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Bayesian Learning

Mathematical Exercises 4

Try to solve the problems before class and don't worry if you fail. The important thing is to try. You should not hand in any solutions.
This part of the course is not obligatory and is not graded.

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1. Laplace approximation

   (a) Let $x_1, \ldots, x_n | \theta \overset{iid}{\sim} Bernoulli\,(\theta)$. Assume the prior $\theta \sim Beta\,(\alpha, \beta)$. Derive the marginal likelihood for this model.

   (b) Compute the marginal likelihood of the model in (a) using the Laplace approximation.

   (c) Is this approximation accurate if $\alpha = \beta = 1$ and you have observed $s = 7$ success in $n = 11$ trials?

2. Missing marginal likelihoods

   (a) Derive the marginal likelihood for the Geometric model with Beta prior on Slide 7 at Lecture 10.

   (b) Derive the marginal likelihood for the Poisson model with Gamma prior on Slide 7 at Lecture 10.

3. The Pareto family

   (a) Let $x_1, \ldots, x_n | \theta \overset{iid}{\sim} Uniform\,(0, \theta)$. Assume the prior $\theta \sim Pareto\,(\alpha, \beta)$, which is given by

   $$p\,(\theta) = \frac{\alpha \beta^{\alpha}}{\theta^{\alpha+1}},\ \theta \geq \beta.$$

   Show that this is a conjugate prior to this Uniform model and derive the posterior for θ. [Hint: Don't forget to include an indicator function when you write up the likelihood function. The $Uniform\,(0, \theta)$ distribution is zero for outcomes larger than θ.]

   (b) Derive the predictive distribution of x_{n+1}, given $x_1, \ldots, x_n$. [Hint: It is wise to break up the integrals in two parts.]

Good luck!

Best, Bertil