(\mathbf{i} - \boldsymbol{\varepsilon}_1)^T \hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}}$ to get: $(\mathbf{i} - \boldsymbol{\varepsilon}_1)^T \hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}}\boldsymbol{\varepsilon}_2 = (\mathbf{i} - \boldsymbol{\varepsilon}_1)^T \hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \left(r_1 \sum_{i \neq 1}^{3} vby_{i1} \quad 0 \quad r_3 \sum_{i \neq 1}^{3} vby_{i3} \right) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$\rightarrow (\mathbf{i} - \boldsymbol{\varepsilon}_1)^T \hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}}\boldsymbol{\varepsilon}_2 = r_3 \sum_{i \neq 1}^{3} vby_{i3}$$

The eliminator vector $\boldsymbol{\varepsilon}_2$ has a value of 1 for the row corresponding to the industry from which the digital industry purchases fixed capital, except itself. Excluding own-account capital formation of the digital industry from the calculation is required to prevent double counting of a portion of the backward linkage of the digital industry in $GDP_{digital}$. Therefore, in the illustration, the element of $\boldsymbol{\varepsilon}_2$ corresponding to digital industry, $\boldsymbol{\varepsilon}_{2_1}$, is set to zero, as well as $\boldsymbol{\varepsilon}_{2_2}$. Only $\boldsymbol{\varepsilon}_{2_3} = 1$ because Industry 1 only purchases fixed capital from Industry 3. The term $r_3 \sum_{i \neq 1}^{3} vby_{i3}$ corresponds to the backward linkage of fixed capital goods consumed by digital Industry 1 from nondigital Industry 3.[23]

Quantifying the Digital Economy in an *n*-Industry Economy

The three-industry case is generalizable to an economy with *n* industries. To illustrate, the dimension of the vector of ratios, **r**, is redefined to $n \times 1$. Correspondingly, this is diagonalized as $\hat{\mathbf{r}}$, to form an $n \times n$ matrix.

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}; \quad \hat{\mathbf{r}} = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & r_n \end{bmatrix}$$

Likewise, the $\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}}$ matrix will have a dimension of $n \times n$, as shown below.

$$\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}} = \begin{bmatrix} v_1 b_{11} y_1 & v_1 b_{12} y_2 & \cdots & v_1 b_{1j} y_j & \cdots & v_1 b_{1n} y_n \\ v_2 b_{21} y_1 & v_2 b_{22} y_2 & \cdots & v_2 b_{2j} y_j & \cdots & v_2 b_{2n} y_n \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ v_j b_{j1} y_1 & v_j b_{j2} y_2 & \cdots & v_j b_{jj} y_j & \cdots & v_j b_{jn} y_n \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ v_n b_{n1} y_1 & v_n b_{n2} y_2 & \cdots & v_n b_{nj} y_j & \cdots & v_n b_{nn} y_n \end{bmatrix} \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & r_j & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & r_n \end{bmatrix}$$
$$= \begin{bmatrix} r_1 v_1 b_{11} y_1 & r_2 v_1 b_{12} y_2 & \cdots & r_j v_1 b_{1j} y_j & \cdots & r_n v_1 b_{1n} y_n \\ r_1 v_2 b_{21} y_1 & r_2 v_2 b_{22} y_2 & \cdots & r_j v_2 b_{2j} y_j & \cdots & r_n v_2 b_{2n} y_n \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ r_1 v_j b_{j1} y_1 & r_2 v_j b_{j2} y_2 & \cdots & r_j v_j b_{jj} y_j & \cdots & r_n v_j b_{jn} y_n \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ r_1 v_n b_{n1} y_1 & r_2 v_n b_{n2} y_2 & \cdots & r_j v_n b_{nj} y_j & \cdots & r_n v_n b_{nn} y_n \end{bmatrix}$$

Now, suppose Industry 1 is a digital industry and that it purchases fixed capital from both Industry *j* and itself. Assume that only Industry 1 is digital, while the rest of the industries are nondigital. The $(\mathbf{i} - \boldsymbol{\varepsilon}_1)^{\mathrm{T}}\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}}$ equation becomes

$$(\mathbf{i} - \boldsymbol{\varepsilon}_1)^{\mathrm{T}}\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}} = \left(r_1 \sum_{i \neq 1}^{n} vby_{i1} \quad r_2 \sum_{i \neq 1}^{n} vby_{i2} \quad \cdots \quad r_j \sum_{i \neq 1}^{n} vby_{ij} \quad \cdots \quad r_n \sum_{i \neq 1}^{n} vby_{in} \right)$$
$$\rightarrow (\mathbf{i} - \boldsymbol{\varepsilon}_1)^{\mathrm{T}}\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}} = \left(r_1 \sum_{i \neq 1}^{n} vby_{i1} \quad 0 \quad \cdots \quad r_j \sum_{i \neq 1}^{n} vby_{ij} \quad \cdots \quad 0 \right)$$

