



Universidad Técnica Nacional



Electronic Engineering

Automatic Control Project

Magnetic Levitation System

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Introduction (D)

A magnetic levitator allows to sustain a magnetic load in suspension, it can be obtained by applying current to an electromagnet that generates repulsion between magnetic poles, starting from the control of the electromagnet it's possible to maintain the load in the air at certain position. In this investigation, the study of many tools in the field of engineering is incorporated. However, this field is still under study, for the application in different areas of science and engineering.

The project consists of making a design of a magnetic levitator, carrying out the study of all the elements that are part of it, such as: the mathematical analysis and use of the transfer function, the study and application of magnetic fields, development of PID controller, design of electronic circuits, creation of code to handle variables such as height position, object stabilization and processing of different signals. The set of all these parts belongs to the implementation of a physical model of the magnetic levitation system, considering the difficulties and challenges generated using a great variety of fields of study in the project, as well as the application of their respective solutions.

The main purpose of the project is to give the system the ability to control the position of the levitating object, it must be able to make all the necessary regulations and measurements automatically, making mathematical calculations that help its proper functioning.

The characteristic of the system is that it will have a single sensor that measures the height of the floating magnet is located, this magnet will be located just above the electromagnet and permanent magnets will surround it, which between both components will apply upward force and repulsion to maintain the permanent magnet in the air. The floating magnet will be ideally centered and the only thing that will be regulated will be the height.

Electromagnetic fields are force fields created because of the movement of electric charges. The electromagnets will fulfill this function, since a coil will cause a concentration of magnetic flux, more powerful, more stable, and controllable. This control oversees modifying the electric current that circulates through it.

To effectively apply the stabilization of the levitator, it is necessary to consider its position, to be processed, through code and the information contained in mathematical analyzes such as the transfer function, the linearity that exists between the magnitude of the current and magnitude of the magnetic field, in other words, the force suffered by the load as a function of the current. Thanks to obtaining this data, a precise controller can be designed numerically, however, a series of tests allows the control to have a quick answer, damping changes and to provide stability.

Throughout the project there were a series of changes in the design of the project because they were found convenient during the realization of the mathematical analysis. Changes such as the number of electromagnets, at the beginning of the project it was projected with 4 electromagnets, by simplifying the mathematical and physical calculations, thus improving the operation of the project with the proposed design.

It is worth mentioning that 4 coils with different cable gauges were created, which were changed at convenience due to the current conditions in which the magnetic levitation system was working.

For the design of the levitator, the possible situations that could happen were considered so that the performance could not be the desired. The heat conditions that the coils can produce, as has been described the flux of current generates a magnetic field, when placing the load, parasitic currents can be produced that increase the amount of energy released in the form of heat, affecting the components and the structure. In addition, the precision during the construction of the electromagnet can be a fundamental aspect for the correct operation of the system, also there are other components to consider, including: sensor reliability, signal noise, etc.

During the project, the team is organized into subgroups consisting of two people, each of whom must oversee at least two topics, generally related to support among members, of the same or different groups. As mentioned above, the important aspects of the tasks to be performed are code developing, mathematical analysis, electronic circuit design, electromagnet, and magnetic force, as well as position and current measurement.

The idea in the distribution of tasks is the support between the members, however, it is necessary that the specialists in each subject can correctly carry out their tasks and also provide help in research and physical assembly of the system elements. Therefore, the design, analysis, prototyping, construction or assembly and testing are the 5 main stages to successfully achieve the project performance.

To implement the project on a company the economic part is important, since it needs to render accounts to the creditor to identify if the projects is feasible or not to a company, according to the obtain benefits and the economic cost of this. If the expenses exceed the benefits, then the project is not economically viable for a company. The economic expectations can vary at the beginning of the project compared to the end, since there is little information about the final designs proposed.

Literature review:

Circuit of a magnetic levitation system: (D)

The current control of the electromagnet was carried out with the use of an N-channel MOSFET in a configuration like a step down or buck circuit that helps to reduce the value of the output voltage in relation to the input. Adding protection elements such as a reverse polarity diode in parallel to the electromagnet.

It is possible to find that the frequency of 10kHz was used to switch the MOSFET type transistor, guaranteeing current to the coil and providing smoothness in the operation of the levitator and avoiding voltage peaks. Some problems with the heating of the MOSFET transistor and the electromagnet are discussed, which were solved by using heat sinks. [3]

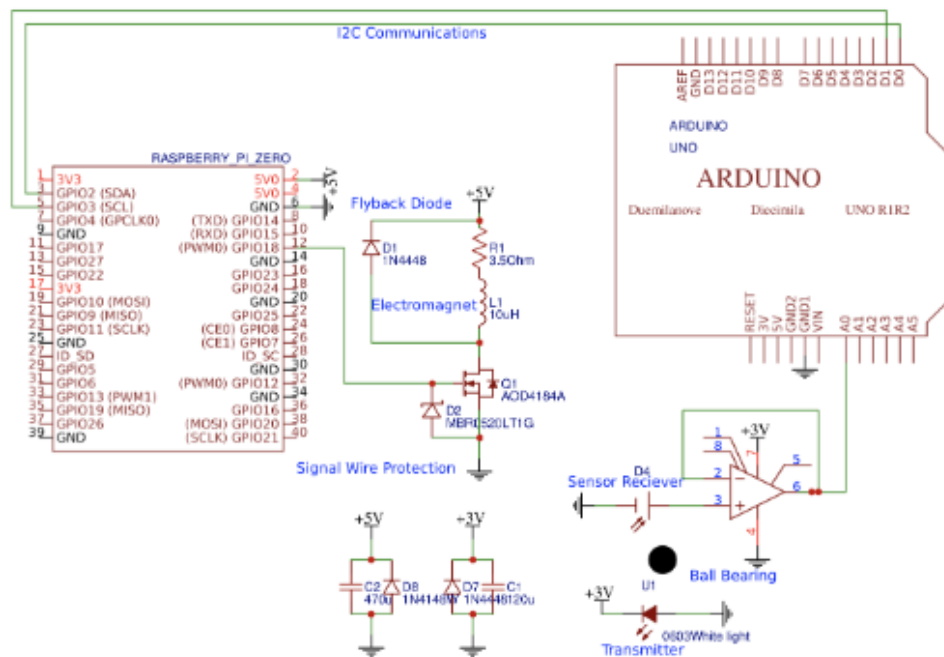


Figure 1. Magnetic levitation system circuit diagram. [3]

Controller design: (E)



Figure 2. Design and construction of a magnetic levitation system controlled by a PID algorithm. [7]

The PID controller exerts a control signal that depends on an error value calculated as the difference between the desired signal and the system output. There must be a signal proportional to the error if the output is far from the reference signal ($P = K_p e(t)$).

In addition, a control action is needed that acts before the accumulation of the error, when it becomes constant, which causes the system to act to improve the response of the system in a steady state with a tendency to the reference signal

$$I = K_i \int_0^t e(t) dt.$$

Finally, the derivative control action acts against changes in the system output over time and tries to anticipate these changes $D = K_d \frac{d}{dt} e(t)$.

So, the general form of the PID is:

$$u(t) = K_p e(t) + K_i \int_0^t e(t) dt + K_d \frac{d}{dt} e(t)$$

$$\frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s$$

Figure 3. PID equations. [7]

Design Two-Dimensional Magnetic Levitation Control System: (S)

In this thesis they analyze 2 different systems, one is a simple levitator that has only 1 electromagnet and a steel ball levitating. The other system is with 2 electromagnets, applying 2 different currents for each electromagnet. They also analyze the stability of the system and how can linearize this system and maintain the ball levitating.

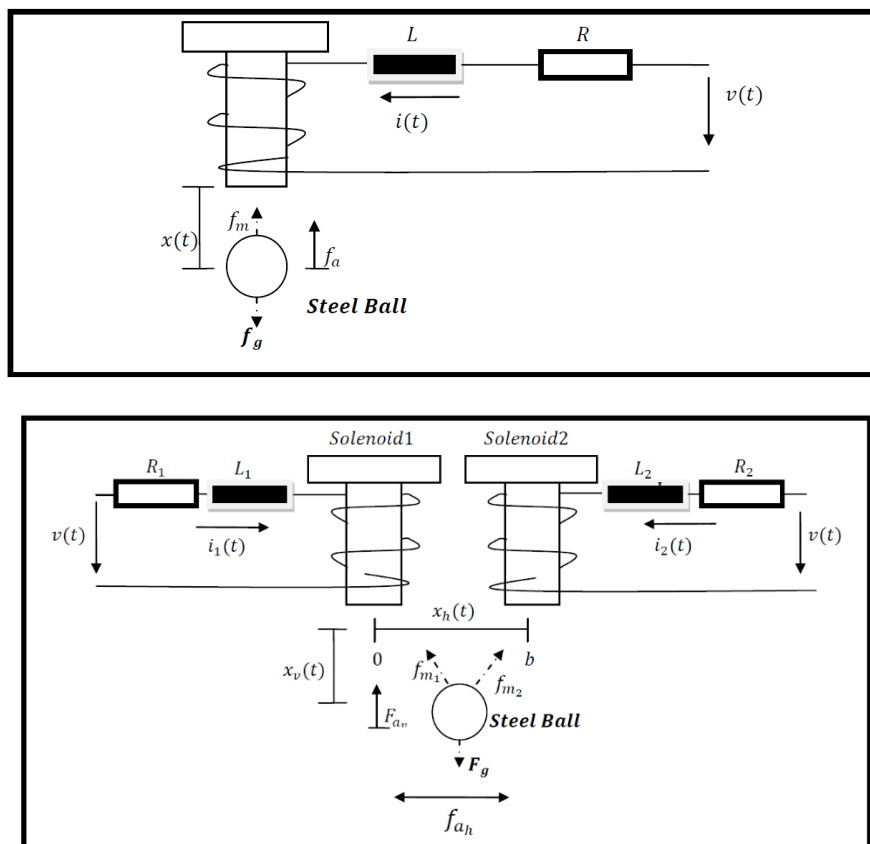


Figure 3. System diagram.

