IN3190 - Exam Preparation Questions H23

University of Oslo

Daniel Tran

November 2023

Contents

1 Essential Topics

1.1 Discrete time

1.1.1 Sine, cosine and exponential functions

Mathematical formula that establishes the fundamental relationship between the trigonometric functions and the complex exponential function.
Euler's formula states that for any real number x:

$$e^{ix} = \cos(x) + i\sin(x) \tag{1}$$

When $x = \pi$, Euler's formula yields Euler's identity:

$$e^{i\pi} + 1 = 0 \tag{2}$$

1.1.2 Elementary discrete signals

Unit impulse Also known as the dirac delta function.

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \qquad \text{may also be written as:} \qquad \delta[n] = u[n] - u[n-1] \tag{3}$$

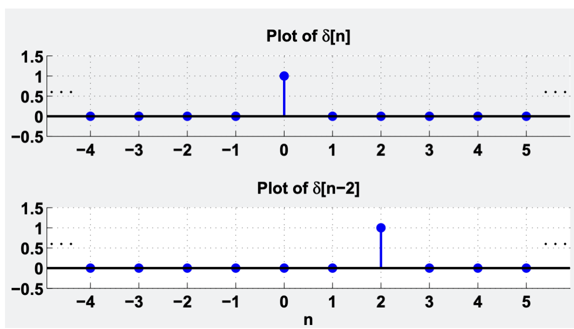

Figure 1: Unit impulse

Step function Also known as unit step, unit step function or heaviside function. The value which is zero for negative arguments and one for positive arguments.

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases} \qquad \text{may also be written as:} \qquad u[n] = \sum_{k=0}^{\infty} \delta[n-k] \tag{4}$$

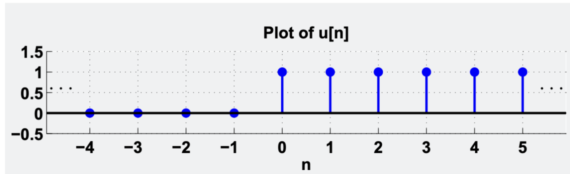

Figure 2: Unit step function

Ramp function Also known as the unit-ramp or unit ramp function. Graph shaped like a ramp.

$$u_r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases} \tag{5}$$

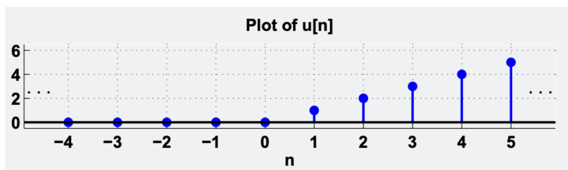

Figure 3: Unit ramp function

Periodic sequences $x[n]$ is periodic if and only if $x[n] = x[n + N]$

- **Fundamental period**:
  Smallest positive integer N which fulfills the relation above
- **Sinusoidal sequences**:
  $x[n] = A\cos(\omega_0 n + \phi)$, where
  - A is amplitude,
  - ω_0 is the angular frequency
  - and ϕ is the phase of $x[n]$.

  $x[n]$ is periodic if and only if $\omega_0 N = 2\pi k$, for N and k as positive integers.

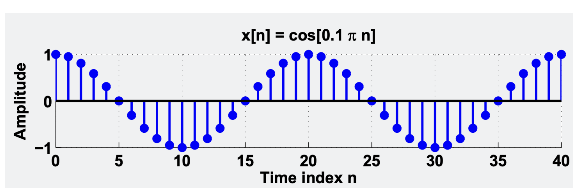

Figure 4: Sinusoidal sequences

1.1.3 LTI systems, including characteristics via the transformation between input and output

Linearity: The relationship between the input $x(t)$ and the output $y(t)$, both being regarded as functions. If a is a constant then the system output to $ax(t)$ is $ay(t)$. Linear system if and only if it $H\{\cdot\}$ is both additive and homogeneous, in other words: If it fulfills the superposition principle. That is:

$$H\{ax_1(t) + bx_x(t)\} = aH\{x_1(t)\} + bH\{x_2(t)\} \tag{6}$$

Time-invariance: A linear system is time-invariant or shift-invariant means that wether we apply an input to the system now or T seconds from now, the output will be identical except for a time delay of S seconds. In other words, if $y(t)$ is the output of a system with a input $x(t)$, then the output of the system with input $x(t-T)$ is $y(t-T)$. The system is invariant becasue the output does not depend on the particular time at which the input is applied.

The fundamental result in LTI system theory is that any LTI system can be characterized entirely by a single function called the system's impulse response $h(t)$. The output of the system $y(t)$ is simply the convolution of the input to the system $x(t)$ with the systems's impulse response $h(t)$.

LTI system can also be characterized in the frequency domain by the system's transfer function $H(s)$, which is the Laplace transform of the system's impulse response $h(t)$. Or Z-transform in the case of discrete time systems.

