Mixed Models 1

Random intercepts

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Learning Objectives

Today we will learn…

- what linear mixed models are
- how to fit a random-intercepts model
- how to inspect and interpret a mixed effects model

Resources

- this lecture covers
  - Chapter 14 'Mixed Models 1: Conceptual Introduction’ (until Section 14.8; Winter, 2019)
  - Winter (2014) (until page 16)
  - Sections 8.1-8.3 in Sonderegger (2023)
- we will be using the data from Biondo et al. (2022)

Set-up

```r
# suppress scientific notation
options(scipen=999)
```

▶ Code for a function to format p-values

Load packages

```
# load libraries
pacman::p_load(
               tidyverse,
               here,
               broom,
               janitor,
               ggeffects,
               sjPlot,
               # new packages for mixed models:
               lme4,
               lmerTest,
               broom.mixed,
               lattice)
```

Resolve conflicts

- both `lme4` and `lmerTest` have a function `lmer()`
  - for now we want to use the `lme4` version

```
lmer <- lme4::lmer
```

Load data

- data from Biondo et al. (2022)

```r
df_biondo <-
  read_csv(here("data", "Biondo.Soilemezidi.Mancini_dataset_ET.csv"),
           locale = locale(encoding = "Latin1") ## for special characters in Spanish
           ) |>
  clean_names() |>
  mutate(gramm = ifelse(gramm == "0", "ungramm", "gramm")) |>
  mutate_if(is.character,as_factor) |> # all character variables as factors
  filter(adv_type == "Deic") |>
  droplevels()
```

And take a look at the data:

```r
head(df_biondo)
```

```
# A tibble: 6 × 13
  sj     item adv_type adv_t verb_t gramm   roi label        fp    gp    tt    ri
  <fct> <dbl> <fct>    <fct> <fct>  <fct> <dbl> <fct>     <dbl> <dbl> <dbl> <dbl>
1 1        54 Deic     Past  Past   gramm     1 En la c…   1173  1173  1173     0
2 1        54 Deic     Past  Past   gramm     2 ayer te…    474   474   474     0
3 1        54 Deic     Past  Past   gramm     3 los car…    910   910   910     0
4 1        54 Deic     Past  Past   gramm     4 encarga…   1027  1027  1027     0
5 1        54 Deic     Past  Past   gramm     5 muchas …    521   521   521     0
6 1        54 Deic     Past  Past   gramm     6 al prov…   1029  1029  1029     0
# ℹ 1 more variable: ro <dbl>
```

Set contrasts

```r
contrasts(df_biondo$verb_t) <- c(-0.5,+0.5)
contrasts(df_biondo$gramm) <- c(-0.5,+0.5)
```

```r
contrasts(df_biondo$verb_t)
```

```
       [,1]
Past   -0.5
Future  0.5
```

```r
contrasts(df_biondo$gramm)
```

```
         [,1]
gramm    -0.5
ungramm   0.5
```

Linear mixed (effects) models

- mixed models allow for varying intercepts and slopes per level of some grouping factor
- recall that intercepts (can) represent the grand mean of the data
- slopes represent a change in y for a 1-unit change in x ($\frac{\Delta y}{\Delta x}$, “rise over run”)
  - i.e., the difference between two categories, or for a 1-unit change of a continuous predictor
- random intercepts take into account that each level of a grouping factor can vary in their mean
- random slopes take into account that each level of a grouping factor can vary in the effect of a predictor

Random intercepts vs. random slopes

- Biondo et al. (2022) used a within-participant, a.k.a. repeated measures design
  - 60 participants saw 96 items, rotated throughout the conditions in a Latin square design
- we would expect some participants to be faster readers than others
  - this would be reflected in a shorter mean reading time
- some participants will tend to have a stronger effect of e.g., grammaticality than others
  - this would be reflected in a steeper slope for `gramm`aticality
- the same could be said for certain experimental items
  - reading times will vary by item e.g., for word length or familiarity
  - some items will also tend to have a stronger effect than others

By-participant variance

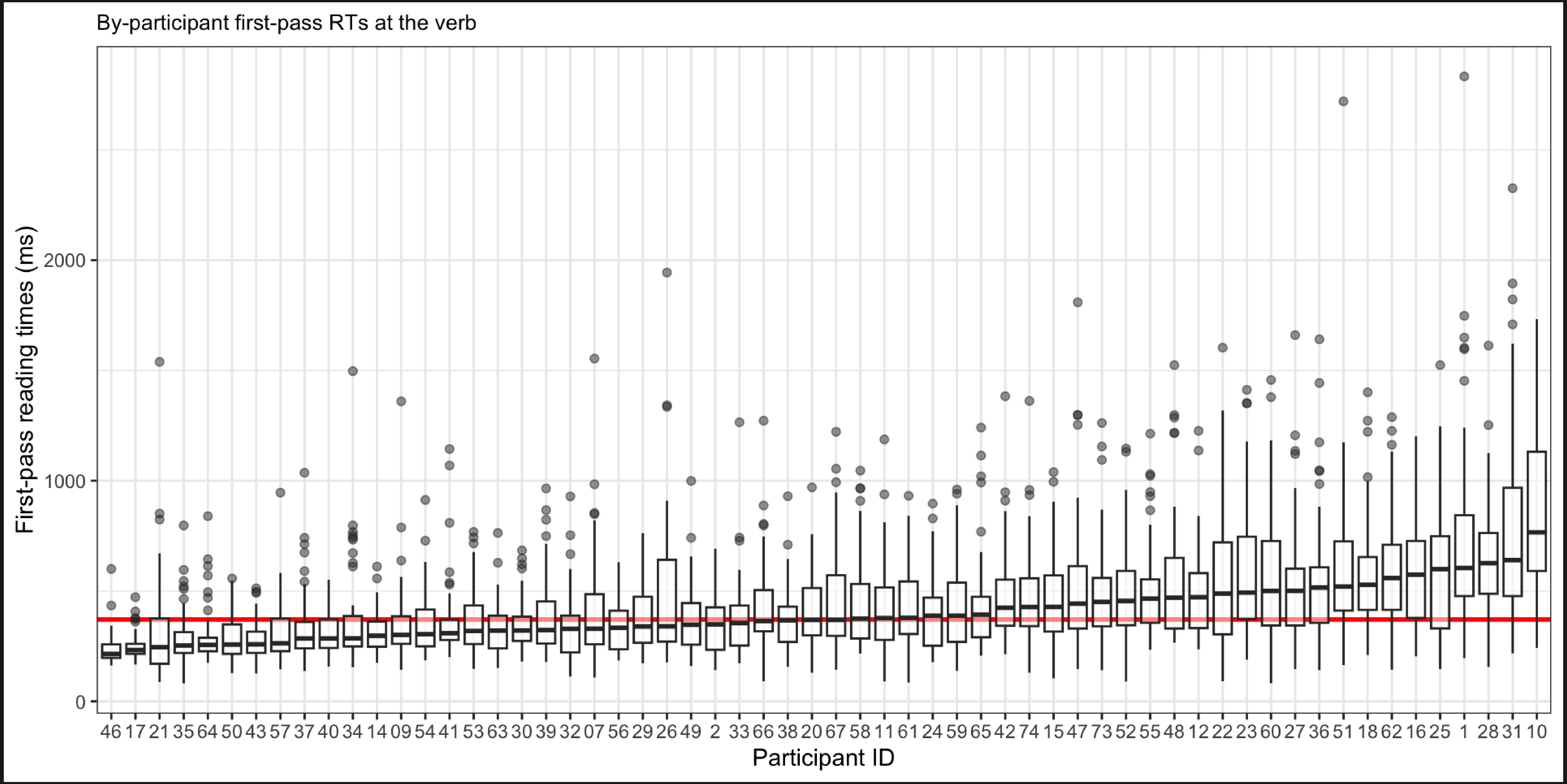


Figure 1: By-participant boxplot of first-pass RTs at the verb region with overall median FP value in red