Project Management

Organization of work teams: (D)

The team was organized in such a way that each one of the members shared at least one of the tasks with another member, with the aim of dividing the workload and having support among colleagues due to the high complexity of the project.

Member	Task		
Daniela Méndez	Software	Case	Position sensor
Keylor Garcia	Software	Mathematical analysis	Position sensor
Shinji Rodriguez	Simulation	Mathematical analysis	
Emerson Molina	Circuit Design	PID	
William Sánchez	Electromagnet	PCB design	Magnetic force

Table 1. Task division by person.

Gantt Diagram: (D)

The Gantt Diagram schedules each of the tasks to be carried out by the team in the expected time to project final presentation, it is also possible to have control of late tasks and tasks to be carried out. The blue color indicates the activities in the planned time, on the other hand, the red color indicates delayed tasks.

Task	Beginning	End	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7
State of the art	Week 1	Week 2							
Task distribution	Week 1	Week 2							
Materials and their costs	Week 1	Week 2							
Circuit Design	Week 2	Week 4							
Mathematical analyzes of the system	Week 2	Week 6							
Electromagnet research	Week 2	Week 4							
Electromagnet making	Week 4	Week 5							
First presentation	Week 4	Week 5							
Week			1	2	3	4	5	6	7

Table 2. Gantt Diagram from week 1 to week 7.

Task	Beginning	End	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9
First assembly (Control current electromagnets)	Week 4	Week 5						
PCB design and printing	Week 5	Week 6						
PCB assembly	Week 6	Week 7						
Second presentation	Week 6	Week 7						
Code Developing	Week 7	Week 8						
PCB second test	Week 7	Week 8						
Third presentation	Week 8	Week 9						
Week			4	5	6	7	8	9

Table 3. Gantt Diagram from week 4 to week 9.

Task	Beginning	End	Week10	Week11	Week12	Week13	Week14
Structure design	Week 9	Week10					
PCB third test	Week 9	Week10					
Fourth presentation	Week10	Week11					
PCB fourth test	Week11	Week12					
Project documentation	Week 2	Week13					
Final presentation	Week12	Week13					
Week			10	11	12	13	14

Table 4. Gantt Diagram from week 10 to week 14.

There were some tasks that were delayed because of the changes to the design of the project and some complications mainly generated because of mathematical and physical calculations. Due to the complexity of the project and the time the research took to find good and reliable information that could be applied to the needs of the project.

Methodology:

The creation of a levitron is requested, which will fulfill the function of keeping a magnet floating, it must have a regulation of its height and stability by using a PI, PD or PID controller.

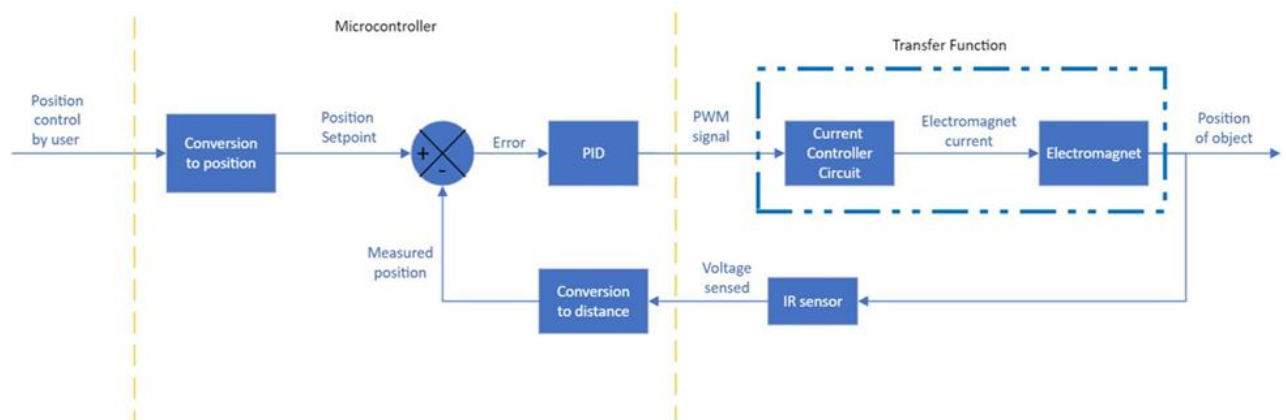
To carry out what was requested, a mixed research methodology was used where both the qualitative and quantitative parts were used. This is because qualitative aspects of the materials to be used and the physic-mathematical aspect of situations that influenced the levitron system had to be fulfilled.

Specifications of the magnetic levitation system:

Description	
Number of coils turns	470
Cable gauge	24AWG
Electromagnet size	30mmx25mm
Magnets	Neodymium N35
Number of coils	1
Number of magnets	8x5
Distances between magnets and electromagnet	49mm
Maximum current	3A
Maximum separation distance in air	10mm
Voltage	12V

Table 5. Specifications of the magnetic levitation system

The previous table will show the specifications that the levitation system and the electromagnet must comply with. The latter must have these specifications for proper functioning since the magnet will be levitated on it.

Block Diagrams: (K)

The block diagram shows the different parts that allow the magnet levitation height to be controlled, the section consists of two large blocks: the microcontroller and the transfer function. The levitation height is defined in the microcontroller, and it is the output of the PWM signal to control the current variation with respect to the position, the position of the magnet is compared with the position defined by the user by means of an infrared sensor that determines

the distance to which the magnet is located, and the error will be the difference between these aspects.

Economic Cost: (D)

The cost of the materials was registered to keep track of the components used for future implementations and improvements of the system. To implement the project on a company the economic part is important, since it needs to render accounts to the creditor to identify if the projects is feasible or not to a company, according to the obtain benefits and the economic cost of this.

Material	Amount	Cost
Hall effect sensor	1	\$8,70
Optical sensor	1	\$7,95
Capacitor	4	\$0,72
Heatsink	1	\$3,95
Resistors	3	\$0,18
Base socket	1	\$0,50
Magnets	2	\$5,90
Cable	1	\$5,90
Cooper plate	1	\$1,25
Terminal block	5	\$4,75
Case	1	\$31,95
Transistors	2	\$0,54
Crystal Oscillator	1	\$1,50
ATMega328	1	\$8,95
12V 10A source	1	\$20,95
5V regulator	1	\$1
Total		\$83,74

Table 6. List of materials and their cost.

Description	Engineer hour	Cost
Design	15	€ 2.898.000,00
Analysis	20	€ 3.864.000,00
Prototype	10	€ 1.932.000,00
Construction	10	€ 1.932.000,00
Testing	30	€ 5.796.000,00
Total		€ 16.422.000,00

Table 7. Cost of the engineer hour.

Final Economic Cost	
Materials	€ 51.054,60
Engineer hour	€ 16.422.000,00
Total	€ 16.473.054,60

Table 8. Final economic cost of the project.

As you can see the cost of the material is not that high, however, the most expensive thing is the hours dedicated to the project, due to the cost of the engineer hour, being €32.200. Owing to the complexity and the time that it takes to do all the calculations, measurements, the design and the testing considering changes doing to simplify.

Final Economic Expectation		
Materials	₺	40.379,11
Engineer hour	₺	16.422.000,00
Total	₺	16.462.379,11

Table 9. Final economic expectation of the project.

At the beginning of the project approach, an estimate of the costs was calculated, however, since there is little information about the final designs proposed, the economic expectation varies from the beginning and at the end of the project. For this reason, if we compare Table 8 and Table 9, it's possible to observe different totals between each of the tables, however, the difference varies by approximately ₺10,000 in material costs.

Theoretical concepts:

For the development of the project an electromagnet was used because it has the properties experienced by electrical conductors, a current flow through the conductor and a magnetic field is generated. In addition to being a magnetic levitation system, this device was used as a pillar for its development.

The electromagnet is a type of magnet in which the magnetic field is generated by the circulation of an electric current through a conductor. Its superiority over the permanent magnet lies in the fact that the intensity of the field generated depends on the amount of current flowing through it, and it is possible to control its behavior. However, in applications in which it is not necessary to modify the magnetic field or make it disappear, the permanent magnet is superior, since it can generate larger fields at the same size.

It was invented in 1825 by a British electrician named William Sturgeon. He based it on the studies of Hans Christian Ørsted, a Danish physicist who discovered that the flow of an electric current through a conductor generates a magnetic field around it. Sturgeon's electromagnet consisted of a horseshoe-shaped (or U-shaped, to bring the poles closer together and concentrate the lines of magnetic force) insulated piece of metal wrapped in a coil: a spiral of conductor with many turns (the more turns the coil has, the more powerful the electromagnet). An important advance in the evolution of the electromagnet was thanks to Joseph Henry, an American scientist who changed the iron insulation for conductor insulation, achieving better results than Sturgeon. In addition, he discovered the principle of electromagnetic induction in parallel with Faraday, although it was published earlier by Faraday. He put into practice his knowledge of electromagnetism to help Morse develop his telegraph based on the electromagnet. However, it is not necessary to insert a metal part into the coil to make an electromagnet. An "empty" coil is called a solenoid, with each pole at one end. The reason for introducing a metallic material (usually iron) inside the coil is that the field generated is much stronger at the same current intensity. This effect, known as ferromagnetism, occurs because materials of this type contain small magnetized zones (domains) that are initially disordered. The magnetic field of the solenoid arranges them, so that the iron becomes a magnet, and its effect is added to that of the coil. When the coil field disappears, the domains

usually become disordered again, but some materials are remanent, i.e., they remain magnetized for a while.

