A Simplified Computer Model of Virus Spread

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Abstract

This is a paper describing the creation of a computer simulation of virus spread and attempts to fit the produced results with an aproximated iterative version of the SIR model of virus transmission.

1 Introduction

Many real world situations can be modelled and approximated using simplified models, with simple assumptions being made for little cost in accuracy. These models can be applied to real world scenarios to inform on predicted outcomes and behaviours. For example, virus spread can be modelled programmatically, such as is presented below. Data from these models can potentially help advise governments about the actions they need to take, if any, to control the spread of a virus.

Virus modelling typically relies on given data about the current situation:

$\mathbf{N}_t$ - The total number of people in the population at time t.

$\mathbf{S}_t$ - The number of people who are succeptable to the disease in the population at time t.

$\mathbf{I}_t$ - The number of currently infected people in the population at time t.

$\mathbf{R}_t$ - The number of people who have been removed from the population at time t.

‘Removed’ refers to the set of the population that cannot become infected. (Assuming a person can only be infected once, $\mathbf{R}_t$ is equivalent to the sum of the number of deceased and the number of recovered patients at time t).

Since the above sets of people are mutually exclusive and assuming that everyone in the population $\mathbf{N}$ can become succeptible, $\mathbf{N}_t = \mathbf{S}_t + \mathbf{I}_t + \mathbf{R}_t$.

The following variables are also used in the model:

α - The probability of catching the disease per time step. $\alpha \in [0, 1] \subseteq \mathbb{R}$

γ - The probability of recovering from the disease per time step. $\gamma \in [0, 1] \subseteq \mathbb{R}$

‘Time step’ can refer to any positive, discrete unit of time, provided α and γ are adjusted accordingly.

2 Model Assumptions

For this model, assumptions were made about the behaviour of a virus in a population, both to simplify code but also reduce unnecessary complexity that adds little additional accuracy to the produced data.

- Everyone in the population is initially succeptible to the disease: $\mathbb{R}_0 = 0$
- The number of births equals the number of deaths during the modelling period, hence $\mathbf{N} = \mathbf{N}_t = k \in \mathbb{N}$
- Virus spread can be modelled to a sufficient accuracy on a 2-dimensional finite grid.
- Virus spread can only occur within a fixed positive radius of an infected person, with probability α.
- γ is constant.
- $\mathbf{I}_0 = 1$.
- Once a person has been removed (through death or recovery), this person cannot later catch the virus for a second time.
- Every person intially starts off at a random location on the grid.
- People travel, for each timestep, in a displacement according to a normal distribution $X \sim (0, \sigma)$ with $\sigma = k \in \mathbb{R}_0^+$ across the population, in a random direction in the 2d plane. This movement ignores the position of others surrounding the person.
- People begin at a random location on this finite grid.

3 Implementation

The simulation was produced using Java and graphics library 'Processing' [1], with giCenter charts [2] used to plot graphs inside the GUI window. Note that a person is named as a 'unit' in the simulation, since virus spread isn't specific to humans. While the radius of infection and standard deviation of a unit's walk distance can be changed dynamically during a simulation, they were left, for sections 3 & 4, at:

Infection Radius $= 10px$

Standard Deviation Of Walk Distance $= 5px$

Allowing a unit to walk in a random direction can sometimes lead to units attempting to exceed the borders of the grid. If a calculated step would lead to a unit exceeding the boundaries of the grid, this movement was ignored and the unit simply remains in it's previous position.

Through simulating a fixed number of units inside a 2d grid of size 800×800 pixels, with colours corresponding to the infection status of a unit, we can plot graphs of the number of units with a specific status against the time step. Colours used were as follows:

S - Black.

I - Red.

R - Grey.

Simulating with the following parameters yields the results in Figure [1].

$$\mathbf{N} = 1'000$$

$$\alpha = 0.4$$

$$\gamma = 0.1$$

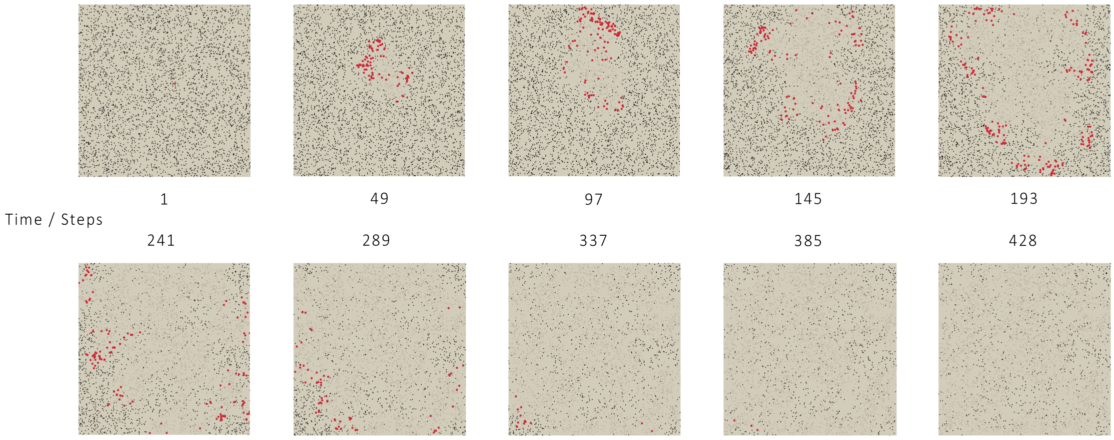


Figure 1: Running a simulation with the above parameters.

