TELE2001 Reguleringsteknikk: Oblig 2

Daniel Carton

Del 1. Logaritmer

oppgave 1.1

$$\begin{aligned}
0.01 &= \frac{2}{2000} \\
log(0.01) &= log(\frac{2}{2000}) = log(2) - log(2 \cdot 1000) = log(2) - log(2) - log(10^3) \\
log(2) - log(2) + log(10^3) &= 0.301 - 0.301 - 3 \\
log(0.01) &= \textbf{-3}
\end{aligned}$$

$$\begin{aligned}
0.2 &= \frac{2}{10} \\
log(0.2) &= log(\frac{2}{10}) = log(2) - log(10) = 0.301 - 1 \\
log(0.2) &= \textbf{-0.699}
\end{aligned}$$

$$\begin{aligned}
0.8 &= \frac{2 \cdot 2}{5} = \frac{2 \cdot 2}{\frac{10}{2}} \\
log(0.8) &= log(\frac{2 \cdot 2}{\frac{10}{2}}) = log(2 \cdot 2) - log(\frac{10}{2}) \\
log(0.8) &= log(2) + log(2) - (log(10) - log(2)) = 0.301 + 0.301 - 1 + 0.301 \\
log(0.8) &= -\mathbf{0.097}
\end{aligned}$$

$$log(2) = \mathbf{0.301}$$

$$\begin{aligned}
5 &= \frac{10}{2} \\
log(5) &= log(\frac{10}{2}) = log(10) - log(2) = 1 - 0.301 \\
log(5) &= \mathbf{0.699}
\end{aligned}$$

$$
\begin{aligned}
8 &= 2^3 \\
log(8) &= log(2^3) = 3log(2) = 3 \cdot 0.301 \\
log(8) &= \mathbf{0.903}
\end{aligned}
$$

$$
\begin{aligned}
12.5 &= \frac{25}{2} = \frac{5^2}{2} \\
log(12.5) &= log(\frac{5^2}{2}) = log(5^2) - log(2) = 2log(5) - log(2) = 2log(\frac{10}{2}) - log(2) \\
log(12.5) &= 2log(10) - 2log(2) - log(2) = 2log(10) - 3log(2) = 2 - 0.903 \\
log(12.5) &= \mathbf{1.097}
\end{aligned}
$$

$$
\begin{aligned}
20 &= 10 \cdot 2 \\
log(20) &= log(10 \cdot 2) = log(10) + log(2) = 1 + 0.301 \\
log(20) &= \mathbf{1.301}
\end{aligned}
$$

$$
\begin{aligned}
50 &= 5 \cdot 10 = \frac{10}{2} \cdot 10 \\
log(50) &= log(\frac{10}{2} \cdot 10) = log(\frac{10}{2}) + log(10) = log(10) - log(2) + log(10) = 1 + 1 - 0.301 \\
log(50) &= \mathbf{1.699}
\end{aligned}
$$

$$
\begin{aligned}
100 &= 10^2 \\
log(100) &= log(10^2) = 2log(10) \\
log(100) &= \mathbf{2}
\end{aligned}
$$

