

# A new genetic algorithm approach to smooth path planning for mobile robots

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## Abstract

**Purpose** – The purpose of this paper is to consider the smooth path planning problem for a mobile robot based on the genetic algorithm (GA) and the Bezier curve.

**Design/methodology/approach** – The workspace of a mobile robot is described by a new grid-based representation that facilitates the operations of the adopted GA. The chromosome of the GA is composed of a sequence of binary numbered grids (i.e. control points of the Bezier curve). Ordinary genetic operators including crossover and mutation are used to search the optimum chromosome where the optimization criterion is the length of a piecewise collision-free Bezier curve path determined by the control points.

**Findings** – This paper has proposed a new smooth path planning for a mobile robot by resorting to the GA and the Bezier curve. A new grid-based representation of the workspace has been presented, which makes it convenient to perform operations in the GA. The GA has been used to search the optimum control points that determine the Bezier curve-based smooth path. The effectiveness of the proposed approach has been verified by a numerical experiment, and some performances of the obtained method have also been analyzed.

**Research limitations/implications** – There still remain many interesting topics, for example, how to solve the specific smooth path planning problem by using the GA and how to promote the computational efficiency in the more grids case. These issues deserve further research.

**Originality/value** – The purpose of this paper is to improve the existing results by making the following three distinctive contributions: a rigorous mathematical formulation of the path planning optimization problem is formulated; a general grid-based representation ( $2n \times 2n$ ) is proposed to describe the workspace of the mobile robots to facilitate the implementation of the GA where  $n$  is chosen according to the trade-off between the accuracy and the computational burden; and the control points of the Bezier curve are directly linked to the optimization criteria so that the generated paths are guaranteed to be optimal without any need for smoothing afterwards.

**Keywords** Robotics, Genetic algorithms

**Paper type** Research paper

## 1. Introduction

With the rapid development of modern industry, mobile robots have been widely used in a wide range of applications such as manufacturing, assembly, logistics and transportation (Li *et al.*, 2015; Robinson *et al.*, 2015). The path planning is one of the most important topics in mobile robotics whose objective is to find a feasible and optimal path from a start position to a target position. A path is said to be “feasible” and “optimal” if the mobile robot moving along it could avoid collisions with obstacles and also satisfy certain optimal criteria. In other words, the path planning can be considered as an optimization problem on certain indices (e.g. shortest

distance) under certain constraints (e.g. collision-free route). As shown in Manikas *et al.* (2007), the path planning problem of a mobile robot is an non-deterministic polynomial (NP)-hard optimization one that can only be solved by heuristic algorithms such as evolutionary computation techniques. Among various algorithms capable of handling NP-hard problems, the genetic algorithm (GA) has proven to be the simple yet effective one that has been frequently used in industry especially in mobile robotics (Manikas *et al.*, 2007).

Recently, a variety of GA-based approaches have been developed for the mobile robot path planning problems (Pol and Murgugan, 2015). A problem-specific GA for the path planning of a mobile robot has been proposed in Hu and Yang (2004) that incorporates the domain knowledge into its

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specialized operators. A new mutation operator has been presented in [Tuncer and Yildirim \(2012\)](#) for the GA and applied to the path planning problem of mobile robots in dynamic environments. In addition to the ordinary crossover, another new mutation operator has been developed in [Naderan-Tahan and Manzuri-Shalmani \(2009\)](#) as a subset of mutation to manipulate an individual. A vibrational GA has been put forward in [Pehlivanoglu et al. \(2007\)](#) to reduce the possibility of premature convergence and therefore help the candidate solution to reach the global optimum. A parallel elite GA has been proposed in [Tsai et al. \(2011\)](#) along with a migration operator to maintain better population diversity, avoid premature convergence and keep parallelism in comparison with the conventional GA.

In almost all aforementioned literature, the GA-based path planning approaches have been concerned with the issue of planning a feasible path with certain simple optimal criterion (e.g. the minimum length of the path). Actually, a collision-free shortest path is often not sufficient for the planned movement of a mobile robot. For example, it is quite common that a traditional path planning algorithm gives rise to a path that contains some polygonal lines or even sharp turns. When moving along the polygonal lines, a mobile robot would have to switch between different modes (e.g. stop, rotate and restart) frequently, and such a switching process is both time- and energy-consuming. Such undesirable switches are even impermissible when the smoothness of the movement is a requirement for some service tasks ([Zhou et al., 2011](#)). Therefore, in addition to the distances, some other optimization criteria should be included such as the path smoothness, energy evaluation, time consumption and robot speed (see ([Mahjoubi et al., 2006](#); [Wang et al., 2007](#); [Yuan et al., 2007](#) for more details). Note that, apart from the length of the path, the path smoothness has been considered to be another important criterion because the smoothness is a closely related to other optimization criteria ([Castillo et al., 2007](#)).