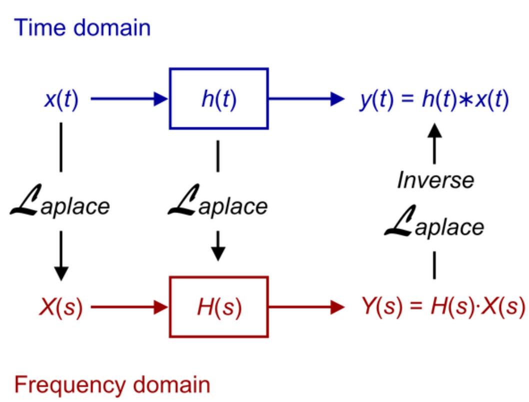

Figure 5: Relationship between the time domain and the frequency domain

1.2 Z-transform

1.3 Frequency analysis and DFT

1.4 Filter design

1.4.1 Advantages/Disadvantages of FIR and IIR filters

Advantages, FIR filters:

- Can have exact linear phase, which makes h(n) zero points symmetric about $|z| = 1$
- Filter structures are always stable for quantized coefficients.
  - Stable IIR filters can become unstable due to rounding errors in coefficients.
- Design methods are generally linear.
- Start transient has finite length.
- Can be realized efficently in HW (but not as efficient as IIR filters)

Disadvantages, FIR filters: Generally require a higher order than IIR filters to meet the same specifications. This often ensure the FIR filter having greater calculation burden/complexity and larger group delay than the corresponding IIR filter.

1.4.2 Common ways to design IIR filters

- Specify the filter as a analog/continuous time (CT) equivalent to the dicrete IIR filter you want to design.
- Design the analog lowpass filter in continuous time.
- Convert the analog lowpass filter to a digital filter using filter transformation.
- Convert the digital lowpass filter to the desired filter using a frequency band transformation.
- Convert the digital filter to a discrete time (DT) filter (From s-domain to z-domain).
- You could also just use an optimization algorithm (e.g. least-square method)

Note on standard approximations for IIR filter design:

- Conversion of the digital filter specification to an anolog low-pass filter specification
- Determination of analog low-pass transfer function $H_a(s)$
- Transformation of $H_a(s)$ to the desired digital transfer function $G(z)$ This method is preferred because:
  - Approximation of an analog transfer function is well established and suitable.
  - Closed expressions for the analog approximation are often obtained.
  - Large tables for analog filter design are available.
  - In many cases, digital simulation of an analogue system is sought, i.e. the analogue specification exist.

1.4.3 Typical IIR filters

1.4.4 Common ways to design FIR filters

1.4.5 Window functions and the consequences of applying them

1.4.6 Linear phase and linear-phase filters

1.4.7 Grapihcal representation of filter structures and operations

1.4.8 Gibb's phenomenon

1.5 Sampling and reconstruction

1.5.1 Sampling theorem

The sampling frequency $\Omega_T = 2\Omega_H$ is called the Nyquist frequency. If this term is satisfied, then the original signal can be recovered from the sampled signal. In practice we often use oversampling (sampling at a higher rate than the Nyquist frequency) to get an appropiate reconstruction.

1.5.2 Synthesizing an arbitrary signal as a sum of unit impulses

Sampling corresponds to multiplying with a Dirac comb with distance T between the unit impulses. In the Fourier domain, this corresponds to convolving with a Dirac comb with distance F_T between scaled unit impulses. (The distance is Ω_T if we look at the angular frequency axis)

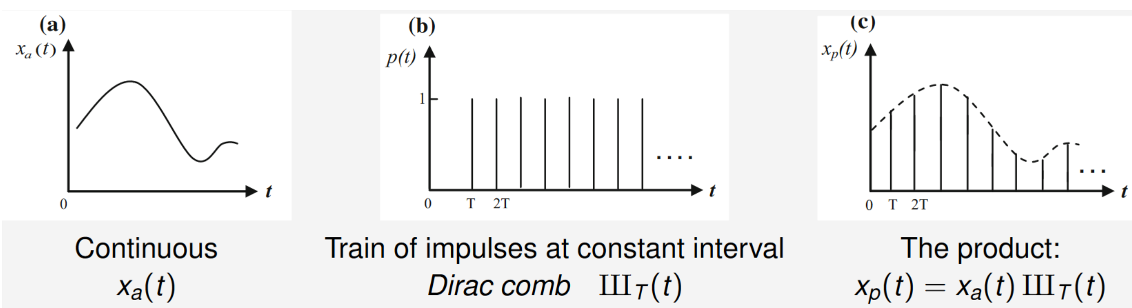

Figure 6: Sampling with dirac impulses

$$x_p(t) = x_a(t)III_T(t) = \sum_{n=-\infty}^{\infty} x_a(t)\delta(t - nT) = \sum_{n=-\infty}^{\infty} x_a(nT)\delta(t - nT) \triangleq x(n) \tag{7}$$

