P600 Pset #4

1.a Start from eqn. of n_{eq} for non. rel particles.
since during recombination e^- and p^+ are non-rel.

$$n_{eq} = g_* \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

$$\left(\frac{n_H}{n_e n_p}\right)_{eq} = \frac{g_H}{g_e g_p} \left(\frac{m_H}{m_e m_p} \frac{2\pi}{T}\right)^{3/2} e^{-m_H + m_p + m_e / T}.$$

$g_H = 4$, $g_e = 2$, $g_p = 2$

$-E_I = m_H - m_p - m_e$

Assume $n_e = n_p$, $m_H \sim m_p$, as the $e^{E_I/T}$ term dominates

Then, $$\left(\frac{n_H}{n_e n_p}\right)_{eq} = \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{E_I/T}$$

$$X_e = \frac{n_e}{n_p + n_H}, \quad \frac{1-X_e}{X_e^2} = \frac{n_H}{n_e^2}(n_p + n_H) = \frac{n_H}{n_e^2} \eta n_\gamma, \quad \eta = \frac{n_B}{n_\gamma}$$

Thus, $$\frac{1-X_e}{X_e^2} = \frac{n_H}{n_e^2} \eta n_\gamma = \frac{n_H}{n_e n_p} \eta n_\gamma$$

$$= \left(\frac{2\pi}{m_e T}\right)^{3/2} \eta n_\gamma e^{E_I/T}$$

$$n_\gamma = \frac{\zeta(3)}{\pi^2} g_\gamma T^3, \quad g_\gamma = 2.$$

$$\boxed{\frac{1-X_e}{X_e^2} = \frac{2\zeta(3)}{\pi^2} \eta \left(\frac{2\pi T}{m_e}\right)^{3/2} e^{E_I/T}}$$

b. Set A equal to the RHS, so

$$\frac{1-X_e}{X_e^2} = A.$$

$$AX_e^2 + X_e - 1 = 0,$$

$$X_e = \frac{-1 + \sqrt{1+4A}}{2A}$$, where the positive result is chosen as $X_e \geq 0$.

Plot in Notebook.

c. At $X_e = 0.1$, $z = 1258$.
At $X_e = 0.5$, $z = 1377$.
While different, X_e decreases dramatically over this redshift range of ~ 100.

d. At $z = 1258$, $t = \frac{1}{H_0} \int_0^a \frac{da}{a^{-1/2}}$. matter domination

$$= \frac{2}{3H_0} \left(a^{3/2} \right)$$

$$= \frac{2}{3H_0} \left(\frac{1}{1+z} \right)^{3/2} = \frac{9.79}{h} \text{ Gyr} \left(\frac{1}{1+1258} \right)^{3/2}$$

$$\boxed{= \frac{220000}{h} \text{ years.}} \approx 300{,}000 \text{ yrs}$$

e. $\Gamma = n_e \sigma_T = n_b X_e(T_{dec}) \sigma_T$

$$= \frac{2\zeta(3)}{\pi^2} \eta \, \sigma_T X_e(T_{dec}) T_{dec}^3$$

$$H(T_{dec}) = H_0 \sqrt{\Omega_{m,0}} \left(\frac{T_{dec}}{T_0}\right)^{3/2}, \quad \text{matter domination.}$$

$$\frac{2\zeta(3)}{\pi^2} \eta \sigma_T X_e(T_{dec}) T_{dec}^3 = H_0 \sqrt{\Omega_{m,0}} \left(\frac{T_{dec}}{T_0}\right)^{3/2}.$$

$$X_e(T_{dec}) = \frac{-1+\sqrt{1+4A}}{2A},$$

$$A = \frac{2\zeta(3)}{\pi^2} \eta \left(\frac{2\pi T_{dec}}{m_e}\right)^{3/2} e^{\epsilon_I/T_{dec}}.$$