Oppgave 1.2

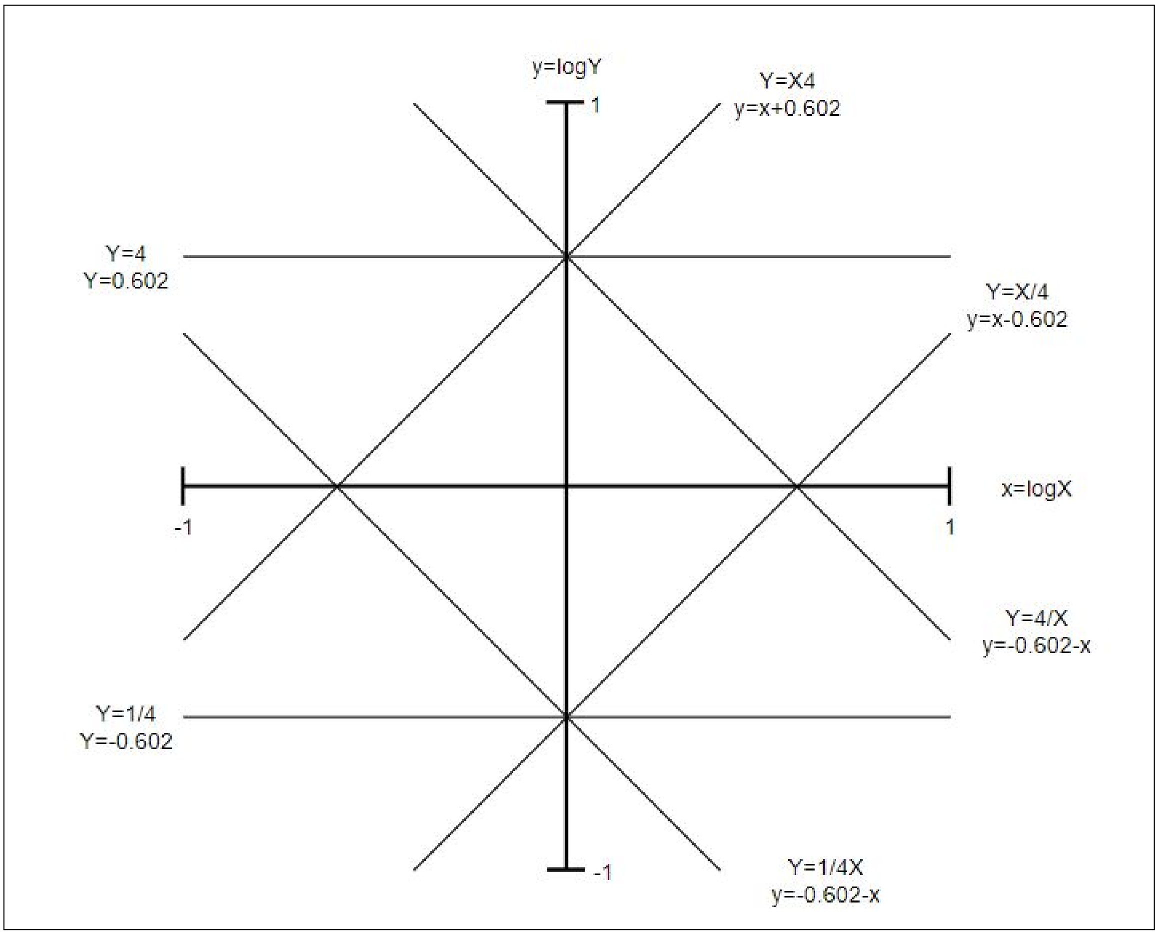
I en dekade av 50 mm, vil første desimal være 5mm lang, så vil andre desimal være 0.5mm lang. Dette betyr at å avrunde til nermeste andre desimal vil gi en maksimal feil på 0.25mm.

Oppgave 1.3

$$logA + logA^3 + logAB - logB^{-1}$$
$$logA + 3logA + logA + logB + logB$$
$$5logA + 2logB = \mathbf{5a{+}2b}$$

$$logA + 2log\sqrt{AB}$$
$$logA + logAB$$
$$logA + logA + logB$$
$$2logA + logB = \mathbf{2a{+}b}$$

