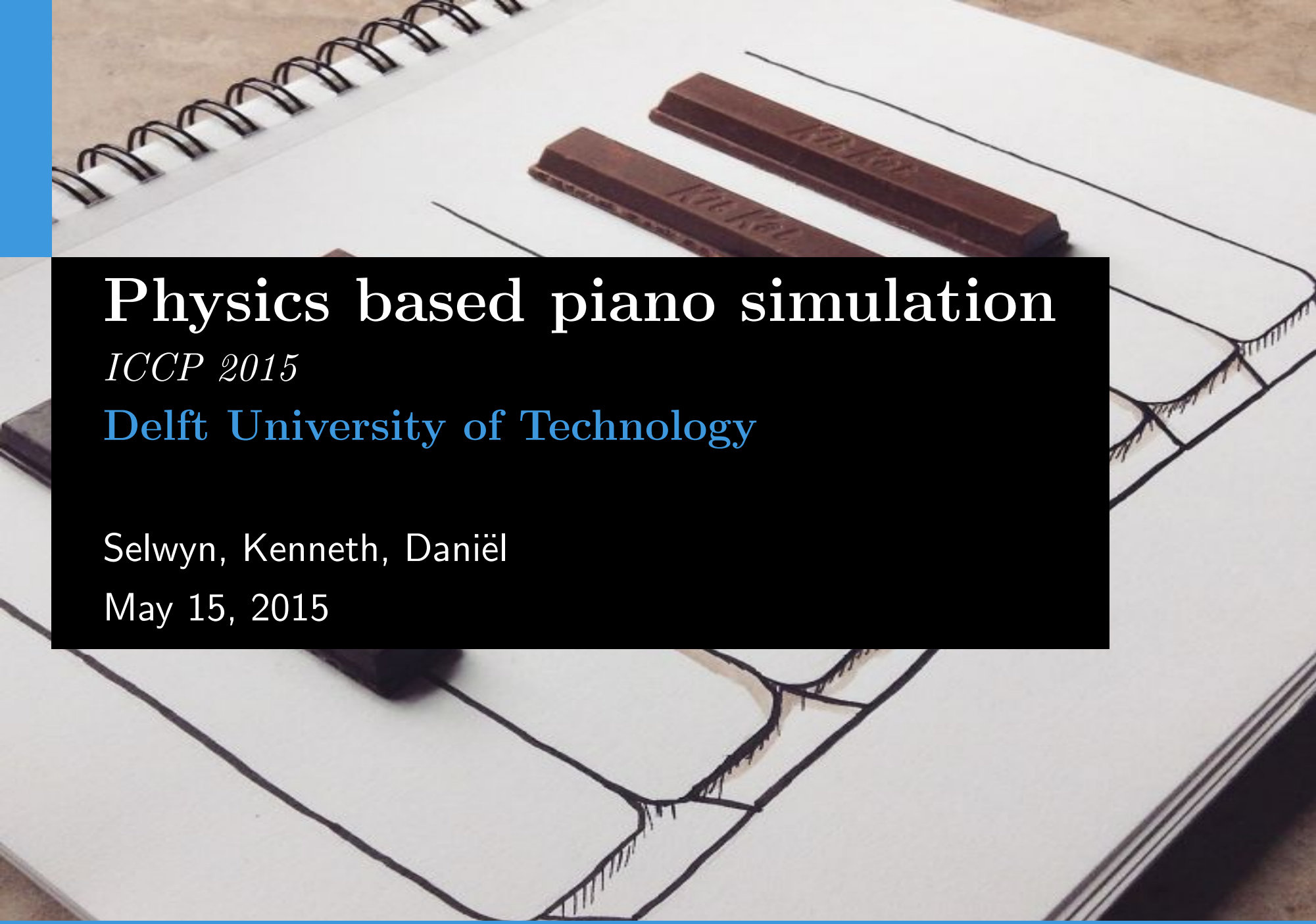


Physics based piano simulation

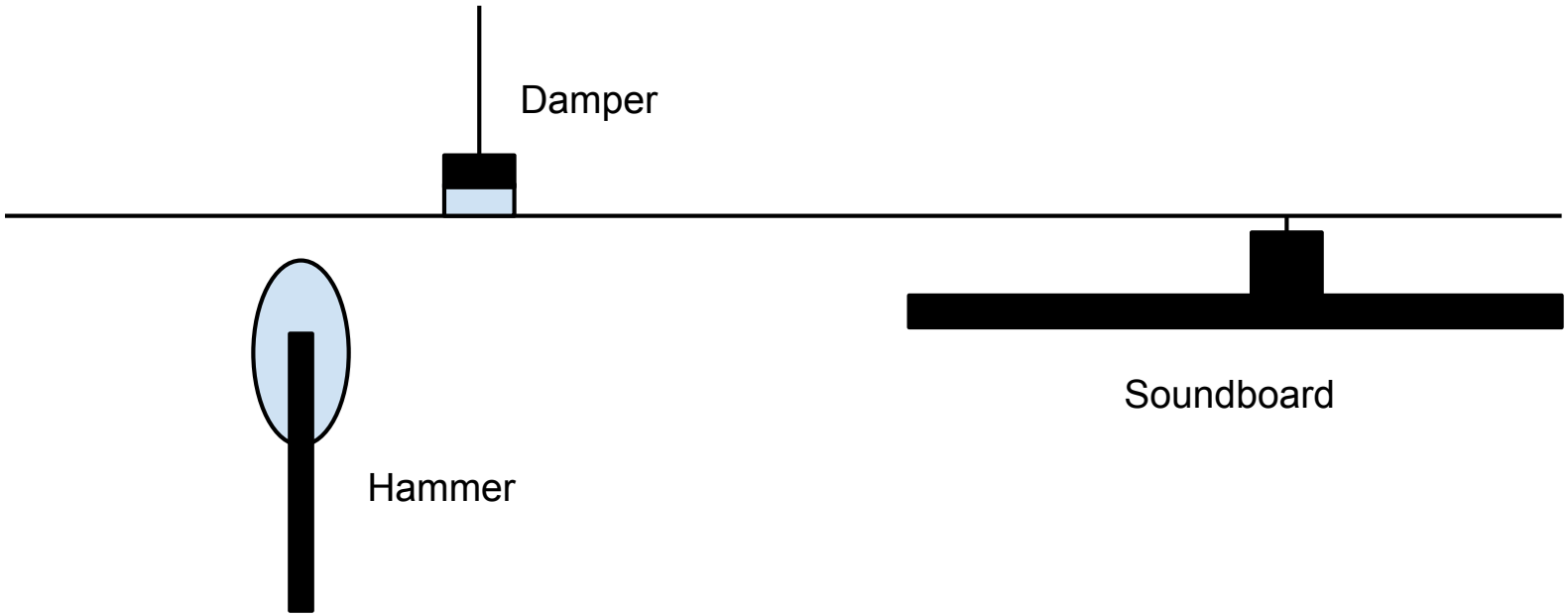
ICCP 2015

Delft University of Technology

Selwyn, Kenneth, Daniël

May 15, 2015

Simplified piano string interaction



The wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} - \kappa^2 \frac{\partial^4 y}{\partial x^4} - 2b_1 \frac{\partial y}{\partial t} + 2b_2 \frac{\partial^3 y}{\partial x^2 \partial t}$$

Finite difference wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} - \kappa^2 \frac{\partial^4 y}{\partial x^4} - 2b_1 \frac{\partial y}{\partial t} + 2b_2 \frac{\partial^3 y}{\partial x^2 \partial t}$$

$$y_n^{t+1} = a_1 \left(y_{n+2}^t + y_{n-2}^t\right) + a_2 \left(y_{n+1}^t + y_{n-1}^t\right) + a_3 y_n^t$$
$$+ a_4 y_n^{t-1} + a_5 \left(y_{n+1}^{t-1} + y_{n-1}^{t-1}\right)$$

$a_i depends on string stiffness, Young's modulus, tension, length, cross-sectio$

Hammer strike

Hammer-string interaction:

$$L = T - V = \sum_i \frac{1}{2} M_H \dot{x_i}^2 + \sum \frac{1}{2} \rho \Delta x \dot{\eta}^2 - (\eta_n - x_n)^{b+1} \frac{K}{b-1}$$

$$+ V_{string}\left(\eta_1, \eta_2, \cdots, \eta_n\right)$$

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{x}_n} = M_H \ddot{x} = \frac{\partial L}{\partial x} = \sum_n - (\eta_n - x_n)^b K$$

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{\eta}_n} = \rho \delta x \ddot{\eta}_n = \frac{\partial L}{\partial \dot{\eta}_n} = k(\eta_n - x_n)^b - \frac{\partial V_{string}\left(\eta_1, \eta_2, \cdots, \eta_n\right)}{\partial \eta_n}$$