In recent years, the Bezier curve has been increasingly applied in the smooth path planning problems ([Arana-Daniel, N., Gallegos, A.A., Lopez-Franco, C. and Alanis, A.Y., 2014](#); [Neto et al., 2010](#); [Song et al., 2010](#)). For example, a Bezier-curve-based approach has been proposed in [Jolly et al. \(2009\)](#) for the path planning of a mobile robot in a multi-agent robot soccer system which is compatible with the velocity and acceleration limits. A collision-free curvature-bounded smooth path planning technique has been presented in [Ho and Liu \(2009\)](#) whose idea is to divide the nodes on the piecewise linear path into control point subsequences so as to generate a collision-free composite Bezier curve under the curvature constraint. A new cooperative collision-avoidance method has been developed in [Skrjanc and Klančar \(2010\)](#) for multiple and non-holonomic robots based on the Bernstein-Bezier curves, and a model-predictive trajectory tracking algorithm has been used to drive the robots on the obtained reference paths. To form a smooth path based on the path points, the Bezier curve and other parameter curves are usually produced by the Voronoi diagram, the Dijkstra algorithm, the  $A^*$  algorithm, the  $D^*$  algorithm, etc.

To date, some scattered results have been available in literature on the smooth path planning problem of mobile

robots or multi-agent systems by combining the heuristic intelligent optimization algorithm (e.g. GA) with the path smoothing approach (e.g. Bezier curve). For example, in [Yang et al. \(2008\)](#), the path planning algorithm has been developed for obstacle avoidance problems of mobile robots by adopting the Bezier curve based on the GA, but the general mathematical description of the optimal path planning problems as well as the representation issue of the workspace have not been thoroughly discussed, and this makes it inconvenient to implement the GA in practice. In [Sahingoz \(2014\)](#), the Bezier curve-based flyable trajectories have been generated for multi-unmanned aerial vehicle (UAV) systems with parallel GA where the Bezier curve has been used for smoothing the obtained path and, as such, it is difficult to guarantee the optimality of the eventually planned path. The purpose of this paper is to improve the existing results by making the following three distinctive contributions:

- 1 a rigorous mathematical formulation of the path planning optimization problem is formulated;
- 2 a general grid-based representation ( $2n \times 2n$  grids) is proposed to describe the workspace of the mobile robots to facilitate the implementation of the GA where  $n$  is chosen according to the trade-off between the accuracy and the computational burden; and
- 3 the control points of the Bezier curve are directly linked to the optimization criteria so that the generated paths are guaranteed to be optimal without any need for smoothing afterwards.

In this paper, a new approach is proposed to solve the smooth path planning problem of a mobile robot based on the GA and the Bezier curve. A new grid-based representation of the workspace is presented in this paper which simplifies the population initialization, chromosome encoding and the genetic operators in existing literature ([Tuncer and Yildirim, 2012](#)). The workspace of a mobile robot is divided into several orderly numbered square grids, and the center of a grid is defined as a candidate control point of a Bezier curve. A sequence of binary number represented control points, namely, the genes, represent a chromosome of GA, and ordinary genetic operators are used to search the optimal chromosome. The optimization criterion of the GA is the length of all piecewise collision-free Bezier curves determined by the control points. The numerical experiment results verify the effectiveness of the proposed smooth path planning approach.

The remainder of this paper is organized as follows. In Section 2, the Bezier curve is introduced and some preliminaries are briefly outlined. In Section 3, the key stages of the smooth path planning approach are presented based on the GA. In Section 4, the effectiveness of the obtained results is illustrated by a simulation example and the expected performance is evaluated as well. Concluding remarks and future work are given in Section 5.

## 2. Preliminary of Bezier curve

The Bezier curve is contained within the convex hull of a sequence of control points. In this case, the Bezier curve is different from the traditional interpolation-based curves such as polynomials and cubic splines. The control points, which

define the Bezier curve, are not on the curve except the start and end points. The high-order derivative continuity of the Bezier curve can guarantee the smooth variation of the curve between the start point and the end point.