Because of their versatility, electromagnets have enabled the emergence of numerous electronic devices, and important industrial applications have arisen. The invention of the electric telegraph was made possible precisely thanks to the electromagnet, and electric motors transform electrical energy into kinetic energy through the operation of several electromagnets together. Thanks also to their ability to vary the magnetic field, they are used in loudspeakers and storage devices. However, the most powerful applications are industrial ones, such as the separation of materials, the cleaning of contaminated water, magnetic levitation trains, current generators, or the transport of heavy materials. [11]

As mentioned, the electromagnet is a magnet, where the magnetic field is generated by electric current, and the current passing through a wire creates a magnetic field around it, these have advantages in their use and over other similar devices, although it is also important to have certain considerations in their use, such as heat.

Unlike common magnets, electromagnets heat up. These artificial devices do everything a magnet can do and much more; and they are particularly useful because they can be made to have any desired field strength and can be made to become stronger or weaker or even turn off. Basically, electromagnets are coils of wire wrapped around a metal core, which in turn are connected to a battery. While they are easy to make, they can have a problem with overheating if given more voltage than their wires can handle. Fortunately, with careful design, this problem can be avoided. [12]

Electromagnet is called the device that has the property of acquiring magnetic properties when its coil is crossed by an electric current. [8] Therefore, it easily transforms electrical energy into mechanical energy.

Used in the project as the central magnet, on which the object to be placed in the project will levitate. This has an external circuit that allows the regulation of the current that enters the winding of the electromagnet created.

Magnetism is called the part of electricity that studies the properties of magnetic fields and the behavior of bodies that are subjected to these fields. [8]

This phenomenon should have been taken into consideration when placing the magnets that are around the central electromagnet, the attraction or repulsion that the object has on each of the base magnets is essential in the levitation that can occur.

The lines of force of a magnet are invisible lines that can be seen by scattering iron filings in a magnetic field, leaving one pole of the electromagnet and going to the other pole in a beam. [8] These lines become unobservable, but they occur since a magnetic field is created in the magnetic levitator.

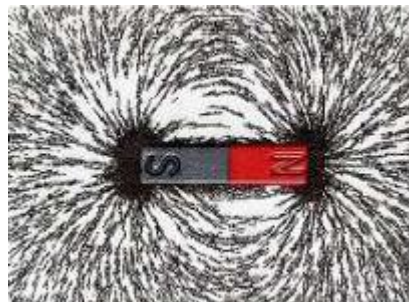


Figure 4. Representation of lines of force [9]

Magnetic flux is the total number of lines of force that cross any section of a magnetic circuit.[8] In the case of the magnetic levitator, the field lines pass through the object that will be found hovering over the electromagnet.

Magnetic permeability is the relationship between magnetic induction and the corresponding magnetic field intensity determined on the magnetization curve. [8]. It is used in the force formula for the object to be held in levitation.

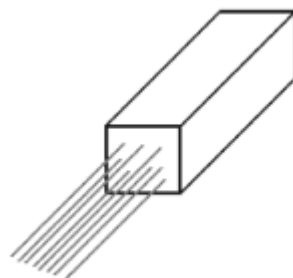


Figure 5. Representation of the magnetic flux [8]

Magnetic flux. It is defined as the surface integral over the normal component of the magnetic field where Φ is the magnetic flux, B the magnetic induction, gives the surface area differential and n the norm. The magnetic flux is analogous to the electric field, it is also represented as arrowhead lines, which run from the north to the south pole of the magnet. For a magnetic induction B and a constant area A . [13]

Magnetic susceptibility. In many isotropic and linear materials there is a relationship between magnetization (M) and magnetic field strength (H), where χ_m is the magnetic susceptibility. According to their magnetic susceptibility materials are classified into paramagnetic if $\chi_m > 0$, being the magnetic induction enhanced and into diamagnetic if $\chi_m < 0$, in which the magnetic induction is weakened, χ_m can vary drastically with temperature, in general χ_m is quite small ($|\chi_m| \ll 1$) for diamagnetic and paramagnetic materials. [13]

Self-inductance. Occurs in a coil when the current flowing through it is varied, inducing an electromotive force on itself. In a coil of N turns, through which circulates a current I , the inductance, where l is the length of the coil and Φ the magnetic flux present in it. Considering Faraday's law, the self-inductance in a coil gives rise to a counter electromotive force [13].

Magnetic circuits. If we consider a well-defined path for a magnetic flux (as in the case of ferromagnetic materials), a magnetic circuit can be considered. A closed circuit of ferromagnetic material excited by a series of wire loops through which a current flow represents a magnetic circuit. The magnetomotive force is given by $\mathcal{F}_m = NI$, where N is the number of turns of the coil and I , the current flowing through it. The reluctance is defined as $\mathcal{R} = \frac{l}{\mu A}$, where dl is the length differential and A the cross-sectional area in question. Analogous to Ohm's circuit law and Φ being the magnetic flux. [13]

Ferromagnetic material: are substances that possess a great magnetism, capable that a magnet remains adhered to it.

Electromagnetic field: These are a combination of invisible electric and magnetic force fields. They occur both naturally and due to human activity.

Inductance: Inductance is the property of an electrical circuit to resist changing current. A current flowing through a wire has a magnetic field around it.

Magnetism: Magnetism is a physical phenomenon whereby objects exert attractive or repulsive forces on other materials.

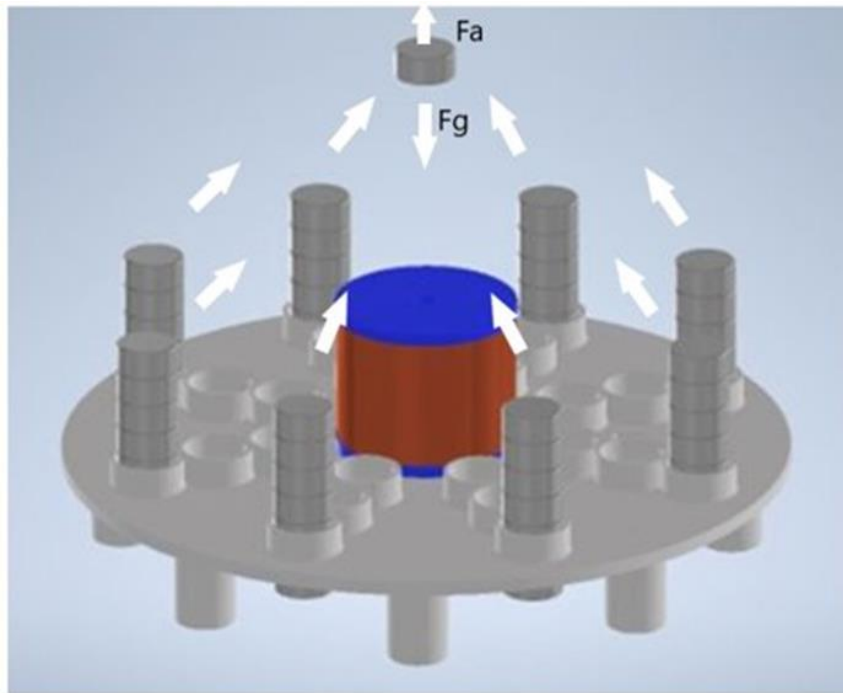


Figure 6: Force diagram.

Each magnet has independent forces, which when united in a single mass, in this case according to the previous image is observed as a tower of magnets placed on the base that would act as permanent magnets.

These permanent magnets exert independent forces oriented to the same direction on the magnet that is in a state of levitation, where this magnet has its forces such as gravity, for example.

So, in the circuit sketch was made use of an electromagnet, which is a device capable of generating magnetic fields using electric currents in its windings.

The electromagnet together with the permanent magnets will allow the interaction of the forces and the regulation of the height of the levitating magnet.

Mathematical calculations and measurement

Electromagnetic field measurements: (D)

The electromagnetic field measurements were made using a hall sensor, where it can measure the fields at different distances. This sensor works by generating an output voltage which will be proportional to the product of the strength of the magnetic field and the current.

If the value of the current is known, then the strength of the magnetic field can be calculated; If the magnetic field is created by using a current flowing through a coil or a conductor, then the value of the current in the conductor or coil can be measured.[10] Applying this concept, the measurement of the magnetic field of the electromagnet used was carried out, relating at the same time the magnitude of the magnetic field to different measured currents and different distances.

For the measurement of the magnetic field of permanent magnets, the same concept was used, but applying the relationship between the magnetic field and the distance, this was measured to obtain the magnetic moment of the permanent magnets found in the base. Whose equation is:

$$\mu = \frac{B \cdot X^3}{2 \times 10^{-7}}$$

Equation 1. Dipole moment of a magnet.

This dipole moment constant will help to apply the formula for the force exerted by the 8 permanent magnets. This will help in mathematical calculations, since it's intended to obtain the value of the forces that will be exerted on the magnet that will be in the air.

Obtaining these values, was written down with their distance, current and magnetic field variations to obtain a representative equation whose variable would be height and current in the case of the electromagnet.

For this measurement, a printed system was built, which can be seen in Figure 7. It helped to obtain the highest possible precision in the data, since this will determine if the height adjustments that the levitator needs will be precise.

