

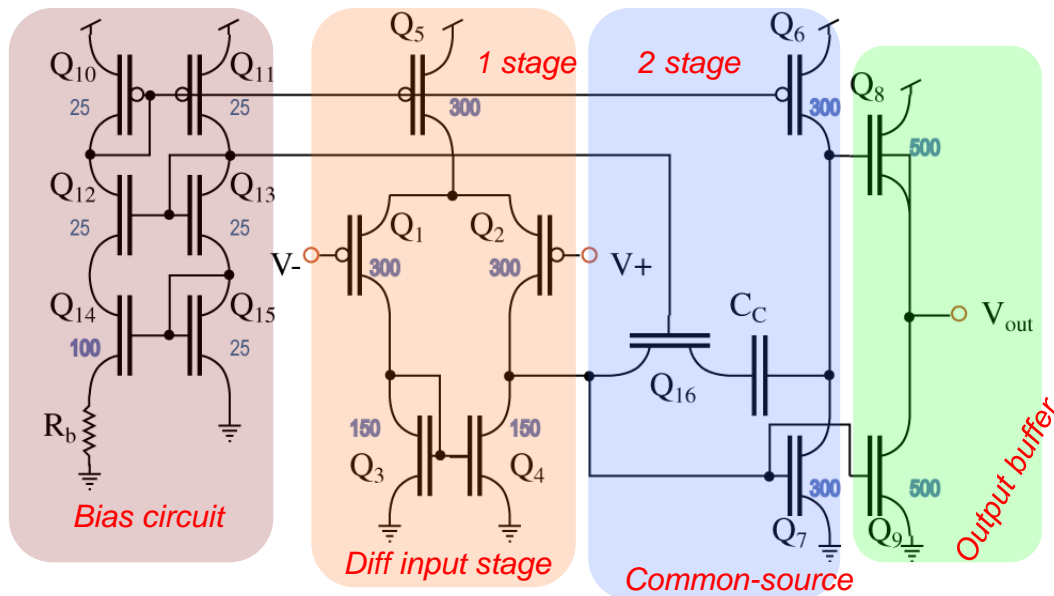
## **IN4180 - Analog Microelectronics Design**

# **Basic Operational Amplifier Design and Compensation - Part 2** **Compensation and stability**

Kristian G. Kjelgård

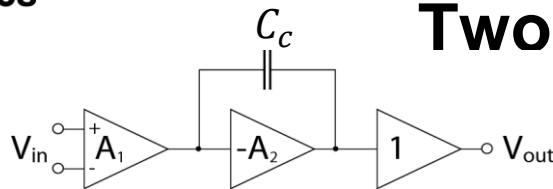


# CMOS OPAMP topology



- PMOS diff input stage
- Numbers realistic transistor widths
  - Length 1-2 times minimum
- Output buffer may not be needed for capacitive loads

# Two stage opamp gain



- Gain for diff pair – 1. stage

$$A_{v1} = g_{m1}(r_{ds2} || r_{ds4})$$

- Typical gain 50-100

- Gain of common source – 2. stage

$$A_{v2} = -g_{m7}(r_{ds6} || r_{ds7})$$

- Typical gain 50-100

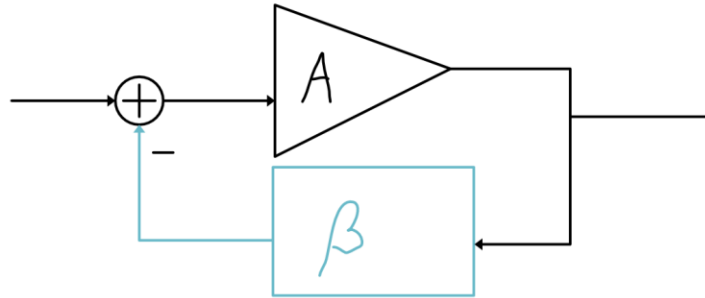
- Gain of source follower – output buffer

$$A_{v3} = \frac{g_{m8}}{G_L + g_{m8} + g_{s8} + g_{ds8} + g_{ds9}}$$

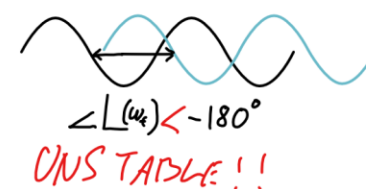
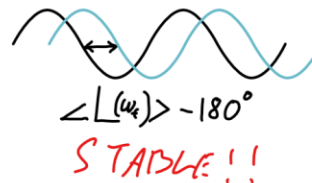
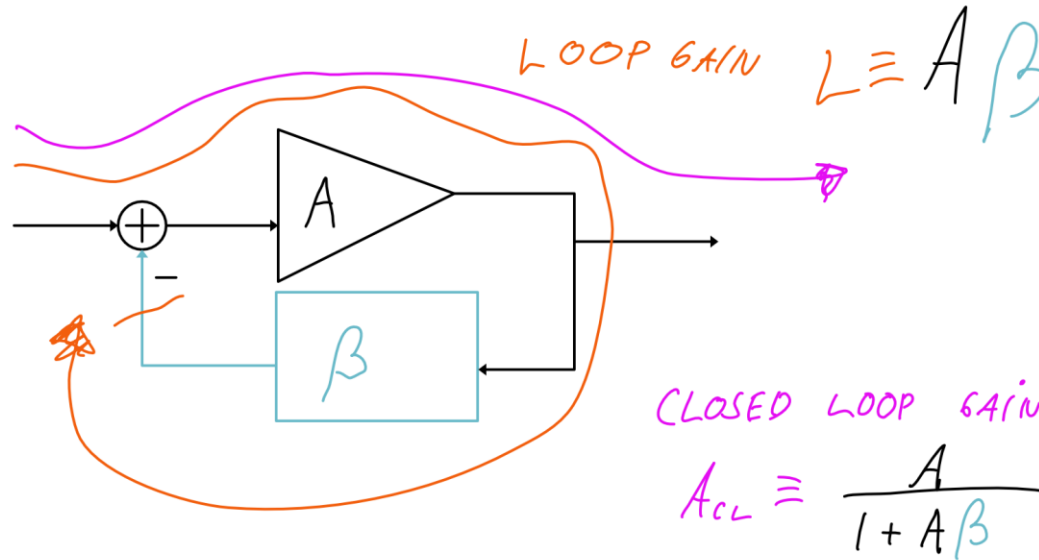
- Gain  $\approx 1$
- Not needed for capacitive loads

$$\begin{aligned} g_{m1} &= \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \sqrt{2\mu_n C_{ox} \frac{W}{L} \frac{I_{bias}}{2}} \\ &\lambda \\ &= \frac{k_{ds}}{2L\sqrt{V_{DS} - V_{eff}} + \Phi_0} \\ k_{ds} &= \sqrt{\frac{2K_s \epsilon_0}{qN_A}} \\ r_{ds} &\cong \frac{1}{\lambda I_D \gamma g_m} \\ g_{s8} &= \frac{1}{2\sqrt{V_{SB} + |2\phi_F|}} \end{aligned}$$