Given a set of control point vectors  $P_0, P_1, \dots, P_n$ , the corresponding Bezier curve is defined as:

$$P(t) = \sum_{i=0}^n B_i^n(t) P_i, \quad 0 \leq t \leq 1, \quad (1)$$

where  $t$  is the normalized time variable,  $B_i^n(t)$  is a Bernstein polynomial and  $P_i = (x_i, y_i)^T$  stands for the coordinate vector of the  $i$ th control point with  $x_i$  and  $y_i$  being the components corresponding to the  $X$  and  $Y$  coordinate, respectively. The Bernstein polynomial is the base function in the expression of Bezier curve, which is defined as:

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}, \quad i = 0, 1, \dots, n. \quad (2)$$

The derivatives of a Bezier curve can also be determined by the control points. The first derivative of a Bezier curve is expressed as:

$$\dot{P}(t) = \frac{dP(t)}{dt} = n \sum_{i=0}^{n-1} B_i^{n-1}(t) (P_{i+1} - P_i) \quad (3)$$

and the higher-order derivative of a Bezier curve can be obtained by repeatedly using equation (3). For example, the second derivative of a Bezier curve is expressed as:

$$\ddot{P}(t) = n(n-1) \sum_{i=0}^{n-2} B_i^{n-2}(t) (\ddot{P}_{i+2} - 2\ddot{P}_{i+1} + \ddot{P}_i). \quad (4)$$

Accordingly, in the two-dimensional plane, the curvature of a Bezier curve with respect to  $t$  can be represented as:

$$\kappa(t) = \frac{1}{R(t)} = \frac{\dot{P}_x(t)\ddot{P}_y(t) - \dot{P}_y(t)\ddot{P}_x(t)}{(\dot{P}_x^2(t) + \dot{P}_y^2(t))^{3/2}}, \quad (5)$$

where  $R(t)$  is the radius of curvature,  $\dot{P}_x(t)$ ,  $\dot{P}_y(t)$ ,  $\ddot{P}_x(t)$  and  $\ddot{P}_y(t)$  are  $X$  and  $Y$  coordinate components of the first and second derivatives of the Bezier curve  $P(t)$ , respectively.

In this paper, piecewise Bezier curves are connected to form a complete path in the smooth path planning of a mobile robot, where second- and lower-order continuities are considered for the smoothness of the path. The zero-order continuity (i.e. continuous position) is held by coincident end and start points of the connected Bezier curves. The first-order continuity is ensured by equivalent tangent vectors at the connection of two curves, and the second-order continuity is ensured by equivalent curvatures. For simplicity, the curvature at the connection of two connected Bezier curves is usually set to zero, i.e. a few adjacent points of the connected Bezier curves are on the same line. Therefore, it is easy to satisfy the continuity requirements in most cases.

### 3. Smooth path planning based on genetic algorithm

In this paper, the GA is combined with the Bezier curve for the smooth path planning of a mobile robot. The GA is applied to search the optimal control points which are used to define the smooth Bezier curve. The feasible and shortest Bezier curve path is the optimum solution for the smooth path planning problem of a mobile robot. Several key stages of the approach are presented in this section as follows.

#### 3.1 A problem description

In this paper, the working environment of a mobile robot is supposed to be a two-dimensional workspace. The proposed GA based smooth path planning is used to find a feasible and optimal Bezier curve path for a mobile robot, which include three conditions: first, the path should be collision-free; second, the smooth movement of a mobile robot should be considered by satisfying the second-order continuity of the path; and third, the path should be the shortest distance from a start point to a target point under the above two constraints. The formal expressions of above conditions are as follows:

$$\begin{aligned} \min \quad & \|P(t)\|, \quad 0 \leq t \leq 1, \\ \text{s.t.} \quad & P(t) \in C^2, \\ & P(t) \in P_{\text{free}} \end{aligned} \quad (6)$$

where  $t$  is normalized time variable,  $\|P(t)\|$  represents the length of the Bezier curve path,  $C^2$  is a set of second-order differentiable function and  $P_{\text{free}}$  indicates a set of collision-free path. As the Bezier curve is defined by its control points, the above optimization problem is to find a sequence of control points which determine the Bezier curve path under constraint conditions.