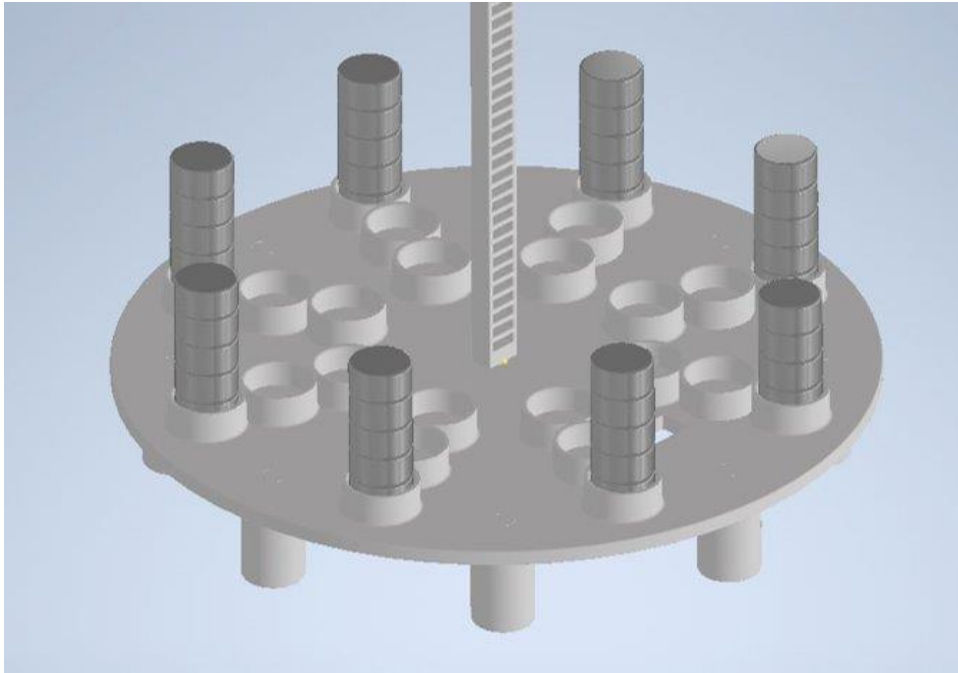


Figure 7. The design was made to measure the magnetic field at different distances.

As can be seen, the measurements were made at different distances and heights, to obtain the distance referring to the position of the permanent magnets with respect to the center of the electromagnet and to analyze at what determined distance is the ideal magnetic field to be able to provide the floating magnet a greater difference in height to which can be regulated.

The measurement of the magnets was also carried out at different positions with respect to the center of the magnet to obtain a measurement that was closer to the ideal behavior of a magnetic field at different heights, since this, as mentioned above, would help mathematical calculations to be more accurate and therefore the height control was more precise.

Current measurements: (K)

For current control, it was determined that the most appropriate way to do it is by implementing a PWM-controlled mosfet transistor to vary the current that reaches the electromagnet and limit it so that it does not exceed levels that could compromise the cables and the electromagnet.

This control is essential since it will allow you to vary the height of the magnet that is levitating, pushing or attracting it as necessary.

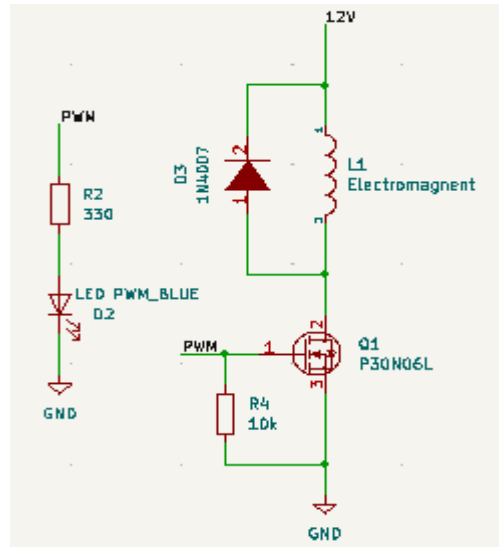


Figure 8. Schematic of the current control for the electromagnet.

A simulation was carried out in the LTspice software to observe the reaction of the circuit with respect to the input PWM signal, as shown in figure 8, the current (red signal) increases according to the variation of the pulse width. PWM (green signal). The simulation contains the final design for current control because with the progress of the project it was modified according to the needs, the frequency of the pwm signal was defined at 31kHz.

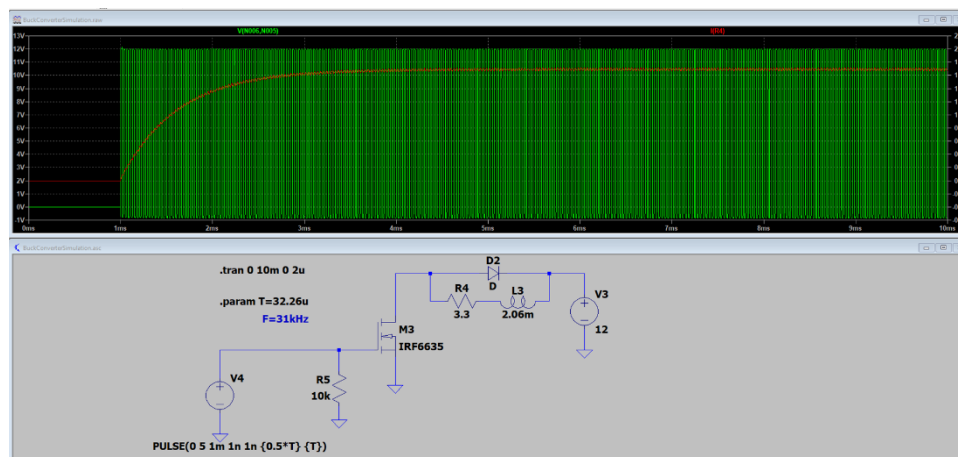


Figure 9. Current simulation in LTspice.

Mathematical and physical analysis: (S)

After measuring the magnetic field of 8 permanent magnets, we obtained the equation $y=0.0338*\exp(-96.24x)$; where y is the representation for the magnetic field and x the distance away from the magnet. And for the electromagnet using 1A is $y= 0.0695*\exp(-129.02x)$.

And calculating the force between the 8 permanent magnets and levitating magnet, and force between electromagnet and levitating magnet applying this formula:

Where:

$$f_m = 2 \frac{\mu_b}{bh_b} \int_{d-h_b/2}^{d+h_b/2} B_\rho(z) dz$$

f_m : magnetic force

μ_b : magnetic permeability of the levitating magnet

h_b : levitating magnet height

b : levitating magnet radius

d : distance between magnets in z axis

$B_\rho(z)$: magnetic field of 8 magnets of electromagnet

For mathematical analysis we will need net forces:

$$f_a = f_m - f_g$$

Where:

- *Magnetic force (using the previous integral equation for the force):*

$$f_m = f_{8m} + f_{em}$$

f_{8m} : 8 fixed magnets force

f_{em} : electromagnet force

$$f_m = 1.9112e^{-96.42z} - i * 3.9598e^{-129z}$$

- *Gravitational force:*

$$f_g = mg$$

- *Acceleration force*

$$f_a = m \frac{d^2 z}{dt^2}$$

The equation of motion for the levitating magnet is given by

$$m \frac{d^2 z}{dt^2} = f_m - mg$$

Substituting magnetic force in the equation:

$$\begin{aligned} m \frac{d^2 z}{dt^2} &= 1.9112e^{-96.42z} - i * 3.9598e^{-129z} - mg \\ \rightarrow \frac{d^2 z}{dt^2} &= \frac{1.9112e^{-96.42z}}{m} - \frac{i * 3.9598e^{-129z}}{m} - g \end{aligned}$$

While the equation for the electrical circuit is given by:

$$L \frac{di}{dt} = -Ri + V_{in_{avg}} \rightarrow \frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}V_{in_{avg}}$$

For the equilibrium point we have:

$$\frac{d^2 z}{dt^2} = 0 \text{ and } \frac{di}{dt} = 0$$

Substituting the acceleration:

$$\begin{aligned} 0 &= \frac{1.9112e^{-96.42z}}{m} - \frac{i * 3.9598e^{-129z}}{m} - g \\ \Rightarrow i_e &= \frac{1.9112e^{-96.42z_e} - mg}{3.9598e^{-129z_e}} \end{aligned}$$

And substituting di/dt

$$\begin{aligned} 0 &= -Ri + V_{in_{avg}} \\ V_{in_{avg}} &= Ri_e = R \left[\frac{1.9112e^{-96.42z_e} - mg}{3.9598e^{-129z_e}} \right] \end{aligned}$$

These equations are nonlinear equation, so we need to initialize them, in order to obtain a linear model of the system, we will use Taylor Series Expansion of the first two terms

$$\frac{d^2 z}{dt^2} = a_1(z - z_e) + a_2(\dot{z} - \dot{z}_e) + a_3(i - i_e) + b(v - v_e)$$

$$a_1 = \frac{\partial f_1}{\partial z} \Big|_{equil} = \frac{\partial}{\partial z} \left(\frac{1.9112e^{-96.42z}}{m} - \frac{i * 3.9598e^{-129z}}{m} - g \right) \Big|_{equil} \\ = \frac{-184.278e^{-96.48z_e}}{m} + \frac{i_e * 510.8142e^{-129z_e}}{m}$$

$$a_2 = \frac{\partial f_1}{\partial \dot{z}} \Big|_{equil} = \frac{\partial}{\partial \dot{z}} \left(\frac{1.9112e^{-96.42z}}{m} - \frac{i * 3.9598e^{-129z}}{m} - g \right) \Big|_{equil} = 0$$

$$a_3 = \frac{\partial f_1}{\partial i} \Big|_{equil} = \frac{\partial}{\partial i} \left(\frac{1.9112e^{-96.42z}}{m} - \frac{i * 3.9598e^{-129z}}{m} - g \right) \Big|_{equil} = \frac{3.9598e^{-129z_e}}{m}$$

$$b = \frac{\partial f_1}{\partial v} \Big|_{equil} = \frac{\partial}{\partial v} \left(\frac{1.9112e^{-96.42z}}{m} - \frac{i * 3.9598e^{-129z}}{m} - g \right) \Big|_{equil} = 0$$

We note that the equation for the electrical part of the system is already linear.

