1.) Yes

2.) (i) CSCI 3521, 5561

(ii) STAT 3021 - Intro to Prob. and Stats

(iii) CSCI 2033 Elementary Computational Linear Algebra

(iv) No

3.) $X \in \mathbb{R}^{n \times p}$ $y \in \mathbb{R}^n$ Find $w^* \in \mathbb{R}^p$

$$w^* = \underset{w \in \mathbb{R}^p}{\arg\min} \frac{1}{2}\|y - Xw\|^2 + \frac{\lambda}{2}\|w\|^2$$

$$\frac{\partial}{\partial w}\left[\frac{1}{2}\|y - Xw\|^2 + \frac{\lambda}{2}\|w\|^2\right] = 0$$

$$-2 \cdot \frac{1}{2} \cdot X^T(y - Xw) + 2 \cdot \frac{\lambda}{2} w = 0$$

$$-X^T y + X^T X w + \lambda w = 0$$

$$X^T X w + \lambda w = X^T y$$

$$(X^T X + \lambda I) w = X^T y$$

$$\boxed{w^* = (X^T X + \lambda I)^{-1} X^T y}$$

when $n < p$, expression is still valid (assuming $\lambda > 0$)

$X^T X + \lambda I$ is always invertible

why?

$X^T X$ is positive semi-definite

λI is positive definite if $\lambda > 0$

$\therefore$ $X^T X + \lambda I$ is positive definite $\therefore$ $X^T X + \lambda I$ is invertible

4.)
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w = np.linalg.inv(X.T @ X + λ *
    np.eye(X.shape[0])) @ X.T @ y
```

5.) $A \in \mathbb{R}^{n \times n}$ is positive definite

$$\max_{\substack{w \in \mathbb{R}^n \\ \text{st } w^T w = 1}} w^T A w \quad \text{and} \quad \min_{\substack{w \in \mathbb{R}^n \\ \text{st } w^T w = 1}} w^T A w$$

are the max and min eigenvalues of A