# Feedback stability



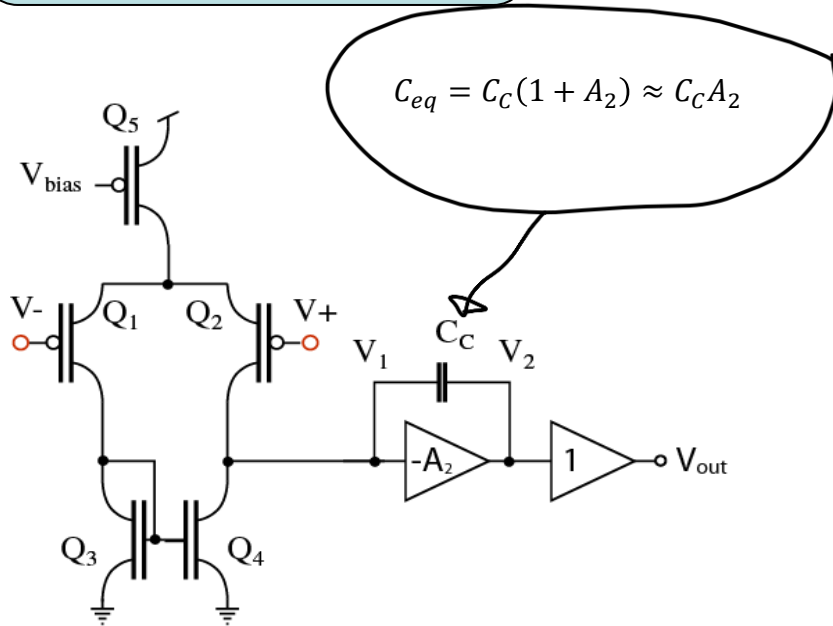
# Feedback stability



# Frequency response – First order model

## Midband frequencies

- $C_{eq}$  dominates



$$A_1 = g_{m1} Z_{out1}$$

$$= g_{m1} \left( r_{ds2} || r_{ds4} || \frac{1}{s C_{eq}} \right)$$

at midband freq  $C_{eq}$  dominates

$$A_1 = g_{m1} \frac{1}{s C_{eq}} = g_{m1} \frac{1}{s C_C A_2}$$

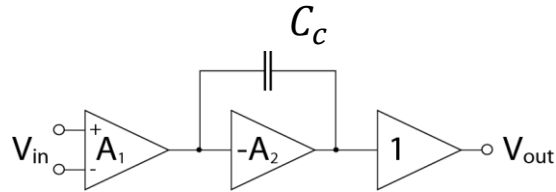
$$A_v = \frac{v_{out}}{v_{in}} = A_1 A_2 A_3 \approx g_{m1} \frac{1}{s C_C A_2} \cdot A_2 \cdot 1 = \frac{g_{m1}}{s C_C}$$

Unit-gain frequency proportional to  $g_m$  assuming  $A_3=1$

setting  $|A_v(j\omega_{ta})| = 1$  and solve

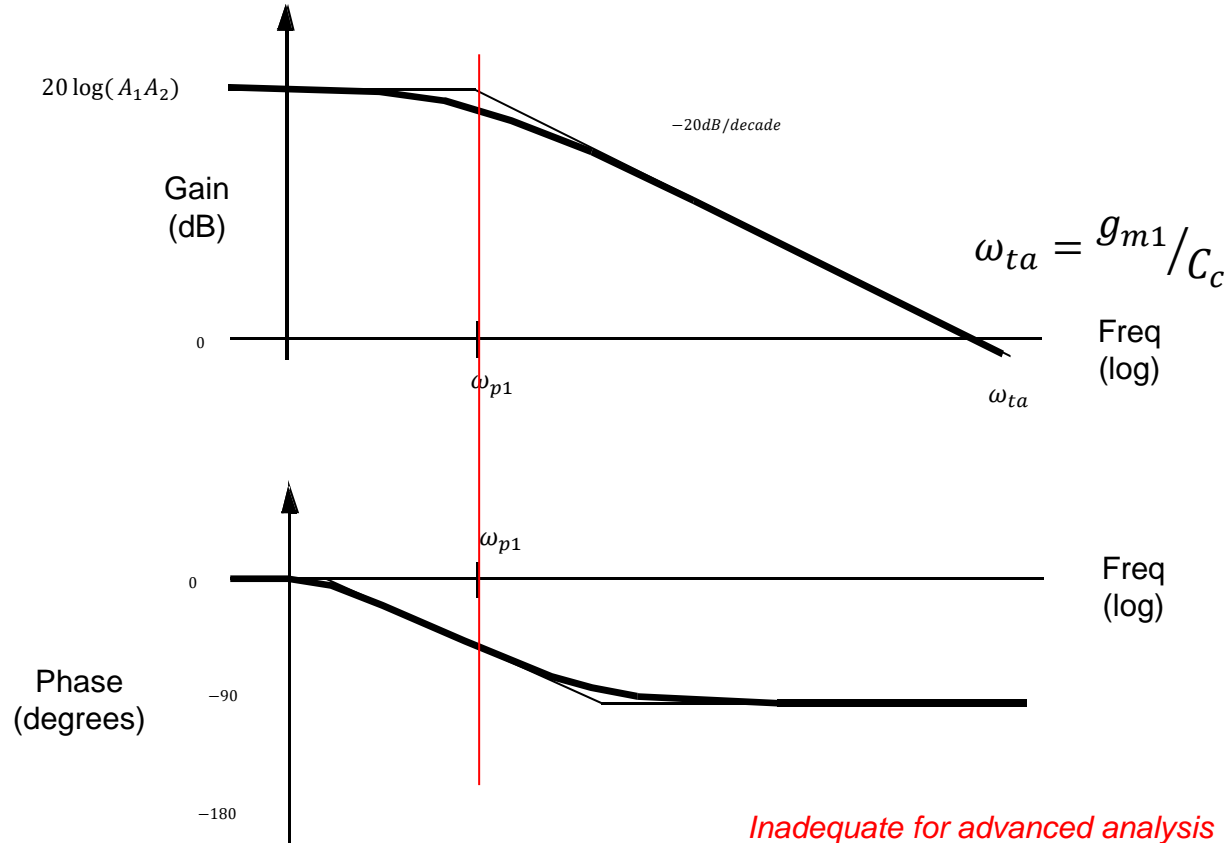
$$\omega_{ta} = \frac{g_{m1}}{C_C} = \frac{I_{D5}}{V_{eff1} C_C}$$

