Principles of Robot Motion

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Principles of Robot Motion

Theory, Algorithms, and Implementation

Howie Choset, Kevin Lynch, Seth Hutchinson,
George Kantor, Wolfram Burgard, Lydia Kavraki,
and Sebastian Thrun

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To our families

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Foreword

THIS IMPRESSIVE book is the result of a serious undertaking of distinguished motion planning researchers led by Howie Choset. Over the years, motion planning has become a major research theme in robotics. The goal is to enable robots to automatically compute their motions from high-level descriptions of tasks and models acquired through sensing. This goal has recently become even more crucial. On the one hand, robotics has expanded from a largely dominant focus on industrial manufacturing into areas where tasks are less repetitive and environments less structured, for instance, medical surgery, ocean and space exploration, assistance to the elderly, and search-and-rescue. In these areas, it is impractical to explicitly program the robots for each new goal. On the other hand, the need for automatic motion-planning capabilities has expanded outside the realm of robotics, into domains such as computer animation (e.g., to generate motions of avatars), computer-aided design (e.g., to test that a product can be assembled or serviced), verification of building codes (e.g., to check access of key facilities to the disabled), exploration of virtual environments (to help the user navigate in geometric models made of tens of millions of triangles), or even computational biology (to help analyze important molecular motions, like folding and binding). Today, progress in motion planning is increasingly motivated by these new applications.

By confronting novel and difficult problems, researchers have made considerable progress in recent years. Not only have faster and more robust algorithms been developed and tested, but the range of motion-planning problems has continuously expanded. In the ’80s and part of the ’90s, finding collision-free paths was the main or only goal. Today, while obstacle avoidance remains a key issue, other important constraints are considered as well, for instance, visibility, coverage, kinodynamic, optimality, equilibrium, and uncertainty constraints. These constraints make problems more interesting and lead to more useful algorithms. In addition, while research in motion planning used to be neatly divided between theory and practice, this distinction has now largely disappeared. Most recent contributions to the field combine effective

algorithms tested on significant problems, along with some formal guarantees of performance.

Although journal and conference papers in motion planning have proliferated, there has not been any comprehensive reference text in more than a decade. This book fills this gap in outstanding fashion. It covers both the early foundations of the field and the recent theoretical and practical progress that has been made. It beautifully demonstrates how the enduring contributions of early researchers in the field, like Lozano-Perez (configuration space) and Reif (complexity of motion planning), have led to a rich and vibrant research area, with ramifications that were unsuspected only a decade ago.

I am usually suspicious of books in which chapters have been written by different authors. But, to my good surprise, this book is more than a standard textbook. The fact that seven authors collaborated on this book attests to the diversity of the research going on in motion planning and the excitement associated with each research topic. Simultaneously, the authors have done excellent work in providing a unified presentation of the core concepts and methodologies, and thus the book can be used as a textbook. This book will serve well the growing community of students, researchers, and engineers interested in motion planning.

Jean-Claude Latombe
Stanford, California

Preface

PEOPLE HAVE always dreamed of building intelligent machines to perform tasks. Today, these machines are called robots, derived from the Czech word *robota* meaning servitude or drudgery. Robots have inspired the imagination of many, appearing in mythology, literature, and popular movies. Some popular robotic characters include Robby the Robot, R2D2 and C3P0, Golem, Pushpack, Wanky and Fanny, Gundam and Lt. Cmdr. Data. Just like their literary counterparts, robots can take on many forms and constructing them involves addressing many challenges in engineering, computer science, cognitive science, language, and so on. Regardless of the form of the robot or the task it must perform, robots must maneuver through our world. This book is about automatic planning of robot motions. However, the approaches developed in this book are not limited to robots: recently, they have been used for "designing" pharmaceutical drugs, planning routes on circuit boards, and directing digital actors in the graphics world.