1.5.3 Sampling of band-limited signals

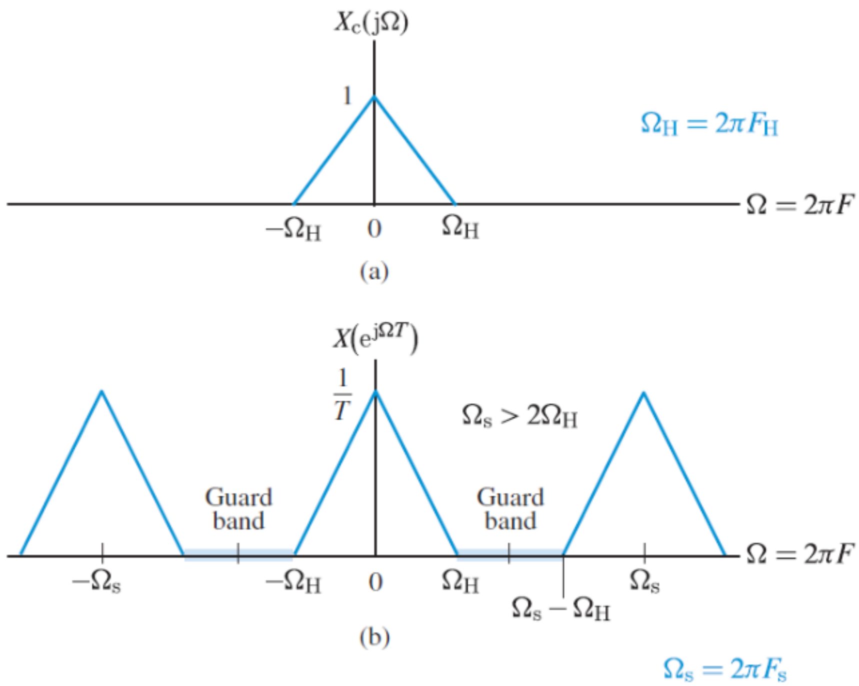

Figure 7: Periodic sampling in freq. domain

The reconstructed signal will repeat itself in the frequency domain. As long the sampling frequency (Ω_T)is equal or greater than the Nyquist rate ($2\Omega_H$), we will be able to seperate the original signal from the aliases. The reconstruction in fig. 7 can be written as a convolution:

$$x_r(t) = \sum_{n=-\infty}^{\infty} x_a(nT) g_r(t - nT) \tag{8}$$

Where $g_r(t)$ is the interpolation reconstruction function (I.e. impulse response). The

1.5.4 Ideal reconstruction

The fundamental copy of the Fourier-domain representation of the sampled signal can be found by the 'ideal' filter for the convolution operation is:

$$G_r(j\Omega) = \begin{cases} T, & |\Omega| \leq \Omega_T/2 \\ 0, & |\Omega| > \Omega_T/2 \end{cases} \tag{9}$$

This gives the ideal: $X_r(j\Omega) = X_a(j\Omega)$
(Reconstructed Fourier domain representation equals the original Fourier domain representation).

1.5.5 Upsampling and downsampling

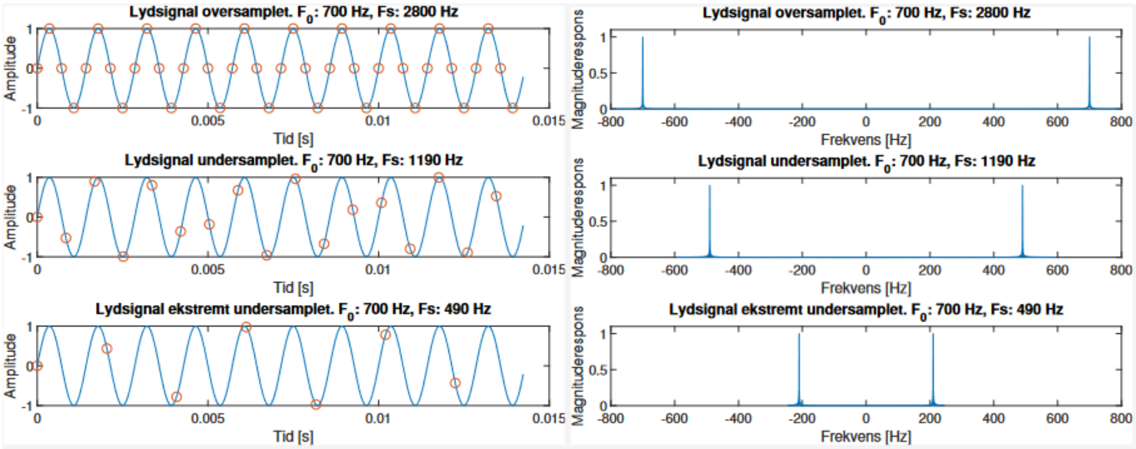

Figure 8: Example of over/undersampling

2 Additional Topics

2.1 Discrete time

2.1.1 Symmetrical signals

2.2 LTI systems and characteristics

2.3 Convolution and correlation

2.4 Z-transform

2.5 Frequency analysis and DFT

2.6 Filter design

2.7 Sampling and reconstruction