Hammer strike

$$\ddot{x}_n = -\frac{k}{M_H}\sum_n (\eta_n - x_m)^b H(\eta_n - x_m)\ddot{\eta}_n = \frac{k}{\rho\Delta x}(\eta_n - x_n)^b + \frac{1}{\rho\Delta x}F_n$$

$$x_n(t+\Delta t) = x_n(t) + v_n(t)\Delta t + \frac{k}{2M_H}(\eta_n(t) - x_n(t))^b H(\eta_n, x_n)$$

$$\dot{x}_n(t+\delta t) = \dot{x}_n(t) + \delta t\frac{(\eta_n - x_n)^b + (\eta_n - x_n(t))^b}{2}$$

$$\eta_n(t+\Delta t) = \eta_n(t) + v_n(t)\Delta t + \frac{k(\eta_n - x_n(t))^b}{2\rho\Delta x}(\Delta t)^2$$

Hammer strike

Hammer release from the string, important for 'plucking' or 'striking' the string.

Damper simulation

Cutoff sounds unnatural

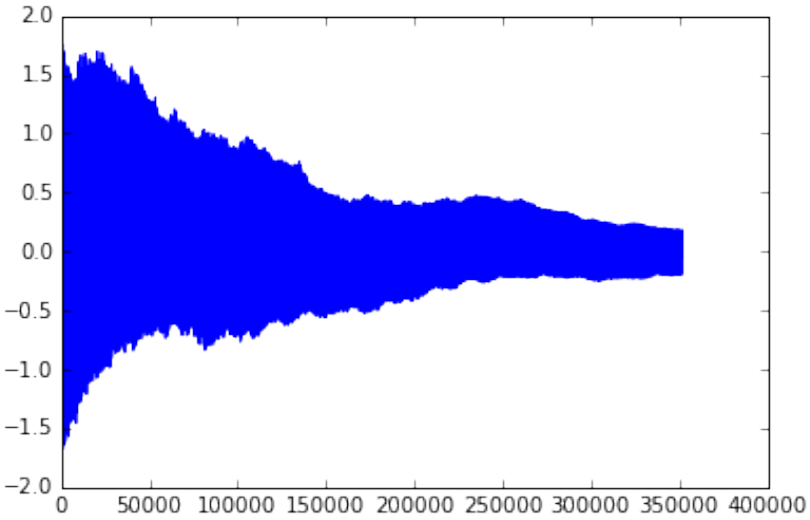
Damper simulation

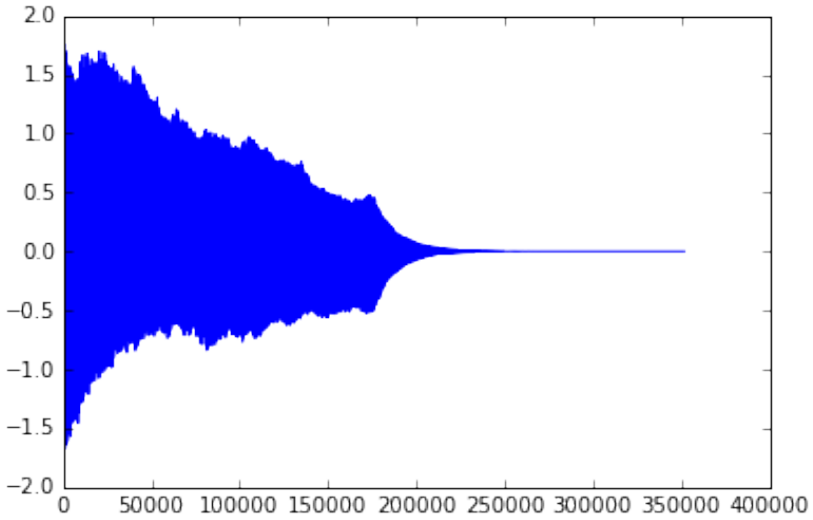
Cutoff sounds unnatural $\rightarrow$ add damper suppression

Damper simulation

Cutoff sounds unnatural$\rightarrow$ add damper suppression
Suddenly increase stiffness

Damper simulation





Examples

Time for some 'music'!

Considerations

- Add more notes

Considerations

- Add more notes
- Real-time playback

Considerations

- Add more notes
- Real-time playback
- Simulate three strings of same pitch with slightly different parameters

Last Page

Thank you!