# Frequency response - First order model

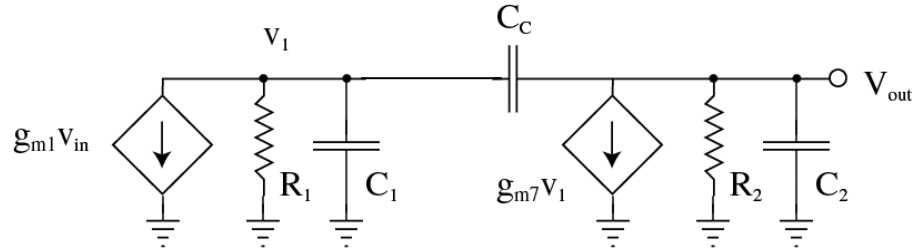


## Midband frequencies

- Below unit-gain frequency
- Above frequencies without compensation effects
- Ignore all C except  $C_c$
- Ignore  $R_c$  which only has effect at  $\omega_{ta}$

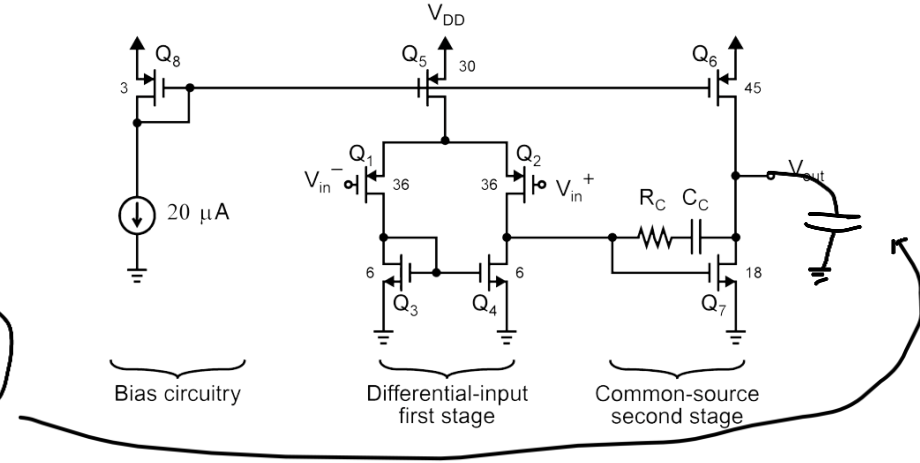


# Frequency response Second order model



$$R_1 = r_{ds4} || r_{ds2} \text{ and } C_1 = C_{db2} + C_{db4} + C_{gs7}$$

$$R_2 = r_{ds6} || r_{ds7} \text{ and } C_2 = C_{db7} + C_{db6} + C_{L2}$$



- Assume  $R_C=0$  give transfer function

$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}g_{m7}R_1R_2 \left(1 - \frac{sC_C}{g_{m7}}\right)}{1 + sa + s^2b}$$

$$a = (C_1 + C_C)R_2 + (C_1 + C_C)R_1 + g_{m7}R_1R_2C_C$$

$$b = R_1R_2(C_1C_2 + C_1C_C + C_2C_C)$$



- Assume widely separated poles

$$D(s) = \left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right) \approx 1 + \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

- Dominant pole

$$\begin{aligned} \omega_{p1} &= \frac{1}{R_1[C_1 + C_C(1 + g_{m7}R_2)] + R_2(C_1 + C_C)} \\ &\approx \frac{1}{R_1C_C(1 + g_{m7}R_2)} \\ &\approx \frac{1}{g_{m7}R_1R_2C_C} \end{aligned}$$

- Non-dominant pole

$$\begin{aligned} \omega_{p2} &= \frac{g_{m7}C_C}{C_1C_2 + C_1C_C + C_2C_C} \\ &\approx \frac{g_{m7}}{C_1 + C_2} \end{aligned}$$

- Increasing  $g_{m7}$   
→ increased pole distance
- Pole splitting compensation
- $C_C$  may decrease  $\omega_{p1}$

