

# West & Harrison Chapter 10, Problem 7

The Dans

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The quarterly sales and cost index of a confectionary product are given in figure 1. The goal is to fit a DLM to the sales

Year	SALES				COST			
	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 1	Quarter 2	Quarter 3	Quarter 4
1975	157	227	240	191	10.6	8.5	6.7	4.1
1976	157	232	254	198	1.9	0.4	1.1	1.9
1977	169	234	241	167	0.2	2.9	4.1	-1.4
1978	163	227	252	185	-4.0	-4.5	-5.3	-8.4
1979	179	261	264	196	-12.8	-13.2	-10.1	-4.6
1980	179	248	256	193	-1.1	-0.1	0.0	-2.5
1981	186	260	270	210	-5.1	-6.4	-8.0	-6.5
1982	171	227	241	170	-3.7	-1.3	6.1	16.5
1983	140	218	208	193	22.9	23.9	18.0	8.3
1984	184	235	245	209	2.9	0.7	-2.4	-7.0
1985	206	260	264	227	-9.8	-10.6	-12.3	-13.2

Figure 1: The data associated with the problem

with first-order polynomial, regression on cost, and full seasonal components. Based on previous information,

- the underlying level of the series when cost is zero is 220 with a nominal standard error of 15
- the regression coefficient of cost is estimated as -1.5 with a standard error of about 0.7
- the seasonal factors for the four quarters of the first year are expected to be -50, 25, 50, -25, with nominal standard errors of 25, 15, 25, and 15, respectively
- the trend, regression, and seasonal components are initially assumed to be uncorrelated
- the observational variance is estimated as 100, with initial degrees of freedom of 12

Using this information, we are to analyze the series along the following lines:

- (a) **Using the information provided for the seasonal factors above, apply Theorem 8.2 to derive the appropriate initial prior that satisfies the zero-sum constraint.**

*Theorem 8.2: Imposing the constraint  $\mathbf{1}'\phi = 0$  on the prior  $(\phi_0|\mathcal{D}_0^*) \sim N(\mathbf{m}_0^*, \mathbf{C}_0^*)$  and writing  $U = \mathbf{1}'\mathbf{C}_0^*\mathbf{1}$  and  $\mathbf{A} = \mathbf{C}_0^*\mathbf{1}/U$  gives the revised joint prior*

$$\begin{aligned}
 (\phi_0|\mathcal{D}_0) &\sim N(\mathbf{m}_0, \mathbf{C}_0), \\
 \mathbf{m}_0 &= \mathbf{m}_0^* - \mathbf{A}\mathbf{1}'\mathbf{m}_0^*, \\
 \mathbf{C}_0 &= \mathbf{C}_0^* - \mathbf{A}\mathbf{A}'U.
 \end{aligned}$$

Using the given information for the seasonal components,

$$\mathbf{m}_0^* = (-50, 25, 50, -25)' \quad \mathbf{C}_0^* = \begin{pmatrix} 625 & 0 & 0 & 0 \\ 0 & 225 & 0 & 0 \\ 0 & 0 & 625 & 0 \\ 0 & 0 & 0 & 225 \end{pmatrix}.$$

Applying Theorem 8.2 yields

$$\begin{aligned}
U &= 1700 \\
A &= (0.3676471, 0.1323529, 0.3676471, 0.1323529)' \\
\mathbf{m}_0^s &= (-50, 25, 50, -25)' \\
\mathbf{C}_0^s &= \begin{pmatrix} 395.22 & -82.72 & -229.78 & -82.72 \\ -82.72 & 195.22 & -82.72 & -29.78 \\ -229.78 & -82.72 & 395.22 & -82.72 \\ -82.72 & -29.78 & -82.72 & 195.22 \end{pmatrix}.
\end{aligned}$$

This leads to the initial prior satisfying the zero-sum constraint for the seasonal components to be  $(\phi_0|\mathcal{D}_0) \sim N(\mathbf{m}_0^s, \mathbf{C}_0^s)$ . Given a state vector  $\boldsymbol{\theta}_t = (\theta_{1,t}, \theta_{2,t}, \phi_{1,t}, \dots, \phi_{4,t})'$ , the initial prior for the DLM is of the form  $(\boldsymbol{\theta}_0|\mathcal{D}_0) \sim N(\mathbf{m}_0, \mathbf{C}_0)$ , where

$$\begin{aligned}
\mathbf{m}_0 &= (220, -1.5, -50, 25, 50, -25)' \\
\mathbf{C}_0 &= \begin{pmatrix} 225.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.49 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 395.22 & -82.72 & -229.78 & -82.72 \\ 0.00 & 0.00 & -82.72 & 195.22 & -82.72 & -29.78 \\ 0.00 & 0.00 & -229.78 & -82.72 & 395.22 & -82.72 \\ 0.00 & 0.00 & -82.72 & -29.78 & -82.72 & 195.22 \end{pmatrix}.
\end{aligned}$$

- (b) **Identify the full initial prior quantities for the 6-dimensional state vector  $\boldsymbol{\theta}_1$  and the observational variance  $V$ . Write down the defining quantities  $\mathbf{a}_1, \mathbf{R}_1, n_0$ , and  $S_0$  based on the above initial information.**

The DLM model in this case is defined as  $\{\mathbf{F}_t, \mathbf{G}, V, \mathbf{W}_t\}$ , with

$$\begin{aligned}
\mathbf{F}_t &= (1, \text{Cost}_t, 1, 0, 0, 0)' \\
\mathbf{G} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}.
\end{aligned}$$

Using this information,

$$\begin{aligned}
\mathbf{a}_1 &= \mathbf{G}\mathbf{m}_0 = (220, -1.5, 25, 50, -25, -50)' \\
\mathbf{R}_1 &= \mathbf{G}\mathbf{C}_0\mathbf{G}' + \mathbf{W}_t = \begin{pmatrix} 225.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.49 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 195.22 & -82.72 & -29.78 & -82.72 \\ 0.00 & 0.00 & -82.72 & 395.22 & -82.72 & -229.78 \\ 0.00 & 0.00 & -29.78 & -82.72 & 195.22 & -82.72 \\ 0.00 & 0.00 & -82.72 & -229.78 & -82.72 & 395.22 \end{pmatrix} + \mathbf{W}_t
\end{aligned}$$

$$n_0 = 12$$

$$S_0 = 100,$$

and  $(\boldsymbol{\theta}_1|\mathcal{D}_0) \sim T_{12}(\mathbf{a}_1, \mathbf{R}_1)$