The program also allows graphs to be rendered and updated alongside the simulation window (Figure 2). These graphs show **S** in black, **I** in red and **R** in grey. This allows later analysis of the curves produced when **S** , **I** and **R** are plotted against the timestep t.

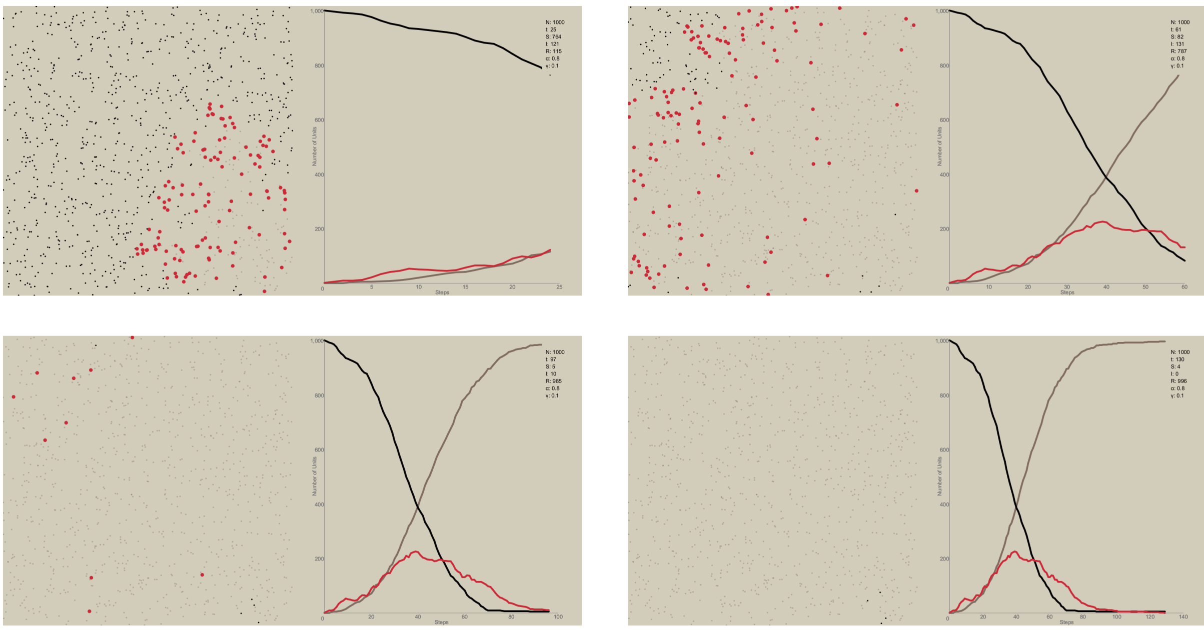


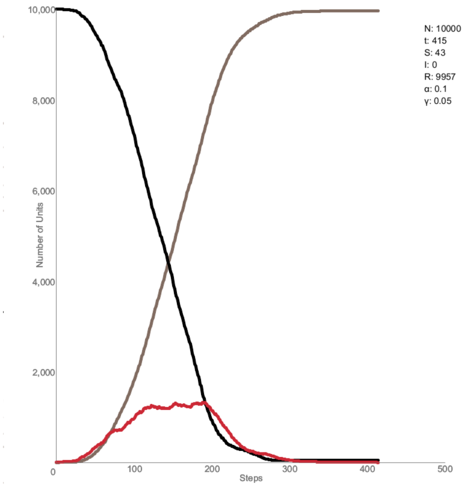
Figure 2: An example simulation with dynamically updating graph plots.

4 Results

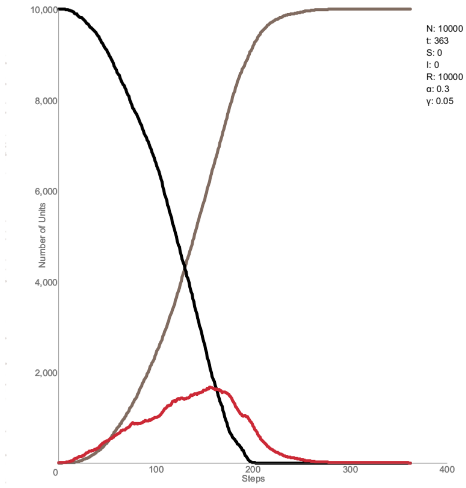
The following pairs of parameters were chosen: (Note that γ is significantly less than α in general, to allow $\mathbf{I}$ to increase at $t = 0$)