To eliminate the double counting of the backward linkage of own-account fixed capital formation in the digital industry, the $n \times 1$ eliminator vector $\boldsymbol{\varepsilon}_2$ is post-multiplied to $(\mathbf{i} - \boldsymbol{\varepsilon}_1)^{\mathrm{T}}\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}}$ to arrive at a value for the backward linkage of fixed capital goods consumed by the digital industry.

$$(\mathbf{i} - \boldsymbol{\varepsilon}_1)^{\mathrm{T}}\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}}\boldsymbol{\varepsilon}_2 = \left(r_1 \sum_{i \neq 1}^{n} vby_{i1} \quad 0 \quad \cdots \quad r_j \sum_{i \neq 1}^{n} vby_{ij} \quad \cdots \quad 0 \right) \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$$

[23] Only domestic purchases of fixed capital are included within the framework estimates, as imported gross fixed capital formation is not produced within the domestic economy.

The Core Digital Economy Equation

The core digital economy equation (Equation 10) is derived by consolidating Equation 9 with the value of the backward linkage of fixed capital goods consumed by the digital industry. In Equation 10, the "agg" subscripts are suppressed for notational simplicity, but note that aggregation (as discussed in Appendix 2) was done prior to calculations.

$$\begin{aligned} \mathrm{GDP}_{\mathrm{digital}} &= \mathbf{i}^{\mathrm{T}}\hat{\mathbf{v}}_{\mathrm{agg}}\mathbf{B}_{\mathrm{agg}}\hat{\mathbf{y}}_{\mathrm{agg}}\boldsymbol{\varepsilon}_1 + \mathbf{i}^{\mathrm{T}}\left(\hat{\mathbf{v}}_{\mathrm{agg}}\mathbf{B}_{\mathrm{agg}}\hat{\mathbf{y}}_{\mathrm{agg}}\right)^{\mathrm{T}}\boldsymbol{\varepsilon}_1 - \left[\mathrm{diag}\left(\hat{\mathbf{v}}_{\mathrm{agg}}\mathbf{B}_{\mathrm{agg}}\hat{\mathbf{y}}_{\mathrm{agg}}\right)\right]^{\mathrm{T}}\boldsymbol{\varepsilon}_1 \\ &\quad + (\mathbf{i} - \boldsymbol{\varepsilon}_1)^{\mathrm{T}}\hat{\mathbf{v}}_{\mathrm{agg}}\mathbf{B}_{\mathrm{agg}}\hat{\mathbf{y}}_{\mathrm{agg}}\boldsymbol{\varepsilon}_2 \end{aligned}$$

$$\mathrm{GDP}_{\mathrm{digital}} = \mathbf{i}^{\mathrm{T}}\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\boldsymbol{\varepsilon}_1 + \mathbf{i}^{\mathrm{T}}(\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}})^{\mathrm{T}}\boldsymbol{\varepsilon}_1 - [\mathrm{diag}(\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}})]^{\mathrm{T}}\boldsymbol{\varepsilon}_1 + (\mathbf{i} - \boldsymbol{\varepsilon}_1)^{\mathrm{T}}\hat{\mathbf{v}}\mathbf{B}\hat{\mathbf{y}}\hat{\mathbf{r}}\boldsymbol{\varepsilon}_2 \tag{10}$$

In Equation 10, the four terms respectively refer to the digital economy's (i) backward linkages; (ii) forward linkages; (iii) the double-counted term (i.e., the aggregate digital industry's value-added contribution to its own final products), and (iv) the nondigital products it capitalizes.

Methodological Requirements

Supply and Use Tables and Input-Output Tables

The principal sources of data for the digital economy framework are national supply and use tables (SUTs) and IOTs. The supply table details how goods and services are supplied in an economy, either by domestic production or imports. On the other hand, the use table demonstrates how these outputs are used in the same economy, either as intermediate consumption, final consumption (which includes household final consumption expenditure, non-profit institutions serving households final consumption expenditure, and government final consumption expenditure), capital formation, or exports. SUTs are the main bases for national economic accounting systems, as a dataset that describes interactions within an economy and as a balancing framework for GDP calculations. This makes it an attractive source for various kinds of analytical uses and satellite systems (UN 2018).

