Pilot Tone Artifact Analysis

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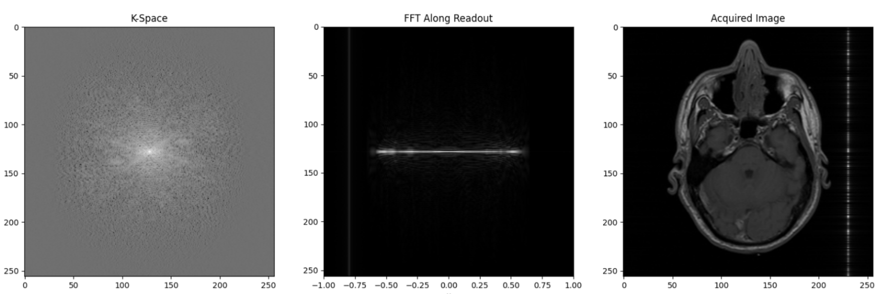
1 Background

A Pilot Tone is simple a constant RF signal that is being transmitted near (or inside) an MR machine. This pure RF tone is typically used as a non-contact motion detection technique. When there is a pure tone playing in the scanning room, the scanner will pick up this pilot tone in the resulting image.

Suppose that the one were to perform a scan in the presence of a pilot tone. This is what the acquisition would look like: The K-space image (left) on the left



looks as it normally would. It is hard to see a Pilot Tone in the K-Space image. The FFT along the readout direction (middle) clearly shows the presence of a constant RF tone in each readout. The acquired image (right) shows the pilot tone artifact in the resulting image. We are interested in finding out why the pilot tone creates this 'zipper' artifact in the acquired image.

The goal here will be to find out why this is a zipper artifact. We will do a mathematical analysis as well as a python simulation in order to confirm our findings.

2 K-Space Modeling

First let us introduce all the necessary equations and constants that will govern the MR acquisition process:

F_s - The sampling frequency during readout

F_{PT} - The frequency of the pilot tone relative to the center frequency

T_R - The time between readouts

N_r - The number of readouts

N_p - The number of phase encodes

$K(p, r)$ - The 2D K-space matrix indexed by p (phase encode) and r (readout sample)

$I(a, b)$ - The 2D image matrix indexed by a (row of image) and b (column of image)

The transmitted pilot tone will appear as a complex exponential to the scanner. That is,

$$PT(t) = \exp\{j2\pi F_{PT}t\}. \tag{1}$$

The scanner will sample this continuous time pilot tone signal on top of the signal received from the subject inside the scanner. However, to make the analysis simpler, we will ignore signals that are not from the pilot tone. That is, this would be like acquiring a scan when there is no RF or gradient on. We are just interested in the pilot tone.

We can model the measured K-space matrix with the following relationship;

$$K(u, v) = \exp\{j2\pi F_{PT}(\frac{r}{F_s} + pT_R)\} \tag{2}$$

$$p = u + N_p/2,\ r = v + N_r/2$$
$$u \in [-N_p/2, N_p/2 - 1],\ v \in [-N_r/2, N_r/2 - 1]$$

There are a few things to keep in mind for equation(2). r is the 0-indexed readout number and p is the 0-indexed phase encode number. So, for any given r and p, the time elapsed since the first sample is just $\frac{r}{F_s} + pT_R$. Notice that the only difference between (1) and (2) is that we replaced t with the time elapsed as a function of r and p.