γ \ α	0.10	0.30	0.80	1.00
0.05	Y	Y	Y	Y
0.10		Y	Y	Y
0.30		Y	Y	Y

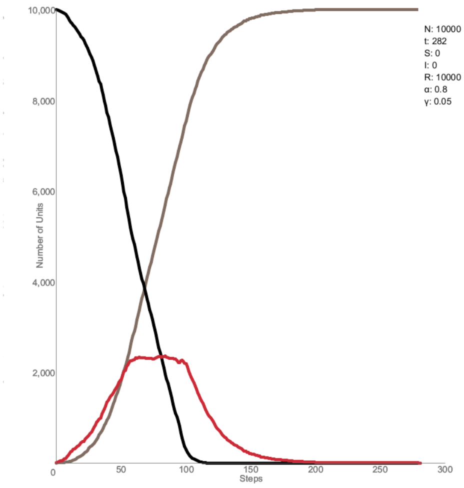
The following results were produced with $\mathbf{N} = 10'000$.



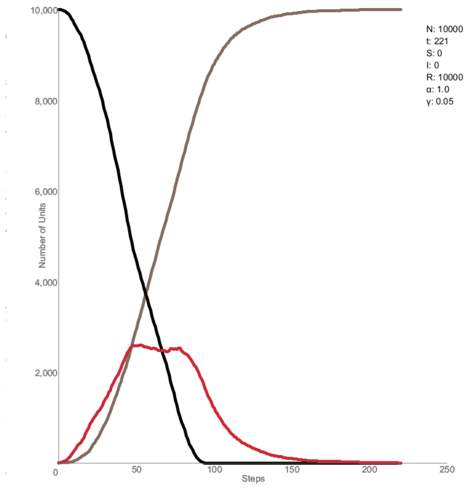
$\alpha = 0.10, \gamma = 0.05$



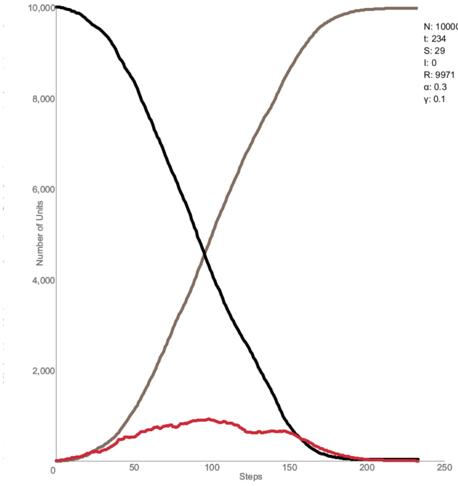
$\alpha = 0.30, \gamma = 0.05$



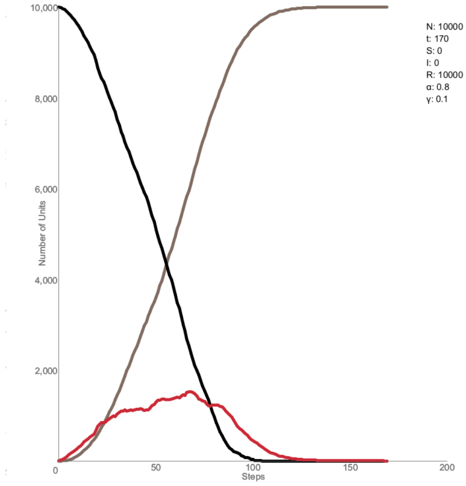
$\alpha = 0.80, \gamma = 0.05$



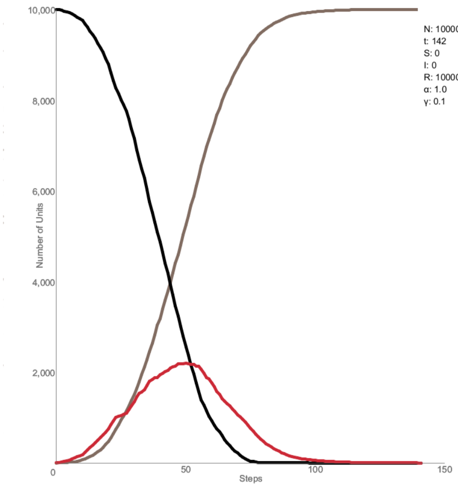
$\alpha = 1.00, \gamma = 0.05$



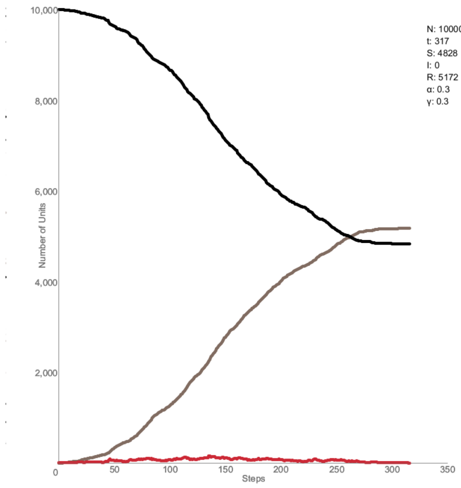
$\alpha = 0.30, \gamma = 0.10$



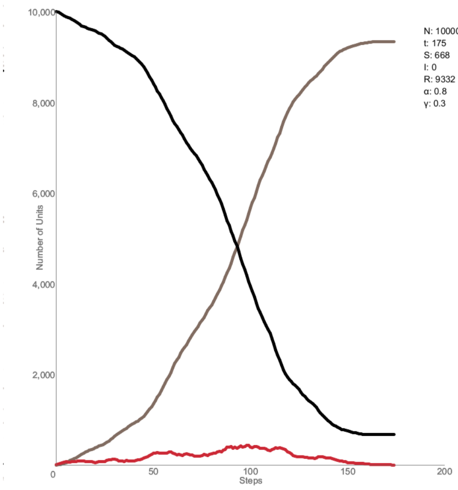
$\alpha = 0.80, \gamma = 0.10$



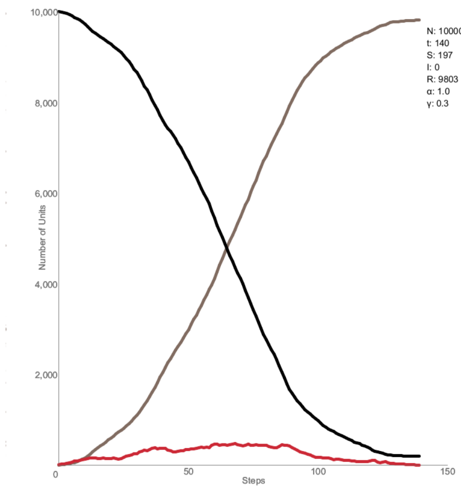
$\alpha = 1.00, \gamma = 0.10$



$\alpha = 0.30, \gamma = 0.30$



$\alpha = 0.80, \gamma = 0.30$



$\alpha = 1.00, \gamma = 0.30$

5 The SIR Model

'The most popular mathematical model is the SIR model' [3]. It relies on the three following differential equations:

$$\frac{d\mathbf{S}}{dt} = -\rho\mathbf{SI} \tag{1}$$
$$\frac{d\mathbf{I}}{dt} = \rho\mathbf{SI} - \sigma\mathbf{I} \tag{2}$$
$$\frac{d\mathbf{R}}{dt} = \sigma\mathbf{I} \tag{3}$$

$\rho, \sigma \in [0, 1] \subseteq \mathbb{R}$, where ρ is the rate of infection and σ is the rate of recovery. **S**,**I** & **R** are the same as before.

There are a few interesting things to note here:

1. Changes from one set of people to another cancel out. This makes sense as the total number of people **N** is assumed not to change, so we can expect $\frac{d\mathbf{N}}{dt} = 0$. Notice that:

   $$\frac{d\mathbf{N}}{dt} = \frac{d\mathbf{S}}{dt} + \frac{d\mathbf{I}}{dt} + \frac{d\mathbf{R}}{dt} = -\rho\mathbf{SI} + \rho\mathbf{SI} - \sigma\mathbf{I} + \sigma\mathbf{I} = 0$$

   This relationship is easier to see in a flowchart:

   $$\mathbf{S} \xrightarrow[\rho]{} \mathbf{I} \xrightarrow[\sigma]{} \mathbf{R}$$

   Figure 3: The flow of people between states for each timestep

2. Equations 1 & 2 contain a non-linear term **SI**, so they are non linear differential equations.
3. For an epidemic to spread, $\frac{d\mathbf{I}}{dt} > 0$. (The number of infected people is increasing with time).

   Rearranging equation 2, with the assumed initial conditions from the model, $\mathbf{S}_0 = \mathbf{N} - \mathbf{I}_0$, $\mathbf{R}_0 = 0$, $\mathbf{I}_0 = 1$ & $\mathbf{R}_0 = 0$ gives us the following inequality:

   $$\frac{d\mathbf{I}}{dt} = \rho(\mathbf{N} - 1) - \sigma > 0$$
   $$\rho(\mathbf{N} - 1) > \sigma$$

   This inequality is useful for predicting if an epidemic will spread at $t = 0$ and whether the number of cases will continue to increase at a given time t.

Exact analytical solutions to the SIR Model are not needed for the purposes of fitting simulation data to the SIR equations, as they exceed the requirements needed to plot graphs in the next section.

6 Fitting Simulation Results to the SIR Model

This section focuses on the attempt to fit a specific simulation result to a modified, iterative version of the SIR model.

Conditions used for the following simulation were as follows:

$\mathbf{N} = 10'000$

$\alpha = 0.2$

$\gamma = 0.1$

Infection Radius $= 30px$

Standard Deviation of Walk Distance $= 5px$

While the SIR equations use a continuous time interval, the simulation produced relies on a discrete time interval. Hence, an iterative model was made to allow a direct comparison between the simulation and SIR model predictions, using subscript i to later differentiate between the iterative and simulation values.