The IOT combines the identities in the supply table and the use table into a single identity (UN 2018). As discussed, the proposed framework methodology requires matrices and vectors directly extracted from IOTs. SUTs may be easily transformed into IOTs using a transformation model prescribed by Eurostat (2008). For this report, the "fixed product sales structure" assumption was used to transform SUTs to IOTs, which converts a product-by-industry SUT to an industry-by-industry IOT.[24]

[24] Known as "Model D," this assumes that each product has its own specific sales structure, irrespective of the industry where it is produced (Eurostat 2008).

While IOTs allow a more organized application of Leontief's insight in analyses, SUTs provide greater detail on dynamics between products and industries at the rudimentary level. Thus, SUTs are particularly useful to capture the fourth term in the central formula (Equation 10), which incorporates the dependence of digital sectors on fixed capital. They may also be used for analyses concerning specific product-industry relationships, such as in assessing the digitalization of industries based on the use of digital products.

Uniformity Across National Tables

To ensure consistency with published aggregates, SUTs and/or IOTs are sourced from each economy's published tables on its respective national statistics office (NSO) website. Oftentimes, this entails further data collection and adjustment to apply the methodology as uniformly as possible across different economies. Three main concerns are considered to ensure uniformity and comparability of data: correspondence in classification systems, harmonization of SUT and IOT presentation format, and comparability in price and valuation.

Correspondence in classification systems

One major point of consideration is that different product and industry classification systems may be adopted by different economies. As such, identifying the exact same digital products and industries across economies requires close inspection and harmonization of these classification systems. For example, Canada uses the North American Industry Classification System, while Singapore uses its own Singapore Standard Industries Classification. Ensuring comparability between estimates of Canada's digital economy and that of Singapore requires an accurate correspondence between two different classification systems.

Another consideration is the varying levels of disaggregation of product and industry classification in SUTs or IOTs. Even when two economies adopt the same classification system, further data manipulation is necessary when disaggregation levels are not the same.

Harmonization of table presentation format

Another main concern is possible differences in the format by which SUTs and IOTs are presented for each economy. While presentation formats, in general, do not pose any real issue, problems arise when the variance pertains to difference in values contained in the $\mathbf{Z}$ matrix and $\mathbf{y}$ vector. For example, in the case of Japan, competitive imports are included in the intermediate consumption matrix. In the framework, the $\mathbf{Z}$ matrix only includes domestically produced inputs. Thus, appropriate adjustments must be made in such cases.

Comparability in price and valuation

Values in SUTs and IOTs may also be expressed in different prices (i.e., current prices or constant prices) and/or in different valuations (i.e., basic prices, producer's prices, or purchaser's prices). Tables at current prices are the bases of the main estimates produced by the framework. However, tables at constant prices are also employed when temporal analyses are made, such that only real changes are measured.

Furthermore, assuming that taxes, subsidies, and trade and transport margins are proportionately distributed across the products in an economy, estimates of $GDP_{digital}$ as a percentage of GDP calculated using tables valued in either basic, producer's, or purchaser's prices should not significantly differ from each other. Otherwise, when comparing across economies or time, it is preferred that the tables follow the same valuation.

The aforementioned are the most common differences observed across national tables. However, others may be encountered and should be appropriately addressed, especially when the inconsistency has a pervasive effect on the estimates. As long as the same methodology is applied given the available data, overall results per economy may be used for comparative analyses.

Disaggregating Products and Industries

Given the varying levels of product and industry disaggregation that economies present in their SUTs and IOTs, it is necessary to conduct a thorough evaluation of product and industry classification, then appropriately disaggregate the data. This poses a key challenge for tables with less than the desired level of detail, for which isolation of the exact digital activities identified for this methodology is crucial. As an example, software publishing is often combined with all publishing activities, and this needs to be extracted from other nondigital publishing activities.

Consing et al. (2020), a study that employed the same theoretical framework, examined and compared several data sources based on merits and drawbacks as a basis for disaggregation. Table 2 lists the established rankings of the top sources of data, from highest to lowest in terms of degree of reliability.

Table 2: Data Sources for Disaggregating Sectors

Source of Data	Merits	Drawbacks and/or Caveats
National statistics office	Highly reliable data consistent with the construction of SUT	Dependent on public availability of data or the NSO's responsiveness to queries
Relevant journals and published reports	Alternative of sourcing out if primary data are not available	Finding consistent and reliable data may be time-consuming, if even available
Supply table	Readily available in the SUT	Applies only if the desired degree of disaggregation among sectors is present
Operating revenue data from credible data resources	Readily available given permissions to access certain databases	May be limited by the amount of data collected by the resource
Data from donor economy	Based on an actual economy's industry disaggregation	Requires some degree of similarity in terms of structure between the two economies
Number of establishments from credible data resources	Readily available given permissions to access certain databases	Bias from an assumption of homogeneity

NSO = national statistics office, SUT = supply and use table.
Source: R. Consing III, M. Barsabal, J. Alvarez, and M. Mariasingham. 2020. The Wellness Economy, A Comprehensive System of National Accounts Approach. *Asian Development Bank Economics Working Paper* Series. No. 631. Manila: Asian Development Bank.