#### 3.2 Representation of workspace

The workspace is an environment where the mobile robot and obstacles both exist. The grid-based model is usually used to represent the workspace in the path planning of a mobile robot, as it is easy to calculate distances and represent obstacles. The boundary of obstacles is formed by their actual boundary plus minimum safety distance considering the size of a mobile robot, so that a mobile robot could be treated as a point in the workspace (Hu and Yang, 2004). The whole workspace is represented by orderly numbered grids, and the size of the grids determines how many numbers there is. For each grid, it is defined to be either empty (i.e. the white square grid in the workspace) or occupied (i.e. the black square grid in the workspace), which depends on whether the boundary of obstacles is in the grid.

The whole workspace is divided into  $M \times N$  grids in this paper, where both  $M$  and  $N$  are positive integer power of Number 2.  $M$  could be equal to  $N$  or not in practice, because a workspace with  $M \times N$  grids could be treated as a workspace with  $M \times M$  grids and some of the grids are occupied. Figure 1(a) and (b) show a workspace with  $M \times M$  grids ( $M = 16, N = 16$ ) and  $M \times N$  grids ( $M = 15, N = 16$ ), respectively. This representation approach simplifies the chromosome encoding and population initialization of GA. For example, if the workspace is divided into  $10 \times 10$  grids, then the numbers of the grids are from 0 to 99. Binary encoding is used for chromosome in this paper, and this

**Figure 1** (a) The workspace with  $16 \times 16$  grids; (b) the workspace with  $15 \times 16$  grids

240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255
224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239
208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223
192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207
176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191
160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175
144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159
128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143
112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127
96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95
64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

(a)

240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255
224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239
208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223
192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207
176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191
160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175
144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159
128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143
112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127
96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95
64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

(b)

means a 7 bit binary number is necessary to represent one grid. However, some of the numbers (e.g. from 100 to 127) exceed all of the grid numbers in the workspace, so that additional process has to be added to check whether the randomly generated chromosomes (defined in next section) are reasonable.

### 3.3 Chromosome encoding and population initialization

A sequence of grid numbers is used to represent the chromosomes of the GA in this paper, and chromosomes are encoded by binary numbers for easier bit operations of genetic operators. Take the  $160 \times 160$  units workspace with  $16 \times 16$  grids (i.e. each grid is a  $10 \times 10$  units square) in Figure 2, for example, hereinafter, each grid is assigned a number between 0 and 255, which could be represented by a 8 bit binary number. All of the binary numbers are connected orderly to form a chromosome. This kind of chromosome encoding is suitable for both  $M \times M$  and  $M \times N$  grids. Moreover, genetic

operators will not bring unreasonable chromosome by using this approach because both crossover and mutation operators will keep a grid number in the chromosome between 0 and 255. This is another advantage of the workspace representation approach mentioned above.

All of the control points are defined at the center of the grids in the workspace. The transformation from grid numbers to coordinate values is expressed as:

$$\begin{aligned} P_x(t) &= (\text{Number} \% 16) \times 10 + 5, \\ P_y(t) &= \lfloor \text{Number} / 16 \rfloor \times 10 + 5, \end{aligned} \quad (7)$$

where percentage indicates the complementation,  $\lfloor \cdot \rfloor$  denotes the rounding down,  $P_x(t)$  and  $P_y(t)$  are  $X$  and  $Y$  coordinate components of the center of the grid, respectively. On the other hand, the transformation from coordinate components of any point on the path to the number of a grid that contains the point is expressed as:

$$\text{Number} = \lfloor P_x(t) / 10 \rfloor + \lfloor P_y(t) / 10 \rfloor \times 16. \quad (8)$$

**Figure 2** The workspace of a mobile robot in this paper

240	241	242	243		246	247	248	249	250	251	252	253	254	255
224	225	226	227		230	231	232	233	234	235	236	237	238	239
208	209	210	211		214	215	216	217	218	219	220	221	222	223
192	193	194	195		198	199	200	201						
176	177	178	179	180	181	182	183	184	185	186	187	189	190	191
								168	169	170	171	173	174	175
								152	153	154	155	157	158	159
128	129			132	133	134	135	136	137	138	139	141	142	143
112	113			116	117	118	119	120	121	122	123	125	126	127
96	97			100	101	102	103	104	105	106	107	108	109	110
80	81	82	83	84	85	86		89	90	91				
64	65	66	67	68	69	70		73	74	75				
48	49	50	51	52	53	54		57	58	59	60	61	62	63
32	33	34	35					41	42	43	44	45	46	47
16	17	18	19	20	21	22		25	26	27	28	29	30	31
0	1	2	3	4	5	6		9	10	11	12	13	14	15

Considering the representation of the workspace and chromosome encoding approaches, it is very easy to implement the population initialization of the generic algorithm. For example, if a chromosome has  $n$  grid numbers, then the initialized population is a set of  $8 \times n$  bit binary numbers.