By-item variance

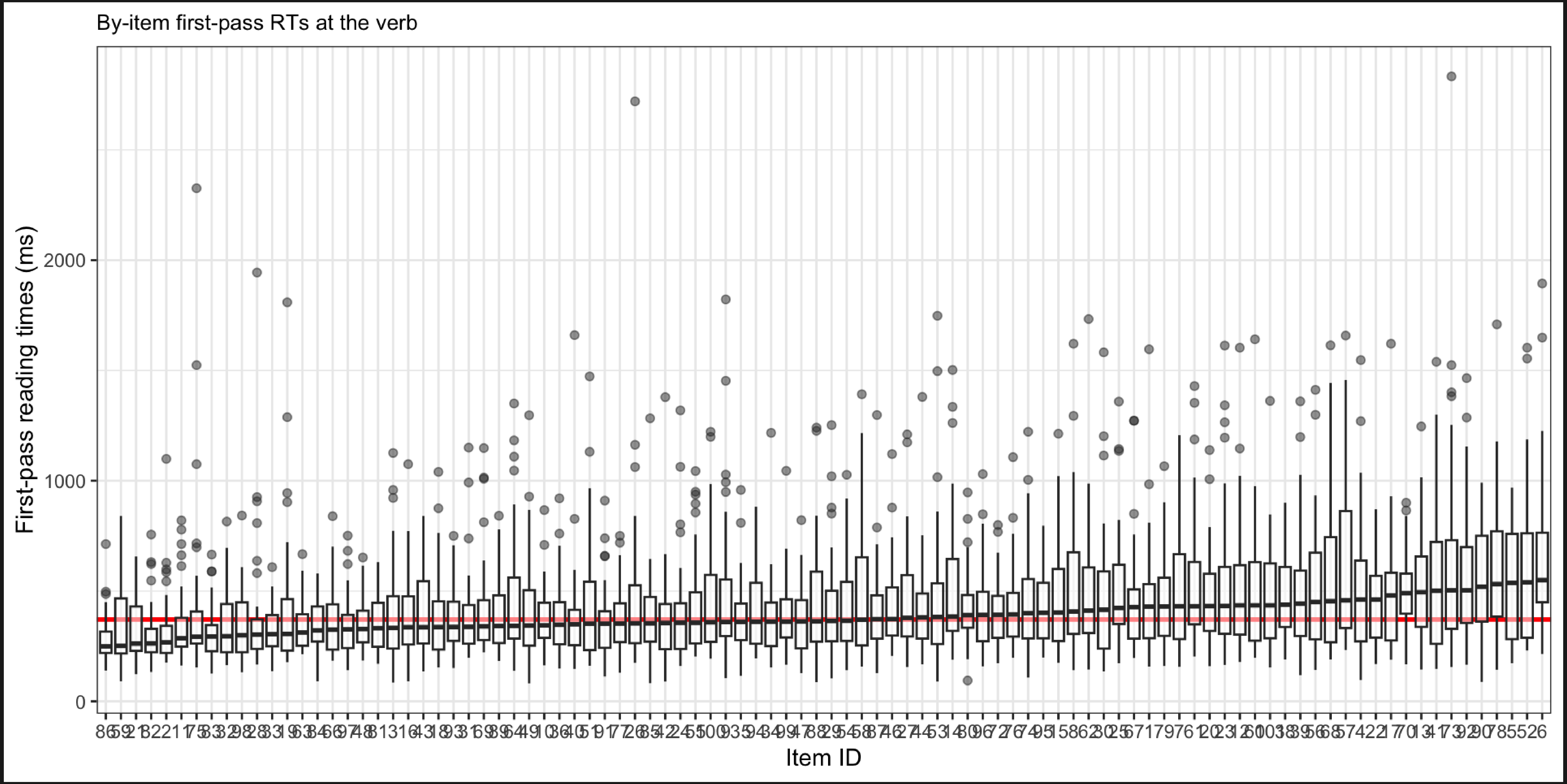


Figure 2: By-item boxplot of first-pass RTs at the verb region with overall median FP value in red

By-participant varying intercepts and slopes

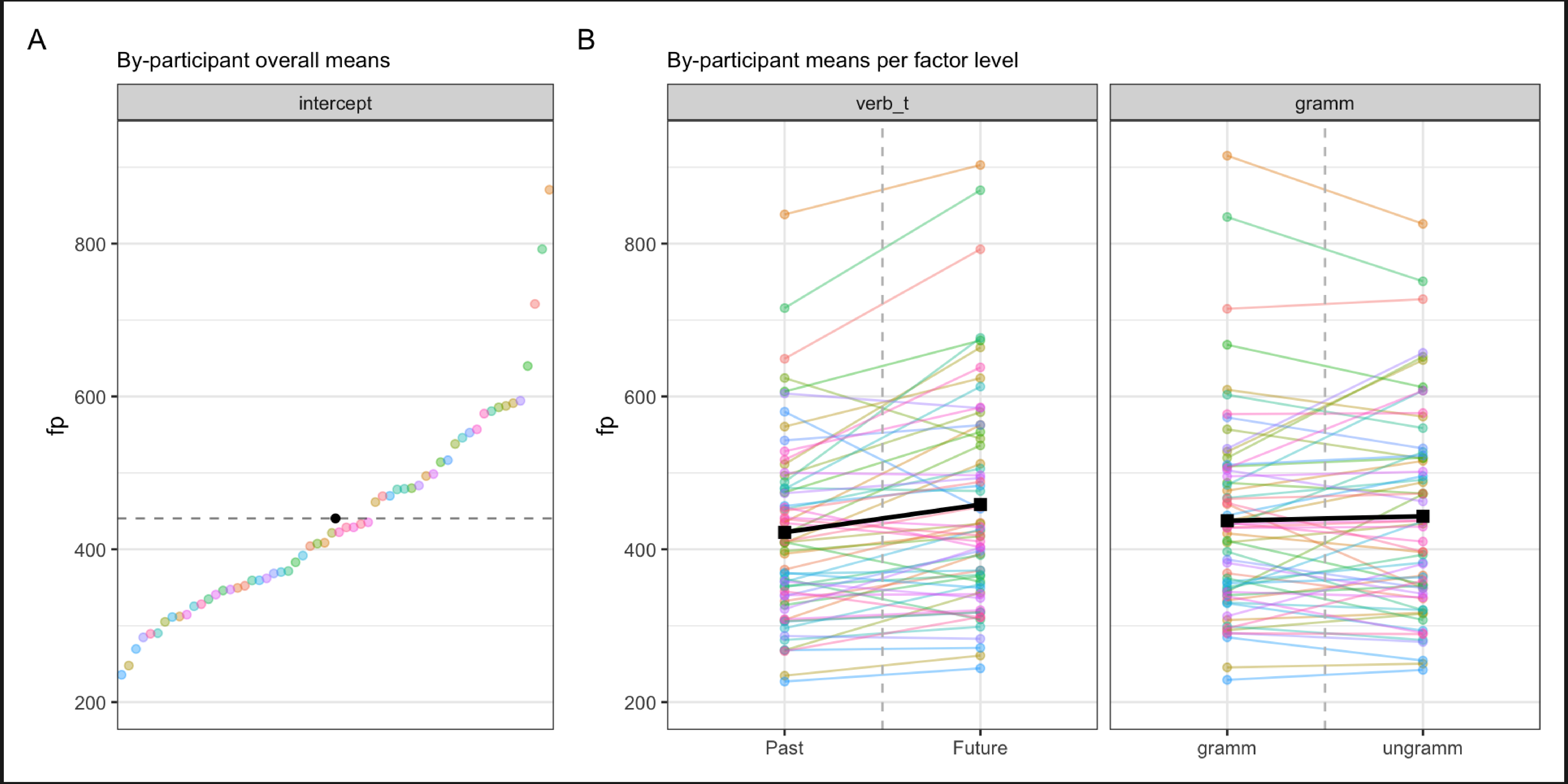


Figure 3: Predicted by-participant varying intercepts (A) and slopes (B) with overall effects in black

