Mathematical Exercises 1: problem 4

$$p(\theta) \propto \theta^{\alpha-1} e^{-\beta\theta}$$

$$p(x|\theta) = \prod_{i=1}^{n} p(x_i|\theta) = \prod_{i=1}^{n} \theta e^{-\theta x_i} = \theta^n \cdot e^{-\sum_{i=1}^{n} x_i} \cdot \theta$$

$$p(\theta|x) \propto p(x|\theta) \cdot p(\theta) \propto \theta^n \cdot e^{-\sum_{i=1}^{n} x_i} \cdot \theta \cdot \theta^{\alpha-1} e^{-\beta\theta} = \theta^{\alpha+n-1} e^{-(\beta+\sum_{i=1}^{n} x_i)\theta}$$

- Hence, $\theta|x \sim \text{Gamma}\left(\alpha+n, \beta+\sum_{i=1}^{n} x_i\right)$, i.e.
- the gamma prior for θ is the conjugate prior.