Oppgave 1.4



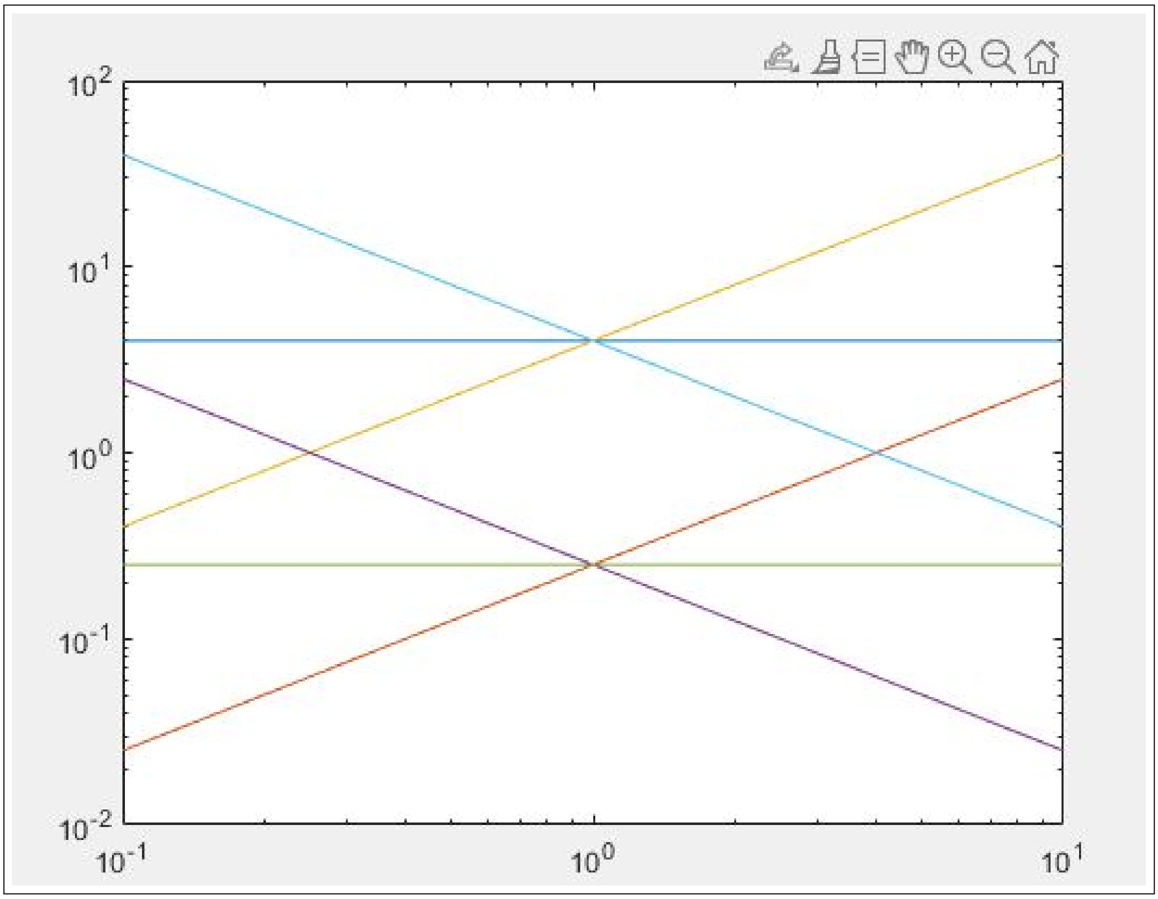
Oppgave 1.5

```
X = logspace(-1, 2);
% Lager en vector med 50 punkter mellom 10^-1 og 10^2

Y2 = ones(size(X)).*4;
% lager vektoren over til en matrise

Y3 = X./4;
% reger at y3 er en funksjon av X ganger skalaren 1/4

Y4 = X.*4;
% regner at y4 er en funksjon av X ganger skalaren 4

Y5 = 1./(4*X);
% regner at y5 er en funksjon av 1 delt på X ganger skalaren 4

loglog(X, Y2, X, Y3, X, Y4, X, Y5);
% plotter funksjonene over i et logaritmisk space
```

Oppgave 1.6

```
X = logspace(-1, 1);
Y2 = ones(size(X)).*4;
Y3 = X./4;
Y4 = X.*4;
Y5 = 1./(4*X);
Y6 = ones(size(X)).*1/4;
Y7 =4./X;
loglog(X, Y2, X, Y3, X, Y4, X, Y5, X, Y6, X, Y7);
```

Oppgave 1.7



Del 2: Linearisering av dynamisk modell

Oppgave 2.1

$$
\begin{aligned}
m\dot{v} &\stackrel{\mathsf{def}}{=} m\Delta\dot{v}(t) \\
F_x &= \frac{d}{dt}(F_{xa} + \Delta F_x) = \Delta F_x(t) \\
Cv(t)^2 &= \frac{d}{dt}(v_a(t)^2) = 2v_a(v_a + \Delta v) = 2v_a\Delta v(t) \\
m\Delta\dot{v}(t) &= \Delta F_u(t) + \Delta F_f(t) - 2Cv_a\Delta v(t)
\end{aligned}
$$