### 3.4 Fitness function and selection method

The purpose of the GA-based smooth path planning in this paper is to find an optimal path under constraint conditions [equation (6)]. The fitness function is defined as:

$$f = \begin{cases} \frac{1}{\sum_{i=1}^n \|P_i(t)\|}, & \text{for feasible paths} \\ \frac{1}{\sum_{i=1}^n \|P_i(t)\| + \text{penalty}}, & \text{for infeasible paths} \end{cases} \quad (9)$$



where  $P_i(t)$  is the  $i$ th segment of the piecewise Bezier curves with  $n$  segments and  $penalty$  is added when the Bezier curve passes through an occupied grid. A shorter path will have a larger fitness value and the optimal path is a shortest feasible Bezier curve path.

The proportional selection strategy is used in the selection method of the GA, i.e. the probability that the selected chromosome is proportional to the fitness value. Suppose the fitness value of the  $i$ th chromosome is  $f_i$  and the population size is  $S_p$ , the selection probability of the  $i$ th chromosome can be expressed as:

$$P_i = \frac{f_i}{\sum_{i=1}^{S_p} f_i}, \quad (10)$$

where  $P_i$  is the selection probability and the roulette wheel method is used for selection operation afterwards.

### 3.5 Genetic operators

Crossover and mutation are two heart genetic operators in the GA. The crossover operator is to combine the features of two parent chromosomes to produce two offspring chromosomes. The crossover probability is randomly generated to determine whether the crossover operator is implemented on two parent chromosomes. The single point crossover operator is used in this paper, i.e. the genes of two chromosomes after a randomly generated crossover point are swapped.

The mutation operator is implemented after the crossover operator on randomly selected chromosomes in the population. The binary complement operation is implemented on a randomly generated mutation point of the chromosomes. Above genetic operators will not produce genes (i.e. grid numbers) out of the workspace by using the proposed approaches in Sections 3.2 and 3.3:

- *Remark 1:* The grid-based representation of the workspace is common in the literature of mobile robot path planning. However, a general grid-based representation is proposed in this paper. The whole workspace is divided into  $2^n \times 2^n$  grids despite whether the workspace is a square (the workspace could be treated as a square with some occupied grids if it is not a square). This approach facilitates the implementation of the GA in the processes of initialization, crossover and mutation.
- *Remark 2:* In this paper, the control points of the Bezier curve are directly linked to the optimization criteria, which is different from most Bezier-curve-based path planning approaches where Bezier curve is used to smooth the path produced by some path planning methods (Sahingoz, 2014). Although the path planning adopting the Bezier curve based on the GA has been developed in Yang *et al.* (2008), the details of the implementation have not been thoroughly discussed.

## 4. Numerical experiments and performance evaluation

In this section, the smooth path planning based on the GA will be applied to the workspace in Figure 2 to demonstrate the effectiveness of the proposed approach. The parameters of the

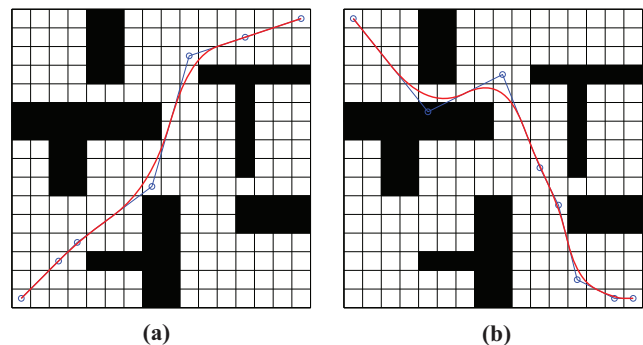
GA are as follows: the population size is taken as 200, the maximum generation is taken as 100, the crossover probability is taken as 0.5, the mutation probability is taken as 0.1 and the  $penalty$  is taken as 10 for each infeasible point of the Bezier curve path.