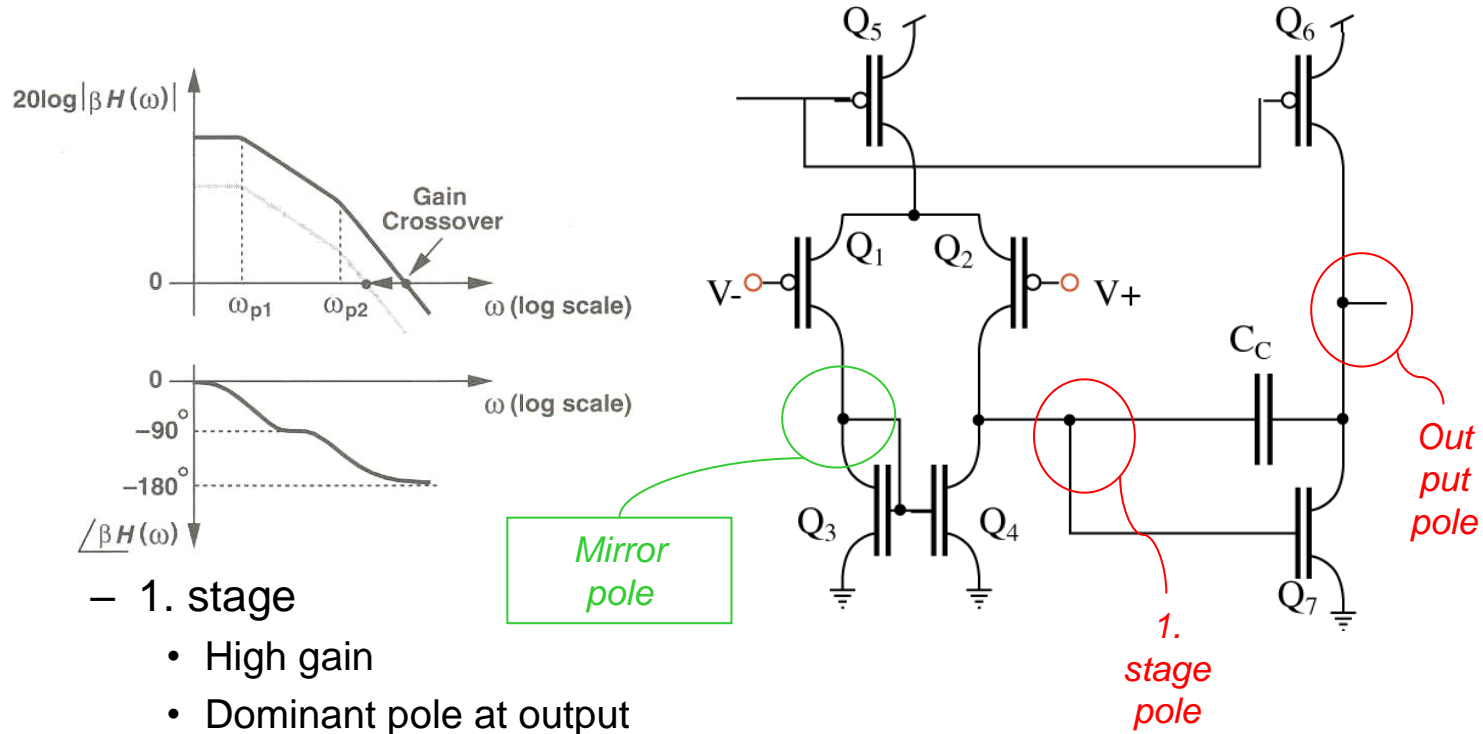
- Additional zero

$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}g_{m7}R_1R_2 \left(1 - \frac{sC_C}{g_{m7}}\right)}{1 + sa + s^2b} \quad \Rightarrow \omega_Z = -\frac{g_{m7}}{C_C}$$

- Right half-plane → negative phase shift with decreased PM
- Stability issues
- Hard to get rid of, but pole distance is increased with  $g_{m7}$