Oppgave 2.2

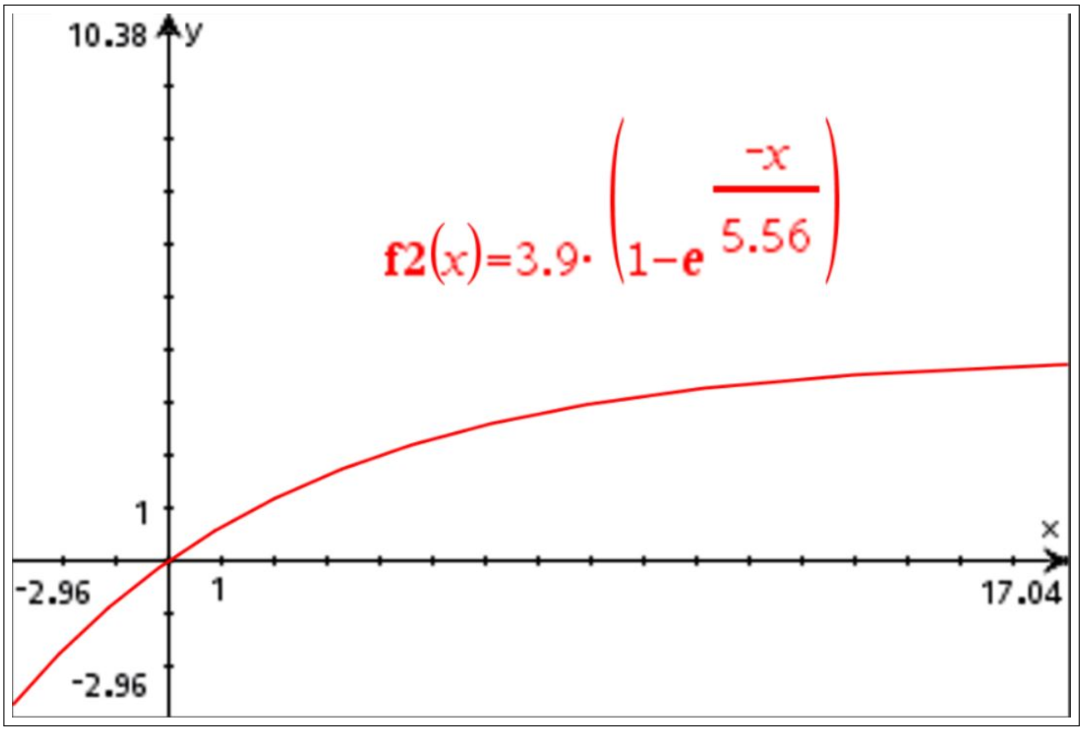
$$
\begin{aligned}
m\Delta\dot{v}(t) &= \Delta F_u(t) + \Delta F_f(t) - 2Cv_a\Delta v(t) \\
m\Delta\dot{v}(t) &= -2Cv_a\Delta v(t) + \Delta F_u(t) + \Delta F_f(t) \\
m\dot{y} &= -2Cv_a y(t) + u(t) + v(t) \\
m\dot{y} &= -D_a y(t) + u(t) + v(t) \\
D_a &= 2Cv_a = 2 \cdot 3 \cdot 30 = 180
\end{aligned}
$$

Del 3: Laplace-transformasjon og overføringsfunksjoner

Oppgave 3.1

$$\begin{aligned}
m\dot{y} &= -D_a y(t) + u(t) + v(t) \\
1000\dot{y} &= -180y(t) + 700 + 0 \\
\text{laplacetransformerer} & \\
1000sY(s) &= -180Y(s) + \frac{700}{s} \\
1000sY(s) + 180Y(s) &= \frac{700}{s} \\
Y(s)(1000s + 180) &= \frac{700}{s} \\
Y(s) &= \frac{700}{(1000s + 180)s} \\
Y(s) &= \frac{\frac{700}{1000}}{(1000s + 180)\frac{s}{1000}} \\
Y(s) &= \frac{0.7}{(s + 0.18)s} \\
\text{ønsker i form:} & \quad \frac{k\alpha}{s(s + a)} \\
k \cdot 0.18 &= 0.7 \rightarrow k = 3.9 \\
Y(s) &= \frac{3.9 \cdot 0.18}{(s + 0.18)s} \\
y(t) &= 3.9(1 - e^{-0.18t}) = 3.9(1 - e^{-\frac{t}{5.6}})
\end{aligned}$$

