Measuring voter satisfaction in various voting systems with complex voter opinion space and campaign strategy

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Democratic voting strategies seek to maximize a utility function governed by voter satisfaction. In this work, we present a model for community elections that enables comparison of voting strategies with respect to voter satisfaction maximization and campaign transparency.

I. INTRODUCTION

Our main interests in this research are the translation of complex voter opinions into the voting arena and the influence of different voting systems.

Most opinion models, such as the classic Voter model [1], limit agent opinion space to a single dimension (i.e, there is a single issue at hand) and often binary. While this perspective on opinion may be useful to investigate dynamic opinion propagation, it fails to encapsulate the universal truth that socio-political stance is derived from many opinions. Although models for opinion-election interaction exist [2], there is a lack of rudimentary, agent-based techniques. Using this perspective, we see a unique opportunity to extend agent-based voter opinion dynamics to a more complex space.
Our contributions are

1. Complex opinion, a model for voter political stance.
2. A comparison of three models for democratic voting that make use of complex opinion.
3. A discussion on our findings and suggestions on their implications.

We seek to compare three different contemporary democratic voting systems, each utilized in political systems around the world. These voting systems are

Plurality: A familiar method analogous to traditional voting in the United States in which each citizen casts a single vote for their ideal candidate.

Ranked-Choice: Implemented in Ireland and Maine, USA. In its simplest form, each citizen ranks each candidate from most to least ideal.

Approval: Each citizen casts a vote for as many candidates as they would like. Vote percentages may exceed 100%. This method is commonly utilized at academic or research institutions.

In exploring each voting systems, we examine idea of campaign strategy by way of opinion transparency by enabling candidates to expose less than all of their political views to the population.

Exploring complex opinion space and comparing voting systems enables evaluation on real world political systems and could lend insight on how to improve current political systems and shed light on strategies with which the systems can be gamed.

II. VOTING SYSTEMS

A voting system V is an algorithm that takes as input N submitted ballot sheets and k candidates from a population of individuals and returns a single winner of an election. Two voting systems U and V may have unique input formats for their ballot sheets, (e.g, plurality and ranked-choice, see I for a short description of each). For a certain population k individuals are chosen as candidates. Citizens are asked to cast their votes with respect the input requirements of V and a voting system is employed across the population.

Evaluation of a voting system is performed by computing the distribution of happinesses/satisfaction/agreeability across a population with the winner of an election. This metric is described in III.

The voting systems that will be tested are plurality, ranked-choice, and approval voting, to be described in the following sections.

A. Plurality

Each individual casts a single vote for the candidate with whom they believe their opinion is aligned most. Votes are tallied and the candidate with the largest share of votes wins the election.

Algorithm 1 Plurality Voting System Algorithm

Input: Set of k candidates $C = \{c_1, \dots c_k\}$, ballot box B
  return $\text{argmax}_{c \in C} \text{TALLY}(c, B)$
  function TALLY(candidate, ballots)
    return $\sum_{b \in B} \mathbb{1}_{b=\text{candidate}}$

The `plurality` algorithm runs by finding the candidate that maximizes the `tally` function, a counter for the votes for that candidate.

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B. Ranked-Choice Voting

Each individual ranks all candidates from most to least favorite. Votes are tallied to compute the proportion of the population that voted for each candidate. If the maximum proportion is less than majority ($\geq 51\%$), the bottom candidate is removed from the race and their votes are redistributed. Redistribution takes place by observing the next highest ranked candidate in each of the rank sheets submitted by those who casted votes to the removed candidate. This process goes on until one candidate has attained the majority share of votes.

In ranked-choice voting, the algorithm takes as input a ballot box B of arrays b of size k, where $b_i[0] =$ the ideal candidate for citizen i.