By-item varying intercepts and slopes

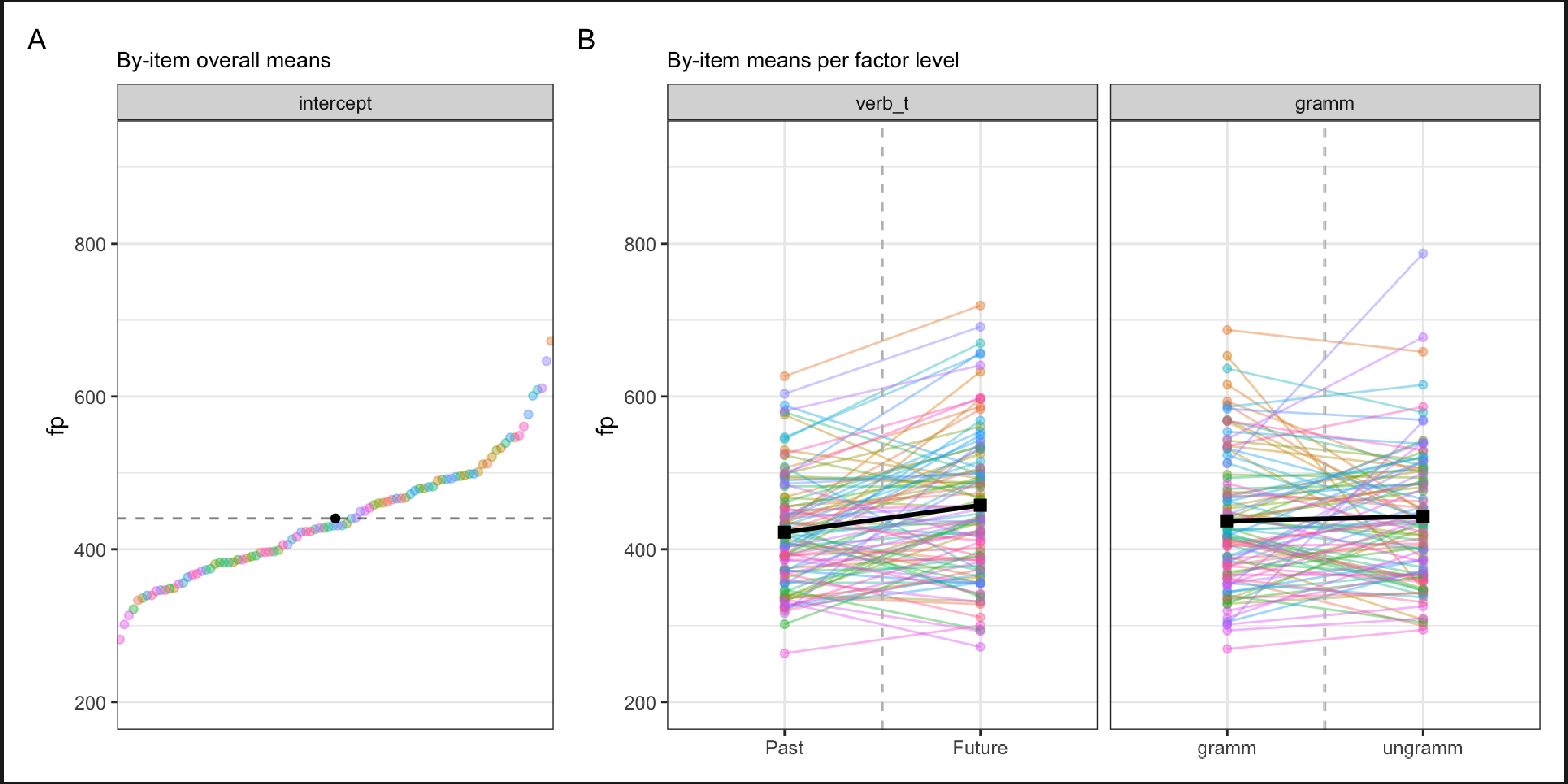


Figure 4: By-item varying intercepts (A) and slopes (B) with overall effects in black

Comparing participant and item

- just by eye-balling our data we see there was more variability in intercepts and effects by-participant than by-item
  - this is typical: People tend to vary more than our highly controlled experimental items
- how can we take this variability of both item and participant into account?
  - mixed models!
  - today we’ll focus on varying intercepts, first for by-participants and then by-item

Random intercepts

- random intercepts = taking group-level variance in overall tendencies into account

Random intercepts: one grouping factor

- below is our first *mixed effects model*

```
fit_lmm_fp_sj <-
  lmer(log(fp) ~ verb_t*gramm +
         (1|sj),
       data = df_biondo,
       subset = roi == 4)
```

1. create an object `fit_lmm_fp_sj`, which contains…
2. a mixed model (`lmer()`): log first-pass RTs as a function of our fixed effects, plus…
3. varying intercepts (`1`) by-participant (`|sj`)
4. from our dataset
5. subsetted to only include the verb region
    - can also be done above the model using `filter(roi == 4)`

Summary

- we can use the `summary()` function, just as we did with `(g)lm()`

```r
summary(fit_lmm_fp_sj)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: log(fp) ~ verb_t * gramm + (1 | sj)
   Data: df_biondo
 Subset: roi == 4

REML criterion at convergence: 4479.1

Scaled residuals:
    Min      1Q  Median      3Q     Max 
-4.0560 -0.6427 -0.0419  0.6168  4.0901 

Random effects:
 Groups   Name        Variance Std.Dev.
 sj       (Intercept) 0.06573  0.2564  
 Residual             0.18030  0.4246  
Number of obs: 3795, groups:  sj, 60

Fixed effects:
                Estimate Std. Error t value
(Intercept)     5.957102   0.033809 176.199
verb_t1         0.062209   0.013787   4.512
gramm1          0.003466   0.013787   0.251
```

Model info

```
Linear mixed model fit by REML ['lmerMod'].                ①
Formula: log(fp) ~ verb_t * gramm + (1 | sj)               ②
   Data: df_biondo                                          ③
 Subset: roi == 4                                           ④

REML criterion at convergence: 4479.1                       ⑤

Scaled residuals:                                           ⑥
    Min      1Q  Median      3Q     Max
-4.0560 -0.6427 -0.0419  0.6168  4.0901
```

① Model description (object class = `lmerMod`)

② Model formula

③ Data

④ Any subsetting

⑤ REML: Restricted Maximum Likelihood; important for model comparison, but not for us today

⑥ Residuals: do these look normally distributed?

Fixed effects

```
Fixed effects:                                              ①
                 Estimate Std. Error t value                ②
(Intercept)      5.957102   0.033809 176.199                ③
verb_t1          0.062209   0.013787   4.512                ④
gramm1           0.003466   0.013787   0.251                ⑤
verb_t1:gramm1 -0.015741   0.027573  -0.571                 ⑥

Correlation of Fixed Effects:                               ⑦
            (Intr) vrb_t1 gramm1
verb_t1      0.000
gramm1       0.000 -0.002
vrb_t1:grm1  0.000  0.002  0.000
```

① Our fixed effects:

② Estimate (coefficient), standard error, t-value (no p-value…)

③ Intercept (grand mean)

④ Effect of tense

⑤ Effect of grammaticality

⑥ Interaction Effect

⑦ Correlation matrix of fixed effects

Random effects

```
Random effects:                                  ①
 Groups   Name        Variance Std.Dev.          ②
 sj       (Intercept) 0.06573  0.2564            ③
 Residual             0.18030  0.4246            ④
Number of obs: 3795, groups:  sj, 60             ⑤
```

① Random effects

② Grouping factor, effect name, overall variance and standard deviation

③ By-participant random intercepts

④ Residual error not accounted for by our random effects

⑤ Number of observations and grouping factor levels

Interpreting random effects

- we can also selectively print our random effects (variance components) using the `VarCorr()` function from `lme4`
  - only gives us the standard deviation, which is the square root of the variance

```
VarCorr(fit_lmm_fp_sj)
```

```
 Groups   Name        Std.Dev.
 sj       (Intercept) 0.25638 
 Residual             0.42462 
```

- to also get the variance

```
print(VarCorr(fit_lmm_fp_sj),
      comp = c("Variance", "Std.Dev"))
```

```
 Groups   Name        Variance Std.Dev.
 sj       (Intercept) 0.065731 0.25638 
 Residual             0.180305 0.42462 
```

- if we compute the mean, variance, and SD of the by-participant intercepts we get

▶ Code

```
# A tibble: 1 × 3
   mean    var    sd
  <dbl>  <dbl> <dbl>
1  5.96 0.0630 0.251
```