Assuming t refers to the timestep, we have:

$$\begin{aligned}
\mathbf{S}_{i_{n+1}} &= \mathbf{S}_{i_n} + \Delta t \frac{d\mathbf{S}}{dt}\bigg|_{t=n} \\
\mathbf{I}_{i_{n+1}} &= \mathbf{I}_{i_n} + \Delta t \frac{d\mathbf{I}}{dt}\bigg|_{t=n} \\
\mathbf{R}_{i_{n+1}} &= \mathbf{R}_{i_n} + \Delta t \frac{d\mathbf{R}}{dt}\bigg|_{t=n}
\end{aligned}$$

To match with the single timesteps used in the simulation, let $\Delta t = 1$. Then the iterative model is as follows:

$$\begin{aligned}
&\text{For n} \in \mathbb{N}_0 : \\
\mathbf{N}_{i_0} &= 10'000 \\
\mathbf{I}_{i_0} &= 1 \\
\mathbf{R}_{i_0} &= 0 \\
\mathbf{S}_{i_0} &= 9'999 \text{ since } \mathbf{N} = \mathbf{S}_n + \mathbf{I}_n + \mathbf{R}_n \\
\mathbf{S}_{i_{n+1}} &= \mathbf{S}_{i_n} - \rho \mathbf{S}_{i_n} \mathbf{I}_{i_n} \\
\mathbf{I}_{i_{n+1}} &= \mathbf{I}_{i_n} + \rho \mathbf{S}_{i_n} \mathbf{I}_{i_n} - \sigma \mathbf{I}_{i_n} \\
\mathbf{R}_{i_{n+1}} &= \mathbf{R}_{i_n} + \sigma \mathbf{I}_{i_n}
\end{aligned}$$

While an iterative form does not perfectly represent the analytical solution of the SIR model, I believe that this form is sufficient to recreate the general shapes of the curves expected from the SIR equations.

Next, simulation data was imported into Excel and SIR values were calculated based on the above iterative model: (Values are rounded to 1 d.p but only for display purposes. Actual values used are not rounded significantly).

t	Model S	Model I	Model R	SIR S	SIR I	SIR R	SIR delta S	SIR delta I	SIR delta R
25	3702	2999	3299	6168.9	2899.4	931.7	-810.1	577.9	232.1
26	3291	3112	3597	5274.6	3503.8	1221.6	-894.3	604.4	289.9
27	2832	3266	3902	4350.5	4077.4	1572.0	-924.0	573.7	350.4
28	2393	3387	4220	3463.6	4556.7	1979.8	-887.0	479.2	407.7
29	2020	3415	4565	2674.5	4890.1	2435.4	-789.1	333.5	455.7
30	1658	3444	4898	2020.5	5055.0	2924.4	-653.9	164.9	489.0
31	1276	3512	5212	1509.9	5060.2	3429.9	-510.7	5.2	505.5
32	965	3479	5556	1127.8	4936.2	3936.0	-382.0	-124.0	506.0
33	736	3356	5908	849.5	4720.9	4429.6	-278.4	-215.3	493.6
34	548	3199	6253	649.0	4449.4	4901.7	-200.5	-271.6	472.1
35	381	3058	6561	504.6	4148.8	5346.6	-144.4	-300.6	444.9

Figure 4: Sample Excel values.

Next, an error value, ϵ was calculated for specific values of ρ & σ. This error is as follows:

$$\epsilon = \frac{1}{\mathbf{N}} \sum_{t=0}^{T} (|\mathbf{S}_t - \mathbf{S}_{i_n}| + |\mathbf{I}_t - \mathbf{I}_{i_n}| + |\mathbf{R}_t - \mathbf{R}_{i_n}|)$$

Where subscript i_n denotes the iterative data value at timestep t. T denotes the total number of timesteps where $\mathbf{I}_t > 0$. (e.g. the duration of the simulation).

The error, ϵ was then minimised through multivariate non-linear regression in Excel, yielding the following final values for ρ & σ:

$\rho \; = 0.00005$

$\sigma \; = 0.1001$

$\epsilon \; = 1527$ (0 d.p)

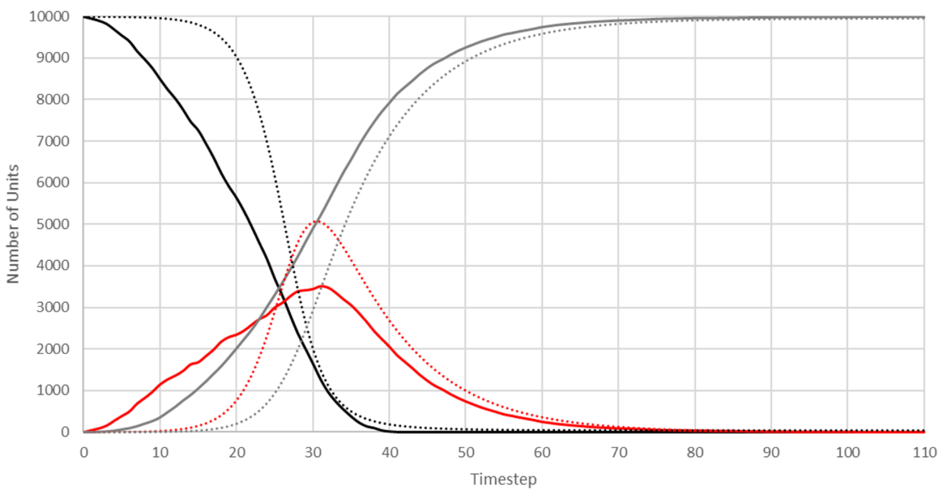


Figure 5: Fitted graph. $\rho = 0.00005, \sigma = 0.1001$. Simulation data is shown with solid lines and iterative values with dotted lines.