Oppgave 3.2



Oppgave 3.3

$$
\begin{aligned}
msY(s) &= -D_aY(s) + U(s) + V(s) \\
msY(s) + D_aY(s) &= U(s) + V(s) \\
Y(s) &= \frac{U(s) + V(s)}{ms + D_a}
\end{aligned}
$$

U(s) er lik 0 i det tilfelle når vi løser for v(s), motsatt når vi løser for U(s)

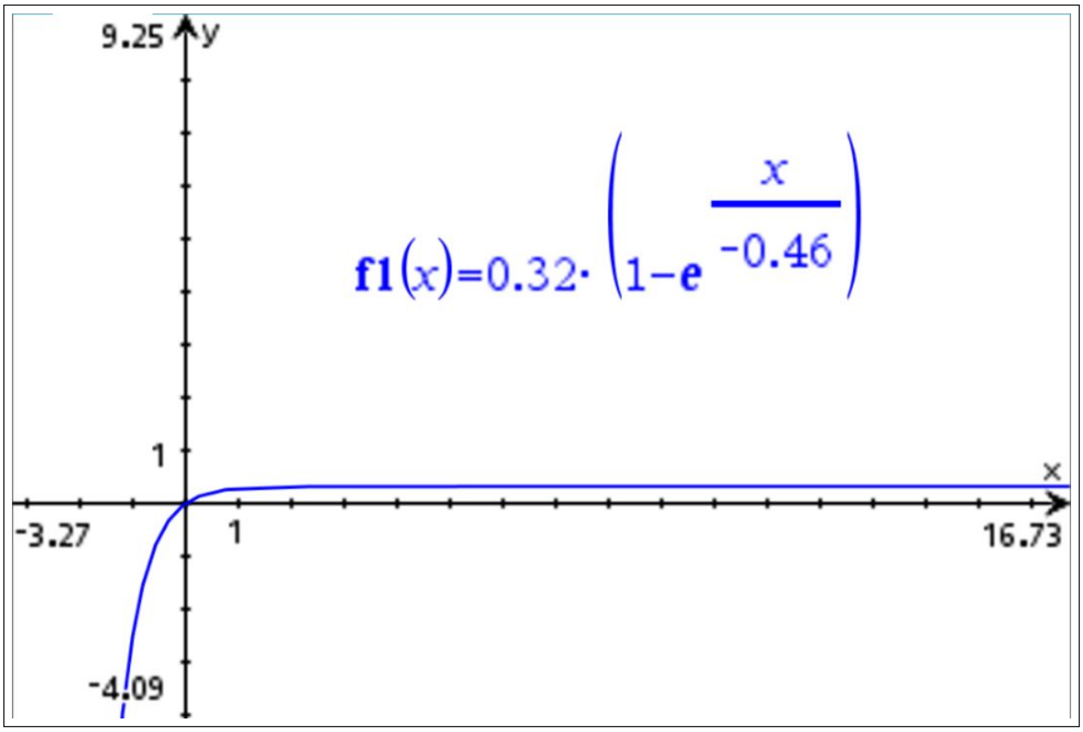
$$
\begin{aligned}
H_v &= \frac{Y(s)}{V(s)} = \frac{\frac{V(s)}{ms+D_a}}{V(s)} = \frac{1}{ms + D_a} \\
H_v &= \frac{Y(s)}{U(s)} = \frac{\frac{U(s)}{ms+D_a}}{U(s)} = \frac{1}{ms + D_a}
\end{aligned}
$$

Del 4: Regulert system

Oppgave 4.1

$$
\begin{aligned}
m\dot{y} &= -D_a y(t) + K_p(r(t) - y(t)) + v(t) \\
1000\dot{y} &= -180y(t) - 2000 \cdot y(t) + 700 \\
\text{laplacetransformerer} & \\
1000sY(s) &= -180Y(s) - 2000 \cdot Y(s) + \frac{700}{s} \\
1000sY(s) &= -2180 \cdot Y(s) + \frac{700}{s} \\
1000sY(s) + 2180 \cdot Y(s) &= \frac{700}{s} \\
Y(s)(1000s + 2180) &= \frac{700}{s} \\
Y(s) &= \frac{700}{(1000s + 2180)s} \\
Y(s) &= \frac{\frac{700}{1000}}{(1000s - 1820)\frac{s}{1000}} \\
Y(s) &= \frac{0.7}{(s + 2.18)s} \\
\text{ønsker i form:} & \quad \frac{k\alpha}{s(s+a)} \\
k \cdot 2.18 &= 0.7 \rightarrow k = 0.32 \\
Y(s) &= \frac{0.32 \cdot 2.18}{(s + 2.18)s} \\
y(t) &= 0.32(1 - e^{-2.18t}) \rightarrow y(t) = 0.32(1 - e^{-\frac{t}{0.46}})
\end{aligned}
$$