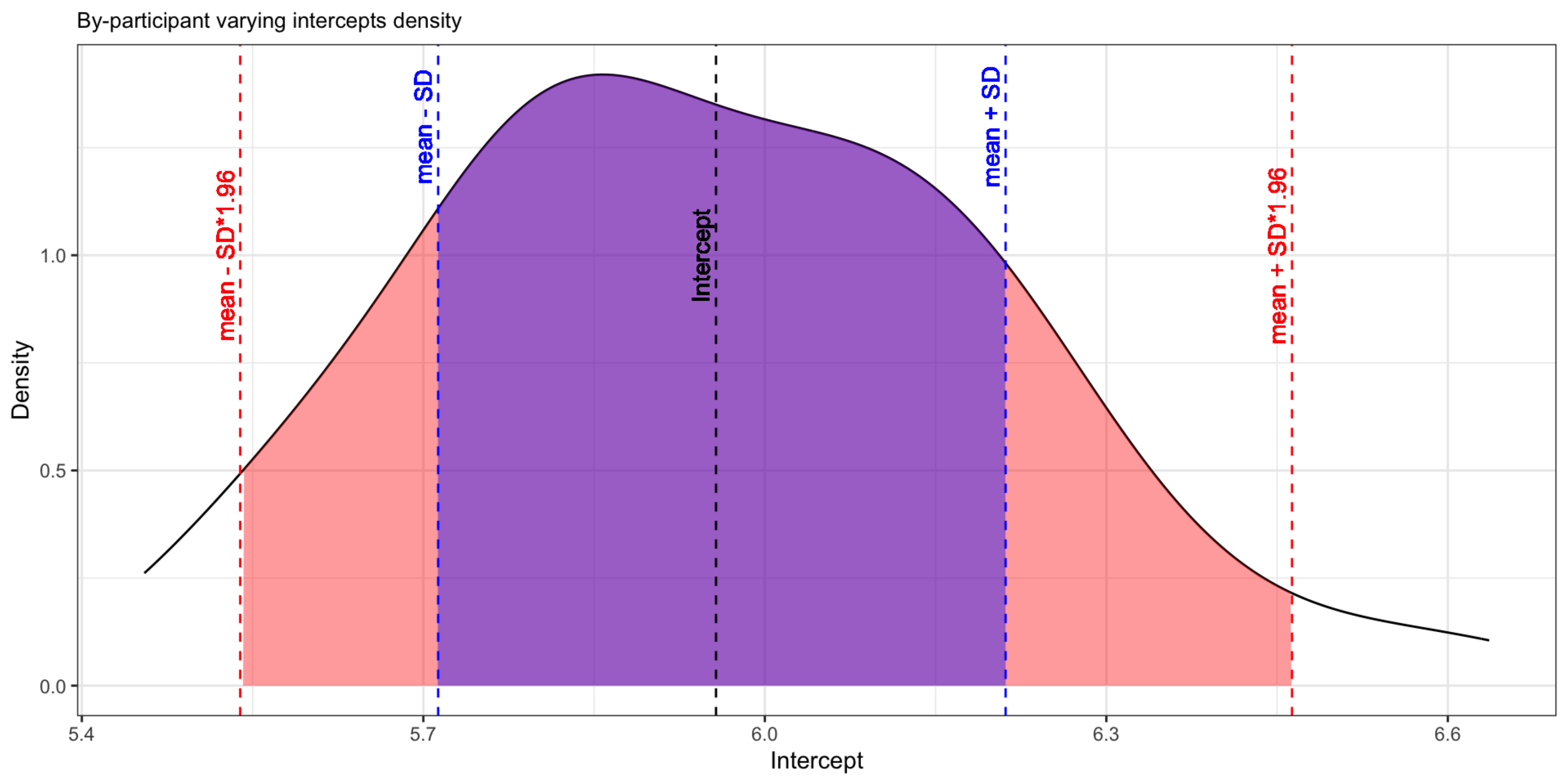
68-95% rule

- in a normal distribution, 68% of the data will lie within +/-1 SD of the mean, and 95% will lie within +/-2 SDs of the mean
- our model intercept is 5.957102, so 68% of our participant intercepts will lie roughly between 5.7007211 and 6.2134829

```
Groups   Name        Variance Std.Dev.
sj       (Intercept) 0.065731 0.25638
Residual             0.180305 0.42462
```

- so we can interpret the standard deviation as quantifying the variability of the by-participant intercepts around the mean (i.e., intercept)

Figure 5: Density plot of by-participant intercepts with 95% (+/-SD*1.96) and 68% (+/-SD) ranges



`lmerTest::lmer()`

- recall we had a conflict for the function `lmer()` between `lme4` and `lmerTest`, and we set `lme4` to be our default for this function
- let's refit our model using `lmerTest::lmer()`

```
fit_lmm_fp_sj <-
  lmerTest::lmer(log(fp) ~ verb_t*gramm +
          (1|sj),
        data = df_biondo,
        subset = roi == 4)
```

- the only change is the addition of `lmerTest::`, which specifies which package to retreive the `lmer()` function from

- what’s different in the summary?

```r
summary(fit_lmm_fp_sj)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: log(fp) ~ verb_t * gramm + (1 | sj)
   Data: df_biondo
 Subset: roi == 4

REML criterion at convergence: 4479.1

Scaled residuals:
    Min      1Q  Median      3Q     Max
-4.0560 -0.6427 -0.0419  0.6168  4.0901

Random effects:
 Groups   Name        Variance Std.Dev.
 sj       (Intercept) 0.06573  0.2564
 Residual             0.18030  0.4246
Number of obs: 3795, groups:  sj, 60

Fixed effects:
                  Estimate  Std. Error          df t value              Pr(>|t|)
(Intercept)       5.957102    0.033809   58.991899 176.199 < 0.0000000000000002
verb_t1           0.062209    0.013787 3732.065822   4.512           0.00000662
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method ['lmerModLmerTest']   ①
...

Fixed effects:
                  Estimate  Std. Error          df t value               Pr(>|t|)   ②
(Intercept)       5.957102    0.033809   58.991899 176.199 < 0.0000000000000002 ***
verb_t1           0.062209    0.013787 3732.065822   4.512             0.00000662 ***
gramm1            0.003466    0.013787 3732.032139   0.251                  0.802
verb_t1:gramm1   -0.015741    0.027573 3732.037124  -0.571                  0.568

...
```

① New: `Satterthwaite's method` and object class (`lmerModLmerTest`)

② Fixed effects include `df` (degrees of freedom) and *p*-values (`Pr(>|t|)`)

- `lme4::lmer()` doesn’t provide degrees of freedom or *p*-values
  - defining degrees of freedom (and therefore calculating *p*-values) is more complex and not trivial in mixed models
  - `lmerTest` uses the Satterthwaite method, which is fine for our purposes
- importantly, everything else is exactly the same as when we use `lme4::lmer()`

Fixed effects for `lm()` and `lmer()`

- let's compare our fixed effects to those from a model without random effects

```
fit_lm_fp <-
  lm(log(fp) ~ verb_t*gramm,
          data = df_biondo,
          subset = roi == 4)
```

Comparing fixed effects

Table 1: Fixed effects for `lm_fp_sj`

term	estimate	std.error	statistic	p.value
(Intercept)	5.957	0.008	741.568	0.0000000
verb_t1	0.061	0.016	3.809	0.0001415
gramm1	0.003	0.016	0.193	0.8469596
verb_t1:gramm1	-0.015	0.032	-0.474	0.6352122

Table 2: Fixed effects for `lmm_fp_sj`

term	estimate	std.error	statistic	df	p.value
(Intercept)	5.957	0.034	176.199	59	0.0000000
verb_t1	0.062	0.014	4.512	3732	0.0000066
gramm1	0.003	0.014	0.251	3732	0.8015026
verb_t1:gramm1	-0.016	0.028	-0.571	3732	0.5681183

- so far we see that our model estimates are still descriptively similar, there are only some slight quantitative differences
- your fixed effects will typically be unchanged with the addition of random effects
  - what changes will be usually be the standard error, *t*-value (or *z*-value for generalised linear (mixed) models), confidence intervals, and *p*-values
  - the magnitude of this change will depend on whether the inclusion of the random effects better accounts for variability in your data than your fixed effects alone

Comparing residual error

- the residual error for our fixed-effects-only model was is 0.49

```
glance(fit_lm_fp)$sigma
```

```
[1] 0.494879
```

- for our by-participant varying intercepts model it goes down to 0.42

```
glance(fit_lmm_fp_sj)$sigma
```

```
[1] 0.424623
```

- this tells us that our inclusion of by-participant varying intercepts accounts for some of the variance in the model that was not accounted for in fixed-effects-only model
- are there any other possible sources of variance that we haven’t taken into account?

Crossed random effects: two grouping factors

- we still haven’t taken by-item variance into account, let’s now include *crossed* random effects in our model

```
fit_lmm_fp_sj_item <-
  lmerTest::lmer(log(fp) ~ verb_t*gramm +
                   (1|sj) +
                   (1|item),
                 data = df_biondo,
                 subset = roi == 4)
```

- crossed random effects refer to a property of your data (/experimental design), i.e., repeated measures for items *and* participants
  - one level of a grouping factor contains observations from all levels from another grouping factor (e.g., each item has an observations from each participant and vice versa)

```r
summary(fit_lmm_fp_sj_item)
```

```
Linear mixed model fit by REML. t-tests use
Satterthwaite's method [
lmerModLmerTest]
Formula: log(fp) ~ verb_t * gramm + (1 | sj) + (1 | item)
   Data: df_biondo
 Subset: roi == 4

REML criterion at convergence: 4220.3

Scaled residuals:
    Min      1Q  Median      3Q     Max
-4.1568 -0.6169 -0.0257  0.6006  4.0422

Random effects:
 Groups   Name        Variance Std.Dev.
```

Comparing random effects

Table 3: By-participant varying variance component

group	term	estimate
sj	sd__(Intercept)	0.2563809
Residual	sd__Observation	0.4246230

Table 4: By-participant and -item variance components

group	term	estimate
item	sd__(Intercept)	0.1392928
sj	sd__(Intercept)	0.2579514
Residual	sd__Observation	0.4011073

