SMLP 2022 Notes

Daniela Palleschi

2022-08-31

Ch. 1 - Intro

- given some data, how to use Bayes' theorem to ***quantify uncertainty about our belief*** regarding a scientific question of interest
- topics to be understood:
  - the basic concepts behind probability
  - the concept of random variables
  - probability distributions
  - the concept of likelihood.

Probability

Frequency-based versus uncertain-belief perspective of probability:

1. repeatable events, like rolling a die and getting a 6, are *frequentist* because **probability** is related to the *frequency* at which we'd observe an outcome given repeated observations
2. one-of-a-kind events, like earthquakes, don't work with this idea of probability

- the probability of an earthquake expresses our *uncertainty* about an event happening
- we also be *uncertain* about how probable an event is: being 90% sure something is 50% likely to happen
- this is what we're interested in: how uncertain we are of an estimate

In Bayesian analysis, we want to express our uncertainty about the probability of observing an outcome (*prior distribution*).

Conditional probability and Bayes' rule

- A = "the streets are wet"
- B = "it was raining"
- P(A|B) = the probability of A given B
- P(A,B) = P(A|B)P(B) (the probability of A and B happening)

Law of total probability

- dunno

Discrete random variables

Generating random sequences of simulated data with a binomial distribution. Imagine a cloze task, where we consider a particular word a success (1) and any other word a failure (0). If we run the experiment 20 times with a sample size of 10, the cloze probabilities for these 20 experiments would be:

```
rbinom(10, n = 20, prob = .5)
```

[1] 7 5 7 5 4 4 3 3 5 5 3 4 5 5 3 6 4 6 4 4

For discrete random variables such as the binomial, the probability distribution *p(y|theta)* is called a probability mass function (PMF) . The PMF defines the probability of each possible outcome. With $n = 10$ trials, there are 11 possible outcomes (0, 1, 2,... 10 succeses). Which outcome is most probable depends on the parameter theta that represents the probability of success. Above, we set theta to `0.5`.

The mean and variance of the binomial distribution In real exerimental situations we never know the true value of theta (probability of an outcome), but it can be derived from the data: *theta hat* $= k/n$, where k = number of observed successess, n = number of trials, and *theta hat* = observed proportion of successes. *Theta hat* = ***maximum likelihood estimate*** of the true but unknown parameter *theta*. Basically, the **mean** of the binomial distribution. The **variance** can also be estimated by computing *(n(theta))(1 - theta)*. These estimates can be be used for statistical inference.

Compute probability of a particular outcome (discrete)

`dbinom` calculates probability of k successes out of n given a particular *theta*.

```
dbinom(5, size = 10, prob = .5)
```

[1] 0.2460938