3 Image Modeling

Now that we have modeled our K-space matrix, we can perform the Inverse Two Dimensional Discrete Fourier Transform to obtain the resulting acquired image. It is the 2DFT that governed the relationship between K-space and the Image. This Fourier relationship is given by

$$I(a, b) = \sum_{u=N_p/2}^{N_p/2-1} \sum_{v=N_r/2}^{N_r/2-1} K(u, v) \exp\{j2\pi(\frac{au}{N_p} + \frac{bv}{N_r})\}. \tag{3}$$

We can substitute (2) into $K(u,v)$ to simplify (3):

$$I(a,b) = \sum_{u=N_p/2}^{N_p/2-1} \sum_{v=N_r/2}^{N_r/2-1} \exp\{j2\pi F_{PT}(\frac{r}{F_s} + pT_R)\} \exp\{j2\pi(\frac{au}{N_p} + \frac{bv}{N_r})\}$$

$$I(a,b) = \sum_{u=N_p/2}^{N_p/2-1} \sum_{v=N_r/2}^{N_r/2-1} \exp\{j2\pi(u(F_{PT}T_R + \frac{a}{N_p}) + v(\frac{F_{PT}}{F_s} + \frac{b}{N_r}) + \frac{1}{2}(C+a+b))\}$$

$$I(a,b) = \exp\{j\pi(C+a+b)\} \sum_{u=N_p/2}^{N_p/2-1} \sum_{v=N_r/2}^{N_r/2-1} \exp\{j2\pi(u(F_{PT}T_R + \frac{a}{N_p}) + v(\frac{F_{PT}}{F_s} + \frac{b}{N_r})\}$$

$$\text{Where } C = F_{PT}(T_R N_p + \frac{N_r}{F_s})$$

Since we only care about $|I(a,b)|$:

$$|I(a,b)| = |\sum_{u=N_p/2}^{N_p/2-1} \sum_{v=N_r/2}^{N_r/2-1} \exp\{j2\pi(u(F_{PT}T_R + \frac{a}{N_p}) + v(\frac{F_{PT}}{F_s} + \frac{b}{N_r})\}|$$

$$|I(a,b)| = |\sum_{u=N_p/2}^{N_p/2-1} \exp\{j2\pi u(F_{PT}T_R + \frac{a}{N_p})\}|| \sum_{v=N_r/2}^{N_r/2-1} \exp\{j2\pi v(\frac{F_{PT}}{F_s} + \frac{b}{N_r})\}| \tag{4}$$

4 Artifact Analysis: In A Vacuum

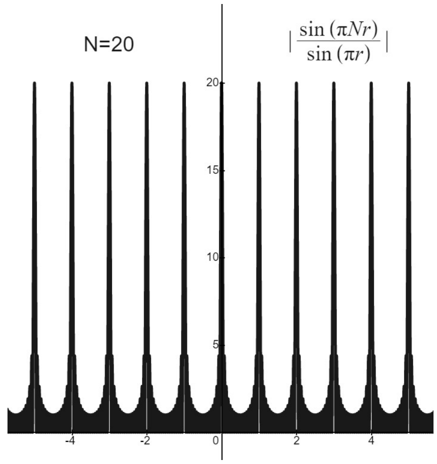
Now it is important to answer the following question: **For what values of a, b does $I(a,b)$ have a significant magnitude?** To answer this, we must first understand when the sum of complex exponentials has a significant magnitude. Let's start with a simpler case. Looking at the equation below, we will take N to be some even number and r to be a real number:

$$A(r) = \sum_{k=-N/2}^{N/2-1} \exp\{j2\pi kr\}$$

We can recognize A(r) as the N-DFT of a length N rectangular function. We have the final magnitude as:

$$|A(r)| = |\frac{\sin(\pi N r)}{\sin(\pi r)}|$$

By searching A(r) for critical points, we can verify that the most significant points (by their magnitude) in this function occur when r is an integer. A graph of the function below shows exactly this relationship:



Finally we can apply this to both magnitude terms in (4). Starting with the left term in equation (4), we see that instead of r, we have $F_{PT}T_R + \frac{a}{N_p}$. In order for $I(a,b)$ to have a significant value, we require $F_{PT}T_R + \frac{a}{N_p}$ to be an integer. We also notice that

$$a \in [-\frac{N_p}{2}, \frac{N_p}{2} - 1] \implies \frac{a}{N_p} \in [-1/2, 1/2).$$

