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1 Question 1 - Lena

1.1 Approximations

Approximations with rank 10, 20, 50, 100, 200, 400

(a) r = 10

(b) r = 20

(c) r = 50

(d) r = 100

(e) r = 200

(f) r = 400

1.2 Error Discussion

From inspection of the program output we can see that the error computed using the **frobenius** norm is equal to the square root of the sum of eigen values that are zeroed out to achive the approximation. We see this is true from the folowing. We have $A = U\Sigma V^T$ by SVD, and then we get the rank k approximation of k as $\widetilde{A} = U\widetilde{\Sigma}V^T$. Now considerthe folowing

$$\begin{aligned} A - \widetilde{A} &= U\Sigma V^T - U\widetilde{\Sigma}V^T \\ &= U(\Sigma - \widetilde{\Sigma})V^T \\ &= U\Sigma' V^T \end{aligned} \tag{1}$$

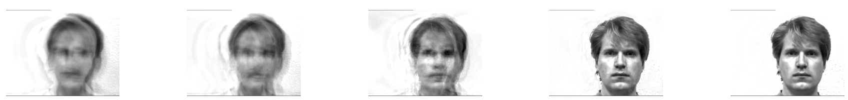
Where the diagonal of Σ' contains all singular values not in $\widetilde{\Sigma}$.

So finally $\left\|A-\widetilde{A}\right\|_{Fro} = \sqrt{\sum_{i=k+1}(\sigma'_i)^2}$
As we saw experementally

2 Question 2 - Yalefaces

2.1 Approximations

Approximations with rank 10, 20, 40, 80, 120



(g) r = 10 (h) r = 20 (i) r = 40 (j) r = 80 (k) r = 120

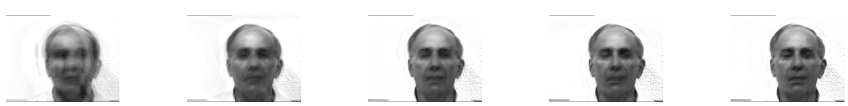
Figure 1: Image 5

(a) r = 10 (b) r = 20 (c) r = 40 (d) r = 80 (e) r = 120

Figure 2: Image 10

(a) r = 10 (b) r = 20 (c) r = 40 (d) r = 80 (e) r = 120

Figure 3: Image 14



(a) r = 10 (b) r = 20 (c) r = 40 (d) r = 80 (e) r = 120

Figure 4: Image 50

(a) r = 10 (b) r = 20 (c) r = 40 (d) r = 80 (e) r = 120

Figure 5: Image 67

(a) r = 10 (b) r = 20 (c) r = 40 (d) r = 80 (e) r = 120

Figure 6: Image 100

2.2 Error Discussion

See section 1.2

3 Question 3 - SVD - PCA Relationship

Take X as our data matroix (we assume X has been mean centered). Then PCA is asking us to compute the eigenvalues and eigenvectors of the covariance matrix of X which is XX^T. As XX^T is symmetric by the spectral theorem we have $X = KAK^T$. Where A is diagonal with entries corosponding to the eigenvalues of XX^T and K is orthonormal with columns corosponding to the eigenvectors of XX^T.

Now we consider the SVD of X, $X = U\Sigma V^T$, take $XX^T = (U\Sigma V^T)(U\Sigma V^T)^T = U\Sigma^2 U^T$. So the singular values of X corospond to the square roots of the eigen values of XX^T