- we now have the variance of by-item random slopes (Table 4)
- the *variance component* for the by-subjects random slopes is not much changed
  - the value is slightly different because it now also takes by-item variability into account
- the *residual error* also goes down in `fit_lmm_fp_sj_item` (0.4), because by-item intercepts account for some of the variance that was unaccounted for in `fit_lmm_fp_sj` (0.42)
- note that there is most by-participant than by-item variance, this is typical and reflects what we saw in our boxplots

Comparing fixed effects

Table 5: Fixed effects for `lmm_fp_sj`

term	estimate	std.error	statistic	df	p.value
(Intercept)	5.957	0.034	176.199	59	0.0000000
verb_t1	0.062	0.014	4.512	3732	0.0000066
gramm1	0.003	0.014	0.251	3732	0.8015026
verb_t1:gramm1	-0.016	0.028	-0.571	3732	0.5681183

Table 6: Fixed effects for `lmm_fp_sj_item`

term	estimate	std.error	statistic	df	p.value
(Intercept)	5.956	0.037	161.903	79	0.0000000
verb_t1	0.062	0.013	4.752	3637	0.0000021
gramm1	0.003	0.013	0.247	3637	0.8052568
verb_t1:gramm1	-0.014	0.026	-0.550	3637	0.5826522

- again we see there isn’t much change to our coefficient estimates

Comparing predictions

▶ Code for plots

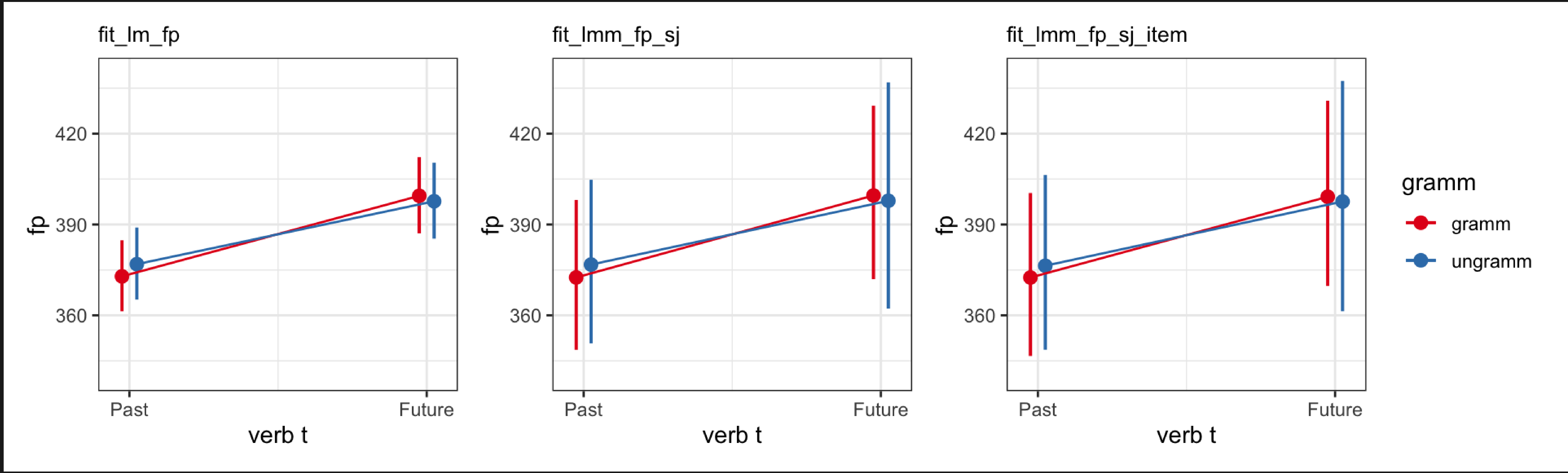


Figure 6: Comparison of predicted estimates and 95% confidence intervals for the three models

- if we plot the results from all three models we've fit so far we see the estimates are similar but the confidence intervals are wider for the mixed models
  - this is despite the fact that the *p*-values are significant for all three

Model comparison

- we can use the `anova()` function to compare model fit

```r
anova(fit_lmm_fp_sj, fit_lmm_fp_sj_item)
```

```
Data: df_biondo
Subset: roi == 4
Models:
fit_lmm_fp_sj: log(fp) ~ verb_t * gramm + (1 | sj)
fit_lmm_fp_sj_item: log(fp) ~ verb_t * gramm + (1 | sj) + (1 | item)
                   npar    AIC    BIC  logLik deviance  Chisq Df
fit_lmm_fp_sj         6 4467.3 4504.8 -2227.7   4455.3
fit_lmm_fp_sj_item    7 4210.3 4254.0 -2098.2   4196.3 258.98  1
                               Pr(>Chisq)
fit_lmm_fp_sj
fit_lmm_fp_sj_item < 0.00000000000000022 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- here we see that the AIC, BIC, and logLik are all lower for our model with by-participant and -item varying intercepts
  - *lower* AIC and BIC indicate better model fit
  - *higher* logLik indicates better fit
- the inclusion of by-item random intercepts significantly improves the fit of our model

Exploring our random effects estimates

- what we saw in our model summary were the variance components
  - a description of the variance of our by-item and by-participant random intercepts
- our model also contains intercept estimates for each level of item and participant
  - we can extract the intercept estimates
  - or we extract their deviance from the model intercept

Extracting fixed effects

- we've already used `coef()` to extract fixed effect estimates from `lm` objects

```
coef(fit_lm_fp)
```

```
  (Intercept)        verb_t1         gramm1 
verb_t1:gramm1 
  5.957251870    0.061204153    0.003101061 
-0.015245374 
```

- to extract our fixed effect estimates from `lmer` objects we need `fixef()`

```
fixef(fit_lmm_fp_sj_item)
```

```
  (Intercept)        verb_t1         gramm1 
verb_t1:gramm1 
   5.95640363     0.06189237     0.00321152 
-0.01431578 
```

- or we can append `$coefficients` to the model summary

```
summary(fit_lmm_fp_sj_item)$coefficients |>
  as_tibble()
```

```
# A tibble: 4 × 5
  Estimate `Std. Error`    df `t value` `Pr(>|t|)`
     <dbl>        <dbl> <dbl>     <dbl>      <dbl>
1   5.96         0.0368  79.2   162.      1.31e-101
2   0.0619       0.0130 3637.     4.75    2.09e-  6
3   0.00321      0.0130 3637.     0.247   8.05e-  1
4  -0.0143       0.0260 3637.    -0.550   5.83e-  1
```

Extract random intercept estimates

- `coef()` behaves very differently with `lmer` objects, extracting the random effects estimates per level

```r
coef(fit_lmm_fp_sj_item) |> pluck("item") |>
  rownames_to_column(var = "item") |> head()
```

```
  item (Intercept)    verb_t1      gramm1 verb_t1:gramm1
1    1    6.022184 0.06189237 0.00321152    -0.01431578
2    2    5.761268 0.06189237 0.00321152    -0.01431578
3    3    5.854873 0.06189237 0.00321152    -0.01431578
4    4    6.056862 0.06189237 0.00321152    -0.01431578
5    5    6.138213 0.06189237 0.00321152    -0.01431578
6    6    6.331058 0.06189237 0.00321152    -0.01431578
```

- which outputs a `list` object, with one data frame for `item` and one for `sj`
  - in the code above I've 'plucked' just the by-item coefficients

- we can extract just one or the other (`head()` is for presentation purposes):

```
coef(fit_lmm_fp_sj_item) |> pluck("item") |>
  rownames_to_column(var = "item") |> head()
```

```
  item (Intercept)    verb_t1      gramm1 verb_t1:gramm1
1    1    6.022184 0.06189237 0.00321152    -0.01431578
2    2    5.761268 0.06189237 0.00321152    -0.01431578
3    3    5.854873 0.06189237 0.00321152    -0.01431578
4    4    6.056862 0.06189237 0.00321152    -0.01431578
5    5    6.138213 0.06189237 0.00321152    -0.01431578
6    6    6.331058 0.06189237 0.00321152    -0.01431578
```

```
coef(fit_lmm_fp_sj_item) |> pluck("sj") |>
  rownames_to_column(var = "sj") |> head()
```

```
  sj (Intercept)    verb_t1      gramm1 verb_t1:gramm1
1  1    6.401777 0.06189237 0.00321152    -0.01431578
2  2    5.794179 0.06189237 0.00321152    -0.01431578
3 07    5.869627 0.06189237 0.00321152    -0.01431578
4 09    5.782527 0.06189237 0.00321152    -0.01431578
5 10    6.621081 0.06189237 0.00321152    -0.01431578
6 11    5.913712 0.06189237 0.00321152    -0.01431578
```

- why do our intercepts vary, but not `verb_t1`, `gramm1`, or `verb_t1:gramm1`?

Extract deviations from the intercept

- the `ranef()` function provides the deviance from the model intercept and each random intercept estimate
  - the output is a `list` with a one element per grouping factor