The following inequality also holds:

$$\begin{aligned} \rho(\mathbf{N}-1) &> \sigma \\ 0.00005 \times (10'000-1) &> 0.10010 \\ 0.49995 &> 0.10010 \end{aligned}$$

Hence the predicted virus spread shown by this inequality agrees with the iterative SIR model's resultant increase in **I**.

7 Evaluation

While I am satisfied that simulation results roughly fit the adapted iterative SIR model, I do believe that with some improvements, a simulation design which yields outputs closer to the SIR model can be produced. Because of a large number of model assumptions, the simulation may have been oversimplifed and hence the two produced graphs differ substantially. I am pleased that my model value of recovery rate, γ, matches up almost perfectly with the SIR iterative model's recovery rate σ. (0.10 & 0.1001 respectively). However, the model's infection rate $\alpha = 0.20$ is far off the SIR model's infection rate $\rho = 0.00005$.

Further improvements could be made to the simulation model:

- A unit could attempt to maintain a fixed distance between other units, to help reduce virus spread. (Sometimes known as 'social distancing' [4].)
- The proposed model assumes that everyone inside a fixed radius of an infected unit has an equal chance, α, of infection. However, virus spread could be better modelled as a non-constant function of distance. For example:

$$\alpha(r) = \left\{ \begin{array}{ll} k(r-d), & \text{for } 0 \leq n \leq r \\ 0, & \text{for } n > r \end{array} \right\}$$

  where r represents the infection radius used, d is the distance between an infected unit and a neighbouring susceptible unit and k is an arbitrary constant.
- Units could attempt to periodically travel to a common location. In reality, this could represent a supermarket, train station, park etc.

Improvements in my data analysis could also be made:

- Different walk distance standard deviations could be analysed.
- The infection radius could be changed.
- Other simulation data could be analysed to be able to more confidently conclude whether results and the iterative SIR model match up sufficiently.

8 References

[1] Foundation, P. (n.d.). Processing.org. Retrieved May 19, 2020, from `https://processing.org/`

[2] Chart. (n.d.). Retrieved May 19, 2020, from `https://www.gicentre.net/utils/chart`

[3] Luz, Paula & Struchiner, Claudio & Galvani, Alison. (2010). Modeling Transmission Dynamics and Control of Vector-Borne Neglected Tropical Diseases. Pg 1. PLoS neglected tropical diseases. 4. e761. 10.1371/journal.pntd.0000761

[4] Coronavirus: What are social distancing and self-isolation rules? (2020, May 22). Retrieved May 22, 2020, from `https://www.bbc.co.uk/news/uk-51506729`

9 Appendices

9.1 Source Code

Simulation source code can be found at `https://github.com/danielbatchford/VirusSimulation`