# Two-pole amplifier

- Dominant poles of two-stage amps

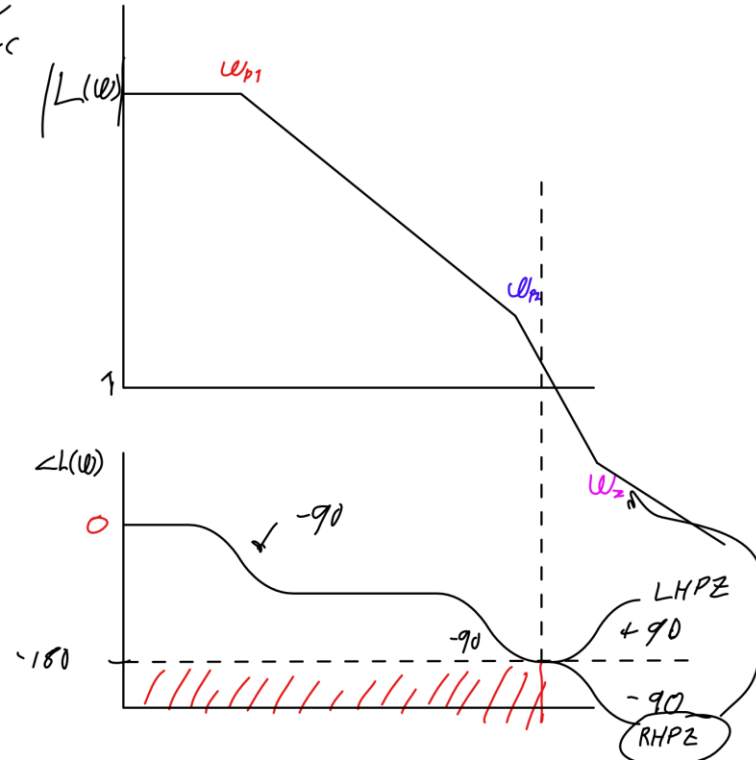


- 1. stage
  - High gain
  - Dominant pole at output

$$\omega_{p1} = \frac{1}{g_m R_1 R_2 C_c}$$

$$\omega_{p2} = \frac{g_m}{C_1 + C_2}$$

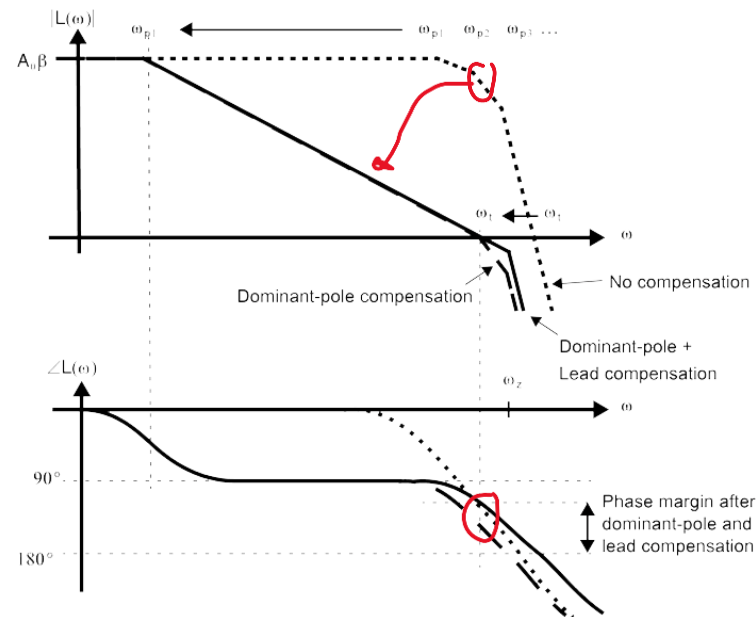
$$\omega_z = -\frac{g_m}{C_c}$$



# Opamp compensation

- Dominant-pole compensation
  - Forcing a feedback system to have 1. order response up to loop unit-gain frequency  $\omega_t$
  - First order system unconditional stable with  $> 90$  phase margin
- Lead compensation
  - Adding zero,  $\omega_z$ , just above  $\omega_t$
  - May improve PM with  $20^\circ$

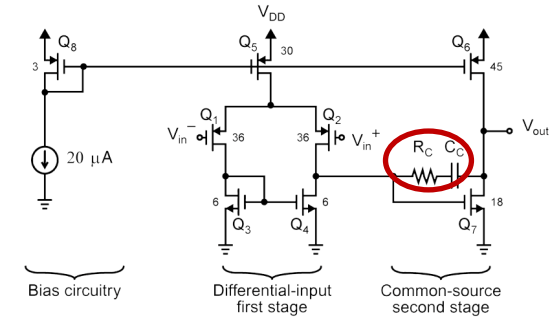
## Dominant pole comp using miller $C_c$



## Lead comp using $R_c$

$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}g_{m7}R_1R_2 \left(1 - \frac{sC_C}{g_{m7}}\right)}{1 + sa + s^2b}$$

$$\Rightarrow \omega_Z = -\frac{g_{m7}}{C_C}$$



- Have to make  $R_C > 0$ 
  - Zero with some resistive element

$$\omega_Z = -\frac{1}{C_C(1/g_{m7} - R_C)}$$

$$R_C = \frac{1}{g_{m7}}$$

- May eliminate that zero by setting
- Alternatively try to cancel  $\omega_{p2}$  with  $\omega_Z$

$$\frac{g_{m7}}{C_1 + C_2} = -\frac{1}{C_C(1/g_{m7} - R_C)} \Rightarrow R_C = \frac{1}{g_{m7}} \left(1 + \frac{C_1 + C_2}{C_C}\right)$$

- “Overcompensation” might even be wise:  $\omega_Z = 1.7\omega_t$

$$R_C \gg 1/g_{m7} \Rightarrow \omega_Z \approx \frac{1}{R_C C_C} \quad \omega_t \approx g_{m7}/C_C \text{ gives } R_C = \frac{1}{1.7g_{m7}}$$

# Compensation procedure

## Dominant pole

- From **first order** model  $C_C$  and  $\omega_t$  is given as:

$$\omega_t = L_0 \omega_{p1} = \beta \frac{g_{m1}}{C_C}$$

UGF

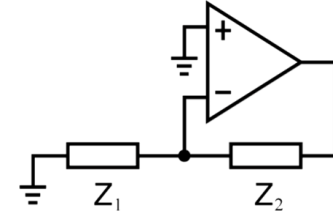
- Find **initial**  $C_C$  setting unit-gain frequency close to second pole

$$\beta \frac{g_{m1}}{C_C} = \frac{g_{m7}}{C_1 + C_2} = \frac{g_{m7}}{C_L}$$

$$C'_C = \left( \beta \frac{g_{m1}}{g_{m7}} \right) C_L$$

UGF  $C_C \rightarrow \omega_{p2}$

$\beta = 1$  (max feedback)

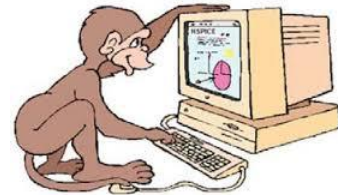


$$L(s) \approx A(s) \frac{Z_1}{Z_1 + Z_2}$$

$$\beta = \frac{Z_1}{Z_1 + Z_2}$$

1. Start with  $C'_C = \left( \beta \frac{g_{m1}}{g_{m7}} \right) C_L$  setting unit-gain frequency close to second pole
2. By simulation (SPICE, CADENCE) find frequency with  $-125^\circ$  phase shift ( called gain A')  
- This is our unit gain frequency  $\omega_t$  target
3. Choose new  $C_C$  such that  $\omega_t$  is unit-gain freq of L(s)
  - $C_C = C'_C A'$  giving  $55^\circ$  phase margin
  - A couple of simulation iterations may be necessary
4. Choose  $R_C$  : 
$$R_C = \frac{1}{1.7 \omega_t C_C}$$
 *Almost optimum lead compensation for any opamp*
  - Giving phase margin of  $85^\circ$  ( $+30^\circ$ ) leaving  $5^\circ$  for variations

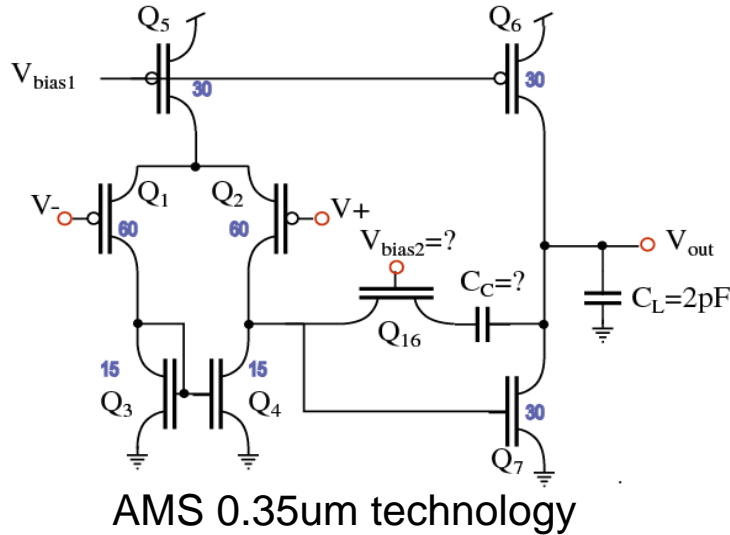
1. Sometimes phase margins are not adequate, then increase  $C_C$
2. Replace  $R_C$  with a transistor 
$$R_C = \frac{1}{\mu_n C_{ox} \left( \frac{W}{L} \right)_{16} V_{eff16}}$$





