Exam in
Neural Networks and Learning Systems
TBMI26 / 732A55
Home exam - Part II

Date:	2020-08-29
Time:	8.00 - 10.00 (part 1) and **9.30 - 12.00** (part 2)
Teacher:	8.00 - 10.00: Magnus Borga, Phone: 013-28 67 77
	10.00 - 12.00: Martin Hulman, Phone: 013-28 68 57

Read the instructions before answering the questions!

Part 2 Consists of four 5-point questions. These questions test deeper understanding and the ability to apply the knowledge to solve problems. All assumptions and calculations made should be presented. Reasonable simplifications may be done in the calculations. **This part needs to be submitted before 12:00**

Write your answers by hand and then scan them using a scanner or mobile phone, or write the answers in a separate file using a word processor. The answers may be given in English or Swedish. **If you write by hand, please write clearly using block letters! (Do not use cursive writing.) Answers that are difficult to read, will be dismissed.** The exam should be submitted before the deadline in PDF format. Each part should be handed in as one single PDF file. The PDF files should be named with your LiU-ID followed by a the number of the part of the exam, e.g. "abcde132-2".

The maximum sum of points is 20 on each part. To pass the exam (grade 3/C) at least 13 points are required on part 1. For grade 4/B, an additional 10 points on part 2 are required and for grade 5/A, 15 points are required on part 2, in addition to pass part 1.

Note that all forms of collaboration or communication with any person except the course staff is strictly forbidden during the exam!

The result will be reported at 2020-09-18 at the latest. The exams will then be available at "studerandeexpeditionen" at IMT.

GOOD LUCK!

AID:	Exam Date: 2020-08-29
Course Code: TBMI26 / 732A55	Exam Code: TEN1

1. The correlation of a 2D image $f(x, y)$ and a kernel $h(x, y)$ is defined as

$$g(x,y) = (f \Box h)(x,y) = \sum_{\alpha=-\infty}^{\infty} \sum_{\beta=-\infty}^{\infty} f(\alpha - x, \beta - y) h(\alpha, \beta).$$

a) 2p Perform the correlation below, i.e. calculate the image C. All values outside the image array A are equal to zero. In the arrays A and B, the respective number written in bold face is at position $(x, y) = (0, 0)$. Note that C is only a part of the correlation result.

$$\begin{array}{|c|c|c|c|} \hline \mathbf{0} & 1 & 0 & 1 \\ \hline 2 & 0 & 1 & 0 \\ \hline 0 & 2 & 0 & 0 \\ \hline \end{array} \;\Box\; \begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 1 & \mathbf{2} & 2 \\ \hline 0 & 2 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|c|} \hline \mathbf{?} & ? & ? & ? \\ \hline ? & ? & ? & ? \\ \hline ? & ? & ? & ? \\ \hline \end{array}$$

image A * kernel B = image C

b) 1p In practice, when implementing correlation, e.g. `correlate2d(A,B)`, no part of the kernel can be placed outside the image. Consequently, the resulting 2D array has size 1×2 and is equal to the central part of the image array C above. By extending the image array A to a new image array AA in a suitable way, `correlate2d(AA,B)` will be equal to C.
Give the image array AA.

c) 2p A CNN consists of N complex layers. Each complex layer consists of a correlation with a 3×3 kernel, a sigmoid activation, and a max pooling layer with stride 2 in each dimension. The input image is 256×256. Compute the (spatial) size of the layer after the first and second pooling. When is the image of size 1×1, i.e. what is the maximum value for N?

2. You have the following training samples:

$$
\begin{array}{lcccccccccc}
\mathbf{X} & = & \text{-1.8} & \text{-1.5} & \text{-1.2} & \text{-0.8} & \text{-0.5} & \text{-0.2} & 0.2 & 0.6 & 0.8 \\
\mathbf{Y} & = & \text{-1} & \text{-1} & \text{-1} & 1 & 1 & 1 & \text{-1} & \text{-1} & \text{-1}
\end{array}
$$

where $\mathbf{Y}$ is the class for the samples in $\mathbf{X}$.

Your task is to use a two-layer neural network to separate the data. The output layer should have two nodes, using one-hot-encoding of $\mathbf{Y}$ as desired output from the network, i.e.

$$
\begin{aligned}
\mathbf{Y}_i = 1 \quad &\implies \mathbf{D}_i = \begin{pmatrix} 1 & 0 \end{pmatrix} \\
\mathbf{Y}_i = -1 \quad &\implies \mathbf{D}_i = \begin{pmatrix} 0 & 1 \end{pmatrix}
\end{aligned}
$$

for sample i. Note that the orientation of $\mathbf{D}$ here assumes samples are in rows, i.e. $\mathbf{X}$ is a column-vector.