Using the best data disaggregation source available, a disaggregation ratio is calculated as the proportion of estimated digital activity (output) from the aggregate industry activity (output). The resulting percentage is then multiplied to all values in both the row and the column corresponding to the particular aggregate industry in the IOT. In effect, two subindustries replace the aggregate industry, expanding the dimension of the original IOT, but without changing its total measures and symmetry.

To illustrate, suppose there is the following 2 × 2 IOT:

	Industry 1	Industry 2	Final Demand	Gross Output
Industry 1	z_{11}	z_{12}	f_1	x_1
Industry 2	z_{21}	z_{22}	f_2	x_2
GVA	gva_1	gva_2		
Gross Output	x_1	x_2		

Suppose further that Industry 1 is an aggregate sector that contains both digital and nondigital subsectors. It is therefore necessary to disaggregate Industry 1 into two subindustries. Given the following revenue shares, derived from credible sources:

α which stands for the share of digital Industry 1a to Industry 1's total revenue, and

β which stands for the share of nondigital Industry 1b to Industry 1's total revenue.

where $\alpha + \beta = 1$, a disaggregated 3 × 3 IOT is obtained as follows:

	Industry 1a	Industry 1b	Industry 2	Final Demand	Gross Output
Industry 1a	$\alpha\alpha z_{11}$	$\alpha\beta z_{11}$	αz_{12}	αf_1	αx_1
Industry 1b	$\beta\alpha z_{11}$	$\beta\beta z_{11}$	βz_{12}	βf_1	βx_1
Industry 2	αz_{21}	βz_{21}	z_{22}	f_2	x_2
GVA	αgva_1	βgva_1	gva_2		
Gross Output	αx_1	βx_1	x_2		

Several checks have to be implemented to ensure the accuracy of disaggregation. First, the resulting 3 × 3 IOT should be symmetric with respect to its gross output, as in the original 2 × 2 IOT. Second, total gross output must be exactly the same for the two tables.[25] Last, the sum of the technical coefficients for Industry 1a and Industry 1b should be the same as the technical coefficient of aggregate Industry 1.[26] Note that this disaggregation method can be extended to an *n*-industry setting.

Construction of the Multiregional Input-Output Tables with Digital Sectors

When measuring international linkages, particularly global value chains (GVCs) in the context of the digital economy, credible regional or inter-economy IOTs should be used instead of individual national IOTs. One useful resource in conducting such analyses is the Multiregional Input-Output Tables (MRIOTs) produced by the Asian Development Bank. However, the main hurdle prior to conducting any GVC analyses for the digital economy is the aggregation level of the MRIOTs. As such, one of the key efforts of this project is the construction of these tables with industries disaggregated up to the level required in the framework.

The MRIOT database contains information on the production, consumption, and trade linkages of 62 economies, and an aggregated economy for "the rest of the world." Each economy has 35 sectors[27] and five final demand components.[28] The MRIOTs generally follow the sources and methods used to construct the World Input Output Database (WIOD), handled by the University of Groningen.[29]

25 To show that gross output is the same for the 2 × 2 and 3 × 3 IOTs:
$x_1 + x_2 = \alpha x_1 + \beta x_1 + x_2 \Rightarrow x_1 + x_2 = (\alpha + \beta)x_1 + x_2 \Rightarrow x_1 + x_2 = x_1 + x_2$ ∎

26 To show that the sum of technical coefficients of Industries 1a and 1b is equal to the technical coefficient of Industry 1:
$$\frac{z_{11}}{x_1} = \frac{\alpha\alpha z_{11}}{\alpha x_1} + \frac{\beta\alpha z_{11}}{\alpha x_1} = \frac{\alpha\beta z_{11}}{\beta x_1} + \frac{\beta\beta z_{11}}{\beta x_1} \Rightarrow \frac{z_{11}}{x_1} = \frac{\alpha z_{11}}{x_1} + \frac{\beta z_{11}}{x_1} \Rightarrow \frac{z_{11}}{x_1} = \frac{\alpha z_{11} + \beta z_{11}}{x_1} \Rightarrow \frac{z_{11}}{x_1} = \frac{(\alpha + \beta)z_{11}}{x_1} \Rightarrow \frac{z_{11}}{x_1} = \frac{z_{11}}{x_1} \blacksquare$$