9.2 Excel Data

t	$\mathbf{S}_t$	$\mathbf{I}_t$	$\mathbf{R}_t$	$\mathbf{S}_{i_t}$	$\mathbf{I}_{i_t}$	$\mathbf{R}_{i_t}$	$\Delta\mathbf{S}_{i_t}$	$\Delta\mathbf{I}_{i_t}$	$\Delta\mathbf{R}_{i_t}$	ϵ
0	9999	1	0	9999.0	1.0	0.0				0
1	9955	45	0	9998.5	1.4	0.1	-0.5	0.4	0.1	87
2	9902	91	7	9997.8	2.0	0.2	-0.7	0.6	0.1	192
3	9827	157	16	9996.8	2.7	0.4	-1.0	0.8	0.2	340
4	9691	275	34	9995.4	3.8	0.7	-1.4	1.1	0.3	609
5	9546	389	65	9993.5	5.4	1.1	-1.9	1.5	0.4	895
6	9403	501	96	9990.8	7.5	1.6	-2.7	2.1	0.5	1176
7	9169	688	143	9987.1	10.5	2.4	-3.8	3.0	0.8	1636
8	8972	823	205	9981.8	14.7	3.4	-5.3	4.2	1.1	2020
9	8761	971	268	9974.5	20.6	4.9	-7.4	5.9	1.5	2427
10	8497	1148	355	9964.2	28.8	7.0	-10.3	8.2	2.1	2934
11	8247	1271	482	9949.8	40.3	9.9	-14.4	11.5	2.9	3406
12	8028	1361	611	9929.8	56.3	13.9	-20.1	16.0	4.0	3804
13	7767	1473	760	9901.8	78.7	19.5	-28.0	22.3	5.6	4270
14	7475	1631	894	9862.8	109.8	27.4	-39.0	31.1	7.9	4776
15	7264	1684	1052	9808.7	152.9	38.4	-54.1	43.2	11.0	5089
16	6948	1832	1220	9733.7	212.7	53.7	-75.0	59.7	15.3	5571
17	6612	1985	1403	9630.2	294.9	74.9	-103.5	82.2	21.3	6036
18	6237	2177	1586	9488.2	407.4	104.4	-142.0	112.5	29.5	6502
19	5918	2287	1795	9294.9	559.9	145.2	-193.3	152.5	40.7	6754
20	5641	2346	2013	9034.7	764.1	201.1	-260.2	204.2	56.0	6787
21	5316	2445	2239	8689.5	1032.9	277.6	-345.2	268.8	76.4	6747
22	4946	2586	2468	8240.7	1378.4	380.9	-448.8	345.5	103.3	6589
23	4554	2721	2725	7672.8	1808.5	518.7	-568.0	430.1	137.8	6238
24	4165	2818	3017	6979.0	2321.5	699.5	-693.8	513.0	180.9	5628
25	3702	2999	3299	6168.9	2899.4	931.7	-810.1	577.9	232.1	4934
26	3291	3112	3597	5274.6	3503.8	1221.6	-894.3	604.4	289.9	4751
27	2832	3266	3902	4350.5	4077.4	1572.0	-924.0	573.7	350.4	4660
28	2393	3387	4220	3463.6	4556.7	1979.8	-887.0	479.2	407.7	4480
29	2020	3415	4565	2674.5	4890.1	2435.4	-789.1	333.5	455.7	4259
30	1658	3444	4898	2020.5	5055.0	2924.4	-653.9	164.9	489.0	3947
31	1276	3512	5212	1509.9	5060.2	3429.9	-510.7	5.2	505.5	3564
32	965	3479	5556	1127.8	4936.2	3936.0	-382.0	-124.0	506.0	3240
33	736	3356	5908	849.5	4720.9	4429.6	-278.4	-215.3	493.6	2957
34	548	3199	6253	649.0	4449.4	4901.7	-200.5	-271.6	472.1	2703
35	381	3058	6561	504.6	4148.8	5346.6	-144.4	-300.6	444.9	2429
36	236	2873	6891	399.9	3838.6	5761.5	-104.7	-310.2	414.9	2259
37	143	2666	7191	323.2	3531.5	6145.3	-76.8	-307.1	383.9	2091
38	102	2437	7461	266.1	3235.4	6498.5	-57.1	-296.1	353.1	1925
39	44	2246	7710	223.1	2954.9	6822.0	-43.0	-280.5	323.5	1776
40	16	2062	7922	190.1	2692.4	7117.5	-33.0	-262.5	295.5	1609

t	$\mathbf{S}_t$	$\mathbf{I}_t$	$\mathbf{R}_t$	$\mathbf{S}_{i_t}$	$\mathbf{I}_{i_t}$	$\mathbf{R}_{i_t}$	$\Delta\mathbf{S}_{i_t}$	$\Delta\mathbf{I}_{i_t}$	$\Delta\mathbf{R}_{i_t}$	ϵ
41	5	1856	8139	164.5	2448.7	7386.8	-25.6	-243.6	269.2	1504
42	0	1700	8300	144.4	2224.0	7631.6	-20.1	-224.7	244.9	1337
43	0	1545	8455	128.3	2017.7	7854.0	-16.1	-206.3	222.4	1202
44	0	1379	8621	115.4	1828.8	8055.8	-12.9	-188.8	201.8	1130
45	0	1237	8763	104.8	1656.5	8238.7	-10.5	-172.3	182.9	1049
46	0	1115	8885	96.1	1499.5	8404.3	-8.7	-157.0	165.6	961
47	0	1024	8976	88.9	1356.8	8554.3	-7.2	-142.7	150.0	843
48	0	917	9083	82.9	1227.1	8690.0	-6.0	-129.6	135.7	786
49	0	823	9177	77.8	1109.5	8812.7	-5.1	-117.6	122.7	729
50	0	741	9259	73.5	1002.9	8923.6	-4.3	-106.6	111.0	671
51	0	669	9331	69.8	906.3	9023.9	-3.7	-96.6	100.3	614
52	0	598	9402	66.6	818.8	9114.5	-3.2	-87.5	90.6	575
53	0	540	9460	63.9	739.7	9196.4	-2.7	-79.2	81.9	527
54	0	487	9513	61.6	668.1	9270.4	-2.4	-71.6	74.0	485
55	0	430	9570	59.5	603.3	9337.2	-2.1	-64.7	66.8	466
56	0	395	9605	57.7	544.8	9397.5	-1.8	-58.5	60.3	415
57	0	356	9644	56.1	491.9	9452.0	-1.6	-52.9	54.5	384
58	0	323	9677	54.7	444.1	9501.2	-1.4	-47.8	49.2	352
59	0	289	9711	53.5	400.9	9545.6	-1.2	-43.2	44.4	331
60	0	249	9751	52.5	361.9	9585.7	-1.1	-39.0	40.1	331
61	0	224	9776	51.5	326.6	9621.9	-0.9	-35.2	36.2	308
62	0	198	9802	50.7	294.8	9654.5	-0.8	-31.8	32.7	295
63	0	180	9820	49.9	266.1	9684.0	-0.7	-28.7	29.5	272
64	0	162	9838	49.3	240.1	9710.6	-0.7	-25.9	26.6	255
65	0	148	9852	48.7	216.7	9734.6	-0.6	-23.4	24.0	235
66	0	132	9868	48.1	195.6	9756.3	-0.5	-21.1	21.7	223
67	0	120	9880	47.7	176.5	9775.9	-0.5	-19.1	19.6	208
68	0	110	9890	47.2	159.2	9793.5	-0.4	-17.2	17.6	193
69	0	100	9900	46.9	143.7	9809.4	-0.4	-15.5	15.9	181
70	0	90	9910	46.5	129.7	9823.8	-0.3	-14.0	14.4	172
71	0	86	9914	46.2	117.0	9836.8	-0.3	-12.7	13.0	154
72	0	78	9922	46.0	105.6	9848.5	-0.3	-11.4	11.7	147
73	0	71	9929	45.7	95.3	9859.0	-0.2	-10.3	10.6	140
74	0	63	9937	45.5	85.9	9868.5	-0.2	-9.3	9.5	137
75	0	55	9945	45.3	77.5	9877.1	-0.2	-8.4	8.6	136
76	0	49	9951	45.1	70.0	9884.9	-0.2	-7.6	7.8	132
77	0	47	9953	45.0	63.1	9891.9	-0.2	-6.8	7.0	122
78	0	42	9958	44.8	57.0	9898.2	-0.1	-6.2	6.3	120
79	0	39	9961	44.7	51.4	9903.9	-0.1	-5.6	5.7	114
80	0	34	9966	44.6	46.4	9909.0	-0.1	-5.0	5.1	114
81	0	34	9966	44.5	41.8	9913.7	-0.1	-4.5	4.6	105
82	0	31	9969	44.4	37.7	9917.9	-0.1	-4.1	4.2	102
83	0	30	9970	44.3	34.1	9921.6	-0.1	-3.7	3.8	97
84	0	29	9971	44.2	30.7	9925.0	-0.1	-3.3	3.4	92
85	0	27	9973	44.2	27.7	9928.1	-0.1	-3.0	3.1	90
86	0	24	9976	44.1	25.0	9930.9	-0.1	-2.7	2.8	90
87	0	20	9980	44.1	22.6	9933.4	-0.1	-2.4	2.5	93
88	0	18	9982	44.0	20.4	9935.6	0.0	-2.2	2.3	93
89	0	16	9984	44.0	18.4	9937.7	0.0	-2.0	2.0	93