Now define the following variables

$$x_1 = (z - z_e); x_2 = (\dot{z} - \dot{z}_e); x_3 = (i - i_e); u = (v - v_e)$$

Where:

$$x_1 = \text{Magnet position}$$

$$x_2 = \text{Magnet velocity}$$

$$x_3 = \text{Drive current}$$

$$u = \text{Input average voltage}$$

Whit this equation we can define a transfer matrix to obtain the transfer function of the behavior of this system.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ a_1 & 0 & a_2 \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} u(t)$$

$$y = x_1 = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0$$

And after calculating this transfer matrix we obtained this transfer function:

$$G(s) = \frac{\frac{a_2}{L}}{s^3 + \frac{R}{L}s^2 - a_1s - a_1\frac{R}{L}}$$

We specify the following design parameters:

$$R = 3.3\Omega; \quad L = 0.00206H; \quad m = 0.0028kg; \quad g = 9.81\frac{m}{s^2}$$

$$\text{For } z_e = 0.01m + 0.0225m(\text{offset from center of the electromagnet}) = 0.0325m$$

$$i_e = \frac{1.9112e^{-96.42z_e} - mg}{3.9598e^{-129z_e}}$$

$$i_e = \frac{1.9112e^{-96.42*0.01} - 0.0028 * 9.81}{3.9598e^{-129*0.01}}$$

$$i_e = 0.6433383A$$

$$V_{in_{avg}} = Ri_e = R \left[\frac{1.9112e^{-96.42z_e} - mg}{3.9598e^{-129z_e}} \right]$$

$$V_{in_{avg}} = 3.3 * 0.6433383$$

$$V_{in_{avg}} = 2.123V$$

$$a_1 = \frac{-184.278e^{-96.48z_e}}{m} + \frac{i_e * 510.8142e^{-129z_e}}{m} = 7228.70173$$

$$a_2 = \frac{3.9598e^{-129z_e}}{m} = 389.291874$$

$$G(s) = \frac{\frac{a_2}{L}}{s^3 + \frac{R}{L}s^2 - a_1s - a_1\frac{R}{L}}$$

$$G(s) = \frac{188976.6379}{s^3 + 1601.941748s^2 - 7228.70173s - 11579959.08}$$

For apply a PID control, we will use this equation for the PID:

$$C(s) = K_p + \frac{K_i}{s} + K_d s$$

$$C(s) = \frac{K_d s^2 + K_p s + K_i}{s}$$

Calculating the close loop transfer function:

$$\begin{aligned}
 CLTF(s) &= \frac{C(s)G(s)}{1 + C(s)G(s)} \\
 CLTF(s) &= \frac{\left(\frac{K_d s^2 + K_p s + K_i}{s}\right) \left(\frac{\frac{a_2}{L}}{s^3 + \frac{R}{L} s^2 - a_1 s - a_1 \frac{R}{L}}\right)}{1 + \left(\frac{K_d s^2 + K_p s + K_i}{s}\right) \left(\frac{\frac{a_2}{L}}{s^3 + \frac{R}{L} s^2 - a_1 s - a_1 \frac{R}{L}}\right)} \\
 CLTF(s) &= \frac{\frac{\frac{a_2}{L} (K_d s^2 + K_p s + K_i)}{s(s^3 + \frac{R}{L} s^2 - a_1 s - a_1 \frac{R}{L})}}{s(s^3 + \frac{R}{L} s^2 - a_1 s - a_1 \frac{R}{L}) + \frac{\frac{a_2}{L} (K_d s^2 + K_p s + K_i)}{s(s^3 + \frac{R}{L} s^2 - a_1 s - a_1 \frac{R}{L})}} \\
 CLTF(s) &= \frac{\frac{\frac{a_2}{L} (K_d s^2 + K_p s + K_i)}{s(s^3 + \frac{R}{L} s^2 - a_1 s - a_1 \frac{R}{L})}}{\left(s^4 + \frac{R}{L} s^3 - a_1 s^2 - a_1 \frac{R}{L} s\right) + \frac{\frac{a_2}{L} (K_d s^2 + K_p s + K_i)}{s(s^3 + \frac{R}{L} s^2 - a_1 s - a_1 \frac{R}{L})}} \\
 CLTF(s) &= \frac{\frac{\frac{a_2}{L} (K_d s^2 + K_p s + K_i)}{s(s^3 + \frac{R}{L} s^2 - a_1 s - a_1 \frac{R}{L})}}{s^4 + \frac{R}{L} s^3 + \left(K_d \frac{a_2}{L} - a_1\right) s^2 + \left(K_p \frac{a_2}{L} - a_1 \frac{R}{L}\right) s + K_i \frac{a_2}{L}}
 \end{aligned}$$

Where:

$$2\delta\omega_n = K_p \frac{a_2}{L} - a_1 \frac{R}{L}$$

$$\omega_n^2 = K_i \frac{a_2}{L}$$

We will omit K_d since it is not within the current analytical capabilities, so we can rewrite the equation as follows

$$\begin{aligned}
 C(s) &= \frac{K_p s + K_i}{s} \\
 CLTF(s) &= \frac{\left(\frac{K_p s + K_i}{s}\right) \left(\frac{\frac{a_2}{L}}{s^3 + \frac{R}{L} s^2 - a_1 s - a_1 \frac{R}{L}}\right)}{1 + \left(\frac{K_p s + K_i}{s}\right) \left(\frac{\frac{a_2}{L}}{s^3 + \frac{R}{L} s^2 - a_1 s - a_1 \frac{R}{L}}\right)}
 \end{aligned}$$

$$CLTF(s) = \frac{\frac{a_2}{L}(K_p s + K_i)}{\left(s^4 + \frac{R}{L}s^3 - a_1 s^2 - a_1 \frac{R}{L}s\right) + \frac{a_2}{L}(K_p s + K_i)}$$

$$CLTF(s) = \frac{\frac{a_2}{L}(K_p s + K_i)}{s^4 + \frac{R}{L}s^3 - a_1 s^2 + (K_p \frac{a_2}{L} - a_1 \frac{R}{L})s + K_i \frac{a_2}{L}}$$

$$t_s = \frac{3}{\delta \omega_n} \rightarrow \delta \omega_n = \frac{3}{t_s} \rightarrow \omega_n = \frac{3}{t_s \delta}$$

$$2\delta \omega_n = K_p \frac{a_2}{L} - a_1 \frac{R}{L} \rightarrow \frac{6}{t_s} = \frac{K_p a_2 - a_1 R}{L}$$

$$K_p = \frac{6L + a_1 R t_s}{t_s a_2} \Rightarrow K_p = 61.276$$

$$\left(\frac{3}{t_s \delta}\right)^2 = K_i \frac{a_2}{L}$$

$$K_i = \left(\frac{L}{a_2}\right) \left(\frac{3}{t_s \delta}\right)^2 \Rightarrow K_i = 132.289; \text{ para } \delta = 0.6$$

$$K_i = 97.2; \text{ para } \delta = 0.7$$

$$K_i = 74.4; \text{ para } \delta = 0.8$$

$$K_i = 58.8; \text{ para } \delta = 0.9$$

Designs

Schematic Design: (K)

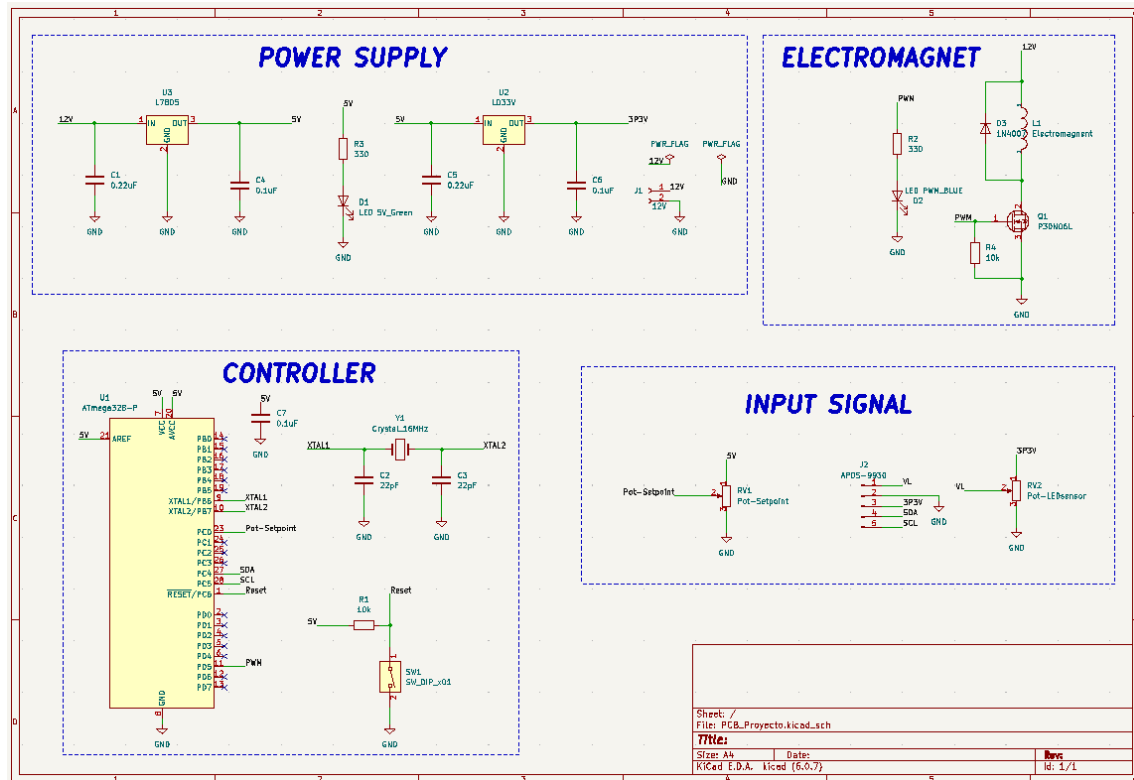


Figure 10. Schematic Design

The circuit design was divided into four parts, a control part, a power part, the electromagnet and the input signal, where each one fulfills a specific and important function within the circuit.