# Opamp compensation Cadence example

- Find best compensation network  $C_c$  and  $R_c$  for:



- New simulation with  $C_c=1.9\text{pF}$  give

–  $\omega_t=44.7\text{MHz}$  with  $A'=1.32$

$$C_C = C'_C A' = 1.3\text{pF} \cdot 1.32 \approx 2.5\text{pF}$$

- New simulation with  $C_c=2.5\text{pF}$  give

–  $\omega_t=41\text{MHz}$  with  $A'=1.2$

$$C_C = C'_C A' = 2.5\text{pF} \cdot 1.2 \approx 3.1\text{pF}$$

- New simulation with  $C_c=3.1\text{pF}$  give

–  $\omega_t=37.7\text{MHz}$  with  $A'=1.00$

- Finding  $R_c$

$$R_C = \frac{1}{1.2\omega_t C_C} = \frac{1}{1.2 \cdot 37.7 \cdot 10^6 \cdot 3.1 \cdot 10^{-12}} \approx 7132\Omega$$

Marker at  
55 deg  
phase  
margin

# Compensation procedure

## Lead compensation - controlling Zero

$$\omega_z \approx \frac{-1}{C_C \left( \frac{1}{g_{m7}} - R_C \right)}$$

Several possibilities for  $R_C$  :

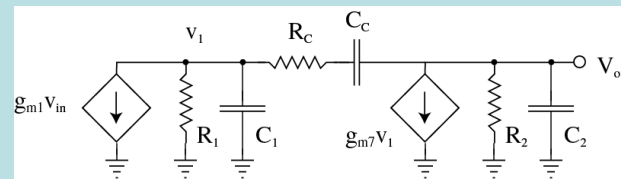
$$R_C = \frac{1}{g_{m7}} \rightarrow \omega_z = \infty$$

$$R_C > \frac{1}{g_{m7}} \quad \text{RHPZ} \rightarrow \text{LHPZ and cancel } \omega_{p2}$$

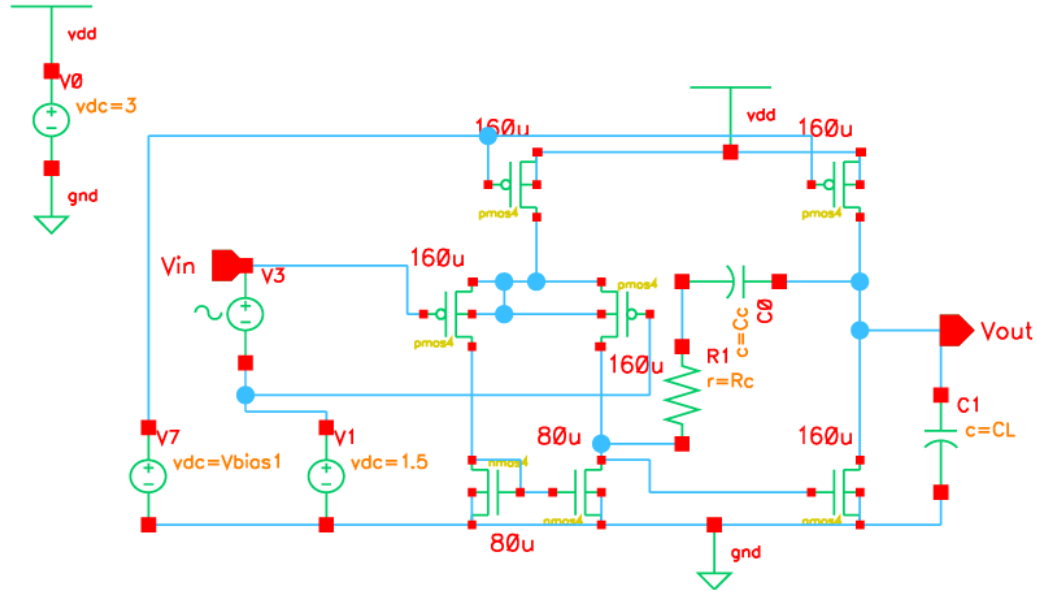
$$R_C \gg \frac{1}{g_{m7}} \quad \begin{array}{l} \text{Moving LHPZ to a frequency slightly higher than } \omega_t \text{ (wo } R_C) \\ \text{Recommended to get more PM (20-30 degrees)} \end{array}$$

$$\omega_{p2} = \frac{g_{m7}C_C}{C_1C_2 + C_1C_C + C_2C_C} = \frac{-1}{C_C \left( \frac{1}{g_{m7}} - R_C \right)} \Rightarrow R_C = \frac{1}{g_{m7}} \left( 1 + \frac{C_1 + C_2}{C_C} \right)$$

Two stage opamp small signal model



Find bias voltage:

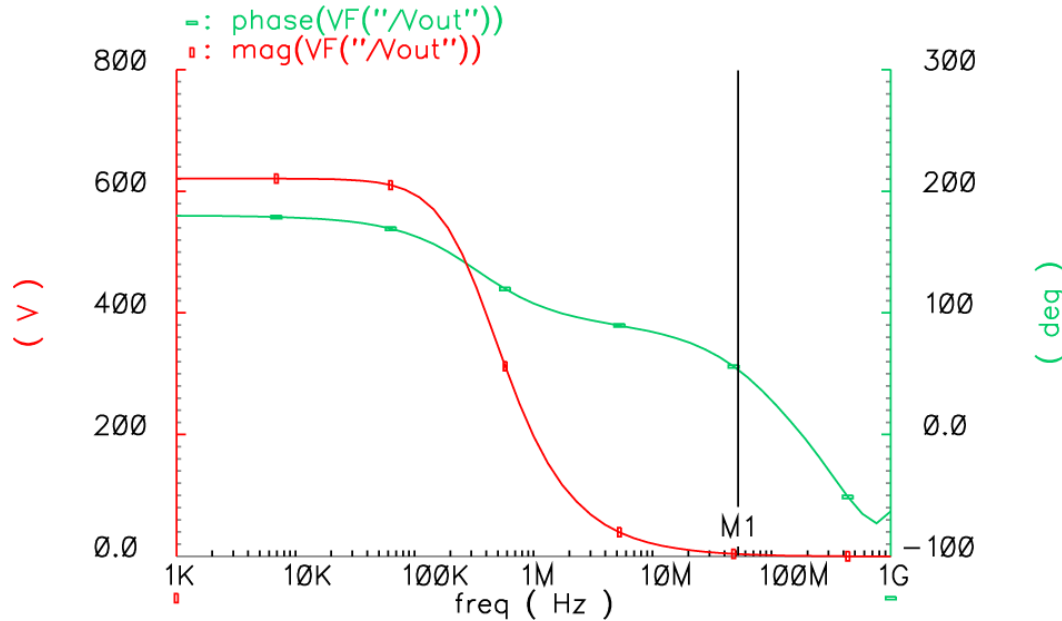


*$V_{bias1}=2.3V$  give  $84\mu A$  tail current*

*Found by simple simulation run displaying  
tail current*

- Start with  $C_c = 0.5\text{pF}$  and  $R_c = 0$

Expressions



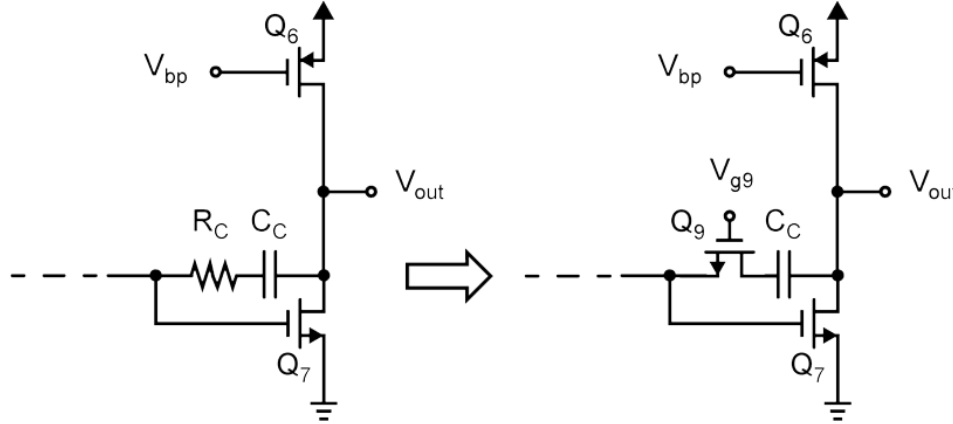
0° phase in  
CADENCE  
display is -180°  
actual phase shift

*Find  $(180 - 125) = 55^\circ$  phase shift at  $\omega t = 50.1\text{MHz}$  with gain  $A' = 3.7$*