Algorithm 2 Ranked-Choice Voting System Algorithm

Input: Set of k candidates $C = \{c_1, \dots c_k\}$, ballot box B of ranked choices.

```
function TALLY(C, B)
    votes ← {b[0] | b ∈ B}
    if max{VOTESHARE(c,votes)|c ∈ C} ≥ 0.51 then
        return argmax_{c∈C} VOTESHARE(c,votes)
    else
        bottom ← argmin_{c∈C} VOTESHARE(c,votes)
        C ← C \ {bottom}
        for b ∈ B do
            if b = bottom then
                b ← b[1 :]          ▷ Remove the first entry of b
        return TALLY(C, B)
function VOTESHARE(candidate, votes)
    return (Σ_{b∈B} 1_{b=candidate}) / |votes|
```

The algorithm proceeds by recursively calling `tally` on its candidate and ballot sheet sets until a candidate has met the desired popular vote threshold of 0.51.

C. Approval Voting

In approval-based voting, an approval threshold is first chosen. Our work focuses on an approval threshold of 1, the reason this being that our metric for the disagreement (or dissimilarity) between two opinions is bounded in $[0, 2]$ and a threshold of 1 implies that no voter will agree with a candidate "across the aisle". Given the approval threshold, the citizen submits as an unordered set of candidates that meet that threshold with respect to dissimilarity. The winner of the election is selected by finding the candidate with the most votes. Hence, the algorithm for approval is almost exactly the same as for plurality with the main difference being that approval requires a collapsing step in which all ballot sheets are submitted and aggregated into one long list of votes. One characteristic of approval based voting is that vote percentages may not add up to 100%. This does not change the way the election is held but should be taken into mind when computing vote related statistics.

III. MODEL SET-UP, ASSUMPTIONS, AND PARAMETERS

We define "the complex opinion of voter v" $o^{(v)}$ as a real-valued vector of of d components $o_i \in [-1, 1]$, where $o_i \approx 1$ implies that v has strong support towards issue i and $o_i \approx -1$ implies the opposite. This encapsulates the notion (and assumption) that an individual's final vote for a candidate is parameterized by the interactions between their views on several different issues.

Our model is non-spatial, agent-based, and not temporally influenced. Complex opinions are stochastically generated, yet each voting algorithm is deterministically run. The model (the combination of voter opinions and election scheme) takes a random population of size N where each person p has an opinion vector $o^{(p)}$ of dimension d.

Assumption 1: We consider each dimension (issue) of $o^{(p)}$ to be independent of one another $\mathbf{Pr}(o_i^{(p)} | o_j^{(p)}) = o_i^{(p)}$ $\forall i, j \in d$.

This means that the sampling method for an individual's opinions is consistent across the topic space and the distribution from which opinion o_i is chosen is not influenced by any other opinion already drawn. The opinion vector $o^{(p)}$ is generated for every person, p in the population, N at random, where each $o_i^{(p)}$ is drawn from the uniform distribution, $\mathrm{U}(-1, 1)$.

We measure the dissimilarity of two opinion vectors, D by computing the mean absolute error, shown in Eq. 1. This dissimilarity is computed between a person and each candidate and the choice or rank of the candidates is based on that value. At the end of the voting scheme, this dissimilarity score is used again to quantify voter satisfaction with the final elected candidate.

$$\mathrm{D}(o^{(p1)}, o^{(p2)}) = \frac{\sum_{i=1}^{d} \left| o_i^{(p1)} - o_i^{(p2)} \right|}{d} \tag{1}$$

Minimizing dissimilarity maximizes happiness in the system and therefore the "End-of-Election" happiness of the population, H_P is calculated by taking the average "happiness", as in Eq. 2,

$$\mathrm{H}_P = \frac{\sum_{p=1}^{N} \left(2 - \mathrm{D}\left(o^{(p)}, o^{(c_w)}\right)\right)/2}{N} \tag{2}$$

where o^{c_w} is the full opinion vector of the candidate that won the election. Since 2 is the maximum dissimilarity score that can happen between two opinion vectors, happiness is renormalized as 0 being not at all happy and 1 being as happy as possible.

A. Opinion transparency and campaign strategies

In order to model the effect of campaign strategies and the realistic scenario where every person in the population does not know every opinion of every candidate, we add an opinion transparency parameter to the model.