In the control stage all the power supplies of the circuit, there you can find the components responsible for feeding or regulating the other components so that they can operate properly.

The control stage is responsible for the operation of the circuit, this part uses the energy generated by the previous stage to achieve the actuation of the components, controlling their functions within each stage of the device, in our case the main component of the control stage is an ATmega.

The electromagnet part is responsible for generating a magnetic field that arises from the circulation of an electric current through a conductor. This, together with the permanent magnets, manages the intensity of the field, which depends on the amount of current flowing.

The signal input stage is responsible and has the function of generating a wave at a certain frequency to communicate the components, this works together with the other stages to achieve the proper operation of the device.

PCB Design: (W)

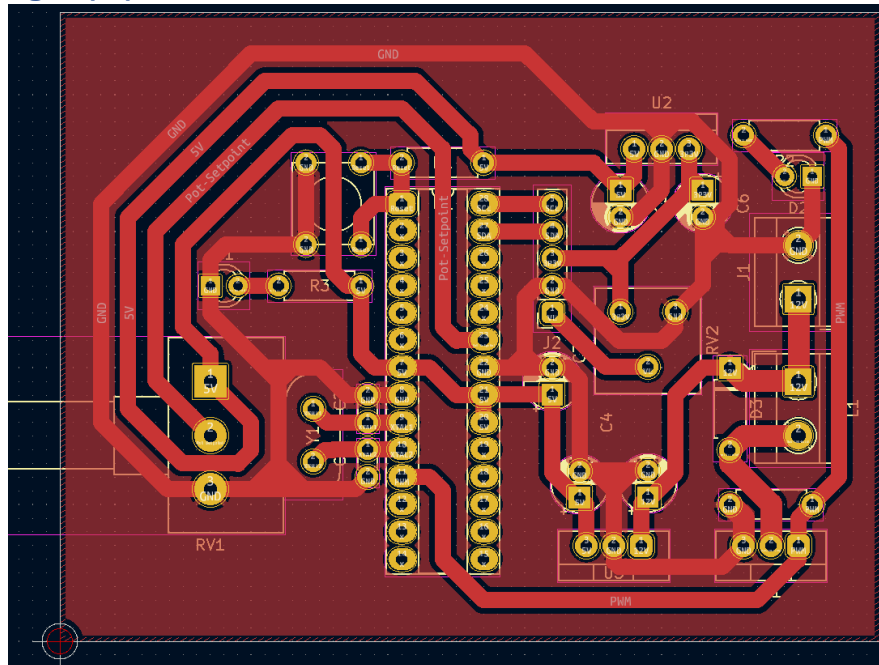


Figure 11. PCB design

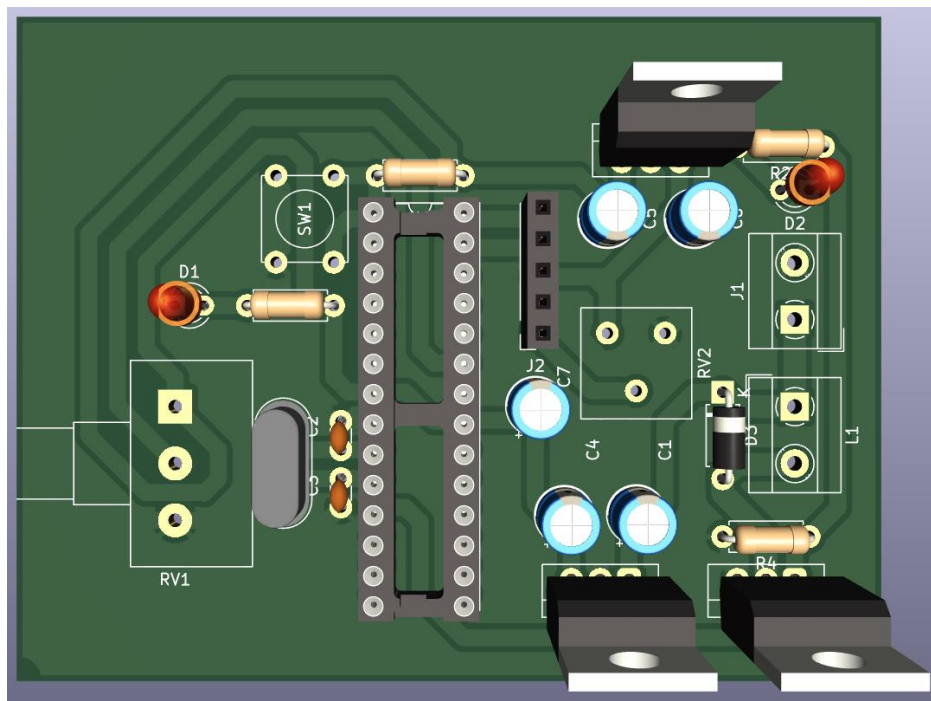


Figure 12. View of the PCB Design.

For the design of the PCB was used KiCad software because it is a very versatile software and with a very wide gallery of components, then once designed the circuit allows you to create the PCB as such, where once placed all the pieces can accommodate your own taste and design the respective tracks; these tracks can be expanded to the thickness this in order that when printed on the CNC machine avoids any malformation of this.

In figure 11 you can see one of the different layers of the PCB, this shows the design of how the connections will look like, the holes where each component will go, the parts that will have copper, those that will not and the actual size of the piece.

In Figure 12 you can see a schematic of how the circuit will be with the assembled components, this is remarkable of KiCad software because it allows users a realistic view of the project they are working on, so it is easier to make changes of positions or different modifications.

Structure design: (D)

The physical structure was designed with the Autodesk company's Inventor program, which allowed the design of all the structures made during the project, both in tests during the process, measurement structures for precision, and the final structures that provide the project, stability, magnet location facilities, space for cables, sensor support, circuit boards, likewise it's part of the electromagnet support.

These were designed and printed with the objective of resisting the temperatures generated by all the circuitry, therefore, they were printed in PETG, whose resistance provides the project with the conditions not to melt or damage elements in the process.

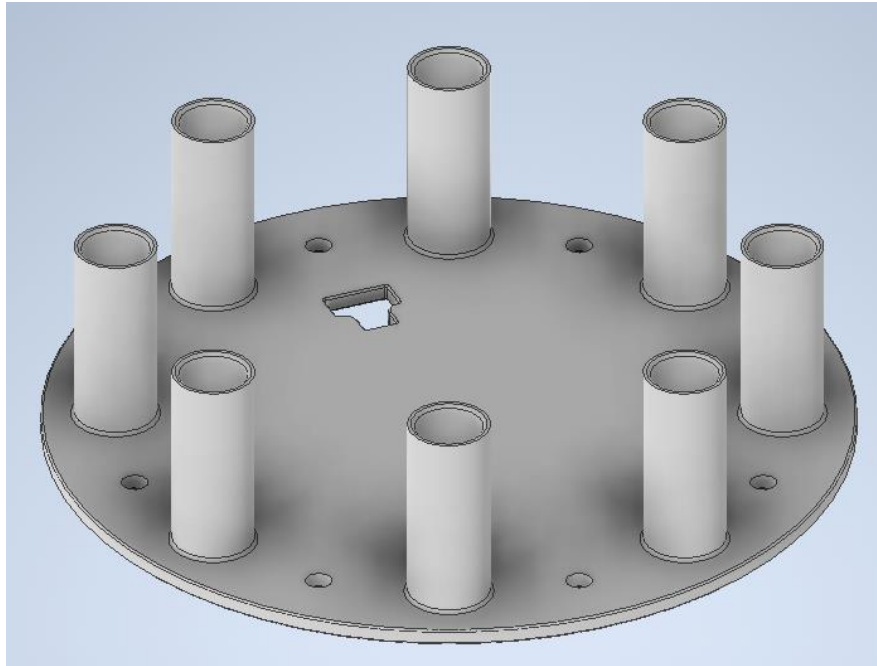


Figure 13. Design of the base of the magnetic levitator.

As can be seen in image 13, the vertical spaces are the spaces intended to support the permanent magnets, located in a fixed manner according to the conditions that provide the system with greater control due to the measurements made and, therefore, more precision.