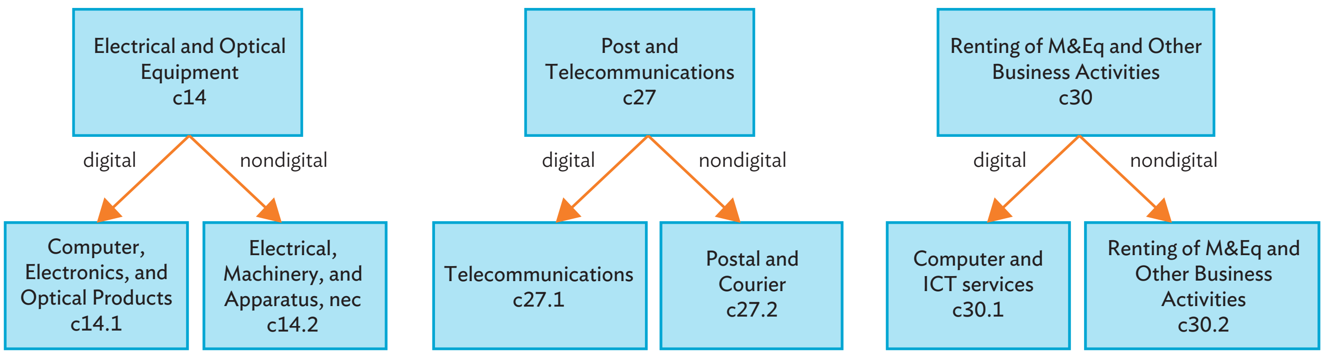
27 Table A3.1 (Appendix 3) outlines the 35 MRIO sectors.

28 The five final demand components include household final consumption expenditure (FCE), nonprofit institutions serving households (NPISHs) FCE, government FCE, gross fixed capital formation, and changes in inventories.

29 See Timmer et al. (2012) for details on constructing the WIOD.

The MRIOT sectors Electrical and Optical Equipment (c14), Post and Telecommunications (c27), and Renting of Machinery & Equipment and Other Business Activities (c30) include the digital sectors identified in the framework, and were therefore each split into two subsectors to isolate these digital subsectors. Thus, instead of the usual 35 sectors, this study uses a 38-sector MRIOT for 2017–2019. Figure 3 shows the six new sectors as a result of isolating digital industries. For the MRIOT, the authors had to disaggregate the three sectors for each of the 63 economies (Figure 4).

Figure 3: Isolating Digital Sectors in the Multiregional Input-Output Tables



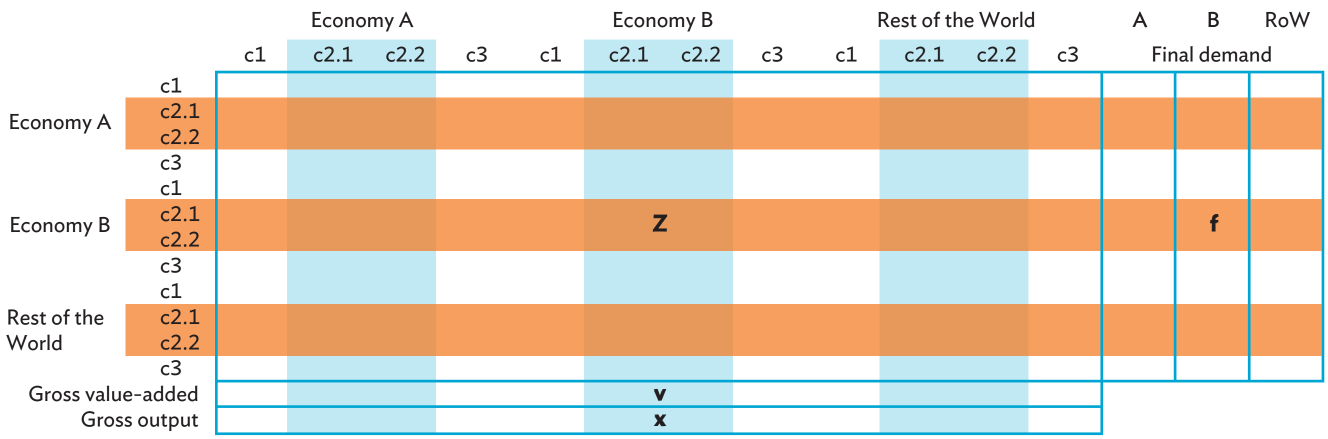
ICT = information and communication technology, M&Eq = machinery and equipment, nec = not elsewhere classified.
Source: Methodology of the Digital Economy Measurement Framework study team.