For each population, each voting system is run $d-1$ times, starting where all people can "see" and base their vote(s) off of each candidate's full opinion vector. Each subsequent time the voting system is run, the candidates "mask" 1 more dimension of their vector than during the previous iteration, making them $d-1$ less transparent.

Each candidate also chooses to mask different components of their vector, symbolizing a campaign strategy, where the candidates are attempting to leverage certain topics and hide others from the general population. In our algorithm, this choice is made by each candidate in a random manner, but with each iteration they add an additional dimension to the mask while still hiding the previously hidden dimensions. As an example for $k=3, d=6$, on iteration 4, 3 components will be masked:

$$o^{(c1)} = \begin{bmatrix} \square \\ -0.99 \\ 0.71 \\ \square \\ \square \\ 0.10 \end{bmatrix} \quad o^{(c2)} = \begin{bmatrix} -0.54 \\ 0.79 \\ \square \\ \square \\ -0.11 \\ \square \end{bmatrix} \quad o^{(c3)} = \begin{bmatrix} 0.58 \\ \square \\ 0.45 \\ \square \\ 0.99 \\ \square \end{bmatrix}$$

and then on the next iteration the vectors could look like:

$$o^{(c1)} = \begin{bmatrix} \square \\ \square \\ 0.71 \\ \square \\ \square \\ 0.10 \end{bmatrix} \quad o^{(c2)} = \begin{bmatrix} \square \\ 0.79 \\ \square \\ \square \\ -0.11 \\ \square \end{bmatrix} \quad o^{(c3)} = \begin{bmatrix} 0.58 \\ \square \\ 0.45 \\ \square \\ \square \\ \square \end{bmatrix}$$

With these masks, the dissimilarity scores are calculated only on the visible components of each candidates opinion vector, but the final end voter satisfaction is calculated based on the full opinion vector dissimilarity. Continuing from the above example, if person $p1$ has opinion vector,

$$o^{(p1)} = \begin{bmatrix} 0.05 \\ 0.50 \\ -0.99 \\ -0.33 \\ 0.89 \\ 0.01 \end{bmatrix}$$

on iteration 4, their vote for the general election would be cast for candidate 2, since their dissimilarity score is the lowest based off of the visible components.

It could, and likely will, occur where the candidate that a person votes for (has minimum dissimilarity to) based off the unmasked vectors might not actually be the least similar to them, based off of the full vectors of all candidates. Since all of this is implemented randomly, there is not actual "strategy" occurring and thus *bad* strategies will likely be implemented.

IV. EXPERIMENTS & METHODS

We create 100 different sample populations, each of size $N = 10,000$, where each opinion vector of $d = 7$ for each person is drawn at random from $\mathrm{U}(-1,1)$. We iterate through each population and run through the each algorithm presented in Section II. For this initial trial, we randomly select $k = 10$ nominees from each population to hold an election. The masking of opinions for a single candidate in a sample population is determined and then kept constant as each voting system is run. Thus for a transparency level of t, for a population P_i, the voting system is the only variable changing in the system.

V. RESULTS

Here we compare the performance of each voting algorithm with respect to the average happiness achieved by the elected candidate and the effect of opinion transparency.

In Figure 1, we show for varying threshold values the distributions of satisfaction for each voting algorithm. This demonstrates the efficacy of each voting scheme with respect to satisfaction. We see that plurality is shown to have the minimum happiness in all three scenarios with approval based voting tending towards a higher mean. Although the differences in mean happiness are slight, we believe that the fact that these distributions are realized over 100 separate simulations indicates that indeed approval-based and ranked-choice voting seek a more optimal candidate solution than plurality. The