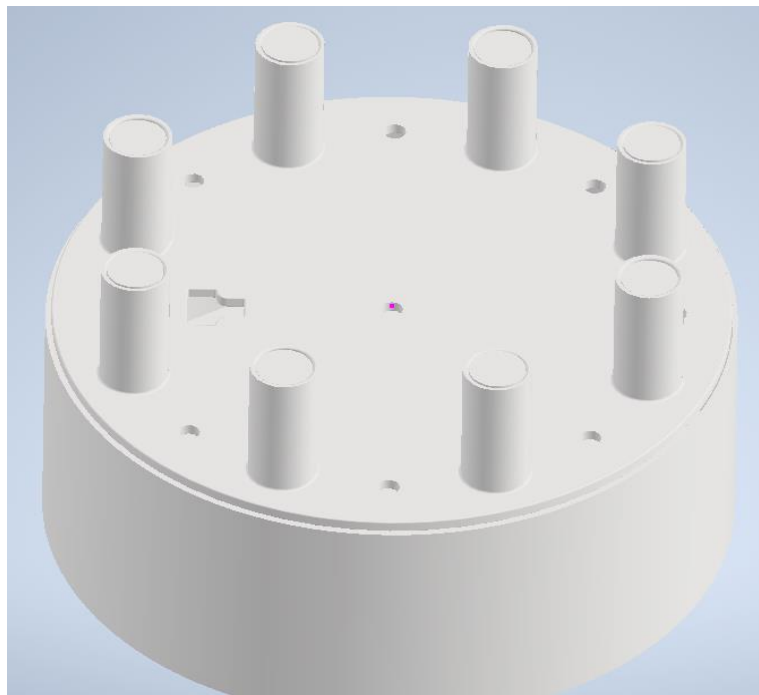


Figure 14. Design of the magnetic levitator case and all its components.

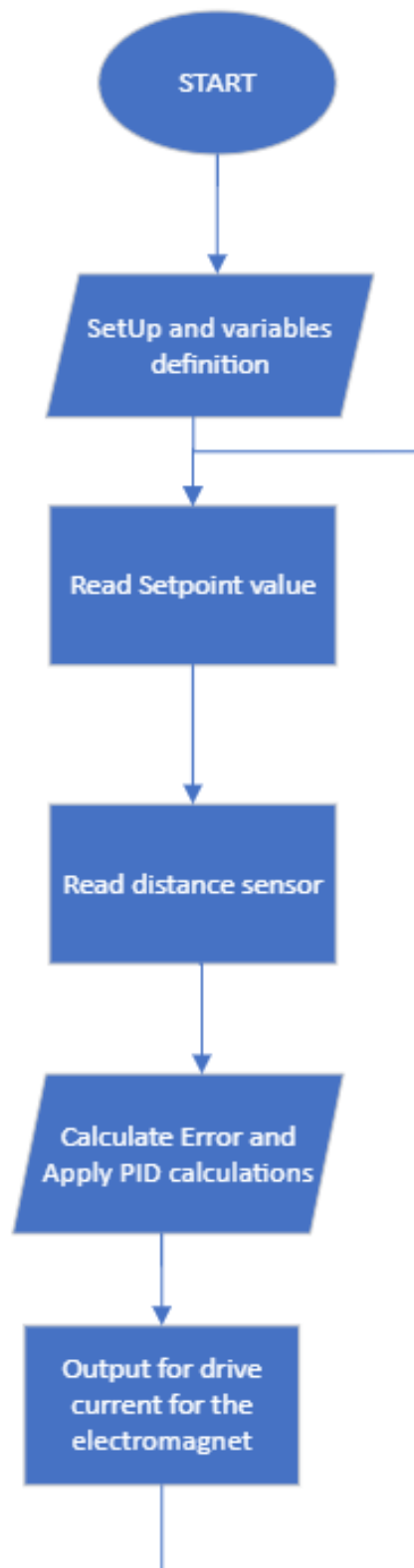
Code

Figure 15. Flow diagram of the code.


```
//https://playground.arduino.cc/Code/PIDLibrary/

int potentiometer_pin = A0; //From the main potentiometer
int PWM_pin = 5;

//PID:
#include <PID_v1.h>
#define PIN_INPUT 0
//#define PIN_OUTPUT 5
double Kp = 1, Ki = 0, Kd = 0;
double Setpoint, In, Out;
PID myPID(&In, &Out, &Setpoint, Kp, Ki, Kd, DIRECT);

//Sensor
#define DUMP_REGS
#include <Wire.h>
#include <APDS9930.h>

#define AVERAGES 75

float calcNewMovingAverage(uint8_t averages);
uint16_t movingAverageBuffer [AVERAGES];
// Global Variables
APDS9930 apds = APDS9930();
uint16_t proximity_data = 0;
```

Figure 16. Code development.

```
void setup() {
  Serial.begin(115200);
  pinMode(potentiometer_pin, INPUT);
  pinMode(PWM_pin, OUTPUT);

  //PID
  Setpoint = 0.005; //initial value before read the pot; 5mm
  //turn the PID on
  myPID.SetMode(AUTOMATIC);
  //myPID.SetOutputLimits(0,max current);

  //PWM
  TCCR2B = TCCR2B & B11111000 | B00000001; // pin 3 and 11 PWM frequency of 31372.55 Hz
  //APDS9930
  setup_APDS9930();
}

void loop() {
  read_Setpoint();

  apds.readProximity(proximity_data);
  In = input_feedback(calcNewMovingAverage(AVERAGES));

  //myPID.Compute();

  int out_PWM = 0;
  if (In < Setpoint) out_PWM = 0;
  else out_PWM = 255;
  //int out_PWM = 88.8*Out + 63.5;
  //if(out_PWM < 0) out_PWM = 0;
  //if(out_PWM > 255) out_PWM = 255;
  if (In*1000 > 10) out_PWM = 0;
  Serial.print(Setpoint*1000);
  Serial.print(" ");

  Serial.print(In*1000);
  Serial.print(" ");

  Serial.print(out_PWM);
  Serial.println(" ");

  analogWrite(PWM_pin, out_PWM);
}
```

Figure 17. Code development.

```

void read_Setpoint() {
    double measure = analogRead(potentiometer_pin);
    Setpoint = measure * 0.0000083088954 + 0.0015;
}

double input_feedback(float sensor) {
    //double conversion = double(-0.0776 * pow(sensor, 3) + 3.9312 * pow(sensor, 2) - 68.807 * sensor + 1115.6);
    double conversion = double(2.1842460593*exp(sensor*-0.0071420962));
    return conversion;
}

//Sensor
/*****
void setup_APDS9930() {
    // Initialize APDS-9930 (configure I2C and initial values)
    if ( apds.init() ) {
        Serial.println(F("APDS-9930 initialization complete"));
    } else {
        Serial.println(F("Something went wrong during APDS-9930 init!"));
    }
    // Start running the APDS-9930 proximity sensor (no interrupts)
    if ( apds.enableProximitySensor(false) ) {
        Serial.println(F("Proximity sensor is now running"));
    } else {
        Serial.println(F("Something went wrong during sensor init!"));
    }
}
#endif
#define DUMP_REGS
/* Register dump */
uint8_t reg;
uint8_t val;

for (reg = 0x00; reg <= 0x19; reg++) {
    if ( (reg != 0x10) && \
        (reg != 0x11) )
    {
        apds.wireReadDataByte(reg, val);
        Serial.print(reg, HEX);
        Serial.print(": 0x");
        Serial.println(val, HEX);
    }
}
apds.wireReadDataByte(0x1E, val);
Serial.print(0x1E, HEX);
Serial.print(": 0x");
Serial.println(val, HEX);
#endif

```

Figure 18. Code development.

```

}

float calcNewMovingAverage(uint8_t averages) {
    static uint32_t movingAverageSum = 0;
    static uint32_t bufferPointer = 0;
    uint16_t newSample = 0;
    if ( !apds.readProximity(proximity_data) ) {
        Serial.println("Error reading proximity value");
    } else {
        newSample = proximity_data;
    }
    movingAverageSum = movingAverageSum - movingAverageBuffer[bufferPointer] + newSample;

    movingAverageBuffer[bufferPointer] = newSample;
    bufferPointer++;

    if (bufferPointer >= averages) {
        bufferPointer = 0;
    }
    return (movingAverageSum / averages);
}

```

Figure 19. Code development.

Results

The first experimental tests were carried out with other types of cable thickness for the winding of the electromagnet, such as thickness 31, which did not obtain expected results since it had a very low current and said current did not fulfill the purpose of levitation.

After various tests, the winding is carried out with a total of 470 turns with a 24-gauge wire. The main advantage was that it was a winding capable of withstanding high currents of up to 3A; Another advantage was that at the time of winding (which is done by hand) there is greater comfort due to the thickness.

For the development of the project, we resorted to many documents with information on the subject and the relevant concepts for its development, where the properties of the magnets, electromagnets and components to be used were studied in order to choose the most suitable for its operation and the development of the PCB.

For the development of the PCB circuit tests were made in the software and in physical, mainly once the tests in the protoboard were completed, we proceeded to make the design of the PCB, in this the chosen components were placed, the parameters of each physical component were considered, such as the spaces that covered in the proto, the size. Also, the tracks were adjusted to avoid that when printing any track suffered any damage, also the different views of KiCad were used to visualize each part of the PCB and how they work together giving an expectation of the actual physical design.

We also opted for the design of this in CNC machine to obtain a better precision in the development of this, although PCB designs were also made with acid.

The measurement of electromagnetic fields, in addition to providing important information that would later be used in mathematical calculations, whose purpose was to feedback the errors that occurred in the system to correct it.

