Chapter1: Induction.

1. Prove that $1^2+2^2+3^2\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$

2. Prove that $1^3+2^3+3^3\ldots+n^3=(1+2+3+\ldots+n)^2$

3. Prove that $x^n-y^n=(x-y)(x^{n-1}+x^{n-2}y+\ldots+xy^{n-2}+y^{n-1})$

4. Porve that $1\cdot 2+2\cdot 3+3\cdot 4+\ldots+n(n+1)=\frac{n(n+1)(n+2)}{3}$

5. Prove that $1+3+5+\ldots+(2n-1)=n^2$

6. Prove that $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

7. Take the Fibonacci Sequence where $F_1=1, F_2=1, F_3=2$ and subsquently $F_n=F_{n-1}+F_{n-2}$. Prove that $F_1+F_2+F_3+\ldots+F_n=F_{n+2}-1$

8. Prove that $F_1+F_3+F_5+\ldots+F_{2n-1}=F_{2n}$

9. Prove that $F_2+F_4+F_6+\ldots+F_{2n}=F_{2n+1}-1$

10. Prove that $F_{n+1}^2-F_nF_{n+2}=(-1)^n$

11. Prove that $F_1F_2+F_2F_3+F_3F_4+\ldots+F_{2n-1}F_{2n}=F_{2n}^2$

12. Prove that $F_1F_2+F_2F_3+F_3F_4+\ldots+F_{2n}F_{2n+1}=F_{2n+1}^2-1$

13. Take the Lucas Numbers $L_n$ which are the difined by the equations $L_1=1$ and $L_n=F_{n+1}+F_{n-1}$ for each $n\geq 2$. Prove that $L_n=L_{n-1}+L_{n-2}\quad(n\geq 3)$

14. What is wrong with the following arguement: TODO
15. Prove that $F_{2n}=F_nL_n$

16. Prove that $L_1+2L_2+4l_3+8L_4\ldots+2^{n-1}L_n=2^nF_{n+1}-1$

17. Prove that $n(n^2-1)(3n+2)$ is divisible by 24 for each positive integer
18. Prove that if $n$ is an odd positive integer, then $x^n+y^n\mid x+y$