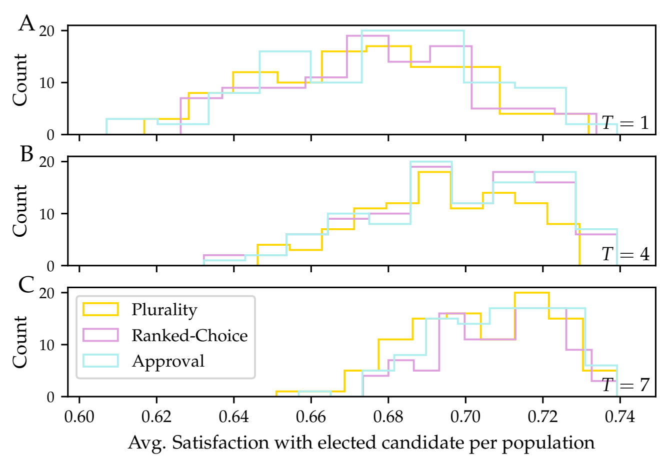

FIG. 1. Average Happiness for all populations for each voting system and transparency level

FIG. 2. Histograms of the average satisfaction/happiness/agreement of the populations with the elected candidate. Each panel represents the a simulation for opinion transparency levels T found in the bottom right corner.

notion of the plurality method's ill-performing nature in finding the most utilitarian candidate is further indicated by visualizing voter satisfaction as a function of opinion transparency in Figure V. Here we see that plurality clearly performs the worst among the three, barely overlapping in standard error with the latter two. Meanwhile, ranked-choice and approval-based voting show a higher trend.

Figure **??** further reveals evidence of the effect of transparency of voting systems in the complex opinion space. The figure suggests the intuitive claim that a candidate's revealing their full position in opinion space results in a increase in voter satisfaction post election.

We can further draw the conclusion that if candidates are imagined to stop at nothing to attain office, their best case scenario to win an election is the one where they reveal the least about themselves, leaving the vote up to uniform chance. This claim clearly comes with various caveats and may be biased through the model.

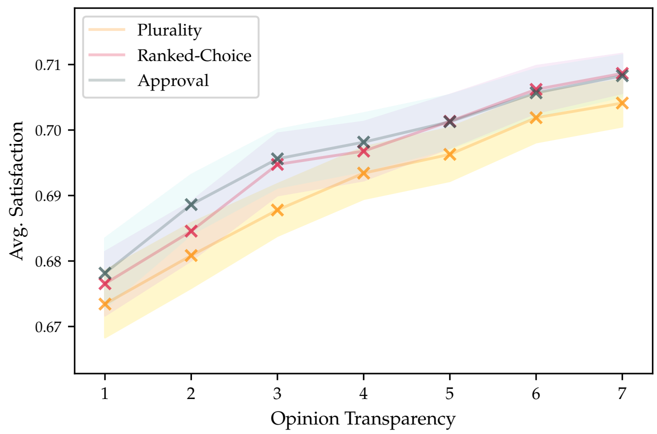

FIG. 3. Average citizen satisfaction for all populations for each voting system and transparency level shown with 95% confidence interval of standard error.

VI. DISCUSSION AND CONCLUSIONS

We have presented a model for a more complex opinion than binary, single dimensional vectors. Furthermore, we compared three common voting algorithms and showed sensible evidence in the suboptimality of plurality voting with respect to maximization of each voter utility function, the happiness metric. Furthermore, we have illustrated the effect of opinion transparency on elections in the complex opinion space; namely that a candidate can "fool" their way into office by masking their opinion of select issues, thereby misleading voters.

A. Voting strategies as search algorithms

As a final note, we observe that in this fomulation of opinion as a d-length vector, voting algorithms function as search algorithms that search the space of candidates and attempt to select the candidate that is the least distance in opinion from each voter in the population. This idea is illustrated in Figure 4. From the figure, we observe that both ranked choice and approval based voting have determined more centrally located candidates with respect to the opinion space.

VII. FUTURE WORK

In future work, we would like to further explore this model by informing our opinion vectors with data. We think it could lead to even more interesting results if the opinions were drawn from different distributions for the different topic dimensions, possibly distributions representative of actual polling data on a variety of political topics.

For instance, the effect of the candidate opinion transparency would likely be much greater if all topics were not distributed the same in the opinion space, such that if one topic was heavily polarized and bi-modal and was masked by a candidate, it could lead to a much larger change in happiness outcome.