All the measurements carried out were made in such a way that the precision errors were minimal, however, it is impossible to give the necessary conditions so that there are no measurement errors, in addition to this, in the operation of the magnetic levitator, unexpected errors can occur.

Even when trying to characterize the system with precise measurements and avoiding many variations in the process, many of the calculations in the end were approximations that were made since by taking several measurements of the same set up or configuration of magnets that we used, variations were obtained, It could be due to external magnetic fields that we could not control, failures in the measuring instruments or even, after doing several tests, the magnets were damaged, so this can also affect the results obtained.

Conclusions

- In conclusion one of the points that caused the system to not work properly was in the final design by leaving the permanent magnets in a fixed location, since regulating the distance of the magnets facilitated the creation of ideal magnetic field conditions to levitate the magnet.
- When making the changes in the sensors, it was detected that the IPR was affected by the hand of the person who placed the magnet to float, on the other hand, the hall effect sensor could produce not so precise measurements due to all the magnetic fields that surrounded it.
- Apart from the problem with the sensors, the circuit design was found to be correct, since it correctly regulated the current according to the system specifications. Having the correct response speed and commutation, verifying with the tests that they did work.
- The use of a correct gauge and a number of turns in the electromagnet provides a better magnetic field in the system, since the magnet is levitating on the electromagnet.
- In the physical-mathematical calculations, changes are observed with respect to those obtained in the real measurements, this is due to external factors such as the manual placement of the magnet that was levitating, the placement of the permanent magnets, changes in the current, which affects directly the electromagnetic field, among others.
- For the PCB design we chose the KiCad application because it is very complete software, which has the necessary components or allows us to create them.

- With the development of the project, you will acquire knowledge about magnetism, magnetic fields, properties of electromagnets and how to develop a project of this type.
- Calculating a PID for a complex system can be difficult if we don't know the canonical form of the transfer function, and the lack of knowledge to apply approximations to complex systems such as this affects when performing mathematical calculations.
- The lack of considerations and experience when using some materials and components makes it difficult to develop a product capable of performing its function correctly, and the learning curve slows down the process.
- The control of current was fundamental in the project, since although the proper operation was not perfected, in the end it was possible to make the magnet levitate in a controlled manner and varying the height thanks to the designed current control circuit.
- The magnet was able to levitate and vary the levitation height in a controlled way using a tube that prevented it from leaving the central position, this is because the magnet could only influence the rise and fall position and not the attraction towards the ends.
- When placing the magnet on the electromagnet it was always done inaccurately because when placing it by hand it would always have different positions, even though the fixed magnets were in exact positions, depending on where the magnet was placed it always tended to go to one of the fixed magnets.
- Initially, in the tests where it was possible to levitate the magnet without any external help apart from the system, there was the option of moving the fixed magnets, however, this operation had to be carried out each time the magnet was placed on the electromagnet, the same operation as could not be repeated when modifying the design with exact positions for the fixed magnets.

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Appendix

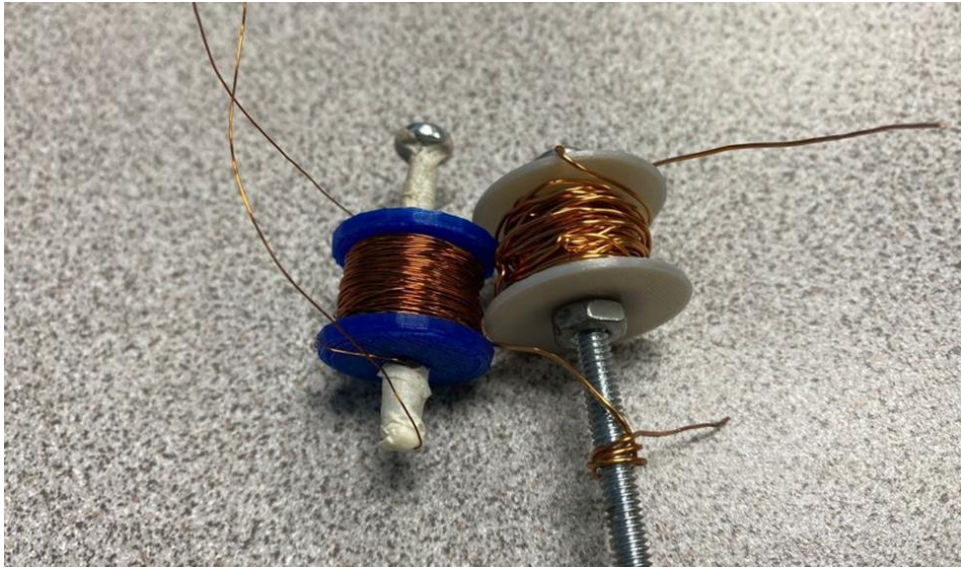


Image 1. Tests with different coils



Image 2. Physical design of the measurement of magnetic fields

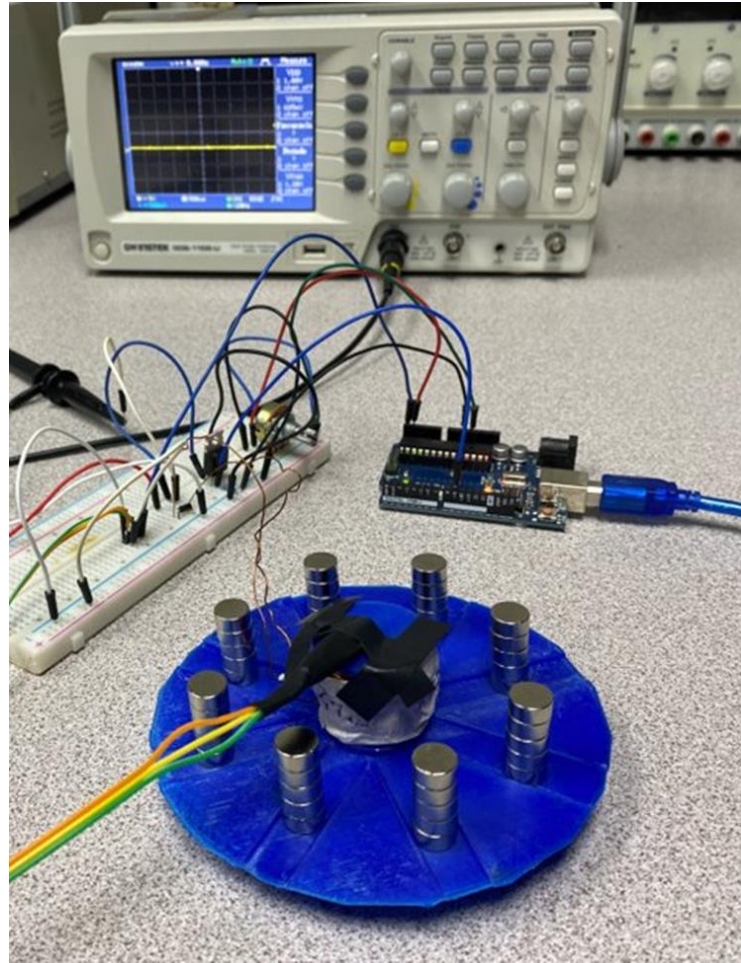


Image 3. Protoboard tests

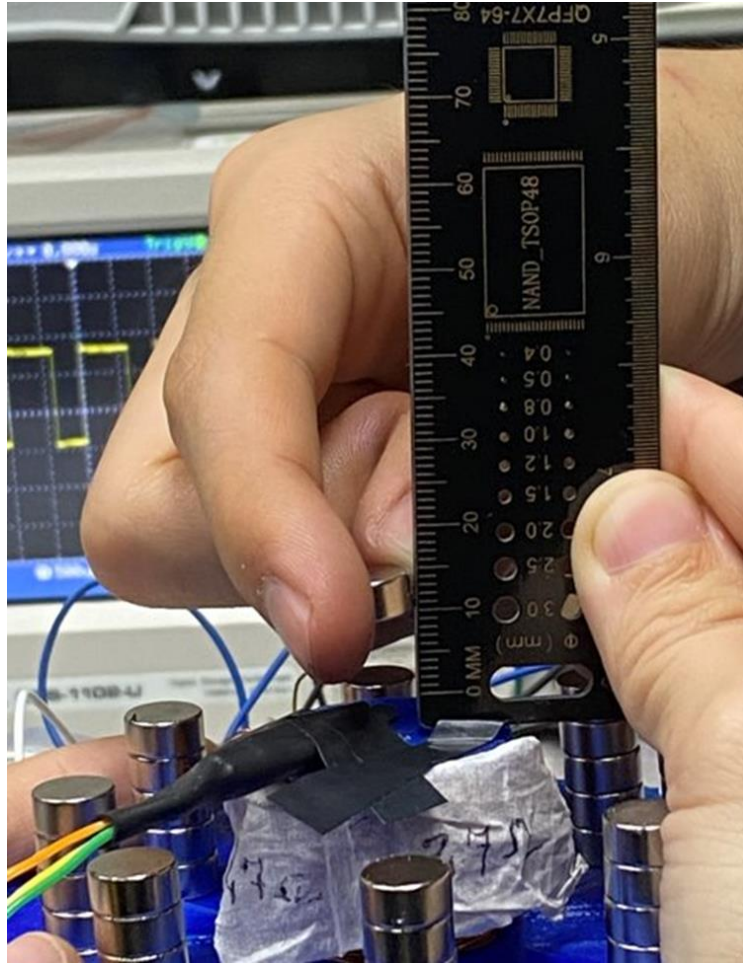


Image 4. First tests to measure the magnetic field of the coil at different distances