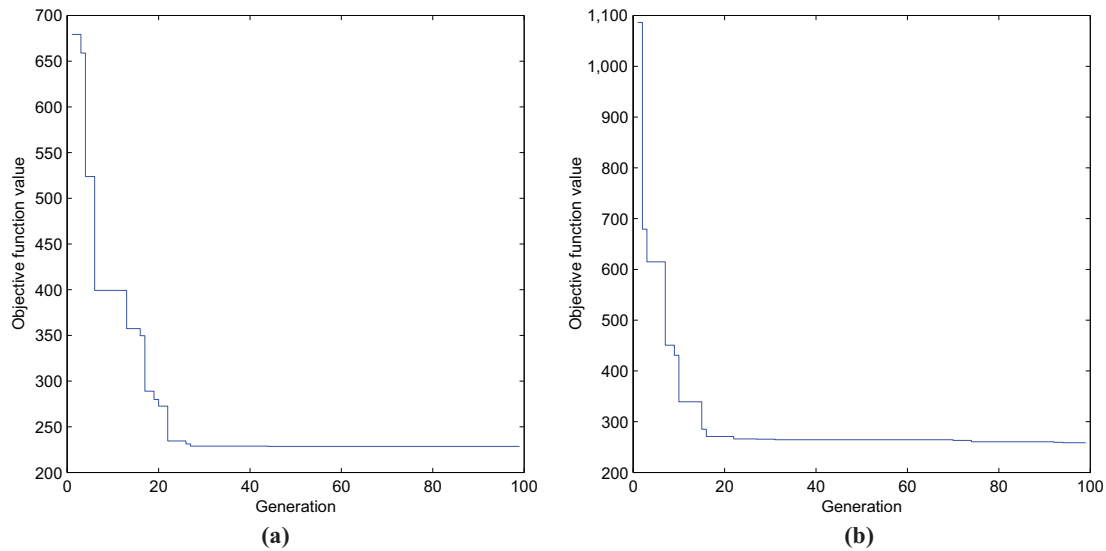
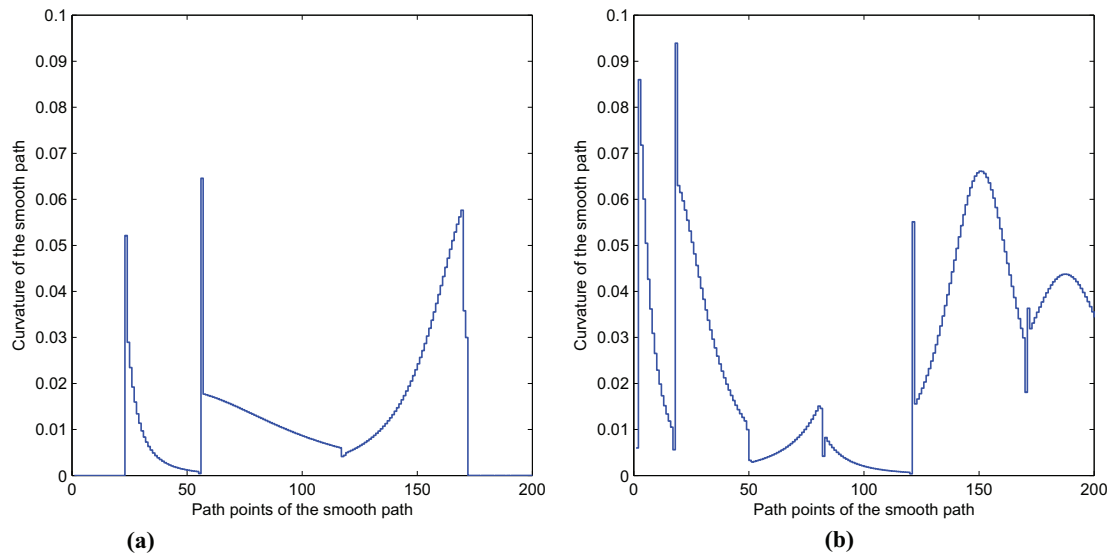
Figure 3(a) and (b) shows the numerical experiment results with different start positions and target positions in the workspace, where blue circles indicate the control points of the Bezier curve path, blue solid lines compose the convex hull and red solid lines is the optimum smooth path. Eight control points (i.e. eight grid numbers for each chromosome) are used for the Bezier curve path in this paper. Despite the difficulty in the two cases, the proposed approach can accomplish the smooth path planning task successfully. Define the objective function as the reciprocal of the fitness function  $f$  in equation (9). The objective function value of the optimum chromosome in each generation is depicted in Figure 4, which shows the fast convergence of the GA in this problem.