To isolate the digital component from c14, c27, and c30, column and row disaggregators were generated using multiple data sources. Column disaggregators gathered information from the WIOD and national SUTs to disaggregate the digital component in gross output, GVA, and imported inputs. The digital components for intermediate consumption and domestic inputs were calculated as a residual. Meanwhile, row disaggregators made use of bilateral exports and imports data by Broad Economic Categories classification[30] from the United Nations Commodity Trade (UN Comtrade) database (2017).[31] This information was converted into shares, which were subsequently used to split the rows and columns of the MRIO 63 × 35 tables into 63 × 38 tables. The authors then compared the resulting MRIOT values with the published NSO figures to ensure data consistency, and checked whether the table was balanced and/or symmetric.

30 The broad economic categories fall under intermediate use, final consumption, or capital goods.

31 The Comtrade database provides international trade data with variety in specification of product type, classification, year, and trade flow, among others (UN 2017).

Figure 4: The Multiregional Input-Output Disaggregation Process
(disaggregating the c2 sector)



RoW = Rest of the World.
Note: **Z** = intermediate consumption matrix, **v** = value-added vector, **x** = gross output vector, and **f** = final demand matrix.
Source: Methodology of the Digital Economy Measurement Framework study team.

Limitations of the Framework

The framework presented in this study aims to be entirely data-driven and based on economically and statistically sound approaches. Data collection and analysis adopted a mainly top-down strategy, relying on secondary data published by official and credible sources. As such, a range of data limitations arise.

First, the accessibility to granular data is often limited. Therefore, to disaggregate high-level data, direct inquiries to the appropriate NSOs are necessary, further supplemented by subordinate methods to extrapolate the required data. Where there are available data, the format, structure, and statistical compilation methods used may vary widely by economy, thus requiring a significant amount of data cleaning and processing. Therefore, a constraint exists in ensuring consistency and accuracy of all data.

Second, exclusions from what is defined to be the digital economy may be interpreted as limitations in completeness. This framework considers the narrowest possible definition of digital products. For example, the entire value of an online sale of a nondigital commodity is not considered. Instead, only the value contribution of the digital products (or the digital industries producing these) involved in such a transaction is captured. A narrow definition is employed in order to avoid ambiguities that require some arbitrary judgment. As the scope of digital products is at the narrowest level, it excludes the digitally dependent economy, which comprises the value-added of the sectors that are critically dependent on digital sectors. Nonetheless, the measurement framework is flexible to accommodate the calculation of this.

Appendix 1: A Standard Input-Output Table

A standard input-output table (IOT) is generally comprised of three quadrants. The first quadrant contains the $\mathbf{Z} = [z_{ij}]$ matrix, which is a matrix of interindustry flows of output from industry *i* (row) to industry *j* (column). The second quadrant contains the $\boldsymbol{y} = [\boldsymbol{y}_i]$ vector, which is a column vector of the final consumption of output from industry *i*. The vector of final demand is comprised of the aggregated final consumption of households, nonprofit institutions serving households, and government; and gross capital formation. The third quadrant contains the $\mathbf{gva'} = [gva_j]$ vector, which is a row vector of the gross value-added of industry *j*.

One of the important features of a standard IOT is its symmetry. Put simply, in an IOT, total output of industries (i.e., summing columns under intermediate consumption along the rows or **x'**) is equal to the total output used by industries and by final users (i.e., summing rows along columns or **x**). Table A1.1 shows the structure of a standard *n*-industry IOT.

Table A1.1: Standard Industry Input-Output Table

<table>
<tr><th rowspan="2"></th><th colspan="4">Intermediate consumption</th><th rowspan="2">Final demand</th><th rowspan="2">Gross output</th></tr>
<tr><th>Industry 1</th><th>Industry 2</th><th>⋯</th><th>Industry n</th></tr>
<tr><td>Industry 1</td><td colspan="4" rowspan="4">Quadrant I:
$\mathbf{Z}$</td><td rowspan="4">Quadrant II:
$\mathbf{y}$</td><td rowspan="4">$\mathbf{x}$</td></tr>
<tr><td>Industry 2</td></tr>
<tr><td>⋮</td></tr>
<tr><td>Industry n</td></tr>
<tr><td>Value-added</td><td colspan="4">Quadrant III:
$\mathbf{gva'}$</td><td></td><td></td></tr>
<tr><td>Gross output</td><td colspan="4">$\mathbf{x'}$</td><td></td><td></td></tr>
</table>