The robot motion field and its applications have become incredibly broad, and this is why the book has seven co-authors. This type of book requires a broad spectrum of expertise. However, it should be stressed that this is indeed a textbook and not a collection of independent chapters put together in an edited volume. Each author participated in writing each of the chapters and all of the chapters are integrated with each other.

This book is aimed at the advanced undergraduate or new graduate student interested in robot motion, and it may be read in a variety of ways. Our goal in writing in this book is threefold: to create an updated textbook and reference for robot motion, to make the fundamental mathematics behind robot motion accessible to the novice, and to stress implementation relating low-level details to high-level algorithmic concepts.

Since the robot motion field is indeed broad, this book cannot cover all the topics, nor do we believe that any book can contain exhaustive coverage on robot motion. We do, however, point the reader to Jean-Claude Latombe's *Robot Motion Planning* [262].

Latombe's book was one of the first text and reference books aimed at the motion-planning community and it certainly was a guide for us when writing this book. In the decade since Latombe's book was published, there have been great advances in the motion-planning field, particularly in probabilistic methods, mechanical systems, and sensor-based planning, so we intended to create a text with these new advances. However, there are many topics not included in our text that are included in his, including assembly planning, geometric methods in dealing with uncertainty, multiple moving obstacles, approximate cell decompositions, and obstacle representations.

We also believe that concepts from control theory and statistical reasoning have gained greater relevance to robot motion. Therefore, we have included an appendix briefly reviewing linear control systems which serves as background for our presentation on Kalman filtering. Our description of Kalman filtering differs from others in that it relies on a rich geometric foundation. We present a comprehensive description of Bayesian-based approaches. Concepts from mechanics and dynamics have also had great impact on robot motion. We have included a chapter on dynamics which serves as a basis for our description of trajectory planning and planning for underactuated robots.

This book can be read from cover to cover. In doing so, there are four logical components to the book: geometric motion planning approaches (chapters 2 through 6), probabilistic methods (chapters 7, 8, and 9), mechanical control systems (chapters 10, 11, and 12), and the appendices. Covering the entire book could require a full year course. However, not all of the topics in this book need be covered for a course on robot motion. For semester-long courses, the following themes are suggested:

Theme	**Chapter and Appendix Sequence**
Path Planning	3, 4, G, 5, 7, and 6
Mobile Robotics	2, H, 3, 4, 5, D, and 6
Mechanical Control Systems	3, 10, 11, and 12
Position Estimation	I, J, 8, and 9

The algorithms and approaches presented in this book are based on geometry and thus rest on a solid mathematical basis. Beyond anything superficial, in order to understand the many motion-planning algorithms, one must understand these mathematical underpinnings. One of the goals of this book is to make mathematical concepts more accessible to students of computer science and engineering. In this book, we introduce the intuition behind new mathematical concepts on an "as needed" basis to understand both how and why certain motion planning algorithms work. Some salient

concepts are formally defined in each chapter and the appendices contain overviews of some basic topics in more detail. The idea here is that the reader can develop an understanding of motion planning algorithms without getting bogged down by mathematical details, but can turn to them in the appendices when necessary. It is our hope that the reader will gain enough new knowledge in algebra, graph theory, geometry, topology, probability, filtering, and so on, to be able to read the state of the art literature in robot motion.

We discuss implementation issues and it is important to note that such issues are not mere details, but pose deep theoretical problems as well. In chapters 2, 4, 5, and 6, we discuss specific issues on how to integrate range sensor information into a planner. The Kalman Filtering (chapter 8) and Bayesian-based (chapter 9) approaches have been widely used in the robot motion field to deal with positioning and sensor uncertainty. Finally, we discuss in chapters 11 and 12 issues involving kinematic and dynamic contraints that real robots experience.

We have also included pseudocode for many of the algorithms presented throughout the book. In appendix H, we have included a discussion of graph search with detailed examples to enable the novice to implement some standard graph search approaches, with applicability well beyond robot motion. Finally, at the end of each chapter, we present problems that stress implementation.

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