Figure 5 shows the curvature of each point on the Bezier curve path, and it is obvious that the maximal curvature is lower than 0.1 according to the coordinates of the workspace. The low curvature values reflect the smoothness of the obtained path. Table I presents some comparisons on smooth path planning and non-smooth path planning both based on the GA. The experiment results with different start positions and target positions indicate that, by using the proposed smooth path planning approach, we can derive an optimal path with the similar length as in the non-smooth path planning. Moreover, there is little difference on the minimal convergence generation between smooth and non-smooth path planning:

- *Remark 3:* It is very important to choose the control point vectors in practical applications. Generally speaking, more control point vectors are required if the workspace is more complicated (e.g. with more obstacles). The GA could be implemented with different numbers of control points in practice. It is worth mentioning that the redundant control point, which is coincide with the other, will be rejected in the optimum control point sequence at the end of the procedure (e.g. eight control points are used for the GA and seven control points are remained in Figure 3(a)).
- *Remark 4:* Compared with other smooth path planning methods (Arana-Daniel, N., Gallegos, A.A., Lopez-Franco, C. and Alanis, A.Y., 2014; Neto *et al.*, 2010; Song *et al.*, 2010), one of the advantages of the proposed

**Figure 3** (a) Smooth path planning from Grid 0 to Grid 255; (b) smooth path planning from Grid 15 to Grid 240



**Figure 4** (a) Optimum objective function value in the case of Figure 3(a); (b) optimum objective function value in the case of Figure 3(b)**Figure 5** (a) Path curvature in the case of Figure 3(a); (b) path curvature in the case of Figure 3(b)**Table I** Comparison of genetic algorithm based smooth path planning and non-smooth path planning

Start position and target position	Minimal generation non-smooth path	Objective value non-smooth path	Minimal generation smooth path	Objective value smooth path
Grid 0 to 15	26	225.4982	49	221.0383
Grid 15 to 255	88	211.2393	67	218.5059
Grid 240 to 255	76	178.3131	47	195.3404
Grid 0 to 240	69	244.9847	73	247.8467

approach is the general grid-based representation of the workspace, which facilitates the implementation of the GA. The other but more meaningful one is that this approach is a “real” smooth path planning approach combining a heuristic intelligent optimization algorithm (e.g. the GA in this paper) with a path smoothing approach (e.g. Bezier curve in this paper). In this paper, the control

points of the Bezier curve are directly linked to the optimization criteria of the GA, so that the generated paths are guaranteed to be optimal instead of smoothing the paths after some path planning processes.

- *Remark 5:* In equation (9), the *penalty* is added when the Bezier curve passes through an occupied grid. In some special cases, a path will have larger fitness value, even its

control points lie in the occupied grids and the *penalty* is added in the fitness function. For example, it can be found that optimum control points can be in the *occupied* grids as shown in Figure 3(b). Moreover, how to select the *penalty* to improve the performance of the algorithm is still an open problem for further discussion.

## 5. Conclusions and future work

This paper has proposed a new smooth path planning for a mobile robot by resorting to the GA and the Bezier curve. A new grid-based representation of the workspace has been presented, which makes it convenient to perform operations in the GA. The GA has been used to search the optimum control points that determine the Bezier curve-based smooth path. The effectiveness of the proposed approach has been verified by a numerical experiment, and some performances of the obtained method have also been analyzed. There still remain many interesting topics, for example:

- how to solve the specific smooth path planning problem by using the GA;
- how to promote the computational efficiency in the more grids case;
- how to select the number of control points and value of the “*penalty*”; and
- how to apply the developed algorithms to more complicated situations (e.g. mobile navigations in networked environments (Ding *et al.*, 2015a, 2015b, 2015c, 2015d, 2015e, 2015f; Dong *et al.*, 2015, 2014; Hu *et al.*, 2016; Liu *et al.*, 2014, 2015; Luo *et al.*, 2015; Shen *et al.*, 2016; Zou *et al.*, 2016)).

These issues deserve further research.

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