<table>
<tr><th rowspan="2"></th><th colspan="4">Intermediate consumption</th><th rowspan="2">Final demand</th><th rowspan="2">Gross output</th></tr>
<tr><th>Industry 1</th><th>Industry 2</th><th>⋯</th><th>Industry n</th></tr>
<tr><td>Industry 1</td><td>z_{11}</td><td>z_{12}</td><td>⋯</td><td>z_{1n}</td><td>y_1</td><td>x_1</td></tr>
<tr><td>Industry 2</td><td>z_{21}</td><td>z_{22}</td><td>⋯</td><td>z_{2n}</td><td>y_2</td><td>x_2</td></tr>
<tr><td>⋮</td><td>⋮</td><td>⋮</td><td>⋱</td><td>⋮</td><td>⋮</td><td>⋮</td></tr>
<tr><td>Industry n</td><td>z_{n1}</td><td>z_{n2}</td><td>⋯</td><td>z_{nn}</td><td>y_n</td><td>x_n</td></tr>
<tr><td>Value-added</td><td>gva_1</td><td>gva_2</td><td>⋯</td><td>gva_n</td><td></td><td></td></tr>
<tr><td>Gross output</td><td>x_1</td><td>x_2</td><td>⋯</td><td>x_n</td><td></td><td></td></tr>
</table>

Source: Construction of the Digital Economy Measurement Framework study team.

The symmetry of the IOT provides an organized visual model of the circular flow resources in any economy. We show below how this can be approached in a similarly organized but more concise manner from a mathematical perspective.

Consider an economy with n industries, as in the IOT in Table A1.1. Each industry i produces its own output, x_i, where i = 1, 2, .., n. Each x_i can either be used as inputs to industrial production or finally consumed by households, government, nonprofit institutions serving households, and even other industries (the interactions within an IOT are discussed in detail under "Methodological Requirements" on p. 23 of the main text). Let z_{ij} represent the monetary value of industry j's purchases of industry i output for intermediate use, and y_i be the total amount of purchases from industry i intended for final consumption. As is customary in traditional input-output analysis, we will assume that interindustry flows from i to j contemporaneously depend entirely on sector 's total output (Miller and Blair 2009), which implies that final demand is exogenous. Given this information, in Equation (1), we describe the gross output of each industry i to be broken down across its intermediate users and final users.

$$x_i = z_{i1} + z_{i2} + \cdots + z_{in} + y_i, \quad i = 1, 2, \ldots., n \tag{1}$$

Given that Equation (1) is a system of n equations, we express it in matrix notation in Equation (2).

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1n} \\ z_{21} & z_{22} & \cdots & z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{nn} \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$\mathbf{x} = \mathbf{Zi} + \mathbf{y} \tag{2}$$

We derive a technical coefficient, a_{ij}, to describe the ratio between the amount of industry i's output used by j and the amount of industry j's output; that is, $a_{ij} = z_{ij}/x_j$. Following the Leontief insight, each a_{ij} is assumed to be unchanging over the course of an accounting period. Stating z_{ij} in terms of a_{ij}, the gross output in Equation (1) becomes

$$x_i = a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n + y_i, \quad i = 1, 2, \ldots., n$$

which may be re-expressed as

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1n} \\ z_{21} & z_{22} & \cdots & z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{nn} \end{bmatrix} \begin{bmatrix} x_1 & 0 & \cdots & 0 \\ 0 & x_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & x_n \end{bmatrix}^{-1} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$\mathbf{x} = \mathbf{Ax} + \mathbf{y} \tag{3}$$

Rearranging Equation (3):

$$\mathbf{x} = \mathbf{Ax} + \mathbf{y} \rightarrow \mathbf{x} - \mathbf{Ax} = \mathbf{y} \rightarrow (\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{y}$$

Assuming that $\mathbf{I} - \mathbf{A}$ is nonsingular, we have the fundamental Leontief identity:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} \tag{4}$$

We refer to $(\mathbf{I} - \mathbf{A})^{-1}$ as the Leontief inverse, which gives the total output requirements from each industry in order to meet final demand for a specific time period. Note that $\mathbf{x}$ from Equation (4) would yield the exact same vector of gross output $\mathbf{x}$ in a standard n-industry IOT. Therefore, we could mathematically represent Table A1.1 through Equation (4).

Reference:

R. Miller and P. Blair. 2009. *Input-Output Analysis: Foundations and Extensions*. Cambridge: Cambridge University Press.