```
ranef(fit_lmm_fp_sj_item)
```

```
$item
     (Intercept)
1     0.065780608
2    -0.195135717
3    -0.101530802
4     0.100458122
5     0.181809783
6     0.374654251
7     0.092819196
8     0.136954752
9     0.058102873
10   -0.054265683
11   -0.149873360
12    0.110751479
13    0.147096084
```

- `ranef()$grouping_factor` or `pluck("grouping_factor")` selects the relevant grouping factor

```r
ranef(fit_lmm_fp_sj_item)$sj |>
  head()
```

```
   (Intercept)
1   0.44537367
2  -0.16222459
07 -0.08677692
09 -0.17387701
10  0.66467739
11 -0.04269124
```

```r
ranef(fit_lmm_fp_sj_item) |>
  pluck("sj") |> head()
```

```
   (Intercept)
1   0.44537367
2  -0.16222459
07 -0.08677692
09 -0.17387701
10  0.66467739
11 -0.04269124
```

Compare estimates and deviances

- the values extracted by `ranef()` (`sj_dev` in Table 7) equal the difference (`difference`) between the model intercept (`model_intercept`) and the by-participant random intercept estimates (`sj_est`)
- so we can either look at each participant's (or item's) estimate, or look at how much it deviates from the model intercept

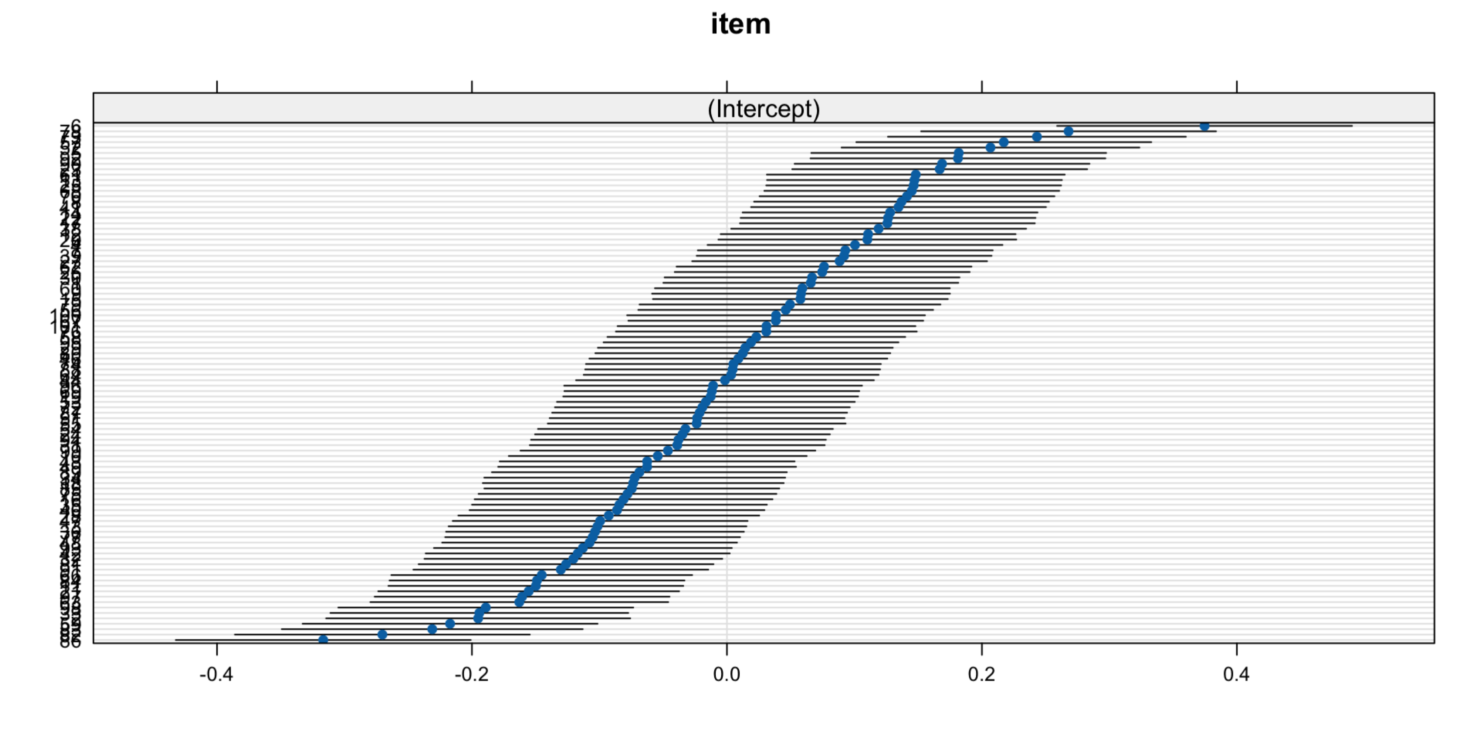
Table 7: Random intercept estimates versus deviance

sj	sj_est	sj_dev	est_minus_dev	model_intercept	est_minus_intercept
1	6.402	0.445	5.956	5.956	0.445
2	5.794	-0.162	5.956	5.956	-0.162
07	5.870	-0.087	5.956	5.956	-0.087
09	5.783	-0.174	5.956	5.956	-0.174
10	6.621	0.665	5.956	5.956	0.665
11	5.914	-0.043	5.956	5.956	-0.043

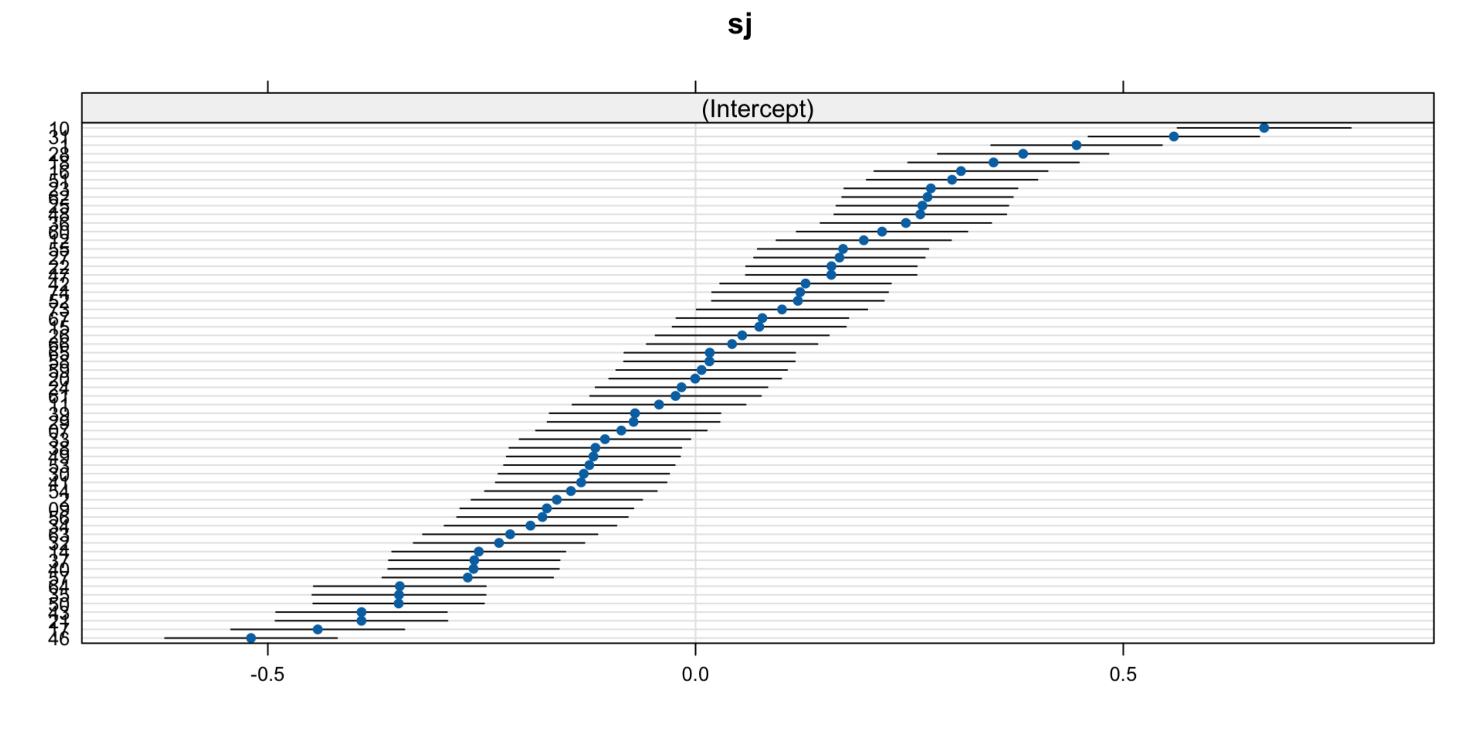
Visualise random effects

- the `lattice` package automatically produces plots of random effects estimates