Oppgave 4.2

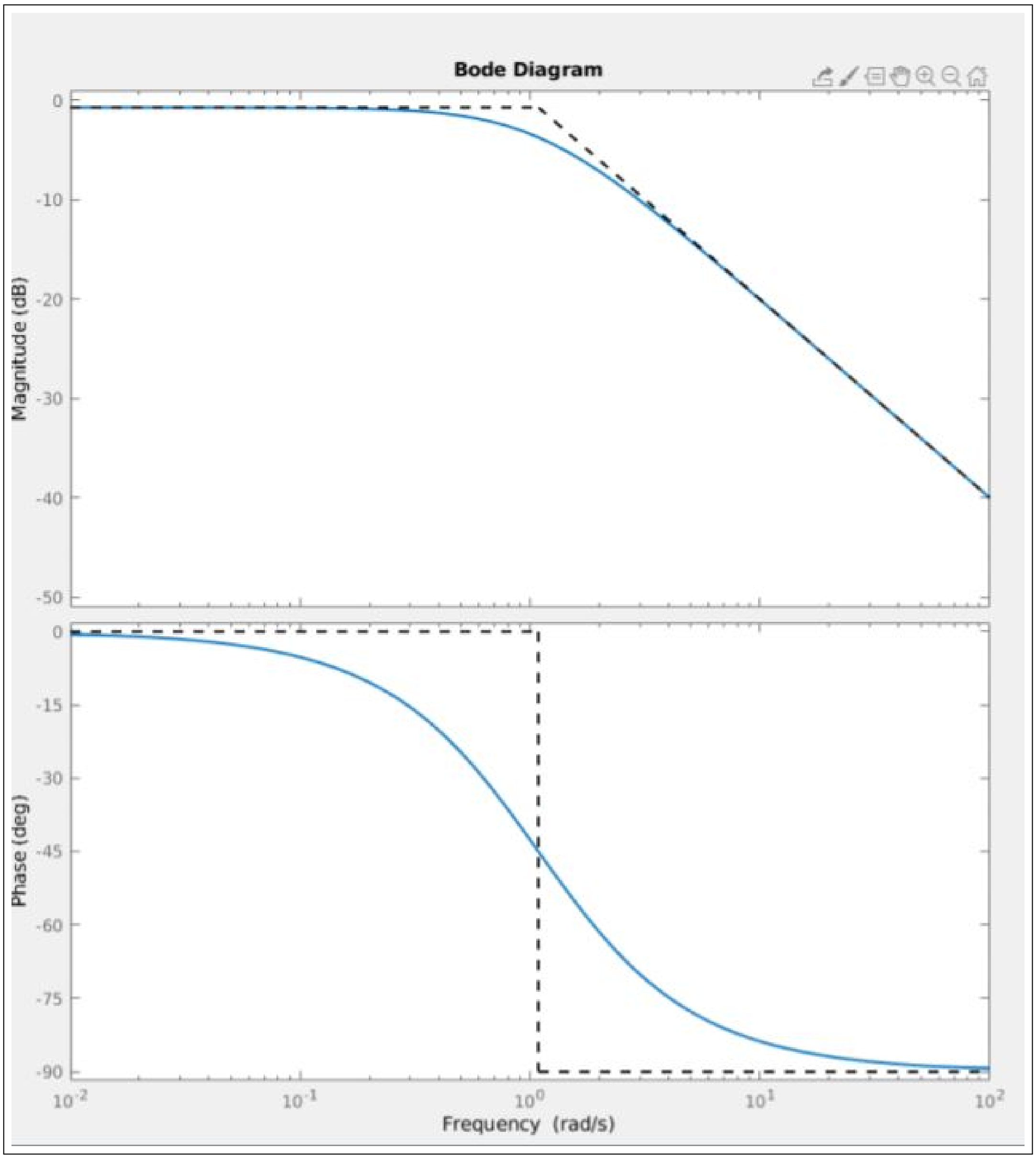


Del 5: Frekvensplott

Oppgave 5.1

$$
\begin{aligned}
m\dot{y} &= -D_a y(t) + u(t) + v(t) \\
u(t) &= K_p(r(t) - y(t)) \\
m\dot{y} &= -D_a y(t) + K_p(r(t) - y(t)) + v(t) \\
m\dot{y} &= -D_a y(t) + K_p r(t) - K_p y(t) + v(t) \\
m\dot{y} &= -D_a y(t) - K_p y(t) + K_p r(t) + v(t) \\
m\dot{y} &= -y(t)(D_a + K_p) + K_p r(t) + v(t) \\
\text{laplacetransformerer} & \\
msY(s) &= -Y(s)(D_a + K_p) + K_p R(s) + V(s) \\
\text{(Kan sete V(s) = 0 her pga supersposisjon)} & \\
msY(s) + Y(s)(D_a + K_p) &= K_p R(s) \\
Y(s)(ms + D_a + K_p) &= K_p R(s) \\
Y(s) &= \frac{K_p R(s)}{ms + D_a + K_p} \\
H_r &= \frac{Y(s)}{R(s)} = \frac{K_p R(s)}{(ms + D_a + K_p)R(s)} \\
H_r &= \frac{K_p}{ms + D_a + K_p} \\
H_r &= \frac{2000}{1000s + 180 + 2000} \rightarrow \frac{2000}{1000s + 2180} \cdot \frac{\frac{1}{2180}}{\frac{1}{2180}} \\
H_r &= \frac{0.92}{1 + 0.42s}
\end{aligned}
$$

Oppgave 5.2



Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
-50
Phase (deg)
0
-15
-30
-45
-60
-75
-90
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/s)

