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Logical Neural Networks: Opening the black box

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Abstract

Neural networks have been shown to be universal function approximators and have been employed to solve a number of non-trivial problems. The downside to these networks is that they are very difficult (if not impossible) to understand. We aim to build on previous research showing that Logical Neural Networks applied to the MINST dataset showing comparable performance to standard NN's and a more understandable network by formally proving a foundation for LLN's, investigating where LLN's are best applied and whether we can approximate our LLN as a discrete logic circuit.

1. Introduction

Neural Networks (NN's) perform exceptionally well over a wide range of different problems. However once trained these networks become a black box, near impossible for a human to understand what features the network is using to solve the problem presented to it. Logical Neural Networks (LLN's), that is NN's with logic based activation functions, have previously [REFERENCE PREV HONORS PROJECT] been shown to provide a more understandable representation while still performing well (in the context of the MINST dataset). Despite the promise of LLN's not alot investigation into them has occurred.

2. The Problem

The problem with standard NN's is that once trained they are very difficult (if not impossible) for a human to understand. Previously LLN's have been shown to have promise as an alternative to standard NN's with the added benefit of having sparse weights and being easier to understand. We suggest this problem still hasn't been completely solved as we have yet to develop a rigorous and formal foundation for LLN's. First we ask, can it be proven that like standard NN's, LLN's are also universal function approximators? Before we can hope to answer this key question we also must ask whether we have the right parameterisation of noisy gates as these form the foundation of an LLN.

Regardless of our answer to the first question a next logical step for our investigation is to ask, what types of problems is an LLN best suited to? Which ones can it not only perform well on but also provide a useful learned representation. This will help build the foundation of LLN's, giving us a better understanding of how to configure LLN's for different problems, i.e. network layout and which logical neurons to use.

Finally we pose a concept which LLN's seem uniquely qualified to solve given they are based on logic gates. Being able to create hardware NN's would be incredibly powerful (as hardware is alot faster). There doesn't seem to be any intuitive way to achieve this with standard NN's but approximating the continuous logic gates in our LLN as discrete ones would make it possible. But is it possible to create an approximation in a feasible way without sacrificing too much accuracy?

3. Proposed Solution

Research done previously into LLN's has shown that using a network with layers consisting of Noisy-OR and Noisy-AND neurons can achieve a reasonable accuracy on the MINST dataset while also providing a more understandable network. To begin this project we take a step back to ensure we have the right approach. A network built out of neurons that resemble logic gates we would expect should be able to learn any given boolean formula. This would show we have good parameterisations for our noisy gates and be a fundamental step towards proving LLN's are universal function approximators. First we must address a fundamental issue with our existing setup, namely our choice of "gates" is not a functionally complete set. Consider the issue of representing some known boolean expression using discrete logic, but we only have access to AND and OR gates, AND combined with OR is not functionally complete. We propose to implement a number of different sets of functionally complete neurons, such as NAND and NOR. These logical neurons would be parameterised so we can observe the same nice properties we see with discrete gates, i.e. $\text{NOT}(\text{AND}) = \text{NAND}$. If an LLN consisted of multiple layers of NAND's or NOR's we would expect to

find groups of gates learning to act like some of our other gates.

The first 4 weeks of this project would be taken to fully understand the concept of Noisy gates, using the bibliography of the previous research along with other resources. Along with developing our parameterisation of logical gates. The gates we aim to parameterise and implement are AND, OR, NOT, NAND, NOR.

Our choice of gates have parameterisations which maintain convenient properties and are functionally complete, so intuitively we would expect our LLN to be able to learn any known boolean function, the results here will be interesting as we might find that theoretically our LLN can approximate any boolean function but in practice some can not be learnt so easily. As part of experiments with learning boolean functions it would be interesting to experiment with the training data given to the network, how can we train a network which generalises well. Say we trained a network with one of the boolean inputs fixed at 0, then would the network be able to generalise to the case where that input is 1? We expect to spend 6 weeks investigating at the conclusion of which we aim to have a proof demonstrating that LLN's can approximate any known boolean function along with strong evidence showing which collections of neurons work well together, give the most understandable trained network and in what configuration yields the best results.

Next we address a fundamental question, are LLN's universal function approximators? Working towards a proof or strong evidence showing this in either direction we can start by experimenting with an LLN learning some general functions and comparing its performance to a standard NN. These experiments are not equivalent to a proof but will reveal which direction we should pursue. At the conclusion of this part we expect to have strong justification for answering this question, we allocate 6 weeks for this section.

Now that we have explored the foundation of LLN's, using this knowledge we want to use our LLN to solve standard benchmark problems and compare against performance of standard NN's. We would use K-Fold Cross Validation, which is commonly used for comparisons like this. It would then be useful to examine the trained LLN and see if we can understand how the network has learned to represent the problem as this really is the motivation behind LLN's. We allocate 4 weeks for this.

We have essentially constructed NN's out of neurons which resemble continuous logic gates, something to explore is can we feasibly approximate these continuous gates with discrete ones as this would essentially allow us to create hardware NN's. Performing such a task would certainly require more gates than neurons but would this blowup result in something that could feasibly be printed on a circuit board. We allocate any remaining time to investigating this problem and developing/implementing methods to solve it.

An overview of the proposed solution follows

- Produce bibliography and parameterisation of gates (est: 4 weeks)
- Proving LLN's can/can not approximate any known boolean function (est: 6 weeks)
- Proving LLN's are/are not universal function approximators (est: 6 weeks)
- Evaluate LLN's on benchmark problems (est: 4 weeks)
- Explore approximating LLN as discrete logic circuit (est: remaining time)

4. Evaluating your Solution

A solution to this problem should build the foundation to LLN's. Giving a proof that LLN's can approximate any known boolean function and strong evidence/proof showing that LLN's are universal function approximators. This would require an indepth investigation into various paramaterisations of logic gates. Folowing this it should identify where LLN's are most powerful and where a standard NN should be used insted. Finally a study investigating approximating an LLN as a descrete logic circuit.

5. Resource Requirements

I do not forsee any requirements I dont allready have access to.