```
lattice::dotplot(ranef(fit_lmm_fp_sj_item))$item
```




```
lattice::dotplot(ranef(fit_lmm_fp_sj_item))$sj
```



Reporting your model

- according to Sonderegger (2023) (p. 297), we should report:
  1. model definition (sometimes in ‘Data Analysis’ section)
  2. Fixed effects
  3. Random effects
  4. Sample size (number of observations, number of levels for each grouping factor)
  5. one or more quantitative summaries of the model, e.g., AIC, BIC, or logLik (although they’re only informative in comparison to another model fit to the same data)

Model definition

> We conducted the analysis by fitting `linear mixed-effect models` to our data, using the R package lme4 (Bates et al., 2014). We included Time Reference (past, future), and Verb Match (match, mismatch) as `fixed-effect factors` […] by adopting `sum contrast coding` (Schad et al., 2020): past and match conditions were `coded as -.5`. while future and mismatch conditions were `coded as .5`. […] Moreover, we included `crossed random intercepts` and random slopes for all fixed-effect parameters `for subject and item` grouping factors (Barr et al., 2013) in all models. […] Logit mixed-effect models were employed (Jaeger, 2008) for the analysis of the probability of regression measure. […] P-values were derived by using the lmerTest package (Kuznetsova et al., 2017).

— Biondo et al. (2022), p. 9

- could also explicitly mention method used for *p*-values, an example:

> P-values for individual predictors were computed using `lmerTest`, with the `Satterthwaite` option for denominator degrees of freedom for F statistics.

— Troyer & Kutas (2020), p. 9

- but here they don't cite the package
  - so you see, there's alway something you miss…
- FYI, to get a package's citation, run `citation("lmerTest")` in the Console

Results

- a combination of tables, figures, and in-text coefficient estimates is always key
- in-text, the *t*- and *p*-values should be included at minimum, Estimate and standard error (*Est* = …, *SE* = …,) could also be included if you aren't reporting many effects but must at least be included in a table
- figures will typically only show the distribution of raw observations and model predictions for fixed effects

In-text

A main effect of tense was found in first-pass reading times at the verb region (*Est* = 0.062, $t = 4.8$, $p < .001$), with the future tense (M = 458ms, SD = 274ms) eliciting longer first-pass reading times than the past tense.

Tables

Fixed effects

▶ Code for table

Table 8: Table of fixed effects from fit_lmm_fp_sj_item

Coefficient	$\hat{\beta}$	SE	t	df	p
Intercept	5.956	0.037	161.903	79.2	< .001
Tense	0.062	0.013	4.752	3637.1	< .001
Grammticality	0.003	0.013	0.247	3637.2	0.805
Tense x Gramm	-0.014	0.026	-0.550	3637.1	0.583

Random effects

▶ Code for table

Table 9: Table of random effects from fit_lmm_fp_sj_item

Group	Term	Variance	SD
item	(Intercept)	0.019	0.139
sj	(Intercept)	0.067	0.258
Residual	NA	0.161	0.401

Figures

- we don’t usually include plots of our random effects in publications, but these can be useful for model exploration and can be included in supplementary materials

lattice

- as already mentioned, we can simply use the the `lattice` package

```r
library(lattice)