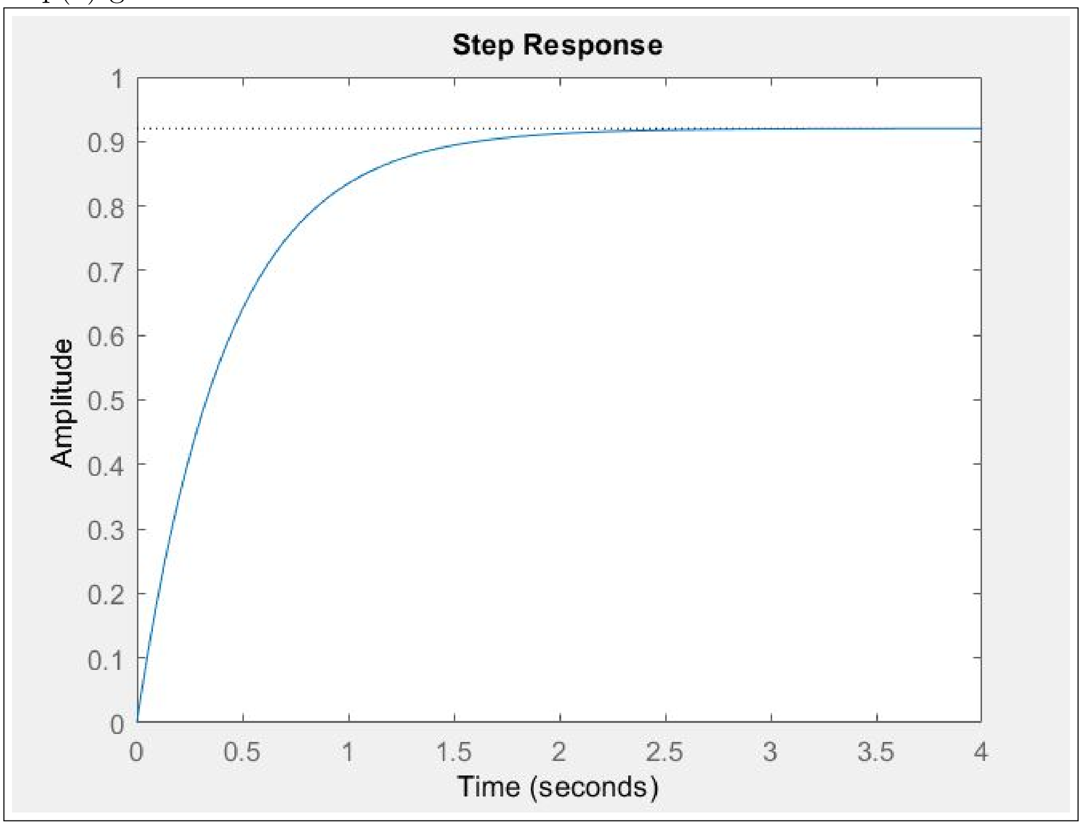
Del 6: Overføringsfunksjoner i Matlab

Oppgave 6.1

```
>> a=tf(0.92, [0.42 1])
    a =

     0.92
  ----------
  0.42 s + 1

Continuous-time transfer function.