Much of the weakness of this study is in the purely stochastic set-up, where all seems to average out in the end. But even though it was not heavily significant, the plurality voting system was the worst for overall happiness in every scenario run, and surprisingly approval voting, which is similarly simple came out ahead a couple times.

Furthermore, we would like to investigate the idea of utilizing a different metric for disagreeability. We belive that though our results are sound, the proposed metric function makes little use of its entire range as see in Figures V and 1. We would also like to further investigate the notion of opinion transparency and trace its effect on the system.

Additionally, there are many more branches to expand this research. Incorporating the notion of no-show voters, contrarians, and opinion-polarizing candidates.

[1] P. Clifford and A. Sudbury, Biometrika **60**, 581 (1973).
[2] R. W. Hoyer and L. S. Mayer, American Journal of Political Science , 501 (1974).

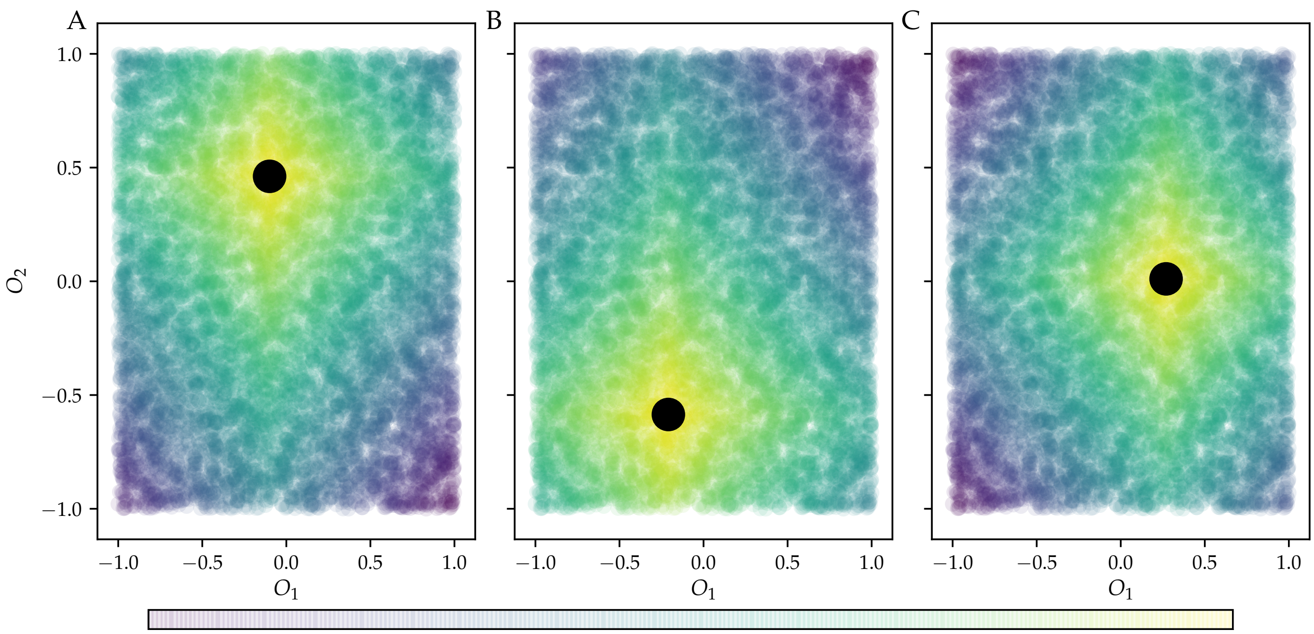

FIG. 4. Elected candidate in a 2-dimensional opinion space for a single sample population, comparing plurality (**A**), ranked-choice (**B**), and approval-based (**C**) voting outcomes with average happiness ratings $\bar{H}_{\mathrm{plural}} = 0.7$, $\bar{H}_{\mathrm{ranked}} = 0.72$, $\bar{H}_{\mathrm{approve}} = 0.74$, Each point is an opinion vector (o_1, o_2) and its color is governed by the happiness of the voter with the winning candidate.