Appendix 2: Aggregating Matrices

Suppose one has a 4 × 4 input matrix, **Z**:

$$\mathbf{Z} = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} \\ z_{21} & z_{22} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ z_{41} & z_{42} & z_{43} & z_{44} \end{bmatrix}$$

The dimensions of the aggregator matrix are [n-(q-1)] × n, where n is the original number of industries and q is the number of industries to be aggregated into one sector. Thus, to aggregate two industries, one needs a (4 – 2 +1) × 4 or a 3 × 4 aggregator matrix. To aggregate column vectors, one only needs to pre-multiply the aggregator matrix to them, while matrices (**Z** in this case) have to be pre- and post-multiplied with the aggregator matrix and its transpose, respectively. Letting **Q** denote the aggregator matrix, these steps are given by the equations:

$$\mathbf{x}_{\mathrm{agg}} = \mathbf{Qx}$$

$$\mathbf{f}_{\mathrm{agg}} = \mathbf{Qf}$$

$$\mathbf{gva}_{\mathrm{agg}} = \mathbf{Qgva}$$

$$\mathbf{Z}_{\mathrm{agg}} = \mathbf{QZQ}^{\mathrm{T}}$$

The logic behind aggregator matrices is discussed with the aid of some examples. To aggregate Industries 1 and 2, the following aggregator matrix is needed:

$$\mathbf{Q} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For Industries 1 and 3:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For Industries 1 and 4:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

For Industries 2 and 3:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And for Industries 2 and 4:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

In aggregating Industries 1 and 2, the contents of the first row of **Q** depend on whether the first industry should be aggregated with any other industry. Since 1 and 2 will be aggregated, the entries of the first and second columns take the value of 1. Entries for the third and fourth columns are set to zero since Industry 1 will not be grouped with any of those industries. In the second row, only the entry in the third column corresponding to Industry 3 is set to 1, since Industry 3 will not be grouped with any other industry. Lastly, after accounting for the first three industries, the third row of **Q** should be altered depending on whether Industry 4 will be grouped with any other industry. Since this is not the case, only the entry in the fourth column is set to 1, with everything else being zero.

In aggregating Industries 1 and 3, the entries in the first row of **Q** depend on whether the first industry will be aggregated with any other industry. Since 1 and 3 will be grouped together, the entries in the first and third columns are set to 1 and zero to the second and fourth columns. Since Industry 2 is not yet accounted for, the second row should consider if Industry 2 will be grouped with any other industry. Since this is not the case, the second column is set to 1, with all other entries set to zero. Finally, since Industries 1, 2, and 3 have already been accounted for, the third row of **Q** should consider if Industry 4 will be grouped with any other industry. Since this is not the case, the fourth column is set to 1 and all others to zero.

Thus, the sequence of industries in an input-output table (IOT) is crucial when it comes to the use of aggregator matrices. Columns still correspond to the exact order of industries in an IOT, but rows will be adjusted whenever industries are grouped together. However, inputting values to rows of **Q** is still based on the sequence of industries in an IOT, with skips occurring when an industry has already been lumped with another that appeared prior to it.

Table A4.2: Main Digital Industries by International Standard Industrial Classification of All Economic Activities Revision 4

Main Activity Group	Code	Industry
Hardware	2620	Manufacture of computers and peripheral equipment
	2680	Manufacture of magnetic and optical media
Software publishing	5820	Software publishing
Web publishing	6312	Web portals
Telecommunications services	61	Telecommunications services
Specialized and support services	62	Computer programming services, consulting, and other related services
	6311	Data processing, hosting and related activities

Source: United Nations. 2008. *International Standard Industrial Classification of All Economic Activities (ISIC), Rev. 4*. New York: United Nations.

Table A4.3: ADB Multiregional Input–Output 35-Sector Classification

Code	Sector	Code	Sector
c1	Agriculture, hunting, forestry, and fishing	c19	Sale, maintenance, and repair of motor vehicles and motorcycles; retail sale of fuel
c2	Mining and quarrying	c20	Wholesale trade and commission trade, except of motor vehicles and motorcycles
c3	Food, beverages, and tobacco	c21	Retail trade, except of motor vehicles and motorcycles; repair of household goods
c4	Textiles and textile products	c22	Hotels and restaurants
c5	Leather, leather products and footwear	c23	Inland transport
c6	Wood and products of wood and cork	c24	Water transport
c7	Pulp, paper, paper products, printing, and publishing	c25	Air transport
c8	Coke, refined petroleum, and nuclear fuel	c26	Other supporting and auxiliary transport activities; activities of travel agencies
c9	Chemicals and chemical products	c27	Post and telecommunications
c10	Rubber and plastics	c28	Financial intermediation
c11	Other nonmetallic minerals	c29	Real estate activities
c12	Basic metals and fabricated metal	c30	Renting of M&Eq and other business activities
c13	Machinery, n.e.c.	c31	Public administration and defense; compulsory social security
c14	Electrical and optical equipment	c32	Education
c15	Transport equipment	c33	Health and social work
c16	Manufacturing, n.e.c.; recycling	c34	Other community, social, and personal services
c17	Electricity, gas and water supply	c35	Private households with employed persons
c18	Construction		

M&Eq= machinery and equipment, n.e.c.= not elsewhere classified.
Source: Asian Development Bank Multiregional Input-Output Database.