```

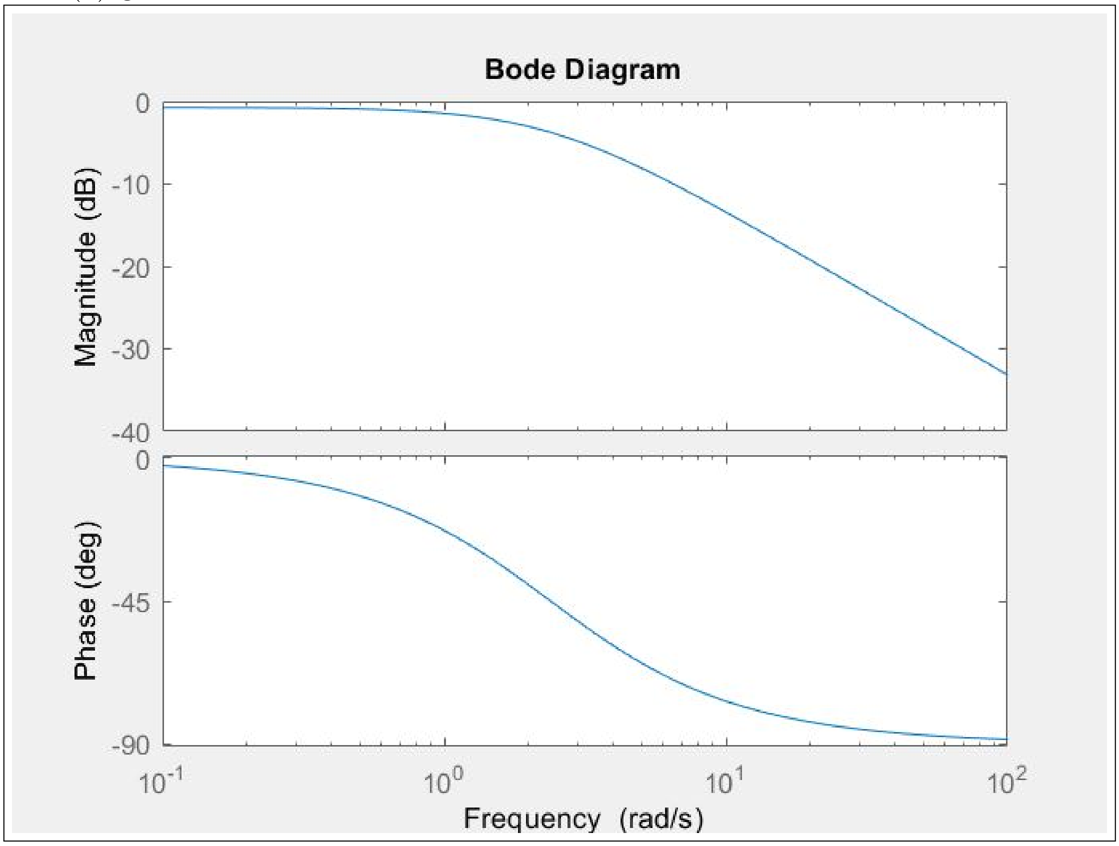
Oppgave 6.2

step(a) gir:



Oppgave 6.3

bode(a) gir:



Dette gir det samme diagrammet vi lagde i 5.2, bare dette diagrammet mangler asymptoter.