Solving for T_{dec}, $T_{dec} \approx 0.27\,eV$

$$\text{At } z_{dec} = \frac{0.27\,eV}{0.235\,eV} \approx 1149$$

$$z_{dec} = 1149.$$

$$t_{dec} = \frac{2}{3H_0} a^{3/2} \approx 380{,}000 \text{ yrs.}$$

At this time, $X_e(T_{dec}) = \frac{-1+\sqrt{1+4A}}{2A} \approx 0.01.$

2. $$X_n \equiv \frac{n_n}{n_n+n_p}.$$

$$n_x = g_x \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}, \quad g_n = g_p = 2.$$

$$X_n = \frac{g_n \left(\frac{m_n T}{2\pi}\right)^{3/2} e^{-m_n/T}}{g_n \left(\frac{T}{2\pi}\right)^{3/2} \left(m_n^{3/2} e^{-m_n/T} + m_p^{3/2} e^{-m_p/T}\right)}.$$

$$= \frac{m_n e^{-m_n/T}}{m_n e^{-m_n/T} + m_p e^{-m_p/T}}$$

$\{ m_n \sim m_p, \quad Q \equiv m_n - m_p = 1.3\,\text{MeV}$

$$= \frac{1}{1 + e^{-m_p + m_n/T}} = \frac{1}{1 + e^{Q/T}} = \boxed{\frac{e^{-Q/T}}{1 + e^{-Q/T}}}$$

b. Assuming $T = 0.8\,\text{MeV}$,

$$X_n = \frac{e^{-1.3/0.8}}{1 + e^{-1.3/0.8}} = \boxed{0.1645}$$

c. Assume all neutrons converted into He4.

$2n + 2p \rightarrow He$.

Thus, $Y_p = \frac{4 n_{He}}{n_H} = \frac{4 \cdot (\frac{1}{2} n_n)}{n_p - n_n}$ ← pn used 1:1.

$$\approx \frac{2X_n}{1 - X_n} = \boxed{0.3938}$$

d. $Y_p \approx \frac{2X_n}{1 - X_n}$, $X_n = \frac{e^{-2.6/0.8}}{1 + e^{-2.6/0.8}} = 0.03733$.

$$Y_p \approx \boxed{0.07155}$$

3 $\sigma \to \chi\chi$. $S_{n\chi} = -2\delta n_0$.

Γ does not depend on number density

a. $$\frac{1}{a^3}\frac{d(na^3)}{dt} = 2\Gamma h(t)\, n_{0,eq}(t).$$

$$h(x) \simeq \frac{x}{x+2}, \quad x \equiv \frac{m_0}{T}. \quad Y = \frac{n}{s}$$

$$s \propto T^3 \propto a^{-3}, \quad x \propto T^{-1} \propto a$$

$$\frac{1}{a^3}\frac{d(na^3)}{dt} = s\frac{dY}{dt} \propto \Gamma h(t)\, n_{0,eq}(t).$$

$$\frac{dY}{dt} \propto \Gamma h(t)\frac{n_{0,eq}(t)}{s} = \Gamma h(t)\, Y_{eq}(t).$$

$$\frac{d}{dt} = H\frac{d}{d\ln a}$$ (P. 106 Huterer).

$$= Ha\frac{d}{da}.$$

$$\frac{dY}{dt} \propto a\frac{dY}{da} \propto x\frac{dY}{dx}\frac{dx}{da} \propto x\frac{dY}{dx}, \quad \boxed{\frac{dY}{dx} = \lambda_i\, x\, h(x)\, Y_{eq}(x).}$$

b. Plot on Notebook

c. For the freezeout scenario the abundance follows the equilibrium abundance then flattens out. In the freeze-in scenario, the abundances flattens out much earlier by around $x \sim 1$. Lambda is approximately proportional to the freeze-in abundance. It does not affect the time at which freeze-in is achieved, unlike the freezeout scenario. Finally, as we are tracking the ~~ff~~ particle being created, it does not follow Y_{eq}, but rather increases from $Y_0 = 1e\text{-}20$ in the plot.