$$C_c = C'_c A' = 0.5\text{pF} \cdot 3.7 \approx 1.9\text{pF}$$

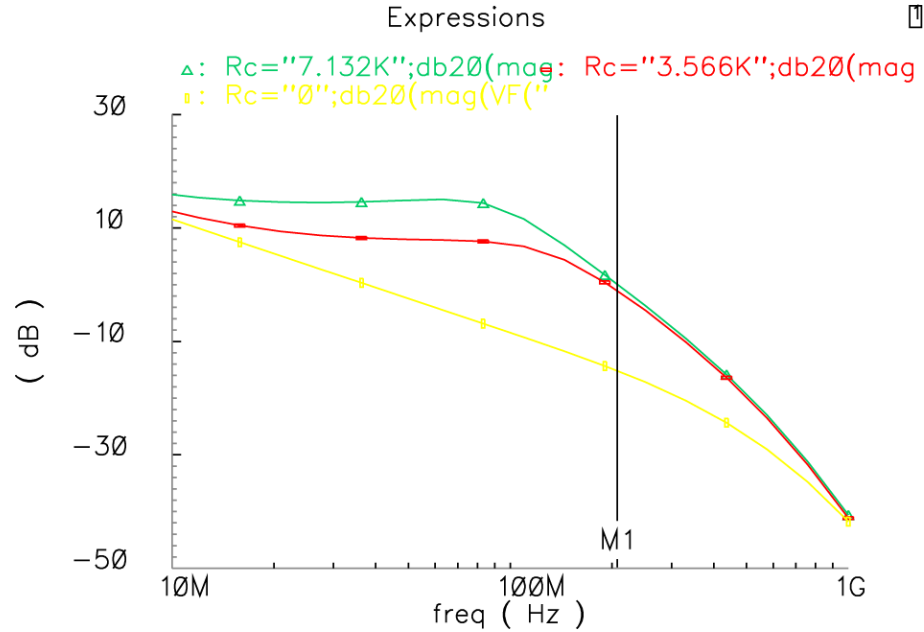
## $R_C$ as transistor

- Compensation resistor
  - Replaced by transistor in triode region

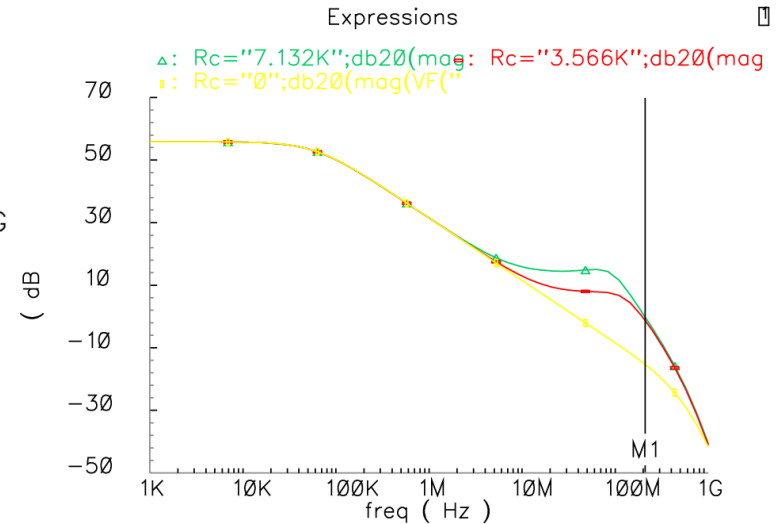


$$R_C = r_{ds} = \frac{1}{\mu_n C_{ox} \frac{W}{L} V_{eff}}$$

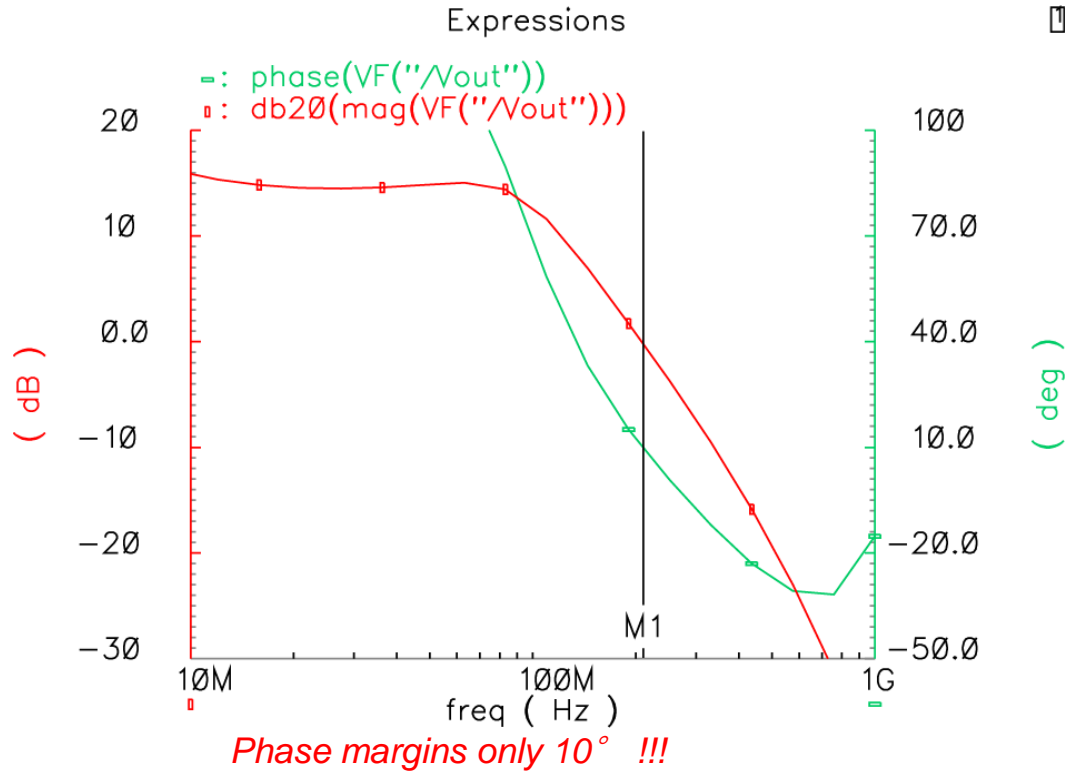
- Adding compensation resistor  $R_c$



*Give unit-gain freq of 209MHz*

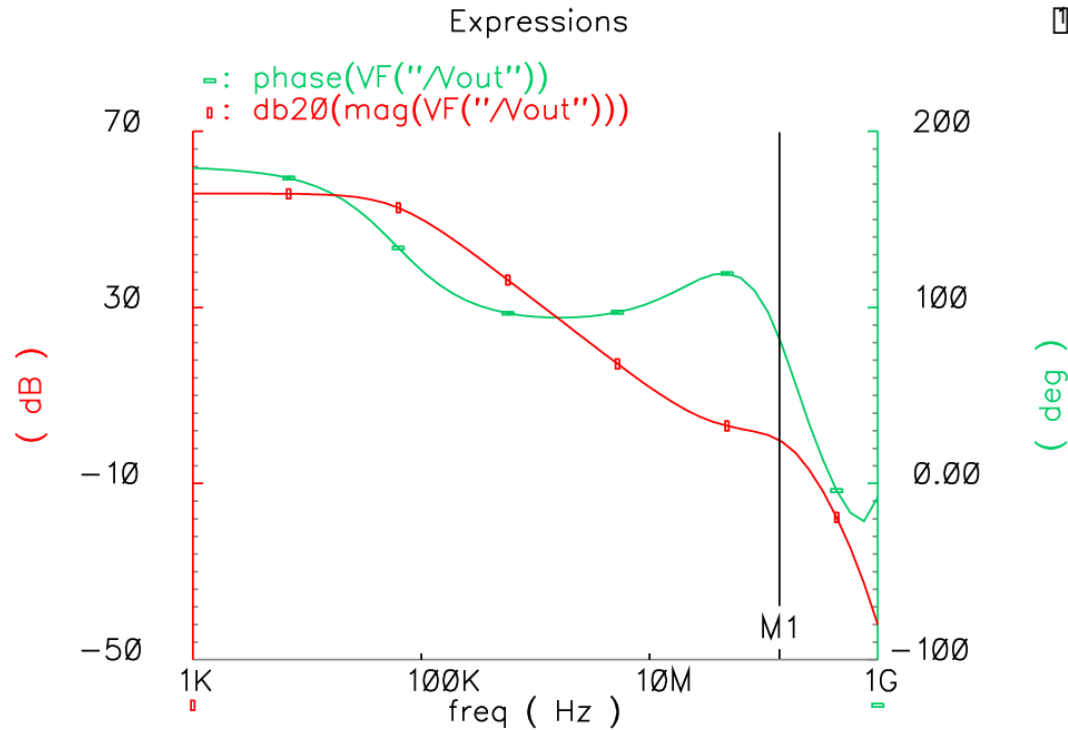


## Phase margins?





- What to do?
  - Book: increase  $C_c$
  - Try to decrease  $R_c$



*Give unit-gain freq of 133MHz with  $PM=84^\circ$  with  $R_c=2050\Omega$*