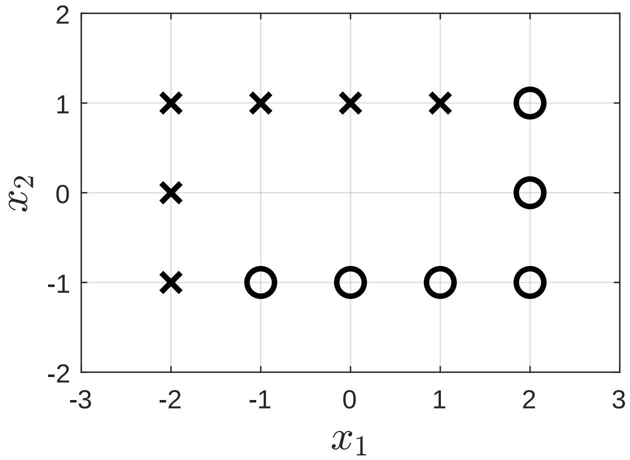
Use *step* as activation function, both in the hidden layer and the output layer, where *step* is the function given by:

$$
step(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}
$$

Sketch the network and assign $\mathbf{W}$ (weights in the hidden layer) and $\mathbf{V}$ (weights in the output layer) so that the predicted output exactly matches the desired output for all samples in $\mathbf{X}$ (5p).

AID:	Exam Date: 2020-08-29
Course Code: TBMI26 / 732A55	Exam Code: TEN1

3. The data points in the figure have two features (x_1 and x_2) and belong to either the class "crosses" or the class "circles":



Perform Linear Discriminant Analysis (LDA) on the data to reduce the dimensionality to one dimension that separates the two classes optimally. Draw the reduced data. (5p)

Hint: The inverse of a 2×2-matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $\frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

4. Figure 1 shows a reinforcement learning problem with eight states in which the valid actions are *right* and *up*. State S_8 is terminal and moving into it results in a reward of 15. Moving into state S_5 results in a reward of -5. All remaining states result in a reward of 0.

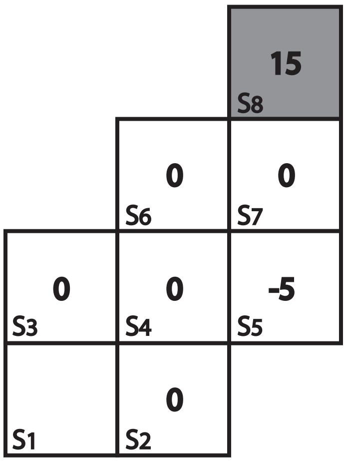


Figure 1: All possible states and rewards

Two possible sequences of action are shown in Figure 2.

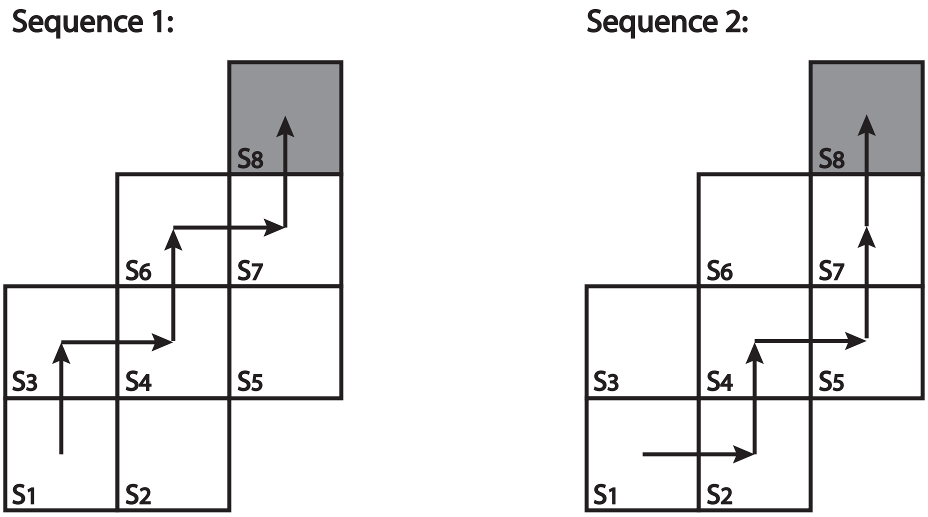


Figure 2: Sequences of action

Show how the Q-values are modified by the Q-learning algorithm if sequence 1 is used once, followed by sequence 2, and then a final use of sequence 1.

Give the results as a function of γ (discount factor) and α (learning rate). All Q-values are initialized at 0 (5p).