We can repeat the same logic for the right term of equation (4) to get that

$$b \in [-\frac{N_r}{2}, \frac{N_r}{2} - 1] \implies \frac{b}{N_r} \in [-1/2, 1/2).$$

This means that in order for $F_{PT}T_R + \frac{a}{N_p}$ to be an integer, we choose a such that $\frac{a}{N_p}$ will 'round off' $F_{PT}T_R$ to the nearest integer. The same is true of $\frac{b}{N_r}$ and $\frac{F_{PT}}{F_s}$ That is, only when a, b are

$$a = N_p(round(F_{PT}T_R) - F_{PT}T_R) \tag{5}$$

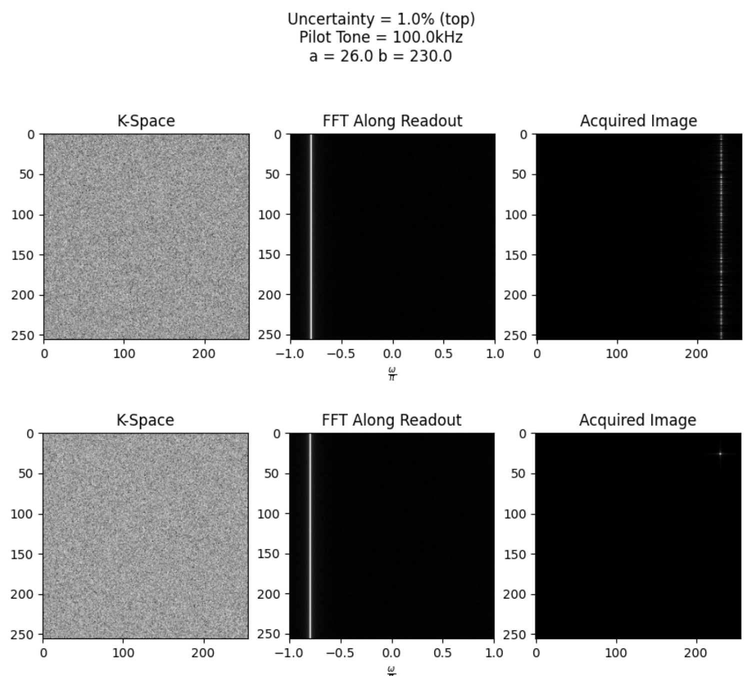
$$b = N_r(round(\frac{F_{PT}}{F_s}) - \frac{F_{PT}}{F_s}) \tag{6}$$

is $I(a,b)$ of significant magnitude. **Equations (5) and (6) conclude that the artifact due to a pilot tone is roughly (since we are only considering significant magnitudes) a point in the image.**

5 Artifact Analysis: The Real World

Remember, the conclusion of the last section was that the pilot tone artifact is simply a bright point in the final image. But this is not at all what we expect! Remember, the pilot tone is typically shown as a zipper artifact as shown in the background section. So what is going on here? It turns out that equation (2) is actually incorrect! **We assumed that the time elapsed between one readout to another is T_R. However, this is not always true. The scanner is not perfect and will not always have the exact same T_R between readouts.**

We will illustrate this correlation between the randomness of the scanner and the zipper artifact by using a Python Pilot Tone simulation. The simulation will generate a random K-Space matrix, this part is not too important. Then, the pilot tone will be added to each readout. The first Pilot Tone altered K-space will assume the phase accumulated between readouts depends on a constant T_R. The second Pilot Tone altered K-space will instead change T_R slightly between readouts. The way this will be done is by adding some uncertainty to the true T_R. This uncertainty is labeled in the graphs below as well as the location of the pilot tone:



We showed that under no noise, the Pilot Tone cannot cause a zipper artifact. Then, once noise in T_R was simulated, we saw the zipper artifact. For those reasons, we believe that it is the uncertainty in T_R that causes the zipper artifact in the final image.