dotplot(ranef(fit_lmm_fp_sj_item))["sj"]
```

```
$sj
```

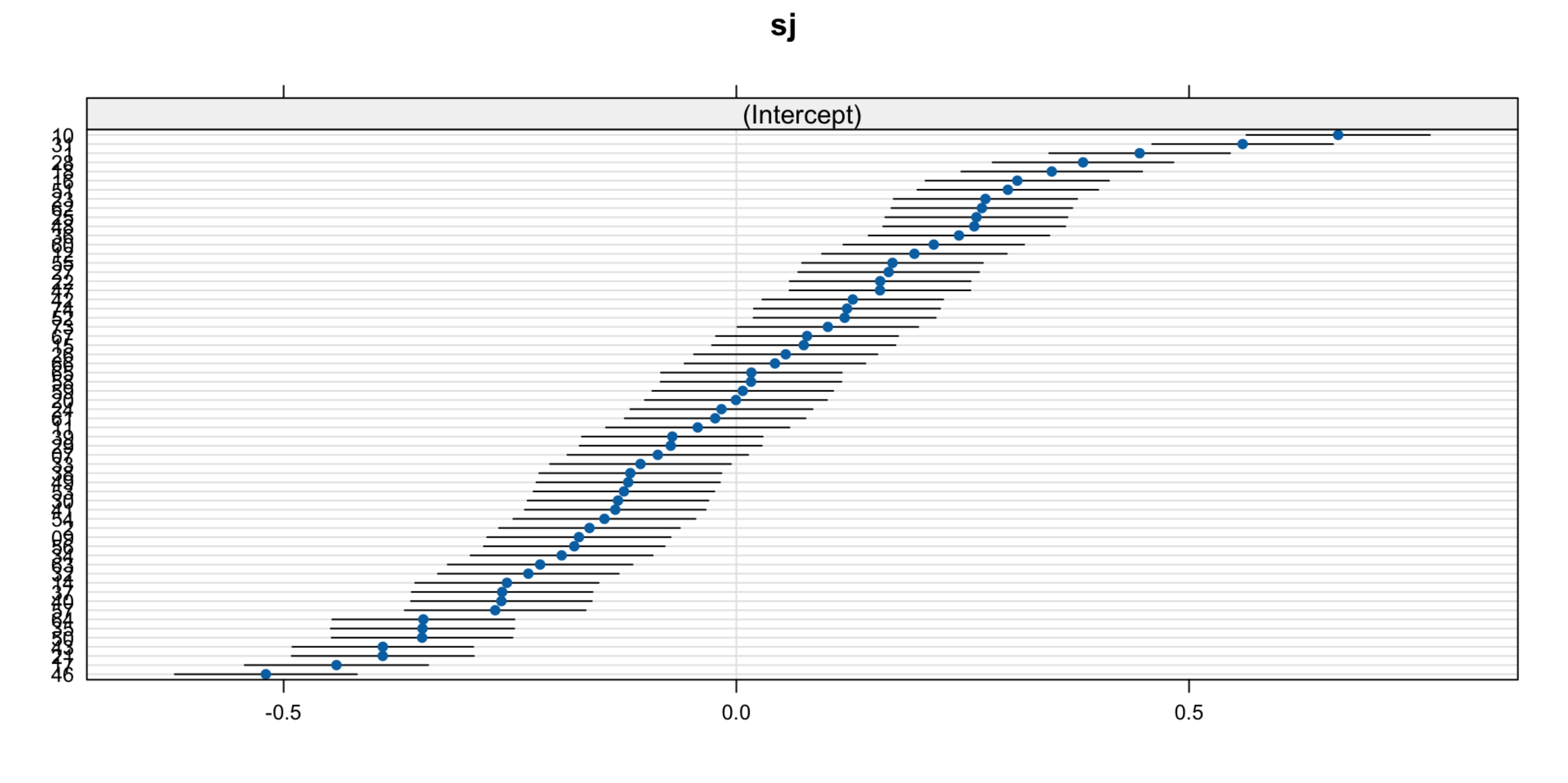


Figure 7: By-participant varying slopes (`lattice::dotplot(res(model))`)

broom.mixed

- or we can also generate the same plots using `tidy()` from the `broom.mixed` package + `ggplot()` (Figure 8 A)
- and we can add the model intercept to get each by-participant estimate, i.e., the values we get with `coef()` (Figure 8 B)

▶ Code for plot

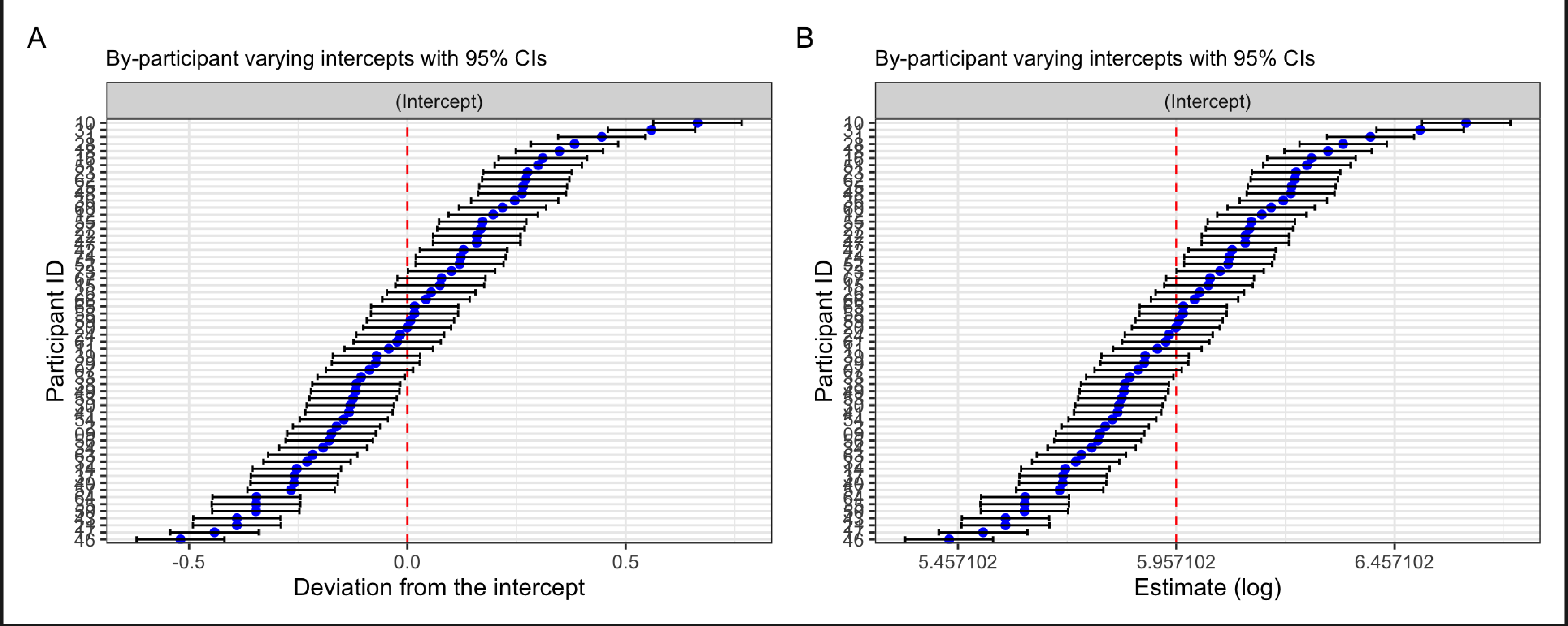


Figure 8: Back-transformed first-pass reading times (ms) at the verb region with 95% CIs

- and we can back-transform these values to milliseconds by exponentiating the estimates in the log scale (Figure 9 A)
- and we can back-transform deviances by subtracting the exponentiating model estimate from the back-transformed estimates (Figure 9 B)

▶ Code for plot

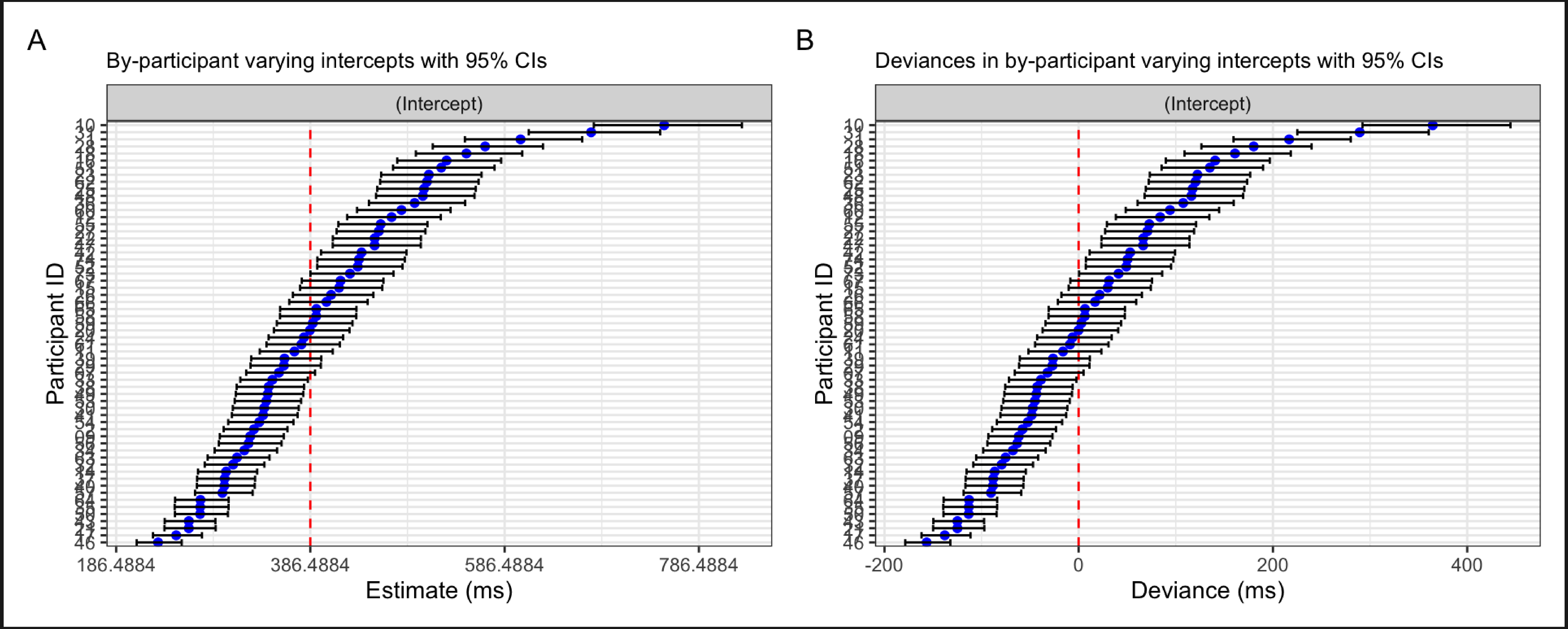


Figure 9: By-participant estimates back-transformed to milliseconds

Learning objectives 🏁

Today we learned…

- what linear mixed models are ✅
- how to fit a random-intercepts model ✅
- how to inspect and interpret a mixed effects model ✅

Important terms

Term	Definition	Equation/Code
linear mixed (effects) model	NA	NA

Task

1. Fit a linear mixed model (`lm()` function) to log-transformed total reading times (`tt`) at the adverb region (`roi == 2`), with adverb time reference (`adv_t`) and gramm (`gramm`) and their interaction as fixed effects and by-participant and by-item varying intercepts. Use sum contrast coding (`Past` and `gramm` = `-0.5`, `Future` and `ungramm` = `+0.5`). Save this model as `fit_lmm_adv_tt`.

3. Inspect the fixed effect of your model.

5. Plot the fixed effects for `fit_lm_adv_tt` and `fit_lmm_adv_tt`.

```
coef_fixed <-
  broom.mixed::tidy(
  fit_lmm_adv_tt,
  effects="fixed",
  conf.int = T
)

pred_back <-
  tibble(
  tense = c(rep("Past",2),rep("Future",2)),
  gramm = rep(c("gramm","ungramm"),2)
  )
```

4. Inspect the random effects for `fit_lmm_adv_tt`. Describe what you see.
5. Plot the random effects per participant and item.
6. Write up a description of your model as if for a publication (model formula, contrasts, random effects structure, packages/methods used).
7. Write up the results (coefficient estimates, etc.).

References

Biondo, N., Soilemezidi, M., & Mancini, S. (2022). Yesterday is history, tomorrow is a mystery: An eye-tracking investigation of the processing of past and future time reference during sentence reading. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *48*(7), 1001–1018. https://doi.org/10.1037/xlm0001053

Kuznetsova, A., Brockhoff, P. B., & Christensen, R. H. B. (2017). lmerTest package: Tests in linear mixed effects models. *Journal of Statistical Software*, *82*(13), 1–26. https://doi.org/10.18637/jss.v082.i13

Sonderegger, M. (2023). *Regression Modeling for Linguistic Data*.

Troyer, M., & Kutas, M. (2020). To catch a Snitch: Brain potentials reveal variability in the functional organization of (fictional) world knowledge during reading. *Journal of Memory and Language*, *113*(August 2019), 104111. https://doi.org/10.1016/j.jml.2020.104111

Winter, B. (2014). *A very basic tutorial for performing linear mixed effects analyses (Tutorial 2)*.

Winter, B. (2019). Statistics for Linguists: An Introduction Using R. In *Statistics for Linguists: An Introduction Using R*. Routledge. https://doi.org/10.4324/9781315165547