t	$\mathbf{S}_t$	$\mathbf{I}_t$	$\mathbf{R}_t$	$\mathbf{S}_{i_t}$	$\mathbf{I}_{i_t}$	$\mathbf{R}_{i_t}$	$\Delta\mathbf{S}_{i_t}$	$\Delta\mathbf{I}_{i_t}$	$\Delta\mathbf{R}_{i_t}$	ϵ
90	0	15	9985	43.9	16.6	9939.5	0.0	-1.8	1.8	91
91	0	13	9987	43.9	14.9	9941.2	0.0	-1.6	1.7	92
92	0	12	9988	43.8	13.5	9942.7	0.0	-1.5	1.5	91
93	0	10	9990	43.8	12.2	9944.0	0.0	-1.3	1.3	92
94	0	7	9993	43.8	11.0	9945.2	0.0	-1.2	1.2	96
95	0	6	9994	43.8	9.9	9946.3	0.0	-1.1	1.1	95
96	0	6	9994	43.7	8.9	9947.3	0.0	-1.0	1.0	93
97	0	6	9994	43.7	8.1	9948.2	0.0	-0.9	0.9	92
98	0	6	9994	43.7	7.3	9949.0	0.0	-0.8	0.8	90
99	0	6	9994	43.7	6.6	9949.7	0.0	-0.7	0.7	89
100	0	6	9994	43.7	5.9	9950.4	0.0	-0.6	0.7	87
101	0	5	9995	43.7	5.3	9951.0	0.0	-0.6	0.6	88
102	0	4	9996	43.7	4.8	9951.5	0.0	-0.5	0.5	89
103	0	4	9996	43.6	4.3	9952.0	0.0	-0.5	0.5	88
104	0	4	9996	43.6	3.9	9952.4	0.0	-0.4	0.4	87
105	0	4	9996	43.6	3.5	9952.8	0.0	-0.4	0.4	87
106	0	3	9997	43.6	3.2	9953.2	0.0	-0.3	0.4	88
107	0	3	9997	43.6	2.9	9953.5	0.0	-0.3	0.3	87
108	0	2	9998	43.6	2.6	9953.8	0.0	-0.3	0.3	88
109	0	1	9999	43.6	2.3	9954.1	0.0	-0.3	0.3	90
110	0	1	9999	43.6	2.1	9954.3	0.0	-0.2	0.2	89
111	0	0	10000	43.6	1.9	9954.5	0.0	-0.2	0.2	91
112	0	0	10000	43.6	1.7	9954.7	0.0	-0.2	0.2	91